THE TIME OF PERCEPTION AS A MEASURE OF DIFFERENCES IN SENSATIONS

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I. INTRODUCTION—THE METHOD

The extensive literature which has sprung up since the time of Weber and Fechner on the measurement of differences in sensations is an evidence of the interest of the psychologist in this problem and the importance attached to it. In well-nigh all the investigations made to measure such differences the unit of measurement has been the just noticeable difference. The theoretical difficulties involved in the assumption of this unit and in the methods used to determine it are numerous and have been frequently pointed out. It will be the purpose of this introductory discussion to call attention briefly to some of these difficulties. This will perhaps serve to introduce the method of the time of perception as a measure of differences in sensations, which is used in this study, and to suggest its *raison d'être*. This method proposes the substitution of the time of perception as a unit of measurement for the just noticeable difference.

The possibility of the measurement of sensations rests on the fact that quantitative terms can be applied to them. Common observation teaches that one sensation can be judged to be more or less intense than another or equal to it. When such estimates, however, go beyond the mere judgment of equality or difference, they are at best vague and inexact. Sensations, being themselves variable and lacking a standard of measurement, can not be determined directly, and it is only by virtue of the connection of every sensation with a stimulus that measurement becomes possible at all. Moreover, this measurement is limited to the estimation of differences in sensation. A direct measurement of the relation of stimulus and sensation is not possible, because of the impossibility of establishing a direct measurable connection between them. A direct connection between the physical processes accompanying a sensation and the sensation itself can not be made out. Further, the sensation is a unitary experience, incapable of subdivision or addition such as would be necessary to measure it directly as related to physical states.

The problem, then, is to determine the relation of stimulus differences to sensation differences. As noted above, the judgment of sensation differences can not with any exactness go further than the mere judgment of difference. The unit of sensation difference, therefore, is that difference which can just be perceived, or the just noticeable difference. This is the unit used by Weber in his experiments, which led him to conclude that not equal absolute differences, but equal relative differences, are perceived, or that the just noticeable difference is relatively constant. Fechner gave a mathematical expression to this law by the assumption that the just noticeable difference is a constant quantity, and, further, that what applies to finite or sensed differences applies also to infinitely small differences. This being assumed, Weber's law becomes: the sensation varies as the logarithm of the stimulus, or, in other words, if sensations are to increase in an arithmetical ratio, the stimuli must increase in a geometrical ratio. These two assumptions have given rise to much controversy, which is by no means settled. It is argued by Fechner, Wundt and many others that it is a contradiction to assert that the just noticeable difference is not a constant. If the difference between sensations is just noticeable, it can not be more or less than this difference or it would not then be just noticeable. To which it is replied that this is a confusion of the difference in sensations with the sensation of being different, and that there is in consciousness no such thing as a difference in sensations which can be obtained by the subtraction of the magnitude of one sensation from another. Moreover, experiments show that the just noticeable difference varies widely with different individuals and with the same individual at different times, with the different senses, with varying degrees of practise, etc. The second assumption leads necessarily to the assumption of negative sensations, which has likewise occasioned much debate. It is assumed by Fechner, Wundt and those who support Fechner's law that the zero-point on the scale of sensations is the point of just noticeable difference. It is necessary to assume further, however, that there are differences in sensation below the threshold, and that these sensations, negative as contrasted with the positive sensations above the threshold, must also increase proportionally to the stimulus. This gives rise to many mathematical and other difficulties which will not be taken up here.

Again, in his discussion of errors of observation, Professor Cattell[1] says, concerning the just noticeable difference: "Another case in which German psychologists have run counter to the theory of probability is in the assumption of a just noticeable difference. According to the theory of probability the apparent difference in sensation and the probability of correct judgment tend to increase continuously as the differences between the stimuli are made greater, but it is entirely arbitrary to choose one difference and call it just noticeable. A difference in the stimuli can be found which will be obscured by the error of observation 1 time in 10 or 1 time in 1,000, but no difference can be called just noticeable, meaning that it and larger differences will be correctly distinguished, while smaller differences will be indistinguishable."

[1] J. McKeen Cattell, 'On Errors of Observation,' *Am. Jour. of Psychol.*, **5**: 284-293, 1893.

In addition to the difficulties involved in the unit of measurement, there are likewise methodological difficulties in determining it. The psychophysical methods, according to the classification of Wundt, fall into two classes, the gradation methods and the error methods. The gradation methods are the method of just noticeable difference (method of least differences [Müller], method of minimal changes [Wundt]) and the method of mean gradations. The error methods are the method of right and wrong cases and the method of average error. It is not our purpose to give a statement of the principles and methods of procedure in the use of these methods. This is given in the text-books on psychology. It is proposed to merely note the objections urged against them as means of measuring differences in sensations.

The method of just noticeable difference, in the old form in which it was used by Weber and Fechner, which consisted of adjusting a difference after various trials until it seemed just noticeable, was soon found to be unsystematic and unsatisfactory. Refinements in the mode of procedure in the application of the method were made by Müller (method of least differences), by Wundt (method of minimal changes) and by Jastrow (method of gradual increments). In spite of these improvements many investigators have found much to criticize in the method. Jastrow points out in connection with Weber's experiments the ambiguity of the term just noticeable. He says:[1] "This expression may have as many as four practically distinct meanings. It may refer to (1) two pressures sufficiently different to enable the subject *sometimes* to tell which is which; or (2) to differences that will *always* be correctly recognized; or (3) such as will only accidentally fail to be recognized; or finally (4) it may refer to differences which will be correctly judged any characteristic proportion of the times between these extremes." With any of these meanings, the difference fixed upon will vary with individuals and at different times. As Fullerton and Cattell, who found the method unsatisfactory, state it:[2] "The clearness with which a difference is distinguished varies gradually from complete doubt to complete certainty. The variation is continuous and no point can be taken and called the 'just noticeable difference' and kept constant for different observers and for the same observer at different times. If complete certainty be taken as the standard a difference in the stimuli will be required much greater than that which can ordinarily be distinguished and the standard will be found to differ greatly with different observers, measuring, if anything, rather their character than their fineness of sensation" The influence of ex-

[1] Joseph Jastrow, 'Psychophysical Methods,' *Am. Jour. of Psychol.*, 1: 274, 1888.

[2] Fullerton and Cattell, *On the Perception of Small Differences*, p. 11, 1892.

pectation often leads a subject to judge that there is a difference in two sensations when no such difference is present. What is to a subject a just noticeable difference is, furthermore, often determined by association and not at all by real differences in sensation.

The method of mean gradations in its various forms deals with overnoticeable differences and some investigators are of the opinion that if this method could be freed from disturbing factors of association and the like, it would in many respects be the best method. Cattell[1] thinks that this method may really measure differences in sensation in contrast to the other methods which determine the error of observation. The difficulties of the method lie in the fact that in adjusting a sensation midway between two other sensations the subject is perhaps influenced more by association with objective differences than by differences in sensation.

The error methods are generally held to be the best of the psychophysical methods. They are based upon the supposition that the mathematical theory of errors can be applied directly in psychophysics. The probable error, or the error of observation, is then taken as a measure of sensitivity on the supposition that the error of observation is proportional to the just noticeable difference. Difficulties and objections in plenty have been raised against the principles on which the methods are founded and the mode of procedure. As to the latter, much controversy has centered about the rejection or admission of judgments of equality and their treatment when admitted. As to the former, it is by no means established that Gauss's law of error can be applied as it is in physical measurements. The errors which the psychologist studies are perhaps not the simple accidental errors, distributed uniformly, positively and negatively, according to the theory of probability. Further, it is not necessarily true that the probable error, which is taken as a measure of discriminativeness, is proportional to the just noticeable difference. That it bears some relation is, of course, apparent, but, as Müller, Ebbinghaus and others have pointed out, the two measures depend on very different conditions, and their variation may not be a proportional one. Moreover, the whole question of the application of the error of observation to the measurement of the intensity of sensations is questioned. Thus, Cattell says:[1] "I entirely question the application of the error of observation to the measurement of the intensity of sensations." And again: "I conclude, consequently, that we can not measure the intensity of sensations and its relation to the energy of the stimulus either by determining the error of observation or by estimating amounts of difference."

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[1] *Op. cit.*, p. 293.

This brief statement of the chief objections and difficulties in psychophysical measurements has aimed only to show that the entire question of the measurement of differences in sensation is still unsettled. They seem to show the desirability of approaching the problems of psychophysics from other points of view or by other methods. This is attempted in the investigation here reported, in which differences in sensations are measured, not by just noticeable differences, but by the time it takes to perceive them. It represents an application of a method first proposed by Cattell for such measurement. It is believed that this study gives evidence of the value of the method and a further confirmation of its validity.

Differences in sensation should be equal if it takes equal time to perceive them. Again, if the differences are unequal, the greater the difference the shorter the time necessary to perceive it, and, on the other hand, the smaller the difference the longer the time. The times of perception for intermediate differences should be inversely proportional to these differences. It is believed that the experimental evidence shows that this is the case. If, therefore, we have a measure of the time it takes to perceive a difference, we have a measure of the difference itself. But experiments have shown that the time of perception can be rather accurately measured, and we should thus have a unit for the measurement of sensations. In this way it would be possible to arrange in accurate series groups of differences in every department of our experience, successively or simultaneously perceived. These differences may be differences in quality as well as in intensity. On the one hand, we can take equal objective differences and discover with what differences for consciousness these are correlated. On the other hand, we can take equal relative differences and thus test the psychophysical law. If this law, which holds that we perceive not absolute, but relative, differences, be true, then the time necessary to perceive equal relative differences should be equal, *e. g.*, the time taken to perceive the difference between two lines 10 mm. and 12 mm. long should be equal to that for perceiving a difference between lines 20 and 24 mm. or 30 and 36 mm. Or, again, if the square root formula of Cattell holds, it should be possible to correlate the increase in time of perception inversely with the increase in the error of observation.

The determination of the actual time of perceiving the difference in a pair of stimuli is possible. In the application of the method, however, this is not necessary. What we are interested in is the nature of the increase in the time of perception with the increase in the intensity of stimuli, or in other words with the decrease in the differences between the stimuli. Now, in determining the time of reaction to a pair of stimuli, *e. g.*, red and orange, this includes the time of perception, the time of volition, and that of the simple reaction. But with another pair, *e. g.*, red and an orange-red containing 50 per cent. of red, the time of volition and that of the simple reaction remain presumably the same, and any difference in the times is a difference in the time of perception or discrimination. We can, then, correlate directly objective differences with subjective differences as measured in terms of mental time.

The method has this in common with the method of mean gradations, that it deals with a determination of overnoticeable differences. It was noted above that this method is held by some investigators to determine actual differences in sensation, while the other methods determine the errors of observation, but can not be said to measure the intensity of the sensation. The difficulties attendant upon the use of the method were found to lie in the disturbing influences of association and the tendency to determine estimates on a basis of known quantitative relations or by measurement, as in the case of lines, by familiar units. The psychometric method makes use of these overnoticeable differences and measures differences in sensation, while it is freed from these disturbing factors.

The method also gives a valuable means of comparing the differences in sensations from the various senses with considerable accuracy, something which can not now be done. We could by this means determine that the difference between two colors is equal to such and such a difference in the lengths of lines or such and such an interval in pitch. It is possible then to say, for instance, that for consciousness, in some individuals at least, a difference of four vibrations in pitch is equal to a difference of 0.5 mm. in the lengths of lines, and that the increase in difference as we approach the threshold is about the same. Further, we can say that a difference of two wave-lengths in color in a certain part of the color scale represents approximately a difference of one vibration in pitch. A comparison of the tonal and color scales is thus possible. The method opens up here a wide range of problems of considerable interest.

Individual differences in discrimination can be measured better perhaps than by any other method. The usual psychophysical methods determine such differences at or about the threshold. But this gives a measure over only a limited portion of the sensation scale. The threshold values for differences in, for instance, red, yellow and orange may not differ greatly, but for various individuals the differences between red and yellow and red and orange may and do in fact vary considerably, representing by no means equivalent differences for consciousness. Likewise in sensations of tone the threshold may give little clue to what a difference of four or eight vibrations in pitch means for different individuals.

Lastly, this method may serve to discover sense defects, such as ability to discriminate differences in tone or color. The determination of color deficiency is practically important. The methods of detecting it have been entirely based on the confusion of colors, but it appears that the time factor is likewise significant. The person who is deficient in color sense will take a longer time to distinguish the colors in which he is deficient, and if we have a measure of the time of discrimination, we should get a measure of his color perception. This usually takes the form of determining deficiency in distinguishing the reds and greens. If the person be red-green blind, it will take him a longer time to distinguish these colors than the blues and yellows, even if he make no errors or confusions, and any defect will thus be disclosed.

The following pages give an account of the applications of this method, the results obtained and the conclusions drawn from them. First is given an historical account of the method. Then there follows a statement of the experiments made. The first group deals with qualitative differences in color, with differences between various colors and black, and between various pairs of colors. There is also proposed a test for color-blindness, based on the time of perception.

The second group is concerned with the measurement of linear magnitudes, using both equal absolute differences and equal relative differences in lengths of horizontal lines.

The third group consists of experiments on the perception of pitch.

II. HISTORICAL ACCOUNT OF THE METHOD

The history of mental measurement shows that two apparently independent lines of research in psychology have been pursued with a diligence rarely if ever equaled in the history of any science. On the one hand is the measurement of the time-relations of mental phenomena; on the other, the measurement of the relation of the stimulus to the sensation, the psychophysical law, which has concerned itself chiefly with relations of intensity. These lines of inquiry, mental time and mental intensity, appear to converge in the method here applied, one mental quantity being used as a measure of the other. The method proposes the use of mental time to measure mental intensity. It would, therefore, be in place to consider the history of the measurement of these quantities, which has run a parallel and independent course with occasional evidences of interrelations and points of contact. However, it would carry us too far afield to give a detailed account of the investigations of reaction-times, or, more particularly, that part of the work which has to do with perception- and will-time, or, as it is perhaps more often termed, the time of discrimination and choice. This would bear more or less directly on this study. Again, the method here used and the results obtained bear directly on the problems of psychophysics, an account of which is beyond the limits of this paper. Summaries there are in plenty, and a detailed account is here out of the question. I shall content myself with giving, after some introductory considerations, a historical account of the psychometric method as a measure of differences in sensations.

From the measurement of the velocity of the nerve impulse by Helmholtz in 1850 it was not a far cry to the determination of the time of reaction, or the time elapsing between the giving of the stimulus and the execution of an appropriate and predetermined movement. The elements that enter into the processes which take place have been variously analyzed, and attempts have been made to isolate them for purposes of determining especially the time taken up by the purely cerebral processes. There is the time taken to overcome the inertia of the sense organs, the time of centrifugal and centripetal conductions in the sensory and motor nerves and in the cord, and the latent time of a muscle. Moreover, the factors which influence the reaction-time, such as the quality and intensity of the stimulus, the effects of practise, fatigue, attention, mode of reaction, and racial and individual differences, have been studied in detail.

The simple reaction-time, aside from the analysis of the conditions above noted, is of especial interest and value from the fact that there may be added to it mental processes such as discrimination, choice, recognition and association, and these again may be qualitatively and quantitatively analyzed out. The work of Donders, von Kries and Auerbach, Friedrich, Merkel, Tischer, Cattell, Münsterberg and others indicates that these processes can be isolated and measured and how it is to be done. Cattell has covered the ground more thoroughly than any one else, both as to the simple reaction and as to the times of perception, volition and association. In his psychometric studies there are numerous evidences of measurement, not only of the time of mental processes, which is the main purpose of the investigation, but also of differences in sensations. Thus, in the time it takes to distinguish various colors, ten in all, from white, the time is shortest for black and white and longest for white and yellow, then for white and pink, green, gray, brown, red, blue, orange and violet, in the order named. The differences in time of perception are in the main what one might expect from the differences in the sensations. It suggests that it may be possible to arrange by this method the colors in a definite scale between white and black, and that it would be a means of investigating rather accurately individual differences in color perception. Similarly, in various pairs of colors the differences in time of perception indicate the differences in the sensations.

To *Cattell*, also, is due the method that is here under consideration. In concluding his discussion of the time of association of ideas, he says:[1] "We have lastly to consider the time it takes to form a judgment or opinion. I choose three cases in which the results could conveniently be averaged. In the first case, the subject estimated the length of a line drawn horizontally on a card 10 cm. long, fifty lines being used, varying in length from 1 to 50 mm. In the second case, the subject estimated the number of short perpendicular lines on a card, the number varying between 4 and 15. In the third case, the names of two eminent men were shown, and the subject decided which of them he thought to be the greater. I made rather a large number of determinations with the lines, as I wished to find the ratio between the length of the lines and the average error (psychophysical law) and between the error and the time taken up in coming to a decision. I think it, however, desirable to still further increase the number of experiments before publishing the results."

Again, in a report of studies in progress from the Psychological Laboratory of Columbia University, among others is one[2] 'On Differences in Sensation and Time of Perception.' I quote the abstract: "Lights (whose intensities are measured by the angle of incidence of

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[1] J. McKeen Cattell, *Philos. Studien*, **4**: 250; and *Mind*, **12**: 74, 1887.

[2] *Proc. of the Am. Psychol. Assoc.*, 1893.

the rays) and other stimuli are being used. The time of perception is measured when the differences between two lights are altered and when the absolute intensity and areas are altered. Equal differences in sensations are thus determined by the fact that they require equal times for discrimination."

In a later report he says:[1] "The method used by the writer is to measure the time it takes to perceive the difference in the intensity of two lights. The greater the difference the less is the time of perception, and the times are equal when differences are equally great for consciousness. It is thus possible to find a gray usually midway between black and white and to measure equal increments in sensations. Disparate sensations can thus be compared, and color-blindness and other defects be discovered."

The publication of results by the method thus indicated bore on the measurement of differences in the intensity of light.[2] The stimuli were gray surfaces reflecting a known percentage of light. "These surfaces were washed with India ink. The ink was made as black as possible and one piece of drawing paper was washed; then a drop of water was added to the ink, another piece was washed and the process continued till the wash became imperceptible." The grays obtained were measured and used for various purposes. For the experiments which bear on the psychometric method, cards were prepared and exposed in the manner described below. Inasmuch as I used the same apparatus and methods of exposing stimuli in many of my experiments, I refer for the details to the discussion of the experiments on color in the next section. It should be noted that for white the surface was painted with zinc oxid and for black a hole in a black box was used, which gave nearly an absolute black. Grays, reflecting 25, 50 and 75 per cent. of light, and black and white were used in various pairs.

The experiments showed that the time of perception increased as the differences decreased uniformly with the two subjects. The number of experiments was, however, not sufficiently large to determine the exact nature of this increase. The investigation was concerned, as is stated, more with the establishing of the validity of the method than with the definite measurement of differences in intensity.

Münsterberg[3] has applied this method to the measurement of linear magnitudes. The chain method of reaction was used. One person reacted and gave the stimulus to the second and the second to the third, and so on till the last person again gave the stimulus to the first. The intervening time, deducting the time of reaction of the first person,

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[1] *Proc. of the Am. Assoc. for the Adv. of Sci.*, p. 95, 1899.

[2] J. McKeen Cattell, *Philos. Studien*, **19**: 63-68, 1902.

[3] H. Münsterberg, *Psychol. Rev.*, **1**: 45-51, 1894.

gives the time of discrimination and movement of the chain. Five persons besides the first acted as subjects. The stimuli were exposed by a complicated apparatus which can not be described here. The stimuli were lines, 2.5-5-7.5 mm., 4-5-6 mm. and 4.5-5-5.5 mm. with four multiples of each. Before the subject was placed a disc with a line equal in length to the second line in each of these groups. Three fingers of his right hand rested on keys which were to be pressed down according to the nature of the stimulus exposed, the first if the line were shorter than the standard, the second if equal and the third if longer.

It is not difficult to note objections to the method and the complicated conditions involved. Especially is this true if any application is to be made to the psychophysical law. Whatever may be the advantages of the chain reaction in eliminating individual differences and in the saving of time, it certainly has obvious disadvantages for accurate measurements of differences in sensations. The averaging together of reactions of many individuals may or may not give a valid measure. The three possible judgments further complicate the conditions and must have introduced disturbing factors, which would make the times of reaction irregular. The times are, as a matter of fact, both long and irregular. What their variability was, does not appear in the table. The number of cases for each group of stimuli is further too small to admit of any definite conclusions.

The conclusion drawn from the investigation is that the psychophysical law holds approximately. The time decreases somewhat with the increasing absolute differences in the length of the lines whence the qualification is made that 'for our subjective discrimination, therefore, the stronger effect of the relative differences of stimuli is constantly influenced by the weaker effect of the absolute differences in stimuli.'

These two studies, which are both perhaps to be regarded as preliminary, are the only ones that have been made by this method. The relation of the time of a judgment to its accuracy has, however, been noted in several investigations.[1]

Warren,[2] in his study of the reaction-time of counting, determined the limits of perceptive counting, and further suggests that the rate of progressive counting, or the law by which the time of progressive counting increases with the increase in number, might have been taken up. This was not done, owing to the complexity of the problem and the number of experiments required for its solution.

[1] G. E. Müller, *Die Gesichtspunkte und die Tatsachen der Psychophysischen Methodik*, Wiesbaden, pp. 15-16, 1904.

[2] H. C. Warren, *Psychol. Rev.*, 4: 569-591, 1897.

Martin and Müller,[1] in their experiments with weights, determined in a rough way with a metronome the time required to make a judgment. They noted that the recognition of equality required a longer time than the recognition of a difference. They noted, further, that the greater the difference between two stimuli the shorter the time, and that the immediate judgments were more likely to be correct and certain.

Whipple,[2] in his study of the discrimination of clangs and tones, took account of the relation of the speed of judgment to certainty and correctness of judgment as well as the relation of speed and immediacy to correctness. The judgments fell into at most four categories as to speed, introspectively noted. The immediate judgments were found to be usually both correct and certain, while compared judgments were more often uncertain and incorrect.

Angell,[3] in a study of discrimination of shades of gray for different intervals of time, measured to tenths of a second the time of delivering the discriminative judgment. The times were determined for the four categories of judgment, 'sure,' 'fairly sure,' 'like' and 'doubtful,' for the four intervals of time 5, 15, 30 and 60 seconds. The results showed that 'like' judgments are longest, 'sure' judgments shortest, with 'fairly sure' intermediate.

This historical summary of the applications of the method indicates that the method has been used systematically in but two investigations and that these are preliminary and concerned more with the method as such than with the measurement of differences in sensations. The present research, it is believed, gives not only certain positive results by the method, but also a definite confirmation of its validity and value.

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[1] Lillie J. Martin and G. E. Müller, *Zur Analyse der Unterschiedsempfindlichkeit*, Leipzig, p. 196 ff., 1899.

[2] G. M. Whipple, *Am. Jour. Psychol.*, **12**: 445-446, 1901.

[3] Frank Angell, *Philos. Studien*, **19**: 16-21, 1901.

III. EXPERIMENTS ON COLOR

1. *Differences in sensations of color in the red end of the spectrum*

The first application of this method in the investigation here reported had for its purpose the measurement of qualitative differences in color. The colors chosen as standards were yellow, orange and red, such as can be produced from the pigments chrome yellow, mineral orange and the best English vermilion. These pigments were mixed with gum arabic and applied with a brush to the surface of a white egg-shell cardboard. Care was exercised to get the mixture of the same degree of consistency as far as this is possible, and to distribute the pigments over the surface evenly. The result was a uniformly colored surface of a high degree of saturation. The wave-lengths of the colors given by these pigments had been determined in the physical laboratory of Columbia University and are, respectively, $585\mu\mu$, $614\mu\mu$ and $644\mu\mu$. Instead of the standard red it was, however, found convenient to use the lighter shade of vermilion, it being better suited for mixture. This pigment gives an orange-red containing 85 per cent. of the standard red and 15 per cent. of the standard orange. The wave-length of the resulting color is thus $639.5\mu\mu$, allowance being, of course, made for a slight variation either above or below this measurement, owing to the possible error of observation. The above equation, which I obtained from the colors used, agrees with that obtained by Rood. Mention is made of this fact because of the slight but noticeable differences in various supplies of the pigments. This is especially to be observed in mixtures. For instance, in securing a certain color by the mixture of chrome yellow and vermilion, it is quite useless to give a statement of the quantities of each pigment used, because, with a new supply of pigments these quantities may and do in fact vary not a little. Any color produced by the mixture of pigments must, therefore, be determined empirically. In the early attempts to produce equal intermediate color-tones between orange and red, the quantities of pigment of each color were measured accurately, and the consistency of the mixture was gauged as well as possible in order to get definite values for each color and thus facilitate reproduction. For the reasons stated this was found to be impossible. It was therefore necessary to resort to numerous trials in order to obtain the desired colors.

Equal intermediate color-tones between orange and red, representing, respectively, 25 per cent., 50 per cent. and 75 per cent. of red or orange, as the case may be, were produced from the pigments mentioned.

For convenience, I shall call them hereafter orange-red^{25}, orange-red^{50} and orange-red^{75}, using in the tables the abbreviations OR^{25}, OR^{50} and OR^{75}. The first, OR^{25}, was secured from a mixture of vermilion and mineral orange; the remaining two from vermilion and chrome yellow. This gives a very satisfactory series of colors. Aside from the normal difference in brightness or luminosity, the only difference that could be noted was that of color-tone. There were no other differences apparent which might serve as cues in discrimination.

The determination of the position of each color-tone in the scale between orange and red was made with color wheels and is thus subject to a small error of observation. In order to reduce this to a minimum, the equations were determined on a basis of the judgments of at least five persons connected with the psychological laboratory, and were repeated until it seemed certain that the proper hues had been obtained. Tests were made on different days and with the normal varying light conditions of diffuse daylight. The wave-lengths of these colors, intermediate between orange, $614\mu\mu$, and red, $639.5\mu\mu$, are thus $620.375\mu\mu$, $626.75\mu\mu$ and $633.125\mu\mu$. This statement of the wave-lengths is based on calculations from the standards and not from actual determinations.

From the papers secured squares 3x3 cm. were cut and mounted side by side on black cards, each together with a corresponding square of standard red. The colored surfaces were thus 6x3 cm. in area. The series consisted of red together with yellow, orange, orange-red^{25}, orange-red^{50} and orange-red^{75}.

The apparatus by means of which these cards were exposed was that devised by Cattell and used by him in the experiments referred to above. A brief description will be given here. It consisted of a black wooden frame, 39x39 cm., in the center of which is an aperture 6 cm. in width and 3 cm. in height. That part of the frame which surrounds the aperture is so cut as to prevent shadows being thrown on any stimuli exposed. The drop-screen has an especial feature which makes it very valuable for experiments such as are to be described. The screen by means of which the aperture is exposed is of aluminium and is drawn down by a spiral spring. The exposure is practically instantaneous, the eye being unable to follow the screen as it drops. The rate of motion when the exposure is made is about 10 m. per second, and the card is thus uncovered in 3σ. The cards are fitted into a shutter which runs in a horizontal track behind the screen. The screen, when the aperture is closed, is held in place by a lever to which a spring is attached. A small rod is fitted on the screen between the prongs of a fork, which breaks or closes the circuit as the screen is raised or dropped. Simultaneously with the exposure of the card, the electric circuit is closed.

A detailed description of the further arrangement of the apparatus and electrical connections for reaction-time with discrimination and choice seems superfluous inasmuch as it has been so frequently given. It may suffice to state that the circuit was made in the usual manner and the times of reaction recorded by the Hipp chronoscope. Tests with the Cattell control-hammer were made before and after each series to insure accuracy of the measurements. Frequent records show the errors of the chronoscope to be below 1σ. In the early experiments a current from a small dynamo was employed, but later five gravity cells were substituted. These were used in all the experiments made except the first series on color. In these series on observer H, I used the dynamo current, in the corresponding series on observer W, storage batteries, in all the other series, the gravity cells. The subject was seated 55 cm. before the screen and reacted by lifting his hand or finger from a telegraph key. Owing to the position naturally taken by the subject, he was about 50 cm. from the screen, which seemed a convenient distance and entirely satisfactory for the perception of the stimuli. All the experiments were made in a room with a north exposure on the third floor of the psychological laboratory. The illumination was, therefore, very constant. Most of the experiments were made in the morning between 9 o'clock and 11 o'clock, though occasionally they were made in the afternoon between 2 o'clock and 3 o'clock. The light conditions appeared to be uniform. On especially dark and cloudy days, when this seemed not to be the case, no series were taken.

Throughout this investigation extended series of experiments have been made on two subjects, either W and H, or S and H. However, numerous preliminary experiments were made on various other subjects, using combinations of red together with green, orange, orange-red^{50} and orange-red^{75}. These were made to determine the validity of the method and to eliminate the effects of practise. In this way, subject H had made approximately 3,000 reactions. The effects of practise in reaction had, therefore, been almost entirely eliminated in his case. Similarly on subject W, 1,200 reactions to the orange-red series of stimuli had been taken before the results which appear in the tables were obtained. The relative values in the practise series do not differ from the later results, except in the general reduction of the time of reaction and the greater constancy. This is shown in the comparatively small mean variation and the resulting increase in the reliability of the determinations in the later series.

The method of experimentation for the first series on color and in general for all the experiments made was in accordance with the following plan. The subject seated before the screen with his hands on two telegraph keys, reacted by lifting his right hand or his left hand from the key according as the stimulus to which reaction was to be made appeared to the right or to the left. Thus, if the card on which red and orange were mounted was exposed and reaction was to be made to the orange, the subject would react with his right or his left hand according as orange appeared to his right or to his left. In each series 200 reactions were taken at one sitting running through the entire set of cards. There were thus 40 reactions to each pair of stimuli and 20 to each of the two. More specifically, the method was to take, *e. g.*, 20 reactions to red, then 20 to yellow, etc., throughout the series, an equal number being taken with right and left hands. The order of presenting the stimuli, *i. e.*, as to their appearing at the right or the left, was a chance one, and the element of expectation was eliminated so far as this is possible with two stimuli. The introspective testimony of both observers indicates that this factor has been eliminated and that it does not affect the results obtained. In order to equalize the conditions for each pair of stimuli the order of presentation alternated from series to series. Thus, in one series the order would be red and yellow, red and orange, red and orange-red^{25}, etc. In the next succeeding series the order would be red and orange-red^{75}, red and orange-red^{50}, etc. Further, in each pair of series the order of reacting to the stimuli was varied. Thus first red and then yellow in the next group, first yellow and then red. These precautions were taken to guard against possible effects of practise and fatigue. As a matter of fact, no noteworthy effects of either practise or fatigue are shown, as a study of the tables indicates.

A factor which tends to influence the time of reaction is the rapidity with which the stimuli are presented and more especially the interval between the giving of the warning signal and the exposure of the stimuli. The time required to take 200 reactions was approximately fifty minutes. The recorders had by the time these series were begun fallen into a uniform rate of speed, and the interval between each reaction was quite constant, approximately 5 reactions per minute being made. More important, however, is regularity in the time between the warning signal and the giving of the stimulus. In order to secure constancy and reliability in the results, it is necessary to take a favorable interval for adjustment of attention and preparation for reaction. It is further necessary to keep to this interval as uniformly as possible. In the preliminary experiments it became apparent to the writer, who was then acting as recorder, that he could by altering the interval make the times shorter or longer, especially the latter. In order to equalize the conditions, a uniform method has been followed by all the recorders. The warning signal consisted of the starting of the chronoscope, and after an interval of approximately a second and a half the screen was released. The rhythmic character of the movement into which the recorder fell after some practise gave a satisfactory regularity.

Tables I and II give the results of 4,400 experiments on two subjects, H and W. There are thus 2,200 reactions by each subject, 440 to each pair of stimuli. Each figure in the table, except the total average, represents an average taken from 40 reactions. Column 1 gives the number of the series. The order of presentation in the first series was red and yellow, then red and orange, and so on throughout the series. In the second series the order was the reverse and from then on the order alternated. The remaining columns give the times of reaction and the mean variations, the values being in thousandths of a second and the usual symbol σ used. At the foot of the tables appear the averages, the mean variations and the probable errors of the averages, the differences and the probable errors of the differences, and the number of false reactions distributed with reference both to the series and to the pairs of stimuli.

The tables show clearly that the equal objective differences between any two pairs of stimuli, except between red and yellow and red and orange, are correlated with increasingly greater differences for consciousness. Taking the time of perceiving red and orange as a standard, we note that the time of discriminating the differences between these two stimuli and making the proper movement is for H

TABLE I

Subject H. 2200 Reactions. III & IV, 1904.

Series.	R & Y	M. V.	R & O	M. V.	R & OR[25]	M. V.	R & OR[50]	M. V.	R & OR[75]	M. V.	False
I.	212.4	*16.9*	244.9	*18.2*	251.3	*17.1*	258.5	*19.2*	275.0	*20.8*	4
II.	210.2	*16.9*	240.5	*19.8*	251.5	*19.6*	261.5	*14.7*	270.1	*20.8*	4
III.	223.3	*23.4*	249.5	*17.1*	253.3	*18.9*	259.7	*23.2*	272.6	*19.1*	1
IV.	212.4	*16.3*	248.0	*17.5*	247.8	*19.0*	252.4	*20.3*	268.6	*19.1*	4
V.	231.4	*13.3*	249.6	*13.7*	255.8	*16.6*	266.5	*19.5*	270.8	*17.0*	2
VI.	217.3	*17.7*	250.2	*19.5*	253.8	*20.1*	262 5	*19.0*	273.8	*20.5*	0
VII.	215.1	*15.6*	239.7	*15.0*	247.7	*18.8*	256.0	*20.7*	268.6	*15.8*	4
VIII.	219.1	*17.7*	244.6	*15.5*	250.1	*17.1*	254.6	*21.4*	260.8	*18.7*	1
IX.	220.3	*18.0*	252.8	*18.2*	259.1	*22.6*	265.3	*20.1*	270.6	*23.0*	3
X.	214.9	*17.9*	245 3	*23.1*	250.3	*20.7*	264.7	*22.6*	276.1	*18.8*	3
XI.	209.3	*22.3*	242.7	*23.0*	250.0	*20.9*	256.0	*20.4*	272.4	*20.8*	1
Av.	216.86	*17.8*	246.16	*18.2*	251.88	*19.2*	259.79	*20.1*	270.85	*19.5*	
P. E. of Av.	.72		.73		.77		.81		.78		
Diff.	29.3				5.72		7.91		11.06		
Diff. from R & O	29.3				5.72		13.63		24.69		
P. E. of Diff.	1.02				1.06		1.11		1.13		
P. E. of Diff. from R & O	1.02				1.06		1.09		1.07		
False	2		6		4		7		8		27

TABLE II

Subject W. 2200 Reactions. X, 1904.

Series.	R & Y	M. V.	R & O	M. V.	R & OR[25]	M. V.	R & OR[50]	M. V.	R & OR[75]	M. V.	False
I.	253.7	*23.6*	257.9	*28.0*	259.3	*28.2*	266.2	*39.8*	286.6	*24.5*	14
II.	246.3	*24.7*	256.8	*28.1*	267.3	*22.4*	278.9	*30.3*	290.7	*27.9*	10
III.	258.3	*23.1*	267.1	*27.6*	272.8	*26.3*	283.3	*24.7*	287.9	*27.9*	10
IV.	258.5	*24.9*	264.1	*25.4*	276.8	*24.4*	284.1	*23.7*	291.9	*21.2*	11
V.	256.6	*21.3*	280.3	*28.7*	281.6	*35.0*	289.1	*31.9*	295.9	*29.8*	6
VI.	263.9	*19.8*	266.7	*23.0*	277.3	*26.2*	280.9	*22.3*	295.8	*31.8*	5
VII.	249.7	*20.2*	268.3	*26.6*	275.6	*25.8*	285.9	*24.9*	295.7	*26.3*	7
VIII.	254.6	*20.4*	267.2	*21.8*	270.7	*24.3*	279.5	*20.6*	290.6	*27.5*	4
IX.	265.5	*19.3*	281.8	*20.6*	286.4	*18.1*	289.0	*22.8*	295.3	*22.2*	3
X.	262.9	*25.2*	267.9	*22.1*	275.4	*19.2*	280.9	*22.3*	297.8	*20.1*	3
XI.	251.5	*21.6*	265.5	*23.7*	271.2	*28.9*	275.0	*19.0*	281.5	*27.3*	5
Av.	**256.50**	*22.1*	**267.59**	*25.0*	**274.03**	*25.3*	**281.16**	*25.6*	**291.79**	*26.0*	
P. E. of Av.	.89		1.00		1.02		1.03		1.04		
Diff.	**11.09**				**6.44**		**7.13**		**10.63**		
Diff. from R & O	**11.09**				**6.44**		**13.57**		**24.20**		
P. E. of Diff.	1.34				1.42		1.45		1.47		
P. E. of Diff. from R & O	1.34				1.42		1.44		1.44		
False	8		10		18		16		26		78

246.16σ and for W 267.59σ. These figures represent an average of 440 reactions for each subject. The mean variation is for H 18.20 and for W 25.0σ. The probable errors of the averages are for H .73σ and for W 1.00σ, indicating the reliability of the measurements. It is thus practically certain that the true times from an infinite number of experiments would not deviate from the averages obtained by more than 3σ and 4σ, respectively. The reaction-time to red and orange-red[25] was for H 251.88σ, and for W 274.03σ, or 5.72σ longer in the case of H, and 6.44σ longer in the case of W, than when reaction was made to red and orange. It is unquestionably to be assumed that the processes of reaction in the two cases are identical and that the differences in the times obtained are differences in the time of perception. That is to say, expressed in terms of time, the difference between orange and orange-red[25] is for subject H represented by 5.72σ and for W by 6.44σ. We thus have a measure of what differences for consciousness, or what differences in sensation, the given objective differences in stimuli are correlated with. Further, that this is a real difference and that the measurements are not due to chance, the probable errors and the close agreement between the two observers indicate. The probable errors of the differences between the time of perceiving red and orange and red and orange-red[25] are for H 1.06σ and for W 1.42σ. Taking such a value as two and one-half times the probable error, it is reasonably certain that the true difference in the case of H will not fall below 3.07σ nor rise above 8.37σ, and in the case of W will not fall below 2.89σ nor rise above 9.99σ.

The reaction-times to red and orange-red^{50} are for H 259.79σ and for W 281.16σ. The differences in the time of reaction to red and orange-red^{25} and to red and orange-red^{50} are thus for H 7.91σ and for W 7.13σ, though objectively the difference between these pairs of stimuli is equal to that between red and orange and red and orange-red^{25}. This indicates that the orange-red which objectively stands midway between orange and red is not so situated for consciousness. The determination of that orange-red which is subjectively midway between orange and red will be discussed later. As in the previous case, we have here a measure of the difference between orange-red^{25} and orange-red^{50}, which is thus as much greater than the difference between orange and orange-red^{25} as the differences in time are longer. Approximately, the same statements hold for the reliability of the differences as before. The probable errors are but very slightly larger. The differences from red and orange are for H 13.63σ, and for W 13.57σ. It will be noticed that though the time of reaction for W is approximately 21σ longer than for H the differences agree very closely.

The reaction-times to red and orange-red^{75} are for H 270.85σ and for W 291.79σ. The differences between this pair of stimuli and red and orange-red^{50} show a marked increase, being for H 11.06σ and for W 10.63σ, or, expressed in relation to red and orange, are for H 24.69σ and for W 24.20σ. The differences are thus almost twice as great as those between red and orange and red and orange-red^{25}, though, as before noted, the objective differences remain equal. This is indicative of a decreasing sensitivity to color differences in the scale between orange and red, and that this decrease becomes much more rapid as we near the end of the spectrum.

The reaction-times to red and yellow are for H 216.86σ and for W 256.50σ, and the corresponding differences from red and orange are, respectively, 29.3σ and 11.09σ. This pair of stimuli is not defined in relation to the remainder of the series. It was included, however, to determine the relations of yellow, orange and red, and to serve as a control series. The close agreement of the two observers in the orange-red series, as far as differences in time of reaction are concerned, has been referred to. In the case of red and yellow there is a striking difference, which points to individual differences in perception. It will be noticed that the differences between red and orange and red and yellow are for H 29.3σ and for W 11.09σ. The probable errors indicate that it is certain, the chances are 999 to 1, that the true difference for W will not be greater than 17.12σ and that the true difference for H will not be less than 24.71σ.

In the preliminary experiments made on other subjects, similar differences were obtained. A summary of these results, excluding four or five practise series, is given to illustrate this point.

Subject.	No. of React.	R. & G.	M. V.	R. & O.	M. V.	Diff.
B	260	288	*28*	305	*28*	17
M	620	250	*27*	280	*27*	30
H	832	244	*25*	265	*26*	21

Experiments on the same observers, using as stimuli red and orange-red^{50} and red and orange-red^{70}, indicate a much closer agreement.

Subject.	No. of React.	R. & OR^{50}.	M. V.	R. & OR^{70}.	M. V.	Diff.
B	260	285	*29*	295	*28*	10
M	432	266	*20*	275	*22*	9
H	520	276	*19*	283	*19*	7

These results show interesting individual differences in color perception and the value of this method in disclosing them. It seems that these differences hold for apparently very dissimilar colors, while within the red series the differences in the sensations appear very constant. In none of these subjects was there any evidence of color deficiency.

Tables III and IV give the differences between the various pairs of stimuli as they appear in the individual series. The first four

TABLE III

Subject H.

Series.	Differences. R.-Y. & R.-O.	R.-O. & R.-OR^{25}.	R.-OR^{25}. & R.-OR^{50}.	R.-OR^{50}. & R.-OR^{75}.	Differences from R. & O. R.-O. & R.-OR^{50}.	R.-O. & R.-OR^{75}.
I.	32.5	6.4	7.2	16.5	13.6	30.1
II.	30.3	11.0	10.0	8.6	21.0	29.6
III.	26.2	3.8	6.4	12.9	10.2	23.1
IV.	35.6	—.2	4.6	16.2	4.4	20.6
V.	18.2	6.2	10.7	4.3	16.9	21.2
VI.	32.9	3.6	8.7	11.3	12.3	23.6
VII.	24.6	8.0	8.3	12.6	16.3	28.9
VIII.	25.5	5.5	4.5	6.2	10.0	16.2
IX.	32.5	6.3	6.2	5.3	12.5	17.8
X.	30.3	5.0	14.4	11.4	19.4	30.8
XI.	33.4	7.3	6.0	16.4	13.3	29.7
Av. Diff.	**29.3**	**5.72**	**7.91**	**11.06**	**13.63**	**24.69**
M. V.	*4.08*	*1.96*	*2.28*	*3.60*	*3.47*	*4.66*
P. E.	1.04	.50	.57	.91	.88	1.19

TABLE IV

Subject W.

Series.	Differences. R.-Y. & R.-O.	R.-O. & R.-OR^{25}.	R.-OR^{25}. & R.-OR^{50}.	R.-OR^{50}. & R.-OR^{75}.	Differences from R. & O. R.-O. & R.-OR^{50}.	R.-O. & R.-OR^{75}.
I.	4.2	1.4	6.9	20.4	8.3	28.7
II.	10.5	10.5	11.6	11.8	22.1	33.9
III.	8.8	5.7	10.5	4.6	16.2	20.8
IV.	5.6	12.7	7.3	7.8	20.0	27.8
V.	23.7	1.3	7.5	6.8	8.8	15.6
VI.	2.8	10.6	3.6	14.9	14.2	29.1
VII.	18.6	7.3	10.3	9.8	17.6	27.4
VIII.	12.6	3.5	8.8	11.1	12.3	23.4
IX.	16.3	4.6	2.6	6.3	7.2	13.5
X.	5.0	7.5	5.5	16.9	13.0	29.9
XI.	14.0	5.7	3.8	6.5	9.5	16.0
Av. Diff.	**11.09**	**6.44**	**7.13**	**10.63**	**13.57**	**24.20**
M. V.	*4.25*	*2.99*	*2.51*	*3.99*	*4.05*	*5.75*
P. E.	1.08	.76	.64	1.02	1.03	1.21

columns give the differences between each pair and the next above it. In the last two columns the differences from red and orange appear for red and orange-red^{50} and red and orange-red^{75}. The differences in the first two columns, of course, remain the same, and hence are not repeated.

These tables indicate the constancy of the differences maintained throughout the series. The mean variations and the probable errors show their variability and reliability.

Before entering into a further discussion of the results, a brief historical summary of the results of experiments made by various investigators to determine the sensitiveness of the eye to differences in color-tone will be given. This will facilitate an appreciation of the meaning of the data obtained in this investigation.

Aubert,[1] using colored discs, made an extended study of sensitiveness to differences in intensity, saturation and color-tone. It is only with the last that we are concerned here. He used three colored discs, red, orange and blue, in the six possible combinations. Thus, a small amount of red was added to orange and blue, and these colors again to red, and so on. He concluded that the addition of from 1/360 to 1/100 of one color to another was sufficient to produce a noticeable difference. The addition of 1/360 of orange to ultramarine blue produced a perceptible change. For the other colors greater additions must be made.

[1] H. Aubert, *Physiologie der Netzhaut*, Breslau, p. 151 ff., 1865.

Mandelstamm,[1] using the Helmholtz ophthalmometer, determined the just perceptible differences in color-tone at Fraunhofer's lines and four undefined intermediate points. The method was to observe the solar spectrum through the ophthalmometer plates so placed as to give two adjacent colored fields. By rotation of the plates, alteration of the color-tone could be brought about. A slit in the ocular gave approximately monochromatic light. The maximum sensitivity was found in the yellow region (D), a second maximum in blue (F), and a third in blue-green (E-F). The sensitivity was considerably less for indigo (G), green (E), violet (G-H), and least of all for red (C).

Dobrowolsky[2] used the same apparatus as Mandelstamm with some alterations. In the carrying out of the experiments greater care was exercised to exclude differences in brightness. The method was as before to determine just noticeable differences. The results agreed with those obtained by Mandelstamm, though a much greater sensitiveness was shown. The results are indicated by the following figures taken from Aubert's *Grundzüge der Physiologischen Optik*.[3]

B	C	C-D	D	D-E	E	E-F	G	G-H
1/115	1/167	1/331	1/772	1/246	1/340	1/740	1/272	1/146

B. O. Peirce[4] used a large spectroscope, into the collimator of which was inserted a sheet of vulcanite dividing the tube into an upper and a lower half. The slit in the upper part through which the light entered was movable to the right or to the left. The apparatus was further adjusted so that when the movable collimator slit was displaced, the color of the observer's lower slit was changed without a change of position in the field. The spectrum was produced by a light from an Argand burner falling upon a Rutherford diffraction grating.

The object of the experiment was to determine the smallest displacement 'that could be infallibly detected and named in direction by the observer.' The amount of the displacement was found to vary with different colors and with different observers. The largest displacement necessary for any observer was $5\mu\mu$ and the smallest $.5\mu\mu$. The greatest sensitivity was found at a point near D, varying with the observers, 'being in some cases more orange and in others more yellow.' The second maximum was found at F. Between B and F there was a decrease. A third slight increase was found at Li. Dobrowolsky had found a similar increase at B, but later discovered that this was due to alteration in brightness.

[1] E. Mandelstamm, *Gräfes Archiv f. Ophth.*, **13**, 2: 399 ff., 1867.

[2] W. Dobrowolsky, *Gräfes Archiv f. Ophth.*, **18:** 102 ff., 1867.

[3] H. Aubert, *Grundzüge der physiologischen Optik*, Leipzig, p. 530, 1876.

[4] B. O. Peirce, *Am. Jour. Sci.*, 3rd Series, **26:** 299–302, 1883.

König and Dieterici[1] made the most complete investigation of the sensitivity of the normal eye to differences in wave-lengths. A more detailed account of this research will be given because of its bearing upon this study and the possibility of a comparison of results. For their purposes they employed the Helmholtz spectrophotometer, consisting of an equilateral flint-glass prism about which are arranged three collimators. Two of the collimators could be rotated about the axis of the apparatus, the amount of rotation being indicated on finely graduated scales. The other, which served as a telescope for observation, was stationary and directed toward one edge of the prism. It was so adjusted that the edge of the prism formed the vertical diameter of the objective, and an observer looking through the diaphragm saw the two prismatic surfaces side by side. The diaphragm situated in the focal plane of the telescope contained a movable slit parallel to the slits of the collimators through which light from gas-lamps with Argand burners was allowed to enter. Light passing through the collimators formed in the plane of the diaphragm, the ocular in the telescope being removed, two spectra, one on each of the surfaces of the prism. By proper adjustment of the width of the diaphragm slit, nearly monochromatic light was visible to the observer. An alteration of position of one of the collimators brought about a change in its color, the amount of the alteration being registered on the scales. The determinations of the measurements on these scales were made by using lithium, thallium, natrium, etc.

The method of average error was followed in the experiments. An experiment consisted in adjusting one collimator with reference to the other until the colored fields, which appeared side by side in the telescope, seemed to be alike in color. Twenty-two places in the spectrum, equidistant from each other and ranging from $430\mu\mu$ to $640\mu\mu$, were investigated. Beyond these limits differences in brightness only, and not differences in color-tone, could be perceived. Fifty experiments were made at each place by each of the two subjects, K and D, with two intensities of illumination. For the region from $640\mu\mu$ to $520\mu\mu$, the average errors were the same for the two intensities and hence are grouped together. From $520\mu\mu$ to $430\mu\mu$, the different intensities caused divergences in the course of the curve, as will be pointed out below.

The curve of sensitivity beginning with $640\mu\mu$ fell gradually to the line D, where the first maximum was obtained. This maximum point appeared for K at $590\mu\mu$ and for D at $570\mu\mu$. There was a slight rise again in the greens and then a fall at F, where the greatest sensitivity was found. Mandelstamm, Dobrowolsky and Peirce had

[1] A. König and C. Dieterici, *Wiedemann's Annalen*, **22**: 579–589, 1884, and A. König, *Abhandlungen zur physiologischen Optik*, Leipzig, pp. 23–33, 1903.

all found the maximum to lie in the yellows, or near the line D. König considers that the curve obtained by Peirce is due to the fact that he grouped the results of observations on many individuals, which obscured the individual differences that appeared in his study, but the nature of which is not given in detail. The influence of the different intensities is shown by a greater sensitivity at F with the weaker intensity. This difference is marked in the case of K and is very slight in the case of D. At 450$\mu\mu$, likewise, there is greater sensitiveness with the weaker intensity, while beyond this point greater sensitiveness accompanies the greater intensity. It is to be noted that in this part of the spectrum calcium light and gas light were employed for the two intensities, and Brodhun has pointed out that the distribution of brightness in the two spectra may be the source of the deviations. The third point of maximal sensitivity is at 450$\mu\mu$, the point of transition from indigo-blue to violet. A more detailed consideration of the red end of the spectrum which most concerns the present investigation will be given after the work of two other investigators has been presented.

Brodhun,[1] who is green-blind, investigated in his own case the region lying between 570$\mu\mu$ and 440$\mu\mu$, and repeated the experiments of König and Dieterici on the former. The apparatus and methods were identical with those used by König and Dieterici. With the results of experiments on dichromates we are not here concerned, and the red end of the spectrum was of course not investigated by Brodhun, so for a consideration of that part of the research reference is made to the original article. He noted the great influence of a decrease in intensity at either end of the spectrum and made experiments on himself in which the intensity was constant and in another series where the intensity was varied. In the latter series, before each experiment not only the color, but also the brightness of the field, was changed by an alteration in the width of the collimator slit. The result was that throughout the spectrum there was shown a decrease in sensitivity. This led to a repetition of the experiments of König for the entire spectrum from 640$\mu\mu$ to 440$\mu\mu$. The curve obtained was in general the same as that obtained in the earlier investigation. There was a marked reduction in sensitivity at the ends of the spectrum. The third maximal point between indigo-blue and violet disappeared, and there was a gradual rise in the curve from 490$\mu\mu$ to 440$\mu\mu$. A comparison of these second results on K appears below.

Uthoff[2] determined the just noticeable differences for the normal eye from 650$\mu\mu$ to 450$\mu\mu$. The apparatus and methods of procedure

[1] E. Brodhun, *Verhandlungen d. physiol. Gesell.*, Berlin, Nos. 17 and 18, 1885–86, and *Zeitschrift f. psychol. und Physiol. der Sinnesorgane*, **3**: 97–107, 1892.

[2] W. Uthoff, *Gräfes Archiv f. Ophth.*, **34**, 4: 1 ff., 1888.

were those used by Brodhun. The curves show the same general characteristics as those of the three last-named investigators. The just noticeable differences which were taken as measure of sensitivity were approximately 2.2 times greater than the average errors of König and Dieterici and Brodhun. In the red end of the spectrum there appeared relatively smaller values than those of König and Dieterici, which may have been due to individual differences in the subjects.

In order to show the relation of the results of the three last-mentioned researches to those obtained in this study a table is given of the results of König and Dieterici, Brodhun and Uthoff for the region of the spectrum lying between 610$\mu\mu$ and 640$\mu\mu$, 650$\mu\mu$ in the case of Uthoff. To facilitate comparison between my results and these, the average errors and just noticeable differences for the wave-lengths corresponding to the stimuli used in this research have been interpolated. These appear enclosed in parentheses. The interpolations are calculated on the assumption that the line connecting the average errors and just noticeable differences of the separate wave-lengths is a straight one. This, while not strictly true, of course, gives a sufficiently close approximation to the real errors and differences for purposes of general comparison, which is all that is attempted. The table requires no other explanation than the statement that the second column, headed K^a, gives the average errors as obtained on subject K in the research of König and Dieterici. The fourth column, headed K^b, gives the errors obtained on the same subject in Brodhun's research.

TABLE V

SUMMARY OF RESULTS FOR KÖNIG AND DIETERICI, BRODHUN AND UTHOFF FOR THE RED END OF SPECTRUM

Wave-length.	K^a Av. E.	D Av. E.	K^b Av. E.	U J. N. D.
650$\mu\mu$				4.70$\mu\mu$
640	1.28$\mu\mu$	1.82$\mu\mu$	2.37$\mu\mu$	2.97
(639.5	1.278	1.803	2.32	2.90)
(633.125	1.12	1.576	1.765	2.08)
630	1.05	1.47	1.35	1.68
(626.75	.92	1.316	1.125	1.58)
(620.375	.686	1.015	.685	1.257)
620	.68	1.00	.67	1.24
(614	.608	.87	.60	1.145)
610	.56	.78	.55	1.08

The general tendency of the curve of sensitivity in these results is identical. The same tendency is shown in my results as the differences in the times of perception have shown. It was pointed out above that the equal objective differences were not correlated with equal differences for consciousness. From the results, however, it is possible to determine what stimuli are necessary to give such equal subjective differences. In the table below is given such a series of increments in stimuli which will produce equal increments in sensation.

TABLE VI

SERIES OF INCREMENTS IN STIMULI CORRELATED WITH EQUAL INCREMENTS IN SENSATION

Wave-length.	Per cent. of red. H.	Incr. in wave-length.	Wave-length.	Per cent. of red. W.	Incr. in wave-length.
614μμ			614μμ		
620.375	25	6.375μμ	620.375	25	6.375μμ
624.99	43.1	4.615	626.045	47.3	5.670
628.84	58.2	3.85	630.218	63.6	4.163

The time of perceiving the difference between red and orange and red and orange-red[25] being given, the difference in these times of perception will represent the difference between orange and orange-red[25] as it appears for consciousness. This was found to be for H 5.72σ and for W 6.44σ. With this as a basis and the curve of increase being known, it is possible to determine what stimulus will give an equivalent increase in time. Thus, that orange-red, corresponding to orange-red[50], which will stand midway between red and orange for consciousness or, in other words, will require for H 11.44σ and for W 12.88σ longer to perceive than the difference between red and orange, is an orange-red which contains for H 43.1 per cent. and for W 47.3 per cent. of red. Similarly, the next equivalent increase, corresponding to the objective orange-red[75], is an orange-red containing for H 58.2 per cent. of red and for W 63.6 per cent. In like manner it is possible to subdivide further these increments and determine approximately any number of equal increments in stimuli and sensations. Thus, the orange-red which stands 3/8 of the way between red and orange would contain approximately 35 per cent. of red for H and for W approximately 36 per cent. This subdivision has not been attempted except in a general way. The corresponding increments in wave-lengths for the three differences used appear in the tables. As will be noticed, the curve of increase, while it takes the general course followed by those of König, Dieterici, Brodhun and Uthoff, does, however, not rise so rapidly. Of the results given in those researches the only ones with which mine are strictly comparable are those of K[a] and D, where the intensity was kept constant except in so far as the results represent an average of series with a higher and a lower intensity. The normal differences in brightness in the spectrum, therefore, influence the average errors and tend to indicate a greater sensitivity to differences in color-tone than is actually the case. Now where colored surfaces are used brightness differences, of course, enter in. The orange is brighter than the red. This difference can, however, not exercise the influence in discrimination that it does exercise where the observer looks through a diaphragm of a dark telescope at small colored fields, light from all other sources being prevented from entering the eye. When colored surfaces, such as have been used here, are observed in diffused daylight, the differences in brightness are not nearly so marked. That they do, however, affect the discrimination of the colors seems unquestionable, but to what extent does not appear. The brightnesses of the colors which I have used have been determined by Rood and Abney. According to the calculations of Rood, the standard red reflects about 25 per cent. of light, white paper being taken as 100 per cent. Abney places the luminosity of an orange disc ($615\mu\mu$) at 64 per cent., an orange-red ($624\mu\mu$) at 52 per cent., another ($633\mu\mu$) at 40 per cent., and a red ($652\mu\mu$) at 21 per cent. It thus seems that the difference in brightness is such that it must appear as a function in discrimination.

Another important reason for the relatively slower rise in the curve of my results is without doubt to be found in the smallness of the area of the colored fields to be compared in the spectrophotometer. It is, of course, well known that area is an important function in the discrimination of colors. Common observation teaches that differences which can scarcely be perceived in small areas become very apparent when the areas are increased. To determine this influence in these experiments several series were taken, using as stimuli red and orange and red and orange-red[75]. The colored surfaces used in all the above-described experiments were 3 cm. square. In these experiments the areas were 1 cm., 5 mm. and 1 mm. square, respectively. I do not give the details of the results inasmuch as only 50 reactions were taken to each pair of stimuli, or 300 reactions in all, by each subject. The results, however, showed not only a very considerable increase in the times of reaction, but also that the increase was disproportionately much greater in reaction to red and orange-red[75] than to red and orange. This was naturally to be expected.

In taking the experiments of the main series it will be remembered that red appeared in all the pairs of stimuli as a constant while the other member in the pair was a variable. It might be expected that differences would appear, according as reaction was made to the constant red or to the variable orange or orange-red. It seemed to the subjects at times that reaction was, if anything, slightly easier to the variable stimulus than to the constant one. To discover whether any difference of this sort was present the times have been calculated separately for each stimulus. The result appears below. V indicates the variable stimulus and R the constant red.

	Subject H.		Subject W.	
	R.	V.	R.	V.
R. & Y.	217.7	215.9	259.8	252.8
R. & O.	245.2	247.1	269.5	265.7
R. & OR^{25}.	252.3	251.5	274.7	273.3
R. & OR^{50}.	261.2	258.3	280.9	281.3
R. & OR^{75}.	269.5	272.3	290.5	292.7

It appears that the difference is very slight. In the case of both subjects, except for H in the orange and for W in orange-red^{75}, the reaction-time is shorter for the variable stimulus until orange-red^{75} is reached, when it becomes longer. The differences are so small that they may well be due to chance.

Likewise, the great number of reactions made would cause any difference in the reactions of the right and left hands to appear. It has been found by Tischer, Merkel, Cattell and others that the simple reaction is the same for the right and left hands. In order to determine this for my experiments, the times were, as above, calculated separately. This is a matter that is only incidental to the experiments themselves. The results are as follows:

	Subject H.		Subject W.	
	R.	L.	R.	L.
R. & Y.	214.1	219.2	257.9	254.9
R. & O.	243.7	248.6	268.5	266.7
R. & OR^{25}.	251.6	251.9	276.0	272.0
R. & OR^{50}.	259.3	260.2	281.1	281.0
R. & OR^{75}.	271.1	270.8	294.0	289.5

From these results the curious fact appears that generally the reaction-time is shorter with the left hand than with the right, though the difference is inconsiderable. Both subjects are right-handed. Subject W thought that his times would be shorter with his right than with his left, while the results show the contrary to be the case.

In addition to the method as above described, the attempt was made to measure the time of perception by the serial method of reaction. For this purpose the colored surfaces 3x3 cm. were mounted on ordinary playing-cards. Twenty-five such cards were prepared for each color and put in the same combinations as above. The time of distribution of the fifty cards, placed in a chance order, was taken accurately with a stop-watch by a recorder. As far as results are concerned, a considerable difference in time was obtained between the red and yellow combination and the remainder of the series. The time for red and orange-red[75] was, likewise, noticeably longer than the other combinations of red and orange-red. After a great number of trials the writer finally gave up the method, and this for several reasons which seem to apply generally to this method when it is used to measure the time of discrimination. In the first place, it seems impossible to distribute a set of cards of this sort so rapidly that it measures the time of perception. It may serve as a measure of a certain complex process, but can hardly be said to measure the time required to discriminate colors. Furthermore, in distributing the reds and yellows the writer, who had attained considerable skill and practise in distribution, found that he could not make the distribution of the cards in less than 17 seconds. This would then make his reaction-time to a single stimulus 340σ while by the more exact method his time is 216σ and gets a minimum below 150σ. Obviously, though the card reaction may measure something it can not measure discrimination-time. The conditions are, of course, not entirely identical with those of the method previously described.

In the next place, the numerous disturbances and irregularities that are found to occur if great rapidity is attempted make measurements unreliable. The throwing of a card into a wrong pile tends to invalidate the whole series, that is, for the measurement of discrimination-time. If such a time is taken as to avoid such difficulties, it certainly does not determine the time of discrimination, whatever else it may measure. The introspective evidence goes to show that only a few out of many distributions do approach to a measurement such as the more exact method gives.

The general conclusion of most importance that can be drawn from the results of this series of experiments is the validity and the applicability of the method in the measurement of differences in sensations. That we do get a measure of the differences in sensations seems unquestionable. In securing it we are freed from the ambiguities of just noticeable or just unnoticeable differences, and the measure of sensitivity or precision in the method of average error. We, likewise, escape the difficulties, which have been noted in the discussion of methods, attendant upon the various methods used to measure over-noticeable differences.

As far as the measurement of color differences is concerned, the results are in accord with those obtained by other methods except as pointed out. The sensitivity to differences in the red-orange series of colors decreases as we approach the red end of the spectrum, but this decrease does not appear to be as rapid as previous results have shown.

I go on now to give results of some minor investigations into the relations of sensations of color.

2. *Differences between black and various colors*

The second series of experiments on color aimed to measure the differences between a series of colors and black. The colors used and their wave-lengths were red (644$\mu\mu$), red (639.5$\mu\mu$), orange (614$\mu\mu$), chrome yellow (585$\mu\mu$), emerald green (521$\mu\mu$), ultramarine blue (452$\mu\mu$) and white. These colors were mounted on cards side by side with black. The black squares were taken from the blackest paper obtainable. Rood calculates that paper of this sort reflects about 5 per cent. of light, and it is to be assumed that this is the luminosity of the paper here used. For white the whitest paper that could be secured was mounted in the same manner. The method was as in the previous experiments. A series taken at one sitting consisted of 140 reactions, 20 reactions to each of the colors. No reaction to black was made. Tables VII and VIII give the results of 1,080 reactions by subject H and 700 by subject S. Diff. W. gives the differences from black and white taken as standard and F. R. gives the false reactions.

In taking these series it had seemed that the method here used might, in addition to determining the differences between black and white and black and the various colors, which is a matter of interest in itself, serve as a measure of their relative brightnesses as they appear to consciousness. The relative brightnesses of these colored surfaces as estimated from the determinations by Abney and Rood[1] are on a basis of white paper as 100 as follows: reds approximately 25 and 30;

TABLE VII

Subject H. 1080 Reactions. XI, 1904.

Series.	W	M. V.	Y	M. V.	G	M. V.	O	M. V.	R^1	M. V.	R^d	M. V.	Bl	M. V.
I.	192.4	*14.3*	209.5	*23.2*	213.6	*17.0*	205.3	*20.0*	210.6	*22.4*	233.5	*15.0*	227.8	*19.5*
II.	198.2	*13.9*	215.7	*18.0*	209.1	*23.0*	215.1	*19.0*	218.7	*20.7*	220.4	*21.8*	227.8	*23.0*
III.	199.0	*19.0*	200.7	*20.3*	199.1	*18.9*	214.1	*18.2*	209.7	*23.3*	209.0	*21.6*	213.5	*17.4*
IV.	195.6	*24.3*	195.0	*24.6*	204.2	*14.8*	197.8	*19.2*	205.4	*26.0*	216.4	*25.0*	226.0	*14.7*
V.	195.7	*19.7*	191.7	*20.5*	189.0	*19.0*	195.2	*22.0*	184.3	*13.8*	197.8	*26.0*	198.2	*20.0*
VI.	182.5	*22.4*	181.4	*20.1*	185.9	*15.0*	189.5	*18.0*	204.8	*20.6*	201.0	*15.0*	204.2	*16.1*
VII.			180.6	*18.7*	196.3	*21.8*	187.5	*25.9*	206.7	*27.0*	206.4	*17.7*	198.5	*23.0*
VIII.			180.7	*24.5*	193.1	*23.2*	193.4	*25.0*	207.4	*20.9*	199.3	*15.6*	204.2	*16.8*
Av.	193.9	*18.9*	194.4	*21.2*	198.7	*19.1*	199.7	*20.9*	205.9	*21.8*	210.6	*19.7*	212.6	*18.8*
P. E.	1.45		1.41		1.28		1.40		1.46		1.32		1.18	
Diff. W.			.5		4.8		5.8		12.0		16.7		18.7	
F. R.			1		2				1				2	

[1] O. N. Rood, *Modern Chromatics*, New York, 1879.

TABLE VIII

Subject S. 700 Reactions. XII, 1904.

Series.	W	M. V.	Y	M. V.	O	M. V.	G	M. V.	R^d	M. V.	R^l	M. V.	Bl	M. V.
I.	209.6	*29.4*	213.6	*29.1*	238.2	*30.6*	240.8	*18.8*	237.1	*15.7*	238.7	*32.6*	231.3	*28.3*
II.	216.2	*23.6*	219.6	*24.0*	227.4	*37.1*	237.3	*28.2*	234.6	*30.4*	231.4	*29.5*	244.8	*24.4*
III.	212.3	*26.7*	234.3	*22.1*	244.9	*18.4*	237.8	*26.7*	246.3	*34.8*	243.3	*19.1*	246.7	*33.6*
IV.	213.3	*18.6*	242.0	*26.3*	235.4	*23.1*	242.0	*33.4*	234.7	*24.1*	241.5	*27.0*	242.2	*17.6*
V.	222.9	*28.1*	232.6	*22.2*	232.8	*20.6*	221.6	*19.8*	236.1	*22.4*	236.4	*20.1*	229.7	*31.0*
Av.	214.9	*25.3*	228.4	*24.7*	235.7	*25.9*	235.9	*25.4*	237.8	*25.5*	238.2	*25.6*	238.9	*27.0*
P. E.	2.14		2.09		2.19		2.14		2.15		2.16		2.28	
Diff. W.			13.5		20.8		21.0		22.9		23.3		24.0	
F. R.	1		1		1						2		2	

orange 64; yellow 80; green 48; blue 10. This is admittedly but an approximation, but the results did not warrant a closer determination of the brightnesses. As appears from the differences, the time of perception does increase regularly, or nearly so, with the decrease in brightness. Whether this is due to the difference in brightness or not does not appear. Aubert[1] long ago called attention to the fact that we have no means of determining how far the different brightnesses of the different regions of the spectrum depend on the intensity and how far they are dependent on a greater or less sensitivity to one color or another.

Moreover, the differences are small and in the case of one subject, S, are obscured by the probable errors which are such as to make any conclusions hazardous. The time of perception for black and white is markedly less than for the colors. Yellow, likewise, is more easily perceived than the remaining colors, where the times are practically the same. Observer H, who has taken a great number of such experiments, reacts more constantly and the differences are more marked. It is to be noticed that the difference in time for black and white and black and yellow is very small. This, in the opinion of the writer, is not so much due to the smallness of the differences between the two stimuli, though it may be that, as to the fact that where the differences are so great as in the case of black and white one is not able to register the rapidity of the perception. The introspective evidence goes to show this. The proper movement can not be executed quickly enough to measure the actual time of perception. The experiments with the remaining colors show small increase with the decrease in brightness. The colors appear to fall into two groups, a

[1] H. Aubert, *Grundzüge der physiologischen Optik*, Leipzig, p. 530, 1876.

lighter, including yellow, green and orange, and a darker, including the reds and blues.

3. *Differences between various pairs of colors*

Experiments of a similar character to those just described were made to measure the differences in various combinations of the four principal colors and again black and white. The combinations used were black and white, red and green, blue and yellow, red and blue and green and yellow. The series were taken as before and on the same subjects H and S, except that to each pair of stimuli 40 reactions were made, 20 to each stimulus. Each figure in the table represents an average of 40 reactions.

TABLE IX

Subject H. 1200 Reactions. XII, 1904.

Series.	B. & W.	M. V.	R. & G.	M. V.	Bl. & Y.	M. V.	R. & Bl.	M. V.	G. & Y.	M. V.
I.	199.4	*17.7*	202.1	*24.9*	195.9	*21.3*	206.3	*18.3*	219.4	*22.9*
II.	198.8	*14.9*	208.4	*20.9*	210.2	*25.4*	214.9	*20.9*	206.3	*25.3*
III.	205.2	*20.9*	208.0	*19.4*	200.5	*20.9*	217.5	*21.1*	216.2	*23.1*
IV.	193.5	*26.1*	201.7	*25.8*	196.9	*24.9*	216.8	*25.0*	206.7	*22.1*
V.	195.4	*19.0*	199.8	*27.8*	198.6	*26.4*	206.5	*25.3*	203.1	*20.9*
VI.	187.1	*22.2*	199.4	*24.6*	196.5	*25.8*	214.0	*22.0*	218.6	*24.4*
Av.	**196.6**	*20.8*	**203.2**	*22.2*	**199.8**	*24.1*	**212.1**	*22.1*	**211.7**	*23.1*
P. E.	1.09		1.21		1.31		1.20		1.26	
False	2		1		3		2		3	

TABLE X

Subject S. 600 Reactions. XII, 1904.

Series.	B. & W.	M. V.	R. & G.	M. V.	Bl. & Y.	M. V.	R. & Bl.	M. V.	G. & Y.	M. V.
I.	228.5	*23.5*	240.0	*26.6*	244.8	*26.4*	251.7	*24.1*	251.5	*33.5*
II.	238.6	*21.0*	238.4	*26.8*	247.1	*22.2*	244.6	*26.1*	250.4	*26.8*
III.	234.6	*27.1*	241.5	*29.8*	240.9	*26.3*	256.0	*25.1*	256.7	*30.9*
Av.	**233.9**	*23.8*	**239.9**	*27.7*	**244.2**	*24.9*	**250.7**	*25.1*	**252.9**	*30.4*
P. E.	1.85		2.14		1.93		1.94		2.26	
False	7		4		3		4		7	

The results of experiments on other subjects, which have been referred to, led the writer to believe that in reactions of this sort individual differences would be likely to appear. We are prone to assume that the differences between any two colors, for instance, are the same for all individuals unless there is color deficiency; that for the normal person the scale of colors and the distances between them are more or less uniform. It is probable, however, that this is not the case, but that for consciousness the color differences produce noticeably different effects on different individuals. Such differences appear in these results as in the previous ones, but are not particularly marked. The differences one would naturally not expect to be great.

We are further interested in knowing what the differences of the various colors are for consciousness in addition to discovering individual differences. We have no means except by this method of comparing the difference between, for instance, red and green and red and blue. Any comparison we might make by observation is inexact and vague, and it is furthermore doubtful whether we can ever make such a comparison at all. The writer attempted to make such comparisons with the combinations of colors before the reactions were taken, in order to compare such estimates with the reaction-times. He found it well-nigh impossible to do this. The introspective evidence did not warrant the belief that the difference between red and green and red and blue would be as marked as it is.

To return to a consideration of results. The time of reaction to blue and yellow is shorter for H than to red and green. With subject S, the differences are about the same, but in his case the difference between red and green appears to be most readily perceived. The differences are small and may be due to chance. In general, at least as far as these subjects are concerned, the time of discrimination is about the same for red and green as for blue and yellow. The colors, it may be stated, are not complementary, though it would have been desirable to have them so. It was not thought necessary to cope with the difficulty of getting pigments or mixtures where this would have been the case for experiments such as these.

The differences between these two combinations and red and blue and green and yellow are marked in both subjects. It should be expected from the position of red and blue in the color scale that the differences here would have been quite as great as in the case of red or green, or greater. Why this is not so it is difficult to say. It is hardly necessary to state that the colors were in each case rich, saturated colors. There is, however, the normal difference in brightness, and as in the previous experiments it is the opinion of the writer that the differences in the time of perception are due in some degree to the differences in brightness in the case of red and blue. Of the two variables present in the stimuli the color-tone should have given, if anything, a shorter time, and therefore the reasonable assumption is that the brightness causes the difference.

In the green and yellow the times are again longer, in the case of S longer than for red and blue, and for H slightly shorter. The proximity of the two colors in the scale makes it appear that the color-tone causes the difference here. The differences in brightness are greater than in any of the other combinations except blue and yellow.

In order to analyze out the difference between the various colors it would be desirable in further experiments to eliminate the influence of brightness, but in that case we would be dealing with colors not as they are usually experienced but under artificial conditions. We are interested in discovering the differences as they appear under normal conditions where brightness is naturally a function in color perception.

For the discovery of individual differences a greater number of subjects would be necessary than I have used. However, the possibilities of the method, it is believed, have been brought out.

4. *A method of testing for color-blindness*

The method here used furnishes a valuable means of measuring differences in sensibility in individuals and in the various sense departments, and of determining sense deficiencies such as ability to discriminate tones or colors.[1] In the case of color vision it is of importance on account of its practical bearings in the railway and marine service to determine defects. More than forty methods have been devised either for scientific or for practical purposes to determine defective color vision most expeditiously and with the greatest certainty. All of the methods with which I am acquainted are based on the naming or matching of colors. The principle of confusion is in reality the basis of all of them.

Now it may well happen that a person deficient in color sense can distinguish the various colors and pass successfully such a test as that of Holmgren, which in some form or other is the one usually employed for practical purposes, without confusion of colors and also without showing such hesitancy that it would be safe to conclude from it that there is defective color sense without more exact determination than the observations of the one making the test. Mauthner reports just such a case in advocating his test with pigments. If the one tested is blue-yellow blind, however, it will take him a longer time to distinguish these colors than the reds and greens, and, on the other hand, if he is red-green blind the reverse will be the case. The person of normal color vision, as the experiments of the next preceding section have shown, takes about as long to distinguish one pair of colors as the other. If then we have a measure of the time of perceiving differences between each of these pairs of colors, we have an indication of the nature of the color sense. If the time is markedly longer for one pair of colors than for the other, it is almost certain that there is defective color vision or color-blindness. Furthermore, in the railway service not only the fact of discrimination but also the time of discrimination

[1] J. McKeen Cattell, *Op. cit.*, p. 65.

and reaction is important. When it is recalled that the engineer is required to see the signals at a limited distance and that it requires a considerable time to stop a rapidly moving train, the time element in color perception may be of practical significance. In fact, a test based on the time of perception would seem to approach more nearly to the actual conditions to be met, where it is necessary to determine defects, than methods based merely on the inability to distinguish colors.

In devising a test to determine the time of perception, it is possible to use the apparatus and methods employed in the experiments described in the preceding sections or again by the use of colored lights or the lanterns actually found in the railway service to simulate these conditions exactly. The time required in both of these methods makes the application cumbersome. It is necessary to use a variety of reds and greens, for as is well known the reds and greens that are confused or are difficult to distinguish vary widely in color-blind persons. The following method used in this study seemed to be entirely satisfactory. One hundred and thirty-two cards were prepared, on sixty-six of which were mounted three blues and three yellows, on the remainder three reds and three greens. The colored surfaces were, as in all the previous experiments, 3 cm. square. The Milton-Bradley papers were used, it being the aim to make this test a practical one and these papers are readily obtained. With pigments it would be possible to get greater purity in the colors and one could define their relations more exactly. Such surfaces, however, can not be handled, as some of the pigment quickly rubs off, for instance, emerald green. The other papers are quite as efficient. In each color I used the Milton-Bradley standard together with one tint and one shade, selecting them so that the normal person could distinguish them as rapidly as he could distribute the cards. More specifically, as denominated in the Milton-Bradley color book, the colors were as follows: Red, red tint No. 1, red shade No. 1; green, green tint No. 1, green shade No. 2; blue, blue tint No. 1, blue shade No. 2; yellow, yellow tint No. 2, yellow shade No. 1. With these differences the person of normal color vision should have no difficulty in discrimination. With smaller differences he may unless he has had considerable experience with colors. The differences are by no means equal, of course. Thus, the yellow shade No. 1 is apparently further removed from yellow than green shade No. 2, and so on with the other shades and tints. The selection made answers all purposes, however. A second set of reds was also prepared made up of the Milton-Bradley orange-red series. These colors are orange-red, orange-red tint No. 1 and orange-red shade No. 1. This series is in some respects better than the standard red series. The remainder of the colors were the same as in the other test. The method of giving the test is to place before the subject seated at a table one each of the blues and yellows or of the reds and greens. In arranging these it was found to be desirable to keep to a certain order to avoid interference in associations. I have used the following order, beginning at the left of the subject: light red, red, dark red, light green, green, dark green, and, for the blues and yellows, the yellows on the left and the blues on the right. This was found to be better than an alternate or a chance arrangement, since either of these methods would have lengthened the time somewhat and would have given no new information as to color vision. The time required to distribute the sixty cards, first the blues and yellows and then the reds and greens, and so on alternately, was taken with a stop-watch. The alternate order provides for the equalizing of the effects of practise in distribution.

The results of experiments on five color-blind persons, ranging in degree of deficiency from what would be termed a reduced color sense to extreme confusion of reds and greens, appear in Table XI. Ten series were taken on each subject with the standard reds and greens and blues and yellows, and five with the orange-reds substituted for the standard reds except in the case of subject T_2. There are thus 1,800 reactions to color by each of four subjects and 1,200 by one. The tables of the standard series appear first. The first column gives the number of the series, the second, third, fourth and fifth the times of distribution in seconds and the mean variations. The sixth column gives the gross differences between the time of distribution of the blues and yellows and the reds and greens, and the last column gives this difference in terms of percentage of increase in time for the reds and greens which makes possible a comparison of the results of the various subjects. The practise effect in distribution made it advisable to take the average of the first five series and the last five. The mean variations would otherwise be larger and indicate greater variability and, hence, less reliability than the measurements justify. A plotting of the practise curve for each subject and the determinations from it would in reality be a better indication of the reliability of the results. The orange-red series for four subjects follows the first series. Following this is given the median percentage of difference in time of distribution for the different subjects and for the group, together with the mean variation and probable error for the median of the group. The median seems a better measure than the mean in a case of this kind because it is not influenced by extreme values such as do appear for reasons which will be pointed out later. The median for the group is based on the supposition that the percentages are comparable, which, though not strictly true, is warranted because the differences in absolute time of distribution are not great. A summary of the gross differences in time is also given.

TABLE XI

STANDARD RED SERIES.

	Subject K.						Subject L.					
Series.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.
I.	50.0	5.6	60.0	1.1	10.0	20	54.0	10.2	60.4	3.5	6.4	12
II.	45.0	.6	70.2	9.1	25.2	56	47.0	3.2	64.0	7.1	17.0	36
III.	45.8	1.9	62.5	1.4	16.7	36	42.0	1.8	57.2	.3	15.2	36
IV.	42.2	2.2	60.0	1.1	17.8	42	35.0	8.8	48.0	8.9	13.0	37
V.	38.8	5.6	53.0	8.1	14.2	37	41.0	2.8	54.8	2.1	13.8	34
Av.	44.4	3.1	61.1	4.2			43.8	3.6	56.9	4.4		
VI.	37.0	.2	47.6	3.2	10.6	25	39.4	.4	59.0	4.9	19.6	50
VII.	36.2	.6	43.0	.6	6.8	19	40.0	.2	52.9	1.2	12.9	32
VIII.	35.8	1.0	41.8	2.6	6.0	17	37.2	2.6	60.8	6.7	23.6	63
IX.	35.0	1.8	43.8	.6	8.8	25	44.0	4.2	50.4	3.7	6.4	15
X.	40.0	3.2	46.0	1.6	6.0	15	38.2	1.6	47.4	6.7	9.2	24
Av.	36.8	1.4	44.4	1.7	12.2	29.2	39.8	1.8	54.1	4.6	13.7	33.9

	Subject T_1.						Subject B.					
Series.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.
I.	46.8	7.1	75.4	17.4	28.6	61	66.2	1.5	77.6	3.8	11.4	17
II.	41.0	1.3	50.8	7.2	9.8	24	62.8	3.1	78.8	5.0	16.0	24
III.	40.6	.9	54.6	3.4	14.0	34	57.6	2.1	68.8	5.0	11.2	19
IV.	35.8	3.9	54.6	3.4	18.8	53	59.0	.7	76.0	2.2	17.0	39
V.	34.2	5.5	54.8	3.2	20.6	60	53.0	6.7	68.0	5.8	15.0	28
Av.	39.7	3.7	58.0	6.9			59.7	3.8	73.8	4.4		
VI.	34.2	1.5	49.0	3.9	14.8	43	53.0	2.5	65.4	.1	12.4	23
VII.	34.0	1.3	53.0	7.9	19.0	56	53.6	1.9	65.0	.5	11.4	21
VIII.	33.2	.5	42.0	3.1	8.8	27	60.2	4.7	66.8	2.3	6.6	11
IX.	32.2	.5	41.8	3.3	9.6	30	51.0	4.5	62.4	2.1	11.4	22
X.	29.8	2.9	39.8	4.3	10.0	34	59.6	4.1	63.0	1.5	3.4	6
Av.	32.7	1.3	45.1	4.5	15.4	40.2	55.5	3.5	64.5	1.3	11.6	21.0

Subject T_2.

Series.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.
I.	55.0	*1.6*	69.2	*4.8*	14.2	26
II.	60.6	*4.0*	58.8	*5.6*	—1.8	—3
III.	62.0	*5.4*	65.8	*1.4*	3.8	5
IV.	52.4	*4.2*	63.4	*1.0*	11.0	21
V.	53.0	*3.6*	65.0	*.6*	12.0	23
Av.	56.6	*3.8*	64.4	*2.7*		
VI.	50.2	*3.6*	55.8	*4.9*	5.6	11
VII.	49.0	*2.4*	52.0	*1.1*	3.0	6
VIII.	44.2	*2.4*	49.2	*1.7*	5.0	10
IX.	43.8	*2.8*	51.6	*.7*	7.8	18
X.	46.0	*.6*	46.0	*4.9*	.0	0
Av.	46.6	*2.4*	50.9	*2.7*	6.1	12.7

ORANGE-RED SERIES.

	Subject K.						Subject L.					
Series.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.	B. & Y.	M. V.	R. & G.	M. V.	D.	Per Cent. of D.
I.	33.8	*.8*	50.8	*6.1*	17.0	50	39.8	*4.7*	53.4	*5.3*	13.6	34
II	31.2	*1.8*	46.2	*1.5*	15.0	48	33.2	*1.9*	47.0	*1.1*	13.8	42
III.	32.0	*1.0*	42.6	*2.1*	10.6	33	32.8	*2.3*	47.2	*.9*	14.4	16
IV.	34.0	*1.0*	44.0	*.7*	10.0	29	38.2	*3.1*	44.4	*3.7*	6.2	16
V.	34.0	*1.0*	40.0	*4.7*	6.0	18	31.6	*3.5*	48.4	*.3*	16.8	53
Av.	30.0	*1.1*	44.7	*3.0*	11.7	35.6	35.1	*3.1*	48.1	*2.7*	13.0	37.4

	Subject T_1.						Subject B.					
I.	30.4	*.2*	41.0	*3.4*	10.6	35	50.6	*6.9*	66.0	*3.6*	15.4	31
II.	29.8	*.4*	37.0	*.6*	7.2	24	41.4	*2.3*	59.8	*2.6*	18.4	44
III.	30.4	*.2*	40.0	*2.4*	9.6	32	42.6	*1.1*	61.4	*1.0*	18.8	44
IV.	29.8	*.4*	36.0	*1.6*	6.2	21	42.0	*1.7*	64.0	*1.6*	22.0	52
V.	30.8	*.6*	34.2	*3.4*	3.4	11	41.2	*2.5*	61.0	*1.4*	19.8	48
Av.	30.2	*.4*	37.6	*2.7*	7.4	24.6	43.6	*2.9*	62.4	*2.0*	8.8	43.8

SUMMARY.

	K.	L.	T_2.	B.	T_1.	Group	M. V.	P. E.
Med. Per Cent. Diff.	25.0	35.0	43.5	21.5	10.5	24.5	*11.5*	1.31
Diff. O.—R. Series	33.0	42.0	24.0	44.0		33.5	*10.6*	2.00
Gross Diff.	12.2	13.7	15.4	11.6	6.1	12.0		
Gross Diff. O.—R. Series	11.7	13.0	7.4	18.8		12.7		

In order to show that the times of distribution for persons of normal color vision do not differ for the blues and yellows and reds and greens, a table of results from three subjects is given. Tests have been made on various other persons as well, but it seemed inadvisable to multiply results in order to show this point, which seems well established from the table given. The averages and mean variations only are given as the differences were insignificant.

TABLE XII

STANDARD RED SERIES.

	Subject H.				Subject F_1.				Subject F_2.			
Series.	B. & Y.	M.V.	R. & G.	M. V.	B. & Y.	M. V.	R. & G.	M. V.	B. & Y.	M. V.	R. & G.	M. V.
I.	36.8	1.4	36.6	1.1	55.8	6.2	66.0	14.7	48.6	6.6	51.4	6.9
II.	36.4	1.0	35.0	.5	53.4	3.8	55.0	3.7	45.4	3.4	48.8	4.3
III.	35.4	.0	37.0	1.5	46.8	2.8	51.4	.1	43.2	1.2	46.6	2.1
IV.	34.8	.6	37.2	1.7	51.4	1.8	51.2	.1	42.2	.2	47.6	3.1
V.	34.0	1.4	33.4	2.1	48.2	1.4	45.0	6.3	41.2	.8	46.6	2.1
VI.	35.8	.4	35.0	.5	50.0	.4	53.2	1.9	39.0	3.0	39.3	5.2
VII.	33.4	2.0	34.8	.7	44.0	5.6	46.4	4.9	40.4	1.6	43.4	1.1
VIII.	35.5	.1	33.8	1.7	50.0	.4	50.4	.9	41.4	.6	40.2	4.3
IX.	34.9	.5	36.0	.5	46.8	2.8	49.6	1.7	40.2	1.8	41.6	2.9
X.	37.2	1.8	36.4	.9	49.4	.2	44.8	6.5	38.2	3.8	39.2	5.3
Av.	35.4	.9	35.5	1.1	49.6	2.5	51.3	4.1	42.0	2.3	44.5	3.7

ORANGE-RED SERIES.

I.	30.2	1.0	32.8	.4	41.6	.8	42.6	.0	38.9	3.2	44.8	2.0
II.	33.0	1.8	32.6	.2	42.8	.4	47.0	4.4	44.8	2.7	44.9	2.1
III.	32.0	.8	31.0	.6	44.8	2.4	42.4	.2	41.2	.9	40.6	2.2
IV.	31.0	.2	33.2	.8	40.8	1.6	40.2	2.4	38.6	3.5	41.2	1.6
V.	29.6	1.6	32.4	.0	42.2	.2	40.8	1.8	47.2	5.1	42.4	.4
Av.	31.2	1.1	32.4	.4	42.4	1.1	42.6	1.8	42.1	3.1	42.8	1.7

The results of the experiments go clearly to show the validity of the test. The person of normal color vision takes no longer to distinguish the reds and greens than the blues and yellows. The color-blind person takes much longer, approximately 12 seconds when the time for the blues and yellows is approximately 40 seconds. On the basis of the results it can be safely said that if a person takes markedly longer to distribute the reds and greens there is evidence of a reduction in color sense or of color-blindness. With a large number of cases on individuals of normal color vision and on color-blind persons the limits can be definitely fixed, and this will then make possible a convenient method of measuring the color sense. This is one of the valuable features of the method that it furnishes a means of measuring rather exactly individual differences in color perception, and not only the fact of color deficiency, but the degree of the deficiency as well. Tests, which do not appear in the tables, on persons of supposedly normal color vision show this and give ground for the belief that a reduction in the color sense is more common than is usually supposed, and that there are all degrees of ability to discriminate colors, ranging from the highest discriminativeness down through the reduction in color sense to the extreme forms of color-blindness where the confusions are numerous and marked. It is, I think, mainly held that the color-blind form a separate species marked off from those of normal color vision. It seems to me that this is not the case, but that the color-blind form the lower end of a normal distribution curve.

This method combines the methods based on the confusion of colors with a measure of the time of perception. This becomes apparent when one correlates with the results in Table XI the errors made. Wherever in the table the differences are small between the times for the blues and yellows and the reds and greens this is due in a large measure to the errors and confusions in the reds and greens, which would disclose a defect. Thus, in Series I on subject K, who is red-green blind, there were many errors both in the discrimination of the reds or greens and in the confusion of reds and greens, especially the light red, which is a rose red approximating Holmgren's standard, and the dark green. In Series III, however, on the same subject all the reds and greens were correctly distributed, but the time is much longer. In all cases with this subject the differences could be noticed with careful inspection. Of the fifteen series taken, in eight no errors were made and in all but three the errors seemed to be of a chance character. On running over the results of a distribution he could readily detect an error when the colors were side by side.

Subject L is distinctly red-blind. There is no confusion of the reds and greens, but a difficulty in discriminating reds, which are apparently not distinguished from other colors of equal brightness. Thus lavender, purple and pink seem to him to be shades of blue. A light green-blue, Milton-Bradley's tint No. 2, is not distinguished from pink.

In the case of subject T_1, the difference in time is greater than in the two subjects K and L, which would appear to indicate a greater degree of deficiency. This is due to the fact that no errors were made after the third series. In the first three there was some difficulty in distinguishing shades of red. The fact of deficiency is, however, shown in the time differences.

Subject B is red-green blind with numerous confusions. Thus pink and light green, blue and purple, are frequently confused. In all series some errors occurred.

In subject T_2 the smallness of the difference in time is due largely, according to his introspective testimony as well as to the results themselves, to the difficulty in discriminating blue from dark blue. There was also some difficulty with the red and dark red.

These explanations are given to account for any seeming inconsistencies in the results. In the orange-red series no errors except an occasional chance one appeared, while the differences in time were quite as marked as in the other series. It is believed that these results show the value of the test. That is all that is attempted in this discussion, which aims only to establish the method and indicate its possibilities. The test is of such a character that it can be given by any one, and it requires but a short time. If in the first series the time differences do not appear, the test need not be continued. If they do appear, five or more series should give a sufficiently accurate measure for all practical purposes. Improvements in the test can be made by defining the colors more exactly with reference to each other, as has been suggested above. In the present state of color-printing this is difficult to do. In order that equal differences be obtained throughout, all that it would be necessary to do would be to apply the methods used in the previous sections. The time of discriminating the standard blue and the standard yellow and the corresponding standard red and standard green is about equal. We can determine a difference similarly between blue and a blue tint which will be equal to the difference between red and a red tint for consciousness. To take equal objective differences or equal percentages of white light would, of course, not give us such differences for consciousness. The preparation of such a series of colors would give us the conditions for the very best application of this method.

IV. EXPERIMENTS ON LINEAR MAGNITUDES

1. *Equal absolute differences in lengths of horizontal lines*

The experiments to be reported in this section aimed to measure differences in various lengths of lines or the extensive sensations of sight. It is a matter of special interest to apply this method to the measurement of these sensations because of its relation to the psychophysical law. Since the first experiments of Weber with such stimuli, numerous investigations have gone to show that Weber's law holds perhaps more closely here than in any other sense department. This will be pointed out in the brief historical summary which will follow a presentation of the experiments made and the results obtained.

Two sets of experiments were made on horizontal lines, in one of which equal absolute differences in stimuli were used, in the other equal relative differences in accordance with Weber's law. The order in which these experiments were made will be followed in the discussion of the methods and results, hence the experiments with equal absolute differences will be taken up first.

The apparatus was that used in the experiments on color and the lengths of the lines were adjusted accordingly. This is, however, no limitation, as the lengths of the lines taken for the experiments would have been as they were in any case, for various reasons. The standard line was 10 mm. in length and the variable lines 10½, 11, 11½, 12, 12½ and 13 mm. With these lengths the effect of eye movements was largely if not entirely eliminated. They are, furthermore, easily perceived at the distance of 50 cm., which was the distance from the subject at which the apparatus was situated. The lines were drawn on white cardboard with India ink, due care being exercised to have a uniform width of 2 mm. in all the lines. In order to secure the greatest accuracy I caused the lines to be prepared for me by an expert draughtsman. The distance between the lines was uniformly 1 cm. The cards, of which there were six in all, 10–10½, 10-11, 10–11½........10–13, were used just as in the experiments previously described, and the experiments were, in general, made according to the same methods.

A series consisted of 240 reactions, 40 to each pair of stimuli. It might appear that a series of this length would introduce an element of fatigue, but such did not prove the case. The differences between the stimuli could be readily perceived, unless one should except the 10–10½ pair, and were not found to be trying to the eye, hence I felt warranted in taking series of this length. The time required for each series was approximately fifty-five minutes, varying from fifty to sixty minutes. The same methods were pursued as in the experiments on color as to order of presentation of pairs of stimuli, which alternated, and likewise in the order of reaction to each one of a pair of stimuli, which also alternated from day to day. The cards were so adjusted

TABLE XIII

Subject H. 2400 Reactions. X, 1904.

Series.	10-13.	M. V.	10-12½.	M. V.	10-12.	M. V.	10-11½.	M. V.	10-11.	M. V.	10-10½.	M. V.	F. R.
I.	292.1	*23.6*	296.5	*17.6*	301.7	*23.2*	308.7	*23.0*	323.3	*25.3*	345.1	*24.8*	7
II.	293.2	*17.1*	296.8	*16.4*	303.2	*25.4*	312.3	*25.5*	327.4	*24.2*	348.2	*25.2*	10
III.	295.3	*21.8*	297.6	*17.8*	306.4	*16.6*	313.5	*17.4*	324.1	*25.1*	335.3	*20.0*	7
IV.	303.9	*19.6*	302.9	*21.6*	313.3	*19.0*	317.8	*23.0*	330.3	*24.0*	347.4	*27.1*	12
V.	291.9	*24.1*	292.9	*18.2*	294.3	*17.6*	301.7	*19.1*	314.4	*23.5*	337.4	*23.5*	12
VI.	304.0	*20.5*	307.2	*16.9*	312.6	*19.8*	315.5	*26.8*	320.7	*24.9*	335.7	*26.8*	3
VII.	290.9	*25.4*	292.5	*17.5*	298.1	*20.2*	310.7	*21.4*	322.2	*20.6*	350.1	*23.2*	3
VIII.	289.5	*16.6*	292.7	*19.8*	295.5	*18.2*	303.9	*23.5*	321.3	*19.8*	346.6	*19.8*	4
IX.	299.5	*19.3*	302.9	*17.5*	310.7	*21.7*	318.6	*20.0*	327.2	*21.9*	350.7	*22.9*	5
X.	298.6	*17.6*	302.1	*23.2*	311.2	*18.6*	322.8	*21.4*	328.5	*19.2*	350.6	*19.9*	3
Av.	**295.89**	*20.6*	**298.41**	*19.1*	**304.70**	*20.0*	**312.55**	*22.1*	**323.96**	*22.9*	**344.71**	*23.3*	
P. E. of Av.	.87		.81		.84		.93		.97		.98		
Diff.			**2.52**		**6.29**		**7.85**		**11.41**		**20.75**		
Diff. from 10-13.			**2.52**		**8.81**		**16.66**		**28.07**		**48.82**		
P. E. of Diff.			1.19		1.17		1.25		1.34		1.38		
P. E. of Diff. from 10-13.			1.19		1.21		1.27		1.30		1.31		
False Reactions.	5		7		12		14		15		13		66

in the drop-screen that on exposure a shifting of fixation was unnecessary, the midpoint between the lines falling always exactly at the center or so nearly so that a deviation was undetected.

Tables XIII and XIV give the results of 4,800 reactions by two

TABLE XIV

Subject S. 2400 Reactions. XII, 1904.

Series.	10–13.	M. V.	10–12½.	M. V.	10–12.	M. V.	10–11½.	M. V.	10–11.	M. V.	10–10½.	M. V.	F. R.
I.	339.4	*35.7*	337.2	*33.2*	331.7	*43.7*	341.9	*31.4*	345.3	*36.3*	353.7	*36.5*	18
II.	321.2	*30.2*	323.3	*35.1*	338.9	*34.1*	342.5	*39.1*	350.8	*43.6*	368.2	*30.9*	17
III.	333.0	*31.6*	328.8	*37.4*	332.5	*33.6*	342.2	*33.8*	351.3	*29.0*	360.2	*34.4*	16
IV.	332.9	*27.6*	323.8	*34.3*	328.8	*27.9*	340.1	*29.6*	342.0	*31.7*	356.4	*35.9*	18
V.	347.3	*26.4*	342.2	*31.9*	338.2	*27.5*	344.8	*39.7*	354.8	*26.3*	364.7	*30.2*	12
VI.	323.5	*26.1*	335.6	*28.0*	334.4	*23.0*	338.0	*34.7*	347.8	*26.8*	359.6	*34.5*	13
VII.	320.2	*31.2*	318.8	*26.3*	330.0	*28.6*	325.0	*28.9*	350.6	*28.5*	358.9	*33.7*	15
VIII.	316.0	*29.0*	318.8	*31.7*	325.3	*25.5*	335.8	*27.7*	341.0	*32.5*	352.8	*35.8*	16
IX.	330.8	*22.1*	326.5	*27.0*	331.5	*31.7*	341.7	*27.2*	341.2	*35.9*	353.3	*40.4*	8
X.	322.5	*32.4*	330.7	*33.0*	328.5	*33.9*	337.7	*29.3*	339.0	*34.1*	355.0	*38.8*	15
Av.	**328.68**	*29.2*	**328.57**	*31.8*	**331.98**	*30.9*	**338.97**	*32.1*	**346.38**	*32.4*	**358.28**	*35.1*	
P. E. of Av.	1.23		1.34		1.30		1.35		1.37		1.48		
Diff.			**—.11**		**3.41**		**6.99**		**7.41**		**11.90**		
Diff. from 10-13.			**—.11**		**3.30**		**10.29**		**17.70**		**29.60**		
P. E. of Diff.			1.81		1.87		1.87		1.92		2.01		
P. E. of Diff. from 10-13.			1.81		1.79		1.83		1.85		1.92		
False Reactions.	14		22		14		19		42		37		148

subjects H and S, 2,400 reactions by each subject, or 400 to each pair of stimuli. The lengths of lines in millimeters in each pair appears at the head of the column giving the reaction-times. The averages, mean variations and probable errors are given as in the preceding tables.

Before entering into a discussion of the results I give the tables of differences in time of perception for the individual series. This will serve to indicate the reliability of the differences in the final averages. For each subject the tables give the differences between each pair of stimuli, which, to repeat what has already been stated, represent equal objective differences from the 10–13 mm. pair of stimuli. The tables give the final average differences, which appear in Tables XIII and XIV and their mean variations and probable errors.

Table XV

Differences. Subject H. Differences from 10–13.

Series.	10-13 / 10-12½	10-12½ / 10-12	10-12 / 10-11½	10-11½ / 10-11	10-11 / 10-10½	10-13 / 10-12½	10-13 / 10-12	10-13 / 10-11½	10-13 / 10-11	10-13 / 10-10½
I.	4.4	5.2	7.0	14.6	21.8	4.4	9.6	16.6	31.2	53.0
II.	3.6	6.4	9.1	15.1	20.8	3.6	10.0	19.1	34.2	55.0
III.	2.3	8.8	7.1	10.6	11.2	2.3	11.1	18.2	28.8	40.0
IV.	—1.0	10.4	4.5	12.5	17.1	—1.0	9.4	13.9	26.4	43.5
V.	1.0	1.4	7.4	12.7	23.0	1.0	2.4	9.8	22.5	45.5
VI.	3.2	5.4	2.9	5.4	14.8	3.2	8.6	11.5	16.9	31.7
VII.	1.6	5.6	12.6	11.5	27.9	1.6	7.2	19.8	31.3	59.2
VIII.	3.2	2.8	8.4	17.4	25.3	3.2	6.0	14.4	31.8	57.1
IX.	3.4	7.8	7.9	8.6	23.5	3.4	11.2	19.1	27.7	51.2
X.	3.5	9.1	11.6	5.7	22.1	3.5	12.6	24.2	29.9	52.0
Av. Diff.	**2.52**	**6.29**	**7.85**	**11.41**	**20.75**	**2.52**	**8.81**	**16.66**	**28.07**	**48.82**
M. V.	*1.24*	*2.21*	*2.07*	*3.07*	*3.83*	*1.24*	*2.21*	*3.41*	*3.75*	*6.91*
P. E.	.33	.58	.55	.82	1.02	.33	.59	.91	1.00	1.84

The results show that as the differences in the lengths of the lines decrease the time of perception steadily increases, and that this increase is much more rapid than the corresponding decrease in the differences in the lengths of the lines. The question that we are interested in is to discover the nature and law of this increase. If a law of direct proportionality obtained, the differences should remain constant. Thus, for H, the differences in times of perception should be constant, or increase by a constant amount, since the objective differences are equal, instead of being 2.52σ, 6.29σ, 7.85σ, etc., or 2.52σ, 8.81σ, 16.66σ, etc.

TABLE XVI

Differences. Subject S. Differences from 10-13.

Series.	10-13 / 10-12½	10-12½ / 10-12	10-12 / 10-11½	10-11½ / 10-11	10-11 / 10-10½	10-13 / 10-12½	10-13 / 10-12	10-13 / 10-11½	10-13 / 10-11	10-13 / 10-10½
I.	—2.2	—5.5	10.2	3.4	8.4	—2.2	—7.7	2.5	5.9	14.3
II.	2.1	15.6	3.6	8.3	17.4	2.1	17.7	21.3	29.6	47.0
III.	—4.2	3.7	9.7	9.1	8.9	—4.2	—.5	9.2	18.3	27.2
IV.	—9.1	5.0	11.3	1.9	14.4	—9.1	—4.1	7.2	9.1	23.5
V.	—5.1	—4.0	6.6	10.0	9.9	—5.1	—9.1	—2.5	7.5	17.4
VI.	12.1	—1.2	3.6	9.8	11.8	12.1	10.9	14.5	24.3	36.1
VII.	—1.4	11.2	—5.0	25.6	8.3	—1.4	9.8	4.8	30.4	38.7
VIII.	2.8	6.5	10.5	5.2	11.8	2.8	9.3	19.8	25.0	36.8
IX.	—4.3	5.0	10.2	—.5	12.1	—4.3	.7	10.9	10.4	22.5
X.	8.2	—2.2	9.2	1.3	16.0	8.2	6.0	15.2	16.5	32.5
Av. Diff.	—.11	3.41	6.99	7.41	11.90	—.11	3.30	10.29	17.70	29.60
M. V.	*5.13*	*5.31*	*3.83*	*5.15*	*2.46*	*5.13*	*7.44*	*5.95*	*7.82*	*8.62*
P. E.	1.37	1.42	1.02	1.38	.66	1.37	1.98	1.59	2.09	2.30

Taking the reaction-time to the pair of stimuli most easily discriminated, 10–13 mm., as a standard for subject H, it is to be noted that when the difference between the stimuli is decreased by ½ mm. to 10–12½ mm. the time of reaction increases by 2.52σ. When the difference is decreased by 1 mm., or to 10-12 mm., the time increases by 8.81σ; when decreased by 1½, 2 and 2½ mm. the times increase, respectively, by 16.66σ, 28.07σ and 48.82σ. For S, the increase in time of perception with the corresponding decrease in differences between the stimuli is represented by —.11σ, 3.30σ, 10.29σ, 17.70σ and 29.60σ. In the case of both subjects the approximation is, it seems, sufficiently close to warrant the general statement that as the differences between the pairs of stimuli decrease in an arithmetical progression the differences in the times of perception increase in a geometrical progression. The only marked deviation from this general tendency is for H in the smallest difference in time, 2.52σ. This difference is manifestly too small, as the relations of the remaining differences would indicate. When the differences in the stimuli are large the differences in the times of perception are, of course, small, and it is then difficult to register the actual differences for consciousness, one or two marked deviations tending to affect the average greatly. In the present case for H the table of differences shows that the median difference is 3σ and should possibly be higher, or the difference 8.81σ be somewhat lower. A deviation at the lower end when the differences in stimuli are great may simply indicate that the reaction can not be made rapidly enough to register the real difference for consciousness from the pair of stimuli next above it. This was suggested in the discussion of the times of perceiving the differences between various colors and black. The times, it will be noted, are considerably longer than in the case of the colors, which is due to the longer time the line requires to act upon the retina and not necessarily to a greater difficulty in discrimination. It is possible that when the differences are large the factor called by Müller the *absolute Eindruck* enters in. It may be that when the difference between stimuli attains a certain magnitude a real act of discrimination in the sense in which it takes place with the other pairs of stimuli is not present.

A consideration of the results obtained from subject S brings out more clearly this point. The larger difference in stimuli in his case gives a minus difference in time, indicating that the real difference has not been registered. The reactions on subject S, it should be stated, were taken before those on color reported in the preceding section. The results which appear in the table were preceded by practise series amounting in all to 720 reactions. The reaction-process had not become as automatic as in subject H, and the mean variations are therefore larger and the differences much less constant. The only difference that is maintained with some constancy is the difference in time of perceiving the pair of stimuli most difficult to discriminate and the next below it, or 11.90σ.

The general conclusion, then, which seems warranted for the perception of differences in linear magnitudes is that stated above, viz., that as the differences between stimuli decrease in an arithmetical progression the differences in the times of perception increase in a geometrical progression.[1] It is not implied that this ratio would hold beyond the limits of the differences used. If the differences between the lines were decreased below ½ mm. the times would presumably increase much more rapidly than in a geometrical progression until the differential threshold was reached. On the other hand, differences greater than 3 mm. would affect the reaction-times but little.

[1] It seems to the writer that it should be possible to apply to these results Fechner's *Unterschiedsformel* and correlate the increase in the magnitude of the stimuli inversely with the increase in time of perception, and thus test the psychophysical law. In these results we are concerned with a difference in the intensity of two sensations and obviously only a difference in the stimuli will serve as such a measure. The difference in sensations is a function not of the stimulus differences but of the ratios of the stimuli. A development of the formula of difference gives the formula for the differences between sensation differences (*Elemente*, **2**: 89-96). This formula, it appears, should be applicable here. The mathematical difficulties involved in this application are such that the writer prefers to publish the results of the experiments with the general indication of their relations rather than to attempt an application and interpretation of doubtful validity.

2. *Equal relative differences in lengths of horizontal lines*

The second set of experiments was concerned with a measurement of the time of perceiving differences in the lengths of lines which were relatively equal. Owing to the fact that it was desired to retain the same conditions as obtained in the series described above, the number of differences possible was not as large as would have been desirable. The limitations of the size of the aperture in the drop-screen made it impossible to use longer lines than 24 mm., and the distance at which the apparatus was situated from the subject likewise made it undesirable to use lengths smaller than 5 mm., owing to the effects of irradiation, fatigue, etc., and the fact that the distance between the two lines on any card was 10 mm. The standard distances were thus 5, 10, 15 and 20 mm. and the variable distances 6, 12, 18 and 24 mm. Cards were prepared similar to those used in the experiment above described, with the same distances between the lines (10 mm.) and the same width of the lines (2 mm.). The cards were thus 5–6, 10–12, 15–18 and 20–24 mm. These were exposed to the subjects as before, the same principles likewise governing in the alternating order of presentation and reaction. Tables XVII and XVIII give the results of 2,560 reactions by subjects H and S. As before, each figure represents an average of 40 reactions, 320 in all to each pair of stimuli.

If the psychophysical law, which holds that we perceive not equal absolute but equal relative differences as equal is valid, then the reaction-times to the various pairs of stimuli obtained in the above experiments should remain constant. It should take no longer time to perceive the difference 5–6 mm. than the difference 10–12 mm.,

Table XVII

Subject H. 1280 Reactions. III, 1905.

Series.	20-24.	M. V.	15-18.	M. V.	10-12.	M. V.	5-6.	M. V.	False
I.	312.9	*28.9*	314.4	*28.8*	324.7	*24.4*	324.3	*23.0*	4
II.	297.6	*27.3*	316.9	*25.6*	320.4	*28.0*	330.1	*29.8*	2
III	314.2	*20.6*	307.8	*23.5*	305.1	*26.6*	306.1	*18.5*	2
IV.	308.4	*25.6*	315.7	*24.8*	307.5	*25.5*	321.9	*25.0*	2
V.	297.4	*27.0*	308.3	*26.6*	306.2	*21.1*	310.1	*27.9*	3
VI	304.3	*19.0*	295.2	*14.3*	301.8	*19.4*	299.6	*21.0*	5
VII.	290.4	*19.4*	296.3	*23.6*	297.9	*24.7*	307.7	*23.5*	2
VIII.	305.6	*21.7*	307.2	*19.4*	314.1	*28.9*	320.1	*20.4*	2
Av.	303.85	*23.7*	307.73	*23.3*	309.71	*24.8*	314.98	*23.6*	
P. E. of Av.	1.12		1.10		1.17		1.11		
P. E. of Diff.			1.56		1.60		1.61		
False	6		4		3		9		22

TABLE XVIII

Subject S. 1280 Reactions. III, 1905.

Series.	20-24.	M. V.	15-18.	M. V.	10-12.	M. V.	5-6.	M. V.	False
I.	328.3	*37.5*	327.1	*40.5*	337.3	*31.4*	349.2	*30.7*	12
II.	315.1	*36.5*	328.8	*30.9*	338.4	*33.4*	343.2	*33.6*	14
III.	330.3	*28.9*	333.8	*28.8*	334.9	*28.2*	337.3	*26.8*	8
IV.	319.9	*39.6*	330.8	*32.3*	333.3	*29.3*	328.6	*25.5*	8
V.	333.1	*25.1*	319.8	*25.8*	314.6	*26.4*	343.2	*27.7*	8
VI.	312.8	*30.6*	317.1	*26.1*	333.2	*23.8*	332.1	*26.6*	10
VII.	326.3	*26.3*	317.6	*28.2*	311.5	*31.4*	338.1	*30.7*	9
VIII.	318.5	*22.4*	313.1	*27.5*	324.6	*28.9*	327.9	*29.5*	5
Av.	323.0	*28.3*	323.5	*30.0*	328.5	*29.1*	337.4	*28.9*	
P. E. of Av.	1.34		1.41		1.37		1.36		
P. E. of Diff.			1.94		1.97		1.94		
False	16		26		16		29		74

15–18 mm., or 20–24 mm. That there is approximate constancy in the times for the limited area covered by these experiments is apparent. This constancy is, however, only approximate, for as the absolute difference decreases the time of perception gradually increases. This increase is slight except in the case of the difference between 5–6 mm. and especially for subject S. The subject introspectively noted the greater difficulty of discrimination with this pair of stimuli, and this difficulty was apparently due more to irradiation than anything else.

Münsterberg[1] says, in concluding the discussion of his experiments by the psychometric method on linear magnitudes, that, while the psychophysical law holds approximately, 'the stronger effect of the equal relative differences in stimuli is constantly influenced by the weaker effect of the absolute differences in stimuli.' The results here under consideration bear out this statement, as do well-nigh all results in psychophysical experimentation. They show, however, that the influence of the absolute difference is not nearly so great as in his results. The conditions, as has been set forth in the section on the history of the method and its applications, were not the same and this doubtless accounts for the differences. In the opinion of the writer those experiments were little adapted to testing the psychophysical law both on account of the complicated conditions and also the use of the chain reaction, which, though it may have advantages, has obvious disadvantages if any application to the psychophysical law is to be made.

The results of the experiments on equal relative differences, as

[1] H. Münsterberg, *Psychol. Rev.*, **1**: 50–51, 1894.

far as they go, show the approximate validity of Weber's law. A continuation of the experiments with a wider range of distances would be necessary in order to obtain conclusive data. The value of the method for psychophysical experiments, it is believed, has been demonstrated.

A brief historical summary of the numerous investigations made to determine the ability of the eye to estimate linear magnitudes will be given here in order to facilitate a comparison of these results with those which I have obtained by quite a different method. This summary will be based largely on the discussions of *Augenmass* by Fechner, Müller, Helmholtz, Wundt and Münsterberg. A brief systematic presentation of the methods and results in a chronological order is all that is attempted. The investigations which will be considered are those which bear on linear magnitudes, omitting the effects of illusions on such estimates.

E. H. Weber[1] observed that persons of exceptionally keen vision could detect differences in lines that were to each other as 50: 51 or even 100: 101, whatever might be the absolute length of the lines. The lines to be estimated, varying in length from 100 to 105 mm. by differences of .5 mm., were drawn on separate sheets of paper. The subject was to judge which was the longer line when these stimuli were presented successively within short intervals. The lengthening of the interval caused an increase in the just noticeable difference. Persons not especially trained in eye-measurements could distinguish differences in lines which differed by about 1/25 of their length. Weber placed this fraction for himself at about 1/20. These observations are admittedly of no great value except in so far as they aroused an interest in such experiments and led to the numerous investigations which have followed.

F. Hegelmayer[2] made experiments on horizontal and vertical lines by the method of right and wrong cases, which appears to have the first application of this method in psychophysics. His problem was to determine the influence which the length of time between the presentation of stimuli exerted on the discrimination of their differences, and also to compare discriminativeness with horizontal and vertical lines. As far as these observations went, they showed that the relation of the number of right and wrong cases was dependent not on the absolute, but on the relative differences in the lines, and that horizontal lines could be judged more accurately than vertical lines. The number of cases was, however, very small, from one to three judgments being made in each series. From such results, of course, no conclusions can be drawn. As Müller says, the only conclusion

[1] E. H. Weber, *Programmata Collecta*, p. 142, 1851.

[2] F. Hegelmayer, *Vierordts Archiv.* **12:** 844–853, 1859.

warranted is that the discrimination of differences between two lines decreases in accuracy as the interval of time between their presentation increases.

Fechner's[1] experiments were made by the method of average error and the method of just noticeable difference. He used compass-points placed before him on the table at a distance of one Paris foot. To prevent the size of the angle of the compasses from affecting the estimates all but the points were concealed. He states that with the greater distances this concealment was not entirely successful, but considers that it introduces a constant error that does not affect the variable error, which measures discrimination. The standard distances were 10, 20, 30, 40 and 50 half Paris decimal lines (1.624 mm.). The results show that the variable errors, made in adjusting a second pair of compasses to these distances, increased proportionally to the distances with as great exactness as could be expected. The number of experiments with each distance was 120, 60 with the variable stimulus to the left and 60 to the right. The following summary of results shows the constancy of the relative discriminativeness, S/D being the sum of the average errors in the two space positions divided by the distance.[2] The fraction in the last column gives the measure of discriminativeness.

D	10	20	30	40	50	
S/D	3.864	3.843	4.030	3.862	3.708	1/62.1

Fechner also made a few experiments by the method of just noticeable difference. He placed compass-points, whose distances were 1 inch and $1^{1}/_{44}$ inches, in a horizontal position. Without knowing which was which, he found that after a considerable practise he could correctly judge which was the longer. When the lengths of the lines were doubled and quadrupled, the same relative discriminativeness obtained. These three series were repeated at different times with the same results. He noted, moreover, that the distance from the eye had no effect on the estimates as long as this distance was within the limits of accommodation. Fechner makes no claim in these experiments to have determined the discriminativeness, inasmuch as the just noticeable difference was taken somewhat greater than it should have been. He says it is probable that he could have distinguished smaller differences, to determine which the method of right and wrong cases should have been used.

[1] G. T. Fechner, *Elemente*, Leipzig, pp. 211–236, 1860. *Revision der Hauptpunkte der Psychophysik*, Leipzig, pp. 335–358, 1882.

[2] G. T. Fechner, *Revision*, p. 336.

Volkmann[1] has made perhaps the most extended investigation of eye-measurement of large as well as of very small distances. For his experiments he used three parallel threads suspended by weights from a horizontal bar before a dark background. The distances between the threads could be accurately adjusted and measured directly to 1 mm. and estimated to 1/10 mm. The threads were 220 mm. long and situated 800 mm. from the eye in the case of Volkmann, 343 mm. in the case of Solger. The distances for which the average errors in adjustment were determined ranged from 10 to 240 mm. The results of these experiments are shown as follows, S/D as before giving the average error divided by the distance.

D			10	20	40	80	120	160	200	240
S/D	V	I	1.150	.905	1.105	1.019	1.119	1.136	.917	1.039
		II	1.160	1.005	.730	.924	1.010	.962	.945	.900

D			10	20	30	40	50	60
S/D	S	I	2.380	2.266	1.998	1.806	1.914	1.879
		II	2.258	1.612	1.805	1.715	1.634	1.617

From these investigations it appears that Fechner correctly judges a difference in lines which amounts to 1/60 of the length of the line, and Volkmann, differences of 1/90 and 1/100, which differences are constant. As Münsterberg says, "It can not be denied that within the limits of 10 to 240 mm. the values in these series are so constant that they furnish a model illustration of Weber's law."

Volkmann's[2] experiments with very small distances, ranging from .2 mm. to 1.4 mm., showed marked deviations from the above results. The distances were secured by three very fine silver threads, 11 mm. long and .445 mm. thick, suspended before an opalescent glass screen or the clear sky. These distances were adjusted by means of a micrometric screw which could be read directly to .01 mm. and estimated to .001 mm. The threads were observed from a distance of 333 mm., and in the case of Appel, who possessed an extraordinarily keen ability in eye-measurement, 370 mm. and 300 mm. The errors in these experiments did not increase proportionally to the distances, but indicated rather an approach to a lower limit where the absolute differential threshold was constant. With the two smallest distances, .2 mm. and .4 mm., the irradiation was so strong that in the case of Volkmann the estimates were very uncertain. The deviations from Weber's law are explained by Fechner by resolving the average error into two components, one of which, the so-called Volkmann's constant,

[1] A. W. Volkmann, *Physiologische Untersuchungen im Gebiete der Optik*, Leipzig, 1863.

[2] A. W. Volkmann, *Berichte der königⁱ. sächs. Gesell.*, p. 140 ff., 1858.

due to the size of the retinal elements, remains constant for the various distances; the other, Weber's variable, which is proportional to the distances. The discussion of this does not concern us here and only the values for Weber's variable in these series for Volkmann and Appel as calculated by Fechner are given.

Volkmann		I	1/79.1
"	(vertical ser.)	II	1/45.1
Appel		I	1/164.5
"		II	1/85.3

Volkmann and Krause later took up the measurement of very small distances by the method of just noticeable difference. The apparatus was that above described. The distance from the eye was 200 mm. The result of these experiments as given by Müller was as follows:

D (in mm.)	0.3	0.6	1.2	2.4	4.8
V u	1/11	1/15	1/16	1/21	1/29

D (in mm.)	0.5	0.9	1.3
K u	1/35	1/66	1/90

Volkmann also made experiments to determine the accuracy in estimating unequal distances. This consisted of adjusting a movable line between two others 1/10, 2/10, 3/10, 4/10 and 5/10 of the whole distance. The distance to be divided was one Paris line. The mean variable error is given by Helmholtz as follows:

For .1 and .9 = 2.6
For .2 and .8 = 5.6
For .3 and .7 = 7.9
For .4 and .6 = 6.5
For .5 = 2.8

Chodin[1] determined the average error and just noticeable difference for seven distances from 2.5 mm. to 160 mm., and arrived at results departing considerably from those of Fechner and Volkmann, whence he concluded that Weber's law does not hold as claimed by them. His method consisted in marking off equal distances on a line drawn by a lead-pencil. From a point on the line, horizontal or vertical, equal distances to the right and to the left or above and below the standard distance were determined to within .1 mm. The reciprocals of the average errors indicating the discriminativeness appear in the following table.

D (in mm.)	2.5	5	10	20	40	80	160
I Horizontal	1/39	1/52	1/64	1/76	1/69	1/73	1/65
II "	1/50	1/78	1/90	1/112	1/94	1/88	1/71
III Vertical	1/32	1/44	1/60	1/53	1/57	1/48	1/56

[1] A. Chodin, *Gräfes Archiv für Ophth.*, **23**: 92–108, 1877.

In the experiments by the method of just noticeable difference the procedure was the same as in the above experiments except that the variable distance was to be made not equal, but just noticeably greater or less. The results are as follows:

D (in mm.)	2.5	5	10	20	40	80	160
I	1/17	1/29	1/37	1/53	1/44	1/39	1/43
II	1/26	1/32	1/45	1/57	1/36	1/32	1/30

The deviations from the results of earlier observations are, according to Fechner, to be attributed to the method and to the inaccuracies of the hand in measuring off small distances. The method is certainly unsystematic and inferior to that of Volkmann particularly. Münsterberg, likewise, calls attention to the inaccuracies in measurement where micrometric methods are not employed, especially where the distances are 2.5 and 5 mm. The estimation to hundredths of a millimeter is manifestly impossible with any degree of accuracy.

Wundt's[1] experiments to determine the discriminativeness to change of convergence of the eyes have a bearing on the determination of the accuracy of eye-measurement on account of the muscular sensations which accompany eye-movements and fixations. The method of determining this was the following. The subject looked with both eyes through a dark box at a ground-glass surface. Between this surface and the eye was suspended a thread which could be moved to and fro. The displacement necessary in either direction in order that the eye, which was fixated on the thread, could detect it was determined. It was found that the sensitivity to changes in convergence was approximately constant with varying distances, the relative magnitude of the just noticeable displacement of the line of fixation being about 1/51.

Münsterberg[2] goes over the whole subject of eye-measurement historically and critically. He calls attention to the various sources of error and the methodological difficulties in such experiments which it is necessary to eliminate and which have not always been guarded against in the earlier investigations. The diverse methods which have been employed and the complicated conditions under which observations have been made make comparisons of results well-nigh impossible. The psychophysical methods have been, moreover, used in a loose and unsatisfactory manner. He concludes that the question of the relation of the average error to Weber's law has not been as yet satisfactorily answered. His problem was to isolate the various con-

[1] W. Wundt, *Beiträge zur Theorie der Sinneswahrnehmung*, Leipzig and Heidelberg, p. 195, 1862.

[2] H. Münsterberg, *Beiträge zur experimentellen Psychologie*, Freiburg, 2: 125–181, 1889.

ditions affecting the estimation of distances between points and the lengths of lines and to determine the influence of eye-movements in visual judgments.

The apparatus consisted of a wooden frame, 600 mm. long and 500 mm. wide, covered by a green cloth. This color is least fatiguing to the eye and does not give such strong effects of contrast and irradiation. Points, 1 mm. square, and lines, vertical and horizontal, under many variations of condition, 36 in all, were exposed on this screen at a distance of 600 mm. from the subject, with and without eye-movement, simultaneously and successively, with varying intensities, etc. These points and lines, further, were viewed monocularly and binocularly. The 20,000 experiments made with a modification of the method of average error were on 20 distances, ranging from 10 mm. to 200 mm. by differences of 10 mm. It would carry us too far afield to enter into all the details of the experiments and their results, hence a summary of the main points only will be given. (1) Every change of eye-movement, fixation or use of the eyes markedly affects the estimation of distances, whence it follows that muscular sensations or their memory-images are necessary to an explanation of errors. (2) Distances to the left are overestimated, to the right underestimated, which is ascribed to the ease of movement from left to right, due to habits formed in reading and writing. (3) Monocular vision with eye-movement shows that the right eye overestimates distances to the right, the left overestimates distances to the left. This is the reverse of results obtained by Kundt. (4) Distances reproduced after varying intervals are usually overestimated. (5) Linear distances do not seem longer than distances between points when an interval is interposed between the two distances. (6) The variable line as compared with the standard line is overestimated. (7) The overestimation of vertical distances takes place only under three conditions when the distances are between points, when the vertical is above the horizontal distance with which it is compared, and when the eyes move freely. (8) In estimation of simultaneously presented distances where the eyes are free to move, the average error varies between 1.1 per cent. and 2.3 per cent., while without eye-movements the average errors vary from 3.7 per cent. to 4.9 per cent. or an average of 2.6 per cent. more in the latter than in the former case. This is taken as a proof positive that estimations of magnitudes are really estimations of the intensity of muscular sensation. (9) Weber's law is approximately valid, but the law applies not to extensive sensations, but to the intensity of motor sensations.

R. Fischer[1] investigated the accuracy of the estimates of lengths

[1] R. Fischer, *Gräfes Archiv für Ophth.*, **37**, pts. 1 and 3: 97–136, 55–85, 1891.

of lines and angles. The lines were observed binocularly in the right visual field at a distance of 200 mm. The distances were marked off by a pointer on the various arms of a cross, the center of which was situated before the right eye and fixated in the various experiments. The estimates in the different space positions were made alternately to avoid effects of immediate repetition. The method consisted of adjusting a variable distance equal to a standard distance or halving the standard distances. Without entering into the details of the results under the numerous conditions, it may suffice to state the course of the average error which for our present purposes is the matter of interest. The relative average error showed a very great constancy both with horizontal and with vertical distances. This error was smaller in the halving than in the comparison of distances.

In the experiments on angles he used a circle, 360 mm. in diameter, divided into sectors by threads. The apex of the angle was fixated at a distance of 180 mm. The method consisted of halving a given angle. This could be done with a sector of 180° with great exactness in both the horizontal and vertical directions, less exactly with smaller angles. The average errors departed considerably from Weber's law.

Higier,[1] in his experiments to determine the relations of the different psychophysical methods, used extensive magnitudes as stimuli inasmuch as all the methods can there be conveniently applied. The method of average error and the method of right and wrong cases were used, the latter in such a way as to make possible at once a test of the method of minimal changes. In the application of the method of right and wrong cases the suggestion of Jastrow that 'equal' cases be excluded was followed, though a later series of experiments with the three possible judgments was made. The method of doubling the stimulus was also used to a limited extent.

The apparatus consisted of a frame covered by a dark cloth on the surface of which was a horizontal line of light, ½ mm. in width, produced by a mirror. The distances were given by a fine thread suspended by a weight and slides or pointers on either side, from which distances could be read off on a scale above. The distances were observed monocularly at a distance of 500 mm. The results of the experiments by the method of average error appear in the following table.

D	10	20	50	100	150	200	250
Δ	.3245	.6085	.8673	1.8791	3.3744	5.1103	6.8357
Δ/D	.0324	.0304	.0173	.0188	.0225	.0255	.0273

The results show that the average error increases with the increasing distances, but not as Weber's law requires. Higier finds a close agree-

[1] H. Higier, *Philos. Studien*, 7: 232–297, 1892.

ment with the results of Chodin and concludes that Weber's law does not hold.

The experiments by the method of right and wrong cases show approximately the same curve for the distances used, viz., 50, 100, 150 and 200 mm., and further show the approximate correspondence of the measures by the two methods. The differential threshold obtained by the method of average error does, however, not agree with that obtained by the method of minimal changes.

Merkel[1] made an extended study of the method of average error based on experiments in eye-measurements. The average errors (M) in making variable lines equal to the standard lines 1, 2, 5, 10, 20 and 50 mm. and the measure of the validity of Weber's law according to his method of calculation (F/M), which should be constant if the law holds, appear in the following table, which I abstract from the complete table of results.

D	1	2	5	10	20	50
M	.987	1.962	4.984	10.00	19.64	48.79
F/M	.0050	.0047	.0042	.0040	.0043	.0041

Correspondingly, the results by the method of just noticeable differences.

D	1	2	5	10	20	50
M	.984	1.974	4.970	9.898	19.62	48.68
F/M	.0057	.0044	.0034	.0031	.0033	.0034

With the method of dividing the stimulus, the following results were obtained. This method consists of adjusting a stimulus twice as great as a standard stimulus, beginning in one case with a difference clearly greater than this amount and in the next with a difference clearly less.

D	1	2	5	10	20
M	1.72	3.74	9.20	18.48	36.93
F/M	.0074	.0047	.0033	.0035	.0033

In the last series reported by Merkel, the method was to halve the stimulus, beginning, in the one case, with a difference clearly greater than one half, in the other, with a difference clearly less. The following results, in summary, were obtained:

D	2	4	10	20	40
M	1.175	2.145	5.485	10.84	21.15
F/M	.0066	.0062	.0054	.0059	.0060

The upshot of all these experiments of Merkel is that, aside from the deviations with very small differences, Weber's law holds very closely. Merkel's recalculation of Higier's results seems to show that these again do not show the deviations which Higier claimed.

[1] J. Merkel, *Philos. Studien*, 9: 400 ff., 1894.

This summary has not reported the work of Kundt, Messer and others, inasmuch as these investigations relate to the estimation of magnitudes as affected by illusions. The investigation of Münsterberg by the psychometric method has been taken up elsewhere in this paper under the historical discussion of the method used throughout this research. In the above presentation we have been concerned to give the facts of experiments and results obtained, omitting the critical consideration of their value in the individual investigations. The point which has been emphasized is the nature of the increase of the average error with increasing distances and what the law of such increase may be, in order to compare with them the increase in the time of perception as the differences in the lengths of the lines decrease. It appears from the summary that Weber's law expresses rather closely the relation of the stimulus to the sensation in this department. Of all the researches, only two show very marked deviations, viz., those of Chodin and Higier, and, as pointed out above, Merkel's recalculation of Higier's results show greater approximation to the law than the results at first blush appear to indicate.

V. EXPERIMENTS ON PITCH

The experiments on sound aimed to measure differences in sensations of tone or pitch. Such experiments are of interest for several reasons. It may be possible by the method here used to arrive at a more definite comparison of discriminativeness of color quality with that of tone quality than by any other method. The many attempts to subdivide the tonal scale and the color scale by a common principle, and thus to make them directly comparable, are evidences of the interest in this matter. Experiments have shown, also, that in this province individual differences are perhaps more prominent than in any other. Furthermore, such experiments may have a bearing on the theory of tone perception and the theory of hearing generally. And lastly, there is the question of the validity of Weber's law for the perception of pitch. The experiments so far have shown that the law does not hold, if the stimulus is measured by the rate of vibrations per second. The number of investigations into the measurement of discriminativeness to differences in pitch is not large, due no doubt to the very great difficulty of dealing adequately with sound stimuli. This is especially true of tones, where uniformity in intensity, pitch and quality is so essential and so difficult to secure.

In the experiments to be described, apparatus and methods were employed, which seemed to meet these difficulties very well. The tones were produced by means of three König tuning-forks mounted on resonance boxes. Two of the forks were of 512 vibrations and the third, of 516 vibrations. The mounted forks were encased in rubber tubing and placed side by side in a horizontal position in which they were held firmly by clamps, adjustable by means of three set-screws. The rubber tubing permitted the forks to be held in a constant position when firmly clamped and did not dampen the tone. The set-screws allowed of such an adjustment as to eliminate any differences in the quality of the tone produced by varying pressures. Uniformity in height was also made possible.

The variations in pitch were secured by means of three steel riders of uniform size and weight. The standard fork, one of 512 vibrations placed between the two variables, was kept at a constant pitch. Its vibration rate was 499.75 vibrations. The differences of 4, 8, 12 and 16 vibrations, in the one case higher, in the other lower, were produced on the other two forks, the one of 516 vibrations and the other, 512 vibrations. The positions of the riders for these differences were determined after many measurements at different times and by three individuals, one of whom had had a great deal of practise in such work.

In the shifting of the riders, the distance could be measured to .25 mm. and estimated to .1 mm. or less.

Constancy in the intensity of the tone was obtained by means of an apparatus devised by Cattell[1] for experiments on sensations of impact, which was modified for the present purpose. It consists essentially of a hammer, the handle of which is an aluminium tube 25 cm. long, one end fitting into a brass axis, 6 cm. long, at the base of the apparatus, which again is firmly adjusted, permitting the hammer to fall freely with a minimum of friction. At the other end, the tube passes through a solid India rubber cylinder, 22.5 mm. long and 11.5 mm. in diameter. An insulated wire passes through the aluminium tube and terminates on the further side of the cylinder in a platinum point. On the lower side of the tube and hammer runs a brass spring which carries a second very delicate spring that breaks and closes the electrical circuit. When the hammer falls, it closes the circuit instantaneously and with but slightly greater noise than the tick of a watch. On the upper end of the cylinder was a brass knob of such weight as to give a sound of satisfactory intensity, but without producing disturbing overtones. On the under side of the cylinder was a piece of felt 1 cm. in diameter and 1 cm. thick. The total weight of the hammer was 42 grams, and it fell upon the forks from a height of 24 cm., or from nearly a vertical position. Just before striking the fork, the hammer fell upon a weak spiral spring, situated 9 cm. from the end, and 2 cm. above the level, of the fork. This spring was surmounted by a rod carrying a piece of felt, which prevented any noise from the striking of the hammer. The spring allowed the hammer to strike the fork a quick blow and then rebound, to be caught up by the operator, who, for this purpose as well as for the stopping of the vibrations of the forks, held in his hand a rod encased in rubber tubing. The spring made it unnecessary to catch the hammer, but unless this was done, its vibrations on the spring caused a slight noise. The hammer struck at a distance of 1½ cm. from the end of the forks.

The hammer was attached to a wooden base, about 20 cm. long and 10 cm. wide, which was fitted into a box before the forks. The bottom of the box was covered with chamois-skin, and the apparatus could be shifted from side to side with but little noise. In the operation of the instrument, the standard fork was always sounded first, and then the apparatus shifted either to the right or to the left and the variable fork struck. After the striking of the standard fork the larger spring in the hammer was raised, opening the circuit. When the apparatus was moved, it made the contacts at either side when it

[1] J. McKeen Cattell and C. S. Dolley, *On Reaction Time and the Velocity of the Nervous Impulse*, National Academy of Sciences, Vol. VII, 2nd Memoir.

was in position for the striking of the second. These spring contacts on the base of the apparatus, simplified its manipulation and made possible greater constancy in the intervals between the giving of the two stimuli.

The experiments to be reported were made in a large dark room in the interior of the psychological laboratory. The apparatus was situated 2½ meters from the subject, who sat at a table at the end of the room with his back to the apparatus. The chronoscope was in an adjacent room. It could just be heard in the room where the reactions were taken, and the starting of the chronoscope served as a ready signal for the operator and for the subject. This signal was found to be better for the two subjects than any other, as they had become accustomed to it as a signal in thousands of reactions and it was therefore the most natural stimulus. The sound of the chronoscope could not be heard when the tuning-fork was struck, for the tones were of considerable intensity and rang out full and clear.

A series consisted of 160 reactions, or forty to each stimulus. The differences in rates of vibration were, as stated above, 4, 8, 12 and 16 vibrations. Of the forty reactions, 20 were with the second or variable stimulus higher, and 20 with this stimulus lower. The standard stimulus was always sounded first after the starting of the chronoscope by the recorder. After an interval of from 2 to 2½ seconds during which this tone was sounded, and during which the apparatus was adjusted, the vibrations of the fork were stopped. Then after an interval of about two seconds the variable tone, either higher or lower, was given. A pendulum swinging near the table on which the apparatus rested, enabled the operator to make this interval sufficiently constant. If the second or variable tone was higher than the standard, the subject reacted with his right hand; if lower, with his left hand. After the practise series, the association was so complete that the appropriate movement was made automatically. The series were all taken in the evening between 7.30 and 9 o'clock, except the first three series on subject H, which were taken at another time when there were no distracting noises in the building. The evening was selected in order to secure, as nearly as possible, absolute quiet.

Tables XIX and XX give the results of 2,560 reactions on two subjects, H and W. Before this, three practise series were taken. In these series H and W alternately acted as subject and operator. Each figure, as before, represents an average of 40 reactions. The times, mean variations, differences and probable errors appear as in the previous tables.

TABLE XIX

Subject H. 1280 Reactions. III, 1905.

Standard 499.75 v.

Series.	16 v.	M. V.	12 v.	M. V.	8 v.	M. V.	4 v.	M. V.	False.
I.	296.6	*30.8*	300.9	*21.1*	309.6	*22.3*	330.4	*34.0*	3
II.	288.4	*25.8*	291.0	*32.7*	325.8	*33.2*	356.5	*25.9*	6
III.	284.8	*31.1*	296.7	*28.8*	306.8	*30.7*	348.1	*32.1*	2
IV.	287.4	*21.1*	302.9	*30.1*	307.3	*26.6*	331.7	*30.2*	3
V.	297.6	*26.4*	301.8	*25.8*	311.3	*31.5*	331.1	*27.1*	2
VI.	290.2	*26.8*	303.0	*24.9*	313.9	*23.1*	326.9	*27.9*	1
VII.	282.1	*26.9*	289.5	*23.6*	300.9	*31.1*	324.1	*33.0*	3
VIII.	294.7	*23.6*	304.1	*16.2*	312.9	*28.4*	323.7	*24.9*	4
Av.	**290.22**	*26.6*	**298.73**	*28.0*	**311.06**	*28.4*	**334.06**	*29.4*	
P. E.	1.28		1.32		1.34		1.38		
Diff.			**8.51**		**12.33**		**23.00**		
Diff. from 16 v.			**8.51**		**20.84**		**43.84**		
P. E. of Diff.			1.84		1.88		1.92		
P. E. of Diff. from 16 v.			1.84		1.85		1.88		
False	4		4		6		10		24

TABLE XX

Subject W. 1280 Reactions. III, 1905.

Standard 499.75 v.

Series.	16 v.	M. V.	12 v.	M. V.	8 v.	M. V.	4 v.	M. V.	False.
I.	360.8	*35.2*	368.0	*43.9*	400.0	*43.0*	534.6	*87.9*	2
II.	364.1	*45.3*	368.9	*51.2*	444.1	*48.6*	505.8	*79.0*	7
III.	360.9	*33.9*	365.4	*36.9*	380.3	*53.1*	421.8	*58.2*	7
IV.	348.5	*44.7*	368.5	*47.4*	420.4	*62.5*	454.2	*58.5*	5
V.	343.7	*37.1*	346.1	*36.9*	369.1	*47.9*	464.1	*89.1*	7
VI.	323.1	*45.2*	330.3	*46.7*	375.8	*59.7*	460.4	*68.6*	6
VII.	331.3	*44.7*	340.0	*42.8*	416.8	*44.8*	471.2	*61.9*	9
VIII.	321.8	*41.8*	350.9	*57.0*	358.9	*44.8*	449.5	*77.9*	2
Av.	**344.27**	*40.9*	**354.76**	*45.3*	**395.67**	*50.5*	**470.20**	*72.6*	
P. E.	1.93		2.14		2.38		3.42		
Diff.			**10.49**		**40.91**		**74.53**		
Diff. from 16 v.			**10.49**		**51.40**		**125.93**		
P. E. of Diff.			2.88		3.20		4.16		
P. E. of Diff. from 16 v.			2.88		3.06		3.93		
False	4		7		10		24		45

The differences in the individual series between each pair and between these and the pair of stimuli most easily perceived are given in Tables XXI and XXII.

TABLE XXI
Subject H.

Series.	Differences. 16–12 v.	12–8 v.	8–4 v.	Differences from 16 v. 16–12 v.	16–8 v.	16–4 v.
I.	4.3	8.7	20.8	4.3	13.0	33.8
II.	2.6	34.8	30.7	2.6	37.4	68.1
III.	11.9	10.1	41.3	11.9	22.0	63.3
IV.	15.5	4.4	24.4	15.5	19.9	44.3
V.	4.2	9.5	19.8	4.2	13.7	33.5
VI.	12.8	10.9	13.0	12.8	23.7	36.7
VII.	7.4	11.4	23.2	7.4	18.8	42.0
VIII.	9.4	8.8	10.8	9.4	18.2	29.0
Av. Diff.	8.51	12.33	23.00	8.51	20.84	43.84
M. V.	*3.88*	*5.62*	*6.90*	*3.88*	*5.14*	*11.04*
P. E.	1.16	1.68	2.05	1.16	1.53	3.29

TABLE XXII
Subject W.

Series.	Differences. 16–12 v.	12–8 v.	8–4 v.	Differences from 16 v. 16–12 v.	16–8 v.	16–4 v.
I.	7.2	32.0	134.6	7.2	39.2	173.8
II.	4.8	75.2	61.7	4.8	80.0	141.7
III.	4.5	14.9	41.5	4.5	19.4	60.9
IV.	20.0	51.9	33.8	20.0	71.9	105.7
V.	2.4	23.0	95.0	2.4	25.4	120.4
VI.	7.2	45.5	84.6	7.2	52.7	137.3
VII.	8.7	76.8	54.4	8.7	85.5	139.9
VIII.	29.1	8.0	90.6	29.1	37.1	127.7
Av. Diff.	10.49	40.91	74.53	10.49	51.40	125.93
M. V.	*7.03*	*21.44*	*26.67*	*7.03*	*21.13*	*22.69*
P. E.	2.09	6.39	7.95	2.09	6.30	6.76

The results of these experiments show considerable individual differences which are somewhat difficult to account for. Subject W is more musical than H, but he apparently does not perceive the differences as readily as the latter. It was expected that the reverse would be the case, and that it is not so is possibly due to the great amount of practise which H obtained in adjusting the forks, to eliminate differences in timbre as well as in measuring the rates of vibration. The normal reaction of W is for colors, as pointed out in Section III, about 21σ longer, while here it is even for the differences of 16 vibrations about 50σ longer. His times are, moreover, very irregular, as is shown by the large mean variations. This is, however, due to a considerable extent to a mixture of species in the averages. It took for W a very much longer time to perceive the differences, when the second tone was lower than when it was higher. Luft had noted this difference in his subjects while Max Meyer found no such difference. It is evidenced here very strikingly for W, but not at all for H, which seems to show that this constant error is a purely individual matter. The reaction-times for W with the second higher, and for the second lower for each of the differences, are as follows:

Subject W.

	16 v.	12 v.	8 v.	4 v.
Lower	356.6	363.4	415.7	492.9
Higher	331.9	346.3	375.6	447.5
Diff.	24.7	17.1	40.1	45.4
Diff. Lower		6.8	52.3	77.2
Diff. Higher		4.4	29.3	72.9

The differences are, as will be noticed, very marked, while the differences between the various pairs of stimuli do not differ very greatly except in the case of 12 to 8 vibrations. The mean variations are reduced by about 30σ when the averages are taken separately. I have not thought it worth the time and trouble to determine all the variations accurately.

The above differences do not appear for H, as the following shows:

Subject H.

	16 v.	12 v.	8 v.	4 v.
Lower	288.9	296.6	311.9	335.6
Higher	292.4	300.9	310.2	332.5
Diff.	—3.5	—4.3	+1.7	+3.1

We had also considerable difficulty with the difference of 4 vibrations, as is indicated by the false reactions which are due, in most cases, to wrong judgments. The other false reactions are not due to this cause. False reactions will occur, as appears in other sections, even with black as the stimuli to be discriminated from white, and are not due to inability to discriminate, but rather to an occasional associative preparation for a certain reaction which then irresistibly takes place.

Such false reactions are relatively very few in number. They are, however, seemingly bound to occur, and at times they can be introspectively accounted for, at other times not.

The reactions of subject H show greater constancy, though this is less here than in the case of visual stimuli. This may be due, to some extent, to the less natural character of the movement to be made. To react to the higher tone with the right hand and to a lower with the left is not as natural an association as the rightness and leftness in the case of the colors or lines exposed as explained above. Further successive discrimination is not as readily made as simultaneous, at least such appears to be the case. For introspection, which is worth very little here, the difference of 16 vibrations seemed to H to be quite as great as the difference between red and yellow, or red and orange, but the time of reaction is considerably longer.

As regards the differences and their increase, they do not show what previous investigations might perhaps lead one to expect. The researches on the perception of pitch differences have indicated in, as yet, a rather incomplete way, that the absolute discriminativeness remains, within certain limits, constant. If this were true, we should expect that the differences in time of perception would remain constant. This is not the case, as the differences in time of perceiving equal absolute differences are for H, 8.51σ, 12.33σ and 23.00σ, and for W the increase is very much more marked. With the differences which I have used, it is difficult to determine what the significance of the increase is. The difference of 4 vibrations between the various pairs is rather too large. Introspectively, the difference between 4 vibrations and 8 vibrations seems very great, while that between 16 vibrations and 12 vibrations, on the other hand, seems very small. If I should ever repeat these experiments or continue them, I should use differences of 2 vibrations, beginning with the smallest difference at 4 vibrations. There is, apparently, an inflection point on the curve between 4 vibrations and 8 vibrations, at least for subject H, or a point where the process of discrimination changes from one of considerable difficulty to one of comparative ease. In his case, one extreme value in the second series caused the difference of 8.51σ and 12.33σ in the difference between 16 vibrations and 12 vibrations, and 12 vibrations and 8 vibrations. Excluding that series, the average differences would be 9.3σ and 9.1σ, while the median values are 8.0σ and 9.6σ, showing approximate constancy.

A comparison of the differences for consciousness of the various differences of pitch with those of other series, show some interesting correspondences. The correspondence in the case of the differences between the last four pairs of lines and the differences in pitch is, for H, close. A difference of ½ mm. in lines seems, therefore, as great as a difference of 4 vibrations in pitch, and the times of perception with the varying differences increase in the same manner. Similarly, a change of 1 wave-length in the red end of the spectrum corresponds in a general way with 2 vibrations in pitch. Numerous detailed comparisons could, of course, be made showing the interrelations of sensations from the different senses.

The general conclusion that can be drawn from the study seems to be that as the difference in stimuli decreases the difference in sensation is disproportionately increased. The exact nature of this increase can hardly be analyzed out by these experiments. To do this, at least for subject H, would necessitate a further study of the range of differences lying between 8 vibrations and 4 vibrations especially. Indications point to a rapid rise in the curve of increase at some point intermediate between these differences, as pointed out above. The study further shows the very considerable individual differences which are especially brought out in dealing, as is here done, with overnoticeable differences, rather than with the differential threshold. The threshold values for the two subjects appear to differ but slightly.

As in the previous studies, I give here a brief summary of the results of experiments on the discrimination of pitch. The upshot of the results has been to show the great discriminativeness of the ear to differences in quality, as compared with differences in intensity. This discriminativeness varies widely with individuals. Within the range of the human voice, the absolute threshold of difference remains very constant, and not the relative threshold, as Weber's law requires. The discriminativeness appears to increase in the lower register, and in the middle register remains constant, then again in the higher register increases rapidly.

The results of the experiments of *Delezenne*,[1] *Seebeck*[2] and *Preyer*[3] are brought together by Preyer in the form of a table, and we can consider them jointly. Delezenne used a monochord and aimed to measure the ability to distinguish purity of interval. Seebeck used tuning-forks and determined the least noticeable difference that could be detected and named in direction. Preyer used metal reeds varying in tone by 1/10 of a vibration, and found that difference which could always be detected as different without its being indicated as higher or lower. The results are, therefore, not strictly comparable, both because of the difference in instruments which gave different timbres, and because of the difference in method. The subjects were musical and highly practised individuals. Preyer's results were obtained from twelve such subjects, and more than 1,000 determinations. The

[1] M. Delezenne, *Recueil des travaux de la soc. des Sci. de Lille*, 1827.

[2] A. Seebeck, *Pogg. Annalen*, **68**: 462.

[3] W. Preyer, *Ueber die Grenzen der Tonwahrnehmung*, Jena, 1876.

following gives a summary of the results. v denotes the vibration rate, d the just noticeable difference, a the absolute discriminativeness, e the relative discriminativeness.

	v	d	a	e
Delezenne,	120	.418	2.39	287
Seebeck,	440	.363	2.75	1,212
Preyer,	500	300	3.33	1,666
Preyer,	1,000	.500	2.00	2,000

On a basis of the results of the first two investigators, Weber had claimed that the discrimination of differences in pitch depended not on the number of vibrations, but on the relation of the number of vibrations compared. Preyer, with his own results and those of Delezenne and Seebeck, concludes that the absolute discriminativeness is approximately constant, and that within the region lying between 120 vibrations and 1,000 vibrations, Weber's law does not hold at all. Preyer's methods, the statistical treatment of the results and the conclusions, have been criticized by Fechner, Wundt and G. E. Müller. The grouping of his own results with those of Delezenne has been attacked particularly. It seems obvious that only a general comparison can be made between them, and any application to the problem of the validity of Weber's law would be out of the question.

Stumpf[1] made experiments on entirely unmusical persons, while the experiments just spoken of were on highly practised individuals. He sounded on an organ or a piano, in three different regions of the scale, two tones which differed by a fifth, a major or a minor third, and a major or a minor second. The subject was asked to state which tone was the higher. The astonishing result was obtained that even with such overnoticeable differences, the number of right cases was on the average as 3: 4. Preyer had, in similar experiments, found that the judgment as to the direction of a tone was more uncertain than the mere judgment of the likeness or difference, and that the number of right cases was to the total number of cases as 3: 5. He used, however, much smaller differences than Stumpf. The analysis of the results does not concern us here inasmuch as they have but an indirect bearing upon our problem.

Luft,[2] in the Leipzig laboratory, made experiments by the method of minimal changes on a wider range of tones. He used tuning-forks of 64, 128, 256, 512, 1,024 and 2,048 vibrations. The variable fork carried two riders of equal weight which could be shifted up and down, the amount of their displacement being registered on a millimeter scale.

[1] C. Stumpf, *Tonpsychologie*, Leipzig, p. 313, 1883.

[2] Edward Luft, *Philos. Studien*, 4: 511–540, 1887.

The results of the experiments on two subjects are summarized as follows:

	L.			v. Tch.		
v	d	a	e	d	a	e
64	.149	6.71	430	.433	2.31	147
128	.159	6.28	805	.333	3.00	384
256	.232	4.31	1,103	.229	4.36	1,118
512	.251	3.98	2,040	.233	4.29	2,197
1,024	.218	4.58	4,697	.202	4.95	5,069
2,048	.362	2.76	5,657			

v denotes the vibration rate, d the just noticeable difference, a the absolute discriminativeness, e the relative discriminativeness.

As compared with earlier results reported by Preyer, a much finer discrimination is shown. The differential threshold increases gradually from the tone of 64 vibrations to 512 vibrations, then decreases at 1,024 vibrations and again increases at 2,048 vibrations. The increase, except in the last, is slight. The judgment of differences appears to be somewhat more certain when the second stimulus is higher than when lower. Luft concludes that within the region from 64–1,024 vibrations the psychophysical law does not hold; on the contrary, the differential threshold within these intervals is approximately constant, being about .2 of a vibration.

The experiments of Delezenne, Preyer, Schischmanow and Stumpf on the least noticeable difference in the purity of interval, are not directly related to this investigation, hence they are not taken up.

Lorenz[1] used the method of mean gradations, which consisted of determining the position of a tone *m*, midway between two others *h* and *l*. That form of the method called by Wundt the irregular variation of the mean stimulus was employed. The apparatus was the Appunn tonometer. The experiments covered a range of five octaves from 32 vibrations to 1,024 vibrations, with possible differences of 4 vibrations, except between 32 and 128 vibrations, where the differences were 2 vibrations. The intervals were either harmonic or unharmonic. The method consisted in giving one standard, then the variable and again the second standard, with intervals of about one second. The time order was varied thus, *h*, *m*, *l* or *l*, *m*, *h*, etc. The subject was required to judge whether the variable stimulus represented the mean or was higher or lower than the mean. The results showed that the judgment of the mean was most accurate when the standard stimuli differed by no more than two octaves, and further that the estimation of equal distances is most certain when they correspond to harmonic intervals. The estimated mean was in each case very near the arith-

[1] C. Lorenz, *Philos. Studien*, 6: 26–103, 1890.

metical mean, and the conclusion was drawn that Weber's law does not hold for tones; on the contrary, that a direct proportionality between the absolute difference in tone sensation and the difference in rate of vibration obtains.

Meyer[1] made experiments by the method of right and wrong cases on highly practised subjects and especially on Professor Stumpf. The tones were produced by tuning-forks of 400, 600 and 1,200 vibrations. The pitch differences were not produced by riders, but a hole was bored in one prong of the fork and into this a steel screw fitted. The sound was produced by a wooden hammer covered by rubber or felt. The method of experimentation was to sound the standard fork before each experiment and later the variable. The interval between the stimuli is not given nor was it constant throughout the entire series of experiments. Whenever the subject desired it, the variable stimulus was repeated, the results therefore give a maximal discriminativeness rather than an average. The number of cases, further, was very small for the method of right and wrong cases. As far as the results go, they show the constancy in the absolute discriminativeness. Differences in discriminativeness such as Luft found, when the second tone was higher, did not appear. The results of the experiments on Professor Stumpf are summarized below. The table gives the differences in rate of vibration, the rate of vibration and the percentage of right cases.

	100	200	400	600	1200
D	%.	%.	%.	%.	%.
.35	71	83	80	84	67
.65	74	91	92	90	70

[1] Max Meyer, *Zeitschrift für Psychol. u. Physiol. der Sinnesorgane*, **16**: 352–372, 1898.

VI. CONCLUSION

The most important general conclusion to be drawn from this investigation is the confirmation of the validity of the method, of which it is believed unquestionable and ample evidence has been given. That the method is not only valid, but also of considerable value for the measurement of differences in sensation likewise appears.

The method has been applied in this study to the measurement of differences in sensations of color, linear magnitudes and pitch, together with an application to testing for color-blindness. The main results of the investigations in each of these fields will be briefly summarized.

(1) The experiments on the perception of qualitative differences in colors from yellow and orange to red, show that equal objective differences in this part of the spectrum are correlated with a marked increase in the differences for consciousness. The curve of increase in the time of perception agrees very well with that obtained by the methods of average error and just noticeable difference. The transition from orange to red does not show as rapid an increase as in the earlier results, due no doubt to the difference in conditions as to method of observation and as to area, under which those determinations were made. The curve of increase has been determined here by the psychometric method, and, further, those differences which represent equal differences for consciousness have been found for the subjects in the experiments.

Minor studies have been made in the measurement of differences between the various colors and black and between various pairs of the principal colors.

A method of testing for color-blindness based on the principle of the time of perception as a measure of differences in sensations is suggested and applied. If a person be red-green color-blind, it will take him a longer time to distinguish the reds and greens, than the blues and yellows. If we have a measure of the time of discrimination of these colors, we have a measure of the color sense of the one tested. The red-green color-blind person takes a much longer time to discriminate the reds and greens and thus discloses his defect. The test has all of the advantages of the methods based on confusion, and in addition provides for determining deficiencies which escape these methods. It serves, moreover, as a convenient measure of reduced color sense and of individual differences.

(2) The experiments on linear magnitudes with equal absolute differences in stimuli show that as the differences between the pairs

of stimuli decrease in an arithmetical progression the differences in the time of perception increase approximately in a geometrical progression. In this generalization it is not implied that this relation holds except for the region investigated.

With equal relative differences in the lengths of lines the times of perception tend to remain constant, indicating the approximate validity of Weber's law for the range of lengths used. As the absolute differences decrease, however, the time of perception increases and the law holds only approximately even for the small extent covered by these experiments.

These experiments illustrate, it is believed, the value of the method in psychophysics.

(3) The experiments on discriminativeness of differences in pitch bring out, prominently, individual differences, and show the value of the method for the measurement of such differences. The absolute discriminativeness does not remain constant, as the results of other investigations have seemed to show; on the contrary, the time of perception increases more rapidly than the decrease in differences in pitch. The exact nature of this increase is not entirely made out. There is apparently a region varying with individuals in which the absolute discriminativeness is constant, and beyond this a sudden decrease. Further experiments would be necessary to disclose this. A comparison of differences in various sense departments, which this method makes possible in a definite manner, shows that in a general way a difference of 4 vibrations in pitch is relatively as great as a difference of ½ mm. in lines, or again that a difference of one vibration in pitch is approximately equivalent to two wave-lengths in color in the red end of the spectrum.

In conclusion, I desire to express my great obligations to Professor Cattell, who suggested to me the problem, and who has followed my work with kindly interest and with many helpful suggestions; to Professor R. S. Woodworth and Professor E. L. Thorndike, for valuable aid; to Mr. F. L. Wells (W), of the Department of Psychology, who has kindly acted as subject and recorder in many of my experiments at a great sacrifice of time, which experiments such as these require; to Mr. A. R. Shelander (S), of the Department of Philosophy, who has likewise kindly acted as both subject and recorder; and to the many others who have assisted me as subjects, or otherwise, in this investigation. Without their valuable assistance this study would have been impossible.

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