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STUDIES ON THE VARIATION AND CORRELATION OF SKULL MEASUREMENTS IN BOTH SEXES OF MATURE ALBINO RATS (MUS NORVEGICUS VAR. ALBUS).

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WITH 1 FIGURE AND 10 TABLES

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With 1 Figure and 10 Tables.

The present investigation was undertaken to determine the size of the skull and the relative development of its constituent parts in the adult albino rat. To this end the biometric method was employed with the idea that in this way it would be possible to obtain more precise results and also with the idea of later comparing these skulls with those of the hybrids between Mus norvegicus and Mus rattus.

Materials and Methods of Measurement.

For the present study 53 male and 51 female skulls of mature rats (rats more than 150 days old) were measured. The following measurements were made with vernier calipers: (1) the length of the entire skull; (2) the fronto-occipital length; (3) the zygomatic width; (4) the length of the nasal bone; (5) the height of the skull; (6) the width of the cranium or the squamosal distance. In every case the maximum length alone was recorded.

In the present paper the horizontal straight line joining the tip of the nasal bone to the end of the occipital bone is called the length of the entire skull. This however is not exactly equal to the sum of the length of the nasal bone and that of the fronto-occipital.

The fronto-occipital length was determined in the following way: Since the length measured with the calipers from the tip of the nose to the posterior end of the inter-parietal bone is not always equal to the length measured from the tip of the nose to the end of the occipital bone, both measurements were taken (See Fig. 1). The latter measurement is usually the longer. The difference between the two measurements was added to the length from the tip of the frontal bone to the end of the inter-parietal bone, and the sum was called the fronto-occipital length.

The width of the cranium (squamosal distance) was determined by taking the maximum distance between the two points (right and left) where the zygomatic bones rest on the lateral walls of the cranium. The height of the skull was determined by measuring a perpendicular distance between the greatest convexity of the parietal bone in the median line and the junction line between the basi-occipital and the basi-sphenoidal bones on the ventral surface.

The cranial capacity was determined in the following way: The skull was held vertically, with the nose downwards and was filled with fine shot (No. 11) to the upper level of foramen magnum and then the nose of the skull gently struck twice against the palm of the hand. The space

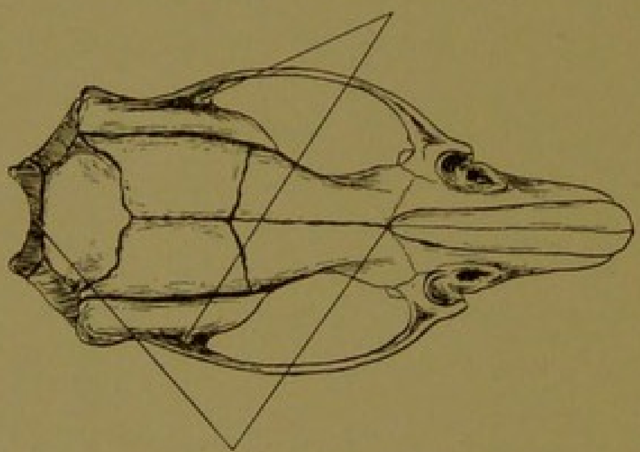

FIG. 1. Diagram of the skull of the adult albino rat, seen from above.

thus formed was again filled. Although this is a simple procedure yet it needs the greatest care and much practice in order to produce uniform results. The slightest variation will easily cause differences of more than one gram in the weight of the shot. The greater the experience of the observer the more uniform are the results. By practice the author has been able to reduce the difference between the first and second filling to less than one decigram in eight cases out of ten. The distribution of the errors in this work was found to follow the Gaussian normal curve and therefore it is inferred that the number of minus errors is the same as the number of plus errors. As a matter of fact the average difference between the first and second fillings did not exceed one per cent. The cranial capacity thus determined was finally transformed into cubic centimeters of brain substance (see page 435).

TABLE I.

		Mean.		Standard deviation.		Coefficient of variation.		No. of Rats.
			Difference.		Difference.		Difference.	
Length of the entire skull.	♂	43.255 ± 0.166	1.706 ± 0.204	1.786 ± 0.117	0.530 ± 0.144	4.129 ± 0.271	1.113 ± 0.338	53
	♀	41.549 ± 0.119	3.944 %	1.256 ± 0.084		3.016 ± 0.202		51
Zygomatic width.	♂	21.745 ± 0.109	0.820 ± 0.137	1.177 ± 0.077	0.301 ± 0.064	5.412 ± 0.356	1.226 ± 0.453	53
	♀	20.925 ± 0.083	3.771 %	0.876 ± 0.059		4.186 ± 0.280		51
Length of the nasal bone.	♂	16.958 ± 0.096	1.266 ± 0.122	1.038 ± 0.068	0.245 ± 0.086	6.121 ± 0.403	1.068 ± 0.526	53
	♀	15.692 ± 0.075	7.465 %	0.793 ± 0.053		5.053 ± 0.338		51
Fronto-occipital length.	♂	27 264 ± 0.093	0.911 ± 0.126	1.007 ± 0.066	0.103 ± 0.090	3.693 ± 0.242	0.256 ± 0.334	53
	♀	26.373 ± 0.085	3.341 %	0.904 ± 0.060		3.427 ± 0.229		51
Squamosal distance.	♂	15.273 ± 0.010	0.217 ± 0.040	0.338 ± 0.022		2.213 ± 0.145		53
	♀	15 056 ± 0.039	1.420 %	0.409 ± 0.027	−0.071 ± 0.036	2.716 ± 0.181	−0.503 ± 0.232	51
Height of skull.	♂	11.493 ± 0.049	0.354 ± 0.065	0.526 ± 0.034	0.151 ± 0.043	4.576 ± 0.300	1.210 ± 0.375	53
	♀	11.139 ± 0.035	3.080 %	0.375 ± 0.025		3.366 ± 0.225		51
Cranial capacity.	♂	10.896 ± 0.068	0.528 ± 0.098	0.735 ± 0.048		6.745 ± 0.444		53
	♀	10.368 ± 0.070	4.845 %	0.743 ± 0.050	−0.008 ± 0.069	7.166 ± 0.481	−0.421 ± 0-655	51
Body-weight.	♂	214.886 ± 5.318	47.541 ± 5.982	52.887 ± 3.760	32.413 ± 4.088	25.076 ± 2.675	12.841 ± 2.847	45
	♀	167.345 ± 2.739	22.170 %	20.474 ± 1.605		12.235 ± 0.974		37
(Length × width × height)$^{\frac{1}{3}}$.	♂	16.875 ± 0.044	0.452 ± 0.056	0.478 ± 0.031	0.096 ± 0.040	2.832 ± 0.186	0.506 ± 0.242	53
		16.423 ± 0.036	2.678 %	0.382 ± 0.026		2.326 ± 0.155		51

Determination of the Means and Variability of the Several Measurements.

In Table I are exhibited the means of the several measurements, the standard deviations, the coefficients of variation and the differences between the two sexes, with their respective probable errors. These values were determined by the usual biometric formulæ (Davenport, 04). As one would expect, the mean values in the males are always higher than in the females. Since in the present investigation the total number of measurements of both sexes was not large, it is important to compare the differences between the sexes with the corresponding probable errors in order to see whether or not the differences here found are to be considered significant. Table I shows clearly that in all cases the differences are greater than three times the probable errors. The maximum difference occurs in the nasal bone; ten times the probable error, and the minimum occurs in the width of the cranium, five times the probable error. This indicates that the characters under consideration are really greater in the male. The maximum percentage differences occurs in the nasal bone (7.5 per cent) and minimum in the width of the cranium (1.4 per cent) while the remaining differences are nearly similar (3.1 per cent to 3.9 per cent).

From the percentage differences here found it would seem probable that if we compared the mature male and female skulls having *the same total length,* the length of the nasal bone in the male would be longer than that in the female, and since the length of the entire skull depends on the combined length of the nasal bone and the fronto-occipital length, it would follow that the fronto-occipital length or the length of the cranium, would be less in the male than in the female. In order to test this conclusion in any individual case it is necessary to determine whether or not the total length of the skull and of the nasal bone on one hand, and the total length of the skull and fronto-occipital length on the other hand, are closely correlated. This point will be more fully discussed after the coefficients of correlation have been determined.

(a) *Range of variates and rate of increase of the various characters associated with changes in the size of the entire skull.*—The two extremes of the various characters, as well as the rate of increase of those characters associated with the increase in the length of the entire skull, is somewhat different according to sex as is shown in Table II.

As is shown in Table II, the absolute range between the two extremes is always slightly greater in male than in female except in the case of the width of the cranium or squamosal distance and in the fronto-occipital length where they are nearly alike in the two sexes. Generally speaking the changes associated with the increase in the length of the entire skull are relatively greater in female than in male. This is especially evident in the length and width of the cranium and the zygomatic width, although the absolute amount of change is considerably less in the female than in the male. In the female the relative change is very slightly greater in the nasal bone and is the same in the height of the skull. The same table

TABLE II.

	Male.			Female.		
	Minimum mm.	Mean* mm.	Maximum mm.	Maximum mm.	Mean* mm.	Minimum mm.
Length of the entire skull.	39.4	43.3	47.4	44.5	41.5	38.9
Rate	100	100	100.	100.	100.	100.
Zygomatic width.	19.6	21.7	24.8	23.4	20.9	18.9
Rate	49.8	50.2	52.3	52.5	50.3	48.5
Length of the nasal bone.	14.7	17.0	18.7	17.8	15.7	14.4
Rate	37.3	39.2	39.3	40.0	37.7	37.0
Fronto-occipital length.	24.9	27.3	28.8	28.2	26.4	24.9
Rate	63.2	63.0	60.7	63.3	63.5	64.0
Squamosal distance.	14.6	15.3	16.2	16.2	15.1	14.4
Rate	37.0	35.3	34.1	36.4	36.2	37.0
Height of Skull.	10.4	11.5	13.0	12.2	11.1	10.3
Rate	26.4	26.5	27.4	27.4	26.8	26.4

shows us clearly that in respect to the length of the entire skull, the width and length of the cranium are greater in female than in male although in both sexes the absolute amount of change is less than for any other characters measured. The relatively slight increase in the width of the skull (zygomatic width) associated with the increase in the length of the entire skull, has also been noticed by Allen, 94, in the case of Neotoma micropus.

When the mean values (Table II) are treated in the same manner as the two extremes, additional light is thrown on the changes following the increase in the length of the entire skull. In all characters, except the

* Taken from Table I.

length of the nasal bone, the female gives relatively greater values than the male. Although the excess shown in the female is not large in the case of the zyomatic width, and in the case of the height and length of the cranium yet the width of the cranium or squamosal distance is decidedly greater in the female than in the male, as has already been seen from the measurements of the extremes. When mean values for the length of the entire skull are reduced to the same standard and the associated measurements are compared, all the three diameters of the cranium of the female are seen to be relatively greater than those of the male in respect to the length of the entire skull. The cube root of product of these three diameters in the case of the male is 39.01 per cent of the length of the entire skull and in the case of the female 39.52 per cent, thus indicating that we might expect the relative capacity of the female cranium would be sensibly greater than that of the male. This apparent superiority of the female cranium over that of the male is not due, however, to the relatively greater lengths of the three cranial measurements of the female, but is due to the fact that the nasal bone in the male skull is considerably longer, thus producing a somewhat less percentage value for cube root of the product of the three diameters of the male cranium. As a matter of fact, when the length of the entire skull is equated either to male or to female standard by means of the characteristic equations and the resulting measurements of the cranium in the two sexes are compared, the size of the cranium in the two sexes is almost identical. We shall discuss this point later (page 436). It is therefore enough at the present moment to see that the somewhat greater percentage values obtained from the three diameters of the cranium, when mean values for the length of the entire skull are reduced to 100, indicate that the nasal bone is much shorter in the female, and vice versa.

(b) *Relative variability in the two sexes.*—The relative variability in the two sexes is a question which has passed through various phases during the last century. The several opinions held by different investigators are fully summarized by Havelock Ellis in his book on "Man and Woman," 94. The history of this question may therefore be omitted. It is, however, important to note here that the quantitative investigation of this question has been made for the most part on the human subject.

As is shown in Table I, the standard deviation is in every case greater in the male than in the female except in the case of the squamosal distance. Since the standard deviation measures the amount of concentration of the variates about the mean, the greater the standard deviation the less will be the concentration and consequently the greater will be the variation in the character under consideration. Therefore the greater values obtained for the male means simply that the characters in question vary more in the male than in the female.

The absolute value of the standard deviation is widely different throughout the table. This is due to the fact that in these cases the standard deviation is a concrete number and therefore the variabilities can not be directly compared with one another since the magnitude of the characters as well as the unit taken for grouping is never the same. The coefficient of variation, however, enables us to compare the relative amount of variability of the characters measured in different units since it is one hundred times the quotient of the standard deviation divided by the means. From Table I it was found that the values of the coefficients of variation in the male are always greater than that in the female, except in the case of the squamosal distance in which the reverse is true.[1] Here also the length of the nasal bone shows the greatest variation and the zygomatic width comes next while the least variation is found in the width of the cranium. Brewster, 97, also noticed a greater variation in both the length of the nasal bones and zygomatic width than in any other measurements made on the different parts of the skull. His studies were made on the lynx (Lynx canadensis); cat (Felis domesticus); and red fox (Vulpes fulvis). The methods employed by Brewster for determining his coefficients of variation are so different from those used in the present investigation that two sets of figures can not be directly compared. Except the length of the cranium, the remaining characters tend to show the existence of a sexual difference as to the relative variability, that is the male tends to vary more than in the female.

The mean values obtained from the cranial capacity, body-weight, and cube root of the product of the height, length and width of the cranium are also greater in the male than in the female. The standard deviations as well as the coefficients of variation indicates a relation similar to that found in the skull measurements, that is, the male tends to show in these characters a greater variability than the female.

(c) *Coefficients of variation in man and rat.*—The following table was compiled in order to show the variation in the human skull as compared with that for the skull of the albino rat:

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[1] Slightly greater variability in the female is also found in the cranial capacity, nevertheless the result is insignificant owing to the greater probable errors in this case.

TABLE III.

	Male.	Female.	Investigator.
SKULL CAPACITY.			
Etruscan	9.58	8.54	Pearson and Lee.
Modern Italian	8.34	8.99	Pearson and Lee.
English	8.28	8.68	Macdonell.
Egyptian mummies	8.13	8.29	Pearson and Lee.
Modern German	7.74	8.19	Pearson and Lee.
Naquada	7.72	6.92	Fawcett.
Parisian French	7.36	7.10	Pearson and Lee.
Aino	7.07	6.90	Pearson and Lee.
Albino rat	6.75	7.17	Hatai.
SKULL HEIGHT.			
English	4.21	3.96	Macdonell.
Aino	3.67	3.18	Pearson and Lee.
German	4.47	3.91	Pearson and Lee.
Albino rat	4.58	3.37	Hatai.
SKULL LENGTH.			
English	3.31	3.45	Macdonell.
Naquada	3.09	2.96	Fawcett.
Aino	3.20	3.08	Pearson and Lee.
German	3.37	3.57	Pearson and Lee.
English (base of skull)	4.07	4.11	Macdonell.
Albino rat	3.69	3.43	Hatai.
SKULL BREADTH.			
English	3.75	3.54	Macdonell.
Naquada	3.42	3.42	Fawcett.
Aino	2.76	2.68	Pearson and Lee.
German	3.89	3.39	Pearson and Lee.
Albino rat	2.21	2.72	Hatai.

As will be seen from Table III, so far as the cranial measurements are concerned the coefficients of variation in the albino rat are slightly less than in man, though the difference is by no means large. It is interesting to note that the magnitude of variability both in man and rat is in the same order for corresponding characters, that is the variability of the cranial capacity is considerably greater than the remaining three linear measurements both in man and rat. This is perhaps due to the fact that the cranial capacity is itself highly variable and in addition the technical difficulties of determination of the capacity influence the results further.

Pearson, 97, 01, with reference to variation in the several dimensions of the human skull, thinks that with advance in civilization woman tends to gain in variability on man (see on Aino and Naquada races). Nevertheless, if we examine the data recently obtained by Fawcett, 02, and Macdonell, 02, Pearson's conclusion, so far as the measurements of the skulls are concerned, is not well supported. For instance the total averages of the coefficients of variation in man (Aino and Naquada races are excluded) are 4.99 per cent while in woman it is 4.84 per cent. Even if we include Aino and Naquada, the relative variability is still in favor of man. The same is true also for the albino rat (4.31 per cent male, and 4.17 per cent female). Therefore so far as the data at hand are concerned the several measurements of the male skulls show a general trend to a greater variability than those of the female. Since in all cases, the number of the skulls examined is not large, it is evident that this point needs still further study.

Determination of the Coefficients of Correlation.

The degree of correlation between any two characters is usually determined by the formula: $Y = \frac{\varepsilon(x.y.)}{n.\ \sigma_1\ \sigma_2}$, where x, y are deviations from the means of the two correlated characters and δ_1, δ_2 the respective standard deviations. It will be advantageous to discuss the coefficients of correlation under five headings.

(a) *Correlation between the length of the entire skull and the other cranial measurements.*

TABLE IV.

	Male.	Female.	Difference.
Length of entire skull and zygomatic width	.948 ± .011	.836 ± .029	.113 ± .031
Length of entire skull and fronto-occipital length	.946 ± .010	.956 ± .008	—.010 ± .013
Length of entire skull and length of nasal bone	.845 ± .027	.890 ± .020	.044 ± .034
Length of entire skull and squamosal distance	.582 ± .061	.309 ± .085	.273 ± .105
Length of entire skull and height of skull	.555 ± .064	.314 ± .085	.241 ± .107

As one would expect, the degree of correlation in the first three cases is very high in both sexes. Since the length of the skull depends on the length of the nasal bone and the fronto-occipital length, any change in the length in the entire skull must involve a change in the length of either the nasal bone or the fronto-occipital length or in both. The correlation shows, however, that change in the length of the skull is associated with changes in both the nasal bone and the fronto-occipital length; the latter change being the better correlated. The high correlation between the length of the skull and the zygomatic width means a regular enlargemen in the transverse diameter associated with a change in length. The correlation between the length of the entire skull and width (squamosal distance) and height is comparatively low in both sexes. Therefore it is concluded that change in both width and height of the cranium corresponding to the change in the length of the entire skull is less regular than in the other three characters already discussed. It is of interest to note that although the correlations between the length of the entire skull and the width and height of the cranium are not high, yet the correlation between the width and height of the cranium is high, especially in the male. (See Table V.)

(b) *Correlation between the height, length, and width of the cranium.*—Table V shows the coefficients of correlation between the three linear measurements just mentioned.

TABLE V.

	Male.	Female.	Difference.
Fronto-occipital length and squamosal distance	.665 ± .052	.397 ± .080	.268 ± .095
Fronto-occipital length and height of skull	.571 ± .063	.387 ± .080	.184 ± .102
Squamosal distance and height of skull	.707 ± .046	.425 ± .077	.282 ± .090

As the table indicates the width (squamosal distance) and height of the cranium give the highest correlation in both male and female, the length and width come next, while length and height give the lowest correlation. Although in every case the male sex gives the higher correlation, the comparison between the differences and probable errors shows that we can not lay much weight on this apparent superiority, except in the case of the height and width, since in the two other cases the differences are smaller than three times the probable error. For comparison there are no data available except the observation by Pearson and Lee, 01, on the human skull, their results are given in Table VI.

TABLE VI.

		Male.	Female.
Length and breadth	German	.286 ± .062	.488 ± .052
	Aino	.432 ± .059	.377 ± .073
Length and height	German	—.098 ± .067	.314 ± .061
	Aino	.501 ± .054	.349 ± .075
Breadth and height	German	.071 ± .067	.276 ± .063
	Aino	.345 ± .064	.178 ± .082

Pearson and Lee's data show that the correlation in the first two instances (length and breadth, and length and height) are about the same and are much higher than that of the last case (breadth and height). In the albino rat, length and width, and length and height also give nearly the same degree of correlation as in man, but the values obtained are smaller than for the width and height. Thus in this respect the rat and man show reverse relation. The difference is probably due to differences in relative development of the several bones of the cranium depending on the skull form characteristic for the two species.

Whether or not it is a general phenomenon in the lower mammals that the coefficient of correlation is higher in the male than in the female, as is shown in the albino rat, needs further observation. It has been pointed out by Pearson, 97, that in the human race with advancing civilization woman tends to gain in correlation on man. It is clearly seen from the data given above that in German skulls the coefficients of correlation tend to be higher in the female than in the male while in the Ainos the reverse relation is true, favoring the view maintained by Pearson. Pearl, 06, who compared a large number of brain records with other body characters in the case of Bohemians, Bavarians, Hessians, and Swedes found that the weight of the brain also tends to be more highly correlated with other characters in the female than in the male.

(c) *Correlation between cranial capacity and other cranial measurements.*—The cranial capacity as determined with the shot gives high degree of correlation with the length of the entire skull as well as with the three diameters of the cranium. This is shown in the following table:

TABLE VII.

	Male.	Female.	Difference.
Cranial capacity and length of the entire skull	.678 ± .050	.484 ± .072	.194 ± .088
Cranial capacity and fronto-occipital length	.761 ± .039	.577 ± .062	.184 ± .073
Cranial capacity and squamosal distance	.838 ± .028	.632 ±.057	.206 ± .063
Cranial capacity and height of skull	.760 ± .039	.666 ± .053	.094 ± .066
Cranial capacity and (height × length × width)⅓	.836 ± .028	.854 ± .026	—.018 ± .038

As is shown in Table VII the coefficients of correlation are in general higher in the male than in the female, but when compared with their respective probable errors the differences are not large enough to warrant laying much stress on the apparent superiority in male sex, except in the one case of the cranial capacity and squamosal distance. It is therefore safe to say that according to this test the degree of correlation is nearly the same in both sexes. It is interesting to note that the cranial capacity is best correlated with the width (male and female) and the height (male) of the cranium. Therefore a prediction of brain weight based on the cranial measurements would give least error when based on the width. The length of the entire skull gives the lowest degree of correlation in both sexes. This is of course what one would expect since the length of the entire skull is least correlated with both the height and width of the cranium, especially in the case of the female. The product of the height, length, and width of the cranium is highly correlated with the cranial capacity and the correlation, although a trifle higher in the female, is nearly the same in the two sexes. This is also an anticipated result since roughly speaking the capacity should be closely related to the product of three diameters. In this case one might expect to find the correlation almost unity, but remembering that the cranial cavity has an irregular shape and is bounded by curved surfaces, the value shown in the table can be considered satisfactory.

(d) *Correlation between cranial capacity and body-weight.*—Despite the fact that in the human subject the coefficient of correlation between brain and body-weight is extremely low (0.167 in male and 0.226 in female, Pearl, o6,) an intimate relation between these two characters found in the rat (Donaldson) suggests that in the rat at least it would be higher.

As a matter of fact the following coefficients[2] of correlation have been obtained:

TABLE VIII.

	Male.	Female.	Difference.
Body weight and cranial capacity...	.516 ± .074	.692 ± .058	.176 ± .094

Assuming that the regression between the body and brain weights can be expressed by a straight line with a given angle, the following equations were formulated and used to determine the correspondence between the predicted and observed values.

(1) Brain-weight, male = (0.0072 × (body-weight, male) + 9.349) ÷ 5.980.

(2) Brain-weight, female = (0.0251 × (body-weight, female) + 6.168) ÷ 6.009.

[2] Unfortunately in this series, several rats which were found dead are included and thus the correlation here obtained may be slightly less than it should normally be. It is however true that the normal fluctuation in body-weight is rather wide and therefore the including of several self-dead rats should not affect the results to any great extent.

5.980 and 6.009 are two factors for the transformation of the observed weight obtained from shot, into the estimated brain-weight. In other words, if we apply the characteristic equations which enable us to compare the cranial capacity in terms of the weight of shot from the observed body-weight, directly to the adult rats with known body- and brain-weight. (We have a large number of records which give the observed brain-weight accompanied by the corresponding body-weight) then the observed brain-weight was found to be 1/5.98 of the weight of shot in the case of male and 1/6.009 of the weight of shot in the case of female. This simple numerical relation between the observed brain-weight and shot-weight was found to be quite consistent and indeed the characteristic equations with these two new factors have given very satisfactory results in the test thus far.

(e) *Coefficients of correlation in man and rat.*—The coefficients of correlation between the cranial capacity and other cranial measurements in man have been determined by several investigators. The following table shows these relations in man as well as in rat:

TABLE IX.

	Male.	Female.	Difference.	Investigator.
CORRELATION BETWEEN CRANIAL CAPACITY AND LENGTH OF SKULL.				
Aino	.893 ± .016	.663 ± .053	.230 ± .055	Pearson and Lee.
English	.597 ± .051	.691 ± .040	.094 ± .065	Macdonell.
German	.515 ± .050	.687 ± .037	—.172 ± .062	Pearson and Lee.
Naquada	.501 ± .054	.599 ± .039	—.098 ± .067	Fawcett.
Albino rat ...	.761 ± .039	.577 ± .062	.184 ± .073	Hatai.
CORRELATION BETWEEN CRANIAL CAPACITY AND BREADTH OF SKULL.				
Aino	.561 ± .053	.502 ± .070	.059 ± .083	Pearson and Lee.
English	.631 ± .048	.646 ± .044	—.015 ± .065	Macdonell.
German	.672 ± .037	.707 ± .034	—.035 ± .050	Pearson and Lee.
Naquada	.434 ± .058	.532 ± .044	—.098 ± .072	Fawcett.
Albino rat ...	.838 ± .028	.632 ± .057	.206 ± .063	Hatai.
CORRELATION BETWEEN CRANIAL CAPACITY AND HEIGHT OF SKULL.				
Aino	.544 ± .054	.521 ± .068	.023 ± .087	Pearson and Lee.
German	.243 ± .064	.451 ± .054	—.208 ± .084	Pearson and Lee.
Albino rat ...	.760 ± .039	.666 ± .053	.094 ± .066	Hatai.
CORRELATION BETWEEN CRANIAL CAPACITY AND (HEIGHT × BREADTH × LENGTH).				
Aino	.795 ± .039	.780 ± .037	.015 ± .054	Pearson and Lee.
German	.701 ± .034	.814 ± .023	—.113 ± .041	Pearson and Lee.
Naquada	.674 ± .044	.793 ± .025	—.119 ± .051	Pearson and Lee.
Albino rat (height × breadth × length) $^{1/3}$	.836 ± .028	.854 ± .026	.018 ± .038	Hatai.

As is shown in the table, the coefficients of correlation are higher in the rat than in the averages from the human subject in every case, except the cranial capacity and length of skull where the female rat is slightly low. The correlation is decidedly greater in the rat in the case of the capacity as related to the width and height, while in the case of the length and the product of the three diameters the results for the rat are close to those for man. Here again we notice that in the female the coefficients of correlation are slightly greater than in male in the case of the human skulls (except Aino) while in the rat the reverse is true. We can not lay too much stress on this relation, however, since as is shown in the column marked " difference " the size of the probable errors shows the differences to be without significance. Thus although, the general tendency is to show that in the human skull, except Aino, the coefficients of correlation are slightly greater in female nevertheless any definite statement must be postponed until we have data sufficiently abundant to further diminish the value of the probable errors.

(f) *Comparison between the observed and predicted values of skull measurements.*—In Table I we noticed that the mean values of the male characters are always greater than those of the female, the differences always being more than three times the probable errors. The greatest difference was found in the length of nasal bone, the least in the width of the cranium, while the remaining characters gave intermediate values. The question now arises: How the several characters will be related if the length of the entire skull of the male is reduced to that of the female? In other words, is the male skull to be considered as an overgrown female skull, or the female skull an undersized male skull? To answer this question a number of characteristic equations were prepared. These equations will enable us to determine the probable values of the characters in both sexes. The form of the characteristic equation is as follows:

$$Y = \bar{y} + \gamma \frac{\sigma_y}{\sigma_x}(X - \bar{x})$$

where X, Y are the two characters under consideration, $\bar{x}$, $\bar{y}$ are the two respective means of the arrays, σ_x, δ_y are also the two respective standard deviations and γ is the coefficient of correlation. The following table was made in order to show the values of the characters when the lengths of the entire skulls were equated.

As is shown in Table X, when the length of the entire skull of the male is equated to the observed length of the female skull and vice versa, the sexual differences become very small. The closeness of agreement between observed and predicted values of the several characters varies with the standard taken. This is due to the fact, as Tables IV.-VIII show, that the correlations is higher in male in some cases and in female in others. Five out of seven, characters in the male are absolutely greater than those in female even when the length of the entire skull in two sexes is equated. However the differences are too small to be significant, except in the case of the length of the nasal bone and perhaps the zygomatic width. The nasal bone is significantly longer in the male while the zygomatic width is slightly greater in the female. On the other hand if we equate the

TABLE X.

Probable values of male characters with mean skull length equal to that of the observed female.				Probable values of female characters with mean skull length equal to that of the observed male.			
Characters and No. of equations.	ob-served ♀	Prob-able ♂	Differ-ence %	Differ-ence %	Prob-able ♀	ob-served ♂	Characters and No. of equations.
Fronto-occipital length. I.	26.37	26.36	.07	—.89	27.51	27.26	Fronto-occipital length. II.
Squamosal distance. III.	15.06	15.09	—.19	.30	15.23	15.27	Squamosal distance. IV.
Height of skull. V.	11.14	11.21	—.67	1.69	11.30	11.49	Height of skull. VI.
Length of nasal bone. VII.	15.69	16.12	—2.64	1.81	16.65	16.96	Length of nasal bone. VIII.
Cranial capacity. IX.	10.37	10.42	—.50	.37	10.86	10.90	Cranial capacity. X.
Zygomatic width. XI.	20.93	20.68	1.17	—.80	21.92	21.75	Zygomatic width. XII.
(Height × length × width)$^{\frac{1}{3}}$. XIII.	16.43	16.43	—.06	.53	16.79	16.88	(Height × length × width)$^{\frac{1}{3}}$. XIV.

length of the female skull to that of the male then the greatest difference is noticed in the length of the nasal bone also and the height of the skull comes next. Besides these two the remaining characters give differences which are always less than 1 per cent. Taking all the results together, we reach to the conclusion that aside from the nasal bone, and perhaps the zygomatic width and height of skull too, the actual sexual differences in the remaining characters are inconsiderable, being less than 1 per cent. This suggests that the nasal bone in the rat may be considered as one of the secondary sexual characters. Consequently the female skull can not be considered as an undersized male skull nor the male skull an overgrown female skull since these two skulls show at least one significant difference, i. e., in the length of the nasal bone (perhaps zygomatic width also). On the other hand the female cranium, i. e., if we disregard the length of the nasal bone and zygomatic width, may be considered as an undersized male cranium and vice versa, since the differences observed from the three measurements of the cranium in the two sexes are too small to be significant.

According to general belief the female brain is relatively heavier than that of the male although absolutely lighter. Blakeman, 05, found however, that "The Englishman of the same age, stature and diametral product as the mean woman has 1235 grs. brain-weight, or only 10 grs. more than the average woman. The Englishwoman of the same age, stature, and diametral product as the mean man has 1315 grs. brain-weight, or only 13 grs. less than the average man." He concludes from the above that "as far as present evidence goes, we can safely conclude that there is no sensible relative difference in the brain-weights of man and woman, the absolute differences observed are quite compatible with the differences which result from the relative size of the two sexes." The same conclusion, as has been given by Blakeman, may be drawn from the present study on the albino rats. It was found (see Table X) that when the length of the entire skull of the male rat is equated into the length of the entire skull of female, and vice versa, the resulting values for the cranial capacity in the two sexes are almost identical. The difference is in average less than 0.5 per cent, indicating that the sexual difference found in the cranial capacity is entirely accounted for the difference in the size of body. It is also interesting to note in our case that only one character has been equated and therefore if we took a multiple regression-equation the difference would probably almost disappear.

(g) *Characteristic equations.*—I have put together on the opposite page all characteristic equations which have been used in the course of the present study. Equations 1-14 will enable us to find the probable values of the other characters of the skull in two sexes when we know the length of the entire skull, while from the equations 15 and 16 we can obtain the probable brain-weight from the observed body-weight.

The characteristic equations show clearly that the relation between the length of the entire skull and the other characters of the skull, and brain and body-weight can not be determined by simple arithmetical proportion but require in each case the introduction of two or more of the necessary constants which are specific for the character chosen. It follows therefore that in general if the relation existing between the two characters turns out to be skew (that is a non-linear regression) then the relation should be more complicated and a simple proportion would fail to correctly express the relations existing between the two characters under consideration. On the other hand if the regression is linear the relation may sometimes, but not always be shown by a simple proportion just as well as by a characteristic equation.

(1)	Fronto-occipital length,	♂ = 0.5326	Entire skull length,			♂ + 4.226
(2)	Fronto-occipital length,	♀ = 0.6663	"	"	"	♀ − 1.311
(3)	Squamosal distance,	♂ = 0.1100	"	"	"	♂ + 10.515
(4)	Squamosal distance,	♀ = 0.1004	"	"	"	♀ + 10.884
(5)	Height of skull,	♂ = 0.1634	"	"	"	♂ + 4.425
(6)	Height of skull,	♀ = 0.0937	"	"	"	♀ + 7.246
(7)	Length of nasal bone,	♂ = 0.4911	"	"	"	♂ − 4.285
(8)	Length of nasal bone,	♀ = 0.5619	"	"	"	♀ − 7.654
(9)	Capacity of cranium,	♂ = 0.2790	"	"	"	♂ − 1.172
(10)	Capacity of cranium,	☿ = 0.2863	"	"	"	☿ − 1.527
(11)	Zygomatic width,	♂ = 0.6243	"	"	"	♂ − 5.259
(12)	Zygomatic width,	☿ = 0.5830	"	"	"	☿ − 3.298
(13)	(Height × length × width)$^{\frac{1}{3}}$,	♂ = 0.2590	"	"	"	♂ + 5.672
(14)	(Height × length × width)$^{\frac{1}{3}}$,	☿ = 0.2119	"	"	"	☿ + 7.619
(15)	Brain-weight,	♂ = (0.0072	Body-weight,		♂ + 9.349)	+ 5.980
(16)	Brain-weight,	☿ = (0.0251	"	"	☿ + 6.168)	+ 6.009

Conclusions.

1. All the characters here examined are absolutely greater in the adult male than in the adult female.

2. The coefficients of variation reveal the fact that the male characters show a general trend to a greater degree of variability than those of the female. The length of the nasal bone and the zygomatic width show a much greater variation than any other skull characters in the two sexes.

3. The coefficients of correlation are always positive and tend to be higher in male than in female. The correlation between cranial capacity and body-weight was found to be quite high (0.516 in male and 0.692 in female).

4. The brain-weight corresponding to the observed body-weight may be calculated from the following two characteristic equations:

Brain-weight, male = (0.0072 × (body-weight, male) + 9.349) ÷ 5.980.

Brain-weight, female = (0.0251 × (body-weight, female) + 6.168) ÷ 6.009.

5. The observed sexual differences are considerably reduced when the length of the entire skull is equated to either the male or female standard. When the lengths of the entire skull in the two sexes are equated and

the remaining characters are compared, the greatest difference is found in the length of the nasal bone (mean differences amount to more than 2 per cent), the height of skull and width of zygoma come next (slightly over 1 per cent), while the smallest differences are found in the remaining characters (less than 1 per cent). From the relation shown above the writer inclines to consider the relative development of the nasal bone in the rat as one of the secondary sexual characters.

6. From the above it is clear that the female skull can not be considered as an undersized male skull, nor the male skull as an overgrown female skull, since there is at least one significant difference in the skulls of the two sexes; i. e., the length of the nasal bone.

7. The female cranium on the other hand may be considered as an undersized male cranium, and vice versa, since the differences found from the three cranial measurements in the two sexes are too small to be significant.

8. The relation between the coefficients of correlation and regression is linear.

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