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The New Flicker Photometry.

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THE NEW FLICKER PHOTOMETRY.

By F. L. Tufts.

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Historical Introduction.

The methods at present used to compare the luminosities of two sources of light nearly all depend upon the ability of the eye to detect very slight differences in the luminosities of two optically adjacent surfaces. The following is, in outline, the general method employed.

Two optically adjacent surfaces are illuminated, the one by a source of light, A, the other by a source B, to be compared with A. The light radiated to its surface by B is then varied according to some known law until the two surfaces appear to the eye to be of the same luminosity. The ratio of the luminous intensity of B to that of A can then be calculated according to the law of variation used.

The many forms of photometers in which the general method just outlined is used differ as to the nature and position of the luminous surfaces and also as to the method employed to vary the amount of light radiated to these surfaces.

Bouger* used two white surfaces illuminated respectively by the two sources of light to be compared. The surfaces were at different distances from the observer, but so placed as to be optically adjacent. The distance between one of the surfaces

* Essai d' Optique. 1729.

and its source of light was then varied until the two surfaces appeared to be of the same luminosity. The relative intensities of the two lights were then calculated by the law of inverse squares.

Lambert * made use of the principle of two shadows to obtain two adjacent surfaces, one illuminated by a source A, the other by a source B, to be compared with A. The distance of the light B, from the screen was varied until the luminosities of the two shadows appeared to be the same. The ratio of the luminous intensities of the two lights could then be calculated by the law of inverse squares.

Rumford † used the same principle as Lambert but made many improvements in its application.

Potter ‡ modified Bouger's photometer by replacing the opaque white screens by a single translucent one and putting the lights to be compared behind the screen.

Richie§ made still further changes in the Bouger photometer. The lights to be compared were placed at opposite ends of a table and two mirrors forming a wedge, the faces of which were inclined at angles of forty-five degrees to the line connecting the lamps, reflected the light to a translucent screen placed in front of the wedge. The next modification in this form of photometer was to replace the mirrors by a wedge of an angle of ninety degrees, covered with white paper. The two faces of the wedge occupied the positions of the two mirrors and the translucent screen was dispensed with.

In all the photometers above referred to, the dependence upon what I will call the principal of equal luminosities of adjacent surfaces is very evident. In nearly all the boundary between the two surfaces is quite distinct even when equality of illumination is obtained. This detracts from the accuracy with which slight differences of luminosity can be detected. To avoid this source of error a number of devices have been resorted to, all of which more or less conceal the use of the principle of equal luminosities. A slight inspection of the photometer, however reveals it.

Pernot ‖ increased the accuracy of the Rumford photometer by using a translucent screen and viewing the shadows from behind it. Instead of judging directly concerning the equality in luminosities of the two shadows, he brings a third source of light up from behind the screen and if both shadows disappear

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* Photometria Sine de Mensura et Gradibus Luminis, Colorum et Umbræ. 1760.
† Philosophical Transactions of the Royal Society of London. Vol. LXXXIV. 1794.
‡ Edinburgh Journal of Science. New Series, III, page 284.
§ Annals of Philosophy. Third Series. Vol. I. Page 174. 1826.
‖ Dinglers Polytech. Journal, CXIX. 1851.

at the same time they are equally luminous. The shadows disappear when the difference of illumination between that portion of the screen occupied by them and the adjacent portion is inappreciable and the method evidently depends upon the ability of the eye to detect slight differences in luminosity.

The Bunsen photometer is the type of a large number of photometers depending upon the disappearance of a spot on the illuminated surface when equality of illumination is obtained. The Bunsen screen originally consisted of a disk of white paper having a central translucent grease spot. As originally used, a compensating light was placed at a certain distance behind the screen. One of the two lights to be compared was placed in front of the screen at such a distance as to cause the central spot to vanish. The distance from the light to the screen having been measured, this light was replaced by the light to be compared with it and the distance adjusted until the spot again vanished. The relative intensities of the two lights can then be calculated from the law of inverse squares. The condition for the disappearance of the spot is, evidently, that that portion of the disk shall be of the same luminosity as the adjacent portions and the accuracy of the measurements depends upon the accuracy with which the eye can detect slight differences in luminosity. That it is a comparison of luminosities is made very evident if the light on one side of the screen is of a different color from the light on the opposite side. Under this condition the spot can not be made to disappear.

The polarization photometers of Arago,* Babinet,† and Wild ‡ still further disguise the principle of equal luminosities. The following is a brief outline of the method employed in this class of photometers.

The light from one of the sources is polarized in a given plane and the light from the other source in a plane perpendicular to this. The two rays after polarization travel in the same direction, and by viewing the combined ray with suitable apparatus, certain phenomena are observed which disappear when the quantities of light polarized in planes perpendicular to each other are equal. With photometers of this class, as with the other classes mentioned, the investigator seeks ultimately to obtain a uniform field, and it is, therefore, evident that the accuracy depends, as in the other photometers considered, upon the accuracy with which the eye can detect slight differences in luminosity. The polarization photometers are open to the same objection as the

* Comptes Rendus, XIII., pages 840, 967. 1841.
† Comptes Rendus, XXXVII., page 774. 1853.
‡ Pogg. Ann., XCIX.

others when the two sources of light differ in color. Under this condition it is impossible to obtain a uniform field.

The accuracy of the photometric methods, so far considered, is dependent upon the sensitiveness of the eye to slight inequalities of illumination. Various experiments * for the purpose of determining the degree of sensitiveness show that for white light the eye can detect differences in luminosity of from 3 per cent. to about 0 6 per cent. of the total luminosity of the field, the sensitiveness being less for high and low illuminations than for a field of medium luminosity. If, however, the surfaces compared are of different colors, a judgment concerning the equality of their luminosities is subject to a very much greater error. Indeed Helmholtz † remarks:

"I must explain that personally I put no confidence in my judgment concerning the equality in luminosity of differently colored surfaces. I admit, however, that of two differently colored fields one can be so much darkened that there remains no doubt that the other is brighter.

"For myself I have the impression that in the comparison of the luminosities of differently colored fields I am not dealing with a single magnitude but with the effect of two, luminosity and color, for which I do not know how to form a simple sum, and which I cannot scientifically define."

There have, nevertheless, been many attempts made to compare directly the luminosities of differently colored surfaces. These attempts have, in general, led to slight modifications in the use or construction of the photometers belonging to the types already described.

The difficulty of judging when two differently colored surfaces are of the same luminosity, a difficulty encountered in the use of photometers belonging to any of the classes above described, is, to some extent, obviated by taking the mean of two measurements. A measurement is taken when one of the surfaces is evidently more luminous than the other, another measurement when it is less luminous by the same amount. The average of the two measurements gives a determination of the luminosity. This method was employed by Professor O. N. Rood in measuring the luminosities of colored discs.‡

Another method very commonly used is to continually vary the luminosity of one of the surfaces so that it will be alternately darker and lighter than the other, the amounts of the variation being gradually diminished. By this means the observer can

* Grundzüge der Physiologishen Optik von Hermann Aubert. Page 488. 1876.
† Handbuch der Physiologishen Optik. Second edition, page 440.
‡ American Journal of Science, 1878.

more easily form a judgment concerning the equality of the two luminosities.

Mace de Lepinay and Nicati in their experiments* used a Rumford photometer in which the surfaces to be compared were very small. The angle subtended by them at the eye was about forty-five minutes. They found that the accuracy with which the luminosities of differently colored surfaces could be compared was thus increased. The results, however, were now found to vary with the absolute size of the retinal image.

Professor Mayer† uses the effect of simultaneous contrast to destroy the difference in color between the two surfaces to be compared. This method, however, is only applicable when the difference in color is not very great.

The methods of Virerordt‡ and Draper§ avoid the necessity of comparing the luminosities of two differently colored surfaces. The validity of the methods, however, depend upon the assumption that the quantity of light which must be added to a luminous, surface in order to be just perceptible is, under all conditions, the same fraction of the total luminosity. The assumption is no longer considered valid.

I will now consider the photometric methods which do not depend upon what I have called the principle of equal or uniform luminosity. The first of such methods used depended upon the extinction of the source of light by the introduction of translucent media. This method seems to have been employed as early as 1700 by Francois Marie. He introduced pieces of glass between the light and the observer until the light was extinguished. The value of the luminosity was then judged by the number of pieces of glass used. The same method but with various refinements comes up again and again in the history of photometery. To all such photometers there is this serious objection, that the limit of sensitiveness of the eye to light is so variable that the error thus introduced may be three or more times the quantity to be measured.

A photometric method depending upon the distinctness of vision was first suggested by Celsius (1701–1744). It has been very carefully worked up by Maci de Lepinay and Nicati.‖ As a measure of visual acuteness ("accuete visuelle") they take the angle subtended at the eye by a black character, as a printed

* Annales de Chimie et de Physique, 5th series, Vol. 30, page 151. 1883.
† American Journal of Science. Vol. 46. 1893.
‡ Pogg. Ann. 1869.
§ Phil. Mag. 5th series. Vol. VIII.
‖ Annales de Chimie et de Physique. 5th series, Vol. 24, page 30, and Journal de Physique 1882-1883.

letter, upon a white ground when seen with the greatest distinctness. In the first part of their research they regarded two quantities of light as equal if, when illuminating successively the same object (black udon a white ground) placed at the same distance from the observer, the details could be distinguished with the same minuteness. The ordinary definition of equality is, that two quantities of light are considered equal if, when illuminating two portions of a given white surface, the two portions appear of the same luminosity. Their investigations showed that the two definitions are not equivalent, therefore photometers in which the method of distinctness of vision is employed do not give correct results if we adopt the latter definition of the equality of two lights.

Schafhautl * introduces a still different factor in photometric measurements, viz., the time interval which may elapse between two equal light impulses without the eye noticing any interruption. His photometer consists essentially of a pendulum to which is attached a black screen with a single opening in it. The source of light is placed behind the screen and the opening observed through a binocular tube. The light can only reach the eye when the epening in the screen is opposite one of the openings in the binocular tube. The interval of time elapsing between the two impressions can then be regulated by changing the effective length of the pendulum. The intensity of the source of light, he assumes, is inversely proportional to the square of the times of swing when there is no flickering, or to the fourth power of the length of the pendulum. No data were attainable from which the accuracy of the method could be determined.

Charpantier † employs a method for determining the luminosity of colored lights which obviates the difficulties introduced by differences in color. He makes use of a phenomenon previously pointed out by Landolt that any source of colored light, if sufficiently weakened, gives rise to a sensation of gray. Charpantier decreases the illumination until all sensation of color vanishes and then compares the luminosities. It has, however, been pointed out by Purkinje that as the illumination is decreased the luminosities of the colors in the more refrangible parts of the spectrum are increased relative to those in the less refrangible part, therefore determinations of relative luminosities made under feeble illumination would not hold under normal conditions of illumination.

In the *American Journal of Science* for September, 1893, Professor Rood describes a photometric method which is indepen-

* Münchener Abhandl. Vol. VII. 1855.

† Comptes Rendus, LXXXVIII., page 301. 1879.

dent of color. The method depends upon the shock which the retina experiences when one surface is withdrawn from view and replaced by another of different luminosity. If, for example, a disk, one-half of which is of different luminosity from the other, is rotated before the eye a flickering is observed with appropriate rates of rotation. Professor Rood states that the flickering disappears if the two halves are of equal luminosity, no matter what their differences in color may be. For the determination of the luminosity of a colored disk a series of one hundred gray disks were used. The gray disks of the series varied quite uniformly in luminosity from the luminosity of white bristol board to the luminosity of lamp black. The luminosity of a given colored disk was then determined by successively combining it "pair-wise" with various gray disks until one would be found with which it would give no flicker on rotation. The luminosity of the gray disk could then be determined in the ordinary way and this would represent the luminosity of the colored disk. The number thus obtained to represent the luminosities of the different colored disks were controlled in the following way: Two disks of complementary colors were selected. Their luminosities were measured by the flicker method. These disks were then combined so as to form a gray on rotation. The luminosity of this gray was then measured in the ordinary way and also calculated from its color equation. In all cases it was found that the measured luminosity of the gray agreed with the calculated luminosity, the latter being dependent upon the flicker, within two per cent. of the luminosity of white bristol board.

The flicker method has been adapted to use on the optical bench by Frank P. Whitman.* In the summary of the results obtained by his investigations he remarks:

"The flicker photometer used to compare lights of any color approximates in convenience and accuracy any of the ordinary photometric appliances used with lights of the same color. Different observers whose vision is normal obtain like results.

"The instrument gives a true measurement of the luminosity comparable with that obtained in other trustworthy ways."

The results obtained by Professor Rood and Mr. Whitman promise a solution of one of the most difficult problems of photometry, viz., the comparison of the luminosities of differently colored surfaces. While the ordinary photometric methods have, in general, been most thoroughly worked over, the application of the phenomena of flickering to photometric measurements being entirely new opens a very promising field for investigation. The present research was undertaken at the suggestion of Pro-

*Physical Review. Vol. III. No. 4. Page 241. 1896.

fessor Rood for the purpose of studying more elaborately the phenomena of flickering and finding out, if possible, whether or not it is entirely independent of differences in color.

Before the phenomenon of flickering is extensively used in photometric work it is necessary that the phenomenon should be elaborately studied with a view of distinguishing it from all allied phenomena with which it is at all likely to be confounded. It is also necessary that the effect, if there is any, of difference in color upon the phenomenon of flickering should be subjected to as rigorous experimental investigation as possible. And finally, the advantages of having results verified by more than a single pair of eyes should not be lost sight of.

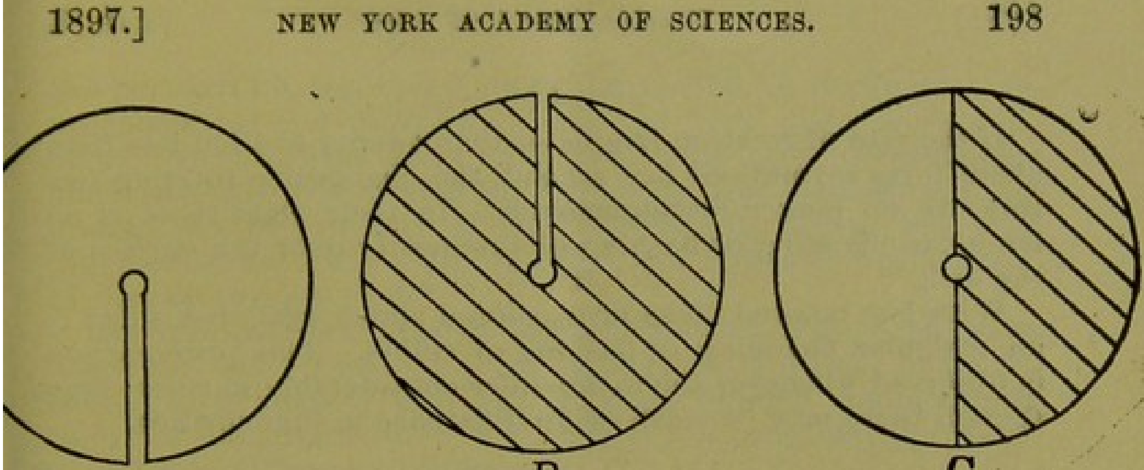
For the study of flicker phenomena there is perhaps no more convenient means than that afforded by a graduated series of gray disks and, as the experiments of Professor Rood had, so far as I knew, never been verified by another pair of eyes, a series of gray disks similar to the series used by Professor Rood was prepared and used throughout the following investigation.

The Apparatus Used.

Two graduated sets of gray disks were used, a large set five and one-half inches in diameter, and a small set three inches in diameter. The disks of the large set were numbered 7, 8, 9,.... etc., from the lightest to the darkest shades. The set included one hundred and twenty-nine disks. The disks of the small set were similarly numbered 7, 8, 9,....etc., and contained the same number of disks. The luminosities of these disks were determined in the ordinary way by the use of a black and a white disk and are given as so many per cent. of the luminosity of white bristol board which is taken as one hundred per cent. The determined luminosity of the lightest disk of either set was 89 per cent., of the darkest disk, 4 per cent. Whenever in the succeeding portions of this article the difference in luminosity between two disks, gray or colored, is given as so many per cent., by that is meant a difference of so many per cent. of the luminosity of white bristol board.

Two sets of colored disks of the same sizes as the gray disks were kindly furnished me by Professor Rood. As the corresponding large and small colored disks were cut from the same piece of paper, they are of the same tint and luminosity.

Each disk was slit radially and perforated in the center (Fig. I. A. and B.). They could thus be combined pair-wise (Fig. I. C.) and placed on the axis of the rotation apparatus. The rotation machine was pla‹ed at a distance of about one and a-half meters from a window of southern exposure. The disks, when



A B C

FIG. I.

placed on the axis of the machine, were screened from all side illumination by black screens suitably adjusted. The observer was placed at a distance of about one meter in front of the rotation machine and so seated that his head was just below the level of the disks.

The total luminosity of the window during observations was between one thousand and two thousand candle power. The variation in luminosity of the more refrangible colors with respect to the less refrangible ones, due to a variation in illumination, an effect described by Purkinje,* was found to be inappreciable so long as the illumination did not vary beyond the above limits.

THE PHENOMENON OF FLICKERING.

If a white disk and a black disk or two gray disks differing by more than about 40 per cent. in luminosity are combined pairwise, as in Fig. I. C., and are rotated, the following phenomena may be observed under good daylight illumination :

I.

With a rate of rotation between ten and twenty-five revolutions per second, the compound disk seems to be divided into a number of rotating sectors alternately light and dark. The apparent number of these sectors increases with the rate of rotation, and the phenomenon is rather trying on the eyes.

If the two disks are of different colors, as well as of different luminosities, the rotating sectors will differ alternately in color as well as in luminosity.

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* Zur Physiologie der Sinne. Vol. II. Page 109.

II.

If the rate of rotation is greater than twenty-five but less than about forty revolutions per second, the changeable rotating sectors are no longer distinguishable. In their place there is an irregular changing or flashing of luminosity over the surface of the disk.

With two colored disks or a colored and a gray disk there is an irregular changing or flashing of colors. This phenomenon is observed whenever the disks differ considerably in color, even though there may be small or no difference in luminosities.

III.

With a rate of rotation between forty and sixty revolutions per second the irregular flashing just described disappears, the disk assumes a background of a uniform hue, and over this uniform background one notices a flickering very similar to that which is noticed by a person looking at a picket fence while running past it.

If two disks of different colors are used, or a colored and a gray disk, the phenomenon of flickering is the same in all cases after the rotating compound disk has assumed a uniform background.

If the two disks differed in color but only slightly in luminosity, it was found that there was very little true flicker after the background had become uniform.

IV.

If the speed is increased beyond about sixty revolutions per second the flickering disappears and we have only a uniformly tinted disk.

It should be remarked that for the phenomena of rotating sectors described under I, and of flashing described under II, the disk has no background of uniform hue or luminosity. As soon, however, as the speed is sufficiently rapid to produce true flickering the disk assumes a background of uniform hue and luminosity.

The transition from any one of the above phenomena to another is not in general very sharply defined. The transition from the flickering described under III. to the uniform background without flicker described under IV. is, however, quite abrupt. Next in definiteness is the transition from flashing to flickering and this transition is much more sharply defined when the difference in luminosity is more than ten per cent. than when it is less. When the difference was very small (less than five per cent) it was found almost impossible to distinguish between flashing and flickering when two gray disks were combined. If two disks differing considerably in color were combined pair-wise and rotated the transition from the flashing background to a background of uniform hue with flicker was very well marked even when the difference in luminosity was as small as one or two per cent.*

The experiments seemed to indicate that, while the phenomena described under I and II could be obtained for little or no difference in luminosity provided the disks differed sufficiently in color, for the production of flickering it was necessary that there should be a difference in luminosity.

The rates of rotation given in the above description of the various phenomena hold true only when the difference in luminosity is more than about forty per cent. If the difference in luminosity between the two disks used is less than this amount the rates of rotation at which the various phenomena occur will, in general, be lower than the rates above given.

In the application of the phenomenon of flickering to photometric work it may be of importance to know the rate of speed at which the disks must be revolved to give the most accurate results. The speed required for the greatest accuracy will evidently be the speed at which the true flickering due to very slight differences in luminosity is the most violent, and this, in general, was found to be the lowest rate at which true flickering could be observed. For the determination of this speed a number of pairs of disks were selected so that the disks of each pair differed in color but did not differ in luminosity by more than two per cent. It was found that if the illumination was very low, about sixteen candle power at a distance of one meter, the lowest speed at which the flashing vanished was about fourteen revolutions per second. The lowest speed at which the flicker disappeared was about twenty-five revolutions per second. The best speed, therefore, for observation at this illumination would be about fourteen revolutions per second. When the illumination was about fifteen hundred candle power at a distance of one meter the lowest speed at which all flashing disappeared was about twenty-four revolutions per second and the lowest rate required to extinguish the flickering was about thirty-four revolutions per second. Therefore, under this illumination a speed of about twenty-four revolutions per second would give the best result.

*For the methods employed in measuring the luminosities of colored disks see next section.

The Measurement of the Luminosities of Colored Disks.

In this section the methods employed in measuring the luminosities of colored disks will be described. The first method which I have called the method of Least Flicker is essentially the same as was described by Professor Rood in the *American Journal of Science* for September, 1893. The validity of the method depends on the assumption that the violence of the flicker, observed when two disks are combined pair-wise and rotated before the eyes, decreases if the difference between the luminosities of the two disks thus combined is diminished, no matter what the difference in color may be, and that for zero difference in luminosity all flicker vanishes.

In applying this method to the determination of the luminosity of a given colored disk various gray disks are successively combined pair-wise with the colored disk until one is found which on rotation gives no flicker. The luminosity of the gray disk can then be determined in the ordinary way and this will be the luminosity of the colored disk if the above assumption is correct. In the practice of the method two difficulties will in general be encountered.

First: As Professor Rood has pointed out, the entire series of gray disks may not contain one of exactly the same luminosity as the colored disk and we will on this account be unable to get rid of the last trace of the flicker.

Second: Unless the gray disks are very carefully prepared, slight differences in the luminosities of different parts of the same disk may give rise to a greater flicker than that due to the difference in luminosity between the gray and the colored disk.

On account of these two difficulties I was in general unable to find a gray disk which would give absolutely no flicker when rotated in a pair-wise combination with a colored disk and had, therefore, to be contented with the gray disk which gave the least flicker. The selection of the gray disk giving the least flicker with the colored disk was made as follows:

A large gray disk giving only a slight flicker, when combined pair-wise with the colored disk and rotated, was selected. The flicker thus produced was used as a standard for comparison and all the small gray disks giving a less violent flicker, when rotated in a pair-wise combination with the colored disk, were selected. From these small gray disks the one judged to give the least violent flicker with the colored disk was chosen, and this flicker used as a second standard for comparison in the selection of all large gray disks giving a less violent flicker with the colored disk. From the large gray disks thus selected the one judged to give the least violent flicker with the colored disk was chosen, and this flicker used as a third standard for comparison in the second selection of small gray disks giving a less violent flicker with the colored disk. This method of successive selection of gray disks giving less violent flickers with the colored disk than preceding gray disks have given was continued till all but a single large and small gray disk had been excluded. The luminosities of these two disks were then measured and the average of the two luminosities taken as the luminosity of the colored disk.

The comparison of the flicker given by a small gray disk combined pair-wise with the colored disk, with that given by a large gray disk similarly combined was made in the following way. The small gray disk was placed concentrically on the large gray disk as in Fig. II. A. and the two combined pair-wise with the

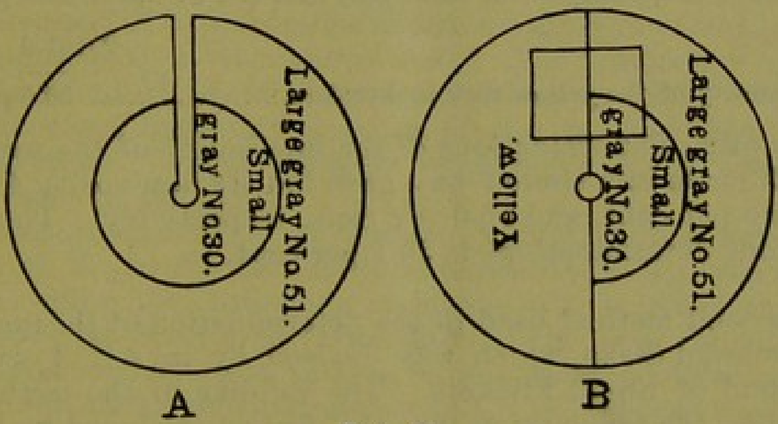

FIG. II.

colored disk as in Fig. II. B. This combination could be placed on the axis of the rotation apparatus, rotated, and the flickers inside and outside compared. The direct comparison of the relative violence of the two flickers was found very difficult to make. It was observed, however, that the flicker which required the highest rate of rotation to extinguish it was always, as near as could be judged, the most violent. In comparing the relative violence of the inside and outside flickers produced by such a combination as is shown in Fig. II. B. the rate of rotation of the disks was increased till all flickering disappeared. The speed was then gradually diminished and the disk was observed carefully so as to detect whether the flickering began first on the inside or outside. The flicker which appeared first was the most violent. The flickering was observed through a rectangular opening cut in black cardboard, the cardboard being held in such a position as to exclude from view all of the disk except the portion inclosed by the rectangle in Fig. II. B.

The following is an example of the application of the method. It being desired to find the luminosity of a yellow disk, various large gray disks were successively combined pair-wise with it and rotated, and the large gray disk No. 40 was found to give only a slight flicker when rotated with the yellow disk. The small gray disks Nos. 44, 43, 39 and 37 respectively gave less violent flickers with the colored disk than the flicker given by the large gray disk, No. 40, and the colored disk. Of the small gray disks, No. 43 seemed to give the least flickering with the colored disk. Only one large gray disk, No. 39, was found to give as little flicker with the colored disk as the small gray disk, No. 43, had given.

The measured luminosity of the large gray disk No. 39 was.....51.9 per cent.
The measured luminosity of the small gray disk No. 34 was.....50.0 per cent.

2)101.9

The luminosity of the yellow disk is therefore...................... 50.9 per cent.

Three other determinations of the luminosity of the same yellow disk made in a similar way gave for its luminosity 49.5 per cent., 50.0 per cent. and 50.0 per cent. respectively. The mean of the four determinations is 50.1 per cent.

The second method used in the determination of the luminosities of colored disks, which was devised by myself, I will call the method of Equal Flickers. The validity of the method depends upon the assumption that the flicker produced by the rotation of a pair-wise combination of a gray and colored disk in which the gray disk is, to a moderate extent, more luminous than the colored disk, is of the same violence as the flicker produced by the rotation of a pair-wise combination of the same colored disk with a gray disk less luminous than the colored disk by the same amount.

In the practice of the method a large gray disk is selected which is somewhat more luminous than the colored disk. This is combined pair-wise with the colored disk, and various small gray disks somewhat less luminous than the colored disk are successively combined with the large gray and colored disks, as in Fig. II. B, until a combination is found which on rotation gives outside and inside flickers of equal violence. The violence of the two flickers is compared in the way already described. When the two flickers are of the same violence the large gray disk should be as much more luminous than the colored disk as the small gray disk is less luminous if the above assumption is correct. Therefore the luminosity of the colored disk is found by taking the mean of the luminosities of the large and small gray disks.

The application of this method to the determination of the luminosity of the yellow disk used in the previous method gave the following results. After a few trials it was found that the yellow disk, when combined pair-wise with the large gray disk No. 51, and the small gray disk No. 30 (see Fig. 2, A and B) and rotated, gave outside and inside flickers of equal violences.

The measured luminosity of the large gray disk No. 51 was.....	48.0 per cent.
The measured luminosity of the small gray disk No. 30 was.....	55.5 per cent.
	2)103.5
The luminosity of the yellow disk is, therefore, according to this method	51.7 per cent.

The average of seven similar determinations gave 50.4 per cent as the luminosity of the yellow disk. The luminosity obtained by the method of Least Flicker was 50.9 per cent. The maximum difference between the mean of the seven determinations by the method of Equal Flickers and any one of them was about 4.0 per cent. of the mean.

A comparison of a large number of determinations, by the method of Equal Flickers of the luminosity of a given colored disk, in which the difference in luminosity between the large and small gray disks used varied from about 5 per cent. to about 85 per cent., showed that so long as this difference was less than 20 per cent., the luminosities obtained for the colored disk agreed with each other and with the luminosity obtained by the method of Least Flickers. For greater differences than about 30 per cent., the results did not show so good an agreement. So long, however, as the difference was kept less than 20 per cent., it was found that the figures obtained by the method of Equal Flickers for the luminosities of various colored disks agreed with those obtained by the method of Least Flicker, as is shown in the following table:

Description of the colored disks.	Luminosity of colored disks as determined by the Method of Least Flicker.	Method of Equal Flickers.
Bright Red	16.1%	16.3%
Dark Blue	6.6%	6.5%
Green	46.5%	46.5%
Bright Yellow	50.1%	50.4%
Light Orange	55.0%	54.6%

Each of the luminosities given under the method of Least Flicker is the average of not less than two determinations and each of the luminosities given under the method of Equal Flickers is the average of not less than five determinations. The difference between the luminosities of a given color determined by the two methods is in no case greater than 0.4 per cent. The advantage of the method of Equal Flickers over that of Least Flicker is that it enables one to work with a smaller number of gray disks.

Since the preparation of a graduated series of gray disks involves considerable labor, every one wishing to carry on experiments in flicker photometry may not care to prepare them. Professor Rood in his article in the *American Journal of Science* for September, 1893, suggested that the luminosity of a given colored surface might be determined by matching the color using only a few standard disks, the luminosities of which had been previously carefully determined by someone possessing a graduated series of gray disks. In applying this method to the determination of the luminosities of colored surfaces I selected for my standard disks the following.

Description of the standard disks.	Abbreviations used to designate the standard disks.	Determined luminosities of the standard disks.
White	W.	92.3%
Black	Bk.	4.5%
Bright Yellow	Y.	50.3%
Dark Blue	B.	6.5%
Green	G.	46.5%
Bright Red	R.	16.2%

Experiments to test the applicability of the method were made upon the following four disks: a straw yellow, (S. Y.), a light orange, (O.), a dark brown, (Br.), and a very dark green, (D. G.). Each of the following four color equations is the mean of ten determinations:

100 per cent. S. Y. = 15.6 per cent. G. + 30.4 per cent. Y. + 11.3 per cent. R. + per 9.3 cent. W. + 33.4 per cent. Bk.

The luminosity of the straw yellow disk calculated from this equation is 34.3 per cent.

82.2 per cent. O. + 17.8 per cent. Bk. = 40.5 per cent. R. + 39.1 per cent. Y. + 20.4 per cent. W.

The luminosity of the light orange disk calculated from this equation is 56.6 per cent.

100 per cent. Br. = 18.0 per cent. R. + 27.0 per cent. Y. + 3.9 per cent. W. + 51.1 per cent. Bk.

From which we obtain for the luminosity of the dark brown disk 23.0 per cent.

98.1 per cent. D. G. + 1.9 per cent. W. = 15.3 per cent. G. + 7.7 per cent. B. + 77.0 per cent. Bk.

The luminosity of the dark green disk calculated from this equation is 9.6 per cent.

The luminosity of each of the above four colored disks was then independently determined by the method of Equal Flickers, the average of six determinations for each disk being taken. The agreement between the results obtained by the two methods is shown in the following table.

Description of the colored disks.	Luminosity determined by the method of Equal Flicker.	Luminosity calculated from the color equations.
Straw Yellow	34.4%	34.3%
Light Orange	54.6%	56.6%
Dark Brown	23.2%	23.0%
Dark Green	10.1%	9.6%

The greatest difference between the luminosities of any one of the disks determined by the two methods is 1.1 per cent.

The determination of the luminosity of a colored disk by matching it, using a few standard disks, is based upon the assumption that the luminosities obtained by the flicker method satisfy Grassman's law for color mixtures. The experiments of Professor Rood * and Mr. Whitman † show that the assumption is correct within the experimental error which was about 2 per cent. of the luminosity of white bristol board. Similar experiments made by myself gave like results. The luminosities of several grays, formed by combinations of three colors, were measured in the ordinary way and also calculated from the color equations. The difference between the numbers obtained by the two methods for the luminosity of a given gray was in no case over two per cent. of the luminosity of white bristol board. The agreement between the luminosity of a given colored disk determined by the method of matching and the luminosity of the same disk determined directly by the flicker method is another proof of the correctness of the assumption. The experiments referred to in this paragraph constitute, so far as I know, the only experimental proofs so far given of the correctness of Grassman's law for color mixtures.

Flickering is Found to be Independent of Color.

In the preceding section it was stated that for the determination of luminosities by means of the phenomenon of flickering, the assumption must be made that the violence of the

* American Journal of Science, September, 1893.
† Physical Review, Vol. III., No. 4. 1896.

flicker depends upon the difference in luminosity and is independent of difference in color. The agreement among the results obtained by the three different methods employed would seem indirectly to warrant the assumption. The experiments now to be described were undertaken for the purpose of obtaining a direct proof.

For each of four colored disks, yellow, green, blue and red, respectively, a large and small gray disk was selected of the same luminosity as the color. The selection was made by the method of Least Flicker. In the experiments to be described both large and small colored disks were used, but since the large and the small disks of each color were cut from the same piece of paper they are of the same luminosity. The numbers of the large and small gray disks, and the colored disks of equal luminosities are given in the following table. Gray and colored disks of the same luminosity are placed in the same horizontal line.

Numbers of the large gray disks.	Numbers of the small gray disks corresponding to the large in luminosity.	Colored disks of the same luminosities as the gray disks.
120	123	Large and small red disk.
39	36	Large and small yellow disk.
50	51	Large and small green disk.
129	122	Large and small blue disk.

Various pair-wise combinations were made among the large and small disks respectively. The disks were so chosen that the difference in luminosity between two large disks combined pair-wise should be the same as the difference in luminosity between two small disks similarly combined. In the following table the colors and numbers in the two columns on the left refer to the large colored and gray disks of different luminosities which were combined pair-wise. The corresponding colors and numbers in the two columns on the right refer to small colored and gray disks presenting an equal difference in luminosity. Thus: the luminosities of the disks referred to in column I. are the same as the luminosities of the corresponding disks of column III., the disks referred to in column II. are of the same luminosity as the corresponding disks referred to in column IV.

	Large colored and gray disks which were combined pair-wise.		Small colored and gray disks which were combined pair-wise.		
	I.	II.	III.	IV.	
1.	120	and blue,	red	and blue.	(See Fig. III)
2.	red	and blue,	123	and 128.	
3.	red	and yellow,	123	and 36.	
4.	120	and 39,	red	and yellow.	
5.	red	and green,	123	and 50.	

6.	120 and green,	red and green.
7.	red and green,	123 and green.
8.	39 and green,	yellow and green.
9.	blue and green,	128 and green.
10.	red and yellow,	123 and yellow.

Each pair-wise combination of small disks was placed concentrically upon the corresponding pair-wise combination of large disks. (See Fig. III.) The four disks thus combined were

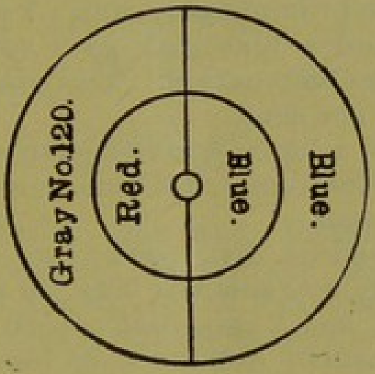

FIG. III.

placed upon the axis of the rotation machine and rotated at a sufficiently high speed to cause all flickering to vanish. The speed was then gradually diminished and in each case, as near as could be judged, the flickering began inside and outside simultaneously. Now in each of the above combinations the difference in luminosity between the disks opposed pair-wise is practically the same for the small as for the large disks. On the one hand, however, the difference is between two differently colored disks, on the other hand, an equal difference in luminosity occurs between two gray disks or between a gray and a colored disk.

From these experiments we may, therefore, conclude that, under ordinary daylight illumination, the violence of a flicker, due to a combination of two gray disks of different luminosities, is not affected if for one or both of the gray disks a colored disk of equal luminosity is substituted. It will be seen that the above set of experiments constitute a rather severe test of the assumption that under ordinary daylight illumination flicker depends on luminosity and is independent of color.

To further test the assumption the following experiments were made: From a number of colored disks the luminosities of which were unknown to me, I selected by the method of Least Flicker a number of pairs which gave only a very feeble flickering when rotated in pair-wise combination. Now we would expect the differences in measured luminosity to be very small if the violence of the flicker is dependent on difference in luminosities. The luminosities of the disks were then measured by the method of Equal Flickers, the average of five determinations being taken for each disk, and the difference between the measured luminosities of the disks of each pair was found in no case to be over two per cent. The actual results were as follows:

A red disk and a dark blue disk were found which gave very little flickering when rotated in pair-wise combination. Their measured difference in luminosity was 1.7 per cent.

A yellow disk and a blue disk gave very little flicker when rotated in pair-wise combination. Their measured difference in luminosity was 1.6 per cent.

A light green and a light blue disk gave less flicker than either of the above combinations. The measured difference in luminosity was 1.2 per cent.

A red and a yellow disk gave very little flicker. Their measured difference in luminosity was 1.0 per cent.

A very dark red and a very dark green disk gave very little flicker. Their measured difference in luminosity was 1.1 per cent.

The above results show that where the flickering between differently colored disks is very slight the measured difference in luminosity is small. Expressing the general result in a few words one can say that, in the matter of flicker, under daylight illumination differently colored disks behave in the same way with each other as they do with gray disks.

To still further test the matter a number of combinations (See Figs. IV. to VI.) of colored disks, the luminosities of which had been determined, were so made that on the inside two colors of the same hue but of different lúminosities were opposed, while on the outside two colors of different hues but having a less difference in their measured luminosities were opposed to each other. Each combination was then placed on the axis of the machine and rotated at such a speed that all flickering disappeared. Then, as the speed was gradually diminished, the disk was observed through a rectangular opening in black cardboard and the relative violence of the flickering outside and inside thus compared. The following is a record of the results obtained.

A large green disk and a large blue disk which I will call Green 1. and Blue 1. respectively (See Fig. IV.) were opposed outside. Their measured difference in luminosity was 1.2 per cent. On the inside a small green disk, Green 2, was opposed to the large green disk and the measured difference in luminosity was 4.4 per cent. This combination was rotated on the machine and the flickering inside was found to be more violent than the

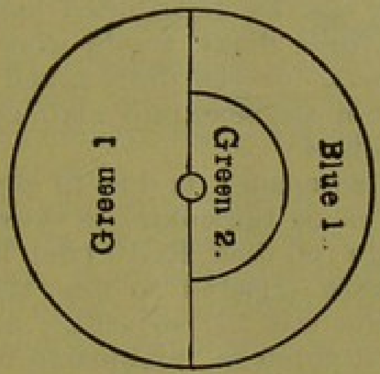

FIG. IV.

flickering on the outside. We may, therefore, conclude that if the difference in color on the outside causes any flickering such flickering cannot amount to as much as the flickering due to a difference in luminosity of 4.4 per cent. — 1.2 per cent. = 3.2 per cent.

To find out if a more precise conclusion could not be drawn, the following experiment was tried. From my set of colored disks I selected a large yellow disk, Yellow 1 (See Fig. V.), and

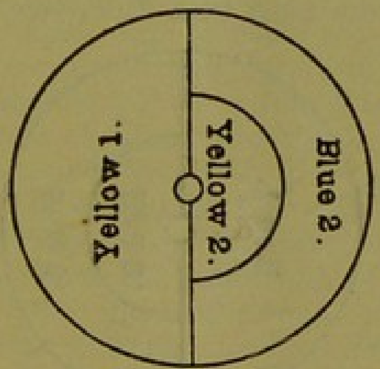

FIG. V.

a large blue disk, Blue 2, such that their measured difference in luminosity was 2.1 per cent. A small yellow disk, Yellow 2, was found such that the difference between the luminosity of it and that of Yellow 1. was 2.2 per cent. These three disks were combined as shown in Fig. V, rotated on the axis of the machine and the inside and outside flickers compared. We would expect the inside flicker to be slightly more violent than the outside flicker. It was found, however, that the outside flicker was very slightly the more violent of the two. The discrepancy can be readily explained if we assume that the measured differences in luminosity are in error by a little less than one per cent. This error, since it can be distributed among the determined luminosities of the three disks, would give a probable error of less than 0.5 per cent. for the determined luminosity of each disk. This is within the accuracy claimed for the determined luminosities of the disks.

That the above discrepancy is due to slight errors in the determined luminosities of the colors is shown by the following experiment. A large red disk, Red 1 (See Fig. VI.), and a large blue disk, Blue 3, were selected so that the difference in their measured luminosities was 2.1 per cent. A small red disk, Red 2, was found which differed in luminosity from the disk, Red 1, by 1.7 per cent. These three disks were combined as is shown in Fig. VI., and rotated on the axis of the machine. The inside flicker was found to be more violent than the outside flicker, although on the inside there is no difference in color and a less difference in measured luminosity than on the outside. The discrepancy is, however, readily explained by the assumption of a slight error in the determined luminosities of each of the three disks.

The last three experiments seem to show, beyond a doubt, that under daylight illumination, difference in color cannot, of itself, produce an appreciable flicker and that for the production of this phenomenon there must be a difference in luminosity.

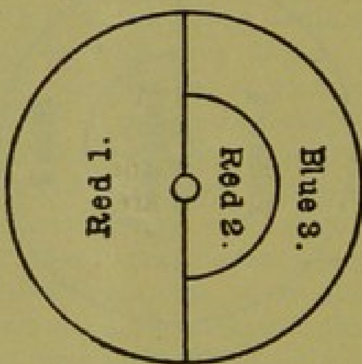

FIG. VI.

SUMMARY.

The results of the present research may be summed up briefly as follows:

In the application of the phenomenon of flickering to photometric measurements it was found necessary to distinguish carefully between true flickering which occurs only on a background of uniform hue and the allied phenomenon of flashing which presents no uniform background. True flickering was found to be no more tiring on the eyes than ordinary photometric work. The phenomenon of flashing, on the other hand, was quite painful.

It has been proved that under ordinary daylight illumination true flickering cannot be produced by differences in color. For the production of this phenomenon a difference in luminosity was found to be necessary, and the experiments also showed that as the difference in luminosity was decreased the violence of the flicker was also diminished.

Furthermore, the experiments showed that when two flickers are produced, the one by the rotation of a surface B, in combination with a surface A, more luminous than B, the other by the rotation of the same surface B, in combination with a surface C, less luminous than B by the same amount, the two flickers will be of practically the same violence so long as the difference in luminosity between A and C is not greater than about 20 per cent.

Upon the last two conclusions are based two general methods for the application of the phenomenon of flickering directly to photometric work. The two methods are : the method of Least Flicker and the method of Equal Flickers, respectively. The results obtained by the two methods have been shown to be harmonious. The relative advantages of the two methods will depend largely upon the nature of the work in hand.

The experimental proof of Grassman's law which the new flicker photometry enables one to make has been verified. The application of this law to the determination of the luminosities of colored surfaces by the aid of a few standardized disks was suggested but not tried by Professor Rood. This method was tested experimentally and found to give results which corresponded with those obtained by either of the direct flicker methods. Consequently, investigators are enabled by the use of a few standard disks to determine the luminosity of any colored surface.

The experiments also indirectly showed that the flicker method can, with great convenience and delicacy of results, be applied to the comparison of the luminosities of surfaces of the same color. In brief, the results of the experiments show that the new flicker photometry will enable us to compare the luminosities of differently colored surfaces with the same ease and exactness as was previously only attainable in the comparison of surfaces of like hue.

I wish here to express my thanks to Professor Rood and Professor Hallock for assistance very kindly furnished during the investigation.

Physical Laboratory of Columbia University,
April 2, 1897.