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With the Author's respects.

RESULTS OF EXPERIMENTS

ON THE

Vibrations of Pendulums

WITH

DIFFERENT SUSPENDING SPRINGS.

BY W. J. FRODSHAM, F.R.S.

WHITING, LONDON.

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BEING

THE SUBSTANCE OF A PAPER BY W. J. FRODSHAM, F. R. S.,

READ BEFORE THE ROYAL SOCIETY, JUNE 21, 1838.

The experiments of which I am about to give an account, and from which I propose to draw some practical conclusions, were undertaken with a view to determine whether some particular condition of the suspending spring of the pendulum, with respect either to its length, its strength, or both, might not cause it, with a lighter maintaining power, to produce a given arc of vibration, or, with a given maintaining power, to produce a greater arc of vibration than any other; and at the same time to ascertain whether some practical means might not be devised for making unequal arcs of vibration in the ordinary pendulum, correspond to equal intervals of time.

My attention was drawn to the subject many years ago, when, having replaced the spring of a turret-clock by a stronger one, I found the arc of vibration materially altered.

Having often reflected upon the subject, I at length resolved to make some experiments to satisfy my mind respecting it; and I accordingly had made for the purpose a lenticular pendulum bob of about fourteen pounds weight, a cylindrical rod passing through it, with a nut working on a screw at the lower end, and supporting the bob.

The upper end of the rod was slit to receive the spring; and the spring and the rod were attached to each other by a pin passing through a hole in both.

But before fixing the pin, what I call an *isochronal piece* was slid over the top of the rod, and if this part of the apparatus had served only to attach the rod and spring more firmly together, and prevent any wavering motion of the pendulum, it would have rendered an important service. This, however, was but a secondary and incidental effect of its application.

The piece, which I have so named, is a brass tube about five inches long, fitting the pendulum rod very nicely, and slit to form a spring for about an inch at the bottom, so as to slide rather stiffly on the rod. At the upper end of the tube is a *clip*, which is made to embrace the suspending spring firmly by means of two screws; so that after the pendulum has been brought to the proper length by the adjusting nut at the lower end of the rod, the length of the acting part of the suspending spring may be varied at pleasure, without in the least altering the length of the pendulum, by merely sliding the isochronal piece up or down the rod, and tightening the screws of the *clip*.

I also provided five springs of different degrees of strength, and a silken string, by which, in the first experiments, the pendulum was suspended.

The pendulum used was an uncompensated one, but in each experiment it was adjusted to nearly the proper length for mean time.

Commencing with the silken thread, or rather two parallel threads, one behind the other, I suspended the pendulum within the case of a clock, perfectly detached from the works, no maintaining power being applied.

Each degree of the scale on which the arcs of vibration were noted, was nearly ·8 of an inch in length, and a degree was subdivided into twenty equal parts.

I drew the bob aside 2°, and leaving it to vibrate by its own gravity, I found the arc of vibration was reduced from 2° to 1°, and from 1° to $\frac{1}{2}$°, in the times noted as under:

Arc of vibration	from	2° to 1°	in 20m.	15s.
Do.	do.	1 to $\frac{1}{2}$	23	6

On repeating the experiment, the results were—

Arc of vibration	from	2° to 1°	in 21m. 0s.
Do.	do.	1 to $\frac{1}{2}$	24m. 0s.

Drawing the pendulum aside 1°, I found from five successive trials that the arc of vibration was reduced to half a degree in the times following:

From 1° to ½°	in	21m.	45s.
Do.	do.	22	45
Do.	do.	22	0
Do.	do.	22	30
Do.	do.	23	0
Mean	. .	22	24

The mean of the two preceding corresponding results is 23m. 12s. The difference may be satisfactorily accounted for, by the difficulty of setting off the pendulum at the precise point intended, and of noting the time when the arc is diminished to the proposed quantity.

It is apparent from these experiments, that when a pendulum is freely suspended, and left to vibrate from its own gravity, the arc of vibration is sooner reduced from 2° to 1°, than from 1° to ½°, as might indeed be anticipated from the increased resistance experienced by the bob, while moving through a greater space in the same time.

I attached the pendulum, suspended as before, to a clock, with a maintaining power of 6lb. 8oz., but the clock stopped in 39 minutes; and setting it off again, it stopped in 43 minutes; but on applying a weight of 6lb. 11oz., the clock continued to go; thus showing that a weight of 6lb. 11oz. was sufficient to keep the pendulum in vibration, while one only 3oz. lighter was not.

The arcs of vibration in the preceding experiments being smaller than is desirable in practice, I proceeded to experiment with heavier weights, the pendulum being still suspended by the parallel silk threads, noting in each case the arc of vibration and the *rate* of the clock, viz., its gain or loss in 24 hours.

In the following experiments each succeeding pair is to be considered as giving the results for two consecutive days, though more than one day occasionally elapsed between the times at which the sets were taken.

Weight.		Arc of Vibration.		Rate.
14lb.	6oz.	2°	3′	— 9s ·0
8	0	1	30	+ 0 ·7
14	6	2	3	—10 ·0
8	0	1	30	0 ·0

Weight.		Arc of Vibration.	Rate.
11lb.	2oz. 1	1° 45′	— 7s.
8	0	1 30	+ 1
= 19	0	2 15	— 13

It hence appears, that when a pendulum is suspended by a flexible string, a heavier weight and a consequent greater arc of vibration, causes the clock to lose.

The following are the dimensions of the springs which were experimented with:

Number.	Breadth.	Thickness.
1	·350 inch	·001 inch.
2	·390	·002
3	·395	·003
4	·395	·004
5	·400	·005

The pendulum being suspended by the weakest spring, No. 1, the times were noted as before, in which the arcs of vibration were reduced from 2° to 1°, and from 1° to ½°, no maintaining power being applied.

Arc reduced from	2° to 1° in	1h.	58m.
Do.	do.	1	57
Do.	1 to ½	2	8
Do.	do. ½	2	10

With the same spring, and a maintaining power of 4lb. 1oz. and 2lb. 2oz., the following arcs of vibration and rate of the clock resulted from two consecutive days, the effective length of the spring being ·92 inches.

Weight.		Arc.	Rate.
4lb.	1oz.	2° 3′	—9s ·6
2	2	1 30	—6 ·1

The pendulum being suspended with spring No. 2, and clipped at ·92 inch, without maintaining power, the arcs of vibration were reduced as follow:

From	2°	to	1°	in	2h.	20m.	0s.
Do.	2	—	1	—	2	20	44
Do.	1	—	½	—	2	26	0
Do.	1	—	½	—	2	26	0

Applying 4lb. 1oz. and 2lb. 2oz. in succession, as a maintaining power, I found as under:

Weight.	Arc.	Rate.
4lb. 1oz.	2° 9′	—0s ·2
2 2	1 36	+2 ·5

With spring No. 3, and effective length ·92 inch, the following results were obtained on two consecutive days:

Weight.	Arc.		Rate.
4lb. 1oz.	2° 15′	—	2s ·5
2 2	1 39	—	2 ·8

Reducing the effective length of the spring to ·80 inch, the following results were obtained on consecutive days:

Weight.	Arc.		Rate.
4lb. 1oz.	2° 9′	—	0s ·0
2 2	1 30	—	0 ·0
4 1	2 9	—	0 ·5
2 2	1 30	—	0 ·2
4 1	2 9	—	0 ·2

Hence, with either of these lengths of this spring, the rate does not appear to be perceptibly influenced by the extent of the arcs of vibration. In fact, the vibrations of the pendulum may, for all practical purposes, be considered as isochronous.

The effective length of the spring was then increased to ·92 inch, and the following results were noted, without maintaining power:

Arc reduced from		2° to	1° in	2h.	26m.	0s.
Do.	do.	2	1	2	25	45
Do.	do.	1	½	2	37	0
Do.	do.	1	0	2	36	40

On three other occasions, with the same spring, and effective length, ·92 inch, the following comparative results were obtained:

Weight.		Arc.		Rate.
4lb.	1oz.	2°	15′ —	4s ·0 }
2	2	1	39 —	4 ·2 }
4	1	2	15 —	5 ·0 }
2	2	1	39 —	5 ·2 }
4	1	2	15 —	5 ·0 }
4	1	2	15 —	5 ·0 }

Showing that even with different lengths of this spring, the vibration of the pendulum may be considered as isochronous, with considerably different arcs of vibration; and also that with this spring, a greater arc of vibration is produced with the same maintaining power, than with any other spring that has been tried.

Spring No. 4 was next applied without maintaining power.

With it the arc of vibration was reduced from

2° to 1° in	1h.	47m.
do.	1	48
do.	1	50
1° to ½°	1	54
do.	1	55
do.	1	58
do.	2	0

Applying maintaining power of 4lb. 1oz. and 2lb. 2oz. respectively, with ·97 inch effective length of the spring, the following results were noted:

Weight.		Arc.		Rate.
4lb.	1oz.	2°	6′	— 2s ·5
2	2	1	30	+ 1 ·2

Even with this comparatively stiff spring, the arc of vibration is greater with a maintaining power of 4lb. 1oz., than it was with 14lb. 6oz., when the pendulum was suspended by two parallel silk threads. But the rate appears to vary more with the arc of vibration, than it did when No. 3 was used.

Reducing the length of this spring to ·66 inch, the following results were obtained:

Weight.	Arc.	Rate.
4lb. 1oz.	2° 3′ —	14s ·1
2 2	1 27 —	11 ·5

Sliding up the isochronal piece still further, till the length of the effective part of the spring was reduced to ·50 inch, the following were the results:

Weight.	Arc.	Rate.
4lb. 1oz.	2° 3′ —	18s ·0
2 2	1 12 —	14 ·5

This further shortening of the spring appears to have had a perceptible effect on the arc of vibration, when the lighter weight was applied.

I lastly attached the strongest spring, No. 5, and with effective length 1·0 inch.

Weight.	Rate.
4lb. 13oz. —	15s ·5
2 10 —	13 ·5

Reducing the length of this spring to ·8, the following results were obtained:

Weight.	Rate.
6lb. 3oz. —	14s ·6
2 10 —	12 ·4

Sliding up the isochronal piece still further, till the length of the effective part of the spring was reduced to ·50 inch, the following were the results:

Weight.	Rate.
4lb. 13oz. —	12s ·0
2 10 —	8 ·2

The lighter weight, 2lb. 2oz., employed on experimenting with the weaker springs, was found insufficient to keep the pendulum in vibration with No. 5; 2lb. 10oz. was found adequate to the purpose, and it was therefore employed.

In experimenting with this spring, the arcs of vibration were not noted, as I found that both it and No. 4 were too strong for the weight of the bob I was using, and to which the experiments indicate that No. 3 was excellently adapted.

The arc of vibration with the spring, No. 3, (viz. 2° 15′) using a weight of 4lb. 1oz., required 19lb. weight to produce it when the pendulum was suspended by the silken threads.

It appears then, from the preceding experiments on suspending springs differing in length and strength, that there is one which, with a given maintaining power, produces a greater arc of vibration than others, and gives the same arc of vibration with a smaller maintaining power; and, further, that with this same spring the vibrations may, in point of time, be all considered as isochronous, whether the arcs are large or small. And with the aid of the *isochronal* piece, a spring of the proper length and thickness may easily be selected in a very few trials.

It may be noticed too, that unless the pendulum is first *isochronized* by some such method as that which has been pointed out, anomalies may be imputed to *imperfect compensation*, which have their origin in a very different source.

In fine, it may be stated in conclusion, that if the pendulums of astronomical clocks were furnished with what I have called an isochronal piece, any person possessing a few springs of different degrees of strength, may with very little difficulty determine what spring is best adapted to the weight of the pendulum, and also what part of the spring may be most advantageously employed in action; and I shall not think that the attention which I have given to this subject has been misspent, if any thing that I have done may contribute to the advancement of an art, to which I have been professionally devoted during the whole of my life.

Change Alley, Cornhill.
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