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Note on the Formulas Used for Calculating the Weight of the Brain in the Albino Rats

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NOTE ON THE FORMULAS USED FOR CALCULATING THE WEIGHT OF THE BRAIN IN THE ALBINO RATS.

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In previous papers (Hatai, '08; Donaldson, '08) the formulas for calculating the weight of the brain and of the spinal cord in relation to the body weight were determined on the assumption that the amount of increment to the weight of these parts is proportional to the reciprocal of the body weight plus a constant or

$$\frac{dy}{dx} = h\frac{1}{(x + a)} \qquad (1)$$

where y is the weight of the brain or spinal cord in grams and x the weight of the body in grams.

Integration of (1) gives at once the value of y.

Thus:

$$y = h\int\frac{1}{(x + a)}dx = h \log (x + a) + c$$

or in our previous notation (Donaldson, '08):

$$y = A + C \log (x + \beta) \qquad (2)$$

This type of logarithmic formula has been used by the present writer ('08) and Donaldson ('08) and was found to be very satisfactory for representing the relation between the body weight and the weight of the brain or spinal cord.

This type was further employed by Donaldson ('09) to represent the relation between the body weight and body length and was proved by him to be satisfactory.

The formula in each case was as follows:

Brain weight	or $y = .569 \log (x - 8.7) + 0.554$	(3)
Spinal cord weight	or $y = .585 \log (x + 21) - 0.795$	(4)
Body length	or $y = 143 \log (x + 15) - 134$	(5)

Although the formulas (4) and (5) are entirely free from theoretical objections within the interval x = (5 grams, 325 grams), the formula (3), however, has two defects when we apply it to the case of $x < 8.7$. The first defect, which appears when x, the body weight, is less than 8.7 grams, is due to the fact that the resulting value of $(x — 8.7)$ becomes a negative quantity and the logarithm of such a quantity is necessarily imaginary. The difficulty thus presented is, however, merely a theoretical one, since for the purpose of computation the following method may be employed.

Let us consider the two cases when x is greater than a and when x is less than a then we have

$$(A)\quad \frac{dy}{dx} = \frac{h}{(x - \alpha)} \text{ when } x > \alpha$$

$$(B)\quad \frac{dy}{dx} = \frac{h}{(\alpha - x)} \text{ when } x < \alpha$$

Then integration of (A) leads to the formula (3) which we have already obtained, that is

$$y = A + C \log (x - B) = .554 + .569 \log (x - 8.7)$$

while the integration of (B) becomes

$$(C)\quad y = A - C \log (\beta - x) = .554 - .569 \log (8.7 - x)$$

The formula (C) thus obtained gives results identical with those obtained when we compute the value of y from the formula

$$(D)\quad y = .554 + .569 \log (-C_1)$$

In this case, of course, with an understanding that $\log (— C)$ should be treated as equivalent to $— \log C_1$.

As long as the results obtained by the formula (C) agree with those obtained by the formula (D), the following procedure is justified.

In the formula (3), when the variable x is less than a constant C, or in this case 8.7, we can take the logarithm of the real positive number (C_1) and put a negative sign before it, *i. e.*, $.569 \log(-C_1) = -.569 \log C_1$ where $-C_1 = (x - 8.7)$.

With the foregoing understanding, the formula can thus be applied even in the case of a rat, the body weight of which is less than 8.7 grams.

There remains, however, a second defect in this formula (3) which cannot be overcome.

When the value of x lies between 7.7 and 9.7 grams, the formula fails to represent the observed values on account of sudden change in the course of the resulting curve. Although this interval is very small when we consider the whole extent of the curve, yet it prevents the general application of the formula.

In Chart I, Plate II, in the paper by Donaldson, '08, the curve representing the change in the brain weight between the body weights of 5 and 10 grams was completed by simply joining the two points, both of which had been carefully calculated by the formula (3), and it was not until we came to consider the formula in another connection that we appreciated the impossibility of applying it to this interval.

I have now obtained a revised formula which is free from the foregoing objections. At the same time it should be stated that the values obtained by this new formula do not differ from the values so far as computed by the previous formula (3), or as given by the ideal line by which the curve was previously completed.

I shall present first the theoretical considerations touching the revised formula. Let us consider the series

$$S = \frac{\psi(z) + \phi(z)}{2} + \sum_{0}^{\infty} [\psi(z) - \phi(z)] \left[\frac{1}{1+z^n} - \frac{1}{1+z^{n-1}} \right] \ldots\ldots (6)$$

where $\psi(z)$ and $\phi(z)$ are (some) functions of z. The sum of the first n terms of S becomes obviously

$$S_n = \phi(z) + \frac{\psi(z) - \phi(z)}{1+z^n}.$$

When $(z) < 1$, the limit of z^n is zero for $n = \infty$ and consequently $s = \psi(z)$.

On the other hand, where $(z) > 1$, z^n tends to ∞ and therefore in this case $S = \phi(z)$. (See Jordan "Course d'analyse," Tome I, p. 320.)

We have shown already that the brain weights in rats in which the body weights are greater than 10 grams, can be calculated by the formula

$$y = .569 \log(x - 8.7) + 0.554 \quad \ldots\ldots\ldots\ldots (3)$$

Later we found that the brain weights in rats in which the body weight lay between 5-10 grams may be calculated by a special formula for this portion of the curve, namely:

$$y = 1.56 \log(x) - .87 \quad \ldots\ldots\ldots\ldots (7)$$

and therefore in the two formulas (3) and (7) y can be considered as the function of log x.

The values calculated by the latter formula (7) agree perfectly with the ideal line which completes the brain-weight curve between 5 and 10 grams of body weight.

As has been shown already, the formula (6) is perfectly general in its application when two conditions are satisfied; namely, when $|z| > 1$ in one case and $|z| < 1$ in the other.

We also found that not only are the two formulas (3) and (7) functions of log x, but that (1) is applicable to rats in which the body weights are more than 10 grams or $|\log x| > 1$, while formula (7) is applicable to rats in which the body weight is less than 10 grams or $|\log x| < 1$. This satisfies all the necessary conditions.

Thus a combination of the two formulas (3) and (7) will enable us to calculate the brain weight for any given body weight from 5 grams to 320 grams. (Extrapolation may be used towards the upper end of the curve).

The final formula is represented by the following:—

$$y = \frac{\log x^{1.56} (x - 8.7)^{.569} - 0.316}{2} + \sum_{0}^{\infty} \left(\log \frac{x^{1.56}}{(x - 8.7)^{.569}} + 1.424 \right) \left[\frac{1}{1 + (\log x)^n} - \frac{1}{1 + (\log x)^{n-1}} \right] \quad \ldots\ldots\ldots\ldots (8)$$

in which y represents the brain weight and x the body weight.

As to the actual use of the above formula, I may add the following remarks.

As was mentioned already, the series reduces to

$$\phi\ (\log x) = .569 \log (x - 8.7) + 0.554$$

when (log x) is greater than 1; while, on the other hand, the series reduces to

$$\psi\ (\log x) = 1.56 \log x - .87$$

when (log x) is less than 1. Therefore it is only necessary to note whether we are treating rats in which the body weights are greater or less than 10 grams.

If the body weight is greater than 10 grams, we can simply use

$$\phi\ (\log x) \text{ or } y = .569 \log (x - 8.7) + 0.554$$

and if it is less than 10 grams, the other formula

$$\psi\ (\log x) \text{ or } y = 1.56 \log x - .87$$

Of course, one can determine the brain weight directly from the formula (8) after some laborious calculation; nevertheless such a procedure has no advantage over the simpler process described above.

The present formula (8) is desirable simply, first, because it is free from the theoretical objections; and, second, because by it we can express the complicated relations existing between the body and brain under a single generalized form.

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