Emma Williams.

YORK.

1837.

MENSURATION.

OF

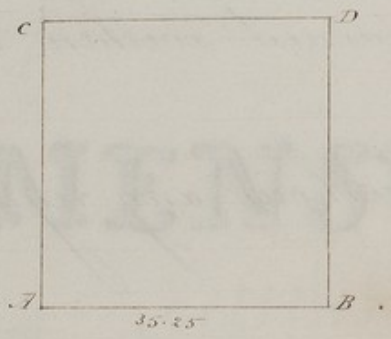
Superficies.

The area of any figure is the measure of its surface, or of the space contained within the bounds of that surface, without any regard to the thickness.

Problem 1st

To find the area of a parallelogram, whether it be a square, a rectangle, a rombus, or a rhomboid.

What is the area of a square, whose side is 35.25 chains?



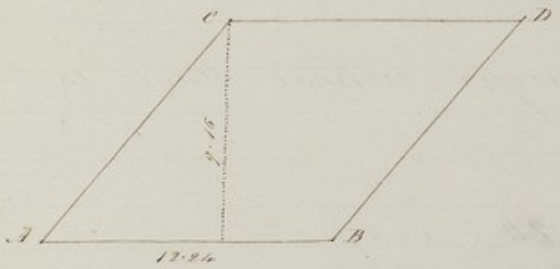

```
      35.25
      35.25
     17625
    70500
   17625
  10575
10 | 1242.5625
     124,25625
                4
     1,02500
                40
     1,00000
```

Ac.	ro.	po.
124	1	1 Ans.

Required the area of a rectangular board whose length is $12\frac{1}{2}$ feet, and breadth 9 inches.

$$\frac{25}{2} \times \frac{3}{4} = \frac{75}{8} = 9\frac{3}{8} \text{ Ans.}$$

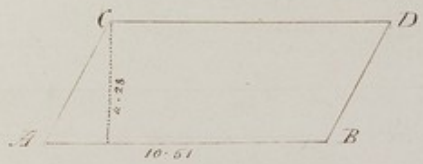
Required the area of a rhombus, the length of whose side is 12·24 feet, and height 9·16 feet.



Fe.	pa.	in.
112	1	5 Ans.

```
   12·24
    9·16
   7344
  1224
 11016
112,1184
      12
1,4208
      12
5,0496
```

Required the area of a rhomboid whose length is 10·51 chains, and breadth 4·28 chains.



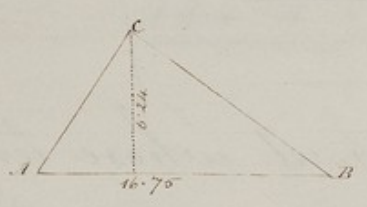
Ac.	ro.	po.
4	1	39 Ans.

```
   10·51
    4·28
10,449828
 4,49828
       4
1,99312
      40
39,72480
```

Problem 2nd

To determine the area of a triangle.

What is the area of a triangle, whose base is 16.75 feet and height 6.24 feet.




```
   16.75
    6.24
   6700
  3350
 10050
104,5200
        12
 6,2400
        12
 2,8800
```

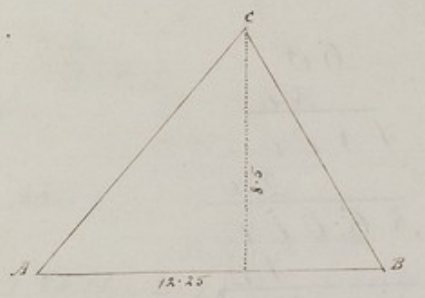
```
   Fe.  pa.  in.
2 | 104 „ 6 „ 2
    52 „ 3 „ 1 Ans.
```

If the base of a triangle be $17\frac{1}{5}$ yards, and its perpendicular height $11\frac{1}{7}$ yards; what is its area in square feet?

$17\frac{1}{5} = \frac{86}{5}$ $\quad 11\frac{1}{7} = \frac{78}{7}$ $\quad \frac{86}{5} \times \frac{78}{7} = \frac{6708}{35}$

$\frac{6708}{35} \div 2 = \frac{6708}{70} = \underline{862\frac{16}{35} \text{ fe. Ans.}}$

Required the area of a triangle, whose base is 12·25 chains, and height 8·5 chains.



$$\begin{array}{r} 12\cdot25 \\ 8\cdot5 \\ \hline 6125 \\ 9800 \\ \hline 10\,\big|\,104125 \\ \hline 10,4125 \\ 4 \\ \hline 1,6500 \\ 40 \\ \hline 26\cdot0000 \end{array}$$

$$\begin{array}{r|l} 2 & 10 \text{ ,, } 1 \text{ ,, } 26 \\ \hline & 5 \text{ ,, } 0 \text{ ,, } 33. \end{array}$$

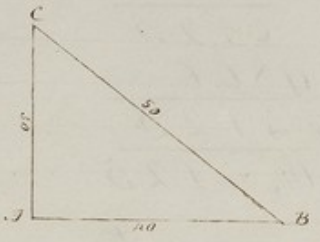
PROBLEM 3rd

To find the area of a triangle, when the three sides only are given.

Required the area of a right angled triangle, whose hypotenuse is 50, and the other two sides 30 and 40.

$$\frac{50+40+30}{2} = \frac{120}{2} = 60$$

60	60	60
50	40	30
10	20	30




```
   60
   30
 1800
   20
 36000
   10
 360000 | 600
 36
 0000
```

Required the area of a triangle, whose three sides are 20, 30 and 40 feet:

$$\frac{20+30+40}{2} = \frac{90}{2} = 45$$

45	45	45
20	30	40
25	15	5

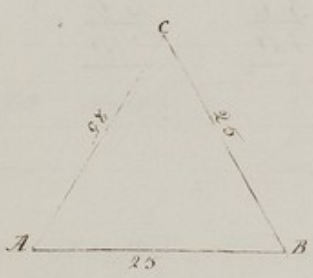
```
      45
      25
     225
    90
    1125
      15
   16875
       5
| 843750000000 | 290·4737
  4
  441 +
5804 = 27500
       23216
58087 = 428400
        406609
580943 = 2179100
         1742829
5809467 = 43627100
          40866259
           2960831
```

60
30
30

Required the area of an equilateral triangle, each of whose equal sides is 25 chains.

$$\frac{25 + 25 + 25}{2} = \frac{75}{2} = 37.5$$

```
37.5
25
12.5
```

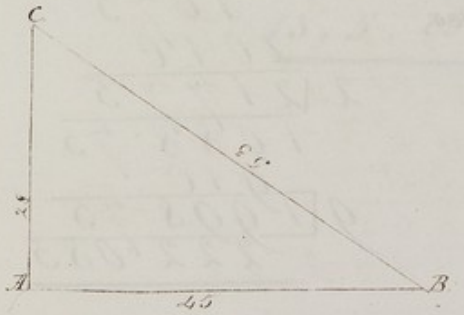



```
      37.5
      12.5
      1875
     750
    375
    468.75
        12.5
   234375
   93750
  46875
  585937.5
      12.5
  2929687.5
 11718750
 5859375
 73242187500 | 10 | 270.632
 4                  27.0632
 47332
  329
 5406 = 34218
       32436
 54123 = 178275
       163269
 541262 = 1590600
        1082524
         508076
```

Problem 4.

Any two sides of a right angled triangle being given to find the third side.

What must be the length of a scaling ladder that will reach the top of a wall, whose height is 28 feet, and the breadth of the ditch before it 45 feet?



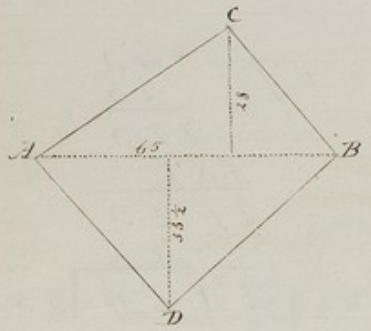

```
  28
  28
 224
 56
 784

  45
  45
 225
 180
2025
 784
2809 | 53
25
103 = 309
      309
```

Examination 6th Mo 16th 1837.

Problem 5th

To find the area of a trapezium.

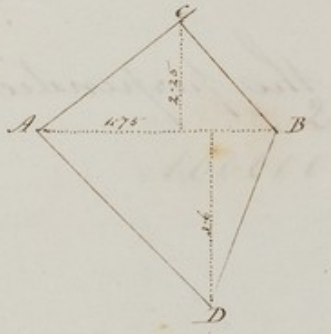
How many square yards of paving are there in a trapezium, whose diagonal is 65 feet and the two perpendiculars let fall on it from its opposite angles 28 and 33½ feet respectively.




```
   65
   28
  520
2 1820   130
  910
```

```
   33·5
    65
  1675
 2010
2 2177·5
  1088·75
   910
9 1998·75
  222·083
```

How many acres are there in the trapezium whose diagonal is 4·75 chains and the two perpendiculars falling on it, from its opposite angles 2·25 and 3·6 chains respectively.




```
   2·25
   3·6
   ----
   5·85
   4·75
   ----
   2925
  4095
 2340
 ------------
2 | 27·7875
  ----------
  13·89375
           4
  ----------
  3·57540
           40
  ----------
  23·01600
```

Ac.	ro.	po.
13	3	23 Ans.

```
  33·5
  65
  ----
  1675
 010
 ------
 177·5
 ------
 188·75
 010
 ------
 198·75
 ------
 222·083
```

PROBLEM 6th

To find the area of a trapezoid or a quadrangle which has two of its opposite sides parallel to each other.

Required the area of a trapezoid, whose two parallel sides are 20½ and 12¼ and the perpendicular distance between them 10¾


```
   20·5
   12·25
   32·75
   10·75
  163·75
 2292·5
 32·750
2|35·20625
 1760·3125 Ans.
```

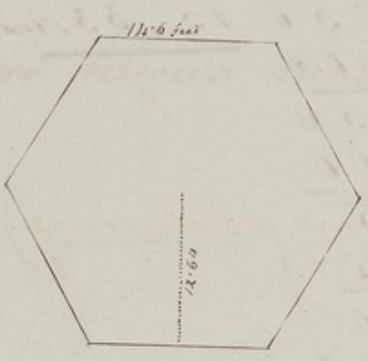
Required the area of a trapezoid, whose two parallel sides are 750 and 1225 links, and the perpendicular distance between them 1540 links.

```
                1225
                 750
                1975
                1540
               79000
              09875
             19750
          2 | 30415000
100000      15207500      Ac. 15 „ po 33 Ans.
            100000
             5207500
             5000000
              207500
                   4
              830000
                  40
             33200000
```

Problem 7th

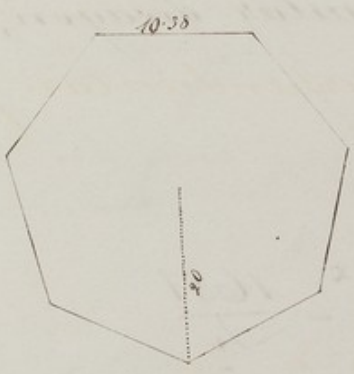
To find the area of a regular polygon.

Required the area of a hexagon, one of whose equal sides is 14·6 feet and the perpendicular from the centre 12·64 feet.



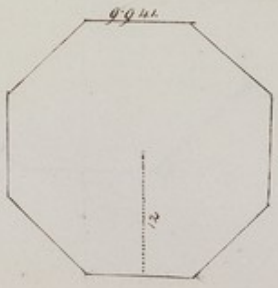

```
      14·6
       6
      ----
       876
      1264
      ----
      3504
     5256
    10512
2 | 1107·264
  |  553632
```

Required the area of a heptagon one of whose equal sides is 19·38 and the perpendicular from the centre 20.



$$
\begin{array}{r}
19\cdot38 \\
7 \\
\hline
135\cdot66 \\
20 \\
\hline
2\,\big|\,2713\cdot20 \\
\hline
1356\cdot6
\end{array}
$$

Required the area of an octagon one of whose equal sides is 9·941 and the perpendicular from the centre 12.

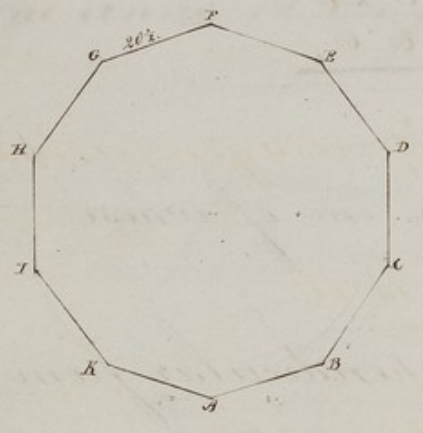


$$
\begin{array}{r}
9\cdot941 \\
8 \\
\hline
79\cdot528 \\
12 \\
\hline
2\,\big|\,954\cdot336 \\
\hline
477\cdot168
\end{array}
$$

PROBLEM 8th

To find the area of a regular polygon when one of its equal sides only is given.

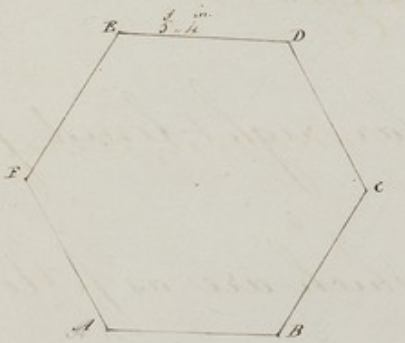
Required the area of a regular decagon, either of its equal sides being 20½.



$$20\tfrac{1}{2} = \frac{41}{2}^2 = \frac{1681}{4}$$

```
      7·6942088
           1681
      ---------
      7694208 8
    61555670 4
   46165252 8
   7694208 8
4 | 12933·964092 8
    3233·4912482 Ans.
```

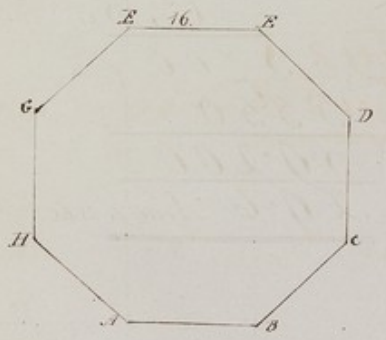
Required the area of a regular hexagon, either of the equal sides of which is 5 feet 4 inches.



$5\frac{1}{3}^2 = \frac{256}{9}$

```
      2·5980762
            256
      15588457 2
     129903810
     51961524
9 ) 665·1075072
      73·9008341
```

Required the area of a regular octagon, either of the equal sides of which is 16.



$16^2 = 256$

```
      4·8284271
            256
      289705626
     241421355
    96568542
    1236·0773376
```

PROBLEM 9th

To find the area of an irregular right-lined figure of any number of sides.

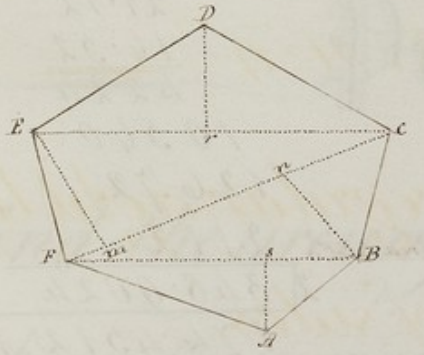
Required the area of the irregular right-lined figure ABCDEF. the dimensions of which are as follows:

FB 10·375, FC - 13·74, EC - 9·25. Bn 7·125. Em 4·675 Dr - 6·4

and As - 4·3

```
      10·375 F.B
        4·3 As.
      -------
      31125
     41500
   2 | 44·6125
     22·306 25 Area Δ ABF.
```

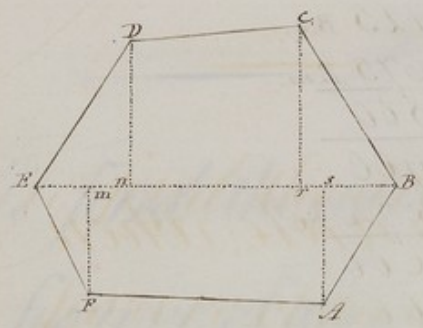
```
      9·25 E.C.
        6·4 D.e.
      -------
      3700
     5550
   2 | 59·200
     29·6 Area Δ DEC.
```



7·125 B.n.
4·675 E.m.
2 | 11·800
5·900
13·74 F.C.
23600
41300
17700
5900
81·06600 Area of the trapezium FBCE.

22·30625
29·60000
81·06600
132·97225 Area of the whole figure.

Required the area of the irregular right lined figure ABC DEF, the dimensions of which are as follows: F.n 9·08 E.m 16·52 E.r 40·02 E.s 52·44 E.B 60·30 D.m 21·12 C.r 24·48 E.n 17·12 and A.s 18·52.



17·12
9·08
13696
154080
2 | 155496
77·7248 Area △EFn

21·12
16·52
4224
10560
12672
2112
2 | 348·9024
174·4512 Area △EDm.

24·48
20·28
19584
4896
48960
2 | 496·4544
248·2272 Area △CBr

18·52
7·86
11112
14816
12964
2 | 145·5672
72·7836 Area △ABs

21·12
24·48
45·60
23·51
228000
13680
9120
2 | 1071·6000
535·8 Area △Cm

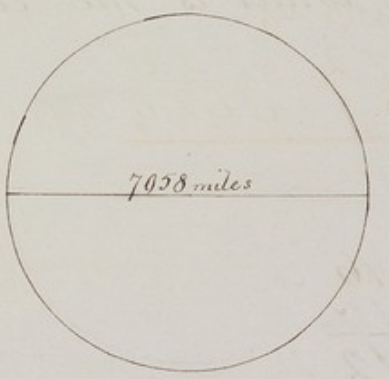
17·12
18·52
35·64
43·36
12384
10692
10092
14256
2 | 1545·3504
772·6752 Area trap. FS

77·7248
174·4512
248·2272
72·7836
535·8
772·6752
1881·6620. Area of the whole.

Problem 10th

To find the circumference of a circle, when the diameter is given or the diameter when the circumference is given.

What is the circumference of the globe of the earth supposing its diameter to be 7958 miles which it is very nearly?

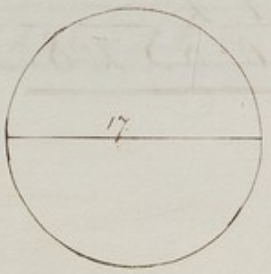



```
      3·1416
        7958
    251328
   157080
  282744
 219912
 25000·8528 Ans.
```

If the circumference of a circle be 354, what is the diameter?

```
3·1416 | 354·0000000 | 112·681 Ans.
       | 31416
        39840
        31416
         84240
         62832
         214080
         188496
          255840
          251328
           45120
           31416
           13704
```

If the diameter of a circle be 17 what is the circumference.

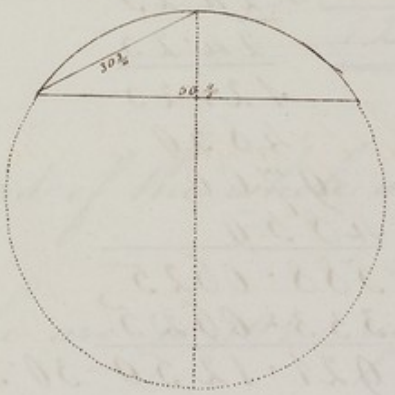



```
3·1416
    17
53·4072
```

Problem 11th

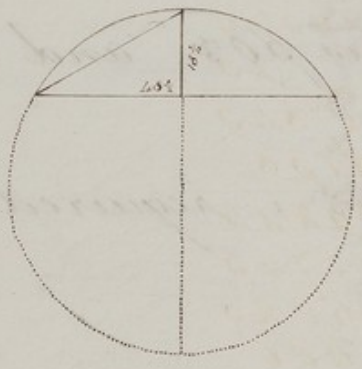
To find the length of any arc of a circle.

The chord of the whole arc is 50⅔, and the chord of half the arc is 30⅔; required the length of the arc.




```
   30·6
      8
  244·8
   50·8
3 | 1920
   64·0 Ans.
```

The chord of the whole arc is 48½ and its height or versed sine of half the arc 18¼ what is the length of the arc?

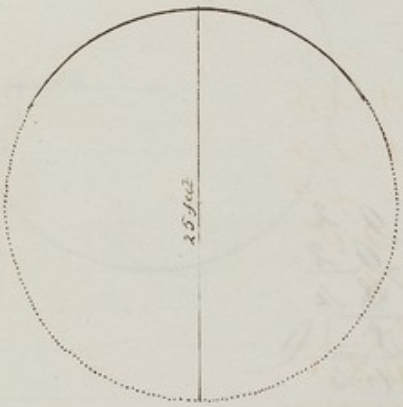



```
   18·25
   18·25
   91 25
  3650
 14600
18 25
333·0625
```

```
   30·35
       8
  242·80
   48·5
3 194·30
   64·76 Ans.
```

```
2 | 48½
    24·25
    24·25
    121 25
    4850
   9700
  4850
  588·0625
  333·0625
  921·1250 = 30·35
```

Required the length of an arc of 57° 17′ 44½″ the diameter of the circle being 25 feet.




```
60 | 44·5
60 | 17·7416
   | 57·29569
                25
          28647845
         11459138
       1432·39225
           ·01745
        71619612 5
       57495690 0
      10026745 75
      14323925
2 | 24·9952447625
    12·49762238125 - 12½ feet nearly.
```

The chord of the whole arc is 50⅘, and the chord of half the arc is 30⅓: required the length of the arc.

```
      30·6
         8
     244·8
      50·8
3 | 194·0
     64·6 Ans
```

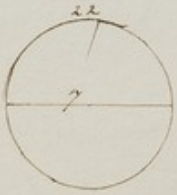
PROBLEM No 12th

To find the area of a circle.

Required the area of a circle, the circumference of which is 9½.


```
      9·2
      9·2
      184
     828
     8464
    ·07958
    6·77712
   42320
  76176
 59248
 6·7356512 Ans.
```

What is the area of a circle whose diameter is 7 and circumference 22.




```
      22
      7
  4  154
     38½ Ans.
```

How many square yards are there in a circle whose radius is 15¼ feet.


```
        30·5
        30·5
        1525
       9150
      930·25
       ·7854
     372100
    465125
   744200
  651175
9 ) 730·618350
     81·179816 Ans.
```

It is required to find the radius of the circle whose area is an acre.

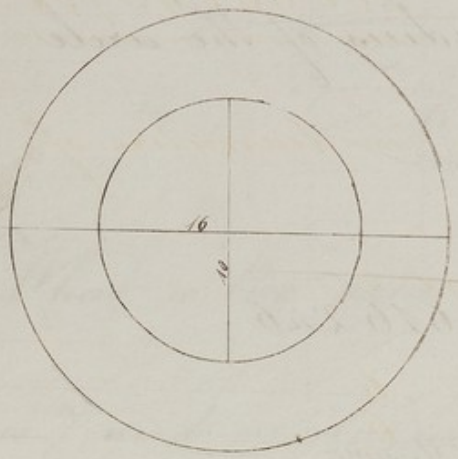
4840·000000 ÷ ·7854 = 6162·46

```
6162·46 | 2 ) 78·5
  49            39·25 Answer.
148 ) 1262
      1184
1565 )  7846
        7825
          21
```

Problem 13th

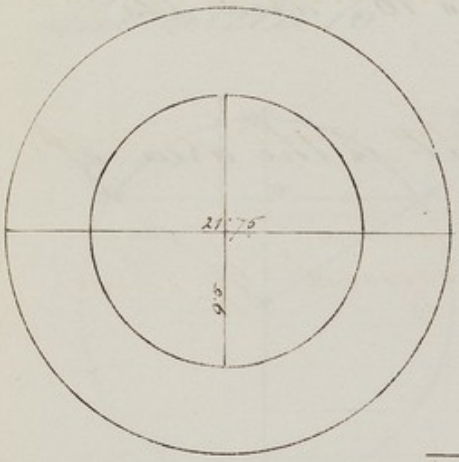
To find the area of a circular ring or the space included between two concentric circles.

The diameters of two concentric circles are 16 and 10; what is the area of the ring formed by those circles?



	16
	16
	256
	.7854
	4 7124
	39270
	15708
	201.0624
	78.5400
	122.5224 Ans.
10	
10	
100	
.7854	
78.5400	

The diameters of the two circles are 21·75 and 9·5; required the area of the ring.




```
      9·5
      9·5
     475
    855
    90·25
     ·7854
   70·8823 50
```

```
      21·75
      21·75
      10875
     15225
     2175
    4350
    473·0625
      ·7854
   1802 2500
  2365 3125
 3784 5000
3311 4375
371·5432 8750
 70·8823 50
300·6609 3750 Ans.
```

Required the area of the ring, the diameters of whose bounding circles are 6 and 4.

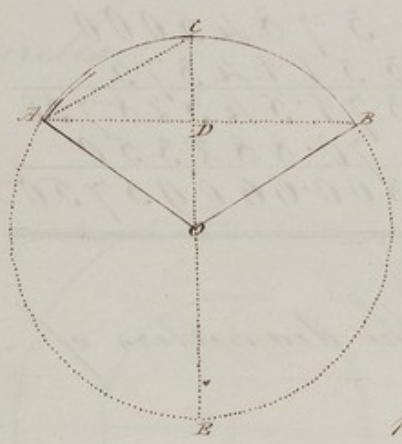
```
      4
      4
     16
    ·7854
  12·5664
```

```
      6
      6
     36
    ·7854
   4712 4
  2356 2
  28·2744
  12·5664
  15·7080 Ans.
```

PROBLEM 14th

To find the area of a sector of a circle.

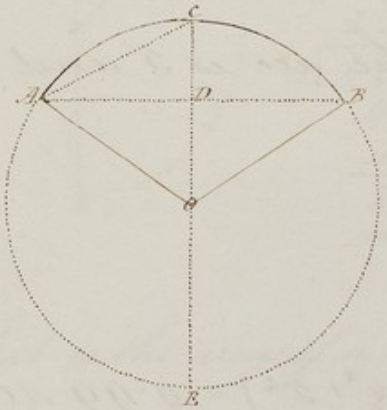
The chord of the whole arc is 16, and its height or versed sine is 6; what is the area of the sector?



$6^2 = 36$
$8^2 = 64$
$\sqrt{100} = 10$ A.C.
8
80
16 AB.
3 64
21·3 Length of the Arc.

$10^2 \div 6 = 16·6$ The Diameter
1278
1278
213
4 353·58
88·395 Area of the Sector.

Required the area of the sector, whose height or versed sine is 4, and the diameter of the circle 16.



$8^2 = 64$

4

$4^2 = 16$

$\sqrt{48} = 6.928$

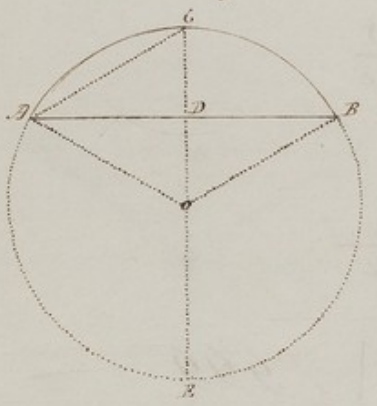
```
      6.928
      6.928
     -------
     55424
    13856
   62352
  41568
  ---------
  47.997184
  16.
  ---------
  63.997184 | 7.999
  49
  ---------
  149 1499
      1341
  ---------
  1589 15871
       14301
  ---------
  15989 157084
        143901
        ------
        13183
```

```
      7.999
          8
     -------
     63.992
     13.856
     -------
  3  50.136
     16.712
         16
     -------
4 | 267.392
     -------
     66.848 Area of Sector.
```

PROBLEM 15th

To find the area of a segment of a circle

Required the area of a segment of a circle whose height or versed sine of half the arc is 5 and the diameter of circle 20.



$10^2 = 100$

5

$5^2 = \underline{25}$ $\sqrt{(8.66^2 + 5^2)} = 9.99$ AC

$\sqrt{75} = 8.66$ AD

9.99 AC.

$\underline{8}$

$\sqrt{79.92}$

17.32 AB

$3 \overline{\big)\, 62.60}$

20.86

$\underline{20}$

$4 \overline{\big)\, 417.20}$

104.3 Area of Sector

$\underline{43.3}$

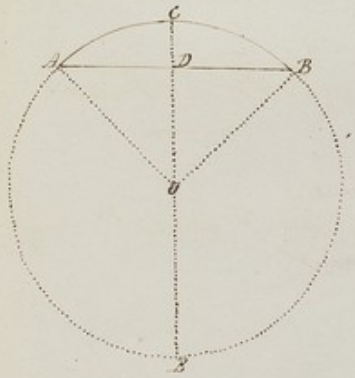
61.0 Answer.

8.66

$\underline{5}$

43.30 Area $\triangle$ OAB.

What is the area of the segment of a circle, whose arc is a quadrant, the diameter being 18 feet?



$18^2 = 324$

$\cdot 7854$

31416

15708

23562

4 | 254·4696 Area of the circle.

63·6174 Area of the sector.

40·5

23·1174 Area of the segment.

9

9

2 | 81

40·5 Area of the triangle.

What is the area of a segment of a circle, whose height or versed sine is 5, and the diameter of the circle 25?

25 | 50

2 the tabular versed sine.

·111823 - tabular segment.

625 - 25^2

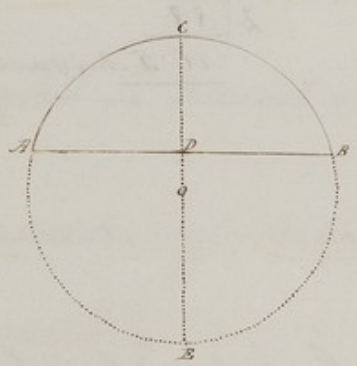
559115

223646

670938

69·889375 Area of the segment.

What is the area of the segment of a circle, whose height, or versed sine, is 3, and the diameter of the circle 8?



8 3

·375 tabular versed sine

·269013 - tabular Segment

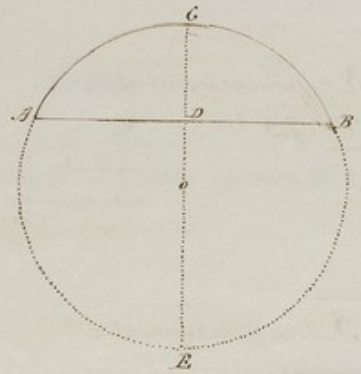
64 - 8^2

1076052

1614078

17·216832 Area of Segment

What is the area of the segment of a circle, whose height or versed sine, is 6 and the diameter 21?



21 = 3 6

7 2000

·285 tabular versed sine.

·184521 Tabular Segment.

441 - 21^2

184521

738084

738084

81·373761 Area of the Segment.

Problem 16th

To find the area of a circular zone, or the space included between two parallel chords, and their intercepted arcs.

One of the parallel chords of a circular zone is 48, and the other 30, and its breadth is 13; what is the area of the zone?

$$\sqrt{\left\{13^2+\left(\frac{48^2+30^2}{2}\right)+\left(\frac{48^2-30^2}{52}\right)^2\right\}}=50 \text{ Diameter.}$$

$$\frac{50}{2}-\frac{1}{2}\sqrt{\left\{\left(\frac{48+30}{2}\right)^2+\left(\frac{48^2-30^2}{52}\right)^2\right\}}=1\cdot28295 \text{ Versed Sine.}$$

50) 1·28295

·025659 Tabular versed sine

48
30
78
13
2) 1014
507 area of trapezoid

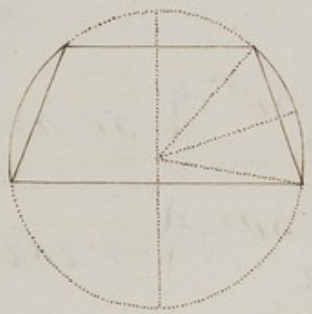
·005230 Tabular segment
2500 = 50^2
026150000
10460
130750000 area of segment
2
261500000 double segment
507
533·15 Area of the Zone.

Required the area of a circular zone, each of whose parallel chord are 50, and their perpendicular distance 30.

$\sqrt{(30^2 + 50^2)} = 58{\cdot}309519$ = Diameter.

$\frac{1}{2}(58{\cdot}309519 - 50) = 4{\cdot}154759$ = Versed Sine.

$4{\cdot}154759 \div 58{\cdot}309519 = {\cdot}071253$ Tabular versed sine.

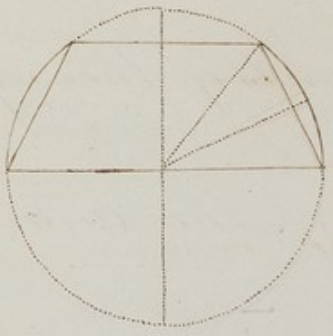


$$\begin{array}{r} 50 \\ \underline{56} \\ 100 \\ \underline{30} \\ 2\,\boxed{3000} \\ \hline 1500 \end{array}$$ Area of Trapezoid.

$$\begin{array}{r} {\cdot}024680 \\ \underline{3400} \\ 8872000 \\ \underline{74040} \\ 82{\cdot}912000 \\ \underline{2} \\ 1{\cdot}658240\,00 \\ \underline{1500{\cdot}} \\ 1665{\cdot}824 \end{array}$$

Tabular segment. $3400 = 58{\cdot}309519^2$ area of segment double segment. Answer.

It is required to find the area of a circular zone, the greater chord of which, being equal to the diameter of the circle, is 40, and the less 20.



$20^2 = 400$
$10^2 = 100$

```
        300 | 17·3205
          1
27      | 200
        189
343     | 1100
        1029
3462    | 7100
        6924
346405  | 760000
          732025
          27975
```

```
17·3205
     10
173·2050 Area of △ COD
```

```
        ·7854
40² =   1600
6 | 12566400 = Area of the Circle
    209·44   = Area of the Sector
    173·205  = Area of △ COD
    36·235   = Area of the segment CED.
```

```
628·32 = Area of Semicircle
 36·235
592·085 Area of Zone.
```

PROBLEM 17th

To find the area of a lune, or the space included between the intersecting arcs of two eccentric circles

The chord is 20, and the heights of the segments 10 and 2; required the area of the lune.

2 : 10 :: 10

10

2) 100

50 + 2 = 52 Diameter of less seg.

52) 2

.0384 Tabular versed sine

.7854

400 = 20^2 Diameter

2 314·1600

157·08 Area of greater Segment.

.009763 Tabular segment

2704 = 52^2

39052

683410

19526

26·399152 = Area of less segment

157·08

130·680848 Ans.

Supposing the length of the chord to be 48, and the heights of the segments to be 18 and 7; what is the area of the lune?

18 : 24 :: 24

24
96
48
18 | 576
32 + 18 = 50 Diameter of less seg.

50 | 18·
36 Tabular versed sine

254550
2500 = 50^2
127275000
509100
636·375000 Greater seg.

7 : 24 :: 24

24
96
48
7 | 576
$82\frac{2}{7} + 7 = 89\frac{2}{7}$

$\frac{7}{89\frac{2}{7}} = \frac{40}{625} = 0'784$

636·375
226·052295
410·322705 Answer

028356
$625^2 =$ 390625
441780
56742
170136
255204
85068
$49 = \frac{7}{1}$ | 110076·562500
| 1582·366071
220·052295

PROBLEM 18th

To find the area of a mixtilineal figure, or one formed by right lines and curves.

The length of an irregular mixtilineal figure is 47 and its breadths, at 6 equidistant places beginning at the left hand extremity of the base, 5·7; 4·8; 7·5; 5·1; 8·4; and 6·5; what is its area?

```
        5·7
        4·8
        7·5
        5·1
        8·4
        6·5
6 )     38·0
        6·3333
              47
        44333 1
       25333 2
       297·6051 Answer
```

The length of an irregular mixtilineal figure, of which the curvilinear boundary meets the base, is 37½, and its breadth at 7 equidistant places is 4·9; 5·6; 4·5; 8·2; 7·3; 5·9, and 8·5; what is its area?

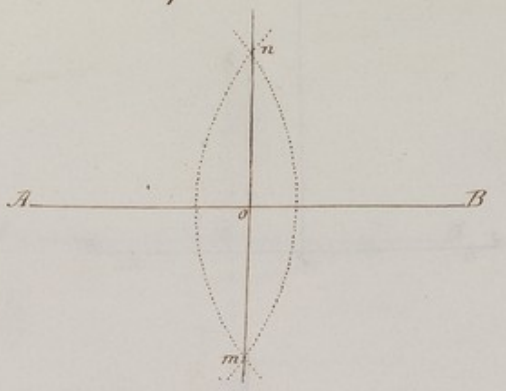
```
        4·9
        8·5
2 )    13·4
        6·7
        5·6
        4·5
        8·2
        7·3
        5·9
6 )    38·2
        6·3666
           37·5
       31·8330
      445662
     190998
    238·74750 Answer.
```

PRACTICAL

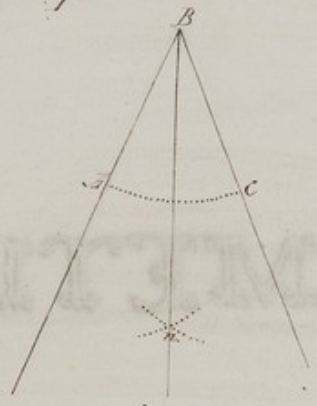
GEOMETRY.

Problem 1st

To divide a given line AB into two equal parts.

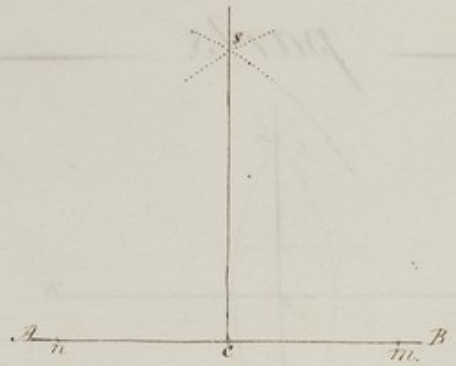


2nd To divide a given angle ABC into two equal parts.

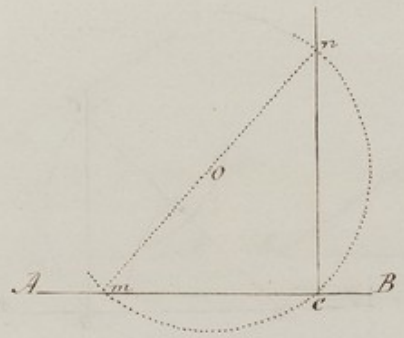


3rd From a given point C in a given right AB to erect a perpendicular.

Case 1st When the point is near the middle of the line.

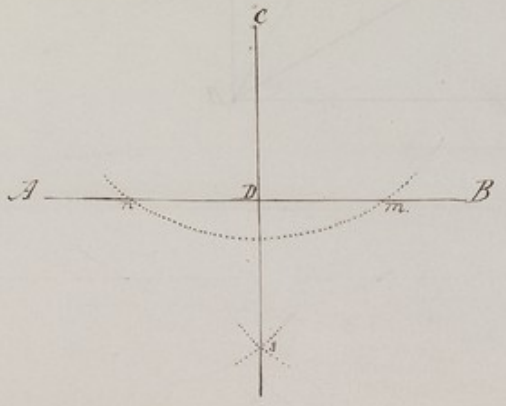


Case 2nd When the point is opposite, or nearly opposite to the end of the line.

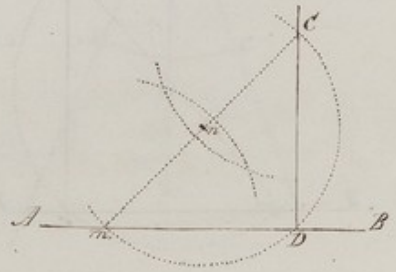


4th From a given point c, without a given line AB to let fall a perpendicular.

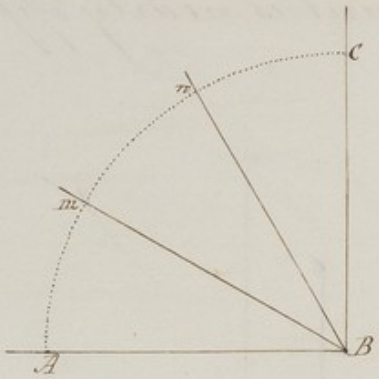
Case 1st When the point is nearly opposite to the middle of the line.



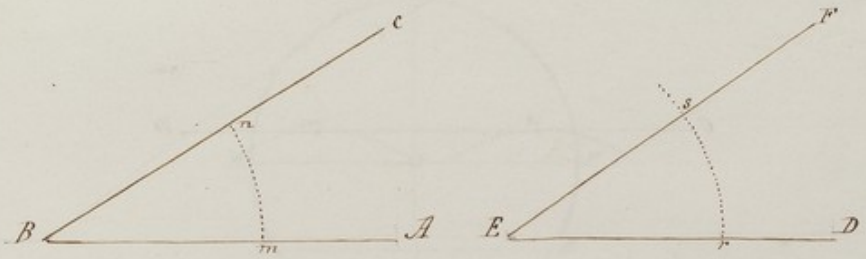
Case 2nd When the point is opposite, or nearly opposite to the end of the line.



5th To trisect, or divide a right angle ABC into three equal parts.

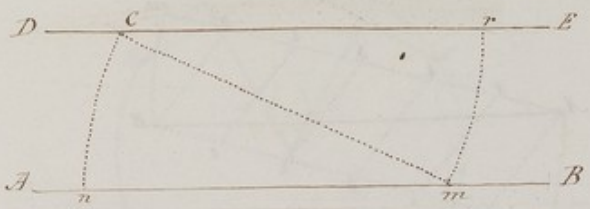


6th At a given point E, to make an angle equal to a given angle ABC.

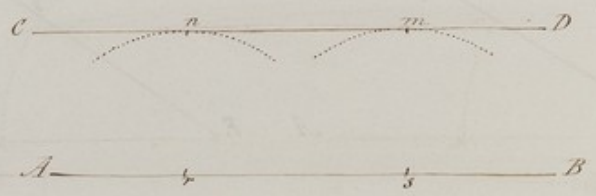


7th To draw a line parallel to a given line AB.

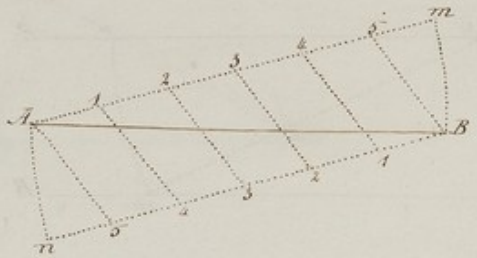
Case 1st When the parallel line is to pass through a given point C.



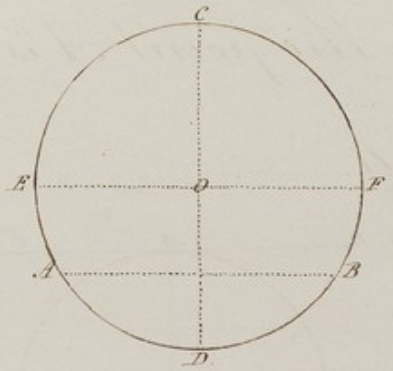
Case 2nd. When the parallel line is to be at a given distance from AB.



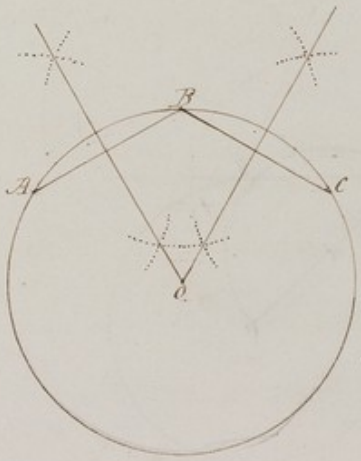
8th. To divide a given line AB into any proposed number of equal parts.



9th To find the centre of a given circle, or of one already described.

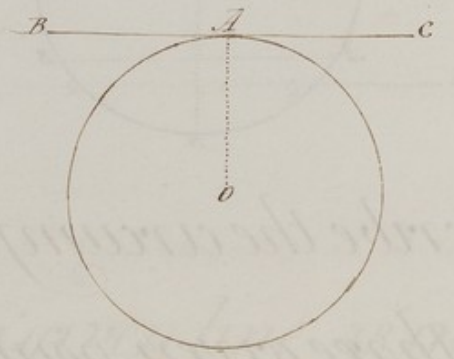


10th To describe the circumference of a circle through any three given points A.B.C. provided they are not in a right line.

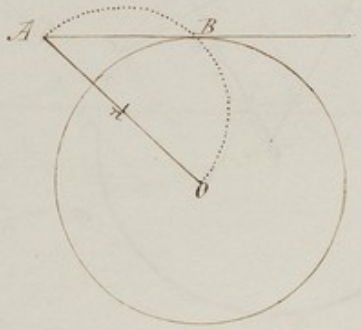


11th To draw a tangent to a given circle, that shall pass through a given point A.

Case 2nd When the point A is in the circumference of the circle.

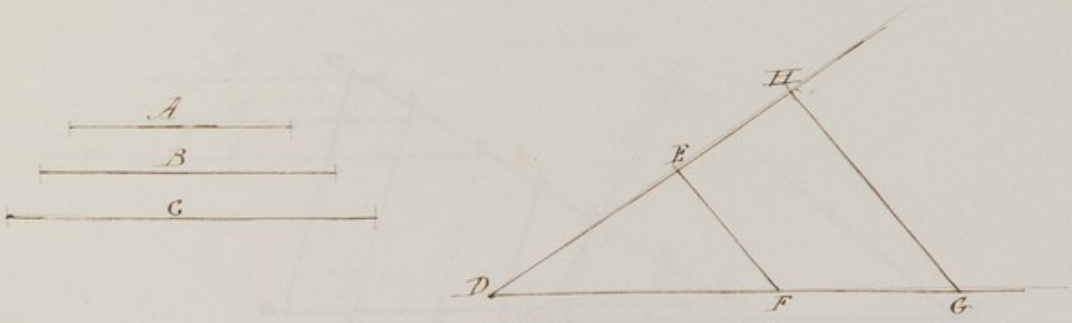


Case 2nd When the given point is without the circle.

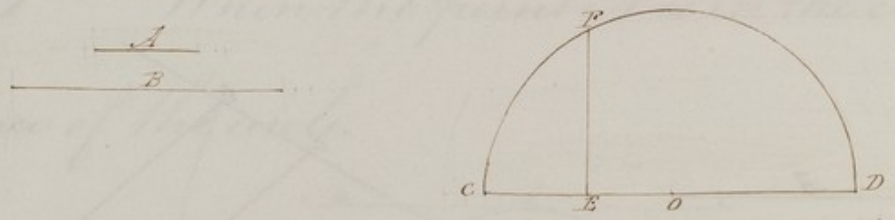


12th To two given right lines, A.B. to find a third proportional.

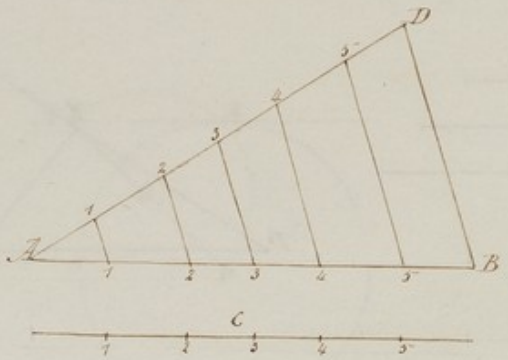
13th To three given right lines, A.B.C to find a fourth proportional.



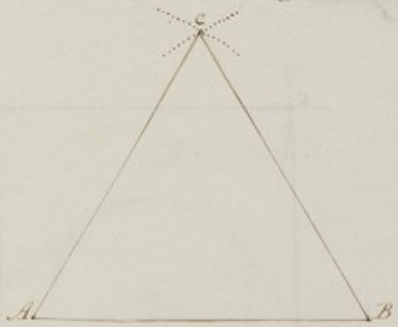
14th Between two given right lines AB to find a mean proportional.



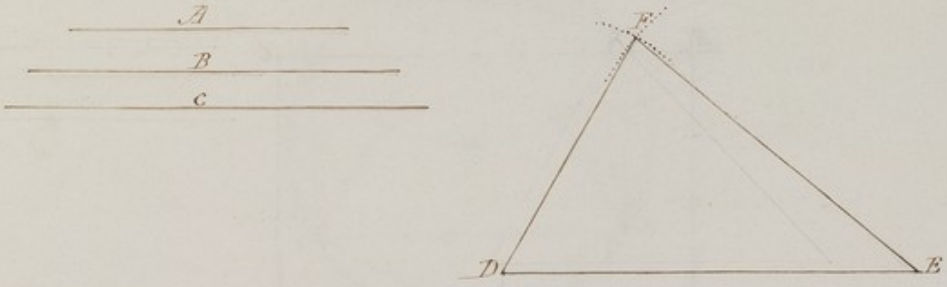
15th To divide a given line AB into the same number of proportional parts, that another given line C is divided into.



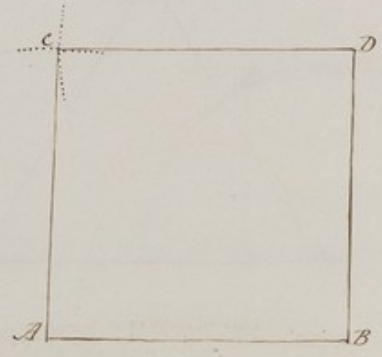
16th Upon a given right line A.B. to make an equilateral triangle.



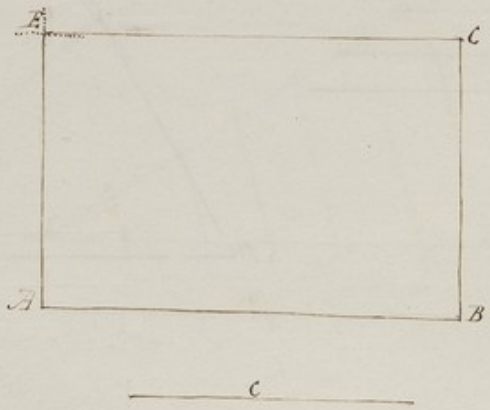
17th To make a triangle, the three sides of which shall be respectively equal to three given lines A,B,C.



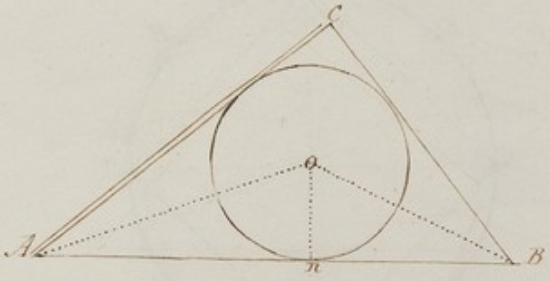
18th Upon a given line AB. to describe a square.



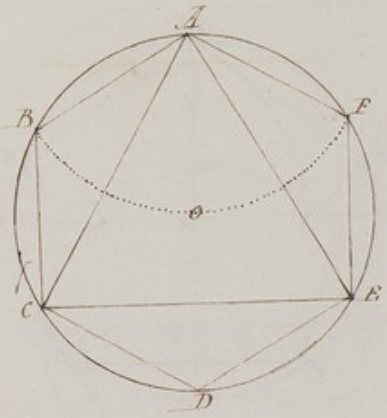
19th To describe a rectangle, whose length and breadth shall be, respectively, equal to two given lines AB and C.



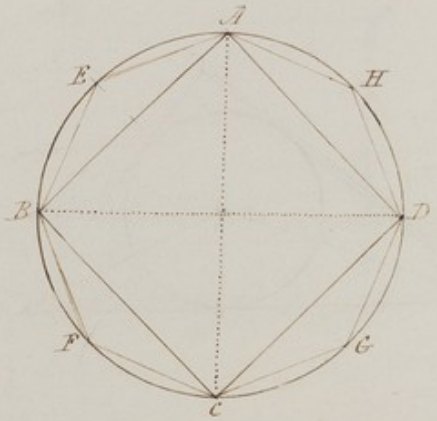
20th In a given triangle ABC. to inscribe a circle.



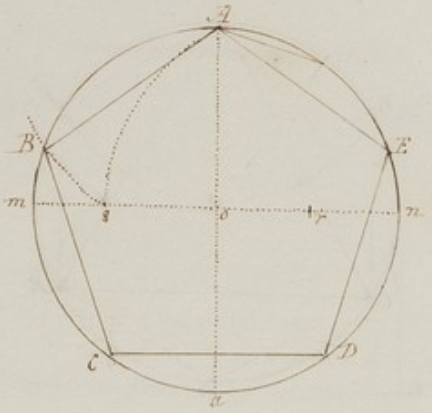
21st In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon.



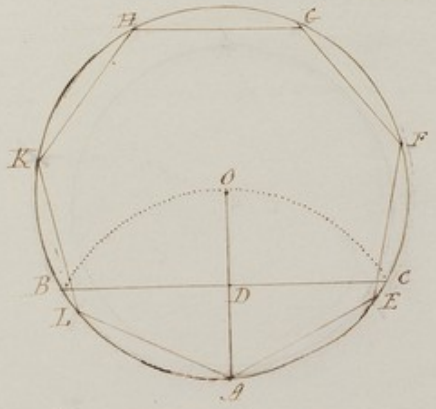
22nd. To inscribe a square, or an octagon in a given circle.



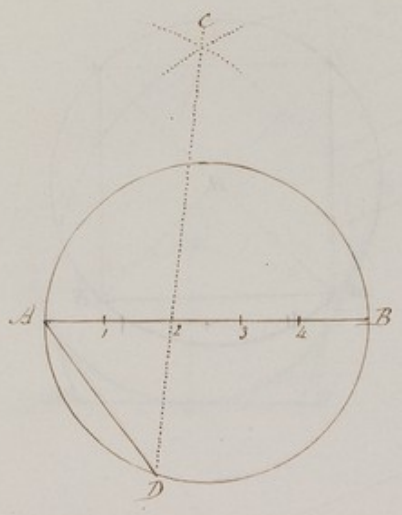
23rd. To inscribe a regular pentagon or decagon in a given circle.



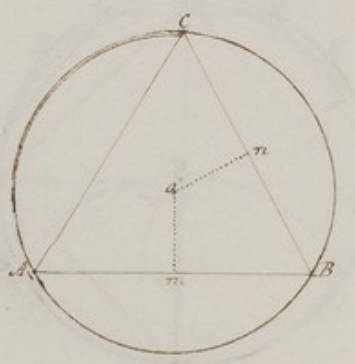
24th In a given circle to inscribe a regular heptagon.



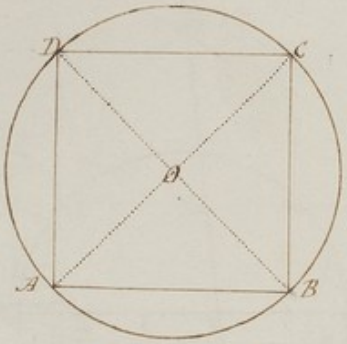
25th In a given circle to inscribe any regular polygon.



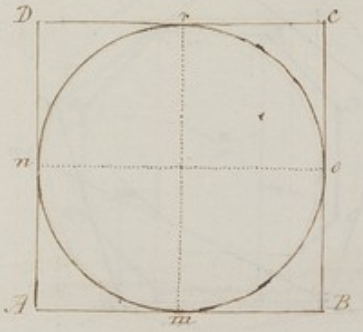
26th About a given triangle ABC to circum-scribe a circle.



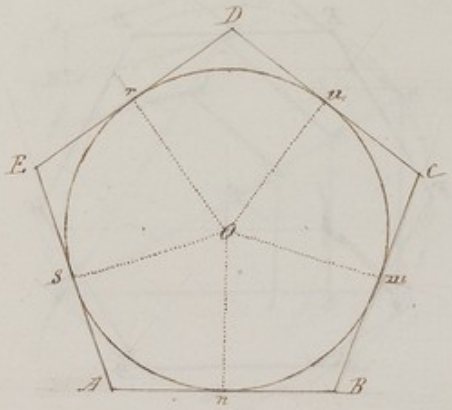
27th About a given square ABCD to circum-scribe a circle.



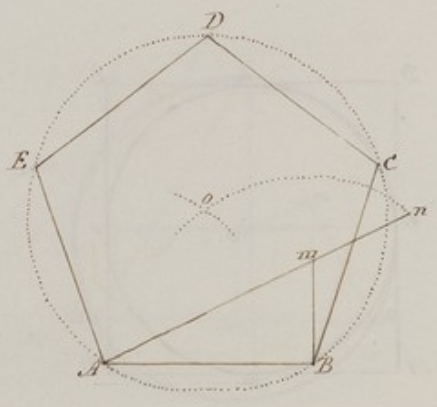
28th To circumscribe a square about a given circle.



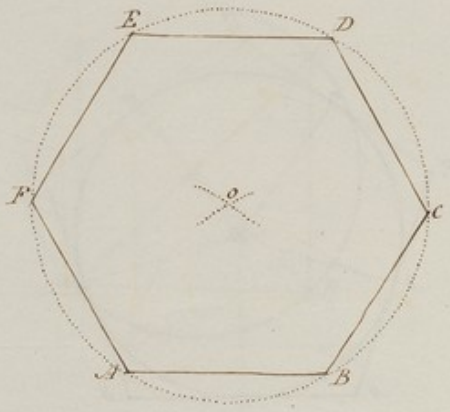
29th About a given circle to circumscribe a regular pentagon.



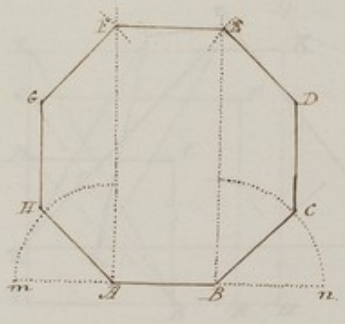
30th On a given line AB to make a regular pentagon.



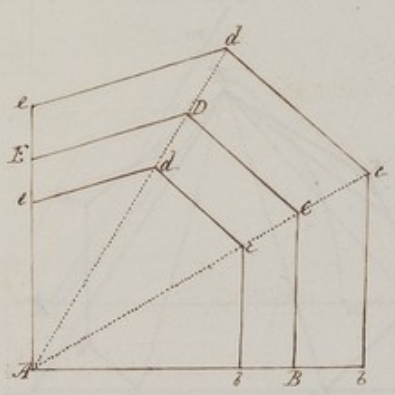
31st On a given line AB to make a regular hexagon.



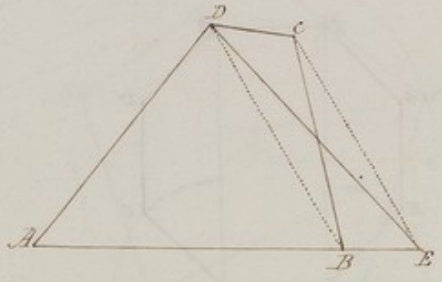
32nd. On a given line AB to form a regular octagon.



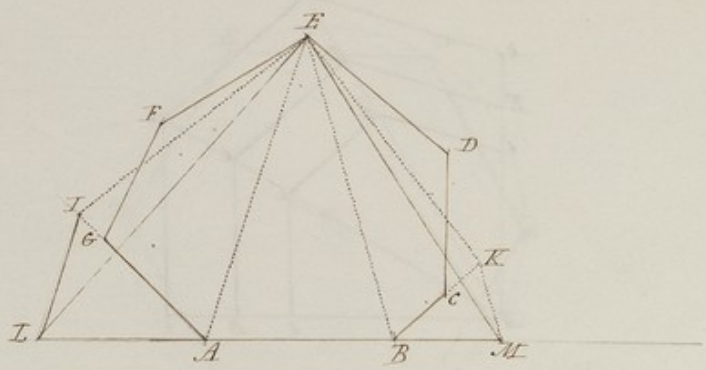
33rd. To make a figure similar to a given figure ABCDE.



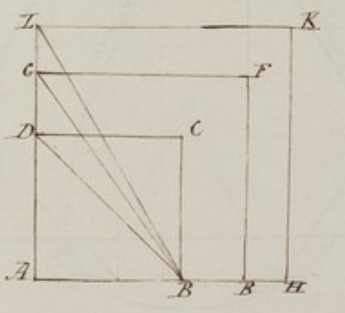
34th To make a triangle equal to a given tra-pezium ABCD.



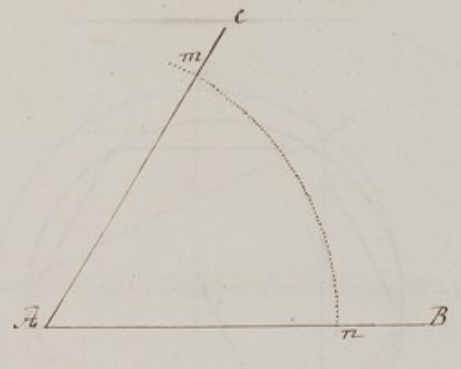
35th To make a triangle equal to any right-lined figure ABCDEFGA.



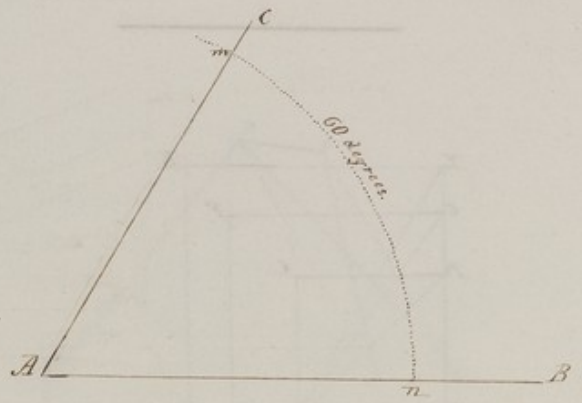
36th To describe a square that shall be any multiple of a given square ABCD.



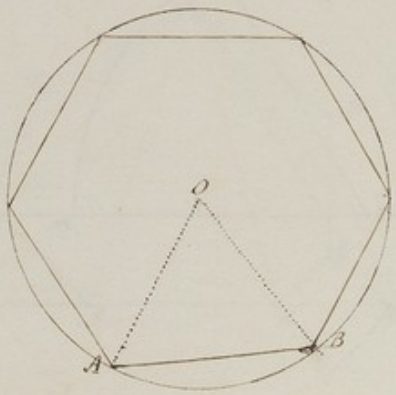
37th To make an angle of any proposed number of degrees.



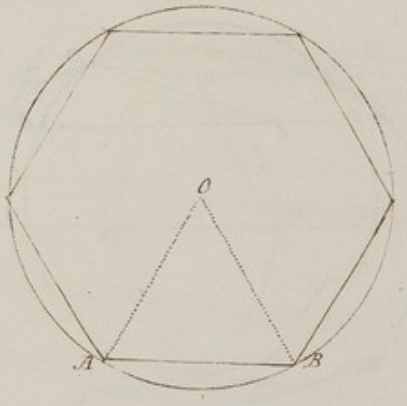
38th Any angle C.A.B. being given, to find the number of degrees it contains.



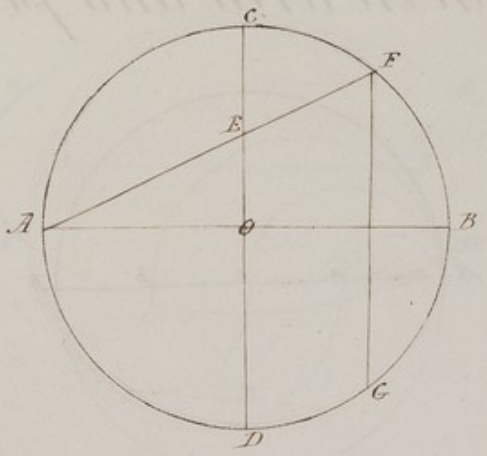
39th In a given circle to inscribe a polygon of any proposed number of sides.



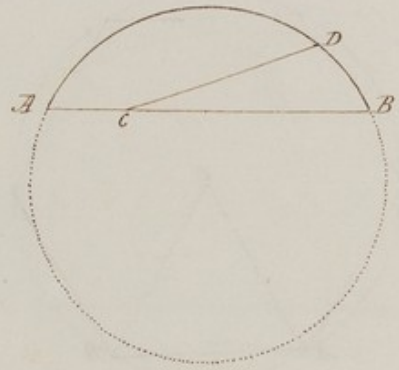
40th On a given line AB. to form a regular polygon of any proposed number of sides.



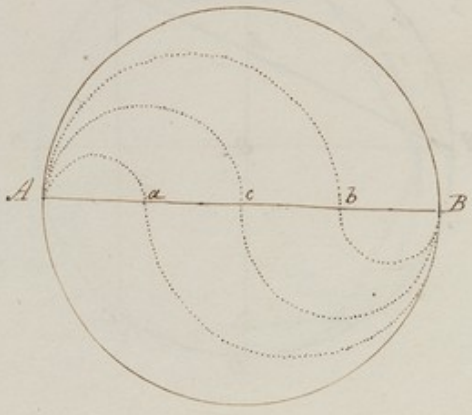
41st To make a square that shall be nearly equal to a given circle.



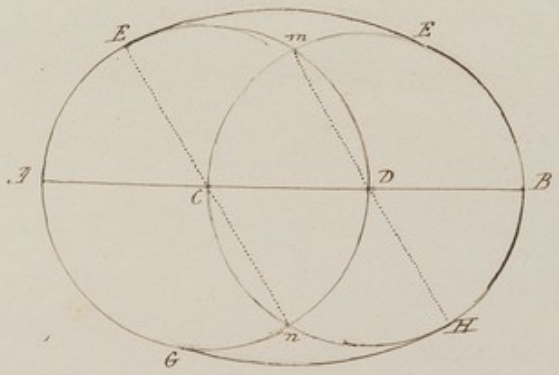
42nd To find a right line that shall be nearly equal to any given arc ADB of a circle.



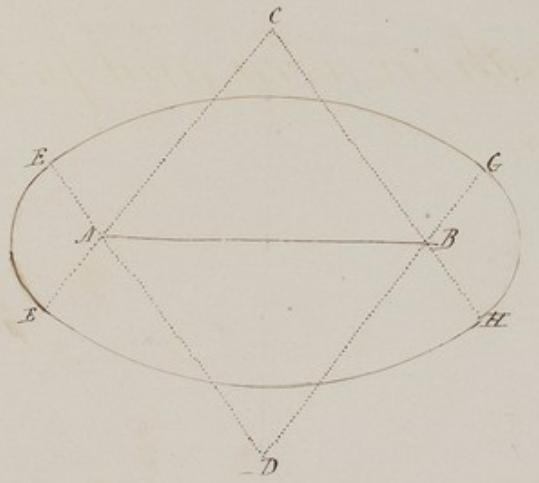
43rd To divide a given circle into any proposed number of parts, that shall be mutually equal to each other, both in area and perimeter.



44th Upon a given line AB, to describe an oval, or a figure resembling an ellipse.



The same otherwise.



Ex. 6mo 1838.