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THE DISTANCES OF THE MOON, THE PLANETS, AND THE SUN, DEDUCED THEORETICALLY.

BY G. PINNINGTON,
ACTUARY TO ORDER OF DRUIDS, &c., &c.

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ON

THE DISTANCES

OF

The Moon, the Planets, and the Sun,

DEDUCED THEORETICALLY.

By G. PINNINGTON.

DATA :—

In cases of Circular Motion ;

1—Angular velocity *constant* "centrifugal force" is as distance.

2—Angular velocity *varying*, distance constant, "centrifugal force" is as velocity squared.

3—Linear velocity *constant*, "centrifugal force" is as distance, inversely.

4—Linear velocity *varying*, distance constant, "centrifugal force" is as velocity squared.

THE MOON'S DISTANCE.

DATA :—

Earth's Polar diameter = 7899·5 miles.

Do. circumference = 24817.011167 miles.

Polar value of g. = 32·255 feet.

Descent of body at Earth's surface in 1 min. = 10·996 miles.

Then, in order that "centrifugal force" at the Equator should counterbalance gravity there, $\frac{v^2}{\text{dia.}}$ must equal 10-996 miles and v^2 therefore = 10·996 × 7899·5 = 86862·902, and v in 1 min. . ∵ = 294·725295 miles.

But, the equatorial v of the earth = 17·280693402 miles per min. of a sidereal day, and this velocity would therefore require to be $\frac{294 \cdot 725}{17 \cdot 28069}$ times greater, in order that centrifugal force and gravity should be equal and counterbalanced at our equator, or, 17·05517 times greater than at present.

Then, present centrifugal force at the equator is $\frac{1}{17 \cdot 05517}^{2}$th of gravity there, that is, it is $\frac{1}{290 \cdot 8788237289}$th of gravity.

At 290·878 of the earth's semi-diamaters distance—linear velocity remaining the same as at our equator—centrifugal force (as per Datum 3) would be $\frac{1}{290 \cdot 878}$ of the $\frac{1}{290 \cdot 878}$ experienced at our equator and gravity would also at those semi-diameters distant be $\frac{1}{290 \cdot 872}^{2}$. Gravity and centrifugal force would, in such case, therefore, be equal, and a Satellite having lin. vel. of revolution equal to Earth's equatorial velocity of rotation would revolve at 290·878 semi-diameters of the earth's distance, and would have a period of 290·878 sidereal days.

But, our Satellite, the Moon, revolves in 27·49645670508 sidereal days, her angular velocity is therefore $\frac{290 \cdot 878}{27 \cdot 4964}$ = 10·57877 times greater than if her lin. vel. were the same as earth's equatorial vel. and she were at a distance at which centrifugal force and gravity were equal and coanterbalanced under this latter condition.

She revolves, then, 10·5787 times as fast, and if she were 290·878 semi-diameters of the earth distant, her centrifugal force would be $10 \cdot 578^2 = 111 \cdot 9103647129$ times too great for that place.

The problem then is, to find a place where decrease of the centrifugal force of 111·91 (which is as decrease of distance when angular vel. remains constant, as per Datum 1) would be equal to and counterbalanced by increase of gravity—which is as distance ^{2} inversely.

Let x be the distance required.

Then, because $\frac{1}{x}$th of any distance causes $\frac{1}{x}$th centrifugal force, and, at th e same time, causes increased gravity of $(\frac{1}{x})^2$ inversely, and centrifugal force and gravity must be equal; therefore, $\frac{111 \cdot 919}{x} = x^2$, and $x = \sqrt[3]{111 \cdot 918} = 4 \cdot 8189964$.

Then, $\frac{290 \cdot 878}{4 \cdot 8189964}$ = 60·340125 semi-diameters of the earth = 238,328 miles = the Moon's distance from the centre of the Earth.

With regard to the employment of the Polar diameter of the earth, and the Polar value of g., full explanations of the reasons for their use may be afforded when the author comes to treat of the refinements of the subject; in the meantime, the foregoing will exhibit his method, and prepare the way for its application to the finding of the Sun's distance.

THE DISTANCES OF THE INFERIOR PLANETS.

ARGUMENT.

At the Earth's distance from the Sun, a body having angular velocity equal to that of Mercury, would travel 4·152792 too quickly and have $4 \cdot 152^2$ too great centrifugal force; then at distance $\frac{1}{x}$ (angular velocity remaining the same) centrifugal force would be $\frac{1}{x}$th (as per Datum 1) and gravity would be $(\frac{1}{x})^2$ inversely, and both would be equal; $\frac{1}{x}$ = therefore, $\sqrt[3]{\frac{1}{4 \cdot 152792}^{2}} = \frac{1}{2 \cdot 5837}$ and Mer ury's distance = ·38665, the Earth's being 1.

The distance of Venus, obtained similarly = $\sqrt[3]{\frac{1}{1 \cdot 63259}^{2}} = \frac{1}{1 \cdot 38264}$ = ·7232331.

THE DISTANCES OF THE SUPERIOR PLANETS.

ARGUMENT.

Mars, if at the Earth's place of revolution, would have $\frac{1}{1\cdot86344}$th of due v. for such place and $\frac{1}{1\cdot86344^2}$th due centrifugal force, then at x times the Earth's distance (angular v. remaining the same) centrifugal force would be x times as much (as per Datum 1) and gravity would be $(\frac{1}{x})^2$ of that at Earth's place, and as centrifugal force and gravity must be equal, $\frac{1}{1\cdot86344^2} \times x$ must equal $(\frac{1}{x})^2$.

Then $x \frac{1}{1\cdot863444^2} = \frac{1}{3\cdot4754086336} \times x = (\frac{1}{x})^2$ and $x = \sqrt[3]{3\cdot475} = 1.5141$.

Therefore, at distance 1·5141, centrifugal force $= \frac{1}{3\cdot4754} \times 1\cdot5141 = \frac{1}{2\cdot29524988}$th and gravity would also equal $\frac{1}{2\cdot292} = \frac{1}{1\cdot5141^2}$ Mar's distance therefore = 1·5141, the Earth's being 1.

The proportionate distances of the other superior Planets agree with those found by Kepler's 3rd law.

THE DISTANCE OF THE SUN.

DATA :—

Moon's period = 1.
Earth's period = 13·3687.

If the Moon, at her present distance, were to move, with $\frac{1}{13\cdot3687}$th her present angular velocity, then she would have $\frac{1}{13\cdot3687^2}$ (as per Datum 2) of due centrifugal force.

But, if she were placed at $13\text{-}3687^2 = 178\cdot77213969$ times her present distance whilst having the same linear vel. of $\frac{1}{13\cdot3687}$th, she would then have (as per Datum 3) $\frac{1}{178\cdot722}$ of $\frac{1}{178\cdot722} = \frac{1}{178\cdot722}^2$th the centrifugal force, and gravity would also be $\frac{1}{178\cdot722^2}$th, and at such distance gravity and centrifugal force would therefore be equal.

Angular v. in this case would be $\frac{1}{178\cdot722}$ of $\frac{1}{13\cdot3687}$.

But, a body having 178·722 times this angular velocity, or $\frac{1}{13\cdot3687}$th of the actual ang. v. of the Moon, if placed at 178·722 times the Moon's actual distance, and having true circular motion at such place, would require to be controlled by a central mass $178\cdot722^2$ times (as per Datum 2) greater than that of the Earth, or, 31941·603036649733 8561 times greater.

Or, the body might be at twice this distance and under control of eight times the mass. For, ang. v. remaining constant the deviation from a straight path to a circular motion would be as 2, but gravity would but he as ¼th, and, therefore, the central mass would require to be increased eight times to allow of circular motion at such double distance under such circumstance.

Or, generally, in such cases of increased distance, whilst ang. v. remains constant, mass is required to be as distance^3.

Now, by the usual methods of finding the Sun's mass, we have it, at distance of 92,000,000 miles, equal to 322,700 times that of the earth.

And by comparing the distance of the Sun obtained by the parallactic method with the distance obtained by multiplying his $\sqrt[3]{\text{mass}}$ by 178·722 times the Moon's distance, we may find if his parallactic, observed, distance is correct.

The estimated mass of the sun being $\frac{332700}{31041\cdot608}$ = 10·0880973 times a mass which would control a body at 178·722 times the Moon's distance and having the Earth's angular velocity, then the Sun's distance would equal $\sqrt{10.0880973}$ times 178.722 of the Moon's actual distance, or, = 2·160742 × 178·72213969 × 238328·4087 = 92035861 miles.

The estimated mass of the Sun, at a distance of 92 millions of miles, is obtained by employment of different values of the Earth's diameter and of g, than is used in this paper in finding the Moon's distance, but, allowing for the same values of Earth's diameter and of g. as are herein used in the case of the Moon, we would have the Sun's mass increased in proportion as $\sqrt[3]{10\cdot139869}$ is greater than $\sqrt[3]{10\cdot0880973}$, and, consequently, would have a greater difference over and above 92 millions of miles than has been previously arrived at.

But, it must be noted, that 92 millions of miles would be very nearly the true distance of the Sun, and that the distance cannot be greater, for if we add to that distance and find his mass which would correspond to increased distance, we would also add to the difference, or "error," of the two methods.

The "error" named, can be adjusted, and the true distance of the Sun correctly deduced, but the process involves the consideration of much preliminary theoretical matter, and will, therefore, be the subject of another paper.

Allowing for the mass of the Moon being ·0123, the Earth's mass being 1, the Moon's distance will be 238328 × $\sqrt{1\cdot0123}$ = 239305·5 miles.

But, in finding the Sun's distance, the Moon's mass is not, and need not, be taken into account, and the Sun's distance and its error remain, as stated.

In this paper the name of centrifugal force is given to the tendency to move from a circular path in a tangential direction, which may be taken as opposite and equal in intensity to the deviation from a straight path to the centre of a circle in a minute period of time under the influence of a central pull; both the central pull and the centrifugal tendency are measured by the formula $\frac{\textit{chord}^2}{\textit{dia}}$ = space of deviation from straight path to circular motion.

Or, we may more properly view centrifugal power in this light: that a body *would* fall to the centre through space = $\frac{\textit{chord}^2}{\textit{dia}}$ in a given time, but has "power" accompanying its motion, which causes it to maintain its original distance from the centre of a circular motion. The first view is that derived from the mere *picture* of the geometry of the matter.

An Independent Deduction of the Moon's Mass.

1st—THE DISTANCE AT WHICH TWO OF OUR EARTHS WOULD REVOLVE ABOUT ONE ANOTHER.

If the Earth attracts any minute mass and causes a descent of the latter, in 1 minute of time, through 10·996 miles, then, two Earths would, if at 21·992 miles apart, meet together in 1 minute of time.

Then, in order that centrifugal force should counterbalance gravity when two Earths are in contact, $\frac{v^2}{dia.}$ must equal 21·992 miles, $v^2 =$ 173725·8 $\therefore$ $v =$ 416·8046 miles per minute.

But, the equatorial velocity of the Earth = 17·2807 miles per min. of a sidereal day, and this velocity would therefore require to be $\frac{416{\cdot}8}{17{\cdot}28} =$ 17·05517 × 1·414214 times greater, in order that centrifugal force and gravity should be equal and counterbalanced, there.

Then, present centrifugal force at equator is $\left(\frac{1}{17{\cdot}055 \times 1{\cdot}414}\right)^2 = \frac{1}{581{\cdot}7576}$ of gravity there, in such case.

At 581·7576 of the Earth's semi-dias. distance—lin. vel. remaining same as at our equator—centrifugal force (as per datum 3) would be $\frac{1}{581{\cdot}7}$th of the $\frac{1}{581{\cdot}7}$th experienced at our equator, and gravity would, also, at those semi-dias. distances be $\left(\frac{1}{581{\cdot}7}\right)^2$

Gravity and centrifugal force would, in such case, therefore be equal, and a mass equal to that of the Earth would revolve at 581·7576 semi-dias. distance, and would have a period of 581·7576 sidereal days.

But, the Earth revolves about the Sun in 365·25634 sidereal days, her ang. vel. is $\therefore$ $\frac{581{\cdot}76}{365{\cdot}26} =$ 1·59276 times greater than if her lin. vel. were the same as that at the Earth's equator and she were balanced at 581·7576 semi-dias. distance.

She revolves, then, 1·59276 times as fast, and if she were at 581·7576 semi dias. distance, her centrifugal force would be $1{\cdot}59276^2 = 2{\cdot}53588$ times too great for that place.

The problem, then, is, to find a place where decrease of centrifugal force of 2·5358 (which is as decrease of distance when ang. vel. remains constant, as per datum 1) would be equal to and counterbalanced by gravity, which is as distance^2, inversely.

Let x be the distance required,

Then because $\frac{1}{x}$th of distance causes $\frac{1}{x}$th of centrifugal force, and, at the same time causes increased gravity of $\left(\frac{1}{x}\right)^2$ inversely, and centrifugal force and gravity must be equal, therefore, $\frac{2{\cdot}5358}{x} = x^2$, and $x = \sqrt[3]{2{\cdot}5358} = 1{\cdot}362$.

Then $\frac{581{\cdot}7567}{1{\cdot}362} =$ 427·13 semi-dias. of the Earth = 1687056·7 miles = distance at which two of our Earths would revolve about one another.

And this conclusion is capable of being proved by the method (which may be more minutely worked out) as under :

If at distance = 92,000,000 miles, the sum of the masses of the Sun and Earth = 322700.

Then, if two Earths revolve about each other at distance of 1687056·7 miles, masses whose sum is as 161350 times that of the two Earths would revolve (ang. vel. being same in each case) at $\sqrt[3]{161350} \times$ 1687056·7 miles = about 92,000,000 miles.

2ND—THE MOON'S MASS.

But if two Earths were to revolve 13·3687 times as quickly, then, they would have $13{\cdot}3687^2 = 178{\cdot}722$ times too great centrifugal force for their place.

Then, at distance $\frac{1}{x}$th (ang. vel. of 13·3687 remaining the same) centrifugal force would be $\frac{1}{x}$th and gravity $(\frac{1}{x})^2$ inversely, and these must be equal $\therefore \frac{1}{13{\cdot}3687^{\frac{2}{3}}} = \sqrt[3]{\frac{1}{178{\cdot}722}} = \frac{1}{5{\cdot}632} = \frac{1687056{\cdot}7175}{5{\cdot}632} = 299548{\cdot}43$ miles = two Earths distance of revolution with Moon's ang. vel.

But two Earths distance is $\frac{299548{\cdot}43}{238328{\cdot}5} = 1{\cdot}2568$ times greater than Earth's and Moon's distance from each other—and ang. vel. being the same, distance is as $\sqrt[3]{\text{mass}}$—therefore, two Earths mass $= 1{\cdot}2568^3 =$ 1·9851737 times greater than Earth's + Moon's mass, $\therefore \frac{2}{1{\cdot}9851737} =$ 1·00746 $\therefore$ Moon's mass = ·00746 = $\frac{1}{133{\cdot}9}$ th of the Earth's mass.

The masses of the Satellites of the other Planets may be obtained similarly.

PRELIMINARY CORRECTIONS OF ALL THE FOREGOING PAPERS.

On reviewing the arguments upon which the estimated mass of the Moon has been arrived at ; it is found, that, as attraction is as the sum of two masses, and in the case of a falling body at our earth's surface we obtain the value of gravity by experiment on mass, which, compared with that of the earth, may be considered as infinitesimal ; in the previous calculations we have first been dealing with Mass = Earth's + o, or of the earth and a minute mass so small as, practically, to have an infinitesimal effect on our calculations, our deduction of 238328 miles as the distance of the moon is therefore, in reality, that of the distance of a small mass of, say, a few pounds weight for instance ; and, when we find though we have been really but dealing with so small a mass revolving about the earth, yet, our calculations make that small mass = $\frac{1}{133{\cdot}9}$th that of the earth, it is at once evident that in dealing with the actual perlod of the Moon we have been using too great a value of ang. vel. in our calculations, and that we should have used an ang. vel. which would give, as result, the distance of revolution of the earth and infinitesimal mass about each other.

The correction for this will be, Earth + o = M, and Earth + Earth = 2 M, $\therefore$ if E + o revolve at 238328·5 miles E + E would revolve at $\sqrt[3]{2} = 1{\cdot}259921$ times that distance = 300275·08 miles, with proper ang. vel. for that distance, and, $\frac{1687056\cdot7172}{300275\cdot082} = 5\cdot61837$ and $5\cdot61837^3 = 177\cdot35$, and $\sqrt{177\cdot35} = 13\cdot3183$ = *true* period of the Earth with regard to that of the Moon.

But, the Moon's period is *actually* $\frac{1}{13\cdot3687}$th that of the Earth's; it is therefore shortened $\frac{1}{271}$th. The reason of this will be explained further on.

Then, taking, for the present, the parallactic distance of the Moon as being correct, and = 238828 miles; if Masses 1·00746 revolve at 238328·5 miles distance with the Moon's *actual* period, then, the mass which will revolve at 238828 miles, will be $(\frac{238828}{238328\cdot5})^3 = 1\cdot00214^3 = 1\cdot006434$, and ∴ Earth + Moon's mass = $1\cdot006434 \times 1\cdot00746 = 1\cdot013942$ and Moon's mass = $\cdot013942 = \frac{1}{72\cdot7221}$ of the Earth's mass.

Her specific gravity ∴ = 3·76.

In order to complete the corrections, of the previous Papers, considerations such as the following arise:—

We may follow either of two methods. We may, as is done in these Papers, take values of *g*. and of the Earth's semi-dia. and deduce the distance at which two Earths would revolve about one another, and pursue the argument to the finding of the Moon's mass, and—as in the first Paper—find the mass about which a body having the Earth's ang. vel. would revolve, and its, and the Sun's distance; or, we may accept an assumed distance of the Sun and find his mass corresponding to that distance and work back and find the distance at which two Earths would revolve about one another and that at which the Earth and Moon would revolve about each other, and find corresponding values of *g*. and the Earth's dia.—in fact, find all the astronomical ratios for one assumed distance of the Sun, and therefore, proportionately, for any distance, and in so doing we would correct some errors which necessarily enter into all calculations based on the method of analysis we are using; viz., the errors arising from taking $\frac{arc^2}{dia.}$ = ver. sine, when in reality $\frac{chord^2}{dia.}$ = ver. sine; and, though with the great circumference of the Earth's orbit the error, at first, may be said to be nil, yet, when we come to deal with the smaller circumference of the Earth, the error, in calculations based on the descent of a body in one minute of time, is appreciable and vitiates to some extent, our after calculations.

Working back from assumed Sun's mass and distance would, then, greatly correct the source of error named, and also another error generally made in accepting gravity as an equable or uniform accellerating force of triangular space surface character, whereas, gravity is a variable accellerating force of hyperbolic space surface character (Morin's apparatus shows it of parabolic space surface character at small distances, but it is truly hyperbolic at greater distances of descents), and this would have appreciable effect in calculations based upon the descent of a small mass to our Earth during one minute of time.

The various ratios for any distance of the Sun would be deduced by the latter mentioned method of correction, and we may yet follow it in extenso; at any rate, it is hereafter followed in part. It would ultimately lead to a discussion of the true values of *g.* and of the Earth's dia., and of the density of the Earth's equatorial belt as well as of the hyperbolic characters of descents under the influence of gravity, bnt these discussions we wish to avoid at present, whilst outlining the rough plan of the work before us.

It has, then, been preferred, for the present, and until a further Paper be presented embodying further important corrections, to work with particular ends in view ; and the first which presented itself, was, the finding of the Moon's mass agreeable with her accepted parallactic distance. And this led, naturally, to the finding of the Moon's true period, and it is seen that it is shortened, instead of being lengthened as is generally believed.

With regard to the shortening of the Moon's period, the author is not surprised, as, he has long found fault with that portion of Newton's argument which *assumes* that because the Sun attracts the full Moon less than the new Moon, the Moon's orbit is therefore enlarged by the effect of the Sun's diminished pull on the full Moon as well as by that of his increased pull on the new Moon; for, at full Moon, the diminished pull of the Sun is more than counterbalanced by the more powerful total pull in the direction of the Sun. It is just, at such time, that additional pull is added in the direction of the Earth, and, therefore, as if the Earth were of *greater mass* at such time.

But, this argument may be made quite clear and be extended ; as, let us suppose an arm revolving about a pivot at one of its ends, and at the other end of the arm let there be a mass whose centrifugal force is balanced by an elastic cord attached to the pivot, then, let us suppose a fixed point outside this system, and from which we may imagine another elastic cord of such a nature, as that when it is stretched, its pull will be slightly diminished, to exert a pull on the revolving mass when it and its system is at "full moon" relative to the fixed point. The mass would, in such case, *certainly* approach its centre of revolution, and its period would be shortened, its lin. vel. would remain constant whilst its ang. vel.—at its diminished radlus of revolution—would be increased. Then, at point corresponding to new Moon the pull from the outer fixed point would counteract to some extent, the pull to the centre of revolution of the mass, and the radius of revolution and the period of the mass would be lengthened. But, in the first case,—as at full Moon,—we would have pull to centre of revolution + pull to outer point, and in the last case, as at new Moon, pull to centre of revolution—lesser pull in opposite direction, and it will be found that (allowing for the differential force of the Sun at each place) Earth's + Sun's pull at full Moon has greater effect than Earth's—Sun's pull at new Moon ; and taking the effect of the Sun's pull at both quadratures also, the Moon's period and distance is diminished by the Sun's pull, and in a similar manner as if the Earth's mass were increased.

In applying this argument it must be remembered that the pull of the elastic cord to the centre of revolution is always greater than that of the cord from the outside point, and so, at new Moon, the pull of the Earth on the Moon is always greater than—even whilst yielding to—

the Sun's pull, and therefore diminishes the effect of the Sun's pull which might otherwise be considerable on the new Moon considered as in a state of equilibrium as regards the Earth's pull and her own centrifugal force—*i.e.* when this equilibrium is disturbed the more powerful pull of the Earth than that of the Sun will control the amount of the disturbance and therefore will diminish the amount of radial and period extensions which would otherwise result from Sun's pull alone. At full Moon, on the contrary, the Sun's pull would have *full* effect, nay, more, as it would be aided by that of the Earth, and that, reciprocally as the diminished radius squared, whilst the lin. vel. of the Moon remaining constant, her centrifugal force would but increase reciprocally as her diminished distance. The diminished period of the Moon is, therefore, satisfactorily accounted for.

Though, on the whole, the Moon's period is permanently shortened, yet, in going to the full, her motion is accellerated, and in going to the new it is retarded, but the retardation will be only from quadrature to new Moon whilst the accelleration will be during the other three quarters of her revolution. Superadded, there will be yet another effect—though a minor one—of the Sun's pull which will retard the Moon's motion from new to full and accellerate it from full to new. And, again, the Earth in her motion about the Moon and in her smaller orbit, will experience the same effects of the Sun's pull, but to a $\frac{1}{71}$th of the extent, inasmuch as the Earth's inertia is 71 times greater than the Moon's.

From the foregoing, it follows that still another correction must be made for the Moon's mass, as, if the Sun diminishes her distance as well as her period, 1·013942 = Earth's + Moon's mass + effective pull of Sun in diminishing the Moon's distance, and, therefore, deducting the Sun's effect—which is as if Earth's mass were increased—the Moon's mass will be greater, relatively to that of the Earth, than we have previously found reason for believing it to be.

A more correct estimate of the Moon's mass may be arrived at when we have deduced the correct mass of the Sun.

In the first Paper on the Sun's distance, the value of the Sun's mass, taking his distance as 92,000,000 miles, was taken from a modern work on Astronomy and accepted without questioning, but, since writing the Paper named, it has been found—by the method (ver. sine of Arc moved through by the Earth in 1 sec.) × (Sun's distance in Earth's semi-dias. × 5280) $^2 \div \frac{1}{2}$ g. that the sum of the masses of the Sun and the Earth = 323890.

Then, the correction of the first Paper, will be, Sun's and Earth's mass = 323890; Moon's period = 13·3183; then, $13{\cdot}3183^2 = 177{\cdot}35$, and $177{\cdot}35^2 = 31453$.

Then, $\frac{323890}{31453} = 10{\cdot}2976$, and $\sqrt[3]{10{\cdot}2976} = 2{\cdot}1756$.

Then, 2·1756 × 177·35 × 238328·4 = 91957100 miles.

This evinces that the assumed distance of the Sun = 92,000,000 miles, and his consequent mass and the first method of the deduction of his distance—after the correction of the Moon's period—do not agree.

Now, with mass of Sun and Earth = 323890, two Earths' mass would equal $\frac{1}{161945}$th of 323890.

Then, if Sun + Earth revolve about each other in a year at a distance of 92,000,000 miles, two Earths would revolve about each other—ang. vel. being in each of the cases the same—at $\frac{92000000}{\sqrt[3]{161945}} = \frac{92000000}{58.8557}$ = 1563143 miles.

But, we previously deduced, from the data g., the Earth's semi-dia. and the lengths of a day and of a year, that two Earth's would revolve about each other at 1687056·7 miles distance.

And, for the present, preferring the data upon which the latter mentioned calculations were based, rather than the assumption that the Sun is 92,000,000 miles distant, we will work from two Earths distance = 1687056·7 miles and deduce the distance of the Sun which $\sqrt[3]{\text{mass}}$ × two Earths distance of revolution, would allow of.

Then, we have two Earths distance of revolution about each other at 1563143 miles, if the Sun's distance were truly 92,000,000 of miles, whereas, the real distance of revolution of two Earths deduced from the latter data aforesaid = 1687056·7 miles, and this latter distance at once indicates that we have under consideration a larger System than one with the Sun at but 92,000,000 of miles from the Earth.

And, as distances are as $\sqrt[3]{\text{mass}}$, and $\frac{1687056.7}{1563143} = 1.07927$, then in the more correctly deduced distance of revolution of two Earths about each other, we are dealing with a System $1.07927^3 = 1.25716$ times greater than that which would have the Sun at 92,000,000 of miles from the Earth, or, with a System having the Sun 115,658,780 miles from the Earth and with the Sun, therefore, having 1.25716^3 times greater mass, and, therefore, having Sun's + Earth's mass =645156 times that of the Earth.

Then, working by an inverse method; $\frac{115658780}{\sqrt[3]{322578}} = 1687056$ = distance of revolution of two Earths about each other, proves the case.

And, recurring to the first method (in first Paper) of finding Sun's distance, we have $177.35 \times 238328.4 \times^3 (\frac{645156}{314553}) = 117492000$ = the Sun's distance, corrected for the newly found mass of the Sun.

We have, now, two different estimates of the Sun's distance, viz: 115,658,780 and 117,492,000 miles, and this leads the Author to bring forward his final Paper on the Sun's distance sooner than he otherwise would have done, though, the said Paper was really the first production in point of time and has been prepared for some years, and, in truth, led to the preceding work herein reproduced.

THE PRINCIPAL PAPER ON THE SUN'S DISTANCE.

In order to arrive at the true distances from each other of the bodies composing our Solar System, we will be obliged, first of all, to have a thorough insight into the bases upon which our investigations are founded, and to lay down a clear method of procedure.

Now, that we may understand circular motions about a centre of attraction whose efficacy is reciprocally as distance^2, we must learn (as indeed we always should do when considering applied mathematics) to constantly bear in mind the Physical facts which are—so to speak—working simultaneously with, and underlying, the workings of the mathematics of the matter.

The *rigid* Geometry of Circular Motion is well known. Huygens gave us the theorem which is expressed, algebraically, by the equation $\frac{chord^2}{dia.}$ = space of deviation from a straight path to a point in a circle.

A modification of this, is, $\frac{chord^2}{rad.}$ = twice the space of deviation, and yet another modification, is, $\frac{arc^2}{rad.} = ?$, and a further modification—more specially to suit cases of circular motion—is, $\frac{v^2}{r} = ?$

Notes of interrogation are put on one side of the latter two equations, for the present, as, we will have to consider, further on, that, though we may say in an infinately small portion of circular motion $\frac{arc^2}{r}$ = what is equivalent to twice space of descent, as arc may be said to be, ultimately, equal to chord; yet, when we come back again to greater portions of a circular motion $\frac{arc^2}{r}$ equals an *incongruous* quantity representative of no line in Geometry, no fact in Nature, and nothing which the mind of man could conceive nor which his tongue could give utterance to. Newton's method of prime and ultimate ratios (which is *entirely* a different method to his fluxions and the more modern differential calculus) is in error in the treatment of the case of circular motion, as, though we may be said to differentiate to the ultimate ratio $\frac{arc^2}{r}$ (which is a *false* ratio supposed to be made true by arrival at equality of arc and chord) we should, in larger matters, integrate—so to speak—again to *rigid* geometry, $\frac{chord^2}{r}$, which is correct in itself, and is also a *true* ratio.

An example of this will show the glaring errors which the use of $\frac{arc^2}{r}$ leads us into.

Let us take the case of a quadrant of a circular motion of dia. = 2, then $\frac{chord^2}{rad.} = 2s$, and, numerically, $\frac{1{\cdot}414214^2}{1} = 2$; but $\frac{arc^2}{r}$ would give us a *very* different result and would be working with a *false* ratio (the ratio of unlike things) and not, then, be working geometrically, but with numerical values of the false ratio, merely, and would have as result, a number indicative of the value of no geometrical line and of no fact in Nature such as twice space of descent, or deviation, which latter is obtainable by working with rigid geometry and *true* ratio $\frac{chord}{r}^2$

The *rigid* geometry, then, affording us *true* result and a Physical fact, at a quadrant of a circular motion and at *all* points included in that quadrant (we need not consider the application of $\frac{chord}{r}^2$ to cases greater than quadrants, for the purposes of the present Paper) whilst the *false* ratio gives us truth but at *one* point where arc = 0 and deviation = 0 and circular motion = 0 and where we are considering *nothing at all*, and so might save ourselves useless trouble! We will work, then, with rigid geometry.

The fact is, Newton *manufactured* his lemmas which treat of prime and ultimate ratios, because Kepler's 3rd law *requires* that $\frac{arc}{r}^2$ should equal twice space of deviation, and because gravity at distances from a centre of attraction has always been treated erroneously as a constant or uniform accellerating force, whereas it is not so, but is a variable accellerating force.

To distinguish between geometry and Physical fact.

It must be, first, noted, that in an actual case of circular motion there is *no* descent from a straight path, and still less, is there any *accelleration* of descent.

The ratio $\frac{chord}{r}^2$ is used in the idea that, by its means force is found, and force being supposed to be measurable only by vel. acquired during a descent, $\frac{chord}{r}^2$ is supposed to give us twice the numerical value of space of descent, to give us velocity acquired during a *uniformly* accellerated descent, to give us "force."

Newton's false teaching concerning prime and ultimate ratios—the subtlety of which (as in his 6th lemma) consists in the fact that we reduce a question to nothingness, when in such case all the nothings are equal, but afterwards we work as if things (as arcs, chords, and tangents) when unreduced to nothingness, or when having absolute extension, are *still* equal, (which teachings must not be confounded with his teachings with regard to fluxions, the latter being but an *empirical* method which allows of statements being so made of any case of which we have had *previous* knowledge derived from geometry and the arithmetic of infinites that we may work back from *supposititious* infinitesimal values and make *true* summations to *suit* each case and the *desired* end to which we work) now steps in and allows of us to work with $\frac{arc}{r}^2$ in lieu of the *rigid* $\frac{chord}{r}^2$ and, so, to work with the desired $\frac{v^2}{r}$ also, in cases of circular motion.

Several considerations arise here; we will take them in the order in which they arise.

1st. In a case of *actual* circular motion, we have a constant and uniform pull or pressure from a centre, which constant pull counteracts the tendency of a revolving body to proceed in a straight line at any and all points in the motion, and which constant pressure during any time is *equivalent* to a force which—acting at a right angle with what we may call the prime straight line of the natural tendency of motion of a body at the point at which we commence to consider actual case of circular motion, and at which we commence to apply our geometry—would cause the body to move from the straight line to the circular motion in the same time. There is, then, an *equivalency* of constant pressure or pull and of that which would, otherwise, cause space of deviation to circular motion, and this is the way in which to interpret and wed our geometry to cases of circular motion; there is *no accelleration* of descent to take into account at all, but there is a geometrical line which is *equivalent* space of descent, merely, or, which is representative of space of deviation from a straight path to a circular motion under the influence of any imaginary force exactly *equivalent* to the constant pull from centre which actually causes the deviation.

2nd. As to "force" and its measure, we must note that in a case of circular motion, the "pressure" or pull of central attraction is the only force with which we are *actually* concerned (the force which gave velocity to the body having circular motion and which is conserved in or by the body need not, at present, be considered), that is, we have no force acting at a right angle with a straight path and causing a descent through space to a circle, we have but a constant pressure preventing deviation to straight path. A body is not travelling in a straight path whilst a "force" acting at a right angle to that path is causing it to descend during time to a point in a circle, but a body is restrained by constant pressure from deviating from a circular to a straight path each instant of time. We must get rid of all the impressions made by the mere "picture" the geometry of circular motions, on the mind.

But *pressure* is truly *force*, and, abstracting that which causes pressure to be experienced, (as by abstracting anything upon which a body rests) motion, and accelleration of motion ensues, and accellerations are caused by mutations of pressure proportionate to the mutations of velocity of motion during accellerations, and the whole of the said mutations of pressure are conserved in the accelleration they have caused. A total accelleration (not merely a final velocity of accelleration) is a quantity of motion accompanied by the quantity of pressure which has caused it, and this quantity of pressure may be learned by reference to our standard of comparison for the admeasurement of quantities of pressure, viz., the descent of a body at the earth's surface. The formula $\frac{v^2}{2g}$ = space of descent during time of acquiring final velocity of any accelleration, gives us the quantity of pressure conserved in *any* accelleration, as it gives us the space through which a body having velocity due to any accelleration would rise as against the influence of gravity pull, as pressing against gravity pull. It gives us then the total quantity of *pressure* conserved in any motion, and therefore, the total quantity of pressure which has caused the motion.

Force, then, is pressure, and it requires the same total quantity of mutations of pressure to cause any accelleration no matter during what length of time, or under what circumstances, any accelleration is acquired. And a reference to our terrestrial standard of comparison, to "work" at the Earth's surface will apply universally to all accellerations and their accompanying or conserved pressures—to all cases of vis viva, to all cases of "energy."

Quantities of mutations of pressure cause proportional quantities of mutations of velocity and motions over space, and are as spaces passed over during accellerations. Space will, therefore, be representative of quantity of pressure no matter whether mutations of pressure have been equable and uniform or whether they have been variable, and mere final velocity acquired during an accelleration is but proportional to final mutation of pressure of the accelleration and does not represeut and is not proportional to "force" or quantity of pressure, vis viva, energy, or work.

A line representative of space passed over in acquiring an accelleration will then be representative of force, of quantity of pressure, of vis viva, of energy, of work, and this latter "work" is our reference of quantity of pressure, vis viva, energy, to our standard of reference, the space through which any accelleration and its accompanying quantity of pressure, vis viva, or energy, would press its way against the influence of gravity at our Earth's surface. And we thus see that a line representative of space of deviation in the geometry of circular motion, is representative of quantity of pressure or force *equivalent* to the constant equable pressure or pull from centre which *actually* prevents deviation from circular motion to straight path, or which, from another point of view, may be said to cause constant deviation to circular motion. $\frac{Chord^2}{dia.}$ is then applicable to the *equivalent* physical fact, the central pull, causing circular motion, whilst $\frac{chord^2}{rad.}$ would represent but final velocity acquired during an equable or uniform accelleration and would not be representative of quantity of pressure, of force, equivalent to constant pressure or central pull causing the constant deviation to a circular motion, which latter is the *actual* physical fact underlying our mathematics of circular motion.

Still less, therefore, would $\frac{arc^2}{rad.}$ apply to any case of circular motion.

Again, Gravity, is, and always has been, too coarsely treated as a constant, equable, or uniform accellerating force instead of being truly treated as a variable accellerating force. The difference between a uniform and a variable accellerating force may be readily perceived if we consider that at, say, 4 semi-dias. distance from the Earth's centre gravity would cause a pull of $\frac{1}{16}$th that experienced at the Earth's surface, and, therefore, a descent of one foot during first second of time, whilst at 2 semi-dias. distance pressure or pull would be ¼th and descent four feet. If a body, then, commenced to fall from 4 semi-dias. distance under the influence of a constant, equable, or uniform accellerating force this would be capable of communicating two feet additional velocity in each second of time throughout the duration of descent to the Earth's surface, but, in actual fact, when the same body, starting its descent from 4 semi-dias. distance arrives at 2 semi-dias. distance, it now reaches a point at which eight feet additional velocity would be communicated to it in each second of time of its remaining portion of descent, even if gravity were to remain a uniform accellerating force from such point. But, gravity, before the body reaches that point gradually communicates the additional velocity up to eight feet, and after, it still further increases the velocity that is additional to that of a uniform increase of two feet per second, and which we have seen a *constant* or equable accelleration would but communicate throughout the whole descent.

This is the difference betwixt a constant and a variable accellerating force.

A constant accelleration may be represented by a right angled triangle the perpendicular side of which is representative of time of descent and the base of which is representative of final velocity acquired during the accelleration, the surface of the triangle will be representative, then, of the mutations of velocity throughout the accelleration, the total quantities of mutations of velocity will be as the space passed over and the surface of the triangle will be representative of this space, and, further, as mutations of velocity are proportionate to the mutations of pressure which cause them, the triangular surface will be representative of quantity of pressure causing the accelleration and conserved by, or accompanying, the accelleration.

Morin's Apparatus for experimenting on descents at the Earth's surface indicates a *variable* accelleration of a parabolic instead of a triangular surface space character, but, in reality, descents from distances to the Earth's surface are of hyperbolic surface space characters. For, it stands to reason, that if one centre of attraction (as, say, our earth) alone existed in Nature, no mass could be projected from it to an infinite distance except it had an infinite velocity of motion to commence with, and, conversely, any mass would acquire an infinite velocity in falling from an infinite distance to the Earth, and this, when gravity is a variable accellerating force. Now, if gravity were a uniform accellerating force, the same infinite velocity of projection would allow of mass to move to a—so to speak—still greater infinite distance, or, conversely, the mass, in this case, would fall through greater space to acquire the same final velocity (as we have previously seen in the consideration of descents from different semi-dias. distance), but, as when treating of different infinite descents with uniform and with variable accellerations, when the space fallen through with uniform accelleration would be greater than that fallen through with variable accelleration to obtain the like, or nearly like, velocities, and, as infinite motions would never end or be determinate, the only representative that would satisfy such cases would be that of an assymptote and continued major axis of a hyperbola as the extremities of increasing velocity lines of the *uniform* accelleration, and the contained half of a hyperbola as bounding the increasing velocity lines of the *variable* accelleration.

And, when we have a definite or short of infinite velocity of projection, or of descent, through definite space we may represent such definite variable accellerated motion by one half of a definite hyperbola.

Descents under the influence of a force which varies reciprocally as $distance^2$, are then, of hyperbolic surface space characters, and, as space of descent represents total mutations of velocity during time of descent, which mutations are proportional to mutations of pressure which cause them, a line representative of space of descent is representative of quantity of pressure, of force, of vis viva, of energy, of work causing and conserved by or accompanying any motion acquired under the influence of a variable accellerating force, and we are, thus, enabled to represent quantity of pressure, quantity of force, geometrically as by a line.

The first case in which we will utilise the fact that gravity is a *variable* accellerating force, is, the following :—

Now, if gravity were a *constant* accellerating force, in cases of descents from different distances from an attracting centre, the squares of the times of descents would be as the cubes of the distances.

Let us take two cases of descents to our Earth's surface ; one of them from a distance of 2 semi-dias. from Earth's centre, and the other from 4.

Then, at the first named distance, gravity would cause four feet of space to be moved over during first second of time.

At the second distance, one foot of space would be moved over during the first second.

Say (for facility of calculation) that the Earth's semi-dia. = 4000 miles, then,

1st distance = 42240000 feet
2nd do. = 84480000 feet

1st distance $T^2 \times 4$ ft. = 42240000, $T^2 = 10560000$
2nd do. $T^2 \times 1$ ft. = 84480000, $T^2 = 84480000$

∴ 1st distance is as 1 ; 1st $time^2$ is as 1
2nd do. is as 2 ; 2nd $time^2$ is as 8

and the squares of the times are as the cubes of the distances.

This *fact* has an important bearing on what follows, as, Kepler's 3rd law, is ; The squares of the periodic times of the Planets are as the cubes of their distances.

This law of Kepler's, like the foregoing deduced law of the times and distances of descents, *could only be true if gravity were a uniform accellerating force*, and, gravity is not so, but is a variable accellerating force.

And, that Kepler's 3rd law would only apply if gravity were a uniform accellerating force, may be proved, for, assuming for the present, that Kepler's 3rd law be true, the following tabulation of distances and periodic times, viz:

Dis. 1, $\frac{v^2}{r}=2$, $v=\sqrt{2}=1{\cdot}414$ Per. time=4·4, or as 1

Dis. 2, $\frac{v^2}{r}=\frac{1}{2}$, $v=1$ Per. time=12·5664, or as 2·8284

Dis. 3, $\frac{v^2}{r}=\frac{2}{3}$, $v^2={\cdot}666$, $v={\cdot}8164965$, Per. time=23·085, or as 5·19615

(Note, the 1st $\frac{v^2}{r}$ is taken for a quadrant of the motion.)

Shows, that Repler's 3rd law *requires* that the *false* ratio $\frac{v^2}{r}$, which is neither a geometrical truth nor a Physical fact (as has been previously shown) *must* be worked to, and that gravity must be treated as a *constant* accellerating force in order that times^2 may be as distances^3.

And, as the quadrant of each circular motion is as ¼th the periodic time, the equivalent deviations from a straight path to the circular motions, are equal to the distances of revolution, and, therefore, the squares of the equivalent times of descents from straight paths to circular motions are as the cubes of the distances, and this, as has been seen, could only occur under the influence of a uniform accellerating force.

But, knowing, as we do, that gravity is a *variable* accellerating force, how then are we to approach cases of actual circular motions?
We approach such cases, thus:

The pull or pressure of gravity is as $\frac{1}{distance^2}$ and, as we have no cases of actual descents to consider, but only spaces of deviation from straight paths to circular motions, which said spaces are representative of quantity of force *equivalent* to constant pressure which actually causes deviation to any point in a circular path, and we can always find the pressure or pull at any distance, we can, therefore, always find its equivalent quantity of pressure and the representative space line of the latter.

As, for instance, at a distance of one from a centre of attraction, the pull of attraction would be constantly as 1, and, during the quadrant of a circular motion at this distance, the total constant pull to centre would be equivalent to a quantity of pressure which would cause descent from a straight line to the circular path in same time, and which would be represented by a line equal to the radius of the motion; then, the following table will show the time of revolution at distances at which the constant pull of gravity equals 1, $\frac{1}{4}$, $\frac{1}{9}$ and $\frac{1}{16}$, and the equivalent spaces of descent or deviation from straight path to circular motions are also as 1, $\frac{1}{4}$, $\frac{1}{9}$ and $\frac{1}{16}$, in the same or equal time.

Dis. 1, $\frac{chord^2}{dia.}=1$, chord = 1·4142, arc = 1·5708, p. time = 1

Dis. 2, $\frac{chord^2}{dia.}=\frac{1}{4}$, chord = 1. arc = 1·05848, p. time = 3·10868

Dis. 3, $\frac{chord^2}{dia.}=\frac{1}{9}$, chord = ·8164765, arc = ·8190145, p. time = 5·749

Dis. 4, $\frac{chord^2}{dia.}=\frac{1}{16}$ chord = ·70710678, arc = ·70802904, p. time = 8·7329

This Table is worked true to the Physical facts of the case, *i.e.*, to the diminishment of gravity as $\frac{1}{distance^2}$; and to the lines which represent this fact by *equivalently* representing quantities of pressure and constant pressure, and, it is, therefore, a *true* application of mathematics to Physics. In its construction, rigid truth is adhered to, throughout, and *rigid* geometry, viz., $\frac{chord^2}{dia.}$ = space of deviation from straight path to circular motion is relied upon. It is worked true to the fact that gravity is a *variable* accellerating force inasmuch as the constant pressure from centre in each case of circular motion and the force represented by the *space* lines of deviation from straight to circular paths are equivalent.

Then, as Kepler's 3rd law does not truly apply to actual cases of circular motions, and our Tables of circular motions agree in every way with rigid mathematics and actual physical facts underlying the working of mathematics; how then are we to deduce the proportionate distances of the Planets?

And how are we, by the Transit of Venus to find the Sun's true distance?

An inspection of the two Tables will show that the actual periodic times are in every case about 1·1 times greater than the periodic times which would be true if gravity were a constant accellerating force, and, therefore, dividing the periodic times in the last Table by 1·1 will reduce them to times whose squares are as the cubes of distances, and we can, thus, make Kepler's 3rd law to apply. By dividing the true periodic times by 1·1 we, in fact, alter cases of true circular motions under the influence of a variable accelleration to those of fictitious cases of circular motion under the influence of a theoretical uniform accellerating force and, then, Kepler's 3rd law applies.

Then, to apply this to the Sun's distance as found by the Transit of Venus.

By Kepler's law, the proportionate distances of the Earth and Venus from the Sun, would be as 1 and ·723332, and the distance of the Sun deduced from the Transit of Venus based upon such proportional distances of the Earth and Venus = 92,000,000 miles.

Then, by our correction ; and taking the distance of Venus as 1 :

Earth's time =	365·25636 days	log =	2·5625978
	1·1	log =	·0413927
			2·5212091
Venus's time =	223·7008 days	log =	2·3516046
			·1696055
			2
			3) ·3392110
			·1130703

Proportionate distance of Venus = ·770779 = Arith. Comp. log ·1130703 = log ·08869296

Then 1 — ·723332 = ·276667
and 1 — ·770779 = ·22922
and $\frac{·723332}{·276667}$ = 2·61445,
and $\frac{·770779}{·22922}$ = 3·362616,

Then $\frac{92000000 \times 3·362616}{2·61445}$ = 118·32000000 of miles = the Sun's distance after correction of Kepler's 3rd law.

Errors of observation, (although these Papers prove these to be marvellously small, considering the difficulties of observing actual contact of Venus at contact with both limbs of the Sun, or at ingress or egress,) cause the correction of the Sun's distance founded upon the Transit of Venus to be less reliable than it otherwise would be.

And, on the whole, the deduction of the Sun's distance which is based upon the values of g. and the Earth's semi-dia. and the distance at which two Earth's would revolve about each other, is the most correct. This last named deduction makes the Sun's distance from the Earth = 115,658,780 miles.

Further minor corrections have yet to be made, and a more correct division of periodic times than 1·1 will be employed.

In order not to clash too much with the prejudices of some who have (or rather *think* they have,) " refined away " " centrifugal force," it must be noted that the writer gives the name of centrifugal force to that "power," which at the Earth's present rate of revolution, causes weights of bodies and descents at the Equator to be $\frac{1}{289}$th less than they would be if the Earth did not rotate about its axis, and, which would take away *all* weight of bodies and stop all descent if the Earth were to rotate 17 times faster than at present. To do away with the notion that centrifugal force implies action at a right angle, the writer has shown in a Treatise on Dynamics, (published in the "Carpenter and Builder," 1880—1884,) that the bequeathed or conserved force (pressure), in a tangential direction of each *previous* instance. causes the "centrifugal pressure" of each instant of time, in circular motions.

It must be noted that whilst a body *has* circular motion, it but tends to move outward from the centre in a truly *radial* direction, and would describe a spiral; it is only on *arrestment* of the motion that the body would fly off in a tangential direction. Circular motion is accompanied with a *pressure* in a radial direction, and this is truly "centrifugal force." Pressure is "force," and is the cause of motion.

The same treatise contains explanation of the reason for the use of the Polar value of g, in this paper; it also contains a true theory of Heat, and the source of the Heat the Earth receives from the Sun; also a true theory of the "medium" in the universe; a correction of Meyer's and Joule's false Mechanical Equivalent of Heat, and a thorough explanation of "force" (pressure) cumulative pressure, (energy and work); the parallelogram of pressure accompanying motion, in lieu of the false parallelogram of velocities; true theories of the orbital motions of the Planets; of Inertia; of Elasticity (and the cause of the rapid development of force in explosions); the conversion of terms of energy into terms of pressure (on arrestment of motions); the work done by descending bodies (whilst raising others) and the velocity of descent in such cases; a true theory of Geodetical operations, and the shape of the Earth; rigid mathematics is applied throughout the work, and no "accommodations" are accepted, and the Geometrical, in lieu of the Algebric, analysis is generally used. The theory of the medium in space has been "*adopted*" by Dr. Hunt, of America.

This Treatise is about to be published in one volume, at 2/6; those willing to take this work are requested to send their names and addresses.

N.B.—The portion of this Pamphlet, after page 5, has not been offered to the R. A. Society.

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