A second study of the statistics of pulmonary tuberculosis : marital infection / by the late Ernest G. Pope ; edited and revised by Karl Pearson ; with an appendix on assortative mating from data reduced by Ethel M. Elderton.

Pope, Ernest G.
Pearson, Karl, 1857-1936.
Elderton, Ethel Mary, 1878-1954.
Royal College of Surgeons of England

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DEPARTMENT OF APPLIED MATHEMATICS,
UNIVERSITY COLLEGE, UNIVERSITY OF LONDON

DRAPERS' COMPANY RESEARCH MEMOIRS

III. A SECOND STUDY OF THE STATISTICS OF PULMONARY TUBERCULOSIS: MARITAL INFECTION.

BY THE LATE
ERNEST G. POPE,
ADIRONDACK COTTAGE SANITARIUM, SARANAC LAKE, N.Y.

EDITED AND REVISED
BY
KARL PEARSON, F.R.S.

WITH AN APPENDIX ON ASSORTATIVE MATING FROM DATA
REDUCED BY

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UNIVERSITY OF LONDON

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Marital Infection in Tuberculosis.

[1. *Introduction.* At the time of his death Mr Ernest G. Pope was engaged on the investigation by modern statistical methods of the problem of marital infection in tuberculosis. His memoir was nearly but not entirely completed. He had demonstrated that there was a sensible correlation between the presence of tuberculosis in husband and wife, and he had reached the conclusion that the interpretation of this result depended on the extent to which assortative mating prevailed with regard to what we may term the tuberculous diathesis. Such a conclusion clearly indicates that marital infection is not of that marked character which makes itself manifest on the slightest inspection of the statistics. It is indeed of that subtle nature which requires very delicate handling to differentiate it from other sources of resemblance in husband and wife. The constitution which is peculiarly liable to tuberculosis—the phthisical constitution or what I have ventured to term the tuberculous diathesis—is one very familiar to the medical man. It is not infrequently associated with special mental or physical traits, which present undoubted sexual attraction. It is conceivable accordingly that the resemblance in phthisical character between husband and wife may be wholly or in part due to the tendency of like to mate with like, and not at all or not wholly to post-marital infection. This point was fully recognised by Mr Pope, but he was not in a position to estimate definitely the intensity of assortative mating in man for characters more or less closely resembling those constitutional factors with which the tuberculous diathesis is obviously associated. More than a year ago the coefficients of assortative mating for a number of cases of health, temperament and intelligence had been worked out from Pearson's *Family Records* by Miss E. M. Elderton and were preserved among the unpublished material of the Galton Laboratory for National Eugenics. They are published in an Appendix to this memoir and enable me to some extent to supplement Mr Pope's data. Even with this material we have a very difficult problem before us, depending not only on the variable intensity of assortative mating as we pass from one character to a second, but also on the fact that with few exceptions the whole of the available tuberculous data is obtained from *selected populations*. And in this lies the heart of the difficulty; the great bulk of the material collected from various sources by Mr Pope—a fairly exhaustive collection indeed of the available data—provides us with the relationship as to tuberculosis of husband and wife, when they were not a mere random sample of the general population*, but selected because they were parents of tuberculous offspring. Now if we start from the standpoint that there is no inheritance of the tuberculous diathesis we conclude at once that there is no selection in such parents; they would form a mere random sample of the general population. On this assumption the average value of the resemblance between husband and wife in the tuberculous character falls below the value that it takes for general health, for insanity, for eye-colour, or for intelligence; it is much the same as Miss Elderton has found it to be for temper and some types of temperament. It is impossible to suppose that eye-colour is due to infection, nor at the present stage of our knowledge to attribute insanity to a communicable bacillus. Thus there is grave difficulty in assuming that the resemblance in tuberculosis—associated as the tuberculous diathesis is with noteworthy physical and mental traits—is wholly due to infection. On the other hand, if we assume—as I think we are now justified in assuming—that the tuberculous diathesis is inherited, we must treat our parents of tuberculous offspring as a selected group, and correct for this selection. This correction is a difficult but possible step, and we then find that the resemblance of husband and wife for the tuberculous character is carried at least to the verge if not sensibly beyond the limit of what we can attribute to the average action of assortative mating. We may probably assert a sensible but not very large effect of infective origin. The assertion must be made, however, with reserve and not dogmatically. If this view be accepted, then a good deal of light is thrown on the very different results as to marital infection reached by different authorities. It appears on the bulk of the existing material difficult to reach a definite demonstration of the existence of marital infection, and that demonstration largely depends on the assumption that the tuberculous diathesis is an inherited character. Admitting this we do, I think, reach a definite but not at all influential action of marital infection.

I have stated these points in the belief that they represent the conclusions that would have been reached by Mr Pope had he lived to develop fully the section of his paper on assortative mating, which Dr Lawrason Brown tells me he had much on his mind.

The exact history of Mr Pope's memoir should be recorded here. Shortly after Mr Pope's death, Dr Lawrason Brown of the Adirondack Sanitarium placed it in my hands for comment and suggestion. The paper was not wholly completed and in particular the questions of selected populations and of assortative matings required full treatment. Dr Lawrason Brown gave me finally full power to handle the memoir as I thought best. I have accordingly added the entire sections on selection and assortative mating. All paragraphs between square brackets [ ] as well as square bracket insertions in non-bracketed paragraphs are entirely due to me. In the paragraphs due to Mr Pope I have left statements which I should probably not have

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* I have added an additional "general population" sample provided by the Galton Laboratory material.

written myself, but it did not seem to me fitting that I should alter or add more than was absolutely needful in order to elucidate the question under consideration.

Two further points should be referred to here. In all future data as to marital infection—especially when rural populations are dealt with—it is of vital importance that a strict record should be made of the number of consanguineous marriages among the tuberculous and non-tuberculous pairs. If consanguineous marriages are found to be in excess among the tuberculous we shall have another factor coming in which will need very careful consideration.

Further, definite statements ought to be made of the dates at onset of the disease in both husband and wife as well as the dates of death or recovery in both cases, if the history be complete. It is clear that a perfectly definite estimate ought to be formed of the number of tuberculous pairs in which infection was really possible.

Finally I have most heartily to thank Dr Lawrason Brown for the faith he has had that a recent controversy would not cloud my judgment of the great merit of Mr Pope's work nor my appreciation of the way in which he has handled his material. I can only join in the general regret that the medical profession should have lost at such an early stage a statistician so capable of throwing light on the numerical aspect of medical problems.]

2. When an infectious disease is as widely prevalent as tuberculosis, we must expect to find frequently several members of the same family, household or workshop, dying from this cause, without taking into account either heredity or direct infection as causative factors.

Where, for instance, one person in every ten dies from tuberculosis, if we take 100 families or groups of five persons each, we shall expect, on the doctrine of chances, to find two or more members thus dying in each of eight groups out of the 100 groups, simply as a result of a chance distribution of the disease. Lists of cases, where several or many deaths have occurred in the same family, house or shop, are therefore valueless as proofs of infection or heredity unless we are also given the sizes and numbers of the groups from which these cases have been taken, so as to enable us to determine whether a chance distribution will account for the facts or not.

The marital relationship affords an example of close association without any ties of blood[*] so that we can consider most other factors as eliminated and investigate the problem of infection almost alone. The necessity for the qualification in this statement due to assortative mating will appear later.

We know of no investigations of marital infection which fulfil the conditions necessary for obtaining a definite solution of the problem. Longstaff's classical inquiry was along correct lines but lacked sufficient data[†]. A very ingenious discussion has been given by Weinberg, but there is one fatal flaw in his most painstaking work,

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[* Consanguineous marriages in the professional classes amount possibly to 5 to 8 per cent.; in the working classes the percentage is probably lower: see *British Medical Journal*, June 6th, 1908, p. 1395.]

[† *Studies in Statistics*, London, 1891, p. 384.]

viz. that he has compared the death-rate from tuberculosis amongst the survivors of tuberculous partners with that among all adults of corresponding ages. It was necessary, to make a valid proof, that the comparison should have been made with widowed persons who had lost their partners through other diseases[*].

It has not been forgotten that a belief in infection between man and wife has been accepted by many on the grounds of their clinical experience, but until such experience has been definitely recorded and analysed, it can in no way be considered as a scientific demonstration.

It may, therefore, be said that, up to the present, no logical proof of marital infection in tuberculosis has been given.

It is the object of this paper to attack the problem with the aid of modern statistical methods.

A simple illustration will make the basis of the methods clear. Suppose we have 1000 married couples and that one person in every ten dies of tuberculosis. Amongst the 1000 husbands we should expect to find 100 dying from this cause and 900 from other causes. Among the wives of the 100 tuberculous men we should expect to find one in every ten, that is 10, die from tuberculosis and 90 from other causes. Among the wives of the 900 non-tuberculous men we should have 90 tuberculous and 810 non-tuberculous deaths. Summing up we should expect our 1000 couples to die as follows[†]:

$$\begin{array}{lr} H+,\ W+ & 10 \\ H+,\ W- & 90 \\ H-,\ W+ & 90 \\ H-,\ W- & \underline{810} \\ & 1000 \end{array}$$

Of course there would always be variations from these exact numbers, but the limits of such variations can easily be calculated. If now we find, instead of the above, the following results:

$$\left.\begin{array}{r} 20 \\ 80 \\ 80 \\ 820 \end{array}\right\} \text{ or } \left\{\begin{array}{r} 10 \\ 85 \\ 85 \\ 820 \end{array}\right.$$

we note that something has disturbed the random or chance distribution of the disease and, where the differences are greater than can be accounted for by random sampling, we are forced to conclude that the marital relationship is the cause of the disturbance.

[3. The problem of the extent to which random sampling will produce an apparent disturbance of the chance distribution is, however, a very delicate one which can

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[* The question of assortative mating must also be considered. For Weinberg's papers see *Beiträge für K. der Tub.* Bd. v. 1906; and *Württemb. Jahrbücher für Statistik*, u.s.w. Jg. 1907, S. 195–7.]

[†] Throughout this memoir $H+$, $W+$, signify Husband tuberculous, Wife tuberculous; and $H-$, $W-$, Husband non-tuberculous, Wife non-tuberculous.

only be treated by the exact methods of modern statistics. Taking the two cases given above, we may write them as follows:

	$H-$	$H+$	Totals
$W-$	820 (+10)	80 (−10)	900
$W+$	80 (−10)	20 (+10)	100
Totals	900	100	1000

	$H-$	$H+$	Totals
$W-$	820 (+10)	85 (−5)	905
$W+$	85 (−5)	10 (0)	95
Totals	905	95	1000

I have placed the deviations from the chance distributions in brackets. Now let the following quantity be formed:

$$\chi^2 = \text{Sum for each class of } \frac{\text{Difference between observed and expected frequency squared}}{\text{Expected frequency}}.$$

In our first case this equals

$$\frac{10^2}{810}+\frac{10^2}{90}+\frac{10^2}{90}+\frac{10^2}{10}=12\cdot 3,$$

and for our second case equals

$$\frac{10^2}{810}+\frac{5^2}{90}+\frac{5^2}{90}+\frac{0^2}{10}=\cdot 68.$$

Now if these values of χ^2 be looked up in the Tables* for the probability of such deviations in four groups, we find that the chance of such a set of deviations as the first occurring by random sampling is only ·0065, i.e. would only occur 6 or 7 times in 1000 trials. It is therefore almost certainly significant. In the second case the probability is about ·86, i.e. in a hundred random samples 80 to 90 would deviate more than this from the chance distribution. In other words there is no significance at all in the deviations. It is this important distinction which must be insisted on in all problems of this kind. It deserves to be illustrated further on the actual material of this memoir. Thus I take the results reached by Dock and Chadbourne and again by Riffel from Table I. The reader will notice at once the high value of the marital correlation, ·49, given in both these cases.

Dock and Chadbourne

	$H-$	$H+$	Totals
$W-$	88 (+1·51)	5 (−1·51)	93
$W+$	5 (−1·51)	2 (+1·51)	7
Totals	93	7	100

Riffel

	$H-$	$H+$	Totals
$W-$	606 (+13·49)	31 (−13·49)	637
$W+$	60 (−13·49)	19 (+13·49)	79
Totals	666	50	716

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* *Biometrika*, Vol. I. p. 155.

TABLE I.

1	2	3	4	5	6	7	8	9	10	11	12	13
Series	Source	Kind of cases	All dead or living and dead	Number of couples	$H-$ $W-$	$H+$ $W-$	$H-$ $W+$	$H+$ $W+$	Total % +	Correlation r_5	Excess of cases beyond chance	Percentage of + + cases beyond chance
Riffel, E. & J. I *a*	Erblichkeit & Infectionstat, Karlsruhe, 1890	General population	L & D	716	606	31	60	19	9	·49	14·8	78
,, ,, I *b*	,, ,, ,,	,,	D only	152	88	22	25	17	27	·36	8·3	49
Riffel, S. & K. II *a*	Schwindsucht & Krebs, Karlsruhe, 1905	,,	L & D	288	194	45	35	14	19	·19	4·9	35
,, ,, II *b*	,, ,, ,,	,,	D only	159	85	28	32	14	28	·11	2·5	18
Reiche	Muench. Med. Woch. 1903, L. 135	Parents of −	L & D	2450	2058	239	114	39	9	·30	23·7	61
Dock & Chadbourne	Phila. Med. Jl. 1898, II. 966	,,	L & D	100	88	5	5	2	7	·49	1·6	80
Kuthy (Schwartzkopf)		,,	L & D	108	84	10	11	3	13	·26	1·5	50
Schwartzkopf	Deut. Archiv Klin. Med. LXXVIII. 73	,,	L & D	194	157	28	7	2	10	·14	0·7	35
Fischer	Beit. z. Klin. d. Tub. 1904, III. 19	,,		360	315	11	26	8	7	·55	6·7	83
A. C. S.	Adirondack Cottage Sanitarium Records	Parents of +	L & D	1897	1471	199	190	37	12	·12	10	27
,,	,, ,, ,, ,,	,,	D only	155	96	23	28	8	22	·07	1	13
Squires	Essays on Consumption, London, 1900	,,		1000	673	148	135	44	19	·14	11·8	27
Williams	Pulm. Consumption, Phila. 1887	,,		1000	880	43	67	10	7	·28	6·3	63
Reiche	Muench. Med. Woch. 1903, L. 135.	,,		2476	1659	449	270	98	18	·10	20·4	21
Dock & Chadbourne	Phila. Med. Jl. 1898, II. 966	,,	L & D	100	74	13	12	1	14	− ·25	0	—
Kuthy (Schwartzkopf)		,,		432	329	50	43	10	13	·14	3	30
Schwartzkopf	Deut. Archiv Klin. Med. LXXVIII. 73	,,		160	106	26	25	3	18	− ·26	0	—
Brompton	Brompton Hospital Report, 1903	,,		563	438	55	51	19	13	·28	9·2	48
Fischer	Beit. z. Klin. d. Tub. 1904, III. 19	,,		59	44	6	3	6	18	·75	5	83
Krankenheim	Tub. Arbeit. K. & G. 1904, Heft 2, 240	,,		3831	2779	610	283	159	17	·22	63·3	40
Basler Heilstätte	E. Finkheimer—Dissertation, 1904	,,		1010	580	224	150	56	24	− ·01	0	—
Cottbus	Beit. z. Klin. d. Tub. 1906, V. 455	,,		1081	650	211	147	73	23	·16	19·7	27
Grabowsee	Tub. Arbeit. 1904, Heft 2, 169	,,		2047	1483	311	209	44	15	·01	·1	·2
Planegg	,, ,, ,, 215	,,		861	636	116	87	22	14	·11	5·3	24
Ruppertshain	,, ,, ,, 269	,,		1795	1272	274	191	58	16	·12	14·2	25
Sokolowski	Z. f. T. 1901, II. 210	,,		8074	6825	545	605	99	8	·20	46·7	47
Gabrilowitsch	Z. f. T. 1902, III. 205	,,		1000 .	619	148	189	44	21	− ·01	− ·9	− 2·4
Fuller	Diseases of the Lungs, London, 1867	,,		385	286	48	40	11	14	·16	3·7	33·6
Friedrichsheim	Tub. Arbeit. 1905, Heft 4, 37	,,		686	451	134	73	28	19	·07	4·9	18
Albertsberg	,, ,, ,, 87	,,		1197	829	204	127	37	17	·06	4·8	13
Belzig	,, ,, ,, 140	,,		500	358	89	42	11	15	·02	0·5	5
Sülzhayn	,, ,, ,, 193	,,		687	511	102	57	17	14	·13	4·9	29
Oderberg	Tub. Arbeit. 1906, Heft 5, 35	,,		1352	989	186	141	36	15	·10	8·1	23
Glückauf	,, ,, ,, 79	,,		432	243	95	68	26	25	− ·01	− ·6	− 2·3
Harlaching	,, ,, ,, 120	,,		252	158	40	38	16	22	·18	5·1	32
Königsberg	,, ,, ,, 160	,,		419	316	62	28	13	14	·28	6·5	50
Kreises Altena	,, ,, ,, 202	,,		734	552	103	64	15	13	·08	2·7	18
Loslau	,, ,, ,, 245	,,		804	653	81	61	9	10	·05	1·3	14
Sophienheilstätte	,, ,, ,, 287	,,		536	420	69	36	11	12	·19	4·5	41
Friedmann	Deut. Med. Woch. 1901, XXVII. 813	,,		1734	751	503	323	157	33	− ·13	0	—

Again the deviations from the expected numbers are placed in brackets. By the same process and tables we find :

Dock and Chadbourne : $\chi^2 = 5{\cdot}4$, Probability = ·25.
Riffel : $\chi^2 = 39{\cdot}8$, Probability = ·000000.

Thus Dock and Chadbourne's results might arise from random sampling once in about four samples. Standing alone therefore they have no weight as showing relationship in tuberculosis between husband and wife. Riffel's data on the other hand could not on any reasonable assumption be expected to arise from random sampling; on an average not once in a million samples would such a discrepancy be shewn, if husband and wife were affected merely by chance. The casual reader might easily notice the high value ·49 for the resemblance of husband and wife given by both Riffel's and Dock and Chadbourne's data and suppose them of equal value, or at any rate of value determined merely by the total of cases dealt with. This is very far from the fact; standing alone Dock and Chadbourne's data would not enable us to form any conclusion at all, Riffel's from the statistical standpoint *only* would shew a high degree of relationship. On the other hand Riffel's results have to be taken in close conjunction with the circumstances under which they were collected. In the 1890 material he worked through an entire German village selecting no individuals at all. He thus had no random sample of the village, but the whole of its population. Can we, however, treat this village as a random sample of the general population ? It is hardly possible. In the 1905 material we really find the answer to our question. In this village as in the former one phthisis must have been very rife, because in both 27 to 28 per cent. of the married population appeared to have suffered from it ! One in four of all married persons was affected. Yet the second result so far from confirming the early one more or less discredits its claim to be considered as a sample of the general population. When living and dead members are included, the coefficient of phthisical resemblance between husband and wife has fallen from ·49 to ·19 less than half its value and for completed histories from ·36 to ·11 less than a third its value. It will appear in the sequel that while ·49 and ·36 would enable us to assert definitely marital infection ·19 and ·11 would fall well into the limits of assortative mating and could not be made the basis of any statement whatever. Yet the material of Riffel is all that we can classify in Table I under the heading of sample of the "General Population." In every other case allowance for selection has to be made before we can reach any definite results.]

[**4.** In Table I are collected together all the data Mr Pope had been able to find regarding the prevalence of tuberculosis in married couples. In the first column is given the name of the recorder, or the Sanatorium where the data were obtained ; in the second the locus of publication. The third column gives the class of cases ; the first group were as we have seen the entire population of two villages ; the next group contains the cases where the parents were selected, being the parents of non-tuberculous offspring, while the third and last group, probably containing the most reliable data, is that in which inquiries were made as to the parents of tuberculous patients. Column 4 states whether the histories were complete or not, and it will be seen that there are only three short series in which all the pairs involved were dead. Column 5 gives the numbers dealt with; Column 6 gives the number of pairs in which neither husband nor wife were tuberculous; Column 7 gives the number of cases in which husband was, and wife was not tuberculous; Column 8 gives the number in which the husband was not tuberculous, but the wife was; Column 9 gives the number of cases in which both husband and wife were tuberculous. Column 10 gives in round figures the percentage of tuberculous individuals in the total married population, which is of course double the number in Column 5. Column 11 gives the correlation between husband and wife with regard to tuberculosis.]

5. With regard to the series taken from the *Tub. Arbeit.*, it has to be noted that the parents are divided into "Known to be tuberculous" and "Probably tuberculous." For instance taking the Grabowsee series we have

Neither parent tuberculous		1483
Father known to be tuberculous	208	311
,, probably ,,	103	
Mother known to be ,,	141	209
,, probably ,,	68	
Both parents known to be tuberculous	35	44
,, ,, probably ,,	9	
	Total	2047

In deciding how to treat the probably tuberculous, we have been guided by the desire to be as conservative as possible and have considered all the probably tuberculous as being tuberculous and get the following figures:

$$r_5 = \cdot 01, \quad \text{Percentage } 0\cdot 2\ {}^\circ/_\circ.$$

If we consider them all as being non-tuberculous we get

$$r_5 = \cdot 21, \quad \text{Percentage } 44\cdot 6\ {}^\circ/_\circ.$$

If we exclude the uncertain cases we get

$$r_5 = \cdot 18, \quad \text{Percentage } 38\cdot 9\ {}^\circ/_\circ.$$

It is therefore clear that the course adopted has led to minimum values for these series*.

The same holds for the Friedmann series.

6. The general problem has been approached in two ways. The first way is by a study of the correlation between husbands and wives in regard to tuberculosis.

[* It is also clear how a little bias in dealing with "probably tuberculous" cases on the part of the recorder will substantially modify values.]

The idea of a numerical estimate of closeness of relationship, expressed in numbers between + 1 and − 1 (+ 1 representing complete dependence or causation, 0 representing independence and − 1 representing complete opposition or negative dependence) is of supreme importance in the discussion of a subject such as the present. The *British Medical Journal* gives a popular exposition of this idea in its issue of July 15, 1907, in which references are given to complete discussions. For the purposes of this paper the method of calculation used was that given by Professor Pearson (*Phil. Trans.* Vol. 195 A. p. 16) as Q_5 in a series of approximations. The reasons for the use of that approximation in this case are those given in the memoir cited. For several cases the values were calculated by the use of the normal probability integral tables and agreed within the limits of the probable errors.

[7. It may be of interest to compare the values found by Mr Pope from my approximate formula with those calculated from fourfold division tables. The following series were worked out for me by the kindness of Dr Alice Lee:

Series	By Fourfold Table	Approx. Formula
Riffel I *a*	·48	·49
„ I *b*	·35	·36
Riffel II *a*	·18	·19
„ II *b*	·10	·11
Reiche	·29	·30
Dock and Chadbourne	·51	·49
Kuthy	·25	·26
Schwartzkopf ...	·12	·14
Fischer	·56	·55

After the agreement demonstrated in these first nine cases, it will be admitted that Mr Pope's use of the approximate formula is justified. He thus confirms the conclusion as to the suitability of the formula for such work previously reached by Dr J. Brownlee.]

[8. Before we discuss the correlation results, it is desirable to consider another method of approaching the problem, which has been suggested by Mr Pope. I have replaced his discussion by the present one, as I think there is a point at which his treatment is not wholly valid, but the idea of the method is entirely his. While, I believe, there are reasons why it cannot be applied to the case of marital infection, I think that it deserves reproduction because it forms the natural method of approaching the problem of general infection for any disease, when the infected and infecting persons are thrown together, without blood relationship or assortative mating, in school, office, workshop or domicile.

Let the table of pairs in the general population be:

1st member A

2nd member B

	$A+$	$A-$	Totals
$B+$	a	b	$a+b$
$B-$	c	d	$c+d$
Totals	$a+c$	$b+d$	$N=a+b+c+d$

Now since A and B do not owe their condition to each other: $a \times d = b \times c$.

Suppose now that A and B are brought into special contact in home or school or workshop, and let a fraction of the $B+$ associated with the sound $A-$ infect the latter, i.e. let us suppose pb of the b $A-$'s to become infected. We shall further suppose qc of the sound B's $(B-)$ to become affected by their association with the tainted $A+$'s. We have now the table:

1st member A

2nd member B

	$A+$	$A-$	Totals
$B+$	$a+pb+qc$	$b(1-p)$	$a+b+qc$
$B-$	$c(1-q)$	d	$c+d-qc$
Totals	$a+c+pb$	$b+d-pb$	N .

Key

a'	b'	$a'+b'$
c'	d'	$c'+d'$
$a'+c'$	$b'+d'$	N

This is the table as it would present itself if we collected statistics for the general population of specially associated couples. If we form the cross product difference $= N\epsilon$, say, we have

$$\begin{aligned} N\epsilon &= d(a+pb+qc) - bc(1-p)(1-q) \\ &= ad - bc + (db+bc)p + q(dc+bc) - bcpq \\ &= ad - bc + bp(c+d-qc) + cq(b+d-pb) + bp \times cq. \end{aligned}$$

Now $c+d-qc$ and $b+d-pb$ are known frequencies $=c'+d'$ and $b'+d'$ say. bp is the number of cases β in which B infects A, and $cq=\alpha$ the number of cases in which A infects B. Further $N\epsilon = a'd' - b'c'$ is known.

Accordingly $\quad a'd' - b'c' = ad - bc + \beta(c'+d') + \alpha(b'+d') + \alpha\beta.$

Mr Pope assumes that $ad-bc$ is zero for husband and wife. This is to assert that there is no assortative mating for those characters on which a tuberculous tendency depends, and indirectly amounts to saying the whole extent of the correlations given in Column 11 of Table I are due to infection. As Mr Pope in another paragraph of his paper endeavours to allow for assortative mating, I think he would have modified this view had he lived to complete his memoir.

But it is quite clear that $ad-bc$ would be zero in the case of workshop or school association, and this is the case where the above formula is likely to be of service. In such a case

$$a'd'-b'c'=\beta(c'+d')+\alpha(b'+d')+\alpha\beta.$$

Mr Pope puts $\alpha=\beta$ and thus obtains the formula

$$a'd'-b'c'=\alpha(b'+c'+2d)+\alpha^2$$

to determine the number of cases in which husband or wife infect wife or husband. Now the equality of α and β does not seem to me a reasonable hypothesis, when the number of infected wives is not equal to the number of infected husbands. It appears a better hypothesis to consider that the *percentage* of infected wives who infect their sound husbands will be the same as that of infected husbands who infect their sound wives. There appears to be no ground for supposing the wife or husband to be a more active centre of infection, which is one of the results which would follow from the equality of α and β applied to cases in which the group $H+$, $W-$ occurs, as it does in so many of the recorded series, with very different frequency to $H-$, $W+$.

Accordingly I take $p=q$. Hence

$$\beta=bp=\frac{p}{1-p}b', \qquad \alpha=cq=\frac{p}{1-p}c'.$$

Or, putting $p/(1-p)=u$,

$$u^2b'c'+u(2b'c'+d'(b'+c'))-(a'd'-b'c')=0 \quad \text{..................(i)},$$

which may be read

$$p^2d'(a'+b'+c')-pd'(2a'+b'+c')+a'd'-b'c'=0 \quad \text{...............(ii)},$$

or again in the still simpler form

$$(1-p)^2d'(a'+b'+c')-(1-p)d'(b'+c')-b'c'=0 \text{...............(iii)}.$$

Let us illustrate this on Riffel I *a*.

We have $\qquad d'=606, \quad b'=60, \quad c'=31, \quad a'=19,$

$$\therefore \quad (1-p)^2\,606\times 110-(1-p)\,606\times 91-60\times 31=0,$$

$$(1-p)^2\,66660-(1-p)\,55146-1860=0.$$

We find at once $\qquad p=\cdot 1403.$

Hence $\qquad \beta=\frac{p}{1-p}b'=9\cdot 79, \qquad \alpha=\frac{p}{1-p}c'=5\cdot 06.$

Thus we learn that if there were no assortative mating there would be 9·79 cases in which a tainted wife infected a sound husband and 5·06 cases in which a tainted husband infected a sound wife, or in all a total of 14·85 cases of infection. Before infection there would be 69·79 cases of $H-$, $W+$ and 36·06 cases of $H+$, $W-$, and in 14 per cent. of these there would be infection. The transfer of 14·85 cases to the $H+$, $W+$ category means that only 4·15 of the 19 cases in that class are due to chance. Or, on this hypothesis, 78 per cent. of the observed $H+$, $W+$ cases are due to infection.

The figures in Columns 12 and 13 of Table I were obtained by Mr Pope on the assumption that $\alpha = \beta$, or that with different frequency in Columns 7 and 8, still the wives infected as many husbands as husbands infected wives. This as we have seen is not legitimate, but there is rarely a difference of 1 per cent. in Columns 12 and 13 from the figures calculated on the more correct theory, and accordingly I have not reworked the whole series and tabled p. I have been the less anxious to do this as it is perfectly easy to get numbers quite within the probable errors of those recorded in Columns 12 and 13 by a far simpler process. Thus the chance in Riffel I a of $H+$ is $\frac{50}{716}$ and of $W+$ $\frac{79}{716}$ and there would be on the combined chance $716 \times \frac{50}{716} \times \frac{79}{716}$ cases of $H+$ and $W+$, or 5·52, that is to say there is an excess of 13·48 instead of 14·8. The difference being due to the slight increase in the $H+$ and $W+$ due to marital infection. Again for Reiche's data we get an excess of 21·6, as against Mr Pope's 23·7. For Fischer's data we find 6·0 as against Mr Pope's 6·7. This simple process thus differs in results by 1 to 2 per cent. But I think the calculation of these excesses by any of these processes, and above all of the percentages in Column 13, is open to criticism, if the probable errors in each case are not given. For example in Dock and Chadbourne's data, the return of two couples with both husband and wife tuberculous is subject to a probable error of ± ·9. Thus within the limits indicated by the probable error, the percentage of cases beyond the chance number might really be zero instead of 80 per cent. Similarly for many of the high percentages beyond chance a very considerable allowance has to be made on the score of the probable error. It appears better accordingly to reason on the total drift of the correlation coefficients, than on the percentages in Columns 12 and 13. I have replaced, however, Mr Pope's title of excess and percentage due to infection, by the words "beyond chance*."]

[9. Yet even when we come to examine these series minutely we are at once struck with certain remarkable discrepancies among them. The four series that give high values of the correlation coefficients and of the percentages are:

(*a*) The first Riffel series with ·49,
(*b*) The Dock and Chadbourne series with ·49,
(*c*) The first Fischer series with ·55,
(*d*) The second Fischer series with ·75.

Now we have already seen that the Dock and Chadbourne series consists of so few couples that with random sampling of a chance population it might occur once in four samples. Little again can be based on Fischer's second series of 59 couples only! Examining more closely the first Riffel and first Fischer series we note a very curious point: the number of cases of sound husband and tuberculous wife in both of them is *double* the number of cases of sound wife and tuberculous

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* I.e. this suffices to indicate that the excess may in whole or in part be due to assortative mating.

husband. On the other hand in nearly all German data the number of sound husbands and tuberculous wives is in a ratio of about 2 to 3 to the number of tuberculous husbands with sound wives. We naturally ask why the series which give these exceptional values of the husband and wife correlation, are also exceptional in the fact that the ratio of 2 to 3 has been changed into one of 2 to 1? If there be no explanation forthcoming, then at least we must conclude that we are not dealing with random samples of the general population. A noteworthy fact is that in the Riffel series, if we confine our attention to completed histories, we find this ratio has risen from 2 to 1 to about 2 to 2. I suggest therefore that in the case of both Riffel and Fischer the number of tuberculous males, 5 and 7 per cent. respectively, has been for some reason* vastly underestimated. Both writers give the percentage of tuberculous wives as about 11 per cent. The percentage in mothers of the tuberculous in Germany at large is about 12 per cent. and in the fathers of the tuberculous about 16 per cent. The former is very little in excess of the value for the general German population of wives given by Riffel and Fischer; the latter is two to three times their values for the general population of husbands. One cannot avoid supposing that Riffel and Fischer are dealing with some very exceptional conditions, and that we must be cautious of giving too much weight to the evidence of these series.

Taking the series as they stand we have the following results for these very divergent values. It would, however, be quite reasonable *a priori* to reject or give but slight weight only to such series as those of Dock and Chadbourne and Fischer II:

Mean of Riffel's incomplete History series: ·34,
Mean of Riffel's complete History series: ·23,
Mean of series for Parents of the tuberculous: ·35,
Reiche's long but incomplete History series of Parents of the tuberculous: ·30.

If we can judge from Riffel's two series, the effect of completing histories in the other series also would reduce their values to something between ·20 and ·25. I have not attempted to correct these results for selection in the case of parents of sound offspring. Such parents will of course be much nearer the general population than parents of tuberculous offspring. These first results therefore seem to indicate that for completed histories the general population will hardly give for resemblance of husband and wife with regard to tuberculosis a higher value than ·25.

To test this Miss Elderton took out of the *Family Records* of the Eugenics Laboratory 634 cases of Husband and Wife. These records had been formed without regard to special diseases and may be taken as a random sample of the professional middle classes. In 567 cases neither husband nor wife were tuberculous; in 23 cases the husband was definitely tuberculous and the wife was not; in 30 cases the wife was and the husband was not tuberculous; in three cases both were tuberculous; in six cases there was a doubt as to the presence of tuberculosis, in one pair in the case of a tuberculous husband the wife was doubtful; in one pair the wife was in doubt,

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* Confession of tuberculous symptoms might, for example, affect the employment of the males.

but the husband was sound; in four pairs the wife was sound, but the husband doubtful; in three cases the wife was tuberculous, but the husband was marked "not known"; in one case the husband was tuberculous and the wife was marked as "not known"; and in one case the wife was doubtful and the husband was marked as "not known."

The problem of course is to determine what to do with the six cases in which there was some doubt as to whether one or both members had possibly shewn tuberculous signs, and again with the five cases of "not known."

If we include the doubtful with the tuberculous and the "not known" with the sound, the correlation is ·20; this is also the value we reach if the doubtful are treated as non-tuberculous and we leave out the "not known" altogether. To reach the other limit all the "not knowns" and the "doubtfuls" were taken as tuberculous, and the correlation rose to ·47. This is, of course, an incorrect procedure, because the mere fact that nothing was known about the cause of death or the ailments in life of an individual, does not justify us in assuming that that individual was tuberculous. If we leave out the "not knowns" and make all the "doubtfuls" tuberculous the correlation is ·24.

It will be seen that these results are singularly in accordance with Riffel's complete history series, and that while their possible maximum is just the height of his first incomplete history series, it is very unlikely that its actual value rises to anything like this height. We see further that the classification of a few doubtful cases makes little real difference, but that a highly improbable distribution of the "not known" cases makes considerable difference.

As a result of this consideration of "random samples" and of the only slightly selected parents of the non-tuberculous, I think, we may say that the correlation in tuberculosis between husband and wife amounts certainly to more than ·2, is very unlikely to be as large as ·4, and might possibly reach ·30 to ·35.]

[10. I now pass to Mr Pope's data for the parents of tuberculous offspring. This group of series I believe to be much more reliable than any other material at present collected, and it is far more extensive. If we take the average value of the 31 series collected in Table I, we find that the correlation of husband and wife is ·11. The material, however, seems to be of a kind which is fairly homogeneous, and quite worth working out as a whole. I find the following combined table:

	$H-$	$H+$	Totals
$W-$	30,806	5,586	36,392
$W+$	4,098	1,296	5,394
Totals	34,904	6,882	41,786

The correlation worked for this table is

$$r = \cdot 17.$$

Now the mean of Mr Pope's separate 31 values, or the value for this big table, are both distinctly below the average value of assortative mating in man. Hence if there be assortative mating for the tuberculous constitution, or rather for its closely allied physical and mental characters, we should be compelled to conclude that marital infection can play but a small part in the case of tuberculosis.

Such a conclusion, however, assumes that the above table may be taken as a *random* sample of the general population. This it certainly would be, if in selecting the parents of the tuberculous, we were not making a differentiated population. The answer to this question depends in the main on whether we grant that the tuberculous diathesis is inherited. If we allow that it is, then we must remember that the above is a stringently selected element of the population, and the correlation value has to be corrected for this selection. If we do not accept the inheritance of the tuberculous constitution then it is very difficult to look upon the above result as denoting any appreciable marital infection. There may, of course, be a chance of parental infection, but remembering that in many cases the parents are dead or through the danger zone before the offspring reach it, this parental infection must give a still lower correlation than marital infection. It could hardly exceed ·1, and the correction of ·17 for a correlation of this order is not worth consideration*.]

[11. I pass now to the consideration of the correction to be made owing to the fact that tuberculous offspring are more likely than the general population to be offspring of tuberculous parents. We will inquire what correction must be made on the three assumptions (a) that tuberculous offspring are 1 in 15 of all offspring, (b) 1 in 10, and (c) 1 in 5. (a) to (b) would be true according to the class in England. Probably (b) to (c) is more nearly true for Germany from which the bulk of Mr Pope's data are drawn.

Let the subscripts 1 and 2 refer to father and mother and 3 to the offspring, who are selected as tuberculous. Then assuming a normal distribution for the tuberculous diathesis† the following formulae for selection are proved in Appendix I:

$$\bar{x}_1 = \frac{\sigma_1}{\sigma_3} r_{13} \bar{x}_3 \quad \text{...(iv)},$$

$$\bar{x}_2 = \frac{\sigma_2}{\sigma_3} r_{23} \bar{x}_3 \quad \text{...(v)},$$

$$\bar{\sigma}_1^2 = \sigma_1^2 \left\{ 1 - r_{13}^2 + r_{13}^2 \frac{\bar{\sigma}_3^2}{\sigma_3^2} \right\} \quad \text{....................................(vi)},$$

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* As will be shewn in the sequel, the correction depends on the square of the correlation, i.e. on $(\cdot 1)^2 = \cdot 01$ in this case and is insignificant.

† The formulae are really true if the distribution be not normal, but the method of determining $\bar{x}_3$ and $\bar{\sigma}_3$ discussed below would require modification.

$$\bar{\sigma}_2^2 = \sigma_2^2 \left\{1 - r_{22}^2 + r_{23}^2 \frac{\bar{\sigma}_3^2}{\sigma_3^2}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)}.$$

$$\bar{r}_{12} = \frac{r_{12} - r_{23}r_{13} + r_{23}r_{13}\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}{\sqrt{1 - r_{13}^2 + r_{13}^2\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}\sqrt{1 - r_{23}^2 + r_{23}^2\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)}.$$

Here r_{13} is the resemblance between father and offspring, r_{23} between mother and offspring; r_{12} is the resemblance between husband and wife in the general population, $\bar{r}_{12}$ in the selected population of parents of tuberculous offspring. The correlation between both parents and the offspring may be taken equal and equal to the average value of inheritance of any physical character, say ·46, if we accept the inheritance of the tuberculous diathesis*. $\bar{r}_{12}$ has been found above and is equal to ·17, or more exactly ·1710. r_{12} is what we wish to determine, i.e. the correlation between husband and wife in a non-selected or general population.

$\bar{x}_1$, $\bar{x}_2$ are the distances of the means of the parents of tuberculous offspring from the means of parents in general, owing to their offspring being tuberculous; and $\bar{x}_3$ is the shift of the mean of tuberculous offspring from the mean of the general population. σ_1, σ_2, σ_3 are the variabilities of father and mother and offspring in the general population with regard to the tuberculous character; $\bar{\sigma}_1$, $\bar{\sigma}_2$ are the variabilities of parents of tuberculous offspring and $\bar{\sigma}_3$ the variability of tuberculous offspring themselves.

We have first to determine $\bar{x}_3$ and $\bar{\sigma}_3$ on the assumptions that the "tuberculous tail" of the normal distribution of the general population for the tuberculous character contains 1 in 15, 1 in 10 and 1 in 8 of the population. Let h_3 be the deviation from the mean of the normal population at which the tuberculous character becomes manifest as tuberculosis, i.e. the point at which the constitutional resistance to the disease breaks down. Then if the tail be $\frac{1}{n}$th of the total population:

$$\frac{1}{n} = \frac{1}{\sqrt{2\pi}\,\sigma_3}\int_{h_3}^{\infty} e^{-\frac{1}{2}x^2/\sigma_3^2}\,dx,$$

$$\frac{1}{n} = \frac{1}{\sqrt{2\pi}}\int_{h_3/\sigma_3}^{\infty} e^{-\frac{1}{2}x'^2}\,dx' = \frac{1}{2}(1 - \alpha),$$

or,

$$1 - \frac{1}{n} = \frac{1}{2}(1 + \alpha),$$

where $\frac{1}{2}(1 + \alpha)$ is tabled for h_3/σ_3 in tables of the probability integral, e.g. Sheppard's†. Hence for the three cases:

$$\tfrac{1}{2}(1 + \alpha) = \cdot 80, \ \cdot 90 \text{ and } \cdot 9\dot{3}\,;$$

these give respectively:

$$h_3/\sigma_3 = \cdot 84162, \ 1\cdot 28155 \text{ and } 1\cdot 50109.$$

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* See Pearson, "A First Study of the Statistics of Pulmonary Tuberculosis," *Drapers' Research Memoirs*, Dulau and Co., Soho Square, London.

† *Biometrika*, Vol. II. pp. 174—190.

We have next to determine the centroid $\bar{x}_3$ of this portion of the tail

$$\frac{1}{n}\bar{x}_3 = \frac{1}{\sqrt{2\pi}\,\sigma_3}\int_{h_3}^{\infty} x e^{-\frac{1}{2}x^2/\sigma_3^2}\,dx,$$

$$\frac{1}{n}\frac{\bar{x}_3}{\sigma_3} = \frac{1}{\sqrt{2\pi}}\int_{h_3/\sigma_3}^{\infty} x' e^{-\frac{1}{2}x'^2}\,dx'$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(h_3/\sigma_3)^2}.$$

This is given at once by Sheppard's Tables and we find respectively:

$$\frac{\bar{x}_3}{\sigma_3} = 1{\cdot}39981,\ 1{\cdot}76195 \text{ and } 1{\cdot}94593.$$

Thus from Equations (iv) and (v) we find the average deviation of the parents of tuberculous offspring from the mean of the normal population to be:

$$\frac{\bar{x}_1}{\sigma_1} = \frac{\bar{x}_2}{\sigma_2} = {\cdot}6439,\ {\cdot}8105, \text{ and } {\cdot}8951.$$

The next point is to find the variability of the selected population. We have in the same manner as above:

$$\frac{1}{n}\left(\bar{\sigma}_3^2 + \bar{x}_3^2\right) = \frac{1}{\sqrt{2\pi}\sigma_3}\int_{h_3}^{\infty} x^2 e^{-\frac{1}{2}x^2/\sigma_3^2} dx,$$

$$= \frac{\sigma_3^2}{\sqrt{2\pi}}\int_{h_3/\sigma_3}^{\infty} x'^2 e^{-\frac{1}{2}x'^2} dx',$$

or, integrating by parts, $= \dfrac{\sigma_3^2}{\sqrt{2\pi}}\dfrac{h_3}{\sigma_3} e^{-\frac{1}{2}\frac{h_3^2}{\sigma_3^2}} + \dfrac{\sigma_3^2}{\sqrt{2\pi}}\displaystyle\int_{h_3/\sigma_3}^{\infty} e^{-\frac{1}{2}x'^2} dx'.$

Thus: $$\frac{\bar{\sigma}_3^2}{\sigma_3^2} + \frac{\bar{x}_3^2}{\sigma_3^2} = \frac{h_3}{\sigma_3}\frac{\bar{x}_3}{\sigma_3} + 1,$$

or, $$\frac{\bar{\sigma}_3^2}{\sigma_3^2} = 1 + \frac{h_3\bar{x}_3}{\sigma_3^2} - \frac{\bar{x}_3^2}{\sigma_3^2} = 1 - \frac{\bar{x}_3(h_3 - \bar{x}_3)}{\sigma_3^2}.$$

This gives for our three cases:

$$\frac{\bar{\sigma}_3^2}{\sigma_3^2} = {\cdot}2186,\ {\cdot}1536 \text{ and } {\cdot}1344,$$

or, $$\frac{\bar{\sigma}_3}{\sigma_3} = {\cdot}4675,\ {\cdot}3919 \text{ and } {\cdot}3666,$$

as measuring the stringency of the selection.

We then have by Equation (iv):

$$\frac{\bar{\sigma}_1^2}{\sigma_1^2} = \frac{\bar{\sigma}_2^2}{\sigma_2^2} = 1 - ({\cdot}46)^2 + ({\cdot}46)^2\frac{\bar{\sigma}_3^2}{\sigma_3^2}$$

$$= {\cdot}8347,\ {\cdot}8209 \text{ and } {\cdot}8168,$$

or, $$\frac{\bar{\sigma}_1}{\sigma_1} = \frac{\bar{\sigma}_2}{\sigma_2} = {\cdot}9136,\ {\cdot}9060 \text{ and } {\cdot}9038.$$

We are now in a position to ascertain what percentage of the parental population would be tuberculous, if we selected only the parents of tuberculous offspring. Clearly, if we suppose the degree of constitutional weakness at which the disease appears to be the same in the two generations, we should have the parental generation in the case of the selected parental population centering round $\bar{x}_1$ (or $\bar{x}_2$) as mean and therefore at a distance $h_3 - \bar{x}_1$ or $h_3 - \bar{x}_2$ from the constitutional limit. Taking $\sigma_1 = \sigma_2 = \sigma_3$ we shall call these tails $\bar{h}_1$ and $\bar{h}_2$ and we have:

$$\frac{\bar{h}_1}{\sigma_1} = \left(\frac{h_3}{\sigma_3} - \frac{\bar{x}_1}{\sigma_1}\right)\frac{\sigma_1}{\bar{\sigma}_1}$$

$$= \frac{\cdot 1977}{\cdot 9136}, \quad \frac{\cdot 4711}{\cdot 9060} \text{ and } \frac{\cdot 6060}{\cdot 9038},$$

or $= \cdot 2164$, $\cdot 5200$ and $\cdot 6705$ for the three cases respectively. In round numbers these give us 41 °/o, 30 °/o and 25 °/o of the parental generation tuberculous. Now although there occur cases with 25 °/o among Mr Pope's series, this is too high on the average, the mean being 14·7 °/o. Must we therefore suppose the number of cases in the unselected population to be less than 1 in 15? I do not think this is the legitimate inference. I believe it is higher than this in Germany, but that the married population is already a selection of the general population, before we still further select it by choosing only the parents of tuberculous offspring. That is, the tuberculous element of the general population does not live to marry and have offspring in its full quota. I have already shewn that this is true for the United States, namely that the deaths from tuberculosis in the married population are fewer than in the general population over 15 years of age. If so the mean of the married population must be taken further on the sound side of the tuberculous limit than in the case of the general population. In fact h_3' must take the place of h_3 in the equation for $\bar{h}_1/\bar{\sigma}_1$ above, and possibly σ_3' smaller than σ_3, the place of σ_3. $\bar{x}_1$ will of course remain unchanged being the selection from the mean wherever it may be. We have thus

$$\frac{\bar{h}_1}{\bar{\sigma}_1} = \left(\frac{h_3'}{\sigma_3'} - \frac{\bar{x}_1}{\sigma_1}\right)\frac{\sigma_1}{\bar{\sigma}_1}.$$

For example, if 1 in 5 of the general population were tuberculous and 14·7 °/o of the parents of tuberculous offspring, we should have

$$\frac{h_3'}{\sigma_3'} = \frac{\bar{x}_1}{\sigma_1} + \cdot 9136 \times 1\cdot 05^*$$

$$= \cdot 6439 + \cdot 9593 = 1\cdot 6052,$$

or, there would be 5 °/o or 1 in 20 of the general married population tuberculous.

Thus in our illustration we have the following course of affairs. In the generation of offspring 1 in 5 are tuberculous. Of these not all marry, so that in the general

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* I.e. value of $\bar{h}_1/\bar{\sigma}_1$ corresponding to 14·7 °/o of population tuberculous.

married population only 1 in 20 are tuberculous. If we select out of this general married population the parents of the tuberculous offspring, we have a married population of which about 1 in 7 (14·7 °/₀) are tuberculous. Now I do not lay any particular stress upon this actual example, except as illustrating the need for bearing in mind the very different tuberculosis rates we shall find, if we work with :

(1) a random sample of the general population,

(2) a random sample of the married population,

(3) a selected sample of the married population, i.e. the parents of tuberculous offspring.

We cannot determine these ratios more fully than is suggested in the above example, as for really accurate calculations the sexes with differentiated tuberculosis rates must not be grouped together as in the data of Table I.

I now proceed to find from Equation (viii) the value of r_{12} from the known value ·1710 of $\bar{r}_{12}$. We may, writing $r_{13} = r_{23}$, put that equation in the form :

$$r_{12} = 1 - (1 - \bar{r}_{12}) \frac{\bar{\sigma}_1^2}{\sigma_1^2} = 1 - {}^{\cdot}8289 \frac{\bar{\sigma}_1^2}{\sigma_1^2}.$$

We find for the three cases :

·31, ·32, and ·32,

corresponding to 1 in 5, 1 in 10 and 1 in 15 of the general population tuberculous. It will be very evident that the exact extent of the tuberculous makes little difference in the result. But that by selecting the parents of the tuberculous only we have reduced the resemblance between husband and wife from the real value ·32 to the apparent value ·17. This value ·32 agrees excellently with the values found for Riffel's and for Reiche's incomplete series, and I think we may safely conclude that the different methods of approaching the problem lead to the same result—a marital correlation of about ·30.]

[12. We have next, supposing this value to be correct, to ask whether it is reasonable to suppose it due to infection or in whole or part to be due to assortative mating, i.e. the marriage of persons of like constitution.

The following values for the coefficient of assortative mating have been found by various investigators. It will be seen that while varying considerably, they are quite substantial.

TABLE II. *Assortative Mating in Man.*

Character	Correlation	Method	Authority	Remarks
Eye-colour	·26	Contingency	Pearson[1]	Galton's Data
Stature	·28	Correlat^{n} proper	Pearson & Lee[2]	Pearson's Family Measurements
Span	·20	,,	,, ,,	
Forearm	·20	,,	,, ,,	
Length of life, Yorkshire	·20	,,	Weldon & Pearson[3]	Data collected by C. Fawcett, M. Beeton, Weldon & Pearson
,, ,, Oxfordshire	·25	,,	,, ,,	
Length of life, Society of Friends	·20	,,	,, ,,	
Alcoholism	·27	Contingency	Schuster & Elderton[4]	Heymans' & Wiersma's Data
General health	·27	,,	E. M. Elderton[5]	Pearson's Family Records
Mean physical characters	·24	—	—	—
Intelligence	·33	Contingency	E. M. Elderton[5]	Pearson's Family Records
Truthfulness	·22	,,	Schuster & Elderton[4]	Heymans' & Wiersma's Data
Temper	·18	,,	E. M. Elderton[5]	Pearson's Family Records
Temperament, excitable	·11	,,	,,	
,, sympathetic	·15	,,	,,	
,, reserved ...	·27	,,	,,	
Success in career ...	·48	,,	,,	
Neglect of duty	·20	,,	Schuster & Elderton[4]	Heymans' & Wiersma's Data
Tone of voice	·26	,,	,, ,,	
Mean psychical characters*	·24	—	—	—
Insanity	·30	Fourfold tables	E. M. Elderton[5]	Pearson's Family Records

[1] *Biometrika*, Vol. v. p. 475. [2] *Biometrika*, Vol. II. p. 373. [3] *Biometrika*, Vol. II. pp. 487–8.
[4] *Biometrika*, Vol. v. pp. 467–8. [5] Data published for the first time in Appendix II. to this memoir.

Now of the results in this Table, those for the physical characters are admittedly the better. When we turn to the more subtle phases of temperament and character, I believe that all we can say is, that human matings are not mere random matings. In many factors there may actually be two opposed currents, one giving a tendency of like to mate with like and the other marked by the fascination of extremes. As our knowledge stands to-day I should be inclined to say that there is a marked association of characters in husband and wife represented by a relationship of ·20 to ·25. Unless therefore any characteristics shew a relationship between husband

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* The agreement between the means of the physical and psychical characters is purely fortuitous. I selected from Heymans' and Wiersma's material the characters which seemed to me to be likely to be reliably determined and to be appropriate to the matter in hand.

and wife markedly greater than ·20 to ·25, it would be very difficult to assert that this resemblance is due to other causes than those assortative processes which have just been shewn to produce quite a sensible degree of resemblance in husband and wife. An average degree of resemblance with regard to tuberculosis of ·17 is quite compatible with and explicable by the tendencies which lead the truthful, the intelligent or the sympathetic man to marry a woman with like characteristics. The tuberculous tendency may not be equally well definable, but it is undoubtedly closely associated with marked mental and physical characters. "The tendency to disease," remarks Dr R. E. Thompson* in the case of tuberculous constitutions, "is counterbalanced by delicacy of feeling, and by intellectual capacity which results in the best moral and scientific work."

There are examples of assortative mating where the correlation reaches as high a value as ·30 to ·35. Insanity, indeed, where we can hardly suppose infection, is one of them. But they are not the best established; they are cases in which we can almost always appeal to a strong possibility of co-environment influencing the result†. We have then, I think, reason for asserting that if the tuberculous coefficient be as low as ·17 no inference as to marital infection can be drawn; if on the other hand it be as high as ·30, it is possible that 20 to 30 per cent. of the resemblance is due to marital infection. Now whether we say that the coefficient of resemblance is ·17 or ·30 appears to me to depend on whether we allow that the tuberculous diathesis is non-heritable or heritable.

At this point I would draw attention to another feature of the case. Let us assume that the whole degree of resemblance as to the tuberculous condition is due to infection. Then we are confronted with the remarkable result, that while the degree of resemblance between husband and wife is not more than ·17 or possibly ·25 at a maximum, that between parent and child is about ·4 to ·6 and that between brothers about ·4 to ·5‡. When we consider how intimate is the relationship of husband and wife; when we remember the modal age of tuberculosis, and how the co-environment of many parents and children, of many a brother and brother has ceased before the date of onset, it seems impossible to suppose the effect of infection to be more than double as great between parent and child as between husband and wife. To rationalise such results we are bound to consider that the inheritance of the constitution is the vital matter, and that infective action plays a subordinate roll.

This discussion of the intensity of assortative mating, absolutely necessary as it is if we are to measure the intensity of the factor of marital infection, is undoubtedly a difficult one. I should be prepared to accept with some reservation a sensible but probably not very large infective action. A husband or wife with constitution not

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* *Family Phthisis*, p. 193.

† The environment summed up in an insane husband or wife produces in itself a great mental strain on the sound.

‡ Pearson, "A First Study of the Statistics of Pulmonary Tuberculosis," *Drapers' Research Memoirs*, Dulau and Co., Soho Square, London.

very far removed from the tuberculous limit might escape the disease if not brought into very close relations with an infected person. That seems to me all the existing statistics will admit of our predicating. Even here we must refrain from dogma, until the insanity results have been amplified and confirmed or explained.]

[13. It has already been pointed out by Mr Pope that in the small data we have for completed histories the correlation of husband and wife is less than for incomplete histories*. I should be inclined to account for this by supposing that an individual constitutionally inclined to tuberculosis will develop it earlier if mated with a tuberculous individual; if mated with a sound individual the risk may not be so immediate, but will probably come in the course of life. Thus when the histories are completed there will be a considerably increased passage of individuals from $H-$, $W-$ to the $H+$, $W-$ and $H-$, $W+$ quadrants of the table.

TABLE III. *Frequency of Tuberculosis and Degree of Resemblance of Husband and Wife.*

Percentage of tuberculous in population	Rank	Correlation between Husband and Wife	Rank
7	1	·28	29
8	2	·20	26
10	3	·05	9
12	4·5	·19	25
12		·12	17·5
13	7	·14	20·5
13		·28	29
13		·08	13
14	11	– ·25	2
14		·11	16
14		·16	22·5
14		·13	19
14		·28	29
15	15	·01	7
15		·02	8
15		·10	14·5
16	17	·12	17·5
17	18·5	·25	27
17		·06	10
18	21	·10	14·5
18		– ·26	1
18		·75	31
19	23·5	·14	20·5
19		·07	11·5
21	25	– ·01	5
22	26·5	·07	11·5
22		·18	24
23	28	·16	22·5
24	29	– ·01	5·5
25	30	– ·01	5·5
33	31	– ·13	3

Another point which was emphasised by Mr Pope, is that in the case of the parents of tuberculous offspring, there exists a *negative* correlation between the

* This is confirmed by Miss Elderton's Table, pp. 16 and 30.

percentage of tuberculous parents and the degree of resemblance between them. He does not appear to have worked out the actual correlation. I have supplied the value for the 31 cases, working by the method of "bracketed ranks"* shewn in Table III. The variate correlation thus determined is $-\cdot41$.

$$\text{Correlation of ranks } \rho = -\cdot394.$$

$$\text{Correlation of variants } r = 2 \sin \left(\frac{\pi}{6}\rho\right) = -\cdot41.$$

Mr Pope interpreted the correlation in the following manner :]

Is it not probable that this points to the necessity for a certain diathesis being necessary to the presence of the tuberculosis bacillus? So that where there is a large amount of tuberculosis in a community, all with the diathesis get infected independent of husband and wife, but where there is a smaller amount the influence of the exposure to the conjugal infection is the determining factor. The series of Williams which is taken from a very select social class seems to be a striking instance of this tendency.

[It would I think be equally reasonable to assert that the assortative tendency would probably be weakened in a community with much tuberculosis ; the fact that many married persons are tuberculous would probably signify that there was less hesitation for a person already attacked to marry, a grosser view of marriage thus prevailing. In such cases the finer view we have endeavoured to emphasise in the factor of assortative mating, i.e. the marriage of those constitutionally inclined to tuberculosis, not of those actively tuberculous would be disregarded and the coefficient of assortative mating undoubtedly weakened. The fact that with very high percentages the assortative mating becomes *negative*, may even indicate that where tuberculosis is rife, the tuberculous select sound mates†. It could not be explained on any idea of the weakening of infective action.

I may conclude in the slightly modified words of Mr Pope :

It would seem probable then (1) that there is some sensible but slight infection between married couples, (2) that this is largely obscured or forestalled by the fact of infection from outside sources, (3) that the liability to the infection depends on the presence of the necessary diathesis, (4) that assortative mating probably accounts for at least 2/3 and infective action for not more than 1/3 of the whole correlation observed in these cases. But the demonstration of this result depends on the acceptance of the inherited diathesis to be effective, and the existence of assortative mating of equal intensity in the case of want of mental balance must prevent dogmatism. In all future collection of statistics with regard both to marital infection and parental infection, it is most important that the age of husband and wife at marriage and the age at onset and death in both should be recorded. Age at onset and death of the

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[*] Pearson, "Further Methods of Correlation," *Drapers' Research Memoirs*, Dulau & Co., Soho Square.

[†] It would be *negative* (see p. 30) even without any assortative mating in the general population for a selection of the parents of the tuberculous.

parent, age of parent at birth of child and age at onset and death of child should also be recorded. It is only by such complete records that we shall ultimately be able to accurately apportion the action of infection, assortative mating and inheritance.

For real light on the problem of assortative mating of the tuberculous, we must wait till we have definite knowledge in each case of the family history of both husband and wife. If we find (i) that the marriage of two ultimately tuberculous persons took place before either were suspected of the disease, and (ii) that there is in such cases a larger percentage of family histories of tuberculosis than in the case of non-married tuberculous individuals, we should have definite evidence of the assortative mating which seems probable. If on the other hand the percentage were smaller we should have definite evidence for the infection theory. (See p. 35.)]

APPENDIX I. *On a Proof of the Fundamental Formulae of p.* 17.

In an earlier memoir* I have given the equations which enable us to determine the influence of selecting q variables out of n on the means, variabilities and correlations of the system. The form of selection adopted in that case was a normal correlation surface for the q variables. In fact we selected round a new mean system with diminishing frequency when we had increasing deviations from this new mean system. I have since obtained a general proof of these selection formulae independent of any assumption as to normality in the original population, or in the directly selected material. It would be tedious to reproduce that proof here, but an interesting special case of it is of such frequent occurrence that it is worth while devoting a few pages to its consideration.

We have three characters with respective means m_1, m_2, m_3, deviations from means x_1, x_2, x_3, standard deviations σ_1, σ_2, σ_3, and correlations r_{12}, r_{23}, r_{31}. We select for consideration the tail of the distribution of the third character. For example if 1 represents father, 2 mother and 3 offspring, we consider how we influence the characters, variabilities and relationship of father to mother, if we deal only with the selected population wherein the offspring suffer from the presence of a certain diseased condition. We cut off the tail, for example, of the population of offspring which has sufficient constitutional weakness to exhibit tuberculosis, and we ask how the parental population for these offspring differs from the general parental population for all offspring.

Let $x_3 = h_3$ mark the intensity of the character at which resistance to the disease breaks down. Then we have to find $\bar{x}_1$, $\bar{x}_2$ the mean deviations, $\bar{\sigma}_1$, $\bar{\sigma}_2$ the standard deviations and $\bar{r}_{12}$ the correlation for parents of diseased offspring. Let $\bar{x}_3$ be the mean of the diseased tail, $\bar{\sigma}_3$ the standard deviation of this tail. Then we have

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* "On the Influence of Selection on the Variability and Correlation of Organs," *Phil. Trans.*, Vol. 200, A. pp. 1–60.

shewn in the text of the present paper how to find the quantities h_3/σ_3, $\bar{x}_3/\sigma_3$ and $\bar{\sigma}_3/\sigma_3$ from a knowledge of the percentage of diseased offspring in the general population.

The general frequency surface for three variables takes the form:

$$z = z_0 e^{-\frac{1}{2R}\left(R_{11}\frac{x_1^2}{\sigma_1^2} + R_{22}\frac{x_2^2}{\sigma_2^2} + R_{33}\frac{x_3^2}{\sigma_3^2} + 2R_{23}\frac{x_2 x_3}{\sigma_2\sigma_3} + 2R_{31}\frac{x_3 x_1}{\sigma_3\sigma_1} + 2R_{12}\frac{x_1 x_2}{\sigma_1\sigma_2}\right)},$$

where R_{11}, R_{22}, R_{33}, R_{23}, R_{31}, R_{12} are expressible as minors of the determinant R of the correlation coefficients*.

If we integrate this value of z for all values of x_3 from h_3 to ∞, we shall have the frequency surface for the selected population of parents, and by multiplying by $x_1^{n_1} x_2^{n_2}$ before integrating we can obtain the various means, standard-deviations and product moments.

(1) The shifted mean of the first parent.

$$M\bar{x}_1 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-h_3}^{\infty} x_1 z\, dx_3 dx_1 dx_2$$

$$= \int_{-h_3}^{\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1 z\, dx_1\, dx_2 dx_3,$$

where M is the number in the selected population, and we have altered the order of integration.

Substituting for z and integrating first with regard to x_2, we have:

$$M\bar{x}_1 = \int_{-h_3}^{\infty} z_0 \int_{-\infty}^{+\infty} x_1 \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\frac{1}{R}\left\{\frac{R_{22}}{\sigma_2^2}\left(x_2 + \frac{R_{12}}{R_{22}}\frac{\sigma_2}{\sigma_1}x_1 + \frac{R_{13}}{R_{22}}\frac{\sigma_2}{\sigma_3}x_3\right)^2 + 2\left(R_{13} - \frac{R_{12}R_{13}}{R_{22}}\right)\frac{x_1 x_3}{\sigma_1\sigma_3} + \left(R_{11} - \frac{R_{12}^2}{R_{22}}\right)\frac{x_1^2}{\sigma_1^2} + \left(R_{33} - \frac{R_{23}^2}{R_{22}}\right)\frac{x_3^2}{\sigma_3^2}\right\}} dx_2 dx_1 dx_3$$

$$= \int_{-h_3}^{\infty} \sqrt{2\pi}\, z_0 \sigma_2 \sqrt{\frac{R}{R_{22}}} \int_{-\infty}^{+\infty} x_1 e^{-\frac{1}{2}\frac{1}{R}\left\{\left(R_{11} - \frac{R_{12}^2}{R_{22}}\right)\frac{x_1^2}{\sigma_1^2} + 2\left(R_{13} - \frac{R_{12}R_{13}}{R_{22}}\right)\frac{x_1 x_3}{\sigma_1\sigma_3} + \left(R_{33} - \frac{R_{23}^2}{R_{22}}\right)\frac{x_3^2}{\sigma_3^2}\right\}} dx_1 dx_3.$$

But

$$z_0 = \frac{N}{(2\pi)^{\frac{3}{2}}\sigma_1\sigma_2\sigma_3\sqrt{R}}, \qquad R_{22} = 1 - r_{13}^2,$$

$$\frac{R_{11}R_{22} - R_{12}^2}{RR_{22}} = \frac{R_{33}R_{22} - R_{23}^2}{RR_{22}} = \frac{1}{1 - r_{13}^2}, \qquad \frac{R_{13}R_{22} - R_{12}R_{13}}{RR_{22}} = -r_{13}.$$

Thus our result is reduced to the simple form:

$$M\bar{x}_1 = \frac{N}{2\pi\sigma_1\sigma_3}\frac{1}{\sqrt{1 - r_{13}^2}}\int_{-h_3}^{\infty}\int_{-\infty}^{+\infty} x_1 e^{-\frac{1}{2}\frac{1}{1 - r_{13}^2}\left(\frac{x_1^2}{\sigma_1^2} - \frac{2r_{13}x_1x_3}{\sigma_1\sigma_3} + \frac{x_3^2}{\sigma_3^2}\right)} dx_1 dx_3$$

$$= \frac{1}{2\pi\sigma_1\sigma_3}\frac{1}{\sqrt{1 - r_{13}^2}}\int_{h}^{\infty}\int_{-\infty}^{+\infty} x_1 e^{-\frac{1}{2}\frac{1}{1 - r_{13}^2}\left\{\left(\frac{x_1}{\sigma_1} - \frac{r_{13}x_3}{\sigma_3}\right)^2 + \frac{x_3^2(1 - r_{13}^2)}{\sigma_3^2}\right\}} dx_1 dx_3.$$

Put

$$x_1 = \sigma_1\left(x_1' + \frac{r_{13}x_3}{\sigma_3}\right),$$

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* Pearson, "Regression, Heredity and Panmixia," *Phil. Trans.*, Vol. 187, A. p. 302.

and we have

$$M\bar{x}_1 = \frac{1}{2\pi}\frac{\sigma_1}{\sigma_3}\int_h^\infty \int_{-\infty}^{+\infty} \frac{\left(x_1' + \frac{r_{13}x_3}{\sigma_3}\right)}{\sqrt{1-r_{13}^2}} e^{-\frac{1}{2}x_1'^2\frac{1}{1-r_{13}^2} - \frac{1}{2}\frac{x_3^2}{\sigma_3^2}} dx_1' dx_3$$

$$= \frac{1}{\sqrt{2\pi}}\frac{\sigma_1}{\sigma_3}\int_h^\infty e^{-\frac{1}{2}\frac{x_3^2}{\sigma_3^2}} \frac{r_{13}x_3}{\sigma_3} dx_3$$

$$= r_{13}\frac{\sigma_1}{\sigma_3} M\bar{x}_3.$$

Thus $$\bar{x}_1 = r_{13}\frac{\sigma_1}{\sigma_3}\bar{x}_3 \text{ ....(iv).}$$

Similarly $$\bar{x}_2 = r_{23}\frac{\sigma_2}{\sigma_3}\bar{x}_3 \text{ ....(v).}$$

These give the displaced means of the parents in terms of the mean of the "tail," or of the selected offspring.

(2) To find the variability of the selected generations, we have only to write $\bar{\sigma}_1^2$ for $\bar{x}_1$ on the left and $(x_1 - \bar{x}_1)^2$ for x_1 on the right, i.e. we find at once

$$M\bar{\sigma}_1^2 = \frac{1}{2\pi}\frac{1}{\sigma_1\sigma_3}\frac{1}{\sqrt{1-r_{13}^2}}\int_h^\infty \int_{-\infty}^{+\infty} (x_1 - \bar{x}_1)^2 e^{-\frac{1}{2}\frac{1}{1-r_{13}^2}\left(\frac{x_1}{\sigma_1} - \frac{r_{13}x_3}{\sigma_3}\right)^2 - \frac{1}{2}\frac{x_3^2}{\sigma_3^2}} dx_1 dx_3.$$

Putting as before $$x_1' = \frac{x_1}{\sigma_1} - \frac{r_{13}x_3}{\sigma_3},$$

we have

$$M\bar{\sigma}_1^2 = \frac{1}{2\pi}\frac{\sigma_1^2}{\sigma_3}\frac{1}{\sqrt{1-r_{13}^2}}\int_h^{+\infty}\int_{-\infty}^{+\infty}\left(x_1' + \frac{r_{13}x_3}{\sigma_3} - \frac{\bar{x}_1}{\sigma_1}\right)^2 e^{-\frac{1}{2}\frac{x_1'^2}{1-r_{13}^2}} e^{-\frac{1}{2}\frac{x_3^2}{\sigma_3^2}} dx_1' dx$$

$$= \frac{1}{\sqrt{2\pi}}\frac{\sigma_1^2}{\sigma_3}\int_h^\infty e^{-\frac{1}{2}\frac{x_3^2}{\sigma_3^2}}\left\{1 - r_{13}^2 + \left(\frac{r_{13}x_3}{\sigma_3} - \frac{\bar{x}_1}{\sigma_1}\right)^2\right\} dx_3$$

$$= \sigma_1^2 M\left(1 - r_{13}^2 + \frac{\bar{x}_1^2}{\sigma_1^2} - \frac{2r_{13}\bar{x}_3\bar{x}_1}{\sigma_3\sigma_1} + \frac{r_{13}^2(\bar{\sigma}_3^2 + \bar{x}_3^2)}{\sigma_3^2}\right).$$

Substituting for $\bar{x}_1$ from the first set of equations we have

$$\bar{\sigma}_1^2 = \sigma_1^2\left(1 - r_{13}^2 + r_{13}^2\frac{\bar{\sigma}_3^2}{\sigma_3^2}\right) \text{ ....(vi).}$$

Similarly $$\bar{\sigma}_2^2 = \sigma_2^2\left(1 - r_{23}^2 + r_{23}^2\frac{\bar{\sigma}_3^2}{\sigma_3^2}\right) \text{ ....(vii).}$$

Thus the selected variabilities of both parents are determined.

(3) To determine the correlation between the selected parents.

We have $$M\bar{\sigma}_1\bar{\sigma}_2\bar{r}_{12} = \int_h^\infty dx_3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) z\, dx_1 dx_2.$$

I now refer the z surface to its centre for a given x_3. If the coordinates of this centre be $\overline{\eta}_1$ and $\overline{\eta}_2$:

$$\overline{\eta}_1 = -\frac{(R_{13}R_{22} - R_{12}R_{23})}{R_{11}R_{22} - R_{12}^2}\frac{x_3\sigma_1}{\sigma_3},$$

$$\overline{\eta}_2 = -\frac{(R_{23}R_{11} - R_{12}R_{13})}{R_{11}R_{22} - R_{12}^2}\frac{x_3\sigma_2}{\sigma_3}.$$

Now $\qquad R_{13}R_{22} - R_{12}R_{23} = -r_{13}R, \quad R_{11}R_{22} - R_{12}^2 = R.$

Therefore:

$$\overline{\eta}_1 = r_{13}x_3\sigma_1/\sigma_3,$$

$$\overline{\eta}_2 = r_{23}x_3\sigma_2/\sigma_3.$$

Whence if $\qquad x_1 = \overline{\eta}_1 + \eta_1, \quad x_2 = \overline{\eta}_2 + \eta_2,$

$$M\bar{\sigma}_1\bar{\sigma}_2\bar{r}_{12} = \int_h^\infty \frac{N dx_3}{(2\pi)^{\frac{3}{2}}\sqrt{R}\sigma_1\sigma_2\sigma_3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\eta_1 - \overline{\eta}_1 - \bar{x}_1)(\eta_2 - \overline{\eta}_2 - \bar{x}_2)$$
$$\times e^{-\frac{1}{2}\frac{1}{R}\left(R_{11}\frac{\eta_1^2}{\sigma_1^2} + R_{22}\frac{\eta_2^2}{\sigma_2^2} + 2R_{12}\frac{\eta_1\eta_2}{\sigma_1\sigma_2}\right)} e^{-\frac{1}{2}\frac{x_3^2}{\sigma_3^2}} d\eta_1 d\eta_2,$$

or, $$M\bar{\sigma}_1\bar{\sigma}_2\bar{r}_{12} = \frac{N}{(2\pi)^{\frac{3}{2}}\sqrt{R}\sigma_1\sigma_2\sigma_3}\int_{h_3}^\infty dx_3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\{\eta_1\eta_2 + (\overline{\eta}_1 + \bar{x}_1)(\overline{\eta}_2 + \bar{x}_2)\}$$
$$\times e^{-\frac{1}{2}\frac{1}{R}\left(R_{11}\frac{\eta_1^2}{\sigma_1^2} + R_{22}\frac{\eta_2^2}{\sigma_2^2} + 2R_{12}\frac{\eta_1\eta_2}{\sigma_1\sigma_2}\right)} e^{-\frac{1}{2}\frac{x_3^2}{\sigma_3^2}} d\eta_1 d\eta_2.$$

A term

$$\frac{1}{2\pi\sqrt{1-\rho_{12}^2}}\frac{N}{\Sigma_1\Sigma_2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\eta_1\eta_2 e^{-\frac{1}{2}\frac{1}{R}\left(R_{11}\frac{\eta_1^2}{\sigma_1^2} + R_{22}\frac{\eta_2^2}{\sigma_2^2} + 2R_{12}\frac{\eta_1\eta_2}{\sigma_1\sigma_2}\right)} d\eta_1 d\eta_2$$

would be the product moment $\Sigma_1\Sigma_2\rho_{12}N$ of a correlation surface for which

$$\rho_{12} = -R_{12}/\sqrt{R_{11}R_{22}}, \quad 1 - \rho_{12}^2 = R/(R_{11}R_{22}),$$

$$\Sigma_1 = \sigma_1\sqrt{R_{22}}, \quad \Sigma_2 = \sigma_2\sqrt{R_{11}}.$$

Thus:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\eta_1\eta_2 e^{-\frac{1}{2}\frac{1}{R}\left(R_{11}\frac{\eta_1^2}{\sigma_1^2} + R_{22}\frac{\eta_2^2}{\sigma_2^2} + 2R_{12}\frac{\eta_1\eta_2}{\sigma_1\sigma_2}\right)} d\eta_1 d\eta_2 = \quad 2\pi\sqrt{1-\rho_{12}^2}\,\Sigma_1^2\Sigma_2^2\rho_{12}N$$
$$= -2\pi\sigma_1^2\sigma_2^2\sqrt{R}\,R_{12}.$$

Again $$\overline{\eta}_1 + \bar{x}_1 = r_{13}\frac{\sigma_1}{\sigma_3}(\bar{x}_3 - x_3),$$

$$\overline{\eta}_2 + \bar{x}_2 = r_{23}\frac{\sigma_2}{\sigma_3}(\bar{x}_3 - x_3),$$

and the integral

$$\frac{1}{2\pi\sqrt{1-\rho_{12}^2}}\frac{N}{\Sigma_1\Sigma_2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}\frac{1}{R}\left(R_{11}\frac{\eta_1^2}{\sigma_1^2} + R_{22}\frac{\eta_2^2}{\sigma_2^2} + 2R_{12}\frac{\eta_1\eta_2}{\sigma_1\sigma_2}\right)} d\eta_1 d\eta_2 = N,$$

or, the second part gives us

$$2\pi\sqrt{1-\rho_{12}^2}\,\Sigma_1\Sigma_2 = 2\pi\sqrt{R}\sigma_1\sigma_2.$$

Thus finally

$$M\bar{\sigma}_1\bar{\sigma}_2\bar{r}_{12} = \frac{N\sigma_1\sigma_2}{\sqrt{(2\pi)}\,\sigma_3}\int_{h_3}^{\infty}\left(-R_{12} + r_{13}r_{23}\frac{\sigma_1\sigma_2}{\sigma_3^2}(\bar{x}_3 - x_3)^2\right) \times e^{-\frac{1}{2}x_3^2/\sigma_3^2}\,dx_3$$

$$= M\sigma_1\sigma_2\left(-R_{12} + r_{13}r_{23}\frac{\bar{\sigma}_3^2}{\sigma_3^2}\right),$$

or, substituting for $\bar{\sigma}_1$ and $\bar{\sigma}_2$ and R_{12}

$$\bar{r}_{12} = \frac{r_{12} - r_{13}r_{23} + r_{13}r_{23}\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}{\sqrt{1 - r_{13}^2 + r_{13}^2\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}\sqrt{1 - r_{23}^2 + r_{23}^2\dfrac{\bar{\sigma}_3^2}{\sigma_3^2}}} \quad \text{..................(viii).}$$

This is the value from which the true correlation r_{12} may be deduced.

In the case of tuberculosis, we note that if no assortative mating existed $\bar{r}_{12}$ would be negative since σ_3 is $> \bar{\sigma}_3$. The apparently negative value of $\bar{r}_{12}$ for the parents of tuberculous offspring may in some cases arise from this result.

The above investigation covers the formulae used in this paper, but they are more general than the proof here given indicates.

APPENDIX II.

In this Appendix the tables of Miss Elderton for assortative mating in man (the reduced constants of which have been cited in Table II. p. 22) are put together with some remarks on their general features.

I. *Tuberculosis in Husband and Wife.*

		Husband				
		Non-tuberculous	Tuberculous	Doubtful	Not known	Totals
Wife	Non-tuberculous	567	23	4	—	594
	Tuberculous	30	3	—	3	36
	Doubtful	1	1	—	1	3
	Not known	—	1	—	—	1
	Totals	598	28	4	4	634

These 634 cases were taken from Pearson's family records, and represent an average sample of the population without any selection of the tuberculous or non-tuberculous.

II. *General Health of Husband and Wife.*

Husband

		Very Robust	Robust	Normally Healthy	Delicate	Totals
Wife	Very Robust	19	17	15	8·5	59·5
	Robust	16	73·5	48	16·5	154
	Normally Healthy	30	92	149·25	25·25	296·5
	Delicate	14	28	55·25	11·75	109
	Totals	79	210·5	267·5	62	619

The value of the assortative mating, ·27, was found by mean square contingency. The general excess of the diagonal column is fairly well marked. The result is in good agreement with those for longevity.

III. *General Intelligence of Husband and Wife.*

Husband

		Very Able and Distinctly Capable	Fair Intelligence	Slow Intelligence	Dull and Defective	Totals
Wife	Very Able & Distinctly Capable	20·25	21·5	3·25	1	46
	Fair Intelligence	68·25	252·25	42	4·25	366·75
	Slow Intelligence	22·5	71·25	34·25	3·75	131·75
	Dull & Defective	6	14·5	7	6	33·5
	Totals	117	359·5	86·5	15	578

The value of the relationship, ·33, was found by mean square contingency. The general excess of the diagonal column is manifest.

IV. *Temper in Husband and Wife.*

Husband

		Quick	Even	Sullen	Weak	Totals
Wife	Quick	65·75	101	8	6·25	181
	Even	135	170·75	7·5	9·25	322·5
	Sullen	9·25	9·75	1	2	22
	Weak	8	9·5	3	3	23·5
	Totals	218	291	19·5	20·5	549

The excesses here are not so direct. The weak (defined as "weak good-nature") and sullen tend to marry the weak and sullen. The quick tempered marry the even tempered in excess and their like in defect. The result appears to indicate the existence of certain cross-tendencies giving a quite sensible relationship, ·18, by mean square contingency, but the attraction is not wholly of like to like.

V. *Success in Career of Husband and Wife.*

Husband

		Marked and Beyond Family Average	Average	Difficult	Failure	Totals
Wife	Marked and Beyond Family Average	71	7·75	9·25	2	90
	Average	99·5	168·25	24·25	3	295
	Difficult	4·5	4·5	11	1	21
	Failure	—	—	—	2	2
	Totals	175	180·5	44·5	8	408

By "marked career" was denoted one which was noteworthy from the standpoint of the community, as well as from that of the individual family. A career prosperous beyond that of the family average was also a fairly well rounded category. The "average" group were those who had not fallen below the family standard of life. Struggling and unprosperous careers were included under "difficult," and bankruptcy, crime or moral failure under "failure." The classification of the wives was naturally more difficult than that of the husbands. But attention was paid to academic successes, to professional and business success when the wives had at any time had a separate career. Further to their reputation as efficient housewives, mothers, or social workers. The "failure" groups are relatively so small that (working by mean square contingency) they were thrown into the "difficult" groups. The diagonal excesses are very marked and the assortative mating rises to ·48. A large part of this is very probably due to common environment and to common influence.

VI. *Excitability in Husband and Wife.*

Husband

		Excitable	Medium	Calm	Totals
Wife	Excitable	41	27	71	139
	Medium	16	27	42	85
	Calm	59	58	112	229
	Totals	116	112	225	453

Medium or "betwixt" covers those cases in which it was not possible to assert that the individual was actually of an excitable or calm temperament. There is a sensible excess of excitable and medium individuals tending to mate with their likes, but the total assortative mating found by mean square contingency is only ·11.

VII. *Sympathetic Temperament in Husband and Wife.*

Husband

		Sympathetic	Betwixt	Self-centered	Totals
Wife	Sympathetic	223	84	25	332
	Betwixt	51	35	5	91
	Self-centered	23	3	2	28
	Totals	297	122	32	451

The sympathetic tend to marry their like and so do the indifferent natures. The relationship found by mean square contingency is ·15.

VIII. *Reserved Temperament in Husband and Wife.*

Husband

		Reserved	Betwixt	Expressive	Totals
Wife	Reserved	66	17	47	130
	Betwixt	39	53	35	127
	Expressive	96	41	48	185
	Totals	201	111	130	442

The reserved and indifferent classes tend to intermarry; but again a cross-current comes in and the expressive tend rather to mate with the reserved than with one another. The deviation from random mating as measured by the coefficient of mean square contingency is marked, being ·27.

IX. *On the Insane Diathesis in Husband and Wife.*

Husband

		Normal	Insane	Nervous	Doubtful	Totals
Wife	Normal	539	36	7	2	584
	Insane	25	7	1	—	33
	Nervous	11	—	—	—	11
	Doubtful	2	—	—	—	2
	Totals	577	43	8	2	630

The doubtful include two cases marked "eccentric" and two of "alcoholism," not amounting to a chronic mania. The nervous cases include "night terrors," slight nervous breakdowns and neurotic conditions. If we throw them all into the normal, we shall clearly much emphasise the assortative mating. The table has accordingly been worked out in two ways (A) by throwing nervous and doubtful into the normal, (B) by throwing them into the insane. We shall thus reach upper and lower limits for the assortative mating with regard to insanity.

(A)

		Husband		
		Normal	Insane	Totals
Wife	Normal	561	36	597
	Insane	26	7	33
	Totals	587	43	630

(B)

		Husband		
		Normal	Insane	Totals
Wife	Normal	539	45	584
	Insane	38	8	46
	Totals	577	53	630

These tables by the fourfold process give respectively:

$$r = \cdot 361, \quad \text{and } r = \cdot 244.$$

Accordingly, as far as these 630 random couples carry us, we may say that the assortative mating in man with regard to insanity lies between ·25 and ·35, or is, say, ·30 ± 05. This result comes perilously near the value found in the body of this paper for tuberculosis, and is somewhat in excess of the value found from the present Appendix data for tuberculosis.

It would appear therefore that stocks which are insane, epileptic, markedly eccentric or alcoholic in a degree amounting to mania do tend to mate together. This result will have to be confirmed on far wider data, but it deserves to be vigorously impressed upon those who dogmatically assert that the association of tuberculous husband and wife beyond the random proportions can only be due to the influence of infection. They must be prepared to admit that insanity in possibly a more marked degree than tuberculosis is transmissible between husband and wife. The data of this Appendix seems to suggest that sexual selection in the form of assortative mating in man which extends from general health and physique to intelligence and temperament must be fully allowed for, before we can be very dogmatic as to the marital influence factor in the case of either insanity or tuberculosis.

X. *On the Tuberculous Diathesis in Husband and Wife.*

I place last here a table which if it contained ten times its present numbers would be of much importance to the present investigation. I asked Miss Elderton to take out of my records all cases of husband and wife, when the parents, brothers and sisters, aunts and uncles of both were fully known. Unfortunately only 221 such cases were available. No attention was then to be paid to the tuberculous or non-tuberculous character of either husband or wife, but only an inquiry made as to whether there had been cases of tuberculosis among their near relatives. Some of the stocks were small and others large, it was not easy to assert when a stock should be considered tuberculous. If we consider a stock sound, when not a single case of tuberculosis was recorded we have Table A. If we count a stock with only one member tuberculous as "sound,"—a not very legitimate assumption, especially for small stocks—we have Table B.

Table (A).

Husband's stock

		Sound	Tuberculous	Totals
Wife's stock	Sound	130	31	161
	Tuberculous	39	21	60
	Totals	169	52	221

Table (B).

Husband's stock

		Sound	Tuberculous	Totals
Wife's stock	Sound	175	14	189
	Tuberculous	29	3	32
	Totals	204	17	221

The correlation given by Table A is ·3, and by Table B is ·1. Not much stress can be laid on such small numbers, but the tables seem to indicate a tendency of tuberculous stocks to intermarry, and they suggest a method by which assortative mating in this character can be ascertained independently of any marital infective action. Of course the presence of tuberculosis in husband or wife was not taken to mark the existence of a tuberculous stock, and this again somewhat widens the category of "sound." On the whole I believe a large random sample of the population thus treated would be the best method of dealing with our problem. As far as our slender sample goes, it indicates that there is genuine assortative mating with regard to the tuberculous diathesis.

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