Minerva instructing Youth in Literature, Science & the fine Arts.

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THE

MODERN PRECEPTOR;

OR,

A GENERAL COURSE OF EDUCATION:

CONTAINING

INTRODUCTORY TREATISES ON

LANGUAGE,
ARITHMETIC,
BOOKKEEPING,
ALGEBRA,
GEOMETRY,
GEOGRAPHY,
ASTRONOMY,
CHRONOLOGY,
NAVIGATION,
DRAWING, PAINTING, &c.
AGRICULTURE,
GEOLOGY,

MORAL PHILOSOPHY.

FOR THE USE OF SCHOOLS.

ILLUSTRATED WITH PLATES AND MAPS.

BY JOHN DOUGALL.

IN TWO VOLUMES.

VOL. I.

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ADVERTISEMENT.

"The importance of education is a point so generally understood and confessed, that it would be of little use to attempt any new proof or illustration of its necessity and advantages.

"At a time when so many schemes of education have been projected, so many proposals offered to the public, so many schools opened for general knowledge, and so many lectures in particular sciences attended; at a time when mankind seems peculiarly intent upon familiarizing the several arts, and when every age, sex, and profession, is invited to an acquaintance with those studies which were formerly supposed accessible only to such as had devoted themselves to literary leisure, and dedicated their powers to philosophical enquiries; it seems rather requisite that an apology should be made for any further attempt to smooth a path so frequently beaten, or to recommend attainments so ardently pursued, and so officiously directed.

"As this book is intended to correspond with all dispositions, and to afford entertainment for minds of different powers, it is necessarily to contain treatises on different subjects. As it is designed for schools, though for the higher classes, it is confined wholly to such parts of knowledge as young minds may comprehend: and as it is drawn up for readers yet unexperienced in life, and unable to distinguish the useful from the ostentatious or unnecessary parts of science, it is requisite that a nice distinction should be made, that nothing unprofitable should be admitted for the sake of pleasure, nor any arts of attraction neglected that might fix the attention upon more important studies.

"In the following pages it must not be expected that a complete circle of the sciences should be found: the object of the compilation is not to enrich the mind with affluence, or to deck it with ornaments, but to supply it with necessaries. The enquiry therefore was not what degrees and kinds of knowledge are desirable, but what are in most stations of life indispensably requisite; and the choice was determined not by the splendor of any part of literature, but by the extent of its use and the

the inconvenience which its neglect was likely to produce."

Such were the statements with which, now sixty years ago, THE PRECEPTOR published by *Robert Dodsley* was ushered into the world: a work the production of various eminent hands, which has long and deservedly maintained a high reputation as an elementary system of general instruction.

In affixing to the following pages the modified title of THE MODERN PRECEPTOR, it has been the compiler's purpose to represent his work as in general an imitation of a valuable original, but at the same time to warn his readers, that the various treatises introduced have been altered in a way to include the numerous improvements which, in a lapse of sixty years, have by modern ingenuity been discovered and adopted. These treatises have been not only adapted to the present state of knowledge falling within the scope of each, but arranged in an order more natural than that chosen by the editors of the old *Preceptor*: the student, therefore, by tracing the connection between the preceding and following chapters, as they stand in this book, will find the course of his studies more engaging, and consequently his progress greatly facilitated.

The

The table of contents shows the nature and distribution of the tracts comprehended in this work, which, notwithstanding the coincidence of many of the titles with those of the original *Preceptor*, are, with the exception of the excellent system of *Moral Philosophy*, and a few other portions occasionally introduced under various heads, compiled anew for the present purpose. Several tracts are also contained in this publication on subjects entirely untouched in the old work.

J. D.

London, 1st Jan. 1810.

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TO

THE FIRST VOLUME.

Ledger

THE

MODERN PRECEPTOR.

INTRODUCTION.

EDUCATION has been defined to be that series of means by which the human understanding is gradually enlightened, and the dispositions of the human heart are formed and called forth, between the earliest infancy and the period when we consider ourselves as qualified to take a part in active life, and when, ceasing to direct our views solely to the acquisition of new knowledge, or the formation of new habits, we are contented to act upon the principles we have already acquired. Under this definition of education, many particulars are comprehended, such as the circumstances of the child in regard to local situation, and the manner in which the necessaries and conveniencies of life are supplied to him; the degree of care and tenderness in which he is nursed in infancy; the examples set before him by parents, preceptors, and companions; the measure of restraint or licentiousness to which he is accustomed; the various bodily exercises, languages, arts, and sciences, which are taught him, and the order and method in which they are communicated; the moral and religious principles instilled into his mind; and even the state of health he enjoys during that period of life.

In different periods of society, in different climates, and under different forms of government, various institutions have naturally prevailed in the education of youth; and even in different families, the children are educated in different manners, according to the various situations, dispositions, and abilities of the parents.

In the infancy of society, little attention can be given to the education of youth. Before men have arisen above the savage state, they are almost entirely the creatures of appetite and instinct: The power of instinct is not even always so strong, as to induce them to preserve and bring up their offspring. But even when their own wants are not so urgent, nor their hearts so destitute of feeling, as to prompt them to abandon their new born infants to the ferocity of wild beasts, or the severity of the elements, yet still their uncomfortable and precarious situation, their ignorance of the laws of nature, their deficiency of moral and religious principles, and their want of skill or dexterity, in any of the arts of life; —all these circumstances together, must render men so situated, unable to regulate the education of their children with much sagacity or attention. The parents may relate the wild inconsistent tales in which are contained all their notions concerning superior beings, and all their knowledge of the circumstances and transactions of their ancestors; they may teach their offspring to bend the bow, to point the arrow, to hollow the trunk of a tree into a canoe, or to trace the almost imperceptible path of an enemy, or a wild beast, over dreary mountains, or through intricate forests; but they cannot impress on their tender minds just ideas concerning their social relations, or their obligations to a Supreme Being, the framer and upholder of nature. They are at no pains to teach the youth to repress their irregular appetites, to restrain the sallies of passion when they exceed just bounds, or are improperly directed; nor can they inform the opening understanding, with accurate

rate or extensive views of the operations and appearances of nature itself. Besides all this, the man in his savage state, knows not how far implicit obedience to the commands of the parent are to be required from the youth, nor how far the youth ought to be left to the guidance of his own reason or humour.

When men have attained to such knowledge and improvement, as to be entitled to a more honorable appellation than that of savages, one part of their improvement generally consists in becoming more judicious and attentive in directing the education of their youth. They have now acquired ideas of dependence and subordinaton; they have arts to teach, and knowledge to communicate; they have moral principles to instil, and have formed notions of their relation and obligations to higher powers; and these they are desirous their children should likewise entertain. Their affection to their offspring, is now also more tender and constant. It is observed even in that state of society in which we are placed, that the poor who can scarce earn for themselves and their children the necessaries of life, are generally less susceptible of parental affection, in all its anxious tenderness, than the rich, or those whom Providence hath placed in easy circumstances.

In this improved state of society, the education of youth is viewed as an object of higher importance. The child is dearer to the parent, and the parent is more capable of cultivating the understanding, and rectifying the dispositions of the child. The parent's knowledge of nature and dexterity in the arts of life, give him more authority over the child than the savage can possess : obedience is now inforced, and a system of education is adopted, by means of which the parent attempts to form the child for acting a part in social life. On some occasions the legislature itself interferes in the education of youth, which is considered as highly worthy of public concern, lest the foolish fondness

or the unnatural caprice of parents should, in the rising generation, blast the hopes of the State.

In reviewing history, we will find that this interference actually took place in several of the most celebrated governments of antiquity. The Persians, the Cretans, and the Lacedæmonians, were all too anxious to form their youth for discharging the duties of citizens, to intrust the education of their children solely to the care of the parents. Public establishments were formed among those nations, and a series of institutions enacted for carrying on, and regulating the education of their youth: not such as the schools and universities of modern times, in which literary acquirements being the only object of pursuit, the student is maintained at his parents' expense, and attends only if his parents think proper to send him; but of a very different nature, and on a much more enlarged plan.

Among the Athenians, and afterwards among the Romans, we no where discover a regular system established by the laws for the education of youth; but we are not thence to conclude that such establishments were either unnatural or improper. The Athenians and Romans gradually rose from rudeness to refinement, so as to become the glory and the wonder of the world, without resorting to such institutions: but though education was managed amongst these and other nations of antiquity, without detaching children from the care and inspection of their parents, still education was every where regarded as an object of the highest importance. As the manners of mankind were improved; as the invention of arts and the discovery of science gradually introduced opulence and luxury, connubial, parental, and filial affection acquired greater strength, and greater tenderness: of consequence children were reared with more care, and that care was directed to form them for acting a becoming part in life. According to the circumstances of each nation, the arts which they cultivated,

ted, and the form of government under which they lived; the knowledge they wished to communicate to their children, and the habits they endeavoured to impress upon them, were different from those of other nations. Again; according to the different circumstances, tempers, abilities, and dispositions of parents, even the children of each family, were brought up in a manner different from that in which those of other families were managed. The Athenians, the Romans, the Carthaginians, conducted each the education of their youth in a different way, because they had each different objects in view.

Many men eminent for virtue and talents, amongst the ancients as well as the moderns, have devoted their time and attention to the theory and the practice of education. Quintilian and Cicero, Milton, Locke, Rousseau, &c. &c. have left us admirable treatises on this most important subject, unfolding systems very opposite indeed one to another; but all of them furnishing such lights as cannot fail to be of infinite service to those entrusted with the precious charge of the instruction of youth.

When a child arrives at the age of five or six years, it will be proper not only to exact obedience from him, and to call his attention for a few minutes, now and then, to those things of which the knowledge is likely to be afterwards useful to him; but we may even venture to require of him, a regular and steady application, during a certain portion of his time, to such things as we wish him to learn. Before this period it would have been wrong to confine his attention to any particular task; the attempt would have produced no other effect, than to destroy the child's natural gaiety and cheerfulness, to blunt the quickness of his powers of apprehension, and to render hateful what you wished him to learn. Now, however, the case is different; the child is certainly not yet sensible of the advantages which he may, for instance, derive from learning to read: but

but such is the disposition of human nature, in every stage of life, to be more influenced by present objects, than by future prospects; that the mere sense of utility alone would not be sufficient to induce a child to apply to learning. Nothing, however, could be more absurd, than on this account to suffer a child to pass his time in idleness or foolish tricks, till views of utility and advantage should prompt him to employ himself in a different manner: on the contrary, he ought to be early habituated to application and the industrious exertion of his powers: besides at this period of life, we can command his obedience, awaken his curiosity, rouse his emulation, gain his affection, call forth his natural disposition to imitation, and influence his mind by the hope of reward and the fear of punishment. When we possess so many means of establishing our authority over the mind of a child, without usurpation or tyranny, it cannot surely be difficult, with prudent and moderate management, to cultivate his powers, by making him begin, even at this early period, to give regular application to something which may afterwards be useful.

The question then arises, what task will be best calculated for a child at this period — that to which children are usually first required to apply —*reading*. Be not afraid that his abillities will suffer from an attention to books at so tender an age; think not it is folly to teach him words before he has gained a knowledge of things: it is necessary, it is the voice of nature herself, that he should at the same time be acquiring a knowledge of things, and an acquaintance with the vocal and written signs by which they are distinguised: these are so intimately connected as tolead the one to the other. When we view an object, we attempt to give it a name, or seek to learn this name from those who know it; and in the same manner when the names of substances or qualities are brought before us, we wish to know what these names signify. At the same

time,

time, so imperfect is the knowledge of nature possessed by children without our aid, that they must constantly have recourse to us for information, to enable them to form any distinct notions of the objects around them. Indeed language cannot be taught, without explaining that it is merely a system of signs, and pointing out the object which each sign is intended to represent. If, therefore, language is not only necessary for facilitating the mutual intercourse of men, but is even indispensable for enabling us to obtain some knowledge of external nature; and if the knowledge of language has a natural tendency to advance our knowledge of things, to acquaint ourselves with language must be regarded as an object of the first importance, and one of the first objects to which the attention of children ought to be directed.

For the very same reasons which evince the necessity of teaching children the use and value of those vocal signs or words, which we employ to denote certain ideas, it becomes proper to teach them the use of those other signs, by which we express the same ideas in writing.

When a boy has made such progress as to be able to read with some correctness and facility, it becomes an object of importance, and of no small difficulty, to determine what books are to be put into his hands, and in what manner his literary education is to be conducted. These ought perhaps to be only such as are descriptive of the actions of men, of the scenes of external nature, and of the forms and characters of animals. With these he is already in some measure acquainted; they are the objects of his daily attention, and beyond them his ideas have never ranged, ; other subjects would, therefore, it is probable, render his task disagreeable to him. Besides, as the present object is to teach him words with their signification, until he shall have acquired a considerable knowledge of language, and laid up a fund of simple ideas, it will be im-

possible

possible for him to speak or read with understanding, profit and pleasure, on any other subject: and instead of being particularly anxious to communicate by instruction, moral or religious information, let this be done by our own example, and by causing the child to act in such a manner as we judge to be proper. In this way, by his own observation, he will soon become capable of understanding all we intend to communicate: but let us not be hasty, the boy cannot long view the actions of mankind, and observe the economy of the animal and vegetable world, without becoming capable of receiving both religious and moral instruction, when judiciously communicated.

As soon as the pupil can read and spell with tolerable facility, and has acquired sufficient strength and management of his hands to govern a pen, he may be introduced to the art of writing. If this art is not made disagreeable to him by the manner in which his application to it is required, he will learn it without difficulty. The natural desire of children to imitate, particularly whatever depends on manual operation, will render this art peculiarly easy and pleasing to them, when they are neither harshly forced to apply to it, nor suffered to perform their task with haste and negligence.

In this, as in all other branches of education, it requires the most cautious prudence, the nicest delicacy, the most artful address, to prevail with children to give a cheerful and attentive application to what they consider as an appointed task. If you be stern and rigid, in enforcing application, you may seemingly obtain your object; the child sits motionless, with his eye fixed on his book or copy; but his attention you cannot command; his mind is beyond your reach, and can elude your tyranny; it wanders from the present objects, and flies with pleasure to those scenes and objects in which it found delight. Thus, you are disappointed of your purpose, and besides inspire the pupil

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with such aversion both to you, and to those objects to which you wish him to apply, that perhaps at no future period of his life will he view learning otherwise than with disgust.

On the other hand, gentleness and the arts of insinuation will not always be successful. If you permit the child to apply just when he chooses to do so, if you readily listen to all his pretences and excuses, in short, if you yourself seem to consider learning as a matter not of the first importance, and treat him with kindness while he pays but little attention and makes but slow progress; the consequences of this behaviour in you, will be scarcely less unfavorable than those which attend imprudent and unreasonable severity. It is impossible to give particular directions how to treat children, so as to allure them to learning, and at the same time to command their attention: but the prudent and affectionate parent, and the zealous and judicious tutor, will not be always unsuccessful, since there are so many circumstances in the condition of children, and so many principles in their nature, which subject them to the will of those around them.

The principles of arithmetic ought to make a part in the boy's education, as soon as his reasoning powers appear to have attained such strength and quickness as will enable him to understand them. Arithmetic affords more exercise to his faculties than any other of those branches of learning which are taught in the early years; and if the child's attention be directed to it at a proper period, if he be allowed to proceed slowly, care being taken to make him comprehend fully, the principles upon which each particular operation proceeds, it will contribute much to increase the strength and acuteness of his understanding.

From arithmetic the pupil may proceed to the practical branches of the mathematics; and in these, as in all other parts of learning, what he is taught will be best remem-

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bered, and more thoroughly understood, if opportunities are afforded of applying his lessons to real use in ordinary life.

Where the acquistion of the learned languages is regarded as an object deserving attention, the boy is generally initiated in them about this period, and frequently earlier: and here another question occurs.—Is the time usually spent in learning the languages usefully employed?—What advantages can our British youth derive from an acquaintance with the languages and the learning of Greece and Rome?—Were we to listen to many parents and tutors of the present day, they would persuade us that the time which is dedicated to grammar-schools, and to Virgil, Cicero, Homer, and Demosthenes, is foolishly thrown away, and that no advantages can be gained from the study of classical learning. They wish their children and pupils to be not merely scholars; they wish them to acquire what may be useful and ornamental, when they come to mingle with the world; and for this purpose they think it much better to teach their young people to smatter out French, to dance, to fence, to appear in company with invincible assurance; and to dress in such a manner as may attract the attention of the world. Besides this, the humanity, the tenderness of these persons are amazing: they are shocked at the idea of the sufferings which the poor boys undergo, in the course of a classical education. The confinement, the harsh language, (to say no more), the burdens laid on the memory, and the pain occasioned to the eyes, during the dreary period spent in acquiring a knowledge of Greek and Latin, affect them with horror, when they think of them as inflicted on children. They, therefore, give a preference to a plan of education, in which less intense application is required, and less severity employed.

Again, there are other persons who are less warm in their commendation of a classical education, and no less industrious in recommending the study of the Greek and Latin, than

than the former are eager in their endeavours to draw neglect on the polished languages of antiquity. With this second class, if an adept in Greek and Latin, you are a learned and a great man; but without those languages, your ignorance is contemptible. They think it impossible to inspire the youthful mind with generous or virtuous sentiments, to teach the boy wisdom, or to animate him with courage, without the assistance of the ancient philosophers, historians, and poets.—Now, with which of these parties shall we agree? or shall we mediate between them? Is it improper to call youth to the study of the languages? Is it impossible to communicate useful knowledge without them? or are they, though highly useful, not always indispensably necessary? To give satisfactory answers to these queries it may be necessary to examine a little into the real utility of acquaintance with the dead languages, and what is called classical learning.

To begin then, it may be observed, that the cultivation of classical learning has a favorable influence on the living languages themselves; it has a tendency to preserve their purity from debasement, and their analogy from irregularities. In studying the dead languages it is necessary to give more attention to the principles of grammar than in acquiring our mother-tongue. We learn our native language without attending much to its analogy and structure. Of the numbers who speak English throughout the British dominions, but few, comparatively speaking, are skilled in the formation of its nouns and verbs, or able to distinguish between adverbs and conjunctions. Desirous only of being mutually understood, they are not anxious about purity or correctness of speech: they reject not an expression which occurs to them, because it is barbarous or ungrammatical: as they grow up they learned to speak from their parents, their nurses, and others about them; they were soon able to make known in words their wants, their wishes, and their

observations: satisfied with this, or called at a very early period to a life of humble industry, they have continued to express themselves in their mother-tongue, without acquiring any accurate knowledge of its general principles. Should these persons be called upon to state their ideas in writing, they are scarcely more studious of correctness and elegance in this way, than in speaking; or if they aspire at these properties, rarely do they attain them. Such speakers and writers, however, can never be expected to refine any language, or reduce it to a regular system; neither can they be qualified to distinguish themselves as the guardians of the purity and regularity of their native tongue, if it should before have attained a high degree of perfection. But those who, in learning a language different from their native tongue, have found it necessary to pay particular attention to the principles of grammar, afterwards apply the knowledge they have thus acquired, in using their own language; and thus becoming better acquainted with its structure, they learn to speak and write it with more correctness and propriety. The languages of Greece and Rome are besides so highly distinguished for their copiousness, their regular analogy, and various other excellencies, that the study of them must naturally tend to the improvement and enrichment of all modern languages.

If we look backwards to the fifteenth century, when learning began to revive in Europe, and that species of learning which began first to be cultivated was classical literature, we find that almost all the languages of Europe were wretchedly poor and barbarous. No knowledge could be communicated, no business transacted, without the aid of Latin: classical learning, of course, soon came to be cultivated by all ranks with enthusiastic eagerness. Not only those intended for learned professions, and men of fortune whose object was a liberal education, but even the lower ranks, and the

the female sex keenly applied to the study of the languages, and the wisdom of Greece and Rome.

This avidity for classical learning produced the happiest effects; but its influence was chiefly remarkable in producing a vast change on the form of the living languages: these soon became more copious and regular, and many of them have consequently attained such perfection, that the poet, the historian, the philosopher, may now clothe his thoughts in these modern tongues, to the greatest advantage. Were we therefore to derive no new advantage from the study of ancient languages, yet would they be deserving of our care and study, as having contributed so much to raise modern languages to their present improved state. The intercourse of nations, the affectation of writers, the gradual introduction of provincial barbarisms, and various other causes, have a tendency to corrupt and debase even the noblest tongues. By such means were the languages of ancient Greece and Rome gradually corrupted, till the language used by a Horace, a Livy, a Xenophon, a Menander, was lost in a jargon unfit for the purposes of composition. If we would not, therefore, disdain to take advantage of them, the classical writings in those languages might prevent our own from experiencing such a decline and subversion. He who knows and admires the excellencies of the dead languages, and the beauties of those writers who have rendered them so celebrated, will be the firm enemy of barbarism, affectation, and negligence, whenever they attempt to debase his mother-tongue. It may, therefore, be asserted, that, when the polished languages of antiquity cease to be studied among us, English will then lose its purity, regularity, and other excellencies, and gradually decline until it be no longer known for the language of Pope and Addison: and this consideration ought certainly to be of great weight in favor of the study of what are called the dead languages.

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But, further, in those plans of education of which the study of the dead tongues forms no part, proper means are seldom adopted for impressing the youthful mind with habits of labour and industrious perseverance; nor do the judgment, the memory, and the other faculties, receive equal improvement, as they are not led through the same exercises as in classical education. Hence, when the pupils who have been educated according to such plans, come to enter the world, and engage in active life, they are found deficient in many qualifications requisite to form a manly character. Though they have grown up to the size of manhood, their understandings are still childish and feeble, they are capricious, unsteady, incapable of industry or fortitude, and unable to pursue any particular object with keen unremitting perseverance. That long series of study and regular application which is required in order to become a proficient in dead languages, produces much happier effects on the youthful mind. He who is permitted to trifle away the earliest part of his life in idleness or in frivolous occupations, can scarcely be expected to display many manly or vigorous qualities, when he reaches a more mature age; in the same way he whose earlier days have been employed in exercising his memory, and furnishing it with valuable treasures, in cultivating his judgment and reasoning powers, by calling the one to make frequent distinctions between various objects, and the other to deduce many inferences from the comparison of such objects; and also in strengthening and improving the acuteness of his moral powers, by attending to human actions and characters, and distinguishing between them as vicious, or virtuous, mean or glorious:—he who has thus cultivated his faculties may surely be expected to distinguish himself when he comes into the world, by prudence, activity, firmness, perseverance, and most of the other noble qualities which can adorn a human character. But it is perhaps in the course of a classical education alone that

that the mind receives this cultivation and discipline; and from it in general these effects are to be expected. The repetitions which are required afford improving exercise to the memory, and store it with the most valuable treasures; the understanding is employed in observing the distinction between words, in tracing words to the substances and qualities in nature, which they are used to represent, in comparing the terms and idioms of different languages, and in observing the laws of their analogy and construction; while the moral faculties are at the same time improved by attending to the characters described, and the events and actions related in those books which we are directed to peruse, in order to gain a skill in the ancient languages.

If, after all, however, the study of classical learning is to be given up, where shall we find the same treasures of moral wisdom, of elegance, of useful historic knowledge, which the celebrated writers of Greece and Rome afford? Shall we content ourselves with the modern writers of Italy, France, and Britain? or shall we be satisfied to survey the beauties of Homer, and Virgil, of which so much has been said by men of learning and taste of all ages, through the medium of a translation? Surely no; but let us at once penetrate to those sources from which modern writers have derived most of the excellencies which recommend them to our notice.

Classical learning has besides been long cultivated among us, and both by the stores of knowledge it has conveyed to the mind, and the habits it has impressed, has contributed in no small degree to form many illustrious characters. In reviewing the annals of our country, we will scarcely find an eminent politician, patriot, general, or philosopher, during the two last centuries, who did not spend his earlier years in the study of the classics.

Yet, although these things are mentioned, and many other circumstances might be enumerated in favor of

classical

classical literature, it is by no means intended to argue that it is impossible to be a good, a wise, or a great man, without a knowledge of Greek and Latin: on the contrary, it is certain that methods are not wanting to inspire the minds of youth with virtuous dispositions, to call forth the powers of their understanding, to impress on them habits of industry and vigorous perseverence, without having recourse to the discipline of a classical education, either at school or in private:—all that is meant to be asserted, is, that a classical education is that which promises most speedily and effectually to produce these desirable fruits.

Those persons whom Providence hath placed in an elevated station, and in affluent circumstances, so that they seem rather to be born to the enjoyment of wealth and honors, than to act in any particular profession or employment, have notwithstanding a certain part assigned them to perform, and many important duties to fulfil. They are members of society, and enjoy the protection of the institutions of that society to which they belong: they must, therefore, contribute in some manner to the support of those institutions. The labours of the poor are necessary to furnish the luxuries of life to the rich; and the rich ought to know how to distribute their wealth, with prudence as well as generosity, to the poor. The rich enjoy much leisure; they ought, therefore, to know how to employ that leisure in a way at once innocent, agreeable, and useful. Besides, as circumstances enable the rich to attract the regard and respect of those in inferior stations, who are ever ready to imitate the conduct of their superiors, it is necessary that the rich endeavour to adorn their wealth and honors, by the most eminent virtues, in order that their example may have a happy influence on the manners of the community: with such views, therefore, their education ought to be conducted. After what has been said respecting a classical education, it will naturally be expected to be recommended

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as peculiarly suited to men of fortune. The youth who is born to the enjoyment of affluence and dignities, cannot employ his early years to more advantage, than in gaining an acquaintance with the elegant remains of ancient literature: the benefits arising from such studies, are peculiarly adapted to a person in such circumstances. Care must be taken to prevent his acquiring a haughty, fierce, imperious temper and demeanour. The submissive attention usually given to the youth of people of fortune, and the foolish fondness with which they are too often treated, have a natural and direct tendency to inspire them with high notions of their own importance, and to render them conceited, passionate, and overbearing. If this temper be acquired in childhood, what may not be expected from them in more advanced life, when their attention will evidently be drawn to the dignity and rank of their station in life, and to the comparative humility of many around them. On this comparison, they are in danger of becoming proud, insolent, resentful, revengeful, and to render themselves disagreeable, and even hateful to all who know them; whilst at the same time they are incapable of the delightful feelings which attend mild, humane, and benevolent dispositions. Let the man of fortune, therefore, as he values the future happiness and character of his son, be no less careful to prevent him from being treated in such a way as to inspire him with caprice, haughtiness, and insolence, than to secure his mind from being depressed and soured by harsh and tyrannical usage.

The manly exercises as favorable to the health, the strength, and even the morals, are highly worthy of engaging the attention of the young gentleman. Dancing, fencing, running, horsemanship, the management of the musket, and the motions of military discipline, all deserve to occupy his time, at proper seasons. It is unnecessary to point out the advantages which may be reaped from dancing:

these seem to be pretty generally understood. Perhaps our men of fortune would be ashamed to make use of their legs for running; but occasions may occur, when even this humble accomplishment may be useful. Though we wish not to see a young gentleman become a jockey, yet to be able to make a graceful appearance on horseback, and to manage his horse with dexterity, will not be unworthy of his station and character. If times of public danger should arise, and the State should call for the services of her subjects, against any hostile attempt, they whose rank and fortune place them in the most eminent stations, will be expected first to stand forward; but if unacquainted with the exercises connected with the military art, what figure must they make in the camp or in the field?

As the man of fortune may perhaps enjoy by hereditary right, or by the voice of his fellow-citizens, a seat in the legislative body of his country, he ought in his youth to be carefully instructed in the principles of her political constitution, and of those laws by which his own rights and those of his fellow-citizens are determined and secured.

Natural philosophy, as being at once useful and entertaining, well deserves the attention of all who can afford to set apart a portion of their time to scientific pursuits. To the man of fortune, a taste for natural philosophy might often procure the most delightful occupation. To trace the wonders of the planetary system, to mark the progress of vegetation, to examine all the properties of the fine element we breathe, to observe the laws by which all these different elements are confined to their proper functions, and above all, to apply the principles of natural philosophy in the cultivation of the ground, are amusements which might agreeably, innocently, and usefully occupy many of the leisure hours of the man who enjoys an independent and splendid fortune.

The history of civil society, and the study of the principles

ciples of morals, it is supposed, are not to be neglected. Without an acquaintance with these branches of knowledge, how can any just or accurate estimate of the laws and constitution of his own, or any other country be formed? History lays before us the actions and the fortunes of other men, and thus in some measure supplies the want of experience: it teaches prudence, and affords exercise to the moral sense. When history condescends to take notice of individuals, they are almost always such as have been eminent for virtue, for abilities, or for the rank they held in life: to the rich and great, therefore, history ought to speak with peculiar efficacy; and they ought to be carefully invited to listen to her voice.

Such is the manner in which the education of youth of rank and fortune ought to be educated, in order that they may be prepared to enjoy their opulence and honors, with credit and delight to themselves, and with advantage and gratification to their country. With such accomplishments, they might adorn their station, and find wealth and rank a blessing and not a curse.

With respect to those who are designed for a mecantile employment, or other occupations in life, not connected with literature, a classical education seems unnecessary; and that chiefly, because the English and French languages are enriched with many eminent productions, poetical, historical, and philosophical. But still the youth who is destined for such a course of life, need not be confined to a mere acquaintance with writing and accounts. These studies, however important to the young merchant, have no great power to restrain the force of evil passions, or to inspire the mind with virtuous and generous sentiments. Though he is not be burdened with Latin and Greek, yet we should strive to give him a taste for useful knowledge and elegant literature. Some of the purest and best of our poets, the excellent periodical works which adorn our language, such

as the Spectator, the Adventurer, the Mirror, and the compositions of our British historians, together with some of the best translations of the ancients, which we possess; all these may, with great propriety, be put into his hands. These will teach him to think justly, and to express himself in conversation or in writing, with correctness and elegance; they will refine and polish his mind, and raise him above low and vulgar pleasures: and as no man who has occasion to write, ought to be entirely ignorant of the nature of grammar, the youth intended for a mercantile life, should be carefully instructed in the grammar of his mother-tongue.

A sacred regard to his engagements, and an honesty which may prevent him from taking undue advantages, or exacting unreasonable profits, are the virtues which a merchant is most frequently called on to exercise: punctuality and integrity are the duties most peculiarly incumbent on those of the mercantile profession. Temptations will occasionally arise to seduce the merchant, and draw him to the violation of these duties: but, if superior to such temptations, he is most likely to be a successful merchant, and certainly will acquire an illustrious reputation. From his earliest years then, let a youth destined for commerce be inspired with a sacred regard for truth and justice, and be accustomed to view deceit and fraud, and the violation of a promise, with abhorrence and disdain. Frugality is a virtue which in the present age seems to be antiquated or proscribed: even the merchant often appears better skilled in the arts of profusion than of parsimony: the miser, a character at no time estimable, is at this day beheld with double contempt and detestation. Notwithstanding all these unfavorable circumstances, fear not to impress on the young merchant habits of frugality; let him know the folly of beginning to spend a fortune before he has realized it; let him be taught to consider a regular attention

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to confine his expenses within due bounds, as one of the chief virtues that can adorn his character.

Frugality and industry are so closely connected in nature, that in recommending the one, the other must be included. Without industrious application, no man can reasonably expect to meet with success in his occupation; and if the merchant leave his business to the management of clerks, and shopkeepers, it is not very probable, that he will soon accumulate a fortune. Hence, it will become as necessary for him who is designed for trade, to acquire early habits of sober application, and to be restrained from levity and volatility, as that he be instructed in arithmetic, writing, and keeping of accounts.

With these virtues and qualifications, the merchant is likely to be respectable, and not unsuccessful in trade; and if he come at last to possess an independent fortune, his acquaintance with elegant literature in his early years, and the various habits of order he has acquired, will enable him to enjoy it with comfort and dignity. Indeed, all the advantages which a man without taste, or knowledge, or virtue, can derive from the possession of even the most splendid fortune, are so inconsiderable, that they can be no adequate reward for the toil he has undergone, and the mean arts he may have practised, in acquiring it. At the head of a great fortune, a fool can only make himself more ridiculous, and a man of wicked and vicious character, more generally abhorred, than if fortune had kindly concealed their folly and their crimes, by placing them in a more obscure station.

A considerable part of the members of society, are placed in such circumstances, that it is impossible for them to receive the advantages of a liberal education. The mechanic and the husbandman, who earn a subsistence by their daily labour, can seldom afford, whatever parental fondness may suggest, to favor their children with many opportuni-

ties of literary instruction: and happily it is not requisite, that those who are destined to spend their days in this sphere of employment, should be furnished with much literary or scientific knowledge. They should, however, be taught to read their native tongue, to write, and to perform some of the most common, and the most generally useful operations of arithmetic: for without an acquaintance with reading, it will scarcely be possible for them to acquire any rational information on the doctrines and precepts of religion, or on the duties of morality; the invaluable volume of the sacred Scriptures will be sealed to them: without the art of writing, how are they to enjoy the sweet satisfaction of communicating accounts of their welfare to their absent friends; and both writing and arithmetic are necessary for the accomplishment of those little transactions in which they are interested. It would be hard indeed, if even to the poorest and lowest among us, these simple and easily acquired branches of education were denied, since happily that degree of proficiency in them, which is requisite for the mechanic and the labourer, may be obtained without greater expense than can be afforded by parents in the lowest situation. Ignorance may, indeed, be a very convenient quality, in those doomed to exist under superstitious and despotic thraldom; but that in these highly favoured Isles, where religion consists in the knowledge and performance of reasonable duties, and where government founds its claims to submission, on a system of public established law, the more the subject is enabled to examine into the nature and foundation of the duties expected from him, the more he will be qualified to appreciate their worth, and the more steadily and zealously will he contend for the preservation of that system, by which such blessings are secured to him.—But to return:—Let the youth who is born to pass his days in a humble station, be carefully taught to consider honest and patient industry, as one of the first

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virtues in life: let him be taught to regard the sluggard as one of the most contemptible of beings: teach him contentment with his lot, by informing him that wealth and honor seldom confer superior happiness. But at the same time, he must be told, (and for his encouragement) that if by honest arts, by abilities virtuously exerted, he can raise himself above the humble condition in which he was born, he may find comfort in affluence, and have reason to rejoice that he has led a virtuous, industrious, and active life. In teaching him the principles of religion, be careful to exhibit them only as intimately connected with morality; lest from an attention to abstruse doctrines, he contract a tendency to enthusiasm, too often found among the ignorant and illiterate; and at last, fall into the grievous persuasion, that he may be pious without being virtuous. Labour to inspire him with an invincible abhorrence for falsehood, deceit, fraud, and theft. Inspire him with the highest value for temperance, in all its branches, and with an awful regard for the duties of a son, a husband, and a father. Thus may man, in the lowest sphere, become respectable and happy; and infinitely superior, in the eyes of God and men, to the rich and great, who pervert their wealth and leisure, in shameful and vicious pursuits.

In the foregoing observations, the education of the *male* sex has been principally kept in view: but that of the *female* sex, whose duties in society are so important, are not less deserving of our serious consideration. In infancy, the instincts, the dispositions, the faculties of boys and girls seem to be nearly the same; they discover the same curiosity, and the same inclination to activity; for a while they are fond of the same sports and amusements; but by and by, when we begin to make a distinction in their dress; when the girl begins to be more confined to a sedentry life, under her mother's eye, while the boys are permitted to ramble

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without doors, the distinction between their characters begins to be formed, and their taste and manners to assume a different appearance. The boy now imitates the arts and active amusements of his father; he digs and plants a garden, builds a miniature house, shoots his bow, or draws his little cart; while the girl, with no less emulation, imitates her mother, knits, sews, and dresses her doll: they are now no longer merely children; they are become boys and girls. This taste for female arts, which the girl so easily and so naturally acquires, affords a happy opportunity for instructing her in a very considerable part of those arts which it is proper she should know: while she is dressing her doll she becomes expert in needle-work, and learns how to adjust her own dress in a becoming manner; and therefore, if she be kindly treated, it will not be a matter of difficulty to engage her to apply to those branches of education. Her mother, or governess, if capable of managing her with mildness and prudence, may teach her to read with facility: for being already more disposed to sedentary application, than the boy of the same age, the confinement to which she must submit, in order to learn to read, will be less irksome to her. Some have pretended that the reasoning powers of girls begin to exert themselves much sooner than those of boys: but as it is evident that these powers unfold themselves much earlier, in both sexes, than we are perhaps willing to allow, it is impossible to lay down any rule on this head. The different amusements and occupations in which we cause children to engage, from their earliest years, naturally call forth their faculties in different ways; and this may perhaps occasion the one to imitate our modes of speaking and acting, sooner than the other. But be this as it may, it is surely desirable that girls should, as soon as they are capable of serious attention, be instructed in reading, writing, and arithmetic, with the principles of religion and morality; in the same order in which these

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branches are communicated to boys. It is unnecessary to assign many reasons for wishing females to possess all these accomplishments: it is our important privilege, as placed in a situation different from that of other animals, to be susceptible of moral and religious notions; religious instruction, therefore, ought to be communicated to both sexes, with equal assiduity. Besides, as the care of children, during their earlier years, belongs in a peculiar manner to their mother, she whom nature has destined to fulfil such important duties, ought to be carefully prepared for the proper discharge of them, by being herself accurately instructed in her youth, in such things as it will be afterwards requisite for her to teach her offspring.

"Though the duties of religion, strictly speaking, are equally binding on both sexes," says a father to his daughters*, "yet certain differences in their natural character and education, render some vices in your sex particularly odious. The natural hardness of our hearts, and the strength of our passions, are apt to render our manners more dissolute, and make us less susceptible of the finer feelings: your superior delicacy, your modesty, and the usual severity of your education, preserve you, in a great measure, from any temptation to those vices to which we are most subjected. The natural softness and sensibility of your dispositions, particularly fit you for the practice of those duties, where the heart is chiefly concerned: and this, along with the natural warmth of your imagination, renders you peculiarly susceptible of the feelings of devotion. There are many circumstances in your situation, that peculiarly require the supports of religion, to enable you to act in them with spirit and propriety: yonr whole life is often a life of suffering: you cannot plunge into business, or dissipate yourselves in pleasure and riot, as men too often do, when under the pressure of misfortunes: you must bear

* Professor Gregory's *Legacy to his Daughters.*

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your sorrows in silence, unknown and unpitied: you must often put on a face of serenity and cheerfulness, when your hearts are torn with anguish, or sinking in despair: then your only resourse is in the consolations of religion. You are also sometimes in very different circumstances, that equally require the restraints of religion. The natural vivacity, and perhaps the natural vanity of your sex, are very apt to lead you into a dissipated state of life, that deceives you under the appearance of innocent pleasure; but which in reality wastes your spirits, impairs your health, weakens all the superior faculties of your minds, and often sullies your reputation. Religion, by checking this dissipation and rage for pleasure, enables you to draw more happiness, even from those very sources of amusement, which, when too frequently applied to, are often productive of satiety and disgust. Religion is rather a matter of sentiment than of reasoning: the important and interesting articles of faith are sufficiently plain: fix your attention on these, and do not meddle with controversy: if you get into that, you plunge into a chaos, from which you will never be able to extricate yourselves: it spoils the temper, and, I suspect, has no good effect on the heart. I wish you to go no farther than the Scriptures, for your religious opinions: embrace those you find clearly revealed: never perplex yourselves about such as you do not understand, but treat them with silent and becoming reverence. I would advise you to read only such religious books as are addressed to the heart, such as inspire pious and devout affections, such as are proper to direct you in your conduct, and not such as tend to entangle you in the endless maze of opinions and systems."

Ladies have often distinguished themselves as prodigies of learning: many of the most eminent writers on the continent have been of the female sex, and among our own countrywomen, are not wanting instances of ladies who have made a respectable figure in the republic of let-

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ters. It does not hence follow that girls should all have a literary education: to acquire the accomplishments more peculiarly belonging to their situation in life, will abundantly occupy their earlier years. If they be instructed in the grammar of their own tongue, and taught to read and speak it with propriety; if they learn to write a fair hand, and to perform with readiness the most useful operations of arithmetic; if they be instructed in the nature of the duties they owe to God, to themselves, and to society; this will be the main literary instruction girls will require. By this, however, it is by no means intended, that girls should be forbid the literature of their country: the periodical writers, who in so elegant and pleasing a manner have taught all the duties of morality, the decencies of life, and the principles of taste, may with the greatest propriety be put into the hands of the female pupil. Neither must she be denied the best historians, the most popular voyages and travels, and such of our British poets as may be perused without corrupting her heart or inflaming her passions. But, were it possible that any advice could have so much influence, we would strive to persuade our countrywomen and countrymen too, to banish from among them the modern tribe of *Novelists*, the propagators of false taste, false feeling, and false morality, with no less determined severity than that with which Plato excluded the poets from his ideal republic, or that with which the converts to Christianity, mentioned in the Acts of the Apostles, condemned their magical volumes to the flames. Unhappily, novels and plays are almost the only species of reading, in which the young people of the present age take delight; and nothing has contributed more effectually to bring on that dissipation and dissoluteness of manners, which so much prevail among all classes of the community. If in the room of these pernicious productions of the press, to which their admirers, however, are seldom

willing to acknowledge their attachment, we were to substitute plain and popular treatises on Geography, Astronomy, Natural History in its various branches, together with the principles of the French and Italian languages, now become almost indispensable, the young female, however elevated in station above labour and care, would find her time abundantly occupied, and to the best of purposes.

It is far, however, from the purpose of these observations, to confine female education to such things as are merely plain and useful: those accomplishments which are only ornamental, and the design of which is to render females amiable in the eyes of the other sex, are not to be forbidden. When we consider the duties for which they are destined by nature, of which the art of pleasing constitutes no inconsiderable part, it would be wrong to deny them those inferior arts, the end of which is to enable them to please. Let them endeavour to acquire a taste in dress, for to dress in a neat and graceful manner, to suit colours to her complexion, and the figure of her clothes to her shape, is no small accomplishment for a young woman. She who is rigged out by the dexterity of her maid and her milliner, is no better than a doll sent abroad to public places, as a sample of their handiwork. *Dancing* is a favorite exercise; nay, it may in some countries, be almost called the favorite study of the fair sex. So many pleasing images are connected with the idea of dancing; dress, attendance, balls, elegance and grace of motion irresistible, admiration: and these are so early inculcated by all around her, that we need not wonder if the young female consider her lessons of dancing as a matter of much greater importance than her book or her sampler. Indeed, though the public in general seem at present to set too high a value on dancing, and though the undue estimation paid to it, seems to be owing to that taste for dissipation, and that rage for public amusements, which naturally prevail amidst such refinement and opulence,

opulence, still dancing is an accomplishment which may be cultivated with advantage by both sexes. It has a happy effect on the figure, the air, and the carriage, and perhaps it may be even favorable to dignity of mind; yet, as to be a first-rate poet or painter, and in particular to value himself on his excellence in those arts, would be no true ornament in the character of a great Monarch, so any very superior skill in dancing, must serve rather to disparage, than to adorn, the lady or the gentleman. There are some arts, in which, though a moderate degree of skill may be useful or ornamental, yet superior taste and knowledge are rather hurtful; as they have a tendency to seduce us from the performance of the important duties we owe to society and to ourselves: of those arts, dancing seems to be one.

Music also is an art in which female youth are generally instructed; and if their voice and ear be such as to enable them to attain any excellence in vocal music, it may conduce greatly to increase their influence over our sex, and may afford a pleasing and elegant amusement to their own leisure hours. The harpsichord, the harp, are instruments often touched by female hands; nor would it be proper to forbid ladies to exercise their delicate fingers in calling forth the enchanting sounds of these instruments: but still if your daughter have no voice nor ear for music, compel her not to make it a study.

Drawing is another accomplishment which generally enters into the plan of female education. Girls are usually taught to aim at a few strokes with a pencil; but when they grow up, they either lay it wholly aside, or else apply to it, with so much assiduity, as to neglect their more important duties. A skill in drawing, like a skill in poetry, cannot be considered as an accomplishment very necessary for females; yet, as far as it contributes to improve their taste in dress, it may not be improper for them to pursue it. They may very properly be taught to sketch a landscape,

to colour a flower, and the like: but let them not throw aside this employment as soon as the drawing-master is withdrawn; on the contrary, let them preserve a taste for it through life, to be an useful guide in the art of personal ornament. Pride certainly never cân be lovely, nor vanity estimable, even in the fairest form; yet ought a young woman to be carefully impressed with a due respect for herself: this will join with her good sense and native modesty, to be the guardian of her virtue, and to preserve her from levity and impropriety of conduct.—Such are the hints it may be proper to follow in the education of females, as far as it ought to be conducted in a manner different from that of males.

It would be improper to close these general observations on education, without taking notice of a question much agitated by men of the best intentions and first-rate abilities, concerning the respective merits of public and private systems of education: whether a young man should be brought up in a private manner at home, or sent to receive his instruction at a public school? As on this topic, nothing new can be expected to be advanced at this day, it may be sufficient just to give a hasty sketch of the principal arguments, adduced by each party, in defence of their opinion.

Those who have considered children as receiving their education in the house, and under the eye of their parents, and as secluded from the society of other children, have been sometimes led to regard this situation as particularly favorable for their acquiring useful knowledge, and being formed to virtuous habits. Though we reap many advantages from mingling in social life, yet there we are also tainted with many vices, to which he who passes his time in retirement is a stranger. At whatever period we begin to mix with the world, we still find we have not acquired sufficient strength, to resist those temptations with which we

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are assailed: but if we are thus ready to be infected with folly and vice, even at any age, surely little argument will be necessary, to show the propriety of confining children from those scenes in which this infection may be so readily caught. Whoever examines the state of public schools, with careful and candid attention, even those under the most virtuous, judicious, and assiduous teachers, will soon find reason to acknowledge, that the empire of vice is there established not less fully than in the world abroad. Nothing, therefore, can be more negligent, more inhuman, than for parents to expose their children to those seductions which a great public school presents, at a time too, when they are particularly disposed to imitate any example set before them, and have not yet learned to distinguish between such examples as are worthy of imitation, and those which ought to be beheld with abhorrence. Even while under the parent's eye, from intercourse with servants and visitors, the native innocence of children may be considerably impaired: still the parent's care will be much more likely to preserve the manners of his child uncorrupted in his own house, than all the assiduity and watchfulness of teachers in a public school.

The morals and dispositions of a child, ought to be the first objects of concern, in conducting his education; his initiation into the principles of knowledge, is but a secondary object; and it is natural to suppose, that in a private system of education, both those great purposes may be more readily and securely attained, than in a public. Thus in fact it happens: when one or two boys are intrusted to the care of a judicious tutor, he can watch for, and seize the most favorable seasons for communicating instruction; he can awake curiosity, and command attention, by the gentle arts of insinuation: though he strive not to inflame their breasts with emulation, which often leads to envy and hatred; yet he will succeed in rendering know-

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ledge pleasing, by other means, less likely to produce unfavorable effects on the temper and dispositions of his pupils. As his attention is not divided among a great number, he can pay more regard to the particular dispositions and turn of mind of each of his pupils: he can encourage the slow, and repress the quick and volatile; he can call forth their powers, by leading them, at one time, to view the face of nature, and the changes she undergoes through the succession of seasons; at another, he calls them to attend to entertaining experiments in natural philosophy, or artfully allures them to perform their literary exercises; with these he may mix some active games, and he may assume so much of the fondness of a parent, as to join in them with his little pupils. These are certainly circumstances favorable to both the happiness and the literary improvement of youth: but they are peculiar to a private education. Besides, as in a private education, children spend much more time in the company of grown people, than in a public; those who receive a domestic education, sooner acquire our manner of thinking, of expressing ourselves, and of behaving, in our ordinary intercourse with one another. For the very same reason for which girls are often observed to be capable of prudence and propriety of behaviour, at an earlier age than boys, those boys who are educated at home, will sooner begin to think and act like men, than those who pass their first years in a public seminary. Although the boy be brought up at home, it is not necessary that he should be more accustomed to domineer over his inferiors, or to indulge a capricious and inhumane disposition, than if he had been educated among fifty or a hundred boys, all his fellows in age, size, and rank. He may also, in a private education, exercise his limbs with the same activity as in public: he cannot, indeed, engage in those sports, for which a number of companions is necessary: but still there are a thousand objects which call forth his activity: if in

the country, he will be disposed to fish, to climb for birds'-nests, to imitate all he sees done by labourers and mechanics; in short, he will run, leap, throw stones, and keenly exert himself in a variety of exercises, which will produce the best advantages on the powers of both his body and his mind. It is true that, by opposing the designs of nature in the boy, you may effectually repress his natural activity, and cause him to pine away his time in listless indolence; but in this you will do violence to his dispositions, as well as to those instincts which nature has planted in his breast: the bad consequences, however, resulting from this management, are not to be considered as the proper effects of a domestic education, but of an education carelessly or imprudently conducted.

There is yet another consideration, which will perhaps be more likely than any of those hitherto urged, to prevail in favour of a private education. As the infant who is abandoned by its mother to the care of a hireling nurse, naturally transfers its affection from the parent to the person who supplies her place and performs her duties; so the boy who is banished from a parent's house, at a time when he has scarcely begun to understand the relation in which he stands to his father and mother, his brothers and sisters, soon ceases to regard them with that fondness he had contracted for them, from living intheir company and receiving kind offices. His respect, his affection, and his kindness, are bestowed on new objects, on his master and his companions; or else his heart, for want of suitable objects, becomes selfish and destitute of every tender and generous feeling: and when the gentle and amiable affections of filial and fraternal love are thus, as it were, torn up by the roots, every evil passion springs up, with rapid growth, to supply their place in his bosom. When the boy returns to his father's house, it is no home to him; there he is a stranger, and is no longer capable of regarding his parents and rela-

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tions with the same tenderness of affection. He is unacquainted with that filial love which rises in the breast of a child, constantly sensible of the tender care of his parents, and which appears rather the effect of instinct than of habit. Selfish views are now the only bond connecting the youth with his parents and relations; and by thus falling under the influence of such views in early life, he runs the hazard of being ever incapable of all the most amiable virtues of his nature.—Such are the arguments commonly adduced in behalf of a *private* and *domestic* education:—But let us listen to what has been advanced by the partizans of a *public* system of tuition. It has been asserted, that a public education is the most favourable to the pupil's improvement in knowledge, and the most likely to inspire him with an ardour for learning. In private, with whatever assiduity and tenderness you labour to render learning agreeable to your pupil, still it will be but an irksome task: you may confine him to his books but for a short space in the course of the day, and allow him regular changes of study and recreation; but you will never be able to render his books the favourite object of his attention; he will apply to them with reluctance, and careless indifference; even while he seems engaged on his lesson his mind will be otherwise occupied: if the period during which you require his application be short, during the first part of it he will be thinking of the amusements he has left, and regretting his confinement, during the last, he will fondly anticipate the moment when he is to return to his sports. Again, if his attendance is required for a longer period, the effects are still more unfavourable: peevishness, dulness, and a determined aversion to all that bears the name of learning, will naturally be impressed on his tender mind. Nor can it well be otherwise; for books possess so few of the qualities which recommend any object to the attention of children, that they cannot be naturally agreeable to them: they have nothing

nothing to gratify the eye, the ear, or any of the senses; they present things with which children are unacquainted, and of which they know not the value. Children cannot look beyond the letters and words to the things these represent; and even if they could do this, it is much more pleasing to view objects and scenes as they exist originally in nature, than to trace their images in a faint and imperfect representation. It is in vain, therefore, to hope that children will be prevailed on to give attention to books, by means of any allurements which books alone can present; other means must be used; but such means, in a private system of education, cannot be commanded. In a public seminary, on the other hand, the situations of pupils and masters are widely different. When a number of boys meet together in the same school, they soon begin to feel the impulse of a principle which enables the master to command their attention without difficulty, and prompts them to apply, with cheerful ardour, to tasks which would otherwise be extremely irksome. This principle is a generous emulation, animating the breast with the desire of superior excellence, without inspiring envy or hatred of a competitor. When children are properly managed in a great school, it is impossible for them not to feel this impulse; it renders their tasks scarcely less agreeable than their amusements, and directs their natural activity and curiosity to proper objects. Behold a scholar at a public school, composing his theme, or turning over his dictionary: how alert! how cheerful! how indefatigable! He applies with all the eagerness, and all the perseverence of a candidate for one of the most honorable places in the Temple of Fame. On the other hand, view and pity the poor youth who is confined to his chamber, with no companion but his tutor, none whose superiority can provoke his emulation, or whose inferiority might flatter him with thoughts of his own excellence, and so move him to

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preserve by industrious application, the advantages he has already required. His book is indeed before him; but how languid, how listless his posture! how heavy, how dull his eye! nothing is expressed in his countenance but dejection or indignation. Let him be examined respecting his lesson; he replies with confusion and hesitation, and soon will show that he has spent his time without making any progress in learning; that his spirits are broken, his native cheerfulness destroyed, and his breast armed with invincible prejudice against all application in the pursuit of literary knowledge. To all this must be added, that in a public school, there is something more than emulation to render learning less disagreeable than it naturally is to children. The slightest observation of life, or attention to our own conduct in various circumstances, will convince us that wherever mankind are placed in situations of distress, or subjected to disagreeable restraints, that which a single person bears with impatience or dejection, will make a much feebler impression on his mind, if a number of companions be joined with him in his restraint or suffering. It is esteemed a much greater piece of severity to confine a prisoner in a solitary cell, than to permit him to mix with others in the same situation: a journey appears far less tedious to a party of travellers, than to him who beats the path alone. In the same manner, when a number of boys in a great school are all busied on the same, or on similar tasks, a spirit of industry and perseverence is communicated from one to another over the whole circle; each of them insensibly acquires new ardour and vigour, even though he felt not the spur of emulation, yet while all are busy around him, he cannot remain idle. These are facts obvious to the most careless observer.

Neither are public schools so unfavourable to the virtue of their members, as they have been represented to be. If the masters be men of virtue and prudence, careful to set

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a good example before their pupils, attentive to the particular character of each individual among them, firm to punish obstinate and incorrigible depravity, and even to expel from the society those who may more probably injure the morals of others, than be themselves reclaimed; and at the same time be eager to applaud and encourage amiable and virtuous dispositions wherever they appear: under the government of such masters, a public school will not fail to be a school of virtue. There will doubtless be particular individuals among the pupils of such a seminary, whose morals may be corrupt, and their dispositions vicious: but this, in all probability, arises from the manner in which they are managed before entering the school, or from some other circumstances, rather than from their being sent for education to a public seminary of education. Again, at a public school, young people enjoy much greater advantages for preparing them to enter the world, than they can posssibly be favoured with, if brought up in a private and solitary manner. A great school is a miniature representation of the world at large: the objects which engage the attention of boys at school, are different from those which occupy their parents: the views of the boys are less extensive, and they are not yet capable of prosecuting them by so many arts, not always perhaps the most laudable or honorable: but in other respects, the scenes and the sets of actors nearly resemble each other: in both you perceive contending passions, opposite interests, weakness, cunning, folly, vice. He, therefore, who has performed his part on the miniature scene, has, as it were, rehearsed it for the greater: if he has acquitted himself well on the one, he may also be expected to distinguish himself on the other: and even he who has not been remarked at school, will at any rate enter the world with superior advantages, when compared with the youth who has spent his earlier days in the ignorance and

solitude

solitude of a private education. Besides, when boys meet at a public seminary of education, separated from their parents and relations, all nearly of the same age, engaged in the same studies, and fond of the same amusements, they naturally contract friendships one with another, which are more cordial and sincere than any that take place between persons of a more advanced age. A friendship is often formed between two boys at school which continues through life, and is productive of the best consequences to both. While at school, they assist and encourage each other in their learning; and their mutual affection renders their tasks less burdensome than otherwise they would be. As they advance in life, their friendship still continues to produce the happiest effects on their sentiments and conduct: perhaps they are mutually useful to each other by interest, or by personal assistance, in making their way in the world; or when they are engaged in the cares and bustle of life, their intercourse and correspondence may contribute much to console them, amid the vexations and fatigues to which they may be exposed.

Such are the principal arguments usually adduced in favor of a *public* education.—When we compare these with those which have been urged to recommend a *private* education, we shall, as in all cases where advantages are stated in the extreme, find each system to be eligible according to circumstances, and that some plan combining the good qualities of both, will be the most desirable. A *public* education is the most favorable to the acquisition of knowledge, to vigour of mind, and to the formation of habits of industry and fortitude. A *private* education when judiciously conducted, will not fail to be peculiarly favourable to innocence and mildness of disposition; and notwithstanding what has sometimes been advanced by the advocates for a public plan of tuition, it is surely wiser to keep youth at a distance from the seductions of vice, until they be sufficiently armed

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against them, than to expose them to those seductions at an age when they know not whither these lead, and are wholely unable to make any effectual resistance to their force. Were we to give implicit credit to the specious language of the two parties in this contest, we should expect from either a public or a private education, beings more like to angels, than to the men we ordinarily find in the world: but these partizans speak with the ardour of enthusiasts; they must therefore be cautiously listened to on so important a subject as the education of youth, both as to the facts they state and to the inferences they draw. Were it possible, without exposing children to the contagion of a great town, to procure for them the advantages of both a public and a private education at the same time, we would by this measure probably be most successful in rendering them both respectable scholars, and virtuous and useful men. Upon the whole, therefore, it would be most desirable, (unless when unavoidable necessity of circumstances compels to the contrary), that parents should not banish their offspring from under their roof until they be advanced beyond their boyish years: let the mother nurse her own child, and let her and the father join in superintending the first principles of its education; then may they expect to be rewarded, if they have rightly acted their parts, by securing the gratitude, the affection, and the respect of their child, while he and they continue to live together. Let matters be so conducted that the boy may reside in his father's house, and at the same time attend a public school; but let the girl be wholly educated under her mother's eye.

Education, in whatever way ordered, must still be imperfect without preparing youth for their appearance on the stage of life. Much has been said concerning the utilty of a knowledge of the world, and the advantage of acquiring this knowledge at an early period: but those who have with the greatest earnestness recommended it, have generally explained

plained themselves in so inaccurate a manner that it is difficult to understand what ideas they affix to the expression. Some seem to wish (but surely it cannot be their serious wish) that, in order to acquire a knowledge of the world, young people should be very early introduced into what is affectedly called *fashionable* company, carried to all public places, and allowed to follow their headlong career through every haunt of folly and vice. Some knowledge of the world may unquestionably be gained by such means; but it is dearly purchased: nor are the advantages expected to be derived from it so considerable as to tempt the judicious and affectionate parent to expose his child to the infection of vanity, folly, and vice, for their sake. Carry a boy or a girl into public life at the age of fourteen or fifteen; show them all the splendid scenes of London or Paris; tell them of the importance of dress, and of the ceremonies of good breeding, and the forms of intercourse; teach them that fashionable indifference and assurance which give the *ton* to the manners of the age. What effects can you expect the scenes into which yon introduce them, and the mysteries you disclose to them, to produce on their tender minds?—they must have a direct tendency to inspire the children with a taste, perhaps never to be changed, for vanity, frivolity, and dissipation. If you wish them to be like the foolish, the gay, the dissipated, you can hardly fail to obtain your end: but if on the contrary your views are to prepare them for discharging the duties of life, you could not take a more improper method to instruct them. They will perhaps become well acquainted with all those things on which you set such value; but they will not thereby have gained any accession of useful knowledge. The children are not now a whit more able than before, to judge of the real value of the objects around them: nay they are now more liable than before to form erroneous judgments and erroneous estimates of the worth of these objects, from the ideal value stamped

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on them by luxury, vanity, and fashion. By mingling at an early age in the scenes where luxury, vanity, and fashion, reign with arbitrary sway, young people must naturally be impressed with all the prejudices which these have a tendency to inspire: instead of acquiring an useful knowledge of the world, they become incapable of viewing the world with an unprejudiced and discerning eye. If possible, therefore, we should labour to restrain young people from mingling in the scenes of gay and dissipated life, till after they have attained a certain maturity of age and judgment: they will then be more able to view them in a proper light; and perhaps be happy enough to escape their contagion.

But fortunately there is another and a more valuable knowledge of the world; and this we ought most industriously to communicate to them, as soon as they are capable of receiving it. When they are made thoroughly acquainted with the distinctions between right and wrong, between virtue and vice, between piety and impiety, and have become capable of entering into our reasonings; we ought then to inform them concerning the various establishments and institutions existing in society; concerning the opinions, customs, and manners of mankind; concerning the various degrees of strength or weakness of mind, of ingenuity or dulness, of virtuous or vicious qualities, which discriminate those characters which appear in society. We ought also to seize every opportunity, of exemplifying our lessons, by instances in real life. We must point out those circumstances which have led mankind to place an undue value on some objects, while others are appreciated much below their real utility and importance. Thus, let us fortify their judgment against that impression which the dazzling novelty of the scene, and force of passion, will be apt to produce; and thus communicatee to them a knowledge of the world, without exposing them imprudently to the infection of its vices or its follies.

At length the period arrives when the youth must be emancipated, set free from subjection, and committed to the guidance of their own conscience and reason, and of those principles which we have laboured to inculcate on their minds: let us, then, warn them of the dangers to which they are about to be exposed; tell them of the glory and happiness to which they may attain; inspire them, if possible, with a hearty disdain for folly, vanity, and vice, whatever dazzling or enchanting forms they may assume; and then set them forward, to enrich their minds with new stores of knowledge, by visiting foreign nations, or to enter immediately on the duties of some useful employment in active life.

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MODERN PRECEPTOR.

CHAPTER I.

ON LANGUAGE.

No inquiry can be more useful or agreeable than that into the nature, the origin, and the principles of language. By language we are enabled to communicate one to another our ideas and feelings, either in conversation or by writing. Conversation furnishes us with information, in the order and rapidity with which conceptions are formed in the mind of the speaker; and writing lays open to us the treasures of science, learning, and experience; the opinions, discoveries, and transactions of the most distant ages, and the most remote situations. It is, in fact, by language that man is chiefly distinguished from the other animals: not that these have not modes of expressing their sensations, by which they are mutually understood; but this species of language seems to be limited entirely to the expression of Passion; whereas in man, language, as the organ of reason, to which it gives its proper activity, use, and ornament, becomes a vehicle for a boundless variety of expression adapted to the various powers and faculties of the human frame. Nay, so much is this the case, that in proportion as language is enlarged, refined,

and polished, the nation where it prevails is justly regarded as exalted above others, in the scale of civilization and improvement of understanding.

So close is the connection between words and ideas, that no learning whatever can be obtained without their interposition and assistance. In proportion as words are studied and examined, ideas become more clear and complete; and according to the fulness and accuracy with which our ideas are conveyed to others, the perplexities of doubt, the errors of misconception, and the cavils of dispute are avoided. For it is always to be remembered, that words are connected with, and received as the representatives of ideas, merely by custom, and not from any natural affinity between them: and that ideas, like rays of light, are liable to be tinged, by the shades of the medium through which they are conveyed to our senses. Had this circumstance been duly attended to, the many ponderous volumes of controversy, which fill the libraries of the learned, would never have existed; the disputants, on giving clear and comprehensible explanations of the terms severally employed, finding their opinions, however contradictory in appearance, to be much more concordant than had been aprehended. Hence arises the importance, and indeed, the necessity of clear and distinct conception in the mind of the speaker, and of correct appropriation of terms in his language, to convey to the hearer adequate impressions of the ideas and notions intended to be communicated. Definitions and explanations are, however, not always sufficient to give precisely the meaning of words; derivation must frequeutly be called in to give its aid: for from derivation we discover the source whence a word springs, and the various streams of signification flowing from it. Much advantage will to the same end, be drawn from history, where the student will find allusions, idioms, figures of speech, illustrated by particular facts, opinions, and institutions. Thus, for example, without

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some acquaintance with the Roman laws, many passages in Cicero's Orations will be unintelligible; the customs of the Greeks throw light on the language of their writers; and many descriptions, allusions, and injunctions in the Sacred Scriptures, are not to be comprehended without a knowledge of the opinions and manners of the East. Furnished, therefore, with such aids, the scholar acquires not partial, but complete information, is enabled to throw upon language all the light collected from his mass of study, and imbibes, as far as can now be done, the genuine sense and spirit of ancient writers.

The student who confines his attention entirely to his native tongue, will never be able to arrive at a perfect knowledge of it, or to ascertain with precision, its riches or its poverty, its beauties or its defects: but he, who, together with his own language, cultivates those of other countries and other times, acquires new means to increase his stock of ideas, and discovers new paths laid open, to conduct him to knowledge. Such a person draws his learning from the purests sources, converses with the natives of other countries, without the need of an interpreter, and peruses original compositions, without being reduced to have recourse to the feeble and often deceptious light of translations. He may unite the speculations of the philosopher, with the acquirements of the linguist, comparing different tongues, and forming just conclusions with respect to their beauties and defects, and their correspendence with the temper, genius, and manners of a people. He may trace the progress of rational refinement, and discover, by a comparison of arts and improvements with their respective terms, that the history of language, considered as unfolding the effects of human genius, and the rise and advancement of its inventions, constitutes a very important part of the history of man.

ORIGIN

Origin of Language.

Various theories have been formed to account for the origin of language, which, however ingenious, have all failed in giving satisfaction: but the only rational method of accounting for it, is to refer speech to the operation of the Great Creator. Not that it is necessary to suppose he inspired the first parents of the human race, with any particular set of terms or primitive language, but that he made them sensible of the power they possessed of forming articulate sounds, and gave them an impulse to exert this faculty, leaving them, however, to their own choice in the application of each articulate sound to its corresponding object. Their ingenuity was left to itself, to multiply names, as new objects arose to their observation, and their language gradually advanced, in process of time, to the different degrees of accuracy, copiousness, and refinement, which it has reached among the various nations of the globe. This theory is conformable to the description given in the Sacred Writings, and agrees very remarkably with the opinions to be collected from profane history. Thus, Plato mentions, that the original language of man was of divine formation; that primitive words proceeded from the immediate communication or suggestion of the Divinity; but that derivatives owed their existence to the wants and the ingenuity of man himself. To whatever part of the world we direct our attention, we shall find additional reasons to conclude, that all the languages now spoken, as well as those which have ceased to exist, but of which memorials still remain, were, notwithstanding their apparent difference and variety, originally derived from one and the same source. When we remark certain words in the Latin tongue resembling others in the Greek, we are not surprised, considering the intimate connection which for many years subsisted between

tween the two nations, and the evident derivation of the former from the latter. It is also natural to suppose, that the modern languages were derived from the ancient, which were spoken in the same conutries: thus, the present languages of Europe may be readily traced back to the Latin, the Teutonic, the Sclavonian, the Celtic, &c. But when we observe words in one quarter of the globe, to be like those used in another quarter extremely remote, and that these corresponding terms have precisely the same signification, and were used in such senses, long before the inhabitants of the one country had any knowledge of, or intercourse with those of the other. When we observe all this, how can we but refer their origin to one ancient common language, flowing in various channels, from one and the same spring?

Language kept pace with the progress of invention, and the cultivation of the mind urged mankind to the increase and improvement of the sounds, by which its suggestions were mutually communicated to the ear. From denoting the perceptions of sense, they proceeded to represent by words the instruments and operations of art, the flights of fancy, the deductions of reason, and the results of observation and experience: hence may be traced the progress of poetry, history, and philosophy. Thus, speech, from being the child of necessity, became the parent of ornament, and words, originally the rude dress of ideas, have been improved, as society has advanced to higher degrees of refinement, into their most splendid and most beautiful decorations.

On Letters or Characters to express Sounds.

To fix the fleeting sounds, as soon as they are breathed from the lips, and to represent them faithfully to the eye, by certain determinate characters; these are the wonderful

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property of letters. Those to whom books have, from their childhood, been familiar, and who view literature only in its present improved state, are utterly unable to form a just estimate of the difficulties which must have attended the first application of symbols or signs to the expression of ideas. The pictures of the Mexicans, and the hieroglyphics of the Egyptians, were, without doubt, very ingenious contrivances, and mark the various efforts made by human ingenuity, towards expressing, by objects of sight, what passes in the mind: but it comes not within the limits of painting, to represent the succession of thoughts; neither are its operations sufficiently rapid or direct: so that such a mode of representation is very ill adapted to the activity and variety of our ideas. The great excellence of letters as representatives of sounds, consists in their simplicity, and the facility and precision with which they can be combined, so as to express every separate thought: they possess a decided superiority over every other artificial vehicle of thought, by communicating, with the utmost ease, every conception of the mind. By their assistance in carrying on epistolary correspondence, the warm effusions of affection and friendship are conveyed to the most remote corners, and the constant intercourse of commerce, science, and learning, is maintained, in defiance of all the obstacles of distance. Learning is indebted to letters for its diffusion and continuance, and to them, genius and virtue owe the rewards of lasting fame. Oral tradition is fleeting and uncertain: it is a stream, which, as it insensibly flows into the ocean of oblivion, is mixed with the impure soil of error and falsehood: but letters furnish the unsullied memorials of truth, and impart to successive generations the perfect records of knowledge. They constitute the light, glory, and ornament of civilized man; and when the voice of the poet, the philosopher, and the scholar, and even the sacred words of the Author of our religion himself, are heard no more,

more, letters record the bright examples of virtue, and teach the inestimable lessons of science, learning, and Revelation, to every age, and to every people.

Various have been the modes of writing adopted by various nations: some, like the Chinese, place their letters in perpendicular columns, and write from the top to the bottom of the page; others, at least all the nations of modern Europe, follow what appears to be a more natural motion of the hand, from left to right; on the contrary, it was the prevailing custom in the East, as among the Egyptians, Phœnicians, Arabians, and Hebrews, to pursue the opposite practice, and write from right to left. These various modes of arrangement, may give some plausibility to the opinion, that each people were the inventors of their own scheme of letters, or their alphabet. A presumption so flattering to national vanity has accordingly prevailed, as the Egyptians attributed the origin of writing to Thoth, supposed to be the same with Mercury, the Greeks to Cadmus, and the Latins to Saturn. This opinion arose from the high reputation acquired by those who first introduced, or made improvements in the art of representing sounds by characters: for, notwithstanding the vast variety now observable in the alphabets of various nations, it is highly probable, that they were all originally formed or derived from the same source, and were carried at various periods of time, into different countries. Can any two sets of letters appear to the eye more dissimilar, than the Hebrew and the English? Yet the affinity of the latter with the former, may with a little ingenuity be evidently traced. Nay, the origin of letters has by some authors been carried much higher than to the Hebrew or Jewish nation, and referred to the Egyptian hieroglyphics. The learned and acute Warburton states, upon the authority of ancient writers, that throughout many of the early ages of the world there was a regular gradation of improvement, in the manner of con-

veying ideas by signs; that pictures were first used as the representatives of thoughts, and that in process of time, alphabetical characters were substituted, as an easier and more compendious mode of communication, than the vague use of arbitrary marks. Moses, the great lawgiver of the Jews, brought letters with the rest of his learning from Egypt, and he simplified their forms, in order to prevent the abuse to which they would have been liable, as symbolical characters, among a people so much inclined to superstition, as those under his direction. From the Jews this simplified or, as we now call it, this alphabetical mode of writing, passed to the Syrians and Phœnicians, or perhaps it was common to them all, at the same time, and from the same source. The Greek authors maintain, that Cadmus and his Phœnician companions, introduced the knowledge of letters into Greece. Herodotus, the most ancient of profane historians, whose works have come down to us, mentions the curious fact that he saw at Thebes in Bœotia, in the temple of Apollo, three tripods inscribed with Cadmean letters, which very much resembled the Ionic of his time. It is well known that the Romans derived their letters from the Greeks. Tacitus has remarked the similarity of the Roman characters to those of the most ancient Greek, or Pelasgic; and the same observation is made by Pliny, and confirmed by an ancient inscription on a tablet of brass dedicated to Minerva. By the Romans, their alphabet was communicated to the Goths, and other nations of modern Europe.—In this way, the descent of alphabetical characters or letters from the earliest times, may be traced down to the present systems in constant use; and by adopting this theory, we simplify objects of curious and important inquiry, extricating ourselves from that perplexity, in which we should be involved, were we to reject an opinion so conformable to reason, and to the surest historical evidence. It is, besides, a pleasing circumstance, that

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while we maintain a system supported by the most respectable profane authorities, we strengthen the arguments in favour of the high antiquity of the Hebrew language, and corroborate, with respect to its origin, the relations given in the Scriptures.

On the English Language.

The impressions made by the conquerors who have settled in any particular nation, are in few respects more closely to be traced, than in the change they have produced in the language of the former inhabitants. This observation may be applied, with peculiar propriety, to our own country: for after the Saxons had subdued the Britons, they introduced into the country their own language, which was a dialect of the Teutonic. From the fragments still extant of the Saxon laws, history, and poetry, we have many proofs to convince us that this language was capaple of expressing, with a great degree of copiousness and energy, the sentiments of a civilized people. In the common language of England, no considerable alteration took place, after this period, for the space of six hundred years, until the conquest by William of Normandy, who promoted another change which had been begun by Edward the Confessor, and caused the Norman French to be used, even in the courts of justice. This language thus, in a short time, became current among the higher classes, and the constant intercourse which subsisted between England and France, for many centuries, introduced a very considerable addition of terms, which were adopted with very slight deviation from their original.——Such were the principal sources of the English tongue, which has from time to time been augmented by the influx of the Latin, and other terms and modes of expression, with which the pursuits of commerce,

the cultivation of learning, and the progress of the arts, have made our ancestors and ourselves acquainted.

From the countries which have supplied us with improvements, we have also drawn the terms belonging to them: thus music, sculpture, and painting, borrowed their terms from Italy; the words used in navigation were received from the inhabitants of Flanders and Holland; the French have supplied the expressions used in fortification and military affairs: the terms of mathematics and philosophy are drawn from the Greek and Latin; and from the Saxon, with a few remains of the original British or Celtic tongue, we have most of the words of general use, as well as those belonging to agriculture, and the mechanical arts. But although the English language has little simplicity to boast of, respecting its origin, yet in its grammatical structure it bears a close resemblance to the Hebrew, the most simple language of antiquity. Its words depart less from the original form, than those of other modern European tongues; and it is only by the different degrees of comparison that any changes take place in the adjectives. There is but one conjugation of the verbs, some of which, indeed, are not varied at all, and others have only two or three changes of termination. Almost all the modifications of time in the verbs, are expressed by the auxiliaries; and the verbs themselves preserve in many instances very nearly, and in some cases exactly, their radical form in the different tenses: the different powers also of these auxiliary verbs, are of great use in expressing the several moods. The article possesses a striking peculiarity, differing from that in most other languages, being indeclinable, and common to all genders. By this simplicity of structue, the English tongue is much easier to a learner than the French, Italian, Spanish, &c., in which the variations of the verbs in particular, are very numerous, complex, and difficult to be retained.

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The English language is uniform in its composition, and its irregularities are far from being numerous. The distinctions in the genders of nouns are agreeable to the nature of things, and are not applied with that caprice which occurs in many other tongues, both ancient and modern. The order of construction is more easy and simple than that of the Greek and the Latin; and it has neither genders of adjectives, gerunds, supines, nor variety of conjugations. These peculiarities give the English a philosophical character; and as its terms are strong, expressive, and copious, few languages seem better calculated to facilitate the intercourse of mankind, as an universal medium of communication. A language which has been so much indebted to others, both ancient and modern, may for abundance of forcible terms and expressions, well be put in comparison with any other now spoken. Since the English tongue has been so much cultivated, and brought to its present excellence, no writer has had reason to complain that his ideas could not be adequately expressed, or clothed in a suitable and becoming dress: none has been obliged to write in a foreign language, on account of its superiority over his own. Whether we open the volumes of our divines, our philosophers, our historians, or our artists, we find them to abound with all the terms necessary to communicate their sentiments, observations, and discoveries, and to give their readers the most complete views of their respective subjects. Hence it appears, that the English language is sufficiently capacious for all the purposes of life, and can furnish proper and adequate expression to variety of argument, delicacy of taste, and fervour of genius; and to evince its sufficient copiousness to communicate to mankind every action, event, invention, and observation, in a full, clear, and elegant manner, it is only necessary to appeal to the authors most admired and esteemed by persons of correct judgment. But the excellence of the English language,

is perhaps in few respects exhibited to such advantage as in the productions of the poets. Whoever reads the works of Shakespeare, Spenser, Milton, Dryden, and Pope, will be sensible that they employ a kind of phraseology that may be said to be sacred to the muses: it is distinguished from prose, not merely by the harmony of numbers, but by the variety of its appropriate terms and expressions. A considerable degree of beauty likewise results from the different measures employed in poetry. The Allegro and Penseroso of Milton, Alexander's Feast by Dryden, the Ode to the Passions by Collins, and the Bard of Gray, are as complete examples of versification, judiciously varied, according to the nature of the subjects, as they are specimens of exquisite sentiment, and original genius. One of the most beautiful figures in poetry, is the personification of inanimate objects: the genius of the English language enables the poet to give the most striking effect to this figure, as the genders of nouns are not arbitrarily imposed, but may be varied according to the nature of the subject. Thus, he may establish the most evident distinction between prose and verse, and communicate to his descriptions that spirit and animation, which cannot fail to strike every reader of taste. It must, however, be acknowledged, that it is chiefly to grave subjects, to the details of the historian, the arguments of the divine and the politician, the speculations of the philosopher, and the invention of the epic and tragic poet, that our language is best adapted. It has energy and copiousness, but it corresponds not so well with the mirth of the gay, or the pathos of the distressed, as some other languages. In describing the pleasantries of the mind, in the effusions of delicate humour, the sallies of wit, and the trifling levities of social intercourse, the French possesses a decided advantage: and in delineating the tender passions, the soothing of pity, the ardour of love,

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it must yield the superiority to the softer cadences of the Italian.

It is but natural to indulge a favour for our native soil, and for every thing belonging to it; it is therefore no wonder that we should be partial to our own language: but this propensity ought not to blind our eyes to the defects of either. It cannot, therefore, be amiss to remark some of the chief imperfections of the English tongue. As many of the words which are not derived from the Greek and Latin, are monosyllables, and terminated in consonants, the pronunciation becomes rugged and broken, very unlike the regular easy flow of classic language. Many words are in themselves harsh and unmusical, and there are even some which the inhabitants of the South of Europe, accustomed to softer expressions, hardly ever learn to utter. It has been remarked, that in the English language, computed to contain thirty-five thousand words, there are not above a dozen which end in *a*, about two dozen in *o*, and in *y* not less than four thousand nine hundred; altogether forming but one-fifth of the language, terminating in vowels; while the remainder ending in consonants, must, to many nations, give it an air of roughness and want of melody.

The want of teminations in the verbs, rendering necessary the introduction of auxiliaries, often obliges us to express ourselves by circumlocutions. There is no distinction in the persons of the plural number, nor in the tenses and persons of the passive voice, which frequently occasions ambiguity, so that foreigners in reading English are often at a loss, unless they give particular attention to the context, to understand the meaning of many of our sentences. Our accents are naturally fitted to give considerable variety to pronunciation; but the pra[illegible] of placing the accents in some cases on the first syllable of a word, in a great degree destroys their use, and leaves only an indistinct, hurried, and almost unintelligible sound, to the other syllables. None of the

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modern languages of Europe are so strongly marked by accents as the English; and their use is very evident in poetry, as they enable us to support with ease the varied numbers of blank verse. But, notwithstanding the zeal of some authors to show the excellence of English, with respect to the quantity of syllables, and to prove that it is in itself harmonious and musical, it must be acknowledged that Greek and Latin, have a great superiority over it, from the regular and constant distinction of long and short syllables. It is certainly true, that we have many syllables and words which are strictly long or short; but still the far greater number cannot well be said to be of any determinate quality.

Orthography, or the mode of spelling words, appears to have been in former times extremely vague and uncertain: it is not uncommon to find in our old writers, the same word spelt differently even in the same page. In the beginning of the last century, orthography began to be more attended to, and attempts were made, with considerable success, to rescue it from uncertainty; yet authors of eminence have differred much from one another in their modes of spelling, adjusting their practice to their several notions of propriety. But as the influence of their authority has had no effect upon general custom, such authors have not escaped the imputation of singularity and affectation. Many other means have been devised for estsablishing English orthography, on a proper foundation, but still without the desired effect: at last, what many had wished, and many had in vain attempted, what seemed, indeed, to demand the united efforts of a number, was performed by the diligence and acuteness of a single man. Dr. Johnson's English Dictionary appeared; and as the weight of truth and reason is irresistible, its authority has nearly fixed the external form of our language, and from its decisions few appeals of importance have been made. The principles on which Johnson

founded

founded his work, were those of etymology and anology: with the former, few men were better acquainted; and if in it, as well as in analogy, some have gone beyond the bounds of his observation, it has chiefly been by following up minute researches, perhaps incompatible with the greatness of his undertaking.

The English language is to be considered not only with a view to its grammatical propriety, but as a subject of taste; and in order to avoid the errors of those who have been led astray by affectation and false refinement, as well as to form a proper opinion of its genius and idioms, it is necessary to peruse the works of the best and most approved writers. In the various departments of religion, history, poetry, and general literature, many writers might be quoted, as deserving to be held up as models of good English; but without any wish to detract from the merits of others, whom the limits of this work, and not insensibility of their qualities, render it necessary to omit.

The student should begin his course of reading with the most distinguished writers of Elizabeth's reign, when the language began to be refined from its original roughness, and to be improved in form and features; thence continuing his course down to the present times. In this plan he must not be deterred, from an apprehension that he will find the old writers clothed in the garb of rude uncouth antiquity: on the contrary, he will find, that the language of his forefathers, in point of structure, formation, and the general meaning of terms, differs not materially from his own.

The substance of a language undergoes but little alteration, even in the course of ages; although from the influx of new customs, and the inventions or improvements of arts, some addition to its terms, and change in its orthography and pronunciation, may be introduced. The dramas of Shakespeare will, therefore, afford to the young student

of English, such instances of style adapted to the grave, the gay, the polished, the rough, the heroic, the vulgar, characters, appearing on the scene, as will prove, that our language was abundantly copious and expressive, to be a proper vehicle for the conceptions of his wonderful genius. The works of Speed, Ascham, Raleigh, Taylor, Clarendon, and Temple, are highly valuable, for the vigour and compass of their language, as well as the knowledge and ablilities there displayed. The common translation of the Bible, abstracting from the important nature of its contents, deserves great attention: its phraseology is such as evinces no less the powers of the language, than the judgment of the translators. The words are, in general, elegant and expressive, conveying the sublime ideas of the original, on the one hand without coarseness or familarity, and on the other without pedantry or affectation. The manly and dignified prose, the rich and sublime poetry of Milton, far from being degraded or fettered, are exalted and adorned by his style; and it was that admirable author's peculiar glory, that, with consummate skill and taste, he was able to apply to the majesty of an epic poem, the flowing and unshackled periods of blank verse. The increasing tribute of praise has at all times been paid to the vast stores of his erudition, and the flights of his transcendant genius.

In the reign of Charles the Second, the reader will find, perhaps, no author more deserving of his attention, than Barrow, whose periods so round and exuberant, give us a very just representation of the eloquence of Cicero; and display to the greatest advantage, the energy and the fertility of Barrow's intellectual powers, employed upon the most important subjects of morality and religion.

The great Locke, in a plain and severe style, well adapted to the precision of his researches, unravelled the intricacies of the most interesting branch of philosophy, by tracing ideas to their source, and unfolding the faculties of the mind.

The

The reign of Anne, which, for the eminent attainments of Britons in arts and literature, may be compared with the ages of Pericles in Greece, and of Augustus in Rome, produced a Swift, who in clear and familiar diction, unaided by flowery ornaments, expressed the dictates of a strong understanding, and a lively invention. Addison, the accomplished scholar, the refined critic, the enlightened moralist, like another Socrates, brought philosophy from the schools, and arranged her in the most engaging attire; calling the attention of his countrymen to virtue, and to taste, in his elegant and entertaining essays. The prefaces of Dryden, are marked by the ease and the vivacity of a man of transcendant genius; and there is a facility in his rhymes, and a peculiar vigour in his poetry, which justly render him the boast of his country. Pope composed his prefaces and letters with singular grace and beauty of style; and his poems present the finest specimens of exquisite judgment, adorned by the most harmonious and polished versification. The works of Melmoth, in particular his letters and translations of Cicero and Pliny, are remarkable for smoothness and elegance of composition. The late Sir Joshua Reynolds, has, in his lectures in the Royal Academy, illustrated the principles of his delightful art, in a manner not less creditable to him as a fine writer, than as an eminent painter, and a skilful connoisseur. The sacred discourses of the amiable Horne, recommend the duties of that religion, of which he was so bright an ornament, in a sweet and lively style. The manly vigour of Bishop Watson, diffuses its animation through all his works, whether philosophical, controversial, or religious.— To conclude; where can we find compositions uniting the politeness of the gentleman, with the attainments of the scholar, blended in juster proportions, than in the Polymetis of Spence the Athenian letters, the dialogues of Lord Littleton and Bishop Hurd, and the papers of the Ad-

 venturer

venturer and Observer? These are some of the sources from which may be derived a knowledge of the purity, the strength, the copiousness of the English language: and such are the examples by which the student ought to regulate his style. In these he may remark the idiomatic structure of sentences, and the proper arrangement of their parts: they present specimens of purity without stiffness, and elegance without affectation: they are free equally from vain pomp and vulgarity of diction; and their authors have the happy art of pleasing our taste, while they improve our understandings, and confirm our principles of morality and religion. In the course of this perusal it will be found, that in proportion as the great controversies in this country upon religion, politics, and philosophy, began to subside, since the Revolution, a closer attention has been paid to the niceties of grammar and criticism; and coarse and barbarous phraseology has been gradually polished into propriety and elegance.

As the practice of writing for public inspection, has been much improved since the period above mentioned, a remarkable change has taken place. The long parenthesis which so frequently occurs in the old writers, to the great embarrassment and perplexity of their meaning, has fallen much into disuse: It has been remarked that the parenthesis occurs in no place of Johnson's writings. Authors have shortened their sentences, which in some of the best writers of the seventeenth, and the beginning of the last century, were extended to an almost immeasurable length; and they have stated their sentiments to much more advantage, by separating their ideas from each other, aud expressing them with greater exactness: whether however, this circumstance may not argue a want of fertility of ideas, and a tardiness of conception, might be a subject of inquiry: it must, however, be allowed, that the practice of writing in short sentences has a tendency to give languagea disjointed appearance,

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and detracts from roundness of period, and dignity of composition; but it certainly contributes so materially to perspicuity, that every reader must be pleased with the change.

The preceding remarks have chiefly been confined to the English as a written language; but books have a much more extensive use than merely to direct the practice of writers, for they are fitted to correct the errors of conversation, and to communicate both accuracy and purity to social intercourse. Less variation, and less vulgarity of speech, will always be found among the natives of different provinces, in proportion as well written books are circulated and perused: but the standard ought always to continue the same; it should consist in a compliance with general rules, and the practice of the polished ranks of society. Such regulations rescue the standard of propriety from the caprice of individuals, and establish a barrier against the encroachments of commercial idioms, professional phraseology, vulgarity, ignorance, and pedantry.

The court speaker rejects local and provincial forms of expression, for those which are general: he converses neither in the dialect of Somersetshire, nor of Norfolk, of Northumberland, nor of Cornwall, but in that elegant phraseology which has received the sanction of the best company. He neither countenances by his approbation, nor authorises by his example, new fashioned phrases or upstart words, having nothing but novelty to recommend them, whether introduced by the great or the little vulgar, the learned, or the ignorant. Upon such occasions as these, a good taste is the surest guide. The correct speaker conforms to idiom and analogy, and at the same time that he confesses his obligation to learned men, for their labours in attempting to reduce his native language to a fixed standard, he remembers what an Englishman should never forget, that the genius of the British language, like the spirit of the British people, disdains to be encroached upon by arbitrary and fo-

reign

reign innovations. Those who write only for the present day, labour to adorn their style with modish phrases: a popular speaker, and in particular a member of the house of commons, enjoys, or at least assumes, a sort of privilege of coining words at pleasure, which no sooner receive his sanction, than they intrude upon us from every quarter, in letters, plays, and periodical publications. Such words, however, in general resemble certain insects, which are seen to flutter for a day, and then entirely disappear.

It is impossible not to join in the complaint made by foreigners, that our language in pronunciation is much at variance with its orthography. The practice of the court and the stage has multiplied these variations, which have been but too eagerly adopted in the higher classes of society. One reason assigned for this practice is agreeableness of sound: but in opposition to this, two consonants are frequently sounded where only one is written, which surely can never contribute to the melody of speech; besides, that the irregularities of the English are already abundantly numerous, without making any such additions. It is fortunate, however, that the great body of the people are not much influenced by these fashionable changes, but steadily adhere to ancient established customs: it is, therefore, among them that we are to look for that regularity of writing and speaking, which some among the higher ranks, would sacrifice to caprice and a love of novelty and distinction. But it is time to close these remarks on the English language, which by the conquests, the colonies, and the commerce of this empire, is now well known in all quarters of the globe. Its reputation is every day increasing, and of late years the study of English is become indispensible, among those foreigners who wish to complete a liberal education: and without partiality or exaggeration, this language may well be said to deserve their attention, as in it are contained some of the richest treasures of the human mind; which for intellectual

vigour,

vigour, energy of thought, depth of erudition, warmth of imagination, and philosophical research, yield to none of any other country. For this is the language which has conveyed to the world the sublime productions of Shakespeare and Milton, the philosophical discoveries of Boyle, the profound disquisitions of Locke, the elegant essays of Addison, the polished numbers of Dryden and Pope, the nervous eloquence of Barrow, Tillotson, and Clarke, the entertaining novels of Fielding and Richardson, and the historical excellence of Clarendon, Hume, Robertson, and Henry. These eminent writers have secured the stability, as well as extended the reputation of the English tongue; and while literary curiosity retains its hold on the mind, the studious of all countries will ardently desire to enjoy such admirable productions of genius and learning, in their original dress.

The prevalence and the flourishing state of our language, do not depend solely upon the inhabitants of the British Isles; in many parts of the West Indies, it is diligently cultivated; our extensive and increasing settlements in the East, promise to insure its preservation, and open a wide field for its farther diffusion. The United States of America must perpetuate the language of their parent country; and the spirit of literary and scientific inquiry, disseminating among them, cannot fail to contribute to the same end; since it will encourage the study of those celebrated productions, from which the Americans have drawn their knowledge of the best system of legislation, and the most correct principles of liberty.

It is impossible, however, when we reflect on the uncertainty of all human affairs, and particularly on the mutability of language, not to give way to the melancholy apprehension, that the period may arrive, when the English tongue, at present in appearance so durable, will become obsolete. From the caprice of fashion, the wide extent of our

commercial

commercial relations with the natives of all quarters of the world, our general intercourse with foreigners, and above all, from the predominancy of the French language, as much from affectation, as from any real utility; from these and other causes, great and material changes in our language, may and must take place; and Hume and Johnson, Pope, and Goldsmith, may become what Speed, and Ascham, Chaucer and Phaer, are at this day. It is, nevertheless, to be hoped, that, for the honour of true taste, and the good sense of mankind, the volumes of English literature will never sink into oblivion; that the language in which they are written, embalmed in the choicest productions of human ingenuity and labour, and fit to be ranked with the classical tongues of Greece and Rome, will be preserved for the improvement and pleasure of mankind; and will convey these master-pieces of genius, learning, and philosophy, to the most distant generations.

Having already stated some of the many advantages to be reaped from an acquaintance with the Latin and Greek languages; it will be sufficient in this place to add only a few remarks on each.

A knowledge of the Latin introduces the student to many works, deservedly esteemed amongst the most elegant productions of human nature, as well as the most perfect models of literary excellence. In comparing its worth and usefulness, we are compelled to assign to this language, a place next to our own, both because the ancient Roman writers have employed it to convey to us their ideas and sentiments, on the most important subjects; and also, that Latin has been used by many of the most eminent modern authors on various occasions. The utility of Latin will be still more apparent, if we consider how much the English owes to it, for terms of art and science, and even for most of our polysyllables. So much is this the case that, without the intervention of Latin, it is almost impossible to

write

write or speak a sentence of elegant English: hence, the student of Latin is, in fact, making great progress in the knowledge of his own language, at the same time that he is laying the foundation for the future acquirement of the French, Italian, Spanish, and Portuguese languages, of all which, the ancient Latin constitutes by far the greatest portion.

Whilst the Romans were masters of the world, and even since the restoration of literature, the Latin has had the best pretensions to be considered as an universal tongue; and as such it has been cultivated by every enlightened nation. There is no branch of learning, no discovery of art, no system of science, nay, we may say, no topic of liberal inquiry, which has not owed its expression, illustration, and ornament, to this language. It has at all times served to carry on communication between men of letters, of whom many, instead of their native tongues, as being either imperfect in themselves, or little known to foreigners, have adopted the language of ancient Rome, to publish to the world their most valuable productions:—in this language were composed the admirable works of Erasmus, Grotius, Puffendorf, Boerhaave, Bacon, Newton and Gravina. These eminent men considered that modern languages are in a state of continual fluctuation, and subject to the caprices of fashion and novelty; whilst the Latin is fixed and permanent. The language once used by the conquerors of the world, is also used at this day to express the feelings of affection, gratitude, honour, and veneration; it is inscribed on public edifices, it distinguishes the monuments and the medals of every country in Europe; and transmits to future ages the memory of scholars, philosophers, patriots, and heroes, in terms which, for precision and dignity, can be equalled by no modern tongue.

Over all languages, the ancient Greek is entitled to the superiority. This assertion will be supported by those who are best acquainted with such matters, and thereby best qualified to make just and extensive comparisons. In the various modes of expression, we find precision without obscurity, and copiousness without redundance: the former property, the Greek owes to the numerous and diversified inflections of its words, and the latter, to the great number of its derivatives: in its general structure and formation, regard is paid to the ear, as well as to the understanding; for its harmony is not less noticeable than its energy: the grammatical system is in every part, exact and complete, at the same time that the rules are no restraint upon its expressions.

Whoever considers attentively the genius of the Greek tongue, will be ready to allow, that it merits to be proposed as a perfect model of expression, and that it fully authorises the praise of those critics and scholars who have celebrated its excellence, in proportion as they were able to relish its beauties, deriving taste, improvement, and pleasure, from the perusal of its incomparable writers.

Among the numerous beauties of the Greek tongue, its sweetness is deservedly celebrated, as well as its variety of sounds, even in our very imperfect mode of pronunciation. By transposing, altering, adding, and taking away letters, the Greek was rendered more soft and pleasing to the ear: the diphthongs, as well as the open vowels, swell and elevate the tones, in a manner superior to modern languages. The various declensions of the nouns, the conjugations of the verbs, the changes of dialects, and the several poetic licenses, furnish a vast variety of terminations: many words end in vowels, and but very few in mute consonants, as is the case in the Oriental, Gothic, and Celtic tongues.

The works of the best Greek authors are admirable, for the skilful arrangement of words, and the beauties of the compositon.

composition. The accurate distinction of genders, and cases in nouns, and of persons in verbs, allowed the writers to choose the most proper place for their words, without affecting the perspicuity of the sentence. But in this respect, modern languages are very defective, in which the nouns and verbs being in general indeclinable, they must be closely connected in the sentence to their respective articles, and auxiliary verbs: whence, an uniform arrangement and position is requisite, in which no change can be made, without injuring the sense, or rendering it equivocal, or unintelligible. Greek authors, on the contrary, could indulge in grand and lofty sentences, composed of members of various extent, and terminated in whatever part of speech might seem most proper. Hence, the ear is gratified with an endless variety of pauses, and a harmonious flow of periods; an emphatic word, like the principal figure in a picture, being placed where it produces the best effect. Unable as we moderns are, to form a competent notion of these beauties, still, from the testimony of ancient critics, whose perfect knowledge and delicacy of taste, qualified them to form a better judgment on the subject, we must yield our assent to the justice of the praises, which in every age, have been bestowed on the Greek language. In one point, however, the English scholar may appreciate the excellence of this language, in as far as his own possesses a similar property, although in an inferior degree: and that is the compounding of words. The prolific power of the Greek was unbounded: verbs were the trees, from which sprung innumerable branches, in endless variety and abundance. These verbs are at one time compounded together, at another time with substantives; nouns are formed from them, and even from different parts of the same verb: but the power of combination with prepositions, was of a much more extraordinary extent: with any one of the Greek prepositions, any

 verb,

verb, unless its meaning made it naturally unfit for such an alliance, could be united. Instances of these combinations are common, and even of verbs and nouns joined with double and treble prepositions. As such compound words possess an unrivalled strength, richness, and significant brevity, they show the creative powers of a language, containing inexhaustible resourses for composition. In poetry, their effect is most particularly felt, supplying it with one of its most striking and beautiful ornaments. To this power of compounding words, so extensive, and indeed indefinite, no resemblance can be traced in the works of art, excepting in the unlimited combination of letters forming words, and the multiplication of numbers in arithmetic.

From the whole of these observations on the English, Latin and Greek languages, may be formed some notion of the characteristics of each, and of their respective merits. To the Greek we must allow the praise of harmony, copiousness, and an amazing ductility, by which it can express in original, derivative, and compound words, every discovery in science, or invention in art: the Latin is recommended by its majesty, precision, and vigour; and the classic writers in both these languages, are to be considered as the best models of learning and taste. But in these acknowledgements of superior excellence, let us not shut our eyes to the many admirable properties of our own tongue: the English language, drawing its stock of words from various sources, and very imperfectly understood, without the help of Greek and Latin, is energetic, rich, and copious: and perhaps, if we were restricted to the use of one modern tongue, to the exclusion of all others, none could be found better calculated for every purpose of social intercourse, more capable of expressing the general sentiments of the mind, or more deserving the study and cultivation of men in every situation.

ENGLISH

ENGLISH GRAMMAR.

GRAMMAR.

GRAMMAR is the Art of rightly expressing our thoughts by words.

Grammar in general, or Universal Grammar, explains the principles which are common to all languages.

The Grammar of any particular Language, as the English Grammar, applies these common principles to that particular language, according to the established usage and custom of it.

Grammar treats of Sentences; and of the several parts of which they are compounded.

Sentences consist of Words; Words, of one or more Syllables; Syllables, of one or more Letters.

So that Letters, Syllables, Words, and Sentences, make up the whole subject of Grammar.

LETTERS.

A LETTER is the first Principle, or least part, of a Word.

An Articulate Sound is the sound of the human voice, formed by the organs of speech.

A Vowel is a simple articulate sound, formed by the impulse of the voice, and by the opening only of the mouth in a particular manner.

A Consonant cannot be perfectly sounded by itself; but joined with a vowel, forms a compound articulate sound, by a particular motion or contact of the parts of the mouth.

A Diphthong, or compound vowel, is the union of two or more vowels pronounced by a single impulse of the voice.

In

In the English there are twenty-six Letters:

A, a; B, b; C, c; D, d; E, e; F, f; G, g; H, h; I, i; J, j; K, k; L, l; M, m; N, n; O, o; P, p; Q, q; R, r; S, s; T, t; U, u; V, v; W, w; X, x; Y, y; Z, z.

J, j; and *V, v*; are consonants; the former having the sound of the soft *g*, and the latter that of a coarser *f*; they are therefore entirely different from the vowels *i* and *u*, and distinct letters of themselves; they ought also to be distinguished from them, each by a peculiar name; the former may be called *ja*, and the latter *vee*.

The names, then, of the twenty-six letters will be as follow; *a, bee, cee, de, e, ef, gee, aitch, i, ja, ka, el, m, en, o, pee, cue, ar, ess, tee, u, v, double u, ex, y, zad*.

Six of the letters are vowels, and may be sounded by themselves; *a, e, i, o, u, y*.

E is generally silent at the end of a word; but it has its effect in lengthening the preceding vowels, as *bid, bide*: and sometimes likewise in the middle of a word; as, *ungrateful, retirement*. Sometimes it has no other effect, than that of softening a preceding *g*; as *lodge, judge, judgement*; for which purpose it is quite necessary in these and the like words.

Y is in the sound wholly the same with *i*; and is written instead of it, at the end of words; or before *i*; as, *flying, denying*: it is retained likewise in some words derived from the Greek; and it is always a vowel.

W is either a vowel, or a diphthong; its proper sound is the same as the Italian *u*, the French *ou*, or the English *oo*: after *o*, it is sometimes not sounded at all; sometimes like a single *u*.

The rest of the letters are consonants; which cannot be sounded alone: some not at all, and these are called Mutes: *b, c, d, g, k, p, q, t*: others very imperfectly, making a kind of obscure sound; and these are called Semi-

Semi-vowels or Half-vowels, *l, m, n, r, f, s,* the first four of which are also distinguished by the name of Liquids.

The Mutes and the Semi-vowels are distinguished by their names in the Alphabet; those of the former all beginning with a consonant, *bee, cee,* &c.; those of the latter all beginning with a vowel, *ef, el,* &c.

X is a double consonant, compounded of *c,* or *k,* and *s.*

Z seems not to be a double consonant in English, as it is commonly supposed: it has the same relation to *s,* as *v* has to *f,* being a thicker and coarser expression of it.

H is only an Aspiration, or Breathing: and sometimes at the beginning of a word is not sounded at all; as, *an hour, an honest* man.

C is pronounced like *k* before *a, o, u*; and soft like *s,* before *e, i, y:* in like manner *g* is pronounced always hard before *a, o, u*; sometimes hard and sometimes soft before *i* and *y:* and for the most part soft before *e.*

The English Alphabet, like most others, is both deficient and redundant; in some cases, the same letters expressing different sounds, and different letters expressing the same sounds.

SYLLABLES.

A Syllable it a sound either simple or compounded, pronounced by a single impulse of the voiee, and constituting a word, or a part of a word.

Spelling is the art of reading by naming the letters singly, and rightly dividing words into their syllables. Or, in writing, it is the expressing of a word by its proper letters.

In spelling, a syllable in the beginning or middle of a word, ends in a vowel, unless it be followed by *x,* or by two or more consonants: these are for the most part to be separated; and at least one of them always belongs to the

preceding

preceding syllable, when the vowel of that syllable is pronounced short. Particles in Composition, though followed by a wowel, generally remain undivided in spelling. A mute generally unites with a liquid following; and a liquid, or a mute, generally separates from a mute following: *le* and *re* are never separated from a preceding mute. Example: *Ma-ni-fest*, *ex-e-cra-ble*, *un-e-qual*, *mis-ap-ply*, *dis-tin-guish*, *cor-res-pond-ing*.

But the best and easiest rule for dividing the syllables in spelling, is to divide them as they are naturally divided in a right pronunciation; without regard to the derivation of words, or the possible combination of consonants at the beginning of a syllable.

WORDS.

WORDS are articulate sounds, used by common consent as signs of ideas or notions.

There are in English nine sorts of Words, or, as they are commonly called, Parts of Speech.

1. The ARTICLE; prefixed to substantives, when they are common names of things, to point them out, and to show how far their signification extends.

2. The SUBSTANTIVE, or NOUN; being the name of any thing conceived to subsist, or of which we have any notion.

3. The PRONOUN; standing instead of the noun.

4. The ADJECTIVE; added to the noun to express the quality of it.

5. The VERB, or word, by way of eminence; signifying to be, to do, or to suffer.

6. The ADVERB; added to verbs, and also to adjectives and other adverbs, to express some circumstance belonging to them.

7. The PREPOSITION; put before nouns and pronouns chiefly,

chiefly, to connect them with other words, and to show their relation to those words.

8. The CONJUNCTION; connecting sentences together.

9. The INTERJECTION; thrown in to express the affection of the speaker, though unnecessary with respect to the construction of the sentence.

EXAMPLE.

1 2 7 2 5 1 2 4 7 2 8
The power of speech is a faculty peculiar to man, and
5 5 7 3 7 3 4 2 7 1
was bestowed on him by his beneficent Creator for the
4 8 6 4 2 8 9 6 6 5
greatest and most excellent uses; but, alas! how often do
3 5 3 7 1 4 7 2
we pervert it to the worst of purposes.

In the foregoing sentence, the Words *the*, *a*, are articles; *power*, *speech*, *faculty*, *man*, *creator*, *uses*, *purposes*, are Substantives; *him*, *his*, *we*, *it*, are Pronouns; *peculiar*, *beneficent*, *greatest*, *excellent*, *worst*, are Adjectives; *is*, *was*, *bestowed*, *do*, *pervert*, are Verbs; *most*, *how*, *often*, are Adverbs; *of*, *to*, *on*, *by*, *for*, are Prepositions; *and*, *but*, are Conjunctions; and *alas* is an Interjection.

The Substantives, *power*, *speech*, *faculty*, and the rest, are General, or Common, Names of things; whereof there are many sorts belonging to the same kind; or many individuals belonging to the same sort; as there are many sorts of power, many sorts of speech, many sorts of faculty, many individuals of that sort of animal called man; and so on. These general or common names are here applied in a more or less extensive signification; according as they are used without either, or with the one, or with the other, of the two Articles *a* and *the*. The words *speech*, *man*, being accompanied with no article, are taken

in their largest extent; and signify all of the kind or sort; all sorts of speech and all men. The word *faculty*, with the article *a* before it, is used in a more confined signification, for some one out of many of that kind; for it is here implied, that there are other faculties peculiar to man beside speech. The words, *power*, *creator*, *uses*, *purposes*, with the article *the* before them, (for *his* Creator is the same as *the* Creator *of him*,) are used in the most confined signification, for the things here mentioned and ascertained: *the power* is not any one indeterminate power out of many sorts, but that particular sort of power, here specified; namely, the power of speech; *the creator* is the One great Creator of man and of all things: *the uses*, and *the purposes*, are particular uses and purposes; the former are explained to be those in particular, that are the greatest and most excellent; such, for instance, as the glory of God, and the common benefit of mankind; the latter to be the worst, as lying, slandering, blaspheming, and the like.

The pronouns, *him*, *his*, *we*, *it*, stand instead of some of the nouns, or substantives, going before them; as, *him* supplies the place of *man*; *his* of *man's*; *we*, of *men*, implied in the general name *man*, including all men, (of which number is the speaker;) it, of *the power*, before mentioned. If, instead of these pronouns, the nouns for which they stand had been used, the sense would have been the same; but the frequent repetition of the same words would have been disagreeable and tedious: as, The power of speech peculiar to *man*, bestowed on *man*, by *man's* Creator, &c.

The Adjectives, *peculiar*, *beneficent*, *greatest*, *excellent*, *worst*, are added to their several substantives, to denote the character and quality of each.

The Verbs, *is*, *was bestowed*, *do pervert*, signify severally, being, suffering, and doing. By the first it is implied, that there is such a thing as the power of speech, and it is affirmed to be of such a kind; namely, a faculty peculiar

liar to man: by the second it is said to have been acted upon, or to have had something done to it: namely, to have been bestowed on man: by the last we are said to act upon it, or to do something to it; namely, to pervert it.

The Adverbs, *most*, *often*, are added to the adjective *excellent*, and to the verb *pervert*, to show the circumstance belonging to them; namely, that of the highest degree to the former, and that of frequency to the latter; concerning the degree of which frequency also a question is made by the adverb *how* added to the adverb *often*.

The Prepositions, *of*, *to*, *on*, *by*, *for*, placed before the substantives and pronouns, *speech*, *man*, *him*, &c. connect them with other words, substantives, adjectives, and verbs; as, *power*, *peculiar*, *bestowed*, &c. and show the relation which they have to those words; as the relation of subject, object, agent, end; *for* denoting the end, *by* the agent, *on* the object; *to* and *of* denote possession, or the belonging of one thing to another.

The Conjunctions, *and*, and *but*, connect the three parts of the sentence together; the first more closely, both with regard to the sentence and the sense; the second connecting the parts of the sentence, though less strictly, and at the same time expressing an opposition in the sense.

The Interjection, *alas!* expresses the concern and regret of the speaker; and though thrown in with propriety, yet might have been omitted, without injuring the construction of the sentence, or destroying the sense.

ARTICLE.

THE ARTICLE is a word prefixed to substantives, to point them out, and to show how far their signification extends.

In English there are but two articles, *a*, and *the*: *a* be-

comes *an* before a vowel, *y* and *w* excepted; and before a silent *h* preceding a vowel.

A is used in a vague sense to point out one single thing of the kind, in other respects indeterminate: *the* determines what particular thing is meaned.

A substantive, without any article to limit it, is taken in its widest sense: thus *man* means all mankind; as,

"The proper study of mankind is man."

Pope.

Where *mankind* and *man* may change places, without making any alteration in the sense. *A man* means some one or other of that kind, indefinitely; *the man* means, definitely, that particular man, who is spoken of: the former, therefore, is called the Indefinite, the latter the Definite Article *.

Example:

* "And I persecuted this way unto *the* death." Acts, xxii. 4. The Apostle does not mean any particular sort of death, but death in general: the Definite Article therefore is improperly used. It ought to be *unto death*, without any Article: agreeably to the Original; αχρι θανάτου. See also, 2 Chron. xxxii. 24.

"When He, the Spirit of truth, is come, he will guide you into *all truth*." John xvi. 13. That is, according to this translation, into all Truth whatsoever, into Truth of all kinds: very different from the meaning of the Evangelist, and from the Original; εις πασαν την αληθειαν, into all *the* Truth; that is, into all Evangelical Truth.

"Truly this was *the* Son of God." Matt. xxvii. 54. and Mark, xv. 39. This translation supposes, that the Roman Centurion had a proper and adequate notion of the character of Jesus, as the Son of God in a peculiar and incommunicable sense: whereas, it is probable, both from the circumstances of the History, and from the expression of the Original, (υἱος Θεου, *a* Son of God, or of *a* God, not ὁ υἱος, *the* Son,) that he only meaned to acknowledge him to be an extraordinary person, and more than a mere man; according to his own notion of the Sons of Gods in the Pagan Theology. This is also more agreeable to St. Luke's account of the confession of the Centurion: "Certainly this was δικαιος, a righteous man;" not ὁ Δικαιος, the Just one. The same may be observed of Nebuchadnezzar's word, Dan. iii. 25. "And the form of the fourth is like *the* Son of God;" it ought to be expressed by the indefinite Article, like *a* Son of God; ὁμοια υἱῳ Θεου, as Theodotion

very

Example: "*Man* was made for society, and ought to extend his good will to all *men:* but *a man* will naturally entertain a more particular kindness for the *men* with whom he has the most frequent intercourse; and enter into a still closer union with *the man* whose temper and disposition suit best with his own."

It is of the nature of both the articles to determine or limit the thing spoken of: *a* determines it to be one single thing of the kind, leaving it still uncertain which; *the* determines which it is, or, of many, which they are. The first, therefore, can only be joined to substantives in the singular number*; the last may also be joined to plurals.

There is a remarkable exception to this rule in the use of the adjectives *few* and *many*, (the latter chiefly with the word *great* before it,) which though joined with plural Substantives, yet admit of the singular Article *a:* as *a few men, a great many men.*

"Told of *a many thousand* warlike French:"
"A care craz'd mother of *a many children.*"

Shakespear.

The reason of it is manifest from the effect which the ar-

very properly renders it: that is, like an Angel; according to Nebuchadnezzar's own account of it in the 28th verse: "Blessed be God, who hath sent his *Angel*, and delivered his servants." See also Luke, xix. 9.

"Who breaks a butterfly upon *a* wheel?" Pope.

It ought to be, *the* wheel; used as an instrument for the particular purpose of torturing Criminals; as Shakespear:

"Let them pull all about mine ears; present me
Death on *the* wheel, or at wild horses' heels."

"God Almighty hath given reason to *a* man to be a light unto him." Hobbes, Elements of Law, Part I. Chap. v. 12. It should rather be, "to man," in general.

* "A good character should not be rested in as an end, but employed as *a means* of doing still further good." Atterbury, Serm. II. 3. Ought it not to be *a mean*? "I have read an author of this taste, that compares a ragged coin to *a* tatterred *colours.*" Addison, Dial. I. on Medals.

ticle

ticle has in these phrases: it means a small or great number collectively taken, and therefore gives the idea of a Whole, that is, of unity. Thus likewise *a hundred*, *a thousand*, is one whole number, an aggregate of many collectively taken; and therefore still retains the Article *a*, though joined as an adjective to a plural Substantive; as, *a hundred years*.

> "For harbour at *a thousand doors* they knock'd;
> Not one of all *the thousand*, but was lock'd."
>
> Dryden.

The Definite Article *the* is sometimes applied to Adverbs in the Comparative and Superlative degree; and its effect is to mark the degree the more strongly, and to define it the more precisely: as, "*the more* I examine it, *the better* I like it. I like this *the least* of any."

SUBSTANTIVE.

A SUBSTANTIVE, or NOUN, is the *Name* of a thing; of whatever we conceive in any way to *subsist*, or of which we have any notion.

Substantives are of two sorts; Proper and Common Names. Proper Names are the Names appropriated to individuals; as the names of persons and places: such are *George*, *London*. Common Names stand for kinds, containing many sorts; or for sorts containing many individuals under them; as *Animal*, *Man*. And these Common Names, whether of kinds or sorts, are applied to express individuals, by the help of Articles added to them, as hath been already shown; and by the help of Definitive Pronouns, as we shall see hereafter.

Proper Names being the names of individuals, and therefore of things already as determinate as they can be made, admit not of Articles, or of Plurality of Number; unless by a Figure, or by Accident: as, when great Conquerors are called *Alexanders*; and some great Conqueror

An

An Alexander, or *The* Alexander of his Age: when a Common Name is understood, as *The* Thames, that is, the *River* Thames: *The* George, that is, the *Sign* of St. George: or when it happens, that there are many persons of the same name: as, *The* two *Scipios.*

Whatever is spoken of is represented as one, or more, in Number: these two manners of representation in respect of Number are called the Singular, and the Plural, Number.

In English, the Substantive Singular is made Plural, for the most part, by adding to it *s*; or *es*, where it is necessary for the pronunciation: as *king, kings; fox, foxes; leaf, leaves;* in which last, and many others, *f* is also changed into *v*, for the sake of an easier pronunciation, and more agreeable sound.

Some few Plurals end in *en*; as, *oxen, children, brethren,* and *men, women,* by changing the *a* of the Singular into *e**. This form we have retained from the Teutonic; as likewise the introduction of the *e* in the former syllable of two of the last instances; *weomen,* (for so we pronounce it,) *brethren,* from *woman, brother*: something like which may be noted in some other forms of Plurals: as, *mouse, mice*; *louse, lice*; *tooth, teeth*; *foot, feet*; *goose, geese.*

The words *sheep, deer,* are the same in both Numbers.

Some Nouns, from the nature of the things which they express, are used only in the Singular others only in the Plural, Form: as, *wheat, pitch, gold, sloth, pride,* &c. and *bellows, scissars, lungs, bowels,* &c.

The English Language, to express different connexions and relations of one thing to another, uses for the most part, Prepositions. The Greek and Latin among the ancient, and some too among the modern languages, as the

* And anciently, *eyen, shoen, housen, hosen:* so likewise anciently *sowen, cowen,* now always pronounced and written *swine, kine.*

German,

German, vary the termination or ending of the Substantive, to answer the same purpose. These different endings are in those languages called cases. And the English being derived from the same origin as the German, that is, from the Teutonic, is not wholly without them. For instance, the relation of Possession, or Belonging, is often expressed by a case, or different ending of the Substantive. This Case answers to the Genitive Case in Latin, and may still be so called; though perhaps more properly the Possessive Case. Thus "*God's* grace:" which may also be expressed by the Preposition; as, "the grace *of God*." It was formerly written; "*Godis* grace;" we now always shorten it with an Apostrophe; often very improperly, when we are obliged to pronounce it fully; as, "*Thomas's* book:" that is, *Thomasis* book," not "*Thomas his* book," as it is commonly supposed*.

When the thing, to which another is said to belong, is expressed by a circumlocution, or by many terms, the sign of the Possessive Case is commonly added to the last term;

* "*Christ his* sake," in our Liturgy, is a mistake either of the Printers, or of the Compilers. "Nevertheless, Asa *his* heart was perfect with the Lord." 1 Kings, xv. 14. "To see whether Mordecai *his* matters would stand." Esther, iii. 4.

"Where is this mankind now? who lives to age
Fit to be made Methusalem *his* page." Donne.
"By young Telemachus *his* blooming years." Pope's Odyssey.

"My paper is *Ulysses his* bow, in which every man of wit or learning may try his strength." Addison, Guardian, N° 98. See also Spect. N° 207. This is no slip of Mr. Addison's pen: he gives us his opinion upon this point very explicitly in another place. "The same single letter *s* on many occasions does the office of a whole word, and represents the *his* and *her* of our forefathers." Addison, Spect. N° 135. The latter instance might have shown him, how groundless this notion is: for it is not easy to conceive how the letter *s* added to a Feminine Noun should represent the word *her*; any more than it should the word *their* added to a Plural Noun; as, the "*children's* bread." But the direct derivation of this Case from the Saxon Genitive Case is sufficient of itself to decide this matter.

as,

as, "The King of Great *Britain's* Soldiers." When it is a Noun ending in *s*, the sign of the Possessive Case is sometimes not added; as, "for *righteousness'* sake *;" nor ever to the Plural Number ending in *s*; as, on "*eagles'* wings." Both the Sign and the Preposition seem sometimes to be used; as, "a soldier *of the king's*;" but here are really two Possessives; for it means, "one *of* the soldiers *of* the king."

The English in its Substantives has but two different terminations for Cases, that of the Nominative, which simply expresses the Name of the thing, and that of the Possessive Case.

Things are frequently considered with relation to the distinction of Sex or Gender; as being Male or Female, or neither the one, nor the other. Hence, Substantives are of the Masculine, or Feminine, or Neuter, (that is, neither,) Gender: which latter is only the exclusion of all consideration of Gender.

The English Language, with singular propriety, following nature alone, applies the distinction of Masculine and Feminine only to the names of Animals; all the rest are Neuter: except when, by a Poetical or Rhetorical fiction, things inanimate and Qualities are exhibited as Persons, and consequently become either Male or Female; and this gives the English an advantage above most other languages in the Poetical and Rhetorical style: for when Nouns naturally Neuter are converted into Masculine and Femi-

* In Poetry, the Sign of the Possessive Case is frequently omitted after Proper Names ending in *s* or *x*; as, "The wrath of Peleus' Son." Pope. This seems not so allowable in Prose: as "Moses' minister." Josh. i. 1. "Phinehas' wife." 1 Sam. iv. 19. "Festus came into Felix' room." Acts, xxiv. 27.

nine *, the Personification is more distinctly and forcibly marked.

Some few Substantives are distinguished in their Gender by their termination: as, *prince*, *princess*; *actor*, *actress*; *lion*, *lioness*; *hero*, *heroine*, *&c.*

The chief use of Gender in English is in the Pronoun of the Third Person; which must agree in that respect with the Noun for which it stands.

PRONOUN.

A PRONOUN is a word standing *instead of a Noun*, as its Substitute or Representative.

* "At his command the uprooted hills retired
Each to *his* place; they heard his voice, and went
Obsequious: Heaven *his* wonted face renew'd,
And with fresh flow'rets Hill and Valley smil'd."
Milton, P. L. b. vi.

"Was I deceiv'd; or did a sable Cloud
Turn forth *her* silver lining on the Night?"
Milton, Comus.

"Of law no less can be acknowleded, than that *her* seat is the bosom of God; *her* voice, the harmony of the world. All things in heaven and earth do *her* homage: the very least, as feeling *her* care; and the greatest, as not exempted from *her* power." Hooker, B. i. 16. "Go to your Natural Religion: lay before *her* Mahomet and his disciples, arrayed in armour and in blood:—shew *her* the cities, which he set in flames; the countries, which he ravaged: when *she* has viewed him in this scene, carry *her* into his retirements; shew *her* the Prophet's chamber, his concubines, and his wives: when *she* is tired with this prospect, then shew *her* the blessed Jesus." See the whole passage in the conclusion of Bp. Sherlock's 9th Sermon, vol. i.

Of these beautiful passages we may observe, that as, in the English, if you put *it* and *its* instead of *his*, *she*, *her*, you confound and destroy the images, and reduce, what was before highly Poetical and Rhetorical, to mere prose and common discourse; so if you render them into another language, Greek, Latin, French, Italian, or German; in which Hell, Heaven, Cloud, Law, Religion, are constantly Masculine, or Feminine, or Neuter, respectively; you make the images obscure and doubtful, and in proportion diminish their beauty.

In

In the Pronoun are to be considered the Person, Number, Gender, and Case.

There are Three Persons which may be the Subject of any discourse: first, the Person who speaks, may speak of himself; secondly, he may speak of the person to whom he addresses himself; thirdly, he may speak of some other Person.

These are called, respectively, the First, Second, and Third Persons: and are expressed by the Pronouns, *I*, *Thou*, *He*.

As the Speakers, the Persons spoken to, and the other Persons spoken of, may be many; so each of these Persons hath the Plural number; *We*, *Ye*, *They*.

The Person speaking and spoken to, being at the same time the subjects of the discourse, are supposed to be present; from which and other circumstances their Sex is commonly known, and needs not be marked by a distinction of Gender in their Pronouns; but the third Person or Thing spoken of being absent, and in many respects unknown, it is necessary, that it should be marked by a distinction of Gender; at least when some particular Person or Thing is spoken of which ought to be more distinctly marked: accordingly the Pronoun Singular of the Third Person hath the Three Genders: *He*, *She*, *It*.

Pronouns have three Cases; The Nominative, the Genitive or Possessive, like Nouns; and moreover a Case, which follows the Verb Active, or the Preposition, expressing the object of an Action, or of a Relation. It answers to the Oblique Cases in Latin, and may be properly enough called the Objective Case.

PRONOUNS;

according to their Persons, Numbers, Cases, and Genders.

PERSONS.

1.	2.	3.	1.	2	3.
	Singular.			Plural.	
I,	Thou,	He;	We,	Ye or You,	They.

CASES.

Nom.	Poss.	Obj.	Nom.	Poss.	Obj.
		First Person.			
I,	Mine,	Me;	We,	Ours,	Us.
		Second Person.			
Thou,	Thine,	Thee;	Ye or You,	Yours,	You *.
		Third Person.			
Mas. He,	His,	Him;	They,	Theirs,	Them.
Fem. She,	Hers,	Her;			
Neut. It,	Its,	It;			

The Personal Pronouns have the nature of Substantives, and, as such, stand by themselves; the rest have the nature of Adjectives, and, as such, are joined to Substantives; and may be called Pronominal Adjectives.

Thy, My, Her, Ours, Yours, Their, are Pronominal Adjectives: but *His,* (that is, *He's,*) *Her's, Our's, Your's, Their's,* have evidently the Form of the Possessive Case: and by Analogy, *Mine, Thine,* may be esteemed of the same rank. All these are used, when the Noun, to which they belong, is understood: the two latter sometimes also

* Some Writers have used *Ye* as the Objective Case Plural of the Pronoun of the Second Person; very improperly, and ungrammatically.

"The more shame for *ye*: holy men I thought *ye*."

Shakespear, Henry VIII.

"But tyrants dread *ye*, lest your just decree
Transfer the power, and set the people free."—Prior.

"His wrath, which one day will destroy *ye* both."

Milton, P. L. ii. 734.

instead

instead of *my*, *thy*, when the Noun following them begins with a vowel.

Beside the foregoing, there are several other Pronominal Adjectives; which, though they may sometimes seem to stand by themselves, yet have always some Substantive belonging to them, either referred to or understood: as *This*, *that*, *other*, *any*, *some*, *one*, *none*. These are called Definitive, because they *define* and limit the extent of the Common Name, or General Term, to which they refer, or are joined. The three first of these are varied, to express Number; as, *These*, *those*, *others*; the last of which admits of the Plural form only when its Substantive is not joined to it, but referred to, or understood: none of them are varied to express the Gender: only two of them to express the Case; as, *other*, *one*, which have the Possessive Case. *One* is sometimes used in an indefinite sense, (answering to the French *on*,) as in the following phrases; "*one* is apt to think;" "*one* sees;" "*one* supposes." *Who*, *which*, *that*, are called Relatives, because they more directly *refer* to some Substantive *going before*, which therefore is called the Antecedent. They also connect the following part of the Sentence with the foregoing. These belong to all the three Persons; whereas the rest belong only to the Third. One of them only is varied to express the three Cases; *Who*, *whose* *, (that is, *who's*,

* *Whose* is by some authors made the Possessive Case of *which*, and applied to things as well as persons.

"The *question*, *whose* solution I require,
Is, what the sex of women most desire." Dryden.

"Is there any other *doctrine*, *whose* followers are punished?"
Addison.

The higher Poetry, which loves to consider every thing as bearing a Personal Character, frequently applies the personal Possessive *whose* to inanimate beings:

"Of man's first disobedience, and the fruit
Of that forbidden Tree, *whose* mortal taste
Brought death into the world, and all our woe." Milton.

who's *,) *whom:* none of them have different endings for the Numbers. *Who*, *which*, *what*, are called Interrogatives, when they are used in *asking questions*. The two latter of them have no variation of Number or Case. *Each*, *every* †, *either*, are called Distributive; because they denote the Persons, or Things, that make up a number, as taken *separately* and singly.

Own, and *self*, in the Plural *selves*, are joined to the Possessives, *my*, *our*, *thy*, *your*, *his*, *her*, *their*; as, *my own* hand; *myself*, *yourselves*: both of them expressing emphasis, or opposition; as, "I did it my *own self*," that is, I and no one else; the latter also forming the Reciprocal Pronoun; as, "he hurt *himself*." *Himself*, *themselves*, seem to be used in the Nominative Case by corruption, instead of *his self*, *their selves*: as, "he came *himself*," "they did it *themselves*;" where *himself*, *themselves*, cannot be in the Objective Case. If this be so, *self* must be in these instances, not a Pronoun, but a Noun. Thus Dryden uses it:

"What I show,
Thy *self may* freely on thyself bestow."

Ourself, the Plural Pronominal Adjective with the Singular Substantive is peculiar to the Regal Style.

Own is an Adjective; or perhaps the Participle *own* ‡, of

* So the Saxon *hwa* hath the Possessive Case *hwæs*. Note, that the Saxons rightly placed the Aspirate before the *w*, as we now pronounce it. This will be evident to any one that shall consider in what manner he pronounces the words *what*, *when*, that is, *hoo-ăt*, *hoo-ĕn*.

† *Every* was formerly much used as a Pronominal Adjective, standing by itself: as, "He proposeth unto God their necessities, and they their own requests, for relief in *every* of them." Hooker, v. 39. "The corruptions and depravation to which *every* of these was subject." Swift, Contests and Dissentions. We now commonly say, *every one*.

‡ Chaucer has thus expressed it:

"As friendly, as he were his *owen* brother."

Cant. Tales, 1654, edit. 1775. And so in many other places; and, I believe always in the same manner.

of the verb *to owe*; to be the right owner of a thing*.

All nouns whatever in Grammatical Construction are of the Third Person; except when an address is made to a Person: then the Noun, (answering to what is called the Vocative Case in Latin,) is of the Second Person.

ADJECTIVE.

An Adjective is a word *added to* a Substantive to express its quality †.

In English the Adjective is not varied on account of Gender, Number or Case ‡. The only variation which it admits of, is that of the Degrees of Comparison.

Qualities for the most part admit of *more* and *less*, or of different degrees, and the words that express such Qualities have accordingly proper forms to express different degrees. When a Quality is simply expressed without any relation to the same in a different degree, it is called the Positive; as, *wise*, *great*. When it is expressed with augmentation,

* "The Man that *oweth* this girdle." Acts, xxi. 11.

† Adjectives are very improperly called *Nouns*; for they are not the *Names* of things. The Adjectives *good*, *white*, are applied to the Nouns *man*, *snow*, to express the Qualities belonging to those Subjects; but the Names of those Qualities in the Abstract, (that is, considered in themselves, and without being attributed to any subject), are *goodness*, *whiteness*; and these are Nouns, or Substantives.

‡ Some few Pronominal Adjectives must here be excepted, as having the Possessive Case; as, *one*, *other*, *another*; "By *one's* own choice." Sidney.

Teach me to feel *another's* woe." Pope, Univ. Prayer.

And the Adjectives, *former*, and *latter*, may be considered as Pronominal, and representing the Nouns, to which they refer; if the phrase in the following sentence be allowed to be just: "It was happy for the state, that Fabius continued in the command with Minucius: the *former's* phlegm was a check upon the *latter's* vivacity."

or with reference to a less degree of the same, it is called the Comparative; as, *wiser*, *greater*: When it is expressed as being in the highest degree of all, it is called the Superlative; as, *wisest*, *greatest*.

So that the simple word, or Positive, becomes Comparative by adding *r* or *er*; and Superlative by adding *st* or *est*, to the end of it. And the Adverbs *more* and *most* placed before the Adjective have the same effect; as *wise*, *more wise*, *most wise* *.

Monosyllables, for the most part, are compared by *er* and *est*; and Dissyllables by *more* and *most*: as *mild*, *milder*, *mildest*; *frugal*, *more frugal*, *most frugal*. Dissyllables ending in *y*, *happy*, *lovely*; and in *le* after a mute, as *able*, *ample*; or accented on the last syllable, as *discrete*,

* Double Comparatives and Superlatives are improper:

"The Duke of Milan,
And his *more braver* Daughter could controul thee."

Shakespear, Tempest.

"After the *most straitest* sect of our religion I lived a Pharisee." Acts, xxvi. 5. So likewise Adjectives, that have in themselves a Superlative signification, admit not properly the Superlative form superadded: "Whosoever of you will be *chiefest*, shall be servant of all." Mark, x. 44. "One of the first and *chiefest* instances of prudence." Atterbury, Serm. IV. 10. "While the *extremest* parts of the earth were meditating a submission." Ibid. I. 4.

"But first and *chiefest* with thee bring
Him, that yon soars on golden wing,
Guiding the fiery-wheeled throne,
The Cherub Contemplation."

Milton, Il.Penseroso.

"That on the sea's *extremest* border stood."

Addison's Travels.

But poetry is in possession of these two improper Superlatives, and may be indulged in the use of them.

The Double superlative *most highest* is a Phrase peculiar to the old Vulgar Translation of the Psalms; where it acquires a singular propriety from the Subject to which it is applied, the Supreme Being, who is *higher than the highest*.

polite;

polite; easily admit of *er* and *est*. Words of more than two syllables hardly ever admit of those terminations.

In some few words the Superlative is formed by adding the adverb *most* to the end of them; as, *nethermost*, *uttermost*, or, *utmost*, *undermost*, *uppermost*, *foremost*.

In English, as in most languages, there are some words of very common use, (in which the caprice of Custom is apt to get the better of Analogy), that are irregular in this respect: as, *good*, *better*, *best*; *bad*, *worse*, *worst*; *little*, *less* *, *least*; *much*, or *many*, *more*, *most*; and a few others. And in other languages, the words irregular in this respect, are those which express the very same ideas with the foregoing.

VERB.

A Verb is a *word* which signifies to be, to do, or to suffer.

There are three kinds of Verbs; Active, Passive, and Neuter Verbs.

A Verb Active expresses an Action, and necessarily implies an Agent, and an Object acted upon; as, *to love*; "I love Thomas."

* *Lesser*, says Johnson, is a barbarous corruption of *less*, formed by the vulgar from the habit of terminating Comparisons in *er*."

"Attend to what a *lesser* Muse indites." Addison.

"The tongue is like a race-horse; which runs the faster, the *lesser* weight it carries." Addison, Spect. N° 247.

Worser sounds much more barbarous, only because it has not been so frequently used.

"Changed to a *worser* shape thou canst not be."

Shakespear, 1 Hen. VI.

"A dreadful quiet felt, and *worser* far
Than arms, a sullen interval of war." Dryden.

The Superlative *least* ought rather to be written without the *a*, being contracted from *lessest*; as Dr. Wallis hath long ago observed. The conjunction of the same sound, might be written with the *a*, for distinction.

A Verb Passive expresses a Passion, or a Suffering, or the Receiving of an Action; and necessarily implies an Object acted upon, and an Agent by which it is acted upon; as, *to be loved*; "Thomas is loved by me."

So when the Agent takes the lead in the Sentence, the Verb is Active, and is followed by the Object; when the Object takes the lead, the Verb is Passive, and is followed by the Agent.

A Verb Neuter expresses Being; or a state or condition of being; when the Agent and the Object acted upon coincide, and the event is properly Neither action nor passion, but rather something between both; as, *I am*, *I sleep*, *I walk*.

The Verb Active is called also Transitive; because the action *passeth over* to the Object, or hath an effect upon some other thing: and the Verb Neuter is called Intransitive; because the effect is confined within the Agent, and doth *not pass over* to any object.

In English many Verbs are used both in an Active and Neuter signification, the construction only determining of which kind they are.

To the signification of the Verb is superadded the designation of Person, by which it corresponds with the several Personal Pronouns; of Number, by which it corresponds with the number of the Noun, Singular or Plural; of Time, by which it represents the being, action, or passion, as Present, Past, or Future, whether Imperfectly, or Perfectly; that is, whether passing in such time, or then finished; and lastly of Mode, or of the various Manners in which the being, action, or passion is expressed.

In a Verb, therefore, are to be considered the Person, the Number, the Time, and the Mode.

The Verb in some parts of it varies its endings, to express, or agree with, different Persons of the same number: as, "I *love*, Thou *lovest*, He *loveth*, or *loves*"

So

So also to express different Numbers of the same person: as, "Thou *lovest*, Ye *love*; He *loveth*, They *love* *.

So likewise to express different Times in which any thing is represented as being, acting, or acted upon: as, "I *love*, I *loved*; I *bear*, I *boer*, I have *borne*."

The Mode is the *Manner* of representing the Being, Action, or Passion. When it is simply *declared*, or a question asked, in order to obtain a *declaration* concerning it, it is called the Indicative Mode; as, "I *love*; *lovest* thou?" when it is *bidden*, it is called the Imperative; as, "*love* thou:" when it is *subjoined* as the end or design, or mentioned under a condition, a supposition, or the like, for the most part depending on some other Verb, and having a Conjunction before it, it is called the Subjunctive; as, "If I *love*; if thou *love*:" when it is barely expressed *without any limitation* of person or number, it is called the Infinitive; as, "*to love*:" and when it is expressed in a form in which it may be joined to a Noun as its quality or accident, *partaking* thereby of the nature of an Adjective, it is called the Participle; as, "*loving* †.

But to express the time of the Verb the English uses also the assistance of other Verbs, called therefore Auxiliaries, or

* In the Plural Number of the Verb, there is no variation of ending to express the different Persons; and the three Persons Plural are the same also with the first Person Singular: moreover in the Present Time of the Subjunctive Mode all Personal Variation is wholly dropped. Yet is this scanty provision of terminations sufficient for all the purposes of discourse, nor does any ambiguity arise from it: the Verb being always attended either with the Noun expressing the Subject acting or acted upon, or the Pronoun representing it. For which reason the Plural Termination in *en*, *they loven*, *they weren*, formerly in use, was laid aside as unnecessary, and hath long been obsolete.

† A Mode is a particular form of the Verb, denoting the *manner* in which a thing is, does, or suffers: or expressing an intention of mind concerning such being, doing, or suffering. As far as Grammar is concerned, there are no more Modes in any language, than there are forms of the Verb appropriated

or helpers; *do, be, have, shall, will*; as, "I *do* love, I *did* love; I *am* loved, I *was* loved; I *have* loved, I *have been* loved; I *shall*, or *will*, love, or *be* loved."

The two principal Auxiliaries, *to have*, and *to be*, are thus varied, according to Person, Number, Time, and Mode.

Time

priated to the denoting of such different manners of representation. For instance, the Greeks have a peculiar form of a Verb, by which they express the subject, or matter of a wish; which properly constitutes an Optative Mode: but the Latins have no such form; the subject of a Wish in their language is subjoined to the Wish itself either expressed or implied, as subsequent to it and depending on it: they have, therefore, no Optative Mode; but what is expressed in that Mode in Greek falls properly under the Subjunctive Mode in Latin. For the same reason, in English the several expressions of Conditional Will, Possibility, Liberty, Obligation, *&c.* come under the Subjunctive Mode. The mere expression of Will, Possibility, Liberty, Obligation, *&c.* belong to the Indicative Mode: it is their Conditionality, their being subsequent, and depending upon something preceding, that determines them to the Subjunctive Mode. And in this Grammatical Modal Form, however they may differ in other respects Logically, or Metaphysically, they all agree. That Will, Possibility, Liberty, Obligation, *&c.* though expressed by the same Verbs, that are occasionally used as Subjunctive Auxiliaries, may belong to the Indicative Mode, will be apparent from a few examples.

"Here we *may* reign secure.
Or of th' Eternal co-eternal beam
May I express thee unblam'd?"
"Firm they *might* have stood,
Yet fell." Milton.
"What we *would* do,
We *should* do, when we *would*."

Shakespear, Hamlet.

"Is this the nature
Which passion *could* not shake? whose solid virtue
The shot of accident, or dart of chance,
Could neither raze, nor pierce?" Ibid. Othello.

These sentences are all either declarative, or simply interrogative; and however expressive of Will, Liberty, Possibility, or Obligation, yet the Verbs are all of the Indicative Mode.

It seems, therefore, that whatever other Metaphysical Modes there may be in the theory of Univeral Grammar, there are in English no other Grammatical Modes than those above described.

As

Time is Present, Past, or Future.

TO HAVE.

Indicative Mode.

Present Time.

Person. 1. I have,	We	have.
2. Thou hast *,	Ye	
3. He hath, or has †;	They	

Past

As in Latin the Subjunctive supplies the want of an Optative Mode, so does it likewise in English, with the Auxiliary *may* placed before the Nominative Case: as, "Long *may he live!*" Sometimes, chiefly when Almighty God is the Subject, the Auxiliary is omitted: as, "*The LORD bless* thee, and keep thee;" Numb. vi. 24. But the phrase with the Pronoun is obsolete: as, "Unto which *he vouchsafe* to bring us all!" Liturgy.

That the Participle is a mere Mode of the Verb, is manifest, if our Definition of a Verb be admitted: for it signifies being, doing, or suffering, with the designation of Time superadded. But if the essence of the Verb be made to consist in Affirmation, not only the Participle will be excluded from its place in the Verb, but the Infinitive itself also; which certain ancient Grammarians of great authority held to be alone the genuine Verb, denying that title to all the other Modes.

* *Thou*, in the Polite, and even in the Familiar Style, is disused, and the Plural *You* is employed instead of it: we say, *You have;* not, *Thou hast.* Though in this case we apply *You* to a single Person, yet the Verb too must agree with it in the Plural Number: it must necessarily be *You have;* not, *You hast. You was,* the Second Person Plural of the Pronoun placed in agreement with the First or Third Person Singular of the Verb, is an enormous solecism: and yet Authors of the first rank have inadvertently fallen into it. "Knowing that *you was* my old master's good friend." Addison, Spect. No. 517. "The account *you was* pleased to send me." Bently, Phileleuth. Lips. Part II. See the Letter prefixed. Would to God *you was* within her reach!" Bolingbroke to Swift, Letter 46. "If *you was* here." Ditto, Letter 47. "I am just now as well as when *you was* here." Pope to Swift, P. S. to Letter 56. On the contrary, the Solemn Style admits not of *you* for a single Person. This hath led Mr. Pope into a great impropriety in the beginning of his Messiah:

"O *Thou* my voice inspire,
Who *touch'd* Isaiah's hallow'd lips with fire."

The Solemnity of the Style would not admit of *You* for *Thou* in the Pronoun; nor the measure of the Verse *touchedst,* or *didst touch,* in the Verb,

Past Time.

1. I had,	We had.
2. Thou hadst,	Ye had.
3. He had;	They had.

Future Time.

1. I shall or will, have;	We shall, or will, have.
2. Thou shalt, or wilt *, have;	Ye shall, or will, have.
3. He shall, or will, have;	They shall, or will, have.

Imperative

Verb, as it indispensably ought to be, in the one or the other of these two forms; *You,* who *touched;* or *Thou,* who *touchedst,* or *didst touch.*

"What art *thou,* speak, *that* on designs unknown,
While others sleep, thus *range* the camp alone?"
Pope's Iliad, v. 90.

"Accept these grateful tears; for thee they flow:
For *thee, that* ever *felt* another's woe." Ib. xix. 316.

"Faultless *thou dropt* from his unerring skill.
Dr. Arburthnot, Dodsley's Poems, vol. i.

Again:

"Just of *thy* word, in every thought sincere;
Who *knew* no wish, but what the world might hear."
Pope, Epitaph.

It ought to be *your* in the first line, or *knewest* in the second.

In order to avoid this Grammatical Inconvenience, the two distinct forms of *Thou* and *You* are often used promiscuously by our modern Poets, in the same Poem, in the same Paragraph, and even in the same Sentence, very inelegantly and improperly:

"Now, now, I seize, I clasp *thy* charms;
And now *you* burst, ah, cruel! from my arms." Pope.

† *Hath* properly belongs to the serious and solemn style; *has* to the familiar. The same may be observed of *doth* and *does.*

"But confounded with thy art,
Inquires her name, that *has* his heart." Waller.

"Th' unwearied Sun from day to day,
Does his Creator's power display." Addison.

The nature of the style, as well as the harmony of the verse, seems to require in these places *hath* and *doth.*

* The Auxiliary Verb, *will,* is always thus formed in the second and third Persons singular: but the Verb *to will,* not being an Auxiliary, is formed regularly in those Persons: I *will,* Thou *willest,* He *willeth or wills.*

"Thou

Imperative Mode.

1. Let me have,	Let us have,
2. Have thou,	Have ye,
or, Do thou have,	or, Do ye have.
3. Let him have;	Let them have.

Subjunctive Mode.

Present Time.

1. I	have;	We	have.
2. Thou		Ye	
3. He		They	

Infinitive Mode.

Present; To have: Past, to have had.

Participle.

Present, Having: Perfect *, Had:

Past, Having had.

TO BE:

Indicative Mode.

Present Time.

1. I am,	We	are.
2. Thou art.	Ye	
3. He is;	They	

Or,

1. I be,	We	be.
2. Thou beest,	Ye	
3. He is †;	They	

"Thou that art the author and bestower of life, canst doubtless restore it also, if thou *will'st*, and when thou *will'st;* but whether thou *will'st* [wilt] please to restore it, or nor, that Thou alone knowest." Atterbury, Serm. I. 7.

* This participle represents the action as complete and finished; and, being subjoined to the Auxiliary *to have*, constitutes the Perfect Time: I call it, therefore, the Perfect Participle. The same, subjoined to the Auxiliary *to be*, constitutes the Passive Verb; and in that state, or when used, without the Auxiliary in a Passive sense, is called the Passive Participle.

† "I think it *be* thine indeed; for thou liest in it." Shakespear, Hamlet. *Be*, in the singular Number of this Time and Mode, especially in the third Person, is obsolete; and is become somewhat antiquated in the Plural.

Past

Past Time.

1. I was,	We	were.
2. Thou wast,	Ye	
3. He was;	They	

Future Time.

1. I shall, or will,	be;	We	shall,
2. Thou shalt, or wilt,		Ye	or will,
3. He shall, or will,		They	be.

Imperative Mode.

1. Let me be,	Let us be,
2. Be thou,	Be ye,
or, Do thou be,	or Do ye be,
3. Let him be;	Let them be.

Subjunctive Mode.

1. I	be;	We	be.
2. Thou		Ye	
3. He		They	

Past time.

1. I were,	We	were.
2. Thou wert*,	Ye	
3. He were;	They	

Infinitive Mode.

Present, To be: Past, To have Been.

* "Before the sun,
Before the heav'ns thou *wert*."—Milton.

"Remember what thou *wert*."—Dryden.

"I knew thou *wert* not slow to hear."—Addison.

"Thou who of old *wert* sent to Israel's court."—Prior.

"All this thou *wert*."—Pope.

"Thou, Stella, *wert* no longer young,
When first for thee my harp I strung.—Swift.

Shall we in deference to these great authorities allow *wert* to be the same with *wast*, and common to the Indicative and Subjunctive Modes; or rather abide by the practice of our best ancient writers; the propriety of the language, which requires, as far as may be, distinct forms for different Modes; and the analogy of it in each Mode; I *was*, Thou *wast*; I *were*, Thou *wert*? all which conspire to make *wert* peculiar to the Subjunctive Mode.

Participle.

Participle.

Present, Being: Perfect, Been.

Past, Having been.

The Verb Active is thus varied according to Person, Number, Time, and Mode.

Indicative Mode.

Present Time.

Person.	Sing.	Plur.	
	1. I love,	We	love.
	2. Thou lovest,	Ye	
	3. He loveth, or loves;	They	

Past Time.

1. I loved,	We	loved.
2. Thou lovedst,	Ye	
3. He loved;	They	

Future Time.

1. I shall, or will,	love;	We	shall, or will, love.
2. Thou shalt, or wilt,		Ye	
3. He shall, or will,		They	

Imperative Mode.

1. Let me love,	Let us love *.
2. Love thou, or, Do thou love;	Love ye, or, Do ye love,
3. Let him love;	Let them love.

Subjunctive Mode.

Present Time.

1. I	love;	We	love;
2. Thou		Ye	
3. He		They	

* The other form of the first Person Plural of the Imperative, *love we*, is grown obsolete.

And,

1. I may	We	may love :
2. Thou mayest	love ; Ye	and
3. He may	They	have loved *.

Past Time.

1. I might	We	might love ;
2. Thou mightest	love ; Ye	and
3. He might	They	have loved *.

And,

I could, should, would ; Thou could'st, &c. love : and have loved.

Infinitive Mode.

Present, To love : Past, To have loved.

Participle.

Present, Loving : Perfect, Loved.
Past, Having loved.

But in discourse we have often occasion to speak of Time, not only as Present, Past, and Future, at large and indeterminately ; but also as such with some particular distinction and limitation ; that is, as passing, or finished ; as imperfect, or perfect. This will be best seen in an example of a Verb laid out and distributed according to these distinctions of Time.

* Note, that the Imperfect and Perfect Times are here put together. And it is to observed, that in the Subjunctive Mode, the event being spoken of under a condition, or supposition, or in the form of a wish, and therefore as doubtful and contingent, the Verb itself in the Present, and the Auxiliary both of the Present and Past Imperfect Times, often carry with them somewhat of a future sense :—as, " if he come to-morrow, I may speak to him :" —" if he should, or would, come to-morrow, I might, would, could, or should, speak to him." Observe also, that the Auxiliaries *should* and *would* in the Imperfect Times are used to express the Present and Future as well as the Past ; as, " It *is* my desire, that he *should*, or *would*, come *now*, or *to-morrow* ;" as well as, " It *was* my desire, that he *should*, or *would*, come *yesterday*." So that in this Mode the precise Time of the Verb is very much determined by the nature and drift of the Sentence.

Indefinite,

Indefinite, or Undetermined,
Time.

Present,	Past,	Future,
I love,	I loved ;	I shall love.

Definite, or Determined,
Time.

Present Imperfect :	I am (now) loving.
Present Perfect :	I have (now) loved.
Past Imperfect :	I was (then) loving.
Past Perfect :	I had (then) loved.
Future Imperfect :	I shall (then) be loving.
Future Perfect :	I shall (then) have loved.

It is needless here to set down at large the several Variations of the Definite Times, as they consist only in the proper Variations of the Auxiliary, joined to the Present or Perfect Participle, which have already been given.

To express the Present and Past Imperfect of the Active and Neuter Verb, the Auxiliary *do* is sometimes used : I *do* (now) love, I *did* (then) love.

Thus, with very little variation of the principal Verb, the several circumstances of Mode and Time are clearly expressed, by the help of the Auxiliaries, *be*, *have*, *do*, *let*, *may*, *can*, *shall*, *will*.

The peculiar force of the several Auxiliaries is to be observed. *Do* and *did* mark the Action itself, or the time of it *, with greater force and distinction. They are also of

* Perdition catch my soul
But I *do* love thee !"
" This to me
In dreadful secrecy impart they *did*.—Shakespear.
" Die he certainly *did*."—Sherlock, Vol I. Disc. 7.

" Yes, I *did* love her ;" that is, at that time, or once ; intimating a negation, or doubt, of present love.

" The Lord called Samuel ; and he ran unto Eli, and said, Here am I, for thou *calledst* me.—And the Lord called yet again, Samuel. And Samuel arose and went to Eli, and said, Here am I, for thou *didst* call me." 1 Sam. iii. 4—6.

 frequent

frequent and almost necessary use in Interrogative and Negative Sentences. They sometimes also supply the place of another Verb, and make the repetition of it, in the same or a subsequent sentence, unnecessary: as,

"He *loves* not plays,
As thou *dost*, Anthony."

Shakespear, Jul. Cæs.

Let does not only express permission; but praying, exhorting, commanding. *May* and *might* express the possibility or liberty of doing a thing; *can* and *could*, the power. *Must* is sometimes called in for a helper, and denotes necessity. *Will*, in the first Person singular and plural, promises or threatens; in the second and third Persons, only foretells; *shall*, on the contrary, in the first Person, simply foretells; in the second and third Persons, promises, commands, or threatens *. But this must be understood of Explicative Sentences; for when the Sentence is Interrogative, just the reverse for the most part takes place: Thus, "I *shall* go; you *will* go;" express event only: but, "*will* you go?" imports intention; and "*shall* I go?" refers to the will of another. But again, "he *shall* go," and "*shall* he go?" both imply will, expressing or referring to a command. *Would* primarily denotes inclination of will; and *should*, obligation: but they both vary their import, and are often used to express simple event.

Do and *have* make the Present Time; *did*, *had* †, the Past;

* This distinction was not observed formerly as to the word *shall*, which was used in the Second and Third Persons to express simply the Event. So likewise *should* was used, where we now make use of *would*. See the Vulgar Translation of the Bible.

† It has been very rightly observed, that the Verb *had*, in the common phrase, *I had rather*, is not properly used, either as an Active or as an Auxiliary Verb; that being in the Past time, it cannot in this case be properly expressive of time Present; and that it is by no means reducible to any Grammatical

Past; *shall, will,* the Future *let* is employed in forming the Imperative Mode; *may, might, could, would, should,* in forming the Subjunctive. The Preposition *to*, placed before the Verb, makes the Infinitive Mode *. *Have*, through its

Grammatical construction. In truth, it seems to have arisen from a mere mistake, in resolving the familiar and ambiguous abbreviation, *I'd rather*, into I *had rather*, instead of I *would rather*; which latter is the regular analogous, and proper expression.

* Bishop Wilkins gives the following elegant investigation of the Modes, in his *Real Character*, Part III. Chap. 5.

"To shew in what manner the subject is to be joined with his Predicate, the Copula between them is affected with a Particle; which, from the use of it, is called *Modus*, the manner or *Mode*.

Now the Subject and Predicate may be joined together either *Simply*, or with some kind of *Limitation*; and accordingly these Modes are Primary, or Secondary.

The Primary Modes are called by Grammarians Indicative and Imperative.

When the matter is declared to be so, or at least when it seems in the Speaker's power to have it be so, as the bare Union of Subject and Predicate would import; then the Copula is nakedly expressed without any variation: and this manner of expressing it is called the Indicative Mode.

When it is neither declared to be so, nor seems to be immediately in the Speaker's power to have it so; then he can do no more in words, but make out the expression of his will to him that hath the thing in his power: namely, to

$$\text{his}\left\{\begin{array}{l}\text{Superior,}\\ \text{Equal,}\\ \text{Inferior,}\end{array}\right\}\text{ by }\left\{\begin{array}{l}\text{Petition,}\\ \text{Persuasion,}\\ \text{Command.}\end{array}\right\}$$

And the manner of these affecting the Copula, (Be it so, or let it be so), is called the Imperative Mode; of which there are these three varieties, very fit to be distinctly provided for. As for that other use of the Imperative Mode, when it signifies *Permission*; this may be sufficiently expressed by the *Secondary Mode* of Liberty; You *may* do it.

The Secondary Modes are such, as, when the Copula is affected with any of them, make the Sentence to be (as Logicians call it) a *Modal Proposition*.

This happens, when the matter in discourse, namely, the being, or doing, or suffering of a thing, is considered, not *simply by itself*, but *gradually in its causes*; from which it proceeds either *contingently* or *necessarily*.

Then a thing seems to be left as *Contingent*, when the Speaker expresses only the *Possibility* of it, or his own *Liberty* to it.

1. The

its several Modes and Times, is placed only before the Perfect Participle; and *be* in like manner, before the Present and Passive Participles: the rest only before the Verb, or another Auxiliary, in its Primary form.

When an Auxiliary is joined to the Verb, the Auxiliary goes through all the Variations of Person and Number; and the Verb itself continues invariably the same. When there are two or more Auxiliaries joined to the Verb, the first of them only is varied according to Person and Number. The Auxiliary *must* admits of no variation.

The Passive Verb is only the Participle Passive, (which for the most part is the same with the Indefinite Past Time, Active, and always the same with the Perfect Participle), joined to the Auxiliary Verb *to be*, through all its Variations: as, " I *am loved*; I *was loved*; I *have been loved*; I *shall be loved*:" and so on, through all the Persons, the Numbers, the Times, and the Modes.

The Neuter Verb is varied like the Active; but having somewhat of the Nature of the Passive, admits in many instances of the Passive form, retaining still the Neuter

1. The *Possibility* of a thing depends upon the power of its cause; and may be expressed,

$$\text{when} \left\{ \begin{array}{l} \textit{Absolute,} \\ \textit{Conditional,} \end{array} \right\} \text{by the Particle} \left\{ \begin{array}{l} \textit{Can;} \\ \textit{Could.} \end{array} \right.$$

2. The *Liberty* of a thing depends upon a freedom from all obstacles either within or without, and is usually expressed in our language,

$$\text{when} \left\{ \begin{array}{l} \textit{Absolute,} \\ \textit{Conditional,} \end{array} \right\} \text{by the Particle} \left\{ \begin{array}{l} \textit{May;} \\ \textit{Might.} \end{array} \right.$$

3. The *Inclination of the Will* is expressed,

$$\text{if} \left\{ \begin{array}{l} \textit{Absolute,} \\ \textit{Conditional,} \end{array} \right\} \text{by the Particle} \left\{ \begin{array}{l} \textit{Will;} \\ \textit{Would.} \end{array} \right.$$

4. The Necessity of a thing from some *external Obligation*, whether *Natural or Moral*, which we call Duty, is expressed,

$$\text{if} \left\{ \begin{array}{l} \textit{Absolute,} \\ \textit{Conditional,} \end{array} \right\} \text{by the Particle} \left\{ \begin{array}{l} \textit{Must, ought, shall;} \\ \textit{Must, ought, should."} \end{array} \right.$$

See also HERMES, Book I. Chap. viii.

signification; chiefly in such Verbs, as signify some sort of motion, or change of place or condition: as, "I *am come*; I *was gone*; I *am grown*; I *was fallen**." The Verb *am, was*, in this case precisely defines the Time of the action or event, but does not change the nature of it; the Passive form still expressing, not properly a passion, but only a state or condition of Being.

Irregular Verbs.

In English both the Past Time Active and the Participle Perfect, or Passive, are formed by adding to the Verb *ed*; or *d* only, when the Verb ends in *e*: as, "*turn, turned; love, loved*." The Verbs that vary from this rule, in either or in both cases, are esteemed Irregular.

The nature of our language, the Accent and Pronunciation of it, incline us to contract even all our Regular Verbs:

* I doubt much of the propriety of the following examples: "The rules of our holy religion, from which we *are* infinitely *swerved*."—Tillotson, Vol. I. Serm. 27. "The whole obligation of that law and covenant, which God made with the Jews, *was* also *ceased*."—Ib. Vol. II. Serm. 52. "Whose number *was* now *amounted* to three hundred."—Swift, Contests and Dissentions, Chap. 3. "This Mareschal, upon some discontent, *was entered* into a conspiracy against his master."—Addison, Freeholder, N°. 31. "At the end of a Campaign, when half the men *are deserted* or killed."—Addison, Tatler, N°. 42. Neuter Verbs are sometimes employed very improperly as Actives; "Go, *flee thee* away into the land of Judah."—Amos, vii. 12. "I think it by no means a fit and decent thing to *vie Charities*, and erect the reputation of one upon the ruins of another."—Atterbury, Serm. I. 2. "So many learned men, that have spent their whole time and pains to *agree* the sacred with the Profane Chronology."—Sir William Temple, Works, Fol. Vol. I. p. 295.

"How would *the Gods my* righteous *toils succeed*?"

Pope, Odyss. xiv. 447.

—"If *Jove this arm succeed*.—Ibid. xxi. 219.

And Active Verbs are as improperly made Neuter: as, "I must *premise* with three circumstances."—Swift, Q. Anne's Last Ministry, Chap. 2. "Those that think to *ingratiate* with him by calumniating me."—Bentley, Dissert. on Phalaris, p. 519.

thus,

thus, *loved, turned,* are commonly pronounced in one syllable, *lov'd, turn'd:* and the second Person, which was originally in three syllables, *lovedest, turnedest,* is become a dissyllable, *lovedst, turnedst:* for as we generally throw the accent as far back as possible towards the first part of the word, (in some even to the fourth syllable from the end), the stress being laid on the first syllables, the rest are pronounced in a lower tone, more rapidly and indistinctly; and so are often either wholly dropped or blended into one another.

It sometimes happens also, that the word which arises from a regular change, does not sound easily or agreeably; sometimes by the rapidity of our pronunciation the vowels are shortened or lost; and the consonants, which are thrown together, do not easily coalesce with one another, and are therefore changed into others of the same organ, or of a kindred species. This occasions a further deviation from the regular form: thus, *loveth, turneth,* are contracted into *lov'th, turn'th,* and these for easier pronunciation immediately become *loves, turns.*

Verbs ending in *ch, ck, p, x, ll, ss,* in the Past Time Active, and the Participle Perfect or Passive, admit the change of *ed* into *t*; as, * *snatcht, chect, snapt, mixt,* dropping also one of the double letters, *dwelt, past*; for *snatched, checked, snapped, mixed, dwelled, passed:* those that end in *l, m, n, p,* after a diphthong, moreover shorten the diphthong, or change it into a single short vowel; as, *dealt, dreamt, meant, felt, slept,* &c. all for the same reason; from the quickness of the pronunciation, and because the *d* after a short vowel will not easily coalesce with the preceding consonant. Those that end in *ve* change also *ve* into *f*; as, *bereave, bereft, leave, left*; because likewise *v* after a short vowel will not easily coalesce with *t*.

* Some of these Contractions are harsh and disagreeable: and it were better, if they were avoided and disused: but they prevail in common discourse, and are admitted into Poetry.

All

All these, of which I have hitherto given examples, are considered not as Irregular but as Contracted only, in most of them the Intire as well as the Contracted form is used; and the Intire form is generally to be preferred to the Contracted.

The formation of Verbs in English, both Regular and Irregular, is derived from the Saxon.

The Irregular Verbs in English are all monosyllables, unless compounded; and they are for the most part the same words which are Irregular Verbs in the Saxon.

As all our Regular Verbs are subject to some kind of Contraction; so the first Class of Irregulars is of those that become so, from the same cause.

I.

Irregulars by Contraction.

Some Verbs ending in *d* or *t* have the Present, the Past Time, and the Participle Perfect and Passive, all alike, without any variation; as beat, burst *, cast †, cost, cut, heat [1]‡, hit, hurt, knit, let, lift [1], light [1] §, put,

* These two have also *beaten* and *bursten* in the Participle; and in that form they belong to the Third Class of Irregulars.

† Shakespear uses the Participle in the Regular Form:

> "And when the mind is quicken'd, out of doubt
> The organs, tho' defunct and dead before,
> Break up their drowsie grave, and newly move
> With *casted* slough, and fresh celerity."— Hen. V.

‡ "He commanded, that they should heat the furnace one seven times more than it was wont to be *heat.*" Dan. iii. 19.

The Verbs marked thus[1], throughout the three Classes of Irregulars, have the Regular as well as the Irregular Form in use.

§ This Verb in the Past Time and Participle is pronounced short, *light* or *lit*: but the Regular form is preferable, and prevails most in writing.

quit [1], read *, rent, rid, set, shed, shred, shut, slit, split †, spread, thrust, wet [1].

These are Contractions from *beated*, *bursted*, *casted*, &c.; because of the disagreeable sound of the syllable *ed* after *d* or *t*.

Others in the Past Time, and Participle Perfect and Passive, vary a little from the present, by shortening the diphthong, or changing the *d* into *t*; as, lead, led; sweat, swet [1] ‡; meet, met; bleed, bled; breed, bred; feed, fed; speed, sped; bend, bent [1]; lend, lent; rend, rent; send, sent; spend, spent; build, built [1]; geld, gelt; gild, gilt [1]; gird, girt; lose, lost.

Others not ending in *d* or *t* are formed by Contraction; have, *had*, for *haved*; make, *made*, for *maked*; flee, *fled*, for *flee-ed*; shoe, *shod*, for *shoe-ed*.

The following, beside the Contraction, change also the Vowel; sell, sold; tell, told; clothe, clad [1].

Stand, stood; and dare, durst, (which in the Participle hath regularly *dared)*, are directly from the Saxon, *standan*, *stod*; *dyrran*, *dorste*.

II.

Irregulars in *ght*.

The Irregulars of the Second Class end in *ght*, both in

* This Verb in the Past Time and Participle is pronounced short; *read*, *red*, *red*; like *lead*, *led*, *led*; and perhaps ought to be written in this manner: our ancient writers spelt it *redde*.

† Shakespear uses the Participle in the Regular Form:

"That self hand,
Which writ his honour in the acts it did,
Hath, with the courage which the heart did lend it,
Splitted the heart itself."— Ant. and Cleop.

‡ "How the drudging goblin *swet*."
Milton, Allegro.

Shakespear uses *sweaten*, as the Participle of this Verb:

"Grease, that's *sweaten*
From the murtherer's gibbet throw."— Macbeth.

In this form it belongs to the Third Class of Irregulars.

the

the Past Time and Participle; and change the vowel or diphthong into *au* or *ou*: they are taken from the Saxon, in which the termination is *hte*.

		Saxon.	
Bring,	brought:	Bringan,	brohte.
Buy,	bought:	Bycgean,	bohte.
Catch,	caught.		
Fight,	fought *:	Feotan,	fuht.
Teach,	taught:	Tæchan,	tæhte.
Think,	thought:	Thencan,	thohte.
Seek,	sought:	Secan,	sohte.
Work,	wrought:	Weorcan,	worhte.

Fraught seems rather to be an Adjective than the Participle of the Verb to *freight*, which has regularly *freighted*. *Raught* from *reach* is obsolete.

III.

Irregulars in *en*.

The Irregulars of the Third Class form the Past Time by changing the vowel or diphthong of the Present; and the Participle Perfect and Passive, by adding the termination *en*; beside for the most part, the change of the vowel or diphthong. These also derive their formation in both parts from the Saxon.

* "As in this glorious and well *foughten* field
We kept together in our chivalry."

Shakespear, Hen. V.

"On the *foughten* field
Michael, and his Angels, prevalent,
Encamping, plac'd in guard their watches round."

Milton, P. L. VI. 410.

This Participle seems not agreeable to the Analogy of derivation, which obtains in this Class of Verbs.

Present.		Past.	Participle.
a changed	into	*e*.	
Fall,		fell,	fallen.
a	into	*o*.	
Awake,		awoke [1],	[awaked.]
a	into	*oo*.	
Forsake,		forsook,	forsaken.
Shake,		shook,	shaken *.
Take,		took,	taken.
aw	into	*ew*.	
Draw,		drew,	drawn †.
ay	into	*ew*.	
Slay,		slew,	slayn †.
e	into	*a* or *o*,	*o*.
Get,		gat, or got,	gotten.
Help,		[helped ‡.]	holpen [1].
Melt,		[melted,]	molten [1].
Swell,		[swelled,]	swollen [1].
ea	into	*a* or *o*.	
Eat,		ate,	eaten.
			o.
Bear,	bare,	or bore,	born.
Break,	brake,	or broke,	broken.
Cleave,	clave,	or clove [1],	cloven, or cleft.
Speak,	spake,	or spoke,	spoken.

* "A sly and constant knave, not to be *shak'd!*"
Shakespear, Cymb.

"*Wert* thou some star, that from the ruin'd roof
Of *shak'd* Olympus by mischance didst fall."
Milton's Poems.

The Regular form of the Participles in these places is improper.

† When *en* follows a Vowel or Liquid, the *e* is dropped: so *drawn, slayn,* (or *slain,*) are instead of *drawen, slayen;* so likewise *known, born,* are for *knowen, boren,* and so of the rest.

‡ The ancient Irregular form *holpe* is still used in conversation.

Swear,

Swear,	sware,	or swore,	sworn.
Tear,	tare,	or tore,	torn.
Wear,	ware,	or wore,	worn.
Heave,	hove [1],		hoven [1].
Shear,	shore,		shorn.
Steal,	stole,		stolen, or stoln.
Tread,	trode,		trodden.
Weave,	wove,		woven.
ee into *e*,			*o*.
Creep,	crope [1],		[creeped, or crept.]
Freeze,	froze,		frozen.
Seethe,	sod,		sodden.
ee into *aw*.			
See,	saw,		seen.
i long into *i* short,			*i* short.
Bite,	bit,		bitten.
Chide,	chid *,		chidden.
Hide,	hid,		hidden.
Slide,	slid,		slidden.
i long into *o*,			*i* short.
Abide,	abode.		
Climb,	clomb,		[climbed.]
Drive,	drove,		driven.
Ride,	rode,		ridden.
Rise,	rose,		risen.
Shine,	shone [1],		[shined.]
Shrive,	shrove,		shriven.
Smite,	smote,		smitten.
Stride,	strode,		stridden.
Strive,	strove [1],		striven.
Thrive,	throve †,		thriven.

* "Jacob *chode* with Laban."— Gen. xxxi. 36.
Num. xx. 3.

† Mr. Pope has used the Regular form of the Past Time of this Verb:
"In the fat age of pleasure, wealth and ease,
Sprung the rank weed, and *thriv'd* with large increase."—Essay on Crit.

Write,

Write*,	wrote,		written.
i long into *u*,			*i* short.
Strike,	struck,		stricken, or strucken.
i short into *a*.			
Bid,	bade,		bidden.
Give,	gave,		given.
Sit †,	sat,		sitten.
Spit,	spat,		spitten.
i short into *u*.			
Dig,	dug [r],		[digged.]
ie	into	*ay*.	
Lie ‡,		lay,	lien, or lain.
o	into	*e*.	
Hold,		held,	holden.
o	into	*i*.	
Do,		did,	done, i. e. doen.

* This Verb is also formed like those of *i* long into *i* short, Write, writ, written: and by Contraction *writ* in the Participle; but, I think, improperly.

† Frequent mistakes are made in the formation of the Participle of this Verb. The analogy plainly requires *sitten*; which was formerly in use: "The Army having *sitten* there so long."—"Which was enough to make him stir, that would not have *sitten* still, though Hannibal had been quiet."— Raleigh. "That no Parliament should be dissolved, till it had *sitten* five months." Hobbes, Hist. of Civil Wars, p. 257. But it is now almost wholly disused, the form of the Past Time *sat* having taken its place. "The court *was sat*, before Sir Roger came."—Addison, Spect. N° 122. See also Tatler, N° 253 and 265. Dr. Middleton hath, with great propriety, restored the true Participle.—"To have *sitten* on the heads of the Apostles; to have *sitten* upon each of them."—Works, Vol. II. p. 30.

‡ This Neuter Verb is frequently comfounded with the Verb active *to lay*, [that is, to *put* or place;] which is Regular, and has in the Past Time and Participle *layed* or *laid*.

"For him, thro' hostile camps I bent my way;
For him, thus prostrate at thy feet I *lay*:
Large gifts proportion'd to thy wrath I bear."

Pope, Iliad, xxiv. 622.

Here *lay* is evidently used for the Present Time, instead of *lie*. "Before they *were laid* down." Josh. ii. 8. "And he *was laid* down." 2 Sam. xiii. 8. It ought to be, *had lien*, or *lain* down. See also Ruth iii. 7. 1 Sam. iii, 2, 3. 1 Kings, xix. 6. xxi. 4.

oo	into	*o*.	
Choose,		chose,	chosen.
ow	into	*ew*.	
Blow,		blew,	blown.
Crow,		crew,	[crowed.]
Grow,		grew,	grown.
Know,		knew,	known.
Throw,		threw,	thrown.
y	into	*ew*,	*ow*.
Fly *,		flew,	flown †.

The following are Irregular only in the Participle; and that without changing the vowel.

Bake,	[baked,]	baken [1].
Fold,	[folded,]	folden [1] ‡.
Grave,	[graved,]	graven [1].
Hew,	[hewed,]	hewen, or hewn[1].
Lade,	[laded,]	laden.
Load,	[loaded,]	loaden [1].
Mow,	[mowed,]	mown [1].
Owe,	[owed, or ought,]	owen [1].

* That is, as a bird, *volare*; whereas *to flee* signifies *fugere*, as from an enemy. This seems to be the proper distinction between *to fly*, and *to flee*; which in the Present Times are very often confounded. Our Translation of the Bible is not quite free from this mistake. It hath *flee* for *volare*, in perhaps seven or eight places out of a great number; but never *fly* for *fugere*.

† "For rhyme in Greece or Rome was never known,
Till by barbarian deluges *o'erflown*.

Roscommon, Essay.

"Do not the Nile and the Niger make yearly inundations in our days, as they have formerly done? and are not the countries so *overflown* still situate between the tropicks?"—Bentley's Sermons.

"Thus oft by mariners are shown
Earl Godwin's castles *overflown*."—Swift.

Here the Participle of the Irregular Verb, *to fly*, is confounded with that of the Regular Verb, *to flow*. It ought to be in all these places, *overflowed*.

‡ "While they be *folden* together as thorns." Nahum, i. 10.

Rive,

Rive,	[rived,]	riven.
Saw,	[sawed,]	sawn [1].
Shape,	[shaped,]	shapen [1].
Shave,	[shaved,]	shaven.
Shew,	[shewed,]	shewn [1].
or		
Show,	[showed,]	shown.
Sow,	[sowed,]	sown [1].
Straw, -ew,	or -ow, [strawed, &c.]	strown [1].
Wash,	[washed,]	washen [1].
Wax,	[waxed,]	waxen [1].
Wreath,	[wreathed,]	wreathen.
Writhe,	[writhed,]	writhen.

Some Verbs, which change *i* short into *a* or *u*, and *i* long into *ou*, have dropped the termination *en* in the Participle.

i short into *a* or *u*,			*u*.
Begin,	began,		begun.
Cling,	clang,	or clung,	clung.
Drink,	drank,	drunk,	or drunken.
Fling,	flung,		flung.
Ring,	rang,	or rung,	rung.
Shrink,	shrang,	or shrunk,	shrunk.
Sing,	sang,	or sung,	sung.
Sink,	sank,	or sunk,	sunk.
Sling,	slank,	or slung,	slung.
Slink,	slunk,		slunk.
Spin,	span,	or spun,	spun.
Spring,	sprang,	or sprung,	sprung.
Sting,	stung,		stung.
Stink,	stank,	or stunk,	stunk.
String,	strung,		strung.
Swim,	swam,	or swum,	swum.
Swing,	swung,		swung.
Wring,	wrung,		wrung.

In many of the foregoing, the original and analogical form

form of the Past Time in *a*, which distinguished it from the Participle, is grown quite obsolete.

i long into *ou*,		*ou*.	
Bind,	bound,	bound,	or bounden.
Find,	found,	found.	
Grind,	ground,	ground.	
Wind,	wound,	wound.	

The following seem to have lost the *en* of the Participle in the same manner:

Hang *,	hung [1],	hung [1].
Shoot,	shot,	shot.
Stick,	stuck,	stuck.
Come,	came,	come.
Run,	ran,	run.
Win,	won,	won.

Hangen and *scoten*, are the Saxon originals of the two first Participles; the latter of which is likewise still in use in its first form in one phrase: a *shotten* herring. *Stuck* seems to be a contraction from *stucken*, as *struck* is now in use for *strucken*. Chaucer hath *comen* and *wonnen*: *becommen* is even used by Lord Bacon. And most of them still subsist intire in the German; *gehangen*, *kommen*, *gerunnen gewonnen*.

To this third Class belong the Defective Verbs, Be, been; and Go, gone; i. e. goen.

From this distribution and account of the Irregular Verbs, if it be just, it appears, that originally there was no exception from the Rule, That the Participle Preterit, or Passive, in English, ends in *d*, *t*, or *n*. The first form included all the Regular Verbs; and those which are become Irregular by contraction, ending in *t*. To the second properly be-

* This Verb, when Active, may perhaps be most properly used in the Regular form; when Neuter, in the Irregular. But in the Active sense of *furnishing a room with draperies*, the Irregular form prevails. The Vulgar Translation of the Bible uses only the Regular form.

longed only those which end in *ght*, from the Saxon Irregulars in *hte*. To the third, those from the Saxon Irregulars in *en*; which have still, or had originally, the same termination.

The same Rule affords a proper foundation for a division of all the English Verbs into Three Conjugations, or Classes of Verbs, distinguished one from another by a peculiar formation, in some principal part of the Verbs belonging to each: of which Conjugations respectively the three different Terminations of the Participle might be the Characteristics. Such of the contracted Verbs, as have their Participles now ending in *t*, might perhaps be best reduced to the first Conjugation, to which they naturally and originally belonged; and they seem to be of a very different analogy from those in *ght*. But as the Verbs of the first Conjugation would so greatly exceed in number those of both the others, which together make but about 117 *; and as those of the third Conjugation are so various in their form, and incapable of being reduced to one plain rule; it seems better in practice to consider the first in *ed* as the only Regular forms, and the others as deviations from it; after the example of the Saxon and German Grammarians.

To the Irregular Verbs are to be added the Defective; which are not only for the most part Irregular, but are also wanting in some of their parts. They are in general words of most frequent and vulgar use; in which Custom is apt to get the better of Analogy. Such are the Auxiliary Verbs; most of which are of this number. They are in use only in some of their Times and Modes; and some of them are a Composition of Times of several Defective Verbs having the same Signification.

* The whole number of Verbs, in the English language, Regular and Irregular, Simple and Compounded, taken together, is about 4300. See in Dr. Ward's Essays on the English Language, the Catalogue of English Verbs. The whole number of Irregular Verbs, the Defective included, is about 177.

Present.

Present.	Past.	Participle.
Am,	was,	been.
Can,	could.	
Go,	went,	gone.
May,	might.	
Must.		
Quoth,	quoth.	
Shall,	should.	
Weet, wit, or wot;	wot.	
Will,	would.	
Wis,	wist.	

There are not in English so many as a hundred Verbs, (being only the chief part, but not all, of the Irregulars, of the Third Class), which have a distinct and different form from the Past Time Active, and the Participle Perfect, or Passive. The general bent and turn of the language is towards the other form; which make the Past Time and the Participle the same. This general inclination and tendency of the language seems to have given occasion to the introducing of a very great Corruption; by which the form of the Past Time is confounded with that of the Participle in these Verbs, few in proportion, which have them quite different from one another. This confusion prevails greatly in common discourse, and is too much authorised by the example of some of our best Writers. Thus it is said, *He begun*, for *he began; he run*, for *he ran; he drunk*, for *he drank:* the Participle being used instead of the Past Time. And much more frequently the Past Time instead of the Participle: as, *I had wrote, it was wrote*, for *I had written, it was written; I have drank*, for *I have drunk*; *bore*, for *borne*; *chose* for *chosen*; *bid*, for *bidden*; *got*, for *gotten*, &c. This abuse has been long growing upon us, and is continually making further encroachments; as it may be observed in the example of those Irregular

Verbs of the Third Class, which change *i* short into *a* and *u*; as, Cling, clang, clung, in which the original and analogical form of the Past Time in *a* is almost grown obsolete; and the *u* prevailing instead of it, the Past Time is now in most of them confounded with the Participle. The Vulgar Translation of the Bible, which is the best standard of our language, is free from this corruption, except in a few instances; as *hid* is used for *hidden*; *held*, for *holden*, frequently; *bid* for *bidden*; *begot*, for *begotten*, once or twice: in which, and a few other like words, it may perhaps be allowed as a Contraction. And in some of these, Custom has established it beyond recovery: in the rest it seems wholly inexcusable. The absurdity of it will be plainly perceived in the example of some of these Verbs, which custom has not yet so perverted. We should be immediately shocked at, *I have knew*, *I have saw*, *I have gave*, &c. but our ears are grown familiar with, *I have wrote*, *I have drank*, *I have bore*, &c. which are altogether as ungrammatical.

There are one or two small Irregularities to be noted, to which some Verbs are subject in the formation of the Present Participle. The Present Participle is formed by adding *ing* to the Verb; as, *turn*, *turning*. Verbs ending in *e* omit the *e* in the Present Participle: as, *love*, *loving*. Verbs ending with a single consonant preceded by a single Vowel, and if of more than one Syllable, having the accent on the last Syllable, double the Consonant in the Present Participle, as well as in every Part of the Verb in which a Syllable is added: as, *put*, *putting*, *putteth*; *forget*, *forgetting*, *forgetteth*; *abet*, *abetting*, *abetted* *.

* Some Verbs having the Accent on the last Syllable but one, as *worship*, *council*, are represented in the like manner, as doubling the last consonant in the formation of those parts of the Verb, in which a Syllable is added; as, *worshipping*, *counselling*. But this I rather judge to be a fault in the spelling; which neither Analogy nor Pronunciation justifies.

ADVERB.

ADVERB.

ADVERBS are *added to Verbs*, and to Adjectives, to denote some modification or circumstance of an action, or quality: as the manner, order, time, place, distance, motion, relation, quantity, quality, comparison, doubt, affirmation, negation, demonstration, interrogation.

In English they admit of no Variation; except some few of them, which have the degrees of Comparison; as, * "often, oftener, oftenest;" "soon, sooner, soonest;" and those Irregulars derived from Adjectives † in this respect likewise irregular; "well, better, best;" &c.

An Adverb is sometimes joined to another Adverb, to modify or qualify its meaning; as, "very much, much too little, not very prudently."

PREPOSITION.

PREPOSITIONS, so called because they are commonly *put before* the words to which they are applied, serve to connect words with one another, and to show the relation between them.

One great Use of Prepositions in English is to express those relations, which in some languages are chiefly marked by Cases, or the different endings of the Noun.

* The formation of Adverbs in general with the Comparative and Superlative Termination seems to be improper; at least it is now become almost obsolete: as, "Touching things which generally are received,—we are *hardliest* able to bring such proof of their certainty, as may satisfy gainsayers."—Hooker, B. v. 2. "Was the *easilier* persuaded."—Raleigh. "That he may the *stronglier* provide."—Hobbes, Life of Thucyd. "The things *highliest* important to the growing age."—Shaftsbury, Letter to Molesworth. "The question would not be, who loved himself, and who not; but, who loved and served himself the *rightest*, and after the truest manner."—Ibid. Wit and humour. It ought rather to be, *most hardly*, *more easily*, *more strongly*, *most highly*, *most right* or *rightly*. But these Comparative Adverbs, however improper in prose, are sometimes allowable in Poetry.

"Scepter and pow'r, thy giving, I assume;
"And *gladlier* shall resign."—Milton, P. L. vi. 73.

† See above, p. 47.

Most Prepositions originally denote the relations of Place, and have been thence transferred to denote by similitude other relations. Thus, *out*, *in*, *through*, *under*, *by*, *to*, *from*, *of*, &c. *Of* is much the same with *from*: "ask *of* me," that is, *from* me: "made of wood;" "Son *of* Philip;" that is, sprung *from* him. *For*, in its primary sense, is *pro*, *loco alterius*, in the *stead*, or *place*, of another. The notion of Place is very obvious in all the rest.*

Prepositions are also prefixed to words in such a manner, as to coalesce with them. Prepositions, standing by themselves in Construction, are put before Nouns and Pronouns; and sometimes after Verbs; but in this sort of Composition they are chiefly prefixed to Verbs: as, *to outgo*, *to over-*

* The Participle *a* before Participles, in the phrases *a* coming, *a* going, *a* walking, *a* shooting, &c.; and before Nouns, as *a*-bed, *a*-board, *a*-shore, *a*-foot, &c.; seems to be a true and genuine Preposition, a little disguised by familiar use and quick pronunciation. Dr. Wallis supposes it to be the Preposition *at*. I rather think it is the Preposition *on*; the sense of which answers better to the intention of those expressions. *At* has relation chiefly to *place*: *on* has a more general relation, and may be applied to *action*, and many other things, as well as *place*. "I was *on* coming, *on* going," &c.; that is, employed *upon* that particular *action*: so likewise those other phrases above mentioned, *a*-bed, &c. exactly answer to *on* bed, *on* board, *on* shore, *on* foot. Dr. Bentley plainly suppossed *a* to be the same with *on*, as appears from the following passage: He would have a learned University make Barbarisms *a* purpose. Dissert. on Phalaris, p. 223. "The depths *on* trembling fell." J. Hopkins, Ps. lxxvii. 16. That is, as we now say in common discourse, "they fell *a* trembling." And the Proposition *on* has manifestly deviated into *a* in other instances: thus, the Saxon compounded Prepositions *on*gean, *on*mang, *on*butan, are become in English, by the rapidity of pronunciation, *a*gainst, *a*mong, *a*bout. Much in the same manner, John *of* Nokes, and John *of* Stiles, by very frequent and familiar use, become John *a* Nokes, and John *a* Stiles: and one *of the* clock, or rather *on the* clock, is written, one o'clock, but pronounced, one *a* clock. The phrases with *a* before Participles are out of use in the solemn style; but still prevail in familiar discourse. They are established by long usage, and good authority; and there seems to be no reason, why they should be utterly rejected.

come,

come, *to undervalue*. There are also certain Particles, which are thus employed in Composition of words, yet cannot stand by themselves in Construction; as, *a*, *be*, *con*, *mis*, &c. in *abide*, *bedeck*, *conjoin*, *mistake*, &c. these are called Inseparable Prepositions.

CONJUNCTION.

THE CONJUNCTION connects or *joins together* Sentences; so as, out of two, to make one Sentence.

Thus, "You, *and* I, *and* Peter, rode to London," is one Sentence, made up of these three by the Conjunction *and* twice employed; "You rode to London; I rode to London; Peter rode to London." Again, "You *and* I rode to London, *but* Peter staid at home," is one Sentence made up of three by the Conjunctions *and* and *but*: both of which equally connect the Sentences, but the latter expresses an Opposition in the Sense. The first is therefore called a Conjunction Copulative; the other a Conjunction Disjunctive.

The use of Copulative Conjunctions is to connect or to continue the Sentence, by expressing an addition, *and*; a supposition or condition, *if*, *as*; a cause, *because* *, *then*; a motive, *that*; an inference, *therefore*; &c.

The use of Disjunctives is to connect and to continue the Sentence; but withal to express Opposition of meaning in different degrees; as, *or*, *but*, *than*, *although*, *unless*, &c.

INTERJECTION.

INTERJECTIONS, so called, because they are *thrown in between* the parts of a sentence, without making any other alteration in it, are a kind of Natural Sounds adopted to express the affection of the Speaker.

* The Conjunction *because*, used to express the motive, or end, is either improper or obsolete: as, "The multitude rebuked them, *because* they should hold their peace."—Matt. xx. 31. "It is the case of some to contrive false periods of business, *because* they may seem men of dispatch."—Bacon, Essay xxv. We should now make use of *that*.

The

The different Passions have, for the most part, different Interjections to express them.

The Interjection *O*, placed before a Substantive, expresses more strongly an address made to that person or thing; as it marks in Latin what is called the Vocative Case.

SENTENCES.

A SENTENCE is an assemblage of words, expressed in proper form, and ranged in proper order, and concurring to make a complete sense.

The Construction of Sentences depends principally upon the Concord or Agreement, and the Regimen or Government, of Words.

One word is said to agree with another, when it is required to be in like case, number, gender, or person.

One word is said to govern another, when it causeth the other to be in some Case, or Mode.

Sentences are either Simple, or Compounded.

A Simple Sentence hath in it but one Subject, and one Finite Verb; that is, a Verb in the Indicative, Imperative, or Subjunctive Mode.

A Phrase is two or more words rightly put together, in order to make a part of a Sentence; and sometimes making a whole sentence.

The most common PHRASES used in simple Sentences, are the following:

1st. Phrase: The Substantive before a Verb Active, Passive, or Neuter; when it is said, what thing *is*, *does*, or *is done*: as, "I am;" "Thou writest;" "Thomas is loved;" where *I*, *Thou*, *Thomas*, are the Nominative * Cases,

* "He, *whom* ye pretend reigns in heaven, is so far from protecting the miserable sons of men, that he perpetually delights to blast the sweetest flowerets in the Garden of Hope."—Adventurer, No. 76. It ought to be *who*, the Nominative Case to *reigns*; not *whom*, as if it were the Objective Case governed

Cases; and answer to the question, *who*, or *what?* as, "Who is loved? Thomas." And the Verb agrees with the Nominative Case in Number and Person; as, *thou* being the Second Person Singular, the Verb *writest* is so too.

2d Phrase: The Substantive after a Verb Neuter or Passive; when it is said, that such a thing *is* or *is made*, or *thought*, or *called*, such *another thing*; or when the Substantive after the Verb is spoken of the same thing or person with the Substantive before the Verb: as, "A calf becomes an ox;" "Plautus is accounted a Poet;" "I am He." Here the latter Substantive is in the Nominative Case, as well as the former; and the Verb is said to govern the Nominative Case: or, the latter Substantive may be said to agree in Case with the former.

3d Phrase: The Adjective after a Verb Neuter or Passive, in like manner: as, "Life *is short*, and Art *is long*." "Exercise *is esteemed wholesome*."

4th Phrase: The Substantive after a Verb Active, or Transitive; as, when one thing is said to *act* upon, or *do* something to, another: as, "to open a door;" to build a house:" "Alexander conquered the Persians." Here the thing acted upon is in the Objective Case *: as it appears plainly when it is expressed by the Pronoun, which has a proper termination for that Case; "Alexander conquered

governed by *pretend*. "If you were here, you would find three or four in the parlour after dinner, *whom* you would say passed their time agreeably."—Locke, Letter to Molyneux.

"Scotland and *Thee* did in each other live."

— Dryden's Poems, Vol. II. p. 220.

"We are alone; here's none but *Thee* and I."

Shakespear, 2 Henry VI.

It ought in both places to be *Thou*: the Nominative Case to the Verb expressed or understood.

* "For *who love* I so much?"—Shakespear, Merch. of Venice.

"*Whoe'er*

quered *them*: and the Verb is said to govern the Objective Case.

5th Phrase: A Verb following another Verb; as, "Boys love to play:" where the latter Verb is in the Infinitive Mode.

6th Phrase: When one thing is said to belong to another: as, "Milton's Poems;" where the thing to which the other belongs, is placed first, and is in the Possessive Case; or else last, with the Preposition *of* before it: as, "the poems of Milton."

7th Phrase: When another Substantive is added to express and explain the former more fully; as, "Paul the Apostle;" "King George:" where they are both in the same case; and the latter is said to be put in Apposition to the former.

8th Phrase: When the quality of the Substantive is expressed by adding an Adjective to it; as, "a wise man;" "a black horse." Participles have the nature of Ajectives; as, "a learned man;" "a loving father."

9th Phrase: An Adjective with a Verb in the Infinitive Mode following it: as, "worthy to die;" "fit to be trusted."

10th Phrase: When a circumstance is added to a Verb, or an Adjective, by an Adverb: as, "You read well;" "he is very prudent."

"*Whoe'er I woo*, myself would be his wife."—Ibid. Twelfth Night.

"*Whoever* the King *favours*,
The Cardinal will find employment for,
And far enough from Court."—Ibid. Hen. VIII.

"Tell who *loves who*; what favours some partake,
And who is jilted for another's sake."—Dryden, Juvenal, Sat. vi.

"Those *who* he *thought* true to his party."
Clarendon, Hist. Vol. I. p. 667, 8vo.

"*Who* should I *meet* the other night, but my old friend?"—Spect. No. 32.

"*Who* should I *see* in the lid of it but the Doctor?"—Addison, Spec. No. 57.

"Laying the suspicion upon somebody, I know not *who*, in the country."
Swift, Apology prefixed to Tale of a Tub.

In all these places it ought to be *whom*.

11th

12th Phrase: When a circumstance is added to a Verb, or an Adjective, by a Substantive with a Preposition before it: as, "I write for you;" "he reads with care;" "studious of praise;" "ready for mischief."

12th Phrase: When the same Quality in different subjects is compared: the Adjective in the Positive having after it the Conjunction *as*; in the Comparative, the Conjunction *than*; and in the Superlative, the Preposition *of*: as, "white as snow;" "wiser than I;" "greatest of all."

The PRINCIPAL PARTS of a Simple Sentence are the Agent, the Attribute, and the Object. The Agent is the thing chiefly spoken of; the Attribute is the thing or action affirmed, or denied of it: and the Object is the thing affected by such action.

In English the Nominative Case, denoting the Agent, usually goes before the Verb, or Attribution: and the Objective Case, denoting the Object, follows the Verb Active; and it is the order that determines the Case in Nouns: as, "Alexander conquered the Persians." But the Pronoun, having a proper form for each of those Cases, sometimes, when it is in the Objective Case, is placed before the Verb; and, when it is the Nominative Case, follows the Object and Verb: as, "Whom ye ignorantly worship, *him declare I* unto you." And the Nominative Case is sometimes placed after a Verb Neuter: as, "Upon thy right hand *did stand the Queen*:" "On a sudden *appeared the King*." And always when the Verb is accompanied with the Adverb *there*: as, "there *was a man*." The reason of it is plain: the Neuter Verb not admitting of an Objective Case after it, no ambiguity of Case can arise from such a position of the Noun; and where no inconvenience attends it, variety itself is pleasing *.

Who,

* It must then be meant of his sins who *makes*, not of him who *becomes*, *the convert*.—Atterbury, Sermons, I. 2.

Who, *which*, *what*, and the Relative *that*, though in the Objective Case, are always placed before the Verb; as are also their Compounds, *whoever*, *whosoever*, &c.: as, "He *whom* you *seek*." "This is *what*, or the thing *which*, or *that*, you *want*." *Whomsoever* you please to *appoint*."

When the Verb is a Passive, the Agent and Object change places in the Sentence; and the thing acted upon is in the Nominative Case, and the Agent is accompanied with a Preposition: as, "The Persians were conquered by Alexander."

The Action expressed by a Neuter Verb, being confined within the Agent, such Verb cannot admit of an Objective Case after it, denoting a person or thing as the Object of action. Whenever a Noun is immediately annexed to a preceding Neuter Verb, it either expresses the notion with the Verb; as, *to dream a dream*;" "*to live* a virtuous *life*;" or denotes only the circumstance of the action, a Preposition being understood; as, "*to sleep* all *night*," that is, *through* all *the night*; "*to walk a mile*," that is, *through* the space of *a mile*.

For the same reason, a Neuter Verb cannot become a Passive. In a Neuter Verb the Agent and Object are the same, and cannot be separated even in imagination; as in the examples, *to sleep*, *to walk*: but when the Verb is Passive, one thing is acted upon by another really, or by supposition, different from it *.

A Noun

"In him who *is*, and him who *finds a friend*."

Pope, Essay on Man.

"Eye *hath not seen*, nor ear *heard*, neither *have entered* into the heart of man, *the things* which God hath prepared for them that love him."—1 Cor. ii. 9.

There seems to be an impropriety in these sentences, in which the same Noun serves in a double capacity, performing at the same time the offices both of the Nominative and Objective Case.

* That some Neuter Verbs take a Passive Form, but without a Passive Signification, has been observed above; *To split*, like many other English Verbs,

A Noun of Multitude *, or signifying many, may have the Verb and Pronoun agreeing with it either in the Singular or Plural Number; yet not without regard to the import of the word, as conveying unity or plurality of idea: as, "My *people is* foolish, *they have* not known me."—Jer. iv. 22. "The assembly of the wicked *have* inclosed me."—Psalm xxii. 16. perhaps more properly than "*hath* inclosed me." "The *assembly was* very numerous:" much more properly than, "*were* very numerous."

Two or more Nouns in the Singular Number, joined together by one or more Copulative Conjunctions †, have Verbs, Nouns, and Pronouns, agreeing with them in the Plural

Verbs, hath both an Active and a Neuter Signification: according to the former we say, "the force of gun-powder *split the rock*;" according to the latter, "the ship *split* upon the rock;" and converting the Verb Active into a Passive, we may say, "*the rock was split* by the force of gun-powder;" or, "*the ship was split* upon the rock." But we cannot say with any propriety, turning the Verb Neuter into a Passive by inversion of the sentence, "*the rock was split upon* by the ship: as in the passage following: "What success these labours of mine have had, He knows best, for whose glory they were designed. It will be one sure and comfortable sign to me, that they have had some; if it shall appear, that the words I have spoken to you to-day are not in vain: if they shall prevail with you in any measure to avoid those *rocks*, which *are* usually *split upon* in Elections, where multitudes of different inclinations, capacities, and judgements, are interested."—Atterbury, Sermons, IV. 12.

* "And restores to his *Island* that tranquillity and repose, to which *they* had been *strangers* during his absence."—Pope, Dissertation prefixed to the Odyssey. *Island* is not a Noun of Multitude: It ought to be, his *people*; or, *it* had been a *stranger*. "What reason *have the Church* of Rome to talk of modesty in this case?"—Tillotson, Serm. I. 49. "There is, indeed, no *Constitution* so tame and careless of *their* own defence, where any person dares to give the least sign or intimation of being a traytor in his heart."—Addison, Freeholder, N° 52. "All the virtues of *mankind* are to be counted upon a few fingers, but *his* foliies and vices are innumerable."—Swift, Preface to Tale of a Tub. Is not *mankind* in this place a Noun of Multitude, and such as requires the Pronoun referring to it to be in the Plural Number, *their*?

† The Conjunction Disjunctive hath a contrary effect; and, as the Verb, Noun, or Pronoun, is referred to the preceding terms taken separately, it must be in the singular Number. The following Sentences are faulty in this respect;

Plural Number: as, "*Socrates and Plato were* wise; *they were* the most eminent *Philosophers* of Greece." But sometimes, after an enumeration of particulars thus connected, the Verb follows in the Singular Number; and is understood as applied to each of the preceding terms: as, "The glorious Inhabitants of those sacred palaces, where nothing but light and blessed immortality, no shadow of matter for tears, discontentments, griefs, and uncomfortable passions to work upon; but all *joy, tranquillity, and peace,* even for ever and ever *doth dwell.*"—Hooker, B. i. 4. "*Sand and Salt, and a mass* of iron, *is* easier to bear, than a man without understanding."—Ecclus. xxii. 15 *.

If the Singulars so joined together are of several Persons, in making the Plural Pronoun agree with them in Person, the second Person takes place of the third, and the first of both: "*He and You and I* won it at the hazard of *our* lives: *You and He* ſhared it between *you.*

The Neuter Pronoun *it* is sometimes employed to express, 1. the subject of any discourse or inquiry: 2. the state or condition of any thing or person: 3. the thing, whatever it be, that is the cause of any effect or event; or any person or persons considered merely as a Cause. Examples:

1. "'*T*was at the royal feast for Persia won
By Philip's godlike son."—Dryden.
"*It* happen'd on a summer's holiday,
That to the greenwood shade he took his way."
Ibid.

respect: "A man may see a metaphor, *or* an allegory, in a picture, as well as read *them* [it] in a description."—Addison, Dial. I. on medals. "It must, indeed, be confessed, that a lampoon, *or* a satyr, *do* not carry in *them* robbery or murder."—Ibid. Spect. N° 23.

* "And so *was* also *James and John the sons* of Zebedee, which *were partners* with Simon."—Luke, v. 10. Here the two Nouns are not only joined together by the Conjunction Copulative, but are moreover closely connected in sense by the part of the sentence immediately following, in which the correspondent Nouns and Verbs are Plural: the Verb, therefore, preceding in the Singular Number is highly improper.

"Who

"Who is *it* in the press that calls on me?"
Shakespear, Jul. Cæs.

2. "H. How is *it* with you, Lady?
Q. Alas! how is *it* with you?"
Shakespear, Hamlet.

3. "You heard her say herself, *it* was not I.—
'Twas I that kill'd her."—Shakespear, Othello.

"*'Tis these*, that early taint the female soul."—Pope.

"*It* rains; *it* shines; *it* thunders." From which last examples it plainly appears, that there is no such thing in English, nor, indeed, in any language, as a sort of Verbs, which are really Impersonal. The Agent or Person in English is expressed by the Neuter Pronoun; in some other languages it is omitted, but understood.

The Neuter Pronoun *it* is sometimes omitted, and understood: thus, we say, "as appears; as follows;" for, "as *it* appears; as *it* follows:" and, "may be," for, "*it* may be."

The Verb *to be* has always a Nominative Case after it; as, "It *was I*, and not *He*, that did it:" unless it be in the Infinitive Mode: "though you thought it *to be Him* *."

The Adverbs, *when*, *while*, *after*, &c. being left out, the Phrase is formed with the Participle, independent on the rest of the Sentence; as, "The doors being shut, Jesus stood in the midst." This is called the Case absolute. And the Case is in English always the Nominative; as, "God

* "*Whom* do men say, that *I am*? —But *whom* say ye, that *I am*?"—Matt. xvi. 13. 15. So likewise Mark, viii. 27. 29. Luke, ix. 18. 20. "*Whom* think ye, that *I am*?"—Acts, xiii. 25. It ought in all these places to be *who*; which is not governed by the Verb *say* or *think*, but by the Verb *am*; or agrees in Case with the Pronoun *I*. If the Verb were in the Infinitive Mode, it would require the Objective Case of the Relative, agreeing with the Pronoun *me*; "*Whom* think ye, or do you think, *me to be*?"

"To that, *which* once *was thee*."—Prior.

It

"God from the Mount of Sinai, (whose grey top
Shall tremble, *He descending*) *, will himself,
In thunder, lightning, and loud trumpet's sound,
Ordain them laws."—Milton, P. L. xii. 227.

To before a Verb is the sign of the Infinitive Mode: but there are some Verbs, which have commonly other Verbs following them in the Infinitive Mode without the sign *to*: as, *bid*, *dare*, *need*, *make*, *see*, *hear*, *feel*; as also *let*, and

It ought to be, *which was thou*; or, *which thou wast*. "It is not *me* you are in love with."—Spect. N° 290. The Preposition *with* should govern the Relative *whom* understood, not the Antecedent *me*; which ought to be, *I*. "It is not *I*, or *I* am not *the person*, with *whom* you are in love."

"Art thou proud yet?
Aye, that I *am* not *thee*."—Shakespear, Timon.
"Time was, when none would cry, that oaf *was me*;
But now you strive about your Pedigree."—Dryden, Prologue.
"Impossible; it *can't be me*."—Swift.

* On which place, says Dr. Bentley, "The Context demands that it be, *Him* descending, Illo descendente. But *him* is not the Ablative Case, for the English knows no such Case; nor does *him* without a Preposition on any occasion answer to the Latin Ablative *illo*. I might with better reason contend that it ought to be "*his* descending," because it is in Greek, αὐτοῦ καταβαίνοντος, in the Genitive; and it would be as good Grammar, and as proper English. This comes of forcing the English under the rules of a foreign Language, with which it has little concern: and this *ugly and deformed fault*, to use his own expression, Bentley has endeavoured to impose upon Milton in several places; see P. L. vii. 15. ix. 829. 883. 1147. x. 267. 1001. On the other hand, where Milton has been really guilty of this fault, he, very inconsistently with himself, corrects him, and sets him right. His Latin Grammar Rules were happily out of his head, and, by a kind of *vernacular instinct*, (so, I imagine, he would call it,) he perceived that his Author was wrong.

"For only in destroying I find ease
To my relentless thoughts: and, *him destroy'd*,
Or *won* to what may work his utter loss,
For whom all this was made, all this will soon
Follow, as to him link'd with weal or woe."
P. L. ix. 129.

It ought to be, "*he* destroyed," that is, *he being* destroy'd. Bentley corrects it, "and *man* destroyed."

Archbishop Tillotson has fallen into the same mistake: "Solomon was of this mind; and I make no doubt, but he made as wise and true Proverbs as any body has done since: *Him* only *excepted*, who was a much greater and wiser man than Solomon."—Serm. I. 53.

perhaps

perhaps a few others; as, "I *bade* him do it; you *dare* not do it; I *saw* him do * it; I *heard* him say it †."

The Infinitive Mode is often made Absolute, or used independently of the rest of the Sentence; supplying the Place of the Conjunction *that* with the Subjunctive Mode: as, "*to confess* the truth, I was in fault;" "*to begin* with the first:" "*to proceed;*" "*to conclude:*" that is, "*that I may confess*; &c."

The Infinitive Mode has much of the nature of a Substantive; expressing the Action itself, which the Verb signifies; as the Participle has the nature of an Adjective. Thus the Infinitive Mode does the office of a Substantive in different Cases; in the Nominative; as, "*to play* is pleasant:" in the Objective; as, "boys *love to play.*" In Greek it admits of the Article through all its cases, with the Preposition in the Oblique cases: in English the Article is not wanted, but the Preposition may be used: "For *to will* is present with me: but *to perform* that which is good I *find* not." "All their works they do *for to be seen* of men." But the use of the Preposition, in this and the like phrases, is now become obsolete.

"For not *to have been dipp'd* in Lethe's lake,
Could save the Son of Thetis *from to die.*"—Spenser.

Perhaps

* "To *see* so many *to make* so little conscience of so great a sin."—Tillotson, Serm. I. 22. "It cannot but be a delightful spectacle to God and Angels to *see* a young person besieged by powerful temptations on either side, *to acquit* himself gloriously, and resolutely *to hold* out against the most violent assaults: to *behold* one in the prime and flower of his age, that is courted by pleasures and honours, by the devil and all the bewitching vanities of the world, *to reject* all these, and *to cleave* stedfastly unto God."—Ib. Serm. 54. The impropriety of the Phrases distinguished by Italic Characters is evident.—See Matt. xv. 31.

† "What know you not,
That, being mechanical, you *ought* not *walk*,
Upon a labouring day, without the sign
Of your profession?"—Shakespear, Jul. Cæs.

Both Grammar and Custom require, "*ought* not *to walk.*" *Ought* is not one of the Auxiliary Verbs, though often reckoned among them: that it

cannot

Perhaps, therefore, the Infinitive, and the Participle, might be more properly called the Substantive Mode, and the Adjective Mode.

The Participle with a Preposition before it, and still retaining its Government, answers to what is called in Latin the Gerund: as, " Happiness is to be attained, by avoiding evil, and by doing good; by seeking peace, and by pursuing it."

The Participle, with an Article before it, and the Preposition *of* after it, becomes a Substantive expressing the action itself which the Verb signifies *: as, " These are the Rules of Grammar, by *the observing of* which you may avoid

cannot be such, is plain from this consideration: that, if we consult custom and our ear, it does not admit of another Verb immediately following it without the Preposition *to*.

* This rule arises from the nature and idiom of our language, and from as plain a principle as any on which it is founded: namely, that a word which has the Article before it, and the Possessive Preposition *of* after it, must be a Noun; and if a Noun, it ought to follow the Construction of a Noun, and not to have the Regimen of a Verb. It is the Participle Termination of this sort of words that is apt to deceive us, and make us treat them, as if they were of an amphibious species, partly Nouns, and partly Verbs. I believe, there are hardly any of our writers, who have not fallen into this inaccuracy. That it is such will perhaps more clearly appear, if we examine and resolve one or two examples in this kind.

" God, who didst teach the hearts of thy faithful people, by *the sending* to them *the light* of thy Holy Spirit:"— Collect, Whitsunday. *Sending* is in this place a Noun; for it is accompanied with the Article; nevertheless it is also a Transitive Verb, for it governs the Noun *light* in the Objective Case: but this is inconsistent; let it be either the one or the other, and abide by its proper construction. That these Participial Words are sometimes real Nouns, is undeniable; for they have a plural Number as such: as, " the *out-goings* of the morning." *The Sending* is the same with *the Mission*; which necessarily requires the Preposition *of* after it, to mark the relation between it and *the light*: *the mission of the light*; and so, *the sending of the light*. The phrase would be proper either way; by keeping to the Construction of the Noun, by *the sending of the light*; or of the Participle, or Gerund, *by sending the light*.

Again:—" Sent to prepare the way of thy son our Saviour, *by preaching of Repentance*."—Collect, St. John Baptist. Here the Participle, or Gerund, hath as improperly the Preposition *of* after it; and so is deprived of its Verbal Regimen,

avoid mistakes." Or it may be expressed by the Participle, or Gerund, " by *observing* which :" not, " by *observing of* which;" nor, " by *the observing* which :" for either of those two Phrases would be a confounding of two distinct forms.

I will add another example, and that of the best authority: " the middle station of life seems to be the most advantageously situated for *the gaining of* wisdom. Poverty turns our thoughts too much upon *the supplying of* our wants; and riches, upon *enjoying our* superfluities."—Addison, Spect. N° 464.

The Participle is often made Absolute, in the same manner, and to the same sense, as the Infinitive Mode: as, " This, generally *speaking*, is the consequence."

The Participle frequently becomes altogether an Adjective; when it is joined to a Substantive merely to denote its quality; without any respect to time: expressing, not an Action, but a Habit; and, as such, it admits of the degrees of Comparison: as, " a learned, a more learned, a most learned man; a loving, more loving, most loving father*."

Simple

Regimen, by which, as a Transitive, it would govern the Noun *Repentance* in the Objective Case. Besides, the phrase is rendered obscure and ambiguous; for the obvious meaning of it in its present form is, " by preaching concerning Repentance, or on that Subject;" whereas the sense intended is, " by publishing the Covenant of Repentance, and declaring Repentance to be a condition of acceptance with God." The phrase would have been perfectly right and determinate to this sense, either way; by the Noun, *by the preaching of repentance*; or by the Participle, *by preaching repentance*.

* In a few instances the Active Present Participle hath been vulgarly used in a Passive sense: as, *beholding* for *beholden*; *owing* for *owen*. And some of our writers are not quite free from this mistake. " I would not be *beholding* to fortune for any part of the victory."—Sidney.

" I'll teach you all what's *owing* to your Queen."—Dryden.

" The debt, *owing* from one country to the other, cannot be paid without real effects sent thither to that value."—Locke.

" We have the means in our hands, and nothing but the application of them is *wanting*."

 " His

Simple Sentences are, 1. Explicative, or explaining: 2. Interrogative, or asking: 3. Imperative, or commanding.

1. An explicative Sentence is, when a thing is said to be, or not to be; to do, or not to do; to suffer, or not to suffer; in a direct manner: as in the foregoing examples. If the Sentence be Negative, the Adverb *not* is placed after the Auxiliary; or after the Verb itself, when it has no Auxiliary; as, "it *did not* touch him;" or, "it *touched* him *not*."

2. In an Interrogative Sentence, or when a Question is asked, the Nominative Case follows the Principal Verb, or the Auxiliary: as, "*was it* he?" "*did Alexander* conquer the Persians?" And the Adverb *there*, accompanying the Verb Neuter, is also placed after the Verb: as, "*was there a man?*" So that the Question depends entirely on the order of the words *.

3. In an Imperative Sentence, when a thing is commanded to be, to do, to suffer, or not; the Nominative case follows the Verb or the Auxiliary: as, "*Go thou* traytor;"

"His estate is dipped, and is *eating* out with usury."—Steele, Spect. N°. 114.

So likewise the Passive Participle is often employed in an Active sense in the word *mistaken*, used instead of *mistaking*.

"You are too much *mistaken* in this King."—Shakespear, Hen. V.

"I mistake;" or, "I am mistaking;" means, "I misunderstand;" but "I am mistaken," means properly, "I am misunderstood."

But in some of these Participles the Abuse is so authorised by Custom as almost to have become an Idiom of the language.

* "*Did* he *not fear* the Lord, and *besought* the Lord, and the Lord *repented* him of the evil, which he had pronounced against them?"—Jer. xxvi. 19. Here the Interrogative and Explicative forms are confounded. It ought to be, "Did he not *fear* the Lord, and *beseech* the Lord? and *did not* the Lord *repent* him of the evil—?" "If a man have an hundred sheep, and one of them be gone astray, *doth* he *not leave* the ninety and nine, and *goeth* into the mountains, and *seeketh* that which is gone astray?"—Matt. xviii. 12. It ought to be, *go*, and *seek*; that is, *doth* he *not go*, and *seek* that which is gone astray?

to ;" or, "*do thou* go ;" or the Auxiliary *let*, with the Objective * case after it, is used : as, "*let us* be gone." †

The

* "For ever in this humble cell
Let Thee and I, my fair one, dwell.—Prior.

It ought to be *Me*.

† It is not easy to give particular rules for the management of the Modes and Times of Verbs with respect to one another, so that they may be proper and consistent ; nor would it be of much use ; for the best rule that can be given is this very general one, to observe what the sense necessarily requires. But it may be of use to consider a few examples, that seem faulty in these respects ; and to examine where the fault lies.

"Some, who the depths of eloquence *have found*,
In that unnavigable stream *were drown'd*."
Dryden, Juv. Sat. x.

The event mentioned in the first line is plainly prior in time to that mentioned in the second ; this is subsequent to that, and a consequence of it. The first event is mentioned in the Present Perfect Time ; it is present and completed ; "They *have* [now] *found* the depths of eloquence." The second event is expressed in the Past Indefinite Time ; it is past and gone, but, when it happened, uncertain : "they *were drown'd*." We observed, that the last mentioned event is subsequent to the first ; but how can the Past Time be subsequent to the Present ? It therefore ought to be, in the second line, *are*, or *have been drown'd*, in the Present Indefinite, or Perfect, which is consistent with the Present Perfect Time in the first line : or, in the first line, *had found* in the Past Perfect ; which would be consistent with the Past Indefinite in the second line.

"Friend to my life, which *did* not you *prolong*,
The world *had wanted* many an idle song."
Pope, Epist. to Arbuthnot.

It ought to be, either, *had* not you *prolonged* ; or, *would want*.

There seems to be a fault of the like nature in the following passage :

"But oh ! '*twas* little that her life
O'er earth and waters *bears* thy fame."—Prior.

It ought to be *bore* in the second line.

Again :

"Him portion'd maids, apprentic'd orphans *blest*,
The young who *labour*, and the old who *rest*."
Pope, Moral Ep. iii. 267.

"Fierce as he mov'd his silver shafts *resound*."—Iliad, B. i.

The first Verb ought to be in the same Time with the following :

"Great Queen of Arms, whose favour Tydeus won,
As thou *defend'st* the sire, defend the son."—Pope, Iliad, x. 337.

It ought to be *defendedst*.

"Had

The ADJECTIVE in English having no variation of Gender or Number, cannot but agree with the Substantive in those respects; some of the Pronominal Adjectives only excepted, which have the Plural number: as, *those*, *these*: which must agree in number* with their Substantives. Nouns

"Had their records been delivered down in the vulgar tongue,—they could not now be understood, unless by Antiquaries, who *made* it their study to expound them."—Swift, Letter on the English Tongue. Here the latter part of the sentence depends entirely on the *Supposition* expressed in the former, "of their records being delivered down in the vulgar tongue;" therefore *made* in the Indicative Mode, which implies no supposition, and in the Past Indefinite Time, is improper; it would be much better in the Past Definite and Perfect, *had made*; but indeed ought to be in the Subjunctive Mode, Present or Past Time, *should make*, or *should have made*.

"And Jesus answered, and said unto him, What wilt thou, that I should do unto thee? The blind man said unto him; Lord, that I *might* receive my sight."—Mark, x. 51. "That I may know him, and the power of his resurrection, and the fellowship of his sufferings, being made conformable unto his death: If by any means I *might* attain unto the resurrection of the dead." —Phil. iii. 10, 11. It ought to be *may* in both places.

"*I thought to have written* last week," is a very common phrase: the Infinitive being in the past Time, as well as the Verb which it follows. But it is certainly vicious; for how long soever it *now* is since I *thought*, *to write* was then present to me; and must still be considered as present, when I bring back that time, and the thoughts of it. It ought to be, therefore, I *thought to write* last week. "I cannot excuse the remissness of those, whose business it *should have been*, as it certainly *was* their interest *to have interposed* their good offices."—Swift. "There were two circumstances, which *would have made* it necessary for them *to have lost* no time."—Ibid. "History Painters *would have found* it difficult, *to have invented* such a species of beings."—Addison, Dial. I. on Medals. It ought to be, "*to interpose*, *to lose*, *to invent*."

* By *this means* thou shalt have no portion on this side the river."—Ezra iv. 16. "It renders us careless of approving ourselves to God by religious duties, and by *that means* seeing the continuance of his goodness."—Atterbury, Sermons. Ought it not to be, by *these means*, by *those means*? or by *this mean*, by *that mean*, in the singular number? as it is used by Hooker, Sidney, Shakespear, &c.

"We have strict statutes, and most biting laws,
Which for *this* nineteen *years* we have let sleep."

Shakespear, Meas. for Meas.

"I have

Nouns of Measure, Number, and Weight, are sometimes joined in the Singular form with Numeral Adjectives denoting Plurality: as, "fifty *foot*; six *score*."

"Ten thousand *fathom* deep."

Milton, P. L. ii. 934.

"A hundred *head* of Aristotle's friends."

Pope, Dunciad, iv. 192.

"About an hundred *pound* weight."—John, xix. 39.

The Adjective generally goes before the Noun: as, "a wise man; a good horse;" unless something depend on the Adjective; as, "food convenient for me," or the Adjective be emphatical; as, "Alexander the Great;" and it stands immediately before the Noun, unless the Verb *to be*, or any Auxiliary joined to it, come between the Adjective and the Noun: as, "happy is the man; happy shall he be." And the Article goes before the Adjective: except the Adjectives, *all, such,* and *many*, and others subjoined to the Adverbs, *so, as,* and *how*: as, "*all the* men;" "*such* a man;" "*many a* man;" "*so good a* man;" "*as good a* man as ever lived;" "*how beautiful a* prospect is here!"

"I have not wept *this* forty *years*."—Dryden. "If I had not left off troubling myself about *those kind* of things."—Swift, Letter to Steele. "I fancy *they are those kind* of Gods, which Horace mentions in his allegorical vessel."—Addison, Dial. II. on Medals. "I am not recommending *those kind* of sufferings to your liking."—Bishop Sherlock, Vol. II. Disc. 11. The foregoing phrases are all improper. So the Pronoun must agree with its Noun; in which respect let the following example be considered. "*It is* an unanswerable argument of a very refined age, the wonderful *Civilities* that have passed between the nation of authors and that of readers."—Swift, Tale of a Tub, Sect. x. As to those wonderful *Civilities*, one might say, that *they are* an unanswerable argument, &c. but as the Sentence stands at present, it is not easy to reconcile it to any grammatical propriety. "*A person* [that is, *one*] *whom* all the world allows to be so much your *betters*."—Swift, Battle of Books. "His face was easily taken either in painting or sculpture; and scarce any *one*, though never so indifferently skilled in *their* art, failed to hit it."—Welwood's Memoirs, p. 68, 6th. Edit.

And

And sometimes, when there are two or more Adjectives joined to the Noun, the Adjective follows the Noun: as, " a man learned and religious."

There are certain Adjectives, which seem to be derived without any variation from Verbs, and have the same signification with the Passive Participles of their Verbs: they are indeed no other than Latin Passive Participles adapted to the English termination: as, *annihilate*, *contaminate*, *elate*;

" To destruction sacred and *devote*.—Milton.

" The alien compost is *exhaust*."—Phillips, Cyder.

These (some few excepted, which have gained admission into common discourse) are much more frequently, and more allowably, used in poetry, than in prose *.

The Distributive Pronominal Adjectives, *each*, *every*, *either*, agree with the Nouns, Pronouns, and Verbs of the Singular number only †: as, " The King of Israel and Je-

* Adjectives of this sort are sometimes very improperly used with the Auxiliary *have*, or *had*, instead of the Active Perfect Participle: as, " Which also King David did dedicate unto the Lord, with the silver and gold that he *had dedicate* of all nations which he subdued."—2 Sam. viii. 11. " And Jehoash took all the hallowed things, that—his fathers, Kings of Judah, *had dedicate*."—2 Kings, xii. 18. So likewise Dan. iii. 19. It ought to be, *had dedicated*. " When both interests of Tyranny and Episcopacy *were incorporate* into each other."—Milton, Eiconoclast. xvii.

† " Let *each* esteem other better than themselves."—Phil. ii. 3. It ought to be, *himself*. " It is requisite that the language of an heroic poem should be both perspicuous and sublime. In proportion as *either* of these two qualities *are* [is] wanting, the language is imperfect."—Addison, Spect. N°. 285.

There is a like impropriety in the following Sentence: " I do not mean by what I have said, that I think *any one* to blame for taking due care of *their* health."

Either is often improperly used instead of *each*: as, " The King of Israel and Jehoshaphat, King of Judah, sat *either* [each] of them on his throne." " Nadab and Abihu, the sons of Aaron, took *either* [each] of them his censer." *Each* signifies *both* of them, taken distinctly or separately: *either* properly signifies *only the one*, *or the other*, of them, taken disjunctively. For which reason the like expression in the following passages seems also improper: " They crucified two other with him, on *either* side one, and Jesus in the midst." " Of *either* side of the river was there the tree of life."

hoshaphat,

hoshaphat, the King of Judah, sat *each* [king] on *his* throne, having [*both*] put on their robes."—1 Kings, xxii. 10. "*Every tree* is known by *his* own fruit."—Luke vi. 44.

"Lepidus flatters both,
Of both is flattered; but he neither loves,
Nor *either cares* for him."

Shakespear, Ant. and Cleop.

Unless the plural Noun convey a Collective idea; as, "that *every twelve Years* there should be set forth two ships."

Bacon.

Every Verb, except in the Infinitive or the Participle, hath its Nominative case, either expressed or implied: as,

"Awake, arise, or be for ever fall'n:"

that is, "Awake *ye*, &c."

Every Nominative case, except the case Absolute, and when an address is made to a Person, belongs to some Verb, either expressed or implied; as in the answer to a Question: "Who wrote this Book? Cicero:" that is, "Cicero *wrote it*." Or when the Verb is understood; as,

"To whom thus Adam:"

that is, *spake*.

Every Possessive case supposes some Noun to which it belongs: as when we say, "St Paul's, or St. James's," we mean St. Paul's *Church*, or St. James's *Palace*.

Every Adjective has relation to some Substantive, either expressed or implied: as, "The Twelve," that is, *Apostles*; "the wise, the elect," that is, *persons*.

In some instances the Adjective becomes a Substantive, and has an Adjective joined to it: as, "the chief Good;" "Evil, be thou my Good *!"

In

* Adjectives are sometimes employed as Adverbs; improperly, and not agreeably to the Genius of the English language. As, "*indifferent* honest, *excellent* well:"—Shakespear, Hamlet. "*Extreme* elaborate."—Dryden, Essay on Dram. Poet. "*Marvellous* graceful."—Clarendon, Life, p. 18. "*Marvellous* worthy to be praised." Psal. cxlv. 3. for so the Translators gave it.

"*Extreme*

In others, the Substantive becomes an Adjective, or supplies its place; being prefixed to another Substantive, and linked to it by a mark of conjunction; as, "sea-water; land-tortoise; forest-tree."

ADVERBS have no Government.

The Adverb, as its name imports, is generally placed close or near to the word which it modifies or affects; and its propriety

"*Extreme* unwilling," "*extreme* subject."—Swift, Tale of a Tub, and Battle of Books. "*Extraordinary* rare."—Addison, on Medals. "He behaved himself *conformable* to that blessed example."—Sprat'sSermons, p. 80. "I shall endeavour to live hereafter *suitable* to a man in my station."—Addison, Spect. No. 530. "The Queen having changed her ministry *suitable* to her own wisdom."—Swift, Exam. No. 28. "The assertions of this Author are *easy* detected."—Swift, Public Spirit of the Whigs. "The Characteristic of his Sect allowed him to affirm no *stronger* than that."—Bentley, Phil. Lips. Remark liii. "If our author had spoken *nobler* and *loftier* than another."—Ibid. "Xenophon says, *express*."—Ibid. Remark xiv. "I can never think so very *mean* of him."—Ibid. Dissertation on Phalaris, p. 24. "Homer describes this river *agreeable* to the vulgar reading."—Pope, Note on Iliad, ii. ver. 1032. So *exceeding*, for *exceedingly*, however improper, occurs frequently in the Vulgar Translation of the Bible, and has obtained in common discourse. "Many men reason *exceeding clear* and rightly, who know not how to make a syllogism."—Locke. "We should live soberly, righteously, and *godly*, in this present world."—Tit. ii. 12. See also 2 Tim. iii. 12. "To convince all that are ungodly among them, of all their ungodly deeds, which they have *ungodly* committed."—Jude, 15. "I think it very *masterly* written."—Swift to Pope, Letter lxxiv.

"O Liberty, Thou Goddess *heavenly* bright."—Addison.

The Termination *ly*, being a contraction of *like*, expresses *similitude* or *manner*; and, being added to Nouns, forms Adjectives; and, added to Adjectives, forms Adverbs. But Adverbs expressing *similitude*, or *manner*, cannot be so formed from Nouns: the few Adverbs, that are so formed, have a very different import: as, *daily*, *yearly*; that is, day by day, year by year. *Early*, both Adjective and Adverb, is formed from the Saxon Preposition *ær*, *before*. The Adverbs, therefore, above noted, are not agreeable to the Analogy of formation established in our Language, which requires *godlily*, *ungodlily*, *heavenlily*: but these are disagreeable to the ear, and, therefore, could never gain admittance into common use.

The word *lively*, used as an Adverb, instead of *livelily*, is liable to the objection; and, not being so familiar to the ear, immediately offends it.

"That

propriety and force depend on its position*. Its place for the most part is before Adjectives; after Verbs Active or Neuter; and it frequently stands between the Auxiliary and the Verb: as, "He made a *very elegant* harangue; he *spake unaffectedly* and *forcibly*; and *was attentively heard* by the whole audience."

Two Negatives in English destroy one another, or are equivalent to an Affirmative†: as,

"*Nor* did they *not* perceive the evil plight
In which they were, or the fierce pains *not* feel."

Milton, P. L. i. 335.

PREPOSITIONS have a Government of Cases: and in

"That part of poetry must needs be best, which describes most *lively* our actions and passions, our virtues and our vices."—Dryden, Pref. to State of Innocence. "The whole design must refer to the Golden Age, which it *lively* represents."—Addison, on Medals, Dial. II.

* Thus it is commonly said, "I *only* spake three words;" when the intention of the speaker manifestly requires, "I spake *only* three words."

"Her body shaded with a slight cymare,
Her bosom to the view was *only* bare."

Dryden, Cymon and Iphig.

The sense necessarily requires this order:

"Her bosom *only* to the view was bare."

† The following are examples of the contrary:

"Give not me counsel;
Nor let *no* comforter delight mine ear."

Shakespear, Much ado.

"She cannot love
Nor take *no* shape *nor* project of affection."—Ibid.

Shakespear uses this construction frequently. It is a relique of the ancient style, abounding with Negatives; which is now grown wholly obsolete:

"And of his port as meke as is a mayde:
He *never* yet *no* vilanie *ne* sayde
In alle his lif unto *no* manere wight.
He was a veray parfit gentil knight."—Chaucer.

"I *cannot* by *no* means allow him, that this argument must prove,—" —Bentley, Dissert. on Phalaris, p. 515. "That we need not, *nor* do *not*, confine the purposes of God."—Ibid. Sermon viii.

English they always require the Objective Case after them: as, "*with him*; *from her*; *to me* *."

The Preposition is often separated from the Relative which it governs, and joined to the Verb at the end of the Sentence, or of some member of it: as, "Horace is an author, *whom* I am much delighted *with*." The world is too well bred to shock authors with a truth, *which* generally their booksellers are the first that inform them *of*." This is an idiom, which our language is strongly inclined to: it prevails in common conversation, and suits very well with the familiar style in writing: but the placing of the Preposition before the Relative is more graceful, as well as more perspicuous; and agrees much better with the solemn and elevated style †.

Verbs are often compounded of a Verb and a Preposition; as, *to uphold*, *to outweigh*, *to overlook*: and this composition sometimes gives a new sense to the Verb; as, *to understand*, *to withdraw*, *to forgive* ‡. But in English the Preposi-

* "*Who* servest thou *under*?"—Shakespear, Hen. V.

"*Who* do you speak *to*?"—As you like it.

"I'll tell you, *who* Time ambles *withal*, *who* Time trots *withal*, *who* Time gallops *withal*, and *who* he stands still *withal*.

"I pr'ythee, *who* doth he trot *withal*?"—Ibid.

"We are still much at a loss, *who* civil power belongs *to*."—Locke.

In all these places, it ought to be *whom*.

"Now Margaret's curse is fall'n upon our heads;
When she exclaim'd *on* Hastings, you, and I."—Shakespear, Rich. III.

It ought to be *me*.

† Some writers separate the Preposition from its Noun, in order to connect different Prepositions with the same Noun; as, "To suppose the Zodiac and Planets to be efficient *of*, and antecedent *to*, themselves."—Bentley, Serm. 6. This, whether in the familiar or the solemn style, is always inelegant; and should never be admitted, but in Forms of Law, and the like; where fulness and exactness of expression must take place of every other consideration.

‡ *With* in composition retains the signification, which it has among others in the Saxon, of *from* and *against*: as, to *withhold*, to *withstand*. So also *for* has a negative signification, from the Saxon; as, to *forbid*, *forbeodan*; to *forget*, *forgitan*.

tion

tion is more frequently placed after the Verb, and separate from it, like an Adverb; in which situation it is no less apt to affect the sense of it, and to give it a new meaning; and may still be considered as belonging to the Verb, and as a part of it. As, *to cast*, is to throw; but *to cast up*, or to compute, *an account*, is quite a different thing: thus, *to fall on*, *to bear out*, *to give over*; &c. So that the meaning of the Verb, and the propriety of the phrase, depend on the Preposition subjoined*.

As

* Examples of impropriety in the use of the Preposition, in phrases of this kind: "Your character, which I, or any other writer, may now value ourselves *by* [upon] drawing."—Swift, Letter on the English Tongue. "You have bestowed your favours *to* [upon] the most deserving persons."—Ibid. "Upon such occasions as fell *into* [under] their cognizance."—Swift, Contests and Dissentions, &c. Chap. iii. "That variety of factions *into* [in] which we are still engaged."—Ibid. Chap. v. "To restore myself *into* [to] the good graces of my fair Critics."—Dryden, Pref. to Aureng. "Accused the ministers *for* [of] betraying the Dutch."—Swift, Four last Years of the Queen, Book ii. "Ovid, whom you accuse *for* [of] luxuriancy of verse."—Dryden, on Dram. Poesy. "The people of England may congratulate *to* themselves, that—"—Dryden. "Something like this has been reproached *to* Tacitus."—Bolingbroke, on History, Vol. I. p. 136. "He was made much *on* [of] at Argos." "He is so resolved *of* [on] going to the Persian Court."—Bentley, Dissert. on Themistocles's Epistles, Sect. iii. "Neither the one nor the other shall make me swerve *out of* [from] the path which I have traced to myself."—Bolingbroke, Letter to Wyndham, p. 252.

"And Virgins smil'd at what they blush'd before:"

"at what they blush'd [at."]—Pope, Essay on Crit. "They are now reconciled by a zeal for their cause to what they could not be prompted to [by] a concern for their beauty."—Addison, Spect. N° 81. "If policy can prevail *upon* [over] force."—Addison, Travels, p. 62. "I do likewise dissent *with* "[from] the Examiner."—Addison, Whig-Exam. N° 1. "Ye blind guides, which strain *at* a gnat, and swallow a camel."—Matt. xxiii. 24. διυλίζοντες, which strain *out*, or take a gnat *out* of the liquor by straining it: the impropriety of the Preposition has wholly destroyed the meaning of the phrase. "No discouragement *for* the authors to proceed."—Tale of a Tub, Preface. "A strict observance *after* times and fashions."—Ibid. Sect. ii. "Which had a much greater share of inciting him, than any regards *after* his father's commands."—Ibid. Sect. vi. "Not from any personal hatred to them, but

in

As the Preposition subjoined to the Verb hath the construction and nature of an Adverb, so the Adverbs, *here, there, where,* with a Preposition subjoined, as, *hereof, therewith, whereupon**, have the construction and nature of Pronouns.

The Prepositions *to* and *for* are often understood, chiefly before the Pronoun; as, "give me the Book; get me some paper;" that is, *to me, for me.*

The Preposition *in,* or *on,* is often understood before Nouns expressing Time: as, "*this day; next month; last year*:" that is, "*on* this day;" "*in* next month;" "*in* last year."

In Poetry, the common Order of words is frequently inverted, in all ways in which it may be done without ambiguity or obscurity.

in justification *to* [of] the best of Queens."—Swift, Examiner, N° 23. In the last example, the Verb being Transitive, and requiring the Objective Case, the Noun formed from it seems to require the Possessive Case, or its Preposition after it. Or perhaps he meant to say, "in *justice* to the best of Queens." Observe also, that the Noun generally requires after it the same Preposition, as the Verb from which it is formed. "It was perfectly in compliance *to* [with] some persons, for whose opinion I have great deference."—Swift, Pref. to Temple's Memoirs. "The wisest Princes need not think it any diminution *to* [of] their greatness, or derogation *to* [from] their sufficiency, to rely upon counsel."—Bacon, Essay xx. So the Noun *aversion,* (that is, a turning away), as likewise the Adjective *averse,* seems to require the Preposition *from* after it; and not so properly to admit of *to* or *for,* which are often used with it.

* These are much disused in common discourse, and are retained only in the Solemn, or Formulary style. "They [our Authors] have of late, 'tis true, reformed in some measure the gouty joints and darning-work of *whereuntos, wherebys, thereofs, therewiths,* and the rest of this kind; by which complicated periods are so curiously strung, or hooked on, one to another, after the long-spun manner of the bar or pulpit."—Lord Shaftesbury, Miscel. V.

"Fra sche *thir* wordis had say'd."—Gawin Douglas, Æn. x.

"*Thir* wikket schrewis."—Ibid. Æn. xii.

That is, "*these* words;" "*these* wicked shrews." *Theyr, these,* or *those,* masculine; *thaer, these,* or *those,* feminine, Islandick. Hence, perhaps, *thereof, therewith,* &c. of, with, *them*; and so, by analogy, the rest of this class of words.

Two or more Simple Sentences, joined together by one or more CONNECTIVE WORDS, become a compounded Sentence.

There are two sorts of words, which connect Sentences: 1. Relatives; 2. Conjunctions.

Examples: 1. "Blessed is the man, *who* feareth the Lord." 2. "Life is short, *and* art is long." 1. and 2. "Blessed is the man, *who* feareth the Lord, *and* keepeth his commandments."

The RELATIVES, *who, which, that*, having no variation of gender or number, cannot but agree with their Antecedents. *Who* is appropriated to persons; and so may be accounted Masculine and Feminine only: we apply *which* now to things only; and to Irrational Animals, excluding them from Personality, without any consideration of Sex: *which* therefore may be accounted Neuter. But formerly they were both indifferently used of persons: "Our Father, *which* art in heaven." *That* is used indifferently both of persons and things: but it would better become the solemn style to restrain it more to the latter, than is usually done. *What* includes both the Antecedent and the Relative: as, "This was *what* he wanted;" that is, "*the thing which* he wanted."

The Relative is the Nominative Case to the Verb, when no other Nominative comes between it and the Verb: but when another Nominative comes between it and the Verb, the Relative is governed by some word in its own member of the Sentence: as, "the God, *who* preserveth me; *whose* I am, and *whom* I serve;" because in the different members of the sentence the Relative performs a different office: in the first member it represents the Agent; in the second, the Possessor; in the third, the object of an action: and therefore must be in the different Cases corresponding to those offices.

Every Relative must have an Antecedent to which it refers,

fers, either expressed, or understood: as, "*Who* steals my purse, steals trash;" that is, "*the man, who—*"

The Relative is of the same person with the Antecedent: and the Verb agrees with it accordingly: as, "Who is *this, that cometh* from Edom; *this, that is* glorious in his apparel? *I, that speak* in righteousness."—Isaiah, lxiii. 1. "O Shepherd of Israel; *Thou, that leadest* Joseph like a flock; *Thou, that dwellest* between the Cherubims."—Psal. lxxx. 1*.

When *this, that, these, those*, refer to a preceding Sentence; *this*, or *these*, refers to the latter member or term; *that*, or *those*, to the former; as,

"*Self-love*, the spring of motion, acts the soul;
Reason's comparing balance rules the whole;
Man, but for *that*, no action could attend;
And, but for *this*, were active to no end."

Pope, Essay on Man.

"*Some* place the bliss in action, *some* in ease:
Those call it pleasure, and contentment *these*."

Ibid.

* I am *the Lord, that maketh* all things; *that stretcheth* forth the heavens alone.—Isaiah, xli v. 24. Thus far is right: *the Lord* in the Third Person is the Antecedent, and the Verb agrees with the Relative in the third Person: "I am the Lord, *which Lord*, or *He that, maketh* all things." It would have been equally right if *I* had been made the Antecedent, and the Relative and the Verb had agreed with it in the First Person: "*I* am the Lord, *that made* all things." But when it follows, "*that spreadeth* abroad the earth by *myself*;" there arises a confusion of Persons, and a manifest Solecism.

"*Thou* great first Cause, least understood;
Who all my sense *confin'd*
To know but this, that *Thou art* good,
And that myself am blind:
Yet *gave* me in this dark estate, &c."

Pope, Universal Prayer.

It ought to be, *confinedst*, or *didst confine*; *gavest*, or *didst give*; &c. in the second Person.

The

The Relative is often understood, or omitted: as, "The man I love;" that is, *whom* I love*.

The accuracy and clearness of the sentence depend very much upon the proper and determinate use of the Relative: so that it may readily present its Antecedent to the mind of the hearer, or reader, without any obscurity or ambiguity. The same may be observed of the Pronoun and the Noun; which by some are called also the Relative and the Antecedent.

CONJUNCTIONS have sometimes a Government of Modes. Some Conjunctions require the Indicative, some the Subjunctive Mode, after them: others have no influence at all on the Mode.

Hypothetical, Conditional, Concessive, and Exceptive Conjunctions, seem in general to require the Subjunctive Mode after them: as, *if*, *though*, *unless*, *except*, *whether*, *or*, &c.: but by use they often admit of the Indicative; and in some cases with propriety. Examples: "*If* thou

* "Abuse on all he lov'd, or lov'd him, spread."

Pope, Epist. to Arbuthnot.

That is, "all *whom* he lov'd, or *who* lov'd him:" or, to make it more easy by supplying a Relative that has no variation of Cases, "all *that* he lov'd, or *that* lov'd him." The Construction is hazardous, and hardly justifiable, in Poetry. "In the temper of mind he was then."—Addison, Spect. N° 549. "In the posture I lay."—Swift, Gulliver, Part I. Chap. 1. In these and the like phrases, which are very common, there is an Ellipsis both of the Relative and the Preposition; which would have been much better supplied: "In the temper of mind *in which* he then was." "In the posture *in which* I lay." "The little satisfaction and consistency [which] is to be found in most of the systems of Divinity [which] I have met with, made me betake myself to the sole reading of the Scripture (to which they all appeal) for the understanding [of] the Christian Religion."—Locke, Pref. to Reasonableness of Christianity. In the following example the antecedent is omitted: "He desired they might go to the author together, and jointly return their thanks to *whom* only it was due."—Addison, Freeholder, N° 49. In general, the omission of the Relative seems to be too much indulged in the familiar style; it is ungraceful in the solemn; and, of whatever kind the style be, it is apt to be attended with obscurity and ambiguity.

be the Son of God."—Matt. iv. 3. "*Though* he *slay* me, yet will I put my trust in him."—Job xiii. 15. "*Unless* he *wash* his flesh."—Lev. xii. 6. "No power, *except* it *were* given from above."—John, xix. 11. "*Whether* it *were* I *or* they, so we preach."—1 Cor. xv. 11. The Subjunctive in these instances implies something contingent or doubtful; the Indicative would express a more absolute and determinate sense *.

That, expressing the motive or end, has the Subjunctive Mode, with *may*, *might*, *should*, after it.

Lest, and *that* annexed to a Command, preceding; and *if* with *but* following it; necessarily require the Subjunctive Mode: Examples; "Let him, that standeth, take heed *lest* he *fall*."—1 Cor. x. 12. "Take heed, *that* thou *speak* not to Jacob."—Gen. xxxi. 24. "*If* he *do but* touch the hills, they shall smoke."—Psalm civ. 32.

Other Conjunctions, expressing a Continuation, an Addition, an Inference, &c. being of a positive and absolute nature, require the Indicative Mode; or rather leave

* The following example may serve to illustrate this observation: "*Though* he *were* divinely inspired, and spake, therefore, as the oracles of God, with supreme authority; *though* he *were* endued with supernatural powers, and could, therefore, have confirmed the truth of what he uttered by miracles: yet in compliance with the way in which human nature and reasonable creatures are usually worked upon, he reasoned."—Atterbury, Serm. IV. 5.

That our Saviour was divinely inspired, and endued with supernatural powers, are positions that are here taken for granted, as not admitting of the least doubt: they would, therefore, have been better expressed in the Indicative Mode; "though he *was* divinely inspired; though he *was* endued with supernatural powers." The Subjunctive is used in like manner in the following example: "Though he *were* a Son, yet learned he obedience, by the things which he suffered."—Heb. v. 8. But in a similar passage the Indicative is employed to the same purpose, and that much more properly: "Though he *was* rich, yet for your sakes he became poor."—2 Cor. viii. 9. The proper use, then, of the Subjunctive Mode after the Conjunction is in the case of a doubtful supposition, or concession: as, "*Though* he *fall*, he shall not be utterly cast down."—Psal. xxxvii. 24. And much the same may be said of the rest.

the

the Mode to be determined by the other circumstances and conditions of the sentence.

When the qualities of different things are compared; the latter Noun, or Pronoun, is not governed by the Conjunction *than*, or *as*, (for a Conjunction has no Government of Cases), but agrees with the Verb, or is governed by the Verb, or the Preposition, expressed, or understood. As, "Thou art wiser than *I* [am]." "You are not so tall as *I* [am]." "You think him handsomer than [you think] *me*; and you love him more than [you love] *me*." In all other instances, if you complete the sentence in like manner, by supplying the part which is understood; the Case of the latter Noun, or Pronoun, will be determined. Thus, "Plato observes, that God geometrizes: and the same thing was observed before by a wiser man than *he*;" that is, than *he was*. "It is well expressed by Plato; but more elegantly by Solomon than *him*;" that is, than by *him*.

But the Relative *who*, having Reference to no Verb or Preposition understood, but only to its Antecedent, when it follows *than*, is always in the Objective Case; even though the Personal Pronoun, if substituted in its place, would be in the Nominative: as,

> "Beelzebub, *than whom*,
> Satan except, none higher sat."
> Milton, P. L. ii. 299.

which, if we substitute the Personal Pronoun, would be,

> "none higher sat *than he*."

The Conjunction *that* is often omitted and understood: as, "I beg you would come to me:" "See thou do it not:" that is, "*that* you would;" "*that* thou do."

The Nominative Case following the Auxiliary, or the Verb itself, sometimes supplies the place of the Conjunction, *if* or *though*; as, "Had he done this, he had escaped:"

 "Charm

"Charm he never so * wisely:" that is, "*if* he had done this;" "*though* he charm."

Some Conjunctions have their Correspondent Conjunctions belonging to them; so that, in the subsequent Member of the Sentence, the latter answers to the former: as, *although, yet,* or *nevertheless*; *whether*—, *or*; *either—or*; *neither*, or *nor*—, *nor*; *as*—, *as*; expressing a Comparison of equality; "*as* white *as* snow:" *as*—, *so*; expressing a Comparison sometimes of equality; "*as* the stars, *so* shall thy seed be;" that is, equal in number: but most commonly a Comparison in respect of quality; "and it shall be, *as* with the people, *so* with the priest; *as* with the servant, *so* with his master:" "*as* is the good, *so* is the sinner; *as* the one dieth, *so* dieth the other:" that is, in like manner: *so*—, *as*, with a Verb, expressing a Comparison of quality; "To see thy glory, *so as* I have seen thee in the sanctuary:" but with a Negative and an Adjective, a Comparison in respect of quantity: as, "Pompey had eminent abilities: but he was neither *so* eloquent and politic a statesman, nor *so* brave and skilful a general: nor was he, upon the whole, *so* great a man *as* Cæsar:" *so*—, *that*, expressing a Consequence; &c.

INTERJECTIONS in English have no Government. Though they are usually attended with Nouns in the Nominative Case, and Verbs in the Indicative Mode; yet the Case and Mode are not influenced by them, but determined by the nature of the sentence.

* *Never so*—"This phrase, says Dr. Johnson, is justly accused of Solecism." It should be, *ever* so wisely: that is, *how* wisely *soever*. "Besides, a Slave would not have been admitted into that Society, had he had *never such* opportunities."—Bentley, Dissert. on Phalaris, p. 338.

PUNC-

PUNCTUATION.

PUNCTUATION is the art of marking in writing the several pauses, or rests, between sentences, and the parts of sentences, according to their proper quantity or proportion, as they are expressed in a just and accurate pronunciation.

As the several articulate sounds, the syllables and words, of which sentences consist, are marked by Letters; so the rests and pauses, between sentences and their parts, are marked by Points.

But, though the several articulate sounds are pretty fully and exactly marked by Letters of known and determinate power; yet the several pauses which are used in a just pronunciation of discourse, are very imperfectly expressed by Points.

For the different degrees of connection between the several parts of sentences, and the different pauses in a just pronunciation, which express those degrees of connection according to their proper value, admit of great variety; but the whole number of Points which we have to express this variety, amounts only to Four.

Hence it is, that we are under a necessity of expressing pauses of the same quantity, on different occasions, by different points; and more frequently, of expressing pauses of different quantity by the same points.

So that the doctrine of Punctuation must needs be very imperfect: few precise rules can be given which will hold without exception in all cases; but much must be left to the judgment and taste of the writer.

On the other hand, if a greater number of marks were

invented

invented to express all the possible different pauses of pronunciation; the doctrine of them would be very perplexed and difficult, and the use of them would rather embarrass than assist the reader.

It remains, therefore, that we be content with the Rules of Punctuation, laid down with as much exactness as the nature of the subject will admit: such as may serve for a general direction, to be accommodated to different occasions: and to be supplied, where deficient, by the writer's judgment.

The several degrees of Connection between Sentences, and between their principal constructive parts, Rhetoricians have considered under the following distinctions, as the most obvious and remarkable: the Period, Colon, Semicolon, and Comma.

The Period is the whole Sentence, complete in itself, wanting nothing to make a full and perfect sense, and not connected in construction with a subsequent Sentence.

The Colon, or Member, is a chief constructive part, or greater division of a Sentence.

The Semicolon, or Half-member, is a less constructive part, or subdivision, of a Sentence or Member.

A Sentence or Member is again subdivided into Commas, or Segments; which are the least constructive parts of a Sentence or Member, in this way of considering it; for the next subdivision would be the resolution of it into Phrases and Words.

The Grammarians have followed this division of the Rhetoricians, and have appropriated to each of these distinctions its mark, or Point; which takes its name from the part of the Sentence, which it is employed to distinguish; as follows:

The Period	is thus marked	.
The Colon		:
The Semicolon		;
The Comma		,

The proportional quantity, or time, of the points, with respect to one another, is determined by the following general rule: The Period is a pause in quantity or duration double of the Colon; the Colon is double of the Semicolon; and the Semicolon is double of the Comma. So that they are in the same proportion to one another, as the Semibrief, the Minim, the Crotchet, and the Quaver, in Music. The precise quantity, or duration, of each Pause or Note cannot be defined: for that varies with the Time; and both in Discourse and Music the sameComposition may be rehearsed in a quicker or a slower Time: but in Music the proportion between the Notes remains ever the same; and in Discourse, if the doctrine of Punctuation were exact, the proportion between the Pauses would be ever invariable.

The Points, then, being designed to express the Pauses which depend on the different degreesof connection between Sentences, and between their principal constructive parts; in order to understand the meaning of the Points, and to know how to apply them properly, we must consider the nature of a Sentence, as divided into its principal constructive parts; and the degrees of connection between those parts, upon which such division of it depends.

To begin with the least of these principal constructive parts, the Comma. In order the more clearly to determine the proper application of the Point which marks it, we must distinguish between an Imperfect Phrase, a Simple Sentence, and a Compounded Sentence.

An Imperfect Phrase contains no assertion, or does not amount to a Proposition or Sentence.

A Simple Sentence has but one Subject, and one finite Verb.

A Compounded Sentence has more than one Subject, or one finite Verb, either expressed or understood; or it consists of two or more simple Sentences connected together.

In a Sentence the Subject and the Verb may be each of them accompanied with several Adjuncts; as the Object, the End, the Circumstances of Time, Place, Manner, and the like: and the Subject or Verb may be either immediately connected with them, or mediately; that is, by being connected with some thing, which is connected with some other; and so on.

If the several Adjuncts affect the Subject or the Verb in a different manner, they are only so many imperfect Phrases, and the Sentence is Simple.

A Simple Sentence admits of no Point by which it may be divided, or distinguished into parts.

If the several Adjuncts affect the Subject or the Verb in the same manner, they may be resolved into so many Simple Sentences: the Sentence then becomes Compounded, and it must be divided into its parts by Points.

For, if there are several Subjects belonging in the same manner to one Verb, or several Verbs belonging in the same manner to one Subject, the Subjects and Verbs are still to be accounted equal in number: for every Verb must have its Subject, and every Subject its Verb; and every one of the Subjects, or Verbs, should or may have its point of distinction.

Examples:

"The passion for praise produces excellent effects in women of sense."—Addison, Spect. No. 73. In this Sentence *passion* is the Subject, and *produces* the Verb: each of which is accompanied and connected with its Adjuncts. The Subject is not passion in general, but a particular passion determined by its Adjunct of Specification, as we may call it; the passion *for praise*. So likewise the Verb is immediately connected with its object, *excellent effects*; and

and mediately, that is, by the intervention of the word, *effects*, with *women*, the Subject in which these effects are produced; which again is connected with its Adjunct of Specification; for it is not meaned of women in general, but of women *of sense* only. Lastly, it is to be observed, that the Verb is connected with each of these several Adjuncts in a different manner; namely, with *effects* as the object; with *women*, as the subject of them; with *sense*, as the quality or characteristic of those women. The Adjuncts therefore are only so many imperfect Phrases; the Sentence is a Simple Sentence, and admits of no Point, by which it may be distinguished into parts.

"The passion for praise, which is so very vehement in the fair sex, produces excellent effects in women of sense." Here a new Verb is introduced, accompanied with Adjuncts of its own; and the subject is repeated by the Relative Pronoun *which*. It now becomes a Compounded Sentence, made up of two Simple Sentences, one of which is inserted in the middle of the other; it must therefore be distinguished into its component parts, by a Point placed in each side of the additional Sentence.

"How many instances have we [in the fair sex] of chastity, fidelity, devotion! How many Ladies distinguish themselves by the education of their children, care of their families, and love of their husbands; which are the great qualities and achievements of womankind: as the making of war, the carrying on of traffick, the administration of justice, are those by which men grow famous, and get themselves a name!"—Ibid.

In the first of these two Sentences, the Adjuncts *chastity, fidelity, devotion*, are connected with the Verb, by the word *instances* in the same manner, and in effect make so many distinct Sentences: "how many instances have we of chastity! how many instances have we of fidelity! how many instances have we of devotion!" They must therefore be

separated

separated from one another by a Point. The same may be said of the Adjuncts, "education of their children, &c." in the former part of the next Sentence: as likewise of the several Subjects, "the making of war, &c." in the latter part; which have in effect each their Verb; for each of these "is an achievement by which men grow famous."

As Sentences themselves are divided into Simple and Compounded, so the Members of Sentences may be divided likewise into Simple and Compounded Members: for whole Sentences, whether Simple or Compounded, may become Members of other Sentences, by means of some additional connexion.

Simple Members of Sentences closely connected together in one Compounded member, or sentence, are distinguished or separated by a Comma: as in the foregoing axamples.

So likewise, the Case Absolute; Nouns in Apposition, when consisting of many terms; the Participle with something depending on it; are to be distinguished by the Comma: for they may be resolved into Simple Members.

When an address is made to a person, the Noun, answering to the Vocative Case in Latin, is distinguished by a Comma.

Examples:

"This said, He form'd thee, Adam; thee, O man, Dust of the ground."

"Now morn, her rosy steps in th' eastern clime
Advancing, sow'd the earth with orient pearl.—Milton.

Two Nouns, or two Adjectives, connected by a single Copulative or Disjunctive, are not separated by a Point: but when there are more than two, or where the Conjunction is understood, they must be distinguished by a Comma.

Simple Members connected by Relatives and Comparatives, are for the most part distinguished by a Comma:

but

but when the Members are short in Comparative Sentences; and when two Members are closely connected by a Relative, restraining the general notion of the Antecedent to a particular sense; the pause becomes almost insensible, and the Comma is better omitted.

Examples:

" Raptures, transports, and extasies, are the rewards which they confer: sighs and tears, prayers and broken hearts, are the offerings which are paid to them---Addison, ibid.

" Gods partial, changeful, passionate, unjust;
Whose attributes were rage, revenge, or lust."—Pope.

" What is sweeter than honey? and what is stronger than a lion?"

A circumstance of importance, though no more than an Imperfect Phrase, may be set off with a Comma on each side, to give it greater force and distinction.

Example:

" The principle may be defective or faulty; but the consequences it produces are so good, that, for the benefit of mankind, it ought not to be extinguished."—Addison, ibid.

A Member of a Sentence, whether Simple or compounded, that requires a greater pause than a Comma, yet does not of itself make a complete Sentence, but is followed by something closely depending on it, may be distinguished by a Semicolon.

Examples:

" But as this passion for admiration, when it works according to reason, improves the beautiful part of our species

in every thing that is laudable; so nothing is more destructive to them, when it is governed by vanity and folly."—Addison, ibid.

Here the whole Sentence is divided into two parts by the Semicolon; each of which parts is a Compounded Member, divided into its Simple Members by the Comma.

A Member of a Sentence, whether Simple or Compounded, which of itself would make a complete Sentence, and so requires a greater pause than a Semicolon, yet is followed by an additional part, making a more full and perfect Sense, may be distinguished by a Colon.

Example:

"Were all books reduced to their quintessence, many a a bulky author would make his appearance in a penny paper: there would be scarce any such thing in nature as a folio: the works of an age would be contained on a few shelves: not to mention millions of volumes, that would be utterly annihilated."—Addison, Spect. No. 124.

Here the whole Sentence is divided into four parts by Colons; the first and last of which are Compounded Members, each divided by a Comma: the second and third are Simple Members.

When a Semicolon has preceded, and a greater pause is still necessary; a Colon may be employed, though the sentence be incomplete.

The Colon is also commonly used, when an Example or a Speech, is introduced.

When a Sentence is so far perfectly finished as not to be connected in construction with the following Sentence, it is marked with a Period.

In all cases, the proportion of the several Points in respect to one another is rather to be regarded, than their supposed precise quantity, or proper office, when taken separately.

Beside

Beside the points which mark the pauses in discourse, there are others which denote a different modulation of the voice, in correspondence with the sense. These are,

The Interrogation Point,	thus marked.	?
The Exclamation Point,		!
The Parenthesis,		( )

The Interrogation and Exclamation Points are sufficiently explained by their names: they are indeterminate as to their quantity or time, and may be equivalent in that respect to a Semicolon, a Colon, or a Period, as the sense requires. They mark an Elevation of the voice.

The Parenthesis incloses in the body of a Sentence a Member inserted into it, which is neither necessary to the Sense, nor at all affects the Construction. It marks a moderate Depression of the voice, with a pause greater than a Comma.

A PRAXIS,

OR EXAMPLE OF GRAMMATICAL RESOLUTION.

1. In the fifteenth year of the reign of Tiberius Cæsar, Pontius Pilate being governor of Judea, the word of God came unto John, the son of Zacharias, in the wilderness.

2. And he came into all the country about Jordan, preaching the baptism of repentance, for the remission of sins.

3. And the same John had raiment of camel's hair, and a leathern girdle about his loins; and his meat was locusts and wild honey.

4. Then said he to the multitude that came forth to be baptized of him: O generation of vipers, who hath warned

you

you to flee from the wrath to come? Bring forth therefore fruits meet for repentance.

5. And as all men mused in their hearts of John, whether he were the Christ, or not; John answered, saying unto them all: I indeed baptize you with water; but one mightier than I cometh, the latchet of whose shoes I am not worthy to unloose: he shall baptize you with the Holy Ghost, and with fire.

6. Now when all the people were baptized, it came to pass, that, Jesus also being baptized and praying, the heaven was opened, and the Holy Ghost descended in a bodily shape, like a dove, upon him; and lo! a voice from heaven saying: This is my beloved Son, in whom I am well pleased.

1. *In* is a Preposition; *the*, the Definite Article; *fifteenth*, an Adjective; *year*, a Substantive, or Noun, in the Objective Case, governed by the Preposition *in*; *of*, a Preposition; *the reign*, a Substantive, Objective Case, governed by the Preposition *of*; *Tiberius Cæsar*, both Substantives, Proper Names, Government and Case as before; *Pontius Pilate*, Proper Names: *being*, the present Participle of the Verb Neuter *to be*; *governour*, a Substantive; *of Judea*, a Proper Name, Government and Case as before; *Pontius Pilate being governour*, is the Case Absolute; that is, the Nominative Case with a Participle without a Verb following and agreeing with it; the meaning is the same as, *when* Pilate was governour; *the word*, a Substantive; *of God*, a a Substantive, Objective Case, governed by the Preposition *of*; *came*, a Verb Neuter, Indicative Mode, Past Time, third Person Singular Number, agreeing with the Nominative Case *word*; *unto*, a Preposition; *John* a proper Name; *the son*, a Substantive, put in Apposition to *John*; that is, in the same Case, governed by the same Preposition *unto*; *of Zacharias*, a Proper Name; *in*, a Preposition; *the wilderness*, a Substantive, Government and Case as before.

2. *And*

2. *And*, a Conjunction Copulative; *he*, a Pronoun, third Person Singular, Masculine Gender, Nominative Case, standing for *John*; *came*, as before; *into*, a Preposition; *all*, an Adjective; *the country*, a Substantive; *about*, a Preposition; *Jordan*, a Proper Name; Objective Cases, governed by their Prepositions; *preaching*, the present Participle of the Verb Active *to preach*, joined like an Adjective to the Pronoun *he*; *the baptism*, a Substantive in the Objective Case following the Verb Active *preaching*, and governed by it; *of repentance*, a Substantive, Government and Case as before; *for*, a Prep.; *the remission of sins*, Substantives, the latter in the Plural Number, Government and Case as before.

3. *And*, (b. that is, *as before*;) *the same*, an Adjective; *John*, (b) *had*, a Verb Active, Indicative Mode, Past Time, third Person Singular, agreeing with the Nominative Case *John*; *his*, a Pronoun, third Person Singular, Possessive Case; *raiment*, a Substantive in the Objective Case, following the Verb Active *had*, and governed by it; *of camel's*, a Substantive, Possessive Case; *hair*, Substantive, Objective Case, governed by the Preposition *of*, the same as, *of the hair of a camel*; *and*, (b.) *a*, the Indefinite Article; *leathern*, an Adj. *girdle*, a Subst. *about*, (b.) *his* (b.) *loins*, Subst. Plural Number, Objective Case, governed by the Preposition *about*; *and his* (b.) *meat*, Subst. *was*, Indicative Mode, Past Time, third Person Singular of the Verb Neuter *to be*; *locusts*, Substantive, Plural Number, Nominative Case after the Verb *was*; *and*, (b.) *wild*, Adjective; *honey*, Substantive, the same Case.

4. *Then*, an Adverb; *said*, a Verb Active, Past Time, third Person Singular agreeing with the Nominative Case *he*, (b.) *to*, a Prep.; *the multitude*, Subst. Objective Case, governed by the Prep. *to*; *that*, a Relative Pronoun, its Antecedent is *the multitude*; *came* (b.) *forth*, an Adverb; *to*, a Prep. and before a Verb the sign of the Infinitive Mode;

Mode; *be baptized*, a Verb Passive, made of the Participle Passive of the Verb *to baptize*, and the Auxiliary Verb *to be* in the Infinitive Mode; *of him*, Pronoun, third Person Sing. standing for *John*, in the Objective Case, governed by the Prep. *of*; *O*, an Interjection; *generation*, Subst. Nominative Case; *of Vipers*, Sub. Plural Number, Objective Case, governed by the Prep. *of*; *who*, an Interrogative Pronoun; *hath warned*, a Verb Active, Present Perfect Time, made of the Perfect Participle *warned*, and the Auxiliary Verb *hath*, third Person Singular, agreeing with the Nominative Case *who*; *you*, Pronoun, second Person Plural, Objective Case, following the Verb Active *warned*, and governed by it; *to flee*, Verb Neuter, Infinitive Mode; *from*, a Prep. *the Wrath*, Subst. Objective Case, governed by the Prep. *from*; *to come*, Verb Neuter, Infinitive Mode; *bring*, Verb Active, Imperative Mode, second Person Plural, agreeing with the Nominative Case *ye*, understood; as if it were, *bring ye*; *forth*, an Adverb; *therefore*, a Conjunction; *fruits*, a Substantive Plural, Objective Case, following the Verb Active *bring*, and governed by it; *meet*, an Adjective joined to *fruits*, but placed after it, because it hath something depending on it; *for repentance*, a Substantive governed by a Preposition, as before.

5. *And*, (b) *as*, a Conjunction: *all*, (b.) *men*, Subst. Plural Number; *mused*, a Verb Active, Past Time, third Person Plural, agreeing with the Nominative Case *men*; *in*, (b.) *their*, a Pronominal Adjective, from the Pronoun *they*; *hearts*, Subst. Plural Nnmber, Objective Case governed by the Prep. *in*; *of John*, (b.) *whether*, a Conjunction; *he*, (b.) *were*, Subjunctive Mode, governed by the Conjunction *whether*, Past Time, third Person Sing. of the Verb *to be*, agreeing with the Nominative Case *he*; *the Christ*, Subst. Nominative Case after the Verb *were*; *or*, a Disjunctive Conjunction, corresponding to the preceding Conjunction *whether*; *not*, an Adverb; *John* (b.) *answered*, a Verb.

Verb Active, Indicative Mode, Past Time, third Person Sing. agreeing with the Nominative Case *John*; *saying*, Present Participle of the Verb Active *to say*, joined to the Substantive *John*; *unto* (b.) *them*, a Pronoun, third Person, Plural, Objective Case, governed by the Preposition *unto*; *all*, (b.); *I*, Pronoun, first Person Singular; *indeed*, an Adverb; *baptize*, a Verb Active, Indicative Mode, Present Time, first Person Singular, agreeing with the Nominative Case *I*; *you*, Pronoun, second Person Plural, Objective Case, following the Verb Active *baptize*, and governed by it; *with*, a Prep.; *water*, Subst. Objective Case, governed by the Preposition *with*; *but*, a Disjunctive Conjunction: *one*, a Pronoun, standing for some person not mentioned by name; *mightier*, an Adjective in the Comparative Degree, from the Positive *mighty*; *than*, a Conjunction, used after a Comparative word; *I*, (b.) the Verb *am* being understood; that is *than I am*; *cometh*, a Verb Neuter, Indicative Mode, Present Time, third Person Sing. agreeing with the Nominative Case *one*; *the latchet*, Subst. *of*, (b.) *whose*, Pronoun Relative, *one* being the Antecedent to it, in the Possessive Case; *shoes*, Subst. Plural, Objective Case, governed by the Preposition *of*; *I*, (b.) *am* Indicative Mode, Present Time, first Person Sing. of the Verb *to be*, agreeing with the Nominative Case *I*; *not* (b.) *worthy* an Adjective; *to unloose*, a Verb Active in the Infinite Mode, governing the Substantive *latchet*, in the Objective Case; *he*, (b.) *shall baptize*, a Verb Active, Indicative Mode, Future Time, made by the Auxiliary *shall*, third Person Sing. agreeing with the Nominative Case *he*; *you*, (b.) *with the* (b.) *Holy*, an Adjective; *Ghost*, a Subst. *and with* (b.) *fire*, a Substantive: this and the former both in the Objective Case governed by the Prep. *with*.

6. *Now* an Adverb; *when* a Conjunction; *all*, (b.) *the people*, a Subst. *were baptized*, a Verb Passive, made of the Auxiliary Verb *to be*, joined with the Participle Passive of the Verb *to baptize*, Indicative Mode, Past Time, third

 Person

Person Plural, agreeing with the Nominative Case Singular, *people* being a Noun of Multitude; *it,* Pronoun, third Person Singular, Neuter Gender, of the Nominative Case; *came* (b.) *to pass,* Verb Neuter, Infinite Mode; *that,* a Conjunction; *Jesus,* a proper Name; *also,* an Adverb; *being,* Present Participle of the Verb *to be*; *baptized,* Participle Passive of the Verb *to baptize*; *and* (b.) *praying,* Present Participle of the Verb Neuter *to pray*; *Jesus being baptized and praying* is the Case Absolute, as before; *the heaven,* Substantive; *was opened,* Verb Passive, Indicative Mode, Past Time, third Person Singular, agreeing with the Nominative Case *heaven,* the Auxiliary Verb *to be* being joined to the Participle Passive, as before; *and the holy Ghost,* (b.) *descended,* Verb Neuter, Indicative Mode, Past Time, third Person Singular, agreeing with the Nominative Case *Ghost*; *in a* (b.) *bodily,* an Adjective; *shape,* a Substantive, Objective Case, governed by the Preposition *in*; *like,* an Adjective; *a dove,* Substantive, Objective Case, the Preposition *to* being understood, that is, *like to a dove*; *upon,* Preposition; *him,* Pronoun, third Person Singular, Objective Case, governed by the Preposition *upon*; *and,* (b.) *lo,* an Interjection; *a voice,* Substantive, Nominative Case, *there was* being understood; that is, *there was a voice*; *from,* Preposition; *heaven,* Substantive, Objective Case, (b.) *saying,* (b.) *this* a Pronominal Adjective, *person* being understood; *is,* Indicative Mode, Present Time, of the Verb, *to be,* third Person Singular, agreeing with the Nominative Case *this*; *my,* a Pronominal Adjective; *beloved,* an Adjective; *Son,* a Substantive, Nominative Case, after the Verb *is*; *in,* (b.) *whom,* Pronoun Relative, Objective Case, governed by the Preposition *in,* the Substantive *Son* being its Antecedent; *I am,* (b.) *well,* an Adverb; *pleased,* the Passive Participle of the Verb *to please,* making with the Auxiliary Verb *am* a Passive Verb, in the Indicative Mode, Present time, first Person Singular, agreeing with the Nominative Case *I.*

CHAP.

THE

MODERN PRECEPTOR.

CHAPTER, II.

ON ARITHMETIC.

WHATEVER is in its nature capable of augmentation and diminution, is termed *quantity:* extent, duration, weight, &c., are all quantities: and whatever constitutes quantity, becomes an object of mathematical investigation. That branch of mathematics which considers quantity, as expressed by numbers, is called *Arithmetic,* from a Greek term signifying *number*; and may hence be considered as the science of the nature and properties of numbers: its object is to discover sure and easy methods of representing, compounding, and decompounding numbers; by certain operations, constituting *calculation.*

As all calculation is founded on a knowledge of what is called *Unity,* it must be observed, that an *Unit* is a quantity assumed at pleasure, to serve as the medium or standard of comparison, between quantities of the same sort. Thus, when we affirm of two bodies, that the one weighs three pounds, and the other five pounds, we make a pound the standard of comparison, or the Unit: but if we say that the first body weighs forty-eight ounces, and the other eighty ounces, we consider the ounce to be the standard or unit.

By numbers we express how many units, or parts of an

unit,

unit, are contained in any given quantity. If the quantity consists of entire units, the number by which it is expresed, is called a *whole* number, as for example, sixteen, fifty-nine, two-hundred and four, &c.: but if the quantity contain only parts of any given unit, as three quarters of a pound, the number is called a *fraction*; and when the quantity consists of entire units and parts of an unit, the number expressing it is said to be *fractional*, as nineteen and nine-tenths.

Arithmetic must have been known from the earliest period of society: but, although, we cannot conceive a nation, nor even a rational individual to have subsisted without a knowledge of numbers, in their most simple application and uses, yet men may have continued for many ages ignorant of the wonderful extent of their powers. The Greeks were the first European nation who cultivated the art of numbers; and some have imagined, from the terms employed by them, and by the Romans after them, that in their Arithmetical operations, they made use of small stones or pebbles: for both the Greek term *Psephizo*, and the Latin *Calculo*, (from which we have our *Calculation*), are derived from the words in those languages, signifying a pebble, or small stone.

However this may have been, we find the Greeks very early making use of the letters of their alphabet, to represent numbers. Thus the twenty-four letters taken as they stand in the alphabet, with three other characters, introduced in certain places, were made to represent the nine *digits*, the nine *tens*, and the nine *hundreds*.

But the difficulty of carrying on arithmetical operations, to much extent, with such characters, may be easily imagined, and is very evident from calculations still remaining in the works of some ancient Greek geometricians.

The Romans who drew from the Greeks the chief part of heir skill in the Sciences; imitated them also in this mode of

of expressing numbers; but adopting a different arrangement of the alphabetical characters, as here shown,

I. V. X. L. C. D.
One, Five, Ten, Fifty, One Hundred, Five Hundred,
M.
One Thousand. &c

By the repetition and combination of these numeral characters, any number may be expressed. 1st. By the repetition of a character, the value is also repeated, as, III. represent three; XX. twenty; CC. two hundred,

2d. When a character is followed by one of inferior value, their values are to be added together, as XII. twelve, LV. fifty five. MDCCCVIII. one thousand eight hundred and eight.

3d. But when a numeral letter of small value is placed before one of a greater, the less is to be subtracted from the greater, in order to have the value of the expression: thus IV. represent four, IX. nine, XL. forty, XC. ninety. CD. four hundred, &c.

In old Books we meet with IↃ. instead of D. for 500. and CIↃ. for 1000, but these characters may perhaps have been only inaccurate representations of D and M.

Thousands are also represented by drawing a short line over the numeral character as $\overline{\text{V}}$. for 5,000. $\overline{\text{L}}$. for 50,000. $\overline{\text{CCC}}$. for 300,000.

About the end of the 2d century, a new species of arithmetic was invented, as is supposed, by the great geometrician and geographer Ptolomy. Its object was to avoid the difficulties occasioned by fractions in the common Arithmetic: and in it the unit was divided into 60 equal parts; each of these into 60 others; each of these last again into 60 other parts, and so on: and from these divisions this kind of Arithmetic was called *Sexagesimal*, or by *Sixties*.

The excellent mode of expressing Numbers now used, came into Europe from the Arabians, by way of Spain;

but

but those Arabians did not pretend to be the inventors of these symbols, on the contrary they owned they were derived from the Indians. The period when these Arabic Symbols were introduced into England is uncertain : but inscriptions have been found as far back as in 1090, where they are employed.

The Introduction of these new characters did not immediately put an end to the Sexagesimal Arithmetic, which having been employed in all Astronomical tables, was on this account still retained, at least in the fractions, until Decimal arithmetic came into use.

The most ancient treatises on Arithmetic are certain Books of the Elements of Euclid, who flourished about 280 years before Christ. About the year 1460 Regiomontanus (or Muller of Koningsberg) in his Tables, divided the Radius into 10,000 instead of 60,000 parts; and so far abolished the former Sexagesimal Arithmetic, of which however a vestige still exists, in the division of time, and of a Degree of a Great Circle; for an hour is divided into 60 Minutes, a Minute into 60 Seconds, a Second into 60 Thirds, and so on; and a Degree is divided and subdivided in the same manner, into parts of the same denominations. The greatest improvement however which any age has produced, in Arithmetical operation, is by the invention of *Logarithms*; a discovery for which the world was indebted to Baron Napier of Merchiston in Scotland, towards the beginning of the 17th century. By these and other means, Arithmetic may now be considered as the science which has attained the nearest to perfection; and in which very important improvements can scarcely be looked for.

NOTATION.

By *Notation* is meant the art of expressing numbers, by a limited set of characters, called *Cyphers* or *Figures*.

The

The Figures now used, and their powers, are the following viz.

1.	2.	3.	4.	5.	6.	7.	8.	9.
one	two	three	four	five	six	seven	eight	nine

To these is added 0 to represent *nought,* or the absence or negation of all number or quantity.

To represent all other numbers by means of these figures, it has been agreed on, that Ten units should be formed into one aggregate sum, to be called *Ten,* with which calculation may be carried on, as by a simple unit; as two tens, three tens, six tens, &c. on to nine tens. To represent these new units the former figures are employed, but placed in a different position, to the left hand of their original place. Thus to represent twenty-four, containing two Tens and four units, we write 24: for Sixty, or six tens, without any simple units, we write 60: for ninety-nine, 99.

For Numbers above ninety-nine, on to, and including nine hundred and ninety-nine, another series of Units is formed in the same way, each of which contains Ten of the preceding series, and one hundred of the simple units. This last series is termed *hundreds;* and by it we express any number, as five hundred and sixty-three; thus, 563: Nine hundred and nine thus, 909: that is, nine hundreds, no odd tens, and nine units. Seven hundred would be, 700, without either tens or units.

Again from nine hundred and ninety-nine, by a similiar process, we can count to nine thousand nine hundred and ninety-nine: forming a fresh series of Units called *Thousands,* each containing ten hundreds. Thus, seven thousand four hundred and thirty-five, will be written 7435; eight thousand and six, that is eight thousands, no hundreds nor tens, and six simple units, 8006. The year One thousand eight hundred and eight, 1808.

For

For the better understanding of the principles of Notation here explained, the following Table is given.

Hundreds of Millions	Tens of Millions	*Millions*	Hundreds of Thousands	Tens of Thousands	*Thousands*	*Hundreds*	*Tens*	*Units*	
								1	One.
							2	1	Twenty-one.
						3	2	1	Three hundred and twenty-one.
					4	3	2	1	Four thousand 321.
				5	4	3	2	1	Fifty-four thousand 321.
			6	5	4	3	2	1	654 thousand 321.
		7	6	5	4	3	2	1	Seven millions 654 thous. 321.
	8	7	6	5	4	3	2	1	Eighty-seven millions 654, 321.
9	8	7	6	5	4	3	2	1	987 millions, 654, 321.

The first column on the right hand contains units, and the figure I in that column, represents the number One. The second line consists of 1 Unit and two Tens, or twenty-one; the third line of 3 hundreds, 2 tens, and 1 unit, or three hundred and twenty-one; and in the same manner the lowest line contains 9 hundreds of millions, 8 tens of millions, and 7 millions; in all 987 millions; also 6 hundreds of thousands, 5 tens of thousands, and 4 thousands; in all 654 thousands; and lastly, 3 hundreds, 2 tens, and 1 unit: so that the whole sum expressed by the 9 figures in the lowest line, is nine hundred and eighty-seven millions, six hundred and fifty-four thousands, three hundred and twenty-one.

In

In the same manner, Numeration may be carried on to any extent, as in the following example.

3	*Trillions*	
2	Hundreds of thousands	
1	Tens of thousands	
2	Thousands	of *Billions*
3	Hundreds	
4	Tens	
5	*Billions*	
6	Hundreds of thousands	
7	Tens of thousands	
8	Thousands	of *Millions*
9	Hundreds	
8	Tens	
7	*Millions*	
6	Hundreds	
5	Tens	of *Thousands*
4	*Thousands*	
3	Hundreds	
2	Tens	
1	Units	

Where 19 figures represent the sum Three *Trillions*, two hundred and twelve thousand, three hundred and forty-five *Billions*, six hundred and seventy-eight thousand, nine hundred and eighty-seven *Millions*, six hundred and fifty-four *Thousands*, three hundred and twenty-one.

OF ADDITION.

THE fundamental operations of Arithmetical calculation are these four; *Addition*, *Subtraction*, *Multiplication*, and

Division: or rather, as quantities are susceptible of no other modification but augmentation and diminution; the two last operations, *Multiplication* and *Division*, are in fact only speedy methods of performing the two first operations, *Addition* and *Subtraction*.

By Addition we assemble and express on paper, the aggregate value of a number of separate quantities. When the quantities or the numbers by which they are expressed, consist of only one place of figures, as when 3, 5, 7, and 9, are to be added together, we say thus; three and five are eight, and seven are fifteen, and nine are twenty-four, writing 2 for the number of tens, and 4 for the remaining units, as in the margin: But when the sums to be added together consist of more than one place of figures, the scholar must be careful to place them so as that Units shall be immediately under units, Tens under tens, Hundreds under hundreds, &c. as in the an-annexed example, where the inhabitants of the principal towns of a certain county, being calculated to be 4,386, 2,285, 7,309, 3,025, and 1766; it is required to know the amount of the population of these five towns.

	9
	7
	5
	3
Sum	24

Write down these several sums, as in the margin; then drawing a line under the whole, say 6 and 5 are 11, and 9 are 20, and 5 are 25, and 6 are 31; that is, 3 tens and 1 unit; then write this 1 in the place of units, and *carrying* (as it is termed) the three tens to the second column of figures, say, 3 and 6 are 9, and 2 are 11, and (passing over the nought) 8 are 19, and 8 are 27: here are 2 tens and 7 units, which units are to be written under the second column, and the 2 tens carried or added to the third column. Then say 2 and 7 are 9, and 3 are 12, and 2 are 14, and 3 are 17; where the 7 units are to be written under the column now

	Thousands.	Hundreds.	Tens.	Units.
	4	3	8	6
	2	2	8	5
	7	3	0	9
	3	0	2	5
	1	7	6	6
1	8	7	7	1

summed

summed up, and the ten is to be carried to the fourth column; saying, 1 and 1 are 2, and 3 are 5, and 7 are 12, and 2 are 14, and 4 are 18. This being the last column, the 8 units are written under the figures added together, and the ten comes to occupy an additional place to the left hand: hence we find the whole amount of the population of the five towns to be eighteen thousand seven hundred and seventy-one persons.

It is of the utmost importance in business to be able to perform Addition with dispatch and accuracy; the learner ought therefore to practise it repeatedly, with sums of various lengths; and if he can readily add two simple units (which are also called *Digits*) together, he will easily add a Digit to a higher number: thus, 6 and 9 are 15, and 36 and 9 are 45.

In summing up a long column of figures, where mistakes may happen, from interruption or other accidents, it is proper to write down the full amount of each column, either on a separate paper, or in the way shown in the margin; by which means, should any error be suspected, each column of figures may be examined separately, without its being necessary to repeat the whole operation.

```
 7652
 .428
56865
 .317
 3509
-----
...31
..14.
.26..
21...
5....
-----
73771
-----
```

Here the 1st. column amounts to 31, the 2d. to 14, of which the 4 is placed under the 5 tens of the 1st column: and so on with the others; and by the last addition of these several amounts, the total 73,771 is obtained, in the same way as if the several numbers of tens had been carried to the succeeding column, as before directed.

To ascertain the accuracy of Addition, several methods have been devised, as the following: 1st. to repeat the operation, beginning at the top of the column, and adding the figures downwards: 2d. to divide the column, if it be long, into several portions, and add each separately; the

total

total of these parts added together, ought to be equal to the total of the whole column taken at once: 3d. to cut off a line, the uppermost for instance, of the account, and then add the remaining lines, the amount of which added to the line cut off, should be the same with the total first found, thus,

	3783769
	645098
	82769
	976856
Total of the 4 lines	5488492
Total of the 3 lower lines. . .	1704723
Total of this last sum and 1st line	5488492

Whatever be the quantity adopted for the unit, it may be supposed to be divided into a number of equal parts; and these parts may be of any determined magnitude: but if, for example we should say, that the Pound of Sterling money is divided into 960 Farthings, it would be found extremely difficult either to reckon or to form a distinct conception of such a number of individual farthings, or of intermediate sums between 1 and 960. For this reason the Pound is first divided into 20 equal parts called Shillings; each shilling into 12 equal parts, called Pence; and each penny into 4 equal parts called Farthings: so that 1 Pound will contain 20 Shillings, or 240 Pence, or 960 Farthings.

When a sum is given consisting of one or more units, together with one or more of these subdivisional parts, it is said to be a complex sum; as 25 Pounds, 14 Shillings, 9 Pence, 3 Farthings; or written in this manner, £. 25 .. 14*s.* .. 9*d.* .. 3 *qrs.* where the mark £. stands for the Latin term *Libra*, a Pound in weight, such a quantity of Silver having originally been the value of a Pound Sterling; *Sh.* for Shillings: *D.* being the first letter of the Latin word *Denarius*, a Denier or Penny; and *Qrs.* for

Quadrans, which in Latin signifies the fourth part of any thing.

Suppose a Tradesman wishes to know the amount of seven Bills, drawn from his books; he writes down the several sums as below;

	£.	20 sh.	12 d.	4 qrs.
A's Bill amounts to . . .	24	.. 18	.. 11	.. 1
B's ditto. . . .	9	.. 13	.. 06	.. 3
C's ditto. . . .	152	.. 00	.. 09	.. 2
D's ditto. . . .	35	.. 08	.. 07	.. 0
E's ditto. . . .	9	.. 12	.. 10	.. 1
F's ditto. . . .	85	.. 19	.. 08	.. 3
G's ditto. . . .	212	.. 13	.. 04	.. 0
Total of Bills . £.	530	.. 07	.. 09	.. 2
Total of the 6 lower lines £.	505	.. 08	.. 10	.. 1
Total of this last and the 1st. line £.	530	.. 07	.. 09	.. 2

In this operation the several denominations must be placed under one another, observing that units come under units, tens under tens, &c.: then beginning with the column of Farthings, on the right hand, say 3 and 1 are 4, and 2 are 6, and 3 are 9, and 1 are 10. Now here are 10 farthings, and as 4 farthings are 1 penny, 8 will be 2 pence, and there will be 2 farthings over: or 10 *qrs.* will be equal to 2*d.* 2 *qrs.* Then in the column of farthings write the 2, and carry on the 2 pence to be added to the column of pence, saying, 2 and 4 are 6, and 8 are 14, and (going first up the units) 7 are 21, and 9 are 30, and 6 are 36, and 1 are 37. Write this number on one side of your paper, and then count up the figures in the tens column, thus, 1 and 1 are 2, and write down 2 under the 3 of the 37, which will make 57 pence. But as 12 pence make 1 shilling, 24 pence will be 2 shillings, 36 pence 3 shillings, 48 pence 4 shillings, and 57 pence, being 9 pence more than 48, the whole sum will be 4 shillings and 9 pence.

This

This sum 9 must therefore be written in the units' place of the pence column (filling up the tens' place, either with a dot or a nought, for uniformity's sake), and carrying the 4 shillings to the column of shillings, begin with the units' place, saying, 4 and 3 are 7, and 9 are 16, and 2 are 18, and 8 are 26, and 3 are 29, and 8 are 37: then write on one side the 7, and carrying the 3 to the ten's column, say, 3 and 1 are 4, and 1 are 5, and 1 are 6, and 1 are 7, and 1 are 8: This 8 written on the left hand of the 7 will make 87 shillings: but as 20 shillings are 1 pound, 40 shillings will be 2 pounds, 60 shillings 3 pounds, and 80 shillings 4 pounds, 87 shillings will therefore be 4 pounds 7 shillings. The 7 shillings are then to be written in the units' place of the shillings column, (filling up the tens' place with a nought or a dot as before), and the 4 pounds must be carried on, to be added to the units' place of the column of pounds, saying, 4 and 2 are 6, and 5 are 11, and 9 are 20, and 5 are 25, and 2 are 27, and 9 are 36, and 4 are 40. This column consisting of integers of the highest denomination, it is to be added up, as the simple integers given in the former examples; by which the sum of the several Bills will be found to be £. 530 .. 07*s.* 09*d.* .. 2*qrs.* which was required to be done.

To prove the accuracy of Addition of sums of money, or any other complex numbers and quantities, the same methods are used, as those pointed out for checking addition of integers.

Again, if it be required to add together a number of quantities expressing what is called *Averdupois* weight; it must be observed, that

1 Ton weight	contains	20 Hundreds
1 Hundred		4 Quarters
1 Quarter		28 Pounds
1 Pound		16 Ounces
1 Ounce		16 Drams (or as it

ought to be *Drachms.*

The

The following example will show how addition of such quantities is performed; where *T.* means Tons, *Cwt.* Hundred weight, *Qrs.* Quarters, *Lbs.* Pounds weight, *Oz.* Ounces, *Dr.* Drams.

		20	4	28	16	16
	Tons	Cwt.	Qrs.	Lbs.	Oz.	Dr.
	52 ..	19 ..	3 ..	25 ..	12 ..	15
	628 ..	10 ..	1 ..	19 ..	06 ..	12
	47 ..	13 ..	2 ..	08 ..	11 ..	07
	9 ..	09 ..	1 ..	26 ..	09 ..	14
	63 ..	11 ..	3 ..	22 ..	13 ..	12
Total weight	802 ..	05 ..	1 ..	19 ..	6 ..	12
Total of 4 lower lines	749 ..	05 ..	1 ..	21 ..	09 ..	13
Total of this and first line	802 ..	05 ..	1 ..	19 ..	06 ..	12

The column of Drams amounts to 60, equal to 3 ounces and 12 Drams, which last sum is written in the column, and the 3 Ounces being carried to the next column, make it amount to 54 Ounces, equal to 3 Pounds and 6 Ounces over. These 6 ounces are written in the column, and the 3 Pounds are carried to the following column, making it 103 Pounds, or 3 Quarters and 19 Pounds. Writing down 19 in the Pounds Column, the 3 Quarters are carried on and added to the column for that denomination, which thus amounts to 13 Quarters, equal to 3 Hundred weight, and 1 Quarter over. Enter this Quarter in the proper column, and proceed with the 3 Hundreds to the column of that denomination, which will then amount to 65 Hundreds, equal to 3 Tons and 5 Hundreds; which last sum being written in its due place, the 3 Tons added to the column of Tons, will give a total of 802 Tons. To ascertain this addition, cut off, by a line, the first row of quantities, and add the remaining 4 rows together, producing a total which when added

added to the first row, will, if the operation be correct, give a total equal to that found by the addition of all the five rows of quantities.

In the practice of keeping accounts, it frequently happens that the whole of an account can not be contained in one page or folio of the book; when this is the case, the custom is to sum up the contents of that page or folio, and write it down at the bottom of the column, with the words *Carried forward* opposite to the sum; and the same sum or total is entered at the top of the money column of the subsequent page or folio, with the words *Brought forward*, leading to it. In this manner each preceding page is summed up, and the amount made the first article in the following page, until the account is closed, when the amount of the last page, as comprehending all the particular totals of the preceding pages, is to be considered as the total amount of the whole account.

OF SUBTRACTION.

Subtraction means that branch of calculation by which we find the difference between two given quantities; or agreeably to the meaning of the term, by which we draw a less sum from a greater, and so discover what quantity will be left as a remainder.

Although Subtraction be an operation, directly opposite to Addition, yet the numbers must be placed under one another as before; that is, the small number must be written under the great, and beginning at the units or first figures on the right hand, which may for instance be 8 and 5, we say if from 8 units, such as pounds, yards, &c, we take away 5, there will remain 3: or the difference between 5 and 8 is 3. Again, when one figure of the least number happens

happens to be greater than the corresponding figure in the great number, as in subtracting 37 from 65; as it is impossible to take 7 units out of 5, we *borrow*, as it is called, 1 ten from the place of tens in the great number, and add it to the 5; thereby making 15; from which if we subtract or take away 7, the remainder will be 8. This 8 is accordingly written under the place of units, and the operation proceeds in this way. By borrowing the ten from the place of tens, the figure 6, may be supposed to be reduced to 5: we have then to subtract the 3 of the less number from the 5 of the greater, and the remainder or difference will be 2; that is in all 28. Or instead of taking 1 away from the 6, if we repay or add 1 to the 3, standing under it, we shall have 4, which again being subtracted from 6 will leave 2 as before.

Example.

An army consisting in all of . . .	62187 men
Detaches a body of	15365
How many remain behind? . . .	46822 men.

The *Subtrahend* or less number being written under the *Minuend* or greater, observing to place units under units, tens under tens, &c, we begin at the right hand, saying, if from 7 men 5 be taken away, 2 will remain behind: this 2 therefore is written in the column of units. Again, if from 8 be taken 6, 2 will remain: Then from the third figure of the greater sum 1, we should take away 3; but this is impossible; we must therefore borrow 1 from the next figure on the left hand 2, which, if the 1 be considered as occupying the place of units, will become a ten, which 10 being added to the 1 will make 11. From this sum 11 we can subtract, as was first proposed, the 3, and the remainder will be 8, which is also written in its due place. Having borrowed 1 from the 2 in the fourth place of figures, this 2 comes to be reckoned as 1, and from it, as has just been done, we subtract the 5, by borrowing another ten from the

 fifth

fifth figure 6, making the 1 to be 11: then 5 taken from 11 will leave 6 for the remainder.—But instead of thus diminishing the number from which you have borrowed, in the upper row of figures, it is more customary to leave that number as it is in fact, and to add the ten which was borrowed, to the number to be subtracted: hence we will have, in the fourth place of figures 5 and 1 equal to 6, which being taken from the 2 in the first row, augmented by another ten borrowed from the 6 in the same row, and called 12, will leave a remainder of 6. Lastly, adding or carrying to the 1 in the Subtrahend, the ten that was borrowed, we have 2, which taken from the 6 in the first row or Minuend, will leave 4, to be written in the remainder; and the operation will be completed, showing the number 46,822, for the army remaining in the camp.

To prove Subtraction, you may either add the remainder and the less number together, which if no error have been committed, will give a total equal to the greater number; or from the greater number subtract the remainder, and the difference will be equal to the less number.

The following examples may be sufficient to shew the method of performing subtraction of complex numbers.

A trader retiring from business wishes to know how much clear property he will possess, when all his debts are paid.

	£.	s.	d.	qrs.
His property is	9875 ..	11 ..	05 ..	3
And his Debts are	2187 ..	15 ..	08 ..	1
Remainder	£ 7687 ..	15 ..	09 ..	2
Proof	£ 9875 ..	11 ..	05 ..	3

Beginning at the right hand, he subtracts 1 farthing from 3, and writes down the difference 2 in the remainder: then proceeding to the pence he finds he cannot take 8 pence from 5 pence; to this 5 therefore he borrows an unit from the next column, or 1 shilling, which is equal to 12 pence, and

and adding this 12 to the former 5 he has 17 pence, from which 8 being taken away, there will be a remainder of 9 pence, to be entered in the proper column. The unit or shilling thus borrowed he now repays or adds to the 15 shillings, thus made 16, which are to be subtracted from the 11 in the upper row: but this being impossible, an unit or pound, equal to 20 shillings must be borrowed and added to the 11 which will thus become 31, from which the 16 of the lower row being subtracted, 15 will remain. The pound borrowed is now added to the figure 7 in the units place of the column of pounds, and the amount subtracted from 15 (that is 5 and 10 borrowed from the adjoining figure) will leave 7 for a remainder :—and proceeding in this manner, as has been already shown, the total remainder will turn out to be £.7687 .. 15s. .. 09d. .. 2qrs. as in the example.

The accuracy of this subtraction will be proved by adding the remainder to the less number, which will give a total equal to the great number.

The following example is performed in the same way.

	20	4	28	16	
	Tons	Cwt.	Qrs.	Lbr.	Oz.
From	362	06	1	13	06
Take	285	12	3	22	12
Remainder	.76	13	1	18	10

MULTIPLICATION.

To multiply one number by another, is to take the value of the one number as often as there are units in the other number: thus to multiply 8 by 5 is to take 8 five times; or in other words, as was before mentioned, it is to add 8 five

8
8
8
8
8
—
Sum 40
—

Multiplicand 8
Multiplier 5
—
Product 40
—

times together, as if the figures stood in a column as in the margin; where the amount of five times 8, is 40. But instead of this repeated addition of a number, which in many cases would be extremely inconvenient, we write down the number to be multiplied as in this example, calling it the *Multiplicand*, and under it, beginning at the right hand, that units may stand under units, &c., we write the number denoting how often the value of the multiplicand is to be taken. This last number is called the *multiplicator* or *Multiplier*: and observing that 8 repeated 5 times will amount to 40, we say 5 times 8 are 40; which sum is written down as in the margin, and is called the *Product.*

The Multiplicand and Multiplicator are also termed *Factors.*

To assist in performing Multiplication, the following Table must be well gotten by heart.

MUTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This

This Table is formed by repeatedly adding to itself each number from 1 to 12 included: that is, the first line contains numbers formed by the successive addition of 1 to itself, thus 1 and 1 are 2, and 1 are 3, and 1 are 4, and so on including 12. Again, the 4th line is formed by the successive addition of 4 to itself; thus 4 and 4 are 8, and 4 are 12, and 4 are 16, &c. The use of the Table is this: Suppose it were required to find the amount of 6 added to itself 8 times; that is, the product of 6 multiplied by 8. Look along the upper row of figures for the number 6, then in the same way finding down the left hand column the number 8, carry the eye along the line of 8, until you come to the column at the top of which stands the number 6, and in the space formed by the intersection of this column with the line of 8, you will find the number 48, which is the product of 6 multiplied by 8; or which is the same thing, of 8 multiplied by 6; for it is of no importance which of the two Factors is called the Multiplicand and which the Multiplicator, as 6 repeated 8 times, and 8 repeated 6 times, give precisely the same product. In the same way, if it be desired to know the product of 12 multiplied by 7, under the number 12 in the last column to the right hand, and on the line opposite to 7, on the left hand margin, you will find 84; or reversing this process, under 7 and opposite to 12 you will likewise find the same number 84.—To see how many 9 times 11 are; under 11 and opposite to 9 you will find 99; and the same will be seen under 9 and opposite to 11.

If it be required to multiply two numbers both 12, or one or both under 12, the operation will be performed by a simple inspection of the preceding Table: but when greater numbers are to be multiplied together, you must write down the multiplicand, as in the margin, and under the units' place the multiplicator, which in this case is 8: then beginning with the first figure on the right-

	83755
	8
Product	670040

hand,

hand, say, 8 times 5 are 40, (see the table) write nought in the units' place as there are none, reserving the 4 tens to be added on the next step: say again, 8 times 5 are 40, and the 4 tens remaining to be added are 44: write down the 4 which are over the tens, and reserve the 4 tens for the succeeding step. Here we have 8 times 7, equal to 56, and 4 are 60; write down the nought, and go on with 8 times 3 are 24, and 6 reserved are 30, write nought again, reserving the 3 for the next step, which is 8 times 8, equal to 64, and 3 are 67, which sum being the last, is written down entire. The Product therefore of 83755 by 8 is the sum 670040

```
      83755
      83755
      83755
      83755
      83755
      83755
      83755
      83755
     ------
Sum. 670040
     ------
```

If the same operation were to be performed by Addition, as in the margin, where the multiplicand is repeated 8 times, both the labour and the chances of error would be much increased.

When the Multiplier consists of several places of figures, you multiply by each figure separately, observing to place the first figure of each product, immediately in the column under that figure by which you multiply. Thus in the example here given, where the number 42876 is to be multiplied by 356. Begin with 6 of the multiplier, saying, 6 times 6 are 36, write down the 6 units and carry the 3 tens to the next product, saying, 6 times 7 are 42, and 3 are 45, where the 5 is written down and the 4 to be carried to the next product, and so on to the end of the multiplicand. Then take the figure 5 of the multiplier, and say, 5 times 6 are 30: as there are

```
   42876
     356
--------
  257256
 214380
128628
--------
15263856
--------
```

are no units in this case, write 0 in the column immediately under the 5 by which you multiply, and go on, saying, 5 times 7 are 35, and 3 are 38; write down 8 and carry 3 to the next product; and proceed in this way to the end of the multiplicand. Lastly take the figure 3 of the multiplier and begin as before with the first figure of the multiplicand, 3 times 6 are 18, writing the 8 units in the column under the multiplier 3, ond carrying the 1 to the next product, and so on. When you have in this manner written down the several sums produced by multiplying the whole multiplicand by each figure of the multiplier, draw a line under the whole, and add the several products together, which will give the amount produced by the multiplication of the multiplicand by the whole multiplier.

The reason for beginning the several lines of product, under the respective figures by which you multiply, is this, that, as in the example given, to multiply by the aggregate sum 356, must be the same thing as to mulitiply by the 3 sums 300, 50, and 6. Let this be done:

42876	42376	42875
300	50	6
12862800	2143800	257256

257256	the product of the multiplier	6.
2143800	. . . .	50
12862800	. . . .	300
15263856	. . . .	356

When you have multiplied in the usual way by 6, and set down the product, you come to multiply by 50; and beginning with the 0 supposing it to be an effective figure, it is evident that nought multiplied by 6, or in other words, nought or nothing repeated, 6, or 60, or 600 times, will never produce any positive quantity; write down therefore 0 in the product under the 0 of the multiplier;

plier; and instead of continuing to write a line of noughts, proceed to the next figure 5, employing it in the usual way. In the same manner with the multiplier 300. Beginning with the first 0, write another 0 under it; do the same with the second 0; and then multiply with the figure 3 as before.

Numbers are divided into *prime* and *composite*: a prime number is that which cannot be produced by the multiplication of any two smaller numbers; as 3, 5, 7, 11, and so on; but a composite number is that which may be produced by the multiplication of two smaller numbers; as 4 which is formed by multiplying 2 into 2; 15, the product of 5 multiplied by 3, &c.

To ascertain the accuracy of multiplication, a common method is, to do what is called *throwing out the nines*; as in the example here given:

```
    76385
     6523
   ------
   229155
  152770
 381925
458310
---------
498259355
```

Make a cross as here shown, then begin to add the figures in the multiplicand, saying, 7 and 6 are 13, and 3 are 16, and 8 are 24, and 5 are 29; which contains three nines, and 2 over. Write this 2 on the left side of the cross, and add up the multiplier in the same way, which will amount to 16, equal to 1 nine and 7 over; this 7 must be written on the opposite side of the cross; and multiplying the two figures of the cross into one another; 7 times 2 are 14, that is 1 nine and 5 over, which 5 is written in the upper part of the cross: Lastly, add up the figures

figures of the product, amounting in this example to 32, (for the two nines are passed over, as in this operation of no consequence), equal to three nines and 5 over: this number being placed in the lower part of the cross, and agreeing with the 5 in the upper part, is a proof that the multiplication is correct; or at least that if there be an error, that error amounts to one or more nines; an accident not very likely to happen.

This casting out of the nines will be farther explained, when we have gone through the practice of Division: in the mean time it will be proper to give some account and examples of Contractions in multiplication, by which the labour of calculation may be considerably diminished:

1st. When the multiplier is a number produced by the multiplication of two others; that is when it is a composite number, you may multiply by the two numbers composing it; thus, to multiply 626 by 48, which is itself the product of 8 by 6, or of 12 by 4, multiply 626 first by 8, and then the product by 6, as in the example.

626	626	626
48	8	12
5008	5008	7512
2504	6	4
30048	30048	30048

2d. To multiply by 5, which is the half of 10, you may first write down nought in the product, and then annex the half of the multiplicand: as 36 multiplied by 5, give 180; that is the half of 36, with a nought annexed. Again, to multiply by 25, or the fourth part of a 100, annex two noughts and write down in the product the 4th part of the multiplicand. For example, 1744 multiplied by 25 give 43600, equal to the 4th part of 1744 with 00 added.

3d. When one figure of a multiplier is twice, thrice, four times, &c. or one half, one-third, one-fourth, &c. of any other figure of the multiplier, then the operation may

be

be shortened, by taking those proportional parts of the products found by the preceding numbers: for instance, in multiplying any sum by 639, when you have wrought with the nine, you may take one-third of the product for the product by 3; and doubling this last sum you will have the product by 6.

4th. In multiplying by 9, add 0 to the multiplicand, and beginning with the first figure of it, subtract it from 0, or 10, and so proceed subtracting each subsequent figure of the multiplicand, from the one preceding it, and setting down the remainder as a product: Thus in multiplying 436 by 9, you may write or suppose 0 on the right hand of 6, and say 6 from 10, and 4 remain, which is written down in the product; and then 3 and 1 carried are 4, which taken from 6 leave 2 for the product; 4 from 13 and 9 remain for the product; and carrying 1 for the ten just borrowed, say 1 from the 4 of the multiplicand, and 3 will remain to complete the product.

Multiplication may be performed not only with integral numbers, as of Pounds, Tons, &c. but of compound quantities such as Pounds, Shillings, Pence, &c. Tons. Hundreds, Quarters &c. in this manner: Suppose that in dividing the value of a prize at sea, between 6 Captains, each got £. 266 .. 17 .. 8 .. 3. What was the value of the whole prize?

	£.	s.	d.	q.
Each Share	266 ..	17 ..	8 ..	3
Shares				6
Total . .	£. 1601 ..	06 ..	4 ..	2

It was already observed that multiplication of Integers is only a short method of adding to itself the multiplicand, as often as there are units in the multiplier; the same is true of Compound Multiplication, as may by seen from this example; for if the share were written down 6 times, and these sums were added together, they would give a total equal to the above product, which is obtained in

in this way :—6 times 3 are 18 farthings, or 4 pence 2 farthings; set down the surplus 2, and carry the 4 to the pence, saying 6 times 8 are 48, and 4 carried are 52 pence, or 4 shillings and 4 pence; write the surplus 4, and carry 4 to the shillings: 6 times 17 are 102, and 4 are 106 shillings, equal to 5 pounds and 6 shillings, which 6 being written in the column of shillings, multiply the 266 pounds by 6, adding the 5 pounds carried, obtaining a product of £. 1601 .. 06 .. 4 .. 2. for the total value of the prze.

When the multiplier exceeds 12, or consists of 2 or more places of figures, find its component parts, and multiply the given quantity by one of these component parts, and the product by the other: for example,

	Tons.	Cwt	Qrs.	lbs.	oz.
Multiply	23 ..	12 ..	2 ..	16 ..	12
By 315					9
	212 ..	13 ..	3 ..	10 ..	12
					7
	1488 ..	16 ..	3 ..	19 ..	04
					5
Tons	7444 ..	04 ..	2 ..	12 ..	04

In this example, the multiplier 315 being a compound number, formed by the successive multiplication of 9, 7, and 5; multiply the quantity given by one of these component parts, as 9; then multiply the product by another compotent part, as 7, and the last product by 5, which will give the result, as if the whole multiplier 315 had been employed at once. It is of no consequence in what order these component parts are used, for 9 multiplied by 7, and the product by 5, will give the same result, as if they had been employed in this order, 5, 7, and 9; or 7, 9, and 5, &c.

It is however, convenient, when practicable, to employ these multipliers in such a way, as to remove some of the

lower divisons of the multiplicand; as in the preceding example, had 8 been the first component part, we would have had 8 times 12 equal to 96 ounces, or exactly 6 pounds, so that there would have been no surplus in the column of ounces, and the rest of the operation would have been so far abridged.

Had the multiplier been a prime number, instead of a composite; that is, for instance, had it in this example been 317, or 313, it would have been proper to have multiplied by the same component parts, and for 317, which is 2 more than 315, to have added to the last product twice the amount of the first quantity given; and in the second case for 313, to have subtracted from the last product, twice the quantity given.

When the mutiplier is very large, you may multiply by 10, and that product again by 10, to obtain 100 times the number given; and if the multiplier is, or exceeds 1000, multiply again by 10; continuing still to do so, as often as may be necessary; then multiply the given number by the figure in the units' place of the multiplier; the first product by the 2 figure or tens' place; the second product by the 3d figure, or that in the hundreds' place, &c. All these products added together will give the total required.

For example. What is the amount of the Pay of 4325 Labourers at £. 36 .. 15 .. 8 each per annum.

	£. s. d.		£. s. d.	
	36 .. 15 .. 8	by 5 give—	183 .. 18 .. 4	— 5 times
	10			
10 times —	367 .. 16 .. 8	by 2 —	735 .. 13 .. 4	— 20 times
	10			
100 times—	3678 .. 06 .. 8	by 3 —	11035 .. 00 .. 0	— 300 times
	10			
1000 times	36783 .. 06 .. 8	by 4—	147133 .. 06 .. 8	—4000 times.
			£. 159087 .. 18 .. 4	—4325 times

An

An application of multiplication which very frequently occurs, is to calculate the amount of any number of articles, in Value, Weight, Measure, or any other mode of reckoning. The multiplicand in such cases expresses the rate of value, &c. of one article, and the multiplier the number of articles to be estimated:, this multiplier is therefore always an abstract number, having no reference to any value or measure whatever. The only exception to this remark, and the objection is but apparent, is in performing operations in *Mensuration*, to find the superficial, or the solid quantity of any body whose dimensions are given in certain determinate measures, such as Inches, Feet, yards, &c. In these operations, yards, feet, and inches may be multiplied into yards, feet and inches, and the product will consist of quantities of similar, but not identical denominations; for feet multiplied by feet, will give not *lineal* but *superficial* feet, and this product again multiplied by feet will give *solid* feet. To assist in these operations, the following Table of superficial measures multiplied together, must be well understood.

Table for multiplying Yards, Feet, and Inches, by yards, feet, and inches.

Inches by inches		Square inches of which	144 are 1296	1 Sq. Foot 1 Sq. yard
Inches by feet, or Feet by inches		a Figure	12 108	1 Sq. foot 1 Sq. yard
Inches by yards, or yards by inches	give	a Figure	4 36	1 Sq. foot 1 Sq. yard
Feet by feet		Square feet	9	1 Sq. yard
Feet by yards, or Yards by feet		a Figure	3	1 Sq. yard
Yards by yards		Square yards.		

The principles on which this Table is founded, will be explained when we come to treat of mensuration of Surfaces;

faces; but in the mean time the following examples will show how this species of multiplication is performed.—How many yards of carpeting will cover a room 25 feet 8 inches long, and 17 feet 6 inches broad?

25 f. .. 8 in.	17 f.	25 f.	8
17 .. 6	8 in.	6 in.	6
175	136	150	12) 48
25	150		
			4
425	12) 286 (		
23 .. 10			
4	23 .. 10		
9)449 .. 02			

49 yards, 8 feet, 2 twelfth parts, equal to 1 sixth part of a foot, superficial measure.

The length and breadth being written in two lines, begin and multiply the 25 feet by the 17, setting down the product 425, which by the Table, must be square feet, as the result of multiplying lineal feet into one another. Then multiply the 17 feet by the 8 inches of the multiplicand, producing as above 136 figure sof a certain description, whose length is 1 foot, and whose breadth is 1 inch; to this add the product of 25 feet, multiplied by 6 inches or 150; and taking the 12th part, according to the table, of the sum 286, you will have 23 square feet, and 10 twelfth parts of 1 foot. Lastly, multiply the 8 inches by 6 inches, and the product 48 will be square inches equal to 4 of the above-mentioned figures, or 1 third part of 1 foot square. The 4 being set down, add the whole together, and you will have 449 square feet, and 2 twelfth parts: but as 9 square feet are 1 yard superficial, take the 9th part of 449, when you will have 49 yards 8 feet, and 1 sixth part, for the quantity of carpeting required.

Division.

DIVISION.

Division is an operation by which we discover how often one given number is contained in another.

The number to be divided is called the *Dividend*, the number by which you divide is the *Divisor*, and that which expresses how often the Divisor is contained in the Dividend, is called the *Quotient*. If it happen that after the Divisor is taken as often as it can out of the Dividend, there be still something over, this is called the *Remainder*.

Of whatever denomination be the Dividend and Divisor, the Quotient is either of the same denomination, or an abstract number; as an expression of the magnitude of the former sum relative to the latter: but the Remainder is always of the same denomination with the Dividend. Thus if we divide £48. amongst 6 men, the Quotient 8 will represent the number of Pounds due to each: but if we wish to know how many yards of silk may be had for £. 1 when 36 yards cost £. 12, we divide the 36 yards by the 12 pounds, and the Quotient 3 shows the number of yards required.

As Multiplication is a short method of performing Addition, so Division is an abridgement of Subtraction: for instance, to know how often 6 is contained in 24, perform the repeated subtractions as here shown:

From	24
take	6
1st Subtraction	18
	6
2d Subtraction	12
	6
3d Subtraction	6
	6
4th Subtraction	0

This operation would be very tedious; it has therefore been agreed that instead of subtracting the Divisor 6 at once from the Dividend 24, we may multiply the Divisor by any number which will give a product not greater than the Dividend;

as

as in this example, we find that 4 times 6, will be just 24; we therefore say that 6 is contained 4 times in 24, and nothing remains over. By this method the operation is much shorter and more expeditious than by repeated subtractions.

To assist in this operation you may make use of the Multiplication Table already given, in this manner: run along the first line for the figure 6, and going down the column under 6, you will find 24, on a line at the beginning of which stands the figure 4, which will be the quotient. Again, to divide 49 by 8; in the column under 8, find the number nearest to, but less than, 49 which will be 48, and going to the beginning of the line on which you see 48, you will find 6 for the quotient: and the difference between 48 and 49, that is 1, will be the remainder: or in other words, 8 will be contained 6 times in 49, and there will be 1 over.

In this manner when the Divisor is 12 or under, the Multiplication Table will be serviceable: but when it exceeds 12, we try any Multiplier that promises to answer; and if the product is greater than the Dividend, the Multiplier or Quotient is too great; on the contrary if the product falls short of the Dividend by a sum greater than the Divisor, or equal to it, the Multiplier is too small; and by repeated trials, at last the proper Multiplier or Quotient is discovered.

Example. Divide 6873 by 8.

```
Divr  Dividend  Quotient
 8  )  6873  (  859
       64··
       ——
       ·47
        40
        ——
       ··73
         72
         ——
         ·1 remainder
```

Write down the sum to be divided, and making a bending line on each side, place the Divisor on the left: then ask how often you can have 8 in 6. This you see is impossible; go then to the next figure, 8, making with the first 68, and ask how often 8 may be had in 68. By the Multiplication Table you will find that 8 times 8 are 64, and

and 9 times 8 are 72, which last being more than the given sum 68, you must take the product below it, that is 8 times 8 or 64. Then write this 8 on the right-hand of the dividend, beyond the separating line, placing the product 64 under 68, drawing a line under these figures, in order to subtract the less from the greater, which will give a remainder of 4. Hence it appears that 8 may be contained 8 times in 68 and 4 will remain. To this 4 bring down the next figure of the dividend, which is 7, placing a dot under it, to prevent mistakes, by showing that it has already been used.

Now we have a new dividend 47, in which 8 will be contained 5 times: this 5 is written in the quotient, following the 8, and multiplying the divisor 8 by 5 we have 40, to be written under 47, and subtracting the less from the greater, the remainder will be 7; to this 7 bring down the next figure of the dividend, 3, making 73, and, as before, ask how often 8 can be taken out of 73. By the Table it will appear that 9 times 8 are 72, the nearest number under 73; and writing the 72, subtract it from 73, which will give 1 for the remainder. All the figures of the dividend being now exhausted, the division is performed, and we find that 8 is contained 859 times in 6873, and that there is 1 remaining as a surplus.

Again in working with a divisor of more than one place of figures, as in the following example, to divide 249295 by 365, write down the divisor and dividend as in the last example, thus,

```
Divr   Dividend   Quotient
365 )  249295  (  683
       2190··
       ------
       ·3029
        2920
        -----
        ·1095
         1095
         ----
         ....
```


and

and counting how many places of figures are in the divisor, take the same number in the dividend from the left hand, and say how often can I take 365 out of 249? As this cannot be done, take in another place of figures and say how often can I have 365 out of 2492? If the divisor consisted only of the figure 3, we should have it 8 times in the 24 of the dividend: but 36 could not be taken 8 times out of 249; and even if it were tried to take 365 seven times out of 2492, it would be found impossible: let 6 then be tried, and multiplying 365 by 6, place the product 2190 under the dividend, and drawing a line, subtract this sum from the figures above, by which a remainder will be found, 302. To this sum bring down the figure 9, which is next after those already employed, and say how often can the divisor 365 be found in 3029: This upon trial will be found to be 8 times, which multiplied into 365 will give 2920; and this subtracted from the former remainder with its additional figure 9, will leave another remainder 109. To this bring down the last figure of the dividend 5, and say how often can the divisor 365 be obtained out of 1095; This will be found to be 3 times; and multiplying 365 by 3, you will have 1095, equal to the sum above it, and leaving therefore no remainder.

A number which divides another into any parts, without leaving a remainder, is called a *Measure* of that number: thus 2, 4, 5, and 10, are measures of 20, because each of them will exactly divide 20, without any remainder: and these parts or measures are also termed *aliquot parts:* hence 1 penny, 2d. 3d. 4d. 6d. are aliquot parts of a shilling, &c.

To assist in discovering the measures of any given numbers, it must be remembered that numbers ending with an even figure, such as 2, 4, 6, 8, or 0, are all measurable by 2:—That every number ending with 5 or 0, may be measured by 5:—That all numbers, whose figures, when added together

together, give an even number of threes or nines, may be measured by 3 or 9 respectively.

When it happens, as in the following example, that the remainder together with the figure brought down from the dividend, is not equal to the divisor, you must write 0 in the quotient, and bring down another figure, to be proceeded with as before. This must be done, and 0 placed in the quotient, as often as the sum to be divided is less than the divisor.

```
5386)80806158(15003.
     5386····
     ————
     26946
     26930
     ————
     ···16158
        16158
        —————
        ·····
```

Here the number to be subtracted from the second dividend, leaves only a remainder of 16, to which 1 being brought down, makes the new dividend 161 which being less than the divisor 5386, it cannot be divided by that divisor; write therefore 0 in the quotient and bring down another figure 5. The sum to be now divided, 1615, being still less than the divisor, cannot be divided by it; you will therefore write another 0 in the quotient, and bring down the last figure of the dividend 8, making a sum of 16158, which is not only greater than 5386, but will contain it 3 times and leave no remainder.

Division may be contracted in various ways; as for instance, in dividing by any of the digits or 10, 11, & 12, write down the divisor and dividend as before:

```
         3  124
Divr. 5)8536295(0 Remr.        9)25603976(2 Remr.
        ———————                  ————————
Quot.   1707259                  2844886. Quot.
        ———————                  ————————
```

and drawing a line under the dividend, say, 5 in 8 once and 3 over:

3 over: write 1 for a quotient under the 8, and placing a small 3 over the next figure of the dividend 5, so as to make 35, say 5 in 35, 7 times, and nothing over; then write the 7 in the quotient; and as 5 cannot be taken out of 3, write 0 in the quotient and adding the following figure 6, take 5 out of 36, 7 times and 1 over; writing the 7 in the quotient, place the 1 over the next figure 2, and take 5 out of 12, twice and two over. Proceeding in this way to the end of the dividend, the quotient will come to be, as in the first example, and there will be no remainder. In the second example where the divisor is 9, the last remainder 2 is written up in the place which, in the long operation of division, would be occupied by the quotient.—The placing of the several remainders, in small characters, over the figures of the dividend, is merely as a help for beginners, and will soon be found unnecessary.

Again, when the divisor and dividend are both compounded of the same nnmber, as in the following example, where both are composed of 8 multiplied by different numbers; divide both divisor and dividend by 8, and then divide the quotient of the dividend by the quotient of the the divisor, and the quotient of this last division, will be the same as if the original numbers had been employed.

8)48(	8)45840(	48) 45840 (955 quotient.
		432
6	6) 5730(	
		264
Quotient	955	240
		240
		240
		...

When the divisor ends with one or more noughts, cut them off, and cut off an equal number of figures, beginning at the units' place of the dividend; then dividing by the remaining integers of the divisor, bring down the figures

figures of the dividend that were cut off, to annex to, or form the remainder; thus

```
56|000) 8379|628 (149
        56··
        ———
        277
        224
        ———
         539
         504
         ———
         35628 remainder.
```

Now, had this division been performed in the ordinary way, as is done below, the quotient, and the remainder would have been the same,

```
56000) 8379628 (149
       56000··
       ———
       277962
       224000
       ———
        539628
        504000
        ———
         35628
```

In dividing any sum by 10, 100, 1000, or any other number composed of 1 with noughts after it; all that is necessary is to cut off from the dividend, beginning at the units' place, as many figures as there are noughts in the divisor; when the figures on the left hand of this separation will be the quotient, and those cut off will be the remainder: thus.

```
 1|00) 487|56(             1|0000) 82|7563(
 ——————————                ——————————
Quotient 487|56 Remainder   Qt. 82|7563 Remr.
```

For neither in Multiplication nor Division does the integer 1 make any change in the number to be muliplied or divided.

To divide by 9, 99, 999, or any other number of nines, write down the dividend under itself, as far as the figures will

will allow of it, beginning with the first figure, as far in on the dividend as there are nines in the divisor, and repeating this operation as long as there are places left in the dividend: then add these several sums together, and cut off from the right hand as many figures as there are nines in the divisor; when the figures on the left hand of this separation, will be the quotient, and those on the right hand the remainder.

If any thing has been carried to that column which comes to be the units' place of the quotient, the same number must be added to the remainder, to give the true result.

99)	352896	999)	612487369(
	3528		612487
	35		612
	3564|59		613100|468
	1 carried		1 carried
Quotient	3564|60 rem.	Quot.	6131004|69 rem.

The reason of this mode of working is merely, that however often 100 may be contained in any given sum, just as often will 99 and 1 be contained in it.

In the 1st preceding example the dividend consists of 3528 hundreds, with a remainder of 96; it therefore contains ninety-nine 3528 times, and 1 as many times, besides the same remainder 96. Again, 3528 contains 35 hundreds, with a remainder of 28; it therefore contains 35 ninety-nines and 35 ones, besides that remainder: consequently the whole dividend contains 99 altogether 3528 and 35 times, that is 3563 times, and the remainders 96 and 28 with the addition of 35 ones, making in all 159: But as this sum consists of three instead of two places of figures, the 1 of the hundreds' place must be added to the 3 of the quotient now found, making it 3564; and as 1 had been carried to the units' place of that quotient, the same number must be added to the remainder; then the quotient will stand as in the example, 3564, and the remainder will be 60.

Division,

Division, like Multiplication, may be proved by casting out the nines, thus:

567)812968(1433	1433 Quot.
567...	567 Divisor
2459	10031
2268	8598
	7165
·1916	812511
1701	457 Rem.
·2158	
1701	812968 Dividend.
·457	

Cast out the nines of the divisor, placing the surplus on the left hand of the cross; do the same with the quotient, setting down the surplus on the right hand: then multiply these two figures together, adding the amount of the remainder, if any, to the product, and place on the top of the cross the surpluss of nines. Lastly, take the nines out of the dividend, and the surplus written in the bottom of the cross should be the same with the figure on the top.

But the best way to ascertain the accuracy of Division, is by reversing the operation, that is, as in the above example, if the divisor and the quotient be multiplied into one another, and the remainder be added to the product, the sum will be equal to the dividend.

Compound Division is an operation by which we divide sums composed of numbers of different denominations; in which case that of the highest denomination is divided as in the preceding examples of whole numbers; and if there be a remainder, you must multiply it by the number of units of the next lower denomination, composing an integer of the higher, adding to the product the figures of this second denomination, given in the dividend; proceeding in the same manner, as long as there are any figures, of whatever value, in the dividend. For instance, should it be required

same

to divide £. 323 .. 07 .. 09 .. 3 equally among 9 men, the operation would be performed in this way.

```
 £.       Sh.   D.   Qrs.  £.    Sh.   D.   Qrs
9)323 .. 07 .. 09 .. 3   (35 .. 18 .. 07 .. 3
  27
  ---
   53
   45
   ---
    8
   20
   ---
9)167(18
   9
   ---
   77
   72
   ---
    5
   12
   ---
9)69(7
  63
  ---
   6
   4
  ---
9)27(3
  27
  ---
```

Write down the sum to be divided, placing thenumber of men, 9, as a divisor; then dividing, as has been shown, the quotient of pounds will be 35, and the remainder will be 8. These 8 pounds can no more be divided by 9; but if we bring them into shillings, they will be divisible. Multiply therefore the remainder 8 by 20, the shillings in a pound, adding the 7 shillngs in the dividend which are likewise to be shared among the 9 men, and we have 167 shillings, which become a new dividend, to which write 9 as a divisor, and the quotient will be 18, with 5 shillings for a remainder. This 5 must next be brought into pence, by multiplying by 12 the pence in 1 shilling, taking down the 9 pence of the dividend, producing 69 pence, which divided by 9, will give 7 for the quotient, and 6 for the remainder. This 6 being multiplied by 4, the farthings in a penny, to bring the whole into farthings, taking down the 3 of the dividend, will produce 27, which divided by 9, will give 3 for the quotient, without any remainder. These 4 quotients, therefore, written out in one line, as in the example, will give £. 35 .. 18 .. 07 .. 3. for the share of each man of the 9, among whom the sum given was to be divided.

This species of Division may also be proved, by multi-

plying

plying the quotient by the divisor, and adding the remainder if any, to the product, thus,

```
 £. 35 .. 18 .. 07 .. 3     Share
                      9     Men
 ______________________
£. 323 .. 07 .. 09 .. 3     Total.
```

In the same manner to divide 837 Tuns, 13 Cwt. 2 Qrs. 17 Lbs. 6 Ozs. by 36.

```
     T.    Cwt.  Qrs. lbs.   ozs.   T.   Cwt    qr.  lbs.  oz.
 36)837 .. 13 .. 2 .. 17 .. 6.   (23 .. 05 .. 1 .. 14 .. 07
    72                                                  6
    ___                          ________________________
    117                          139 .. 12 .. 1 .. 02 .. 10
    108              6                                  6
    ___              6           ________________________
      9              _           837 .. 13 .. 2 .. 15 .. 12
     20             36 Rem.                    1 .. 10
    ___                          ________________________
 36)193(5           Tuns         837 .. 13 .. 2 .. 17 .. 06
    180                          ________________________
    ___
     13
      4
    ___
  36)54(1
     36
    ___
     18
     28
    ___
    144
     36
    ___
 36)521(14
     36
    ___
    161
    144
    ___
     17
     16
    ___
    102
     17
    ___
 36)278(7
    252
    ___
     26 Ounces, or 1 lb. 10 ozs.
```

 After

After dividing the tuns by 36, the remainder 9 tuns must be brought to hundreds, by multiplying by 20, and taking down the 13 cwt. of the dividend, which will give 193 cwt. to be divided, as before, by 36: and in this manner the division of the whole quantity given is performed, producing a quotient of tuns 23 .. 05 .. 1 .. 14 .. 07 and a remainder of 26, which being of the last denomination, ounces, will be equal to 1 lb. 10 oz. This remainder might be still brought into drachms; but in articles of such magnitude, this accuracy may safely be disregarded.

To prove this division, multiply the quotient by 36, employing such factors as will produce that number, as 6 times 6, 4 times 9, 3 times 12; and to the last product add the value of the last remainder of the division, when, if no error has been committed, the total will be equal to the quantity given to be divided.

OF REDUCTION.

By *Reduction* we convert units of one denomination into those of another denomination. When it is required to bring units of a higher denomination into others of a lower, as pounds into shillings, pence, &c. tuns into hundreds, quarters, pounds, &c. the operation is performed by *Multiplication*, and is called *Descending Reduction*, as proceeding from a higher to a lower denomination; but when it is required to bring units of a low denomination, into those of a higher, as pence into shillings and pounds, ounces into pounds, quarters, hundreds, &c. the operation is performed by *Division*, and is called *Ascending Reduction*.

1st. *Reduction by multiplication* is performed by multiplying the sum or quantity given, by the number of units of the next lower denomination, constituting one of the higher; adding to it the units of this lower denomination, if any, in the number given to be reduced; and repeating

this

this operation until the whole be brought to the lowest denomination required.

Example. How many shillings, pence, and farthings, are in £. 63 .. 15 .. 6?

Writing down the given sum, as in the margin, multiply the 63 pounds by 20, the number of shillings in 1 pound; taking in the 15 shillings of the sum given to be reduced; by which the product comes to be 1275 shillings. This sum is next to be multiplied by 12, the pence in 1 shilling, taking down the 6 pence of the given sum; so that the product will be 15306 pence; which is next to be multiplied by 4, the farthings in 1 penny; but as there are no farthings in the given sum, the simple product of this multiplication, will be 61224 farthings, the number contained in £. 63 .. 15 .. 6.

£.	Sh: d.
63 ..	15 .. 6
20	
1275	Shillings.
12	
15306	Pence.
4	
61224	Farthings.

Again, reduce the following quantity into ounces; Tuns 85, 3 Cwt. 1 Qr. 17 lb.

T. Cwt. Qr. lbs.	
85 .. 3 .. 1 .. 17	
20	
1703	Hundreds
4	
6813	Quarters
28	
54504	
13626	
190781	Pounds
16	
1144686	
190781	
3052496	Ounces.

In this operation the multipliers are 20, the hundreds in 1 tun; 4, the quarters in 1 hundred weight; 28, the pounds in 1 quarter; and 16, the ounces in 1 pound:— the odd hundreds, quarters, and pounds are severally added to the products of their respective denominations.

2d. *Reduction by Division* is performed by dividing the given number of units by the number constituting an unit of the next higher denomination; observing that the remainder, if any, is always of the same nature with the dividend. For example, let it be required to reduce the number of farthings found by the reduction of £. 63 .. 15 .. 6 in the former example, back into pence, shillings, and pounds.

```
 Farthings
 4)61224(0 Farthings
   ------
12)15306(6 Pence
   ------
2|0) 127|5(
   ------
 £ 63 .. 15 .. 6
```

Divide the number of farthings by 4, the farthings in 1 penny, and the quotient by 12, the pence in 1 shilling, in which last step of the operation there will be 6 pence remaining, which number is placed as in the example here given: Lastly, divide the shillings thus found, by 20, the number of shillings in 1 pound, when the remainder will be 15 shillings; and the result of this Reduction will be, that 61224 farthings are equal to 15306 pence, 1275 shillings, or £ 63 .. 15 .. 6.

Again, agreeably to the 2d. example, before given, reduce 3052496 ounces, back into pounds, quarters, hundreds, and tuns.

```
       Ounces.     28)        4)
 16 )  3052496  (  190781  (  6813  (  1
       16·····     168···     ----
       --          ---    2|0)170|3
       145         ·227       ----
       144          224    T.85 ·· 3 ·· 1 ·· 17
       ---         ---
       ··124       ··38
         112         28
         ---         --
        ·129        101
         128         84
         ---        ---
       ··16          17
         16
         --
         ··
```

Dividing

Dividing the ounces by 16, the ounces in 1 pound, we have a quotient of 190781 pounds, without any remainder: then dividing this number by 28, the pounds in 1 quarter (writing the divisor 28 over the beginning of the dividend, as in this example) we have a quotient of 6813 quarters, and a remainder of 17 pounds: again dividing this number of quarters by 4, those in 1 hundred-weight, we have 1703 hundreds and 1 quarter over: lastly, dividing these hundreds by 20, the number in 1 tun, we have 85 tuns and 3 qrs. over: hence the whole quantity obtained by this Reduction will be 85 tuns, 3 hundreds, 1 quarter, 17 pounds.

From these examples we may observe that the two kinds of Reduction are mutually checks upon or proofs of each other.

It often happens however in reducing a number of units of one denomination into those of another, whether higher or lower, that both multiplication and division are to be employed. This is the case when the one unit does not contain an entire number of the other, as when it is required to reduce guineas to pounds, or pounds to guineas: for example, if a person's pay be one guinea per day, what is his income in a year, counting 365 days?

Multiply the number of days, 365, by the shillings in 1 guinea, or 21, producing 7665 shillings, which being divided by 20, those in 1 Pound, the result comes to be £383 .. 5, equal to 365 guineas. Or to reverse the case, how many guineas are there in £383 .. 5?

```
    Days
     365
      21
    ----
     365
    730
    ----
2|0)766|5
   ------
£  383 ·· 5.
   ------
```

Here

```
   £  Sh
  383 .. 5
   20
 ———
21)7665(365 guineas
   63··
   —
   136
   126
   ——
   ·105
    105
   ——
   ···
```

Here reduce the pounds to shillings, taking in the odd 5 to the product, and divide this sum by 21, the shillings in 1 guinea, giving a quotient of 365, agreeably to the preceding question.

Again, how many Marks are there in 365 Guineas?

```
                  365
                   21
                  ———
                  365
                 730
                 ————
Sh  d            7665
13 ·· 4            12
12               ————
—        16|0)9198|0(574 marks
160            80··
               ——
               119
               112
               ——
               ··78
                 64
                 ——
             12)14(2
                 —
              1sh. 2d.
```

As 1 Mark is equal to 13 shillings and 4 pence, equal to 160 pence, the guineas must also be reduced to pence, multiplying first by 21, the shillings in 1 guinea, and that product by 12, the pence in 1 shilling. Then divide the pence in the given number of guineas, by those in 1 Mark, and the quotient 574 will be the required number of Marks: the remainder 14, being of the same denomination with the dividend, that is pence, will be equal to 1 shilling and 2 pence.

Other cases which are commonly but unnecessarily referred to the rule of Proportion, may be comprehended under Compound Reduction: as for example, What is the price of 8 Dozens and a half of pairs of Stockings, when 6 pairs cost a guinea?

Here

Here the guinea is first reduced to shillings, and then divided by 6, the number of pairs to be had for one guinea, giving 3s. 6d. for the price of 1 pair. This sum is next multiplied by 100 by means of the 2 factors 10 times 10, for the number of pairs equal to 8 dozens and a half. And the product £17 10sh. is the price required in the question proposed.

```
  £    sh.
  1    1
 20
 ---
6)21(3sh. 6d. each pair.
  18
  ---
  . 3
  12
  ---
6)36(6
  ---
   . .
  sh.  d.
   3 .. 6        10
       10        10
  --------      ----
£1 .. 15 .. 0    100
  10
  ---------
£. 17 .. 10    Total price.
```

OF PROPORTION.

The *Proportion* or the *Ratio* of two quantities, signifies the result obtained by comparing them together. If in comparing two quantities, the object be to discover how much the one exceeds or falls short of the other, the result of this comparison, which is the difference between the quantities, is an expression of their *arithmetical ratio*; thus if we compare 12 with 8, their difference, 4, is their arithmeticalratio. Again, if in making this comparison, it be the object to discover how often the one quantity contains or is contained in the other, the result will be the *geometrical ratio or proportion* between the two quantities: thus in comparing 12 with 4, in order to find how often 12 contains 4, the number 3 which expresses the number of times, represents the geometrical ratio or proportion between 12 and 4.

To

To represent these two modes of comparison of quantities, the numbers by which they are expressed are separated by one or more points: thus the arithmetical proportion between 12 and 8 would be written in this manner, 12 . 8. —this, however, is but seldom employed, as it may be mistaken for the expression of a decimal fraction. On the other hand, the geometrical proportion between two quantities, is expressed by two points placed the one over the other in this way 12 : 4. The two quantities are called the *Terms* of the proportion, the first being the *antecedent* and the last the *consequent*. To obtain the arithmetical ratio, subtract the less number from the greater; and to obtain the geometrical ratio, divide the greater by the less.

The arithmetical ratio continues the same when a quantity is added to, or subtracted from both the terms: thus, the arithmetical ratio of 12 to 8 is 4, and adding six to each term, we have 18 and 14, the difference between which is still 4, as before. The geometrical ratio undergoes no change when the two terms are multiplied or divided by the same sum: thus the geometrical ratio of 12 to 4 is 3; and multiplying both terms by 6, we have 72 and 24, the quotient of which is 3, as before.

When 4 sums or quantities are of such a nature, that the proportion or ratio between the two first, is the same with that between the two last, these 4 quantities are said to be *in proportion*, or *proportional*. Thus the 4 quantities, 5, 10, 8, 13, are arithmetical proportionals, the difference between 8 and 13, being the same with that between 5 and 10, that is 5; but the numbers 5, 10, 8, 16, are geometrical proportionals, because the 1st term is contained in the 2d, just as often as the 3d is contained in the 4th, that is twice. These last proportionals therefore are thus expressed 5 : 10 : : 8 : 16; or in words, 5 are to 10, as 8 are to 16. Of these proportionals,

proportionals the 1st and 4th terms are called the *extremes*, and the 2d and 3d the *means*.

The fundamental property of numbers in arithmetical proportion is that the *sum* of the extremes is equal to the *sum* of the means: thus in 5, 10, 8, 13 the sum of the extremes 5 and 13, or 18, is equal to the sum of the means, 10 and 8, or 18.

The fundamental property of numbers in geometrical proportion is that the *product* of the extremes multiplied together is equal to the *product* of the means, thus in 5 : 10 : : 8 : 16, 5 times 16 are 80, and 10 times 8 are 80.

When the two means of a ratio are the same, as in the arithmetical ratio, 5 are to 7 as 7 to 9, this sort of proportion is called a *continued ratio*; in which case the *sum* of the extremes is *double* the mean term: 5 and 9 are 14, equal to double 7: But when the *continued proportion* is geometrical, as 4 are to 8 as 8 to 16; then the *product* of the extremes is equal to the *square* of the mean term; for 4 times 16 are 64; equal to the square of 8, that is 8 times 8, or 64.

From this property of geometrical proportion it follows that, if we know the three first terms of a proportion, we can discover the 4th by multiplying the 2d and 3d terms into one another, and dividing the product by the 1st term; when the quotient will be the 4th term. Let there be given three quantities 3 : 9 : : 4; to find the 4th term. Multiply the 2d and 3d terms 9 and 4 together, and dividing the product 36 by the 1st term 3, the quotient 12 will be the 4th term required; for 12 contains 4 three times; that is just as often as 9 contains 3.

Hence also, by means of any three terms of a ratio, the 4th may be found: suppose the term wanted be one of the extremes, then multiply the two means together, and the product divided by the given extreme, will give the other for a quotient, as in the preceding example. On the contrary, when the two extremes and one of the means are given, and

the

the other mean is required, multiply the two extremes together, and divide the product by the given mean, and the quotient will be the mean required. For instance, let the numbers be 3 : 9 : : — : 12, where the two extremes and the 1st mean are given, and the 2d mean is required. Multiply the two extremes 3 and 12, producing 36; and this product divided by the given mean 9, will give 4 for the mean required.

When four numbers are proportionals, they will still be so if transposed so that the extremes become the means, and the means the extremes: thus, as in the examples already given, if 3 be to 9, as 4 to 12, by transposition 9 the mean will be to 3 the extreme, as 12 the extreme is to 4 the mean. The same thing will likewise happen, in whatever regular order the numbers are placed: As,

3	:	9	: :	4	:	12
3	:	4	: :	9	:	12
9	:	3	: :	12	:	4
9	:	12	: :	3	:	4
4	:	3	: :	12	:	9
4	:	12	: :	3	:	9
12	:	4	: :	9	:	3
12	:	9	: :	4	:	3

In all these various ways of stating the same proportionals, the product of the extremes multiplied together will be equal to the product of the means.

When two or more ratios or proportions are given, and the antecedents are muliplied together, as also the consequents together, the result is called a *Compound ratio:* thus, the proportion between 1 pound of money and 1 penny is composed, 1st of the ratio between 1 pound and 1 shilling, and 2dly of the ratio between 1 shilling and 1 penny. The first ratio is 1 : 20, and the 2d is 1 : 12; then multiplying the two antecedents 1 by 1, we have 1, and multiplying the two consequents 20 by 12, we have a new proportion or compound ratio 1 : 240.

When the two ratios are equal, the result is a *double* or *duplicate* ratio; when the three ratios are equal, the result is a *triple,* or *triplicate* ratio; when the four ratios are

are equal, the result is a *quadruple* or *quadruplicate* ratio; &c. Thus, 1 : 2, and 1 : 2, give 1 : 4, or a duplicate; 1 : 2, 1 : 2, and 1 : 2, give 1 : 8, or a triplicate ratio; and so on.

It was already stated as a fundamental principle of Geometrical proportion, that the product of the extremes, is equal to the product of the means. The application of this principle is of the most extensive use in Arithmetic, and other branches of Mathematics; from which circumstance it is often called the *Golden Rule:* and as in employing it, there are always three things given, and a fourth is required; it is most commonly termed the *Rule of Three.* This Rule is divided into two parts, called the *Direct* and the *Inverse.*

First, of the Rule of Three Direct. This is so called because that of the four quantities concerned, two have always not only a relation to the two others, but are so intimately connected with them, that how often soever the first of the first set of numbers contains, or is contained in the first of the second set of numbers, just so often will the second of the first set contain or be contained in the second of the second set of numbers.

For instance: If 36 yards of cloth cost £. 32. how much will 90 yards of the same cost?

Yds.		*£.*		*Yds.*		*£.*
36	:	32	::	90	:	
		90				
36)		2880		(80 pounds		
		288				
		...0				

Here are three terms of the proportion given, and a fourth is to be found, which shall bear to the 3d term, the same proportion that the 2d does to the 1st. Write down the three terms as in the margin, with the proper points between them, to be read thus: As 36 yards to 32 pounds, so are 90 yards to a fourth number, which will be a number

ber of pounds. It was already observed, that when one extreme and the two means are given, if the means are multiplied together, and the product divided by the given extreme, the quotient will be the other extreme. Let this be done here; multiply the mean 32, by the mean 90; then divide the product 2880 by the given extreme 36, and the quotient 80 will be the other extreme. Now as the antecedents are always of the same name, as are also the consequents, the 1st and 3d terms being a number of yards, and the 2d being a number of pounds, the 4th term must likewise be a number of pounds. The result of the operation therefore is, that if 36 yards of cloth cost £. 32, 90 yards will cost £. 80; which was the thing to be discovered.

Again, If I pay £ 58 for 12 cwt. 2 qrs. 16 lbr. of sugar, how much should I have for £ 365?

£ cwt. qrs. lbr. £
58 : 12 : 2 : 16 :: 365 :
4
——
50
28
£ —— £
58 : 1416 lbr. :: 365 :
365
——
7080
8496
4248
—— 28)
58) 516840 (8911 (7
464 ——
—— 4) 318 (2
528 ——
522 2|0) 7|9 (
—— ——
64 T. 3 .. 19 .. 2 .. 7
58
——
60
58
——
2
16
——
58)32(0
——

In

In this example it is required to find a quantity which shall bear to the 3d term £365, the same proportion which 12 cwt. 2 qrs. 16 lbs. bear to the 1st term £58. Writing down the three terms as here shown; before proceeding to multiply the means together, as one of them consists of different denominations, it will be necessary to reduce these to one, bringing the hundreds, quarters and pounds, to pounds, as has already been shown; by which step we have a new proportion, viz. as £58 to 1416 pounds weight, so are £ 365 to a 4th proportional. Then multiply this sum 1416 by the 3d term 365, and divide the product by the 1st term 58, when the quotient, being of the same name with the dividend, will be 8911 pounds weight, with a remainder of 2 pounds, to be reduced to ounces, and then divided, if possible, by the 1st term 58, which in this example cannot be done; the remainder may therefore be neglected. The quotient being a number of pounds weight, is next to be brought (as has been shown) back into quarters, hundreds, and tuns; and the result will be that, if I pay £ 58. for 12 cwt. 2 qrs. 16 lbs. of sugar, for 365 I should have 3 tuns, 19 cwt. 2 qrs. 7 lbr.

If the interest of £ 864 .. 16 amount to £ 272 .. 12 .. 6 in a given time, what will be the amount of the interest of £ 365 in the same time?

```
£     Sh.    £    Sh.    d.     £
864 .. 16 : 272 .. 12 .. 6 :: 365 :
 20          20                  20
----        ----                ----
17296       5452                7300
              12
            -----
            65430
             7300
          -------
         19629000
         458010
         ---------- 12)
17296)  477639000 (27615 (3
```

```
                 12)
17296) 477639000 (27615 (3
       34592····  ―――
       ―――      2|0) 230|1
       131719      ―――
       121072    £115..1..3..2
       ―――      ―――――
       ·106470
        103776
        ―――
        ··26940
          17296
          ―――
          .96440
           86480
           ―――
           ·9960
               4
           ―――
     17296)39840(2
           34592
           ―――
           ·5248
```

In this example it is required to find a number or sum which shall bear to £365, the same proportion which £272 .. 12 .. 6 bear to £ 864 .. 16. Write down therefore these sums in the order here shown, stating the proportionals so, that, as the first principal sum is to its interest, so is the second principal sum to its interest. Then, as the 1st term contains both pounds and shillings, reduce the whole to shillings, and do the same with the 3d term, although it is a whole number; because, to make the proportion correct, both the 1st and the 3d term should be of the same denomination. The 2d or middle term, consisting of pounds, shillings and pence, must be reduced to this last denomination; and then multiplying it by the shillings of the 3d term, and dividing the product by those in the 1st, we have a quotient of the same name with the dividend; and the remainder, being also a number of pence, must be brought to farthings and divided by the shillings of the 1st term,

when

when the quotient will be a number of farthings. The quotient of pence is next reduced to shillings and pounds; and the whole sum of interest on £365 turns out to be £115 .. 1 .. 3 .. 2.

2dly. The Rule of Three Inverse differs from the Direct Rule in this circumstance, that of the four quantities, known and unknown, of the question, the two principal quantities are to contain one another, or are related to one another, in an order directly contrary to that of the other two quantities connected with them: so that on examining the statement of the question proposed, and placing the quantities or numbers in such a way as to form a proportion, one of these principal quantities and its relative form the extremes, and the other principal quantity with its relative form the means. In such a position the principal quantities are said to be reciprocally proportional to their relative quantities. This difference however creates none in the way of computation, as there are still three quantities given, and a fourth required.

Suppose, for example, it be required to discover what sum of principal will, in 73 days, gain as much interest as £630 would gain in 365 days. Here it is evident that as the time in which the interest is to accumulate is shortened, the principal must be increased; and therefore that the sum to be found, must bear the same proportion to the sum given 630, as the number of days relative to this last sum, or 365, does to the number of days relative to the sum sought for, or 73. Placing therefore the given sum and its relative number of days as the two means, and the number of days relative to the sum demanded, as the first extreme, we would have this proportion.

```
Days  £    Days    £
 73 : 630 :: 365 :
        365
        ---
       3150
      3780
     1890
     ------   £
73)229950(3150
```

Here in order to have a fourth term greater than the middle term, agreeably to the nature of

```
73)229950(3150.
   219...
   ---
   .109
     73
    ---
    365
    365
    ---
    ...0
```

of the question, we multiply the middle term £630 by the greater or third term 365 days, and divide the product by the less or first term 73 days, and the quotient being of the same name with the middle term, comes to be £3150; a sum *five times as much* as 630, because the 73 days in which the determinate sum of interest is to accumulate, is *one-fifth part* of the given number of days, 365.

Again, The garrison of a fort, consisting of 480 men, are supplied with provisions for 25 days; but apprehending an attack, and wishing to be able to hold out for 60 days, how many men must be turned out of the fort, that the remainder may have the regular allowance of provisions for that time? The question here is, if a given stock can support 480 men for 25 days, how many men will the same stock support for 60 days? In this case, the fourth number must evidently be as much *less* than 480, as the given number of days, 60, is *greater* than 25. Work therefore as in the margin; and 200 will be the number of men who may be maintained 60 days on the stock of provisions which would have supported 480 men for 25 days;

```
Days Men   Days Men
 60 : 480 :: 25 :
          25
        ----
        2400
        960
       -----
  6|0)1200|0(
       -----
     200 Men.
```

and subtracting 200 from 480, we have 240 for the number of men who must be sent out of the fort, agreeably to the terms of the question.

Compound Proportion, or the Double Rule of Three, is an application of the preceding rules, with this difference, that the relation

relation of the quantity sought, to the quantity given in the statement of the question, does not depend solely on the relation between two other quantities given, but on the relation which each of these given quantities bears to two other quanties also given in the question: and when these last compound ratios or proportions are formed, the question is solved as in the Simple Rule of Three. For instance: If 25 men dig 215 yards of a ditch in 14 days, how many yards of the same ditch should 55 men dig in 32 days?

Here there are two sets of proportions: first, if 25 men dig 215 yards, in any given time, how much should be dug by 55 men, in the same time? and secondly, if this quantity of work be performed in 14 days, by any number of labourers, how much should be done in 32 days?

Working as in simple proportion we find, that if 25 men dig 215 yards, 55 men should dig 473 yards. Then again, state the proportion, if in 14 days the number of yards now found, 473, be digged, how many yards will be digged in 32 days; and the quotient 1081 yards, with the remainder 2-fourteenth parts, equal to 1-seventh part of a yard, will be the quantity required by the question.

```
Men.  Yds.   Men.
 25 : 215 : : 55 :
days.          days
 14 :       : : 32 :
-------------------
Men.  Yds.   Men.
 25 : 215 : : 55 :
       55
      ----
      1075
     1075
     -----
25)  11825  (473 yards.
     100
     ---
     ·182
      175
      ---
      ··75
        75
       ---
        ··
```

Days.	*Yds.*	*Days.*
14	: 473	: : 32

```
     32
    ----
     946
   1419
   -----
14) 15136 (1081 yards.
    14
    ---
     113
     112
     ---
     ··16
        4
       --
       ·2
```

```
  Men.   Yds.   Men.
   25 : 215 : : 55
Days 14          32
   ---          ----
   100          110
    25          165
   ---          ----
   350 : 215 : : 1760
         1760
        ------
        12900
       1505
       215
       -------        yards.
35|0(37840|0 (1081
    35
    -----
    ·284
     280
     ---
       40
       35
       --
       ·5
```

But the same result will be procured more readily by working as in the margin, by means of the compound ratios of the 25 men and 14 days, and the 55 men and 32 days; the quotient being the same as before and the remainder 5-thirty-fifth parts, being equal to 1-seventh part, as in the former operation.

Again

Again, if £682 12 gain £187 14 in 5 years six months, what principal will be sufficient to produce £ 500 of interest in 8 years, 9 months?

```
         £    Sh.   £    Sh.    £
First.  187 .. 14 : 682 .. 12 :: 500 :
         Y.  M.                 Y.  M.
         5 .. 6 :            :: 8 .. 9.
       ----------------------------------

          £    Sh.    £    Sh.     £
         187 .. 14 : 682 .. 12 :: 500 :
          20          20           20
         ----        -----       -----
         3754        13652        10000
                     10000
                     -----  2|0)
         3754) 136520000 (3636|6
               11262....  ------
               -----  £ 1818 .. 6 .. 6 .. 2
                23900  ---------------------
                22524
                -----
                 13760
                 11262
                 -----
                  24980
                  22524
                  -----
                   24560
                   22524
                   -----
                    2036
                      12
                    ----
              3754)24432(6
                   22524
                   -----
                    1908
                       4
                    ----
               3754)7632(2
                    7508
                    ----
                     124
```

 Secondly

```
              Y.   M.     £    Sh. d. qrs.   Y.  M.
Secondly      5 .. 6 : 1818 .. 6 .. 6 .. 2 :: 8 .. 9 :
             12            20                  12
             --           ----                 --
             66          36366                105
                            12
                         ------
                        436398
                             4
                        -------
                       1745594
                            66
                       --------
                      10473564
                     10473564
                     ----------   4)
              105) 115209204 (1097230 (2
                   105          ---------
                   ---        12)274307(11
                   ·1020        ---------
                     945      2|0) 2285|8
                   ------        ------
                     ·759      £ 1142 .. 18 .. 11 .. 2
                      735      -----------------------
                     ----
                     ·242
                      210
                     ----
                     ·320
                      315
                     ----
                     ··54
                     ----
```

Or by one operation, thus:

```
          Shillings
Shillings 3754
Months     105
          ----
         18770             10000 Shillings
        37540                 66 Months
        ------ Shillings   ------
        394170 : 13652 :: 660000 :
                 660000
                ---------
              819120000
             81912
            -----------   2|0)
39417|0)  90103200|0 (2285|8
```

394170

```
             2 0)
39417.0) 9010320000.0 (22858
         78834····    ------
         -----     £ 1142 .. 18 .. 11 .. 2.
         112692
          78834
          ------
          338580
          315336
          ------
          ·232440
           197085
           ------
           ·353550
            315336
            ------
            ·38214
                12
            ------
      39417)458568(11
            39417
            ------
            ·64398
             39417
             ------
             24981
                 4
             ------
       39417)99924(2
             78834
             ------
             21090
```

In the first process by two operations we say if £ 187 .. 14 arise as interest from £ 682 .. 12, in any time, from what principal would £ 500 arise in the same time; and we have for answer the sum of £ 1818 .. 6 .. 6 .. 2; for, as the interest in the third term is a greater sum than that in the first term, it must be produced by a greater principal: we therefore multiply the middle term by the greater and divide by the less sum; observing to reduce them all previously to shillings, that the three terms may be all of the same denomination. We next say if the given interest (£ 500) arise from the given principal (£ 682 .. 12) in the given time, that

that is in 5 years 6 months, from what principal would the same sum of interest arise in 8 years 9 months. Here, as the time in which the given sum of annual interest is allowed to accumulate, is greater than that first mentioned, the principal producing such a sum of interest must of course be smaller than that in the middle term, which is therefore multiplied by the least and the product divided by the greatest number of months; and the quotient £1142 .. 18 .. 11 .. 2 is the principal sum which, at the rate expressed in the question, would in 8 years and 9 months produce £500 of interest.

In performing the same example by one operation, the given sum of principal £682 .. 12 (reduced to 13652 shillings) is the middle term; and the number 394170, obtained by multiplying the given interest of this sum, by the time connected with the interest of the sum required by the question, becomes the first term: the third term 660000 being composed of the given interest of the unknown principal multiplied into the time connected with the interest of the given principal. We have now a fresh series of proportionals, as 394170 to 13652 shillings, so 660000 to a fourth number of shillings 22858, which together with the value of the remainder, is equal to £1142 .. 18 .. 11 .. 2.

OF FELLOWSHIP.

This rule, also called *Distributive Proportion*, serves to divide amongst a number of partners the profits or loss arising from a common stock, in proportion to the share which each partner has contributed. From the nature of proportionals it follows that of any series, the sum of all the antecedents is to the sum of all the consequents, as each antecedent is to its consequent: that is, that the sum of all the shares is to the sum of all the sums of profit or loss, as each individual share of the stock is to the profit or loss attached to such individual share.

Suppose

Suppose, for example's sake, it were required to divide 120 into 3 parts, in the proportion to one another of 3, 4, and 5; the first share would be to the second as 3 to 4; the second would be to the third as 4 to 5; and the first would be to the third as 3 to 5. But the sum of all the antecedents being to the sum of all the consequents in the same proportion as each antecedent is to its consequent, we will have the sum of the given antecedents 3, 4, and 5, that is 12, to the sum of the consequents or 120, as each of these antecedents, 3 for instance, to its consequent 30, 4 to its consequent 40, and 5 to its consequent 50; and as the numbers 3, 4, and 5, added together make up 12, the amount of the proportional parts, so the several shares 30, 40, and 50, make up 120, the total number that was to be divided.

Again: Suppose three persons to form a joint stock for the purpose of trade, to which A contributed the value of £390, B £520, and C £650. After some time, on settling their accounts, they found the total gain amounted to £312. How much of this sum would fall to each partner, in proportion to his share of the joint stock?

```
          £
        A 390
        B 520
        C 650   Gain A's Stock
          ----   £       £
Stock £1560 : 312  :: 390
               390
               ---
             28080
             936
             ------ £
      1560)121680(78, or A's gain.
           10920
           -----
            12480
            12480
            -----
            .....
```

B's Stock

```
            B's Stock
   £      £       £
 1560 : 312 :: 520 :
          520
          ---
         6240
        1560
        ------  £
1560)162240(104, or B's gain
       1560··
       ----
       ··6240
         6240
         ----
         ····

            C's Stock
   £      £       £
 1560 : 312 :: 650 :
          650
          ---
        15600
        1872
        ------  £
1560)202800(130, or C's gain.
       1560
       ----
       ·4680
        4680
        ----
        ···0
```

	£
A's Gain	78
B's do	104
C's do	130
	£ 312

The question here is to discover what each partner's share of the profits of the adventure should be, in proportion to his share of the stock; add therefore the three shares £ 390, £ 520, and £ 650, together, and then state the proportion, as the amount of all the shares, £1560, is to the amount of the whole gain, £312, so is a partner's stock, A's, for instance, £390, to his proportion of the gain; which by the operation turns out to be £78. Working in the same way, we find B's gain to be £104; and C's £130: and as a proof that the operation has been rightly performed, by adding these several gains together we have £312, agreeing with the total gain given in the question.

Example 2d.

Example 2d. Three partners made a joint stock: D put in £556, E £368, and F £256: but at the end of three months D withdrew his share, and E at the end of five months; whilst F carried on the adventure for eight months, when the profits, amounting to £285 .. 15, were to be divided amongst the partners. How much ought each to receive, in proportion to his share of the common stock, and the time during which his money remained employed?

£	£	£
D 556	E 368	F 256
Months 3	Months 5	Months 8
1668	1840	2048

1668
1840
2048
5556

	£	Sh.		D.		£	Sh.	d.	qrs.
5556 :	285 ..	15	::	1668	D's gain	85 ..	15 ..	8 ..	3
	20				E's	94 ..	12 ..	7 ..	3
					F's	105 ..	06 ..	7 ..	1
	5715								
	1668					285 ..	14 ..	11 ..	3
					Remainders				1
						£285 ..	15 ..	00 ..	0

45720
34290
34290
5715

5556) 9532620 (171|5(2|0)
5556
39766
38892
8742
5556
31860
27780
4080

£85 .. 15 .. 8 .. 3.
or D's gain

Remainders
1380
2868
1308

5556)5556(1 Farthing.
5556
. . . .

. 4080

```
      ·4080
         12
      ——————
5556)48960(8
      44448
      ——————
      ·4512
          4
      ——————
5556)18048(3
      16668
      ——————
      ·1380
```

In this example there are two series of proportions, the one of the several shares contributed to form the joint stock, and the other of the several periods of time for which each share was employed. Hence as D employed his stock £ 556 for 3 months, his share of the profit would be the same as if he had contributed 3 times that sum for 1 month: E's stock of £ 368, for 5 months would be of the same value as 5 times that sum for 1 month: and F's stock of £ 256 for 8 months, would be of the same value as 8 times that stock for 1 month. Hence it becomes necessary to multiply each stock, as above, by the number of months it was employed in the adventure; by which we obtain the compound quantity 1668 for D, 1840 for E, and 2048 for F; which being added together give for the 1st term of the proportions 5556; for the 2d term we have £ 285 .. 15; the given profit on the whole; and for the 3d these several compound quantities. The first operation of proportion gives £ 85 .. 15 8 .. 3. for D's share of the gain, with a remainder of 1380. The second operation (here omitted, but which the student will easily perform, as well as the third) gives for E's share £ 94 .. 12 .. 7 .. 3, with a remainder of 2868; and the third gives £ 105 .. 06 .. 7 .. 1, and 1308 for a remainder. These sums added together, as above, produce £ 285 .. 14 .. 11 .. 3, wanting one farthing to make up the total sum of gain, given

given in the question: but the three remainders which are all fractions of 1 farthing, being added together, as is here done, and divided by the common divisor of the three operations of proportion, viz. 5556, will precisely quote one farthing, without any remainder. This farthing therefore added to the above sum will give £ 285 .. 15; and thereby show that no error has been committed in the calculation.

RULES FOR PRACTICE.

The preceding rules comprehend the usual system of Arithmetic, and are applicable to every sort of calculation: but as it is always desirable to abridge the labour of computation, particularly in business, both to save time and to diminish the occasions of mistakes; sundry methods have been adopted for shortening the process in reckoning the value of articles, when the price of the unit is given, and in calculating the interest of money.

1st. OF CALCULATING PRICES.

When the price of the unit of any number of articles is given at a pound, a shilling, a penny, or a farthing; then the total price will be so many pounds, shillings, pence, or farthings, as there are units in the given number: hence 25 quartern loaves at 1 shilling each, will cost 25 shillings or £ 1 .. 5.

Again, when the price of the unit is any even or aliquot part of a pound, a shilling, &c. the number of articles given must be divided by the number of such aliquot parts forming an integer, and the quotient will give the price required. Thus the price of 100 pairs of stockings at 5 shillings each pair, will be £ 25, because 5sh. being the 4th part of 20sh. or £ 1, 4 pairs will be worth £ 1; consequently there will be as many pounds as there are fours in 100; that is, 25.

When the rate or price of the unit is not an aliquot part of a pound &c, but is composed of two or more such parts, the number of units is to be divided by each of these ali-

quot parts, and the sum of the quotients will be the price of the whole number given: thus the price of 24 pounds weight of sugar, at 9 pence per pound, will be found to be 18 shillings; in this way: As 9 pence is no aliquot part of a shilling, let it be divided into two parts, as 6 pence and 3 pence; then as 6d are the half, and 3 pence the fourth part of a shilling, by taking the half and the fourth part of the number of pounds of sugar, that is 12 and 6 together, equal to 18, we have the number of shillings for the required price of 24 pounds of sugar.

If the given price or rate be equal to the difference of rates easily computed, they may be separately calculated, and the less subtracted from the greater. If, for instance, the price were 11 pence, we may calculate the amount at 1 shilling and also at one penny, which last subtracted from the former will leave the amount at 11 pence.

And if the rate be a compound number, we may calculate first for one of the component parts, and multiply the amount by the other component part: that is, if the price were 54 shillings, we could multiply the number of articles by 9, and the product by 6, to have an amount corresponding to 54.

In performing computations in this way the following tables of the aliquot parts of a shilling and a pound will be useful.

TABLE of the aliquot parts of a SHILLING;

Pence			
1	is	1 twelfth	part of a shilling.
2		1 sixth	
3		1 fourth	
4		1 third	
6		1 half	

5	is the sum of	4d and 1d, or 3d and 2d.
7		6d and 1d, or 4d and 3d.
8		6d and 2d, or twice 4d.
9		6d and 3d, or thrice 3d.
10		6d and 4d,
11		6d, 3d, and 2d.

Table

TABLE of the aliquot parts of a POUND.

Sh.	d			
1		is	1 twentieth	part of a Pound.
1	3	is	1 sixteenth	part of a Pound.
1	4	is	1 fifteenth	part of a Pound.
1	8	is	1 twelfth	part of a Pound.
2	0	is	1 tenth	part of a Pound.
2	6	is	1 eighth	part of a Pound.
3	4	is	1 sixth	part of a Pound.
4	0	is	1 fifth	part of a Pound.
5	0	is	1 fourth	part of a Pound.
6	8	is	1 third	part of a Pound.
10	0	is	1 half	part of a Pound.

1	6	is ths sum of	1 shilling and its half
3	0	is ths sum of	2sh. and 1sh.
6	0	is ths sum of	5sh. and 1sh.
7	0	is ths sum of	5sh. and 2sh. or 4sh. and 3sh.
8	0	is ths sum of	5sh. and 3sh. or double 4sh.
9	0	is ths sum of	5sh. and 4sh.
11	0	is ths sum of	10sh. and 1sh.
12	0	is ths sum of	10sh. and 2sh. or triple 4sh.
13	0	is ths sum of	double 4sh. and 5sh.
14	0	is ths sum of	10sh. and 4sh.
15	0	is ths sum of	10sh. and 5sh. or triple 5sh.
16	0	is ths sum of	10sh. 5sh. & 1sh. or 4 times 4sh.
17	0	is ths sum of	thrice 5sh. and 2sh.
18	0	is ths sum of	10sh. and twice 4sh.
19	0	is ths sum of	thrice 5sh. and 4sh.

Example. At 4 pence per yard, required the price of 56 yards of tape.

```
   yds
 3)56(2
   ——
  18sh. 8 pence.
```

Here, as 4 pence appear by the first Table to be the third part of a shilling, divide the 56 yards by 3, and the quotient 18 will be a number of shillings; and the remainder 2, will be two third parts of a shilling, equal to two times 4 pence, or 8d.

Again. What is the price of 356 yards of cotton at 11 pence per yard?

6 Pence

Pence			
6 is 1 half	6) 4) 2)356(		356 shillings
3 1 fourth			29 .. 8
2 1 sixth.	178 .. 0		
11d.	89 .. 0		32|6 .. 4
	59 .. 4		
			£ 16 .. 6 .. 4
	32|6 .. 4		
	£ 16 .. 6 .. 4		

In this example the given price being no aliquot part of a shilling, we take such parts as together make up the price: thus 6d, 3d, and 2d, are 11 pence. Then taking for the sixpences one half of the number of yards 365, we have a quotient of 178sh; for the threepences, one fourth of 356 or 89sh; and for the twopences, one sixth or 59sh, and for a remainder two sixths equal to 4d. These three quotients added together, as here shown, amount to £ 16 .. 6 .. 4. for the value required.

The same result will be obtained by calculating the price at the rate of 1sh. per yard, and from this amount subtracting one-twelfth part, corresponding to the penny, by which 1 shilling exceeds the given price 11d. See the example above, where 29sh. 8d. the twelfth part of 356, being subtracted from that sum, leaves £ 16 .. 6 .. 4, as before.

At 7 shillings and 6 pence per yard, required the price of 65 yards of Muslin.

Sh.	*Yards*
5 are 1 fourth of 1 pound	65
2 .. 6 1-eight	
7 .. 6.	16 .. 5
	8 .. 2 .. 6
	£ 24 .. 7 .. 6

In this example the price is computed by the aliquot parts of a pound: 7sh 6d. being equal to 5sh. or one-fourth, and 2sh. 6d, or one-eighth part of a pound. The number of yards

yards, 65, is therefore first divided by 4, quoting 16 and 1 over, which is 1 fourth part of a pound, or 5sh: then 65 is divided by 8, the aliquot part corresponding to 2sh. 6d. quoting 8 and 1 over, equal to 2sh. 6d: and these two quotients together give £ 24 .. 7 .. 6, for the price required.

	yards
6d. is 1 half.	65
	7
	455
	32 .. 6
	48|7 .. 6
	£24 .. 7 .. 6

The same answer would be obtained in working by the aliquot part of a shilling as in the margin, where the yards 65 are multiplied by 7, the shillings in the price of 1 yard, and taking 32sh. 6d. the half of 65 shillings, corresponding to the odd 6d. in the price, which is the half of a shilling.

What is the worth of 5 pieces of cloth, each containing 23 yards, at £ 1 .. 7 .. 8. per yard?

d.	Yards
4 is $\frac{1}{3}$ Shilling	23
4 $\frac{1}{3}$ Shilling	5 Pieces
8d.	115 yards
	27
	805
	230
	3105 shillings
	38 .. 4
	38 .. 4
	318|1 .. 8
	£159 .. 1 .. 8

Here, to find the number of yards to be purchased, multiply the yards in each piece by 5, the number of pieces; and then multiply the product 115 by 27, the shillings in the given price, giving 3105sh; to which add twice 38sh. 4d. the aliquot parts for the 115 yards, corresponding to the two

two 4 pences, or 8d, in the price; and the sum £159 .. 1 .. 8 will be the value required.

Required the price of 36 tuns 15cwt. 3qrs. of sugar at 9½ pence per pound.

Here the quantity given is reduced to pounds, the aliquot parts of which corresponding to 9½d. give the price required.	d 4 is $\frac{1}{3}$ 4 $\frac{1}{3}$ 1½ $\frac{1}{8}$ — 9½d.	Tuns cwt. qrs. 36 .. 15 .. 3 20 — 735 12 — 8823 pounds — 2941 .. 0 2941 .. 0 1102 .. 10½ — 698|4 .. 10½ — £349 .. 4 .. 10½

2d. OF DEDUCTIONS ON WEIGHT, &c.

When goods or merchandize are weighed in the box, cask or other package, the amount is called *the Gross weight:* and proper allowances being made for these packages, the remainder is called *the neat or nett weight:* this allowance is called *the Tare*, and is usually calculated at the rate of so much on the 100lbr. *Tret* is another kind of allowance, granted on certain goods which are liable to lose their weight or measure by keeping, that it may be complete when the goods are resold. *Draught* is also an allowance granted to turn the scale in favour of the purchaser. All these allowances are computed as in the following examples:

Required the nett weight of 26cwt. 3qrs. 16lbr. allowance for tare being made at 16lbr. per cwt.

As

As the Tare is to be calculated at the rate of lbr. 16 to every cwt. or lbr. 112, so which 16 is one-seventh part; we must

	cwt.	*qrs.*	*lbr.*		
7)	26 ..	3 ..	16(	Gross	
	3 ..	3 ..	10 ..	$4\frac{4}{7}$	Tare
Cwt.	23 ..	0 ..	5 ..	$11\frac{3}{7}$	Nett

take the seventh part of the gross weight 26cwt. 3qrs. 16lbr., which is 3cwt. 3qrs. 10lbr. $4\frac{4}{7}$ ounces for the Tare, and subtracting this quantity from the gross weight we have 23cwt. 0qrs. 5lbr. $11\frac{3}{7}$oz. for the nett weight.

Again, required the Tret upon 36cwt. 1qr. 18lbr. at 4lbr. on the 104lbr.

```
         c.  qrs. lbr.
         36 .. 1 .. 18
                     4
        ------------    c.  q.  lbr.  oz.
   104) 145 .. 2 .. 16 (1 .. 1 .. 16 .. 13 Tret
        104             36 .. 1 .. 18 .. 00 Gross
        ---             --------------------
         41          C. 35 .. 0 .. 1 .. 3 Nett.
          4
        ---
   104)166(1
       104
       ---
        62
        28
       ---
   104)1752(16
       104
       ---
       712
       624
       ---
        88
        16
       ---
   104)1408(13
       104
       ---
       368
       312
       ---
        56
```


In

In this case the rate of Tret being as 4lbr. to 104lbr. we multiply the gross weight by 4, and divide the product by 104 to have the quantity to be allowed: but as 4 is the 26th part of 104, the same result may be obtained more speedily by dividing the given gross weight at once by 26.

OF INTEREST.

By *interest* is meant the allowance made by the borrower of a sum of money to the lender, for the use of that money, and it is computed at the rate of so much for the £ 100, for a whole year. The highest rate allowed by the Laws of this country is £ 5, for £ 100, per annum, and proportionally for any other sum, and for any other period of time. The Interest may therefore be calculated by Simple or compound Proportion, in the manner already shown: but for practice shorter methods are convenient, such as to divide the given principal by the number expressing what part of £ 100 is the given interest; for instance, in computing the interest of any sum at £ 5 on 100, or, as it is usually written and expressed, 5 per centum, we have only to divide the principal by 20, because 5 is the twentieth part of 100. In the same way for 4 per cent. we divide the principal by 25, because 25 times 4 are 100.

Example. What is the interest of £768 .. 13 .. 4, for 1 year, at 5 per centum?

```
 £     £      £   Sh.   d.      £
100 : 5 : : 768 .. 13 .. 4 :
 20            20
-----        -----
2000         15373
  12            12
-----        ------
24000        184480
                  5
             ------  £   Sh.  d.
24|000)922|400(38 .. 8 .. 8
```

24|000

```
                    £    Sh.   d.
24|000)922|400(38 .. 8 .. 8
         72
         —
         202
         192
         ———
         ·104          £    Sh.   d.
           20      20)768 .. 13 .. 4
         ———       ——————————————————
  24)208|0(8       £38 ..  8 .. 8 Interest
         192
         ———
          ·16
           12
           —
  24)192(8
         192
         ———
         ...
```

Working first by the rule of Proportion we say, if £100 gain £5, in a given time, what will £768 .. 13 .. 4 gain in the same time; and the result is £38 .. 8 .. 8: but as 5 is the twentieth part of 100, the operation will be abridged by merely taking the twentieth part of the principal, as here shown, and the same result will be obtained.

Again, required the interest of £365, for 3 months, at $4\frac{1}{2}$ per cent.

```
 £      £   Sh.       £
100 : 4 .. 10 : : 365
        20
        —
        90
       365
       ——    £   Sh.  d.
1|00)32|8|50(16 .. 8 .. 6. Interest for 12 months
     ————     ——————————
    16..8|12  £4 .. 2 .. 1½ Interest for 3 months.
        6|00
```

Here the proportion is as £100 to £4 .. 10, (equal to $4\frac{1}{2}$ per cent.) so £365 to a fourth proportional; the middle term being reduced to shillings is multiplied by the given

 principal

principal and the product is divided by 100, producing the sum £16 .. 8 .. 6, as the interest for 1 year or 12 months; of which the fourth part £4 .. 2 .. 1½ is the interest for 3 months, as required.

At 4 per cent. required the interest of £5683 .. 12, for the months of May, June, July and August.

```
   Days             £        Sh
 May 31          25)5683 .. 12(
 June 30          ————————————
 July 31          £ 227 ..  6 .. 10
 Augt. 31              20
   —              ——————
  123                4546
                       12
                  ——————
                  54562 Interest for 1 year
                    123 Days
                  ——————
                  163686
                 654744
                  ———————— 12)
Ds. in 1 year 365)6711126(18386(2
                  365       ————
                  ——        1532
                  3061      ————
                  2920      £76 .. 12 .. 2 Int. for 123 days.
                  ——
                  ·1411
                  1095
                  ———
                  ·3162
                  2920
                  ———
                  ·2426
                  2190
                  ———
                   ·236
```

Here 4 being the twenty-fifth part of 100, we divide the given principal by 25, and obtain the interest for a whole year, or 365 days; but as the given months contain only 123 days, we multiply this interest by 123 and divide the product by 365, whence we have £76 .. 12 .. 2 for the interest required.

Again

Again what is the *Discount* on a Bill for £ 580, for 90 days at $2\frac{1}{2}$ per cent ?

```
           £
       4|0)58|0(
           ———
       £ 14 .. 10
               90
       ——————— £  Sh.  d.
365)1305 .. 00(3 .. 11 .. 6.
    1095
    ————
    ·210
      20
    ————
365)4200(11
    365
    ————
    ·550
     365
    ————
     185
      12
    ————
365)2220(6
    2190
    ————
    ··30
```

As $2\frac{1}{2}$ is the 40th part of 100 we divide the given principal by 40, to obtain the interest for 1 year, which multiplied by the given number of days, and the product divided by the days in a year, we have the amount of the Discount required

OF FRACTIONS.

By a *Fraction* in arithmetic is meant a quantity less than a given unit; thus one-half is a Fraction of a whole, that is, it means, as the term imports, a part broken off from a whole. To have a just conception of fractions we must suppose a unit to consist of a certain number of equal parts, of which parts one or more being taken, and one or more left, the part or parts taken and the part or parts left are equally fractions of that unit. Thus if we divide a pound of tea into 16 equal parts or ounces, each ounce, or any number of ounces less than 16, will be a fraction of the pound; and

one

one ounce would be called one-sixteenth part, five ounces would be five-sixteenth parts of a pound, &c.

Fractions are expressed in different ways; one is to give these fractional parts particular names, and then to employ them as integers; thus 12 ounces, although with regard to a pound as the integer, they may be considered as twelve-sixteenth parts of a pound, may also be expressed as twelve units, when an ounce is the integer. In the same manner 12 shillings are a fraction, when a pound is the integer; but they are 12 units when the shilling is the integer.

The other mode of expressing a Fraction is to write under a line the number of parts into which the unit is divided, and above the line the number of those parts of which the fraction consists; thus 12 shillings would be expressed in a fraction by writing the number of shillings in a pound, 20, under a line, and the number of shillings in the given fraction, 12, above the line, thus $\frac{12}{20}$, to be read twelve-twentieth parts, or briefly twelve-twentieths of a pound: and $\frac{1}{2}$ means that the unit is divided into two equal parts of which one is the given fraction, to be read one-half. Hence in expressing a sum of money, sixteen shillings eight pence and two farthings, for instance, it is customary to write it in this way 16*sh.* $8\frac{1}{2}$*d*, because four farthings forming one penny, two farthings must be one half of a penny: and five shillings and nine-pence three-farthings would be written 5*sh.* $9\frac{3}{4}$*d.*

In this mode of representing a fraction the number under the line is termed the *Denominator*, because it denotes the number of parts into which the unit or integer is divided; and the number above the line is termed the *Numerator*, because it shows how many of such parts are contained in the given fraction.

In working with fractions it often happens that we obtain fractional numbers of which the numerator is greater than the denominator, as $\frac{5}{4}$, or five quarters of a yard: this however

is

is not properly a fraction, but rather a mode of representing a number composed of an integer and a fraction, as $1\frac{1}{4}$ yard, that is one yard and one quarter: and the value of such an improper fraction is obtained by dividing the numerator by the denominator. In the same manner $\frac{27}{5}$ is a way of expressing $5\frac{2}{5}$, and is obtained by multiplying the integer by the denominator of the fractional part, and to the product adding the given numerator.

Fractions of the same real value may be expressed in various ways according to the number of parts into which the integer is supposed to be divided: thus $\frac{1}{2}$ one-half, $\frac{2}{4}$ two-fourths, $\frac{3}{6}$ three-sixths, $\frac{5}{10}$ five-tenths, all equally express the same fraction of any given integer: for if we divide any integer, a pound of tea, for instance, into 2 equal parts and take 1 of them, or into 4 parts and take 2, into 6 parts and take 3, or into 10 parts and take 5 of them, it is evident that we still take precisely the same quantity, that is, one-half of the pound of tea. We may therefore increase or diminish the numerator of any fraction as much as we choose, without in the least affecting the value of the fraction, provided the denominator be increased or diminished in precisely the same proportion: thus $\frac{1}{2}$ and $\frac{5}{10}$ are equal because both the numerator and the denominator of the first fraction are multiplied by the same number 5; and $\frac{6}{15}$ and $\frac{2}{5}$ are also equal, because the numerator and denominator of the first fraction $\frac{6}{15}$ are both divided by the same number 3, giving for quotients $\frac{2}{5}$.

1st. In operations with what are called *Vulgar Fractions*, such as have just been described, the first thing to be known is how to reduce a mixed number to an improper fraction: for example, to reduce $5\frac{2}{5}$, which is a mixed number consisting of an integer or whole number and a fraction, to an improper fraction, we place the figures as in the margin,

$$\begin{array}{ll} & 5\tfrac{2}{3}. \\ & \overline{} \\ \text{Numr.} & 17 \\ & \overline{} \\ \text{Denr.} & 3 \end{array}$$

gin, and multiply the integral part 5, by the denominator of the fractional part 3, producing 15, to which we add the numerator 2, making in all 17 for a numerator to the new fraction, under which we draw a line and write the former denominator 3, as a denominator to the new fraction: hence we have $\frac{17}{3}$ an improper fraction of equal value with the mixed number $5\frac{2}{3}$. The reason of this is, that by the fractional part $\frac{2}{3}$, we observe the integer to be divided into 3 equal parts, in which case 5 integers must contain 15 of such parts, and adding the 2 third parts of the given fraction, we have 17 third parts as the value of the whole mixed number given. In the same way the mixed number $25\frac{11}{12}$ will be reduced to the improper fraction $\frac{311}{12}$, by multiplying the integral part 25 by the denominator of the fractional part 12, and adding the numerator 11 to the product for a new numerator 311, and placing the former denominator 12 under it, as a denominator to the improper fraction thus obtained.

$$\begin{array}{ll} 25\tfrac{11}{12} & \\ \overline{} & \\ 311 & \text{Numr.} \\ \overline{} & \\ 12 & \text{Denr.} \end{array}$$

2d. To reduce an improper fraction to its equivalent mixed or whole number, as, for instance, to bring back the improper fraction of the foregoing example $\frac{311}{12}$, to its equivalent whole or mixed number, we divide, as in the margin, the numerator 311 by the denominator 12, and the quotient 25 is the integral part of the required number, while the remainder 11 assumes the fractional form, by writing it as a numerator, and the divisor 12 as a denominator, making altogether

$$\begin{array}{l} 311 \\ 12)311(25\tfrac{11}{12} \\ 24 \\ \overline{} \\ 71 \\ 60 \\ \overline{} \\ \frac{11}{12}. \end{array}$$

$25\frac{11}{12}$, a mixed number equal in value to the improper fraction $\frac{311}{12}$. Suppose this fraction represented 311 twelfth parts of a shilling, that is 311 pence, the value would, by reduction, be found to be 25 shillings and 11 pence, corresponding to the fractional expression $25\frac{11}{12}$ shillings.

3. To reduce a fraction to its lowest term; we divide both numerator and denominator by any number which will divide both without any remainder, and place the two quotients respectively as the numerator and denominator of a new fraction: thus $\frac{75}{120}$ may be reduced to its lowest term by dividing each quantity by 5, which will give $\frac{15}{24}$, $\frac{75}{120} \mid \frac{15}{24} \mid \frac{5}{8}$; and this fraction again divided by 3, gives $\frac{5}{8}$ for the lowest expression of the given fraction $\frac{75}{120}$, for there is no quantity which will divide both 5 and 8 without remainders. These common divisors of a fraction are called its *measures*, and in practice are generally found by trial; but in order to discover methodically the common measure of any fraction, the rule is to divide the greater number by the less, and turning the remainder into a divisor, by it to divide the former divisor; proceeding in this way until no remainder be left; when the last divisor will be the common measure of the given fraction.

Example. Required the common measure of the fraction $\frac{437}{551}$.

```
           437
           ---
       437)551(1
           437
           ---
           114)437(3
               342
               ---
                95)114(1
                   95
                   --
Common measure     19)95(5
                      95
                      --
                      ..
```

$\frac{16}{19})\frac{437}{551}(\frac{23}{29}$.

 Here

Here agreeably to the rule above given we divide the denominator or the greater number 551, by the numerator or least number 437, and the remainder 114 becomes a divisor to the former divisor: the next remainder 95 becomes, in the same manner a divisor to the preceding divisor 114; and the remainder 19 of the third division being contained in the last divisor 5 times without any remainder, becomes a common measure of the fraction $\frac{437}{551}$; for it is contained 23 times in the numerator, and 29 times in the denominator, without remainders.

This operation is founded on the observation, that whatever number measures two other numbers, will also measure their sum and their difference, as well as any multiple of these numbers; thus the number which measures 437 and 551, will measure their difference 114, as also its multiple 342, and 95 the difference between 437 and and 342: the same number will also measure 19 the difference between 95 and 114. But as this last division leaves no remainder, it shews that the last divisor 19 is the common measure of the fraction $\frac{437}{551}$.

When the common measure turns out to be 1, the fraction is already reduced to its lowest term: for instance, had the fraction of the former example been $\frac{437}{550}$, it would be found by this operation to be already at its lowest terms; or in other words, that no number higher than unity would be its common measure;—See the work here performed, where the last divisor or common measure is 1, which neither in division nor multiplication produces any change on the numbers divided or multiplied.

```
437
---
437)550(1
    437
    ---
    113)437(3
        339
        ---
         98)113(1
             98
             ---
             15                    15)98(6
```

```
            15)98(6
               90
               --
                8)15(1
                   8
                   --
                   7)8(1
                     7
                     -
Common measure       1)7(7
                       7
                       -
```

4. To reduce fractions having different denominators, to others of the same value, but having a common denominator; for instance to reduce $\frac{2}{3}$ and $\frac{3}{4}$ to fractions of the same denomination: multiply each numerator by the denominator of the other fraction, for a new numerator in the place of that by which you multiply: and all the denominators into one another for a new common denominator to all the fractions. Thus the first numerator 2 multiplied by 4 the denominator of the other fraction gives 8 for a new numerator in the place of 2; and the second numerator 3 is multiplied by the first denominator 3, giving 9 for a new numerator in the place of 3: then multiplying the two denominators 3 and 4 together, we have 12 for a new denominator common to both fractions: so that $\frac{2}{3}$ and $\frac{3}{4}$ are reduced to the equivalent fractions $\frac{8}{12}$ and $\frac{9}{12}$.

Example, reduce $\frac{2}{5}$, $\frac{4}{11}$, $\frac{6}{7}$ and $\frac{11}{15}$ to fractions of the same value having a common denominator.

$\frac{2}{5}$ $\frac{4}{11}$ $\frac{6}{7}$ $\frac{11}{15}$

2	4	6	11	5
11	5	5	5	11
22	20	30	55	55
7	7	11	11	7
154	140	330	605	385
15	15	15	7	15
2310	2100	4950	4235	5775
5775	5775	5775	5775	

Here each numerator is multiplied successively by all the denominators except its own; that is, the numerator 2 of the first fraction is multiplied by the denominators 11, 7, and 15; the numerator 4 of the second fraction is multiplied by 5, 7, and 15; the numerator 6 is multiplied by 5, 11, and 15; and the numerator 11 is multiplied by 5, 11, and 7. Hence we have the new numerators 2310, 2100, 4950, 4235, corresponding to the numerators given in the question 2, 4, 6, 11; and for a common denominator we multiply the given denominators 5, 11, 7, and 15 into one another, producing 5775, which being written under each of the new numerators, we have the fractions respectively $\frac{2350}{5775}$, $\frac{2100}{5775}$, $\frac{4950}{5775}$, and $\frac{4235}{5775}$, of equal value with the given fractions $\frac{2}{5}$, $\frac{4}{11}$, $\frac{6}{7}$ and $\frac{11}{15}$. This may be proved by finding common measures for each of the new fractions, agreeably to the preceding rule, as here shown.

```
      2310                      2100
      ----                      ----
 2310)5775(2               2100)5775(2
      4620                      4200
      ----                      ----
C. M. 1155)2310(2          1575)2100(1
           2310                 1575
           ----                 ----
           ....        C. M.     525)1575(3
                                     1575
                                     ----
                                     ....
```

In the same way the common measure for $\frac{4950}{5775}$ will be found to be 825, and that of $\frac{4235}{5775}$ will be 385. These new fractions therefore severally divided by their respective measures, will give for quotients the fraction stated in the question.

ADDITION OF VULGAR FRACTIONS.

This is performed by reducing the several fractions to one denominator, if they are given of different denominators, and adding the numerators together for a new numerator to the

the common denominator: thus to add $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{5}$ together, it is evident, as all the fractions are of the same name, or all fifths, that all that is required is to add the numerators 1, 2, and 3 together making 5 fifths, equal to one integer: but in the following example, where $\frac{2}{3}$, $\frac{4}{9}$ and $\frac{8}{15}$ are to be added together, we must, by the rule already given, reduce the fractions to others having a common denominator, and then add the new numerators together, writing the sum over the common denominator, which in this case will form an improper fraction $\frac{666}{405}$; and this reduced to its equivalent mixed number, will give 1 $\frac{261}{405}$, or reducing the fractional part to its lowest term, $1\frac{29}{45}$.

$\frac{2}{3}$	$\frac{4}{9}$	$\frac{8}{15}$		
2	4	8	3	270
9	3	3	9	180
—	—	—	—	216
18	12	24	27	—
15	15	9	15	666
—	—	—	—	—
270	180	216	405	405)666($1\frac{261}{405}$ | $\frac{29}{45}$
—	—	—		405
405	405	405		—
				261

In the annexed example, consisting of mixed numbers, the fractional parts are to be reduced to a common denominator and then added together producing the improper fraction $\frac{390302}{127776}$, equal to the mixed number $3\frac{6974}{127776}$ | $\frac{317}{5808}$; the fractional part brought to its lowest term, as has been shown, is then written down in the sum, and the 3 integers are carried to the column of units, which with the tens and hundreds are summed up as in common addition.

$$\begin{array}{r} 485\frac{2}{3} \\ 67\frac{15}{16} \\ 368\frac{5}{11} \\ 9\frac{241}{242} \\ \hline 932\frac{317}{5808} \end{array}$$

This operation is common in addition of sums of money, where the farthings instead of being entered in a separate column after that of the pence, are annexed to the pence in a fractional form: thus instead of 13sh. 4d. 3qrs, we write

13sh.

13sh. 4$\frac{3}{4}$d; and the addition is performed as in the margin, where the fractions are brought to farthings and added together, in this way $\frac{1}{2}$d. is 2 farthings, and $\frac{3}{4}$ are 5, and $\frac{1}{2}$d or 2qrs. are 7, and $\frac{1}{4}$ are 8 farthings, equal to 2d, which being carried to the pence column, the addition is performed in the usual way.

	s.	d.
5	12	8$\frac{1}{4}$
3	7	6$\frac{1}{2}$
8	11	2$\frac{3}{4}$
3	13	3$\frac{1}{2}$
£21	4	9

SUBTRACTION OF VULGAR FRACTIONS.

Subtraction is performed by reducing the two given fractions, if necessary, to the same denomination, and then, subtracting the less numerator from the greater, for a new numerator to the common denominator: thus if from $\frac{3}{4}$ we take away $\frac{1}{2}$ equal to $\frac{2}{4}$, it is evident that $\frac{1}{4}$ will remain: and in the same wav to subtract $\frac{5}{22}$ from $\frac{36}{53}$, we first reduce them to fractions of a common denominator, viz. $\frac{5}{22}$ to $\frac{265}{1166}$, and $\frac{36}{53}$ to $\frac{792}{1166}$; then subtracting the less numerator 265, from the greater 792, we have 527, for the numerator of the remainder $\frac{527}{1166}$.

In subtracting mixed numbers, supposing the fractional parts to be either given in or reduced to the same denomination, if the fractional part of the least quantity be greater than that of the greater quantity, then an unit must be borrowed, containing a number of parts equal to those in the common denominator, and added to the less fraction; and this unit must be repaid to the unit of the subtrahend, as in subtraction of integers.

In the annexed case the two fractions must be reduced to the same denomination; but as $\frac{40}{48}$ cannot be taken from $\frac{18}{48}$, we borrow an unit from the integral part of the fraction, and dividing it into the same number of parts with the common denominator

$$\begin{array}{r} 58\frac{3}{8} \text{ or } \frac{18}{48} \\ 37\frac{5}{6} \text{ or } \frac{40}{48} \\ \hline 20\frac{13}{24} \text{ or } \frac{26}{48} \end{array}$$

ator, viz 48, we add 48 to 18, making 66, from which subtracting the numerator of the other fraction 40, we have a remainder of $\frac{26}{48}$, or when reduced to its lowest common measure, $\frac{13}{24}$ be written in the remainder, and carrying the unit borrowed to the integer 7, the rest of the subtraction is performed as in common arithmetic.

	£	s.	d.
From	36	11	$9\frac{1}{2}$
Subtract	7	18	$2\frac{3}{4}$
	28	13	$6\frac{3}{4}$

In the annexed example, the $\frac{1}{2}$d being reduced to $\frac{2}{4}$d in order to be of the same denomination with the $\frac{3}{4}$d of the Subtrahend, we must borrow an unit or $\frac{4}{4}$ that is 4 farthings from the pence, making with the $\frac{2}{4}$ or $\frac{1}{2}$d. $\frac{6}{4}$d, and from this subtracting the $\frac{3}{4}$ given we have a remainder of $\frac{3}{4}$ to be written down, and $\frac{4}{4}$d or 1d to be carried to the column of pence; the rest of the subtraction proceeding as formerly shown.

MULTIPLICATION OF VULGAR FRACTIONS.

This operation consists in multiplying the numerators of the two given fractions into one another for a new numerator, and the denominators together for a new denominator to the product: thus in multiplying $\frac{2}{3}$ by $\frac{3}{4}$, the product of the numerators being 6, and that of the denominators being 12, we have the fractional product $\frac{6}{12}$ or $\frac{1}{2}$, for the result. This will be easily understood by taking $\frac{2}{3}$ of a foot to be multiplied by $\frac{3}{4}$ of a foot: for $\frac{2}{3}$ being 8 inches, and $\frac{3}{4}$ being 9 inches, these two quantities multiplied together will give 72 square inches: but it was formerly mentioned that the square inches in one foot are 144, that is 12 times 12; 72 inches are therefore $\frac{1}{2}$ of 144, or of 1 foot, agreeing to $\frac{1}{2}$, being the product of $\frac{2}{3}$ by $\frac{3}{4}$.

Hence it is to be remembered that fractions multiplied together give products of less numerical value than the factors, and that even units multiplied by fractions give products of less apparent value than the units: for 1 foot multiplied by

$\frac{3}{4}$ foot,

$\frac{3}{4}$ foot, that is 12 inches multiplied by 9 inches will give $\frac{3}{4}$ foot square or 108 inches: but this arises from the difference between the lineal measure of the factors, and the superficial measure of the product, which however are both expressed by the same terms and figures.

When integers are multiplied by integers, their value is increased; when they are multiplied by units or 1, their value remains unaltered; but when multiplied by fractions their value is diminished proportionally to the difference between the fraction and unity.

To multiply an integer by a fraction, you multiply the integer by the numerator and divide the product by the denominator, and if there be any remainder it is annexed to the quotient as the numerator of a fraction with the same denominator as that in the multiplier. See the example.

$$\begin{array}{r} 846 \text{ by } \frac{5}{8} \\ 5 \\ \hline 8)4230(\\ \hline 528\frac{6}{8} \mid \frac{3}{4} \end{array}$$

This however is only an application of the rule already given, for the integer here may be represented by an improper fraction whose numerator is the given integer, and whose denominator is 1: thus in the example, the whole number 846 may be fractionally expressed as $\frac{846}{1}$.

To multiply an integer by a mixed number, first multiply it by the integral part, and to the product add that obtained by multiplying the integer with the fractional part. For instance let it be required to multiply 365 by $6\frac{3}{5}$.

Multiply the given sum 365 by 6, the integral part of the mixed number; then multiply 365 by 3, the numerator of the fractional part, and dividing the product by the denominator 5, add the quotient to the first product, and the sum will be the product required.

$$\begin{array}{r} 365 \\ 6 \\ \hline 2190 \\ 219 \\ \hline 2409 \end{array} \qquad \begin{array}{r} 365 \\ 3 \\ \hline 5)1095(\\ \hline 219 \end{array}$$

In

In multiplying a mixed number by a fraction, first multiply the fraction into the integral parts, and then into the fractional; and the sum of these products will be the product required. For example multiply $35\frac{3}{5}$ by $\frac{7}{8}$.

35 multiplied by	$\frac{7}{8}$ give	$30\frac{5}{8}$
$\frac{3}{5}$	$\frac{7}{8}$	$\frac{21}{40}$
		$31\frac{3}{20}$.

Lastly, in multiplying two mixed numbers together, add the products of the two integral and the two fractional parts alternately multiplied into one another, and their sum will be the product required. Multiply, for example, $18\frac{2}{3}$ by $12\frac{3}{4}$.

18 multiplied by	12	216
18	$\frac{3}{4}$	$13\frac{1}{2}$
12	$\frac{2}{3}$	8
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$
		238 :

To illustrate this example: suppose the quantities to be multiplied are $18\frac{2}{3}$ feet, or 18 feet 8 inches, and $12\frac{3}{4}$ feet, or 12 feet 9 inches: then by the rule given when treating of multiplication of compound quantities, the process will be

```
   Feet Inches
    18 .. 8          12      18      9
    12 .. 9           8       9      8
   ---------         ---     ---     -
   216               96      162  12)72(
    21 .. 6         162               --
         6          ---                6
   ---------     12)258(
Feet. 238 .. 0      --
                   21 .. 6
```

as here shown, where the product of 18*f.* 8*in.* by 12*f.* 9*in.* is 238 feet, as was found in the operation by multiplication of mixed numbers.

DIVISION OF VULGAR FRACTIONS.

Division is performed by multiplying the numerator of the dividend by the denominator of the divisor, for a numerator to the quotient, and the denominator of the dividend by the numerator of the divisor, for a denominator to the quotient. For example, let it be required to divide $\frac{3}{4}$ by $\frac{2}{3}$, we multiply the numerator of the dividend 3, by the denominator of the divisor 3, and the product 9 is the numerator of the quotient: again the numerator of the divisor 2 multiplied into the denominator of the dividend 4 gives 8 for the denominator of the quotient, which then becomes the improper fraction $\frac{9}{8}$ equal to $1\frac{1}{8}$.

In this operation the object being to discover how often $\frac{2}{3}$ are contained in $\frac{3}{4}$, it is evident that two thirds will be contained in any quantity only half the number of times that one third would be contained in it: we therefore divide the dividend by 2 and multiply the quotient by 3, or in other words, we take 3 times the half of the dividend; for the quotient required by the question proposed. Thus in the example given; bring the given dividend $\frac{3}{4}$ to some equivalent fraction that will admit of division by 2, as $\frac{6}{8}$; the half of this is $\frac{3}{8}$ and 3 times the quotient is $\frac{9}{8}$ equal to $1\frac{1}{8}$, as before found. This will be illustrated if we suppose the fractions given to be parts of a foot for instance, in which the divisor $\frac{2}{3}$ will be 8 inches and the dividend $\frac{3}{4}$ will be 9 inches: and here it is evident that 8 will be contained in 9, once with one over; or the quotient will be $1\frac{1}{8}$ as before.

In dividing an integer by a fraction, or a fraction by an integer, you must bring the integer into a fractional form by writing 1 under it, for a denominator, and then working as in the preceding example: thus to divide 5 by $\frac{3}{4}$ the operation would be performed as in the margin, where the

quotient

quotient is found to be $\frac{20}{3}$ equal to $6\frac{2}{3}$: and on the other hand dividing $\frac{3}{4}$ by 5 would give for a quotient $\frac{3}{20}$. In short when an integer is to be divided by a fraction, you multiply the integer by the denominator of the fraction and divide the product by the numerator, and when a fraction is to be divided by an integer, you multiply the integer into the denominator of the dividend, for a new denominator to the numerator of the dividend.

$$\frac{3}{4})\frac{5}{1}(\frac{20}{3})20(6\frac{2}{3}$$

$$\begin{array}{r} 18 \\ \hline 2 \end{array}$$

$$\frac{5}{1})\frac{3}{4}(\frac{3}{20}.$$

When mixed numbers are given either in the divisor or the dividend, or in both, they must be reduced to equivalent improper fractions; and then the division is performed as here shown. Divide $326\frac{2}{3}$ by $15\frac{5}{8}$.

$$\begin{array}{cc} 15\frac{5}{8} & 326\frac{2}{3} \\ \hline \end{array}$$

$$\frac{125}{8})\frac{980}{3}(\frac{7840}{375})7840(20\frac{340}{375} \mid \frac{65}{75}.$$

$$\begin{array}{r} 7500 \\ \hline 340 \end{array}$$

Here the divisor $15\frac{5}{8}$ is reduced to the improper fraction $\frac{125}{8}$, and the dividend $326\frac{2}{3}$ to $\frac{980}{3}$; then multiplying alternately as directed, we have for a quotient $\frac{7840}{375}$, an improper fraction equal to the mixed number $20\frac{68}{75}$.

In common accounts shillings and pence may be considered as fractions; that is the pence as fractions of a shilling and both pence and shillings as fractions of a pound: in the same manner lower denominations of any kind may be considered as fractions of higher denominations, and operations where different denominators occur, may be performed by expressing the higher as integers, and the lower as fractions, to be worked with as in the preceding examples. Thus the lower denomination becomes a fraction by placing it as a numerator, with the value of the higher as the deno-

2 K 2 minator,

minator, 5 pence for instance will be $\frac{5}{12}$ of 1 shilling and $\frac{5}{12}$ of $\frac{1}{20}$ equal to $\frac{5}{240}$, of a pound. Again the value in lower denominations of the fraction of a higher denomination, is found by reduction, that is by multiplying the numerator by the units in the next inferior denomination and dividing the product by the denominator of the given fraction: thus the value of $\frac{11}{12}$ of a pound will be found to be 18sh. 4d.

```
    11
    20
    —
12)220(
   ——
  18 .. 4
```

In calculations it often happens that a quantity may be expressed as the fraction of a fraction; and this manner of expression may be carried through any number of stages. To reduce all these fractions to one of the same value, we multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, thus as in the preceding example, where 5 pence are expressed as $\frac{5}{12}$ of $\frac{1}{20}$ of a pound, Here multiplying the numerator 5 by 1, we have 5, and the denominators 12 by 20, we have 240 to form the new fraction $\frac{5}{240}$ of a pound. In the same way 6 ounces will be represented as $\frac{6}{20}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of $\frac{1}{20}$ of a Ton, and multiplying all the numerators together for a numerator, and all the denominators together for a denominator, we have the fraction $\frac{6}{35840}$ or $\frac{3}{17920}$ of a Ton for the value of 6 ounces.

Having thus shown the method of calculation by *Vulgar Fractions*, it remains to give some explanation of the nature and uses of what are called *Decimal fractions*. In money, weight, capacity, dimensions, &c. the unit or integer has by common consent been divided into various numbers of smaller parts: but as these divisions have been optional

optional and arbitrary, and that calculations by them are frequently tedious and consequently liable to mistake; it has been agreed upon to suppose integers of all kinds to be divided into ten equal parts, which are hence termed *tenths* or *decimals* from the Latin word *decem* signifying *ten*. Each of these decimal parts is again divided into ten other equal parts called hundredth parts; these into ten others called thousandth parts; and so on indefinitely.

These decimal parts with all their subdivisions, are represented by the same numerical characters or cyphers as integers; but they are distinguished from integers by having a comma placed before them, or on their left hand; thus 5 without any point, represents five integers; but ,5 with a comma before it stands for five tenth parts or decimals of an integer: and 8,5 would be read eight integers and five tenths.

The numeration of *Decimals* is just the reverse of *Integers*; for as these last increase in value in proportion as they recede from the right hand to the left; so those diminish in value in proportion as they recede from the left hand to the right. From the nature of a decimal fraction it is evident that the first place after the point of separation between them and integers must be that of tenths, the next to the right hand that of hundredth parts, as being tenths of tenths; the third place is that of thousandths; and so on, as in the following Table.

	Tenth parts.	Hundredth parts.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.
,	1	2	3	4	5	6	7	8

Here

Here the first figure after the point is 1 tenth of an unit; the second is 2 hundredth parts, to which adding the 1 tenth, which is equal to ten hundredth parts, we have, 12 representing 12 hundredths: the third figure 3 is 3 thousandth parts, and joined to the preceding figures we have, 123 thousandths; the fourth figure is 4 tenthousandth parts which being joined to the preceding figures, we have ,1234 ten thousandth parts of an integer: so that it is to be remembered in general that, although the value of decimals diminishes in a tenfold proportion with respect to an integer, as they recede to the right hand from the point of separation, yet in reading them, their value relatively to themselves, is reckoned from right to left as in numeration of integers.

When a nought is placed on the right hand of an integer, the value of the integer is increased tenfold; thus 5 stands for five, but 50 for fifty; but one or more noughts on the left hand have no effect on the value: on the contrary, one or more noughts placed on the right hand of a decimal fraction have no effect on its value; but for every nought on its left hand, the value is diminished ten fold: thus ,5, ,50, or ,5000, are all but five tenths; whereas, ,05 will represent five hundredth parts, ,005, five thousandth parts, and ,000005 will be five hundred thousandth parts. For example the characters 1808 will have very different values according to the position of the separating point: thus

1808, are 1 thousand 8 hundred and 8.

180,8 1 hundred and 80, and 8 tenths

18,08 eighteen and 8 hundredth parts

1,808 one and 808 thousandth parts.

,1808 one thousand 808 ten thousandth parts

,01808 1808 hundred thousandth parts

,001808 1808 millionth parts

1st. To reduce Vulgar fractions to Decimal fractions. To the numerator add a number of noughts, separated from

the

the integers by a point, and divide it by the given denominator. Reduce $\frac{53}{68}$ to a decimal fraction.

```
53/68)53,0000(,7794
      47 6
      ----
       ·540
        476
        ---
        ·640
         612
         ---
         ·280
          272
          ---
          ··8
```

In this division, the quotient is brought into the form of a decimal fraction by counting from the last figure 4, towards the left hand as many figures as there are noughts employed in the division, that is four, and then placing before the fourth figure the distinguishing comma or point, as is done here; and the result of the operation is that the decimal fraction ,7794 is of equal value with the Vulgar fraction $\frac{53}{68}$; but not precisely, because there is a remainder in the division, which is lost: but if the division be carried on, by the addition of more noughts to the dividend, the quotient will approach gradually nearer to the truth.

The reason of this operation is that to reduce a vulgar to a decimal fraction is in fact only to find the proportion between two fractions, of which the first has any given denominator and the second has the constant denominator unity or 1. The example here given may therefore be thus stated: as 68 the given denominator to its numerator 53, so the constant denominator 1, to its numerator:—68 : 53 :: 1 : where if the middle term 53 be multiplied by 1, and the product divided by 68, the quotient will be the fourth proportional required: but as multiplying by 1 produces no effect on the middle term, the operation is abridged by at once dividing this term, with the addition of some noughts, by the given denominator 68; and the quotient may be represented as a vulgar fraction, to correspond to that given in the question; thus $\frac{53}{68}$ equal to $\frac{7794}{10000}$, or 7794.

In

In reducing vulgar to decimal fractions, the result obtains different names according to its nature: thus if the division end without a remainder, the quotient is said to be a *finite* or *terminite* decimal; when the division however far continued still gives a remainder, the decimal is said to be *infinite* or *indeterminate*; when the same remainder is produced in succession, and the same figure is repeated in the quotient, the decimal becomes a *repeater*; if the repetition return only after two three or more figures, the decimal is called a *circulate*; and the circulating figures are pointed out by an asterisk or other mark at the beginning and end of the circle, whilst repeating decimals are distinguished by a dash or inclined line through the repeating figure. The fraction $\frac{12}{75}$ reduced to a decimal is terminate or finite; for the quotient is ,16, and there is no remainder.

```
12/75)1200(,16
      75
      —
      450
      450
      ——
      ...
```

The example before given of $\frac{53}{68}$ reduced to ,7794, is an indeterminate or infinite decimal, in the state to which the division is there carried, because there is a remainder; but if the learner will prosecute the division until the quotient shall be ,77,9411764705882352,941 he will find that the remainder of the 18th. multiplication, by the figure 2 of the quotient is 64, which is the same with the remainder of the 2d. multiplication: the division may therefore be continued for ever and the same figures will regularly return or circulate in the quotient, with the same remainders.

The circulating figures of the quotient are distinguished as above by a comma before the first which is 9, and after the last which is 2, the following figures being 94 &c. as before.

The fraction $\frac{5}{6}$ reduced to a decimal produces a repeater; that

$\frac{5}{6}$)5000(,83333, &c.
48
—
·20
18
—
·20
18
—
2

that is, the figures 3 of the quotient and 2 of the remainder will perpetually recur, as long as the division is carried on; and the result, although continually approaching nearer and nearer to the truth, will yet never arrive at it precisely, as there will always be the remainder 2.

2d. To reduce decimals of a lower to a higher denomination: Divide the decimal given (annexing noughts to it) by the integers in the higher denomination; and when there are several denominations given, begin with the lowest, and add its value to the next in order towards the highest. Thus reduce 15 shillings 8 pence 3 farthings to the decimal fraction of a pound.

4)3,00(	12)8,750000(8	20)15,72916666(
,75	,729166	,786458333

First divide the 3 farthings by 4, the farthings in 1 penny; giving for a quotient ,75; write this fraction after the 8 pence, and divide the whole by 12, the pence in 1 shilling: here the quotient becomes a repeater, the figure continually being 6, and the remainder 8; therefore interminate; but adding this quotient with a convenient number of sixes to the 15 shillings, and dividing by those in 1 pound, the last quotient comes to be ,786458333, &c. as the decimal fraction required.

3d. To find the value of a decimal fraction of a given integer: Multiply the decimal by the integers in the denomination next below that of which the decimal is given as a fraction, and cut off by a point from the product as many figures as there are in the multiplicand; then those to the left of the point will be so many integers equal to the given

fraction. Thus, required the value of ,786458333 of a pound sterling, found in the foregoing example:

	,786458333	
	20	
shillings	15,72916666	
	12	
pence	8,74999992	Rem. 8
	4	4
farthings	2,99999968	32
	32	
farthings	3,........	

Answer is 15sh. 8d. 3qrs.

Addition and Subtraction of Decimals are performed in the same way as in integral arithmetic, observing always to place units over and under units, tens over and under tens, &c.; or in other words, to place the separating points in the same vertical column: thus in the annexed examples, observing to place the units of the integral sums in the same column; the separating points will also be in one column, and the several fractional sums will extend to the right hand according to their number of places. The decimals are added together, beginning at the right hand figure, and the tens carried on to the succeeding columns of decimals and integers, as in common addition, without regarding the point, which is written in the sum, under that of the particulars; or the number of decimals in the sum must be equal to that in the longest line of the particulars; but should the first figure of the

	46,8
	234,37
	82,1956
	4,007
Sum	367,3726

	685,3
	268,583
Diff.	416,717

the sum in the decimals be nought, it need not be written down, but only a point for it, which is however to be reckoned in telling off the places of decimals in the sum: for instance, had the first figure of the sum in the above example been 0, instead of 6, we would have merely made a point (.); but this would have been counted in setting off the four places of decimals.

In the example of subtraction, the number of decimals in the subtrahend being greater than that in the upper line of figures, we suppose the upper places to be filled with noughts, and then subtract from them as if they were tens in common arithmetic.

Multiplication of decimals is performed as in multiplication of integers, and the number of places of decimals to be cut off from the right hand of the product must be equal to those in the multiplicand and multiplicator taken together; as in the annexed example, where the decimals cut off from the right hand of the product are five, equal to the three of the multiplicand with the two of the multiplicator.

```
   86,375
     5,83
 --------
   259125
  691000
 431875
 --------
 503,56625
```

It will however sometimes happen that the number of places of figures in the product is not equal to that of those in the two factors: in this case, one or more noughts must be written on the left hand of the figures in the product, to complete the number required, and then the decimal point prefixed. See the annexed examples:

```
   ,001355        ,11856
     ,0023           ,12
   -------        -------
      4065       ,0142272
     2710
   -------
,0000031165
```

To multiply any decimal by 10, 100, 1000, or the like, we have only to move the point to the right hand one place for every nought in the multiplier: thus

,5	,368	,01345
10	100	1000
5,0	36,800	013,45000
or	or	or
5,·	36,8··	13,45···

Division is done as in integers: but in ascertaining the value of the quotient, we cut off as many figures from the right hand, as together with the decimals, if any, in the divisor, shall be equal to those given or assumed in the dividend.

If there be as many places of decimals in the divisor as in the dividend, the quotient will be all integers: if there be more in the divisor than in the dividend, some noughts must be added to the latter to make up that number, and the quotient will still be all integers: and if the divisor consist only of integers, the decimals of the quotient must be as numerous as those in the dividend. See the following examples:

4,52)70,512(15,6
452 ··
2531
2260
·2712
2712
····

3,36)120,96(36
1008 ·
·2016
2016
····

5,2625)378,9000(72
368375 ·
·105250
105250
······

435)1587,75(3,65
1305 ··
·2827
2610
·2175
2175
····

In

In the annexed example, where the quotient can consist of only three places of figures, and the divisor is all integers, room must be left in the quotient to insert two noughts between the figures and the decimal point, in order to make up five, the number of decimals in the dividend.

```
756)4,29408(,00568
    3780
    ----
    ·5140
     4536
    -----
    ·6048
     6048
    -----
     ····
```

Where integers of various denominations are given in any question of Arithmetic, they may be brought into the form of a decimal fraction, and the necessary operation carried on agreeably to the rules now given: for instance, in the following case of proportion, where it is required to find the interest of £268 15, in the time in which £175 5 would gain £52 10, we bring the three

```
175,25 : 52,5 :: 268,75 :
         268,75
         ------
         134375
         53750
        134375
        -------        £
175,25)14109,375(80,50998
        140200
        ------
        ···89375
           87625
           -----
           ·175000
            157725
            ------
            ·172750
             157725
             ------
             ·150250
              140200
              ------
              ·10050.
```

terms,

terms, as has been already shown, to their equivalent decimal form; and the fourth term required turns out to be £80,50998, the decimal part being interminate and therefore less than the truth; but its value for any useful purpose might have been taken at ,51 which is only 2 ten thousandth, or 1 five thousandth part of a pound, above the real quotient, making the whole interest required £80 10 $2\frac{1}{4}$.

EXTRACTION OF ROOTS.

When a number is multiplied by itself, it is said to be *squared* or raised to the *second* power, the simple value of the number itself being considered as the *root* or *first* power: if the second power or square be again multiplied by the original number, the product is called the *cube*, or the *third* power: this again multiplied by the orginal number produces the *biquadratic* or *fourth* power; which multiplied by the original number, gives the *sursolid* or *fifth* power: and this last multiplied by the root produces the *squared cube*, or *sixth* power of the number. In the same manner the 7th, 8th, and all other powers, may be obtained by successive multiplication of the preceding power by the original number.

The following table shows the second and third powers, or the squares and the cubes of the 9 digits.

1st power or root	1	2	3	4	5	6	7	8	9
2d or square	1	4	9	16	25	36	49	64	81
3d or cube	1	8	27	64	125	216	343	512	729

Thus the square of 6 is 36, and the Cube is 216; and so of any other number: as for instance, to find the square and the cube of 256, we multiply the number or root by itself,

256 256 —— 1536 1280 512 —— 65536 square. 256 —— 393216 327680 131072 —— 16777216 cube.	itself, and the product is the 2d power or square of 256; which being again multiplied by the root, gives the 3d power or cube.

As the nature and properties of the square and cube will be explained when we come to treat of *Mensuration*, it will be necessary in this place only to show how to discover or extract the roots or first powers of any given squares or cubes.

1st. To extract the *square* root of any given quantity, we divide the given square into periods of two figures each, beginning at the right hand and counting to the left, if the figures be integers; but the reverse if they be decimals; and for every period thus set off there will be a figure in the root.

Find what number when squared will be equal to or next under the first period on the left hand; write this number or root in the quotient, and its square under the period mentioned, to be subtracted from it: to the remainder bring down the next period of the given square, and doubling the figure already placed in the quotient, for a divisor to the new dividend, write it as such, leaving however room for one or more places of figures in the units and tens place: next see how often this divisor can be found in the new dividend (excluding from it the units place of figures), and write the number of times in the quotient, and also in the void space left in the divisor, the whole of which is to

be

be multiplied by the figure now placed in the qnotient; and the product subtracted from the new dividend will leave a remainder, to which the next period of the given square is to be brought down, and the extraction carried on as before.

Should it happen that the quotient doubled for a divisor, cannot be taken out of the remainder, after the next period is brought down to it, you must write a nought in the quotient, and also in the void space of the divisor, and take down another period from the square, to be divided by the same divisor as before.

If after all the periods of the given square are taken down, there be still a remainder, the extraction may be prolonged, to obtain a decimal fraction, by writing on the right of the remainder a couple of noughts, as if they had been brought down from the square, and the operation continued as far as may be requisite; observing to separate, by a point, the integral from the decimal part of the root; reckoning one place of decimals for every period or pair of noughts employed in the extraction.

Required the root of the square 462876,1225.

```
        Square.      Root.             Proof.
      462876,1225(680,35
       .  .  .  . .                    680,35
      36                               680,35
      --                              --------
  128)1028                             340175
      1024                            204105
      ----                           5442800
  13603)47612                       408210
        40809                       ----------
        -----                       462876,1225
  136065)680325                     -----------
         680325
         ------
         ......
```

In this example, which exhibits the chief varieties that occur in extracting the square root, we begin to point the units'

units' place of the integers, 6; then passing one figure 7, we point the next 8; and passing the 2 we point the 6: in the same way the decimals are pointed in pairs counted from the units' place of the integers to the right hand. We next inquire what root or number will, when squared, give a product equal to or nearest under the first period on the left hand, which is here 46: this the preceding Table shows to be 6, whose square is 36, for the square of 7 being 49, would be too high: we then write the root 6 in the quotient, and the square 36 under the period 46, to be subtracted from it. To the remainder 10 we bring down the whole of the following period 28, and saying twice or double the figure in the quotient 6, is 12, we write 12 as a divisor to the dividend 1028, leaving room for a figure or two in the units' and tens' places. We then inquire how often this divisor 12 can be contained in the dividend 102 (for the units' place is always excluded), and finding it can be contained 8 times, we write 8 in the quotient after 6, and also in the void space of the divisor after 12, making altogether for a divisor 128. This is multiplied by the 8 of the quotient, and the product 1024 subtracted from 1028 leaves 4 for a remainder, to which bringing down the next period 76, we have a new dividend, and for a divisor, doubling the whole quotient 68, we have 136, to be taken out of 47, (excluding the 6 in the units' place, as before directed), which being impossible, we write 0 in the quotient, and also after the 136 of the divisor. Then bringing down the following period 12, which in this example consists of decimals, we have 47612 for a dividend to be divided by the same divisor increased by the nought, viz 1360: this may be done 3 times, the three being written in the quotient, and after the 0 of the divisor, which multiplied by 3 gives 40809 to be subtracted from 47612, leaving the remainder 6803, to which bringing down the last period or pair of decimals 25, we have a new dividend 680325 to be divided by

 the

the double of the quotient 6803, equal to 13606 : this sum being contained 5 times in the dividend, we write 5 in the quotient and in the void space of the divisor, which being multiplied by 5, the product is equal to the dividend, and nothing remains. In this manner we obtain the quotient or root required : but as part of the given square is a decimal fraction consisting of four figures, forming two periods, we set off from the right hand of the quotient two figures, one for each period, separating them by the decimal point from the integral part of the root, which thus is found to be 680,25. As a proof that the operation is correct, we may square this root, or multiply it by itself, and the product, if all is right, and that there is no remainder, will be the same with the given square, the root of which was to be extracted. Had there been any remainder, it should have been added to the square of the quotient, to have made it coincide with the given square.

2d. To extract the *cube* root of any given quantity, we work as in the following example, where it is required to discover the root or first power of the cubical quantity 43169672,512.

Cube Root.	
43169672,512(350,8	3
	3
27	—
	3 times 9
27)161	3
	—
42875	27
	—
3675)··2946725	35
	35
43169672512	—
	175
....	105
	—
	3 times 1225

3 times

3 times 1225
35

6125
3675

42875

Having written down the given cubical quantity, distinguishing by a point the decimals from the integers, we begin to divide the integers into periods, by placing a dot under or over the units place 2; then passing over two figures we place a dot under or over the fourth figure 9; and again passing over two figures, we dot the seventh figure 3: then returning to the units' place of the integers, we count off three figures to the right hand in the decimals, and place a dot over or under the third figure. By this process we discover that there will be 3 integers and 1 decimal in the root required; since there are always as many figures in the root, as there are dots or periods in the cubic dividend. We now inquire what number, when cubed or multiplied twice successively into itself, will produce a cube equal to or the nearest under the first period on the left hand, which is 43: this upon trial, or on inspecting the foregoing Table will be found to be 3; for 3 times 3 are 9, and 3 times 9 are 27; whereas 4 times 4 are 16, and 4 times 16 are 64, which is a greater sum than 43; we therefore write in the quotient, or where the root is to appear, the 3, and its cube 27 under 43, from which subtracting it we have for a remainder 16. To this 16 we bring down the first figure of the next period, viz. 1, making 161 for a new dividend. For a divisor we take 3 times the square of the root or quotient, that is 3 times 9 are 27, which we write before 161, and finding it can be contained 5 times in that dividend, we write 5 after the 3 in the quotient, making in all 35. This quantity is then cubed, as is here shown, and the product is written under 161, but separated from it by a line; for the

new found cube is not to be subtracted from 161, but from the cube originally given at the head of the operation; care must therefore be taken to place the first figure of the product 4 under the first figure of the top line, the second figure 2 under the second figure 3 of the top line, and so on to the last figure 5 of the new cube, which will come immediately under the 9 of the top; thus occupying two periods, corresponding to the two figures in the root or quotient. Subtracting the new cube from the top, we have for a remainder 294, to which bringing down the first figure of the succeeding period of the given cube, viz. 6, we obtain 2946 to be divided, as before, by 3 times the square of the quotient 35, which is, as shown in the example, 3675: but this divisor being greater than the dividend 2946, it cannot be taken out of it; we therefore write a nought in the root after the 5, making the whole 350. This sum should next be cubed, and the product subtracted from the given cube; but as the addition of a nought to the factors would produce no change on the products, it is needless to take this trouble, we therefore bring down the whole period, of which the first figure 6 was already brought down, together with 5, the first figure of the following period, making the new dividend 2946725, or rather 29467, excluding the two last figures, to correspond to the two noughts; which, had the work been completed, would have made the divisor 367500: we then inquire how often this divisor can be found in the new dividend, and the quotient being 8, we write this figure in the root, and cubing the whole 350,8, the product turns out to be precisely the cubical quantity given in the question; and it consists of three places of integers and one of decimals, agreeably to the number of periods in each portion of the given cube, as was before remarked would be the case.

Having

Having thus gone through the most necessary branches of Arithmetic, integral and fractional, it remains only to apprise the student, for his satisfaction and encouragement, that the rules laid down to govern his practice, in the various operations he has been directed to perform, are by no means either arbitrary or merely mechanical; on the contrary they are all founded in the nature of things and of numbers, and are susceptible of the most accurate and evident demonstration. To give however such demonstrations in this limited work would be impracticable, and fully to comprehend them would require a more extensive knowledge of various parts of Geometry, than can be expected to be possesed by those young persons for whose use these pages have been compiled. Opportunities for illustrating arithmetical operations will, notwithstanding, occasionally occur in the progress of the work; and such opportunities shall not be neglected.

CHAP.

THE

MODERN PRECEPTOR.

CHAPTER, III.

OF BOOK-KEEPING.

By *Book-keeping*, or *Merchants' Accounts*, is meant the art of recording, with order and accuracy, all mercantile or commercial transactions.

By this art the merchant or trader obtains a particular and complete statement of every branch of his affairs, exhibiting the profit or loss arising from each separate transaction, as well as from the whole course of his business, and affording satisfactory information on all matters in which, as a man of business, he is concerned.

A merchant's books should contain an account of the whole amount of his property employed in trade, whether consisting of cash, bills, public funds, goods in hand, ships, houses, lands, debts, &c. &c.; and by comparing at different periods the state of these several kinds of property, he will discover the true situation of his affairs, whether prosperous or the contrary, and thence be enabled to manage them to the best advantage.

Various modes of book-keeping have been adopted in the mercantile world, but that which has acquired the approbation of the best judges is, the *Italian* method of book-keeping by *double entry*.

In keeping books by *single entry*, only such articles are entered as are bought or sold on credit: no account being

kept

kept of such as are bought or sold for ready money or other immediate payment: consequently books kept in this way can furnish no statement of the trader's concerns, unless assisted by his taking an account of the stock of goods remaining unsold, and so calculating the amount of gain or loss on his transactions; *single-entry* is therefore chiefly applicable to small dealings or retail business.

Book-keeping by *double-entry,* on the contrary, is applicable to large, extensive, and complicated transactions; possessing this advantage, that by merely inspecting each account, or the periodical balances of his whole concerns, the merchant can at once form a correct idea of the situation of his affairs.

In keeping accounts by *single entry* two books are required, the *Day-book* and the *Ledger*. The *Day-book* contains an account of the trader's property in business, followed by a regular statement of each transaction in the order in which it occurs. The *Ledger* contains, in one account under the head of each person, the several transactions of the *Day-book* in which that person is concerned; stating on opposite pages the different articles in which he may be debtor or creditor, and thus furnishing a state of affairs with him. The manner of keeping these two books will be readily understood from the following specimen of accounts kept by single entry.

On the 1st of January 1808; I purchase on credit from James Martin 86 yards of Silk at 5sh. per yard, value £21 10; on the 4th of January I sell to Joseph Jones on credit 52 yards of the same silk at 6sh. 8d. per yard value £17 6 8; on the 7th January, Joseph Jones pays me £12 in part of his account; and on the 8th January I pay James Martin £11 10, in part of my debt to him. These articles will appear in the *Day-book* as follow.

DAY-

DAY-BOOK.

Fo. of Ledg.		£.	s.	d.
	————1st January 1808.————			
	James Martin Cr.	£.	s.	d.
	By *Silk*, for 86 yards, at 5*s*. per yard -	21	10	0
	————4*th*.————			
	Joseph Jones Dr.			
1	To *Silk*, for 52 yards, at 6*s*. 8*d*. per yard	17	6	8
	————7*th*.————			
	Joseph Jones Cr.			
1	By *Cash* received in part - -	12	0	0
	————8*th*.————			
	James Martin Dr.			
1	To *Cash* paid in part - -	11	10	0

The same articles will appear in the Ledger, as in the following specimen.

LEDGER.

1808		*James Martin,* Dr.	Fo.	£.	s.	d.	1808		*Contra,* Cr.	Fo.	£.	s.	d.
Jan.	8	To *Cash* paid in part - - -	1	11	10	0	Jan.	1	By *Silk* for 86 yards at 5*s.* per yard	1	21	10	0
		To *Balance* - - - - -		10	0	0							
				21	10	0							
1808		*Joseph Jones,* Dr.					1808		*Contra,* Cr.				
Jan.	4	To *Silk* for 52 yards at 6*s.* 8*d.* per yard	1	17	6	8	Jan.	7	By *Cash* received in part - -	1	12	0	0
									By *Balance* - - -		5	6	8
											17	6	8

Here

Here it is to be observed, that as the person who receives any money or article of merchandize becomes the Debtor, and the person who gives away any money or article of merchandize becomes the Creditor; having purchased silk on credit from James Martin, I am his debtor and he becomes my creditor for the value; I therefore enter his name in the Day-book as Creditor for a certain quantity of silk at a certain value, and place the amount in the money-columns.

In the next transaction, by selling on credit to Joseph Jones a quantity of silk, he is entered as Dr. for the value.

In the following transaction Joseph Jones having paid me a part of his debt, I give him credit for the amount; that is, he is entered as Cr.

And in the last transaction, by paying to James Martin a part of what I owe him; that is, by giving him a sum of money, he is entered as my debtor, or he is debited for the amount.

In this manner is the *Day-book* filled up, but in the *Ledger*, a separate account is allotted for each person with whom I have dealings; room being left to contain entries of all such transactions as may probably occur between him and me. This book is laid out in folios, each folio containing two opposite pages, both numbered with the same figure which expresses the folio.

In the example here given the name of James Martin is written at the head of his account, on the left hand of the folio, appropriated to the Dr. side; and on the right hand is written the Latin word *Contra* (meaning *against* or *on the other hand,*) for the Cr. side; in which, as he is my creditor, I enter the date, nature and value of the transaction.

Again; an account is opened for Joseph Jones similar to that for James Martin; but as by parting with goods to him he becomes my debtor; the date, nature and value of the transaction is entered on the Dr. side of the folio.

When Joseph Jones pays me a part of his debt, I give

him

him credit for it, and therefore enter the payment on the Cr. side of his account already opened: and when I, out of the money thus received, discharge a part of my debt to James Martin, I consider him as debtor (that is, accountable) to me for the amount, and therefore enter the payment on the Dr. side of his account already opened.

The transactions being thus entered in the Ledger, it becomes necessary to balance the accounts: this is done by subtracting the less from the greater side, and placing the difference on the less side; as in James Martin's account, where my debt to him is £21 .. 10, of which having paid £11 .. 10, the remainder unpaid or balance due to him, £10, is placed under the £11 .. 10, making the whole £21 .. 10, equal to the Cr. side.

On the other hand, Joseph Jones being indebted to me £17 .. 6 .. 8 for goods delivered, and having only paid in part £12 .. 0, the difference or balance due to me, £5 .. 6 .. 8, is entered on the Cr. side of his account, making it equal to the Dr. side.

By books kept in this way by *single entry*, I can see that I still owe £10. to James Martin, and that Joseph Jones owes me £5 .. 6 .. 8: but I cannot discover the correct state of my affairs; it will therefore be necessary to *take stock*, as it is termed, that is, to enquire what goods are still upon my hands. Of 86 yards of silk purchased from James Martin, I sold 52 yards to Joseph Jones; 34 yards therefore still remain unsold, which valued at the prime cost or 5s. per yard, will be worth £8 .. 10: and this sum added to £17 .. 6 .. 8 the price of the quantity sold to Joseph Jones, gives £25 .. 10 .. 8, exceeding the prime cost of the whole quantity £21 .. 10, by £4 .. 0 .. 8; which sum I consider as my profit already gained on the adventure. But as I have paid James Martin only £11 .. 10, and therefore still owe him £10, I find that by giving

him the whole of the money I have received from Joseph Jones, I would reduce my debt from £10. to £5 .. 19 .. 4.

From these examples the defects of book-keeping by single entry must be obvious, when we consider the difficulty of taking the stock of a merchant of complicated and extensive dealings, an operation in itself of much labour as well as liable to error and fraud: the method by double entry is therefore in such cases constantly to be adopted, agreeably to the following observations and examples.

In Book-keeping by *double-entry* three books are chiefly requisite, viz. the *Waste-Book*, the *Journal*, and the *Ledger*.

The *Waste-book* contains, in simple clear language, a circumstantial and complete account or narrative of all transactions in business, in the order in which they occur.

The *Journal* contains an account of the same transactions, but expressed in a more artificial way, so as to point out the debtor and creditor in each case, and thereby prepare the articles for being carried to the *Ledger*.

In stating the Dr. and Cr. to an account, the following general rules are to be observed.

1st. That every article received, or every person accountable to us, is the Dr.

2d. That every article delivered, or every person to whom we are accountable, is the Cr.

Or more particularly,—the person to whom or for whom we pay money, or furnish goods, becomes the Dr.; and the person from whom, or for whom we receive money or goods, is the Cr.

Every thing which comes into our possession, or under our direction, is the Dr.; and every thing which goes out of our possession, or from under our direction, is the Cr.

The following cases comprehend the most common occurrences in merchants' accounts.

1st. The person to whom any thing or article is delivered

vered, becomes Dr. to the thing or article delivered, when nothing is received in return.

2d. A thing received is Dr. to the person from whom it is received, when nothing is delivered in return.

3d. A thing received is Dr. to the thing given for it.

4th. Goods and other real accounts, that is such as relate to property of every sort, are Dr. for all charges incurred by them.

5th. When any profits are received from real accounts, as rents of houses or land, freights of ships, bounties or goods, and the like, *Cash* becomes Dr. for the amount received, to the account or article from which the profit is derived.

6th. When any loss is sustained, the account of Profit and Loss, or some other of the same kind is Dr. to *Cash* for the amount.

7th. When any profit or gain arises, but not from any real account, Cash, the article received, or the person accountable for the profit, is Dr. to the account of Profit and Loss, or some other account of the same kind, for the amount.

8th. When one person pays money, or delivers any thing to another person on our account, the receiver is Dr. to the person who pays or delivers.

The *Ledger*.—In this book all transactions belonging to one person, or one article of merchandize, are collected together and entered as they occur, under one head, expressing the person or thing concerned in the account; and as in every transaction there must be a Dr. and a Cr. the occurrence is entered in both accounts on opposite sides; and from this circumstance it is that Book-keeping by *double entry* has obtained its name.

All accounts in the *Ledger* are either *real, personal,* or *fictitious.*

In *personal* accounts the person is entered Dr. or Cr. according

according to the nature of the transaction, as in accounts kept by single entry.

In *real* accounts each article is entered on the Dr. or Cr. side agreeably to the entries in the *Journal.*

In *fictitious* accounts all articles appear which have a relation to Stock, or to Profit and Loss. By *Stock* is meant the merchant himself to whom the books belong, for his name never appears; and on the Dr. side of this account appear the debts owing by the merchant; while on the Cr. side appear the monies due to him, with the cash, goods, and other property belonging to him, in the outset of the books.

By *Profit and Loss* is meant whatever may be gained or lost in business; and the Dr. side contains what is lost, while the Cr. side contains what is gained, upon every transaction.

The following specimen will show the method of keeping the *Waste-book*, *Journal*, and *Ledger*, or Book-keeping by *double entry*.

WASTE-BOOK.

London, 1st. January, 1808.

			£	s.	d.
	Inventory of the money, goods, and debts belonging to me X, Z, as also of what I owe.				
		£ s. d.			
√	I have in *ready money*	750 .. 0 .. 0			
	Bills receivable, one on *David Jones*, due 22d. May, next. - -	280 .. 0 .. 0			
	Cloth, 20 pieces each 24 yards at 16*s*. 6*d*. per yard -	396 .. 0 .. 0			
	Sugar 8 hhds, containing in all 104 cwt. at £ 3 2*s*. per cwt.	322 .. 8 .. 0			
	Richard Wilson owes me -	80 .. 0 .. 0			
			1828	8	0
	I owe as follows :				
√	To *James Andrews*	160 .. 0 .. 0			
	Bills payable for my acceptance of *George Gray's* bill due 1st. March	365 .. 0 .. 0	525	0	0
	4th. January				
√	Sold for *ready money* 8 pieces of Cloth, each 24 yards at 18*s*. 4*d*. per yard		176	0	0
	Jany. 6th				
√	Bought for *ready money* 50 pieces of *Linen*, each 25 yards at 3*s*. 2*d*. per yard.		197	18	4

8ht

WASTE-BOOK.

	January 8th.	£.	s.	d.
√	Bought of *Richard Wilson* 60 gallons of *Rum*, at 15*s*. per gallon -	45	0	0
	--11th.--			
√	Sold *William Brown* 5 hhds. *Sugar*, containing 65 cwt. at £3. 12*s*. per cwt.	234	0	0
	13th.			
√	Sold *Thomas Ellis* 30 pieces of *Linen* each 25 yards at 4*s*. 4*d*. per yard. - Received in part - 62 .. 10 .. 0 The rest to be paid in 2 months - - 100 .. 0 .. 0	162	10	0
	16th.			
√	Bought of *Robert Turner* 36 pieces of *Muslin* each containing 15 yards, at 3*s*. 8*d*. per yard. Paid in part - - 49 .. 0 .. 0 Rest due at 2 months - 50 .. 0 .. 0	99	0	0
	19th.			
√	Sold *George Fanshaw* the following goods. 6 pieces of *Cloth* each 24 yards at 18*s*. 6*d*. per yard } 133 .. 4 .. 0 3 hhds of *Sugar* containing 39 cwt. at £3 14*s*. per cwt. - - } 144 .. 6 .. 0 12 pieces of *Linen* each 25 yards, at 4*s*. 6*d*. per yard - } 67 .. 10 .. 0	345	0	0

Received

WASTE BOOK.

	Jany. 19th.	£.	s.	d.
	Received in part			
√	In *Cash* 45 .. 0 .. 0			
√	A Bill on *Harris and Co.* No. 38, due 22 March 150 .. 0 .. 0	195	0	0
	——22nd.——			
√	Paid to *James Andrews* on account	85	0	0
	——23rd.——			
√	Drawn on *Thomas Ellis* for the balance of his account, payable at two months	100	0	0
	——26th.——			
√	Received for the use of *Richard Wilson* £1800, which I have remitted to him, deducting ½ per cent for my *commission.*	9	0	0
	——28th.——			
√	Received a *Legacy* of -	50	0	0
	——Feb. 1st.——			
√	Paid sundry charges for rent, &c. for the last month - -	33	12	0
	——1st.——			
√	Bought of *John Barnes* the following goods, to pay at two months, viz.			
	46 pieces of *Callicoe* each 22 yards at 2*s.* 8*d.* per yard. £134 .. 18 .. 8			
	2 Bags of *Cotton* valued at £68 .. 12 .. 0			
	1 Pipe of *Port Wine* 78 .. 0 .. 0			
		281	10	8

Shipped,

WASTE-BOOK.

	Feb. 1st.	£.	s.	d.
√	Shipped these goods on board the Nancy, Joseph Aylmer master, for Gottenburgh, for account and risk of William Peters, merchant there, as per Invoice rendered.			
	Amount of goods £ 281 .. 10 .. 8			
	Charges of shipping, &c. 11 .. 4 .. 0			
	Commission at $2\frac{1}{2}$ per cent 7 .. 6 .. 4	300	1	0
	3rd.			
√	Sold for ready money 12 pieces of muslin at £ 3 .. 11 .. 6 per piece. -	42	18	0
	5th			
√	Sold Robert Turner 6 pieces of Cloth, each 24 yards at 18*s.* 8*d.* per yard.	134	8	0
	8th			
√	Lost, a Bank Note of -	15	0	0
	Feb. 12th			
√	Received per the Neptune from Gottenburgh 8 tuns of Hemp, to sell for account of William Peters. - -			
√	Sold James Andrews 5 tuns of the said Hemp to pay at 2 months. } £ 175 .. 0 .. 0			
√	Sold the other 3 tuns for ready money. } 102 .. 0 .. 0	277	0	0
	Commission on do. at $2\frac{1}{2}$ per cent - - } 6 .. 18 .. 6			
	Charges paid at landing - 13 .. 17 .. 6	20	16	0

I owe

WASTE-BOOK.

February 12th.

		£.	s.	d.
√	I owe *William Peters* for net proceeds of the *Hemp*, as per account Sales this day rendered - -	256	4	0
	———13th.———			
√	Sold *James Andrews* 12 pieces of *Muslin* at £3. 15*s*. per piece - -	45	0	0
	———16th.———			
	Sold *David Jones* 8 pieces of *Linen*, each 25 yards, at 4*s*. 7½*d*. per yard. £46 .. 5 .. 0 40 gallons of *Rum* at 17*s*. per gallon - 34 .. 0 .. 0	80	5	0
	———18th.———			
√	Taken up my bill in favour of *George Gray* for - £365 .. 0 . 0 Discount allowed by him at 5 per cent for 12 days 0 .. 12 .. 0	364	8	0
	———20th.———			
√	Received from *Gottenburg* a bill on *Howard and Co.* for the account of *William Peters* due 30th March -	480	0	0
	———22nd.———			
√	Discounted at the Bank William Peters' bill due 30th March - 480 .. 0 .. 0 *Discount* for 38 days at 5 per cent - - 2 .. 9 .. 11½	477	10	0½

WASTE-BOOK.

	February 23rd.	£.	s.	d.
√	*William Brown* being declared insolvent, and his creditors having agreed to a composition of 12 shillings in the pound, I have this day received my dividend on his debt of - - £ 234 .. 0 .. 0	140	8	0
	26th.			
√	Received from *James Andrews* payment for the 12 pieces *Muslin* sold to him on the 13th. - - -	45	0	0
	29th.			
√	Paid Sundry expences this month, not charged to any other account -	29	13	0

JOURNAL.

London, 1st January, 1808.

Fo.			£.	s.	d.
1	*Sundries Drs.* to *Stock.*				
	For the amount of my effects.				
1	*Cash* -	£750 .. 0 .. 0			
1	*Bills receivable,* on *David Jones* -	280 .. 0 .. 9			
2	*Cloth,* 20 pieces, each 24 yards, at 16*s.* 6*d.* per yard - -	396 .. 0 .. 0			
2	*Sugar,* 8 hhds. containing 104 cwt. at £3 2*s.* per cwt. -	322 .. 8 .. 0			
2	*Richard Wilson*	80 .. 0 .. 0	1828	8	0
	Do.				
1	*Stock* Dr. to *Sundries*				
	For the amount of what I owe				
2	To *James Andrews*	£160 .. 0 .. 0			
2	To *Bills payable,* for *George Gray's* bill accepted by me, due 1st. March	365 .. 0 .. 0	525	0	0
	4th.				
$\frac{1}{2}$	*Cash* Dr to *Cloth*				
	For 8 pieces, 192 yards at 18*s.* 4*d.* per yard		176	0	0
	6th.				
$\frac{3}{1}$	*Linen* Dr. to *Cash*				
	For 50 pieces, each 25 yards, at 3*s.* 2*d.* per yard - - -		197	18	4

Rum

JOURNAL.

Fo.		£.	s.	d.
	Jany. 8th.			
3/2	*Rum* Dr. to *Richard Wilson*			
	For 60 gallons at 15s. per gallon	45	0	0
	11th.			
3/2	*William Brown* Dr. to *Sugar*			
	For 5 hhds, 65 cwt. at £3 .. 12 per cwt.	234	0	0
	13th.			
3	*Sundries* Drs. to *Linen*. - -			
	For 30 pieces, each 25 yards, at 4*s.* 4*d.* per yard. - -	162	10	0
1	*Cash* received in part £ 62 .. 10 .. 0			
3	*Thomas Ellis* for the rest 100 .. 0 .. 0			
		162	10	0
	16th.			
3	*Muslin* Dr. to *Sundries*. -			
	For 36 pieces, each 15 yards, at 3*s.* 8*d.* per yard - - -			
1	To *Cash* paid in part - £ 49 .. 0 .. 0			
3	To *Robert Turner* for the rest - - 50 .. 0 .. 0			
		99	0	0
	19th.			
4	*George Fanshaw* Dr. to *Sundries*			
2	To *Cloth* for six pieces each 24 yards at 18*s.* 6*d.* per yard - £133 .. 4 .. 0			
2	To *Sugar* for 3 hhds. 39 cwt. at £ 3. 14*s.* per cwt. - - 144 .. 6 .. 0			

To

JOURNAL.

Fo.	January 19th.	£.	s.	d.
3	To *Linen* for 12 pieces, each 25 yards, at 4*s*. 6*d*. per yard - 67 .. 10 .. 0	345	0	0
	—— Do. ——			
4	*Sundries* Drs. to *George Fanshaw*.			
1	*Cash* - - £ 45 .. 0 .. 0			
1	*Bills receivable, Harris and Co.* due 22d March - 150 .. 0 .. 0	195	0	0
	—— 22d. ——			
$\frac{2}{1}$	*James Andrews* Dr. to *Cash* Paid him on account -	85	0	0
	—— 23rd. ——			
$\frac{1}{4}$	*Bills receivable*, Dr. to *Thomas Ellis* For his acceptance of my bill at two months - - -	100	0	0
	—— 26th. ——			
$\frac{1}{4}$	*Cash* Dr. to *Commission* For receiving and remitting £ 1,800, for *Richard Wilson*, at $2\frac{1}{2}$ per cent	9	0	0
	—— 28th. ——			
$\frac{1}{4}$	*Cash* Dr. to *Profit and Loss* For a *Legacy* received - - -	50	0	0
	—— Feb. 1st. ——			
$\frac{4}{1}$	*Profit and Loss* Dr. to *Cash* Rent and other charges paid for last month -	33	12	0

Merchandise

JOURNAL.

February 1st.

Fo.		£.	s.	d.
4/4	*Merchandise* Dr. to *John Barnes*			
	For the following goods bought of him at two months, viz.			
	Callicoe 46 pieces, each 22 yards, at 2*s.* 8*d.* per yard - - £134 .. 18 .. 8			
	Cotton 2 Bags valued at - 68 .. 12 .. 0			
	Port Wine - 78 . 0 .. 0			
		281	10	8
	1st.			
5	*William Beters* Dr. to *Sundries* -			
	For account of goods shipped for his account on board the Nancy, Joseph Aylmer master, for Gottenburgh, as per Invoice.			
4	To *Merchandise* - £ 281 .. 10 .. 8			
1	To *Cash* for charges on merchandise - 11 .. 4 .. 0			
4	To *Commission* at 2½ per cent. - - - 7 .. 6 .. 4			
		300	1	0
	3rd.			
1/3	*Cash* Dr. to *Muslin* - -			
	For 12 pieces, at £ 3. 11*s.* 6*d.* per piece	42	18	0
	5th.			
3/2	*Robert Turner* Dr. to *Cloth*			
	For 6 pieces, each 24 yards, at 18*s.* 8*d.* per yard - - -	134	8	0

2d

JOURNAL.

February 8th.

Fo.			£.	s.	d.
4/1	*Profit and Loss* Dr. to *Cash* For a *Bank note* lost		15	0	0
	12th.				
5	*Sales per the Neptune* Dr. to *Sundries*				
1	To *Cash* for charges on merchandise, as per Account Sales rendered	£ 13 .. 17 .. 6			
4	To *Commission* on £277..0s. at $2\frac{1}{2}$ per cent.	6 .. 18 .. 6			
5	To *William Peters* for net proceeds	256 .. 4 .. 0	277	0	0
	Do.				
5	*Sundries* Drs. to *Sales per Neptune* For 8 tuns of *Hemp* on account of *William Peters*.				
2	*James Andrews* for 5 tuns, at 2 months	£175 .. 0 .. 0			
1	*Cash* for 3 tuns	102 .. 0 .. 0	277	0	0
	13th.				
2/3	*James Andrews* Dr. to *Muslin* For 12 pieces at £3 .. 15s. per piece		45	0	0
	16th.				
5	*David Jones* Dr. to *Sundries*				
3	To *Linen* 8 pieces 200 yds. at 4s. $7\frac{1}{2}d.$ per yd.	£46 .. 5 .. 0			
3	To *Rum* 40 gallons at 17s. per gallon	34 .. 0 .. 0	80	5	0

Bills

JOURNAL.

Fo.	February 18th.	£.	s.	d.
2	*Bills payable* Dr. to *Sundries*			
	For *George Gray's* bill discounted.			
1	To *Cash* paid him - £ 364 .. 8 .. 0			
5	To *Interest* for discount 0 .. 12 .. 0	365	0	0
	20th.			
$\frac{1}{5}$	*Bills receivable* Dr. to *William Peters*			
	For his bill on *Howard & Co.* due 30th March - -	480	0	0
	22nd.			
1	*Sundries* Dr. to *Bills receivable*			
	For *William Peters's* Bill discounted			
1	*Cash* received - £477 .. 10 .. 0½			
5	*Interest* for discount 2 .. 9 .. 11½	480	0	0
	23rd.			
3	Sundries Drs. to *William Brown*,			
1	*Cash* for composition received - £ 140 .. 8 .. 0			
4	*Profit and Loss*, for loss on his debt to me - 93 .. 12 .. 0	234	0	0
	26th.			
$\frac{1}{2}$	*Cash* Dr. to *James Andrews*			
	For 12 peices *Muslin*	45	0	0
	29th.			
$\frac{4}{1}$	*Profit and Loss* Dr. to *Cash*			
	Charges paid this month	29	13	0

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				Sugar	2
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(1) LEDGER.

1808			Fo.	£.	s.	d.
		STOCK Dr.				
Jan.	1	To Sundries - -		525	0	0
Feb.	29	To Balance for the net of my Estate - -	6	1412	10	6
				1937	10	6
		CASH Dr				
Jan.	1	To Stock - -	1	750	0	0
	4	To Cloth - -	2	176	0	0
	13	To Linen - -	3	62	10	0
	19	To George Fanshaw	4	45	0	0
	26	To Commission -	4	9	0	0
	28	To Profit and Loss -	4	50	0	0
Feb.	3	To Muslin -	3	42	18	0
	12	To Sales per Neptune -	5	102	0	0
	22	To Bills receivable -	1	477	10	0½
	23	To William Brown -	3	140	8	0
	26	To James Andrews -	2	45	0	0
				1900	6	0½
		BILLS RECEIVABLE Dr.				
Jan.	1	To Stock - -	1	280	0	0
	19	To George Fanshaw -	4	150	0	0
	23	To Thomas Ellis -	3	100	0	0
Feb.	20	To William Peters -	5	480	0	0
				1010	0	0

By

LEDGER. (1)

			Fo.	£.	s.	d.
1808		CONTRA Cr.				
Jan.	1	By Sundries -		1828	8	0
Feb.	29	By Profit and Loss -	4	109	2	6
				1937	10	6

			Fo.	£.	s.	d.
		CONTRA Cr.				
Jan.	6	By Linen - -	3	197	18	4
	16	By Muslin - -	3	49	0	0
	22	By James Andrews -	2	85	0	0
Feb.	1	By Profit and Loss -	4	33	12	0
	1	By William Peters -	5	11	4	0
	8	By Profit and Loss -	4	15	0	0
	12	By Sales per Neptune	5	13	17	6
	18	By Bills payable -	2	364	8	0
	29	By Profit and Loss -	4	29	13	0
	29	By Balance - -	6	1100	13	$2\frac{1}{2}$
				1900	6	$0\frac{1}{2}$

			Fo.	£.	s.	d.
		CONTRA Cr.				
Feb.	22	By Sundries -		480	0	0
	22	By Balance -	6	530	0	0
				1010	0	0

To

(2) LEDGER.

1808			Fo.	£.	s.	d.
		CLOTH Dr.				
Jan.	1	To Stock, 20 pieces, 480 yards, at 16*s*. 6*d*. per yard -	1	396	0	0
Feb.	29	To Profit and Loss	4	47	12	0
				443	12	0
		SUGAR Dr.				
Jan.	1	To Stock, 8 hhds, 104 cwt. at £ 3 . 2*s*. per cwt.	1	322	8	0
Feb.	29	To Profit and Loss	4	55	18	0
				378	6	0
		RICHARD WILSON Dr.				
Jan.	1	To Stock -	1	80	0	0
		JAMES ANDREWS Dr.				
Jan.	22	To Cash - -	1	85	0	0
Feb.	12	To Sales per Neptune	5	175	0	0
	13	To Muslin -	3	45	0	0
				305	0	0
		BILLS PAYABLE Dr.				
Feb.	18	To Sundries -		365	0	0

By

LEDGER. (2)

1808			Fo	£.	s.	d
		CONTRA CR.				
Jan.	4	By Cash 8 pieces containing 192 yards, at 18*s*. 4*d*. per yard	1	176	0	0
	19	By George Fanshaw 6 pieces containing 144 yards, at 18*s*. 6*d*.	4	133	4	0
Feb.	5	By Robert Turner 6 pieces containing 144 yards, at 18*s*. 8*d*.	3	134	8	0
				443	12	0
		CONTRA CR.				
Jan.	11	By William Brown 5 hhds. containing 65 cwt. £ 3 12*s*. per cwt.	3	234	0	0
	19	By George Fanshaw 3 hhds. containing 39 cwt. £ 3 14*s*.	4	144	6	0
				378	6	0
		CONTRA CR.				
Jan.	8	By Rum - -	3	45	0	0
Feb.	29	By Balance - -	6	35	0	0
				80	0	0
		CONTRA CR.				
Jan.	1	By Stock	1	160	0	0
Feb.	26	By Cash	1	45	0	0
	29	By Balance	6	100	0	0
				305	0	0
		CONTRA CR.				
Jan.	1	By Stock for George Gray's bill	1	365	0	0

To

(3) LEDGER.

1808			Fo.	£.	s.	d.
		LINEN Dr.				
Jan	6	To Cash, 50 pieces, 1250 yards, at 3*s.* 2*d.* per yard -	1	197	18	4
Feb.	29	To Profit and Loss -	4	78	6	8
				276	5	0
		RUM Dr.				
Jan.	8	To Richard Wilson, 60 gallons, at 15*s.* per gallon -	2	45	0	0
Feb.	29	To Profit and Loss	4	4	0	0
				49	0	0
		WILLIAM BROWN Dr.				
Jan.	11	To Sugar - -	2	234	0	0
		THOMAS ELLIS - Dr				
Jan.	13	To Linen - -	3	100	0	0
		MUSLIN Dr.				
Jan.	16	To Sundries, 36 pieces, each 15 yards, at 3*s.* 8*d.* per yard		99	0	0
Feb.	29	To Profit and Loss	4	21	18	0
				120	18	0
		ROBERT TURNER Dr				
Feb.	5	To Cloth - -	2	134	8	0

By

LEDGER. (3)

			Fo.	£.	s.	d.
1808		CONTRA Cr.				
Jan.	13	By Sundries 30 pieces, containing 750 yards, at 4*s*. 4*d*. per yard		162	10	0
	19	By George Fanshaw 12 pieces, containing 300 yards, at 4*s*. 6*d*.	4	67	10	0
Feb.	16	By David Jones 8 pieces, containing 200 yards, at 4*s*. 7½*d*.	5	46	5	0
				276	5	0
		CONTRA Cr.				
Feb.	16	By David Jones 40 gallons, at 17*s*. per gallon - -	5	34	0	0
	29	By Balance 20 gallons, at 15*s*.	6	15	0	0
				49	0	0
		CONTRA Cr.				
Feb.	23	By Sundries - -		234	0	0
		CONTRA Cr.				
Jan.	23	By Bills receivable -	1	100	0	0
		CONTRA Cr.				
Feb.	3	By Cash 12 pieces, at £ 3 11*s*. 6*d*. per piece - -	1	42	18	0
	13	By James Andrews 12 pieces, at 3£ 15*s* - -	2	45	0	0
	29	By Balance 12 pieces at £3 8*s*	6	33	0	0
				120	18	0
		CONTRA Cr.				
Jan.	16	By Muslin - -	3	50	0	0
Feb.	29	By Balance -	6	84	8	0
				134	8	0

(4) LEDGER.

			Fo.	£.	s.	d.
1808		George Fanshaw Dr.				
Jan.	19	To Sundries per Journal		345	0	0
		Commission Dr.				
Feb.	29	To Profit and Loss -	4	23	4	10
		Profit and Loss Dr.				
Feb.	1	To Cash for Expences	1	33	12	0
	8	To Cash for a Bank note lost	1	15	0	0
	23	To William Brown -	3	93	12	0
	29	To Cash for Expences -	1	29	13	0
		To Stock gained -	1	109	2	6
				280	19	6
		Merchandise Dr.				
Feb.	1	To John Barnes -	4	281	10	8
		John Barnes Dr.				
Feb.	29	To Balance -	6	281	10	8

By

LEDGER. (4)

			Fo.	£.	s.	d.
1808		CONTRA Cr.				
Jan.	19	By Sundries per Journal		195	0	0
Feb.	29	By Balance -	6	150	0	0
				345	0	0
		CONTRA Cr.				
Jan.	26	By Cash -	1	9	0	0
Feb.	1	By William Peters	5	7	6	4
	12	By Sales per Neptune	5	6	18	6
				23	4	10
		CONTRA Cr.				
Jan.	28	By Cash for a Legacy	1	50	0	0
Feb.	29	By Cloth -	2	47	12	0
		By Sugar -	2	55	18	0
		By Linen -	3	78	6	8
		By Rum -	3	4	0	0
		By Muslin -	3	21	18	0
		By Commission	4	23	4	10
				280	19	6
		CONTRA Cr.				
Feb.	1	By William Peters	5	281	10	8
		CONTRA Cr.				
Feb.	1	By Sundries per Journal		281	10	8

(5) LEDGER.

			Fo.	£.	s.	d.
1808		**William Peters** Dr.				
Feb.	1	To Sundries per Journal		300	1	0
	29	To Balance -	6	436	3	0
				736	4	0
		Sales per the Neptune Dr.				
Feb.	12	To Sundries -		277	0	0
		David Jones Dr.				
Feb.	16	To Sundries per Journal		80	5	0
		Interest Dr.				
Feb.	22	To Bills receivable for discounting Wm. Peters' Bill	1	2	9	11½
		Balance Dr.				
Feb.	29	To Cash -	1	1100	13	2½
		To Bills receivable	1	530	0	0
		To Richard Wilson	2	35	0	0
		To James Andrews	2	100	0	0
		To Rum -	3	15	0	0
		To Muslin -	3	33	0	0
		To Robert Turner	3	84	8	0
		To George Fanshaw -	4	150	0	0
		To David Jones -	5	80	5	0
		To Interest -	5	1	17	11½
				2130	4	2

By

LEDGER. (5)

			Fo.	£.	s.	d.
1808		Contra Cr.				
Feb.	12	By Sales per Neptune	5	256	4	0
	20	By Bills receivable -	1	480	0	0
				736	4	0
		Contra Cr.				
Feb.	12	By Sundries -		277	0	0
		Contra Cr.				
Feb.	29	By Balance -	6	80	5	0
		Contra Cr.				
Feb.	18	By Bills payable	2	0	12	0
	29	By Balance -	6	1	17	11½
				2	9	11½
		Contra Cr.				
		By John Barnes -	4	281	10	8
Feb.	29	By William Peters	5	436	3	0
		By Stock -	1	1412	10	6
				2130	4	2

The

The Waste-Book opens with an account of the merchants effects in trades and of the debts he owes to others. The first set of articles compose his stock, and each article is journalised, that is, entered in the Journal, as Dr. to Stock for its value; and, on the other hand, Stock is entered as Dr. to each article, where money is due from the merchant.

The first transaction of the 4th January, where Cloth is sold for ready money, appears thus in the Journal, Cash Dr. to Cloth; that is, Cash received is accountable for Cloth parted with; but in the second transaction of the 6th, where Linen is bought for ready money, Linen is journalised as Dr. to Cash for the money given for it.

On the 8th, Richard Wilson furnishes Rum on credit; the Rum is therefore accountable to him for its value, and in the Journal Rum is entered Dr. to Richard Wilson,

On the 13th, Linen is sold to Thomas Ellis, partly for ready money, and partly on credit; the transaction must therefore have a double entry in the Journal, viz. Cash Dr. to Linen for the money received, and Thomas Ellis Dr. for the balance due at 2 months.

On the 26th, where the merchant acquires a sum of money for his trouble in receiving and transmitting money to Richard Wilson, as no real value is given for this money, a fictitious account is formed, titled Commission, under which must be entered all sums of this nature, either received or paid by the merchant.

On the 28th, a Legacy is received, the amount of which being considered as pure gain, for which no equivalent is given, another fictitious account is formed called Profit and Loss, and Cash is charged Dr. to this account for the amount of the Legacy.

On the 1st February, sundry charges are paid for rent, &c. which are necessary in the course of business, but for which no real value is received in return; the same account of

Profit

Profit and Loss, or some similar fictitious account, is therefore charged as Dr. to Cash for the amount.

In this manner, by applying the several rules formerly given, the method of transferring accounts from the Waste-Book to the Journal may readily be understood; and the next step, or transferring them from the Journal to the Ledger, will be very obvious.

The first folio of the Ledger contains an account for Stock, on the Cr. side of which are entered the several articles of the merchant's effects, or their amount in one article; and on the Dr. side the amount of his debts.

The next account opened is for Cash, as being the first article in the inventory of his effects in trade, stated in the Waste-Book and Journal, Cash being made Dr. to Stock for the ready money in the merchant's hands.

Then follows an account for Bills receivable, which become, in the same manner, Dr. to Stock for the Bill on David Jones.

Then appears the account of Cloth Dr. to Stock for the number of pieces and yards at a given price.

The next account opened is for Sugar, which appears Dr. to Stock for the whole quantity on hand at the opening of the books; and on the other side Cr. by the several quantities disposed of at different times.

Lastly, comes the account of Richard Wilson, who appears Dr. to Stock for the amount of his debt.

Having thus entered or posted in the Ledger all the articles composing the merchant's stock in trade, those of his debts come next to be posted; but with this difference, that as the former were all Drs. to Stock for their amount; the latter are Crs. by Stock for their amount; the account therefore of James Andrews is charged Crs. by Stock for the sum due to him, as is that of Bills payable for the amount of George Gray's Bill.

The transaction of the 4th January is posted in the Ledger

Ledger from the Journal, Cash Dr. to Cloth for the amount received; and the account of Cloth is Cr. by Cash for the quantity and value; thus exhibiting an example of double entry; and in the same way, by comparing the several articles as they appear in the Waste-Book, Journal and Ledger, the Scholar may discover how to transfer any other transactions from one book to another.

The crooked line or dash in the column on the left hand of the Waste-Book opposite to certain articles, shews that these articles have been journalised: and the numbers marked in the left hand column of the Journal indicate the folios of the Ledger where the several articles of an account are posted. When two numbers are written with a line between them, like a fractional number, they show that in the opposite line of the Journal there are two articles entered in the Ledger; thus in the transaction of the 4th January, both Cash and Cloth having accounts in the Ledger, Cash being in folio 1st, and Cloth in folio 2d, the reference is marked $\frac{1}{2}$. In the Ledger is a column immediately preceding the money columns, in which is marked the folio of the account specified in the entry.

To ascertain the accuracy of the books, it is necessary at certain periods to make what is termed a Trial Balance of the Ledger, founded on this observation, that when every account of the Journal is twice posted in the Ledger in one account on the Dr. and in another in the Cr. side, the sum of all the Drs. should be precisely equal to the sum of all the Crs. If therefore all the articles on the Dr. side of the Ledger be added together, as also all those on the Cr. side, and the amounts be the same, then the books are supposed to be free from error. To form the General Balance, however, more is requisite, because each account must be balanced; observing that the accounts of Stock and Profit and Loss cannot be balanced until all the other accounts are closed.

To

To begin with the Cash account, the Dr. and Cr. sides being summed up, and the less subtracted from the greater, the difference is placed on the less side to balance it, or make it even with or equal to the greater: the same is done with the next account for Bills receivable; but in that for Cloth, the whole quantity originally on hand being disposed of at an advanced price, the amount of the Creditor side is consequently greater than the Debtor side; the difference, therefore, is considered as the profit or gain on the article Cloth, which appears Dr. to Profit and Loss for the sum.

Had the amount of the Cr. side been less than that of Dr. side, a loss having been sustained by some accident or defect in the Cloth, the differenee would have appeared on the Cr. side, Cloth then becoming Cr. by Profit and Loss for deficiency, as is the case in the transaction of the 23d of February, where William Brown being insolvent, a loss is sustained by a composition being agreed to, and only twelve shillings being received for every twenty shillings he owed.

When all accounts on which there has been either gain or loss are summed up, and the proper sums are posted in the Dr. and Cr. sides of the account of Profit and Loss, the difference between these two sides is entered in the Stock account of the Ledger, in this way; if the Cr. side be the greatest, then the merchant has gained, and Stock is charged Cr. by Profit and Loss for the amount; but had the Dr. side been greater than the Cr. side, then the merchant would have been on the whole a loser by his business, and Stock would have become Dr. for the amount; or, in other words, the merchant's stock would have been diminished by that amount. But supposing, as in the specimens of books here given, that the trader is on the whole of his concerns a gainer, Profit and Loss will be Dr. to Stock for his gains; and the same being entered

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in the Cr. side of the Stock account, will, together with the other articles for the amount of his effects, make a certain sum, from which subtracting what stands on the Dr. side of the Stock account, the difference, or balance, will show the value of his effects at the closing of the books; and the same being entered in the Cr. side of the Balance account, ought, if no error be committed, to make it even with or equal to the Dr. side.

In conducting very large and complicated mercantile concerns, it is indispensible to simplify many accounts in the Ledger, by employing a number of subsidiary or assisting books, each of which is adapted to a particular branch of business; such as the Cash-Book, the Bill-Book, the Sales-Book, the Invoice-Book, &c. thereby forming parts of the Waste-Book, for such articles as belong to their respective subjects; so that the Waste-Book itself contains merely a general account of such transactions, with the particulars of such others only as do not come under the heads of these subsidiary books. By this method, and by balancing such lesser books at stated periods, as at the end of every month, the entries in the Ledger are greatly diminished in number; besides, that every importer or exporter of goods on commission must keep Invoice and Sales-Books, and Cash and Bill-Books are necessary in every branch of commerce.

The Cash-Book contains an account of all money matters, kept in the form of the Cash account in the foregoing specimen of a Ledger; the Dr. side shewing all monies received, and the Cr. side all monies expended or paid. The Cash-Book contains such payments and receipts as being merely temporary; such as small loans, accommodations, &c. need not appear in the Journal or Ledger.

The Bill-Book forms a register of all Bills of Exchange, whether payable or receivable, being divided into columns appropriated to the several particulars relating to each bill, such

such as, when received, by whom drawn, on whom drawn, the date, to whom payable, the time when due, and the sum, &c.

The Invoice-Book contains copies of all invoices, or statements of goods sent off, or exported on commission, containing the name of the ship and the master, place of destination, and person to whom consigned: then follows an account of the quantities and values of the several articles dispatched, to which are added, the shipping and other charges paid by the merchant, exporter, or factor; and upon the total of these sums the commission is reckoned.

The Sales-Book or Factory-Book, serves to point out the net proceeds upon any cargo or merchandise sold on consignment or commission. This is also called the Account of Sales, or commonly Account Sales, and is divided into two pages, like an account in the Ledger; having the title extending over both pages. On the left hand are entered all charges attending the transaction, such as those for freight, customs, landing, selling &c. with brokerage and factor's commission, both of which are reckoned on the gross amount of the sales. On the right hand are entered the quantity, price, and value of the goods sold, with the name of the buyer, and the time of payment; and the difference between the two pages of the account is the net proceeds, which being entered on the left hand, makes that page equal to the other; and for this article of net proceeds the factor gives his correspondent credit, transmitting to him a copy of the Account Sales, signed with his name, and certified to be correct, *errors excepted.*

BILLS OF EXCHANGE.

By Exchange is meant the paying of a sum of money in one country, or part of a country, for an equivalent sum

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in another country, or part of a country; and this is done by means of Bills of Exchange, which are written orders, directing the payment of a certain sum of money at a given time. Bills are either inland or foreign; inland bills are those drawn and payable within the same country or state, and foreign bills are those drawn in one country or state, but payable in another.

The person who draws the bill is styled *Drawer*; the person on whom it is drawn, that is, to whom it is addressed, is the *Drawee*, and also the *Acceptor*, when he engages to pay the bill; and he to whom the money is to be paid is called the *Payee*. Besides these, other persons are frequently concerned with Bills of Exchange, such as the buyer or remitter, the seller or negotiator, and the holder or possessor of the bill.

When the holder parts with a bill he indorses it, that is, he writes his name on the back, and thus every indorser becomes liable, or a security, for the payment. The first indorser ought to be the payee, who, by a special indorsement, may direct the payment to be made to a third person, not named in the bill, who thus becomes the *Indorsee*. When a bill is presented to the drawee, and accepted by him, he writes his name at the bottom with the word *accepted*: but if the bill be refused acceptance, it is put into the hands of a notary public, to be noted for non-acceptance; and if a bill that has been accepted be refused payment when due, it is also noted or protested, and returned to the drawer, who, or any of the indorsers, becomes liable to pay the value of the bill, and all charges accruing on the business: no time should be lost in thus returning a protested bill, otherwise the holder can have no claim on any one but the acceptor.

The period when a bill is due, which is called its *Term*, varies according to the agreement made between the parties concerned in it, or to the practice of the countries where it is drawn and payable; some being drawn at sight, that is,

they

they must be paid as soon as presented; others are made payable at a certain time, as a number of days after sight, or after the date of the bill; and others are drawn at usance, which is the customary term allowed in different countries for a bill being paid; double usance is double this term, and half usance is half this term.

Besides this usual term of payment, a certain time is also allowed after the term is expired, before payment can be enforced: and this time consists of a number of *days of grace*, which, in these kingdoms, is three; but bills drawn at sight admit of no days of grace.

In exchanges two things are of importance to be ascertained, the *par* and the *course* of exchange: the par means the real value of coin of one country in proportion to that of another; but the course signifies the current value of such coins, as fixed between the two countries. If the coins of different states were subject to no alteration in respect of their several intrinsic proportions in quality and weight, the par would be easily ascertained; but as this is by no means the case, the par must be liable to frequent fluctuations.

By means of bills of exchange, debts may be discharged reciprocally in different places, without any real transmission of money: thus, if John of London owe £ 100 to James of Edinburgh, and Robert of Edinburgh owe an equal sum to Thomas of London, both debts may be discharged by a transfer of Debtor and Creditor in this way: James draws a bill on John, which Robert purchases and remits to his Creditor Thomas, who receives the payment from John. When the debts between two places are unequal, the difference must be remitted in money or bills; and on account of the risk attending remittances in specie, bills are the most convenient medium of payment: but if many bills are required for remittance to any place, the number of purchasers will be increased, and consequently the price of such

bills

bills enhanced; that is, the course of exchange will be raised in favour of the place to which the bills are to be remitted. In this way the fluctuation of the course of exchange is proportioned in general to the balance of debts, or remittances between two places, which again is produced by the balance of trade, or the difference between the values of the exports and imports of both places with respect to each other. Thus, if London export goods to Lisbon to the value of £1000, and import from Lisbon to the value of £1500, the difference or balance of trade is evidently against London, and in favour of Lisbon: and if London remit this balance in bills, their price in the money market must be raised, and consequently more money must be given in London, to procure the payment of a certain sum in Lisbon, than would be expressed by valuing these sums at the par, or just value of the coins of each country; and, on this account, when the course of exchange runs high against any country, remittances from thence ought, if possible, to be made in specie instead of bills, which will tend to reduce the rate of exchange nearer to par.

Exchanges are calculated by the rule of Practice: thus, if it be required to find the value of £420 sterling in French money at par of 25 livres for the pound sterling, the product of 420 by 25, or 10500 livres, is the answer; and the same sum of French money, at the course of 24 livres 2 sous, would be equal to £435 .. 13 .. $8\frac{1}{4}$. sterling.

THE

THE

MODERN PRECEPTOR.

CHAPTER IV.

OF ALGEBRA.

ALGEBRA is an Arabic term of uncertain etymology, but generally supposed to signify the arts of restitution, comparison, resolution, and equation; meanings sufficiently denoting the nature of the art. By algebra we discover a general form of expressing the results of all questions comprehending similar circumstances, relating to magnitude, quantity, or number; or, in other words, by algebra we perform the several operations of addition, subtraction, multiplication, and division, employing certain characters or symbols of no real intrinsic value in themselves, but qualified to represent magnitudes, quantities, and numbers of every description. For example, let us suppose any number, as 3, to be represented by the symbol or character a; 5 to be represented by b; and their sum 8 by the symbol c; then, in algebraic language, a and b added together will be equal to c, or thus, $a+b=c$, that is, in this example $3+5=8$. But the values of the arithmetical symbols 3, 5, and 8, having by long and unvaried usage become determinate, they are not susceptable of any change; whereas the values attributable to the symbols a, b, and c, may be varied indifinitely, and operations by them still give correct results; thus a may represent 12, b 15, and c 27, then $a+b=c$, for $12+15=27$.

Although

Although any letter of the alphabet may be employed to represent quantities in algebraic operations, yet it has been found convenient to use the first letters as a, b, c, d, e, &c. for quantities whose values are known or given, and the last letters, as v, x, y, z, for quantities neither given nor known; hence, as in the former example, where the values of a and b are given, and the value of their sum is required, we would say $a+b=x$.

Algebraic quantities are connected by means of certain signs, as($+$) *Plus*, denoting that the quantities before and after the sign are to be added together, as $3+5$ equal to 8.

The sign($-$) *Minus*, denotes that one of the quantities is to be subtracted from the other, as $8-5$ equal to 3.

The sign($\times$) denotes that the quantities between which it stands are to be multiplied together, as 3×5 equal to 15; or $a\times b=z$. The product is also expressed by writing the symbols close together, as the letters in a word; thus $a\times b=ab$, and $a\times b\times c=abc$.

Division is expressed by writing the dividend above a small line, and the divisor below it, as $\frac{b}{a}$ will signify that b is to be divided by a.

When quantities to be multiplied or divided are compound, a line, called a *vinculum*, is drawn over them thus; $\overline{a\times b-c}$, signifying that a is to be multiplied into the difference between b and c, and $a\times\overline{b+m+c}-\overline{d+e}$, will signify that a is to be multiplied by the difference between the sum of b, m, and c, and the sum of d and e; but, instead of this vinculum, or tye, over the compound quantities, many algebraists inclose these quantities as within a parenthesis; thus $a\times(\overline{b+m+c})-(\overline{d+e})$.

When an arithmetical figure stands before an algebraic symbol, it is called the numeral coefficient, and shows how often the algebraic quantity is to be repeated; thus $3\,a$ will signify three times the value of a.

Equality

Equality is represented by the sign (=), as $a+b=c$, or the sum of a and b is equal to c.

Quantities are said to be like when they consist of the same characters; thus, 3 am and 5 am are like quantities, but 3 am and 5 amm would be unlike quantities.

Quantities having the same signs, whether + or −, are said to have like signs; but one having +, and another having −, have unlike signs.

Quantities having the sign + before them, are termed *positive* quantities; and those having − before them, are *negative* quantities. It is true, that in the nature of things there can be no such thing as a negative quantity, that is, a quantity less than nothing; but the term is used in algebra to express such quantities whose value must be deducted from that of others with which they are connected; for example, the amount of a person's estate may be considered as a positive quantity, and that of his debts as a negative quantity, which being deducted from the former quantity, will show how much the person's real property is.

When no sign is prefixed to an algebraic quantity, it is always considered to be + plus, or that the quantity is positive

Addition of algebraic quantities is performed in three different ways, according to the nature of the quantities.

1st. If the quantities be like, and have like signs, the rule is, to add together the co-efficients, (reckoning every character without a co-efficient for one,) annexing the common letter or letters, and prefixing the common sign.

Examples.

	$+3m$		$-24x$
	$+m$		$-8x$
	$+9m$		$-x$
	$+2m$		$-12x$
Sum	$+15m$	Sum	$-45x$

2d. When the quantities are like, but the signs unlike, add all the positive quantities together, and all the negative quantities together, and subtracting the one sum from the other, the remainder will be the total sum required, having the sign belonging to the greatest sum.

Examples.

$+ 4m$		$+ 8m - 5$
$- 3m$		$+ m + 1$
$- 5m$		$+ 9m + 3$
$+ 9m$		$-15m + 4$
$+13m =$	Sum of +	$+18m + 8$
$- 8m =$	Sum of −	$-15m - 5$
$+ 5m =$	Total	$+ 3m + 3$

To understand these two last examples, let us suppose all the positive quantities to represent the several articles of a person's effects, and the negative quantities to represent his debts; it will then be evident, that, to know the real value of his property, we must subtract the debts from the effects, and the remainder will correspond to the value of the whole positive and negative articles taken together. Hence, in the first example of this case, we have $+4m$ and $+9m = 13m$, for the effects, and $-3m$ and $-5m = -8m$ for the debts; consequently this sum being taken away from $13m$, will leave $5m$ for the value of the property remaining.

3d. When the quantities to be added are all unlike, they are to be written down in succession, with their respective signs and co-efficients, in one line, as in the following examples.

$8a$	$ax + m$
$-5b$	$5np - 36yz$
cd	$ax + m + 5np - 36yz$
Sum $8a - 5b + cd$	

Subtraction

Subtraction of algebraic quantities is performed by changing the sign of the quantity to be subtracted, and then adding the two quantities together, agreeably to the rules of addition.

Examples.

From	$8a + 3m$	$6y - 3m + 5$
Take	$3a - m$	$3y - 5m + 8$
Rem^{r}.	$5a + 4m$	$3y + 2m - 3$

In the first example, where $\overline{3a - m}$ is to be taken from $\overline{8a + 3m}$, if we change the sign of $3a$, which is $+$ into $-$, and then add these two quantities by the second rule of addition, the result will be $+5a$; and in subtracting $-m$ from $+3m$, we change the $-$ into $+$; and then adding their two quantities together, the result is $+4m$. If the question had been proposed, to take away only $3a$ from $8a + 3m$, the remainder would evidently have been $5a + 3m$; but as the sum to be subtracted is less than $3a$ by once the value of m, the remainder must be greater than it would have been, on the first supposition, by an additional value of m; that is, it must be $5a + 4m$, as above shown.

Multiplication of algebraic quantities is performed according to the following rules.

When the quantities to be multiplied have like signs, the sign of the product will be $+$; and when they have unlike signs, it will be $-$.

When the quantities given are simple, find the sign of the product by the above rule; to which annex the product of the co-efficients, if any, and then all the letters, which will give the product required; thus, in the first example following, where $8am$ is to be multiplied by $3x$, we mul-

tiply the co-efficients 8 and 3, giving 24 for the co-efficient of the product, and write down the letters of the multiplicand and multiplicator, prefixing the sign +, as the factors have like signs.

$$\begin{array}{lll} +\ 8am & +\ 6a & -\ 9mn \\ +\ 3x & -\ 5b & -\ 3xz \\ \hline +\ 24amx & -\ 30ab & +\ 27mnxz \end{array}$$

Again, when the factors are compound quantities, each term of the multiplicand is to be multiplied by each term of the multiplicator; and the sum of these several products, collected according to the rules for addition, will give the product required.

Examples.

$$\begin{array}{ll} \text{Multiply } 5a + m - 2c & \\ \text{by } \quad 2a & \\ \hline \text{Prod. } 10aa + 2am - 4ac & \end{array}$$

$$\begin{array}{r} x + z \\ x + z \\ \hline xx + xz \\ xz + zz \\ \hline xx + 2xz + zz \end{array}$$

$$\begin{array}{l} \text{Mul. } a + m - x \\ \text{by } \quad n - c + z \\ \hline an + mn - nx \\ \quad - ac - cm + cx \\ \qquad az + mz - xz \\ \hline \end{array}$$

Prod. $an + mn - nx - ac - cm + cx + az + mz - xz$

In the first of these examples, $5a$ multiplied by $2a$, give $10aa$; and both having the like sign +, the product has also that sign; in the same way m multiplied by $2a$, gives $2am$,

$2am$, also a positive quantity; but the third term, $-2c$, multiplied by $+2a$, gives $4ac$, with the sign $-$, because the signs of the factors are unlike.

In the second example, where the multiplicator consists of two terms, x multiplied by x, gives xx; then z multiplied by x, gives xz; again working with z, the second term of the multiplicator, we have zx, or more properly xz, (in order to place the letters as they stand in the alphabet, for the value is the same,) which is written under the second term, and z by z equal to zz: then summing up these two lines of product, beginning either at the right or the left hand, we have $xx + 2xz + zz$ for the product required.

In the third example, the characters of the multiplicand and multiplicator being all different, none of the products obtained by working with the several terms of the multiplicator can be combined together; consequently the three lines of product must be written down successively in one line, as was directed to be done in performing addition.

These operations may be illustrated by using arithmetical numbers; take, for instance, the second, where $x + z$ is to be multiplied into itself or by $x + z$. Let x stand for 4, and z for 6; then the operation would appear in the following shape:

$$\begin{array}{lll} 4 + 6 & & 4 \\ 4 + 6 & & 6 \\ \hline 16 + 24 & & 10 \\ \quad + 24 + 36 & & 10 \\ \hline 16 + 48 + 36 & = 100 = & 100 \end{array}$$

Here the component parts of the given quantity are multiplied separately together, and the result is the sum of the product of 4 by 4, of twice 4 by 6, and of 6 by 6, making

making 100, which is equal to the product of the whole of these component parts, or 10 multiplied into itself.

$$
\begin{array}{l}
a+m-x \\
a-m+x \\
\hline
aa+am-ax \\
\quad -am-mm+mx \\
\qquad\qquad +ax+mx-xx \\
\hline
aa \quad * \quad * \quad -mm+2mx-xx.
\end{array}
$$

In this example, in summing up the separate products, we find $+am$ and $-am$, which destroy each other; that is, if the one be subtracted from the other, as must be done, since they have unlike signs, there will be no remainder; a point, therefore, or asterisk, is placed in the general product: and in the same manner the two quantities $-ax$ and $+ax$ destroy each other, on which account another asterisk is placed in the product, and the following terms are successively brought down.

To illustrate this example by common arithmetic, let a represent 8, b 6, and x 4, then the question will be thus performed.

$$
\begin{array}{lr}
 & 64 \\
8+6-4 & -36 \\
8-6+4 & \overline{} \\
\hline
 & 28 \\
64+48-32 & +48 \\
\quad -48-36+24 & \overline{} \\
 & 76 \\
\qquad +32+24-16 & -16 \\
\hline
64 \quad * \quad * -36+48-16 & = 60 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
8+6-4=10 \\
8-6+4=\ \ 6 \\
\hline
\qquad 60
\end{array}
$$

The

The products arising from the repeated multiplication of a quantity into itself, and its several products, are called its *powers*; and the quantity itself is the *root*; thus a is the root, and if multiplied into itself, the product, or second power, will be aa; which again multiplied by a, will give the third power, or aaa; the 4th power will be $aaaa$, and so on; but as this manner of expression would soon become inconvenient, and liable to error, it has been the practice to express the different powers by a small figure placed over and towards the right hand of the root; thus the 2d power, aa, may be expressed by a^2, the 3d power aaa by a^3, the 4th power by a^4, &c. This small figure is called the *exponent*, or *index*; and, by adding these exponents, the same expression is obtained as if the root had been repeatedly multiplied; thus $a + a^2 = a^3$, and $a^2 + a^4 = a^6$, &c.

Division of algebraic quantities is performed agreeably to the following rules.

When the signs of the divisor and the dividend are like, the sign of the quotient is $+$; but if they be unlike, the sign is $-$.

1st. When the divisor is simple, and a part, of or found in each term of the dividend, you must divide the co-efficient of each term of the dividend by the co-efficient of the divisor, and expunge, or withdraw from each term, the letter or letters of the divisor, and the result will be the the quotient. Thus, if it be required to divide $18mx$ by $3m$, dividing the co-efficient 18 by 3, we have 6 for the co-efficient of the quotient; and mx being a product of which m is one of the factors, this symbol being taken away or expunged, the remaining symbol x will belong to the quotient, which will then be $6x$.

Again divide $5\,a^3m + 25\,abm - 5\,am^2$ by $5am$, and the quotient will be $a^2 + 5b - m$, thus:

5*am*

$$5\,am)\,5\,a^3\,m + 25\,ab\,m - 5\,am^2\,(a^2 + 5\,b - m$$
$$5\,a^3\,m$$
$$\overline{\quad\quad\cdot}$$
$$+ 25\,abm$$
$$25\,abm$$
$$\overline{\quad\quad\cdot}$$
$$-5\,am^2$$
$$-5\,am^2$$
$$\overline{\quad\quad\cdot}$$

In beginning this operation, the divisor $5am$ is to be taken out of $5a^3m$; if we take the root a from the 3d power of a, we have the 2d power of a, which therefore goes into the quotient; and as 5 is contained once in 5, and m once in m, neither this co-efficient 1, nor the quotient 1, are required to appear; they are, therefore, suppressed or expunged, and the divisor, $5\,am$, being multiplied by the quotient a^2, the product written under the dividend, and subtracted from it, gives no remainder. The next term of the dividend, $+25\,abm$, is brought down, and the 5 of the divisor being contained 5 times in the 25 of this dividend, 5 is placed as a co-efficient in the quotient; then am is contained b times in abm, and $5\,b$ becomes the quotient corresponding to this step of the division; which quotient multiplied into the divisor, gives $25\,abm$ to be subtracted from the dividend. Lastly, comes down the term $-5\,am^2$, out of which, if we take the divisor $+5\,am$, the quotient will be $-m^2$; so that the whole quotient in this operation will be $a^2 + 5\,b - m$.

Or, the same operation may be shortened by expunging the characters of the divisor from the dividend as below.

$$5\,am)\,5\,a^3\,m + 25\,a\,b\,m - 5\,am^2($$
$$\overline{\quad a^2 \quad + 5\,b \quad - m\quad}$$

2d.

2d. When the divisor is simple, but not a factor or portion of any term of the dividend, the quotient must be expressed as a fraction of which the numerator is the dividend, and the denominator the divisor: thus 36 am divided by $5xz$, can only be expressed fractionally in this way $\frac{36am}{5xz}$.

3d. When the divisor is compound, or consists of more than one term, the first term of the dividend is to be divided by the first term of the divisor, and the quotient being put down with the proper sign, is multiplied into the whole of the divisor, and the product subtracted from the dividend; if there be a remainder, this becomes a new dividend, the first term of which is again divided by the first term of the divisor, and the quotient annexed with the proper sign to that already put down; then the whole divisor is multiplied by this last part of the quotient, and the product subtracted from the last dividend; and so on as long as any thing remains, or at least until it appear that there will always be a remainder.

For example, divide $xx + 2xz + zz$, otherwise $x^2 + 2xz + z^2$ by $x + z$.

$$
\begin{array}{rl}
x + z) & x^2 + 2xz + z^2 (x + z \text{ quotient.} \\
& x^2 + xz \\
\hline
& \cdot + xz + z^2 \\
& xz + z^2 \\
\hline
& \cdot \quad \cdot
\end{array}
$$

The question being written down, we first enquire how often x, the first term of the divisor, is contained in x^2, the first term of the dividend; and as x^2 is the second power of x, that is, it is the product of x multiplied by itself, it follows, that if we divide this product by x, the quotient must also be x; x is therefore placed in the quotient, and

by

by it multiplying the whole of the divisor, the product is $x^2 + xz$, which written under and subtracted from the first and second terms of the dividend, we have for a remainder xz; to which bringing down the third term of the dividend, the new dividend becomes $xz + z^2$; then enquiring how often we can have x of the divisor in xz of this new dividend, the quotient will be z, for x being one of the factors of the compound quantity xz, it is evident that z must be the other, which being written in the quotient with the sign $+$, because the signs of the divisor and dividend are like, we are next to multiply the whole of the divisor by z, producing $xz + z^2$, to be subtracted from the last dividend, when nothing will remain, and the division is finished.

Divide $12a^2 + 2am + 4ax - 4m^2 + 5mx - x^2$ by $6a + 4m - x$.

$$
\begin{array}{rrrrr}
)12a^2 + 2am + 4ax - 4m^2 + 5mx - x^2(2a - m + x \\
12a^2 + 8am - 2ax \\
\hline
\cdot\quad -6am + 6ax - 4m^2 \\
-6am \qquad\qquad -4m^2 + mx \\
\hline
\cdot\quad +6ax \quad\cdot\quad +4mx - x^2 \\
+6ax \qquad +4mx - x^2 \\
\hline
\cdot \qquad\qquad \cdot \quad \cdot
\end{array}
$$

Division in algebra may be proved, like that in common arithmetic, by multiplying the quotient into the divisor, when the product, with the remainder, if there be any, will be equal to the dividend: for instance in the preceding example.

Multiply

$$\begin{array}{lr}
\text{Multiply the quotient} & 2a - m + x \\
\text{By the divisor} & 6a + 4m - x
\end{array}$$

$$\begin{array}{r}
-2ax + mx - x^2 \\
8am - 4m^2 + 4mx \\
12a^2 - 6am + 6ax \\
\hline
12a^2 + 2am - 4m^2 + 4ax + 5mx - x^2
\end{array}$$

In the preceding example of division, we begin by asking how often $6a$, the first term of the divisor, is contained in $12a^2$, the first term of the dividend; and finding, that 6 is contained 2 times in 12, and a is contained a times in a^2, we place $2a$ in the quotient, by which multiplying all the terms of the divisor, we place the products under the dividend, and subtracting, we obtain the remainder $-6am+6ax$; to which bringing down the next term of the dividend, $4m^2$, we enquire how often $6a$ of the divisor is contained in $-6am$; am being the product of a multiplied by m, we place m in the quotient without any co-efficient, because 6 being contained only 1 time in 6, the character m itself is sufficient, without any number before it; and, agreeably to the rules already given, the signs of the divisor and dividend being unlike, the sign prefixed to m must be $-$; then multiplying $-m$ into all the terms of the divisor, we have $-6am-4m^2+mx$, which being written under their corresponding quantities in the dividend, although they come beyond the term taken down, and subtracted from that dividend, the remainder with the term x^2 brought down, is $+6ax+4mx-x^2$; lastly, asking how often $6a$ can be taken out of $6ax$, the first term of the new dividend, we write x times in the quotient, with $+$ prefixed because the signs are like; and multiplying the whole

divisor

divisor by x, the product comes to be equal to the new dividend, and nothing remains; the quotient being $2a - m + x$.

Divide $a^2 - m^2 + 2mx - x^2$ by $a + m - x$.

$$\begin{array}{llllll}
a + m - x) & a^2 - m^2 + 2mx - x^2 & (a - m + x & & & \\
& a^2 + am - ax & & & & \\
\hline
& \cdot\ - am + ax - m^2 + 2mx - x^2 & & & & \\
& - am \qquad\quad - m^2 + mx & & & & \\
\hline
& \cdot \qquad ax \ \cdot\ + mx - x^2 & & & & \\
& \qquad\quad ax \quad\ + mx - x^2 & & & & \\
\hline
& \qquad\quad \cdot \qquad \cdot \quad \cdot & & & &
\end{array}$$

In this example, the quotient of a^2, divided by a, being a, the whole divisor is multiplied by a, producing the quantity $a^2 + am - ax$, which is to be subtracted from the dividend given in stating the question: but the terms $am - ax$ having no corresponding quantities from which to be subtracted, they are written in the remainder with their signs reversed, as is necessary in subtraction; then are brought down, the two terms $- m$ $+ 2mx$ of the first dividend, not having yet been used, and lastly comes down the next term of the dividend x^2. Now we are to see how often the first term of the divisor a is contained in the first term of the new dividend $-am$; and the number of times m being placed in the quotient, with the prefixed sign $-$ on account of the unlike signs of $+a$ and $-am$, the whole divisor is multiplied by $-m$, and the product subtracted from the last dividend; and those terms not used are brought down to the remainder, making altogether a new dividend, of which the first term is ax, to be divided by a of the divisor; and the number of the times x being placed in the quotient, the whole divisor is

is multiplied by x, and the product being equal to the dividend, the division is finished without any remainder.

The operation may be proved, as before, by multiplying the quotient by the divisor, when the product will be equal to the dividend given in the question.

It sometimes happens that the division will never come to a termination, without a remainder; in which case the quotient may be considered as infinite, and the rate of its progression may often be easily known; or the quotient may be brought to a conclusion in the shape of a fraction, of which the remainder is the numerator, and the divisor is the denominator.

Example, divide 1 by $1 - a$.

$$
\begin{array}{lllllll}
1-a)\ 1 \text{———} (1 + a + a^2 + a^3 + a^4 + \frac{a^5}{1-a}. \\
\quad 1 - a \\
\hline
\qquad + a \\
\qquad\quad a - a^2 \\
\hline
\qquad\qquad + a^2 \\
\qquad\qquad\quad a^2 - a^3 \\
\hline
\qquad\qquad\qquad + a^3 \\
\qquad\qquad\qquad\quad a^3 - a^4 \\
\hline
\qquad\qquad\qquad\qquad + a^4 \\
\qquad\qquad\qquad\qquad\quad a^4 - a^5 \\
\hline
\qquad\qquad\qquad\qquad\qquad + a^5
\end{array}
$$

Here the 1 of the divisor being contained once in the 1 of the dividend, we place 1 in the quotient, and multiplying the whole divisor by 1, we obtain $1 - a$, which subtracted

tracted from the given dividend, gives for a remainder $+ a$ for a new dividend, and the 1 of the divisor being contained a times in the dividend a, we write a with the proper sign prefixed in the quotient, and then multiplying by it the whole divisor, the product is $a - a^2$, which being subtracted from a, leaves $+ a^2$; this being again divided by 1 of the divisor, the quotient is $+ a^2$, by which multiplying the whole divisor, we have $a^2 - a^3$, to be subtracted as before. In this manner the division may be carried on indefinitely, without ever coming to a termination: but from the rate of progression it is evident, that the quotient will continually advance nearer and nearer to the truth, by an additional power of a; the division, therefore, when this rate of progression is ascertained, may be intermitted, and the last remainder written, as a fraction, at the end of the quotient, as in the example here given.

With respect to the method pointed out for division of algebraic quantities, it may be observed, that, in the course of the operation, every term of the divisor being successively multiplied by every term of the quotient, and the several products subtracted from the given dividend, until nothing remain, or until the progression of the quotient be ascertained, the quotient thus obtained must be correct, as may be proved by multiplying it by the divisor, when the product, together with the remainder, if there be any, will be equal to the given dividend.

In algebraic calculations two operations frequently occur, viz. *Involution* and *Evolution*. Involution means the way to discover any power of any given quantity, whether simple or compound, and is performed by multiplication.

1st. *Involution* of a simple quantity is performed by multiplying the exponents of the letters by the index of the power

power required, and raising the co-efficient to the same power.

Example, raise $2\,a^2 m^3$ to the cube, or 3d power: multiply 2, the exponent of a, by 3, denoting the power, making a^6; and 3, the exponent of m^3, making m^9; and cubing the co-efficient 2, thus making it 8, the 3d power of the given quantity $2\,a^2 m^3$, becomes $8\,a^6 m^9$. If the same be performed by multiplication, as here shown, the result will be the same.

$$\begin{array}{l} 2\,a^2 m^3 \\ 2\,a^2 m^3 \\ \hline 4\,a^4 m^6 = \text{2d power or square} \\ 2\,a^2 m^3 \\ \hline 8\,a^6 m^9 = \text{3d power or cube} \\ \hline \end{array}$$

Required the 6th power of $-3\,m^2 x^3$?

Raising the co-efficient 3 to the 6th power, we have a new co-efficient, 729; multiplying the exponent of m^2 by 6, expressing the power required, we have m^{12}; and multiplying the exponent of x^3 also by 6, we have x^{18}; consequently the 6th power of $-3\,m^2 x^3$ will be $729\,m^{12} x^{18}$; as the student may discover by multiplying the quantity given 6 times into itself.

2nd. When the given quantity is composed of two or more terms, the required power must be found by successive multiplications of the quantity into itself; thus, for instance, to find the 4th power of $a+m$.

$$\begin{array}{ll} a + m = \text{1st power or root} & \\ a + m & \\ \hline a^2 + am & \\ \quad\quad am + m^2 & \\ \hline \end{array}$$

a^2

$a^2 + 2am + m^2 = $ 2d power or square
$a + m$

$a^3 + 2a^2m — am^2$
$\quad a^2m + 2am^2 + m^3$

$a^3 + 3a^2m + 3am^2 + m^3 =$ 3d power or cube
$a + m$

$a^4 + 3a^3m + 3a^2m^2 + am^3$
$\quad a^3m + 3a^2m^2 + 3am3 + m^4$

$a^4 + 4a^3m + 6a^2m^2 + 4am^3 + m^4 =$ 4th power required.

Evolution is the method of discovering the root of any given quantity, simple or compound; its operations are, therefore, the reverse of involution.

In algebra, to denote the root of any quantity, the radical sign ($\sqrt{}$) is used, with a figure over the quantity, to denominate the root required: thus $\sqrt[2]{x}$ is the square root of the quantity expressed by x, $\sqrt[3]{m}$ is the cube root of m, $\sqrt[5]{am}$ is the 5th root of the quantity am. The figure is called the exponent or index of the root; and when the square or 2nd root is meant, this index is frequently omitted, so that $\sqrt[2]{x}$ and $\sqrt{x}$, represent the same square root.

1st. When the quantity of which the root is required is simple, divide the exponents of the letters by the index of the root, and prefix the root of the co-efficient to the letters, and this new found quantity will be the root required.

Example, required the square root of 64 a^2m^2. Here the square root of the co-efficient 64 being 8, and the exponents 2 being divided by the index of the square or 2nd root, which is also 2, the result is 8am: and if this root be squared the product will be 64 a^2m^2.

It

It is to be observed, that the root of any positive quantity may be either positive + or negative −, if the index of the root be an even number; for if $+x$ and $-x$ be both squared, the product will still be the same, or $+x^2$; and the root of $+x^2$ would be expressed thus: $\pm x$; but if the index be an odd number, the root will always be positive: the root of a negative quantity is always negative when the index of that root is an odd number; but if the quantity be negative, and the index an even number, no root can be assigned: thus no square root can be assigned for $-x^2$, for both $+x$ and $-x$ if squared would give $+x^2$.

2dly. When the quantity, the root of which is to be extracted, is composed of more than one term, if it be a square number, proceed as in the following example, where the square root of $a^2 + 2ac + c^2$ is required.

$$
\begin{array}{rl}
 & a^2 + 2ac + c^2\,(a + c \text{ the root,} \\
 & a^2 \\
 & \overline{} \\
2a + c) & \quad 2ac + c^2 \\
 & \quad 2ac + c^2 \\
 & \quad \overline{\quad \cdot \quad \cdot \quad}
\end{array}
$$

Find the square root of the first term a^2, which is a; place this root in the quotient, and its square under the dividend: as there is no remainder, bring down the next period $2ac + c^2$, and doubling the quotient, we have for a new divisor $2a$; then enquiring how often $2a$ can be had in $2ac$, the number of times c is placed in the quotient with the sign + prefixed, both divisor and dividend having the same sign, and also in the divisor, the whole of which multiplied by c gives $2ac + c^2$, equal to the last dividend: consequently the square root of $a^2 + 2ac + c^2$ is $a + c$.

The same operation may be performed with arithmetical

 figures

figures as follows, supposing the quantity represented by a to be 8, and that represented by c to be 5.

$$
\begin{array}{l}
8+5 \\
8+5 \\
\hline
64+40 \\
\quad 40+25 \\
\hline
64+80+25\,(8+5 \text{ root} \\
64 \\
\hline
\end{array}
$$

$$
16+5)\quad 80+25
$$

$$
\quad 80+25
$$

Extraction of the cube root is performed as in the following example, where it is required to find the cube root of $a^3 + 3\,a^2\,c + 3\,a\,c^2 + c^3$.

$$
\begin{array}{l}
a^3 + 3\,a^2\,c + 3ac^2 + c^3\ (a + c \text{ cube root.} \\
a^3 \\
\hline
\end{array}
$$

$$
3\,a^2 + 3\,a\,c + c^2)\quad 3\,a^2\,c + 3\,a\,c^2 + c^3
$$

$$
\quad 3\,a^2\,c + 3\,a\,c^2 + c^3
$$

Find first the cube root of a^3, which is a; this being cubed, the product is placed under a^3 of the given cubic quantity, and being subtracted from the whole quantity, leaves the remainder $3\,a^2\,c + 3\,a\,c^2 + c^3$; then taking three times the square of the quotient or root a, we have for the first term of the divisor $3\,a^2$, by which dividing the first term of the dividend $3\,a\,c$, we have for a second term in the root $+\,c$, by which dividing the other terms of the given quantity $3\,a\,c^2 + c^3$, we have $3\,a\,c + c^2$ for the other terms of the divisor;

sor; and multiplying all these last terms by the c of the root, we obtain $3a^2c + 3ac^2 + c^3$, a product equal to the dividend; consequently the extraction is completed, and the required root is $a + c$; which, as before, may be proved by raising the root $a + c$ to the 3d power, which will be found to be the same with the quantity given in the question.

Another method is, when the second term of the root is found to add together three times the square of the first term, three times the product of that part multiplied by the second term, and the square of this second; the sum will be the divisor for the second step, which multiplied by the second term of the root, will give the product desired.

This will be illustrated by using arithmetical instead of algebraic characters; supposing a to be 8 and c to be 5, the quantity $8 + 5$ will be cubed, and the cube root again extracted, in the algebraic form, as follows.

$$\begin{array}{rl}
 & 8 + 5 \\
 & 8 + 5 \\
\hline
 & 64 + 40 \\
 & \quad\;\; 40 + 25 \\
\hline
 & 64 + 80 + 25 \\
 & 8 + 5 \\
\hline
 & 512 + 640 + 200 \\
 & \qquad\quad 320 + 400 + 125 \\
\hline
 & 512 + 960 + 600 + 125(8 + 5 \\
 & 512 \\
\hline
192 + 120 + 25) & 960 + 600 + 125 \\
 & 960 + 600 + 125 \\
\hline
 & \cdot \quad \cdot \quad \cdot
\end{array}$$

Here

Here the cube root of the first term, 512, being 8, this number becomes the first term of the root, and its cube, 512, subtracted from the given cube, leaves no remainder; then the other terms of the given cube are brought down, and a divisor is formed by taking three times the square of the first term of the root 8, or 192, by which dividing 960, the first term of the new dividend, the quotient 5 becomes the second term of the root; then taking three times the product of these two terms, which is 120, and the square of the second term, 25, we have the whole of the new divisor, which being multiplied by the second term, 5, the product is 960 + 600 + 125, equal to the dividend; consequently the extraction is finished and 8 + 5 is the cube root required.

Or thus, the sum of the two terms 8 and 5 being 13, the cube of 13 is 2197, which is equal to the sum of 512, the cube of the first term 8, together with 960 and 600, the intermediate terms of the cube, and 125, the cube of the last term 5, as is here shown.

$$\begin{array}{rr}
8 + 5 = 13 & \\
13 & \\
\hline
39 & \\
13 & \\
\hline
169 & \\
13 & 512 \\
\hline
507 & 960 \\
169 & 600 \\
 & 125 \\
\hline
2197 = & 2197
\end{array}$$

In

In calculations where algebra is employed, it is generally the object, by means of certain known quantities, to discover others that are unknown, but which are stated to stand in given proportions or relations to those known quantities; and this is done by discovering what portion of such known quantities is equal to those required; this portion is termed the *value* of the unknown quantity, and the statement of such a value is termed an *equation*. Thus, in the equation $a + m - n = x$, if the values of a, m, and n be known, their sum, or x, must also be determined. An equation is said to be *resolved*, when the known quantities are all placed on the one side of the mark of equality $=$, and the unknown quantities on the other side; and the value of the unknown quantities is called the *root* of the equation.

When an equation expresses the value of the first power or an unknown quantity, it is called a *simple* equation; when it contains the second power, or square of the unknown quantity, it is called a *quadratic* equation; when it exhibits the cube or third power, it is a *cubic* equation; the 4th power is a *biquadratic* equation; and so on with respect to all higher powers.

1st. When the unknown quantity is connected with a known quantity, and their conjunct value is known, the unknown quantity may be transported to the one side of the equation, and the known quantities to the other, by what is called *transposition*, as in the following example, where $x - 5$ is given equal to 20; or thus $x - 5 = 20$. Here we see, that if we subtract 5 from the value of x, the remainder will be 20, consequently $20 + 5 = 25$ will be the value of x, as here shown.

$$x - 5 = 20$$

$$x = 20 + 5$$

$$x = 25$$

So

So that any quantity may be removed from the one side of the equation to the other, that is, transposed, merely by changing the prefixed sign from + to −, or from − to +.

Again, let $6x + 4 = 9x - 20$, required the value of x.

$$6x + 4 = 9x - 20$$
$$6x = 9x - 20 - 4$$
$$6x = 9x - 24$$
$$6x - 9x = 3x = 24$$
$$x = 8$$

Here as $6x$ wants 4 to be equal to $9x - 20$, the difference 4 may be removed to the opposite side of the equation, changing its sign to −, thus giving a new equation $6x = 9x - 20 - 4$, that is $= 9x - 24$, which sum, 24, is therefore the difference between $6x$ and $9x$, that is, $3x$; and if $3x$ be equal to 24, x alone must be equal to the third part of 24, or equal to 8, which is the value required, as may be proved by the following statement.

$$6x + 4 = 9x = 20$$
$$6 \times 8 = 48 + 4 = 52$$
$$9 \times 8 = 72 - 20 = 52$$

From this rule it follows, that when any quantity is found on both sides of an equation, it may be expunged without in the least affecting the solution: thus if $a + x = m - c + a$, the quantity a may be taken away from both sides of the equation, and the remainder x will be equal to the remainder $m - c$, or, arithmetically,

$$6 + 4 = 7 - 3 + 6$$
$$4 = 7 - 3$$
$$4 = 4$$

2nd. When the unknown quantity is multiplied by any number or quantity, this number or quantity may be removed by dividing by it the other terms of the equation: thus, if

$5m$ be equal to 25, m will be equal to 25 divided by 5, that is, m will be equal to 5.

$$5m = 25$$
$$m = \frac{25}{5}$$
$$m = 5\,.$$

Again, let mx be equal to $a + b$, required the value of m.

$$mx = a + b$$
$$m = \frac{a + b}{x}$$
$$m = \frac{a}{x} + \frac{b}{x}$$

Or thus, arithmetically, let $m = 5$, $x = 6$, $a = 18$, and $b = 12$, then m or x $5 \times 6 = 30 = 18 + 12 = 30$, and $m = \frac{a + b}{x}$ or $\frac{18 + 12 = 30}{6} = 5 = \frac{18}{6} = 3 + \frac{12}{6} = 2 = 5.$

3rd. When any term of an equation is a fraction, it may be removed by multiplying all the other terms by the denominator of the fractional part: for instance, if $\frac{a}{3} + 6 = 24$, multiplying the other terms by the denominator 3, we have a new equation, $a + 18 = 72$, and consequently $a = 72 - 18 = 54$, the third part of which $= 18$, added to 6, will be equal to 24, agreeably to the statement of the example.

4th. When there are several fractions in an equation, all the denominators may be removed, provided some number can be found which is a multiple of each denominator, by which to multiply all the numerators of the terms of theequation; thus, if $\frac{x}{2} + \frac{x}{4} + \frac{x}{6}$, be equal to 22, we may employ

12,

12, which is a multiple of the three denominators 2, 4, and 6, when the equation will stand as below.

$$\frac{x}{2} + \frac{x}{4} + \frac{x}{6} = 22$$

$$\frac{12x}{2} + \frac{12x}{4} + \frac{12x}{6} = 264$$

$$6x + 3x + 2x = 264$$

$$11x = 264$$

$$x = \frac{264}{11}$$

$$x = 24$$

Multiplying all the terms by the assumed number, 12, which is a multiple of the three denominators, we have $\frac{12x}{2} + \frac{12x}{4} + \frac{12x}{6} = 264$, and reducing these fractions to integers, dividing each numerator, $12x$, by its proper denominator 2, 4, or 6, we obtain $6x + 3x + 2x$, that is, $11x = 264$; consequently x must be equal to the 11th part of 264, that is, x will be equal to 24. This may be proved by taking the fractions of 24, which are given in the example, $\frac{24}{2} = 12$, $\frac{24}{4} = 6$, $\frac{24}{6} = 4$, and $12 + 6 + 4 = 22$, the value given in the equation.

5th. When the side of the equation, containing the unknown quantity, is any complete power, the equation may be reduced to a lower power, by extracting the root corresponding to it, from both sides: thus, if $z^2 = 9m^2$, extracting the square root of both sides of the equation, we have $z = 3m$; or, in this example, $x^3 = a + b$, by extracting the cube root, we have $x = \sqrt[3]{a + b}$.

Having thus briefly pointed out the way to resolve an equation containing one unknown quantity, it is proper in the same manner to show how to resolve one involving more than one unknown quantity.

1st.

1st. Required the values of x and z from the two following equations, $15x - 5z = 45$, and $8x + 2z = 52$.

By the first equation $15x - 5z = 45$

$$15x = 45 + 5z$$

$$x = \frac{45 + 5z}{15}$$

By the second equation, $8x + 2z = 52$

$$8x = 52 - 2z$$

$$x = \frac{52 - 2z}{8}$$

Having thus obtained two values of x, we place them as an equation, thus;

$$\frac{45 + 5z}{15} = \frac{52 - 2z}{8};$$

and multiplying each numerator by the denominator of the opposite side, that is, multiplying $45 + 5z$ by 8 and $52 - 2z$ by 15, we have this equation:

$$360 + 40z = 780 - 30z$$

$$40z + 30z = 780 - 360$$

$$70z = 420$$

$$z = \frac{420}{70}$$

$$z = 6$$

Then, from either of the given equations, we can find the value of x, the other unknown quantity; as, for example,

$$8x + 2z = 52$$

$$8x = 52 - 2z$$

$$x = \frac{52 - 2z}{8}$$

But the value of z being already found to be 6, twice $6 = 12$, deducted from 52, will leave 40, which divided by 8, will give 5 for the value of x.

Had

Had the other equation, $15\,x - 5\,z = 45$, been employed, a similar result would have been obtained, as here shown :

$$\begin{aligned} 15\,x + 5\,z &= 45 \\ 15\,x &= 45 + 5\,z \\ x &= \frac{45 + 5\,z}{15} \end{aligned}$$

But the value of z being found to be 6, 5 times $6 = 30$, added to 45, will give 75, which divided by 15, will quote 5, as before, for the value of x.

The truth of these processes may be ascertained by working with the values now discovered, according to the terms of the question; thus,

$$\begin{array}{rcr} 15\,x - 5\,z = 45 & \qquad & 8\,x + 2\,z = 52 \\ 15 \times 5 = 75 & & 8 \times 5 = 40 \\ -5 \times 6 = 30 & & +\,2 \times 6 = 12 \\ \overline{45} & & \overline{52} \end{array}$$

Or, the same results may be obtained in the following manner; where having found a value for one of the unknown quantities, as x, this value is substituted for x itself, and the value of z thus discovered. The value of x was found by the first operation to be $\frac{45 + 5\,z}{15}$; let this value, therefore, be substituted for x itself, in the second statement, which would then, instead of $8\,x + 2\,z = 52$, be as below:

$$8 \times \frac{45 + 5\,z}{15} + 2\,z = 52$$

$$\begin{aligned} \text{that is, } 360 + 40\,z + 30\,z &= 780 \\ 40\,z + 30\,z &= 780 - 360 \\ 70\,z &= 420 \\ z &= 6 \end{aligned}$$

Again,

Again, another method of resolving the same question, is the following. The values of x and z were given in the two equations $15x - 5z = 45$, and $8x + 2z = 52$: to find the value of x, multiply the first value by 8, the exponent of the second, and the second by 15, the exponent of the first; thus giving these equations:

$$120x - 40z = 360$$
$$120x + 30z = 780$$

Where the quantity involving the value of x being the same in both equations, the quantity $40z + 30z = 70z$ must be equal to the difference between 360 and $780 = 420$, which, divided by 70, will give 6 for the value of z.

The value of x may be discovered in the same way, as follows;

$$15x - 5z = 45$$
$$8x + 2z = 52.$$

Where, multiplying by the exponents of the quantity involving the value of z, we have,

$$40x + 10z = 260$$
$$30x - 10z = 90$$
$$70x = 350$$
$$x = 5.$$

The foregoing rules, with their illustrations, will, it is hoped, be sufficient to give the young student of algebra an idea of the nature and uses of that method of calculation: but as some farther applications of this most useful branch of the science of numbers may be desirable, a few examples of questions, resolvable by algebra, are here annexed.

Example 1st. A person being asked what money he had in his pocket, answered, if to what I have, you will add its half, its third, and its fourth parts, the whole will be just 50 guineas: required the sum in his pocket?

Suppose x to represent the sum of money in the person's pocket, then the question will stand thus:

2 x 2 x +

$$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 50 \text{ guineas,}$$

and finding a number which will be a multiple of all the denominators, which, in this case, may be 12, and by it multiplying all the terms, we have the following equation:

$$12x + \frac{12x}{2} + \frac{12x}{3} + \frac{12x}{4} = 600$$

$$12x + 6x + 4x + 3x = 600$$

$$25x = 600$$

$$x = 24$$

The value of the unknown quantity, x, is thus found to be 24, which will answer the terms of the example; for 24 added to its half, or 12, its third, or 8, and its fourth, or 6, will give just 50.

Example 2nd. Two travellers, A and B, set out at the same time; A from London, and B from Edinburgh; A travels 25 miles every day, and B travels 20 miles every day; in what time will they meet, and at what distance from London and Edinburgh, the distance between the towns being 396 miles?

Let x represent the number of days each travels before they meet; then A's journey will be $25x$, and B's will be $20x$; and as the sum of the two journies must be equal to the distance between the towns, we have

$$25x + 20x = 396$$

$$45x = 396$$

$$x = \frac{396}{45}$$

$$x = 8, 8 \text{ days.}$$

Thus we find the number of days each traveller is on the road to be 8, 8; and if by 8, 8 we multiply 25, the miles A travels in one day, his journey will appear to have been 220 miles before he met B; who travelling at the rate of 20 miles, must in 8, 8 days have gone 176 miles; and these two journies, added together, give 396, the number of miles between London and Edinburgh.

Example

Example 3rd. The sum of two quantities or numbers being given, as also the difference between them, to find the quantities or numbers themselves. Let the sum of two quantities be 20, and their difference be 4; required the quantities.

Call the greater x, and the less z, then we shall have

$$x + z = 20$$
$$\text{and } x - z = 4$$
$$x + z + x - z = 20 + 4$$
$$2x = 24$$
$$x = 12 = 10 + 2$$
$$z = 20 - 12$$
$$z = 8 = 10 - 2$$

Here, if we add both sides of the equations together, we have $x + z + x - z = 20 + 4$

$$2x = 20 + 4$$
$$x = 10 + 2$$
$$x = 12$$

But 10 is half the sum of the given quantities and 2 is half their difference; which two numbers added together, are equal to 12, or x: again, z is equal to $20 - x$ or $12, = 8$, which is the difference between 10, half the sum of the given quantities, and $2 =$ half their difference; consequently half the sum of any two quantities $+$ half their difference $=$ the greater, and half the sum $-$ half the difference $=$ the less.

Or, generally, let the sum be represented by s, and the difference by d; then, we have

$$x + z = s$$
$$x - z = d$$
$$2x = s + d$$
$$x = \frac{s + d}{2}$$
$$z = \frac{s - d}{2}.$$

By

By adding each side of the equation together, we obtain $x + z + x - z = s + d$; that is, $2x = s + d$; and by subtracting $x - z$ from $x + z$, we obtain $2z = s - d$; consequently $x = \frac{s + d}{2}$, and $z = \frac{s - d}{2}$; that is, the greater quantity = half the sum of the two quantities + half their difference, and the less quantity = half the sum — half the difference.

$$x = 12 = \frac{20}{2} + \frac{4}{2} = 10 + 2 = 12$$

$$z = 8 = \frac{20}{2} - \frac{4}{2} = 10 - 2 = 8$$

When an equation contains the second power or square of an unknown quantity, it is called a *quadratic* equation; and these are divided into two classes, the *pure* and the *adfected.*

Pure quadratic equations are those which on the one side exhibit the square of theunknown quantity, and on the other side, only known quantities: thus, $25 = z^2$, is a pure quadratic equation: so is $8 + x^2 = 24$, or $a + x^2 = b$.

Adfected quadratic equations are those which contain in one term the square of the unknown quantity, and in another the first or simple power of the same quantity, whilst the remaining terms of the equation consist wholely of known quantities; such as $z^2 + 2z = 24$, $5z^2 = 80 - 6z$, $mx^2 + nx - x = a$.

In resolving a pure quadratic equation, when the unknown quantity is made to stand alone, on one side having 1 for its co-efficient (that is, without any co-efficient expressed), and the known quantities alone on the other side, if the square root of each side be extracted, we obtain the value of the unknown quantity.

It was before observed, that in extracting the square root of any quantity, the sign of the root may be either + or —; for

for either of these signs, if employed in multiplication, will give + for the product; consequently the root of a positive square may be either positive or negative: hence the square root of x^2 may be thus expressed $\pm x$. From this circumstance it follows, that as no positive quantity multiplied into itself can produce a negative quantity, so to find the square root of a negative quantity is impossible: and should it happen in an algebraic operation that the square root of a negative quantity were to be required, some error or contradiction must have occurred either in the statement of the question, or in the steps of the operation.

When adfected equations have been reduced to their proper values, they must appear in one or other of these three forms: viz.

1st. $z^2 + nz = m$

2nd. $z^2 - nz = m$

3rd. $z^2 - nz = -m$

These three equations being resolved in the same way, the first, or $z^2 + nz = m$, may serve as an illustration of the method of proceeding with the others.

As the given quantity $z^2 + nz$ differs in its appearance from a perfect square consisting of the same terms, we may compare it with such a square composed of $z + a = z^2 + 2az + a^2$; and by supposing the quantity $n = 2a$, or $\frac{n}{2} = a$, the quantities $z^2 + nz$ and $z^2 + 2az$ will be equal: but as to bring $z^2 + 2az$ into the form of a complete square, we must add the square of $a = a^2$; so, to complete the square $z^2 + nz$, we must add the square of $\frac{n}{2} = \frac{n^2}{4}$; when we shall have $z^2 + nz + \frac{n^2}{4} =$ the quantity given in the equation $m + \frac{n^2}{4}$; and extracting the square root of

both

both sides, we have $z + \frac{n}{2} = \pm \sqrt{-\frac{n^2}{4} + m}$, and consequently $z = \pm \sqrt{\frac{n^2}{4} + m} - \frac{n}{2}$.

By observing the various results in this process, the following rules have been formed for the resolution of affected quadratic equations: viz, 1st. when the terms involving the unknown quantity are all transposed to the one side of the equation, and the known quantities to the other, and the term involving the square of the unknown quantity is positive; then if this square be multiplied by a co-efficient, all the other terms must be divided by it, that the co-efficient of this square may be 1.

2d. To both sides of the equation add the square of half the co-efficient of the unknown quantity; when that side which involves the unknown quantity will become a perfect square.

3rd. Lastly, extract the square root of both sides of the equation, thus rendering it simple with respect to the unknown quantity, and transposing, agreeably to the rules formerly given, the unknown quantity will come to stand alone on the one side of the equation, while only known quantities stand on the other side, and so the value of the unknown quantity will be discovered.

It is to be observed, that the square root of the first side of the equation will always be equal to the sum or the difference of the unknown quantity, and half the co-efficient of the second terms; namely, that if the sign of this term be $+$, the root will be equal to the sum; but if the sign be $-$, the root will be equal to the difference.

Example 1. Required the value of z from the quadratic equation $z^2 + 2z = 63$.

The

The co-efficient of the second term being 2, we take its half = 1, and square it, which still gives 1 : this quantity being added to both sides of the equation, we have

$$z^2 + 2z + 1 = 63 + 1 = 64 ;$$

and extracting the square root of both sides, we obtain

$$z + 1 = \pm 8,$$

consequently $z = + 7$, or $= z - 9$,

either of which values of z will answer the conditions of the example; for taking $+ 7$, its square is 49, which added to twice 7, give 63; and on the other hand taking the value $- 9$, its square is $- 81$, from which subtracting twice $9 = 18$, the remainder is still 63, as before.

$$7 \times 7 + 7 \times 2 = 63$$
$$9 \times 9 - 9 \times 2 = 63$$

Had the question been stated so as that the given quantities were $z^2 - 63 = 2z$, the result would have been similar, and the value of the unknown quantity z would have been found $= + 7$, or $- 9$, as in the proceeding example.

Example 2. Given $x^2 + 8 = 6x$, required the value of x.

$$x^2 + 8 = 6x$$
$$x^2 - 6x = - 8$$
$$x^2 - 6x + 9 = 9 - 8$$
$$x - 3 = 1$$
$$x = 4$$
$$4 \times 4 = 16 + 8 = 24 = 6 \times 4.$$

Here, by transposition, we get the equation $- x^2 - 6x = - 8$; and adding to each side the square of 3, which is half the co-efficient 6, $= 9$, we obtain another equation, $x^2 - 6x + 9 = 9 - 8 = 1$; and the square roots of each side are $x - 3 = 1$, agreeably to what was already observed, that the root of the first side of the equation will have the

 sign

sign —, because the sign of the term having the co efficient is —.

The limits of this work will not allow these observations on algebraic calculation to be further enlarged: the student must therefore be referred, for more ample information respecting the nature and uses of this branch of the science of numbers, to the various extensive treatises already published on the subject; and for the understanding of such works the foregoing instructions will, it is hoped, be found of considerable utility.

THE

THE

MODERN PRECEPTOR.

CHAPTER V.

OF GEOMETRY.

GEOMETRY is that branch of science which treats of the nature and properties of *Lines*, *Surfaces*, and *Solids*. The name is formed from two Greek words, signifying to measure the earth, or, simply, land-measuring. The principles of geometry, like those of arithmetic, must have been coeval with human society: it is true, that in the ruder states of existence, the hunter, the fisher, the shepherd, have but little occasion for nice ascertainment of the bounds and extent of their respective ranges of country; yet still they must adopt some standard by which to apportion the space requisite for the maintenance of their families and tribes.

Hence the origin of geometry is, by the most ancient writers, assigned to periods far before their own times. Herodotus, the father of profane history, refers it to the time of Sesostris, king of Egypt, who intersected that country by numerous canals, and divided it amongst the inhabitants, according to fixed proportions. Others have attributed its rise to the necessity existing in Egypt, of making yearly surveys and allotments of the land, after the overflowings of the Nile, which levelled, or otherwise effaced, all boundaries and limits of distinction between the possessions of the several cultivators.

The Greek philosophers agree in tracing their knowledge

of geometry back to Egypt; but to themselves it owed great part of the advances it made to perfection. The writings of *Euclid*, in particular, have constantly possessed the highest reputation among geometricians; and his *Elements of Geometry* are to this day considered as the fittest to be placed in the hands of those who wish to attain a rational acquaintance with this most useful, and most entertaining branch of knowledge. The business of the following short tract on geometry shall be, to give the young student such a taste and idea of the nature and advantages of this science, as may induce him to prosecute the study in a more extended and methodical manner, than can be attempted in this work. In the first place, he must understand the following explanation of certain terms, chiefly borrowed from the Greeks, but in constant use amongst geometricians of all countries.

An *Axiom* is a proposition or assertion of which the truth is at first sight so manifest, than no proof or demonstration can make it more evident: for instance.

1st. Two quantities, whether they be lines, surfaces, or solids, which are both equal to a third quantity, are equal to one another: thus two pounds of silver, accurately weighed in the same scales, and with the same weight, must be equal the one to the other.

2d. The whole of any given quantity is greater than any part of it.

3d. The whole of any quantity is equal to the sum of all its component parts taken together.

4th. Only one straight line can be drawn between two points.

5th. Two magnitudes, whether they be lines, surfaces, or solids, which being applied to one another, perfectly coincide in every point, or which exactly fill the same space, are equal the one to the other.

6th. All right angles are equal.

A *Theorem*

A *Theorem* is a proposition where some truth is to be proved by reasoning, that is, by demonstration; and means a thing to be shown.

A *Problem* is a proposition containing some questions which requires a solution, and means a thing to be done.

A *Lemma* is some truth employed in the demonstration of a theorem, or the solution of a problem. This, as well as a theorem and a problem, is comprehended under the general term, *proposition.*

A *Corollary* is some consequence drawn from one or more foregoing propositions.

A *Scholium* means a remark on some foregoing proposition, showing its application, extent, or connection with other propositions.

An *Hypothesis* is a supposition, or something taken for granted, in the statement otherwise called the enunciation, or in the demonstration of a proposition.

To render geometrical operations more concise, certain marks or signs have been adopted, of which those most generally employed are the following.

(=) Two parallel and horizontal lines signify equality: thus $AB=CD$, means that the quantity or line designated by the letters AB, is equal to that expressed by CD.

(+) The St. George's cross, or two lines horizontal and perpendicular crossing each other, signify that the quantities between which this sign is placed, are to be added together; and the name is *Plus*, a Latin word, signifying *more*: thus $A+B=C$ mean, that the two quantities represented by A and B, when added together, are equal to the quantity represented by C:—$3+5=8$.

(—) A single horizontal line between two quantities, signifies that the one is to be subtracted from the other; and it is named *Minus*, from a Latin word, signifying *less*: thus $A-B=C$, or $8-5=3$. These two signs may occur in the same expression of quantity, thus $D+E-F=G$, or

$22+8-20=10$: for 22 *a* 8 are 30, and subtracting 20, we have 10 for the remainder.

($\times$) The St. Andrew's cross, being two lines crossing each other at right angles, but both inclined to the horizon, stands for multiplication: thus $4\times5=20$, for 4 multiplied by 5 give 20.

The product of two quantities multiplied together is also expressed by writing them close together, as AM standing for the product of $A\times M$. A quantity multiplied into itself, or squared, that is, raised to the 2d power, is thus represented A^2; and the cube of A, or the 3d power, would be represented by A^3.

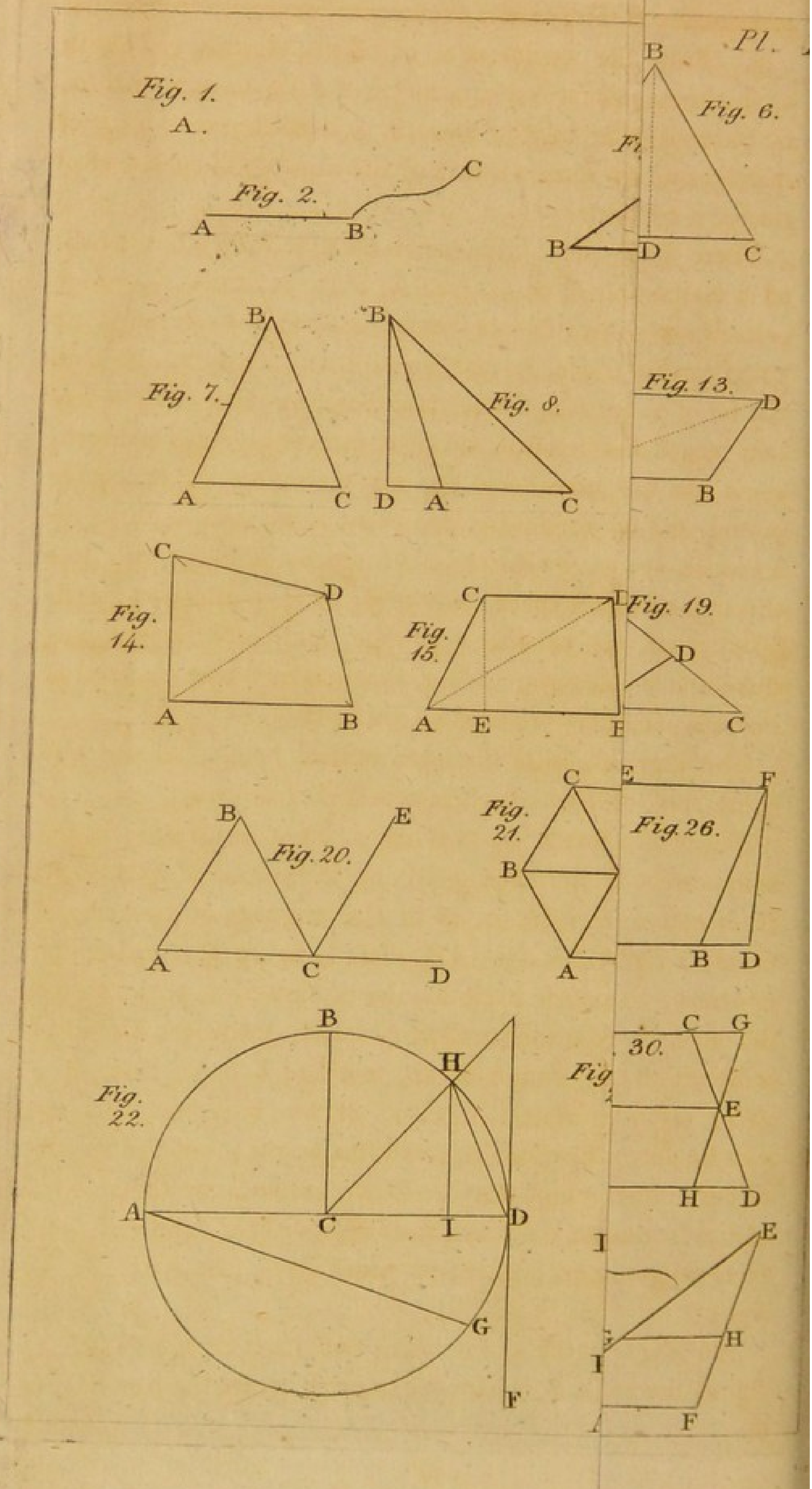
($\surd$) This sign means the root of any number or quantity: thus $\sqrt{A^2}$ is the square root of A, and $\sqrt{\overline{a+m}}^3$ means the cube root of the sum of the quantities a and m.

All bodies, and the spaces they occupy, consist of three dimensions, viz. *length*, *breadth*, and *depth* or *thickness*. Although these three sorts of measure are absolutely inseparable from the idea of a body, yet each of them may be considered by us as existing independent of the others; thus, for instance, if we say the Thames is deep at Woolwich, we are not considering the breadth of the river; and when we say, that it would take 27 yards of carpeting to cover a room, we think only of the length and breadth, but not of the thickness of the carpet.

From these varieties of dimension we have a *line* in which we consider the length alone, although it would be impossible to make or to conceive any line without some breadth, however small.

2. From the consideration of two dimensions, length and breadth, or height and breadth, we have the idea of a superficies or surface: thus, by considering the length and breadth of the carpet, we have a notion of its extent or surface; but the thickness of the cloth is not regarded, although no cloth can be made so thin, as to have no sensible thickness.

3. From


Pl.
Fig. 1.
Fig. 2.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 26.
Fig. 22.
30.

3. From the consideration of length, breadth, and depth or thickness, we form a conception of solid contents; thus, by observing the length, breadth, and thickness of a log of mahogany, we form an idea of the number of solid feet of timber it contains.

These three sorts of dimension all arise from what is called a mathematical point, (see A, Plate 1, figure 1), which is itself not susceptible of measurement: for although no point can be made, by the finest instrument, which shall not be of some magnitude, and therefore must have a certain length and breadth, however minute; yet these dimensions are not considered in our reasoning respecting the point; and by supposing this point to be moved along for a certain space, we produce a line; by moving this line through a certain space sideways, or in a direction parallel to itself, we produce a surface, and lastly, by moving this surface sideways, or in a direction any how inclined to its own place, we produce a solid body.

The extremities of lines are termed *points*, as are also those parts of lines which intersect each other.

The mark made by moving a point constantly in the same direction from the beginning to the end of its motion, is termed a *straight*, or, as in the language of mathematicians, a *right* line, (see AB, Plate 1, figure 2); and on the contrary, the mark made by the motion of a point which deviates, in even the smallest quantity, from the direction with which the motion began, is called a *curve* line, or a *curve*, (see BC, Plate 1, fig. 2). Hence, only one right line can be drawn between two extreme points; but the variety of curve lines which may be drawn between the same extremities is infinite. Lines consisting partly of right and partly of curve lines, are termed *mixed* lines, (see ABC, Pl. 1, fig. 2).

An *Angle* is the point when two lines meet; as (Pl. 1, fig. 3) the point B, where the line AB meets the line BC, is called

called the angle at B, or rather the angle ABC; observing always, that in naming, or, as it is called, reading an angle, by means of three letters, that standing at the angular point is to be placed in the middle of the three; thus, the angle at B may be either read the angle ABC, or CBA. This angular point is also called the *vertex* of the angle. When there are at a point more angles than one, as at the point C, in fig. 4, it is necessary to mention all the three letters denoting the angle intended; for the angles ACF, FCD, DCE, and ECB, are all formed at the point C.

When one right line meets another in such a manner, that the openings formed on each side are equal, or when one line stands upon another placed horizontally, in such a way that, if it were moveable, it would have no tendency to fall over on either side: in these cases, the first line is said to be perpendicular to the second line, and the equal angles on each side of the meeting of the two lines are called *right* angles. Thus, in fig. 4, Pl. 1, the line DC meets the line AB in the point C in such a way, that the opening formed by the line DC, with the part AC, is equal to the opening formed by the same DC with the part CB; or if the line DC were moveable on the horizontal line AB, it would have no tendency to incline or fall towards either the point A or the point B: in such cases the line DC is said to be perpendicular to AB at the point C; and the two angles ACD and DCB are both *right*. But should the line FC (same figure) meet the line AB at C in such a manner, that the angle FCA should be less than the other angle FCB, the angle FCA would be less than a right angle, and termed an *acute* or *sharp* angle; while FCB being greater than a right angle, would be termed an *obtuse* or *blunt* angle. In the same way the angle ECB, which is less than DCB a right angle, is *acute*, and ECA, which is greater than DCA, is *obtuse*.

Angles, like other quantities or magnitudes, are susceptible of

of augmentation and diminution, by addition, subtraction, multiplication, and division: thus, in the same figure 4, the angles ACF and FCD, added together, form the angle ACD; and ACF, FCD, and DCE, are together equal to the angle ACE. Again, if from the angle DCA we subtract the angle DCF, we have remaining the angle FCA. In the same manner, supposing the three angles ACF, FCD, and DCE, to be equal to each other, the whole angle ACE would be three times the angle ACF, and in the same supposition the great angle ACE, divided by 3, would give us the angle ACF.

N.B. It is always to be remembered, that when we speak of an angle, we mean only the opening formed by the meeting of two right lines, without in the least taking into consideration the relative lengths of these lines; for the above angle ACF, would be of precisely the same magnitude whether the forming lines were in length an inch, a yard, a mile, or a thousand leagues.

When two or more right lines lie in the same plane or surface in such a way that, if produced to any imaginable length either to the right or the left, they would never meet, or have a tendency to approach one another, such lines are said to be *parallel*: thus in fig. 5, Pl. 1, the lines AB and CD are so situated with regard to one another, that if produced indefinitely either through A and C, or through B and D, they never would approach one another; that is, that the distance from A to C would be precisely equal to the distance from B to D, or the intermediate distances from *a* to *b*, and from *c* to *d*; in such circumstances the lines AB and CD are said to be parallel to each other.

When a line is parallel to another line, it will also be parallel to any other line which is parallel to the former; thus, in the same figure, the line EF being drawn parallel to the line CD, it will also be parallel to the line AB; for EF being parallel to CD, the distance E *a* will be equal to

 the

the distance F c: but CD and AB being also parallel to each other, the whole distance $a\,b$ will be equal to the whole distance $c\,d$. From these equal quantities therefore, taking away the equal quantities a E and c F, the remainders must be equal; that is, E b must be equal to F d; but two right lines which are so situated as to be, in different points of their extent, equally distant the one from the other, are parallel: therefore the line EF, which was given parallel to CD, is also parallel to AB, which was given parallel to CD.

From what has been said, it is evident that two right lines cannot inclose a space, or form a plane figure; for, to inclose a space, the lines must meet in some point, as in B, (fig. 3,) but they never can meet any more, as the longer they are made, the greater must be the distance between their extremities.

Suppose now these extremities to be joined by another right line, as in fig. 6, Pl. 1, where AC and AB meeting in the point A, have their extremities B and C joined by the right line BC: here we have a figure containing three angles and three sides, and from the former circumstance it obtains the name of a *Triangle.* When the three sides are all of the same length, as in this fig. 6, the triangle is said to be *equilateral,* that is, *equal-sided.* When only two of the sides are equal, as in fig. 7, where AB and AC are of equal length, but the third side AC is shorter, the triangle is said to be *isosceles* or *equal-legged.* Lastly, if the three sides be all of different lengths, as in fig. 8, the triangle is said to be *scalene,* that is, *leaning,* as a ladder against a wall.

If there be a right angle in a triangle, it is called *right-angled,* as fig. 9; and the side opposite to the right angle is called the *hypothenuse,* as BC, while the side AB is also called the *perpendicular,* and the side AC is called the *base*; this last term is also applied to the lower side of all other triangles; as AC in fig. 6, 7, and 8. If a line be let fall perpendicularly on the base of a triangle from the opposite angle, this line will give the *altitude* of the triangle;

triangle; whether it fall within the triangle, as BD in fig. 6, or without it, as BD on the base CA produced in fig. 8; or coincide with one of the sides, as BA in fig. 9.

If a figure has four sides all equal, and the four angles all right; that is, if each side be perpendicular to the two adjoining sides, such a figure is called a *square*, as fig. 10, where the four sides AC, CD, DB, and BA are all equal to one another, and the four angles at A, C, D, and B are all right, and consequently equal to one another. Again, if all the sides be equal, but the angles not right, the figure is termed a *rhombus*, or distorted square; as fig. 11, where the four sides AC, CD, DB and BA are equal, but none of the angles are right.

Four-sided figures, having only the two opposite sides equal and parallel to each other, are in general termed *parallelograms*: but when such a figure contains a right angle (the other three being necessarily right also) it is called a *rectangle*: see fig. 12, where the opposite sides, AB and CD are equal, and CA and DB are also equal; and the four angles at A, C, D, and B, are all equal and right. Again, if the opposites sides of a four-sided figure be respectively equal, but none of the angles right, although respectively equal, such a figure is called a *rhomboides*, or distorted parallelogram. See fig. 13, where AB and CD are equal, and AC and BD are equal, and the opposite angles at A and D, and at C and B are equal, but none of them right angles.

When the four sides are all unequal, and none of them parallel to its opposite side, the figure is called a *trapezium*, or *quadrilateral*, such as in fig. 14; but if two opposite sides be parallel, although all the sides be unequal, the figure is termed a *trapezoid*, as in fig. 15.

A line joining opposite angles in a figure, is termed a *diagonal*; as the dotted lines AD and CB in fig. 10, AD in fig. 11, 12, 13, 14, and 15: and a line let fall perpen-

dicularly from an opposite angle on the base of any parallelogram, gives the altitude of the figure: as the dotted line CE in fig. 11, 13, and 15. When the parallelogram is right angled, the line of altitude coincides with the perpendicular side, as CA or DB in the square fig. 10, and CA in fig. 12.

Figures composed of more than four sides are distinguished by various names, descriptive of their number of sides: thus, one of five sides is called a *pentagon*; one of six sides, a *hexagon*; one of seven sides, a *heptagon*; one of eight sides, an *octagon*; one of nine sides, a *nonagon*; one of ten sides, a *decagon*; one of twelve sides, a *dodecagon*; and so on; but in general they are termed *polygons*, a name merely indicating that they are figures of many sides.

These things being premised, the student will be prepared to understand the following series of propositions, containing some fundamental theorems illustrative of the most important properties of geometrical figures,

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Proposition I. When one right line meets another right line, the angles formed at the point of meeting are either two right angles, or they are, when taken together, equal to two right angles.

(Pl. 1, fig. 4.) Suppose the right line FC to meet the right line AB in the point C, the two angles, FCA and FCB, are either two right angles, or, when taken together, equal to two right angles. If the line FC stood perpendicularly upon AB at the point C, then the angles on each side, as has been already said, would be both right; but in this case, as FC does not stand perpendicularly on AB, let some other line, as DC, be drawn perpendicular to AB, at the same point C; and the angles DCA and DCB will be both right; but the two angles DCA and DCB are equal to the three angles ACF, FCD and DCB, therefore the sum of these three angles is equal to two right angles;

but

but the angles FCD and DCB are equal to the angle FCB, therefore the two angles FCA and FCB are equal to two right angles.

Corollary. However many angles may be formed, on the same side of a line, at any given point, their sum will still be equal to the sum of two right angles. In the same figure the angles ACF, FCD, DCE, and ECB, all formed at the point C, on the same side of the line AB, are, taken together, equal to two right angles; for ACD is a right angle, and it is equal to the two angles ACF, and FCD, and the other right angle DCB, is equal to the remaining two angles DCE and ECB.

2d *Corollary.* From this proposition it follows, that all the angles that can be formed at one point, on both sides of a right line, are equal to four right angles.

Proposition II. If two right lines intersect each other, the vertical or opposite angles are equal to one another. Let the right lines AB and CD (Pl. 1, fig. 16,) cut one another in the point E, then the vertical or opposite angles will be respectively equal to each other: that is, the angle AEC will be equal to the angle BED, and CEB will be equal to AED. For, as the line CE meets AB in the point E, it follows, from the 1st Proposition, that the angles AEC and CEB are equal to two right angles: again, as the line BE meets CD in the point E, the angles CEB and BED are also equal to two right angles; therefore the sum of AEC and CEB is equal to the sum of the same CEB and BED. Now if from equal quantities we take away equal quantities, or a quantity included in, and thereby common to both quantities, the remainder must be equal: take away, therefore, from AEC + CEB, and from CEB + BED, the common angle CEB, and the remaining angle in the one case AEC will be equal to the remaining angle in the other case BED. But these angles are opposite and vertical, agreeably to the proposition; and in the same manner it

may

may be shown that the other opposite angles, CEB and AED, are equal to one another.

Prop. III, fig. 17. Two triangles are equal to one another, when the one has an angle and the two sides containing it, equal to an angle, and the two sides containing it in the other, each to each. If the two triangles ABC and DEF, be of such a nature that the angle ABC in the one is equal to the angle DEF in the other; the side AB in the one equal to the side DE of the other; and the side BC in the one, to the side EF in the other; then these two triangles will be equal the one to the other. For supposing the triangle ABC to be moveable, let it be placed upon the triangle DEF, so that the angular point B shall coincide with the angular point E; the line AB with the line DE, and the line BC with the line EF; then will the point A coincide with the point D, and the point C with the point F, and consequently the line or base AC with the base DF. But it was already stated as an axiom, that magnitudes, whether lines, surfaces, or solids, which, when applied to one another, perfectly coincide in all their parts, are equal to one another; the two triangles, therefore, given in the proposition must be equal to each other; and the remaining angles of the one will be equal to the remaining angles of the other, each to each; that is BAC to EDF, and BCA to EFD, and the remaining side AC to the remaining side DF.

1st *Corollary*. From this proposition it follows that, if two triangles have two sides of the one respectively equal to two sides of the other, but the angle formed by these two sides in the one greater than the corresponding angle in the other, the base of the first will be greater than the base of the second; and if the contained angle in the first triangle be less than the contained angle in the second, the base of the first will be less than the base of the second triangle.

2d *Corollary*. Any two sides of a triangle are together greater

greater than the third side; for, in this figure, the line AC being the shortest distance between the points A and C, it must consequently be shorter than the sum of the two other sides, AB and BC; and, for the same reason, the side AB must be shorter than the sum of AC and BC; and BC must be shorter than the sum of BA and AC.

PROP. IV, fig. 18. In an isosceles triangle, the angles opposite to the equal sides are equal to one another. In the triangle ABC, which is isosceles, that is, which has the side BA equal to the side BC, the angles opposite to these equal sides are equal; that is, BCA is equal to BAC.

Let a line be drawn from the vertical angle at B to the point D, which cuts the base into two equal parts; then we have two triangles, BAD and BCD, having the side BA equal to BC, the base AD equal to DC (the whole AC having been bisected, that is, divided into equal parts in the point D), and the side BD common to both triangles; the angles of the one triangle must therefore be equal to the corresponding angles of the other; that is, the angle BAD or BAC, equal to the angle BCD or BCA.

Corollary. Hence it appears, that equilateral triangles are also equiangular: and hence it is evident, that if two angles of a triangle are equal to another, the two opposite sides must also be equal, and the triangle will be isosceles.

PROP. V, fig. 19. In any triangle the greater side is opposite to the greater angle; that is, in the triangle ABC, the side BC, which is opposite to the greater angle BAC, is greater than the side BA, which is opposite to the less angle BCA, or than AC opposite to the less angle ABC. Let the line AD be drawn, making the angle DAC equal to the angle DCA; then, by the last proposition, will the side AD be equal to DC; by the 2d corollary of Prop. 3, the two sides, BD and DA, of the triangle BAD, are greater than the third side BA: but DA being equal to DC, BD and DA are equal to BD and DC, that is, to the whole line BC,

which

which is therefore greater than BA; and it is opposite to the angle BAC, which is greater than the angle BCA, agreeably to the proposition.

In the same way it may be shown, that the greater angle must be opposite to the greater side.

Prop. VI, fig. 20. If any side of a triangle be produced, the exterior angle it forms with the adjacent side, is equal to the sum of the two interior and opposite angles. If the side AC of the triangle ABC be produced to D, the exterior angle BCD formed by this produced side CD, and the adjacent side of the triangle CB, will be equal to the sum of the two inward and opposite angles CAB and CBA. Let the line CE be drawn from the point C, parallel to the opposite side of the triangle AB. From what was said in Prop. 2, fig. 16, of the properties of lines intersecting each other, it appears that the vertical angles formed by such intersection are equal to each other; and all lines intersecting one of these lines, but parallel to the other, will, with the first line, form angles respectively equal to the former; in fig. 16, the angle BEC is equal to AED; in the same way, in fig. 20, the angle ECD will be equal to ABC; and AB and EC being parallel, the angles formed by them with BC will also be equal, that is, BAC will be equal to ECD. We have now obtained the angle BCE equal to the angle at B, and ECD equal to the angle at A; but BCE and ECD together form the great exterior angle BCD, equal to the sum of the inward and opposite angles at A and B, agreeably to the enunciation of the proposition.

Prop. VII, fig. 20. The three interior angles of any triangle are equal to two right angles. The sum of the three angles at A, B, and C, of the triangle ABC, is equal to two right angles; for let the side AC be produced to D, then by the 6th Prop. the exterior angle BCD will be equal to the two inward and opposite angles at A and B. To these

equal

equal quantities let us add the remaining inward angle BCA, and we shall have the three angles of the triangle equal to the two angles BCA and BCD: but it was already shown in Prop. 1, fig. 4, that one line falling on another makes the angles on each side of the point of meeting, either two right angles, or equal to two right angles; consequently the angles BCA and BCD must be equal to two right angles: but these two angles having been shown to be equal to the sum of the three inward angles of the triangle, it follows, that the sum of these three inward angles must be equal to the sum of two right angles, as affirmed in the proposition.

1st *Corollary*. If two angles of a triangle, or their sum be given, the third can be found by subtracting this sum from that of two right angles.

2d *Corollary*. If one angle of a triangle be right, the two other angles must together be equal to one right angle.

3d *Corollary*. In an equilateral triangle, the angles being all equal, each is one third part of two right angles, or two thirds of one right angle.

Prop. VIII. fig. 21. The sum of all the interior angles of a polygon is equal to twice as many right angles, wanting four, as the figure has sides. Let the figure ABCDEF, be a polygon of six sides, that is, a hexagon. From any point within it, as G, draw lines to the several angles forming the six triangles AGF, AGB, &c. equal to the number of sides of the polygon. Then the sum of all the angles of each triangle (including that at the point G) will, by Prop. 7, be equal to two right angles; therefore the sum of all the angles of all the triangles will be equal to twice as many right angles as there are triangles, that is, as the figure has sides, but the sum of all the angles of all the triangles is equal to the sum of all the angles of the polygon, together with the angles at the central point G, which by 2d Corollary of Prop. 1, are shown to be equal to four right

angles;

angles; therefore, subtracting these four right angles, we have all the angles of the polygon equal to twice as many right angles, excepting four, as the polygon has sides.

Having thus pointed out some of the properties of right lined figures, the next subject of consideration will be those of curve lined figures, particularly of the circle.

The characteristic of a circle is, that every part of its extent, called the *circumference* or *pariphery*, is equally distant from a point within it called the *centre.*

Every right line drawn from the centre to the circumference, is called a *radius*, as CA, CB, CH, and CD, in fig. 22; and any right line passing through the centre, and terminated both ways by the circumference, is called a *diameter*, as ACD, which is double the length of the radius, for AC being equal to CD, the whole AC must be double either of the parts AC or CD.

All straight lines drawn in a circle and terminated at each end by the circumference, but not passing through the centre, are termed *chords;* such as AG, and HD.

An *arch* or *arc* is any portion of the circumference of a circle; as the small portion AG, or the great portion ABHDG: the curve line HD is also an arch of the circle.

A figure contained between an arch of a circle and the chord or line joining the extremities of the arch, is called a *segment* of a circle: thus, the figure comprehended between the curve line AG and the chord AG is a segment; and, on the other hand, the figure contained within the same chord AG and the great curve ABHDG, is also a segment.

If the figure be formed by a line passing through the centre, that is, by a diameter, the segments are equal to one another, each being one half of the circle, or a *semicircle*, as ABD and AGD.

A figure contained within an arch of a circle, as BH, the radius

radius BC, and the radius HC, is called a *sector*. HCD, and ACH, are also sectors; but if the one radius be perpendicular to the other, that is, if they form a right angle at the centre, as BC and DC, then the sector is called a *quadrant*, as being the fourth part of the whole circle, and the half of a semicircle; for AD, the diameter, being equally divided in the centre C, and CB being perpendicular to the diameter at that point, any point in CB will be equally distant from A and B; the arch AB will therefore be equal to the arch BD, and the segment ABC to the segment BDC; each of them will, consequently, be one half of the semicircle ABD, that is, each will be one fourth of the whole circle, or a quadrant.

A right line can meet the circumference of a circle only in two points, as AG or HD.

In the same circle, or in circles equal to each other, the chords of equal arches are equal to each other; and *vice versa*, the arches subtended by equal chords are equal to each other.

In the same or in equal circles, the greater arch is subtended by the greater chord, and the less arch is subtended by the less chord; unless the arch be greater than a semicircle, when the greater the arch the smaller the chord.

Prop. IX. fig. 23. If three points, as ABD, be taken in the circumference of a circle whose centre is C, no other circle can be drawn through the same three points not coinciding with the given circle. Join the three points by the two chords AB and BD, and from the centre C draw CE and CF perpendicular to these chords, and consequently bisecting them, or dividing them severally into two equal parts; in which case the centre of any circle passing through the points A and B, will be somewhere in the line EC; and in the same way the centre of any circle passing through the points B and D must be somewhere in the line

FC:

FC: consequently the centre of a circle passing through the three points AB and D must be where these two lines EC and FC meet, that is, in the point C; but this is the centre of the circle in whose circumference the three given points were taken; therefore only one circle can be drawn to pass through any three given points: and hence it follows, that circles can cut one another only in two points.

Prop. X. fig. 24. The angle formed by the lines AC and BC drawn from the extremities of an arch of a circle AB to the centre C, is double the angle formed by the lines AD and BD drawn from the same extremities to any point, as D in the opposite circumference. From D draw through the centre C the diameter DE. In the triangle DCA the sides DC and CA are equal to each other, each being a radius of the circle; the angles opposite to them are therefore equal, that is, CAD is equal to CDA: but, by the 7th Prop. it appears, that the exterior angle ACE, formed by producing the side DC, is equal to the two inward and opposite angles of the triangle, that is, since they are equal to one another, ACE must be double ADC or ADE; again, in the triangle CDB, by the same Proposition, it appears that the exterior angle ECB is equal to the two inward and opposite angles CDB and CBD; but these angles being opposite to equal sides, must be also equal; and the exterior angle ECB will be double CDB, that is, EDB: it follows, therefore, that the whole angle at the centre ACB, will be double the whole angle at the circumference ADB.

Corollary. All angles, however situated, of the same segment of a circle, or in other words, all angles formed by lines drawn from the extremities of the arch of the segment, and meeting in the circumference, are equal.

From this proposition it follows, that the angles in a semicircle are all right angles; that the angles in a segment greater

greater than a semicircle are less than a right angle; that those in a segment less than a semicircle are greater than a right angle; and that the opposite angles of any quadrilateral figure, drawn within and bounded by the circumference of a circle, are together equal to two right angles.

When two or more figures have their respective angles equal and their respective sides proportionals, these figures are said to be *similar*: and these respective angles and sides are called *homologous*.

When two figures, being applied the one to the other, perfectly coincide in every part, they are said to be *equal*.

When two figures, whatever be their shape, contain surfaces of equal extent, they are said to be *equivalent*.

Two equal figures must be similar, but two similar figures may be very unequal.

Prop. XI. fig. 25. Parallelograms situated on the same or equal bases, and between the same parallels, or of equal altitudes, are equivalent to one another: thus the parallelograms ABDC and AEFB being situated on the same base AB, and being of the same supposed altitude, the sides CD and EF opposite to the base will be in the same parallel CDEF: and from the definition formerly given of a parallelogram, the opposite sides will be respectively equal to one another; AC will be equal to DB, and AE to BF: again, CD and EF being both equal to AB, they will be equal to one another. To these equals add the line DE, and we shall have the whole CE equal to the whole DF; and consequently the triangles CEA and DFB with three sides of the one equal to the corresponding three sides of the other; these triangles will therefore be equal. (See Prop. 3.) If now from the quadrilateral ACFB we take away the triangle CEA, we have remaining the parallelogram AEFB; and if from the same quadrilateral we take away the triangle DFB, we have remaining the parallelogram ACDB:

ACDB: but if from equal quantities we take away equal quantities, the remainders will be equal; we have consequently the parallelogram ACDB equal to the parallelogram AEFB.

Hence it follows, that every parallelogram is equivalent to a rectangle of equal base and altitude, as in this example, when ACDB is a right-angled parallelogram.

In the same way it may be proved that the parallelograms AEFB and CEFD, in fig. 26, are equivalent to each other; for, by the nature of the parallelograms, the opposite sides being respectively equal, the line AB will be equal to the line CD: if from these equals we take away CB, which is common to both, we shall have the part AC equal to the part BD, and consequently the triangle AEC equal to BFD; if, therefore, from the quadrilateral AEFD, we take away the triangle AEC, we have the parallelogram CEFD; and if from the same quadrilateral we take away the triangle BFD, we shall have the parallelogram AEFB, which, from what was before said, must be equal to the parallelogram CEFD.

Prop. XII. fig. 27. Every triangle, as ABC, is one half of the parallelogram ABDC, situated on the same base AC, and of the same altitude BG; for the base AC is equal to BD, the side AB to CD, and the side BC is common to both triangles, which, by Prop. 3, are therefore equal: but these equal triangles form the parallelogram ABDC: the triangle ABC is consequently one half of the parallelogram ABDC.

Prop. XIII. fig. 28. Two right-angled parallelograms, of equal altitudes, are to one another in the proportion of their bases. Let the two rectangles, ABCD and DCEF, have the same altitude DC, these rectangles will be to each other as the base AD of the one is to the base DF of the other. Let the base AD contain two parts of any given measure, and the base DF contain three of the same parts, then

then in the points of division 1, 1, 2, draw dotted lines, as in the figure, perpendicular to the bases, when we shall have the rectangle ABCD divided into two rectangles, and DCEF divided into three rectangles, all of which being of equal bases and altitudes, must, by Prop. 11, be equal to each other: hence the rectangle ABCD, must be to the rectangle DCEF, as two to three; that is, as the base AB to the base DF.

Corollary. Hence it follows, that any two rectangles are in proportion to each other, as the products obtained by multiplying together any numbers proportional to their respective sides.

And hence it follows, that the product of the base of a rectangle, multiplied by its altitude, may be considered as the measure of that rectangle: thus, in fig. 29, the sides of the rectangle ABCD are AB equal to 3, and AD equal to 5; the product of these two numbers is 15, for the measure of the rectangle: for if AB be divided into 3 equal parts in the points 1, 2, and AB into five of the same equal parts by the points 1, 2, 3, 4; and if lines be drawn, through those points, parallel to the sides of the rectangle, the whole will be divided into 15 squares, whose sides are equal to one of the parts into which the sides of the rectangle were divided, as will be evident from inspection of the figure.

Prop. XIV. fig. 27. The area, or surface of a parallelogram, is equal to the product of its base multiplied by its altitude: for the parallelogram ABDC (as was shown by Prop. 11,) is equal to the rectangle AEFC: but the area, or measure of this rectangle, was, by Prop. 13, shown to be equal to the product of its base multiplied by its altitude; the area of the given parallelogram ABDC, will therefore be equal to the product of its base AC multiplied by its perpendicular altitude BG, which is equal to AE or CF.

Prop.

PROP. XV. fig. 27. The area of a triangle is equal to one half of the area of a rectangle, on the same base, and of the same altitude: for the triangle ABC is one half of the parallelogram ABDC, (Prop. 12,) which is equivalent to the rectangle AEFC, constructed on the same base AC, and with the same altitude, AE and CF, both equal to BG, the altitude of the triangle.

Corollary. Triangles on the same base are to each other as their altitudes; and triangles of the same altitude are to each other as their bases.

PROP. XVI. fig. 30. The area or surface of a trapezoid, that is, of an irregular quadrilateral, of which two opposites are parallel, is equal to the product of its altitude multiplied by half the sum of its parallel sides. The area of the trapezoid ABCD, is equal to the product of half of the sum of the two parallel sides AD and BC, multiplied by its altitude BL. Divide the side CD into two equal parts in the point E, and through it draw EF parallel to the two sides AD and BC; also through the same point E draw HG, parallel to the opposite side AB, meeting BC produced in the point G: then in the triangles CEG and HED, the angle CEG is equal to the angle HED, (Prop. 2, fig. 16,) and the side EC is made equal to the side ED, and the angle ECG to the angle EDH, (Prop. 6, fig. 20, and Prop. 2, fig. 16): these two triangles are therefore equal, that is, the triangle ECG, which is added to the given trapezoid, is equal to the triangle EDH, which is taken from the trapezoid: it therefore follows, that the trapezoid ABCD is equivalent to the parallelogram ABGH; but CG being equal to HD, (Prop. 3, fig. 17,) the sum of AD and BC together will be equal to the sum of AH and BG together: these latter lines, as joining equal and parallel lines, must themselves be equal; the sum, therefore, of AH and BG will be double AH or BG, or double FE, and half the sum of AD and BC must be equal to FE; but FE multiplied

by

by the perpendicular BL, is the measure of the parallelogram ABGH, the area of the trapezoid ABCD will consequently be equal to half the sum of the parallel sides BC and AD, multiplied by its altitude BL.

PROP. XVII. fig. 31. If a right line, as AC, be divided into two parts by any point, as B, the square constructed on the whole line AC, will be equal to the squares constructed on the two parts AB and BC, together with twice the rectangle formed by these two parts.

On the right line AC, construct a square ADEC; through B draw BH parallel to AD and CE; make AF in AD equal to AB, and through F draw FI parallel to DE and AC, and intersecting BH in G. From inspection of the figure it will be evident, that the great square ADEC comprehends four figures; 1st, the little square AFGB, being the square of the part AB, for the side AF was made equal to AB: 2dly, the little square HGIE, which corresponds to the square constructed on the remaining part BC, for BC being equal to FD, it is also equal to GI and GH, or to IE and HE; the square HGIE is therefore equal to the square of BC: 3dly, the two rectangles, FDHG and BGIC, whose respective sides, FG and GB, are equal to the part AB; and GH and GI are equal to BC: these two rectangles, therefore, correspond to rectangles constructed on the two parts AB and BC: hence it appears, that the square constructed on the whole line is equal to the sum of the squares constructed on the two parts of the line, together with twice the rectangle contained under these two parts.

The truth of this proportion (like that of many others in geometry) may be illustrated by arithmetic, in this way: suppose the given line AC to be 6 feet long, and that it is divided at the point B, into two parts, AB containing 2 feet, and BC containing 4 feet. The square of the whole

line of 6 feet is 36; the square of the part AB = 2 is 4; the square of BC = 4 is 16; the rectangle, or product of AB = 2 by BC = 4, is 8, and twice this product or rectangle is 16: add, therefore, these two squares and the two products together; $4+16+8+8=36$, which is equal to the square of the whole line AB, or $6\times6=36$.

Prop. XVIII. fig. 1, Plate 2. The square constructed on the hypothenuse of a right-angled triangle is equivalent to the sum of the squares constructed or the two sides containing the right angle. Let ABC be a triangle, having a right angle at B; on the side AB construct the square AFGB, on BC the square BHIC, and on the hypothenuse AC the square ACDE; from the right angle at B, let fall on the hypothenuse the perpendicular BK, and produce it to L, and join BE and FC. The angle BAE is composed of the angles BAC and CAE, of which CAE being the ang'e of a square, must be a right angle: again, the angle FAC is composed of FAB and BAC, of which FAB, the angle of a square, must be a right angle, therefore equal to CAE, and BAC is common to both; consequently the whole angle FAC will be equal to the whole angle BAE. Then AC and AE being sides of a square, they are equal; and for the same reason AB is equal to AF; we have therefore two triangles, CFA and ABE, having two sides of the one equal to two sides of the other, and the contained angle of the one FAC equal to the contained angle of the other BAE; these two triangles must consequently be equal (Prop. 3, fig. 17, Plate 1). But the triangle CFA is one half of the square AFGB, for it stands on the same base, and within the same parallels, AF and GBC, (Prop. 12 and 15, fig. 27); and for the same reason the triangle ABE is one half of the parallelogram AKLE; but if the halves of any magnitudes be equal, the wholes must also be equal; therefore the whole square AFGB must be equal to the whole parallelogram AKLE.

Again,

Again, by drawing the diagonal lines AI and BD, it may in the same way be demonstrated, that the triangle AIC is equal to the triangle BDC; and, consequently, the square BHIC to the parallelogram KCDL: but it was already shown, that AKLE is equal to the square AFGB, and now it appears that KCDL is equal to the square BHIC: and as these two parallelograms, AKLE and KCDL, compose the square ACDE, it follows, that the square ACDE, is equal to the sum of the two squares AFGB and BHIC: or, in other words, that the square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the two sides containing the right angle.

To illustrate this proposition by arithmetic, let the side AB be equal to 3, the side BC to 4, and the hypothenuse AC to 5; the square of 3 is 9; the square of 4 is 16; and the sum of these squares is 25, equal to the square of the hypothenuse 5.

N. B. These three numbers, 3, 4, and 5, or any multiples of them, will always form a right-angled triangle: thus, if the sides were 15, 20, and 25, which are 5 times 3, 4, and 5, the square of 15 which is 225, added to the square of 20, or 400, will give 625, which is just the square of the hypothenuse 25.

Corollary. The square of a side opposite to an angle greater than a right angle, will be greater than the sum of the squares of the two sides containing the obtuse angle: and if the angle be acute, or less than a right angle, the square of the opposite side will be less than the sum of the squares of the containing sides.

Prop. XIX. fig. 32. If in a triangle, as ABC, a right line be drawn, as DE, parallel to one of the sides, as AC, and cutting the two other sides in D and E, this line will cut these sides proportionally, that is, BD will be to BE as BA is to BC. Join AE and DC, and the triangles AED

and CDE being constructed on the same base DE, and of the same altitude (Prop. 12, fig. 27), they are equal: and the triangles BDE, CDE, being of the same altitudes, are to each other as their bases; that is, the triangle BDE is to the triangle CDE, as BE to CE: but CDE has just been shown to be equal to AED, and AED is to BED as the base AD to the base BD: hence we have the four proportionals BE: EC :: BD: DA: and by compounding the proportions BE: BC :: BD : DA, or BE : BD :: BC : BA.

The reverse of this proposition is also true: that is, if two sides of a triangle are cut, proportionally, by a right line drawn within it, such a line must be parallel to the third side of the triangle.

Prop. XX. fig. 33, Plate 1. If a right line BD be drawn in a triangle ABC bisecting, that is, dividing into two equal parts any angle, as that at B, and cutting the opposite side AC, in the point D, then will this side AC be cut into two parts proportionally to the opposite sides of the triangle forming the bisected angle; that is, the part of the base AD will be to the part DC, as the side AB to the side BC.

From A draw AE parallel to BD, and meeting CB produced in the point E; and, by Prop. 19, we shall have CD : DA :: CB : BE : but from the properties of parallel lines formerly explained (Prop. 6, fig. 20), the angle DBA will be equal to BAE, and DBA will also be equal to BEA : BAE and BEA are therefore equal to each other, and consequently the opposite sides BE and BA must be equal; and hence CB : BE :: CB : BA, and CD : DA :: CB : BA; that is, the segments of the base of the triangle, made by a line bisecting the angle opposite to the base, will be proportional to the respective sides forming that angle.

Prop.

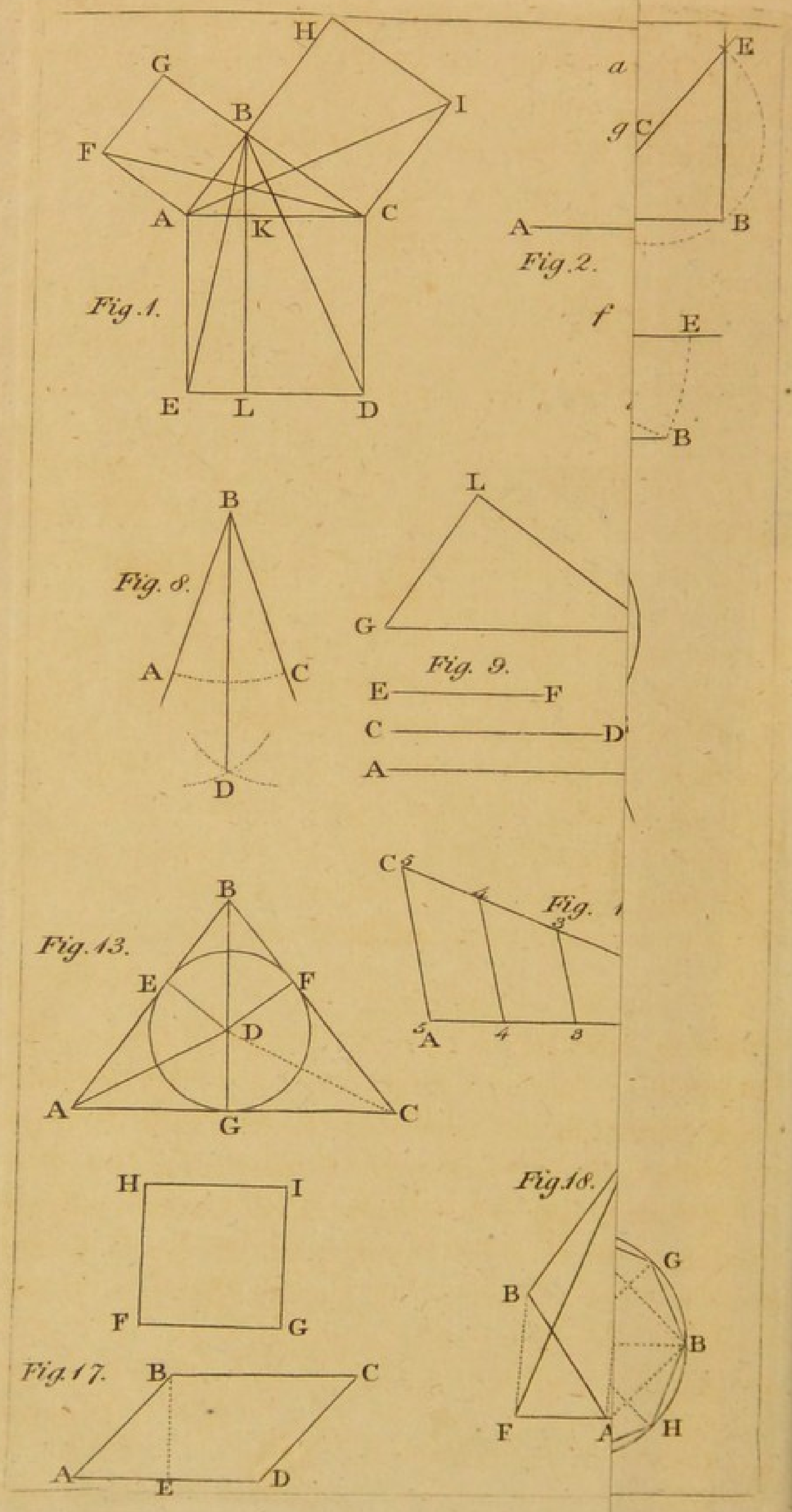
Fig. 1.
Fig. 2.
Fig. 8.
Fig. 9.
Fig. 13.
Fig. 17.
Fig. 18.

PROP. XXI. fig. 34. If two triangles, as ABC and DEF, have any angle of the one equal to an angle of the other, as the angles at B and E, and the sides forming these angles respectively proportional, that is, AB : BC :: DE : EF; then these triangles will be similar to each other. In the greater triangle DEF, make EH equal to BC of the less triangle, and draw HG parallel to the side DF, then will the angle EHG be equal to the angle EFD, and the triangle EGH be equiangular to the triangle EFD; and consequently EF : EH :: ED : EG : but by the proposition EF : BC :: ED : BA, and EH being made equal to BC, EG will be equal to BA, and the triangles EHG and BCA will be equal : but the triangle EHG was already shown to be similar to EFD; therefore the triangle ABC will be similar to the triangle DEF.

From these properties of similar triangles it may be demonstrated, that the areas of such triangles are to each other, in the proportion of the squares of their corresponding or homologous sides; that is, (fig. 34) the area of the triangle ABC, will be to that of the triangle DEF, as the square of AB is to the square of DE, or as the square of BC to the square of EF.

PROP. XXII. fig. 2, Plate 2. To bisect a given right line, AB, that is, to cut it into two equal parts. Upon the extremity A, as a centre, with a radius or opening of the compasses greater than half of AB, make arches on each side of that line, as *a b*, *c d*; from B, as a centre, with the same radius, describe the arches *e f*, *g h*, intersecting the former arches in the points I and K : then lay a ruler, or draw a line from I to K, cutting AB in the point O; so will AB be bisected at O, that is, AO will be equal to OB. For the points I and K being found by intersecting arches described with the same radius, these points must be equally distant from the extremities A and B; and

a right

a right line joining I and K, will also, in every other point, be equally distant from A and B; consequently the point O, where IK cuts AB, must be equally distant from A and B, that is, the line AB must be cut into two equal parts at the point O.

PROP. XXIII. fig. 3. From a given point, C, in the right line, AB, to draw a right line, CF, which shall be perpendicular to AB. Take the points D and E in AB, equally distant from the point C; then from D and E, with any radius greater than DC or CE, make the intersection F, and join FC: then will FC be perpendicular to AB, and it is drawn at the given point C, which was the thing required to be done. See Prop. 22.

PROP. XXIV. fig. 4. From a given point, C, to let fall a perpendicular on a given line, AB. From C, as a centre, with a radius reaching beyond AB, describe an arch cutting AB in two points; and from these points, with any convenient radius, make the intersection D on the side opposite to C: a line drawn from C towards D, till it touch AB, in the point E, will be perpendicular to AB; and it is drawn from the point C, as was required to be done.

PROP. XXV. fig. 5. To erect a perpendicular on the extremity B, of a given right line, AB. Choose any point, C, such that a perpendicular let fall from it, would come within the point B: from C, with the radius CB, describe the arch DBE, cutting AB in D; draw a line joining D and C, and produced till it cut the arch in E; then the segment, or arch, DBE, being a semicircle, the angle at B is a right angle, consequently the line EB is drawn perpendicular to the line AB, at the point B, as was required.

PROP.

Prop. XXVI. fig. 6. At a given point, D, of a given right line, DE, to make an angle which shall be equal to a given angle, CAB. On A, as a centre, with any radius A *b*, draw the arch *b a*; on D, as a centre with the same radius, draw an arch beginning at *e*: next take in the compasses the opening of the arch *b a*, and set the same up from *e* to *d*; and drawing the line D *d* F, this line will form with the given line DE, the angle at D equal to the given angle at A.

Prop. XXVII. fig. 7. Through a given point, C, to draw a line parallel to a given right line, AB. From any convenient point in AB, as B, with the radius BC, describe the arch CD; from C, with the same radius describe the arch BE: take the distance CD, and set it up from B to E; and draw the line CE, which will be parallel to the given line AB: for the line CB falling upon the two lines CE and AB, in such a way that the alternate angles, ECB and CBD, are equal, (Prop. 26,) it follows from what was formerly said of the proportion of parallel lines, that CE must be parallel to AB, and it is drawn through the point C, as was required.

Prop. XXVIII. fig. 8. To bisect a given angle, ABC. On B, as a centre, describe the arch AC: from these points make the intersection D: then join DB, and this line will bisect the given angle ABC.

The same line, DB, will also bisect the arch AC.

Prop. XXIX. fig. 9. To construct a triangle whose sides shall be equal to three given right lines, as AB, CD, EF. Draw the line GH, and make it equal to AB: with CD, as a radius, describe an arch at L: with EF, as a radius, describe another arch, cutting the former in the point

point L, and join LG, and LH; then will the triangle GLH have its sides respectively equal to the three given right lines.

Prop. XXX. fig. 10. To construct a parallelogram, whose opposite sides shall be equal to two given right lines, AB and CD, and which shall contain an angle equal to the given angle at E. Draw FG, making it equal to AB: at F make the angle HFG, equal to the angle at E, (Prop. 26); make FH equal to CD; on H, with a radius equal to FG or AB, and on G, with a radius equal to FH or CD, make an intersection at I, and draw HI, GI; then will the parallelogram, FHIG, have the opposite sides HI and FG, each equal to AB; FH and GI, each equal to CD; and the angles HFG and HIG, each equal to the given angle at E.

Prop. XXXI. fig. 11 and 12. Through a given point to draw a tangent to a given circle. This problem admits of two solutions: the one, when the given point A is in the circumference, as in fig. 11; and the other, when the point A is without the circle, as in fig. 12. In the first case, draw AC to the centre, and on that radius, at A, erect a perpendicular AB, which being produced to D, will touch the circle at the point A, but afterwards fall without it, and will consequently be a tangent drawn through the given point.

But in the second case, where the given point A is without the circle, draw AC to the centre, and bisecting this line in the point B, describe the circle DCE, or so much of it as may cut the given circle in the points D and E: draw the lines AD and AE, and these will both be tangents to the given circle: for if the radii, CD and CE, be drawn, the two angles ADC, AEC, being angles in a semicircle, are right angles; consequently AD and AE being at right angles to the radii, will never fall within the circle, but touch

touch it; they are, therefore, tangents drawn through the given point A, as was required to be done.

PROP. XXXII. fig. 13. To inscribe a circle in a given triangle; ABC. Bisect any two angles, as those at A and B, by the lines AD, BD, meeting in the point D; from which point draw DE and DF perpendicular to the sides AB, BC; then, as the two triangles BDE and BDF have the angles at E and F right, therefore equal, and the angles at B also equal, the remaining angles at D will likewise be equal; and the side BD being common to both, those two triangles will be equal, and consequently the side DE will be equal to the side DF. In the same way, DG may be shown to be equal to DE and DF; and a circle, described with any one of these lines for radius, would pass through the three points E, F, and G; that is, it would touch each side of the given triangle ABC, and therefore be inscribed within it, as required.

PROP. XXXIII. fig. 23, Plate I. To find the centre of a given circle, or of an arch of a circle. Take any three points A, B, D in the given circumference or arch, and drawing the right lines AB, BD, bisect them by the perpendiculars EC, FC, meeting together in the point C: this point will be the centre required.

In the same manner, a circle may be drawn to pass through three given points, provided they are not in a right line; or through the three angular points of a given triangle; that is, it may be thus described about a triangle.

PROP. XXXIV. fig. 14. Plate 2. To divide a right line AB into any number of equal parts, as 5. From either extremity, as B, draw the indefinite line BC, and taking any convenient distance in the compasses, set it off five times from B to the points numbered 1, 2, 3, 4 and 5, at

C;

C; then join CA, and through these points draw lines parallel to CA to the line AB, and cutting it in the points also marked 1, 2, &c. then will this line be divided into five equal parts, as was required to be done.

In the same manner may a line be divided into parts proportional to any given lines or magnitudes.

Prop. XXXV. fig. 15. To find a mean proportional between two right lines, as C and D. Make AE equal to the given line C, and produce it to B, making EB equal to D; upon the middle of the whole line, AB, describe the semicircle AFB; on E draw the perpendicular EF, and join AF and FB.

The angle AFB, in a semicircle, is a right angle, and therefore equal to either of the angles at E: the angle at A is common to the two triangles FAE and FAB; and the angle at B is common to the same triangle FAB and FEB; these three triangles are therefore equiangular and similar; and their corresponding or homologous sides will be proportional; that is, AE will be to EF, as EF to EB; that is, the rectangle under the extremes AE and EB, will be equal to that under the means: but the means in this case being the same quantity repeated, it follows that AE, multiplied by EB, will be equal to the square of EF; therefore EF will be a mean proportional between AE and EF: but AE is equal to C, and EB to D; EF is therefore a mean proportional between the two given lines, C and D.

Prop. XXXVI. fig. 16. To find a fourth proportional to three given right lines, A, B, C. Draw the indefinite lines DE, DF: make DG equal to A, DH equal to B, and DI equal to C: join GI, and draw HL parallel to it: then shall IL be the fourth proportional required; for GI being parallel to HL, the triangles DGI and DHL will be similar, and consequently, their correspond-

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ing sides proportionals: that is DG : DH :: DI : DL; so that DL is a fourth proportional to DG, DH, and DI; that is, to the given lines A, B, and C: or A : B :: C : IL.

PROP. XXXVII. fig. 17. To construct a square equivalent to a given parallelogram, ABCD. Draw BE perpendicular to AD, which will be the altitude of the parallelogram; find FG a mean proportional between BE and AD, (Prop. 35, fig. 15); and on FG construct the square FHIG: then, as BE : FG :: FG : AD, it follows that the square of FG will be equal to the rectangle under AD and BE: but this rectangle is equal to the parallelogram ABCD, consequently the square FHIG is made equal to the given parallelogram ABCD, which was required to be done.

If, instead of a parallelogram, it had been a triangle to which a square was to be made equal, the side of this square must have been found, a mean proportional between the altitude of the triangle and one half of the base, or between the whole base and one half of the altitude.

PROP. XXXVIII. fig. 18. To make a triangle equal to a given polygon, as, for example, to the irregular pentagon ABCDE. Join the angles C and A; through B draw BF parallel to CA, meeting AE produced in the point F, and draw FC: then the triangles CBA, CFA, standing on the same base, CA, and between the same parallels, or being of the same altitude, are equal to one another: again, joining CE, drawing DG parallel to it, and joining CG, we have the triangle CGE equal to the triangle CDE: but CFA and CGE are parts of the great triangle CFG; to these parts add the triangle ACE, which is common to the given pentagon and to the triangle CFG, and we shall have the whole triangle FCG, equal to the whole pentagon ABCDE.

By the same process, may be constructed a triangle

which

which shall be of equal area to a polygon of any other number of sides.

PROP. XXXIX. fig. 19. To inscribe a square in a given circle, ACDB. Draw two diameters, AB and CD, intersecting each other at right angles: join the extremities of these diameters, and these lines will form a square, ACBD, within the given circle. For the angles AOC, COB, BOD, and DOA, being all equal, the chords subtending these angles must also be equal: but the angles at A, C, B, and D, being angles in a semicircle, are all right; consequently the figure ACBD is a square inscribed in the given circle, as was required.

The square may be placed in any position, as GEHF, whose extremities are equally bounded by the circle.

PROP. XL. fig. 20. To describe a regular hexagon in a given circle, whose centre is C. Take the radius of the circle AC, and set it off on the circumference to B and D; draw the diameters AG, BE, and DF; join all the extremities of these lines, and the figure ABFGED will be the hexagon required: for AB and AD being each made equal to AC, which is also equal to BC and CD, these two two triangles, CBA, CAD, are equilateral: and on account of the intersecting lines BE, AG, DF, the opposite triangles must also be equilateral; consequently, EG and GF each equal to the radius of the circle; that is, to AB and AD. In the same way, the remaining sides DE and BF may be proved to be each equal to the radius, or to AB and AD; consequently the six sides of the figure to be all equal; it is, therefore, a regular hexagon inscribed within the given circle, as was required.

If lines be drawn joining the alternate angles of the hexagon A, F, E, we obtain an equilateral triangle, inscribed within a given circle.

PROP.

Prop. XLI. fig. 21. To inscribe a regular octagon within a given circle. By the 39th Prop. construct a square whose angular points touch the circumference of the circle at ADBE; divide the sides AD and DB, into two equal parts; and through the sections and the centre C, draw diameters touching the circumference in the points F, G, H, I: then right lines joining these eight points will form the octagon AFDGBHEI, as was required to be done.

ON TRIGONOMETRY.

Having in the foregoing propositions explained the chief properties of certain plain geometrical figures, as the triangle, the square, the parallelogram, the circle, &c. it is now time to furnish the student with some observations on the application of the properties of one of these figures, namely, the triangle, to sundry very important purposes.

The branch of geometry which regards the properties of triangles, is termed *Trigonometry*, a name borrowed from the Greek language, signifying the art of measuring triangles.

Trigonometry is divided into two parts, *Plane* and *Spherical*. *Plane* trigonometry is employed concerning such triangles as are formed by three sides, or lines, all lying in the same plane, such as those drawn on a sheet of paper, on a smooth even table, or the like. *Spherical* trigonometry relates to those triangles whose sides are portions of circles, such as may be described on the surface of a celestial or terrestial globe, where the sides are all curves, and situated in different planes. This latter branch of trigonometry being founded on a knowledge of parts of geometry that have not been explained in the preceding propositions, it is not intended to enter upon it in this work; the attention

of

of the student will therefore be confined to plane trigonometry alone, and to some of its most common uses in ordinary life.

In every plane triangle there are three sides and three angles, which have all such proportions one to another, that if a certain number of these six parts be given, the remainder may be discovered: for instance, if the three sides be given, the three angles may be found; if two sides, and the angle formed by them, be given, the remaining side, and the other two angles, may be found; if a side, and the angles at each extremity be given, the remaining angle and sides may be found; but if the three angles alone be given, it will be impossible to ascertain any of the sides; their relative proportions, however, may be determined, as bearing a certain ratio to the opposite angles. See the concluding observations in page 353, on fig. 4 of Plate 1.

In speaking concerning fig. 22, Plate 1, (page 361), it was said, that if an angle were formed at the central point C by the radius CD, and the right line CHE, and the arch DH were described, then a line, such as DE, drawn perpendicular to CD, consequently touching the circle in D, and produced until it met CH produced in E, would become the *tangent* of the arch HD, or of the angle HCD; again, that if the right line HD was drawn joining the extremities of the arch, it would become the *chord* of that arch, or of the angle HCD; also, that if from H we let fall HI perpendicularly upon CD, HI would become the *sine* of the same arch HD, or of the angle HCD; and lastly, that if the side CH was produced till it met the tangent at the point E, this line CE would become the *secant* of the same arch HD, or angle HCD. In this manner an infinite number of chords, sines, tangents, and secants, might be found, according to the boundless variety of angles which might be formed by the radius CD

at

at the point C. *Scales*, therefore, may be constructed, containing the chords, sines, tangents, and secants of a convenient number of arches or angles, in the following manner. See Plate 3, fig. 1.

Upon the centre C, with any convenient radius, the larger the better, describe a quadrant, or a semicircle, as AFB; divide the arch, AB, into two equal parts in the point F, and join AF; then will AF be the chord of the angle ACF, or of the arch bounded by the points A and F. The whole circumference of every circle being supposed to be divided into 360 equal parts, called degrees, a quadrant, or fourth part of the circumference, must contain 90 degrees; if, therefore, we divide the arch AF into 9 equal parts, each part will contain 10 degrees. To do this, place one foot of the compasses in the point A, and opening the other to C, turn it round, and cut the arch in the point marked 60 in the figure, which will represent 60 degrees, for the radius of any circle is always equal to the chord of one sixth part of the circumference. Then placing the foot of the compasses in the point F, with the other, at the same opening, cut the arch in the point marked 30 degrees. The given quadrant is thus divided into three equal parts, each of which is again to be divided, by repeated trials, into three other equal parts, when the several points on the arch will be obtained, viz. 10, 20, 30, 40, 50, 60, 70, 80, and 90, which will coincide with the point F. Then placing one foot of the compasses in A, open up the other until it reach the point marked 10; sweep it round, agreeably to the dotted lines on the figure, and cut the right line, or chord, AF, in the point marked 10; again, with the foot fixed in A, and the opening up to 20 on the arch, draw a dotted arch cutting the chord AF in the point marked 20; and in this manner proceed from the fixed centre A, to ascertain the points on the chord, AF, corresponding to all the remaining divisions on the arch; then will the line AF

become

become a line of chords, and so may be transferred for use to a proper instrument or scale.

Having divided the quadrant BF into 9 equal parts, each containing 10 degrees, as was done with FA, numbered from B towards F, lay a ruler from 10 on BF to 10 on AF, and mark the point where it cuts CF to be marked also 10; then will C 10 be the sine of 10 degrees; again, by a ruler laid from 20 on AF to 20 on BF the point 20 will be obtained on CF, and C 20 will be the sine of 20 degrees: in the same way the points 30, 40, &c. will be obtained, and the whole radius or line, CF, will become a line of sines. Or, the same thing may be done by employing the quadrant FB alone: thus, from the point marked 80 on the arch FB, let fall a perpendicular in the radius CB, cutting it in the point marked 10; then from the points of the arch 70, 60, &c. draw other perpendiculars to ACB, marked 20, 30, &c. when the radius CB will also become a line of sines.

Upon B, the extremity of the radius CB, erect the indefinite perpendicular BD; then from the centre C, through the several divisions 10, 20, 30, &c. of the arch FB, draw lines meeting BD in the corresponding points marked 10, 20, 30, &c.; then will the space, B 10, be the tangent of 10 degrees, B 20 the tangent of 20 degrees, and so on; B 80 the tangent of 80 degrees, and the line BD be the line of tangents.

Lastly, the line CF being produced indefinitely to E, with one foot of the compasses in C, open up the other to the point 10 upon the line of tangents BD, thus obtaining the length of the secant of 10 degrees; and turning it round to CFE, agreeably to the dotted arch, mark the point where it cuts FE, with the number 10, (omitted in the figure for want of room); next, from the same centre, C, opening up to the point marked 20 on BD, sweep the compasses round to FE, and mark 20; again, with the distance from

C to

C to 30 on BD, make another sweep to 30 on FE, and so on to 80: then will CFE become a line of secants, to be transferred to a proper scale, as before.

In this manner scales may be constructed for the purpose of measuring the chord, the sine, the tangent, or the secant of any angle whatever, and consequently the angle itself.

For measuring the sides of a triangle, or of any other right-lined figure, other scales are formed, as in fig. 11, Plate 3, where are represented two scales of equal parts, the one divided into spaces of the length of one inch, the other into spaces of half an inch.

The line on the upper scale, marked AA, is divided into three equal spaces, each of one inch, beginning at the point marked O, and proceeding to the left hand to numbers 1, 2, 3; so that the space O 1 is 1 inch, O 2 is 2 inches, O 3 is 3 inches, and so on as far as the scale may be extended. But as it must frequently occur, that the lines or distances required to be measured may not consist entirely of one or more of these spaces, it is necessary to provide smaller divisions of one of the large spaces, agreeably to some determinate ratio, as in the scale in the figure, where the inch is divided into twelve equal parts, called *lines*.

From the point marked O, set off to the right hand the space O 12, equal to O 1, or to 1 inch, and, as was directed in Prop. 34, fig. 14, Pl. 2, divide O 12 into 12 equal parts, to be numbered 1, 2, 3,—12; then will each of those divisions be one twelfth part of one inch, 4 of them will be one third of an inch, 6 will be one half, 9 three quarters, &c.; and, if it were required to take off with the compasses 1 inch and 1 twelfth part, we would place one foot in the great division to the left of O, marked 1, and opening up the other to the first small division to the right of O, the opening of the compasses would be equal to 1 inch and 1 twelfth part. In the same way, should it be required to know the length of a line which, taken in the

compasses, reached from the great division 3 to the left of the beginning of the scale at O, to the 8th small division to the right of O, we would say, the length was 3 inches and 8 twelfth parts, equal to 2 third parts of an inch.

We have now obtained a scale exhibiting inches and twelth parts of an inch; but, as it may be requisite to possess still smaller divisions, such as tenth parts of one twelfth of an inch, we proceed in the following way: parallel to the upper line, AA, and at equal intervals between each other, are drawn ten right lines, numbered at the right hand end of the scale, from top to bottom, 1, 2, 3,—10: then through the divisions of AA, marked O and 12, right lines are drawn across these ten parallels, and perpendicular to AA, cutting the lower line of the scale BB into a space of 1 inch, equal to the space O 12. The inch on BB being divided likewise into 12 equal parts, a right line is drawn from the beginning at O upon the upper line to the 1st division on the under line; then another is drawn from the 1st division on the upper to the 2d division on the lower line; another from the 2d division on the upper to the 3d on the lower line; another from the 3d division on the upper to the 4th on the lower line of the scale; and so on to the 11th division of the upper line, the line drawn from which must go to the 12th or last division on the under line of the scale: by means of which oblique lines crossing the parallels, we shall obtain measures equal to the tenth part of the twelfth part, that is, to the hundred and twentieth part of an inch.

For it is evident that the line which begins at O on the upper line or edge of the scale, and extends down across the ten parallel lines, to the 1st division on the lower line or edge, must, in its course over these ten parallel spaces, have advanced one whole division of the upper line, equal to the distance between O and 1, and, consequently, that in crossing

crossing the space next the upper line, it must have passed through one tenth part of that small division; in passing over the second parallel space, it must have advanced two tenths of the small division; in passing over the third parallel space, it must have advanced three tenths of the small division, and so on, until having crossed the whole ten parallel spaces, it must have advanced the whole of the small division on the upper line.

Were it now required to take in the compasses one inch, four twelfth parts, and three tenths of a twelfth, we would place one foot of the compasses on the upper line at the great division marked 1, and opening up the other towards the right hand to the point marked O, we would have a distance of one inch: then, to obtain the four twelfth parts, the same foot would be opened to the right hand until it extended to the small division marked 4; but to obtain the three tenths of one twelfth part, we move the compasses down across three parallel spaces, and stopping one foot at the line marked 3 at the right end of the scale, where it is cut by the division from one inch, and extending the other towards the diagonal proceeding from the fourth small division, we shall find the opening of the compasses must be a little increased to touch the intersection of that diagonal with the third parallel; when the space contained between the feet of the compasses will be one inch four twelfth parts, and three tenths of one twelfth part of an inch, as was required.

In this example the divisions of the scale have been considered according to their real value, the great divisions being in fact inches, and the small divisions lines, or twelfth parts of an inch; but as in laying down figures on paper such real measures can very seldom be employed, such as are strictly proportional must be adopted; for instance, if instead of considering the space between O and the left hand division 1 to be one inch, we suppose it to represent

 one

one foot, then each of the small divisions on the right of O must be considered as twelfth parts of a foot, that is, as inches, and the distances indicated by the advancing of the diagonal lines across the ten parallel lines will correspond to tenth parts of an inch; so that the opening of the compasses, obtained in the former example, would now represent one foot four inches, and three tenths of an inch. Again, if each of the small divisions on the upper line of the scale from O to the right as far as 12, were supposed to represent one foot, then the great division for O to the left as far as 1, would indicate twelve feet, and each of the advances made by the diagonals across the ten parallels would correspond to one tenth of a foot, in which case the opening of the compasses obtained in the first example would represent 12 and 4, or sixteen feet and three tenths of a foot.

The scale beginning on the upper line AA being divided into twelve equal parts, is purposely adapted for measures of feet, inches, &c.; but the scale beginning on the lower line BB is divided in a different proportion. The great divisions, numbered 1, 2, 3, 4, 5, 6, from left to right, are in fact each one half of an inch, but this real value is not taken into consideration. The small divisions at the left end are tenth parts, from each of which diagonal lines are drawn upwards to the opposite side of the scale, in the same manner as was done in dividing the great division at the right end; for the first diagonal commences at O on the lower line, and is drawn to the first division on the upper line; the second diagonal commences at the first division on the lower line, and is drawn to the second in the upper line, and so on to the end of the divisions.

Now, if we suppose one of the great divisions on the lower line BB to represent an unit of any sort of measure, as an inch, a foot, a yard, a mile, a league, &c. each of the small divisions will represent one tenth part of that unit,

and

and the advances made by the diagonals, across each of the ten parallels, will be one tenth of one tenth, that is, one hundredth part of the unit or great division. Again, let one of the great divisions represent 100, each of the small divisions must represent the tenth part of 100, or 10, and each step of the diagonals across the parallels will be the tenth of a tenth, or the hundredth part of the great division, which is 1 unit. Let it now be required to take off from this lower scale a quantity corresponding to 365 feet. We place one foot of the compasses at the great division marked 3 on the line BB, and opening up the other foot to the left until it come to O, we have 300 feet; next extending the foot still farther to the left to the sixth small division on the same bottom line, we obtain 360 feet; then moving the compasses upwards along the third great division to the fifth parallel, and opening them a little more, so as to touch the point of intersection of that parallel with the diagonal rising from the sixth small division, we at last obtain a space corresponding to 365 feet, as was required. Had the quantity, instead of 365 feet, been 36 feet and 5 tenth parts of a foot, the distance would have been obtained in precisely the same way; for each of the great divisions would have been considered as representing not 100, but 10 units, each of the small divisions as one unit, and each of the diagonal advances would of course have represented tenth parts of an unit: and, on the other hand, had the great divisions been supposed to represent 1,000, each of the small divisions would have been 100, and each diagonal step would have been 10; consequently, on this supposition, the same opening of the compasses would have contained 3,650 feet.

This division of the unit into ten equal parts, is that commonly adopted for constructing scales, on account of the facility of computations in decimal arithmetic, when, by the mere removal to the right or left of the point which dis-

tinguishes

tinguishes the integral from the fractional part of the quantity, the value of the quantity may be augmented or diminished indefinitely in a tenfold proportion, (see page 254 of this work): but however convenient this system may be to the calculator, it is attended with very great trouble to the constructor of instruments of mensuration, and has more than any thing else retarded the progress of such instruments to perfection. For the natural mode of division, by taking successively the half of the quantity given or discovered, is susceptible of the most scrupulous exactness, whereas the division of any given space into ten equal parts can only be accomplished by repeated trials and approximations, or by complicated machines, in the original construction of which the difficulty they are intended to remove, must itself be previously surmounted.

Lines of equal parts, and of sines, tangents, secants, &c. are commonly laid down on the *Gunter's Scale*, so called from the name of the inventor, an eminent English mathematician, who died about 1626.

By supposing the radius of a circle to be divided into a determinate number of equal parts, the respective lengths of the sine, tangent, secant, &c. of any portion of the circumference may be ascertained, and registered in tables for the purposes of proportional calculation: but as calculations by the ordinary process of addition, subtraction, multiplication, division, and the involution or evolution of roots, are liable to become very voluminous, and consequently to be susceptible of errors of great consequence, methods have at various periods been adopted to shorten the labour and diminish the occasions of error in arithmetical operations. Of such methods, by far the most complete is the use of certain artificial numbers called *Logarithms*, from two Greek terms signifying the numbers of the ratios or proportions existing between other numbers with which they are connected.

If

If, for instance, we take a set of numbers increasing by a given geometrical progression, such as that every succeeding number shall be double its predecessor, as 1, 2, 4, 8, 16, 32, 64, 128, &c. and opposite to these place a set of numbers increasing by a given arithmetical progression, such that every succeeding number shall be 1 more than its predecessor, then it will be found that, by simply adding together these last numbers, the same effect will be produced as if we had multiplied the corresponding numbers of the first set; and that by subtracting the last set, the same effect is produced as if the first set had been divided, and so on with all other operations, as may be seen from the following table, where the upper row of numbers are in arithmetical progression differing by 1, and the lower row are numbers in geometrical progression, increasing in a twofold proportion.

0	1	2	3	4	5	6	7	8	9
1	2	4	8	16	32	64	128	256	512

Here the upper numbers express the numbers of the ratio of the lower row; for, if the given quantity at the beginning of the geometrical progression be unity or 1, the first step of the progression, or 2 times $1=2$, is expressed by the figure 1 over the 2; the second step, $2\times2=4$, is expressed by 2 over 4; the third step, $2\times2\times2=8$, is expressed by 3 over 8; the fourth step, $2\times2\times2\times2=16$, is expressed by 4 over 16, and so on to the ninth step, which is expressed by 9 over 512: consequently, the upper row of figures may be considered as the indexes of the proportions between those in the lower row, or, in other words, the upper

upper row contains the *logarithms* of the *natural numbers* in the lower row.

Now, for example, let it be required to multiply together any two of the lower row, as 4 and 8; we find the index or logarithm of the table standing over 4 is 2, and that over 8 is 3: then, by adding together the indexes 2 and 3, we have 5, and observing the natural number placed under 5, we obtain 32, which is the product of 4 multiplied into 8.

Again, to multiply 4 by 16, and the product by 8, we add together the index of $4=2$, the index $16=4$, and the index of $8=3$, in all 9, which is the index of 512, the product of $4\times 16\times 8$.

Let it be required to divide 512 by 8: the index or logarithm of 512 is 9, and that of 8 is 3; subtracting 3 from 9, the remainder is 6, which is the logarithm of 64, the quotient of 512 divided by 8, as was proposed.

To square any number, as 8, we have only to double its logarithm 3, to have 6 the logarithm of 64, the square of 8: and the cube of 8 will be found by tripling its logarithm 3, thus obtaining 9, the logarithm of 512, which is the cube of 8.

Again, to extract the square root of any number, as 64, we have only to take one half of the logarithm 6 or 3, which is the logarithm of 8, the square root of 64: and to extract the cube root of 512, we take one third part of its logarithm 9 or 3, which is the logarithm of 8, the cube root of 512.

From the consideration of such properties of a series of arithmetical proportionals, compared with one of geometrical proportionals, *John Napier, Baron of Merchiston*, in Scotland, gave to the world, in 1614, a collection of tables of natural sines and sines-complement, with their corresponding logarithms, calculated for every minute of the quadrant of a circle; together with directions how, from

those

those tables, to form the logarithms of numbers. No sooner was this most useful application of the properties of numbers made known, than ingenious men in various parts of Europe set themselves with earnestness to understand and improve the discovery; but it is chiefly to *Henry Briggs*, first of Gresham College in London, and afterwards professor of geometry at Oxford, that the world is indebted for their improvement, in consequence of his repeated visits to *Napier*, and concerting with him such alterations, in the form of logarithmic tables, as might best promote the ends of their construction.

In the specimen above given, the natural numbers are geometrical proportionals increased in a twofold ratio; but in the logarithmic tables now used, the progression is in a tenfold ratio; so that 0 being the logarithm or index of 1 or unity, 1 will be the logarithm of 10, 2 the logarithm of 100, 3 the logarithm of 1000, and so on: in the same way the logarithm of the tenth part of an unit will be — 1, of the hundredth part it will be — 2, of the thousandth part it will be — 3, and so on.

When the logarithms of 1, 10, 100, 1000, &c. have thus been determined, it is necessary to discover such as correspond to each intermediate number between 1 and 10, between 10 and 100, &c. Let it be required, for instance, to discover the logarithm of 5 agreeably to this tenfold ratio of the natural numbers, where the logarithm of 1 is 0, and that of 10 is 1. Find a geometrical mean between 1 and 10, that is, multiply them together, and extract the square root of the product, which will be found to be 3,162277; then find the arithmetical mean between 0 and 1, that is, take one half of their sum, which is ,5; consequently the logarithm of the natural number 3,162277 is 0,5.

The number 3,162277 being less than 5, the number for which a logarithm is required, we again proceed to find an-

other geometrical mean nearer to 5, between the extreme now found, 3,162277, and the given extreme 10, by extracting the square root of the product of these numbers multiplied together, which root will be 5,623413. We then find an arithmetical mean between 0,5 and 1, by taking one half of their sum, which will be 0,75, being the logarithm corresponding to the above natural number 5,623413: but this number being greater than 5, and the former number 3,162277 being less than 5, we multiply them together, and extracting the square root of the product, we obtain the number 4,216964; then taking the half of the sum of the logarithms corresponding to these two numbers, namely, 0,75 and 0,5, which is 0,625, we have the logarithm corresponding to the last found number, 4,216964. Again, this last number being less than 5, and the former number, 5,623413, being greater than 5, proceeding as before we find the geometrical mean between them, which is 4,869674; and taking 0,6875, the arithmetical mean between the logarithms of the same numbers, we obtain the logarithm 4,869674. This last sum being the nearest below 5, and the former number, 5,623413, the nearest above 5, we take 5,232990, the geometrical mean between them; and for its logarithm we take 0,71875, the arithmetical mean between their corresponding logarithms 0,6875 and 0,75. Again, the last found number, 5,322920, being the nearest above 5, and 4,869674 being the nearest below 5, we find the geometrical mean 5,0480645, the logarithm of which is 0,703125, being the arithmetical mean between the logarithms of the numbers employed on the occasion, viz. 0,71875 and 0,6875.

By carrying on this process, if the student shall make the calculations himself, he will find that, the results producing natural numbers gradually approaching nearer and nearer to the given number 5, for which a logarithm is required, after the twenty-second operation is performed, the

the logarithm corresponding to 5, will be very nearly 0.698970.

The following table exhibits some steps of this process for discovering logarithms, where the 2d column contains natural numbers having, for distinction's sake, the letters of the alphabet prefixed in the 1st column, by which to refer to the several numbers as they come to be employed in the calculations. The 3d column contains the geometrical mean of the respective pairs of numbers connected together by brackets in the 2d column; and in the 4th column is entered the logarithm corresponding to the mean in the 3d column, which logarithm is the arithmetical mean of the logarithms in the 5th column, corresponding respectively to the numbers on the same line in the 2d column: for example, the 1st number 1, or A, and the 2d number 10, or B, being multiplied together, and the square root of the product being extracted, the result 3,162277, marked C, appears in the 3d column, on a line with which, in the 4th column is 0,500000, its corresponding logarithm, being the arithmetical mean (that is, one half of the sum) of the two logarithms in the 5th column, corresponding to the numbers A and B.

TABLE.

	Numbers.	Geom. mean	Arith. mean	Logarithms.
A	1,000000	C		0,000000
B	10,000000	=3,162277	0,500000=	1,000000
B	10,000000	D		1,000000
C	3,162277	=5,623413	0,750000=	0,500000
D	5,623413	E		0,750000
C	3,162277	=4,216964	0,625000=	0,500000
D	5,623413	F		0,750000
E	4,216964	=4,869674	0,687500=	0,625000
D	5,623413	G		0,750000
F	4,869674	=5,232990	0,718750=	0,687500
G	5,232990	H		0,718750
F	4,869674	=5,0480645	0,703125=	0,687500
H	5,0480645			
F	4,869674	=&c.		

 Having

Having in this manner pointed out one of the various methods employed in the calculation of logarithms, which, however tedious and laborious in practice, is well fitted to give the young geometrician an idea of the nature of such numbers, and of their relation to natural numbers; it will be evident that, by having discovered the logarithm of 5, those of several other quantities may also be discovered, without having recourse to fresh processes of calculation of the same kind with the preceding; for the logarithms of the numbers 1 and 10 being given, and that of 5 now found, those of other numbers related to 1, 5, and 10, may, from the properties of logarithms, be readily discovered. Thus, the logarithm of 10 being 1,000000, and that of 5 being 0,698970, if we subtract this latter sum from the former, the remainder 0,301030 will be the logarithm of 2, which is the quotient of 10 divided by 5: again, knowing the logarithm of 2, if we add it to itself, we have 0,602060, which is the logarithm of 4, or 2 multiplied by itself: if to this logarithm we add 1,000000, the logarithm of 10, we have 1,602060, or the logarithm of 4 multiplied by $10 = 40$: if to the logarithm of 4 we add the logarithm of 2, we have 0,903090, or the logarithm of $8 = 4$ multiplied by 2: if to the logarithm of 5 we add itself, we obtain 1,397940, the logarithm of 5 multiplied by $5 = 25$: and by similar combinations of the logarithms thus discovered, and of others appearing in the course of the operations, a great number of logarithms may be found; it is however to be observed, that in order to give results in this way near the truth, it will be necessary to calculate the first logarithms to many more places of decimals than those given in the foregoing examples.

Description and Use of the Tables of Logarithms.

According to the mode now adopted in constructing tables of logarithms, by which they increase and diminish

in

in a tenfold proportion, the logarithm of unity, or 1, being 0, that of 10 will be 1, of 100 will be 2, of 1000 will be 3, of 10,000 will be 4, and so on; or, in other words, the integral part of the logarithm, called the *index* or *characteristic*, will always be one less than the number of figures of which the natural number consists: and the logarithms of fractional numbers are subject to the same law, for the logarithm of 1 being 0, that of one tenth, or ,1, will be one, with the negative sign before it, thus, — 1, that of one hundredth, or ,01, will be — 2, that of one thousandth, or ,001, will be — 3, and so on. Hence it follows, that any numbers differing the one from the other in a tenfold proportion, will have the fractional part of their logarithms composed of the same figures, but the integral part will vary according to the number of figures in the natural number. Take, for example, the figures 1808, as below.

Nat. numbers.		Logarithms.
1808	=	3,25720
180,8	=	2,25720
18,08	=	1,25720
1,808	=	0,25720
,1808	=	— 1,25720
,01808	=	— 2,25720

Tables of logarithms consist of a set of parallel and perpendicular columns, each subdivided into two other columns, of which the first on the left hand contains the natural numbers, proceeding in order from 1 to the end of the tables; and the second column on the right hand contains the logarithm corresponding to each natural number, as in the specimen here given on the margin, where the logarithm

of

N.	Log.
1	0,00000
2	0,30103
3	0,47712
4	0,60206
5	0,69897
6	0,77815
7	0,84510
8	0,90309
9	0,95424
10	1,00000
11	1,04139
12	1,07918

of 3, for instance, is 0,47712, that of 8 is 0,90309, that of 12 is 1,07918, &c.: but as the index or characteristic from 1 to 10 is always 0, from 10 to 100 always 2, from 100 to 1,000 always 3, &c. it has been customary, in order to save room, to omit this index in the tables, so that many now exhibit the natural numbers and their logarithms as in the annexed specimen; where the index of 5 or 0, the index of 9 also 0, the index of 12 or 1, and the index of 365 or 2, are all omitted, and to be supplied by the calculator as necessary.

N.	Log.
5	69897
9	95424
12	07918
365	56229

The method of using logarithmic tables is very simple: suppose it were required to find in the tables the logarithm of the natural number 365; we look along the left hand column, marked N, for the given number, and on the same line in the right hand column, marked Log. we find 2,56229, or merely the fractional part, 56229, where the index or characteristic must be supplied, namely 2, being 1 less than the 3 places of figures of which the number 365 consists: on the other hand, had it been required to find the natural number corresponding to a given logarithm 2,56229, we look down the column of logarithms for the given number, or the nearest to it, and having met with it, we have in the same line in the left hand column 365, which is the number required. But as it may often occur that the logarithm will be required for a greater number than appears in the tables, as, for instance, if the tables contain logarithms for

for numbers of only three places of figures, and we want to have the logarithm of 3685, which consists of four places, we take first the fractional part of the logarithm of 368 for 3680, which is 56585, and then the fractional part of the logarithm of the next number above it, 369, for 3690, which is 56703, exceeding the former logarithm by 118: then we state this proportion, as the difference between 3680 and 3690, or 10, is to the difference of the logarithms in the tables, or 118, so is 5, the last figure of the given number 3685, to a fourth proportional, 59, which being added to the logarithm for 3680, or 3,56585, will give 3,56644 for the logarithm of 3685, as was required.

Again, when the given logarithm for which the corresponding natural number is sought, is not found precisely in the tables, we proceed as in the following example, where the natural number is wanted, which corresponds to the logarithm 3,56644. The index of this logarithm being 3, the number must of course contain four places of figures, and therefore consist of units, tens, hundreds, and thousands: but the tables being by supposition, calculated for nothing above hundreds, we omit the index 3, and look for the nearest fractional part below the given logarithm, 56644, which is 56585, answering in this case to the number 3680, and also for the nearest above it, which is 56703, answering to 3690: then subtracting the lowest logarithm from the highest, the difference is 118, and subtracting the same from the given logarithm, the difference is 59: hence we state this proportion, as the first difference, 118, to the second 59, so is the difference between 3680 and 3690, or 10, to a fourth proportional, which here will be 5, and this added to the lowest number 3680, will give 3685, the natural number sought for, corresponding to the logarithm 3,56644.

It would exceed the limits proposed for this work to furnish

nish the student with sets of logarithmic tables; he must therefore have recourse for them to those already given to the public, in various treatises on similar subjects, as well as by themselves; but to assist him in acquiring a knowledge of the nature and uses of such tables, the following specimen is annexed, containing all natural numbers, with their corresponding logarithms from 1 to 100, both included.

LOGARITHMS OF NUMBERS.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	00000	26	41497	51	70757	76	88081
2	30103	27	43136	52	71600	77	88649
3	47712	28	44716	53	72428	78	89209
4	60206	29	46240	54	73239	79	89763
5	69897	30	47712	55	74036	80	90309
6	77815	31	49136	56	74819	81	90849
7	84510	32	50515	57	75587	82	91381
8	90309	33	51851	58	76343	83	91908
9	95424	34	53148	59	77085	84	92428
10	00000	35	54407	60	77815	85	92942
11	04139	36	55630	61	78533	86	93450
12	07918	37	56820	62	79239	87	93952
13	11394	38	57978	63	79934	88	94448
14	14613	39	59106	64	80618	89	94939
15	17609	40	60206	65	81291	90	95424
16	20412	41	61278	66	81954	91	95904
17	23045	42	62325	67	82607	92	96379
18	25527	43	63347	68	83251	93	96848
19	27875	44	64345	69	83885	94	97313
20	30103	45	65321	70	84510	95	97772
21	32222	46	66276	71	85126	96	98227
22	34242	47	67210	72	85733	97	98677
23	36173	48	68124	73	86332	98	99123
24	38021	49	69020	74	86923	99	99564
25	39794	50	69897	75	87506	100	00000

Multiplication is performed by logarithms, as in the following

following example, where it is required to multiply 12 by 8. Look in the proper tables, or in the preceding specimen, for the logarithms corresponding to the two factors; add them together, and in the column of logarithms, find one equal, or the nearest to this sum, the natural number opposite to which will be the product required.

$$\left.\begin{array}{l}\text{Logarithm of } 12 = 1{,}07918 \\ \times \text{———————} 8 = 0{,}90309\end{array}\right\}\text{add}$$

$$96 = 1{,}98227$$

Multiply 3 by 5, and the product by 6.

$$\left.\begin{array}{l}\text{Logarithm of } 3 = 0{,}47712 \\ \times \text{———————} 5 = 0{,}69897 \\ \times \text{———————} 6 = 0{,}77815\end{array}\right\}\text{add}$$

$$90 = 1{,}95424$$

Division is performed by subtracting the logarithm of the least quantity from that of the greatest, when the remainder will be the logarithm of the quotient; as in this example, where 96 is to be divided by 12.

$$\begin{array}{r}\text{Logarithm of } 96 = 1{,}98227 \\ - \text{———————} 12 = 1{,}07918 \\ \hline \text{Quotient } 8 = 0{,}90309\end{array}$$

Divide 90 by 6, and the quotient by 5.

$$\begin{array}{r}\text{Logarithm of } 90 = 1{,}95424 \\ - \text{———————} 6 = 0{,}77815 \\ \hline 15 = 1{,}17609 \\ - \text{———————} 5 = 0{,}69897 \\ \hline 3 = 0{,}47712\end{array}$$

Proportion, or the *Rule of Three*, is performed by adding together the logarithms of the 2nd and 3rd terms, and from the sum subtracting the logarithm of the 1st term; as in

in the following example. If for 3*l.* I purchase 5 yards of cloth, how much can I have for 12*l.*?

$$3 : 5 :: 12 :$$

$$\begin{aligned} \text{Log. of } 5 &= 0{,}69897 \\ + \text{——} 12 &= 1{,}07918 \\ 60 &= 1{,}77815 \\ \text{———} 3 &= 0{,}47712 \\ \text{Yards } 20 &= 1{,}30103 \end{aligned}$$

If 64*l.* gain 35*l.* in any given time, how much should 320*l.* gain in the same time?

$$64 : 35 :: 320$$

$$\begin{aligned} \text{Log. of } 320 &= 2{,}50515 \\ + \text{———} 35 &= 1{,}54407 \\ & 4{,}04922 \\ \text{———} 64 &= 1{,}80618 \\ 175 &= 2{,}24304 \end{aligned}$$

In this example, as the number 320 is not found in the preceding specimen of logarithms, we take the logarithm for 32, the fractional part of which is the same with that of the logarithm for 320, and prefixing the index or characteristic 2, because of the three places of figures in the latter number, we obtain 2,50515 for the logarithm of 320: to this adding the logarithm of 35, and from the sum subtracting the log. of 64, we have the remainder 2,24304, for which the natural number is not within the bounds of the specimen; it must, therefore, be discovered in the way before pointed out: we take the nearest number above it in the table, which is, 25527, the log. of 180, and the nearest below it, which is, 23045 the log. of 170; then we state the proportion, as the difference between these two logarithms,

rithms, or 2482, to the difference between their corresponding numbers 170 and 180, or 10, so is the difference between the lowest log. and that above found, or 1259 to a fourth proportional, which will be 5; and this added to the lower number, 170, will give 175, the sum of interest required in the question.

Involution of Roots is performed by multiplying the logarithm of the given number by the exponent of the power to which it is to be raised, when the product will be the logarithm of the power required. For example, raise 8 to the 2nd power, or square.

Log. of 8 =	0,90309
2nd power	× 2
Square 64 =	1,80618

Required the cube of 4?

Log. of 4 =	0,60206
3rd power	× 3
Cube 64 =	1,80618

Evolution of Roots is performed by dividing the logarithm of the given number by the exponent of the power, when the quotient will be the logarithm of the required root. For example, extract the square root of 64.

Log. of 64
Divide by 2)1,80618

Root 8 = 0,90309

Extract the cube root of 64.

Log. of 64
Divide by 3)1,80618

Root 4 = 0,60206

 The

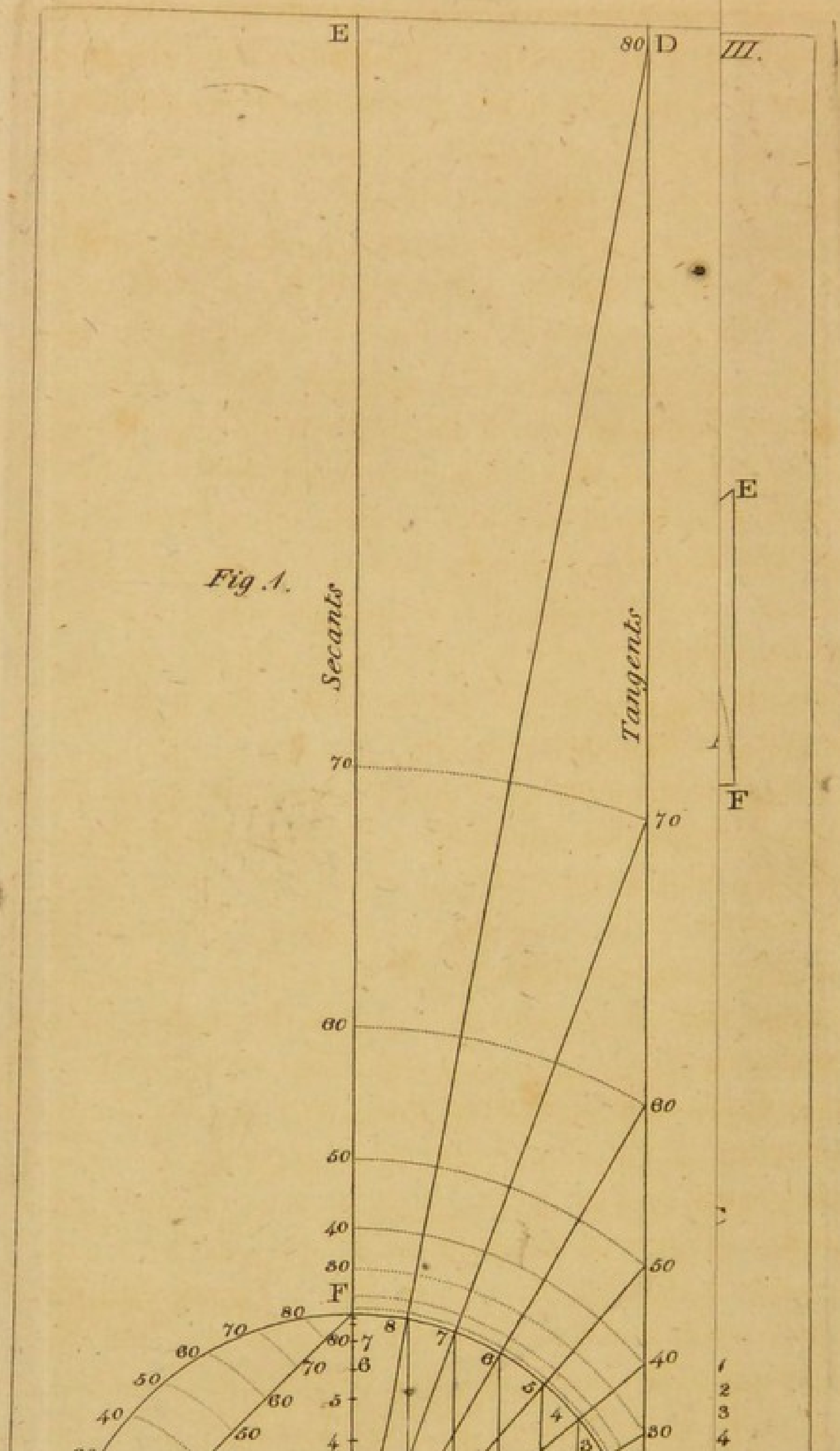
The preceding examples will show the manner of using tables of the logarithms of numbers, and those of logarithmic sines, tangents, secants, &c. are employed precisely in the same way, of which examples will be given in the following illustrations of the several cases of plane trigonometry.

PROP. I. Fig. 2, Plate 3. The radius of every circle is equal to the chord of 60 degrees, and half the radius is equal to the sine of 30 degrees.

Let BDAE be a part of a circle of which C is the centre; at C, with the radius CB, form an angle, BCA, equal to 60 degrees, and join AB, which will then be the chord of 60 degrees. But the sides CA and CB being equal, each being a radius of the circle, the angles at A and B must be equal (Geometry, Prop. 4), and the angle at C being made equal to 60 degrees, the two remaining angles must together be equal to 120 degrees, (page 383, and Geom. Prop. 7), and each of them equal to 60 degrees: the three angles, therefore, of the triangle BAC being all equal, the three sides must also be equal (Geom. Prop. 4), and AB must be equal to CA or CB; but CA and CB are each of them a radius of the circle, consequently AB, the chord of 60 degrees, is equal to the radius of the circle.

Again, if from the centre C, the line CD be drawn bisecting the angle ACB, and consequently bisecting the chord AB in the point F, CD will be a radius, and AF will be the sine of 30 degrees (page 382). But the triangles ACF and FCD having the side AC equal to CB, and the side CF common, as also the angle BCF equal by construction to the angle FCB, the remaining side AF will be equal to the remaining side FB, (Geom. Prop. 3), consequently the sine of 30 degrees will be equal to one half of AB, that is, of the radius of the circle.

PROP.



III.
Fig. 1.
E
D
A
B
C
F
Secants
Tangents
Chords
Sines

PROP. II. Fig. 3, Plate 3. The sides of any plane triangle are to each other in the proportion of the sines of the angles opposite to each side respectively.

In the triangle ABC, the side AB is to the side BC, as the sine of the angle ACB opposite to AB, is to the sine of the angle BAC opposite to BC; and AB is to AC as the sine of the angle ACB to the sine of the angle ABC: again, BC is to AC, as the sine of BAC to the sine of ABC.

Through the three angular points of this triangle let a circle be drawn (Geom. Prop 33), of which the centre will be the point D; join DA, DB, and DC; from D draw the right lines DE, bisecting the angle ADB, and, consequently, (Geom. Prop. 3), the side AB in the point F, the line DG bisecting the angle BDC and the side BC in the point H, and the line DI bisecting the angle ADC and the side AC in the point L.

From the last proposition it is evident, that in the angle BDG, DB and DG being each a radius of the same circle, and BH falling perpendicularly on DG, it will be the sine of the angle BDG, or of the arch BG: but the angle BDG was by construction made one half of the angle BDC, which being an angle at the centre, is double the angle at the circumference BAC, (Geom. Prop. 10), consequently, the angle BDG must be equal to BAC, and the line BH must correspond to the sine of the same angle BAC. In the same way it may be shown, that the line BF must correspond to the sine of the angle BCA, and CL to the sine of the angle ADC; but these lines, BH, BF, and CL, being respectively the halves of BC, BA, and AC, and the whole of any quantities having the same proportions to each other as their halves, it follows, that the three sides of the given triangle ABC must be to each other in the proportion of the sines of the angles respectively opposite to each side.

Corollary. The reverse of this proposition is also true, that

that the sines of the three angles of any plane triangle are to one another, as the respective sides opposite to the angles.

PROP. III. Fig. 4, Plate 3. In any two unequal quantities, one half of their difference, added to one half of their sum, will give the greater quantity; and one half of their difference subtracted from one half of their sum, will give the less quantity.

Let the line AB represent the greater of two quantities, and the continuation of the same line BC represents the less; then the whole line AC will be the sum of the two quantities, which being divided into two equal parts in the point E, (Geom. Prop. 22), the lines AE and EC will each be one half of the sum of the given quantities.

Again, from the extremity A set off AD equal to the less quantity BC, and the space DB will represent the difference between the quantities: and AE being made equal to EC, and AD to BC, these two parts being taken away, the remainder DE will be equal to the remainder EB, but DB being the difference of the quantities, DE or EB will each be one half of that difference.

Now, if to AE half the sum of the given quantities, we add EB, half their difference, the sum is AB, or the greatest quantity; and if from the same AE, or half the sum, we take away DE, or half the difference, the remainder is AD, which was made equal to BC, the less quantity.

The same thing may be shown arithmetically, in this way; let the greater quantity AB be 8, and the less BC be 4, then their sum is 12, and their difference is 4.

$$\begin{array}{rl} \text{half the sum} & = 6 \\ + \text{ half the difference} & = 2 \\ \hline & 8 = \text{greater quantity.} \end{array}$$

$$\begin{array}{rl} \text{half the sum} & = 6 \\ - \text{ half the difference} & = 2 \\ \hline & 4 = \text{less quantity.} \end{array}$$

PROP.

PROP. IV. Fig. 5, Plate 3. In a right-angled plane triangle, the base will be to the perpendicular in the proportion of radius to the tangent of the angle opposite to the perpendicular.

Let the triangle ACB be right-angled at B, then will AB be to BC as radius to the tangent of the angle CAB. From the angular point A, with any convenient radius, describe the arch ED, and at E draw the perpendicular EF, which will be the tangent of the angle at A. The lines BC and EF being both perpendicular to AB, they must be parallel to each other, and the two triangles AEF and ABC must be similar, consequently (Geom. Prop. 19), the side AB will be to BC, as the side AE is to EF: but the side AE was made the radius of a circle to measure the angle BAC, and EF is the tangent of the same angle, therefore the base AB is to the perpendicular BC, as radius to the tangent of the angle CAB; or, conversely, the radius is to the tangent of the angle CAB, as the base AB to the perpendicular BC.

PROP. V. Fig. 8, Plate 3. In a right-angled triangle, as ABC, if the base AC be made radius, the perpendicular BC will become the tangent of the angle at A, and the hypothenuse AB will become the secant of the same angle. Again, if the hypothenuse AB be made radius, the perpendicular BC will become the sine of the angle at A, the base AC will be the sine of the angle at B, and the line EF, drawn touching the circle BF, will be the tangent of the angle at A.

If, therefore, the hypothenuse be given, the other sides will be proportional to the sines of the opposite angles; and if the base be given, the hypothenuse will become the secant, and the perpendicular will become the tangent of the acute angle at the base.

PROP.

Prop. VI. Fig. 6, Plate 3. In any plane triangle, as ABC, the sum of the two sides, AB and BC, opposite to the base AC, is to the difference between these sides, as the tangent of half the sum of the two angles at the base BAC, and BCA is to the tangent of half the difference between these angles.

On the angular point B, with the side BC for radius, describe a circle cutting AB in the point E, and the same line produced in the point D: draw DC, and also EF, parallel to it, and join EC. Then BD being equal to BC, each being a radius of the circle, the whole line ABD will be equal to AB and BC together, that is, AD will be equal to the sum of the two sides opposite to the base; and BE being a radius of the circle, it is equal to BC, consequently, the part AE will be the difference between the two sides AB and BC.

Again, the two triangles BAC and BEC, having a common angle at B, the remaining two angles of the one must be equal to the remaining two angles of the other (Geom. Prop. 7), therefore the two angles BEC and BCE, which are equal to one another, as subtended by equal sides (Geom. Prop. 4), are together equal to the two angles BCA and BAC, the angles at the base: and half these last angles must be equal to BEC, which thus becomes equal to half the sum of the angles at the base of the given triangle; and DCE being the angle of a semicircle, DC must be perpendicular to CE (Geom. Prop. 10), and, consequently, the tangent of the angle BEC, or the tangent of half the sum of the angles at the base.

Again, the difference between the angles BCE and BCA being the angle ECA, and the difference between BEC (which is equal to BCE) and BAC being the same angle ECA, (Geom. Prop. 6), it follows, that the difference between the angles at the base, BCA and BAC, will be equal to twice the angle FCA, and that ECA alone will be equal

equal to half that difference; and EF having been drawn parallel to DC, it will be at right angles to EC, and, consequently, if CE be radius, EF will become the tangent of the angle ECF or ECA, that is, it will be the tangent of half the difference of the angles at the base. But in the triangle ADC, the line EF being drawn parallel to the side DC, the other sides are cut proportionally (Geom. Prop. 19), consequently, AD will be to AE, as DC to EF; AD, however, is the sum of the sides, and AE their difference; while DC was shown above to be the tangent of half the sum of the angles at the base, and EF is the tangent of half their difference; the proposition is, therefore, demonstrated.

Prop. VII. Fig. 7, Plate 3. If in the plane triangle ABC, a line DF be let fall perpendicularly on the base AC, from the opposite angle at B, dividing the base into two segments AF and FC; then will the base AC be to the sum of the opposite sides AB + BC, as the difference between these sides AB — BC, to the difference between the segments of the base AF — FC.

On B, with the length of the side AB for radius, describe a circle cutting AC in the point D, and CB in G, also CB produced in the point E, and draw BD: then BE being equal to BA, the whole EC will be equal to the sum of the sides opposite to the base; and BG being also equal to BD, the space GC will be equal to the difference between the same sides. From the properties of the circle it follows, that all lines drawn cutting it, from any external point, and terminated on the opposite circumference, will be cut reciprocally proportionals: consequently, the whole line CA will be to the whole line CE, as the intercepted portion of this last line CG is to the intercepted portion of the base DC; but AC is the base of the triangle, CE is the sum of

 the

the opposite sides, CG is the difference between these sides, and DC is the difference between AF and FC, the segments of the base made by the perpendicular BF, let fall from the opposite angle, agreeably to the statement of the proposition.

Had the perpendicular been let fall from either of the acute angles at A or C, it would have gone without the triangle, and required the opposite side to be produced, in order to meet it; when the proportion would have been this; as the base is to the sum of the opposite sides, so is the difference of these sides to the *sum* of the segments (of the base when produced) made by the perpendicular.

WHEN the student is acquainted with the properties of triangles exhibited in the preceding propositions, he will be able to understand the process of solving the following cases of plane trigonometry, which are classed into those belonging to right-angled, and those belonging to oblique-angled triangles; and, first, of right-angled triangles.

CASE I. Fig. 9, Plate 3.

Given the two legs, or the base and the perpendicular. Required the angles and the hypothenuse.

In the plane triangle ABC, right-angled at A, are given the base AC = 440 integers of any dimension, as feet, miles, &c. and the perpendicular AB = 308 of the same integers, and the angles at B and C, as also the hypothenuse BC, are required.

By the preceding 4th Prop. of Trigonometry, it was shown, that if the base of a right-angled triangle be made radius, the perpendicular will become the tangent of the opposite angle at the base. In this case, therefore, we state the proportion.

As the base AC = 440,
To the pependicular AB = 308,

So

So is the radius of the tables, which is always equal to the sine of 90 degrees, (pages 382 and 384, Fig. 1, Plate 3,) or 10,00000,

To the tangent of the angle BAC.

Or, thus,

Log. 440 :	Log. 308 ::	Sine 90,00 :	Tang.
2,64345 :	2,48855 ::	10,00000 :	
	+ 10,00000		
	12,48855		
	— 2,64345		
BAC = 35°,00′ =	9,84510		

Having found in the tables the logarithms of the first and second terms, we write them down together with the standing logarithmic sine of radius, which is 10,00000; then, agreeably to what was said of the nature of logarithms, add the second and third terms together, and from their sum subtract the logarithm of the first term. The remainder being a logarithmic tangent, we search in the tables of tangents for the number equal or nearest to it, and in the column of degrees and minutes, we find 35 degrees, without any minutes, to be the quantity required for the angle BCA.

As the three angles of any triangle, are together equal to two right angles (Geom. Prop. 7), and as the angle at A was made a right angle, the remaining two at B and C must together be equal to one right angle, or 90 degrees, (p. 383); subtracting then the angle at C = 35 degrees, from 90 degrees, the remainder, = 55 degrees, must be the quantity of the angle at B, as was required.

Again, to find the length of the hypothenuse BC, we proceed in the following way; supposing the base AC still

to be radius, the hypothenuse will become the secant of the angle at C, whence we have this proportion; as radius to the secant of the angle at C, so is the base AC to the hypothenuse BC; but as the secants are seldom inserted in logarithmic tables, such problems, where they can be introduced, being susceptible of a solution by means of other lines, we proceed with the following proportion, in which the hypothenuse itself being made radius, the perpendicular AB becomes the sine of the angle at C, (Trig. Prop. 5).

As the sine of the angle at C	$= 35°,00' =$	9,75859
To radius	$90°,00' =$	10,00000
So is the side AB	$= 308 =$	2,48855
To the hypothenuse BC	$= 537 =$	2,72996

Here, by adding the logarithms of the second and third terms together, and subtracting from the sum that of the first term, the remainder 2,72996 is found in the tables to be the nearest number to that corresponding to the natural number 537, which is the measure of the hypothenuse, as required.

It was formerly shown, (Geom. Prop. 18), that in all right-angled triangles the square constructed on the hypothenuse is equal to the sum of the two squares constructed on the opposite sides containing the right angle. From this property we may, in the present case of trigonometry, discover the measure of the hypothenuse BC, without having recourse to the logarithmic tables, by means of the following arithmetical operation.

Square

Square the side AC = 440	Square AB = 308
440	308
17600	2464
1760	924
193600	94864
+ 94864	

Extract the square root) 288464 (537 = hypoth. BC, agreeably to what was before found.

```
Extract the square root)288464(537
                        25
                        --
                  103)384
                      309
                      ---
                 1067)·7564
                       7469
                       ----
                        ·95
```

Case II. Fig. 9, Plate 3

Given the hypothenuse and one leg. Required the other leg and the angles.

In the triangle ABC, right-angled at A, are given the hypothenuse BC = 537, and the perpendicular AB = 308; required the base AC, and the two angles ABC and BCA.

Making the hypothenuse BC radius (Trig. Prop. 5), and C the centre, the perpendicular AB becomes the sine of the angle BCA: hence we have the proportion.

As the hypoth. CB	= 537	= 2,72996
To the perpend. BA	= 308	= 2,48855
So is radius	= 90°,00′	= 10,00000
To the sine of BCA	= 35°,00′	= 9,75859
Consequently, ABC	= 55°,00′	
Equal to one right angle	= 90,00.	

Now,

Now, to find the base AC, we may employ the hypothenuse as radius, and make the centre at B, when the base will become the sine of the angle at B.

As radius	$= 90°,00' =$	$10,00000$
To the sine of ABC	$= 55°,00' =$	$9,91336$
So the hypoth. BC	$= 537 =$	$2,72996$
To the base AC	$= 440 =$	$2,64332$

Or, by employing the perpendicular BA as radius, and making B the centre, we have the proportion,

As radius	$= 90°,00' =$	$10,00000$
To the tangent of ABC	$= 55°,00' =$	$10,15477$
So the perpend. BA	$= 308 =$	$2,48855$
To the base AC	$= 440 =$	$2,64332$

Or, the base may be found arithmetically, as was shown in the preceding case, by subtracting the square of the given perpendicular AB = 308, from the square of the hypothenuse BC = 537, and extracting the square root of the remainder, which will be 440, as before.

Case III. Fig. 9, Plate 3.

Given one leg and the angles. Required the other leg and the hypothenuse.

In the triangle ABC, right-angled at A, are given the base AC = 440, and the angle ACB = 35°,00′, consequently the angle ABC = 55°,00′ (Geom. Prop. 7); required the perpendicular AB and the hypothenuse BC.

Making the given base AC radius, the perpendicular AB will become the tangent of the angle at C: hence we have this proportion;

As

As radius	$=90°,00'$	$=$	10,00000
To the tangent of BCA	$=35°,00'$	$=$	9,84523
So is the base AC	$=$ 440	$=$	2,64345
To the perpendicular AB	$=$ 308	$=$	2,48868

Again, if the hypothenuse be made radius, and the angle at B be made the centre, the given base AC will become the sine of the angle at B; from which statement is obtained the following proportion;

As the sine of the angle at B	$=55°,00'$	$=$	9,91336
To radius	$=90°,00'$	$=$	10,00000
So is the base AC	$=440$	$=$	2,64345
To the hypothenuse BC	$=537$	$=$	2,73009

The hypothenuse may also be discovered arithmetically, as was already shown, by adding together the squares of the given base, and the perpendicular when found, and extracting the square root of the sum, which will be the hypothenuse required.

Case IV. Fig. 9, Plate 3.

Given the hypothenuse and the angles. Required the base and the perpendicular.

In the plane triangle ABC, right-angled at A, are given the hypothenuse BC $=537$, and the angle at B $=55°,00'$, consequently the angle at C $=35°,00'$; required the two sides AB and AC.

Supposing an arch of a circle to be described from the point C as a centre, with the radius CB, the perpendicular AB will become the sine of the angle BCA; whence arises the proportion;

As

As radius	$=90°00'=$	10,00000
To the sine of the angle at C	$=35°00'=$	9,75859
So is the hypothenuse BC	$=537$ $=$	7,72997
To the perpendicular AB	$=308$ $=$	2,48856

Again, to find the base AC, with the hypothenuse for radius, and B the centre;

As radius	$=90°,00'=$	10,00000
To the sine of the angle at B	$=55°,00'=$	9,91336
So is the hypoth. BC	$=537$ $=$	2,73009
To the base AC	$=440$ $=$	2,64345

Or, when the perpendicular AB was found as above, another proportion might have been employed, in which AB being radius, the base AC would have become the tangent of the angle at B;

As radius	$=90°,00'=$	10,00000
To the tangent of the angle at B	$=55°,00'=$	10,15477
So is the perpend. AB	$=308$ $=$	2,48868
To the base AC	$=440$ $=$	2,64345

The hypothenuse being given, and one of the sides forming the right angle being found, the other side may be discovered as before, arithmetically, by subtracting the square of the side found from that of the hypothenuse, and extracting the square root of the remainder, which will be the side required.

It was already said, that proportion in logarithms is performed

formed by adding together those of the 2d and 3d terms, and subtracting from their sum the logarithm of the 1st term: but the same effect will be produced if, instead of subtracting the log. of the 1st term, we add what is called its *arithmetical complement* to the sum of the 2d and 3d terms. This arithmetical complement is obtained by subtracting the given logarithm from 10,00000, and thus procuring the *reciprocal* logarithm of the given number: for example, the logarithm of the natural number 248 being 2,39445, its arithmetical complement, or the difference between this logarithm and 10,00000, will be 7,60555. Were it, therefore, required to find a fourth proportional to any three numbers, as 248, 350, and 482, we would to the logarithms of the second and third terms add the arithmetical complement of the logarithm of the first term, when the sum (subtracting from it 10,0000) would be the logarithm of the fourth term required.

248 : 350 : : 482 :

	10,00000	
Log. of 248	= 2,39445	
Arith. comp.	= 7,60555	add
Log. of 350	= 2,54407	
——— 482	= 2,68305	
	12,83267	
	— 10,00000	
4th term, 680	= 2,83267	

The preceding cases comprehend all the varieties of right-angled plane trigonometry: the following are those belonging to oblique-angled triangles, or such as have no right angle.

CASE I. Fig. 10, Plate 3.

Two angles, and a side opposite to one of them, being given, to find the other angle and sides.

In the oblique-angled plane triangle ABC, are given the angle BAC $= 42°, 30'$, the angle ACB $= 56°, 30'$, and the side AB $= 144$; required the remaining angle ABC, and the sides BC and AC.

By the 7th Prop. of Geometry it was shown, that all the angles of any plane triangle are equal to two right angles, or equal to 180 degrees: if, therefore, from 180 we subtract the sum of the two given angles $42°, 30' + 56°, 30' = 99$ degrees, the remainder, $81°, 00'$, is the measure of the required angle at B.

It was shown in Prop. 2 of Trigonometry, that the sides of any plane triangle are one to another in the proportion of the sines of the respectively opposite angles: hence we have this proportion,—as the sine of the angle at C to the sine of the angle at A, so is the log. of the side AB, opposite to the angle at C, to the log. of the side BC, opposite to the angle at A.

$$
\begin{array}{lr}
\text{Sine of ACB} = 56°,30' = & \underline{9{,}92111} \\
\text{——— BAC} = 42{,}30 = & 9{,}82968 \\
\text{Log of AB} = 144 = & \underline{2{,}15836} \\
 & 11{,}98804 \\
 & -\ \underline{9{,}92111} \\
\text{Log. of BC} = 116{,}6 = & 2{,}06693
\end{array}
$$

Or, by taking the arithmetical complement of 9,92111, the sine of $56°, 30' = 0{,}07889$.

$$
\left.
\begin{array}{r}
\text{Arith. comp. of sine of } 56°,30' = 0{,}07889 \\
\text{Log of sine of } 42{,}30 = 9{,}82968 \\
\text{——————— } 144 = 2{,}15836
\end{array}
\right\} \text{add}
$$

$$\text{BC} = 116{,}6 = 2{,}06693$$

Again,

Again, to find the remaining side AC, we may employ either of the other sides with its opposite angle, thus;

Sine of ACB	$= 56°, 30' =$	9,92111
Sine of ABC	$= 81°, 00' =$	9,99462
Log. of AB	$= 144 =$	2,15836
		12,15298
		— 9,92111
Log. of AC	$= 170{,}6 =$	2,23187

Or, by the arithmetical complement:

Arith. comp. of sine of ACB	$= 56°, 30' =$	0,07889
Sine of ABC	$= 81°, 00' =$	9,99462
Log. of AB	$= 144 =$	2,15836
Log. of AC	$= 170{,}6 =$	2,23187

CASE II. Fig. 10, Plate 3.

Two sides, and an angle opposite to one of them, being given, to find the remaining side and angles.

In the oblique-angled triangle ABC, are given the side $AB = 144$, the side $BC = 116{,}6$, and the angle at C, opposite to $AB = 56°, 30'$; required the remaining side AC, and the angles at A and B.

This case being the converse of the preceding, we have this proportion; as the side AB to the side BC, so is the sine of the angle at C, opposite to AB, to the sine of the angle at A, opposite to BC.

Log. of AB	$= 144 =$	2,15836
——— BC	$= 116{,}6 =$	2,06693
Sine of ACB	$= 56°, 30' =$	9,92111
		11,98804
		— 2,15836
BAC	$= 42°, 30' =$	9,82968

Or,

Or, by the arithmetical complement;

Arith. comp. of log. of AB = 7,84164
Log. of BC = 2,06693
Sine of ACB = 9,92111

BAC = 42°, 30′ = 9,82968

Having thus found the angle at A, by adding it to the given angle at C, and subtracting the sum from two right-angles = 180 degrees, (Geom. Prop. 7,) we obtain the angle at B = 81°,00′, which was required.

Then, to find the unknown side AC, we say;

As the sine of the angle BCA = 56°,30′ = 9,92111

To the sine of the angle ABC = 81°,00′ = 9,99462
So is the logarithm of AB = 144 = 2,15836

12,15298
— 9,92111

To the log. of AC = 170,6 = 2,23187

Or, by the arithmetical complement;

Arith. comp. of sine of BCA = 56°,30′ = 0,07889
Sine of ABC = 81°,00′ = 9,99462
Log. of AB = 144 = 2,15836

AC = 170,6 = 2,23187

Case III. Fig. 10, Plate 3.

Two sides, and the angle contained between them, being given, to find the other side and angles.

In the plane triangle ABC, are given the sides AB = 144, and BC = 116,6, as also the angle ABC contained between these two sides = 81°,00′: required the other angles at A and C, and the side AC.

In

In Prop. 6 of Trigonometry it was demonstrated, that in any plane triangle the sum of the two sides opposite to the base is to the difference between these sides, as the tangent of half the sum of the two angles at the base is to the tangent of half the difference between these angles: hence we have this proportion;

As $AB + BC : AB - BC :: \text{Tang.} \frac{BCA + BAC}{2} :$ $\text{Tang.} \frac{BCA - BAC}{2}$.

The sum of $AB = 144$, and $BC = 116{,}6$, is $260{,}6$, and their difference is $27{,}4$; then subtracting the given angle at $B = 81°{,}00'$ from two right angles, or $180°$, we have $99°$, the half of which is $49°.30'$: hence we have this proportion;

Log. of sum of sides	=	260,6 =	2,41597
——— difference	=	27,4 =	1,43775
Tang. of half sum of angles at base.	=	49°,30′ =	10,06850
			11,50625
			— 2,41597
Half difference	=	7°,0′ =	9,09028

In Prop. 3 of Trigonometry it was shown, that in any two unequal quantities, if half their sum be added to half their difference, the sum will be equal to the greater quantity; and that if from half their sum be subtracted half their difference, the remainder will be equal to the less quantity: consequently, knowing now half the sum and half the difference of the two angles at the base, the angles themselves may be found as follows:

Half

Half the sum of BAC and BCA = 49°,30′
+ half the difference found = 7 ,00

BCA = 56, 30

Half the sum of BAC and BCA = 49 ,30
— half the difference 7 ,00

BAC = 42 ,30

And, agreeably to Case 1, the remaining side, or base, AC, will be found by stating the following proportion:

Sine of ACB = 56°, 30′ = 9,92111

—— ABC = 81°, 00′ = 9,99462
Log. of AB = 144 = 2,15836

12,15298
— 9,92111

AC = 170,6 = 2,23187

Or, by the arithmetical complement:

Arith. comp. of sine of 56°,30′ = 0,07889
Sine of 81°,00′ = 9,99462
Log. of 144 = 2,15836

AC = 170,6 = 2,23187

CASE IV. Fig. 10, Plate 3.

The three sides being given, to find the angles.

In the oblique-angled plane triangle ABC, are given the three sides AB = 144, BC = 116,6, and AC = 170,6; required the angles.

Having constructed the triangle agreeably to the given dimensions, from the angle at B, opposite to the base, let fall

fall the perpendicular BD, (Geom. Prop. 24,) cutting the base into two segments, AD and DC. By Prop. 7 of Trigonometry it appears, that a line drawn perpendicular to the base of a triangle from the opposite angle, will cut the base in such a proportion, that the whole base will be to the sum of the two opposite sides, as the difference between these two sides is to the difference between the segments of the base. Hence, in the present case, we have this proportion; as the base AC = 170,6, to the sum of the opposite sides AC = 144 + BC = 116,6 = 260,6, so is the difference of these sides AB — BC = 27,4, to the difference between the segments of the base, AD — DC.

Arith. comp. of log. of AC	= 170,6 =	7,76815
Log. of AB + BC	= 260,6 =	2,41597
Log. of AB − BC	= 27,4 =	1,43775
AD − DC	= 41,867 =	1,62187

Having thus found the difference between the segments of the base, formed by the perpendicular BD, the segments themselves are found in this way. To half the sum of these segments, that is, to half the whole base AC, add half the difference now found, and the sum will be the greater segment, which is always opposite to the greater side; and from half the whole line subtract half the difference, when the remainder will be the less segment opposite to the least side.

Half the base AC	= 170,6 =	85,3
+ half the difference of the segments	= 41,867 =	20,9335
Great segment AD	=	106,2335

Half

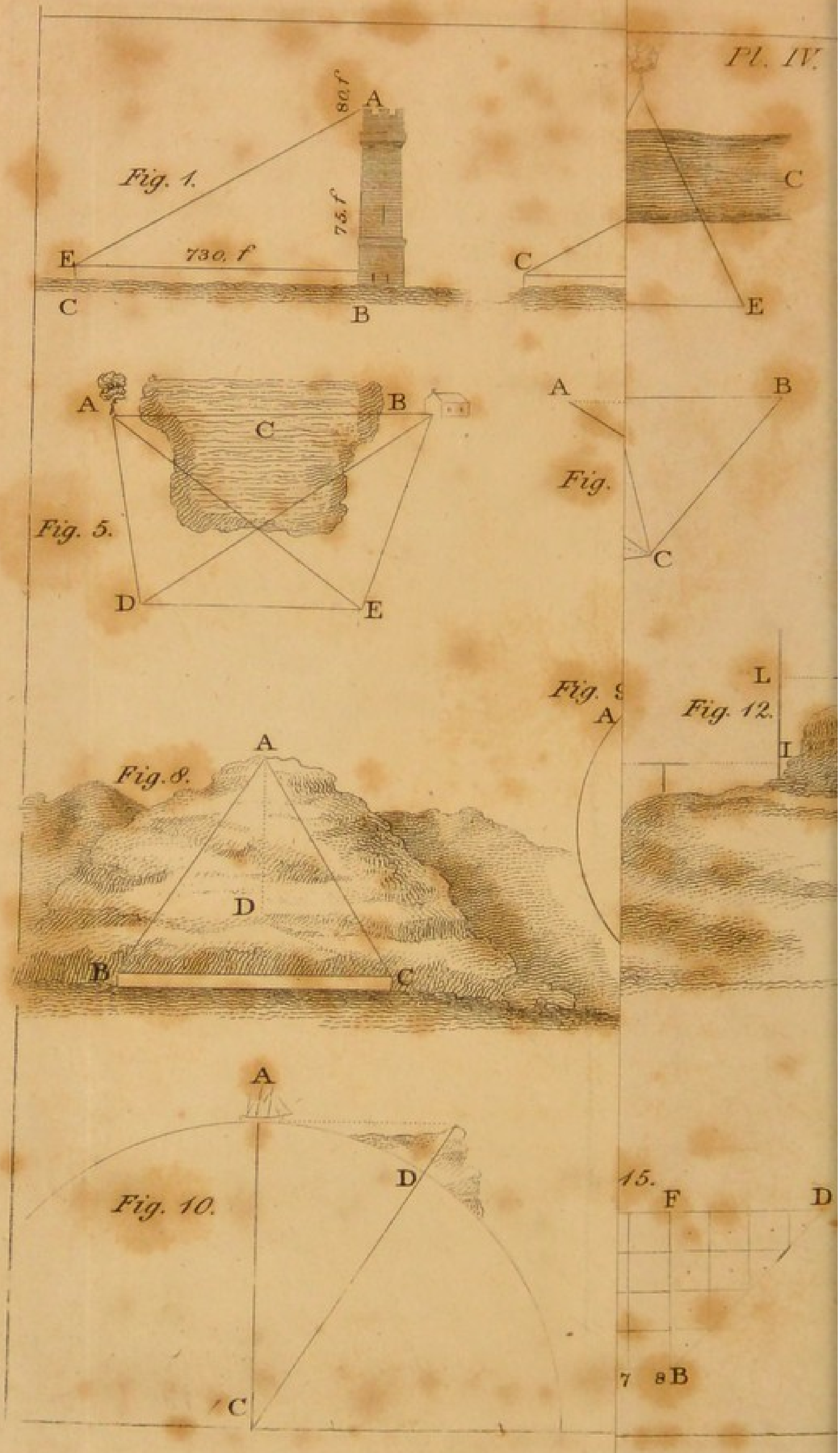
Half the base AC	$=170{,}6$	$=85{,}3$
— half the difference of the segments	$=41{,}867$	$=20{,}9335$
	Less segment DC	$=64{,}3665$

Having thus found the side AD, and the other side AB being given in the triangle ABD, right-angled at D, because BD stands perpendicularly on AD, we have only to apply the the 2d case of right-angled Trigonometry, to find the angle at A.

Log. BA	$= 144 =$	2,15836
Log. AD	$= 106{,}23 =$	2,02612
Radius	$= 90°{,}00' =$	10,00000
		12,02612
		2,15836
Sine of BAD	$= 47°{,}30' =$	9,86776

And subtracting this angle from 90 degrees, we have 42°,30′ for the angle at A. By a similar process, making BC radius, we discover the angle at C to be 56°, 30′; and adding these two angles together, the difference between their sum and 180 degrees is 81°,00′, equal to the great angle ABC.

PRACTICAL


Pl. IV.
Fig. 1.
A
80.f
75.f
730. f
E
C
B
C
E
Fig. 5.
A
B
C
D
E
A
B
Fig.
C
Fig. 8.
A
D
B
C
Fig. 9
A
Fig. 12.
L
I
Fig. 10.
A
D
C
15.
F
D
7
8B

PRACTICAL GEOMETRY.

By *Practical Geometry* is commonly understood, the mensuration of heights and distances of terrestrial objects, by means of plane trigonometry.

Example I.

In Fig. 1, of Plate 4, is represented a tower AB, of which it is desired to ascertain the heighth above the surface of the ground at B.

Suppose the observer placed at the point C, at the distance CB equal to 130 feet, measured horizontally, or on a level from the foot of the tower at B; his eye being situated at E, elevated 5 feet above the line CB. Then with a common quadrant, or other instrument for measuring angles, let the angle AED be measured equal to 29 degrees, 59 minutes, which is formed by EA the line of sight from the observer's eye to the top of the tower, and the level or horizontal line from his eye to a point D elevated 5 feet above the bottom of the tower at B, corresponding to the elevation of the eye above the ground at C.

This line ED being horizontal, and the face of the tower D being supposed perpendicular to the horizon, the angle ADE will be a right angle; consequently we have a right-angled triangle AED, in which are known the base ED = 130 feet, and the angle at E = 29°, 59′, when by Case 3, of right-angled Trigonometry, the perpendicular DE may be found in the following way:

As radius	=	90°, 00′	=	10,00000
To the tangent of AED	=	29°, 59′	=	9,76115
So log. of base ED	=	130	=	2,11394
To log. of perpend. DA	=	75	=	1,87509

But

But as the point D is elevated 5 feet above the bottom of the tower, this quantity, added to the perpendicular just found = 75, will give 80 feet for the whole elevation of the tower, which was required to be known.

Again, by reversing this problem, and supposing that an observer at E found the angle of elevation of the tower AED to be 29°, 59′, and that he knew the height of the tower above the horizontal line ED to be 75 feet, but wished to know the distance between his station and the tower, or the measure of the line ED = CB; by the same Case 3, of right-angled Trigonometry, he would say;

As radius		= 90°, 00′	= 10,00000
To the tang. of EAD		= 60, 01	= 10,23885
So the perpend. AD		= 75	= 1,87506
To the base	ED	= 130	= 2,11391

The angle EAD being the complement of the observed angle AEB to 90 degrees, its measure is 60°, 01′, by means of which the horizontal distance of the observer from the tower is found to be 130 feet, agreeably to the statement in the first part of this case, which is applicable to the measurement of all accessible and perpendicular elevations.

EXAMPLE II. Fig. 2, Plate 4.

Suppose it be required to find the height of the point of a pyramid, or obelisk AB, situated on the top of a hill, at the same time that the observer can approach no nearer than the point D, at the foot of the hill. From D let the level, or horizontal line DC, be measured off equal to 360 feet, in such a direction that the obelisk when observed from C may be seen precisely over and in the direction of D. Then, with a proper instrument, let the angle ACD be measured equal to 30°, 00′, as also the angle ADE equal to 46°, 00′.

When

When these things are ascertained, we have an oblique-angled triangle CAD, of which we know the side CD = 360 feet, the angle ACD = 30°, 00′, and the angle ADC = 134°, 00′ (being the supplement of the measured angle, ADE, to two right angles, or 180 degrees) and, consequently, the remaining angle formed by lines supposed to be drawn from the top of the obelisk to the two stations of the observer, that is, the angle CAD = 16°, 00′.

It was shown in Prop. 2 of Trigonometry, that in any plane triangle the sides are in proportion to the sines of the respectively opposite angles; we may therefore by this proportion discover the length of the sides AC or AD. Let AD then be found as follows:

As the sine of CAD	= 16°, 00′	=	9,44034
To the sine of ACD	= 30, 00	=	9,69897
Log. of side CD	= 360	=	2,55630
			12,25527
		—	9,44034
Log. of side AD	= 653	=	2,81493

Again, in the triangle ADE, having a right angle at E formed by the imaginary lines DE, which is horizontal, and AE a perpendicular let fall from the point of the obelisk at A, we have the hypothenuse AD just found to be 653 feet, and the angle ADE observed to be 46°, 00′; hence the perpendicular EA may be found as follows, (Trig. Prop. 5.)

As radius	= 90°, 00′	=	10,00000
To the sine of ADE	= 46°, 00′	=	9,85693
So is DA	= 653	=	2,81493
To perpend. AE	= 469,7	=	12,67186

But

But the point E being on the level of the eye of the observer, and supposed 5 feet above the ground line, by adding 5 to 469,7 we obtain 474,7 feet for the elevation of the summit of the obelisk AB, above the plain at the foot of the hill on which it is erected.

Example III. Fig. 3, Plate 4.

Suppose it be required to ascertain the length of a flagstaff AB erected on the battlements of a perpendicular tower BC, to the bottom of which we can have access, from the observer's station at the point E. In this case, by measuring the base, or level distance EC, and at E observing the angles formed at the observer's eye by this base, and lines directed to the top and the bottom of the flagstaff at A and B, or AEC and BEC, we can, as in the 1st Example, discover the height BC, and also AC, from which last subtracting BC, the remainder must be the length of the flagstaff AB, which was required. Let, therefore, EC be measured 65 feet, the angle BEC be observed 42°, 30′, and AEC be 52°, 08′: stating this proportion as radius to the tangent of the angle AEC, so is the base EC to the perpendicular height to the top of the flagstaff AC, which will turn out to be 83,6 feet.

Again, in the triangle BEC, having the angle at E, and the base EC, we discover the height of the tower alone BC to be 59,6 feet; consequently, subtracting this quantity from AC = 83,6 feet, the remainder, 24 feet, must be the height of the flagstaff alone, as was required to be discovered.

But on the other hand let it be supposed that the tower is surrounded by a broad ditch, so that the observer can approach no nearer to it than to the point E, then, agreeably to what is said in Example 2, let the horizontal line ED be measured in such a direction that the observer at D will

will see the flagstaff immediately over the point E, and let the length of ED be 62 feet.

At E observe the angles formed with the horizon by lines of sight to the top and the bottom of the flagstaff; the angle AEC being $52°, 08'$, and BEC being $42°, 30'$. In the same way, at D observe the angles ADC $= 33°, 21'$, and BDC $= 25°, 08'$.

In the triangle BDE are given the side ED $= 62$, and the angle BDE $= 25,08$, the angle BED (the supplement of BEC to two right angles) $= 137°, 30'$, and, consequently, (Geom. Prop. 7,) EBD $= 17°, 22'$: and in the triangle ADE are known the side ED $= 62$, the angle ADE $=$ $33°, 21'$, the angle AED (the supplement of AEC) $=$ $127°, 52'$, and, consequently, the angle EAD $= 18°, 47'$. If, therefore, the process pointed out in Example 2 be followed, we shall discover the elevation of the points A and B, the two extremities of the flagstaff, and subtracting the less from the greater of these quantities, the difference will be the length of the staff itself, which was required to be known.

To find the elevation of the point B:

As the sine of EBD	$= 17°, 22' =$	9,47492
To the sine of BDE	$= 25, 08 =$	9,62811
So is the log. of ED	$= 62 =$	1,79239
		11,42050
		− 9,47492
To the log. of BE	$= 88, 22 =$	1,94558

Again, as radius	$= 90°,00' =$	10,00000
To the sine of BEC	$= 42, 30 =$	9,82968
So is the log. of BE	$= 88, 22 =$	1,94558
To the log. of BC	$= 59, 6 =$	1,77526

In

In a similar way the height AC will be found, by means of the triangle EAD, to be 83,6 feet, from which subtracting BC = 59,6 feet, the height of the tower, the difference will be 24 feet for the length of the flagstaff, agreeably to what was discovered in the first case of this example, where the tower was supposed to be accessible from the observer's station at the point E.

Although in this example it has been supposed that the base ED is horizontal, and on a level with the foot of the tower, yet had the point D been elevated above, or depressed below the point E, the length of the flagstaff might have been discovered by a similar application of the rules already given under the head Trigonometry.

EXAMPLE IV. Fig. 4, Plate 4.

It is required to find the distance from the two points D and E, to a tree A situated on the opposite side of a river BC, which can not conveniently be crossed.

Having measured the distance between the two stations D and E, which is 200 yards, and observed the angles ADE = 62°, 18′, and AED = 63°, 23′, and, consequently, DAE = 54°, 19′, by the application of the 2d Prop. of Trigonometry, where it was shown that the sides of plane triangles are proportional to the sines of the respectively opposite angles, we have this proportion:

Sine of DAE = 54°, 19′	=	9,90969
—— AED = 63, 23	=	9,95135
Log. of DE = 200	=	2,30103
		12,25238
	−	9,90969
AD = 220	=	2,34269

Again,

Again, to find the side AE:

Sine of DAE	$= 54°, 19' =$		9,90969
—— ADE	$= 62, 18$	$=$	9,94714
Log. of DE	$= 200$	$=$	2,30103
			12,24817
			— 9,90969
AE	$= 218$	$=$	2,33848

The tree is therefore 220 yards from the station at D, and 218 yards from the station at E, which was required to be known.

EXAMPLE V. Fig. 5, Plate 4.

Required the distance between the tree at A and the house at B, situated on opposite sides of the lake C, at the same time that these objects can be approached no nearer by the observer than at the stations D and E.

Let the distance between these stations be 400 yards, and let the angles formed at each station by lines of sight from the objects at A and B be the following, viz. ADB $=$ 65, 37, BDE $= 33°, 51'$, AED $= 36°, 20'$, and AEB $= 73, 55$.

In the triangle DAE are given the side DE, as also the angle ADE (which is the sum of ADB + BDE) and AED, the supplement of which to 180 degrees, is the angle DAE $= 44°, 12'$: hence, as the sine of DAE $= 44°, 12'$, to the sine of AED $= 36, 20$, so is the given side DE $= 400$ to the side AD, which will be 340.

Again, as the sine of DAE $= 44°, 12'$, to the sine of the supplement of ADE, being greater than a right angle, that is, to the sine of $80°, 32'$, so is the given side DE $= 400$ to the side AE, which will be 566 yards.

Proceeding in the same way with the other triangle DBE, subtracting

subtracting the sum of the two angles BDE and BED (= AED + AEB) from 180 degrees, we obtain 35°, 54′ for the angle DBE: hence, as the sine of DBE to the sine of BDE, so is the given side DE to the side BE, which will be 380.

Again, as the sine of DBE to the sine of the supplement of DEB, which is greater than a right angle, = 69, 45, so is the given side DE to the side DB, which will be 640.

Having proceeded thus far, we obtain the two triangles ADB and AEB, in each of which we know two sides and the angle contained by them, whence by Prop. 6 of Trigonometry we can discover the common base AB. For this purpose let the triangle ADB be employed, in which we have the sides AD found to be 340 yards, and DB = 640, together with the contained angle ADB = 65°, 37′.

Then stating this proportion, as the sum of the sides containing the given angle, AD = 340 + DB = 640 = 980, to the difference between the same sides = 300, so is the tangent of half the sum of the remaining angles (found by subtracting ADB = 65°, 37′ from 180°) = 57°, 11′$\frac{1}{2}$, to the tangent of half the difference between these angles, which will be 25°, 24′; hence, by adding to the half sum this half difference, we obtain the greater angle DAB = 82°, 35′$\frac{1}{2}$, and by subtracting the half difference from the half sum, we have the less angle ABD = 31°, 47′$\frac{1}{2}$. Then by the 2d Prop. of Trigonometry saying, as the sine of DAB = 82°, 35′$\frac{1}{2}$ is to the sine of ADB = 65°, 37′, so is the side DB = 640 to the side AB, which will be 587,8 yards. If the angle ABD and its opposite side AD, had been employed, the result would have been precisely the same: and if the triangle AEB had been used, the common base AB would still have been found = 587,8.

By these several operations we have discovered, that the distance AB across the lake C, between the two given objects, the tree and the house, is 587,8 yards; the distance AD between the tree and the station D is 340 yards; the

the distance AE between the tree and the station E is 566 yards; the distance DB between the house and the station D is 640 yards; and the distance BE between the house and the station E is 380 yards.

EXAMPLE VI. Fig. 6, Plate 4.

Let A and B be two objects situated on opposite sides of a rising ground C, so that the one cannot be seen from the other: it is required not only to measure the level or horizontal distance between these objects, but to determine certain points, as E, F, &c. situated directly in the line between them.

Choose any point, as D, from which both objects can be seen, and measure the lines AD and DB, as also the angle formed by these lines, ADB: then in the triangle ADB, the two sides AD and DB being known, and the contained angle, by the 6th Prop. of Trigonometry, the angles BAD and ABD may be discovered, and, consequently, the remaining side AB, which was required.

Again, in order to ascertain points in the line AB, placing a pole, stake, or other moveable object in any direction, as DE, measure the angle formed by this line and DA, that is, the angle ADE: in the triangle DAE, therefore, are known all the angles and one side; the other side DE may of course be found by applying the rules already laid down, which quantity measured off from D to E will determine the position of E in the line of direction between the given objects A and B.

In the same way determine the position of F, and as many other points as may be requisite; when the direction between the given objects will be ascertained, notwithstanding the high ground which intercepts the view from the one to the other.

EXAMPLE VII. Fig. 7, Plate, 4.

It is required to determine certain points in the line of direction between the objects A and B, which are so situated, that the observer cannot see the one from the other, nor can he conveniently find any one station from which both objects can be perceived.

The point C is such, that from it the object B may be observed; but a hill intercepts the view of the object A: choose, therefore, a position at F, from which may be seen both the object A and the point C. Then measure the lines AF, FC, and CB; also the angles AFC and FCB; and in the triangle FAC, formed by the two lines now measured, and the imaginary horizontal line AC, passing through the intercepting hill, the two sides AF and FC, with the contained angle AFC being known, the remaining angles, and the side AC, will be discovered. Again, in the triangle ACB the side AC being found, and the side CB having been measured, the angle ACB is also known; for it is the difference between FCA, already found, and the measured angle FCB; consequently, all these things being given, the points D, E, &c. in the line AB may be ascertained by the application of the preceding example; because the imaginary line AC has been determined by calculation, and may be employed as if it had been measured on the ground, and that no obstacle had prevented the view of the object at A from the station of the observer at C.

EXAMPLE VIII. Fig. 8, Plate 4.

Let it be required to measure the perpendicular altitude of a mountain whose summit is at A, by means of two stations B and C at the foot of the mountain.

The line BC elevated to the observer's eye, 5 feet above the ground, is to be measured; then with a proper instrument measure the angle CBA formed by the line BC from the

the one station to the other, and the line BA from the station at B to the top of the mountain at A; in the same way the angle BCA is to be measured, when in the triangle BAC knowing the side BC, and the angles ABC and ACB, and of course the angle BAC, the remaining sides BA and CA may be found.

Again, supposing the imaginary line CD to represent the horizon, the angle DCA formed by the horizon and the line CA to the summit of the mountain, may be measured, and AD will represent the perpendicular altitude of the mountain, in which case the angle at D will be a right angle; hence in the right-angled triangle DAC, having the hypothenuse AC and the angle ACD, the perpendicular may be found (Trig. Prop. 5,) to which adding the height of the observer's eye above the surface, the total altitude of the mountain will be ascertained.

EXAMPLE IX. Fig. 9, Plate 4.

Let A, B, and C represent three steeples of a town whose positions and relative distances the one from the other are known, and let an observer at D measure the angles formed at his eye by these objects: it is desired to know how far he is from each of them.

Laying down upon paper, from the plan of the town, the positions of the three steeples, assume any point at pleasure for the observer's station, as at D, through which and the two objects A and B draw a circle: draw lines from D to A and B, and also to C, producing it until it meet the opposite circumference of the circle in E, and draw the lines AE and EB.

From the plan of the town the distances AB, BC, and CA are known, and, consequently, the angles at A, B, and C: the angles ADC = ADE and CDB = EDB measured by the observer at D are also known. In the triangle

BAE the side BA is given, and the angle EAB is known, for it is equal to the angle EDB, being angles in the same segment of a circle, (Geom. Prop. 10,) and standing on the same arch or chord EB; hence the two sides AE and EB may be calculated. Again, in the triangle CEB the sides BE and BC are known, as also the angle CBE, which is composed of the angles CBA which was given, and ABE now found, consequently the angle BCE may be calculated, and its supplement to 180 degrees will be the angle BCD, (Geom. Prop. 1.) Then in the triangle DBC the side BC was given, and the two angles BCD and BDC are known, consequently the angle CBD, and the remaining sides BD and CD may be found, which will give the distance of the observer at D from the two steeples at B and C.

Lastly, in the triangle EAC, the sides EA and AC are known, together with the angle EAC, which is composed of EAB already found, and BAC given in the proposition, consequently the angle ECA may be found, the supplement of which to 180 degress will be the angle ACD: then in the triangle ADC are known the angles ACD and CDA, together with the side AC; the side AD may therefore be found, which will give the distance from the observer at D to the third steeple at A.

In the preceding examples mention has often been made of the level or horizontal line: by such a line, in strictness, should be meant one at all parts equally distant from the centre of the earth; but in ordinary language, by a level or horizontal line, is understood a tangent to the earth's surface at any one place, which, on account of the magnitude of the earth, will, in short distances, have no sensible deviation from the true level or horizon.

Fig.

Fig. 10 of Plate 4, represents a section of part of the earth, in which C is the centre of the globe, and the circular arch is the surface; CA is a radius, or semidiameter, drawn from the centre to the surface of the sea at the ship A; the dotted line AB is the apparent level or horizontal line of an observer in the ship, which touching the surface at the point A, and produced either towards B or in the opposite direction, falls without the circle, or above the surface; it is, consequently, a tangent at the point A, and at right angles, or perpendicular to the semidiameter, or to the axis of the earth. The point B is the summit of a mountain raised above the true circular surface of the earth at D, and the space DB represents the elevation of the mountain above that surface, or, as it is usually termed, above the level of the sea.

Were the surface of the earth a perfect plain, the tangent at the point A would consequently coincide with every part of that plain, and be likewise a tangent at the point D; but experience shows that this is not the case; for an observer in the ship at A, on approaching the land, discovers first the summit of the mountain at B, and as his distance from the mountain continues to decrease, the more of its elevation does he discover; whereas, had the earth been a plain, the whole elevation BD would have been perceptible at once. It is therefore evident, that as the direction of the tangents to the earth's surface at A and D must be very different, varying in proportion to the distance between the points of contact, the line of sight, or the apparent horizontal level, must also be continually changing its direction: hence the level of the point A would, if produced, extend not to the surface of the sea at the bottom of the mountain at D, but to the summit at B: and hence we have a method of ascertaining the elevation of remarkable mountains, provided we know their distance from the place of observation; as also of determining the magnitude of

of the earth, supposing it to be a perfect globe or sphere, which in this case may readily be granted.

Let an observer at A observe the point B, the summit of the *Peak of Tenerif* just appearing along the surface of the sea, (the state of the atmosphere which renders this impossible is not here considered). The height of this mountain he knows to be about 15,400 feet, and that the diameter of the earth, supposing it a perfect sphere, is about 7,944 English miles, or 41,944,320 feet; consequently the radius or semidiameter will be 20,972,160 feet. In the triangle ABC are given the right angle at A, for BA is a tangent to the circle at that point, and consequently perpendicular to the radius or semidiameter AC, (Geom. Prop. 31;) also the side AC = 20,972,160 feet; and the hypothenuse CB is likewise known, for CD being a radius of the same circle, must be equal to CA, to which adding the space DB, the height of the mountain = 15,400 feet, the whole CB, will be = 20,987,560 feet. Then knowing the two sides of a right-angled triangle, the remaining side AB may be found = 729,040 feet, or about 140 English miles, which is the distance at which the summit of the Peak of Tenerif might be perceived just rising from or sinking into the sea, provided the atmosphere would admit of vision along the surface at so great a distance, which is not the case.

On the other hand, by knowing the distance AB, we could calculate the height of the mountain, BD, which is the difference between the hypothenuse BC and the semidiameter DC: and by measuring the angle ABD, formed at the summit of the mountain by the line of vision BA with BD or BC, the vertical line, or that indicated by a plummet as tending to the centre of the earth, we might ascertain the quantity of the angle at the centre ACB, or of the arch AD, and, consequently, of the whole circumference of the globe, to which this arch bears a determined proportion.

From

From the inspection of the figure (10) it will be evident, that if two places, on the surface of the earth, are on the same horizontal line, as A and B, these must be at different distances from the centre, as AC and BC: and the method of ascertaining this difference is termed *levelling*, where in practice the short distances from one station to another, although in fact portions of the spherical surface of the earth, may safely be considered as straight lines, or tangents, to the curvature of the globe: then supposing the horizontal distance AB to be one English mile, or 5280 feet, the line DB which represents the difference between the distances of the points A and B from the centre of the earth will be about ,665 of a foot, or nearly 8 inches, supposing no allowance to be made for the effect produced in the apparent altitude of objects in the horizon by the state of the atmosphere: and if these calculations were continued, it would be found that the difference between the horizontal line AB and the true level AD, might be assumed as sufficiently correct in the proportion of the squares of the distances.

The following rule is near enough the truth, to be used in the practice of levelling. "Multiply the number of chains contained in the distance between the objects, whose difference of level is required, by itself, and this product by 124, a common multiplier in cases of this sort, on account of the curvature of the earth's surface; then divide this last product by 100,000, or cut off five figures from the right hand, when whatever stands on the left of the division will be inches, and the figures cut off will be decimal fractions of an inch.

The following *Table of the Curvature of the Earth* points out the quantity of depression of the true level below the apparent, calculated for every chain's length, or the 80th part of a mile.

Chains

Chains	Inches	Chains	Inches	Chains	Inches	Chains	Inches
1	0,00125	14	0,24	27	0,91	40	2,0
2	0,005	15	0,28	28	0,98	45	2,28
3	0,01125	16	0,32	29	1,05	50	3,12
4	0,02	17	0,36	30	1,12	55	3,78
5	0,03	18	0,40	31	1,19	60	4,50
6	0,04	19	0,45	32	1,27	65	5,31
7	0,06	20	0,5	33	1,35	70	6,12
8	0,08	21	0,55	34	1,44	75	7,03
9	0,1	22	0,6	35	1,53	80	8,0
10	0,12	23	0,67	36	1,62	85	9,03
11	0,15	24	0,72	37	1,71	90	10,12
12	0,18	25	0,78	38	1,80	95	11,28
13	0,21	26	0,84	39	1,91	100	12,5

By applying the 1st Example of Practical Geometry, Fig. 1, Plate 4, the angle of elevation of the top of the tower at A, above the level of the spectator at E, was measured by means of a quadrant, or other proper instrument for taking such angles: but for the practice of levelling, this would require an instrument constructed with the greatest accuracy, because a very small error in the ascertainment of the angle might give a very erroneous result, when the distance between the objects is considerable. On this account another mode of levelling is usually adopted, by means of an instrument represented at Fig. 11, Plate 4. This is a tin pipe bent upwards to a right angle toward each extremity at A and B: into these upright parts are secured glass pipes, and the instrument is filled with water until it appears in the glasses, when the imaginary line ABD passing over the surface in both pipes will be a true level or horizontal line, or a tangent to the surface of the earth, respecting the foot of the instrument C. The manner of using this instrument is shown in Fig. 12 of Plate 4, where the bending line ADBGCK represents the surface of

a

a rising ground, and A and K the lowest and highest points between which the difference of level is required. Having prepared a number of long straight poles, let them be placed perpendicularly at the stations AB and C, at such distances asunder as may suit the surface of the ground and the nature of the levelling instrument, which is to be placed as nearly in the middle between each two poles as can be done, as at D and G. When the instrument is at D, the observer, looking along the horizontal line indicated by the surface of the water in the two glass pipes, directs his assistant to make a mark at E on the pole A, where that line falls on the pole: the distance of this point E above the surface of the ground at A is then accurately measured, and written down. Again, the observer from the other end of the instrument looks along the horizontal line, directing the assistant to mark the spot F where it falls upon the second pole planted at B; the distance between FB being also carefully measured and marked down. Now, supposing the height of the point upon the first pole to be 10 feet above the surface of the ground at A, while that of the point F upon the second pole is only $2\frac{1}{2}$ feet, it is evident that the difference between these two quantities, or $7\frac{1}{2}$ feet, is the difference of elevation of the point B above the point A, that is, B is elevated $7\frac{1}{2}$ feet above the level of A. When this is performed, the instrument is removed from the station at D, and placed at G, when the former operations are repeated to ascertain the point H on the second pole: in measuring its elevation, however, the distance is to be reckoned not from the surface of the ground where the pole is fixed, but from the point F formerly ascertained. Again, the observer, by means of the horizontal line of the instrument, determines the point I on the third pole planted at C. Lastly, removing the instrument to the top of the rising ground at K, the point L is marked on the pole C, and the distance between

L and the point I, formerly determined, will show the elevation of L above the last horizontal line HI.

When as many horizontal lines have been observed, and the distances between them have been measured, as may be requisite for the levelling required, the total of these distances in elevation added together, (deducting the height of the instrument $4\frac{1}{2}$ feet above the surface of the ground at the last station K,) will show the whole difference of level between the two extremities of the ground to be levelled.

Should the ground consist of a succession of heights and hollows, the difference between the levels observed in the hollows, and those on the preceding heights, must not be added to, but subtracted from the amount of the observed elevations.

Besides the instrument here described for levelling, many others are employed, such as the air-level, which points out the horizontal line by means of a bubble of air inclosed with some liquor in a glass tube, of a convenient length, whose ends are so shut up that no air can be admitted or make its escape. When the bubble stands at a mark made exactly in the middle of the length of the tube, the plane or ruler to which the tube is applied is then truly horizontal.

The liquor commonly employed is oil of tartar, which is not liable to freeze like water, nor to be expanded or condensed like spirit of wine. When the tube is not level, the air bubble being specifically lighter than the liquor, will rise to the highest end. The glass tube is set in one of brass, and at each end are placed sights, for the better observing the horizontal line shown by the air-bubble; and the whole is fitted with a ball and socket to a fulcrum, for the purpose of keeping it steady in practice.

But the most accurate levelling instrument is the spirit-level,

level, such as it is now made in London, which is fitted up with a telescope, a mariner's compass, and a set of screws, and other contrivances, by which the instrument can be adjusted and employed in the field with the greatest care as well as correctness.

MENSURATION OF SURFACES.

In the introduction to this tract on geometry (page 350) it was remarked, that all bodies consist of three dimensions, viz. *length*, *breadth*, and *depth* or *thickness*, but that although all these sorts of measure were absolutely inseparable from the idea of a body, yet each of them might be considered by us as existing independently of the others: thus, for instance, if we say that the Thames is deep at Woolwich, we are not considering the breadth of the river; and when we say that it would take 27 yards of carpeting to cover a room, we think only of the length and breadth, but not of the thickness of the carpet. Hence, in treating of the mensuration of surfaces, we are to take into consideration the length and the breadth only of the body to be measured.

Fig. 13 of Plate 4 represents a square, ACDB, the measurement of which is required. Let each side be 5 inches in extent, as divided in the points numbered 1, 2, 3, 4, and 5: through the points in the side AC draw right lines parallel to the sides CD and AB, and upon the points in the side AB erect perpendiculars, which being parallel to the sides AC and BD, will, with the first drawn lines, form a series of squares whose sides will be equal to the

small divisions of the sides of the great square: and as the square of any number or quantity is obtained by multiplying that number or quantity by itself, if we multiply into itself the number of small parts contained in the side of the great square, which is 5, the product 25 will give the number of squares, whose sides are equal to these small parts, contained in the whole surface of the great square ACDB; but the sides being supposed to contain 5 inches, the superficial area of the great square will be 25 square inches.

From this it is evident, that whatever be the denomination of the equal parts by which the sides or other dimensions of a surface are ascertained, the area or superficial content will be composed of squares whose sides are equal to one of such equal parts. If the side AB had been supposed to consist of 5 feet, yards, chains, miles, leagues, &c. the area would have consisted of 25 square feet, yards, &c.

The same reasoning will apply to other plane surfaces besides squares, for in Fig. 14, where the area of a right-angled parallelogram ACDB is required, if the side AC be divided into 4 equal parts, the side AB into 8 of the same parts, and lines be drawn through these divisions respectively parallel to the different sides of the figure, a series of small squares will be formed, equal in number to the product of the number of parts into which one side is divided by those of the other side: thus, by multiplying 8, the number of divisions in the base AB, into 4, the number of parts in AC, the product 32 is the number of small squares continued in the given rectangle ACBD, as may be observed by inspecting the figure.

As this mode of ascertaining the superficial area of any figure, supposes it to be subdivided into a number of small squares, and as squares require to have their sides equal and their angles right, it is evident that the dimensions must

must be taken in directions perpendicular one to another, and, consequently, that the area of the parallellogram ACDB, represented in Fig. 15, Plate 4, is not to be calculated by multiplying the base AB into the side AC, which does not stand perpendicularly upon AB, but into the altitude of the figure indicated by the perpendiculars BF or CA.

It was shown in Geom. Prop. 14, Fig. 27, Plate 1, that the area or surface of a parallelogram is equal to the product of its base multiplied by its altitude, and that the parallelogram ABDC was equal to the rectangle AEFC, which had before been shown to be measured by the product of its base multiplied into its altitude. It was also shown in Geom. Prop. 11, Fig. 25, Plate 1, that parallelograms situated on the same or equal bases, and between the same parallels, and consequently of equal altitudes, were equal to one another. Let then the right-angled parallelogram AEFB be formed, (Fig. 15, Plate 4,) in which the line FE being a continuation of FC, it must be equal and parallel to AB, and consequently EA must be equal and parallel to FB; therefore the right-angled parallelogram AEFB must be equal to the oblique-angled parallelogram ACFD: and from inspecting the figure, the triangle ACE, by which AEFB exceeds ACDB, will be found to contain the same number of small dotted squares and triangles as are contained in the triangle BFD, by which AEFB comes short of ACDB.

The reason for adopting squares as the common measure of surfaces is, that the calculation of the area of a square is the simplest possible, requiring only the multiplication of the side into itself; whereas the areas of all such other figures as are fitted to occupy completely the area of the given surface, as the triangle, the hexagon, the octagon, &c. demand a tedious and complicated process, to be ascertained.

In ordinary language we say that, to obtain the area of a square

a square or a parallelogram, we are to multiply the base into the perpendicular altitude: but as a geometrical line is supposed to have no sensible breadth, it is not susceptible of augmentation by the addition of another line, or of a thousand lines placed parallel to and in contract with it. In the square (Fig. 13, Plate 4,) to multiply the base AB by the perpendicular AC, is in fact to discover how often the 5 squares constructed on that line are contained in the given square ACDB, or how often the parallelogram whose length is AB = 5 inches, and breadth A1 = 1 inch, is contained in the square ACDB: it is evident from the figure that this parallelogram, which consists of 5 square inches, may be repeated 5 times in the great square, which will thus contain 25 inches; hence, to multiply the base by the perpendicular, is to discover the product of the number of squares constructed on the base, multiplied by the number of those constructed on the perpendicular altitude of the given surface.

In treating of Arithmetic (page 189), a reference was made to the explanations now given of the nature of mensuration of surfaces, and it was observed, that in calculating the amount of any number of articles, in value, weight, measure, or in any other mode of reckoning, the multiplicand expressed the rate of value, &c. of one article, and the multiplier expressed the number of articles to be estimated: this multiplier was, therefore, always an abstract number, having no reference to any value or measure whatever. The only objection to this observation, it was there remarked, was in performing operations in mensuration of surfaces or solids, where the dimensions were given in inches, feet, yards, &c; in which case both multiplicand and multiplicator consisted of real and not abstract numbers: but from what has just been said, the multiplication consists in discovering the amount of the square inches, feet, &c. constructed on one side of the given

surface,

surface, repeated as often as there are inches, feet, &c. in the other side.

It was observed, (Geom. Prop. 10, page 365,) that two figures which, when applied the one to the other, perfectly coincide in every part, are said to be equal; and that two figures which, whatever be their form, contain surfaces of equal extent, are said to be equivalent. These assertions are rendered evident by mensuration, for a square of 12 inches a side will contain 144 square inches; and equivalent areas will be contained in right-angled parallelograms of the following dimensions, viz. the base 16 inches, and the perpendicular altitude 9 inches; or of 18 inches by 8 inches, 24 by 6, 36 by 4, 48 by 3, and 72 by 2, since the products of these several dimensions are all 144 square inches: and the two parallelograms, (Fig. 15, Plate 4,) AEFB and ACDB, however unlike in form, are of the same area or superficial content.

These explanations being understood, the student will be prepared to comprehend the following examples of mensuration of surfaces.

Example I.

To measure the area or superficial contents of a square.

Rule.—Square the given side, or, in other words, multiply it into itself, and the product will be the superficial contents: thus, in the square represented in Fig. 13 of Plate 4, the side being given 5 inches in extent, the square of 5, or 25 square inches, is the area required.

Again, had the dimensions of the side of the square been 25 feet 8 inches, the area would, agreeably to the directions for Compound Multiplication contained in pages 189 and 190, have been 658 square feet, 9 inches, and 4 lines, or 4 twelfth parts of an inch.

Example

Example II.

To find the area of a rectangle or right-angled parallelogram.

Rule.—Multiply the base into the altitude, and the product will give the area: thus the rectangle represented in Fig. 14, Plate 4, contains in the base AB 8 equal parts, and in the perpendicular AC, 4 of the same parts; if each of these parts represent a foot, the product of 8 by 4 = 32 will be the number of square feet contained in the given rectangle.

Again, let the base be in extent 33 feet 9 inches, and the altitude 18 feet 5 inches; these two quantities multiplied together, as before directed, (pages 189 and 190,) will give for the superficial content of the rectangle 621 square feet 6 inches, and 9 twelfth parts or lines.

Example III.

To find the area of a rhombus or distorted square, whose sides are all equal but of which none of the angles are right, as represented in Fig. 11 of Plate 1.

Rule.—Multiply the base AB by the perpendicular altitude CE, which must diminish in proportion to the acuteness of the angle at A, and the product will be the area.

Let the side of the rhombus be 36 feet 9 inches, and the perpendicular altitude be 32 feet 6 inches; then the product of these two dimensions, or 1194 square feet, is the area $4\frac{1}{2}$ inches.

Now, had the figure here measured been not only equal-sided, like a square, but also equal-angled, the area would, as in Example I. have been found by squaring the side itself = 36 feet 9 inches, giving an area of 1350 feet $6\frac{3}{4}$ inches, exceeding the area of the rhombus by 156 feet $2\frac{1}{4}$ inches.

EXAMPLE

Example IV.

To measure the superficial contents of a rhomboides or parallelogram, as ACDB, (Fig. 15, Plate 4,) the base of which is 8 feet, and the inclined sides AC and BD each 5,657 feet, but the perpendicular altitude BF, or 4 C, is only 4 feet.

Rule.—Multiply the base into the perpendicular altitude, and the product will be the superficial contents, which in this case will be 32 feet; whereas, had the figure been a rectangle of 8 feet by 5,657 feet, the contents would have been 42 feet 3 inches.

These two last rules for calculating the superficial area of a rhombus and a rhomboides, are derived from the 14th Prop. of Geom. where it is shown that the area of any parallelogram (whether it be a rectangle or not) is equal to the product of the base multiplied into the perpendicular altitude. (See also Propositions 11 and 13 of Geom.)

Example V.

To measure the area of a triangle, Fig. 6, 8, and 9, Plate 1.

Rule.—Multiply the base AC into the perpendicular altitude BD, and take one half of the product; or the whole base into one half of the perpendicular altitude; or one half of the base into the whole altitude: all these methods will give the same result for the area of the given triangle, ABC.

Let the base be 18 inches, and the altitude be 16 inches, the area will be 144 inches.

Base 18		
Alt. 16		
108		
18		
	Base 18	$\frac{1}{2}$ Base 9
2)288	$\frac{1}{2}$ Alt. 8	Alt. 16
144 =	144 =	144

This

This rule is founded on the 12th Prop. of Geometry, Fig. 27, Plate 1, where it is shown that a triangle is one half of a parallelogram, situated on the same base, and of the same altitude; the first method, above pointed out, gives the product of the base multiplied into the altitude for the area of the parallelogram, and its half is that of the given triangle.

It is of no importance whether the triangle be acute, obtuse, or right-angled, as the perpendicular altitude must always be the same, whether it falls within the triangle, as in Fig. 6, Plate 1, or without it to the continuation of the base, as in Fig. 8, or coincides with one of the sides, as in Fig. 9.

EXAMPLE VI.

To find the contents of a trapezium (page 355), or figure of 4 sides, none of which are equal or parallel to any other side, such as appears in Fig. 14, Plate 1.

Rule.—Multiply the diagonal of the figure into half the sum of the two perpendiculars let fall from the opposite angles to the diagonal, and the product will be the superficial area.

In the figure ACDB is a quadrilateral or trapezium, of which all the sides are unequal, and none is parallel to its opposite: if the diagonal AD be drawn, the quadrilateral will be divided into two triangles, ADC and ADB, of which the common base AD may be measured, as also the altitude of each triangle, by letting fall on the diagonal perpendiculars from the angles at the vertex of each at C and B; consequently, as the area of the triangle is equal to the product of the base, multiplied by one half of its altitude, (Example 5,) the area of the trapezium must be equal to the product of the diagonal, multiplied by half the sum of the altitudes of both triangles.

It is evident that the result will be the same from which soever angles the diagonal and perpendiculars to it are drawn; for had it been drawn from C to B, instead of from A to D,

as

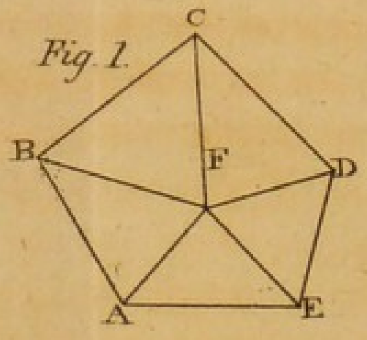


Fig. 1.

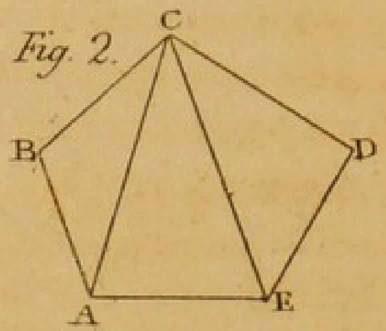


Fig. 2.

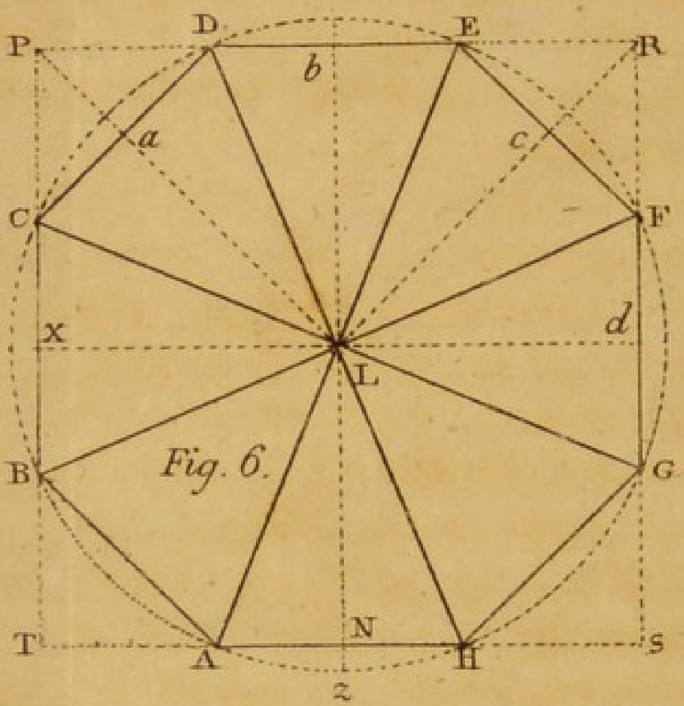


Fig. 6.

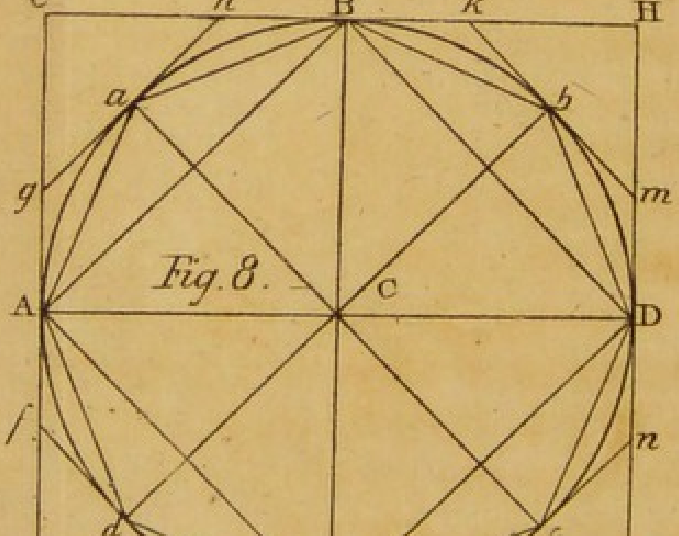
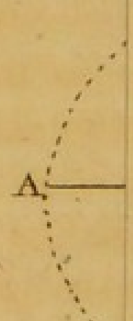


Fig. 8.



as in the figure, the diagonal would have been longer than AD; but in the same proportion the sum of the perpendiculars would have been shorter than in the former case, and consequently the product would still have been the same.

Example VII.

To find the area of a trapezoid, ACDB, (Fig. 15, Plate 1,) or quadrilateral of which the sides are unequal, but two of them, CD and AB, are parallel.

Rule.—Add together the two parallel sides CD and AB, and take half the sum, which is to be multiplied by the perpendicular altitude CE, when the product will be the superficial area required.

In Prop. 16 of Geom. Fig. 30, Plate 1, it was demonstrated, that the area of any trapezoid, ABCD, is equal to a parallelogram ABGH, whose sides BG and AH are each equal to the line FE, there shown to be equal to the half of the sum of the two parallel sides of the trapezoid BC and AD; and whose perpendicular altitude, measured by the line BL, is also that of the trapezoid.

The area of this figure may, however, be calculated like that of a trapezium, by means of a diagonal and two perpendiculars, as pointed out in Example 6, and hence we obtain a method of ascertaining the superficial contents of figures regular or irregular, of any number of sides; because, by drawing lines joining opposite angles, or to each angle from some central point, we throw the polygon into a series of triangles, the contents of each of which may be calculated, and the aggregate of these triangles will give the contents of the given figure.

Thus the pentagon ABCDE, (Fig. 1, Plate 5,) may be measured by assuming any convenient position towards the centre of the figure at F; after which drawing the lines FA, FB, FC, FD, and FE, the pentagon will be thrown into 5 triangles; and by measuring the sides of these trian-

 gles,

gles, their areas may be calculated; the sum of which will be equal to the area of the whole pentagon: but as by drawing lines in this manner from a central point to each angle, the number of triangles will always be equal to the number of sides of the given figure, the trouble of calculation, and the consequent risk of error, must be great, it is in general better in polygons of a moderate number of sides, to form the triangles by lines drawn from some one angle to all the others, as in Fig. 2, where, by means of two lines drawn from the angle at C to those at A and E, the pentagon is divided into 3 triangles, the operation of calculating which will be less tedious than in the way shown in Fig. 1, as the number of triangles formed according to Fig. 1, will always be two more than those formed according to Fig. 2, whatever number of sides the polygon may contain.

In regular pentagons whose angular points are in the circumference of a circle, as Figure. 3, the five triangles into which it is divided, by lines drawn from the centre of the circle at F, being all radii of the same circle, and consequently equal, and the sides AB, BC, CD, DE, and EA being by construction also equal, since the pentagon is regular, it follows that the triangles are all equal, so that if the area of one is found, we have only to multiply it by the number of triangles to obtain the area of the pentagon.

In introducing the subject of Trigonometry it was observed, (page 383,) that the circumference of all circles is supposed to be divided into 360 equal parts, called degrees: if this be taken for the measure of the circle circumscribing the pentagon, (Fig. 3, Plate 5,) the arch subtended by by each side, as AE, must be the fifth part of 360, or 72 degrees, which is also the measure of the angle formed at the centre by lines drawn from the extremities of this arch, that is, the angle AFE, and the two other angles

FAE and FEA, must together be equal to the difference between 72° and 180° = two right angles (Geom. Prop. 7) = 108°; but the sides FA and FE being equal, the angles opposite to each must also be equal, (Geom. Prop. 4,) consequently FAE and FEA will each be an angle of 54°. If, therefore, in the triangle AFE, be drawn the line FG perpendicular to AE, it will bisect the angle at F, as also the side of the pentagon AE, (Geom. Prop. 22 and 28): hence we have two triangles, AFG and EFG, having the base AG = base GE, and the side FG common to both: the angle at A = angle at E, each = 54°, consequently if the side of the pentagon be given, as, for instance, 20 feet, we can, by Case 3 of right-angled Trigonometry, discover the measure of the perpendicular FG = 13,764, and also the two hypothenuses FA and FE, each = 17,013. Then, by Example 1 of Mensuration, knowing the base of the triangle AFE = 20, and the perpendicular altitude GF = 13,764, we discover the superficial area of the triangle to be = 137,64 square feet; which quantity being multiplied by 5, the number of triangles in the pentagon, we obtain 688,2 square feet for the contents of the whole regular pentagon, ABCDE.

The preceding calculation, to ascertain the proportion between the given side AE of the pentagon, and the radius AF of the circumscribing circle, being tedious, although necessary for the due understanding of the process of measuring the area of a regular pentagon, when the external side alone is known, the student may occasionally employ the following standing proportions, which have been calculated on the same principles, viz. that the external side is to the radius of the circle, passing through the angles of a *pentagon*, as one is to ,8506508, &c.; and that the side is to the perpendicular let fall on it from the centre, as one is to ,53884176, &c: or thus, referring to the figure in the plate,

AE

AE : AF :: 1 : ,8506508
AE : FG :: 1 : ,53884176.

So that by multiplying the given side of any pentagon by the first fractional number, may be obtained AF, the radius of the circumscribing circle; and by multiplying the same side by the second fraction, may be obtained the perpendicular FG.

Example IX.

To find the superficial area of a hexagon, Fig. 4, Plate 5.

It was shown in the first Proposition of Trigonometry, (Fig. 2, Plate 3,) that the radius of every circle is equal to the chord of 60 degrees; if, therefore, the hexagon be regular, that is, if all its sides and angles be equal, so that a circle described from the centre of the hexagon would pass through all its angular points; in such a case the side of the hexagon would be equal to the radius of the circumscribing circle; or AG, GF, and FA would form an equilateral triangle. To discover, therefore, the area of this triangle, it would only be necessary to calculate the perpendicular GH, drawn from the vertex to the base.

In explaining Case 1 of Trigonometry, the use was shown of the 18th Prop. of Geometry, which proved that the square of the hypothenuse of a right-angled triangle is always equal to the sums of the squares of the other two sides: from this property, therefore, in the present example, in order to discover the perpendicular GH, we have only to square the hypothenuse AG, which, as was already said, being the radius of the circle, is equal to the given side, and from that square subtract the square of AH = half the side AF, and the square root of the remainder will be the perpendicular GH. Supposing the side of the given hexagon AF to be 20 feet, the radius or hypothenuse AG will also be 20, which squared is 400; and the square of AH = 10 is 100; their difference is 300, and its square root

root is 17,3205 = the perpendicular GH, which multiplied by one half of the base = 10, will give 173,205 square feet for the area of the triangle AGF, and this product again multiplied by 6 = the number of triangles in the hexagon, the last product 1039,23 square feet will be the superficial area of the given hexagon.

Or on account of the triangles in a regular hexagon being always equilateral, the proportions between the side and the perpendicular drawn upon that side from the centre of the figure is as 1 to ,866025; consequently, in order to find the perpendicular in any angular hexagon, we have only to multiply the given side by that fraction, and the product will be the perpendicular required.

Example X.

To find the contents of a regular heptagon, or figure of 7 equal sides and equal angles, such as is represented in Fig. 5, Plate 5.

As the heptagon consists of 7 equal sides, the arch of the circumscribing circle subtended by each side must be the 7th part of the whole circumference, or 360 degrees, and therefore very nearly equal to $51°, 25', 43''$, which is also the measure of the angle at the centre formed by radii drawn to the extremities of the side of the figure. If now the side AG be given = 20 feet, and the angle AHG = $51°, 25', 43''$, we may discover the measure of the perpendicular HL, which bisects the angle AHG as well as the side AG; for in the triangle AHL, right-angled at L, we have the angle AHL = half AHG = $51°, 25', 43''$, = $25°, 42', 51\frac{1}{2}''$, and the side AL = half AG = 20 = 10; hence, making AL radius we have this proportion, as radius : tangent of HAL = $64°, 17', 8\frac{1}{2}''$ :: AG : HL, which will appear to be = 20,76 feet. If the radius of the circumscribing circle were required, it would be found by this statement, as sine of HAL : radius :: HL : HA, =

23,06

23,06 feet: then to find the area of one of the triangles, AHG, by multiplying the perpendicular HL = 20,76 into half the base = 10, we obtain 207,6 square feet for one triangle, which again multiplied by 7, the number of triangles in the pentagon, the total area is 1453,2 square feet.

Or the general proportion between the side of a regular heptagon and this perpendicular let fall upon that side from the centre of the figure is very nearly as 1 to 1,0382683, by which number multiplying the side of any given regular pentagon, the perpendicular, and consequently the area of the figure, will be known.

Example XI.

To find the area of a regular octagon, or figure of 8 equal sides and equal angles, such as ABCDEFGH, Fig. 6, Plate 5.

The circumference of the circle passing through the angular points of the octagon being divided into 8 equal parts, each side, as AH, must subtend an arch = 45 degrees, or in other words, the angle, as ALH, at the centre of an octagon formed by radii drawn to the extremities of the sides, is always = 45 degrees; consequently the angle, ALN = half ALH = 22°, 30′, and its complement to a right angle LAN = 67,30′. Now, knowing the angles of the triangle ALN, right-angled at N, and the base AN = half of the side AH, which may be given 20 feet, we can, in the same way as in the preceding examples, discover the length of the perpendicular LN, which will be = 24,14 feet, as also of the hypothenuse or the radius of the circumscribing circle AL = 26,13 feet. Having proceeded so far, we multiply the perpendicular by half the base, and the product 241,4 square feet is the area of one of the triangles, which again multiplied by 8, the number of

triangles

triangles in the figure, the product 1931,2 square feet is the area of the whole octagon.

In general, if the side of a regular octagon be $=1$, the perpendicular from the centre to that side will be $=1,207107$ very nearly, and the radius of the circumscribing circle will be 1,306563: so that by employing these numbers as multipliers, the perpendicular and the radius of any octagon, of which the side is known, may be found, and of course its superficial content.

If of the octagon (Fig. 6, Plate 5,) the opposite sides BC, DE, FG, and HA should be produced in both directions, as represented by the dotted lines, until they met in the points PRST, these lines would form a square of which the sides would be cut, by the several angular points of the octagon, in such a proportion that the square of the middle portion of the side would be equal to the sum of the squares of the two lateral portions; or the square of DE would be equal to the sum of the squares of PD, and ER, or double the square of either of the portions PD, and ER. Let the side CD be bisected in the point a, and the line L a P be drawn; let DE be bisected by the line L b; and EF be bisected by the line L c R: then the four parts C a, a D, Db, and bE, will be equal to the four parts aD, Db, bE, and Ec; but the first four parts, or the two sides CD and DE, being one fourth part of the octagon, they must comprehend a quadrant of the circumscribing circle, consequently the four parts comprehended between a and c must also be a quadrant, and the angle formed at the centre L by lines drawn through a, and c, that is PLR, must of course be a right angle. But the side DE being bisected by Lb, the angle PLb must be equal to the angle bLR, that is, each must be half a right-angle $=45$ degrees, consequently the angle bPL must also be $=45$ degrees. In the same way it may be shown that the angle bRL is $=$ 45 degrees, and, therefore, the sides opposite to these equal

 angles

angles must be equal, or LP will be equal to LR; likewise LPx may thus be shown to be $=$ 45 degrees; and consequently the whole angle xPb $=$ 90 degrees, or a right-angle, in which case the figure PRST must be a square, as was before said.

Now it was already shown that bP was equal to xP, if from these quantities we take away the equal parts bD and xC, the remainders DP and PC will also be equal; and the square of DC, the hypothenuse, will be equal to the two squares of DP and PC; or double the square of DP: but DC is equal to DE, both being sides of the regular hexagon, consequently the square of DE, the middle portion of the side of the square described by the production of the sides of the octagon, will be double the square of the lateral portions PD or ER.

It is a curious fact in the history of numerical quantities, that no number has yet been discovered, of the square of which the half will be a square number; thus, for instance, let the side of the octagon DE be 20 feet, its square is 400, of which the half is 200, the square root of which will be 14,1421356, with a remainder: and if the side of the octagon DE were $=$ 1, the lateral portions of the side of the square PD and ER would each be $=$,70710678; so that in order to find the side of an octagon whose diameter xd $=$ the side of the square formed by the production of the sides is given, it would only be necessary to state this proportion, as the sum of ,70710678 + 1 + ,70710678 $=$ 2,41421356 is to one, so is the given diameter of the hexagon to its side DE, which being subtracted from the given diameter xd, or side of the coinciding square PR, will leave the sum of the two lateral portions PD, and ER, and the half of this remainder will be one of these portions.

EXAMPLE

Example XII.

To find the superficial area of a regular nonagon, a decagon, an undecagon, a dodecagon, or figures of nine, ten, eleven, twelve, equal sides and equal angles, by following the rules already given, the following proportions will be discovered, viz.

	Side of Triangle	Perpendicular
Nonagon	1,4618	1,37372
Decagon	1,618034	1,5388417
Undecagon	1,77888	1,703
Dodecagon	1,93185	1,86632

In general, to find the area of any regular polygon, divide 360 degrees by the number of sides in the figure, and the quotient will be the angle formed at the centre by lines drawn from it to the extremities of each side; then subtracting the half of this angle from 90 degrees, the remainder is the angle at the base of the triangle of the polygon, formed by the side and the radius of the circumscribing circle: having thus the angles and the base, or side of the polygon, the perpendicular let fall on it from the centre of the figure and the side of the triangle, or the radius of the circumscribing circle, are easily found by the application of the rules of right-angled Trigonometry.

Example XIII.

To find the area of a circle.

From the preceding example the student may have perceived that, in proportion to the increase of the number of the sides of the given polygon, the measures of the side of the triangle, or of the radius of the circumscribing circle, and of the perpendicular from the centre to the

side of the figure, approach nearer and nearer to the same quantity: the consequence of this would be, that in a polygon of an infinite number of sides, the radius and the perpendicular would come to be so nearly of the same magnitude, that their difference would always be less than any assignable quantity, at the same time that they never could absolutely coincide, nor be equal, because, (Fig. 6, Plate 5,) for example, however small the side of the polygon AH, still, as the perpendicular LN must fall upon the middle of that side, and cut it into equal parts, thereby forming right angles upon it, the side LA opposite to the right angle must be greater than the perpendicular LN opposite to the acute angle LAN: but if the perpendicular LN be produced until it touch the circumscribing circle in *z*, then LN and L*z* being radii of the same circle, must be equal; and it is evident, that as the side of the polygon AH is the chord of double the arch A*z*, the line LN which bisects that chord must be less than the radius L*z*, and consequently that the points N and *z* can never entirely coincide.

As, however, all dimensions and distances must be conceived to be measured in right lines, we have no other way of estimating the quantity of the arch A*z*H, however minute it may be imagined to be, but in the directions of the chord ANH: consequently a circle can be measured in no other way, than by supposing it to be a polygon of an indefinite number of sides, each side being the chord of a very minute angle formed at the centre by radii drawn to each extremity of the chord. Suppose a circle to be a polygon of one thousand equal sides, forming with each other a series of equal angles, and that each side was in length one foot, then the whole circuit of the polygon would be 1000 feet, and the area of the whole would be found by multiplying by 1000, the area of each triangle, whose base was 1 foot, and whose altitude was the perpendicular

dicular let fall on that base from the centre of the figure; but, as was already observed, when the polygon consists of a very great number of sides, the difference between this perpendicular, and the side of the triangle, which is the radius of the circumscribing circle passing through the angular points of the polygon, is so minute as not to be sensible; these two lines may therefore in practice be taken to be of the same magnitude, and the radius of the circle may be employed instead of the proper perpendicular, the half of which being multiplied by the base of the triangle, and the area multiplied by 1000 the number of triangles, or, which is the same thing, half the radius multiplied into the whole circumference, or the whole radius into half the circumference, or the whole radius into the whole circumference, and half the product taken, either of these methods will give the superficial contents of the given circle.

The circumferences of circles are to one another in the proportion of their radii; thus, in Fig. 7, Plate 5, about the same centre C are described two circles *abdefghi*, and ABDEFGHI; let these circles be divided into the same number of equal parts, and through the points of division let the radii C*a*A, C*b*B, C*e*E, C*f*F, &c. be drawn; and let chords be drawn in both circles joining the points in which their radii cut their respective circumferences, as *ab*, *bd*, *de*, &c. and AB, BD, DE, &c. then shall we have two similar polygons described within the concentric circles; for in the two triangles *a*C*b* and ACB, the angle at C being common to both, and the sides about this angle being proportionals, *a*C being equal to *b*C, and AC equal to BC, these triangles are therefore similar (Geom. Prop. 21); and in the same way may all the other triangles of the given polygons be shown to be similar; and the whole circuit of the greater polygon will be to that of the less, as one side of the greater is to one

side

side of the less, or as AB to *ab*, or, on account of the similarity of the triangles, as AC to *a*C.

But as this similarity and proportion do not depend on the number of sides in the polygons, they will take place in polygons of an infinite number of sides, and, consequently, in circles which must be regarded as polygons of that description; the circumferences of circles will, therefore, be one to another in the proportion of their radii, that is, the whole circumference ABDEFGHI will be to the whole circumference *abdefghi*, as the radius of the first circle AC is to the radius of the second circle *ac*.

This being established, it is evident that if we could ascertain the proportion between the radius and the circumference of any given circle, we might employ this proportion to ascertain the circumference or the radius of any other circle of which either the circumference or the radius were given; but the precise porportion between the radius or the diameter of a circle, and its circumference, has never yet been discovered, notwithstanding the repeated attempts of geometricians from the earliest times.

Archimedes, the Syracusan, who was born about 280 years before Christ, discovered by means of polygons of 96 sides, that if the diameter of a circle was 7 feet, the circumference would be very nearly 22 feet, so that to find the circumference of any other circle of a different diameter, as 20 feet, it would only be necessary to solve the proportion as 7 to 22, so is 20 to a fourth proportional, which will be 62,857 feet. This proportion is sufficiently exact for common purposes, as far as until the diameter extend to about 800 feet, when its inaccuracy becomes sensible.

Another proportion was discovered by Adrian Metius, which is much more accurate than the preceding, viz. that if the diameter be 113, the circumference will be 355; and even until the diameter of the given circle extend to one million

million of feet, this proportion will not sensibly differ from the truth.

But the proportion subsisting between the diameter and the circumference has been ascertained with the closest approach to accuracy, in various ways, to be the following, viz. if the diameter be unity or 1, the circumference will be 3,14159265358979323846264338327950288, &c. as calculated by Van Keulen, of Leyden.

Various modes have been adopted for determining the proportion between the diameter and the circumference of a circle, of which the following will be sufficient to give the student an idea of this curious and important problem in geometry and mensuration. See Fig. 8, Plate 5.

Let ABDE be a circle of which the centre is C, through which are drawn two diameters AD and BE, cutting each other at right angles: by joining the extremities of these diameters will be formed the inscribed square ABDE. Again, through the angular points of this square, draw tangents to the circle, which meeting each other, will form the circumscribed square GHIF. Now if the diameter of the circle be $= 2$, the side of the inscribed square will be the square root of twice the square of half the diameter, or $= 1,4142135\acute{6}$, and its area will be 2; whereas the side of the exterior square being equal to the radius of the circle $= 2$, its area will be $= 4$; consequently the area of the circumscribing square is always double that of the inscribed square; but it is evident from inspecting the figure, that as the circumference of the circle is situated between the boundaries of these two squares, its area must be greater than that of the interior, and less than that of the exterior square.

Having gone thus far, let us advance another step; if the four sides of the inscribed square be bisected, and lines be drawn through the points of bisection from the centre, they will cut the circumference in the points *a*, *b*, *c*, and *d*, and the chords A*a*, *a*B, B*b*, *b*D, &c. be drawn, a regular octagon,

A*a*

AaBbDcEd will be formed within the circle; as one will of be formed circumscribing the circle, by drawing tangents through the points, a, b, c, and d, joining the sides the exterior square in the points, h, k, m, n, o, e, f, and g.

If now the areas of these octagons be calculated, that of the exterior will be 3,3137085, and that of the interior 2,8284271, somewhere between which quantities must be the area of the interjacent circle.

Again, if the sides of the interior octagon be bisected, and lines supposed to be drawn from the centre through the points of bisection to the circumference, and chords drawn joining these extremities, a polygon of twice the number of sides, or one of 16 sides, will be formed within the circle, and if, through the angular points, tangents be drawn to meet the sides of the exterior octagon, a polygon of 16 sides will be found circumscribing the circle. Then let the areas of these last polygons be calculated, when that of the exterior will be 3,1825979, and that of the interior 3,0614674, differing considerably less from one another than those of the octagons; and between these two quantities must lie the area of the interjacent circle.

By repeated calculations of this nature, founded on the successive bisection of the side of the inscribed polygon, and consequent doubling of the number of sides in each succeeding polygon, the following table has been formed, in which the first column indicates the number of sides in the polygons, the second column shows the area of the inscribed, and the third that of the circumscribed polygon, and the fourth contains the difference between these areas.

No.

N. of Sides	Inse. Polygon.	Circ. Polygon	Difference
4	2,0000000	4,0000000	2,0000000
8	2,8284271	3,3137085	.4852814
16	3,0614674	3,1825979	.1211305
32	3,1214451	3,1517249	..302798
64	3,1365485	3,1441184	...75699
128	3,1403311	3,1432236	8945
256	3,1412772	3,1417504	4732
512	3,1415138	3,1416321	1183
1024	3,1415729	3,1416025	296
2048	3,1415877	3,1415951	74
4096	3,1415914	3,1415933	19
8192	3,1415923	3,1415928	5
16384	3,1415925	3,1415927	1
32768	3,1415926	3,1415926	....

From this table it appears, that the area of a regular polygon of 32768 sides inscribed within a circle, and that of a similar polygon circumscribing the circle, approach so near to one another that, as far as 7 places of decimals, they perfectly coincide: but as the circumference of the circle must be situated between the peripheries of these two polygons, being greater than the interior, and less than the exterior, it must differ less from either than these do from each other, consequently the area of a circle whose diameter is 2, may safely be considered to be 3,1415926: but as the area is the product of the whole circumference into half the radius, or of the whole radius into half the circumference, if the radius be supposed 1, the half of the circumference must be 3,1415926, and consequently double the radius, or the diameter of the circle, must be to the wholecircumference as 1 to 3,1415926.

In order to discover the areas of the several polygons inscribed in a circle, and of those described about it, formed by repeated bisections of the sides of these polygons, the following method may be observed.

 Let

Let XAGBZ (Fig. 9, Plate 5,) be a portion of a circle of which C is the centre, and AGB a quadrant. If the line AB be drawn joining the extremities of the quadrant, it will be the side of a square inscribed within the given circle (Geom. Prop. 39); and if the arch AB be bisected in G, and the line DE drawn a tangent to the circle at G, meeting the two radii CA and CB produced in D and E, then will DE be the side of a square described about the circle. Again, the quadrant AB being bisected in G, if the chords AG and GB be drawn, each of these chords will be the side of an inscribed polygon of double the number of the sides of the first polygon, or of one of 8 sides: and if through A and B are drawn tangents meeting the side of the first external polygon in the points H and I, then will HI be the side of the circumscribed polygon of 8 sides; for in the two triangles CAH and CGH, the angles at A and G being both right, are equal, the sides CA and CG being also equal, and the side CH common, the remaining side AH must be equal to HG: in the same way HG must be equal to GI, consequently HI is double AH, that is, HI is equal to the side of the exterior polygon of the same number of sides with the interior polygon of which AG is the side.

The two triangles ACF and ACG having the same altitude, are to one another in the proportion of their bases CF to CG; they are also in the proportion of the inscribed polygons of which they are relative parts: if, therefore, we represent the area of the polygon of which AB is a side by the character a, and that of the polygon of which DE is a side by b; also x for the area of the polygon of which AG is a side, and z for that of the exterior polygon of which HI is a side; then a will be to b, as CF to CG. But the triangles CGA and CGD being of the same altitude, are in proportion as their bases CA and CD, and also as the polygons of which they are parts, or as x to b, consequently

sequently the areas x and b will be as the bases CA and CD. Again, because FA is parallel GD, CE will be to CG as CA to CD, therefore as the area a to the area x, so is the same area x to the area b, or, in other words, the area x will be a mean proportional between the two first polygons a and b, or $x = \sqrt{a \times b}$, by which the area of the inscribed polygon of which AG is the side will be found.

Further, the triangles CHG and CHD being of the same altitude, they are as their bases GH to HD: but as the angle DCG is bisected by the line CH, HG will be to HD as CG to CD, or as CF to CA, or as a to x; consequently the triangles CGH and CHD will be as the areas a to x, and CGH + CHD = the triangle CGD will be to CGH as $a + x$ to a, and CGD : 2CGH :: $a + x : 2a$: but CGD and 2CGH = CAHG are to one another as the polygons b and z, of which they are proportional parts, consequently $a + x : 2a :: b : z$; but the polygon x having already been found in the first part of this problem, by this last proportion, the area of the external polygon z, of which HI is the side, will also be found; $z = \frac{2a \times b}{a + x}$.

Now, to apply these two rules or forms of expression, let the radius of the given circle be = 1, when the sides of the inscribed square being the hypothenuse of a right-angled triangle whose sides are the radii of the circle, the area of this square will be = 2, and that of the circumscribed square will be = 4. If, therefore, as above shown, we make $a = 2$, and $b = 4$, the first expression $x = \sqrt{a \times b}$, or the square root of $2 \times 4 = 8$ will be 2,8284271, &c. for the area of the inscribed polygon of 8 sides; and the second expression $z = \frac{2a \times b}{a + x}$, or the quotient of $2 \times 2 \times 4 = 16$, divided by $2 + 2{,}82847$, &c. $= 4{,}82847$, will give for the area of the circumscribed polygon of 8 sides 3,137085 &c.

3 o 2 Then

Then, instead of these expressions *a* and *b*, by employing the values now found, and following the same rule, we obtain the areas of the inscribed and circumscribed polygons of 16 sides, and so on through all the other subdivisions, which will give the results set down in the foregoing table.

The result of all these inquires therefore is, as was already said, that in order to find the area of a circle, we must multiply the whole circumference into one half of the radius, or the whole radius into one half of the circumference, or the whole circumference into the whole radius, and take the half of the product.

For instance, required the area of a circular grass-plot of which the diameter is 360 feet. The proportion of the diameter to the circumference of any circle being as 1 to 3,14159, &c. we multiply the given diameter 360 by this last quantity, and the product (the division by the first term 1 making no change upon it) = 1130,9724 feet is the circumference required: then multiplying this quantity by one half of the radius, or one fourth of the diameter = 90, we obtain 101787,516 square feet for the area of the given grass-plot.

Again, supposing the circumference of the globe of the Earth to be 360 degrees, and each degree equal to about $69\frac{1}{3}$ English miles, the circumference will be = 24,960 miles; and dividing this quantity by 3,14159, &c. the quotient will be the diameter of the Earth = 7,945 miles: if now it were supposed that the Earth were cut into two equal portions by a plane passing through its centre, each part would present a flat surface of a circular form, of the above diameter and circumference, and the area or superficial contents, found by multiplying the circumference into one fourth of the diameter = 1986,25 miles, would be = 49,576,800 square English miles.

EXAMPLE

Example XIV.

To find the area of a sector of a circle, as ACBD in Fig. 10, Plate 5.

A sector of a circle was described (Geom. page 362 and 363) to be a figure bounded by a part of the circumference, as ADB and two radii AC and BC.

As this figure is evidently an integral part of the circle ADBE, its area must bear to the area of the entire circle the same proportion as the portion of the circumference belonging to the sector does to the whole circumference of the circle: for if we suppose the line CA to remain fixed, while CB is moveable like the hand of a clock, and that the two lines are in conjunction on CA, if CB be moved round the centre, it will come to occupy the position CD, then that of CB in the figure, afterwards CE, and so on until it again coincide with CA, where it began, describing by its motion round the centre C, sectors whose areas will be respectively equal to the portions of the circumference passed over. As, therefore, to find the area of a circle, we multiply half the radius into the whole circumference, so to find the area of a sector the rule is to multiply half its perpendicular altitude CD, which is the radius of the circle of which it is a part, into the portion of the circumference ADB.

Let, therefore, the perpendicular altitude CD, or the radius of the sector, be given = 10 feet, and the portion of the circumference intercepted between the radii CA and CB, or the arch CDB be = 120 degrees, in order to find the quantity of this arch in such parts as those given in the radii of the sector, we first find the circumference of a circle whose radius is 10 feet and diameter 20, which, by the proportion already given of 1 to 3,14159, will be = 62,83185 feet.

Then

Then saying, as the whole circumference = 360 degrees to the given arch of the sector = 110°, so is the circumference in feet = 62,83185 to the arch also in feet, which will be = 19,19862 feet; and multiplying this quantity into half the radius = 5, the product 95,9931 square feet will be the area of the given sector ACBD.

Had the arch of the sector been given at once in feet instead of degrees corresponding to the angle at the centre ACB, the area would have been found immediately by multiplying the lineal quantity of the arch into one half of the radius.

Example XV.

Fig. 10, Plate 5. To find the area of a segment of a circle, as the space inclosed between the arch ADB and the right line or chord AB. Let the radius of the circle be = 10 feet, the line AB be = 16,388, and the perpendicular FD (which is the portion of the radius bisecting AB, and intercepted between the segment line and the arch, and consequently giving the greatest breadth of the segment) be equal to 4,262. It is evident that this segment ABD is only a portion of the sector ACBD, and that the remainder is the triangle ABC; if then from the area of this sector we subtract that of the triangle, the remainder must be the area of the given segment.

To find the area of this triangle we have given the radius AC = 10, and the side AB = 16,388, the half of which, 8.194, is the side AF of the triangle ACF, as 5,738 (the difference between FD given 4,262 and the radius CD = 10) is the base CF: these things, therefore, being known, we can by the rules of right-angled trigonometry discover the angle ACF, which will be = 55°, 00′, and this doubled will give 110°, 00′, for the great angle ACB formed by radii drawn to the extremities of the given segment ADB.

Having

Having thus obtained the portion of the circumference corresponding to the given segment, and knowing the altitude of the sector CD, its area, by the last example, will be found = 95,9931 square feet.

Again, the three sides of the triangle BAC being given, BA = 16,388, and BC and CA each = 10, as also the perpendicular CF found = 5,738, the area of the triangle will be = 47,017172, which being subtracted from the area of the sector already found = 95,9931, will leave 48,975928 square feet for the area of the given segment AFBD; and if this quantity be taken from the area of the whole circle AEBD, the remainder will be the area of the great segment contained between the line AB and the arch AEB.

If the line AB passed through the centre C, the circle would be divided into two equal segments or semicircles.

In order to abridge the labour of calculating the areas of segments of circles, the following table has been calculated for a circle whose area is unity or 1, and whose diameter is supposed to be divided into 100 equal parts, through each of which the lines forming the several segments are drawn.

TABLE.

TABLE.

VS	Segment	VS	Segment	VS	Segment	VS	Segment
1	0,0017	26	0,2066	51	0,5127	76	0,8155
2	0.0048	27	0.2178	52	0.5255	77	0,8262
3	0,0087	28	0,2292	53	0,5382	78	0,8369
4	0,0134	29	0,2407	54	0,5509	79	0.8474
5	0,0187	30	0,2523	55	0,5635	80	0,8576
6	0,0245	31	0,2640	56	0,5762	81	0,8677
7	0.0308	32	0,2759	57	0,5888	82	0,8776
8	0,0375	33	0,2878	58	0,6014	83	0,8873
9	0,0446	34	0,2998	59	0,6140	84	0,8960
10	0,0520	35	0,3119	60	0,6265	85	0.9059
11	0,0598	36	0,3241	61	0,6389	86	0,9149
12	0,0680	37	0.3364	62	0,6514	87	0,9236
13	0,0764	38	0,3486	63	0,6636	88	0,9320
14	0,0851	39	0,3611	64	0,6759	89	0,9402
15	0,0941	40	0,3735	65	0,6881	90	0.9480
16	0,1032	41	0,3860	66	0,7002	91	0,9554
17	0,1127	42	0,3986	67	0,7122	92	0,9625
18	0,1224	43	0,4112	68	0,7241	93	0,9692
19	0,1323	44	0,4238	69	0,7360	94	0,9755
20	0,1424	45	0,4365	70	0,7477	95	0,9813
21	0,1526	46	0,4491	71	0,7593	96	0,9866
22	0,1631	47	0,4618	72	0,7708	97	0.9913
23	0,1738	48	0,4745	73	0,7822	98	0,9952
24	0,1845	49	0,4873	74	0,7934	99	0,9983
25	0,1955	50	0,5000	75	0,8045	100	1,0000

This table is used in the following way. The first columns contain the *versed sines*, or the heights of the segments of a circle whose diameter is 1, divided into 100 equal parts, and the second columns contain the areas of segments formed by perpendiculars to the diameter drawn through each of these 100 divisions. Suppose now the versed sine or height of the segment were 20, and that the diameter of the circle were 62,5, we state this proportion, as the given diameter 62,5 is to the diameter of the tables 100, so is the given versed sine or height of the segment 20, to a fourth proportional 32; then looking in the tables in the column of versed sines for 32, we have in

in the adjoining column 0,2759, which being multiplied into the area of the circle of which the given segment is a portion, the area of the segment will be known, as in this case, where the area of a circle whose diameter is 62,5 being 6013,21875, if it be multiplied by the above quantity 0,2759, the product 1659,047053125 will be the area of the given segment.

Example XVI.

Fig. 11, Plate 5, To find the area of an ellipse, or oval.

If two points be given in a plain surface, as E and F, and the point G be made to move about them in such a way that the sum of its distances from these points, or GE + GF shall always be the same quantity, wherever D may be situated, as at *g*, when *g*E + *g*F will be equal to GE + GF, then will the tract passed over by the point D be an ellipse.

The points EF, around which the ellipse is described, are called the *foci* of the ellipse, and the point H, which bisects the line joining the foci, is called the centre of the ellipse. The distance between either focus and the centre, as EH or FH, is termed the *excentricity* of the ellipse.

A straight line passing through the centre, and terminated both ways by the ellipse, is a *diameter*, and its extremities are termed *vertices*.

The diameter which passes through the foci, as AB, is called the *transverse*, or the *greater axis*; and a diameter perpendicular to the transverse axis, as CD, is the *conjugate*, or *less axis*.

A line not passing through the centre, but bisected by a diameter, and terminated both ways by the ellipse, is said to be an *ordinate* to that diameter.

The segments of a diameter, intercepted between an ordinate and the vertices, are called *abscisses*.

A right

A right line drawn touching the ellipse in any point, and every where else falling without it, is a *tangent* to the ellipse in that point.

The transverse axis AB is the greatest diameter that can be drawn in an ellipse, and the conjugate diameter CD is the least.

The sum of the distances of any point in the ellipse from the foci, is equal to the transverse axis, or GE + GF = AB.

Every diameter of an ellipse is bisected in the centre.

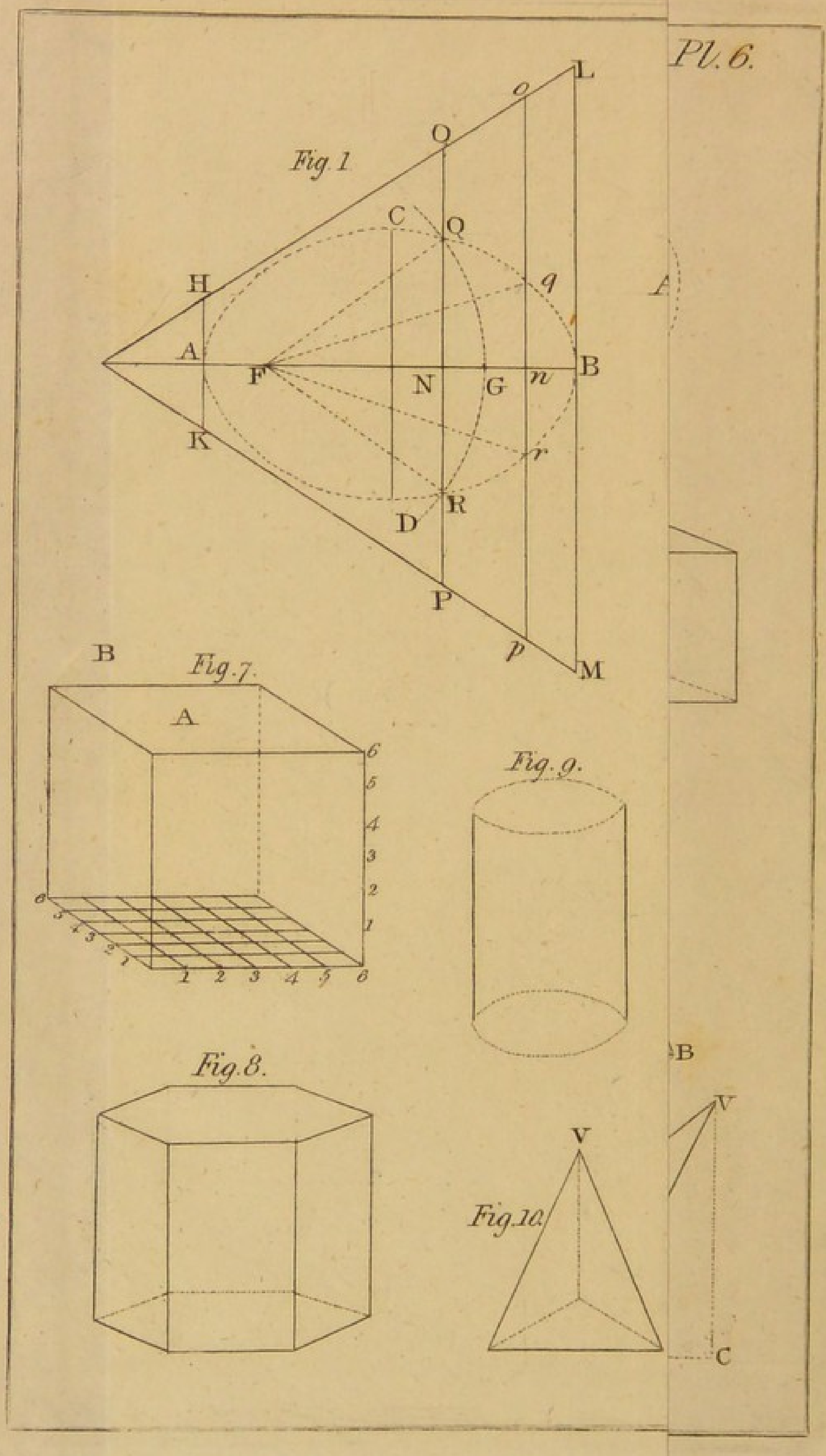
If a circle be described on the transverse axis of the ellipse as a diameter, the area of this circle will be to the area of the ellipse, as the transverse axis to the conjugate axis; consequently, the area of an ellipse is equal to the area of a circle whose diameter is a mean proportional between the transverse and the conjugate axis.

Let, therefore, the transverse axis AB be 24, and the conjugate axis CD be 16; multiplying these quantities together, and extracting the square root of the product 384, we have 19,596 for the diameter of a circle equivalent to the ellipse, and its area will be found, by Example 13, to be 301,6451158464.

In order to describe an ellipse, various methods have been devised, of which the following are the most convenient:

Method I. (Fig 12, Plate 5.)

Let AB and CD be the transverse and conjugate diameters, and E the centre of the ellipse. From C, as a centre, with a radius equal to AE, or EB = half the transverse axis, describe the arch FG cutting that axis in the points F and G, which will be the foci of the ellipse: then take a string equal in length to AB, and fixing the ends


Pl. 6.
Fig. 1
L
o
O
C
Q
H
q
A
F
N
G
n
B
K
r
R
D
P
p
M
B
Fig. 7.
A
1 2 3 4 5 6
1 2 3 4 5 6
Fig. 9.
Fig. 8.
B
V
V
Fig. 10.
C

ends at the points F and G, and stretching the string by means of a pin H, let the pin be moved round, keeping the string tight, when the tract formed by the pin will be the ellipse required; for the sum of the distances of any point, in the circumference from the two foci, will always be equal to the transverse axis.

Method II.

To describe an ellipse by finding a number of points in the curve.

Let AB and CD (Fig. 1, Plate 6,) be the transverse and conjugate diameters of the ellipse required to be described: by the rule just given, find both or either of the foci, F and G; at each extremity of the transverse axis, A and B, erect the perpendiculars HK and LM, making AH and AK equal to AF, and BL and BM equal to BF, and join HL and KM; then take any point, N in AB, and through it draw the perpendicular OP; and on F, as a centre, with a radius equal to NO or NP, describe an arch of a circle cutting OP in the points Q and R, then will these be two points in the curve of the ellipse.

In the same manner, if another point *n* be taken in AB, and through it be drawn the perpendicular *onp*; then, if from F, with a radius equal to *no*, an arch be described cutting *op* in the points *q* and *r*, these points will also be in the curve of the ellipse: and by continuing such a process, a sufficient number of points may be obtained through which the ellipse may be drawn.

Method III. (Fig. 2, Plate 6.)

Let AB be the transverse axis, and CD the conjugate axis of the ellipse intended to be drawn: from the point H, in the conjugate axis, set off the line HG equal to the

difference between the two semi-axes OA and OC, and produce HG, making GP equal to OC; then will P be a point in the curve of the ellipse.

In the same way, let the line EF, made equal to the *sum* of the two semi-axes OA and OC, be placed any where on the conjugate axis, as, for example, at E, and meeting the transverse axis (produced, if necessary, as in this figure) in the point F, and let FK be set off equal to OC, then will the point K be in the curve of the ellipse; and so other points may be found.

On the property of the curve here shown, is founded the construction of the instrument called the *Trammels*, used for describing elliptic curves. This instrument, which is represented in Fig. 3 of Plate 6, consists of two rulers, *ab* and *cd*, firmly secured at right angles in the middle of each, having a groove running lengthwise in their upper surface: a long ruler, or rod *ef*, having two moveable nuts and pins, *g* and *f*, is applied to the rulers, and works in both grooves, while a steel point or pencil at *e* describes the curve of the ellipse by the motion of the rod. Make the distance between the pins *f* and *g* equal to the difference between the semi-axis of the ellipse and the distance between the pin at *g*, and the pencil at *e*, equal to half the conjugate axis, when consequently the distance between the pin at *f* and the pencil at *e* will be equal to half the transverse axis. When this is done, by placing the pins secured by the nuts at *f* and *g* in the grooves of the cross rulers, and moving the rod *ef* gently round, the pencil at *e* will describe the required ellipse.

Other methods have been recommended for describing ellipses, by means of a pair of compasses, but these are all erroneous, and can only give a figure approaching to the ellipse; for the curves described by the compasses must, in every part, however small, be portions of circles, whereas the elliptic curve is in every part different from

a

a circle, as must be evident, when it is onsidered that the circle is described with one fixed radius from one centre, but that the ellipse is described with radii continually changing their dimensions, and from two centres.

If an ellipse be given, and its axis required, the following method is to be followed:

In Fig. 4, Plate 6, Let there be given an ellipse of which it is required to find the axis; within the ellipse, draw the two chords AB and CD parallel to each other, also the line EF, bisecting these chords, and terminated both ways in the curve: bisect this line in the point G, which will be the centre of the ellipse. Then take any point in the curve, as H, and from G, with the radius GH, describe a circle. If this circle fall wholely without the ellipse, the point H will be the vertex of the greater axis; and if it fall wholely within the ellipse, H will be the vertex of the shorter axis; but if the circle falls neither wholely without nor wholely within the ellipse, it must cut it in some other point, as K; then draw HK, and bisecting it in L, draw GL, and produce it both ways to meet the curve in the points M and N, when MN will be the transverse axis of the ellipse, and OP, drawn perpendicular to it through the centre G, will be the conjugate axis, as required.

MENSURATION

MENSURATION OF SOLIDS.

By a *solid body* we understand whatever is described under the three dimensions of length, breadth, and depth or thickness. Solids are bounded by plane surfaces, as a cube, a prism, &c. or by curved surfaces, as a globe, a cone, &c.

All solid figures, whose extremities are similar, equal and parallel planes, and all whose other sides are parallelograms, are in general termed *prisms*, such as are represented in Plate 6, Fig. 5, 6, 7, and 8 ; and the prism may be produced by the motion of one of the planes at its extremity along a right line, in a position always parallel to that which it had at the beginning of the motion.

The two parallel extremities are called the *bases* of the prism, and a perpendicular let fall from one base to the other, gives the *altitude* of the prism.

If a prism be cut across in any place by a plane parallel to the base, this section will be perfectly similar, and equal to the base.

A prism is said to be *right* when the sides are perpendicular to the base ; but it is an oblique prism when the sides are not perpendicular to the base.

Prisms are denominated according to the number of their sides ; thus, one of three sides, as in Fig. 5, is a triangular prism ; Fig. 6 and 7 are quadrangular prisms, their bases consisting of four sides ; and Fig. 8 a hexagonal prism, the base consisting of six sides.

Amongst quadrangular prisms are particularly distinguished the *parallelopipedon* and the *cube* ; the parallelopipedon is a prism whose bases, as well as sides, are parallel-

parallelograms, and, if all the angles are right, the figure is said to be a right-angled parallelopipedon: but if all the angles be right, and all the sides equal to the base, then the figure is termed a *cube*, which is, therefore, a solid comprehended under six equal squares.

The cube is the common measure of the solidity of all other solid bodies, in the same way that the square is the common measure of all surfaces (see page 441). Thus in Fig. 7, Plate 6, let A be a regular cube, each of whose sides is equal to 6 inches; to find the solid contents of this body, we must adopt some standard or unit to which the solidity must be referred, as, for instance, the small figure B, being a cube of 1 inch every side; when it is evident that the whole question is to discover how often the cube B can be contained in the cube A. As the side of A is 6 inches, it may be divided into 6 equal parts, each equal to B, of which the side is but 1 inch. Let then the cube B be cut or sawn through at each 6th division of the side, when it will be divided into 6 equal figures, each 6 inches long, 6 inches broad, and 1 inch thick; and containing 36 solids of 1 inch a side: again, let each of these sections be divided into 6 equal parts by lines drawn through the divisions on one side, when each will be 6 inches long, 1 inch broad, and 1 inch thick, and consequently contain 6 solids of 1 inch a side: lastly, let this section be cut through into 6 equal parts, when each part will be a solid of 1 inch every way, or 1 cubic inch. If now we trace back the operation from this cube, we have in the 3d section a cube of 1 inch; in the 2d a solid figure or parallelopiped of 6 inches; in the 1st section a parallelopiped of 36 inches, and in the whole cube A, a figure containing 216 solid or cubical inches. Hence, in order to find the solid contents of a cube, we multiply its length into its breadth, and the product into its thickness, when the last product will give the contents in solid

solid measures corresponding to those in which the dimensions are taken.

Further, let us suppose a right-angled box A, (Fig. 7, Plate 6,) whose dimensions on the inside are 6 inches in depth, breadth, and length, or that it is a cube of 6 inches a side: let there be a small cube or die B, of 1 inch every way, and let it be required to know how many of these small cubes or dice can be placed within the box A. If the bottom of the box be divided into squares by lines drawn crossing one another at right angles, at the distance of 1 each asunder, it is evident that on each of these squares 1 die may be placed, and that the bottom will contain 6 rows of 6 dice each, or in all $6 \times 6 = 36$ dice.

But as the box is 6 inches in depth, while the die B is only 1 inch deep, each square in the bottom will support a pile or column of 6 dice, and consequently the 36 squares will support each a pile of 6 dice, making in all $36 \times 6 = 216$ dice contained within the given box A: but each die being 1 cubic inch, the box will, of course, contain 216 cubic inches, being the cube of the side of the box, $6 \times 6 \times 6 = 216$.

From what has been said it follows, that in order to have the solid contents of any prism, we must multiply the superficial area of the base by the altitude of the prism, which is always measured at right angles to the base.

Example I.

To find the solid contents of the triangular prism represented in Fig. 5, of which the base is an equilateral triangle 6 inches a side, and the perpendicular altitude is 10 inches.

The area of the base will be found to be 15,588 square inches, (Mensuration of Surfaces, Example 5th,) and this quantity, multiplied by the perpendicular altitude 10 inches, will give 155,88 solid or cubic inches for the solid contents of the given triangular prism.

Example

EXAMPLE II.

To find the solid contents of the rectangular parallelopiped shown in Fig. 6, of which the length is 24 inches, the breadth is 18 inches, and the altitude 12 inches.

In the first place, by the 2d Example of Mensuration of Surfaces, we find the superficial contents of the base, which is 24 inches long by 18 broad, to be 432 square inches; and this multiplied by the altitude 12 inches, will give 5184 solid or cubic inches, for the solid contents of the given parallelopiped.

EXAMPLE III.

To find the solid contents of a cube, which is only a paralellopiped, of which the length, breadth, and altitude, are all equal, we have only to cube the side, and the product will be the solid contents. Thus, in the cube represented, Fig. 7, the sides are all equal, each being given 6 inches, the cube of which, or $6 \times 6 \times 6 = 216$ cubic inches, are the solid contents of the given figure.

EXAMPLE IV.

To find the solid contents of a regular hexagonal prism, such as in Fig. 8, whose base is 6 inches a side, and whose altitude is 10 inches.

Agreeably to the rule laid down in Example 9th of Mensuration of Surfaces, the area of the hexagonal base of the prism will be found to be 93,528 square inches, and this quantity multiplied by the altitude, 10 inches, will give 935,28 cubic inches for the solid contents of the given prism.

EXAMPLE V.

In the same manner the solidity of any prism, whatever be the form of its base, or the number of its sides, may be

 found:

found: and hence we discover the solidity of a *cylinder*, which is only a prism on a circular base, that is, on a base formed by a polygon of an infinite number of sides: so that, to discover the contents of the cylinder shown in Fig. 9, we first find the area of the circular base, (Mensuration of Surfaces, Example 13,) and then multiply that quantity by the perpendicular altitude, to obtain the solidity.

If, therefore, the base of the cylinder were a circle of 10 inches diameter, the circumference of course being 31,4159 inches, the area would be 78,53975 square inches, which quantity multiplied by the perpendicular altitude, say 25 inches, the solid contents of the cylinder would be 1953,49375 cubic inches.

Example VI.

To find the solid contents of a pyramid, such as are represented in Fig. 10, 11, and 12, of Plate 6.

A pyramid is a solid figure of which the base may be any polygon, but not a circle, and the sides are all triangles, whose bases are the sides of this polygon, and whose vertices unite in one point above the base of the figure, which is called its summit or vertex. Thus Fig. 10 represents a pyramid whose base is a triangle, on each side of which is erected a triangle, and the three unite in the vertex V. Fig. 11 represents a pyramid whose base is a square, and consequently the faces of the figure are four triangles meeting together at the vertex. Fig. 12 is a pyramid whose base is a hexagon, and its sides are of course formed by six triangles all meeting at the vertex of the pyramid V.

A perpendicular let fall from the vertex to the base of a pyramid will give the measure of its altitude, whether it fall within the base, as in Fig. 11, or without it, which will be the case when the pyramid is much inclined, as in Fig.

Fig. 13. When this perpendicular falls in the centre of the base, and when the base is a polygon of equal sides and angles, the pyramid is then said to be regular.

In a regular pyramid the triangles forming its exterior faces are isosceles, and equal to one another.

Every pyramid is one third part of a prism on the same base and of the same altitude, as may be seen by examining Fig. 14, Plate 6, which represents a triangular prism whose bases are DEF and ABC. If this prism be cut by a plane in the direction of the dotted lines EA and EB, beginning at the angle of the superior base E, and proceeding to the opposite angles, A and B, of the inferior base, the figure cut off AEBC will be a pyramid having the same base ABC with the prism, and the same altitude EC. Again, when this pyramid has been removed, the remainder of the prism may be cut into two equal pyramids by a plane passing through the angle at E and the dotted line AF, for they stand upon equal bases, DAF and BAF, and their altitude is the same, the angle at E being their common vertex: the prism is thus cut into three equal pyramids, and consequently one of the pyramids is one third part of the prism; but as to find the solidity of a prism we are to multiply the area of its base into its altitude, so for the solidity of a pyramid we must multiply the area of the base into one third part of its altitude.

If, therefore, the base of the pyramid shown in Fig. 11 be a square of 12 inches a side, and the perpendicular altitude be 18 inches, we must multiply the area of the base, 144 square inches, by one third part of the altitude, which is 6, and the product, 864 cubic inches, will be the solid contents of the given pyramid.

Example VII.

To find the solid contents of a cone, as represented in Fig. 15 and 16, of Plate 6.

 A cone

A cone is a solid body contained within the circular base AB and the surface described by the line VA moving round the point V, and following the circumference of the circular base AB. The point V is the vertex or summit of the cone, the line VC is its altitude, whether the figure be a right cone, in which this perpendicular falls on C the centre of the base, as in Fig. 15, or whether it be an oblique cone, as in Fig. 16, where the perpendicular VC does not fall on that centre.

A right cone may also be imagined to be described by the motion of the right-angled triangle VCA, turning round on the perpendicular VC, in which case the hypothenuse VA will describe the surface of the cone.

As a circle is to be considered as a polygon of an infinite number of sides, a cone must of course be considered as a pyramid on a base of an infinite number of sides, and a cylinder being also a prism of an infinite number of sides, it follows that a cone has to a cylinder on the same base and of the same altitude, the same relation that a pyramid has to a prism on the same base and of the same altitude. But it has just been shown (Example 6.) that a pyramid is one third part of a prism of such a description, consequently a cone is one third part of a cylinder on the same base and of the same altitude. If, therefore, it were required to find the solidity of a cone whose base was 20 inches in diameter, and whose altitude was 36 inches, we would first find the area of the circular base (Example 13 of Mensuration of Surfaces,) which will be 314,159 square inches, and multiplying this quantity by one third of the altitude of the cone, or by 12, we have 3769,908 solid inches for the contents of the given cone.

Example VIII.

To find the solid contents of a frustum or portion of a

cone

Pl. 7.

Fig. 1.

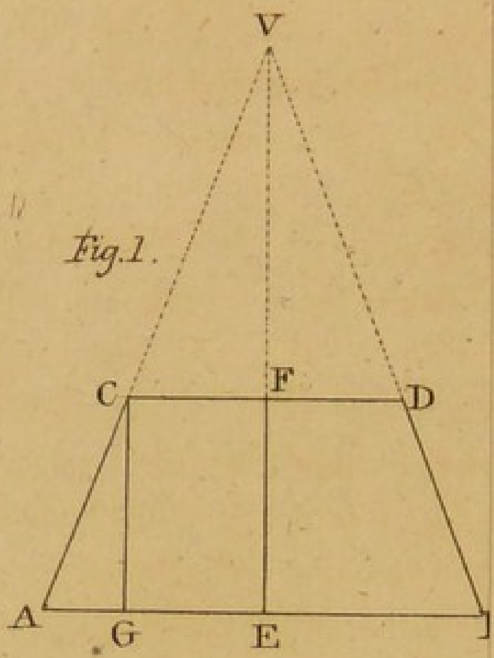
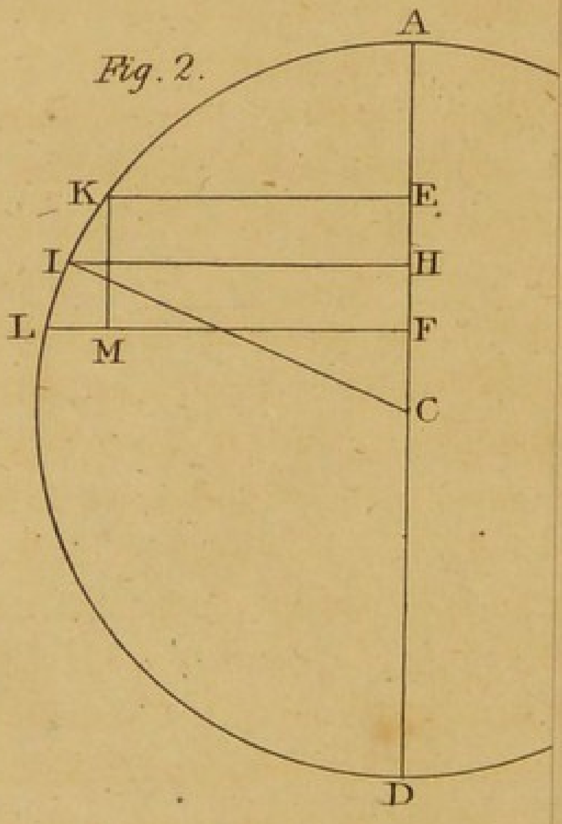


Fig. 2.



cone or a pyramid, as Fig. 1, Plate 7, where AB is the base of a cone whose diameter is 20 inches; and the cone being cut by a plane CD parallel to the base, whose diameter is 12 inches, at the distance FE of 10 inches from the base, the solid figure ACDB is a frustum of a cone. In this case all that is to be done is to calculate the solidity of the whole cone of which the given frustum is a part, and from it subtracting the portion required to complete the cone CDV, the remainder must be the solidity of the given frustum. But as, in order to make these calculations, we must know the perpendicular altitude of the complete cone AVD, which is not given, we proceed in this way: from the point C if a perpendicular be let fall, as CG, on the base, CG will be equal to the altitude of the frustum, and GE being equal to CF the semidiameter of the upper base of the frustum, AG will be the difference between the semidiameters of the two bases; and as the two triangles AGC and CFV are similar, their corresponding sides will be proportionals, (Geom. Prop. 21,) consequently AG will be to GC as CF to FV, that is, the difference between the semidiameters of the two bases of the frustum will be to its altitude, as the semidiameter of the upper base to the altitude of the supplementary portion of the cone. Again, the angles ACG and AVE being similar, their corresponding sides will also be proportionals, AG : GC : : AE : EV; that is, the difference between the semidiameters of the two bases of the frustum will be to the altitude, as the semidiameter of the lower base is to the altitude of the entire cone.

Now if these proportions be employed, we shall find the altitude of the entire cone EV to be 25 inches, and that of the supplementary part of the cone FV to be 15 inches. — Having thus the requisite dimensions of the two cones, the entire one will contain 2617,992 solid inches, and the supplementary one 565,4862 solid inches, consequently the difference

difference between these quantities, 2052,5058 solid inches, will be the contents of the given frustum or portion of the cone ACDB.

The same process is to be followed in calculating the solidity of a frustum of a pyramid.

EXAMPLE IX.

To find the solid contents of a globe or sphere. Fig. 2, Plate 7.

As the circumference of a circle is considered to be a polygon composed of an infinite number of sides, so the external surface of a sphere or globe may be considered as composed of an infinite number of planes, each forming the base of a pyramid whose altitude is the radius of the globe. As, therefore, the solidity of each little pyramid is calculated by multiplying the area of its base into one third part of its altitude, (Example 6,) the total amount of their solidity must be the product of all their bases multiplied by one third of the radius, or, in other words, the solidity of the globe must be the product of its surface multiplied by one third of the radius.

The surface of a sphere is equal to the product of the circumference of a great circle on its being multiplied by the diameter.

Let ALD represent a sphere, and let the half-circumference AKILD be supposed to be divided into an infinite number of small arcs, such as KL, which may therefore be considered as coinciding with its chord. Through the extremities of this arc draw KE and LF perpendicular to the diameter AD, and IH from the middle of KL, also the radius IC, which must be perpendicular to KL; draw likewise KM perpendicular to IH and LF. If now we suppose the half-circumference or semi-circle AKD to be moved round, as a door upon its hinges, on the diameter AD,

AD, it will produce the surface of the sphere, and each of the small arcs, as KL, will produce the surface of a truncated cone, of which KE will be half the upper base, and LF half the lower, (such as is represented in Fig. 1, Plate 7,) forming an integral part of the surface of the sphere; and the surface of this truncated cone will be equal to the product of KM or EF multiplied by the circumference whose radius is IC or AC.

The triangle KML being similar to the triangle IHC, for the three sides of the one are perpendicular to the corresponding sides of the other, we have this proportion, KL : KM :: IC : IH; or, circumferences being to one another in the proportion of their radii, KL : KM :: the circumference described upon IC : the circumference described upon IH, consequently the product of KL multiplied into the circumference on IH is equal to the product of KM multiplied into the circumference on IC, or to the product of EF multiplied into the circumference on AC. Now the first of these products is the surface of the truncated cone formed by the motion of KL, which is therefore equal to the product of EF multiplied into the circumference on AC, or, in other words, to the product of the altitude of the truncated cone, multiplied into the circumference of any great circle of the sphere. In the same way, if we divide the circumference into a number of small arcs, similar to that represented by KL, the same result will be produced, and consequently the whole of these small truncated cones, composing by supposition the surface of the sphere, must be equal to the circumference of a great circle multiplied by the sum of the altitudes of these cones, which will of course form the whole of AD, which is the diameter of the sphere: and hence the surface, or superficial contents of a sphere, is equal to the product,

duct of its diameter multiplied into the circumference of a great circle on it, that is, of a circle whose diameter is that of the sphere.

If we imagine a cylinder circumscribing and in contact with a sphere, that is, a cylinder whose altitude and the diameter of whose base are both equal to the diameter of the sphere, it will follow from what has just been shown, that the surface of the sphere is equal to the interior surface of the circumscribing cylinder; for the surface of a cylinder is equal to the product of the area of its circular base multiplied by its altitude; and in this case the circumference and diameter of the base must be precisely equal to those of the inscribed sphere. Since, therefore, in order to have the superficial area of a circle, we multiply the circumference by one fourth part of the diameter, (Example 13 of Mensuration of Surfaces,) and since, in order to obtain the superficial area of a sphere ,we multiply the circumference by the whole diameter, it follows that the superficial area of a sphere is equal to four times the area of one of its great circles.

From the foregoing demonstrations it will also follow, that in order to obtain the superficial area of a segment or portion cut off from a sphere, we must multiply the circumference of a circle whose diameter is that of the sphere, by the altitude of the segment; and that to obtain the superficial area of a portion of a sphere cut off by two parallel planes, we must multiply the circumference of a great circle by the altitude of the portion included between the two parallel planes.

The rule, therefore, for finding the solidity of a sphere is, to multiply one third part of the radius, which is one sixth of the diameter, by four times the superficial contents of a great circle on a sphere, or four times the third part of the radius by the superficial contents of such a circle,

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or, lastly, two third parts of the diameter by the same superficial area.

It was already shown, (Example 5 of Mensuration of Solids,) that the solidity of a cylinder is the product of the area of its base multiplied into its altitude; hence the solidity of a sphere will be two third parts of that of a circumscribing cylinder, that is, of one whose diameter and altitude are both equal to the diameter of the inscribed sphere.

In comparing solid bodies in general, we inquire how often the number of parts, of a certain determinate magnitude, of which the one of the bodies is composed, may be contained in the number of the same parts of which the other body is composed; or we inquire the proportion between the number of equal parts contained in each of the solid bodies. Thus two prisms and two cylinders, or a prism and a cylinder, are to one another as the product of their respective bases multiplied by their altitudes; consequently prisms and cylinders on equal bases are in proportion as their altitudes, and prisms and cylinders of equal altitudes are in proportion as their bases. In the same way pyramids and cones, each being one third part of their corresponding prisms and cylinders, (Example 6 of Mensuration of Solids,) are to each other in the proportion of their respective bases and altitudes.

The areas of similar *surfaces* are to one another in the proportion of the *squares* of their homologous or corresponding sides or lines: thus the area of an equilateral triangle 10 inches a side is 43,3 square inches; and the area of another equilateral triangle of 20 inches a side is 173,2 square inches: now if we take the square of the side 10, which is 100, and the square of the side 20, which is 400, we shall find that the area 43,3 is to the area 173,2 in the proportion of the square 100 to the square 400.

The same thing holds true, as the student may convince

 himself

himself by a trial, with respect to the areas of all the similar figures: and by analogy, the contents of all similar *solid* figures are to one another in the proportion of the *cubes* of their corresponding or homologous sides or lines. Thus the solid contents of a sphere of 10 inches diameter will be 261,773 nearly, and the solidity of another sphere of 20 inches diameter will be 2094,1829 nearly, then the cube of the diameter 10 being 1000, and that of the diameter 20 being 8000, the solid contents 261,773 will be to the solid contents 2094,1829 as the cube 1000 is to the cube 8000.

If it were required to form a solid body similar to a given solid, and whose contents were to be in a given proportion to the contents of the given solid, we must discover the line or side which, when cubed, would bear to the homologous line or side of the given solid the same proportion with that of their proposed solidities: thus if, for example, it were required to determine the diameter of a sphere whose solidity would be 8 times greater than that of a sphere whose diameter is 10 inches, we would state this proportion; as the solidity of the given sphere which may be expressed by 1 to the solidity of the sphere required, which is 8 times greater, or 8, so is the cube of the given diameter 10 = 1000 to a fourth proportional 8000, which will be the cube of the diameter of the required sphere; and extracting the cube root 20, (Arith. page 266,) this will be the diameter of a sphere whose solidity will be 8 times greater than that of one whose diameter is 10 inches.

As in bodies consisting of the same substance, the weights are in proportion to the quantities of matter in each body, so by knowing the weight of any regular figure, as a ball or sphere of a determinate diameter, if we wish to learn the weight of another ball or sphere of the same substance,

stance, but of a different diameter, we state this proportion; as the cube of the diameter of the first sphere whose weight is known to the cube of the diameter of the second, so is the weight of the first sphere to the weight of the second.

SURVEYING OR LAND-MEASURING.

By *Surveying* is generally understood the art of measuring the contents of a field, an estate, a parish, a county, &c. and it comprehends three several operations, viz. 1st. making the survey, or measuring the dimensions of the ground; 2dly, delineating or laying down upon paper a correct representation of the ground surveyed, every part being exhibited in its due situation and proportion with respect to the others; and, 3dly, calculating the area or superficial contents of the several portions and of the whole. Of these operations the first is what is properly called surveying, the second is called plotting, protracting, or mapping, and the third is termed casting up, or computing the contents.

1st. The first operation, or surveying, consists of two parts, the making of observations for the angles, and the taking of lineal measures for the sides: the former operation being performed with one or other of the following instruments, viz. the theodolite, the circumferentor, the semicircle, the graphometer, the plain-table, the compass, or by the measuring chain alone: the latter operation is performed by the chain or the perambulator.

The *theodolite* is made in various ways, according to the taste and skill of the artist, in order to render it the more simple and manageable, or the more accurate and convenient in use; but in general it consists of a brass circle of 6, 9, or 12 inches diameter, strengthened by four cross bars meet-

ing in the centre. The circular part, or limb, as it is termed, is divided into 360 degrees, each of which is again subdivided into minutes. To the limb are adapted two pillars supporting an axis to bear a telescope for viewing objects at a distance: and on the centre of the instrument moves a circular plate or index, having in the middle a mariner's compass, the meridian line of which corresponds to the fiducial edge of one side of the index; and two other pillars on the index support another telescope, of which the line of collimation corresponds to the fiducial edge on the other side of the index. At each end of these telescopes are also adapted plain sights for viewing objects near the observer; and the ends of the index are cut circularly to suit the limb of the instrument, so that when the limb is divided by means of concentric circles and diagonals, the fiducial line, on the end of the index, points out the degrees and minutes on the limb. The theodolite is also fitted with cross spirit levels to direct the observer in placing the circle truly horizontal, and with a vertical circle for taking angles of elevation and depression. The whole instrument is mounted on a fulcrum, or three-legged staff, with a ball and socket, or other contrivance, for rendering it moveable in all directions without deranging the position of the fulcrum.

The *circumferentor* consists of a brass circle and index all of one piece: the diameter of the circle is about seven inches, the length of the index about fourteen, with a breadth of an inch and a half. On the circle is a chart whose meridian line answers to the middle of the breadth of the index, and the circumference is divided into 360 degrees. On the centre of the instrument is adapted a magnetic needle; and there are two sights to screw on and slide up and down the index, with a contrivance to render the instrument moveable on the head of its fulcrum.

Angles are measured by the circumferentor in this way: the instrument being placed with its centre precisely over the

the angular point, with the flower-de-luce of the compass towards the observer, he directs the sights along one of the lines forming the angle to be measured, and observes what degrees are cut by the south end of the needle : then turning the instrument round until the sights bear along the other line forming the angle, he again observes what degrees are cut by the needle; when subtracting the less from the greater of these two quantities, the difference gives the quantity of the angle in question.

The *graphometer*, or semicircle, is, as its name imports, a figure bounded by the diameter and half the circumference of a circle : the limb or arch is divided into 180 degrees, which are again by diagonals, or otherwise, divided into minutes, &c. On the centre of the instrument are usually fixed a compass-box and needle, together with a moveable index equal to the diameter of the semicircle, on whose extremities, as well as on those of the fixed diameter, are fitted sights for viewing objects at a distance ; the whole instrument being mounted with a proper apparatus on a fulcrum.

In taking an angle with the graphometer, the instrument is to be placed, by means of a plummet, directly over the angular point, with the fixed diameter in the direction of one of the lines forming the angle, then moving the index round until through the sights some distant objects is observed in the direction of the other side of the angle, the number of degrees and minutes on the limb indicated by the extremity of the index will be the measure of the given angle.

These directions are also applicable to the theodolite, for the graphometer is only one half of that instrument, and the index and compass on either of them may be employed instead of the circumferentor.

The *plain-table* is an instrument by which the draught

or

or plot of a piece of ground is taken on the spot, as the survey or measurement goes on, without the necessity of any future protracting or planning. This instrument consists commonly of a plain rectangular board, of any convenient size, fixed by means of screws and other machinery in the centre to the fulcrum, so as to be moveable in all directions vertically and horizontally. To the table are applied, 1st, a wooden frame made to fit close round its edges, for the purpose of fixing on a sheet of paper. One side of the frame is divided into a scale of equal parts, by which to draw lines across the table, parallel or perpendicular to the sides, and the other side is divided into degrees from a centre in the middle of the table, so that the instrument can be used as a theodolite, &c.—2dly, a magnetic needle and compass screwed into the side of the table, to point out the bearings of objects, and serve as a check upon the sights.—3dly, an index, which is a brass two-feet scale, having either a small telescope fitted to it, or sights erected perpendicularly on the ends, the middle of which must be parallel to the fiducial edge of the index.

In using the plain-table a sheet of paper is spread flat on the surface, having been previously moistened to make it expand, and the edges are fastened down by the frame before mentioned: when the paper becomes dry it will shrink a little and become perfectly flat and smooth, so as to be fit to receive the draught of the ground to be surveyed. The surveyor begins his work by setting up the instrument in some convenient spot, making a point on the paper to represent his position; he then fixes in that point the foot of a pair of compasses, or a steel pin, and applying to it the fiducial edge of the index, he moves it round close to the pin until through the sights he observes some object such as the corner of a wall, a pole, or picket set up on purpose, &c.; then from the first station point he draws a faint line along the edge of the index, and turning it round to some other object,

object, he draws another line towards it. Having in this manner laid down from his station as many lines as are necessary, the surveyor then measures from the station the distances to each object, and applies them on the paper taken from a proper scale of equal parts.

When this is performed, the instrument is removed to one of the objects already observed, and whose distance is known, or to some other central position, the distance and bearing of which from the first station have been ascertained, and at this second station repeats the operations performed at the first, laying down all the bearings and distances in their proper places on the paper. In placing the instrument at the second station care must be taken that by means of the needle it be situated precisely in the same direction with respect to the north and south points of the compass as at the first station.

As it must often occur in a survey of great extent that one sheet of paper will not be sufficient to receive the whole draught, the surveyor draws a line in any manner through the farthest point to which the work has been carried, and then taking off the paper, he applies another sheet to the table, drawing on it a line to represent that last drawn on the former sheet, taking care that this new line be laid down precisely in the same direction with the former in respect of the points of the compass and the direction of the sides of the table; for should there be any deviation in the position of the second line from that of the first, it is evident that the two portions of the draught will not correspond one to another, and consequently that the whole, when put together, will not afford a true representation of the ground surveyed, on which account the best way of connecting the lines on the different sheets will be, to draw them both parallel or perpendicular to the same sides of the table, by means of the scales of equal parts laid down on the frame.

Another

Another way of employing the plain-table in surveying is shown in Fig. 3 of Plate 7.—Let a base, as *a m*, be measured within the field or ground to be surveyed, and placing a pole or other object at *m*, let the plain-table ABCD be erected at *a*, then on the paper drawing a line EF in the direction of *a m*, and ascertaining its length from a scale of equal parts corresponding to that measured, observe the bearings of the several objects of importance for determining the extent and figure of the ground to be surveyed, drawing lines on the paper directed to those objects, such as EG, EH, EI, &c. Having in this way observed and laid down all the bearings of the several objects from the station at *a*, the instrument is removed to *m*, where placing it in such a way that the line EF already drawn on the paper shall perfectly coincide with the measured base *a m*, make observations from the point F to each of the objects formerly observed, as FG, FH, FI, &c. and drawing lines on the paper to these several objects, the points of intersection, with those drawn from the station at *a*, will determine the positions *g*, *h*, *i*, &c. corresponding to the several situations on the ground, and the figure on the paper containing angles equal to those on the ground, and the lines of distance being laid down from a proportional scale, the draught will be precisely similar to the ground surveyed.

Angles may also be measured by the chain, by setting off one or more along each of the sides forming the angle to certain fixed points, and then measuring the distance across between these points.

The instruments commonly employed for the measurement of lines and distances are the *chain* and the *perambulator*. The chain generally consists of 100 links; at every tenth link is fastened a small brass plate having a figure engraved on it, or the plate itself is cut into some particular shape, to denote the number of links between the plate and

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the beginning of the chain. Some chains are made 100 feet long, each link being 1 foot; but this chain is used only in the measurement of great distances, or for military surveys, where no regard is paid to the number of acres or other fixed quantities of ground. Other chains contain only 1 pole equal to $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, being the fourth part of an English standard chain to be used in measuring land: this short chain is employed in laying out gardens, orchards, &c. by the pole or rod. But the chain most commonly consists of 4 poles equal to 22 yards or 66 feet English measure; it is called Gunter's chain, from the name of the inventor, who died about the year 1626, and is peculiarly adapted to the purposes of surveying and measuring of land, because 10 square chains of this sort are exactly one English acre; so that the dimensions of a field or an estate being taken in these chains, and the area or contents being computed in square chains, one tenth part of the product will be the number of acres in the ground measured.

Gunter's chain, being in length 66 feet, and divided into 100 links, each link must be equal to 7,92 inches, or nearly two thirds of a foot; hence any number of chains or links may be readily converted into feet and inches, or a number of feet and inches may easily be turned into chains and links, by employing the standing proportion of 100 links being equal to 66 feet, or 55 links to 33 feet.

The *Scotch* acres, &c. being greater than the corresponding divisions of English measure, the Scotch chain is proportionably longer than the English, for although it is divided into 100 links like the latter, yet the whole length of the chain being 24 Scotch ells, equal to 74 English feet, or rather to 74 feet $4\frac{4}{5}$ inches, on account of the small excess of the Scotch foot above the English foot, the links of the Scotch chain are equal to 8,928 English inches.

In measuring with the chain, the surveyor provides 10 small rods of wood or iron, called *arrows*, to stick in the

 ground

ground at the end of each chain-length, as it is applied to the ground. Two persons hold the chain, one at each end, and the 10 arrows are given to him who leads and draws the chain. The other person, or the follower, who holds the other end of the chain, stands at the beginning of the line to be measured, with the ring in his hand, while the leader walks on, drawing the chain by the other ring until it is stretched to its full length; the follower then, by the motions of his hand, directs the leader to move to the right or left until he be precisely in a line with the object at the other extremity, whether this be some tree, corner of a wall, &c. or, as is generally the case, some pole, or other object set up for the purpose. When the chain is thus stretched out in the true direction of the line to be measured, the leader sticks an arrow in the ground at the end of the chain, which he then draws on for another length, until the follower comes up to the arrow in the ground, who again directs the leader to place himself in the proper situation, and in doing which the leader may now direct himself, by looking back and moving to the right or the left, until he bring the follower on a line with the point where the measurement began. When this is done, and the chain is properly stretched, the leader sticks a second arrow in the ground, while the follower takes up the first, and in this manner the operation proceeds to the end of the line to be measured. Should the line not consist of an even number of chains, the number of links counted from the follower's end of the chain to the extremity of the line, must be added to the number of entire chains measured, and this number is known by that of the arrows taken up by the follower, so that if he has in his hand 6 arrows, and that the distance from the end of the 6th chain to the extremity of the line be 57 links, the length of the line will be 6 chains 57 links, thus written 6,57 chains, or simply 657 links.

It happens in measuring long distances, that the whole

10 arrows are expended before the leader arrives at the extremity; when, therefore, he has placed and stretched the chain the eleventh time, he waits until he receive the 10 arrows from the follower, who marks down the change of arrows; and one being stuck in the ground by the leader, the measurement proceeds as before. Should the line be of such a length that the arrows must be changed 5 times, and that besides this the follower has in his hand 7 arrows, with 45 links more from the end of the chain to the extremity of the line, then the 5 changes will give 50 chains, which, with the other quantities, will give for the whole distance measured 57 chains and 45 links, set down thus, 57,45, or simply 5745 links.

The chain usually employed in land-measuring is so frequently met with, as to require no description; but that which was constructed by the very ingenious Ramsden of London, for the late General Roy, to be used in the trigonometrical operations instituted for determining the distance between the meridians of the two great observatories of Greenwich and Paris, this last chain was of so peculiar a construction, and was made susceptible of so much accuracy in service, that the following short account of it cannot fail to be acceptable to the student of mensuration.

This chain was formed of steel, consisting of 100 links, each 1 foot long: the links were connected together in the same way with those of a watch chain; each link being composed of three principal parts; namely, a long plate $\frac{1}{4}$ of an inch in breadth, and $\frac{1}{6}$ of an inch in thickness; two short plates, half the thickness of the former, with circular holes near the extremities of each; and two cast-steel pins, or axes, suited to the diameters of the holes, which serve to connect the adjoining links together. The holes in the short plates were made rough or jagged with a file, so that when they were applied to and embraced the ends of two adjoining long plates, and the pins were passed through the

 holes,

holes, by rivetting the extremities of the pins they became perfectly fast, and, as it were, united to the short plates, while the embraced ends of the long plates turned freely round on the middle part of the pins.

At every tenth link the joint now described had a position at right angles to the former, that is to say, the short plates lay horizontally, and the pins passing through them stood vertically; thus the whole chain containing 200 cast-steel pins, 180 lay horizontally, and 20, including the two by which the handles were attached to the chain, stood vertically. These cross joints, which were chiefly intended for the purpose of folding up the chain in small compass, by returning upon itself at every tenth link, were useful in another way, by presenting a horizontal surface to which small circular pieces of brass were screwed, with the figures 1, 2, 3, &c to 9, engraved on them, denoting the decimal parts of the length, as 10, 20, 30, &c. to 90 links.

The chain, on its first construction, was in length one hundred feet, including the two brass handles, in the extremity of which was a semicircular hole of the same diameter with the steel arrows employed to keep the account of the chains when applied to common measurements: but although the instrument performed its duty to entire satisfaction, yet, as it was afterwards supposed that still greater accuracy might be obtained by making a few slight alterations, the two end links were changed, each being made equal to one foot in length, exclusive of the handles.

The whole chain weighed about 18 pounds, and when folded up is easily contained in a deal box about 14 inches long, 8 inches broad, and the same in depth.

The *perambulator* is an instrument for measuring distances on roads, streets, &c.; it is also called the *surveying-wheel*, and a particular sort is named the *pedometer*, or *way-wiser*. This wheel is fitted to measure out a pole, or 16½ feet, in making two revolutions, consequently its circumference

cumference is $8\frac{1}{4}$ feet, and its diameter 2,626 feet. The instrument is commonly pushed forward by a person walking and holding the handle; but is also frequently connected to a carriage, and so made to indicate the distances travelled on a road. Within the machine are various movements acting on indexes on the face, to point out the space passed over in miles, furlongs, poles, yards, &c. The advantages of the perambulator are its readiness and expedition in measuring distances; but proper allowances must be made to bring the distance shown to the level line, because the instrument in its revolutions falling into hollows, ruts, or other inconsiderable depressions, must always give the distance somewhat greater than the truth.

II. The second branch of surveying, called *plotting*, *protracting*, or *mapping*, is performed by means of the *scale* and the *protractor*.

In the introduction to Trigonometry were given directions for constructing scales of chords, sines, tangents, and secants (p. 382), and also scales of equal parts (p. 385). That principally used in laying down a survey is called the plotting scale, being made of box, brass, or ivory, six, nine, or twelve inches long, by an inch and a half broad. This instrument contains several scales laid out on both sides: on the one are a number of scales of equal parts, the inch being divided into various numbers of such parts; also scales of chords, &c. for laying down angles; and sometimes the degrees of a circle are marked on one edge, answering to a centre on the opposite edge, by which means the instrument may be employed as a protractor. On the other side are several diagonal scales of different sizes, or numbers of divisions in the inch, serving to take off lines, expressed by numbers, to three dimensions, as units, tens, and hundreds, and also a scale where the foot is divided into 100 parts. Some of these scales have also a line of equal parts laid down on both edges, made thin for the purpose,

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so that by applying the scale to the paper, any distances may at once be marked off, without employing compasses to transfer them from the scale to the line to be determined.

In laying down the sides of a field, &c. on paper, it is evident, that if they have been measured on inclined planes, as up or down hill, or both, the angles of elevation and depression must be observed, and from these the true level or horizontal lines calculated, which lines alone are to be set off on the paper. In pages 425 and 436 of Practical Geometry, directions are given for calculating such horizontal distances, or discovering them mechanically by levelling.

The other instrument employed in laying down a survey upon paper is the protractor, which, in its simplest form, consists of a semicircular limb of brass, ivory, &c. divided into 180 degrees, and subtended by a diameter having in the middle of one of its edges a small notch corresponding to the centre of the limb. For the convenience of counting the degrees both ways, they are numbered on the limb both from the right hand to the left, and from the left hand to the right. Sometimes instead of using the protractor itself, the degrees are marked upon the margin of three sides of a plain scale, with the centre in the middle of the fourth side.

To lay down an angle on paper by means of the protractor, we place that side of the diameter in which the centre is marked along the given leg of the angle, placing the centre accurately at the point or extremity of the leg where the angle is to be formed, then counting from the extremity of the limb which touches the given leg, the number of degrees of which the angle is to consist, we make a dot or other mark opposite to this quantity, and removing the protractor, by laying the edge of a plain ruler over the central point and this mark, a line is drawn indefinitely, which will, with that first drawn, form an angle of the quantity required.

required. In consequence of this property the protractor is useful in readily drawing a line perpendicular to another at a given point: for if we place the diameter along the given line and the centre at the given point, by marking off 90 degrees on the semicircular limb, we obtain a point through which a line drawn to the given point will be the perpendicular required.

III. The third branch of surveying, namely, the computing and casting up of the contents or area of the ground surveyed, is performed by reducing the several inclosures, divisions, &c. into right-lined figures, such as the triangle, the trapezium, the parallelogram, &c. but particularly into the two first; then calculating the area or superficial contents of each, and adding these several quantities together, we obtain the total contents of the land surveyed.

In England the contents of a piece of land are computed in acres, roods, perches or poles, square yards, &c. the acre containing 4 roods, the rood 40 square poles, the pole $30\frac{1}{4}$ square yards, the yard 9 square feet, &c. so that 1 acre is equal to 4 roods = 160 square poles = 4840 square yards = 43560 square feet, &c. This regulated acre, however, does not prevail all over England, for in the northern counties, and in some other quarters, as the length of the pole varies between $16\frac{1}{2}$ feet, which is the standard, and 28 feet, the acres, &c. measured by poles of these various lengths must of course vary greatly in their contents, and such acres are termed *customary measure*.

In Scotland the acre contains 4 roods, the rood 40 falls, the fall 36 square ells, the ell 9 $\frac{73}{144}$ feet or 9 feet 73 square inches, so that 1 acre is equal to 4 roods = 160 falls = 5760 ells = 54760 square feet Scotch measure, which, as was already observed, is greater than the English foot in the proportion of 74,4 to 74, or as 37,2 to 37; hence the Scotch acre will contain 55056 English square feet, whereas the English

English acre contains only 43560 square feet: so that 5 English acres are a little less than 4 Scotch acres.

The customary acre of Wales is about 2 English acres; and the Irish acre is equal to 1 English acre, 2 roods 19 $\frac{27}{121}$ poles.

The acre or *arpent* formerly used in France, was equal to $1\frac{1}{4}$ acre English, or 54,450 feet, nearly agreeing with the Scotch acre, which was probably derived from the French measure.

Let ABCDEF represent a field (Fig. 4, Plate 7,) bounded by the winding lines FA and AB, and by the straight lines BC, CD, DE, and EF: let the side BC be measured and found to be in length 5 chains 12 links, CD = 3 chains 88 links, DE = 4 chains, EF = 8 chains 76 links: the side FA being curved outwards, the shortest distance between the extremities F and A must be measured, as is shown by the dotted line AF, which consequently will fall entirely within the figure, and contain 6 chains 48 links; again, the boundary of the figure A *a* B winding in such a manner, that a line joining the extremities A and B falls partly within and partly without this boundary, let the distance A *a* be measured = 4 chains, and let *a* B be measured = 3 chains 84 links, consequently the whole distance AB will be = 7 chains 84 links.

The exterior sides of the field being thus measured, let the whole be thrown into triangles by lines measured between opposite corners, as in the figure, where the distance from D along the dotted line DB to B is = 5 chains 28 links, the distance DA is = 10 chains 48 links, and the distance DF is = 9 chains 76 links. When all this is performed, the ground to be computed is in a general sense divided into 4 triangles, viz. DFE, DAF, DAB, and DBC, of which all the sides are known, and consequently the perpendicular altitude of each triangle may be found, agreeably to the in-

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structions given in the 7th Prop. and in the 4th Case of Oblique-angled Trigonometry Let this be performed with respect to the triangle DBC, where stating this proportion, as the base BD to the sum of the opposite sides BC + CD, so is the difference of these sides BC — CD to the difference of the segments of the base formed by a perpendicular let fall on it from the angle at C, or B *b*—*b* D, which difference will be = 2,114, and, consequently, by adding half this quantity to half the base, we obtain the greater segment B *b* = 3,697, and the less segment *b* D = 1,583: then, by means of either of these segments and the adjoining side of the triangle we discover the perpendicular, or altitude, of the triangle, to be 3,542.

In the 5th Example of Mensuration of Surfaces, the methods were shown for calculating the area of a triangle, according to which the area of the triangle DBC will be 9,3509 square chains, or 9 square chains and 3,509 square links, to be brought into their proper values in land-measure. It was already observed, that the measuring chain both in England and Scotland is so adapted to the acre of each country, that 10 square chains are equal to 1 acre: but the contents of the triangle DBC not amounting to 10 chains, its area will not be an acre; the contents 9,3509 square chains, therefore, multiplied by the next inferior denomination (Reduction, page 202) or 4, the number of roods in an acre, and the product, 37,4036, divided by 10, the square chains in an acre, we obtain 3 roods and a decimal fraction, ,74036, which again multiplied by 40, the poles in 1 rood, we obtain 29 poles with another fraction, 6144, to be multiplied by the number of square yards in 1 pole, or 30,25, producing 18,5856 square yards: and the whole contents of the triangle DBC will be 3 roods, 29 poles, 18,5856 yards of English measure.

Proceeding in the same way with the triangle DFE,

 whose

whose sides are known, its area will be found = 1 acre, roods, 0 poles, and 4,098576 yards : the triangle DFA will be found to contain 3 acres, 0 roods, 13 poles, and 26,609 yards ; and the triangle DAB will contain 2 acres, 0 roods, 0 poles, and 21,6 yards.

In this manner we discover the quantity of land contained within the right-lined figure ABCDEF : but there remains still to be calculated the space inclosed by the measured line FA, and the crooked boundary of the field between the points F and A, consisting of 4 right lines, F *c*, *c e*, *c f*, and *f* A : let perpendiculars be drawn from the three angular points to the line FA, of which *c d* measures 56 links, that from *e* 36 links, and that from *f* 54 links. By inspection of the plate it will appear that the two figures at the extremities F *c d*, and *h f* A, are right-angled triangles, and that the two intermediate figures are trapezoids; consequently if the spaces F *d*, *d g*, *g h*, and *h* A, be measured, we may calculate the areas of the several figures : let, therefore, F *d* be measured = 92 links, *d g* = 1,44 links, *g h* = 1,84 links, and *h* A = 2,12 links. Having now the bases and the altitudes of these triangles and trapezoids, we discover the area of the triangle F *c d* to be 2,576 square links, that of the trapezoid *c e g d* to be 6,552 square links, that of the trapezoid *e f h g* to be 9,108 square links, and that of the triangle *f* A *h* to be 6,784 square links. (See Examples 5th and 7th of Mensuration of Surfaces.) Then, adding these four quantities together, we have, for the area of the whole figure, cut off by the measured line FA, 3 square chains and 3392 links, which being brought into its proper value, will give 1 rood, 13 poles, and 12,9228 yards, to be added to the contents of the 4 triangles already computed.

It was already observed, that the measured line AB falls partly within and partly without the boundary of the field on that side, cutting it in the point *a;* this boundary consisting

sisting not of right lines, but of a sweeping curve, the area comprehended between it and the line AB can only be calculated by an approximation to the truth, as is done with circles and every other sort of curve: but this approximation may be carried so near to the true area, as always to differ from it by a quantity less than any assigned quantity: at the same time that in land-measuring such nicety of computation can never be required.

The line AB falls within the boundary of the field all the way from A to *a*, a distance of 4 chains: beginning at A set off on A *a*, a space equal to a chain or 100 links, and there erecting a perpendicular to the boundary line, let the distance be measured = 40 links; again, setting off another chain on A *a*, erect another perpendicular to the boundary measuring 52 links; lastly, setting off another chain more on A *a*, let the perpendicular there erected to the boundary, measure 36 links. As the boundary line from the angular points to the extremity of the adjoining perpendiculars, and between the extremity of two adjoining perpendiculars, may be considered as consisting of right lines, the first and last divisions of the figure to be measured may be regarded as right-angled triangles, and the intermediate divisions as trapezoids. In such a case the area of the figure cut off by A *a* will be found = 12800 square links = 20 poles, 14,52 yards, to be added to the contents of the field already found.

On the other hand, as the measured line AB passes on the outside of the boundary of the field from *a* to B, the space inclosed between it and the boundary must be measured and calculated in the same way as the foregoing space; and its area, which will be found to be 17900 square links, or 28 poles, 19,36 yards, is to be deducted from the total contents of the field. In this last position the perpendiculars are set off from *a* B at the distance of 1, 2, and 3 chains from *a*, and their lengths are 44, 58, and 76 links.

 The

	A.	R.	P.	Yards.
The area of the triangle DBC =	0	3	29	18,5856
- - - - DFE =	1	3	00	4,098576
- - - - DFA =	3	0	13	26,609
- - - - DAB =	2	0	00	21,6
+ The space cut off by the line FA - - =	0	1	13	12,9228
+ The space cut off by the line A a - - =	0	0	20	14,52
	8	0	38	7,585
— The space included by the line a B - - =	0	0	28	19,36
Area of the figure given to be measured - - =	8	0	09	18,475

In the preceding example mention has been made of setting off and letting fall perpendiculars in the course of a survey: when the object is at a considerable distance from the line on which the perpendicular is to fall, this may be best performed by means of the theodolite, or some other graduated instrument, by placing the centre of the instrument over the point where the perpendicular is to be erected, and the diameter in the direction of that line; then another line traced from the centre, cutting the graduated limb in 90 degrees, will be a perpendicular at that central point. But in ordinary surveying, when the perpendiculars are never of great length, they may be set off with the greatest accuracy by means of the chain alone.

In Prop. 18 of Geometry, (page 370.) it was shown, that the square constructed on the hypothenuse of a right-angled triangle was equal to the sum of the squares constructed on the two other sides, and this was illustrated by employing certain numbers which form a right angle: these numbers

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are 3, 4, and 5, or any equi-multiples of them. Suppose, then, we divide 100, the links in a chain, by the sum of $3 + 4 + 5 = 12$, the nearest quotient 8 will be the greatest multiple which can be obtained in the chain; thus $3 \times 8 = 24$, $4 \times 8 = 32$, $5 \times 8 = 40$, and $24 + 32 + 40 = 96$: if, then, we form a triangle whose sides shall be 24 links, 32 links, and 40 links, such a triangle will contain a right angle opposite to the largest side of 40 links. To perform this in the field, fix an arrow in the ground at the point where the perpendicular is to be erected; over this arrow place the ring connecting the 24th and 25th links of the chain; carry the beginning of the chain along the given line, and fix the handle on it by another arrow; then counting 32 links from the 24th, that is the 66th, and 40 links more, that is the 96th, place this last over the arrow at the beginning of the chain, when another arrow in the 66th link will, when the two portions of the chain are extended to their proper length, point out a spot exactly perpendicular to the arrow fixed in the ground at the 24th link. Again, when it is required to let fall a perpendicular from a point on a given right line, distant from that point less than the length of the chain, fix one handle at the given point, and extending the chain, with the other handle place an arrow at the point where the handle comes on the given line, and sweeping the chain round until it again meet the line, fix there another arrow. In this way two points are obtained equally distant from that on which the required perpendicular should fall, and by taking the half of this distance, the point will be found to which a line drawn from the point originally given will be perpendicular.

When the three sides of a triangle are given, the area may be found in the following manner: add the sides together, take half the sum, from this half sum subtract each of the sides, multiply the half sum by the difference between it and each side successively, and extract the square root of

of the last product: this root will be the area of the triangle. Take, for example, the triangle DBC in Fig. 4, Plate 7, whose sides are 5,28 chains, 5,12, and 3,88; from half the sum of these sides = 7,14 subtract each side, when the remainders will be 1,86, 2,02, and 3,26: multiply the half sum by the first remainder, the product by the second, and that product by the third, producing the quantity 87,45409008, the square root of which (Arithmetic, p. 263) is 9,3516 square chains, or 0 acres, 3 roods, 30 poles, 18,9244 yards, a greater area than that before discovered by means of the base and the perpendicular of the triangle, in which calculation remainders being necessarily lost, the area came out a little less than the truth.

A very essential part of the business of a surveyor is what is termed *the laying out of land*, or the marking out on the ground a space which shall contain a certain quantity of land. As the area, or contents of a field, may be considered as the product arising from the multiplication of certain dimensions according to the form of the field; so when the area is given, if we divide it by one dimension also given, the quotient will be the other: if, for example, it were required to lay down on the ground a square figure which should contain 640 acres, we would first multiply this number by 10, the square chains composing an acre, and then extracting the square root of the product 6400, we would obtain 80 chains for the side of the given square: and as 80 chains of 66 feet each are 5280 feet, the length of an English statute mile, it follows that a square piece of ground one mile every way will contain 640 English acres of land.

Again, were it required to lay down a rectangular parallelogram containing 640 acres, and that the length of the figure is limited to 320 chains; by dividing 6400, the square chains in the given acre, by 320, we obtain 20 chains for the breadth required.

If it be required to know the altitude of a triangle containing

ing a given area, and to be constructed on a given base, we have only to bring the area into chains and links, and divide this quantity by one half of the base, when the quotient will be the altitude.

As the area of a circle is the product of half the circumference multiplied by half the diameter, (Mensuration of Surfaces, Ex. 13th,) if the diameter be unity, or 1, the circumference will be 3,14159 &c. and the superficial contents of the circle will be ,7853975 &c.: but the areas of similar figures being to one another in the proportion of the squares of their homologous sides, or other corresponding parts, as in circles the radii, the diameters, the circumferences, we obtain this fixed proportion, as the area ,7853975 to the square of its diameter 1, so is another given area to the square of its diameter. Let it then be required to ascertain the diameter of a circle capable of containing 640 acres of land, or 6,400 square chains. As neither multiplication, nor division by unity, or by its square, which is still unity, produce any change upon quantities, we have only to divide the given area 6,400 by ,7853975, when the quotient 90,27 chains will be the diameter of a circle containing very nearly 640 acres, the deficiency arising from the diameter, and consequently the circumference, being found a little less than the truth, occasioned by the remainder in the division.

In Example 16 of Mensuration of Surfaces, it was shown, that if a circle be described on the transverse axis of an ellipse as a diameter, the area of this circle will be to the area of the ellipse, as its transverse axis is to its conjugate axis; and also that the area of an ellipse is equal to the area of a circle whose diameter is a mean proportional between the transverse and the conjugate axis: hence we derive a rule for laying out an ellipse on a given transverse or conjugate axis, and containing a given area. Let it be required, for instance, to find the conjugate axis of an ellipse whose transverse axis is 100 chains, and whose area is 640 acres.

It

It was already shown, that the diameter of a circle containing 640 acres is 90,27 chains; if then we multiply this sum into itself, and divide the product by the transverse axis = 100 chains, the quotient 81,486729 chains will be the conjugate diameter of an ellipse containing 640 acres: and with these dimensions, agreeably to the directions already given, the ellipse may be traced out on the ground.

In this manner may any regular figure be laid down on the ground, containing a given quantity of land: but where the boundaries of the figure are irregular curves, the surveyor must exercise his judgment, in arriving at the truth by gradual approximations, which will give results sufficiently accurate for the ordinary purposes of land-measuring.

Another part of the surveyor's business is the making exchanges of ground, where he is required to lay out such a quantity of land at a given value as may be equivalent to another quantity of a different value. In this case he has only to multiply the given quantity by its value, and to divide the product by the value of the land to be given in exchange for it, when the quotient will be the quantity required: thus, if 50 acres of land, valued at 45 shillings per acre, of yearly rent, are to be exchanged for land worth 3*l.* per acre yearly, the product of 50 by 45 = 2,250 shillings, divided by 60 shillings, the value of the other land, will give for a quotient 35,5 acres, or 35 acres 2 roods, the quantity to be given in exchange for the 50 acres at 45 shillings.

In surveying a county or other extensive tract of land, the usual way is to select a number of commanding positions for stations, such as the tops of hills, steeples, towers, &c. which can be easily observed the one from the other, and from which a number of intermediate towns, villages, &c. may also be observed. The greater the distance between the stations for observation, the more convenient will it be to make the survey. Then with the theodolite, or other proper instrument, measuring the angles formed by the

different

different objects observed at the first station, the distance from it to the second station must be carefully and accurately measured and reduced to the true level or horizontal line, by making the proper allowances for the different elevation of the two stations, as well as for the effects produced by the state of the atmosphere. When this is performed, the surveyor observes the angles formed at the second station by those objects which he had observed before from the first station: and then removing the instrument to a third station, he again observes the objects already noticed; and if the points ascertained by the intersections of these three sets of observations shall coincide, it is to be presumed the angles have been accurately measured. In this way the angles, bearings, and intersections formed by the most remarkable objects, are to be determined all over the tract of country to be surveyed: but the distances between the several stations need not be calculated, because the distance between the first and second stations, which ought always, if possible, to be four, five, or six miles, being ascertained, it becomes a sort of standard by which to compute all the other distances for the sides of all plain triangles being proportionally as the angles respectively opposite to each, and having in the first triangle measured all the angles and one of the sides, the other sides are readily calculated to be employed in computing the sides of all the other triangles in sucession. As, however, it can seldom happen that the distance between any two stations, sufficiently elevated to give a proper command of the country round them, is so level as to allow it to be measured with due accuracy, unless with great attention and labour, the practice in surveys of this kind is to choose some open level tract from which the first and second stations can be observed, on which to measure a line, the longer the better, to serve as a base at which the calculation of the sides of the several triangles is to commence; and at the conclusion of the survey, as also in some intermediate

situations, if requisite, to select another level tract on which another base for verification is to be measured, to check the distances obtained by the trigonometrical calculations. In performing the operations for ascertaining the true distance between the meridians of Greenwich and Paris, already noticed, the first base was measured on Hounslow Heath in a direction from north-west to south-east, and extending 27,404,0137 feet, or nearly 5 $\frac{1}{5}$ miles; and the base for verification afterwards measured on Romney Marsh, on the coast of Kent, lying in a direction nearly parallel to that of the first base, extended to 28,535,6773 feet, or a little more than 5 $\frac{2}{5}$ miles.

END OF VOLUME THE FIRST.

ERRATA

IN SECOND VOLUME.

Page 246, line 16, after *heavenly* read *body*.
—— 326, —— 1, for *central* read *external*.

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