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SIBILITY OF POINT SOURCES OF LIGHT

BY

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CONTENTS.

THE consideration and observation of point sources of light has generally been concerned either with—

(*a*) The observation of stars ; or—

(*b*) The observation of distant lights on land and on sea.

The solution of problems connected with either of these fields of work has been the object of most of the investigations into this subject which the authors have been able to trace.

In attempting to classify stars according to a scale of magnitude astronomers have devised many and various forms of apparatus, but the work is not in any way quantitative, one star being compared against and expressed in terms of another, and thus the results of their work are recorded in terms of an arbitrary standard of magnitude.

Some accounts of the most recent work on this subject can be found in the records of 'The Astrophysical Journal,'* and a particularly full account of, and comparison between, German and American results has been given by Prof. PICKERING.†

The observation of signal lights at sea has for many years played an important part in the safe navigation of ships, and the appreciation of distant lights in connection with railway work is a matter of great importance. The laws governing the degree of appreciation of these lights by the human eye have been the subject of several researches. In the laboratory small illuminated apertures have been used by most observers, both in experiments on the visibility of lights as distinct from their colour,

* Vols. ii., iv., xiii., and xxxi.

† Part II., vol. xiv., 'Harvard College Observatory Annual.'

and in experiments devised to test the eye for colour vision. But although so much qualitative work has been done, the authors have been unable to find any reference to work where absolute measurements have been made, in terms of a reproducible standard, of the visibility of a point source of light. From the time of HELMHOLTZ many workers on the Continent have been interested in the visibility of periodic lights, and have experimented with a view to determining to what extent the visibility depends on the time of duration of such a periodic light. Some of the more complete investigations are those of BROCA and SULZER,* and since the completion of the authors' experiments a paper has been published by MM. BLONDEL and REY† in which is discussed the appreciation by the eye of lights of short duration.

This work is outside the scope of the present Paper and deals with a totally different problem, but it is interesting to note that in this Paper reference is made to the limit of visibility or perception, which is given as $\frac{1}{10}$ candle at 1 km. distance for a dark night in the open, or 0·5 to 0·6 of this for observation in a laboratory where all stray light is excluded.

This agrees approximately with the figures obtained by the authors, but it is, of course, a variable quantity depending on the acuity of the eye of the observer.

A short account is given by MM. ANDRE BROCA and POLACK‡ of some experiments made on illuminated pinholes, with a specially constructed apparatus, with a view to determine how different coloured signal lights might be distinguished even though so faint as to make it impossible to distinguish the colour. Their figures do not enable the value of limiting visibilities of the various coloured signals to be computed, but the generalizations given in this Paper have been supported by the author's experiments, and will be further emphasized later.

The Deutsche Seewarte carried out experiments with a view to finding out the minimum candle-power for ships' signal lights, and their report was issued in 1894. Their units for green and red light were a Hefner candle with a green and red glass placed respectively in front of it. No data are given as to the percentage transmission of these glasses, and therefore it has been impossible to compare the results with those of the present authors. It may be deduced from a formula given in the report that for very clear weather 2·1 Hefner candles (white) are required for visibility at two sea miles, and 13·1 Hefner candles for five sea miles.

The suitability of a light for signal purposes can, of course, always be tested by the direct method of setting up the light in the desired position, and ascertaining if it is sufficiently visible at the range of distance for which it is designed. This is the method which has usually been employed to fix a standard of reference, but for the proper comparison of different lights some more universal method is wanted.

* 'Journal de Physiologie et de Pathologie générale,' No. 4. juillet, 1902.

† "Sur la perception des lumières brèves à la limite de leur portée," 'Journal de Physique,' juillet et août, 1911.

‡ 'Comptes Rendus,' November 11, 1907.

The property of light spoken of as its "visibility" needs to be defined in terms of some definite unit, and it should be possible to measure it by comparison with a standard whose value in terms of such unit is known. It has been the object of this work to ascertain if such a unit is generally applicable, and to construct an apparatus by means of which measurements of the visibility of point sources of light in terms of this unit can be made.

Definition of Point Source.—For the purpose of this Paper a point source of light is taken as one whose linear dimensions subtend an angle at the eye less than the resolving power of the eye, *i.e.*, about 30 seconds of arc for a mean wave-length $0 \cdot 5 \times 10^{-3}$ mm. and pupilar aperture $4 \cdot 5$ mm.

The most obvious step in commencing the work was to produce point sources of light whose candle-power could be measured and yet be small enough to be used over the range of the ordinary laboratory dark room, *i.e.*, from 2 to 10 m.

The pinholes were constructed having diameters varying from 0·01 mm. to 0·20 mm. They were pierced in circular brass plates about 2 cm. diameter and 2 mm. thick. The plate was first deeply countersunk on one side, and then rubbed down on the other until the metal at the base of the countersink was very thin. This thin metal was finally pierced by means of needles ground under the microscope—the piercing being also done under the microscope.

These pinholes were mounted in front of a flame so as to be fully illuminated. A very satisfactory flame was found in that given by a lamp sold by the Vacuum Oil Company burning a wax known as "Vaclite." This lamp will give a steady, constant light for several hours without any tendency to smoke, and the wick chars very little.

The candle-power per square millimetre of the white portion of the standard flame used was obtained by measuring the candle power of several small areas of the flame, the smallest of which was 13·5 sq. mm. area. From these measurements it was found that the candle power per square millimetre of the flame was $6 \cdot 6 \times 10^{-3}$.

In the earliest experiments the pinholes were mounted on a metal plate placed in front of the lamp and held in place by spring clips. This enabled changes of pinhole to be rapidly made. The pinholes were viewed direct, no optical arrangements being used. Later a visibility photometer was constructed for use with one eye. The principle was the same as that of a later binocular photometer, although the latter is improved in technical details. The photometer for use with one eye was discarded in favour of the binocular, because the method of employment of the latter more nearly approached the ordinary method of viewing distant lights.

A photograph of the binocular visibility photometer is shown in fig. 1, and fig. 2 is a sketch of the general arrangements of the photometer. The instrument consists of a brass tube A, about 2 m. long, at one end of which are placed absorption wedges B and the pinhole, this latter being held on the wedges by spring clips. The pinhole is illuminated by the standard vaclite lamp C, which is contained in a box arranged with suitable ventilation. The object of the box is to enable the instrument to be used in the open without the wind causing the flame to flicker. At the end of the tube are placed two adjustable mirrors. In one, D, is seen the image of the standard point source, and the other, E, can be arranged to bring the image of the distant source into the same field of vision. To enable the observer accurately to centre his eyes, the

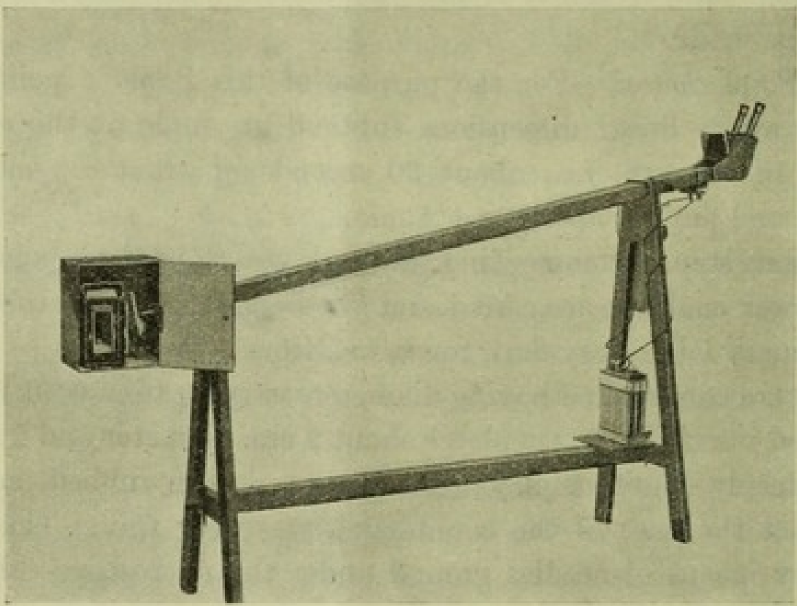


Fig. 1. Binocular visibility photometer.

tubes, F, are capable of a sideway adjustment to suit the distance between the eyes of the observer and of sufficient diameter to enable a full pencil of light to enter the eye.

At G is placed a small metallic filament lamp, suitably screened, by which the field

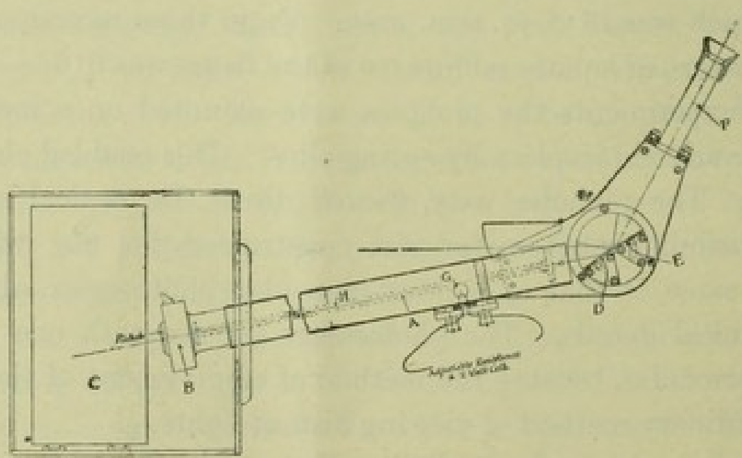


Fig. 2. Diagram of visibility photometer.

surrounding the standard point can be slightly illuminated This was found necessary for outdoor work and is dealt with later.

The visibility of the standard is varied by sliding the wedges relatively to each other. The motion is given to the wedges by means of the rod H and bevel wheels.

In using the apparatus the points of light were viewed in the mirrors, these being arranged so that the images of the sources were separated by a small angular distance. The observer has to look at one point and carry in his mind the memory of it whilst looking at the other. This procedure was necessary because of the considerable brightening of a point source when it is seen obliquely, and, that is to say, when its image does not fall on the centre of the *fovea centralis*. This effect becomes even more marked when the colours are different; in fact, it was found to be impossible to work with the points close together, so that they were viewed at the same time.

Having arranged the mirrors so that an image of each source can be seen, the standard source is varied by means of the wedges until it matches the distant source for visibility.

Before passing on to enumerate the experiments, it may be mentioned that in all the work hereafter described two point sources of light are matched for visibility. This is most easily done by imagining the points, while matching them, to be distant lights on the horizon. It is found that different observers, provided their vision is normal, will agree within close limits as to when lights are equally visible, even though the colours are quite different.

The match made here is purely a physiological one, and when of different colours depends partly, but not entirely, on the luminosity curve* for the eye of the observer.

The principal laws which will first be demonstrated are :—

I. That the visibility of a point source of light is independent of its intrinsic brightness and is proportional to its total candle power.

II. That the visibility of a given source varies inversely as the square of its distance from the eye of the observer.

I. Effect of Intrinsic Brightness.

Experiment 1.—The smallest illuminated pinhole was set up at about 2 m. from the eye, and others of different sizes in turn were set up at the same distance from the eye, but with an adjustable sector disc inserted in the path of the beam.

The sector opening was adjusted in each case till the visibility of the larger pinhole matched the smaller. The pinholes in this experiment subtended an angle of less than 10 seconds of arc, *i.e.*, below the resolving power of the eye. They were at the same distance from the eye, so that the measurements were not complicated by differences of accommodation. In this way a pinhole of large candle power has its brightness reduced until it matches the smaller, whose brightness remains constant. The following tables show the results obtained, and it is seen that when the candle powers of the two holes are equal they are also equally visible, their distances from the eye being equal.

* Sir W. de W. Abney, 'Phil. Trans.,' A, 193 (1900).

TABLE I.

(*a*) A pinhole for comparison purposes was set up at 2 m. distance. It had a candle power $= 5\cdot9\times10^{-6}$.

Pinhole normal candle power $\times 10^{-6}$.	Angle of disc open for equal visibility as reference point.	Intrinsic brightness (original brightness = 1·0).	Equivalent candle power of dimmed pinhole.	Error percentage difference in candle power from the reference source.
162×10^{-6}	$13\cdot_5$	0·037	$6\cdot1_0\times10^{-6}$	$+3\cdot_5$
$84\cdot_5\times10^{-6}$	$25\cdot_5$	$0\cdot07_1$	$6\cdot0\times10^{-6}$	$+1\cdot_5$
$38\cdot_2\times10^{-6}$	$58\cdot_5$	$0\cdot16_2$	$6\cdot2\times10^{-6}$	$+5\cdot_0$
$22\cdot_8\times10^{-6}$	$90\cdot_0$	0·25	$5\cdot7\times10^{-6}$	$-3\cdot_5$
$12\cdot_2\times10^{-6}$	$168\cdot_5$	0·47	$5\cdot7\times10^{-6}$	$-3\cdot_5$

TABLE II.

(*b*) Smaller pinhole used as comparison source candle power $= 3\cdot6\times10^{-6}$.

Pinhole normal candle power $\times 10^{-6}$.	Angle of disc open for equal visibility as reference point.	Intrinsic brightness (original brightness = 1·0).	Equivalent candle power of dimmed pinhole.	Error percentage difference in candle power from the reference source.
162×10^{-6}	$7\cdot_5$	0·021	$3\cdot_4\times10^{-6}$	$-5\cdot_6$
$84\cdot_5\times10^{-6}$	$14\cdot_5$	0·04	$3\cdot_4\times10^{-6}$	$-5\cdot_6$
$38\cdot_2\times10^{-6}$	$33\cdot_0$	$0\cdot09_2$	$3\cdot_5\times10^{-6}$	$-2\cdot_8$
$22\cdot_8\times10^{-6}$	$57\cdot_0$	$0\cdot15_8$	$3\cdot_6\times10^{-6}$	0
$12\cdot_2\times10^{-6}$	$108\cdot_0$	0·3	$3\cdot6_5\times10^{-6}$	$+1\cdot_4$

The last column in the tables indicates the order of agreement between the adjusted candle power and the candle power of the point of reference.

Wedges.—The sector disc, although giving the most absolute method of dimming down the intensity of the light without changing its character, is not suitable where a wide range of variation is desired, and in order to exceed the range over which the last experiment was conducted, the sector disc was replaced by absorption wedges. The wedges used were those belonging to a Féry pyrometer, and although slightly selective, the selectivity has not been found in any way to influence the accuracy of the work. These wedges slide relatively to one another and are fitted with a scale of position.

The wedges were calibrated by using them in the path of a beam of light from an arc, the candle power of the beam being measured. The measurements were made on an ordinary photometer bench using a Lummer-Brodhun photometer head. The wedges were removed from time to time and the candle power of the incident beam measured. In this way a curve of transmission for the wedges was obtained, connecting the relative position of the wedges to one another, and the percentage transmission of the light.

Experiment.—To prove the validity of substituting the calibrated wedges in the place of the sector disc, as a means of reducing the intrinsic brightness of the sources of light.

Illuminated pinholes of diameters varying from 0·03 mm. to 1·00 mm. were set up one after another behind the wedges in the standard apparatus. They were adjusted by means of the absorption wedges until they matched a comparison source for visibility in the manner described earlier. The results are shown in the following tables :—

TABLE III.

(*a*) Reference source candle power $= 4 \cdot 1 \times 10^{-6}$.

Area in sq. mm. of pinhole.	Pinhole candle power $\times 10^{-6}$ (for normal brightness).	Wedge transmission percentage.	Equivalent candle power $\times 10^{-6}$.	Error. Percentage difference from the reference source.
$\cdot 78_5$	4320	$0 \cdot 09_5$	$4 \cdot 1$	0
$\cdot 18_9$	1040	$0 \cdot 4_3$	$4 \cdot 4_5$	$+8 \cdot_5$
$\cdot 029_5$	162	$2 \cdot 6$	$4 \cdot 2$	$-2 \cdot_5$
$\cdot 015_4$	$84 \cdot_5$	$4 \cdot 4$	$3 \cdot 7$	$-9 \cdot_5$
$\cdot 0069_5$	$38 \cdot_2$	$10 \cdot_4$	$3 \cdot 9_5$	$-3 \cdot_5$
$\cdot 0034_5$	$18 \cdot_9$	$23 \cdot_3$	$4 \cdot 4$	$+7 \cdot_5$

(*b*) Reference source $= 1 \cdot 3_5 \times 10^{-6}$ candle power.

Area in sq. mm. of pinhole.	Pinhole candle power $\times 10^{-6}$ (for normal brightness).	Wedge transmission percentage.	Equivalent candle power $\times 10^{-6}$.	Error. Percentage difference from the reference source.
$\cdot 78_5$	4320	$0 \cdot 032_5$	$1 \cdot 4_0$	$+3 \cdot_5$
$\cdot 18_9$	1040	$0 \cdot 13_5$	$1 \cdot 4_0$	$+3 \cdot_5$
$\cdot 029_5$	162	$0 \cdot 7_9$	$1 \cdot 2_8$	$-5 \cdot_0$
$\cdot 015_4$	$84 \cdot_5$	$1 \cdot 5_2$	$1 \cdot 2_9$	$-4 \cdot_5$
$\cdot 0069_5$	$38 \cdot_2$	$3 \cdot 6_0$	$1 \cdot 3_8$	$+2 \cdot_0$
$\cdot 0034_5$	$18 \cdot_9$	$6 \cdot_8$	$1 \cdot 2_9$	$-4 \cdot_5$

Column 3 gives a measure of the intrinsic brightness of the source, the original brightness being represented by 100·0.

In the above test it is interesting to note that the largest pinhole subtended an angle of about 2 degrees, and was therefore greater than the generally accepted limit of the resolving power of the eye. But, in spite of this, it appears that no marked change is seen in the law connecting intrinsic brightness and visibility, although the area of one spot was 3075 times the other. Lord RAYLEIGH* has shown that for areas subtending considerable angles this law does not hold. The corresponding areas of the largest pinhole and the reference pinhole in Table III. (*b*) are shown to scale in the adjacent figure. These had equal candle power when appearing equally visible.

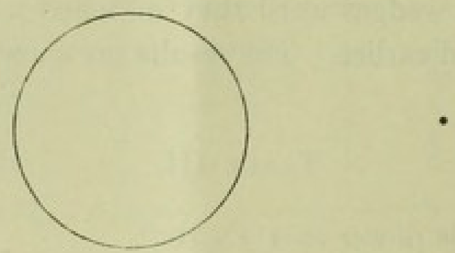


Fig. 3.

The object of giving the two tables is to illustrate the improved accuracy obtained by using rather a faint reference point.

The above experiments show conclusively that for sources subtending angles up to 2 minutes of arc the intrinsic brilliancy of the source does not affect its visibility, the total candle power being the only factor to be considered.

Further experiments were devised and carried out which confirmed the previous results. Lenses were used in the path of the beam from one point, about midway between the eye and the point of light. By this means the intrinsic brightness of this point was kept constant, but the apparent size of the point changed by changing the lenses. Matches made with the standard in the photometer showed that agreement to the order of the previous work was obtained between the theoretical visibility, calculated on the assumptions previously outlined, and the measured visibility.

II. THE INVERSE SQUARE LAW.

Experiment 1.—One pinhole was set up at about 3 m. from the eye and used as a point of reference. The other pinholes in turn were mounted on a travelling carriage and taken to distances up to 20 m., so as to be equally visible as the point of reference.

Standard of reference : $2 \cdot 5 \times 10^{-6}$ candle power, at $2 \cdot 9_5$ m.

Visibility assumed $\propto \frac{2 \cdot 5_0}{(2 \cdot 9_5)^2}$, *i.e.*, $\propto 0 \cdot 287$.

* 'Collected Papers,' vol. II., p. 92.

TABLE IV.

Pinhole candle power $\times 10^{-6}$.	Distance in metres (d).	Visibility $\left(\text{on assumption that Visibility} \propto \frac{1}{d^2}\right)$.	Error. Percentage difference between column 3 and 0·287.
$1\cdot2_2$	$2\cdot1_1$	0·274	$-4\cdot_5$
$6\cdot3_5$	$4\cdot7_5$	·282	$-2\cdot_0$
$16\cdot_2$	$7\cdot4_6$	·291	$+1\cdot_5$
$25\cdot_2$	$9\cdot0_5$	·308	$+7\cdot_5$
$30\cdot_5$	$10\cdot8_0$	·262	$-8\cdot_5$
$52\cdot_2$	$13\cdot2_5$	·297	$+3\cdot_5$
$112\cdot_0$	$20\cdot_4$	·270	$-6\cdot_0$

The agreement of the values in column 3 with that of the comparison source, viz., 0·287, is to be noted.

Experiment 2.—Greater distances were not obtainable in the Laboratory dark room so that the investigation was continued at night in the 550-ft. gangway of the Ship Tank at the National Physical Laboratory. Here the investigation was more complete and was carried out for red and green as well as white lights. The method of procedure was slightly changed when working on longer ranges.

A pinhole was set up as standard with the wedges placed in the beam from it, this giving a means of adjusting the visibility of the standard. Another pinhole was placed at various known distances, and the standard source varied until it matched the distant point for visibility. The visibility of the distant point could then be measured in terms of the size of the standard and the calibration of the absorption wedges.

It is slightly more difficult to match lights of different colours for visibility than lights of the same colour, the principal reason for this being the unequal behaviour of lights of different colour with regard to oblique vision. The authors found the same effects as those mentioned by BROCA and POLACK ('Comptes Rendus,' ref. cit.). A green point viewed slightly obliquely appears to have an intensity of four or five times that by direct vision, and a white point about twice. The green loses its characteristic colour and each point becomes less well defined. A red point becomes much less visible when viewed obliquely, and its colour is also indistinguishable when the light is near the vanishing point.

With regard, therefore, to matching green and white lights for visibility, the unequal brightening seen when looking from one point to another bothers the observer considerably, and his settings become more erratic. It is for this reason that the authors kept the points always at a considerable angular distance from each other, but this does not entirely avoid the above effect. The red is troublesome for a different reason. Very few people seem to see a red point source as a point unless it is very dull, the image always spreading out somewhat and appearing to have dimensions, causing the matching of white and red points to be more difficult than matching two spots of the same colour.

As previously mentioned, the authors tried viewing the two points to be matched simultaneously, but finally gave it up as being less accurate than the method of viewing each point separately, and carrying the memory of its visibility when looking at the other.

A method sometimes used was to shut out the point under observation by means of a screen before looking away from it, and to unscreen the other while the eye is focussed on the place where it will appear, thus avoiding any oblique sight of the points. When working over these long ranges it was necessary to change the size of the pinhole from time to time, owing to the great difficulty of making efficient matches with very bright points, and also to ensure keeping below the limit of the resolving power of the eye. When changes were made, two readings at that station were taken, one with the larger pinhole which served for greater distances, and one with a much smaller pinhole which was to serve for the shorter distances. In general the visibility at a change like this would be reduced or increased three to eight times, but the readings obtained always checked off within the errors of the experiments. This, of course, should follow, in view of the proof that visibility is proportional to the candle power alone.

The results of the tests are shown in Tables V., VI., and VII., where the visibility obtained by assuming the inverse square law is shown in column 3, and the percentage

TABLE V.

White light as distant sources ; white light standard.

Pinhole candle power × 10^{-6}.	Distance in metres (approximately).	Measured visibility in terms of the standard.	Visibility in terms of 10^{-6} candle power at 1 m. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
$101{\cdot}_5$	$9{\cdot}1_5$	$1{\cdot}1_6$	$0{\cdot}120$	$-\ 4{\cdot}_0$
$101{\cdot}_5$	$15{\cdot}_2$	$0{\cdot}41_5$	$0{\cdot}44$	$-\ 6{\cdot}_5$
$101{\cdot}_5$	$24{\cdot}_4$	$0{\cdot}18$	$0{\cdot}17_0$	$+\ 5{\cdot}_0$
1040	$24{\cdot}_4$	$1{\cdot}8_0$	$1{\cdot}7_5$	$+\ 3{\cdot}_0$
1040	$30{\cdot}_5$	$1{\cdot}1_0$	$1{\cdot}1_5$	$-\ 4{\cdot}_5$
1040	$45{\cdot}_7$	$0{\cdot}51_5$	$0{\cdot}51_0$	$+\ 1{\cdot}_0$
1040	$60{\cdot}_9$	$0{\cdot}27$	$0{\cdot}28_0$	$-\ 3{\cdot}_5$
1040	$76{\cdot}_2$	$0{\cdot}18_7$	$0{\cdot}18_0$	$+\ 4{\cdot}_0$
4320	$76{\cdot}_2$	$0{\cdot}76_5$	$0{\cdot}74_5$	$+\ 3{\cdot}_0$
4320	$91{\cdot}_4$	$0{\cdot}60$	$0{\cdot}52_0$	$+15{\cdot}_5$
4320	$122{\cdot}_0$	$0{\cdot}31_5$	$0{\cdot}29_0$	$+\ 7{\cdot}_5$
4320	$152{\cdot}_4$	$0{\cdot}19_8$	$0{\cdot}18_6$	$+\ 6{\cdot}_5$

error in the last column. These results are also plotted in figs. 3, 4, and 5, but in order to show the results effectually the candle power of each pinhole is multiplied up to equal the value of candle power of the largest hole, and its visibility at the various ranges at which it was observed is multiplied in the same ratio. The log of this derived visibility is plotted against the log of the distance. The line drawn indicates the close agreement to the inverse square law.

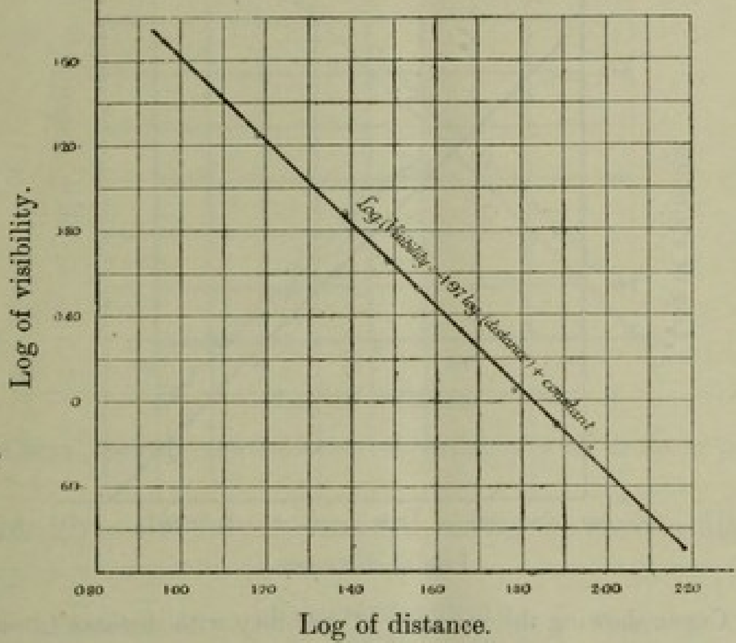


Fig. 4. Curve showing the variation of visibility with distance (white light). (This indicates the close agreement with the inverse square law.)

TABLE VI.

Green light distant source; white light standard.

Candle power × 10^{-6} green light.*	Distance in metres (approximately).	Measured visibility in terms of the standard.	Visibility in terms of 10^{-6} candle power at 1 m. Calculated assuming inverse square law.	Error. Percentage between columns 3 and 4.
8·1	$3\cdot0_5$	0·91	$0\cdot8_7$	+4·5
8·1	$6\cdot1_0$	$0\cdot22_5$	0·22	$+2\cdot_0$
$51\cdot_8$	$6\cdot1_0$	$1\cdot3_6$	$1\cdot4_0$	$-3\cdot_0$
$51\cdot_8$	$9\cdot1_5$	$0\cdot6_1$	$0\cdot61_8$	$-1\cdot_5$
$51\cdot_8$	21·3	$0\cdot11_9$	$0\cdot11_4$	$+4\cdot_5$
216	21·3	$0\cdot4_5$	$0\cdot47_5$	$-5\cdot_0$
216	$30\cdot_5$	$0\cdot2_3$	0·23	0
581	$30\cdot_5$	$0\cdot58_5$	$0\cdot62_5$	$-6\cdot_5$
581	$45\cdot_7$	$0\cdot28_5$	$0\cdot28_0$	$+2\cdot_0$
581	$60\cdot_9$	0·16	$0\cdot15_7$	$+2\cdot_0$
1215	$60\cdot_9$	0·36	0·33	$+9\cdot_0$
1215	$76\cdot_2$	$0\cdot20_5$	0·21	$-2\cdot_5$
1215	$91\cdot_4$	$0\cdot15_5$	$0\cdot14_5$	$+6\cdot_5$
3725	$91\cdot_4$	$0\cdot44_5$	$0\cdot44_5$	0
3725	$122\cdot_0$	$0\cdot24_5$	0·25	$-2\cdot_0$
3725	$152\cdot_4$	$0\cdot16_4$	0·16	$+2\cdot_5$

* The method of obtaining the percentage of light transmitted by a coloured glass is described later.

The slight variation from the square law in the case of the white light (fig. 4) is probably due merely to errors of experiment, and, in virtue of the results in figs. 5 and 6, must not be given weight.

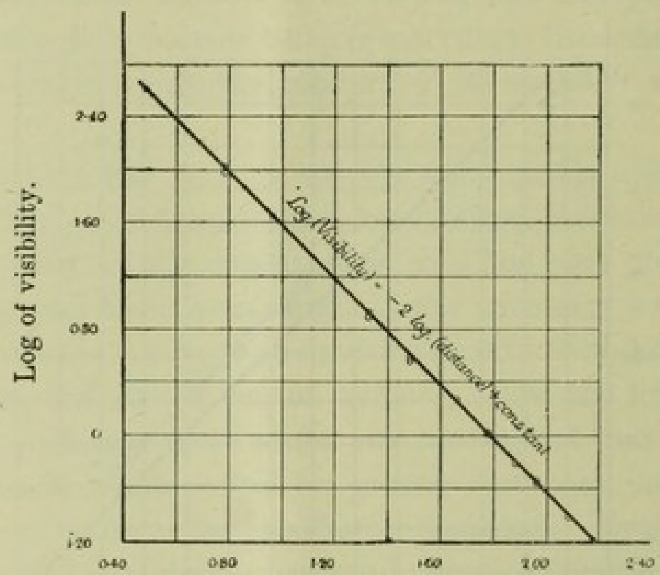


Fig. 5. Curve showing the variation of visibility with distance (green light).

Table VII.

Red light distant source ; white light standard.

Candle power × 10^{-6} red light.*	Diameter in metres (approximately).	Measured visibility in terms of the standard.	Visibility calculated in terms of 10^{-6} candle power at 1 m.	Error. Percentage between columns 3 and 4.
125	$9\cdot1_5$	1·54	1·5	$+2\cdot_5$
125	$15\cdot2_0$	$0\cdot54_5$	0·54	+1·0
125	21·3	0·26	$0\cdot27_5$	$-4\cdot_0$
125	$30\cdot_5$	$0\cdot13_5$	$0\cdot13_4$	0
519	$30\cdot_5$	0·54	0·56	$-3\cdot_5$
519	$45\cdot_7$	0·26	0·25	$+4\cdot_0$
519	$60\cdot_9$	$0\cdot14_5$	0·14	$+3\cdot_5$
2915	$60\cdot_9$	0·76	$0\cdot78_5$	$-3\cdot_0$
2915	$76\cdot_2$	$0\cdot48_5$	0·50	$-3\cdot_0$
2915	$91\cdot_4$	0·33	0·35	$-5\cdot_5$
2915	$122\cdot_0$	$0\cdot19_2$	0·20	$-4\cdot_0$
2915	$152\cdot_4$	$0\cdot12_2$	$0\cdot12_5$	$-2\cdot_4$

* The method of obtaining the percentage of light transmitted by a coloured glass is described later.

Experiment 3.—The result obtained in the last experiment has been confirmed by tests extending over a mile. Two small electric lamps—one a carbon filament and the other a tungsten—calibrated for candle power and current, were set up at various distances and their visibility measured by means of the visibility photometer.

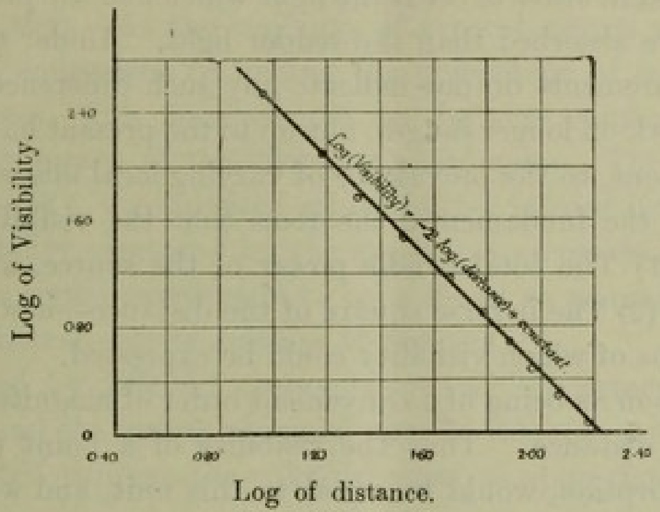


Fig. 6. Curve showing the variation of visibility with distance (red light).

The following table gives the values obtained, using the carbon filament lamp :—

Candle power of lamp.	Distance in metres.	Calculated visibility on inverse square law.	Measured visibility.	Error. Percentage.
0·18	483	$0·7_6$	$0·8_0$	+ 5
0·18	644	$0·4_3$	$0·41_5$	− 4
0·69	1098	$0·5_7$	$0·5_7$	—
0·69	1610	$0·26_5$	$0·24_5$	$-7·_5$

The following table gives the values obtained, using a metallic filament lamp :—

Candle power of lamp.	Distance in metres.	Calculated visibility on inverse square law.	Measured visibility.	Error. Percentage.
$0·18_5$	483	$0·7_8$	$0·7_4$	− 5
$0·18_5$	644	$0·4_4$	$0·4_6$	$+4·_5$
$0·7_0$	1098	$0·57_5$	$0·5_7$	$-1·_0$
$0·7_0$	1610	0·27	$0·2_5$	$-7·_5$

These readings were obtained on dark, clear nights, in April, with the moon behind dense clouds.

The Experiments, 1 to 3 under this section, confirm what would be expected from theory, that the visibility of a point source of light falls off proportionally to the inverse square of its distance from the eye of the observer. Incidentally it should be observed that Experiment 3 shows that, at least, up to ranges of 1 mile atmospheric absorption is not appreciable on a night which would ordinarily be called clear and dark. Similar results were obtained on a 1-mile range for green and red lights. The two different types of lamp were used in order to see if the light which had the preponderance of short wave-lengths was more absorbed than the redder light. Under the conditions of the experiment the measurements do not indicate any such difference. Several attempts have been made to work on longer ranges, but up to the present have been unsuccessful, due, among other reasons, to the prevalence of varying local mists.

Having established the fundamental theorems that the visibility of a point source is proportional to: (1) The total candle power of the source, and is independent of intrinsic brightness; (2) The inverse square of the distance—it was possible to choose a suitable unit in terms of which visibility could be expressed.

The unit decided upon as being of a convenient order of magnitude was one-millionth of a candle at 1 m. distance. Thus the visibility of a point of 1 candle-power at 1 km., neglecting absorption, would be equal to this unit, and would, by the eye, be considered equally visible. It is not suggested that a point whose visibility is equal to 2 is twice as bright to the eye as one of visibility equal to 1; but the former could have its candle-power halved or its distance increased $\sqrt{2}$ and be made equally visible.

Effect of Slight Illumination of the Field of Vision.—When observing in the open, the field of view on which the distant source is seen is rarely as black as that of the standard source, and thus in relation to the standard its visibility is diminished. In order to eliminate this disturbing factor it was found necessary to produce equality of illumination of the two fields of vision. This was achieved by illuminating the background surrounding the standard source by means of a high efficiency electric lamp, fitted into the photometer and suitably screened. The effect of observing a point of light against a faintly illuminated background was more fully investigated by arranging a point of light in the centre of a white screen, which could be illuminated to the same degree as the field of vision in the open.

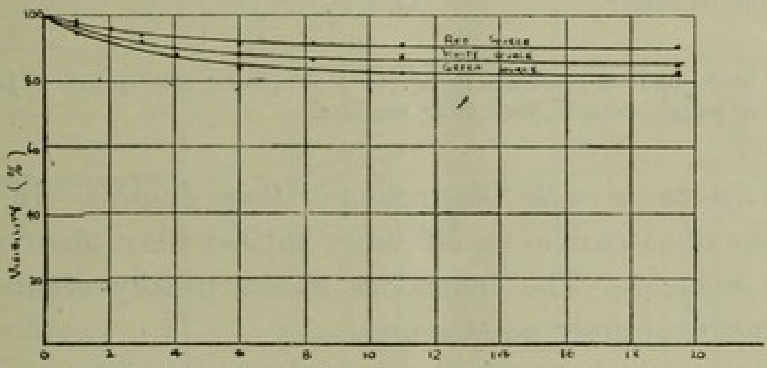
A very bright moonlight night, atmosphere clear, gave a field in the photometer whose illumination was equal to $\frac{5}{1000}$ of a metre candle. Slight mist increased the value of the illumination considerably, a value of $\frac{1}{100}$ of a metre candle having been recorded. Various pinholes were measured in this way for visibility when viewed in a black field and in fields of different illuminations.

Red and green lights were also observed, the results being plotted in fig. 7. The fact that the visibility appears to reach a constant value is rather unexpected, for undoubtedly, when working at a field illumination equal to 1 metre candle the point becomes nearly invisible. The above diagram, therefore, must only be considered as applying to very faint illuminations.

The dimming in each case was about 10 per cent., the green appearing slightly dimmer and the red brighter than the white source, when the field had an illumination equal to $\frac{4}{1000}$ metre candle. This may be considered about an average value for practical observation of lights on clear nights.

Coloured Glasses used in Signal Lanterns.—In connection with signal lights, in which coloured glass screens are placed before a white flame, it is often required to determine by how much the visibility of a point source is reduced by placing a coloured glass screen in front of it. As the visibility of a point source varies directly as the candle power, this is equivalent to finding the percentage of total light transmitted by the coloured screen. This measurement can be very conveniently and comparatively easily made by means of the visibility photometer. The visibility of a white point source is measured ; the coloured screen is then placed before the white source and the reduced visibility measured by the apparatus as previously described. Some green glasses used for signal purposes have been found to transmit only 5 per cent. of the light, whilst others transmit as much as 15 per cent.

The results obtained by different observers are in close agreement so long as they have normal vision, and the figures obtained may be taken as representing the fraction of the light transmitted through the coloured glass, as appreciated by an average eye



Illumination in $\frac{1}{1000}$ metre candles.

Fig. 7. Curves showing the loss of visibility of a point source of light when observed on a slightly illuminated background.

when observing a point source. Cases of defective vision are discussed later. The application of the laboratory tests to predict the visibility of lights in the open over long ranges has been generally confirmed by such tests as the following, made by different observers.

A visibility which was considered as low as could be regarded as satisfactory for practical purposes was fixed in the Laboratory in terms of a white point source of known visibility. A green light was then set up in the open, and observers moving away from the lamp judged when it could be regarded as equally visible. The candle power of the lamp without its green glass screen was measured simultaneously by means of a portable photometer.

Calculating the visibility of this light from the value of the transmission of the green glass determined in the manner described, it was found to agree within narrow limits with the visibility previously determined in the Laboratory. The visibility thus found as the lowest which was considered desirable for practical purposes was $0 \cdot 12 \times 10^{-6}$ candle power at 1 m. distance. Using this figure, the candle power of white light necessary for visibility at any range can be calculated, and by determining the transmission of a glass in the way described earlier the required candle power of the light behind the glass can be also calculated for visibility at any range.

It is interesting to note that the authors' results agree fairly closely with those calculated from the Report of the Deutsche Seewarte of 1894 (ref. cit.) :—

Range.	Candle power (British) of white light required in clear air (from report of Deutsche Seewarte).	Candle power of white light (British units) calculated from results of the authors' experiments.*
1 sea mile = 1855 metres approximately	0·47	0·41
2 sea miles = 3710 metres approximately	$1 \cdot 9_0$	$1 \cdot 6_0$
5 sea miles = 9275 metres approximately	$11 \cdot_8$	$10 \cdot_0$

* No allowance has been made in column three for atmospheric absorption, a factor which probably comes in on the five-mile range, even in very clear weather.

Effects of Using Spectacles in the Observation of Point Sources.—In the course of the work previously described curious results were noticed when observations were made by people using spectacles. The anomalous results usually occurred when making visibility measurements of green point sources.

Experiments were undertaken with a view to determining the cause of such results, but they are not in any way exhaustive. The actual results, however, are given, together with what appears to be an explanation of the phenomena.

Experiment to ascertain the change in the visibility of a point source when viewed through lenses used as spectacles.

The visibility photometer was slightly modified for use in these experiments. The box containing the mirror and vision tubes was removed and an arrangement fitted on top of the standard tube so that a source could be viewed through spectacle lenses. The standard pinhole and wedges were used to measure the visibility of this source. Plain glass was put in the path of the beam from the standard pinhole to balance the absorption and reflection losses at the surface of the lens.

(*a*) Two white point sources were used and the visibility of one measured when viewed through spherical spectacle lenses. The distant source was observed when 15, 9, and 1 m. away. The results are plotted in fig. 8. In all the work with lenses hereafter described, before plotting the results, the observed readings were corrected for the magnification that is produced when an object is observed through lenses. This was only appreciable on the short range of about 1 m., and even then only amounted to 5 per cent. as a maximum correction. Too much emphasis must not be placed upon the actual shape of the curves, but their general form can always be repeated.

The negative lenses have a much less marked effect than the positive, except at the very short range when the curve is nearly symmetrical.

(*b*) In the same figure are shown plotted the results obtained when cylindrical lenses were used instead of spherical, the range in the experiment being 8·5 metres.

It will be seen that the points fall approximately on the curve that was obtained for the visibility of a source 9 m. distant, viewed through spherical lenses; and, in general, it was found that the cylindrical lenses produced the same dimming as the corresponding spherical. When looking at a point at a greater distance than 5 m. the normal eye is almost as negative as it can be, and thus, on adding a positive lens, little or no equivalent negative accommodation can be made, and the focussed image no longer falls on the retina. The curves show that a large dimming occurs even for so small a positive lens as 0·5 diopters. If a negative lens is added the eye can accommodate somewhat, and the visibility falls off less rapidly. When observing a point within the range of accommodation it is seen from the curves that a similar

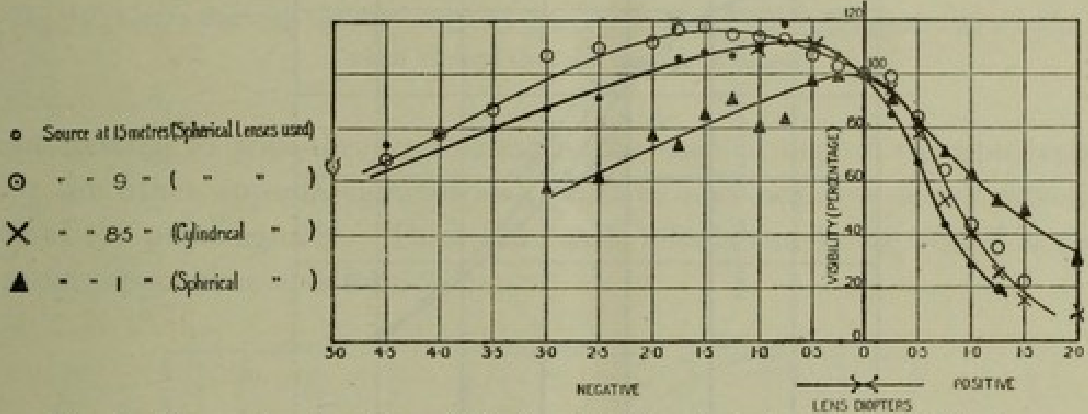


Fig. 8. Curves showing the variation in visibility of point sources of light when viewed through spectacle lenses.

effect occurs. The eye can accommodate a little for the positive lens, and the visibility diminishes less rapidly than at longer ranges, whilst the eye being unable to accommodate positively to such a large extent as before, the negative lenses produced a more marked effect.

The supposition that the dimming was due to the image not being focussed on the fovea was not supported by the appearance of the points of light. They appeared quite well defined, even when viewed through a powerful lens. It was subsequently found, however, that the dimming *was* actually due to the spreading of the light owing to want of focus, but that with the faint sources employed the diffused light was not apparent. If a very bright point is observed under the same conditions the diffused light can be seen without difficulty. The above conclusion was arrived at as a result of the following experiments :—

Spherical Aberration.—It was thought that the spherical aberration of the eye might produce a diffused spreading of the light, especially as the aperture of the eye becomes as large as 5 mm. after working for some time in the dark. In view of this, an experiment similar to the last was carried out, but the aperture of the eye was varied from 2 to 5 mm. by placing stops in front of the pupils of the eyes. To prevent confusion, only the results for 2 mm. and 5 mm. aperture are plotted ; the results for apertures 3 mm. and 4 mm. lie in the neighbourhood of the curves shown. The results are given in fig. 9 and need no comment.

The dimming is not reduced to any appreciable extent by using the smaller apertures. From the experiments it seems conclusively shown that the dimming is due to the image on the retina being thrown out of focus, and that the diffusely spread light around the nucleus of the image is not appreciated, because it is so faint. The cause of it, however, does not appear to be spherical aberration. It was noted in the course of the work that differences in visibility were obtained chiefly by observers who used

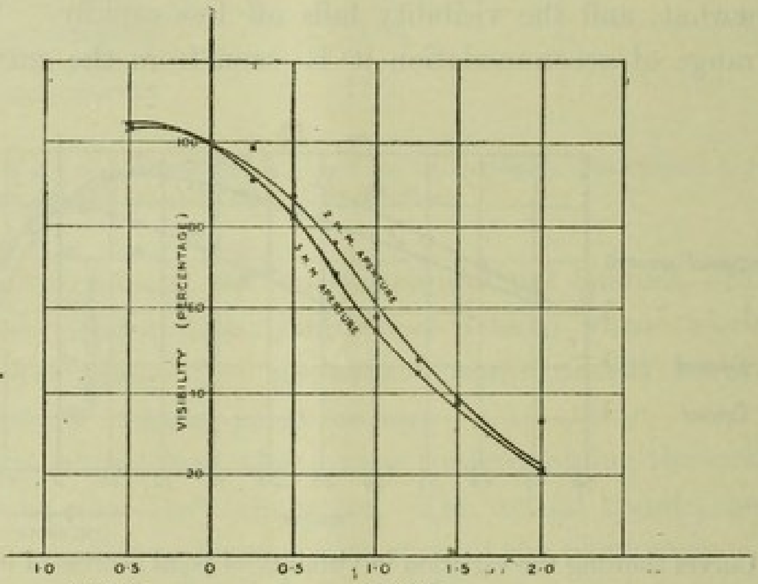


Fig. 9. Curves showing the effect of reducing the pupilar aperture of the eye.

spectacle glasses when viewing coloured lights. It was found that an observer who used positive cylindrical glasses to correct astigmatism saw much less of a green point source than when he observed it unaided by glasses, although without them it was much less well defined. The same observer viewing the green point through a negative cylindrical lens to correct his astigmatism saw no less and sometimes more than he did without the spectacles.

Chromatic Aberration.—To investigate this effect more fully measurements of the dimming of the green and red, relative to white, were made.

The visibility photometer was used in its original form, both points being received in their respective mirrors ; cylindrical lenses were placed at the top of the vision tubes and used when viewing both points. The standard point was the white one, and it was always varied by means of the wedges until it matched the other point under observation. It was found that the green point always appeared to be dimmed, relatively to the white one. The amount of this dimming is shown and plotted on fig. 10. Here, again, the positive lenses produced the greatest dimming, the point observed being outside the range of accommodation. Fig. 11 shows the behaviour of red light relative to white for each of the authors.

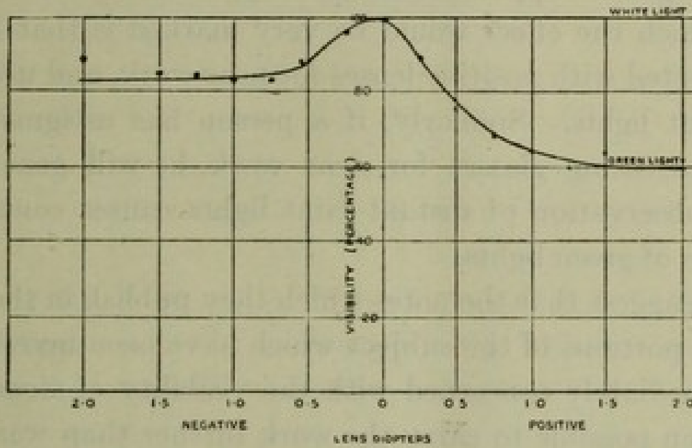


Fig. 10. Curve showing the dimming of a green point source of light relative to a white when viewed through spectacle lenses.

It is interesting to note the brightening in the case of one of the observers. The curve for the other appears symmetrical, positive and negative lenses affecting the visibility of the point equally. These chromatic effects can be explained in the main on the same supposition as before.

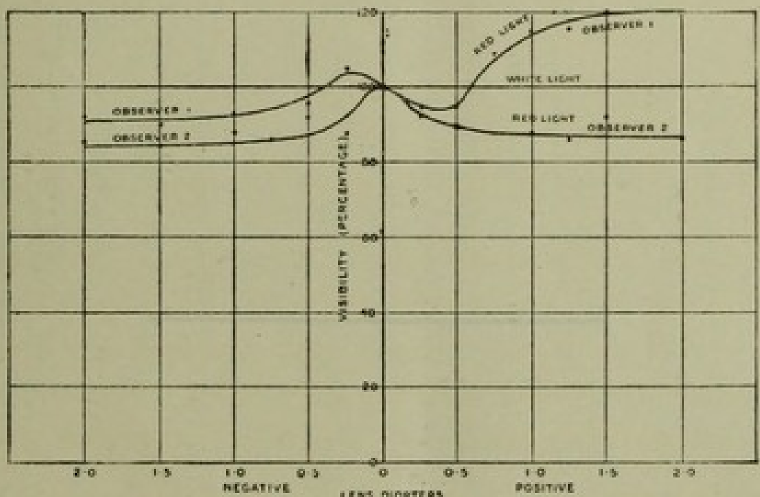


Fig. 11. Curves showing the dimming of a red point source of light relative to a white when viewed through spectacle lenses.

The important point to be borne in mind is that if a person is observing a green point at such a distance that he cannot accommodate any more in the negative direction, the use of a positive lens either spherical or cylindrical will cause the point to become very much less visible, viz., for instance, to the extent of 50 per cent., for a 1 diopter cylindrical lens. An equally powerful negative cylindrical lens would not produce serious dimming, but possibly would produce a brightening. This is of considerable importance in connection with the use of spectacle lenses by men to observe signals, and points to the necessity of testing to what extent such an observer can see a distant light, and how much light he appreciates relatively to an observer with normal eyesight. The case in which the effect would be very marked is that in which a person with long sight is corrected with positive lenses for near work, and uses these spectacles when observing distant lights. Similarly, if a person has astigmatic sight and has positive cylindrical correcting glasses for near work, he will generally find that the use of these for the observation of distant faint lights causes considerable dimming, particularly in the case of green lights.

The authors do not suggest that the notes which they publish in the Paper are in any way exhaustive. The portions of the subject which have been investigated are merely those which were immediately concerned with the visibility of steady signal lights at sea, and it has not been possible to carry the work further than was necessary for the solution of the practical problems in hand.

In conclusion, they desire, in connection with this work, to express their acknowledgments to Dr. Glazebrook, Director of the National Physical Laboratory.