STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES. I. THE RELATIONSHIP BETWEEN "CORRECTED" DEATH RATES AND LIFE TABLE DEATH RATES.

BY

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STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES. I. THE RELATIONSHIP BETWEEN "CORRECTED" DEATH RATES AND LIFE TABLE DEATH RATES.

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(With 2 Diagrams.)

Circumstances in connection with the recent census have again directed my attention to the laws which govern human life. I have long been of the opinion that the old ideas that birth rates and death rates had no biological relationship beyond the obvious ones, that many infants mean more deaths, etc., usually found stated in public health text books, were based on a very imperfect induction. On one aspect of this I published a paper a number of years ago in the Transactions of the Royal Philosophical Society of Glasgow (1), and last summer Sir Shirley Murphy (2) read a paper on another aspect of the same subject before the Sanitary Congress held at York. But I have hitherto refrained from publishing theories because I believe that until quantitative measures are applied no scientific results worthy of discussion can be obtained. Now that such seem to be possible, I propose to discuss in a series of papers the different relationships which I have investigated. No mathematics will be introduced in the earlier papers, but the results of all will be summarised and dealt with in their mathematical and physico-chemical relationships in a concluding communication. The first paper relates to the connection between the "corrected" death rates and the "true" death rates as found by constructing a life table.

Many conclusions in public health work depend on the use of death rates as a means of comparing the state of health in different districts. The death rate commonly used is termed the "crude" death rate, and is obtained by dividing the total number of deaths in a district in one year by the total number of inhabitants. The result is given in parts per thousand. This criterion has long been known to be of doubtful worth. The mortality varies with age and sex, and even in adjacent districts the distribution of persons according to these categories is so different as to preclude the comparison. Thus towns contain a very large number of young adults attracted by the opportunity of obtaining work, and as these constitute the healthiest part of the community the "crude" death rate is correspondingly reduced. To meet this, the refinement of the "corrected" death rate has been introduced. The mortalities for each sex at each age period having been ascertained, these are applied severally to the different age and sex groups of the population in the whole country (termed in this connection a "standard population"). This gives the figure which would be found if the mortalities in the district could be assumed to hold for the whole country. Such figures obviously admit of more certain comparison among themselves than those obtained by the older method. The drawback to this method is evidently the fact that the standard population is not a stationary but an increasing population, in which there is, more or less consistently, a smaller number of persons living as age is approached than would be present in a population in which the death rate equals the birth rate. More infants exist, it is true, than in a stationary population, but the period of the high mortality in childhood is short, not more than five years, and the next few quinquennia have a very low mortality, while at the later ages where the mortality is high there are relatively fewer persons living. Thus a "corrected" death rate is not a real death rate. It may be fictitiously low. This point is very important since many conclusions are daily being drawn from such figures. We hear of garden cities with mortalities of 7 per 1000, etc., though even 12 per 1000 is a death rate to be interpreted only with knowledge and discrimination. Neither, I fancy, can under the best conceivable conditions have any real meaning. This is obvious when we consider that in a stationary population the average age of the population in years at death or the expectation at birth is obtained by dividing the number of persons living by the number of deaths per annum. Thus if the population be 1000 and the annual number of deaths 20, the average age at death is 50 years, a possible result. Twelve per 1000 means the average age of 83 years, 7 per 1000 an average age of 143 years, both sufficiently ridiculous. The usual way to attain the truth is to construct a life table, but that is a process requiring both labour and mathematical skill. Were this the only solution it would be well-nigh impracticable to press its use, but a considerable number of life tables have now been calculated, and by the use of these the true death rate may easily be estimated if the "corrected" death rate be known.

The life tables utilised in the calculations made for this paper are given in the following table. For convenience each is hereafter referred to by the letter placed opposite its description.

TABLE I.

Life Table of England (Farr) 1838–1851	E_1
,, ,, ,, (Ogle) 1871–1881	E_2
,, ,, ,, (Tatham) 1881–1891	E_3
,, ,, ,, (Tatham) 1891–1901	E_4
Healthy District Life Table of England (Farr) 1849–1851 ...	F
,, ,, ,, ,, ,, (Tatham) 1881–1890	H_1
,, ,, ,, ,, ,, (Tatham) 1891–1901	H_2
Brighton Life Table (Newsholme) 1881–1890	B
Manchester Life Table (Tatham) 1881–1890	M
London Life Table (Murphy) 1891–1900	L
Scottish Life Table (Adam) 1891–1900	S
Glasgow Life Table (Chalmers) 1891–1900	G

The number of tables is twelve. The first eleven give in all respects absolutely concordant results. The last, that for Glasgow, shows some differences, due I think to the fact that all the deaths at high ages belonging to Glasgow are not included. At the period for which it was constructed there was no mechanism by which deaths occurring in institutions outside Glasgow could be returned to the city, and the number of institutions outside Glasgow was very considerable. This criticism is borne out by the results of the recent census, and by a comparison with the death rates in Glasgow at the present time.

A life table as usually understood may be defined as the numerical construction of a stable population which possesses the same mortalities at each age as those in the population to be examined. By this means irregularities in the proportions of persons of different ages and sexes, due to varying birth rates and to immigration and emigration, are eliminated. The death rate of any population either "crude" or "corrected" will not be that of the life table. Generally it will be below the latter. Exceptionally, if the death rate is sufficiently high as to annul the natural increase and bring about a stationary or a declining population the "crude" or "corrected" death rate is found to be equal or greater than that obtained when a life table has been constructed.

Two examples will illustrate this. The corrected death rate of the males in the healthy district life table H_2 is found to be 13·49. The actual expectation of life is 52·87 years, giving a death rate of 18·91, both quite conceivable figures. A death rate of 13·49 gives, however, on a stationary population a mean life of $\frac{1000}{30 \cdot 49}$ or 74 years, a figure quite impossible. Manchester on the other hand has a corrected death rate of 28 per 1000. The expectation of life is 34·71 years, giving a death rate of 28·81 on a stationary population. In other words, we do not really have variations of the death rate from 13·5 to 28, but from 18·9 to 28·8, a difference much more easily understood.

For certain reasons, which will be discussed in a later paper, I think that the very highest mean age possible is about sixty years, and this represents a real death rate of 16·3.

How then is this true or life table death rate to be obtained? The method, which partly depends on the biological response of mankind to unhealthy conditions and is partly a pure arithmetical necessity, is expressed in the statement that the relationship between the true death rate and the corrected death rate is linear; that is, given the latter, the former is obtained by multiplying by a constant fraction and adding a definite constant. Thus if D_2 be the real death rate for the whole population of a district and D_1 the "corrected" death rate,

$$D_2 = \cdot 6842 D_1 + 9 \cdot 65.$$

D_2 is thus equal to D_1 when both are equal to 30·5. For diagrammatic purposes the difference between the true and the corrected death rates is the better figure to choose, in which case the above formula may be written

$$D_2 - D_1 = - \cdot 3158 D_1 + 9 \cdot 65.$$

But the theorem is yet more general. It is not necessary to begin at birth, any age is equally appropriate. To obtain the expectations of life at each period, all that it is necessary to know is the "corrected" death rate for all persons above that age.

These are severally calculated in exactly the same manner as the "corrected" death rate itself is calculated. The multiplications are the same, the only difference being that the sum is made by stages. Thus the two products at 75— and 65—75 are added together, then the product at 55—65 to the latter sum, and so on, so that we have a series of sums each to be divided by corresponding numbers obtained by summing the standard population in the same way. Corresponding to this series of death rates we have from the life tables a similar series of real death rate figures obtained by dividing 1000 by the expectation of life at the corresponding ages. At each age the relationship between the two series of figures is linear, the general equations, which are equivalent, being

$$D_2 - D_1 = -m\ D_1 + C$$

and

$$D_2 = (1 - m)D_1 + C.$$

The series of values of m and C from 0 to 55 years is given for both sexes in the accompanying table (Table II).

TABLE II.

	Males			Females		
Age	m	$1-m$	C	m	$1-m$	C
0	·3188	·6811	9·54	·3151	·6849	9·32
5	·5441	·4559	12·68	5246	·4754	12·05
10	·5414	·4586	13·49	·5670	·4330	13·36
15	·4868	·5132	13·53	·5666	·4334	14·35
20	·5140	·4860	14·95	·5400	·4600	15·06
25	·4883	·5117	15·74	·5246	·4754	15·98
35	·4953	·5047	19·16	·4922	·5078	18·05
45	·4953	·5047	23·63	·4397	·5603	20·54
55	·3052	·6948	20·18	·3575	·6425	22·66

The data on which these figures are based are given in Table III. In parallel columns the corrected death rate D_1 the true death rate D_2 obtained from the life table, the actual difference being between the true death rate and the life table death rate, $D_2 - D_1$, and the theoretical value of the latter obtained by fitting the best straight lines, either by the method of least squares or by inspection, are given. The differences found between the real and the theoretical values are tabulated next, and the value of the square root of the mean of the squares of these differences, denoted by Δ, is added. This last figure gives the measure of the difference between the real and theoretical values. It rarely exceeds one per cent. The relationship however is best seen in diagrams. For this purpose the male death rates at 15 years (Diagram I) and at 55 years (Diagram II) have been chosen. Both illustrate well the concordance of fact and theory, and though the divergence is greater in the latter case than in the former the relationship is obviously truly linear.

When the great range of the corrected death rates is considered, *e.g.*, at birth in the case of males, from 13 per 1000 to 28 per 1000, this must be considered a very small error in a prediction of the true death rate. In fact the values obtained by the method here given may with some confidence be taken as probably more accurate than those obtained directly by constructing a life table, inasmuch as they represent the average of many life tables. The figures of one life table are at best

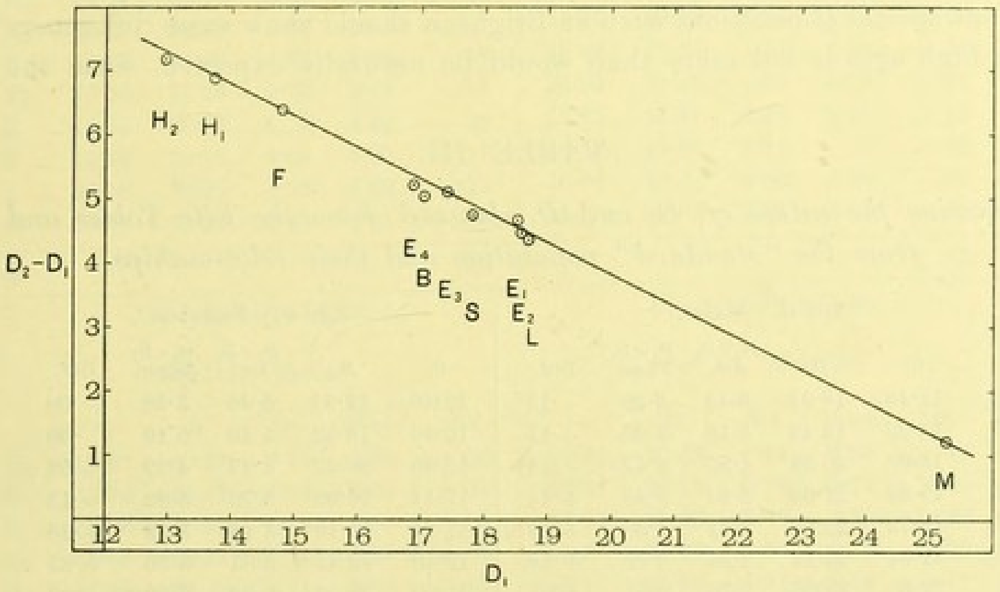

Diagram I.

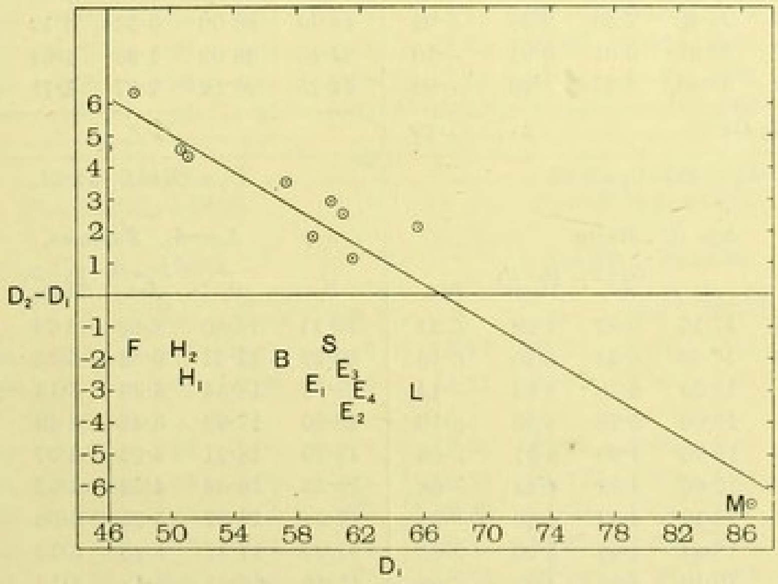

Diagram II.

but a close approximation. So much immigration at certain ages takes place from country to town, *i.e.*, from healthy to unhealthy conditions, that the average death rates at the ages of migration represent not one phenomenon but a mixture of phenomena. The same is more or less true at all ages, since methods of living vary from class to class and all classes are grouped together in the one table. The error due to these factors, however, cannot be great on the whole, but may, as the above tables seem to show, be taken as probably not more than one per cent. That special populations such as Brighton should show some differences at high ages is not more than would be naturally expected, since the

TABLE III.

Showing the values of D_2 and D_1 obtained from the Life Tables and from the "standard" population and their relationships.

	Age 0. Males.					Age 0. Females.				
	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	13·49	18·91	5·42	5·29	−·13	12·49	17·95	5·46	5·38	−·08
H_1	14·26	19·42	5·16	5·05	−·11	13·40	18·50	5·10	5·10	·00
F	16·03	20·59	4·56	4·49	−·13	15·95	20·22	4·27	4·29	+·02
E_4	19·32	22·66	3·34	3·45	+·11	17·14	20·93	3·79	3·92	+·13
E_3	19·79	22·90	3·11	3·30	+·19	17·74	21·19	3·45	3·73	+·28
E_2	21·64	24·18	2·54	2·72	+·18	19·40	22·41	3·01	3·36	+·35
E_1	22·30	25·06	2·76	2·51	+·05	21·00	23·90	2·90	2·90	·00
B	19·75	22·94	3·19	3·31	+·12	16·05	20·41	4·36	4·26	−·10
S	19·21	22·36	3·15	3·58	+·43	17·31	21·06	3·75	3·88	−·13
L	21·82	24·40	2·58	2·66	+·08	18·49	22·06	3·57	3·49	−·08
M	28·00	28·81	0·81	0·91	−·10	24·46	26·02	1·56	1·64	+·08
G	26·43	28·43	2·00	1·20	−·80	24·15	26·52	2·37	1·71	−·66
Δ (excluding G)					·17					·15

$D_2 = \cdot 6811 D_1 + 9\cdot 54.$ $D_2 = \cdot 6849 D_1 + 9\cdot 32.$

	Age 5. Males.					Age 5. Females.				
	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	10·29	17·16	6·87	7·08	+·21	10·11	16·80	6·69	6·74	+·05
H_1	10·99	17·53	6·54	6·70	+·16	10·92	17·24	6·32	6·32	·00
F	12·21	18·39	6·18	6·04	−·14	13·18	18·54	5·36	5·13	−·23
E_4	13·41	18·69	5·28	5·38	+·10	12·50	17·92	5·42	5·49	+·07
E_3	13·98	18·96	4·98	5·07	+·09	13·29	18·21	4·92	5·07	+·15
E_2	15·14	19·66	4·52	4·44	−·08	14·32	18·84	4·52	4·53	+·01
E_1	15·58	20·12	4·54	4·20	−·34	15·94	19·87	3·93	3·68	−·25
B	13·57	18·54	4·97	5·30	+·33	11·29	17·57	6·28	6·12	−·16
S	14·25	19·10	4·85	4·93	+·08	13·38	18·51	5·13	5·12	−·01
L	14·85	19·38	4·53	4·60	+·07	12·85	18·18	5·33	5·31	−·02
M	20·22	21·93	1·71	1·68	−·03	18·43	20·81	2·38	2·38	·00
Δ					·18					·13

$D_2 = \cdot 4559 D_1 + 12\cdot 68.$ $D_2 = \cdot 4754 D_1 + 12\cdot 05.$

Age 10. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	11·37	18·46	7·09	7·33	+·24
H_1	12·07	18·84	6·77	6·96	+·19
F	13·04	19·50	6·46	6·43	+·03
E_4	14·79	20·15	5·36	5·48	+·12
E_3	15·29	20·49	5·20	5·21	+·01
E_2	16·42	21·09	4·67	4·60	−·07
E_1	16·56	21·26	4·70	4·53	−·17
B	14·90	20·09	5·19	5·42	+·23
S	15·69	20·58	4·89	5·00	+·11
L	16·38	20·90	4·52	4·62	+·10
M	22·13	23·39	1·26	1·51	+·25
Δ					·16

$D_2 = \cdot 4586 D_1 + 13\cdot 49.$

Age 10. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	11·08	18·03	6·95	7·16	+·21
H_1	11·92	18·52	6·60	6·60	·00
F	14·04	19·65	5·61	5·40	−·21
E_4	13·64	19·24	5·60	5·62	+·02
E_3	14·42	19·57	5·15	5·18	+·03
E_2	15·46	20·10	4·64	4·59	−·05
E_1	16·93	20·97	4·04	3·76	−·28
B	12·25	18·81	6·56	6·41	−·15
S	14·53	19·85	5·32	5·12	−·20
L	13·92	19·42	5·50	5·46	−·04
M	19·98	22·01	2·03	2·03	·00
Δ					·17

$D_2 = \cdot 4330 D_1 + 13\cdot 36.$

Age 15. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	12·97	20·13	7·16	7·22	+·06
H_1	13·73	20·57	6·84	6·85	+·01
F	14·80	21·19	6·39	6·33	−·06
E_4	16·88	22·12	5·24	5·31	+·07
E_3	17·38	22·49	5·11	5·06	−·05
E_2	18·57	23·05	4·48	4·49	+·01
E_1	18·50	23·16	4·66	4·52	−·14
B	17·03	22·08	5·05	5·24	+·19
S	17·79	22·55	4·76	4·87	+·11
L	18·68	23·04	4·36	4·44	+·08
M	25·26	26·47	1·21	1·23	+·02
Δ					·09

$D_2 = \cdot 5132 D_1 + 13\cdot 53.$

Age 15. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	12·44	19·58	7·14	7·30	+·16
H_1	13·34	20·13	6·79	6·79	·00
F	15·47	21·25	5·78	5·58	−·20
E_4	15·35	21·00	5·65	5·65	·00
E_3	16·17	21·48	5·31	5·18	−·13
E_2	17·27	21·92	4·65	4·56	−·09
E_1	18·71	22·78	4·07	3·75	−·32
B	13·75	20·18	6·43	6·55	+·12
S	16·21	21·62	5·41	5·15	−·26
L	15·68	21·23	5·55	5·46	−·09
M	22·50	24·10	1·60	1·60	·00
Δ					·16

$D_2 = \cdot 4334 D_1 + 14\cdot 35.$

Age 20. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	14·85	22·04	7·19	7·31	+·12
H_1	15·68	22·52	6·84	6·89	+·05
F	...	...	...	...	...
E_4	19·34	24·38	5·04	5·00	−·04
E_3	19·84	24·83	4·99	4·75	−·24
E_2	21·08	25·38	4·30	4·11	−·19
E_1	20·65	25·33	4·68	4·34	−·34
B	19·46	24·34	4·88	4·95	+·07
S	19·60	24·73	5·13	4·88	−·25
L	21·50	25·56	4·06	3·90	−·16
M	28·99	28·90	−·09	−·05	+·04
Δ					·17

$D_2 = \cdot 4860 D_1 + 14\cdot 95.$

Age 20. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	13·97	21·31	7·34	7·51	+·17
H_1	14·91	21·92	7·01	7·01	·00
F	...	...	...	...	...
E_4	17·36	23·02	5·68	5·69	+·01
E_3	18·18	23·57	5·39	5·24	−·15
E_2	19·30	24·00	4·70	4·63	−·07
E_1	20·57	24·82	4·25	3·95	−·30
B	15·61	22·34	6·73	6·63	−·10
S	18·11	23·58	5·47	5·28	−·19
L	17·85	23·38	5·53	5·41	−·12
M	25·52	26·79	1·27	1·27	·00
Δ					·13

$D_2 = \cdot 4600 D_1 + 15\cdot 06.$

Age 25. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	16·94	24·20	7·26	7·47	+·21
H_1	17·77	24·76	6·99	7·06	+·07
F	17·88	24·76	6·88	7·01	+·13
E_4	22·19	27·02	4·83	4·90	+·07
E_3	22·66	27·56	4·90	4·68	−·22
E_2	23·82	28·02	4·20	4·11	−·09
E_1	22·87	27·72	4·85	4·57	−·28
B	22·33	26·94	4·61	4·84	+·23
S	22·14	27·21	5·07	4·93	−·14
L	24·82	28·60	3·78	3·62	−·16
M	33·38	32·58	−·80	−·56	+·24
Δ					·18

$D_2 = {\cdot}5117 D_1 + 15{\cdot}74.$

Age 25. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	15·94	23·33	7·39	7·61	+·22
H_1	16·83	23·98	7·15	7·15	·00
F	18·65	24·89	6·24	6·29	+·05
E_4	19·94	25·40	5·46	5·51	+·05
E_3	20·72	25·97	5·25	5·10	−·15
E_2	21·81	26·33	4·52	4·53	+·01
E_1	22·87	26·99	4·12	3·98	−·14
B	18·06	24·70	6·64	6·50	−·14
S	20·53	25·89	5·36	5·20	−·16
L	20·72	26·00	5·28	5·11	−·17
M	29·41	29·96	·55	·55	·00
Δ					·12

$D_2 = {\cdot}4754 D_1 + 15{\cdot}98.$

Age 35. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	22·78	30·00	7·22	7·88	+·66
H_1	23·63	30·58	6·95	7·46	+·51
F	22·76	30·58	7·82	7·89	+·07
E_4	29·96	34·20	4·24	4·32	+·08
E_3	30·15	34·59	4·44	4·23	−·21
E_2	31·11	34·92	3·81	3·75	−·06
E_1	29·39	34·01	4·62	4·60	−·02
B	29·69	33·96	4·27	4·45	+·18
S	29·41	34·13	4·72	4·59	−·13
L	33·51	36·70	3·19	2·56	−·63
M	44·64	42·09	−2·55	−2·95	−·40
Δ					·35

$D_2 = {\cdot}5047 D_1 + 19{\cdot}16.$

Age 35. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	21·40	28·74	7·34	7·51	+·17
H_1	22·12	29·28	7·16	7·16	·00
F	23·49	29·88	6·39	6·49	+·10
E_4	26·84	31·73	4·89	4·83	−·06
E_3	27·37	32·09	4·72	4·57	−·15
E_2	28·39	32·36	3·97	4·07	+·10
E_1	28·53	32·69	4·16	4·00	−·16
B	24·35	30·79	6·44	6·06	−·38
S	26·88	31·87	4·99	4·81	−·18
L	28·22	32·87	4·65	4·16	−·49
M	39·33	38·02	−1·31	−1·31	·00
Δ					·22

$D_2 = {\cdot}5078 D_1 + 18{\cdot}05.$

Age 45. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	32·59	39·23	6·64	7·49	+·85
H_1	32·28	39·70	7·42	7·64	+·22
F	31·38	39·14	7·76	8·09	+·33
E_4	41·77	45·05	3·28	2·94	−·34
E_3	41·50	45·34	3·84	3·08	−·76
E_2	42·17	45·31	3·14	2·74	−·40
E_1	39·96	42·09	2·13	3·84	+1·71
B	40·40	44·37	3·97	3·62	−·35
S	40·90	44·97	4·07	3·37	−·70
L	45·83	48·43	2·60	·96	−1·64
M	60·67	53·19	−7·48	−6·42	+1·06
Δ					·91

$D_2 = {\cdot}5047 D_1 + 23{\cdot}63.$

Age 45. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	30·16	37·37	7·21	7·28	+·07
H_1	30·53	37·65	7·12	7·12	·00
F	31·52	37·79	6·27	6·68	+·41
E_4	37·10	41·32	4·22	4·23	+·01
E_3	37·33	41·58	4·25	4·13	−·12
E_2	38·34	41·57	3·23	3·68	+·45
E_1	37·81	41·57	3·76	3·92	+·16
B	33·46	39·89	6·43	5·83	−·60
S	36·66	41·20	4·54	4·42	−·12
L	38·66	42·93	3·27	3·54	+·27
M	53·52	50·53	−2·99	−2·99	·00
Δ					·24

$D_2 = {\cdot}5603 D_1 + 20{\cdot}54.$

Age 55. Males.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	50·57	55·19	4·62	5·25	+ ·63
H_1	51·25	55·56	4·31	5·04	+ ·73
F	47·74	54·08	6·34	6·11	− ·23
E_4	61·79	63·33	1·54	1·82	+ ·28
E_3	60·93	63·53	2·60	2·08	− ·52
E_2	61·57	62·70	1·13	1·89	+ ·76
E_1	59·04	60·89	1·85	2·66	+ ·81
B	57·28	60·83	3·55	3·20	− ·35
S	60·10	63·09	2·99	2·34	− ·65
L	65·59	67·75	2·16	·66	− 1·50
M	86·59	80·06	− 6·53	− 5·75	+ ·78
Δ					·72

$D_2 = \cdot 6948 D_1 + 20 \cdot 18.$

Age 55. Females.

	D_1	D_2	D_2-D_1 Act.	D_2-D_1 Theor.	Diff.
H_2	46·14	52·30	6·16	6·16	·00
H_1	46·26	52·47	6·21	6·12	− ·09
F	47·71	51·97	4·26	5·60	+ 1·34
E_4	54·82	58·00	3·18	3·05	− ·13
E_3	54·95	58·04	3·09	3·01	− ·08
E_2	56·35	57·70	1·35	2·50	+ 1·15
E_1	56·65	57·37	0·72	2·40	+ 1·68
B	48·85	54·11	5·26	5·19	− ·07
S	53·66	57·40	3·74	3·47	− ·27
L	55·91	59·81	3·90	2·67	− 1·23
M	76·63	71·89	− 4·74	− 4·74	·00
Δ					·83

$D_2 = \cdot 6425 D_1 + 22 \cdot 66.$

numbers on which the life table is based at these ages are not large enough to permit of certain conclusions. The result of the above investigation is in accordance with the view that the figures on which the Glasgow life table was based were probably not quite trustworthy. Had the life table for Scotland as a whole agreed with that of Glasgow we might have surmised that different conditions held in the two countries of England and Scotland, but the latter falls into line with the other English life tables. The other possibility, that the large numbers of Irish extraction present in Glasgow, approximately one-sixth to one-fourth of the whole population, have had a disturbing effect on the local death rates is of course open to consideration, but the absence of any life table for Ireland itself leaves us without the appropriate data to determine whether the latter hypothesis will bear examination.

In conclusion a few notes are necessary regarding the exact method in which the calculations discussed in the previous portion of the paper should be made. If only the "true" male and "true" female death rate is desired, all that is required is to calculate the corrected death rate for males and females in the usual way, using the proportionate population of England in the years 1891 to 1900 on account of the fact that all the constants of the "corrected" death rates in the above columns have been calculated on these figures. This gives at once the "true" death rates, with a probable error of not more than one per cent.

In order to facilitate the working of the complete method, all the figures necessary for its application are given in Table IV, namely, the proportionate age distribution in the population of England from 1891 to 1900 for males and females, and in parallel columns the

TABLE IV.

Showing the proportions of each sex in the standard population, namely England 1891–1900 *and in parallel columns the sums from each age upwards.*

Numbers in standard population			Sums from each age upwards		
Age period	Males	Females	Age	Males	Females
0– 5	59052	59468	0–	484057	515943
5–10	56000	56289	5–	425005	456475
10–15	53521	53550	10–	369005	400186
15 20	49986	50814	15–	315484	346636
20–25	44106	49419	20–	265498	295822
25–35	74159	81938	25–	221392	246403
35–45	57412	61276	35–	147233	164465
45–55	41980	45629	45–	89821	103189
55–65	27212	31184	55–	47841	57560
65–75	15026	18596	65–	20629	26376
75–	5603	7780	75–	5603	7780

TABLE V.

Table showing the calculation of Life Table for Liverpool Registration District 1891–1900.

ge period	Death rates	Age	Sum products above each age	Corrected death rates above each age	True death rates	Expectation of life in years
			Males.			
0– 5	121·49	0–	18332245	37·872	35·34	28·30
5–10	9·42	5–	11158018	26·253	24·65	40·56
10–15	4·64	10–	10630498	28·808	26·70	37·45
15–20	7·43	15–	10382160	32·908	30·42	32·87
20–25	9·78	20–	10010764	37·705	33·28	30·05
25–35	16·63	25–	9579407	43·268	37·88	25·72
35–45	28·89	35–	8346145	56·686	47·77	20·93
45–55	44·95	45–	6687512	74·453	61·20	16·34
55–65	71·00	55–	4800511	100·34	89·90	11·12
			Females.			
0– 5	107·67	0–	17049658	33·045	31·95	31·30
5–10	8·40	5–	10646738	23·323	23·13	43·25
10–15	4·16	10–	10173910	25·422	24·37	41·04
15–20	5·04	15–	9951142	28·703	26·79	37·30
20–25	6·73	20–	9695040	32·773	30·13	33·19
25–35	13·05	25–	9362450	37·996	34·04	29·39
35–45	24·73	35–	8293159	50·425	43·66	22·90
45–55	38·70	45–	6777804	65·683	57·34	17·44
55–65	60·20	55–	5011961	87·073	78·60	12·72

sum of these figures from birth and any age thereafter up to old age. To illustrate the method the expectations of life at ages 0–55 are calculated for the registration district of Liverpool for the same period. The process is shown in Table V. The death rates at each age for both sexes are given in the first column, next follow the sum of these products from each definite age upwards. Parallel to these are the corrected death rates at each age from 0 to 55, obtained by

TABLE VI.

Expectation of life at different ages.

Males.

	H_2	H_1	F	E_4	E_3	E_2	E_1	B	S	L	M
0	52·87	51·48	48·56	44·13	43·66	41·35	39·91	43·59	44·71	40·98	34·71
5	58·26	57·05	54·39	53·50	52·75	50·87	49·71	52·87	52·36	51·60	45·59
10	54·16	53·07	51·28	49·63	49·00	47·60	47·05	49·12	48·60	47·84	42·75
15	49·67	48·62	47·20	45·21	44·47	43·41	43·18	44·67	44·34	43·40	38·78
20	45·37	44·41	43·40	41·02	40·27	39·40	39·48	40·55	40·43	39·13	34·62
25	41·32	40·39	39·93	37·01	36·28	35·68	36·12	36·51	36·75	34·96	30·69
35	33·32	32·70	32·90	29·24	28·91	28·64	29·40	29·02	29·30	27·25	23·76
45	25·49	25·19	25·65	22·20	22·06	22·07	22·76	22·36	22·24	20·65	17·80
55	18·12	18·00	18·49	15·79	15·74	15·95	16·45	16·48	15·85	14·76	12·49

Females.

	H	H	F	E	E	E	E	B	S	L	M
0	55·71	54·04	49·45	47·77	47·18	44·62	41·85	49·00	47·47	45·33	38·44
5	59·53	58·01	53·93	55·79	54·92	53·08	50·33	56·92	50·02	55·12	48·06
10	55·46	54·01	50·88	51·97	51·10	49·76	47·67	53·15	50·39	51·49	45·43
15	51·06	49·68	47·04	47·61	46·55	45·63	43·90	49·07	46·26	47·10	41·50
20	46·93	45·62	43·50	43·44	42·42	41·66	40·29	47·76	42·41	42·77	37·33
25	42·86	41·71	40·17	39·37	38·50	37·98	37·04	40·48	38·63	38·46	33·38
35	34·79	34·16	33·46	31·52	31·16	30·90	30·59	32·48	31·37	30·42	26·30
45	26·84	26·56	26·46	24·20	24·05	24·06	24·06	25·07	24·27	23·29	19·79
55	19·12	19·06	19·24	17·24	17·23	17·33	17·43	18·48	17·42	16·72	13·91

dividing the former by the corresponding sums from Table IV. The true death rate figures are then calculated by the formulae given in the earlier part of the paper (Table III), and the expectation of life at each age obtained by dividing 1000 by each of the latter. It will be noticed that in Liverpool the true death rate is less than the corrected death rate, in other words that during the years referred to, Liverpool was using up life to a greater extent than she was creating it.

This includes all that need be said in the present communication, but a concluding table (Table VI) showing the expectation of life in all the life tables used above may not be without interest as few persons possess access to all the data. These expectations are limited by the year 55. The ages above that come under a somewhat different law, and will be discussed in a subsequent paper.

The calculations in this article were made with the help of an Arithmometer supplied by the Carnegie Trustees.

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STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES

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STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES.

II.

Density of Population and Death Rate (Farr's Law).

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This subject was first considered statistically by the late Dr Farr. It is one of the brilliant attempts to extract the real meaning of figures so frequent in his work, but though this theory has not shared in the complete neglect that has been the lot of his attempt to put a quantitative measure to the course of epidemics, it has suffered as much from the kind of patronage with which it is usually discussed. On at least one of the great medical officers of health of his time, however,—the late Dr J. B. Russell—the theory exercised a strong fascination. My own copy of Farr's *Vital Statistics* came from Dr Russell's library, and the whole passage referring to the law is lined with his characteristic nervous pencil marks, while in much of his work on vital statistics the influence can easily be traced.

The neglect of the subject is of two-fold origin. In the first place the law appeared quite artificial. In the second place the statistics of the decade on which it was founded happened to be specially suitable for its discovery, while subsequent figures did not appear to afford the same support.

The law itself, if the death rate be denoted by R and the density of population (say the number of persons per square mile) by D, is that

$$R = cD^m,$$

when c and m are constants.

But how is the death rate to be measured? By Farr the crude death rate was used and found to give a good measure of the facts.

Later when corrected death rates were substituted, and that seemed to be the proper course, the law obviously did not hold, and even with crude death rates, its success as a descriptive formula was not nearly so marked. Thus in the absence of any *a priori* justification the law was relegated to a somewhat obscure position. Before proceeding to its justification, however, it is necessary to have a clear idea of what kind of evidence can be produced. The law must be a law of average, for on account of the arbitrary nature of the boundaries of the registration districts, the number of persons living on an acre is merely a rough approximation. The groups of localities which supply the figures must further be large, as some with better conditions will have lower death rates, and others with worse, a higher. Nor can even a large city be divided into small districts and these considered. A city population must be a whole population; the slum is not wholly recruited from the slum by any means. A district consisting chiefly of persons engaged in trades and minor occupations may have a very high density and yet a low death rate. All, or at least the great majority of the inhabitants are respectable, those who are not, are driven elsewhere, yet the latter must be considered as part of the same population: from this class, also, though some ascend in the social scale, they do not constitute a separate population. It is obvious therefore that to obtain a suitable average a few groups only must be chosen. Dr Farr made seven, Dr Tatham sixteen; the former may be too few, the latter seems too many. The effect of density is not merely as density. The country preserves life even in the presence of excess or dissipation: the town does not. Further, in the period of growth, children in the city do not get anything like the same chance as their fellows in the country, even though housing may be better and food more abundant. In addition filth in the country is at its worst in most cases but a local nuisance, spreading enteric fever and diarrhoea at times, but not having the power of rendering a whole district foetid. All these influences act concurrently and cumulatively to depress health the more closely people are crowded together, and as life is a physico-chemical process this effect must be measurable and should be capable of expression in some formula which goes back to chemistry and physics. Such a formula is that of Dr Farr. Nothing comparable to it was known in his day, so that as a mathematical formula can easily be found to describe almost any statistics, his formula seemed just such an one and no better than many others. It is, however, no longer alone.

This subject I investigated many years ago without making any advance. The difficulty of defining a death rate was too great. In my last paper, however, I have given a method for obtaining the death rate on a stationary population, and the application of this method justifies the law.

In order to illustrate the subject as fully as possible, two tables have been constructed, one showing the figures used by Dr Farr which refer to the decade 1861–1870, the second the comparative table for the decade 1891–1900 as given by Dr Tatham. Dr Farr used the crude death rate. Fortunately, he has also published the death rates at each age period for the groups of populations on which he based his law. This allows death rates to be calculated on the same standard population which has been used in framing the figures of the second table, and from these life table death rates, which are strictly comparable with those in the second table, have been calculated. The constants of the curves of the form $R = cD^m$ have been evaluated by the method of least squares for both periods, for the crude death rates, the corrected death rates, and the life table death rates.

It will be noticed that the values of m roughly correspond for each separate case in two periods, but in the case of the life table death rates, they correspond within three places of decimals, the furthest that could be statistically expected. We thus have a quite definite law acting independently of the changes which have taken place due to sanitary progress. Improve all round and the exponent does not vary, but only the multiplying constant. The former constant m therefore represents the law, and the latter c may be called the co-efficient of intensity of unhealthiness in the country. This co-efficient c will vary as sanitary conditions improve or the reverse, though the law will remain the same.

When the columns showing the results obtained by fitting similar curves to the crude and corrected death rates are compared, it is seen that the crude death rate fits less well than the life table death rate and that the corrected death rates are very badly represented by the formula. This is what would be expected from the fact shown in the previous paper that life table death rate can be obtained by multiplying the corrected death rate by one constant and adding a second. It will be noticed that the crude death rate curve of Dr Farr has an exponent of ·1193 which is much nearer the probable true exponent ·100 than that of the crude death rate for the decade 1891–1900 which is ·1276. This is explained by the fact that in the earlier period the crude death rate was 22·42 as against a life table death rate of 24·06, while in the latter period the corresponding figures are 18·19 and 21·77. Dr Farr had thus a better opportunity of formulating a law than his successors. With the crude death rate diverging more and more from the life table death rate it became more and more difficult to accept the relationship demanded by the formula. A law of which the main feature, namely, the exponent *m*, varied could hardly lay claim to be a law at all, any more than the law of gravitation could be justified if the relationship were not constantly the inverse square. Using a death rate which is more comparable between populations, namely, that which would hold if the population were stationary, the exponent takes the same value. It will be noticed that the co-efficient *c* decreases from 12·42 in the first period to 10·83 in the latter. In other words, density has only ·875 times the effect in producing mortality it had in 1860–1870, so much have sanitary conditions improved.

But the law remains apparently. Sanitation may diminish *c*, but the ill effects of concentration do not seem capable of being changed merely by sanitation. What the figures just given clearly mean is, that on the whole, conditions of life in modern England seem to be so uniformly the result of the action of the modern developments of industrialism, etc., as to be comprehended in a formula. The prospect that the town may become as healthy as the country, given proper precautions of living, does not seem possible if any law like that of Farr is found to hold permanently. In any case, decrease of density is essential.

But there is one exception to this law in both periods, namely, that of London. In 1861–70, the life table death rate of London was 26 per mille as against 32 expected by the formula. Unless this can be explained the formula falls. But I think it can be explained. Modern England was in 1860, and still is, a recent phenomenon compared with London. Liverpool, Manchester, etc., are but mushroom growths of yesterday. London began to pay its 'prentice fee' as a city in the middle of the seventeenth century. More than a century ago it had a million inhabitants. Sanitation was unknown. Countless thousands tried to live in it and failed. It was in the contemporary documents the 'wen' or the 'vampyre' that sucked England's blood. It was fifty years later than the rest of England in having a birth rate in excess of its death rate, and now it has its reward, the result of two centuries of natural selection in its crudest form. The death rate of London to-day is in no sense a measure of its sanitation. This will be referred to again in a subsequent paper.

I said earlier in this paper that Farr's law did not stand alone. In later papers certain examples of similar relationships will be referred to, but one specially is mentioned here. It is given in a remarkable communication by Mr A. E. Kennealy[1] entitled "An Approximate Law of Fatigue in the Speeds of Racing Animals." This came into my hands a number of years ago and it immediately suggested "Farr's Law," but the difficulty which was still unsolved was as already mentioned the measure of the death rates. Mr Kennealy's paper contains the results of an investigation into the speeds of animals. It is shown that each racing record whether for horse or man, trotting, pacing, walking, running or swimming, obeys a formula of the same form. The figures compared in each instance are the record times achieved for each different distance. As is well known the rate of running for a hundred yards is greater than that for a mile, but that the record time for 20 yards, 40 yards, 100 yards, one mile, ten miles, etc., for each separate sport for practically all the racing records of the world should be comprehended in the same formula

$$T = cL^{\frac{9}{8}},$$

when T is the time taken to cover a distance L, and c is a constant, was hardly to be expected. The constancy of the value $m = 9/8$ is surprising.

This formula when V is the average velocity can be put in the form

$$V = cL^{-\frac{1}{8}} \text{ since } V = \frac{L}{T}.$$

In this form it may perhaps represent the same kind of relationship as Farr's formula.

These remarks suggest a fact which is fully discussed in a succeeding paper that the death rates of different age periods of life in different districts are really organically connected and cannot be compared without the exercise of great care, though on superficial observation they seem directly significant. It is interesting to note that this fact was perceived by the genius of Dr Farr fifty years before modern statistical methods had been introduced.

[1] *Proc. Amer. Acad. Arts and Sciences*, 1906, p. 275.

TABLE I.

Showing the figures relating to Density and death rate. 1861–70*.

No. of districts	Density (persons per square mile)	Corrected death rate (1)	Do. Fitted by least squares	Crude death rate (2)	Do. Fitted by Farr	Life table death rate (3)	Do. Fitted by least squares
53	166	15·30	16·70	16·75	18·90	19·90	20·73
345	186	17·02	17·00	19·16	19·16	21·07	20·96
137	379	20·52	18·99	21·88	20·87	23·47	22·51
47	1718	24·35	24·03	24·90	25·02	26·09	26·19
9	4499	27·94	27·92	28·08	28·08	28·54	28·84
1	12,357	33·98	32·67	32·49	32·70†	32·67	31·92
1	65,823	40·55	42·39	38·62	38·74	37·17	37·74
			$E\,\% = 3{\cdot}79$ $\Delta = 1{\cdot}17$		$E\,\% = 2{\cdot}70$ $\Delta = {\cdot}90$		$E\,\% = 2{\cdot}01$ $\Delta = {\cdot}61$

(1) $R = 7{\cdot}534\,D^{\cdot 15571}$. (2) $R = 10{\cdot}234\,D^{\cdot 11998}$. (3) $R = 12{\cdot}419\,D^{\cdot 10018}$.

$E\,\%$ = mean percentage error. Δ = square root of the mean of the squares of the errors.

* Farr, *Vital Statistics*, p. 175.

† A misprint in the original of 37·7 has been corrected.

TABLE II.

Showing the figures relating to density and death rate. 1891–1900*.

1 No. of districts	2 No. of inhabitants divided by 1,000	3 Density (persons per square mile)	4 Corrected death rate	5 Do. Fitted by least squares (1)	6 Crude death rate	7 Do. Fitted by least squares (2)	8 Life table death rate	9 Do. Fitted by least squares (3)
27	305	136	11·63	13·06	14·20	14·16	17·38	17·18
112	1676	161	12·54	13·43	15·05	14·51	18·01	18·12
121	2496	181	13·44	13·70	15·44	14·68	18·62	18·33
92	3849	261	14·52	14·56	15·46	15·38	19·36	19·02
53	2272	407	15·53	15·68	16·08	16·28	20·05	19·90
56	2577	457	16·53	15·99	16·67	16·52	20·24	20·13
31	1839	737	17·58	17·32	17·64	17·56	21·45	21·12
40	3690	1303	18·53	19·05	18·04	18·88	22·10	22·31
31	3159	1705	19·42	19·93	18·61	19·54	22·71	22·99
21	2240	2339	20·37	21·00	19·50	20·35	23·36	23·72
18	2777	4424	21·56	23·37	20·21	22·08	24·18	25·31
13	2119	4884	22·36	23·76	20·69	22·35	24·72	25·56
6	801	4194	23·48	23·16	22·05	21·93	25·49	25·10
5	762	2925	24·33	21·80	23·29	20·94	26·07	24·21
5	791	7480	26·54	25·51	24·74	23·60	27·58	26·68
4	288	55,563	34·82	35·66	32·67	30·49	33·25	32·58
				$E\,\% = 4{\cdot}3$ $\Delta = 1{\cdot}05$		$E\,\% = 3{\cdot}8$ $\Delta = 1{\cdot}14$		$E\,\% = 2{\cdot}03$ $\Delta = {\cdot}63$

(1) $R = 12{\cdot}40\,D^{\cdot 16715}$. (2) $R = 13{\cdot}57\,D^{\cdot 12755}$. (3) $R = 10{\cdot}83\,D^{\cdot 10078}$.

* Dr Tatham: *Decennial Supplement, Registrar-General of England and Wales*, Part II. 1908, p. lxxi.

STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES.

III.

The Constitution of a Death Rate.

By JOHN BROWNLEE, M.D., D.Sc.

(With 1 Chart.)

This is a particularly difficult branch of the subject to investigate. It might be thought at first sight that the death rates at different age periods might be compared, and this is often done; but when it is noted that the death rates at different age periods are organically connected, it is obvious that such a comparison is statistical or actuarial and completely neglects the biology of the subject. To illustrate the problem a diagram is given. This is constructed on a principle open to objection, but, if that is remembered, it illustrates a number of points. The healthy district life table H_2 has been taken as a standard of comparison. Now a healthy district life table labours under certain disadvantages. A district may, as the Register General says, be excluded because it contains an institution drawing its inmates from a wider district. This is not a serious objection. A more serious objection is the fact that whether deaths which might be considered to belong to the district are returned to it or not, the result is equally unsatisfactory. Everyone who has had practical experience knows many instances in which it is impossible to allocate a death to the district to which it properly belongs. When all is balanced, I think that probably on the whole the healthy districts get credit for more than their share of deaths, that is, for more deaths than would occur if there was no process of intermingling of town and country. For one death the town gets credit for, the country gets credit for another, in the one case some old person dying of a cancer in a hospital, in the other, some poor young person, who having tried city life, returns broken, to die of phthisis or some similar disease. Thus a healthy district life table is open to objection as a criterion. It is, however, for the purpose at present required, the only one at our disposal.

TABLE I. *Showing relations of death rates at different ages in different districts to healthy districts of England*, 1891–1900.

	Death rates at different ages H_2		Ratio of death rates in several life tables to those of H_2: H_1		E_3		E_4		Scotland		Salford		Manchester	
	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
0	132·074	101·327	0·98	0·98	1·36	1·41	1·46	1·52	1·21	1·26	2·29	2·42	1·66	1·73
1	28·500	26·421	1·18	1·15	2·07	2·05	1·92	1·92	1·91	1·95	3·39	3·59	3·26	3·11
2	10·100	10·355	1·31	1·22	2·39	2·25	2·08	1·96	2·22	2·14	4·00	3·67	3·72	3·50
3	7·386	7·093	1·23	1·22	2·10	2·15	1·80	1·89	1·81	1·91	3·11	3·14	3·31	3·28
4	5·787	5·657	1·23	1·18	1·98	1·96	1·68	1·70	1·62	1·72	2·63	3·21	2·95	3·08
5	4·551	4·489	1·22	1·15	1·84	1·76	1·57	1·58	1·45	1·51	1·94	1·76	2·86	3·06
6	3·680	3·662	1·18	1·11	1·64	1·51	1·42	1·43	1·51	1·61	1·82	1·92	3·01	3·17
7	2·999	3·037	1·14	1·09	1·44	1·27	1·28	1·29	1·59	1·67	1·84	1·99	3·16	3·23
8	2·504	2·588	1·09	1·08	1·26	1·06	1·17	1·18	1·61	1·73	1·84	1·93	3·22	3·21
9	2·165	2·300	1·06	1·09	1·11	0·89	1·10	1·10	1·62	1·68	1·71	1·83	3·21	3·06
10	1·966	2·139	1·05	1·11	1·00	0·78	1·09	1·08	1·62	1·69	1·62	1·64	3·08	2·83
15	2·376	2·913	1·16	1·14	1·21	1·01	1·29	1·05	1·63	1·60	1·52	1·24	1·79	1·36
20	3·793	3·981	1·10	1·17	1·27	1·23	1·21	1·04	1·65	1·43	1·74	1·13	1·46	1·27
25	4·939	4·491	1·10	1·25	1·29	1·39	1·15	1·12	1·47	1·46	1·52	1·25	1·69	1·66
30	5·286	5·021	1·23	1·32	1·58	1·58	1·27	1·23	1·45	1·63	1·54	1·58	2·23	2·03
35	6·261	5·757	1·18	1·27	1·64	1·61	1·44	1·36	1·50	1·64	1·59	1·90	2·50	2·22
40	7·569	6·823	1·15	1·16	1·67	1·55	1·58	1·47	1·54	1·54	2·51	2·14	2·65	2·26
45	9·323	7·834	1·13	1·14	1·69	1·59	1·60	1·50	1·58	1·52	2·67	2·41	2·70	2·45
50	12·545	10·215	1·06	1·09	1·59	1·54	1·56	1·47	1·52	1·53	2·42	2·39	2·53	2·47
55	16·551	14·108	1·07	1·07	1·59	1·50	1·57	1·45	1·71	1·50	2·42	2·41	2·53	2·42
60	24·946	21·894	1·02	1·00	1·47	1·35	1·47	1·36	1·48	1·35	2·49	2·27	2·30	2·16
65	36·387	32·355	1·04	1·03	1·43	1·33	1·40	1·32	1·38	1·37	2·17	2·04	2·20	2·12
70	58·545	52·270	0·99	1·00	1·28	1·23	1·28	1·23	1·23	1·17	1·72	1·80	1·90	1·82
75	95·095	85·099	0·98	0·98	1·16	1·14	1·17	1·15	1·17	1·05	1·99	1·98	1·60	1·56
80	151·442	134·717	1·00	0·99	1·09	1·09	1·09	1·09	0·96	0·99	1·07	0·96	1·36	1·32
85	234·018	206·615	1·03	1·01	1·06	1·05	1·02	1·04	0·95	0·99	0·98	1·52	1·18	1·13
90	349·364	306·353	1·08	1·03	1·06	1·03	0·98	0·99	1·08	1·06	1·06	0·78	1·06	0·96
95	503·268	439·205												
100	694·444	608·696												

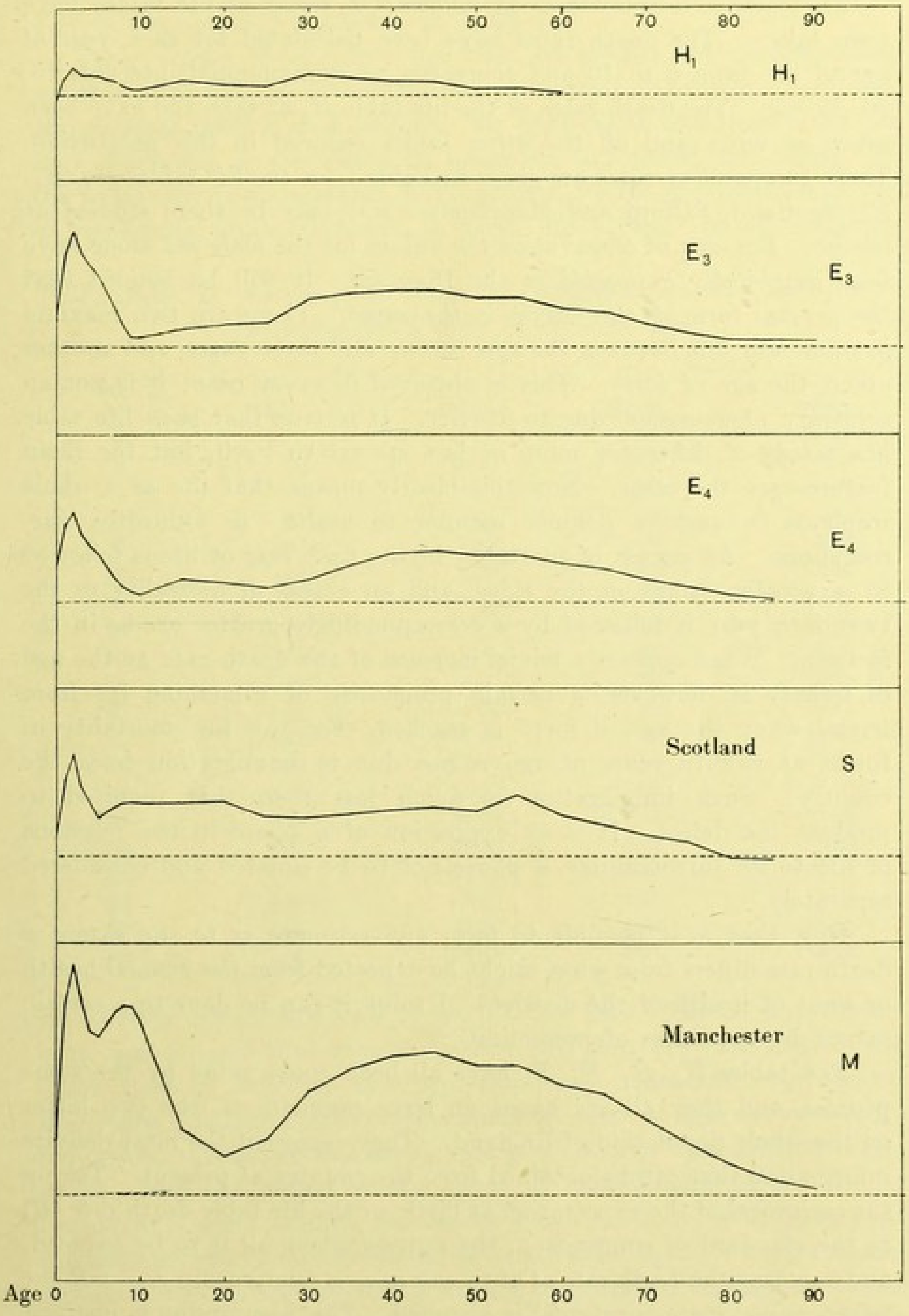

Diagram showing graphically the relationships of some of the figures given in Table I.

The method employed is as follows: A series of life tables have been taken. The death rates have been calculated for each year of age of life from 0 to 10 and thereafter at quinquennial intervals 15, 20, 25, etc. The death rates of the life table H_2 at each age have been taken as unity and all the other tables reduced in this proportion. These comparative rates are given in Table I for the life tables H_1, E_3, E_4, Scotland, Salford and Manchester, and may be there studied at leisure. For ease of observation the values for the male sex alone have been graphically expressed in the Diagram. It will be noticed that the general form of the curves is the same. There are two maxima in each case, one between the ages of two and three years, and another about the age of forty. This is observed in every case; it is not an arbitrary phenomenon due to district. It is true that each life table has points of difference more or less special to itself, but the main features are the same. Now this clearly means that life as a whole responds in quite a definite manner to healthy or unhealthy surroundings. An excess of mortality in the first year of life is followed by a greater excess in the third, and an excess of mortality in the twentieth year is followed by a correspondingly greater excess in the fortieth. What appears a trivial increase of the death rate at the age of twenty is, however, a certain prognostic of something far from trivial when the age of forty is reached. For this low mortality in towns at twenty years of age is not due to immigration from the country. Such immigration is much less than that required to produce the defect. It is an expression of a phase in the relations of life to its surroundings, a phase not to be isolated and considered separately.

How then is it possible to form any estimate as to the extent a death rate differs from what might be expected from the general health or want of health of the district? I think it can be done to a certain extent by a process of prediction.

The tables H_2, H_1, E_4, E_3 have all been constructed by the same process, and they all are based on large populations, the two latter on the whole population of England. They represent the most definite information that can be obtained from the country at present. Taking the reciprocal of the expectation at birth, or the life table death rate (R) as the standard of comparison, the unit to which all is to be reduced, and denoting the death rate at any age x as r_x, a relationship $r_x = mR + c$ (when m and c are constants) is assumed. This assumption is justified by the results; it is only one of many other relationships of the same kind which are found to be described almost truly by straight lines. The values obtained are given in Table II.

These values of *m* and *c* are used to predict the corresponding mortalities at all ages in four different countries and towns: Scotland, Manchester, London and Salford. In predictions like these a 10 per cent. variation might easily be observed, accounted for, firstly, by the roughness of the method, and secondly, by the processes employed in smoothing crude statistics to form life tables. In all instances

TABLE II. *Showing the values of constants* m *and* c *used for predicting death rates at different ages.*

Age	Males		Females	
	m	*c*	*m*	*c*
0	14·95	−155·33	15·46	−179·24
2	3·07	− 47·25	3·45	− 51·10
4	1·18	− 16·10	1·43	− 19·83
6	·459	− 4·81	·517	− 5·59
8	·123	·25	·063	1·61
10	·009	1·85	− ·091	3·89
15	·114	·39	− ·018	3·45
20	·204	·07	·100	2·46
25	·255	·27	·236	·63
30	·484	− 3·44	·473	− 2·78
35	·796	− 8·47	·727	− 6·78
40	1·17	− 14·35	·991	− 10·61
45	1·52	− 19·16	1·26	− 14·50
50	1·89	− 24·00	1·55	− 17·38
55	2·50	− 30·83	2·00	− 21·58
60	3·14	− 34·98	2·55	− 24·35
65	3·97	− 38·98	3·27	− 26·39
70	4·50	− 27·91	4·00	− 20·24
75	4·56	6·75	4·27	7·04
80	3·85	77·39	4·00	61·57
85	2·00	199·22	2·91	154·30

the average percentage error is much below this. The correspondence is in fact so close that it must be assumed that the variations of the death rates at different ages are organically connected. As regards the results it is found that for practically any age, at ages above twenty-five years, the theoretical death rate predicted by this method corresponds with the actual in the range of experimental error. Under twenty-five years certain differences make themselves apparent. If the infantile death rate predicted is found to be equal to that actually observed, the correspondence between the predicted and actual values of the death rate for the whole life is very close. Examples of this are shown in the life tables for London and Salford for the decade 1891–1900. In London the correspondence is almost absolute, except at the age of 10. In Salford it is not nearly so absolute, but as the actual figures for Salford are not so continuous, due apparently to the small numbers on which the table is based, individual differences are larger. The special discrepancy about the age of 10 in both instances is probably due to the fact that in the neighbourhood of this age the minimum death rate exists, and in the neighbourhood of a minimum the methods of life table approximation are open to special error.

Taking the life table for Scotland, a different condition of affairs is seen. Scotland varies markedly from England in the comparative absence of summer diarrhoea, with the result that the infantile death rate predicted from the English life tables is twenty per thousand in excess of the actual infantile death rate of Scotland. For the next few years till the age of six is attained there is no appreciable difference between the actual and the predicted death rates, but this defect in the infantile death rate is balanced by the excess of the actual death rate over the theoretical between that age and the age of thirty, after which the population in Scotland shows essentially the same mortalities as the English tables, used in the manner described, predict.

Of the same phenomenon, Manchester affords a striking example, in curious distinction to the neighbouring town of Salford. In this case the predicted infantile death rate is 55 per thousand in excess of that observed. The period of life at which the compensation begins is from one to two years earlier than that in Scotland, the actual death rate at six years of age being twenty-five per cent. in excess. Matters adjust themselves also at a somewhat earlier age, since by the time twenty years is attained, the actual and theoretical death rates have come into correspondence. More especially when the township of Manchester and the outlying townships are severally examined, the same phenomena are observed, the points of difference not being sufficiently important to require special comment.

It is to be noted that the greater mortalities in Scotland and Manchester at the ages, approximately, of from six years to twenty-five years, though apparently excessive as rates in the sum correspond almost exactly to the number of deaths represented by the deficiency of the actual from the theoretical mortality at the age 0–1.

With regard to the mortalities of the adult population, it is evident from what has been said that practically the same result can be obtained

TABLE III. *Showing actual and predicted death rates, male and female, for London, Salford, Scotland and Manchester.*

	London				Salford			
	Actual		Theoretical		Actual		Theoretical	
	Male	Female	Male	Female	Male	Female	Male	Female
0	207·99	170·31	209·56	161·81	302·50	244·97	297·79	240·81
2	28·01	27·40	27·65	25·01	40·40	38·01	45·76	42·64
4	12·38	12·62	12·57	11·72	15·21	18·16	19·50	19·02
6	6·02	6·19	6·39	5·82	6·71	7·02	9·10	8·46
8	3·15	3·38	3·26	3·00	4·60	5·00	3·98	3·32
10	2·26	2·44	2·06	1·88	3·19	3·52	2·12	1·42
15	3·12	2·79	3·16	3·05	3·60	3·61	3·83	2·96
20	4·13	3·19	5·04	4·67	6·61	4·50	6·24	5·18
25	5·24	4·03	6·48	5·84	7·52	5·63	7·99	7·01
30	7·54	5·82	8·37	7·65	8·15	7·93	11·23	10·07
35	11·26	8·62	10·96	9·25	9·95	10·95	15·66	12·97
40	14·73	11·07	14·25	11·25	18·96	14·58	21·17	16·32
45	18·40	13·52	17·83	13·30	24·92	18·87	26·77	19·73
50	23·91	17·65	22·16	16·81	30·43	24·48	33·32	24·73
55	31·57	23·55	30·25	22·54	40·10	33·97	45·02	32·76
60	42·49	32·14	41·64	31·90	62·10	49·62	60·16	44·93
65	58·16	45·39	57·85	45·75	79·14	66·16	81·26	62·46
70	83·21	67·28	81·97	68·00	100·68	94·28	108·54	88·44
75	120·67	101·16	118·03	101·24	189·38	168·50	144·94	123·06
80	173·26	150·29	171·26	149·81	161·66	129·02	193·96	170·25
85	242·87	218·02	248·02	218·49	228·18	314·40	259·82	233·36

	Scotland				Manchester			
	Actual		Theoretical		Actual		Theoretical	
	Male	Female	Male	Female	Male	Female	Male	Female
0	158·97	127·89	179·05	146·35	219·44	174·97	275·51	223·03
2	22·38	22·13	21·39	21·56	37·53	36·25	41·19	38·67
4	9·39	9·75	10·17	10·29	17·09	17·44	17·75	17·38
6	5·57	5·89	5·45	5·30	11·08	11·63	8·41	7·86
8	4·03	4·47	3·00	2·94	8·07	8·30	3·80	3·25
10	3·18	3·62	2·05	1·97	6·05	6·06	2·10	1·52
15	3·86	4·66	2·93	3·07	4·26	3·97	3·66	2·98
20	6·27	5·68	4·63	4·57	5·54	5·06	5·94	5·06
25	7·27	6·57	5·96	5·60	8·36	7·47	7·61	6·77
30	7·65	8·18	7·38	7·18	11·79	10·20	10·50	9·53
35	9·38	9·45	9·34	8·53	15·64	12·76	14·47	12·14
40	11·68	10·52	11·86	10·26	20·08	15·45	19·42	15·18
45	14·72	11·93	14·73	12·04	25·22	19·22	24·51	18·29
50	19·12	15·60	18·30	15·26	31·79	25·22	30·50	22·95
55	28·33	21·23	25·15	20·54	41·80	34·12	41·29	30·46
60	36·87	29·53	35·23	28·35	57·39	47·27	55·49	42·00
65	50·40	44·20	49·75	42·48	80·12	68·58	75·34	58·70
70	71·82	61·03	72·78	64·00	111·01	94·98	101·83	83·84
75	111·59	89·04	108·73	96·97	152·36	132·37	138·14	118·15
80	145·13	133·73	163·42	145·81	206·31	178·34	188·23	165·65
85	222·09	204·16	243·94	215·58	275·28	233·10	256·84	230·02

in several different ways. On general principles, a high infantile mortality—granted similar environmental conditions—will cut off a good many children, who would otherwise perish at early ages, but the fact which is most evident from the tables given here, is that mortality is specially distributed and that life at all ages is acted on by unhealthy surroundings in a way which is very closely correlated with the sum total of the depressing influences due to the environment. The conditions which produce a high infantile death rate are exactly those conditions which depress the vitality of the whole adult population. All the evidence is against the view that a high infantile mortality produces a healthier population at adult ages. The fact that in the cases of such towns as London and Salford the death rates at all ages can practically be predicted from the knowledge of infantile death rate alone, shows the danger of such a method of reasoning. It is quite true that in certain places, such as Manchester and Scotland, the saving of infantile life is associated with higher death rates in the immediately succeeding ages. But in adult life the influence on health in the case of Manchester is just as adverse as in the case of Salford.

STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES.

IV.

On the Range of Instances in which Geometrical Progressions describe numerically processes of life, i.e. those processes which might be explained by a monomolecular reaction.

By JOHN BROWNLEE, M.D., D.Sc.

In recent years it has been found that *in vitro* many ferments, etc., lose their power of action at a rate corresponding to the monomolecular reaction, *i.e.* if the amount of the substance present at the beginning is denoted by unity, and, if after a certain period of time only one half of the substance remains active, then it may be predicted that at the end of an equal period of time, one half of one half, or only one quarter of the original amount, will still retain its power of action. It is not necessary to give examples, they are sufficiently well known.

In actual living organism, however, this relationship can be just as well demonstrated in a large number of instances, some immediately obvious, others requiring some care to establish. The oldest known example, discovered by Gompertz, is the increase of the death rate with age. Thus the average death rate among males (1858–1901) as given by the Registrar-General is for the age period 55–65, 33·3 per mille, for 65–75, 68·3 per mille, 75–85, 147·4 per mille, and for the ages above 85 as 308·6 per mille.

It will be noticed that the death rate approximately doubles itself with each ten years' increase of age. For high ages in fact this relation is an excellent interpolation formula.

It is not necessary to give in this place cumulative evidence as all that is required for the present purpose is to show that the phenomenon is of some generality. As typical examples it is to be noted that the following sequence of values are described by a geometrical progression:

(1) The rate of decrease in the case mortality of infectious diseases among children as age increases.

(2) The rate of increase in the case mortality of infectious diseases among adults as age increases.

(3) The rate of decay of the protective power of vaccination as age increases.

(4) The decay of the natural immunity of children and the growth of natural immunity of adults towards certain diseases as age increases.

(5) The loss of infectivity of an organism during an epidemic.

Some of the data referring to these instances ("The Relation of the Monomolecular Reaction to Life Processes and to Immunity," *Proc. Roy. Soc. Edin.* 1911) have already been published, but the figures referring to scarlet fever are reproduced.

The case mortality from scarlet fever is highest in children between one and two years, and thereafter declines from year to year. The rate of decline is such that the case mortality during each year of life is three-quarters that of the preceding, as can be easily seen from the table (Table I) in which the case mortalities taken from the statistics of Glasgow and Manchester are given in parallel columns with the appropriate theoretical values, these being the only two cities for which statistics can be given for the several years up to ten. Though the type of scarlet fever prevalent among children in Glasgow is considerably more severe than that in Manchester, it is worthy of special notice that the ratio of decrease is identical. The case mortality for measles varies in the same way, but in this instance the ratio of diminution between the case mortalities of succeeding years of life is not ·75 but ·65; that is, children tend to grow out of the fatal period more quickly. Thus after three years the susceptibility to death in the case of scarlet fever has only fallen to ·42 of that of the epoch of commencement, while in the case of measles it has fallen to ·28.

To these examples the figures relating to diphtheria are added, as they have not been shown to obey the same law. The case mortalities are those obtaining in the city of Manchester for the ten years 1893–1903. In Table I these figures and the progression fitted by the geometrical law are given in parallel columns. The ratio found in this case between the mortality of successive years of age is conspicuously higher, namely, ·86, a figure much higher than in the cases of measles and scarlet fever.

TABLE I.

Showing the case mortalities of scarlet fever and diphtheria fitted to curves of the form $y = ae^{-\kappa x}$.

	Scarlet Fever				Diphtheria	
	Glasgow		Manchester		Manchester	
Age period	Actual	Theoretical	Actual	Theoretical	Actual	Theoretical
1– 2	24·3	22·2	19·3	19·3	58·4	58·1
2– 3	16·5	16·5	14·7	14·2	51·2	50·1
3– 4	12·6	12·4	12·2	10·6	40·9	43·1
4– 5	9·1	9·3	8·7	7·8	36·1	36·9
5– 6	7·0	7·0	5·7	5·8	33·2	31·8
6– 7	4·1	5·2	4·5	4·3	27·9	27·3
7– 8	3·6	3·9	3·4	3·2	24·6	23·6
8– 9	3·1	2·9	2·2	2·3	20·2	20·2
9–10	2·2	2·2	—	—	11·6	17·3
10–15	—	—	—	—	7·2	10·8
15–20	—	—	—	—	4·4	4·4

II. In this group of instances the death rate steadily increases with age. The figures for smallpox and typhus have already been published and the only point of importance is the ratio of increase of the case mortality with age. With smallpox in Gloucester, the case mortality increases with each ten years of age in the ratio 1·30. With regard to typhus the ratio is the same for Glasgow and London and is considerably larger than that just given, namely 1·46.

In more extreme age diarrhoea furnishes a good example.

For the age periods 45–55, 55–65, 65–75, 75– , the death rates per million living during the decade 1891–1900 were respectively 95, 243, 715, 2151 for males, and 76, 226, 703, 2011 for females, showing a rough ratio between successive numbers of nearly 3, the greatest ratio hitherto found. Diarrhoea is an infectious disease which is ubiquitous, and it may be taken that all are alike exposed to infection. This curve thus represents the combined effects of susceptibility to disease and case mortality.

Of a like nature is the phenomenon shown by the influenza statistics. As this disease caused no mortality at all in London in the late eighties, the figures relating to the deaths may be taken as practically true when the years of the great epidemics 1890, 1891 and 1892 are considered. It may be considered that all ages alike succumbed to infection and that the death rates at different ages are thus equivalent to case mortalities. When the ratio of increase is examined, however, it is found to be practically identical in its value with that occurring between the death rates from all causes, given earlier in the paper. With each ten years of age from 45 upwards, the death rate, both for males and females, practically doubles itself. Influenza may thus be looked upon more as a factor accelerating death than as a disease with a special mortality of its own. The figures are given in the adjoining table.

TABLE III.

Death rates per thousand at different ages from influenza for London during the years 1890, 1891 *and* 1892.

	Death rate per thousand	
Age period	Male	Female
45–55	·79	·59
55–65	1·33	1·28
65–75	2·64	2·59
75–	5·61	5·07

III and IV. The figures regarding the decay of the protective power of vaccination treated by the method of correlation have already been published[1], and in this place the ratio of decay is measured in a different manner. An analysis of the susceptibility to certain diseases at each age is made. It involves a double problem; firstly, in youth, either owing to special insusceptibility to disease, *e.g.* enteric fever, or to acquired insusceptibility, *e.g.* smallpox after vaccination, there is a period in which the cases of the disease are few; secondly, as age advances a special immunity developes. It is apparently a fact, and a fact somewhat curious, that these two forms of immunity are additive, or that the protection at any moment is proportional to their sum. Three examples are considered. In the first place the age distribution of susceptibility to smallpox among the vaccinated is examined. The most important data for this are contained in Dr Barrie's report on the epidemic of smallpox in Sheffield in the year 1887. Here owing to the fact that a census was taken as to the numbers of vaccinated in the whole population, the susceptibility to smallpox of persons at different ages can be calculated. This is exceedingly important because owing to the manner in which towns recruit their populations from different districts, very marked variations in the number of vaccinated and unvaccinated at different ages may occur. The drawback to Dr Barrie's Census is that he has not separated between those who are definitely and those who are doubtfully vaccinated as is usually done. There are thus far too many children under five years of age as compared with the statistics of other places. Some correction has therefore to be made to obtain the probable number of those who

[1] *Loc. cit.*

were definitely vaccinated. This is done on the basis of the Glasgow epidemic by assuming that from 0 to 5 years 45 per cent. of the total cases occurred among definitely vaccinated children and from 5 to 10 years 90 per cent. At the other ages correction is immaterial as insusceptibility to smallpox depends much more upon the natural immunity due to age than on the protection due to vaccination.

Two factors determine the distribution of susceptibility, one, the protection due to vaccination great in youth and gradually disappearing, the rate of its disappearance being described by the terms of a geometrical progression: the other factor, the increase of natural immunity with age also described in a like manner. The fitting of this compound curve to the statistics is largely a matter of trial and error. In the accompanying table (Table V) are shown in parallel columns the total number of cases; the susceptibility at each age period; then the reciprocal of the latter: in the next two columns this analysed into its two parts: these two latter columns summed: the number of cases to which they correspond calculated and compared with the number given by observation. The difference is remarkably slight; when the usual calculations are made it is found that $\chi^2 = 3{\cdot}24$, which gives the probability of the analysis $P = {\cdot}95$.

The epidemic of miliary fever in Oise in 1827 is next analysed. For this a table constructed in the manner described above is also given. It is found that $\chi^2 = 6{\cdot}0$, which gives a probability $P = {\cdot}53$. Though not so high as that shown for smallpox, it must be accounted a good fit.

The cases of enteric fever in London are given as the last example and though they do not give a good fit ($\chi^2 = 18{\cdot}16$, due to the very large number of cases analysed, over five thousand) yet the curve is essentially similar and the divergence much less than that found by Prof. Pearson in his analytical graduation of the curve. We thus have the susceptibility to certain diseases described by a curve of different form to any hitherto proposed. The graduation which has been discussed is obviously described by the formula

$$y = \frac{a}{e^{-mx} + e^{nx}}.$$

It is thus possible to describe these three diseases in a manner capable of being explained by the monomolecular reaction. The manner in which the ratios of decay and growth of immunity vary is exceedingly interesting and the collected results are given in the accompanying table.

TABLE IV.

	Rate of decay of the immunity of youth per 5 years	Rate of growth of old age immunity per 10 years
Smallpox	·192	1·91
Miliary fever	·523	1·35
Enteric fever	·182	2·15

It is to be noticed that each disease has its special method of variation.

It may be remarked that though these adult diseases have been thus easily analysed others have more complex immunity phenomena resembling in this respect the diseases of childhood.

These will be considered later after the method of treating a sum of monomolecular reactions has been described.

V. It is not necessary to discuss the curve of an epidemic in this place but only to notice that it can be approximately accounted for if the organism loses its infectivity at a rate something approaching a geometrical progression.

APPENDIX.

On the curve $y = \dfrac{a}{be^{-mx} + ce^{nx}}$.

With change of origin this equation can be put in the form

$$y = \frac{ae^{px}}{e^{-mx} + e^{mx}}.$$

The curve thus contains three constants and is of some interest. It allows finite susceptibility at any age and describes the course of immunity during life in a definite formula. It is therefore superior to graduation curves which seek to smooth merely the numbers of cases and tell nothing of what is going on. To calculate the area and moments the term in the numerator is expanded and each term severally integrated from positive to negative infinity. Thus

$$\int_{-\infty}^{\infty} \frac{e^{px}dx}{e^{-mx} + e^{mx}} = \int_{-\infty}^{\infty} \frac{dx}{e^{mx} + e^{-mx}} + p\int_{-\infty}^{\infty} \frac{xdx}{e^{mx} + e^{-mx}} + \frac{p^2}{1.2}\int_{-\infty}^{\infty} \frac{x^2dx}{e^{mx} + e^{-mx}} + \dots$$

$$= 2\left\{\frac{\pi}{2m} + \frac{p^2}{1.2}\left(\frac{\pi}{2m}\right)^3 E_1 + \frac{p^4}{1.2.3.4}\left(\frac{\pi}{2m}\right)^5 E_2 + \dots\right\}$$

where E_1, E_2, etc., are Euler's numbers,

$$= \frac{\pi}{m}\sec\frac{p\pi}{2m}.$$

TABLE V.

Showing the fitting of curves of susceptibility to the form $y = \dfrac{a}{be^{-mx} + ce^{nx}}$.

(a) *Smallpox, Sheffield*, 1887–8:

Age period	No. of cases	Susceptibility per 1000	Reciprocal	Analysis		Sum	Theoretical no. of cases
0– 5	43[1]	253	88·844	·752	91·221	91·973	41·1
5–10	215·5[1]	603	18·518	1·038	17·546	18·584	214·8
10–15	782	2067	4·837	1·433	3·375	4·808	787·0
15–20	1240	3987	2·508	1·978	·649	2·627	1184·0
20–25	950	3480	2·873	2·731	·125	2·856	956·0
25–35	1019	2194	4·558	4·438	·024	4·462	1041·0
35–45	429	1182	8·460	8·460	·005	8·465	428·0
45–55	156	620	16·129	16·129		16·129	156·0
55–65	66	3033	33·045	30·740		30·740	68·4

[1] Corrected as described in the text.

$\chi^2 = 3{\cdot}24$. $P = {\cdot}91$.

(b) *Miliary fever, Oise*, 1827:

Age period	No. of cases	Susceptibility per 1000	Reciprocal	Analysis		Sum	Theoretical no. of cases
0–10	77	11·3	88·496	2·385	80·000	82·385	82·7
10–20	205	39·3	25·445	3·233	21·909	25·142	207·9
20–30	429	101·9	9·814	4·381	6·000	10·381	405·5
30–40	501	128·3	7·811	5·938	1·642	7·580	516·9
40–50	410	119·9	8·340	8·047	·449	8·496	402·5
50–60	208	98·8	10·111	10·905	·123	11·028	190·7
60–70	101	73·7	13·569	14·778	·033	14·811	92·5
70–80	15	33·0	27·076	20·027	·010	20·037	20·2

$\chi^2 = 6{\cdot}21$. $P = {\cdot}51$.

(c) *Enteric fever, London*, 1871–93:

Age period	No. of cases	Susceptibility per 1000	Reciprocal	Analysis		Sum	Theoretical no. of cases	Prof. Pearson's distribution
0– 5	266	530·3	18·858	·636	18·150	18·786	267	353
5–10	1143	2516·7	3·974	·932	3·300	4·232	1072	1236
10–15	2019	4848·4	2·063	1·386	·600	1·986	2097	1833
15–20	1955	4690·3	2·131	2·031	·090	2·121	1964	1735
20–25	1319	3078·5	3·248	2·975	·019	2·994	1426	1374
25–35	1360	1895·4	5·275	5·275	·003	5·278	1360	1433
35–45	462	889·1	11·248	11·367	·000	11·367	457	470
45–55	138	374·4	27·710	24·495	·000	24·495	156	136
55–65	22	121·9	82·056	51·400	·000	51·400	42	20[1]

Present fitting: $\chi^2 = 22{\cdot}3$, $P = {\cdot}005$; Prof. Pearson's fitting: $\chi^2 = 83{\cdot}1$, $P = 0$.

[1] For 55 to 60 as against 14.

The moments about the origin are obviously obtained by differentiating this with regard to p. Transferred to the centroid vertical, which is at a distance

$$\frac{\pi}{2m} \sec \frac{p\pi}{2m} \tan \frac{p\pi}{2m}$$

from the origin, the values are

$$\mu_2 = \left(\frac{\pi}{2m}\right)^2 \sec^2 \frac{p\pi}{2m},$$

$$\mu_3 = 2\left(\frac{\pi}{2m}\right)^3 \sec^2 \frac{p\pi}{2m} \tan \frac{p\pi}{2m},$$

$$\mu_4 = \left(\frac{\pi}{2m}\right)^4 \left(4 \sec^2 \frac{p\pi}{2m} \tan^2 \frac{p\pi}{2m} + 5 \sec^4 \frac{p\pi}{2m}\right).$$

Whence

$$\beta_1 = 4 \sin^2 \frac{p\pi}{2m},$$

$$\beta_2 = 4 \sin^2 \frac{p\pi}{2m} + 5,$$

$$F = 2\beta_2 - 3\beta_1 - 6$$

$$= 4 - 4 \sin^2 \frac{p\pi}{2m}.$$

The curve would thus be very easily fitted by the moments were it possible to obtain these from the statistics. It has the moment relationship of Type IV or Type VI according as $p <$ or $> \frac{m}{3}$ approximately. The curve in general will be asymmetrical because the ratio of the decay of the natural immunity of childhood is not identical with that at which immunity increases with age.

For maximum of distance from origin be x

$$x = \frac{1}{2n} \log_e \frac{n+p}{n-p}$$

STUDIES IN THE MEANING AND RELATIONSHIPS OF BIRTH AND DEATH RATES.

V.

On the difficulty that in applying the laws of physical chemistry to life processes, indices occur which suggest the actions of fractions of a molecule.

By JOHN BROWNLEE, M.D., D.Sc.

In a large number of cases the curves which describe the rate at which ferments, antibodies, etc., disappear in the body accord in form with the equations of physical chemistry, but not so as to be readily interpretable. If one molecule, two molecules, etc., take part in a reaction then it is clear that the indices in the equations must bear certain relations to whole numbers. Failure to conform to these relations seems at first sight to imply that a fraction of a molecule takes part in a reaction, but this is not necessarily the case. Such a failure has been found to appear when the curves of many vital phenomena, *e.g.* the disappearance of agglutins in an organism, are fitted to physico-chemical equations. A possible meaning, however, can easily be seen.

Firstly, as the monomolecular reaction is so frequent in life phenomena, it is quite reasonable to assume that it is the rule. Secondly, it is highly improbable that such a substance as an agglutin is a simple substance. It is much more probably a group of substances each of which has its own rate of decay. If each member of this group be taken to obey the monomolecular reaction, the amount of agglutin present at a definite time x, instead of being represented by the form ae^{-kx} as is required by the monomolecular theory, will be represented by a sum of such forms, each substance disappearing at a different rate, and each present originally in a different amount.

Now certain curves of frequency exist which may be tried. If the frequency of each value of k be denoted $a\kappa^n e^{-\gamma\kappa}$ (Pearson's Type III, a curve having a very wide range of forms) the amount of the substance after a period x will be represented by

$$a\int_{\kappa}^{\infty} e^{-\kappa x}\,\kappa^n e^{-\gamma\kappa}\,dk,$$

or (M being a constant)

$$\frac{M}{(x+\gamma)^{n+1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (A),$$

which is the form required by a multimolecular reaction where $k+1$ is equal to $\frac{1}{p-1}$, p being an integer. But on the theory here developed the values of n are not circumscribed in this way, and p may be any number, fractional, or integral. Thus a possible explanation of a common phenomenon is obtained.

It is to be noted that the rates of the disappearance of the agglutins or the precipitins formed when the same organism is inoculated into different animals vary greatly, implying that the amount of each separate agglutinin with its special rate of decay may be very different even in nearly allied forms. The experimental working of this subject opens a wide field.

It is not necessary to quote examples that are familiar to all workers in this subject. To illustrate the range, however, an example from the statistics of children's diseases, namely the death rate from whooping cough, has been chosen, as death from whooping cough is due to a considerable variety of causes—convulsions, broncho-pneumonia, etc.

In the accompanying table the numbers of admissions and of deaths, and the case mortality of persons suffering from whooping cough, treated in the City of Glasgow Hospital, Belvidere, between the years 1885 and 1902 are given. The theoretical mortality is calculated on the hypothesis above stated. The theoretical numbers of deaths, calculated from the number of cases, which being the larger number

Table showing that the case mortality of cases of whooping cough from one year to ten is completely described by the curve $y = \frac{4090}{(4 \cdot 7 + x)^{2 \cdot 688}}$, *where* y *is the case mortality and* x *the age in years.*

Age	No. of cases	No. of deaths Actual	No. of deaths Theoretical	Case mortality Actual	Case mortality Theoretical
1– 2	619	233	233	38·0	38·0
2– 3	742	190	183	25·6	24·6
3– 4	779	119	130	15·3	16·7
4– 5	695	81	85	11·7	12·2
5– 6	585	53	53	9·1	9·1
6– 7	420	28	29	6·6	7·0
7– 8	228	13	13	5·7	5·5
8– 9	112	6	5	5·3	4·4
9–10	55	2	2	3·6	3·6

725 733

has the smaller probable error, are added to admit of statistical comparison. The correspondence is exceedingly close since $\chi^2 = 1{\cdot}9$ and $P = {\cdot}99$. This is not a solitary example, and others will be discussed later when some of the problems of special diseases are considered.

In what has gone before, the distribution of the value of k has been assumed to be that of Type III. It may, however, be at least equally probably assumed to be normal. In this case, the resulting equation giving the amount of the original substances present at the time, x is of the form:

$$y = a\int_{-\infty}^{\infty} e^{-\kappa x} e^{-\frac{(\kappa-k)^2}{2\sigma^2}} d\kappa,$$

k being the mean value of κ, or when integration is effected:

$$y = Me^{-kx+\frac{\sigma^2}{2}x^2}.$$

M being a constant. If the range of κ is large, *i.e.* if $\frac{k}{\sigma}$ is small, the deviation from the simple exponential curve is considerable. If, however, the range of κ is small, *i.e.* if $\sigma \fallingdotseq 0$, the resulting formula is approximately the simple exponential. A certain variation in the value of κ may, therefore, take place, without the result of the experiments being found to diverge much from the monomolecular law.

A simple case which may be of interest is that of a mixture of two substances, both decaying according to the monomolecular law, of which the values of κ are somewhat different. To illustrate the point, imagine the sum of two geometrical progressions, one which has a ratio of $\frac{3}{4}$ and the other of $\frac{1}{2}$. The figures are given below:

1·0000	·7500	·5625	·4218	·3146	·2373	·1780	·1335
1·0000	·5000	·2500	·1250	·0625	·0313	·0156	·0078
2·0000	1·2500	·8125	·5469	·3789	·2686	·1936	·1413
2·0000	1·2514	·8138	·5470	·3779	·2677	·1936	·1424

Beneath the sums of the two progressions, a series of theoretical figures are added, which have been obtained by fitting the descending series to an equation of the form:

$$y = \frac{4225600}{(10{\cdot}659 + x)^{5{\cdot}2285}}.$$

The correspondence is as absolute as could be expected in any series of observations made on experiments. It is obvious here that the power to which the denominator is raised is the equivalent of a reaction in which the fractions of a molecule take part.

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