SCIENTIFIC RECREATIONS

IN

PHILOSOPHY AND MATHEMATICS;

INCLUDING

ARITHMETIC,	MAGNETISM,
ACOUSTICS,	OPTICS,
ELECTRICITY,	PNEUMATICS;

TOGETHER WITH

AMUSING SECRETS

IN

VARIOUS BRANCHES OF SCIENCE:

THE WHOLE CALCULATED
TO FORM AN AGREEABLE AND IMPROVING EXERCISE
FOR THE MIND.

BY W. ENFIELD, M.A.
AUTHOR OF NATURAL THEOLOGY, THE YOUNG ARTIST'S ASSISTANT,
PRONOUNCING DICTIONARY, &c.

THE THIRD EDITION.

LONDON:
PRINTED FOR THOMAS TEGG, 73, CHEAPSIDE;
R. M. TIMS, DUBLIN; R. GRIFFIN & CO. GLASGOW;
AND M. BAUDRY, PARIS.

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PREFACE.

To trace out the origin of amusements, it appears that it would be necessary to go back to the earliest ages of the world. For mankind, being exposed to a variety of fatiguing labours, which exhaust both the mind and the body, have at all times exercised their ingenuity in devising means to dispel melancholy, and to revive the depressed spirits.

The remedies pointed out by nature for this purpose are, rest, proper nourishment, and cheerfulness: each day, indeed, exhibits in the same individual a new being, in good or bad spirits, according to the impressions made on the animal economy, by rest, a change of food, and various other circumstances.

The mind is too intimately connected with the body not to participate in the evils by which it is affected; but to the former, rest alone is not sufficient: to revive its powers, and to exhilarate the spirits by a proper stimulus, a change of objects, amusing conversation, agreeable news, and other things of the like kind, are necessary.

Every one knows, that the spirits are depressed by too long application to gloomy or serious objects: to remedy this evil, others more amusing must be substituted in their stead; the least trifle or toy is often capable of giving to the mind the most tranquil and agreeable impressions; and during this state of peace and repose, new spirits are created, which produce a change in the whole frame.

Walking, hunting, dancing, and music, are excellent sources of amusement; but they are not the only ones to which the necessity of unbending the mind, and filling up a vacant hour, have given birth.

A

The game of chess, it is said, had its origin at the siege of Troy; being invented by Palamedes, to amuse the Grecian chiefs, disgusted with the tediousness of the siege.

Cards and tennis were invented by the Lydians, a people of Asia Minor, among whom, according to the antiquarians, all games had their origin: these people, it is well known, were so much addicted to voluptuousness and gaiety, that to express a thoughtless, careless action, it was said proverbially to have been done *Lydio more.*

Amusements, then, are remedies invented to revive the depressed spirits, and to render the mind capable of resuming its usual labours with greater success; but a wise man will employ them with moderation, and will consider them as objects calculated to unbend the mind, and not to occupy it entirely.

Cicero told his son, that amusements ought to be employed like sleep; which, if used to excess, becomes dangerous, and instead of reviving the powers of the mind, renders them torpid.

On this subject, Cassian relates an expression of the Apostle John, which deserves to be recorded.—A hunter, who one day saw him caressing a partridge, seemed astonished that so pious a man should amuse himself with such a trifling object. "My good friend," said the apostle, "what have you got in your hand?"—"A bow," replied the hunter. "And why is it not bent?" added the apostle.—"If it were always bent," returned the hunter, "it would lose its strength."—"Be not then surprised," continued the apostle, "that the mind also should sometimes require relaxation."

Sidronius Hoschius, the Flemish Ovid, has expressed the same thought, with great elegance, in the following lines:

> "Deficiet sensim qui semper tenditur arcus;
> Ferre negat segetes irrequietus ager."

The latter comparison has been employed by Seneca, who says, "The mind of man is like those fields, the fertility of which depends on their being allowed certain periods of rest, at the proper seasons." This philosopher had remarked, that too long and too assiduous labour exhausts the mind, throwing it into a kind of languor; but that, by relaxation, it is revived, and rendered fitter for resuming its occupations.

How often are people difficulted by problems merely of an amusing nature, the whole solution of which depends upon some elementary calculation, the natural properties of certain bodies, or mathematical combinations! We admire the sagacity and pretended knowledge of the person who proposes them; and yet nothing is easier than to comprehend, and even to execute, what thus excites our astonishment and wonder. Why then should not we acquire the knowledge necessary to enable us to propose problems and enigmas ourselves?

Intricate and puzzling questions have, at all times, formed a part of the amusements of the most polished nations; and they have been received with avidity, even by young persons, when presented under the agreeable form of an enigma or recreation. We may even venture to assert, if we are allowed to judge of others by what we experience ourselves, that we are sometimes conducted to the higher parts of the most abstract studies, by the flowery path of some experiment, which we at first considered as an object of mere curiosity.

It is well known, that the high reputation of Solomon induced the queen of Sheba to come from the remotest part of Ethiopia, to admire the wisdom of that great prince—the wonder of his age. She came, says the Scripture, to try his wisdom, by proposing to him enigmas. Solomon satisfied her on every point, and answered all her questions with so much propriety, that the queen returned in the utmost joy, unable to contain the transports of her admiration, excited by the wisdom and magnificence of that great king.

The celebrated Æsop became the favourite of Crœsus, merely on account of his ingenious fables, which contained the most refined morality, and instructions the more delicate, as they conveyed censure without wounding that self-love which is so natural to man. Fable, in the hands of this great genius, seemed a rod dipped with so much art in the gall of satire, as to have none of its bitterness or severity. Nathan represented to David the enormity and injustice of the crime he had committed, in regard to Uriah, only under the veil of an ingenious allegory; which produced a greater and speedier effect on the mind of the monarch, than if the prophet, arming himself with the thunder of his eloquence, had pointed out to him, in a direct manner, the horror of his offence.

Does not Solomon desire the sluggard and the spendthrift to consider the ways of the ant; and does not Æsop seem to have borrowed from this idea his fable of the ant and the grasshopper? That great prince, in delineating the portrait of true wisdom, paints her in saying, "To understand a parable, and the interpretation, the words of the wise, and their dark sayings."

Mental amusements, then, have been esteemed, in all ages, and by persons of every condition; and the pleasure they excite is the purer, as they affect only the more delicate parts of the mind. The human intellect, as is well known, has its peculiar pleasures; every thing that increases knowledge, pleases and exalts it; we are always gratified when we comprehend a difficulty which has checked the progress of others, or have unveiled a mystery, concealed from persons possessed of less penetration than ourselves.

Besides, these amusements, purely intellectual, may be enjoyed at little expense; they do not fatigue the body, on which they make no impression; and, on this account, they ought to be preferred to sensual pleasures, the enjoyment of which creates disgust, injures the health as well as fortune, and almost always deranges the economy of a peaceful and tranquil life.

The class of mathematicians has always arrogated the right of treating of mathematical and philosophical recreations. In compiling the present collection, Ozanam, and those who have written on the same subject, have been our guides; and from their works we have selected the greater part of what we now offer to the public; for these amusements are the production neither of one man, nor one age, but of a great number of the learned, of artists, and of many ages of research and of observation.

By the long experience we have had, we are induced to hope that young persons, who are often disgusted with the formality of study, and who, on that account, sometimes conceive an aversion to the most useful branches of science, will find in the greater part of the amusements which are here presented to them, some things suited to their taste, and easy to be comprehended. When the first difficulties are surmounted, they will become so many steps to conduct them gradually, by the most agreeable path, to the solution of problems, which at first may appear too difficult and abstruse for their age.

As it is impossible to understand properly all these amusements without the knowledge of certain principles, the application of which is often necessary, they are preceded by an introduction, calculated to facilitate the solution of the most difficult problems.

**** *The Reader is requested to observe, that the Figures enclosed within Parentheses, which occur in the course of the following Work, refer to that Section in the Introduction, where the necessary explanation will be found.*

SCIENTIFIC RECREATIONS,

&c.

INTRODUCTION.

1. DIFFERENT symbols or signs, established by general practice, are sometimes employed in order to simplify calculations, and facilitate the resolution of certain problems. Thus,

$+$ signifies plus or more.
$-$ .... minus or less.
$=$ .... equal.
$>$ .... greater.
$<$ .... less.
$\times$ .... multiplied by.
$\div$ .... divided by.

Thus it may be easily conceived that $2+3=5$; that $3-2=1$; that $3>2$; that $2<3$; and that $12\div3$ or $\frac{12}{3}=4$.

OF FRACTIONS.

2. Besides the application of the common rules to whole numbers, with which every body is acquainted, it is sometimes indispensably necessary to perform the same operations with fractional numbers.

A fraction is one or more parts of a whole. Every fraction is expressed by two characters, placed one above the other, with a line between them, in this manner, $\frac{3}{4}$, $\frac{a}{b}$, &c. The upper character, which is called the *numerator*, expresses how many parts are taken of the whole; and the other, called the *denominator*, denotes the quality of these parts. Thus the fraction $\frac{3}{4}$ signifies that the whole is divided into fourths, and that 3 of them are taken.

It hence follows, that a fraction is greater according as the numerator is greater; and, on the other hand, less as the denominator is greater. Thus $\frac{5}{7} > \frac{3}{7}$; $\frac{3}{8} < \frac{3}{5}$. By a necessary consequence, all fractions, the two characters of which are equal, denote exactly the same value: $\frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6}$, &c.

3. To reduce a whole number to a fraction, which shall have a determinate denominator, we must multiply the whole number by the given denominator, and place the product above the latter. Thus 8, reduced to a fraction having 3 for its denominator, is $\frac{24}{3}$; and 5 reduced to a fraction having the same denominator as $\frac{2}{7}$, is $\frac{35}{7}$.

4. To reduce two fractions to the same denominator, the numerator of the first must be multiplied by the denominator of the second, and the numerator of the second by the denominator of the first: these two products will be the numerator of two new fractions; and the product of the two denominators will be the common denominator. Thus $\frac{2}{3}$ and $\frac{3}{5}$, reduced to the same denominator, give $\frac{10}{15}$ and $\frac{9}{15}$. Any number of fractions may, in like manner, be reduced to a common denominator, provided that each numerator be multiplied by the denominators of the other fractions, and that the product of all the denominators be taken for a common denominator. Thus, for example, the three fractions, $\frac{1}{4}, \frac{2}{3}, \frac{3}{7}$, when reduced to the same denominator, give $\frac{21,\ 56,\ 36}{84}$.

5. To add two fractions, we must first reduce them to a common denominator, and then add their numerators. Thus the sum of the two fractions $\frac{3}{5}$ and $\frac{4}{7}$, is $\frac{21 + 20}{35} = \frac{41}{35}$.

6. To subtract one fraction from another, they must first be reduced to the same denominator, and the numerator of the less must then be taken from the numerator of the greater. Hence the difference of the fractions $\frac{3}{7}$ and $\frac{2}{5}$ is $\frac{1}{35}$.

7. To multiply two fractions together, we must make a new fraction, the numerator of which shall be the product of the two numerators, and the denominator. Thus the product of $\frac{2}{3}$ by $\frac{3}{4}$ is $\frac{6}{12}$.

8. To divide one fraction by another, we must make a fraction, the numerator of which shall be equal to the product of the numerator of the first multiplied by the denominator of the second; and the denominator equal to the product of the numerator of the second multiplied by the denominator of the first. The quotient of $\frac{3}{5}$ divided by $\frac{1}{3}$, will therefore be $\frac{9}{5}$.

9. Sometimes it is necessary to simplify a fraction, by reducing it to its simplest expression, or what is called its lowest terms: nothing is necessary for this purpose, but to divide the numerator and denominator by the greatest common measure or divisor. Thus the fractions $\frac{3}{6}$ and $\frac{4}{20}$, reduced to their simplest expression, give $\frac{1}{2}$ and $\frac{1}{5}$.

OF POWERS.

10. By the *power* of a quantity, is understood its product by unity or by itself a certain number of times. Thus, the first power of 2 is 2: its second power or square is 2×2; its third power or cube is $2 \times 2 \times 2$, and so on. Hence it is evident, that to obtain any power whatever of a given quantity, it must be multiplied by itself as many times less 1, as are equal to the number which denotes that power.

The power of any quantity is expressed sometimes algebraically, or numerically, by the figure which denotes its degree, as in the following examples: a^1, a^2, a^3, a^4, &c. 4^1, 4^2, 4^3, 4^4, &c.

11. An algebraic character is sometimes accompanied by two figures, as $2b^3$. The first of these is called the *coefficient*, and the second the *exponent:* the former denotes how many times the quantity is added to itself; and the second indicates the power. Thus, the value of b, being supposed equal to 3, we shall have $2b^3 = 54$.

12. By the *root* of a quantity, is meant a number which being multiplied one or more times by itself, will give that quantity. There is therefore a *square root, cube root*, &c.

The different roots of quantities may be expressed by the following signs: $\sqrt[2]{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, in this manner, $\sqrt[2]{a}$, $\sqrt[3]{a}$, $\sqrt[4]{a}$, $\sqrt[2]{16}$, $\sqrt[3]{27}$.

13. We shall now shew how to extract the square root of any quantity; that is to say, how to find a number, which, being multiplied by itself, will give that quantity, if it be a complete square, or at least the greatest square which it contains.

EXAMPLE I.—Let the number, the square root of which is required, be 1156. First divide this number from right to left into periods of two figures, and then proceed as follows:

```
      11, 56 (34
       9
      ------
  64 | 256
   4 | 256
      ------
```

Find the greatest square contained in 11, which is 9, and write down its root 3, as seen in the annexed example. Square 3, which gives 9, subtract it from 11, and the remainder will be 2. Then bring down the next period, which is 56, that it may serve as a dividend along with the figure 2 on its left. Take 6 as a divisor, that is to say, the double of the root 3 already found; place it on the left, and find how often it is contained in 25; the quotient will be 4, which must be written down in the root after 3, and also after 6, the divisor, which will give 64. Then multiply the last number by the second root 4, and the product will be 256. As there is no remainder, it is a proof that 1155 is a perfect square, the root of which is 34. Had the last product been too large to be subtracted, it would have been necessary to diminish the last figure in the root, in order to make it small enough for that purpose.

EXAMPLE II.—*What is the Square Root of the Number 214369?*

As in the preceding examples, we must first divide the number into periods of two, from right to left, and there will be as many figures in the root as there are periods.

```
       21, 43, 69 (463
       16
  86)   543
   6    516
 923)   2769
   3    2769
```

Then, as the greatest square contained in 21 is 16, the square root of which is 4, write down the 4 in the root; square 4, which will give 16, and having subtracted it from 21, the remainder will be 5. Bring down the following period 43, which, with the preceding figure, must be divided by the double of the root already found, that is to say, by 8. The quotient of 54 divided by 8 is 6, which must be placed after the first root 4, and also after 8 the divisor; then multiply 86 by 6, and subtract the product 516 from 543. Place the remainder 27 under 516, and bring down the next period 69. Take, as the divisor of 276, the double of the two roots already found, which is 92. Divide 27 by 9, and place the quotient 3 in the root after 46, and also after 92. Then multiply 923 by 3, and if the product 2769 be subtracted from the number 2769, there will be no remainder. The truth of this operation may be proved by squaring 463; that is to say, by multiplying it by itself.

After the last subtraction, if any thing remains, it is a proof that, though the root found is not exactly the real root, it does not want unity to be so; but if it were required to approach still nearer to the real root, nothing would be necessary but to reduce the remainder to decimals, and to continue the operation, taking care to separate the whole numbers in the root from the decimals. However, as we propose here only to give a few amusing problems, there will be no necessity for carrying the extraction of the square root beyond whole numbers.

OF EQUATIONS.

14. As certain questions cannot be easily resolved without some knowledge of analysis and equations, we shall here

give a short explanation of them, and such as may be easily understood.

By *equations* is meant the application of numerical and algebraic rules, to the solution of different questions, which may be proposed respecting quantity.

The first and most difficult thing in analysis, is to comprehend properly the state of the question, and the relation which the known quantities bear to the unknown, in order that they may be clearly expressed in an equation.

Every equation is composed of two members, separated by the sign $=$; and each member may consist of several terms. An example of the whole may be seen in the following equations:

$$7 = 3 + 4;\ 8 - 5 = 2 + 1;\ 3 \times 4 = 12;\ \tfrac{9}{3} = 3.$$

There may be equations also consisting of algebraic quantities alone, or in which arithmetical quantities are mixed with algebraic ones, as in the following:

$$x + b = a;\ x - y = a + b;\ 3a - b = 4c - 2x.$$

GENERAL RULES IN REGARD TO EQUATIONS.

RULE I.

15. Any quantity may be transposed from one member of an equation to another, without deranging the equation, provided that the signs be changed.

Thus, as $12 - 3 = 9$, we may write $12 = 9 + 3$. For the same reason, if $a - x + 3b = d - y$, we shall have $a + y - d = x - 3b$.

This method of operation, in regard to equations, is called *transposition*, and is employed when it is necessary to free one member of an equation from any quantity connected with it either by addition or subtraction.

RULE II.

16. When an unknown quantity is involved in an equation either by multiplication or division, it may be disengaged from it, in the first case, by division; and in the second by multiplication. For example,

If $3x = b$, then $x = \frac{b}{3}$; and if $\frac{y}{3} = a$, then $y = 3a$.

This method of disengaging an unknown quantity will be more easily comprehended, if we give determinate values to the quantities a and b. If we suppose, for example, that $b = 12$, and $a = 8$, the two equations above mentioned will be reduced to the following, $x = \frac{12}{3} = 4$, $y = 24$.

17. It appears, therefore, that the whole art of analysis consists, first, in comparing in the equations the unknown with the known quantities, and disengaging them from each other by the means already pointed out, in such a manner, that the known quantity may remain alone in one member of the equation, and the unknown in the other.

To facilitate the solution of algebraic questions, the unknown quantities are generally denoted by some of the last letters of the alphabet, v, y, z; and the known quantities by some of the first, as a, b, c, &c.

OF RATIOS AND PROPORTIONS.

18. *Relation*, or *Ratio*, is what results from the comparison of two quantities. As two quantities may be compared with each other two ways, ratio is distinguished into two kinds, arithmetical and geometrical.

Arithmetical relation, is that of two quantities compared with each other by subtraction.

Geometrical relation, is that of two quantities compared with each other by division.

Thus, for example, the arithmetical ratio of 12 to 4 is 8; and the geometrical ratio of the same quantities is 3; for $12 - 4 = 8$, and $\frac{12}{4} = 3$.

19. *Proportion* is an equality of ratios. As there are two kinds of ratio, there are also two kinds of proportion, arithmetical and geometrical: the first consists in an equality of differences, and the second in an equality of quotients.

Every ratio is expressed by two terms; the first of which is called the *antecedent*, and the second the *consequent*.

Two equal ratios form a proportion; which is either arithmetical or geometrical, according as they contain either the same difference or the same quotient. Thus $3 . 5 \because 7 . 9$, expresses an arithmetical proportion; the meaning of which is, that 3 is arithmetically to 5 as 7 is to 9; and $6 : 3 :: 16 : 8$, expresses a geometrical proportion; the meaning of which is, that 6 is geometrically to 3 as 16 is to 8.

The first and last terms of each of these proportions are called the *extremes;* and the other two the *means.*

20. Proportion is *continued* when the same term is the consequent of that which precedes it, and the antecedent of that which follows it. Thus the two following proportions, one of which is arithmetical and the other geometrical, are continued, viz. $\div 3 . 5 . 7$; $\div\!\!\because 4 : 8 : 16$. The meaning of which is, $3 . 5 \because 5 . 7$; $4 : 8 :: 8 : 16$.

When a continued proportion has more than three terms, it is called a *progression.* Thus $\div 1 . 3 . 5 . 7 . 9$ is an arithmetical progression, and $\div\!\!\because 4 : 8 : 16 : 32 : 64$ is a geometrical progression.

Properties of Arithmetical Proportion and Progression.

THEOREM I.

21. In every arithmetical proposition, the sum of the extremes is equal to that of the means.

$$\text{If } 3 . 5 \because 7 . 9, \text{ or } 9 . 6 \because 8 . 5,$$

$$\text{Then } 3 + 9 = 5 + 7, \text{ and } 9 + 5 = 6 + 8.$$

It hence follows, that when three terms of an arithmetical proportion are known, we may easily find the fourth; for if the unknown term be an extreme, it will be found by subtracting the other extreme from the sum of the means; and if it be one of the means, by subtracting the other mean from the sum of the extremes.

$$\text{If } a . b \because c . x, \text{ or if } 4 . x \because 3 . 8,$$

$$\text{Then } \begin{cases} b + c = + x \\ b + c = = x \end{cases} \text{ and } \begin{cases} 4 + 8 = x + 3 \\ 4 + 8 - 3 = x \end{cases} \quad (15)$$

It hence follows also, that if two terms, as a and b, are given, a third arithmetical proportional to them may be easily found, in order to form an arithmetical progression. For if we suppose the required term to be x, we shall have

$$\because a \,.\, b \,.\, x$$

$$\text{Then} \begin{cases} a \,.\, b \because b \,.\, x \\ a + x = b + b = 2b \ (20) \\ x = 2b - a \quad (24) \end{cases}$$

Consequently, to find a third arithmetical proportional to two given terms, we must subtract the first from double the second. Thus, the third arithmetical proportional to 3 and 7, will be $14 - 3 = 11$; and indeed $\because 3 \,.\, 7 \,.\, 11$.

An arithmetical mean proportional between two given terms, such as a and b, may be found with equal ease; for if the required mean be denoted by x, we shall have

$$\because a \,.\, x \,.\, b \qquad (20)$$

$$\text{Then} \begin{cases} a + b = 2x \\ \dfrac{a + b}{2} = x \quad (15) \end{cases}$$

Which indicates, that an arithmetical mean proportional to two quantities, is equal to the half of these quantities. Thus, the mean proportional between 9 and 13 is 11; for $\because 9 \,.\, 11 \,.\, 13$.

THEOREM II.

22. In every even arithmetical progression, the sum of all the terms, equally distant from the extremes, taken two and two, is equal to that of the extremes; and if it be odd, the sum of the extremes, or of any two terms equally distant from the extremes, is the double of the mean term.

CASE I.

$$\text{If} \div 3 \,.\, 5 \,.\, 7 \,.\, 9 \,.\, 11 \,.\, 13,$$

$$\text{Then} \begin{cases} 5 + 11 = 3 + 13 \\ 7 + 9 = 3 + 13 \end{cases}$$

CASE II.

$$\text{If} \div 2 \,.\, 4 \,.\, 6 \,.\, 8 \,.\, 10,$$

$$\text{Then} \begin{cases} 2 + 10 = 2 \times 6 \\ 4 + 8 = 2 \times 6 \end{cases}$$

In the first case, the sum of an arithmetical progression is equal to the product of the sum of the extremes multiplied by half the number of terms; and in the second, to the product of the mean multiplied by the number of terms.

THEOREM III.

23. In every arithmetical progression, any term whatever is equal to the first and as many times the common difference as there are terms before it.

$$\text{If} \because 2 \,.\, 5 \,.\, 8 \,.\, 11 \,.\, 14,$$

$$\text{Then} \begin{cases} 14 = 2 + \overline{3 \times 4} \\ 11 = 2 + \overline{3 \times 3} \\ 8 = 2 \div \overline{3 \times 2}\,. \end{cases}$$

It hence follows, that we may easily find the value of any term of an arithmetical progression, the first term, the common difference, and the number of terms of which are known.

For example, the 121st term of an arithmetical progression, the first term of which is 5 and the common difference 3, will be 365; for $5 + \overline{3 \times 120} = 365$.

Properties of Geometrical Proportion and Progression.

THEOREM I.

24. In every geometrical proportion, the product of the extremes is equal to that of the means.

$$\text{If} \quad 3 : 6 :: 4 : 8,$$

$$\text{Then } 3 \times 8 = 6 \times 4$$

Consequently, the fourth term of a geometrical proportion, the other three of which are known, may be easily found; for if the required term be an extreme, it will be equal to the product of the means divided by the other extreme; and if it be a mean, it will be equal to the product of the extremes divided by the other mean.

$$\text{If } 2 : 4 :: 3 : x,$$

$$\text{Then} \begin{cases} 2x = 4 \times 3 \qquad (15) \\ x = \dfrac{4 \times 3}{2} = 6 \end{cases}$$

$$\text{If } 2 : y :: 3 : 6,$$

$$\text{Then } \begin{cases} 2 \times 6 = 3y & (15) \\ \dfrac{2 \times 6}{3} = y = 4 \end{cases}$$

Two terms being given, a third, geometrically proportional to them, may be easily found, in order to form a geometrical progression. Let us suppose that a third proportional is required to the terms a and b, and that the term sought is denoted by y. We shall then have

$$\div\, a : b : y$$

$$\text{Then } \begin{cases} a : b :: b : y \\ ay = b^2 \\ y = \dfrac{b^2}{a} \end{cases}$$

Consequently, to find a third term, geometrically proportional, we must divide the square of the second, or its product by itself, by the first term. Thus the third geometrical proportional to 3 and 6, will be $\frac{6 \times 6}{3} = 12$; and indeed $\div\, 3 : 6 : 12$.

A mean geometrical proportional between two terms, as a and b, may be found with equal ease; for if this term be called x, we shall have

$$\div\, a : x : b$$

$$\text{Then } \begin{cases} xx \text{ or } x^2 = ab \\ \sqrt{xx} = \sqrt{ab} \\ x = \sqrt{ab} \end{cases}$$

Thus, if we suppose $a = 2$, and $b = 8$; x will be equal to the square root of 16, which is 4. And indeed $\div\, 2 : 4 : 8$.

THEOREM II.

25. Any term whatever of a geometrical progression, is equal to the product of the first term multiplied by the common ratio, raised to that power the exponent of which is equal to the number of terms before it.

Let the geometrical progression be $\div\, 2 : 4 : 8 : 16 : 32 :$ &c.

The fifth term 32, for example, is equal to the product of

2, the first, multiplied by 16, which is the fourth power of the ratio 2.

THEOREM III.

26. In every geometrical progression, the second term, less the first, is to the first, as the last, less the first, is to the sum of all those which precede it.

$$\text{If} \div 2 : 4 : 8 : 16 : 32 : \&c.$$
$$\text{Then } 4 - 2 : 2 :: 32 - 2 : 2 + 4 + 8 + 16.$$

RULE OF THREE.

27. The *Rule of Three*, is an operation by which, when three terms of a geometrical proportion are known, a fourth, not known, may be found; and it is called *direct*, when the similar terms increase in the same ratio. For example: If four men perform six yards of work in a certain time, it is evident that a greater number must perform more in the same time. On the other hand, if the similar terms, instead of increasing in the same ratio, must decrease, the rule is called *inverse*, as is the case in the following example: If four men perform a certain work in eight days, a greater number of men must perform it in a time proportionally less.

The Rule of Three Direct, and the Rule of Three Inverse, may be expressed by the following formulæ.

Men		Men		Yards		Yards
3	:	6	::	4	:	x
Men		Men		Days		Days
3	:	6	::	x	:	8

The Rule of Three is compound or simple, according as the terms are compound or simple. For example, the above two formulæ express each the simple rule of three. But if it were required to divide the profits of a commercial company among several partners, who have advanced certain capitals, for different periods of time, it would be necessary to multiply the capital of each partner by the time, which would render the rule the compound rule of three.

As the rule of three is only the application of the formulæ of Theorem I. (23.) it is needless to enlarge farther on this subject.

RECREATIONS IN ARITHMETIC.

ARITHMETIC and Geometry, according to Plato, are the two wings of the mathematician; and, indeed, the object of all mathematical questions, is to determine the ratios of numbers or of magnitudes. It may even be said, to continue the comparison of the ancient philosopher, that arithmetic is the mathematician's right wing; for, it is certain, that geometrical determinations would often afford very little satisfaction to the mind, if the ratios thus determined could not be reduced to numerical ratios. This justifies the common practice of beginning with arithmetic.

This science presents a wide field for speculation and curious research; but in the present selection, we shall confine ourselves to such things as are best calculated to excite the curiosity of those who have a taste for the mathematics, and who seek for recreations that may enable them to resume their more serious studies with greater success.

OF OUR NUMERICAL SYSTEM, AND THE DIFFERENT KINDS OF ARITHMETIC.

It has been generally observed, that all the nations with which we are acquainted, reckon by periods of ten; that is to say, after having counted the units, as far as ten, they begin again by adding units to a ten; when they attain to 20, they add units as far as 30, or three tens, and so on in succession, till they come to 100, or ten tens; of ten times a hundred they form a thousand, &c. Did this arise from necessity? was it occasioned by any physical cause? or was it merely the effect of chance?

No one, after the least reflection, will be inclined to ascribe it to chance. It is not only probable, but might also be proved, that this system derives its origin from our physical conformation. All men have ten fingers, a very few excepted, who by some *lusus naturæ* have twelve. The first men began to reckon on their fingers. After having exhausted them by counting the units, it was necessary for them to begin and to count them again, till they had exhausted them a second time; then a third time, and so on. Hence the origin of ten; which, being confined to the fingers, could not be carried beyond the number of ten without forming a new total, called a hundred; then another, consisting of ten hundreds, called a thousand, &c.

A curious consequence hence follows; which is, that if nature, instead of ten fingers, had given us twelve, our numerical system would have been different. After ten, instead of saying ten plus one, or eleven, we should have ascended by simple denominations to twelve, and should have then counted twelve plus one, twelve plus two, &c. as far as two dozen. Our hundred would have been twelve dozens, a thousand, twelve times twelve dozens, &c.

A six-fingered people, in all probability, would have had an arithmetic of this kind, which indeed would not have been inferior to that now in use, or rather would have been attended with some advantages, which our present numerical system does not possess.

This method of numeration would have been as expeditious, and even more so than that now universally received. The number of characters, which would have been increased only by two, to express ten and eleven, would have been as little burdensome to the memory as the present characters; and this system would have possessed some advantages, which ought to give us reason to regret that it was not originally adopted.

This, however, would no doubt have been the case, had philosophy presided when the system was first formed; as it would have readily been seen that 12, of all the numbers almost between 1 and 20, possesses the advantage of being small, and of having the greatest number of divisions; viz. 2, 3, 4, and 6, by which it can be divided without a remainder. Besides, in this system the periods of numeration would have had the advantage of being divisible, the first from one to twelve, by 2, 3, 4, 6; the second, from one to a hundred and forty-four, by 2, 3, 4, 6, 8, 9, 12, 16, 24, 36, 48, 72; whereas, in our system, the first period from one to ten has only two divisors, 2 and 5; the second from one to a hundred has only seven, viz. 2, 4, 5, 10, 20, 25, 50, consequently fractions would have less frequently occurred in numerical operations.

But what would have been particularly convenient in this mode of numeration, is, that it would have introduced into all measures the duodecimal divisions and subdivisions. Thus, as the foot is divided into 12 inches, the inch into 12 lines, and the line into 12 points; the pound, in the like manner, would have been divided into 12 ounces, the ounce into 12 drams, the dram into 12 grains, or other denominations at pleasure; the day would have been divided into 12 portions, called hours; the hour into 12 others, which would have been equal to 10 minutes; and each of these into 12 inferior parts; with so on in succession.

Should it be asked, what would have been the advantage of this division, we might reply as follows:—It is well known, that when it is necessary to divide any measure into 3, 4, or 6 parts, a whole number of measures of the lower denomination cannot always be found, or are found only by chance. Thus, the third, or tenth of a pound avoirdupois, does not always give an exact number of ounces; and the third of a pound sterling does not give an exact number of shillings. The case is the same with the bushel, and the greater part of other measures. These inconveniencies, which render calculations complex, would not have occurred had the duodecimal progression been universally adopted.

Stevin, a Dutch mathematician, proposed to adapt the divisions and subdivisions of all measures to the system of numeration since adopted by the French, making them to decrease in decimal progression. Thus, the fathom would have contained 10 feet, the foot 10 inches, the inch 10 lines, &c. This method, however, though attended with some advantages, is less perfect than the duodecimal, as it gives rise to a greater number of fractions.

A great many systems of arithmetic have been proposed, such as the binary, ternary, quaternary, &c.; and even the duodenary; but there is no great reason to believe that any of them will ever be admitted into practice.

But we shall add nothing farther on the subject; for, as useful recreations are the object of this collection, we must exclude from it every thing too complex, or of too frivolous a nature.

We suppose that the reader is sufficiently well acquainted with arithmetic, both in whole and fractional numbers, (2.) to be able to comprehend every thing that relates to it.

OF SOME PROPERTIES OF NUMBERS.

UNDER this head, we do not comprehend those properties of numbers which engaged so much the attention of the ancients, and to which they ascribed so many mysterious virtues. Every one, whose mind is not tinctured with credulity, must laugh to think of the good canon of Cezene, father Bungo, collecting into a quarto volume, entitled *De Mysteriis Numerorum*, all the ridiculous conceits which Nichomachus, Ptolemy, Porphyrius, and several more of the ancients, childishly published in regard to numbers. How could it enter the minds of reasonable beings to ascribe physical energy to things purely metaphysical? For numbers are mere conceptions of the mind, and consequently can have no influence in nature.

None, therefore, but old women, or persons of weak minds, can believe in the virtues of numbers. Some entertain a notion, that if 13 persons sit at the same table, one of them will die in the course of the year; but there is more probability that one will die in the same time, if the number be 24.

The series 1 2 3 4 5 6 7 8 9 is of such a nature, that it may be multiplied by certain numbers, the product of which shall consist of twos, threes, or fours, &c. at pleasure.

To find a multiplier which shall give similar figures in the product, 9 must be privately multiplied by 2, 3, or 4, according as it is required to have twos, threes, or fours, in the product.

For example, if it be required that the product shall contain only twos, we must multiply 9 privately by 2, which will give 18 for the multiplier of the series. If we multiply the same number by 3, we shall have 27; if by 4, we shall have 36, &c.

If the series, therefore, be multiplied successively by 18, 27, 36, the products will be composed of twos, threes, fours, &c. as may be easily proved by trial.

If the number 37 be multiplied by any of the terms of the arithmetical progression 3, 6, 9, 12, &c. all the products will consist of similar figures.

$$\begin{array}{r} 37 \\ 3 \\ \hline 111 \end{array} \qquad \begin{array}{r} 37 \\ 6 \\ \hline 222 \end{array} \qquad \begin{array}{r} 37 \\ 9 \\ \hline 333 \end{array}$$

We may here observe, that the product 111 is composed of 37 multiplied by 3; or of 7 multiplied by 3, which makes 21, and 30 multiplied by three, which makes 90; the sum-total being one and eleven tens, which, according to the laws of numbers, can be expressed only by three units.

It will not then appear astonishing, that 37 multiplied by 6, should give the double of these three units; and so of the rest.

The number 5 has this peculiar property, that when multiplied by an odd number, the product will always terminate with 5; and if multiplied by an even number, will terminate with a cipher.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \end{array} \qquad \begin{array}{r} 5 \\ 5 \\ \hline 25 \end{array} \qquad \begin{array}{r} 5 \\ 4 \\ \hline 20 \end{array} \qquad \begin{array}{r} 5 \\ 6 \\ \hline 30 \end{array}$$

The number 9 has this property, that the sum of the figures of every number, of which it is a multiple, forms a multiple of 9.

Thus, the product of 17 multiplied by 9 is 153, and the sum of these figures $1 + 5 + 3 = 9$. In the like manner, the sum of the figures 6777, which is the product of 753 multiplied by 9, is equal to 27, a multiple of 9.

This will not appear astonishing when we reflect, that twice 9 is equal to 18, three times 9 = 27, &c.; where it is evident that the tens and units of the product are always reciprocal complements of 9.

If we take any two numbers whatever, one of them, or their sum, or their difference, will always be divisible by three. This may be so easily proved, that it is needless to illustrate it by examples.

Every square number must necessarily terminate with two ciphers, or by 1, 4, 5, 6, 9; and this may enable us to determine, at one view, whether a numerical quantity be a square or not. (10.)

The number 2, of all the whole numbers, is the only one the sum and product of which are equal. Thus $2 + 2 = 4$, as well as 2×2. But in fractional numbers we find other two quantities, the sum and product of which are in like manner equal.

For this purpose, the sum of the two numbers must be divided by each of them separately. The two fractions which thence arise, will give the same quantity, both when added and multiplied. (5. 7.)

If we take the two numbers 2 and 5, and divide their sum by each of them separately, the two fractions $\frac{7}{2}$, $\frac{7}{5}$, will give the same result when added, as well as when multiplied. This may be easily proved by any person in the least acquainted with vulgar fractions. (2.)

Every square number (10.) is divisible by 3, or becomes so when diminished by unity. This may be easily proved with any square number whatever. Thus, $4 - 1$, $16 - 1$, $25 - 1$, $49 - 1$, $121 - 1$, &c. are all divisible by 3; and the case is the same with the rest.

Every square number is divisible by 4, or becomes so when diminished by unity.

Every square number is divisible also by 5, or becomes so when increased or diminished by unity.

Every odd square, diminished by unity, is a multiple of 8.

Every power of 5 terminates with 5, and every power of 6 with 6. (10.)

PROBLEM.—*To find Two Numbers, the Squares of which, if added together, shall form a Square Number.*

If any two numbers whatever be multiplied together, the double of their product will be one of the two numbers sought; and the difference of their squares will be the other.

Thus, if the numbers 2 and 3, the squares of which are 4 and 9, be multiplied together, their product will be 6; if we then take 12, the double of this product, and 5 the difference of their squares, we shall have two numbers, the sum of the squares of which will be a square number; for their squares are 144 and 25, which by addition give 169, the square of 13.

OF ARITHMETICAL AND GEOMETRICAL PROGRESSION, WITH SOME PROBLEMS WHICH DEPEND ON THEM.

SECTION I.

Arithmetical Progression, with an Explanation of its Principal Properties.

ANY series of numbers, continually increasing or decreasing by the same quantity, forms what is called an arithmetical progression. (19.)

Thus, the series of numbers 1, 2, 3, 4, 5, 6, &c. or 1, 5, 9, 13, &c. or 20, 18, 16, 14, 12, &c. or 15, 12, 9, 6, 3, are arithmetical progressions; for, in the first, the difference between each term and the following one, which exceeds it, is always 1; in the second it is 4; it is 2 also in the third, which goes on decreasing, and 3 in the fourth.

From this definition of arithmetical progression, the following consequences may be deduced:

1st, Any term of an arithmetical progression, is equal to the first, plus the common difference taken as many times as there are terms before it. (22.)

2d, The sum of the extremes is always equal to the sum of any two terms equally distant from them; or double the mean term, if the progression contains an odd number of terms. (21.)

3d, If the sum of the extremes be multiplied by half the number of terms, when the terms are even, or the mean by the whole number of terms when the latter are odd, the product will be the sum of the progression.

By considering, with a little attention, the following progressions, the truth of these consequences will be readily perceived.

÷ 2 . 5 . 8 . 11 . 14 . 17
÷ 1 . 3 . 5 . 7 . 9

PROBLEM I.—*If a hundred stones are placed in a straight line, at the distance of a yard from each other, the first being at the same distance from a basket; how many yards must the person walk, who engages to pick them up, one by one, and to put them into the basket?*

It is evident that, to pick up the first stone, and put it into the basket, the person must walk two yards; for the second he must walk 4; for the third 6; and so on, increasing by two, to the hundredth.

The number of yards, therefore, which the person must walk, will be equal to the sum of the progression 2, 4, 6, &c. the last term of which is 200. (22.) But the sum of the progression is equal to 202, the sum of the two extremes, multiplied by 50, or half the number of terms; that is to say, 10,100 yards, which makes more than $5\frac{1}{2}$ miles.

PROBLEM II.—*A gentleman employed a bricklayer to sink a well, and agreed to give him at the rate of three shillings for the first yard in depth, 5 for the second, 7 for the third, and so on, increasing to the twentieth, where he expected to find water: how much was due to the bricklayer when he had completed the work?*

This question may be easily answered by the rules before given, for the difference of the terms is 2, and the number of terms 20; consequently, to find the twentieth term, we must multiply 2 by 19, and add 38, the product, to the first term 3, which will give for the twentieth term 41. (22.)

If we then add the first and last terms, that is to say, 3 and 41, which will make 44, and multiply this sum by 10, or half the number of terms, the product 440 will be the sum of all the terms of the progression, or the number of shillings due to the bricklayer, when he completed the work. (21.)

He would therefore have to receive L. 22.

PROBLEM III.—*A merchant being considerably in debt, one of his creditors, to whom he owed* L.1860, *offered to give him an acquittance, on condition of his agreeing to pay the whole sum in twelve monthly instalments; that is to say,* L.100 *the first month, and to increase the payment by a certain sum each succeeding month to the twelfth inclusive, when the whole debt would be discharged. By what sum was the payment of each month increased?*

In this problem we have given the first term 100, the number of the terms 12, and their sum 1860; but the common difference of the terms is unknown.

This difference may be found in the following manner:— As the sum of the extremes, in an even arithmetical progression, is equal to the sum-total, divided by half the number of terms, if the sum-total 1860 be divided by 6, or half the number of terms, we shall have 310 for the sum of the first and last term, from which if we subtract 100, the first term, the remainder 210 will be the last term; but the last term is always equal to the first and the common difference taken as many times as there are terms before it. If we, therefore, deduct the first term 100, from 210 the last, and divide 110, the remainder, by 11, we shall have 10 as the required difference. The first term being 100, the second therefore will be 110, the third 120, &c. (21.)

PROBLEM IV.—*A gentleman employed a bricklayer to sink a well, to the depth of 20 yards, and agreed to give him* L.20 *for the whole; but the bricklayer happening to die when he had completed only 8 yards, how much was due to his heirs?*

To imagine that two-fifths of the whole sum were due to the workman, because 8 yards are two-fifths of the whole depth, would be erroneous; for as the difficulty must increase arithmetically as the depth, it is natural to suppose that the price should increase in the same ratio.

To resolve this problem, therefore, L.20, or 400 shillings, must be divided into twenty terms in arithmetical progression; and the sum of the first eight of these will express what was due to the bricklayer for his labour.

But 400 shillings may be divided into twenty terms in arithmetical progression a great many different ways, according to the value of the first term, which is here undetermined: if we suppose it, for example, to be 1 shilling, the progression will be 1, 3, 5, 7, &c. the last term of which will be 39; and consequently the sum of the first eight terms will be 64 shillings.

But to resolve the problem in a proper manner, so as to give to the bricklayer his just due for the commencement of the work, we must determine what is the fair value of a yard of work similar to the first, and then assume that value as the first term of the progression. We shall here suppose that this value is 5 shillings; and in that case the required progression will be 5, $6\frac{11}{19}$, $8\frac{3}{19}$, $9\frac{14}{19}$, $11\frac{6}{19}$, $12\frac{17}{19}$, &c. the common difference of which is $1\frac{11}{19}$, and the last term is 35.

Now to find the eighth term, which is necessary before we can find the sum of the first eight terms, multiply the common difference $\frac{30}{19}$ by 7, which will give $11\frac{1}{19}$, and add this product to 5, the first term, which will give the eighth term $16\frac{1}{19}$; if we then add $16\frac{1}{19}$ to the first term, and multiply the sum, $21\frac{1}{19}$, by 4, the product, $84\frac{4}{19}$, will be the sum of the first eight terms, or what was due to the bricklayer for the part of the work he had completed. The bricklayer, therefore, had to receive $84\frac{4}{19}$ shillings, or L.4. 4s. $2\frac{10}{19}$d.

SECTION II.

Of Geometrical Progressions, with an Explanation of their principal Properties.

If there be a series of numbers, each of which is the product of the preceding by a common multiplier, (18.) these numbers form what is called a geometrical progression, Thus, 1 2, 4, 8, 16, &c. form a geometrical progression; for the second is the double of the first, the third the double of the second, and so on in succession. The terms 1, 3, 9, 27, 81, &c. form also a geometrical progression, each being the triple of the preceding.

Progressions may be either increasing, as the two above mentioned, or decreasing, as the two following, 16, 8, 4, 2, 1; 81, 27, 9, 3, 1.

The principal property of geometrical progression is, that if we take any three following terms whatever, as 3, 9, 27, the product 81 of the extremes will be equal to the square of the mean 9. In like manner, if we take any four following terms, as 3, 9, 27, 81, the product 243, of the extremes, will be equal to that of the two means, 9 and 27.

In the last place, if any number of terms whatever of the series be assumed, as 2, 4, 8, 16, 32, 64, the product of the extremes, 2 and 64, will be equal to the product of any two terms equally distant from them, as 4 and 32, or 8 and 16. If the number of terms be odd, it is evident that there will be only one term equally distant from the two extremes; and in that case the square of that term will be equal to the product of the extremes, or of any two terms whatever equally distant from them, or from the mean. (23, 24.)

Between geometrical and arithmetical progression there is a certain analogy, which deserves here to be mentioned, and which is, that the same results are obtained in the former, by employing multiplication and division, as are obtained in the latter by addition and subtraction. When, in the latter, we take the half or the third, we employ, in the former, extraction of the square, cube, &c. roots.

Thus, to find an arithmetical mean between two numbers, for example 3 and 12, we must add the two extremes together, and take the half of 15 their sum, which is $7\frac{1}{2}$; but to find a geometrical mean between two numbers, we must multiply them together, and extract the square root of their product; for example, if a geometrical mean between 3 and 12 be required, we must extract the square root of their product 36, which will give 6; and, indeed, ∺ 3 : 6 : 12. (23.)

A geometrical progression may decrease *in infinitum*, without ever coming to 0; for it is evident that any part of a quantity whatever, greater than 0, can never become 0. A decreasing geometrical progression may be continued, therefore, *in infinitum*, since to find the following term, nothing will be necessary but to divide the last term by the exponent or common ratio. We shall here give two examples of decreasing geometrical progressions.

$$\div\!\!\cdot\, 1 : \tfrac{1}{2} : \tfrac{1}{4} : \tfrac{1}{8} : \tfrac{1}{16} : \tfrac{1}{32} : \tfrac{1}{64}, \text{ \&c.}$$

$$\div\!\!\cdot\, 1 : \tfrac{1}{3} : \tfrac{1}{9} : \tfrac{1}{27} : \tfrac{1}{81} : \tfrac{1}{243}, \text{ \&c.}$$

The sum of an increasing geometrical progression, continued *ad infinitum*, is evidently infinite; but that of a decreasing geometrical progression, whatever be the number of terms supposed, is always finite. Thus, the sum of the terms, continued *in infinitum*, of the geometrical progression $\div\!\!\cdot\, 1 : \frac{1}{2} : \frac{1}{4} : \frac{1}{8} \div \frac{1}{16}$: &c. is only 2. That of the progression $\div\!\!\cdot\, 1 : \frac{1}{3} : \frac{1}{9} : \frac{1}{27} : \frac{1}{81}$: &c. is only $1\frac{1}{2}$.

OF HARMONICAL PROGRESSION.

Three numbers are in harmonical proportion, when the first is to the last as the difference between the first and the second is to the difference between the second and third. Thus the numbers 6, 3, 2, are in harmonical proportion; for 6 is to 2, as 3, the difference between the two first numbers, is to 1, the difference of the two last. This kind of relation is called harmonical, for a reason which will be seen hereafter.

When three numbers in decreasing harmonical proportion are given, it is easy to find a fourth; nothing is necessary but to find a third harmonical to the two last, and this will be the fourth term required. In like manner, the third and fourth may be employed to find a fifth, and so on in succession: this will form what is called a harmonical progression, which by the above method may be always continued decreasing.

If we suppose the two first numbers to be 2 and 1, we shall have the harmonical progression 2, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. It is a remarkable property, therefore, of the series of fractions, having unity for their numerators, and for their denominators the numbers of the natural progression, that they are in harmonical progression.

This series of numbers, indeed, contains all the musical concords possible; for the ratio of 1 to $\frac{1}{2}$, gives the octave; that of $\frac{1}{2}$ to $\frac{1}{3}$, or of 3 to 2, the fifth; that of $\frac{1}{3}$ to $\frac{1}{4}$, or of 4 to 3, the fourth; that of $\frac{1}{4}$ to $\frac{1}{5}$, or of 5 to 4, the third major; that of $\frac{1}{5}$ to $\frac{1}{6}$, or of 6 to 5, the third minor; that of $\frac{1}{8}$ to $\frac{1}{9}$, or of 9 to 8, the tone major; and that of $\frac{1}{9}$ to $\frac{1}{10}$, or of 10 to 9, the tone minor. But this will be explained more at large when we come to treat of harmony.

Let us now return to geometrical progression, and the application of it to a few problems, which may serve as a rule for the solution of all others of the same kind.

PROBLEM I.—*If Achilles can walk ten times as fast as a tortoise, which is a furlong before him, can crawl, will the former overtake the latter; and how far must he walk before he does so?*

This question has been thought worthy of notice merely because Zeno, the founder of the sect of the stoics, pretended to prove by a sophism, that Achilles would never overtake the tortoise; for while Achilles, said he, is walking a furlong, the tortoise will have advanced the tenth of a furlong; and while the former is walking that tenth, the tortoise will have advanced the hundredth part of a furlong, and so on *in infinitum;* consequently, an infinite number of instants must elapse before the hero can come up with the tortoise, and therefore he will never come up with it.

Any person, however, of common sense may readily perceive, that Achilles will soon come up with the tortoise. In what then consists the sophism? It may be explained as follows:—

Achilles, indeed, would never overtake the tortoise, if the intervals of time, during which he is supposed to be walking the first furlong, and then the tenth, hundredth, and thousandth parts of a furlong, which the tortoise has successively advanced before him, were equal: but if we suppose that he has walked the first furlong in 10 minutes, he will require only one minute to walk the tenth of a furlong, and $\frac{1}{10}$ of a minute to walk the hundredth, &c. The intervals of time, therefore, which Achilles will require to pass over the space gained by the tortoise during the preceding time, will go on decreasing in the following manner: 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c.; and this series forms a sub-decuple geometrical progression, the sum of which is equal to $11\frac{1}{9}$, or the interval of time at the end of which Achilles will have reached the tortoise.

Problem II.—*If the hour and minute hands of a clock both begin to move exactly at noon, at what points of the dial-plate will they be successively in conjunction, during a whole revolution of the twelve hours?*

This problem, considered in a certain manner, is in nothing different from the preceding. The minute hand acts here the part which Achilles did in the former; and the hour hand, which moves ten times slower, that of the tortoise. In the last place, if we suppose the hour hand to be beginning a second revolution, and the minute hand to be beginning a first, the distance which the one has gained over the other will be a whole revolution of the dial-plate. When the minute hand has made one revolution, the hour hand will have made one twelfth of a revolution, and so on progressively. To resolve the problem, therefore, we need only apply to these data the method employed in the former case, and we shall find, that the interval from noon to the point where the hands come again into conjunction will be $\frac{1}{11}$ of a whole revolution; or, what amounts to the same thing, one hour and $\frac{1}{11}$ of an hour. They will afterwards be in conjunction at 2 hours and $\frac{2}{11}$; 3 hours and $\frac{3}{11}$; 4 hours and $\frac{4}{11}$, &c. and, in the last place, at 11 hours and $\frac{11}{11}$, that is to say, at 12 hours.

PROBLEM III.—*A sovereign being desirous to confer a liberal reward on one of his courtiers, who had performed some very important service, desired him to ask whatever he thought proper, assuring him it should be granted. The courtier, who was well acquainted with the science of numbers, only requested that the monarch would give him a quantity of wheat equal to that which would arise from one grain doubled sixty-three times successively. What was the value of the reward?*

It will be found by calculation, that the sixty-fourth term of the double progression ∸ 1 : 2 : 4 : 8 : 16 : 32 : &c. is 9223372036854775808. But the sum of all the terms of a double progression, beginning with unity, may be obtained by doubling the last term, and subtracting from it unity. The number of the grains of wheat, therefore, in the present case, will be 18446744073709551615. Now, if a standard pint contains 9216 grains of wheat, a gallon will contain 73728 : and, as eight gallons make one bushel, if we divide the above result by eight times 73728, we shall have 31274997411295 for the number of the bushels of wheat equal to the above number of grains, a quantity greater than what the whole surface of the earth could produce in several years, and which, in value, exceeds all the riches perhaps on the globe of the earth.

Another problem of the same kind is proposed in the following manner :

A gentleman, taking a fancy to a horse, which a horse-dealer wished to dispose of at as high a price as he could, the latter, to induce the gentleman to become a purchaser, offered to let him have the horse for the value of the twenty-fourth nail in his shoes, reckoning one farthing for the first nail, two for the second, four for the third, and so on to the twenty-fourth. The gentleman, thinking he should have a good bargain, accepted the offer : what was the price of the horse?

By calculating as before, the twenty-fourth term of the progression ∸ 1 : 2 : 4 : 8 : &c. will be found to be 8388608, equal to the number of farthings the purchaser gave for the horse. The price, therefore, amounted to L.8738. 2s. 8d. which is more than any Arabian horse, even of the noblest breed, was ever sold for.

We shall conclude this article with some physico-mathematical observations on the prodigious fecundity and progressive multiplication of animals and vegetables, which would take place if the powers of nature were not continually meeting with obstacles.

1st, It is not astonishing, that the race of Abraham, after sojourning 260 years in Egypt, should have formed a nation capable of giving uneasiness to the sovereigns of that country. We are told, in the sacred writings, that Jacob settled in Egypt with seventy persons: now, if we suppose that, among these seventy persons, there were twenty too far advanced in life, or too young, to have children; that of the remaining fifty, twenty-five were males and as many females, forming twenty-five married couples, and that each couple in the space of twenty-five years, produced, one with another, eight children, which will not appear incredible in a country celebrated for the fertility of its inhabitants; we shall find that, at the end of twenty-five years, the above seventy persons may have increased to two hundred and seventy; from which, if we deduct those who died, there will, perhaps, be no exaggeration in making them amount to two hundred and ten. The race of Jacob, therefore, after sojourning twenty-five years in Egypt, may have been tripled. In like manner, these two hundred and ten persons, after twenty-five years more, may have increased to six hundred and thirty; and so on in triple geometrical progression: hence it follows, that, at the end of two hundred and twenty-five years, the population may have amounted to 1377810 persons, among whom there might easily be five or six hundred thousand adults fit to bear arms.

2d, If we suppose that the race of Adam, making a proper deduction for those who died, may have been doubled every twenty years, which certainly is not inconsistent with the powers of nature, the number of men at the end of five centuries, may have amounted to 1048576. Now, as Adam lived about 900 years, he may have seen, therefore, when in the prime of life, a posterity of 1048576 persons.

3d, How great would be the multiplication of many animals, did not the difficulty of finding food, the continual war which they carry on against each other, or the numbers of them consumed by man, set bounds to their propagation! It might easily be proved, that the breed of a sow, which brings forth six young, two males and four females, if we suppose that each female produces, every year after, six young, four of them females and two males, would in twelve years amount to 33554230.

Several other animals, such as rabbits and cats, which go with young only for a few weeks, would multiply with still greater rapidity; in half a century the whole earth would not be sufficient to supply them with food, nor even to contain them.

If all the ova of a herring were fecundated, a very few years would be sufficient to make its posterity fill the whole ocean: for every oviparous fish contains thousands of ova, which it deposits in spawning time. Let us suppose, that the number of ova amount only to 2000, and that these produce as many fish, half males and half females; in the second year there would be more than 200000; in the third, more than 200000000; and in the eighth year, the number would exceed that expressed by 2 followed by twenty-four ciphers. As the earth contains scarcely so many cubic inches, the ocean, if it covered the whole globe, would not be sufficient to contain all these fish, the produce of one herring in eight years!

4th, Many vegetable productions, if all their seeds were put into the earth, would in a few years cover the whole surface of the globe. The hyosciamus, which of all the known plants produces, perhaps, the greatest number of seeds, would for this purpose require no more than four years. According to some experiments it has been found, that one stem of the hyosciamus produces sometimes more than 50000 seeds: now, if we admit the number to be only 10000, at the fourth crop it would amount to a 1 followed by sixteen ciphers. But as the whole surface of the earth contains no more than 5507634452576256 square feet, if we allow to each plant only one square foot, it will be seen, that the whole surface of the earth would not be sufficient for the plants produced from one hyosciamus at the end of the fourth year!

EXERCISES IN THE SINGLE AND COMPOUND RULE OF THREE, BOTH DIRECT AND INVERSE.

(26.) We shall confine ourselves to a small number of examples in this rule, which we briefly explained in the Introduction.

SINGLE RULE OF THREE DIRECT.

EXAMPLE I.—*If 40 pioneers can dig a trench 268 yards long, in a certain time, how many yards can 60 pioneers dig in the same time?*

$$40 : 60 :: 268 : x = 402. \quad (23.)$$

EXAMPLE II.—*If a ship with a fresh breeze sails 200 leagues in three days, how long time will she require to sail 2000 leagues, every other circumstance being the same?*

$$200 : 2000 :: 3 : x = 30.$$

EXAMPLE III.—*If 52 yards 2 feet and 5 inches of mason work cost L.168. 9s. 4d. what will be the expense of 77 yards 2 feet 8 inches of the like kind of work, at the same rates?*

To render the solution of this problem easier, the quantity of each piece of work must be reduced to inches, by multiplying the yards by 3 and 12; and, for the same reason, the price of the work must be reduced to pence. We shall then have the following proportion:—

Inches. Inches. Pence.

$$1901 : 2804 :: 40432 : x. \quad (23.)$$

SINGLE RULE OF THREE INVERSE.

EXAMPLE I.—*If 30 men can perform a certain piece of work in 25 days, how many men will be requisite to perform the same work in 10 days?*

It is here evident, that as the work is to be done in a shorter time, it will require more men. Consequently, the proportion must be expressed in this manner:

Days. Days. Men.

$$10 : 25 :: 30 : x = 75.$$

EXAMPLE II.—*A vessel has provisions for 15 days, but being obliged by certain circumstances to continue at sea for 20 days, to what quantity must the daily ration of each man be reduced, to make the provisions last during that time?*

If the quantity of provisions consumed daily be represented by unity, it is evident that the reduced quantity must be as much below 1 as 15 are less than 20. We shall therefore have

$$20 : 15 :: 1 : x = \tfrac{3}{4}.$$

COMPOUND RULE OF THREE.

EXAMPLE I.—*If 30 men perform 132 yards of work in 18 days, how much will 54 men perform in 28 days?*

Thirty men, working 18 days, will perform the same work as 18 times 30 men = 540, in one day; and, in like manner, 54 men, in 28 days, will perform the same work as 54 times 28 men = 1512, in one day. We have, therefore, the following proportion:

$$540 : 132 :: 1512 : x.$$

EXAMPLE II.—*If a man, walking 7 hours a-day, travels 230 leagues in 30 days, how many days would he require to perform a journey of 600 leagues, walking 10 hours a-day?*

This problem may be reduced to the single rule of three, if we consider, that to travel 30 days, employing 7 hours each day, is the same thing as to travel 30 times 7 hours, or 210 hours. The question, therefore, may be changed in the following manner: If 210 hours are required to travel 230 leagues, how many hours will be requisite to travel 600 leagues? When the number of hours which answer the question have been found, the required number of days may be found by dividing these hours by 10, as the traveller employs 10 hours each day. We must, therefore, find the fourth term of the proportion, the first three of which are as follows;

Leag.	Leag.	Hours.	Hours.
230 :	600 ::	210 :	x.

RULE OF FELLOWSHIP.

As this rule is merely an application of what has been said respecting the rule of three, we shall only give a few examples to illustrate the use of it.

EXAMPLE I.—*A privateer, belonging to three merchants, captured a prize worth L.80,000: What will each partner's share amount to, the first having advanced to purchase and fit out the vessel L. 2000, the second L. 6000, and the third L. 12,000?*

It is here evident, that each partner must have a share of the prize proportioned to the money he advanced.

We must, therefore, make this proportion: As the sum advanced by each partner, is to the whole money advanced, so is the share of each to the whole prize. Hence we shall have the following proportions, where the second and fourth terms, that is to say, the sum-total of the money advanced and the value of the prize, are common to all of them:

$$\left.\begin{matrix}2000\\6000\\12000\end{matrix}\right\} : 20000 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : 80000$$

EXAMPLE II.—*Three persons having entered into partnership, the first advanced L. 3000 for six months, the second L. 4000 for five months, and the third L. 8000 for nine months—at the end of that time they found that their gain amounted to L. 150,000: How much will each partner's share be worth?*

As this problem belongs to the compound rule of three, we shall take, as the first terms, the product of the money advanced by each partner, multiplied by the time it was employed; and for the second, the sum of these products, in the following manner:—

$$\left.\begin{matrix}18000\\20000\\72000\end{matrix}\right\} : 110000 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : 150000$$

It may be readily seen, that by means of the rule of three, any sum, such as the amount of a legacy, for example, may be easily divided among several persons, in such a manner, that the shares shall be in the ratio of certain determinate numbers, as 3, 4, 6. In this case, these numbers must be considered as three sums advanced by three partners, and their sum as the total of the money advanced: if we then call the legacy to be divided a, we shall have the following proportion:

$$\left.\begin{matrix}3\\4\\6\end{matrix}\right\} : 13 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : a.$$

RULE OF ALLIGATION.

Alligation is of two kinds. The first consists in finding the common price of several things, supposed to be mixed together; as if a goldsmith, for example, should make a composition of gold, silver, &c. and be desirous to know the value of an ounce of this mixture. The same rule is employed to determine the mean price of several liquors, or different kinds of merchandise, mixed together.

This rule is exceedingly easy; for nothing is necessary, in solving questions of this kind, but to divide the whole value of the articles by the quantity of each article employed for the mixture, and the quotient will be the answer.

EXAMPLE I.—*A wine merchant mixes together 200 bottles of Madeira at 5 shillings, 500 of Port at 3s., 800 of Malaga at 4s., and 300 of Tokay at 8s.: How much is a bottle of this mixture worth?*

$$\begin{aligned}200 \times 5 &= 1000\\500 \times 3 &= 1500\\800 \times 4 &= 3200\\300 \times 8 &= 2400\\\hline 1800 \qquad & \quad 8100 \text{ shillings.}\end{aligned}$$

Now, if 8100 shillings, or the whole value of the wine, be divided by 1800, the number of the bottles, the quotient will express the value of each bottle of the mixture. Consequently $\frac{8100}{1800}=\frac{81}{18}=4s.\ 6d.$

EXAMPLE II.—*A gentleman employed* 300 *workmen,* 50 *of which were paid at the rate of* 8*s. a-day,* 70 *at the rate of* 6*s., and* 180 *at the rate of* 4*s.* : *How much did each of them, taking one with another, cost him per day ?*

The sum-total, which is 1540 shillings, must be divided by 300, the number of the workmen, and the quotient will be what each of them, taken one with another, cost him per day.

$$\frac{1540}{300}=\frac{154}{30}=5s.\ \frac{2}{15}.$$

The object of the second kind of alligation is to determine in what proportion several things, of different values, ought to be mixed, in order to have an article of a certain mean price.

To obtain this result, the prices of the things to be mixed must be arranged, as seen in the following examples :—

EXAMPLE I.—*A grocer, who has tea at* 3*s.* 4*s.* 7*s. and* 9*s. per pound, is desirous of having a mixture which he can sell at* 5*s. per pound. In what proportions must he mix these four kinds of tea, so as to be able to sell the mixture at* 5*s.* ?

Arrange the prices of the things to be mixed as seen at A ; placing those which are greater than the mean price at the top, and those which are less at the bottom.

	3*s.*		2
	4*s.*		4
Fig. A.		5*s.*	
	7*s.*		2
	9*s.*		1
			9

Then compare in succession with the mean prices, the prices of all the things to be mixed, and place the differences as in the above example.

Thus, the difference between 3 and 5 is 2, which must be set down opposite to 7, and that between 4 and 5 is 1, which must be placed opposite to 9. Then proceed to the prices greater than that of the mean price, and compare them with that price in the following manner: The difference between 7 and 5 is 2, which place opposite to 3; and that between 9 and 5 is 4, which place opposite to 4.

The right hand column, the sum of which is 9, shews that to have 9 pounds of tea, at the mean price of 5*s.* the mixture must consist of 2 pounds at 3*s.*, 4 pounds at 4*s.*, 2 pounds at 7*s.*, and 1 pound at 9*s.*

It may here be readily seen, that 9 pounds of tea, at the mean price of 5*s.* will amount exactly to the value of the quantities mixed.

It may sometimes happen, that the figures, expressing the different values of the things to be mixed, will not be equal in number both below and above the mean price; on this account, if there are, for example, three figures above it, and only two below, the difference of the third figure at the top must be placed opposite to the second at the bottom, along with the difference of the second at the top; and the difference of the second figure at the bottom must be set down twice; that is to say, it must be placed opposite to the second and the third at the top. See fig. B.

	2*s.*		1
	3*s.*		2
	4*s.*		2
Fig. B.		5*s.*	
	6*s.*		3
	7*s.*		$2 + 1 = 3.$
			11

That is to say, to form 11 pounds of tea, of the mean price of 5*s.*, one pound at 2*s.* two pounds at 3*s.* two pounds at 4*s.* three pounds at 6*s.* and three at 7*s.* must be mixed together.

EXAMPLE II.—*A goldsmith has gold of 23 carats fine, and some of 13 carats, which he is desirous of mixing, so as to form gold of 18 carats: What quantity of each must he take?* See fig. C.

	13		5
Fig. C.		18	
	23		5
			10

It hence appears, that he must take an equal quantity of each.

By the same rule, we may find the quantity of alloy in any compound metal, for example bronze, which consists of copper and tin mixed together in a certain proportion. For this purpose, we must take three ingots of the same weight, one of bronze, another of copper, and a third of pure tin. These three bodies, when weighed in water, will each lose a different part of their weight: the ingot of tin will lose more, and that of copper less, than the ingot of bronze. Let us suppose, that the loss of the bronze is 3 ounces, that of copper $2\frac{1}{2}$, and that of the tin $3\frac{1}{4}$. If these three numbers be arranged, according to the above formula, we shall have,

$2\frac{1}{2}$		$\frac{1}{4}$
	3	
$3\frac{1}{4}$		$\frac{1}{2}$
		$\frac{3}{4}$

The sum of the two differences, $\frac{3}{4}$, shews, that in $\frac{3}{4}$ of bronze, there are one of copper and two of tin. This proportion being found, we may thence conclude, that a mass of bronze similar to the ingot, weighing 150 pounds, would contain 100 pounds of tin, and 50 of copper.

A CHRONOLOGICAL PROBLEM.

How many years, months, and days, elapsed between the Battle of Marignan, fought on the 3d of September 1515, and that of Fontenoy, fought on the 11th of May 1745?

The period from the Christian era to the 3d of September 1515, comprehends 1514 years 8 months and 3 days; and that from the same epoch to the 11th of May 1745, comprehends 1744 years 4 months and 11 days.

Consequently, if we subtract the former from the latter, the difference, 229 years 8 months and 8 days, will express the interval of time between the battle of Marignan and that of Fontenoy.

This method may be employed for every other problem of the like kind, and especially when it is necessary, in calculating interest, to know how many years, months, and days, have elapsed between certain dates.

THE RULE OF TARE.

By tare is commonly meant the weight of the cask, box, or bag, in which goods are contained, and which being subtracted, when known, from the gross weight, leaves the real weight of the goods, called the net weight. In general, an allowance is made for it, at the rate of so much per hundred weight; and the quantity to be deducted is found by the Rule of Three, as in the following example:

A merchant purchases a bale of cotton weighing 7 cwt. including the package, and is allowed at the rate of 16 per cwt. of tare: How much ought to be deducted on that account, from the gross weight of the bale of cotton?

As the merchant purchases the goods by the net weight, the seller must give him 16 pounds over and above each cwt.; that is to say, for each 112 pounds he must give him 128. We must, therefore, make the following proportion:

$$128 : 112 : : 700 : x. \quad (23.)$$

The fourth term will express the number of pounds for which the merchant ought to pay.

DISCOUNT.

A merchant purchases goods to the amount of L.1000, to be paid at the end of a year; but the vender offers to *abate* 10 *per cent for ready money: How much must the buyer pay down?*

It might here be supposed, that the abatement ought to be as many times L. 10 as 100 is contained in 1000; that is to say, that L. 100 ought to be deducted, so that the merchant would have to pay only L. 900.

But it is to be observed, that the vender ought to allow the purchaser only 10 per cent on what he will really receive; that is to say, that every L. 110 which the merchant has to pay, ought to be reduced to L. 100. We have, therefore, the following proportion:

$$110 : 100 :: 1000 : x.$$

This is the only true method of estimating discount; for if the vender received only L. 900 ready money, this sum at 10 per cent would produce, at the end of a year, no more than L. 990; consequently it would be much better for him to give a year's credit, and receive L. 1000.

OF COMBINATIONS AND PERMUTATIONS.

BEFORE we enter on this subject, it will be necessary to explain the method of constructing a kind of table, treated of by Pascal and others, called the arithmetical triangle; which is of great use to shorten calculations of this kind.

First form a band A B of ten equal squares, and below it another C D of the like kind, but shorter by one square on the left, so that it shall contain only nine squares; and continue in this manner, always making each successive band a square shorter. We shall thus have a series of squares, disposed in vertical and horizontal bands, and terminating at each extremity in a single square, so as to form a triangle, on which account it has been called the arithmetical triangle.

The numbers with which it is to be filled up, must be disposed in the following manner:

In each of the squares of the first band inscribe unity, as well as in each of those in the diagonal A E.

A	1	1	1	1	1	1	1	1	1	1	B
	C	1	2	3	4	5	6	7	8	9	D
			1	3	6	10	15	21	28	36	
				1	4	10	20	35	56	84	
					1	5	15	35	70	126	
						1	6	21	56	126	
							1	7	28	84	
								1	8	36	
									1	9	
										1	
										E	

Then add the number in the first square of the band C D, which is unity, to that in the square immediately above it, and write down the sum 2 in the following square. Add this number, in like manner, to that in the square above it, which will give 3, and write it down in the next square. By these means, we shall have the series of the natural numbers 1, 2, 3, 4, 5, &c. The same method must be followed to fill up the other horizontal bands; that is to say, each square ought always to contain the sum of the number in the preceding square of the same row, and that which is immediately above it in the preceding. Thus the number 15, which occupies the fifth square of the third band, is equal to the sum of 10, which stands in the preceding square, and of 5, which is in the square above it. The case is the same with 21, which is the sum of 15 and 6; with 35, in the fourth band, which is the sum of 15 and 20, &c.

The different series of numbers, contained in this triangle, have different properties; but we shall here speak only of those which relate to combinations and permutations, as the rest are of too abstract a nature to be employed in arithmetical recreations, the principal object of which is to afford easy and agreeable amusement.

There are two principal kinds of combination. The first is that where the different arrangements of several things are sought, without any regard to their change of place.

The second is that where regard is paid to the different changes of place. For example, the three quantities A, B, C, taken two and two, without regard to the different changes of place, are susceptible of only three combinations A B, A C, B C; but if we pay regard to the changes of place, they are susceptible of six combinations; for, besides the three former, we shall have B A, C A, C B.

In combinations, properly so called, no attention is paid to the order of the things. If four tickets, for example, marked A, B, C, D, were put into a hat, and any one should bet to draw out A and D, either by taking two at once, or one after the other, it would be of no importance whether A should be drawn first or last; the combinations A D and D A ought, therefore, to be here considered only as one.

But, if any one should bet to draw out A the first time, and D the second, the case would be very different, and it would then be necessary to attend to the order in which these four letters may be taken, and arranged together, two and two: it may be readily seen, that the different ways are A B, B A, A C, C A, A D, D A, B C, C B, B D, D B, C D, D C. In like manner, these four letters might be combined and arranged, three and three, twenty-four ways: as A B C, A C B, B A C, B C A, C A B, C B A, A D B, A B D, D B A, D A B, B A D, B D A, A C D, A D C, D A C, D C A, C A D, C D A, B C D, B D C, C B D, C D B, D B C, D C B. This is what is called permutation, or change of order.

PROBLEM I.—*Any number of things whatever being given, to determine in how many different ways they may be com-*

bined, two and two, three and three, &c. without regard to order.

This problem may be easily resolved by making use of the arithmetical triangle. Thus, for example, if there are eight things to be combined, three and three, we must take the ninth vertical band, or, in all cases, that band the order of which is expressed by a number exceeding by unity the number of the things to be combined; then the fourth horizontal band, or that the order of which is greater by unity than the number of the things to be taken together; and in the common square of both will be found the number of the combinations required; which, in the present example, will be 56.

But, as an arithmetical triangle may not always be at hand, or as the number of things to be combined may be too great to be found in such a table, the following simple method may be followed:

The number of things to be combined, and the manner in which they are to be taken, viz. two and two, or three and three, being given:

1st, Form two arithmetical progressions, one in which the terms go on decreasing by unity, beginning with the given number of things to be combined, and the other consisting of the series of the natural numbers 1, 2, 3, 4, &c.

2d, Then take from each as many terms as there are things to be arranged together, in the proposed combination.

3d, Multiply together the terms of the first progression, and do the same with those of the second.

4th, In the last place, divide the first product by the second, and the quotient will be the number of the combinations required.

Problem II.—*In how many ways can 90 things be combined, two and two?*

According to the above rule, we must multiply 90 by 89, and divide the product, 8010, by the product of 1 and 2, that is 2; the quotient 4005 will be the number of combinations resulting from 90 things, taken two and two.

Should it be required to determine, in how many ways the same things can be combined three and three, the problem may be answered with equal ease; for we have only to multiply together 90, 89, 88, and to divide the product, 704880, by that of the three numbers 1, 2, 3; the quotient 117480 will be the number required.

In like manner, it will be found, that 90 things may be combined, four and four, 2555190 ways; for if the product of 90, 89, 88, and 87, be divided by 24, the product of 1, 2, 3, 4, we shall have the above result.

Were it asked how many conjunctions the seven planets could form with each other, two and two, we might reply 21; for, according to the general rule, if we multiply 7 by 6, which will give 42, and divide that number by the product of 1 and 2, that is 2, the quotient will be 21.

If we wished to know the number of all the conjunctions possible of these seven planets, two and two, three and three, &c.; by finding separately the number of the conjunctions two and two, then those three and three, &c. and adding them together, it will seem that they amount to 120.

PROBLEM III.—*Any number of things being given, to find in how many ways they can be arranged.*

This problem may be easily solved by following the method of induction; for,

1st, One thing can be arranged only in one way; in this case, therefore, the number of arrangements is = 1.

2d, Two things may be arranged together two ways; for with the letters A and B we can form the arrangements A B, and B A; the number of arrangements, therefore, is equal to 2, or the product of 1 and 2.

3d, The arrangements of three things, A, B, C, are in number six; for A B can form with C, the third, three different ones B A C, B C A, C B A, and there can be no more. Hence it is evident, that the required number is equal to the preceding multiplied by 3, or to the product of 1, 2, 3.

4th, If we add a fourth thing, for instance D, it is evident that as each of the preceding arrangements may be combined with this fourth thing four ways, the above number 6 must be multiplied by 4, to obtain that of the arrangements resulting from four things; that is to say, the number will be 24, or the product of 1, 2, 3, 4.

It is needless to enlarge further on this subject; for it may be easily seen, that whatever be the number of the things given, the number of the arrangements they are susceptible of may be found, by multiplying together as many terms of the natural arithmetical progression as there are things proposed.

The following table will shew the immense number of permutations, or different arrangements, of which only 12 things are susceptible. We shall afterwards give the result of the permutations arising from the twenty-four letters of the alphabet.

Number of things.	Permutations.
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600

Let it be required to assign what space would contain all the permutations of the twenty-four letters of the alphabet, supposing each of them to occupy a square line.

If we first suppose that each letter occupies a square line, a square inch will contain 144 letters; if we then multiply 144 by itself, the product 20736 will be the number of letters which can be contained in a square foot; and if the last number be multiplied by 9, the product 186624 will be the number of letters which might be contained in a square yard.

Now, as a mile is equal to 1760 yards, a square mile will contain 3097600 square yards; and if we multiply this number by 9, or the square miles in a league, we shall have 27878400 for the number of square yards in the square league, and this number multiplied by 186624 will give 5202778521600 for the number of letters which could be contained in a square league.

If this last product be multiplied by 21951022, or the square leagues on the surface of the earth, allowing the diameter of it to be 7930 miles, we shall have 114206305788769075200 for the number of letters which, according to the above supposition, could be contained in this surface.

If we now employ the method already given to find the number of the permutations of the 24 letters, by successively multiplying all the terms of the arithmetical progression from 1 to 24, we shall have for the number of these permutations 620448401733239439360000, which is above five thousand times greater than the number of letters that could be contained on the surface of the earth; and, as each permutation consists of 24 letters, it thence follows, that to contain them, a space 120000 times greater would be necessary. In attempting to form an idea of this immense surface, the imagination is, as it were, lost; and it could hardly be believed that such an extent would be required, were it not fully demonstrated by calculation.

PROBLEM IV.—*A club of seven persons agreed to dine together every day successively, as long as they could sit down to table differently arranged. How many dinners would be necessary for that purpose?*

It may be easily found, by the rules already given, that the club must dine together 5040 times, before they would exhaust all the arrangements possible, which would require above 13 years.

If any word be proposed, such as AMOR, and it be required to know how many different words could be formed of these four letters, which will give all the possible anagrams of that word, we shall find, by multiplying together 1, 2, 3, and 4, that they are in number 24, as represented in the following table.

AMOR	MORA	ORAM	RAMO
AMRO	MOAR	ORMA	RAOM
AOMR	MROA	OARM	RMAO
AORM	MRAO	OAMR	RMOA
ARMO	MAOR	OMRA	ROAM
AROM	MARO	OMAR	ROMA

The number of all the anagrams possible to be formed of one word, may be found in the same manner; but it must be confessed, that, if there were a great many letters in the word, the arrangements thence resulting would be so numerous, as to require a long time to find them out.

APPLICATION OF THE DOCTRINE OF COMBINATIONS TO GAMES OF CHANCE AND TO PROBABILITIES.

THOUGH nothing, on the first view, seems more foreign to the province of mathematics, than games of chance, the powers of analysis have, as we may say, enchained this Proteus, and subjected it to calculation: it has found means to measure the different degrees of the probability of certain events, and this has given rise to a new branch of mathematics, the principles of which we shall here explain.

When an event can take place in several different ways, the probability of its happening in a certain determinate manner is greater, when, in the whole of the ways in which it can happen, the greater number of them determine it to happen in that manner. In a lottery, for example, every one knows that the probability of obtaining a prize is greater, according as the number of the prizes is greater, and as the whole number of the tickets is less. The probability of an event, therefore, is in the compound ratio of the number of cases in which it can happen taken directly, and of the total number of those in which it can be varied, taken inversely; consequently, it may be expressed by a fraction, having for its numerator the number of the favourable chances, and for its denominator the whole of the chances.

Thus, in a lottery, containing 1000 tickets, 25 of which only are prizes, the probability of obtaining a prize will be represented by $\frac{25}{1000}$, or $\frac{1}{40}$: if there were 50 prizes, the probability would be double; for in that case it would be equal to $\frac{1}{20}$; but if the number of tickets, instead of 1000, were 2000, the probability would be only one-half of the former, or $\frac{1}{80}$; and if the whole number of the tickets were infinitely great, the prizes remaining the same, it would be infinitely small, or 0; while, on the other hand, it would become certainty, and be expressed by unity, if the number of the prizes were equal to that of the tickets.

Another principle of this theory, the truth of which may be readily perceived, and which it is necessary here to explain, is as follows:

A person plays an equal game, when the money staked, or risked, is in the direct ratio of the probability of winning; for to play an equal game, is nothing else than to deposit a sum so proportioned to the probability of winning, that, after a great many throws, the player may find himself nearly at par; but for this purpose, the stakes must be proportioned to the probability which each of the players has in his favour. Let us suppose, for example, that A bets against B on a throw of the dice, and that there are two chances in favour of the former, and one for the latter: the game will be equal, if, after a great number of throws, they separate nearly without any loss. But, as there are two chances in favour of A, and only one for B, after 300 throws A will have won nearly 200, and B 100. A, therefore, ought to deposit 2, and B only 1; for by these means A in winning 200 throws, will get 200; and B in winning 100, will get 200 also. In such cases, therefore, it is said, that there is two to one in favour of A.

PROBLEM I.—*In tossing up, what probability is there of throwing a head several times successively, or a tail? or, in playing with several pieces, what probability is there that they will all come up heads at one throw?*

As this game is well known, it is needless here to give an explanation of it; we shall therefore proceed to analyze the problem.

1st, It is evident, that as there is no reason why a head should come up rather than a tail, or a tail than a head, the probability of one of them coming up is equal to $\frac{1}{2}$, or an equal bet may be taken on either side.

But if any one should bet to bring heads successively in two throws, to know what in this case ought to be staked on each side, we must observe, that all the combinations possible of head and tail, which can take place, in two successive throws with the same piece, are *head, head; head, tail; tail, head; tail, tail;* one of which only gives *head, head.* Here then there is only one case in four favourable to the person who bets to throw a head twice in succession; the probability, therefore, of this event will be only $\frac{1}{4}$; and he who bets that it will take place, ought to deposit only a crown, while his antagonist deposits three: for the latter has three chances of winning, whereas the former has only one. To play an equal game, the money deposited by each ought to be in this proportion.

It will be found, in like manner, that he who should bet to bring a *head*, for example, three times successively, would have in his favour only one of the eight combinations of *head* and *tail*, which might result from three successive throws of the same piece. The probability, therefore, of this event would be $\frac{1}{8}$, while that of his adversary would be $\frac{7}{8}$; and consequently, to play an equal game, he ought to bet only 1 to 7.

It is needless to go over all the other cases; for it may be readily seen, that the probability of throwing a head four times successively would be only $\frac{1}{16}$, and so on. We shall say nothing farther, therefore, on the different combinations which might result from head and tail; as in all such cases the following general rule may be employed.

When the probability of two or more individual events are known, the probability of their taking place all together may be found, by multiplying together the probabilities of those events considered individually.

Thus, the probability of throwing a head, considered individually, being expressed at each throw by $\frac{1}{2}$, that of throwing it twice successively, will be $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$: that of throwing it three times successively, will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$, and so on.

2d, The probability of throwing all *heads*, or all *tails*, with two, three, or four pieces, may be determined in the same manner. When two pieces are employed, there are four combinations of head and tail, only one of which is both *heads:* when three pieces are tossed up, at the same time, there are 8, one of which only is all *heads:* and so on. The probability, therefore, in each of these cases, is similar to those already examined.

It may be seen, indeed, without the help of analysis, that these two questions are absolutely the same, as may be proved in the following manner:—To toss up the two pieces A and B at the same time, or to throw up the one after the other, when A the first has had time to settle, is certainly the same thing. Let us suppose, then, that when A the first has settled, instead of tossing up B the second, A is taken from the ground in order to be tossed up a second time: this will certainly be the same thing as if the piece B had been employed; for, by the supposition, they are both equal and similar, at least in regard to the chance of a *head* or a *tail* coming uppermost; consequently, to toss up the two pieces A B at once, or to toss up twice successively the piece A, is the same thing.

3d, If it were asked, how much a person might bet to bring a *head* at least once in two throws, it may be found by the above method that the chance is 3 to 1. In two throws there are four combinations, three of which give at least one *head*, while there is only one which gives two *tails:* and hence it follows, that there are three combinations in favour of the person who bets to bring a *head* once in two throws, and only one against him.

Problem II.—*Any number of dice being given, to determine the probability of throwing with them an assigned number of points.*

We here suppose that the dice are of the usual kind, that is to say, having six faces marked with the numbers 1, 2, 3, 4, 5, 6, This being premised, we shall analyze some of the first cases of the problem, that we may proceed gradually to those which are more complex.

1*st*, *It is proposed to throw a determinate point, for example, 6, with one die.*

As the die has six faces, one of which only is marked six, and as any one of these may come up as readily as another, it is evident that there are 5 chances against the person who undertakes to throw 6 at one throw, and only one in his favour. To play an equal game, he ought, therefore, to bet no more than 1 to 5.

2d, *Let it be proposed to throw the same point 6, with two dice.*

To analyze this case, it must first be observed, that two dice give 36 different combinations; for each of the faces of the die A, for example, may combine with each of those of B, which will produce 36 combinations. We must next examine in how many ways the point 6 can be thrown with two dice. 1st, It will be found, that it can be thrown by 3 and 3; 2dly, By throwing 2 with the die A, and 4 with the die B, or 4 with A and 2 with B, which, as may be readily seen, forms two distinct cases; 3dly, By throwing 1 with A and 5 with B, or 1 with B and 5 with A, which likewise forms two cases. These are evidently all the ways that can be found: Consequently, there are 5 cases favourable in the 36; and therefore, the probability of throwing 6 with two dice, is $\frac{5}{36}$, and that of not throwing it $\frac{31}{36}$; hence it appears, that the money staked by the players ought to be in the ratio of these two fractions.

By analyzing the other cases it will be found, that of throwing 2 with two dice, there is 1 chance in 36; of throwing 3, there are 2; of throwing 4, there are 3; of throwing 5, there are 4; of throwing 6, there are 5; of throwing 7, there are 6; of throwing 8, there are 5; of throwing 9, there are 4; of throwing 10, there are 3; of throwing 11, there are 2; and of throwing 12, or sixes, there is 1.

If three dice were proposed, with which the least point that could be thrown is evidently 3, and the greatest 18, it will be found, by a similar analysis, that in 216 different throws possible, with three dice, there is one chance of throwing 3; three of throwing 4; six of throwing 5; and so on, as may be seen in the annexed table, the use of which is as follows:—

If it be required, for example, in how many different ways 13 can be thrown with three dice; look in the first vertical column, on the left, for the number 13, and at the top of the table for that indicating the number of the dice, and in the common square of both, opposite to 13, will be found 21, or the number of ways in which 13 can be thrown with 3 dice. It will be found, in like manner, that with 4 dice it may be thrown 140 ways; with 5 dice, 420 ways; and so on.

A TABLE of the different ways in which any point can be thrown with one, two, three, or more dice.

NUMBER OF POINTS.	NUMBER OF THE DICE. I.	II.	III.	IV.	V.	VI.
1	1					
2	1	1				
3	1	2	1			
4	1	3	3	1		
5	1	4	6	4	1	
6		5	10	10	5	1
7		6	15	20	15	6
8		5	21	35	35	21
9		4	25	56	70	56
10		3	27	80	126	126
11		2	27	104	205	252
12		1	25	125	305	456
13			21	140	420	756
14			15	146	540	1161
15			10	140	651	1666
16			6	125	735	2247
17			3	104	780	2856
18			1	80	780	3431
19				56	735	3906
20				35	651	4221
21				20	540	4332
22				10	420	4221
23				4	305	3906
24				1	205	3431
25					126	2856

When once it is known in how many ways any point can be thrown with a certain number of dice, it will be easy to determine the probability of throwing it. Nothing will be necessary but to form a fraction, having for its numerator the number of ways in which the point can be thrown, and for its denominator the number 6, raised to that power denoted by the number of dice; for example, the cube of 6, or 216, for 3 dice; the biquadrate, or 1296, for 4; and so on.

Thus, the probability of throwing 13 with 3 dice is $\frac{21}{216}$; that of throwing it with 4 is $\frac{140}{1296}$.

Various other questions of the like kind might be proposed, some of which we shall here analyze.

PROBLEM III.—*When two persons are playing, to determine the advantage or disadvantage on the side of the one who undertakes to throw a certain face, for example, that marked 6, in a certain number of throws.*

Let us first suppose that the person undertakes it at one throw. To determine the probability of his succeeding, we must consider, that he who holds the die has only one chance of winning, and that there are five of his losing; consequently, to undertake it at one throw, he ought to bet only 1 to 5. There is, therefore, great disadvantage in undertaking on an even bet to throw 6 at one throw.

To determine the probability of throwing at least one face marked 6, in two throws, with the same die, we must observe, as has been already said in regard to tossing up, that this is the same thing as to undertake to bring one face marked 6, by throwing two dice at the same time. In this case, he who holds the dice has only 11 chances, or combinations, in his favour; for he may bring 6 with the first die, and 1, 2, 3, 4, or 5, with the second; or 6 with the second die, and 1, 2, 3, 4, or 5, with the first; or 6 with each die. But there are 25 chances, or combinations, against his winning, as may be seen in the following table;

1 . . 1	2 . . 1	3 . . 1	4 . . 1	5 . . 1
1 . . 2	2 . . 2	3 . . 2	4 . . 2	5 . . 2
1 . . 3	2 . . 3	3 . . 3	4 . . 3	5 . . 3
1 . . 4	2 . . 4	3 . . 4	4 . . 4	5 . . 4
1 . . 5	2 . . 5	3 . . 5	4 . . 5	5 . . 5

Hence it may readily be concluded, that he who undertakes to throw a 6 with two dice, ought to bet only 11 to 25; and consequently, that there is a disadvantage in undertaking it on an even bet.

We must here observe, that 36, or the whole of the chances possible with one throw of two dice, is the square of the given number 6, the number of the faces of one die; and that 25, the number of the chances unfavourable to the person who undertakes to throw a determinate face, is the square of the number 6 diminished by unity, that is, of 5; for this reason, the number of the favourable chances in the present case, is the difference of the squares of 36 and 25, or of the square of the number of the faces of the die, and of that of the faces of the same die less 1.

To determine the probability of throwing a 6 in three throws of the same die; we must in like manner consider, that this is the same thing as to undertake to bring at least one 6 by throwing three dice; but of the 216 different combinations produced by 3 dice, there are 125 in which there is no 6, and 91 where there is at least one 6: consequently, he who bets on throwing one 6 either in three throws with one die, or in one throw with three dice, ought to bet no more than 91 to 125; and it would be disadvantageous to undertake it on an equal bet.

We must again observe, that 91 is the difference of the cube of the number of the faces of one die, viz. 216, and of 125, the cube of the same number diminished by unity, that is to say, of 5. Hence it may seem, that to determine, in general, the probability of throwing any assigned face in a certain number of throws, or in one throw with a certain number of dice, we must raise 6, the number of the faces of one die, to that power indicated by the number of throws given, or of the dice to be thrown at once, and that we must then raise to the same power 6 less unity, that is to say 5, and subtract it from the former: the remainder, and this power of 5, will be the respective number of chances for winning or losing.

For example, if a person should bet to bring at least one 3 with four dice; we must raise 6 to the biquadrate or fourth power, which is 1296, and subtract from it 625, which is the fourth power of 5: the remainder 671 will be the number of chances favourable for winning; and 625 will be those of losing: consequently, there will be an advantage in laying an even bet.

There will be still more advantage in undertaking, on an even bet, to throw a determinate point, for example 3, in five throws, or with five dice; for if we deduct the fifth power of 5, which is 3125, from 7776, the fifth power of 6, the remainder 4651 will be the number of the favourable chances, and 3125 that of the unfavourable.

Consequently, to play an equal game, he who bets ought to deposit 4652 to 3125, or about 3 to 2.

PROBLEM IV.—*In how many throws may a person, with an equal chance of winning, bet to bring a determinate doublet, for example, sixes, with two dice?*

We already know, that the probability of not throwing sixes with two dice is $\frac{35}{36}$; consequently, the probability of not throwing them in two throws, will be as the square of that fraction; in three throws, as the cube; and so on. But as the powers of any number ever so little greater than unity, go on always increasing, those of a number ever so little less go on always decreasing; consequently, the consecutive powers of $\frac{35}{36}$ will go on always decreasing. Let us conceive $\frac{35}{36}$ raised to such a power that it shall be equal to $\frac{1}{2}$; now it will be found that the twenty-fourth power of $\frac{35}{36}$ is a little greater than $\frac{1}{2}$; and that the twenty-fifth power of the same fraction is a little less than $\frac{1}{2}$: a person may then lay an even bet, with some advantage, that another will not bring sixes in 24 throws, but an even bet cannot be laid with advantage that sixes will not come up in 25 throws. Consequently, there is a disadvantage in laying an even bet to bring sixes in 24 throws; and, on the other hand, he who lays an even bet to throw sixes in 25 throws, does so with advantage.

PROBLEM V.—*What probability is there of throwing a determinate doublet, for example, two threes, at one throw, with two or more dice?*

To determine the probability in this case, we must consider, that in undertaking to throw two threes with 2 dice, there is only 1 favourable chance in the 36 given by 2 dice: whence it follows, that the person who undertakes it, ought to bet only 1 to 35. If 3 dice were proposed, we should find that the bet ought to be 16 to 216; for the number of chances or combinations possible with 3 dice is 216: but when it is required to throw two threes with 3 dice, it may be done 16 different ways; for of the 36 combinations of the dice A and B, all those in which there is only one 3, as 1 3, 3 1, &c. which are 10 in number, by combining with the face marked 3 of the die C, will consequently give two threes. Besides, the combination 3 3 of the dice A B, by combining with one of the six faces of the third C, will also give two threes; and hence there are 16 different ways of throwing two threes with 3 dice, which gives 16 favourable chances in 216. Consequently, the probability of throwing two threes with 3 dice, is $\frac{16}{216}$; and therefore the person who undertakes it, ought to bet no more than 16 to 216, or nearly 2 to 27.

If the probability of throwing two threes with 4 dice be required, we shall find that it is expressed by $\frac{171}{1296}$; for, of the 1296 combinations arising from the faces of 4 dice, there are 150 which give 2 threes, 20 which give 3 threes, and 1 which gives 4, making altogether 171 throws, in which there are either 2, 3, or 4 threes. Consequently, a person ought to bet no more than 19 to 144, or about 1 to $7\frac{1}{2}$, on throwing at least 2 threes with 4 dice.

In the last place, if the probability of throwing any doublet with ten dice, or more, at one throw, be required, it will be easy to determine it by the same method of calculation. For, in the case of an indeterminate doublet, it is evident that the probability is six times as great as when a particular doublet is assigned; and therefore nothing will be necessary but to multiply the above probabilities by 6. The probability therefore with 2 dice, is $\frac{6}{36}$ or $\frac{1}{6}$; with 3 dice, $\frac{96}{216}=\frac{4}{9}$; with 4 dice $\frac{576}{216}=\frac{8}{3}$; so that there is an advantage in laying an even bet, to bring at least one doublet with 4 dice.

Problem VI.—*Two persons deposit a certain sum of money, and agree, that he who first gets a certain number of games, for example 3, shall have the whole: one of them has got two games, and the other one: but being unwilling to continue their play, they resolve to divide the stake—in what manner must this be done?*

This problem is one of the first which engaged the attention of Pascal, when he began to study the calculation of probabilities. He proposed it to M. de Fermat, a celebrated geometrician of that period, who resolved it by a different method, viz. that of combinations. We shall here give both.

It is evident that each of the players, when he deposited his money, resigned all right to it; but, on the other hand, each had a right to that which chance might give him; consequently, when they give over playing, the stake ought to be divided according to the probability each had of winning.

Case I.—This proportion may be determined by the following mode of reasoning: Since the first player wants one game to be out, and the second two, it may be readily perceived, that if they continued their play, and if the second won one game, he would want, in the same manner as the first, one game to be out; and if both players were equally advanced, their hopes of gaining the whole would be equal: in this supposition, therefore, they would have an equal right to the stake, and consequently, each ought to have an equal share of it. It is evident, therefore, that if the first wins the game about to be played, the whole stake will belong to him; and if he loses it, he will be entitled only to one-half. As the one case is as probable as the other, the first has a right to the half of these sums taken together; but together they make $\frac{3}{2}$, the half of which is $\frac{3}{4}$. Such is the portion of the stake belonging to the first player, and, consequently, that belonging to the second is only $\frac{1}{4}$.

Case II.—The solution of the first case will enable us to resolve the second, in which we suppose, that the first player wants one game to be out, and the second three; for if the first should win one game, the whole of the stake would belong to him, and if he should lose one, so that the second should want only two games to be out, $\frac{3}{4}$ of the money would belong to the former, since they would then be in the situation alluded to in the first case. But as both these events are equally probable, the first ought to have the half of these two sums taken together, or the half of $\frac{7}{4}$, that is to say $\frac{7}{8}$: the remainder $\frac{1}{8}$, will be what ought to belong to the second.

Case III.—It will be found, by reasoning in the same manner, if we suppose two games wanting to the first player, and three to the second, that on ceasing to play they ought to divide the stake in such a manner, that the first may have $\frac{11}{16}$, and the second $\frac{5}{16}$.

Case IV.—If they had agreed to play four games, and if the first wanted only two games, while the second wanted four, the stake ought to be divided in such a manner, that the first might have $\frac{13}{16}$, and the second $\frac{3}{16}$.

We shall now explain the second method of resolving questions of this kind, which is that of combinations.

To resolve, for example, the fourth case, in which we suppose that the first player wants two games to be out, and the second four, so that both together want six games; if we subtract unity from that sum, we shall have 5, which indicates, that we must take these five similar letters *aaaaa*, favourable to the first player, and the five following, *bbbbb*, favourable to the second. These must be combined together, as seen in the following table, where, of 32 combinations, the first 26 towards the left, where *a* occurs at least twice, indicate the number of chances favourable to the first, and the 6 last towards the right, where *a* is found at most only once, indicate the number of chances favourable to the second.

a a a a a	*a a a b b*	*a a b b b*	*a b b b b*
a a a a b	*a a b b a*	*a b b b a*	*b b b b a*
a a a b a	*a b b a a*	*b b b a a*	*b a b b b*
a a b a a	*b b a a a*	*a b a b b*	*b b a b b*
a b a a a	*a a b a b*	*a b b a b*	*b b b a b*
b a a a a	*a b a a b*	*b b a a b*	*b b b b b*
	b a a a b	*b a a b b*	
	b a a b a	*b a b b a*	
	b a b a a	*b b a b a*	
	a b a b a	*b a b a b*	

Thus, the hope of the first player will be to that of the second, as 26 to 6, or as 13 to 3.

To resolve the case in which we suppose that one of the players has won three games, and the other none; as he will be the winner who soonest gets four games, he must take unity from 5, the number of games wanting to both, which will give 4, and then examine in how many ways the letters *a* and *b* can be combined, four and four. These ways are, in number, 16, viz.

a a a a	*a a b b*	*a b b b*	*b b b b*
a a a b	*a b a b*	*b a b b*	
a a b a	*b a a b*	*b b a b*	
a b a a	*a b b a*	*b b b a*	
b a a a	*b a b a*		
	b b a a		

But, of these 16 combinations, it is evident there are 15 in which *a* is found at least once; and hence it appears, that there are 15 combinations or chances favourable to the first player, and only one to the second; consequently they ought to share the stake in the ratio of 15 to 1; or the first ought to have $\frac{15}{16}$, and the second $\frac{1}{16}$.

PROBLEM VII.—*A mountebank at a country fair amused the populace with the following game: He had 6 dice, each of which was marked only on one face, the first with* 1, *the second with* 2, *and so on to the sixth, which was marked* 6 *: the person who played, gave him a certain sum of money, and he engaged to return it a hundred fold, if, in throwing these six dice, the six marked faces should come up only once in* 20 *throws.*

Though the proposal of the mountebank does not, on the first view, appear very disadvantageous to those who intrusted him with their money, it is certain that there were a great many chances against them.

It may indeed be seen, that of the 46656 combinations of the faces of 6 dice, there is only one which gives the 6 marked faces uppermost; the probability therefore of throwing them, at one throw, is expressed by $\frac{1}{46656}$: and, as the adventurer was allowed 20 throws, the probability of his succeeding was only $\frac{20}{46656}$, which is nearly equal to $\frac{1}{2332}$. To play an equal game, therefore, the mountebank should have engaged to return 2332 times the money deposited.

Problem VIII.—*The same mountebank offered a new chance to the person who had lost, on the following conditions: to deposit a sum equal to the former, and to receive both the stakes in case he should bring all the blank faces, in 3 successive throws.*

Those unacquainted with the method to be pursued in order to resolve such problems, are liable to reason in an erroneous manner respecting dice of this kind; for, observing that there are five times as many blank as marked faces, they thence conclude that it is 5 to 1 that the person who throws them will not bring any point. They are, however, mistaken, as the probability, on the contrary, is 2 to 1 that they will not come up all blank.

If we take only one die, it is 5 to 1 that the person who holds it will throw a blank; but if we add a second die, it may be readily seen, that the marked face of the first may combine with each of the blank faces of the second, and the marked face of the second with each of the blank faces of the first; and, in the last place, the marked face of the one with the marked face of the other: consequently, of the 36 combinations of the faces of these two dice, there are 11, in which there is at least one marked face. But, as we have already observed, this number 11 is the difference of the square of 6, the number of the faces of one die, and of the square of the same number diminished by unity, that is to say, of 5.

If a third die be added, we shall find, by the like analysis, that, of the 216 combinations of three dice, there are 91 in which there is at least one marked face; and 91 is the difference of the cube of 6 or 216, and the cube of 5 or 125: the result will be the same in regard to the more complex cases; and hence we may conclude, that of the 46656 combinations of the faces of the 6 dice in question, there will be 31031 in which there is at least one marked face, and 15625 in which all the faces are blank; consequently, the chance is 2 to 1 that some point, at least, will be thrown; whereas, by the above reasoning, it would appear that 5 to 1 might be betted on the contrary being the case.

Problem IX.—*In how many throws, with six dice, marked on all their faces, may a person engage, for an even bet, to throw* 1, 2, 3, 4, 5, 6?

We have just seen that there are 46655 chances to 1 that a person will not throw these 6 points with dice marked only on one of their faces; but the case is very different with 6 dice marked on all their faces; and to prove it, we need only to observe, that the point 1, for example, may be thrown by each of the dice, as well as the 2, 3, &c. which renders the probability of these six points, 1, 2, 3, &c. coming up, much greater.

But to analyze the problem more accurately, we shall observe, that there are two ways of throwing 1, 2, with 2 dice; viz. 1 with the die A, and 2 with the die B; or 1 with the die B, and 2 with A. If it were proposed to throw 1, 2, 3, with 3 dice; of the whole of the combinations of the faces of 3 dice, there are 6 which give the points 1, 2, 3; for 1 may be thrown with the die A, 2 with B, and 3 with C; or 1 with A, 2 with C, and 3 with B; or 1 with B, 2 with A, and 3 with C; or 1 with B, 2 with C, and 3 with A; or 1 with C, 2 with A, and 3 with B; or 1 with C, 2 with B, and 3 with A.

It hence appears, that to find the number of ways in which 1, 2, 3, can be thrown with 3 dice, 1, 2, 3 must be multiplied together. In like manner, to find the number of ways in which 1, 2, 3, 4 can be thrown with 4 dice, we must multiply together 1, 2, 3, 4, which will give 24; and, in the last place, to find in how many ways 1, 2, 3, 4, 5, 6, can be thrown with 6 dice, we must multiply together these six numbers, the product of which will be 720.

If the number 46656, which is the combinations of the faces of 6 dice, be divided by 720, we shall have $64\frac{4}{5}$ for the chances to 1, that these points will not come up at one throw; and, consequently, a person may undertake for an even bet to bring them in 64 throws.

In the last place, as the dice may be thrown 130 times, and more, in a quarter of an hour, a person may, with advantage, bet more than 2 to 1, that they will come up in the course of that time.

He who engages for an even bet to throw these points in a quarter of an hour, undertakes what is highly advantageous to himself, and equally disadvantageous to his adversary.

ARITHMETICAL AMUSEMENTS IN DIVINATION AND COMBINATION.

PROBLEM I.—*To tell the number thought of by a person.*

Desire the person, who has thought of a number, to triple it, and to take the exact half of that triple if it be even, or the greater half if it be odd. Then desire him to triple that half, and ask him how many times it contains 9; for the number thought, if even, will contain twice as many units as it does nines, and one more if it be odd.

Thus, if 5 has been the number thought of, its triple will be 15, which cannot be divided by 2 without a remainder. The greater half of 15 is 8; and if this half be multiplied by 3, we shall have 24, which contains 9 twice; the number thought of will therefore be 4 plus 1, that is to say 5.

II. Bid the person multiply the number thought of by itself; then desire him to add unity to the number thought of, and to multiply it also by itself; in the last place, ask him to tell the difference of these two products, which will certainly be an odd number, and the least half of it will be the number required.

Let the number thought of, for example, be 10, which multiplied by itself gives 100; in the next place, 10 increased by 1 is 11, which multiplied by itself makes 121; and the difference of these two squares is 21, the least half of which being 10, is the number thought of.

This operation might be varied by desiring the person to multiply the second number by itself, after it has been diminished by unity. In this case, the number thought of will be equal to the greater half of the difference of the two squares.

Thus, in the preceding example, the square of the number thought of is 100, and that of the same number less unity is 81; the difference of these is 19, the greater half of which, or 10, is the number thought of.

III. Bid the person take 1 from the number thought of, and then double the remainder; desire him to take 1 from this double, and to add to it the number thought of: in the last place, ask him the number arising from this addition, and if you add 3 to it, the third of the sum will be the number thought of.

The application of this rule is so easy, that it is needless to illustrate it by an example.

IV. Desire the person to add 1 to the triple of the number thought of, and to multiply the sum by 3; then bid him add to this product the number thought of, and the result will be a sum, from which if 3 be subtracted, the remainder will be decuple of the number required. If 3 therefore be taken from the last sum, and if the cipher on the right be cut off from the remainder, the other figure will indicate the number sought.

Let the number thought of be 6, the triple of which is 18; and if unity be added it makes 19; the triple of this last number is 57, and if 6 be added it makes 63, from which if 3 be subtracted the remainder will be 60: now if the cipher on the right be cut off, the remaining figure 6 will be the number required.

V.—*Another method of telling the number any one has thought of.*

These operations, by which a person seems to guess the thoughts of another, may be introduced very opportunely in company, when any one asserts that all amusing tricks are performed by slight of hand. The following method may be found in Ozanam, but we have here made some additions to it. 1st, Desire any person to think of a number, but, that we may not speak in too abstract a manner, it will be best to desire him to think of a certain number of guineas. 2d, Tell the person that some one of the company lends him a similar sum, and request him to add them together, that the amount may be known. It will here be proper to name the person who lends him a number of guineas equal to the number thought of, and to beg the one who makes the calculation to do it with great care, as he may readily fall into an error, especially the first time. 3d, Then say to the person, I do not lend you, but give you 10, add them to the former sum. 4th, Continue in this manner:—Give the half to the poor, and retain in your memory the other half. 5th, Then add:—Return to the gentleman, or lady, what you borrowed, and remember that the sum lent you was exactly equal to the number you thought of. 6th, Ask the person if he knows exactly what remains? he will answer Yes: you must then say, And I know also the number that remains, it is equal to what I am going to conceal in my hand. 7th, Put into one of your hands 5 pieces of money, and desire the person to tell how many you have got. He will reply 5: upon which open your hand, and shew him the 5 pieces. You may then say—I well knew that your result was 5; but if you had thought of a very large number, for example, two or three millions, the result would have been much greater, and I should not have been able to put into my hand a number of pieces equal to the remainder. The person then supposing that the result of the calculation must be different, according to the difference of the number thought of, will imagine that it is necessary to know the last number, in order to guess the result: but this idea is false; for, in the case which we have here supposed, whatever be the number thought of, the remainder must always be 5. The reason of this is as follows:—The sum, the half of which is given to the poor, is nothing else than twice the number thought of plus 10; and when the poor have received their part, there remains only the number thought of plus 5; but the number thought of is cut off when the sum borrowed is returned, and consequently there remains only 5.

It may be thence seen that the result may be easily known, since it will be the half of the number given in the third part of the operation; for example, whatever be the number thought of, the remainder will be 36, or 25, according as 72 or 50 have been given.

Remark 1*st*, If this trick be performed several times successively, the number given in the third part of the operation must be always different, for if the result were several times the same, the deception might be discovered.

2*d*, When the five first parts of the calculation for obtaining a result are finished, it will be best not to name it at first, but to continue the operation to render it more complex, by saying, for example, Double the remainder, deduct two, add three, take the fourth part, &c. and the different steps of the calculation may be kept in mind in order to know how much the first result has been increased or diminished.—This irregular process never fails to confound those who attempt to follow it.

PROBLEM II.—*To tell two or more numbers which a person has thought of.*

I. When each of the numbers thought of does not exceed 9, they may be easily found in the following manner:—

Having made the person add 1 to the double of the first number thought of, desire him to multiply the whole by 5, and to add to the product the second number. If there be a third, make him double this first sum and add 1 to it; after which desire him to multiply the new sum by 5, and to add to it the third number. If there be a fourth you must proceed in the same manner, desiring him to double the preceding sum; to add to it unity; to multiply by 5, and then to add the fourth number; and so on.

Then ask the number arising from the addition of the last number thought of, and if there were two numbers, subtract 5 from it; if there were three, 55; if there were four, 555; and so on; for the remainder will be composed of figures of which the first on the left will be the first number thought of, the next the second, and so on.

Suppose the number thought of to be 3, 4, 6: by adding 1 to 6, the double of the first, we shall have 7, which being multiplied by 5, will give 35; if 4, the second number thought of, be then added, we shall have 39, which doubled gives 78;

and if we add 1, and multiply 79, the sum, by 5, the result will be 395. In the last place, if we add 6, the number thought of, the sum will be 401; and if 55 be deducted from it, we shall have for remainder 346, the figures of which, 3, 4, 6, indicate in order the three numbers thought of.

II. If one or more of the numbers thought of be greater than 9, we must distinguish two cases: that in which the number of the numbers thought of is odd, and that in which it is even.

In the first case, ask the sum of the first and the second; of the second and third; the third and the fourth; and so on to the last; and then the sum of the first and the last. Having written down all these sums in order, add together all those the places of which are odd, as the first, the third, the fifth, &c.; make another sum of all those the places of which are even, as the second, the fourth, the sixth, &c.; subtract this sum from the former, and the remainder will be the double of the first number. Let us suppose, for example, that the 5 following numbers are thought of, viz. 3, 7, 13, 17, 20, which, when added two and two as above, give 10, 20, 30 37, 23: the sum of the first, third, and fifth is 63, and that of the second and fourth is 57; if 57 be subtracted from 63, the remainder 6 will be the double of the first number 3. Now if 3 be taken from 10, the first of the sums, the remainder 7 will be the second number; and by proceeding in the same manner, we may find all the rest.

In the second case, that is to say, if the number of the numbers thought of be even, you must ask and write down as above the sum of the first and the second; that of the second and third; and so on, as before; but instead of the sum of the first and the last, you must take that of the second and last; then add together those which stand in the even places, and form them into a new sum apart; add also those in the odd places, the first excepted, and subtract this sum from the former: the remainder will be the double of the second number; and if the second number, thus found, be subtracted from the sum of the first and second, you will have the first number; if it be taken from that of the second and third, it will give the third; and so of the rest. Let the numbers thought of be, for example, 3, 7, 13, 17: the sums formed as above, are 10, 20, 30, 24; the sum of the second and fourth is 44, from which if 30, the third, be subtracted, the remainder will be 14, the double of 7 the second number. The first, therefore, is 3, the third 13, and the fourth 17.

PROBLEM III.—*A person having in one hand an even number of shillings, and in the other an odd, to tell in which hand he has the even number.*

Desire the person to multiply the number in the right hand by any even number whatever, such as 2; and that in the left by an odd number, as 3; then bid him add together the two products, and if the whole sum be odd, the even number of shillings will be in the right hand, and the odd number in the left; if the sum be even, the contrary will be the case.

Let us suppose, for example, that the person has 8 shillings in his right hand, and 7 in his left; 8 multiplied by 2 gives 16, and 7 multiplied by 3 gives 21: the sum of which, 37, is an odd number.

If the number in the right hand were 9, and that in the left 8, we should have $9 \times 2 = 18$, and $8 \times 3 = 24$; the sum of which two products is 42, an even number.

PROBLEM IV.—*A person having in one hand a piece of gold, and in the other a piece of silver; to tell in which hand he has the gold, and in which the silver.*

For this purpose, some value represented by an even number, such as 8, must be assigned to the gold, and a value represented by an odd number, such as 3, must be assigned to the silver; after which, you may proceed exactly in the same manner as in the preceding example.

1st, To conceal the artifice better, it will be sufficient to ask whether the sum of the two products can be halved without a remainder; for in that case the total will be even, and in the contrary case odd.

2d, It may be readily seen, that the pieces, instead of being in the two hands of the same person, may be supposed to be in the hands of two persons, one of whom has the even number, or piece of gold, and the other the odd number, or piece of silver. The same operations may then be performed in regard to these two persons, as are performed in regard to the two hands of the same person, calling the one privately the right, and the other the left.

PROBLEM V.—*The game of the ring.*

This game is nothing else than an application of one of the methods employed to tell several numbers thought of, and ought to be performed in a company not exceeding 9, in order that it may be less complex. Desire any one of the company to take a ring, and to put it on any joint of whatever finger he may think proper. The question then is, to tell what person has the ring, and on what hand, what finger, and what joint.

For this purpose, you must call the first person 1, the second 2, the third 3, and so on. You must also denote the 10 fingers of the two hands, by the following numbers of the natural progression, 1, 2, 3, 4, 5, &c. beginning at the thumb of the right, and ending at that of the left, that by this order the number of the finger may at the same time indicate the hand. In the last place, the joints must be denoted by 1, 2, 3, beginning at the points of the fingers.

To render the solution of this problem more explicit, let us suppose that the fourth person in the company has the ring on the sixth finger, that is to say, on the little finger of the left hand, and on the second joint of that finger.

Desire some one to double the number expressing the person, which in this case will give 8; bid him add 5 to this double, and multiply the sum by 5, which will make 65; then tell him to add to this product the number denoting the finger, that is to say 6, by which means you will have 71; and, in the last place, desire him to multiply the last number by 10, and to add to the product the number of the joint 2: The last result will be 712; if from this number you deduct 250, the remainder will be 462; the first figure of which, on the left, will denote the person; the next, the finger, and consequently the hand; and the last, the joint.

It must here be observed, that when the last result contains a cipher, which would have happened in the present example had the number of the finger been 10, you must privately subtract from the figure preceding the cipher, and assign the value of ten to the cipher itself.

The same formula, as may be readily conceived, will answer for all cases whatever.

PROBLEM VI.—*To guess the number of spots on any card which a person has drawn from a whole pack.*

Take a whole pack, consisting of 52 cards, and desire some person in company to draw out any card, at pleasure, without shewing it. Having assigned to the different cards their usual value, according to their spots, call the knave 11, the queen 12, and the king 13. Then add the spots of the first card to those of the second; the last sum to the third; and so on, always rejecting 13, and keeping the remainder to add to the following card. It may be readily seen that it is needless to reckon the kings, which are counted 13. If any spots remain at the last card, you must subtract them from 13, and the remainder will indicate the card that has been drawn: if 12 remains, it has been an ace; but if nothing remains, it has been a king.

Demonstration.—Since a complete pack contains 13 cards of each suit, the values of which are 1, 2, 3, &c. as far as 13, the sum of all the spots of each of the different suits will be 7 times 13 (21), which is a multiple of 13; consequently the quadruple is also a multiple of 13: if we add the spots of all the cards, always rejecting 13, the remainder at last must be 0. Hence it is evident, that if a card, the spots of which are less than 13, be drawn, the difference between its spots and 13 will be what is wanting to complete the number. If, at the end, then, instead of attaining to 13, we attain only to 10, for example, it is plain, that the card wanting is a 3; and if we attain exactly to 13, the card missing must be equivalent to 13; that is, it must be a king.

PROBLEM VII.—*A person having a certain number of counters in each hand, to find how many he has altogether.*

Desire the person to convey 4, for example, from one hand to the other; and then ask him how many times the less number is contained in the greater? Let us suppose that he says the one is the triple of the other; in this case multiply 4, the number of counters conveyed from one hand into the other, by 3, and add the same number, which will make 16. In the last place, from the same number 3 subtract unity, and if you divide 16 by 2, the remainder, the quotient 8 will be the number contained in each hand; and consequently the whole number is 16.

Let us now suppose, that when 4 counters are conveyed from one hand to the other, the less number is contained in the greater $2\frac{1}{3}$ times: in this case, we must, as before, multiply 4 by $2\frac{1}{3}$, which will give $9\frac{1}{3}$; to which if 4 be added, we shall have $13\frac{1}{3}$, or $\frac{40}{3}$. Then if unity be taken from $2\frac{1}{3}$, the remainder will be $1\frac{1}{3}$ or $\frac{4}{3}$; by which if $\frac{40}{3}$ be divided, the quotient 10 will be the number of counters in each hand, as may be easily proved on trial.

Problem VIII.—*Several cards being given, to tell which of them a person has thought of.*

Desire the person to remember the card, and its place in the pack, counting from the bottom. Then take the cards, and in a dexterous manner, so as not to be perceived, convey a certain number of them from the top to the bottom; and subtract them in your mind from the pack, with the number of which you are acquainted. If the pack, for example, consists of 52 cards, and you have conveyed 8 to the bottom, tell the person that the card he has thought of will be the forty-fourth, reckoning from the card the place of which he is going to name. Thus, if he says it is the ninth, you go on counting 9, 10, 11, &c. and the card he thought of will be exactly the forty-fourth, as you announced.

Problem IX.—*Having spread out on the table 20 cards, arranged two and two, and desired one or more persons to think of two, provided they lie close to each other, to tell which cards they have thought of.*

You must retain in your memory the four following words, with the arrangement of the letters which compose them.

m	*i*	*s*	*a*	*i*
t	*a*	*t*	*l*	*o*
n	*e*	*m*	*o*	*n*
v	*c*	*s*	*u*	*l*

Collect all the cards into the left hand, two by two, as they lay on the table, and then place them, one by one, in the same order as the preceding letters, taking care to place the two first as the two *m*, the two next as the two *i*, the two following as the two *s*, and so on.

Ask each person in which horizontal row his two cards are. If he says they are both in the same row, for example the third, they will be pointed out by the letters *n* and *n*, contained in that row; if they are in two different rows, as the first and last, the letters *s* and *s* will indicate the place which they occupy.

PROBLEM X.—*To make all the cards of the same kind to be found together, however often the pack may have been cut.*

Have in readiness a pack, all the cards of which are arranged in successive order; that is to say, if it consist of 52 cards, every 13 must be regularly arranged, without a duplicate of any one of them. After they have been cut as many times as a person may choose, form them into 13 heaps of 4 cards each, with the coloured faces downwards. When this is done, the 4 kings, the 4 queens, the 4 knaves, and so on, must necessarily be together.

PROBLEM XI.—*The four indivisible kings.*

Take four kings, and place between the third and fourth any two common cards whatever, which must be neatly concealed; then shew the four kings, and place the six cards at the bottom of the pack; take one of the kings, and lay it on the top, and put one of the common cards into the pack nearly about the middle; do the same with the other, and then show that there is still one king at the bottom: desire any one to cut the pack, and as three of the kings were left at the bottom, the four will therefore be found together in the middle of the pack.

Problem XII.—*Two heaps of cards being displayed on a table, to write on a piece of paper that heap which a person will choose.*

Place in a heap 2 or 3 sevens; and in another 7 cards. Write on a bit of paper the word *seven*, and invert it, that what you have written may be concealed: then desire any one to choose, and when that is done, turn up the heap chosen, and prove the truth of your prediction by shewing what you wrote; but you must take care to shew only the heap which has been chosen.

Problem XIII.—*Several cards being presented in succession to several persons, that they may each choose one at pleasure: to guess that which each has thought of.*

Shew as many cards to each person as there are persons to choose; that is to say, 3 to each, if there are 3 persons. When the first has thought of one, lay aside the three cards in which he has made his choice. Present the same number to the second person, to think of one, and lay aside the three cards in the like manner. Having done the same in regard to the third person, arrange all these cards in three rows, with their faces turned downwards, and then put them together in order. If you take the 3 first, and present them successively to the different persons, and do the same thing with the others, you may easily guess the cards, by observing, that the card thought of by each person will have the same place among the cards as the person has in regard to the other two; that is to say, the card thought of by the first person, will be first of that packet in which he discovered it; that thought of by the second, will be the second in the packet where he recognized it; and that of the third, will be the last and in the last packet.

The operation is exactly the same when the number of persons is greater. If, instead of 3, there are 4 or 5 persons, four or five cards must be presented to each.

Problem XIV.—*Three cards being presented to three persons, to guess that which each has chosen.*

As it is necessary that the cards presented should be distinguished, we shall call the first A, the second B, and the third C. Let the persons, whom we shall distinguish by first, second, and third, choose privately whichever of the cards they think proper, and when they have made their choice, which is susceptible of six varieties, give the first person 12 counters, the second 24, and the third 36: then desire the first person to add together the half of the counters of the person who has chosen the card A; the third of those of the person who has chosen B; and the fourth part of those of the person who has chosen C; and ask the sum, which must be either 23 or 24, 25 or 27, 28 or 29, as in the following table:

First.	Second.	Third.	Sums.
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

This table shews, that if the sum is 25, for example, the first person must have chosen the card B, the second the card A, and the third the card C; and that, if it be 28, the first person must have chosen the card B, the second the card C, and the third the card A; and so of the rest.

Problem XV.—*To tell the number of spots on all the bottom cards of several heaps, arranged on a table.*

Arrange each heap of cards in such a manner that the spots on the bottom one, added to the cards above it, may always amount to 12; continue to make as many heaps as possible, in the manner above prescribed, and place the remaining cards on one side. Then separate in your mind four heaps, and multiply the heaps which remain, after these are deducted, by 13; this product, added to the number of cards, will be that of the spots required. We shall give the solution of this problem by an analysis in another place.

Problem XVI.—*To name all the cards of a pack.*

Have a complete pack of 52 cards, and arrange them according to the order of the following words, which you must retain in your memory.

Unus	*quinque*	*novem*	*famulus*	*sex*	*quatuor*	*duo*
Ace	five	nine	knave	six	four	two
Rex	*septem*	*octo*	*fœmina*		*trina*	*decem*
King	seven	eight	queen		three	ten

Besides this first order, you must arrange them also according to the order of the colours, spades, hearts, clubs, and diamonds; so that the 52 cards may be disposed as follows:

ORDER OF THE CARDS.

1	Ace of spades	27	Ace of clubs
2	Five of hearts	28	Five of diamonds
3	Nine of clubs	29	Nine of spades
4	Knave of diamonds	30	Knave of hearts
5	Six of spades	31	Six of clubs
6	Four of hearts	32	Four of diamonds
7	Two of clubs	33	Two of spades
8	King of diamonds	34	King of hearts
9	Seven of spades	35	Seven of clubs
10	Eight of hearts	36	Eight of diamonds
11	Queen of clubs	37	Queen of spades
12	Three of diamonds	38	Three of hearts
13	Ten of spades	39	Ten of clubs
14	Ace of hearts	40	Ace of diamonds
15	Five of clubs	41	Five of spades
16	Nine of diamonds	42	Nine of hearts
17	Knave of spades	43	Knave of clubs
18	Six of hearts	44	Six of diamonds
19	Four of clubs	45	Four of spades
20	Two of diamonds	46	Two of hearts
21	King of spades	47	King of clubs
22	Seven of hearts	48	Seven of diamonds
23	Eight of clubs	49	Eight of spades
24	Queen of diamonds	50	Queen of hearts
25	Three of spades	51	Three of clubs
26	Ten of hearts	52	Ten of diamonds

This order is of such a nature, that, by knowing any one of the 52 cards, that which follows it may be also known.

Thus, for example, if it were required to know what card follows the king of spades, it will be sufficient to recollect that *septem*, in the two Latin lines above given, which follows that of *rex*, denotes that it is a seven; and as the colour which follows the spades is hearts, it is the seven of hearts, and so of the rest.

Every thing being thus arranged, having retained in your memory the above words, and the order of the colours, desire any person to cut the pack as many times as he chooses; for it will be easy to name all the cards in order, provided you have found means, by some dexterous manœuvre, to observe that one which is at the top of the pack.

The same arrangement of the cards may be employed for various amusements.

1st, To make a person believe that you can distinguish the cards by their smell.

The pack being disposed in the above order, present it to any one, that he may choose a card at pleasure; open the pack at the place where it has been drawn out, and dexterously observe that which precedes it, by seeming to smell the place from which it was taken. It will then be very easy to name it, as it can be only that which follows in the order already indicated.

2d, A pack of cards being divided into two parts, to discover whether the number in each be odd or even.

First, find out whether the last card in the pack be black or red; then, on the pack being cut into two parts, if the card found at the bottom of the upper division is of the same colour as that at the bottom of the pack, the two parts which have been separated, contain each an even number; on the other hand, if it be of a different colour, they contain each an odd number.

3d, To tell the number of spots on several cards which any person has chosen.

Having presented the pack, that the person may choose several succeeding cards at pleasure, privately observe the card which is above those he has chosen, and how many he has drawn from the pack; it will then be easy to count how many spots they ought to contain.

For example, if the observed card be a nine, and four cards have been drawn, it may readily be seen that those drawn must be a knave, equivalent to 10 spots; a six, a four, and a two. You may then announce, that the cards in the person's hand contain 22 spots.

PROBLEM XVII.—*Having desired a person to draw four cards from a pack, and to think of one of them, to tell the one he has thought of.*

Suffer the person to draw four cards from the pack at pleasure, and desire him to think of one of them; then take these four cards back, and place two of them at the top and two at the bottom of the pack, in a dexterous manner, so as not to be perceived: under the two last, place any four cards whatever; then display the lower part of the pack on the table, shewing only 8 or 10 cards, and ask the person whether the one he thought of be among them. If he says No, you may be sure that it is one of the two which you put at the top of the pack; in that case you must transfer them to the bottom, and then, shewing the bottom of the pack, say, Is not this your card? If he replies No, turn aside that card with your third finger, which you must have previously moistened, and desire him to draw out his card himself from the bottom of the pack.

If the person should say, that the card he thought of is among the first shewn to him, dexterously remove the four cards put at the bottom of the pack, in order that the two, one of which is the card he thought of, may be the lowermost of the pack, and you may then either shew him his card, or make him draw it out himself, as above explained.

PROBLEM XVIII.—*Three things being privately distributed to three persons, to guess that which each has got.*

Let the three things be a ring, a shilling, and a glove. Call the ring A, the shilling E, and the glove I; and in your own mind distinguish the persons, by calling them first, second, and third. Then take 24 counters, and give one of them to the first person, two to the second, and three to the third. Place the remaining 18 on the table, and then retire, that the three persons may distribute among themselves the things proposed, without your observing them. When the distribution has been made, desire the person who has the ring to take from the 18 remaining counters, as many as he has already; the one who has the shilling to take twice as many as he has already; and the person who has the glove to take four times as many. By these different combinations, the counters left can be only 1, 2, 3, 5, 6, or 7. When this is done, you may return, and by the number of counters left, you can discover what thing each has got, by employing the following words.

1	2	3	5	6	7
Par fer	*César*	*jadis*	*devint*	*si grand*	*prince.*

To make use of these words, you must recollect what has been already said, viz. that the number of the counters which remain, can be only 1, 2, 3, 5, 6, or 7, and never 4: you must observe also, that each syllable contains one of the vowels which we have made to represent the three things proposed, and that the above line must be considered as consisting only of six words: the first syllable of each word must also be supposed to represent the first person, and the second syllable the second person. This being comprehended, if there remains only one counter you must employ the first word, or rather the two first syllables *par fer*, the first of which, that containing A, shews that the first person has the ring represented by A; and the second syllable, that containing E, shews that the second person has the shilling, represented by E; from which you may easily conclude, that the third person has the glove. If two counters remain, you must take the second word *César*, the first syllable of which, containing E, will shew that the first person has the shilling, represented by E; and the second syllable, containing A, will indicate that the second person has the ring, represented by A; you may then easily conclude that the third person has the glove.

PROBLEM XIX.—*To tell, by inspecting a watch, at what hour a person has resolved to rise next morning.*

1*st*, When the person has thought of an hour, bid him touch some other hour on the dial-plate, and then desire him to add 12 to it privately in his own mind, which will form a certain number.

2*d*, Then desire him to proceed backwards, and to count the above number, beginning with the hour which he thought of.

Let the hour thought of, for example, be 8, and that touched be 3; as 12 added to 3 makes 15, desire the person to count that number, in a retrograde order from the hour touched, beginning with 8, the hour thought of; counting 8 on the hour 3, 9 on 2, 10 on 1, and so on, by which means 15 will fall upon the hour of 8.

The person will be surprised to find that he has fallen on the hour he thought of.

PROBLEM XX.—*Two persons agree to take alternately numbers less than a given number, for example* 11, *and to add them together till one of them has reached a certain sum, such as* 100*: By what means can one of them infallibly attain to that number before the other?*

The whole artifice of this problem consists in immediately making choice of the numbers 1, 12, 23, 34, and so on, or of a series which continually increases by 11, up to 100.

Let us suppose, that the first person, who knows the game, makes choice of 1; it is evident that his adversary, as he must count less than 11, can at most reach 11, by adding 10 to it. The first will then take 1, which will make 12; and whatever number the second may add, the first will certainly win, provided he continually adds the number which forms the complement of that of his adversary to 11; that is to say, if the latter takes 8, he must take 3; if 9, he must take 2, and so on. By following this method, he will infallibly attain to 89; and it will then be impossible for the second to prevent him from getting first to 100; for whatever number the second takes, he can attain only to 99; after which the first may say, And 1 makes 100. If the second takes 1, after 89, it would make 90; and his adversary would finish by saying, And 10 make 100.

It is evident, that when two persons are equally well acquainted with the game, he who begins must necessarily win.

PROBLEM XXI.—*Sixteen counters being disposed in two rows, to find that which a person has thought of.*

The counters being arranged as follow, desire the person to think of one, and to observe well in which row it is:

A	B	C	D	E	F	H	I
0	0	0	0	0	0	0	*
0	0	0	0	*	0	0	0
0	0	*	0	0	0	0	0
0	0	0	0	0	0	0	0
*	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Let us suppose that the counter thought of is in the row A: take up the whole row in the order in which it now stands, and dispose it in two rows C and D, in such a manner, that the first counter of the row A may be the first of the row C; the second of the row A, the first of the row D; and so on, transferring the 16 counters from A and B, to C and D. This being done, again ask in which of the vertical rows the counter thought of stands. We shall suppose it to be in C; remove that row as well as D, observing the same method as before; and continue in this manner until the counter thought of becomes the first of the row I. If you then ask in which row it is, it may be immediately known, because after the last operation it will be the first in the row said to contain it; and as each row has a distinguishing character or sign, you may cause them all to be mixed with each other, and still be able to discover it by the sign you have remarked.

Instead of 16 counters, 16 cards may be employed. After you have discovered the one thought of, you may cause them to be mixed, which will conceal the artifice.

If a greater number of counters or cards be employed, disposed in two vertical rows, the counter or card thought of will not be at the top of the row after the last transposition: if there are 33 counters or cards, 4 transpositions will be necessary; if 64, there must be 5; and so on.

PROBLEM XXII.—*A certain number of cards being shown to a person, to guess that which he has thought of.*

To perform this trick, the number of the cards must be divisible by 3; and to do it with more convenience, the number must be odd.

The first condition, at least, being supposed, the cards must be disposed in three heaps, with their faces turned upwards. Having then asked the person in which heap is the card thought of, place the heaps one above the other, in such a manner that the one containing the card thought of, may be in the middle. Arrange the cards again in three heaps, and having asked in which of them is the card thought of, repeat the operation as before. Arrange them a third time in three heaps, and having once more asked the same question, form them all into one heap, that containing the card thought of being in the middle. The card thought of must then necessarily be the middle one; that is to say, if 15 cards have been employed, it will be the eighth from the top; if 21, the eleventh; if 27, the fourteenth; and so on. When the number of the cards is 24, it will be the twelfth, &c.

PROBLEM XXIII.—*To arrange 30 criminals in such a manner, that by counting them in succession, always beginning again at the first, and rejecting every ninth person, 15 of them may be saved.*

Arrange the criminals according to the order of the vowels, in the following Latin verse,

4 521 3 1 12 231 221
Populeam virgam mater regina ferebat.

Because *o* is the fourth in the order of the vowels, you must begin by four of those whom you wish to save; next to these place five of those whom you wish to punish; and so on alternately, according to the figures which stand over the vowels of the above verse.

In a company consisting of several persons, the following game may be introduced by way of amusement.

We shall suppose that there are 13 ladies in the company; in that case, provide 12 nosegays, and in order to mortify one of them, without shewing any appearance of partiality, announce that you mean to let chance decide which of them is to go without one. For this purpose, make the 13 ladies stand up in a ring, allowing them to place themselves as they please; and distribute to them the 12 nosegays, counting them from 1 to 9, and making the ninth retire from the ring and carry with her a nosegay. It will be found, that the eleventh, reckoning from the one by whom you began, will remain the last; and consequently will have no share in the distribution.

The following table will shew the person, before her whom you wish to exclude, with whom you must begin to count 9, supposing always that the number of the nosegays is less by 1 than that of the persons.

For 13 persons, the	11th before.
12	2d.
11	5th.
10	7th.
9	8th.
8	8th.
7	7th.
6	5th.
5	3d.
4	3d.
3	2d.
2	1st.

PROBLEM XXIV.—*A man has a wolf, a goat, and a cabbage, to carry over a river, but, as he is obliged to transport them one by one, in what manner is this to be done, that the wolf may not be left with the goat, nor the goat with the cabbage?*

He must first carry over the goat, and then return for the wolf; when he carries over the wolf, he must take back with him the goat, which he must leave, in order to carry over the cabbage; he may then return, and carry over the goat. By these means the wolf will never be left with the goat, nor the goat with the cabbage, but when the boatman is present.

PROBLEM XXV.—*In what manner can counters be disposed in the eight external cells of a square, so that there may always be 9 in each row, and yet the whole number shall vary from 20 to 32?*

This problem may be proposed in the following manner:—A wine merchant caused 32 casks of choice wine to be deposited in his cellar, giving orders to his clerk to arrange them in the annexed figure, so that each external row should contain 9.

1st Order.

1	7	1
7		7
1	7	1

The clerk, however, took away 12 of them, at three different times; that is, 4 each time; yet when the merchant went into the cellar, after each theft had been committed, the clerk always made him count 9 in each row. How was this possible?

This problem may be easily solved by inspecting the following figures:

2d Order.

2	5	2
5		5
2	5	2

3d Order.

3	3	3
3		3
3	3	3

4th Order.

4	1	4
1		1
4	1	4

PROBLEM XXVI.—*To distribute among 3 persons 21 casks of wine, 7 of them full, 7 of them empty, and 7 of them half full, so that each of them shall have the same quantity of wine, and the same number of casks.*

This problem admits of two solutions, which may be clearly comprehended by means of the two following tables:

	Persons.	Full Casks.	Empty.	Half-full.
I.	1st	2	2	3
	2d	2	2	3
	3d	3	3	1

	Persons.	Full Casks.	Empty.	Half-full.
II.	1st	3	3	1
	2d	3	3	1
	3d	1	1	5

Problem XXVII.—*A schoolmaster, to amuse his scholars, shewed them a number, which he said was the sum of 6 rows, each consisting of 4 figures: he then desired them to write down 3 rows of figures, to which he would add 3 more, and assured them that the sum of the whole should be equal to the number he shewed them.*

To solve this problem, multiply in your own mind 9999 by 3; and the product, 29997, will be the number which the schoolmaster shewed to his scholars.

Rows of the scholars,	7285
	5829
	3456
Rows of the master,	2714
	4170
	6543
Total,	29997

It may be here seen, that each figure set down by the master is the complement to 9 of that set down by the scholars; and consequently the sum, though written down beforehand, must be exact.

Problem XXVIII.—*Having desired any person to multiply, for example, one of the three following numbers, by any figure at pleasure, and to tell you the product, after suppressing one figure of it, and even changing the order of the rest, to guess the figure that has been suppressed.*

Let the three given numbers be

364851
234765
823644

If we suppose the person to multiply the third number by 6, the product of which will be 4941864, whatever figure be effaced, it may be easily discovered by that wanting to complete the product, as the sum of its figures must necessarily be a multiple of 9. If the 6, for example, be suppressed, the sum will not be a multiple of 9; for it amounts only to 30: as 6 therefore is wanting to 30 to make it a multiple of 9, you may boldly assert that 6 has been suppressed.

As the sum of the figures would still be a multiple of 9 if a cipher were suppressed, and as it would consequently have no need of being complete, you must make it a condition of the problem that the person shall suppress only one significant or effective figure; and if you find that the sum has no need of being completed, you may conclude that the figure suppressed has been a 9.

A mountebank, to give the greater air of the marvellous to this sport, pretended to discover by the smell what figure had been suppressed; but it may easily be supposed, that while he pretended to smell the figures, he privately added them together, so as to discover their sum.

There is another method of guessing the suppressed figure, even when the person himself has been allowed to write down the sum to be multiplied; but in this case you must stipulate to have permission to add any one figure you choose: you must observe what figure is wanting to complete the sum, and set down that figure; if nothing is wanting, you may add 0, or 9.

PROBLEM XXIX.—*A person having made choice of two numbers, and multiplied them together; to tell the product provided you know only the last figure of it.*

Have in readiness a small bag with two divisions, and put into one of them 12 square bits of card, each inscribed with the number 73; and into the second 9 other pieces, inscribed with the terms of the arithmetical progression, 3, 6, 9, 12, 15, 18, 21, 24, 27.

Present that aperture of the bag which contains the numbers 73, and desire the person to draw out one; then dexterously change the side of the bag, and having desired another person to draw any number from the second division, bid him multiply the number he has taken by that drawn out by the first person: the product will necessarily be one of the nine numbers 219, 438, 657, 876, 1095, 1314, 1533, 1752, 1971. You may then easily tell the product of the multiplication, if you know only the last figure of it.

It must here be observed, that this recreation requires a good memory; as it will be necessary to know by heart the above nine products. The following, founded on the same principle, is much easier.

PROBLEM XXX.—*A person having chosen two numbers, and divided the greater by the less, to tell the quotient; that is to say, how many times the less is contained in the greater.*

Put into the first division of the bag the nine numbers, 219, 438, o57, 876, 1095, 1314, 1533, 1752, 1971; and into the second, the cards inscribed with the number 73. Desire the person to draw a number from each division, and to divide the one by the other; then ask him to tell you the last figure of the greater of the numbers, as it will enable you to discover which of the nine numbers of the above arithmetical progression is the quotient: thus, if it be a 9, the number 3 is the quotient; if it be an 8, the quotient is the number 6; and so on. For the quotient 3, 6, 9, 12, 15, 18, 21, 24, and 27, will be in the ratio of the figures 1, 2, 3, 4, 5, 6, 7, 8, and 9, with which the greater number must necessarily terminate.

POLITICAL ARITHMETIC.

SINCE politicians have acquired juster ideas respecting what constitutes the real strength of states, various researches have been made in regard to the number of the inhabitants in different countries, in order to ascertain their population. Besides, as almost all governments have been under the necessity of making loans, for the most part on annuities, they have naturally been induced to examine according to what progression mankind die, that the interest of these loans may be proportioned to the probability of the annuities becoming extinct. These calculations have been distinguished by the name of *Political Arithmetic*; and as it exhibits several curious facts, whether considered in a political or philosophical point of view, we have thought it our duty to give it a place here, to amuse and instruct our readers.

SECTION I.

Of the proportion between the males and the females.

Many people imagine that the number of the females born exceeds that of the males; but it has long since been proved, that the contrary is the case. More boys than girls are born every year; and since the year 1631, a small interval excepted, we have a register of births in regard to sex; and it has never been observed, that the number of the females born ever even equalled that of the males. It is found, by taking a mean or average term in a great number of years, that the number of the males born is to that of the females as 18 to 17. This proportion is nearly that which prevails throughout all France; but, to whatever reason owing, it seems at Paris to be as 27 to 26.

This kind of phenomenon is observed, not only in England and France, but in every other country. We may be convinced of the truth of it by inspecting the calendars, which, at the commencement of every year, give a table of the births that have taken place in most of the capital cities of Europe; it will be there seen, that the number of the males born always exceeds that of the females; and, consequently, it may be considered as a general law of nature.

We may here observe a striking instance of the wisdom of Providence, which has thus provided for the preservation of the human race. Men, in consequence of the active life for which they are naturally destined, by their strength and their courage, are exposed to more dangers than the female sex; war, long sea voyages, occupations laborious or prejudicial to health, and dissipation, carry off great numbers of the males; and it thence results, that if the number born of the latter did not exceed that of the females, the males would rapidly decrease, and soon become extinct.

SECTION II.

Of the mortality of the human race, according to the different ages.

In this respect, there is apparently a considerable difference tween large towns and the country; but this arises from e women in towns rarely suckling their own children; and, nsequently, the greater part of their children being put out nurse in the country, as it is in the period of childhood at the greatest mortality prevails, it is most apparent in the untry. To make an exact calculation, it ought to be unded on the deaths which happen in the towns, as well as the country; and this M. Dupré de St Maur endeavoured do, by comparing the registers of three parishes in Paris, d twelve in the country.

According to the observations of this author, in 23994 aths, 6454 of them were those of children not a year old. ter carrying his researches on this subject as far as possi-, he concludes, that of 24000 children born, the numbers o attain to different ages, are as follow:

Ages.	Number.
2 years	17540
3	15162
4	14177
5	13477
6	12968
7	12562
8	12255
9	12015
10	11861
15	11405
20	10909
25	10259
30	9544
35	8770
40	7929
45	7008
50	6197
55	5375
60	4564

Ages.	Number.
65	3450
70	2544
75	1507
80	807
85	291
90	103
91	71
92	63
93	47
94	40
95	33
96	23
97	18
98	16
99	8
100	6 or 7.

Such, then, is the condition of the human species, that, of 24000 children born, scarcely one-half of them attain to the age of 9; and that two-thirds are in their grave before the age of 40. About a sixth only remain at the expiration of 62 years; a tenth after 70 years; a hundredth part after 86; about a thousandth part attain to the age of 96; and six or seven individuals to that of 100.

By means of this table we may ascertain, pretty nearly, what probability there is of a new-born child attaining to a certain age; for this probability must be to the contrary probability, as the number of those who attain to that age is to the number of those who die before it.

For example, as 4564 stands opposite to 60, it indicates, that as of 24000 children born, there remain no more than the above number of individuals at the end of 60 years, 19436 must have died; the probability, therefore, of a child attaining to the age of 60, is to the probability of its not attaining to it, as 4564 is to 19436. In this case, the proportion of those living to those dead, is nearly as 1 to 4; from which we may conclude, that the chance is 4 to 1 that a new-born child will not attain to the age of 60.

If the probability of a person, of any determinate age, living to another age, be required; for example, that of a hild of eight years of age attaining to the age of 60; we ust compare the number of those who attain to the age of 0, with that of those who attain to the age of 8; and the ifference, 7691, will give the number of those who die etween these two periods. We shall then have this anagy:—as 4564 is to 7691, so is the probability that a child f 8 years will attain to the age of 60, to the probability of s not attaining to it. If 7691 be divided by 4564, it will e found that the former contains the latter nearly twice; nd we may therefore say, that the chance is almost 2 to 1, at a child of 8 years of age will not live to that of 60.

SECTION III.

Of the number of men of different ages in a given number.

It may be deduced from the preceding observations, that hen the inhabitants of a country amount to a million, the umber of those of the different ages will be as follows:

Between	0 and	1 year complete	38740
	1	5	119460
	5	10	99230
	10	15	94530
	15	20	88673
	20	25	82380
	25	30	77650
	30	35	71665
	35	40	64205
	40	45	57230
	45	50	50605
	50	55	43940
	55	60	37110
	60	65	28690
	65	70	21305
	70	75	13195
	75	80	7065
	80	85	2880
	85	90	1025
	90	95	335
	95	100	82
Above 100 years			3 or 4.

Thus, in a country peopled with a million of inhabitants, there are about 536350 between the age of 15 and 60; and, as nearly one-half of them are men, consequently, this number of inhabitants could, on an emergency, furnish 250,000 men capable of bearing arms, even if an allowance be made for the sick, lame, &c. who may be supposed to be among that number.

SECTION IV.

Of the proportion of the births and deaths to the whole number of the inhabitants of a country—The consequences thence deduced.

As it would be difficult to number the inhabitants of a country, and much more so to repeat the enumeration as often as it might be necessary to ascertain the population, means have been devised for accomplishing the same object, by determining the proportion which the births and deaths bear to the whole number of the inhabitants; for, as registers of births and deaths are regularly kept in all the civilized countries of Europe, we may judge, by comparing them, whether the population has increased or decreased; and, in the latter case, can examine the causes which have produced the diminution.

The proportion of the births to the whole population in three generalities of France, which differ from each other as much by the nature as the form of the soil, give the mean ratio of 1 to $25\frac{1}{2}$, without including the large towns; so that in this country we may reckon 51 inhabitants for two births.

But, as in towns of any magnitude, there are several classes of citizens who spend their lives in celibacy, and who contribute either nothing or very little to the population, it is evident that the proportion between the births and the effective inhabitants must be more considerable. It has been ascertained, by various comparisons, that the proportion nearest the truth, is that of 1 to 28; and it is this ratio which ought to be employed, in order to deduce from the births, in a large city, the number of the inhabitants.

But there is reason to believe, that in regard to cities of the first class, or capitals, such as London, Paris, Amsterdam, &c. which are frequented by multitudes of strangers, invited thither either by pleasure or business, and where great luxury prevails, which increases the number of those who live in voluntary celibacy, the above proportion must be raised, and carried at least to that of 1 to 30 or 31.

SECTION V.

Of some other proportions, in regard to the inhabitants of a country.

We may deduce, by approximation, from the observations of various authors in England, France, Holland, and Germany;—

1st, That the number of the inhabitants of a country is to that of the families as 1000 to $222\frac{1}{2}$; so that 2000 inhabitants give, in general, 445 families.

2d, That the number of the male children exceeds that of the female; and that this excess continues for more than 14 years, according to the proportion of nearly 30 to 29. After 14 years, however, the number of the females exceeds that of the males, in the proportion of about 19 to 18, on account of the considerable decrease of the males by war, navigation, laborious occupations, and intemperance.

3d, That the number of the marriages is annually to that of the inhabitants as 1 to 112.

4th, That the proportion of married men or widowers, to the number of wives or widows, is nearly as 125 to 140; and the whole number of this class is to the whole of the inhabitants as 53 to 126.

5th, That the number of widowers is to that of widows nearly as 1 to 3. This at least is the proportion deduced from the enumeration of the people made in Holland and in England. And it ought not to appear astonishing, if it be considered, that most men marry at a later period of life than the women, and that their laborious occupations, the maritime and land wars in which they are engaged, and the diversity of the climates which they frequent for the sake of commerce, must increase the number of the widows in the bills of mortality.

6th, That, admitting the above proportion of widowers and widows, it follows, that among 651 inhabitants, there are 118 married couples, from 7 to 3 widowers, from 21 to 22 widows; and the rest are composed of children, persons in a state of celibacy, domestics, and passengers.

7th, That 1870 married couples produce annually 357 children; for a town having 10000 inhabitants would contain that number of married couples, and give annually 357 births; from which it is concluded, that 5 married couples, of all ages, give annually, one with another, one birth.

8th, That the number of servants is to the whole number of inhabitants, nearly as 136 to 1535; which is a little more than the eleventh part. The number of male domestics is nearly equal to that of the female; being in the proportion of 67 to 69; but it is probable, that in large cities, where great luxury prevails, the proportion must be different.

The above observations will enable us to solve the following problem, and may serve to facilitate the solution of others relating to the same subject.

PROBLEM I.—*The age of a man being given, suppose that of 30 years, what probability is there that he will be living at the end of a determinate number of years, for example 15?*

To resolve this problem, seek in the table of the second section for the given age of the person, viz. 30, and observe the number opposite to it, which is 9544; then find in the same table the number opposite to 45, which is 7008, and make the latter number the numerator of a fraction, having for its denominator the former number. This fraction $\frac{7008}{9544}$ will express the probability of a person of 30 attaining to the age of 45.

The demonstration of this rule will be evident to those who understand the theory of probabilities.

PROBLEM II.—*A young man, aged 20, borrows L. 1000, to be paid with the interest when he attains to the age of 25: but in case he dies before that period, the debt to be cancelled: What sum ought the lender to receive at the proposed term of payment?*

It is here evident, that if it were certain that the young man would live to complete his twenty-fifth year, the sum to be paid would be the capital increased with 5 years' interest, which we shall suppose to be at the rate of 5 per cent, or L. 1250. But on account of the risk which the lender runs, by the chance of the borrower dying before the time of payment, the sum ought to be increased in the inverse ratio of the probability of his being alive. But this probability is expressed by the fraction $\frac{10259}{10909}$; and therefore the above sum must be multiplied by this fraction inverted, or by $\frac{10909}{10259}$, which will give nearly L. 1329, that is to say, about L. 79 more for the risk of losing the money, which certainly cannot be accounted usurious.

PROBLEM III.—*A state, or an individual, having occasion to borrow a sum of money on an annuity, what interest ought to be given for the different ages, the legal interest being 5 per cent?*

The vulgar, who are accustomed to burdensome loans, entertain no doubt, that an annuity, at the rate of 10 per cent for the age of fifty, is a good bargain; and that this method of borrowing is advantageous to the state: but they are egregiously mistaken; for it appears by the tables of Parcieux, calculated from the foregoing data, that 10 per cent ought not to be given before the age of 56. According to the same table, no more than $6\frac{1}{5}$ per cent ought to be given for the age of 20; $6\frac{1}{2}$ for the age of 25; $6\frac{4}{5}$ for that of 30; $7\frac{2}{3}$ for 40; $8\frac{4}{5}$ for 50; 10 at 56; $11\frac{1}{10}$ at 60; $16\frac{2}{5}$ at 70; $27\frac{2}{5}$ at 80; and $39\frac{1}{10}$ at 85.

It is therefore a great mistake to imagine, that on account of the great number of persons who sink money in these loans, on annuities made by governments, the latter are soon freed from paying a part of the annuities by the death of a part of the annuitants. The slow increase of annuities in tontines is a sufficient proof of the falsity of this idea; besides, the greatness of the number of the persons is precisely the cause, why the extinction of the annuities takes place more in conformity to the laws of probability already explained. A lucky chance, at the end of a few years, may free a person from the payment of an annuity, established on the life of a man 30 years of age; but if this annuity were shared out on 300 different lives, the ages being nearly the same, it is certain that he would not be liberated from it before nearly 65 years; and after 32 or 33, nearly one-half of the annuitants would be living. This Parcieux has clearly shewn, by examining the lists of the tontines.

MAGIC SQUARES.

A MAGIC square is a series of figures arranged in the cells of a square, in such a manner, that the figures in each band, whether vertical, horizontal, or diagonal, form exactly the same sum. They are divided into two kinds—odd and even.

These squares have been called *magic*, because the ancients ascribed to them great virtues, and because this arrangement of numbers formed the basis and principle of several of their talismans.

One square, containing unity, was, according to them, the symbol of the Deity, on account of his unity and immutability; for they observed, that this square, by its nature, was single and immutable; the product of unity by itself being always unity.

A square containing four divisions or cells, was the symbol of imperfect matter, on account of the impossibility of arranging figures in it so as to form a magic square.

The square, with 9 divisions, was consecrated to Saturn; that with 16 to Jupiter; that with 25 to Mars; that with 36 to the Sun; that with 49 to Venus; that with 64 to Mercury; and that with 81 to the Moon.

Those who can find any relation between the arrangement of numbers and the planets, must be indeed not a little visionary; but such was the spirit of the mysterious philosophy of Iamblichus and Porphyry, and of all their disciples, vho were slaves to the most stupid superstition, and to all he absurdities of judicial astrology.

We shall here confine ourselves to the mechanical method f forming a magical square, either even or odd.

Method of constructing an odd Square.

1st, Place unity below the middle cell.

2d, Place the following number in the cells which descend liagonally from left to right.

3d, When you come to the last diagonal cell, go up to he highest cell of the next following band.

4th, When the diagonal cell is filled up, carry the next gure to the most distant cell on the left of the lower band.

5th, In following the diagonal, if you meet with a cell lready filled up, pass over that cell, and place the figure in le diagonal from right to left. See the following figures, ne of which represents a square of 9 divisions, and the other ne of 25.

4	9	2
3	5	7
8	1	6

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Method of constructing an even Square.

We shall apply this method to a square of 16 cells, which ; filled up in the following manner:

1st, Place 1 in the cell A (fig. M.) of the vertical band n the left; then pass the two next, and place 4 in the upper ell of the perpendicular band on the right.

2d, Omit 5, and place 6, 7, and the other figures, as seen 1 fig. M.

The remaining 8 divisions, which are left vacant, must be filled up after the manner of fig. N. Reckon 1 in the cell B without writing it down, and place 2 and 3 in the two next cells; then omit 4, and set down 5 in the first cell of the next band; omit 6 and 7, and write down 8, and so on. If you then fill up each of these squares from the other, you will have a square of 16 divisions.—See the figures.

A Geometrical Square.

FIG. M.

A	1			4
		6	7	
		10	11	
	13			16

FIG. N.

	15	14		
12			9	
8			5	
	3	2		B

To arrange in a square, consisting of 9 cells, the 9 terms of a geometrical progression, in such a manner, that the product arising from the continued multiplication of the numbers in each band shall be always the same, and equal to the cube of the middle term.

Let the terms of the progression be 1 : 2 : 4 : 8 : 16 : 32 : 64 : 128 : 256. If you arrange these 9 terms in a square of 9 cells, in the manner as you did the 9 terms of the arithmetical progression of the natural numbers, 1, 2, 3, &c. you will find that the product of them, in every direction, amounts to 4096; which is exactly the cube of the middle term 16, as may be seen in the annexed figure.

8	256	2
4	16	64
128	1	32

To make the knight pass over all the squares of the chess-board, one after the other, without passing twice over the same.

As the reader may perhaps be unacquainted with the movement of the knight in the game of chess, we shall here describe it. If the knight be placed in the square A, he cannot be moved into any of the squares immediately around him, as those marked 1, 2, 3, 4, 5, 6, 7, 8; nor into the squares 9, 10, 11, 12, which are directly above or below, or on one side; nor into the squares 13, 14, 15, 16, which are in the diagonals, but only into one of those which, in the figure, are left vacant.—See fig. B.

FIG. B.

13		10		14
	1	2	3	
9	8	A	4	11
	7	6	5	
16		12		15

Several celebrated men, who amused themselves with this problem, have given solutions of it; but the following is the simplest of them all, and the easiest to be remembered.

34	49	22	11	36	39	24	1
21	10	35	50	23	12	37	40
48	33	64	57	38	25	2	13
9	20	51	54	63	60	41	26
32	47	58	61	56	53	14	3
19	8	55	52	59	62	27	42
46	31	6	17	44	29	4	15
7	18	45	30	5	16	43	28

The method consists in filling up, as much as possible, the exterior bands, which form, as it were, a border, without entering the third, until there are no other means of passing from the square at which you have arrived to one of the two first; a rule to which the knight is necessarily subjected, in the most evident manner, from his first step to the fiftieth. When he arrives at the 50th, there is no other choice than 51 or 63; but the 51st square being nearer the border, ought to be preferred; and then his progress must necessarily be through 52, 53, 54, 55, 56, 57, 58, 59, 60, 61. When he arrives at the last, it is a matter of indifference whether he be made to pass through the three remaining squares, by directing his progress upwards or downwards; for in either case he will arrive at the last.

APPLICATION OF ANALYSIS

TO

THE SOLUTION OF VARIOUS PROBLEMS.

As the object of this work is to unite instruction with amusement, we shall confine ourselves to such problems as are sufficiently easy to be solved by the application of those rules which we have explained in the Introduction. (14.)

Problem I.—*A lady lamenting that her age was triple that of her daughter; the latter consoled her by observing, that in 15 years it would be only double: What was the age of each?*

Put a to denote the 15 years, and let x represent the age of the daughter; then by the conditions of the problem, the ages of the daughter and mother, which at present are x and $3x$, at the end of 15 years will be $x + a$ and $3x + a$; but as the age of the mother will then be double that of the daughter, we must multiply the age of the latter by 2, to have the following equation:

$$2x + 2a = 3x + a \quad (14)$$

Then by transposition $2a - a = 3x - 2x$ (15.)
And then by reduction $a = x$.

Consequently, the age of the daughter is 15, and that of the mother 45; which will answer all the conditions of the problem.

PROBLEM II.—*A father, on his death-bed, gave orders in his will, that if his wife, who was then pregnant, brought forth a son, he should inherit $\frac{2}{3}$ of his property, and the mother the remainder; but if she brought forth a daughter, the latter should have only $\frac{1}{3}$, and the mother $\frac{2}{3}$. As the widow, however, was delivered of twins, a boy and a girl, what share ought each to have of the property left by the father?*

The only difficulty in this problem is to determine what would have been the will of the testator, had he foreseen that his wife would be delivered of twins. It has generally been explained in the following manner: As the testator desired that in case his wife brought forth a son, he should have two-thirds of his property, and the mother one-third, it hence follows, that his intention was to give his son a sum double to that of the mother; and as he desired, in the other case, that if she brought forth a daughter, the mother should have two-thirds of his property, and the daughter one-third, there is reason to conclude, that he intended the share of the mother to be double that of the daughter. Consequently, to unite these two conditions, the heritage must be divided in such a manner, that the son may have twice as much as the mother, and the mother twice as much as the daughter.

If a, therefore, be supposed to represent the father's property, and x the share of the daughter, then $2x$ will express that of the mother, and $4x$ that of the son. But, as all these shares together are equal to the father's property, we shall have the following equation:

$$x + 2x + 4x = a$$

By reduction $7x = a$

And by division $x = \frac{a}{7}$. (16.)

Hence, if we suppose the whole property to be L. 30000, the daughter's share will be L. $4285\frac{5}{7}$; that of the mother L. $8571\frac{3}{7}$, and that of the son L. $17142\frac{6}{7}$.

Sometimes the following difficulty is proposed in regard to this problem. In case the mother should be brought to bed of two sons and a daughter, in what manner must the property be divided.

In our opinion, no other answer can be given than what would be given by the gentlemen of the gown, viz. that in this case the will would be void; for as no provision was made in it for a third child, its nullity would be established according to all the laws hitherto in existence. Because, 1st, The law is precise: 2d, Because it is impossible to determine what would have been the dispositions of the testator if two sons had been born to him, or if he had foreseen that his wife would be delivered of two.

PROBLEM III.—*A captain being asked how many soldiers he had in his company, replied—One-half of them are in camp, one-third in the trenches, one-eighth in the hospital, and four in prison. Of how many men did his company consist?*

If the number of soldiers be expressed by x, and the four in prison by a, we shall have the following equation:

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{8} + a = x.$$

Then by mult. $24x + 16x + 6x + 48a = 48x$

By reduct. and transp. $48a = 48x - 46x = 2x$

Consequently, $x = \frac{48a}{2} = 96.$

PROBLEM IV.—*The head of a fish is 9 inches in length, its tail is as long as the head and half the body, and the* *body is as long as the head and the tail. What is the length of the fish?*

Let the head be expressed by a, the tail by x, and the body by y. By the conditions of the problem we shall then have the following two equations:

$$x = a + \frac{y}{2}$$

$$y = a + a + \frac{y}{2}$$

(16.) By multiplication, $\left\{\begin{array}{l} 2x = 2a + y \\ 2y = 2a + 2a + y \end{array}\right.$

By transpos. and reduction, $\quad y = 4a = 36.$

By this result the problem is solved; for the head being supposed equal to 9, the body denoted by $y = 36$, and the tail, being equal to the head and half the body, must necessarily be 27, which answers all the conditions of the problem.

Problem V.—*A person who had a lease of a house for 99 years, being asked when it would expire, replied, that two-thirds of the time he had possessed it were exactly equal to four-fifths of the time unexpired. How many years of the lease were still remaining?*

If we call the time elapsed x, and the 99 years a, the time unexpired will be $a - x$. Therefore, by the conditions of the problem,

$$\frac{2x}{3} = \frac{4a - 4x}{5}$$

By multip. $\quad 10x = 12a - 12x$

By transp. and reduction, $\quad 22x = 12a$

Consequently, $\quad x = \frac{12a}{22}$ or $\frac{6}{11}a = 54.$

Hence it appears, that as the time elapsed is 54 years, the period of the lease unexpired must necessarily be 45; and this solution agrees with the conditions of the problem.

Problem VI.—*It is proposed to divide the number 50 into two such parts, that the sum of three-fourths of the one, and five-sixths of the other, may be equal to 40.*

Let $50 = a$, and $40 = b$; and if one of the parts of a be denoted by x, the other must necessarily be $a - x$.

By the conditions of the problem we shall then have the following equation:

$$\frac{3x}{4} + \frac{5a - 5x}{6} = b$$

By multiplication, $18x + 20a - 20x = 24b$

By transp. and reduct. $20a - 24b = 20x - 18x = 2x$

By division, $\dfrac{20a - 24b}{2} = x = 10a - 12b = 20.$

One of the parts of 50 then is 20, and the other 30; which answers the conditions of the problem; for 15, the three-fourths of 20, added to 25, the five-sixths of 30, is just equal to 40.

Problem VII.—*It is proposed to divide 100 into two such parts, that if a third of the one be taken from a fourth of the other, the remainder shall be 11.*

Let $100 = a$ and $11 = b$; also let one of the parts be expressed by x, and the other by $a - x$. Then $\frac{x}{3}$ will denote the third of the one part, and $\frac{a - x}{4}$ the fourth of the other; and by the conditions of the problem we shall have the following equation:

$$\frac{a - x}{4} - \frac{x}{3} = b$$

By multip. $3a - 3x - 4x = 12b$

By transp. and reduct. $3a - 12b = 7x$

By division, $\dfrac{3a - 12b}{7} = x = 24.$

The two parts of 100 then are 24 and 76; for if 8, the third of 24, be taken from 19, the fourth of 76, the remainder will be 11.

PROBLEM VIII.—*Two persons sat down to play, one of whom had 72 guineas, and the other only 52; after a certain number of games they separated, the former carrying with him three times as many guineas as the other. How much did he win?*

Let a represent the 72 guineas of the former, b the 52 guineas of the latter, and x the loss of the second player.

The money of the first player when they give over play will therefore be $a + x$, and that of the other $b - x$; but as $a + x$, by the question, is three times as great as $b - x$, we shall have,

$$a + x = 3b - 3x$$

By transp. $$4x = 3b - a$$

By division, $$x = \frac{3b - a}{4} = 21.$$

As it here appears that the loss of the second player was 21 guineas, leaving him only 31, the first must have carried off 93 guineas, which answers the conditions of the problem.

PROBLEM IX.—*The minute hand of a clock being at 12, and the hour hand at 1, at what point between 1 and 2 will they both be in conjunction?*

If x represent the space between the hours of 1 and 2 passed over by the hour hand before it is overtaken by the minute hand, and a the interval between 12 and 1; as the space passed over by the minute hand will be twelve times as great as that passed over by the hour hand, $a + x$ will be equal to $12x$; and we shall have the following equation:

$$a + x = 12x$$

By transp. and reduct. $$a = 11x$$

By division, $$\frac{a}{11} = x.$$

From which we may conclude, that the minute hand will overtake the hour hand after the latter has passed over $\frac{1}{11}$ part of the space between the hours of 1 and 2.

PROBLEM X.—*If two bodies move towards each other with unequal velocities, the ratio of which is known, as well as the distance between the bodies, to determine the point at which they will meet.*

Let the velocities be as 12 to 1, and let a represent the distance between the bodies, and x that part of it passed over by the body having the least velocity, when they meet.

The space then passed over by the body which has the greatest velocity, will be $a - x$, and we shall have the following proportion:

$$12 : 1 :: a - x : x$$

By equation, $12x = a - x$

By transp. and reduct. $13x = a$

By division, $x = \frac{a}{13}$.

The solution of this problem is general; and consequently applicable to all cases where the distance of the bodies and the ratio of the velocities are known.

PROBLEM XI.—*To divide 90 into two parts, which shall be to each other in the same ratio as 2 to 3.*

Let 90 be represented by a, the least of the two parts by x, and the other by $a - x$. We shall then have the following proportion:

$$2 : 3 :: x : a - x$$

By equation, $2a - 2x = 3x$

By transp. and reduct. $2a = 5x$

By division, $\frac{2a}{5} = x = 36.$

Consequently, the least of the numbers will be 36, and the other 54; and indeed $36 + 54 = 90$, and $36 : 54 :: 2 : 3$.

PROBLEM XII.—*Application of Analysis to the solution of the 11th problem of Divining Arithmetic, in which it is proposed to tell the number of spots on all the bottom cards of several heaps arranged on a table.*

It is here supposed, that a complete pack of 52 cards is employed; and that as many cards are placed over the first of each heap as are necessary to make the sum of the spots and cards together to amount to 12.

Let a represent 52, the whole number of cards, and b that of the remaining cards. The number of cards in all the heaps will then be $a-b$. If the number of spots to be guessed be expressed by x, and the sum of all these spots and the cards over them, as they are known, by c; we shall have the following equation:

$$x+a-b=c$$

By transp. $x=c+b-a.$

That is to say, if four heaps which are equivalent to a be deducted, x will be equal to the sum of the remaining cards, and the number of the spots and cards which are in the other heaps. The truth of this operation may be easily proved.

PROBLEM XIII.—*What number is that, the $\frac{2}{3}$ of $\frac{1}{4}$ of which is equal to 1?*

Let x be the required number.

Then $\frac{2}{3}$ of $\frac{3x}{4}=1$

But $\frac{2}{3}$ of $\frac{3x}{4}=\frac{6x}{12}=\frac{x}{2}$

Consequently $\frac{2}{x}=1$, or $x=2$.

Proof. $\frac{3}{4}$ of 2 are $\frac{3}{2}$; and $\frac{2}{3}$ of $\frac{3}{2}=\frac{6}{6}=1$.

PROBLEM XIV.—*What number is that $\frac{3}{4}$ of $\frac{2}{3}$ of which $+\frac{1}{2}$ of $\frac{5}{6}$ of it, is equal to 11?*

Let x as before be the required number.

Then $\frac{3}{4}$ of $\frac{2}{3}$ of x are $\frac{x}{2}$

And $\frac{1}{2}$ of $\frac{5}{6}$ of x is $\frac{5x}{12}$

But by the supposition $\frac{x}{2}+\frac{5x}{12}=11$

Therefore $11x=11\times 12$, and $x=12$.

Proof. $\frac{2}{3}$ of 12 are 8, and $\frac{3}{4}$ of 8 are $6=\frac{3}{4}$ of $\frac{2}{3}$ of 12; $\frac{5}{6}$ of 12 are 10, and the half of 10 is $5=\frac{1}{2}$ of $\frac{5}{6}$ of 12: but $5+6=11$. Therefore, &c.

PROBLEM XV.—*What number is that $\frac{2}{3}$ of $\frac{4}{5}$ of which $-\frac{1}{2}$ of $\frac{3}{4}$ of it are equal to 19?*

First, $\frac{1}{2}$ of $\frac{3}{4}$ of x is $\frac{3x}{8}$

And $\frac{2}{3}$ of $\frac{4}{5}$ of x are $\frac{8x}{15}$

Then $\frac{8x}{15}-\frac{3x}{8}=\frac{64x}{120}-\frac{45x}{120}=\frac{19x}{120}$.

But $\frac{19x}{120}$ equal 19 by the problem. Therefore, $19x=19\times 120$; and x equal 120.

Proof. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{120}$ are 64, and $\frac{1}{2}$ of $\frac{3}{4}$ of 120 is 45; but $64-45=19$. Therefore, &c.

PROBLEM XVI.—*What number is that of which $\frac{2}{3}$ of $\frac{3}{4}$ multiplied by $\frac{1}{2}$ of $\frac{1}{6}$ of it will be equal to 6?*

$\frac{2}{3}$ of $\frac{3}{4}$ of x are $\frac{x}{2}$

And $\frac{1}{2}$ of $\frac{1}{6}$ of x is $\frac{x}{12}$, and $\frac{x}{2}\times\frac{x}{12}=\frac{x^2}{24}$

Then by the conditions of the problem, $\frac{x^2}{24}=6$

Therefore $x^2=144$; and consequently $x=12$.

Proof. $\frac{2}{3}$ of $\frac{3}{4}$ of 12 are 6; and $\frac{1}{2}$ of $\frac{1}{6}$ of 12 is 1; but $6\times 1=6$. Therefore, &c.

PROBLEM XVII.—*What number is that of which $\frac{1}{2}+\frac{3}{4}$ are equal to 1?*

Let x be the number required.

Then $\frac{x}{2}+\frac{3x}{4}=1$, or $\frac{5x}{4}=1$

Therefore $5x=4$

Consequently $x=\frac{4}{5}$.

Proof. $\frac{1}{2}$ of $\frac{4}{5}=\frac{2}{5}$; and $\frac{3}{4}$ of $\frac{4}{5}=\frac{3}{5}$; but $\frac{2}{5}+\frac{3}{5}$ or $\frac{5}{5}=1$. Therefore, &c.

PROBLEM XVIII.—*What number is that the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of which make 12?*

Let x be the required number.

Then $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=12$

Or $12x+8x+6x=24\times 12$

Therefore $26x=288$

And $x=\frac{288}{26}=11\frac{1}{13}$.

Proof. $\frac{1}{2}$ of $11\frac{1}{13}$ is $5\frac{7}{13}$; $\frac{1}{3}$ of $11\frac{1}{13}$ is $3\frac{9}{13}$; and $\frac{1}{4}$ of $11\frac{1}{13}$ is $2\frac{10}{13}$; but $5\frac{7}{13}+3\frac{9}{13}+2\frac{10}{13}=12$.

PROBLEM XIX.—*The triple, the half, and the fourth of a certain number, are equal to 104: What is the number?*

Let x be the number required. We shall then have, by the conditions of the problem:

$$3x+\frac{x}{2}+\frac{x}{4}=104$$

Therefore $30x=104\times 8=832$

Consequently $x=\frac{832}{30}=27\frac{11}{15}$.

Proof. $27\frac{11}{15}\times 3=83\frac{3}{15}$

$\frac{1}{2}$ of $27\frac{11}{15}=13\frac{13}{15}$

$\frac{1}{4}$ of $27\frac{11}{15}=\ \ 6\frac{14}{15}$

The sum, 104

PROBLEM XX.—*If $\frac{3}{4}$ and $\frac{1}{6}$ of the hull of a ship be immersed in the sea, and only 4 feet of it above the surface of the water; what is the depth of the vessel?*

Let x be the depth of the vessel.

Then $\frac{3x}{4}+\frac{x}{6}+4=x$

Or $18x+4x+96=24x$

Therefore $2x=96$

And $x=48$ feet, the depth of the vessel.

Proof. $\frac{3}{4}$ of $48=36$

$\frac{1}{6}$ of $48=\underline{8}$

44

Feet above water $\underline{4}$

48

PROBLEM XXI.—*A banker, at his death, being desirous to reward 10 of his clerks, gave orders in his will, that 5500 guineas should be divided among them, in such a manner, that the first 5 should have each an equal share of the whole legacy; that the next 3 men should have shared among them one-half of what was bequeathed to the first 5; and that the 2 last should have divided between them one-third of that sum: What was the share of each?*

Let x be the share of each of the first five clerks, and $a=$ the 5500 guineas.

Then, by the conditions of the problem, the share of the first five will be $5x$; that of the next three $\frac{5}{2}x$; and that of the two last $\frac{5}{3}x$.

But as these three quantities are equal to a, or the whole, we have the following equation:

$$5x+\tfrac{5}{2}x+\tfrac{5}{3}x=a$$

By multip. and reduct. $55x=6a$

By division, $x=\frac{6a}{55}=600$ guineas.

Each of the first five then had 600
Each of the next three 500
And each of the two last 500

Proof. $5 \times 600 = 3000$
$3 \times 500 = 1500$
$2 \times 500 = 1000$
$\overline{5500}$

The application which we have here made of Analysis to the solution of a few problems, evidently shews that this method, by its precision, brevity, and extent, is far superior to arithmetic. The latter confines our attention to determinate quantities, and, if we may use the expression, enchains it by the slowness of its progress; while the other, more rapid, enables us to pass over the intermediate operations, and to direct our attention to the real point of difficulty.

The chief advantages, therefore, derived from this science are, that it facilitates the discovery and comprehension of mathematical truths, and that it supplies us with easy methods, and general rules, for resolving all problems that may be proposed respecting quantities.

When we have obtained a result by the rules of arithmetic, there is nothing indeed that exhibits to the mind the chain of operations which conducted to it. When, after a few arithmetical operations, we have obtained 12 for result, we see nothing in 12 which can indicate whether this number has arisen from the multiplication of 3 by 4, of 2 by 6, or by the addition of 5 to 7, or of 2 to 10; or, in general, from the combination of any other operations. Arithmetic gives rules for finding certain results, but these results of themselves can furnish no rules.—Algebra, however, or that mode of calculation which employs indeterminate characters, preserves, as we may say, the traces of all the intermediate operations, which conduct to the last result,

TABLES OF CHANCES ON GAMES OR PLAY.

The following Tables contain the odds or chances, for winning any number of games, in a great variety of cases, either when the chance is equal in every game or throw, or when it is unequal according to any odds or proportion; and the games may be of any kind whatever, either at dice, or cards, or hazard, or billiards, or racing, or cocking, &c.

I. *When the Chances or Bets on each game are equal.*

Against winning			
21 times running,	the odds are . .	2097151	to 1
20 out of 21, ...	are . . .	95324	to 1
19 out of 21, ...	are . . .	9038	to 1
18 out of 21, ...	are . . .	1341	to 1
17 out of 21, ...	are . . .	276	to 1
16 out of 21, ...	are	74	to 1
15 out of 21, ...	are	$24\frac{1}{2}$	to 1
14 out of 21, ...	are	$9\frac{1}{2}$	to 1
13 out of 21, ...	are	$4\frac{1}{5}$	to 1
12 out of 21, ...	are	2	to 1
20 games running, ...	are . . .	1048575	to 1
19 out of 20, ...	are . .	49931	to 1
18 out of 20, ...	are	4968	to 1
17 out of 20, ...	are	775	to 1
16 out of 20, ...	are	168	to 1
15 out of 20, ...	are	$47\frac{1}{4}$	to 1
14 out of 20, ...	are	$16\frac{1}{3}$	to 1
13 out of 20, ...	are	$6\frac{1}{2}$	to 1
12 out of 20, ...	are	$2\frac{9}{10}$, or very near 3	to 1

11 out of 20, . the odds are . .		4 to 3, or $1\frac{1}{3}$ to 1
even games in 20, are 4s. 8d. to 1s. or . .		$4\frac{2}{3}$ to 1
19 games running, the odds are		524287 to 1
18 out of 19, ...	are	26213 to 1
17 out of 19, ...	are	2745 to 1
16 out of 19, ...	are	450 to 1
15 out of 19, ...	are	103 to 1
14 out of 19, ...	are	$30\frac{1}{3}$ to 1
13 out of 19, ...	are .	near 11, or $10\frac{9}{10}$ to 1
12 out of 19, ...	are	$4\frac{1}{2}$ to 1
11 out of 19, . . .	are	{ 2s. 1d. to 1s. or near 2 to 1
18 games running . .	are . . .	262143 to 1
17 out of 18, . . .	are	13796 to 1
16 out of 18, . . .	are	1523 to 1
15 out of 18, . . .	are	264 to 1
14 out of 18, . . .	are	63 to 1
13 out of 18, . . .	are	$19\frac{3}{4}$ to 1
12 out of 18, . . .	are	$7\frac{1}{3}$ to 1
11 out of 18, . . .	are	$3\frac{1}{7}$ to 1
10 out of 18, . . .	are	{ near 10 to 7, or nearer $1\frac{5}{11}$ to 1
even games in 18, . .	are .	$4\frac{2}{5}$, or nearer $4\frac{7}{18}$ to 1
17 games running . .	are . . .	131071 to 1
16 out of 17, ...	are	7280 to 1
15 out of 17, . . .	are	850 to 1
14 out of 17, . . .	are	156 to 1
13 out of 17, . . .	are	$39\frac{3}{4}$ to 1
12 out of 17, . . .	are .	$12\frac{9}{10}$ or near 13 to 1
11 out of 17, . . .	are	5 to 1
10 out of 17, . . .	are . .	13 to 6, or $2\frac{1}{6}$ to 1

16 games running, the odds are	. . . .	65535 to 1
15 out of 16,	. . . are	3854 to 1
14 out of 16,	. . . are	477 to 1
13 out of 16,	. . . are	93 to 1
12 out of 16,	. . . are	25 to 1
11 out of 16,	. . . are	$8\frac{1}{2}$ to 1
10 out of 16,	. . . are	$3\frac{1}{3}$ to 1
9 out of 16,	. . . are	{ near 7 to 5, or nearer $1\frac{10}{21}$ to 1
even games in 16,	. . are .	$4\frac{1}{11}$ or near 4 to 1
15 games running	. . are	32767 to 1
14 out of 15,	. . . are	2047 to 1
13 out of 15,	. . . are	269 to 1
12 out of 15,	. . . are	55 to 1
11 out of 15,	. . . are	$15\frac{7}{8}$ to 1
10 out of 15,	. . . are	$5\frac{1}{2}$ to 1
9 out of 15,	. . . are .	23 to 10, or $2\frac{3}{10}$ to 1
14 games running	. . are	16383 to 1
13 out of 14,	. . . are	1091 to 1
12 out of 14,	. . . are	153 to 1
11 out of 14,	. . . are .	$33\frac{6}{7}$, or near 34 to 1
10 out of 14,	. . . are	$10\frac{1}{8}$ to 1
9 out of 14,	. . . are	$3\frac{2}{3}$ to 1
8 out of 14,	. . . are .	near 3 to 2, or $1\frac{1}{2}$ to 1
even games in 14,	. . are .	near $3\frac{33}{43}$, or $3\frac{7}{9}$ to 1
13 games running	. . are	8191 to 1
12 out of 13,	. . . are	584 to 1
11 out of 13,	. . . are	88 to 1
10 out of 13,	. . . are	$20\frac{2}{3}$ to 1
9 out of 13,	. . . are	$6\frac{1}{2}$ to 1
8 out of 13,	. . . are .	near 9 to 4, or $2\frac{1}{4}$ to 1

12 games running, the odds	are	4095 to 1
11 out of 12,	are	314 to 1
10 out of 12,	are	$50\frac{67}{79}$ or near 51 to 1
9 out of 12,	are	$12\frac{2}{3}$ to 1
8 out of 12,	are	$4\frac{3}{20}$, or near $4\frac{1}{7}$ to 1
7 out of 12,	are	near 8 to 5, or near $1\frac{3}{5}$ to 1
even games in 12,	are	near $3\frac{10}{23}$, or near $3\frac{4}{9}$ to 1

11 games running,	are	2047 to 1
10 out of 11,	are	169 to 1
9 out of 11,	are	$29\frac{1}{2}$ to 1
8 out of 11,	are	$7\frac{24}{29}$ or near $7\frac{5}{6}$ to 1
7 out of 11,	are	near 13 to 5, or $2\frac{3}{5}$ to 1

10 games running	are	1023 to 1
9 out of 10,	are	92 to 1
8 out of 10,	are	$17\frac{1}{4}$ to 1
7 out of 10,	are	53 to 11, or near $4\frac{4}{5}$ to 1
6 out of 10,	are	near 13 to 8, or $1\frac{5}{8}$ to 1
even games in 10,	are	near $3\frac{1}{16}$ to 1

9 games running	are	511 to 1
8 out of 9,	are	50 to 1
7 out of 9,	are	$10\frac{3}{23}$, or near $10\frac{1}{8}$ to 1
6 out of 9,	are	$2\frac{61}{63}$, or near 3 to 1

8 games running,	are	255 to 1
7 out of 8,	are	$27\frac{4}{9}$ to 1
6 out of 8,	are	$5\frac{34}{37}$, or near 6 to 1
5 out of 8,	are	near 7 to 4, or $1\frac{3}{4}$ to 1
even games in 8,	are	near 8 to 3, or $2\frac{2}{3}$ to 1

7 games running, the odds are	. . . . . .	127 to 1
6 out of 7, ...	are	15 to 1
5 out of 7, ...	are	near 17 to 5, or $3\frac{2}{5}$ to 1
6 games running, . .	are	63 to 1
5 out of 6, ...	are	$8\frac{1}{7}$ to 1
4 out of 6, ...	are	21 to 11, or near 2 to 1
even games in 6, . .	are . .	11 to 5, or $2\frac{1}{5}$ to 1
5 games running, . .	are	31 to 1
4 out of 5, ...	are	near 13 to 3, or $4\frac{1}{3}$ to 1
4 games running, . .	are	15 to 1
3 out of 4, ...	are . .	11 to 5, or $2\frac{1}{5}$ to 1
even games in 4, . .	are . .	5 to 3, or $1\frac{2}{3}$ to 1
3 games running, . .	are	7 to 1
2 games running, . .	are	3 to 1

II. *When the Odds or Chances on each game are 6 to 5.*

Against winning		
10 games running, the odds are	. . . . . .	427 to 1
9 out of 10, . . .	are	44 to 1
8 out of 10, . . .	are	$9\frac{1}{2}$ to 1
7 out of 10, ...	are .	$2\frac{9}{10}$, or near 3 to 1
6 out of 10, ...	are	nearly equal, or $1\frac{1}{127}$ to 1
even games in 10, . .	are	$3\frac{1}{5}$ to 1
9 games running, . .	are	232 to 1
8 out of 9, ...	are	$26\frac{1}{2}$ to 1

7 out of 9, . .	the odds are .	very near 6 to 1
6 out of 9, ...	are	near 15 to 8, or $1\frac{7}{8}$ to 1
5 out of 9, ...	are	near 14 to 9, or $1\frac{5}{9}$ to 1
8 games running, ...	are . . .	126 to 1
7 out of 8, ...	are . . .	$15\frac{1}{2}$ to 1
6 out of 8, ...	are	15 to 4, or $3\frac{3}{4}$ to 1
5 out of 8, ...	are	11 to 10, or $1\frac{1}{10}$ to 1
even games in 8, ...	are	14 to 5, or $2\frac{4}{5}$ to 1
7 games running, ...	are . . .	68 to 1
6 out of 7, ...	are	$9\frac{1}{5}$ to 1
5 out of 7, ...	are .	9 to 4, or $2\frac{1}{4}$ to 1
4 out of 7, ...	are	near 3 to 2 or $1\frac{1}{2}$ to 1
6 games running, ...	are	$36\frac{45}{46}$, or near 37 to 1
5 out of 6, ...	are	$5\frac{1}{4}$ to 1
4 out of 6, ...	are	13 to 10, or $1\frac{3}{10}$ to 1
even games in 6, ...	are .	9 to 4, or $2\frac{1}{4}$ to 1
5 games running, ...	are . . .	$19\frac{2}{3}$ to 1
4 out of 5, ...	are	rather better than 3 to 1
3 out of 5, ...	are	near 7 to 5, or $1\frac{2}{5}$ to 1
4 games running, ...	are . . .	$10\frac{1}{4}$ to 1
3 out of 4, ...	are	near 8 to 5, or $1\frac{3}{5}$ to 1
even games in 4, ...	are	near 5 to 3, or $1\frac{2}{3}$ to 1

3 games running, . the odds are	near 31 to 6, or $5\frac{1}{6}$ to 1
2 out of 3, ... are	near 10 to 7, or $1\frac{3}{7}$ to 1
2 games running, ... are	near 7 to 3, or $2\frac{1}{3}$ to 1
1 game each, out of 2, . . are	near 61 to 60, or $1\frac{1}{60}$ to 1

III. *When the Odds or Chances on each game are 5 to 6.*

Against winning		
10 games running,	the odds are	2654 to 1
9 out of 10, ...	are	203 to 1
8 out of 10, ...	are	33 to 1
7 out of 10, ...	are	$8\frac{1}{4}$ to 1
6 out of 10, ...	are	18 to 7, or $2\frac{4}{7}$ to 1
9 games running, ...	are	1206 to 1
8 out of 9, ...	are	101 to 1
7 out of 9, ...	are	$17\frac{3}{4}$ to 1
6 out of 9, ...	are	$4\frac{3}{4}$ to 1
8 games running, ...	are	547 to 1
7 out of 8, ...	are	$50\frac{3}{4}$ to 1
6 out of 8, ...	are	$9\frac{3}{4}$ to 1
5 out of 8, ...	are	near 19 to 7, or $2\frac{5}{7}$ to 1
7 games running, ...	are	248 to 1
6 out of 7, ...	are	$25\frac{1}{2}$ to 1
5 out of 7, ...	are	$5\frac{1}{4}$ to 1

6 games running,	the odds are	112 to 1
5 out of 6, ...	are	$12\frac{3}{4}$ to 1
4 out of 6, ...	are	near 14 to 5, or $2\frac{4}{5}$ to 1

5 games running, ...	are	$50\frac{1}{2}$ to 1
4 out of 5, ...	are	$6\frac{1}{3}$ to 1

4 games running, ...	are	$22\frac{1}{3}$ to 1
3 out of 4, ...	are	very nearly $3\frac{1}{25}$ to 1

3 games running, ...	are . .	near $9\frac{2}{3}$ to 1
2 games running, ...	are . .	near $3\frac{4}{5}$ to 1

IV. *When the Odds or Chances on each game are 5 to 4.*

Against winning

10 games running,	the odds are . . .	356 to 1
9 out of 10, ...	are	$38\frac{1}{2}$ to 1
8 out of 10, ...	are	$8\frac{1}{3}$ to 1
7 out of 10, ...	are	$2\frac{1}{2}$ to 1
6 out of 10, ...	are	14 to 13, or $1\frac{1}{13}$ to 1
even games in 10, ...	are	13 to 4, or $3\frac{1}{4}$ to 1

9 games running, ...	are . . .	197 to 1
8 out of 9, ...	are	24 to 1
7 out of 9, ...	are	$5\frac{1}{3}$ to 1
6 out of 9, ...	are near	5 to 3, or $1\frac{2}{3}$ to 1
5 out of 9, ...	are .	12 to 7, or $1\frac{7}{5}$ to 1

8 games running,	the odds are . . .	109 to 1
7 out of 8, ...	are	$13\frac{3}{4}$ to 1
6 out of 8, ...	are	$3\frac{1}{3}$ to 1
5 out of 8, ...	are	near 22 to 21, or $1\frac{1}{21}$ to 1
even games in 8, ...	are near	17 to 6, or $2\frac{5}{6}$ to 1
7 games running, ...	are	60 to 1
6 out of 7, ...	are	$8\frac{1}{4}$ to 1
4 out of 7, ...	are near	5 to 3, or $1\frac{2}{3}$ to 1
6 games running, ...	are	33 to 1
5 out of 6, ...	are	near 39 to 8, or $4\frac{7}{9}$ to 1
4 out of 6, ...	are near	6 to 5, or $1\frac{1}{5}$ to 1
even games in 6, ...	are	near 11 to 5, or $2\frac{1}{5}$ to 1
5 games running, ...	are	$17\frac{8}{9}$ to 1
4 out of 5, ...	are	near 14 to 5, or $2\frac{4}{5}$ to 1
3 out of 5, ...	are near	3 to 2, or $1\frac{1}{2}$ to 1
4 games running, ...	are .	very near $9\frac{1}{3}$ to 1
3 out of 4, ...	are near	3 to 2, or $1\frac{1}{2}$ to 1
even games in 4, ...	are	near 17 to 10, or $1\frac{7}{10}$ to 1
3 games running, ...	are near	$4\frac{5}{6}$, or near 5 to 1
2 out of 3, ...	are near	4 to 3, or $1\frac{1}{3}$ to 1

2 games running,	the odds are	56 to 25, or near $2\frac{1}{5}$ to 1
1 game each out of 2,	. . . are	41 to 40, or $1\frac{1}{40}$ to 1

V. *When the Odds or Chances on each game are 4 to 5.*

Against winning		
10 games running,	the odds are . . .	3324 to 1
9 out of 10,	. . . . . . are . . .	245 to 1
8 out of 10,	. . . . . . are . . .	$38\frac{1}{2}$ to 1
7 out of 10,	. . . . . . are	$9\frac{1}{3}$ to 1
6 out of 10,	. . . . . . are	3 to 1
9 games running,	. . . are . . .	1476 to 1
8 out of 9,	. . . . . . are . . .	119 to 1
7 out of 9,	. . . . . . are	$20\frac{1}{3}$ to 1
6 out of 9,	. . . . . . are	$5\frac{1}{3}$ to 1
8 games running,	. . . are	655 to 1
7 out of 8,	. . . . . . are	58 to 1
6 out of 8,	. . . . . . are . .	near 11 to 1
5 out of 8,	. . . . . . are	3 to 1
7 games running,	. . . are . . .	290 to 1
6 out of 7,	. . . . . . are . . .	$28\frac{3}{4}$ to 1
5 out of 7,	. . . . . . are .	$5\frac{6}{7}$, or near 6 to 1
6 games running,	. . . are	128 to 1
5 out of 6,	. . . . . . are . .	near $14\frac{1}{4}$ to 1
4 out of 6,	. . . . . . are	3 to 1

5 games running,	the odds are	$56\frac{1}{2}$ to 1
4 out of 5, ...	are	near $6\frac{71}{74}$, or 7 to 1
4 games running, . . .	are	$24\frac{1}{2}$ to 1
3 out of 4, ...	are	3 to 1
3 games running, . . .	are	$10\frac{25}{64}$, or near $10\frac{2}{5}$ to 1
2 games running, . . .	are	65 to 16, or $4\frac{1}{16}$ to 1

VI. *When the Odds or Chances on each game are 6 to 4.*

Against winning

6 times running,	the odds are	near $20\frac{3}{7}$ to 1
5 out of 6, ...	are	near $3\frac{2}{7}$ to 1
4 out of 6, ...	are	1601 to 1424, or near $1\frac{1}{8}$ to 1
even games in 6, . .	are	near $2\frac{5}{8}$ to 1
5 times running, . .	are	near $11\frac{5}{6}$ to 1
4 out of 5, ...	are . . .	very near 2 to 1
3 out of 5, ...	are	near 31 to 15, or $2\frac{1}{15}$ to 1
4 times running, . .	are	near $6\frac{5}{7}$ to 1
3 out of 4, ...	are	328 to 297, or 11 to 10
even games in 4, . .	are .	409 to 216, or 15 to 8
3 times running . .	are . . .	$3\frac{1}{7}{}^{7}$ or near $3\frac{2}{3}$ to 1
2 out of 3, ...	are	81 to 44, or near $1\frac{5}{6}$ to 1

2 times running, the odds are . .	16 to 9, or $1\frac{7}{9}$ to 1
1 game only in 2 . . are .	13 to 12, or $1\frac{1}{12}$ to 1

VII. *When the Chances on each game are* 4 *to* 6.

Against winning		
6 times running, the odds are		243 to 1
5 out of 6, ...	are . . .	near $23\frac{2}{5}$ to 1
4 out of 6, ...	are	. near 32 to 7, or $4\frac{4}{7}$ to 1
5 times running, . .	are	96 to 1
4 out of 5, ...	are	near $10\frac{1}{2}$ to 1
4 times running, . .	are	$38\frac{1}{16}$ to 1
3 out of 4	are	near $4\frac{1}{2}$ to 1
3 times running, . .	are	$14\frac{5}{8}$ to 1
2 times running, . .	are . .	21 to 4, or $5\frac{1}{4}$ to 1

VIII. *When the Odds on each game are* 7 *to* 4.

Against winning		
6 times running, the odds are		$14\frac{1}{18}$ to 1
5 out of 6, ...	are	. near 12 to 5, or $2\frac{2}{5}$ to 1
4 out of 6, ...	are	near 109 to 67, or 5 to 3
equal times in 6 . .	are . . .	very near 3 to 1
5 times running, . . .	are	near $8\frac{4}{7}$ to 1
4 out of 5, ...	are	. near 97 to 65, or 3 to 2
3 out of 5, ...	are	near 29 to 10, or near 3 to 1

4 times running, the odds are	.	51 to 10, or near 5 to 1
3 out of 4, ...	are .	near 19 to 9, or $2\frac{1}{9}$ to 1
3 times running, . .	are	near 23 to 8, or near 3 to 1
2 out of 3, ...	are . . .	7 to 3, or $2\frac{1}{3}$ to 1
2 times running, . .	are . .	72 to 49, or $1\frac{8}{17}$ to 1
1 in 2, or even in 2, .	are . . .	65 to 56, or 7 to 6

IX. *When the Chance on each game is 4 to 7.*

Against winning

6 times running, the odds are	. . . . . .	431 to 1
5 out of 6, ...	are	$36\frac{1}{2}$ to 1
4 out of 6, ...	are	$6\frac{1}{2}$ to 1
5 times running, . .	are	156 to 1
4 out of 5, ...	are	$15\frac{1}{8}$ to 1
4 times running, . .	are	56 to 1
3 out of 4, ...	are	$6\frac{1}{7}$ to 1
3 times running, . .	are	$19\frac{4}{5}$ to 1
2 times running, . .	are	$6\frac{9}{16}$ to 1

X. *When the Odds on each game are 2 to 1.*

Against winning

6 times running, the odds are	. . . . .	near $10\frac{1}{3}$ to 1
5 out of 6, . . .	are .	473 to 256, or 11 to 6
4 out of 6, . . .	are .	near 17 to 8, or $2\frac{1}{8}$ to 1
3 out of 6, or even, .	are .	near 7 to 2, or $3\frac{1}{2}$ to 1

5 times running, the odds are	.	near 33 to 5, or $6\frac{3}{5}$ to 1
4 out of 5, . . .	are	. 131 to 112, or near 7 to 6
3 out of 5, . . .	are	. near 34 to 9, or $3\frac{7}{9}$ to 1
4 times running, . .	are	. 65 to 16, or near 4 to 1
3 out of 4, . . .	are	. . 16 to 11, or $1\frac{5}{11}$ to 1
2 out of 4, or even in 4,	are	. . . 19 to 8, or $2\frac{3}{8}$ to 1
3 times running, .	are	. . . 19 to 7, or $2\frac{5}{7}$ to 1
2 out of 3, . . .	are	. . 20 to 7, or near 3 to 1
2 times running, . .	are	. . . 5 to 4, or $1\frac{1}{4}$ to 1
1 in 2, or even in 2, .	are	. . . 5 to 4, or $1\frac{1}{4}$ to 1

XI. *When the Chance on each game is $\frac{1}{3}$, or 1 to 2.*

Against winning

6 times running, the odds are	. . . .	728 to 1
5 out of 6, . . .	are	55 to 1
4 out of 6, . . .	are . .	$8\frac{72}{73}$, or near 9 to 1
5 times running, . .	are	242 to 1
4 out of 5, . . .	are .	$21\frac{1}{11}$, or near 21 to 1
4 times running, . .	are	80 to 1
3 out of 4, . . .	are	8 to 1
3 times running, . .	are	26 to 1
2 times running, . .	are	8 to 1

ACOUSTICS AND MUSIC.

The ancients seem to have considered sounds under no other point of view than that of music; that is to say, as affecting the ear in an agreeable manner: it is even very doubtful whether they were acquainted with any thing more than melody, and whether they had any art similar to that which we call composition. The moderns, however, by attending to the philosophy of sounds, have made many discoveries in this department, so much neglected by the ancients; and hence has arisen a new science, distinguished by the name of *Acoustics*. Acoustics have for their object the nature of sounds considered, in general, both in a mathematical and philosophical view. This science, therefore, comprehends music, which considers the ratios of sounds, so far as they are agreeable to the ear, either by their succession, which constitutes melody or by their simultaneity, which forms harmony. We shall here give a brief account of every thing most curious and interesting in regard to this science.

Definition of sound—how diffused and transmitted to our organs of hearing—Experiments on this subject—different ways of producing sound.

Sound is nothing else but the vibration of the particles of the air, occasioned either by some sudden agitation of a certain mass of the atmosphere, violently compressed or expanded, or by the communication of the vibration of the insensible parts of a hard and elastic body.

These are the two best known ways of producing sound. The explosion of a pistol, or of any other kind of fire-arms produces a report or sound, because the air or elastic fluid contained in the gunpowder, being suddenly dilated, compresses the external air with great violence: the latter, in con equence of its elasticity, re-acts on the surrounding atmos- here, and produces in its moleculæ an oscillatory motion, vhich occasions the sound, and which extends to a greater r less distance, according to the intensity of the cause that ;ave rise to it.

The other method of producing sound is to excite, in an lastic body, vibrations sufficiently rapid to occasion, in the urrounding parts of the air, a similar motion. Thus an ex- ended string, when struck, emits a sound; and its oscillations, hat is to say, its motion backward and forward, may be dis- inctly seen. The elastic parts of the air struck by the string, uring the time it is vibrating, are themselves put into a state f vibration, and communicate this motion to the neighbour- ng ones. Such also is the mechanism by which a bell pro- uces its sound: when struck, its vibrations are sensible to he hand which touches it.

That air is the vehicle of sound, may be proved by the fol- owing experiment: If a bell be suspended in the receiver of n air-pump, the sound of it decreases in proportion as the ir is exhausted, and at last becomes totally insensible when complete vacuum has been formed.

Sound always ceases when the vibrations of the sonorous ody cease, or become too weak. This may be proved also y an experiment; for when the vibrations of a sonorous body re damped by any soft body, the sound seems suddenly to ease; in a piano-forte, therefore, the quills are furnished with its of cloth, that by touching the strings when they fall down, ey may damp their vibrations. On the other hand, when e sonorous body is, by its nature, capable of continuing its ibrations for a considerable time, as is the case with a large ell, the sound may be heard for a long time after.

)f the velocity of sound—Experiments for determining it—method of measuring distances by it.

Light is transmitted from one place to another with incon- eivable velocity; but this is not the case with sound: the elocity of sound is very moderate, and may be measured in he following manner.

Let a cannon be placed at the distance of several thousand yards, and let an observer, with a pendulum that vibrates seconds, or rather half-seconds, put the pendulum in motion as soon as he sees the flash, and then count the number of seconds or half-seconds which elapse between that period and the moment when he hears the explosion. It is evident, that if the moment when the flash is seen be considered as the signal of the explosion, nothing will be necessary to obtain the number of yards which the sound has passed over in a second or half-second, but to divide the number of the yards between the place of observation and the cannon, by the number of the seconds or half-seconds which have been counted.

Now the moment when the flash is perceived may be considered as the real moment of the explosion; for so great is the velocity of light, that it employs scarcely a second to traverse 70000 leagues.

By this method it has been found, that sound moves at the rate of about 1142 feet in a second.

This method may be employed to determine the distance of ships at sea, or in a harbour, when they fire guns, provided the flash can be seen, and the explosion heard. During a storm also, the distance of a thunder-cloud may be determined in the same manner. But, as a pendulum is not always to be obtained, its place may be supplied by observing the beats of the pulse; for when in its usual state, each interval between the pulsations is almost equal to a second.

How sounds may be propagated in every direction, without confusion.

This is a very singular phenomenon in the propagation of sounds; for if several persons speak at the same time, or play on instruments, their different sounds are heard simultaneously, or all together, either by one person, or by several persons, without being confounded in passing through the same place in different directions. Let us endeavour to account for this phenomenon.

The moleculæ of the air contiguous to the sonorous body, receive from it an oscillatory and vibratory motion, which, in consequence of their elasticity, is successively transmitted to a certain distance. As the sonorous body is the centre from which the motion is communicated in every direction, the sound must necessarily become weaker in proportion as the mass of air, which receives it, becomes greater. The different sounds, of whatever nature, must be heard, because they are transmitted to the organ of hearing by analogous moleculæ of the air, in the same manner as when a certain tone is emitted in an apartment, it cannot be repeated but by those strings of the instrument which are in unison with it. Sounds of greater intensity cannot be propagated with more velocity, though the vibrations of the aerial moleculæ which transmit them be stronger, because they are always isochronous, like those of pendulums more or less removed from the vertical line, or strings more or less bent.

Of echoes—how produced. Account of the most remarkable echoes, and of some phenomena respecting them.

Echoes are well known; but however common this phenomenon may be, it must be allowed that the manner in which it is produced is involved in considerable obscurity; and that the explanation given of it does not sufficiently account for all the circumstances attending it.

All philosophers almost have ascribed the formation of echoes to a reflection of sound, similar to that experienced by light when it falls on a polished body; but, as D'Alembert observes, this explanation is false; if it were not, a polished surface would be necessary for the production of an echo;—but it is well known that this is not the case. Echoes indeed are frequently heard opposite to old walls, which are far from being polished; near shapeless masses of rock; and in the neighbourhood of forests, and even of clouds. This reflection of sound, therefore, is not of the same nature as that of light.

It is evident, however, that the formation of an echo can be ascribed only to the repercussion of sound; for echoes are never heard but when sound is intercepted and made to rebound by one or more obstacles.

Sound, as already said, is propagated in every direction by the vibration of the particles of the air; but if any column of air rests against some obstacle that prevents the direct movement of the elastic globules which serve as the vehicle of sound, it must rebound in a contrary direction, and striking the ear, if it meets with one in the line of repercussion, convey to it a repetition of the same sound, provided the original sound does not affect that organ at the same instant.

But we are taught by experience, that the ear does not distinguish the succession of two sounds, unless there be between them the interval of at least one-twelfth of a second; for during the most rapid movement of instrumental music, each measure of which cannot be estimated at less than a second, twelve notes are the utmost that can be comprehended in a measure, to render the succession of the sounds distinguishable; consequently the obstacle, which reflects the sound, must be at such a distance, that the reverberated sound shall not succeed the direct sound, till after one-twelfth of a second; and as sound moves at the rate of about 1142 feet in a second, and consequently about 95 feet in the twelfth of a second, it thence follows, that to render the reverberated sound distinguishable from the direct sound, the obstacle must be at the distance of no more than about 48 feet.

There are single and compound echoes. In the former, only one repetition of the sound is heard; in the latter, there are 2, 3, 4, 5, &c. repetitions. We are even told of echoes that can repeat the same word 40 or 50 times.

Single echoes are those where there is only one obstacle; but double, triple, or quadruple echoes, give us reason to suppose several obstacles disposed in such a manner, that the different reflected sounds strike the ear at times sensibly different.

There are some echoes that repeat several words in succession; but this is not astonishing, and must always be the case when a person is at such a distance from the echo, that there is sufficient time to pronounce several words before the repetition of the first has reached the ear.

There are certain echoes which have been much celebrated on account of their singularity, or of the number of times that they repeat the same word. Misson, in his description of Italy, speaks of an echo, in the vineyard of Simonetta, which repeated the same word 40 times.

At Woodstock, in Oxfordshire, there is an echo which repeats the same sound 50 times.

The description of an echo still more singular, near Roseneath, some miles distant from Glasgow, may be found in the Philosophical Transactions for the year 1698. If a person, placed at the proper distance, plays 8 or 10 notes of an air with a trumpet, the echo faithfully repeats them, but a third lower: after a short silence, another repetition is heard, in a tone still lower; and another short silence is followed by a third repetition, in a tone a third lower.

A similar phenomenon observed in some places is, that if a person stands in a certain position, and pronounces a few words with a low voice, they are heard only by another person standing in another determinate place: this arises from the elliptic form of arches, which have the property of collecting in one of their *foci* the rays that proceed diverging from the other.

The following phenomenon depends on the same theory.

To construct two figures, to be placed at the two ends of a hall, one of which shall repeat to the ear of a person what has been whispered into the ear of the other figure, without being heard by any other person in the hall.

Provide two heads or busts, made of pasteboard, resting on pedestals, and place them in a hall at such a distance from each other as you may think proper. Then convey a tube of tin-plate, an inch in diameter, from the ear of one of the figures, through the pedestal on which it rests, and below the flooring, till it reach the mouth of the other figure, passing through its pedestal in the same manner as that of the former; this tube must be a little wider, at each of its extremities, somewhat in the form of a funnel.

When it is necessary to bend this tube, care must be taken to cover the interior angles with a piece of tin-plate inclined at an angle of 45 degrees, that the voice may be directly reflected from one part of the tube to the other, and that the sound may be conveyed distinctly to the ear.

This construction will produce the following effect. If a person whispers into the ear of one of these figures, the words he pronounces will be distinctly heard by a second person who applies his ear to the mouth of the other figure.

The secret of the magic mirror, as it is called, depends on the same theory. The construction of this mirror is as follows:—

Fix, in a vertical position, a concave mirror, two feet in diameter, and of such a degree of curvature, that the focus of the rays which fall upon it, in a parallel direction, may be at the distance of twelve or fifteen inches from the reflecting surface. At this distance place a small figure, but in such a manner, that its head may be exactly in the focus.

This mirror must be placed at the distance of eight or ten feet from a wall opposite to it, and parallel to its surface: the wall must have in it an aperture, equal to the surface of the mirror, concealed by a very fine curtain, that the sound may easily pass through it. Provide also a second mirror of the same form, with a similar figure, and place it behind the wall at the distance of two or three feet from it, and opposite to the former, with the figure in its focus. It may be readily conceived, that when a person only whispers into the ear of the small figure behind the wall, a person standing near that placed in the focus of the opposite mirror, will hear very distinctly the words whispered into the ear of the former. In this manner, the person who asks a question, standing near the first figure, hears the answer which is whispered into the ear of the other behind the wall.

In order to conceal entirely the apparatus which produces this effect, and to render it much more extraordinary, the pretended concave magic mirror may be covered with a piece of gauze, which will not prevent the transmission of the sounds from the one focus to the other.

The memoirs of the Academy of Sciences at Paris, for the year 1692, speak of a very remarkable echo in the court of a gentleman's seat, called Le Genetay, in the neighbourhood of Rouen. It is attended with this singular phenomenon, that a person who sings or speaks in a low tone does not hear the repetition of the echo, but only his own voice; while, on the other hand, those who listen hear only the repetition of the echo, but with surprising variations; for the echo seems sometimes to approach and sometimes to recede, and at length ceases when the person who speaks removes to some distance in a certain direction. Sometimes only one voice is heard, sometimes several, and sometimes one is heard on the right, and another on the left. An explanation of all these phenomena, deduced from the semicircular form of the court, may be seen in the above collection.

EXPERIMENTS RESPECTING THE VIBRATIONS OF MUSICAL STRINGS, WHICH FORM THE BASIS OF THE THEORY OF MUSIC.

If a string of metal or cat-gut, such as is used for musical instruments, made fast at one of its extremities, be extended in a horizontal direction over a fixed bridge, and a weight be suspended from the other extremity, so as to stretch it; this string, when struck, will emit a sound produced by reciprocal vibrations which are sensible to the sight.

If the part of the string made to vibrate be shortened, and reduced to one-half of its length, any person who has a musical ear will observe, that the new sound is the octave of the former; that is to say, twice as sharp.

If the vibrating part of the string be reduced to two-thirds of the original length, the sound it emits will be the fifth of the first.

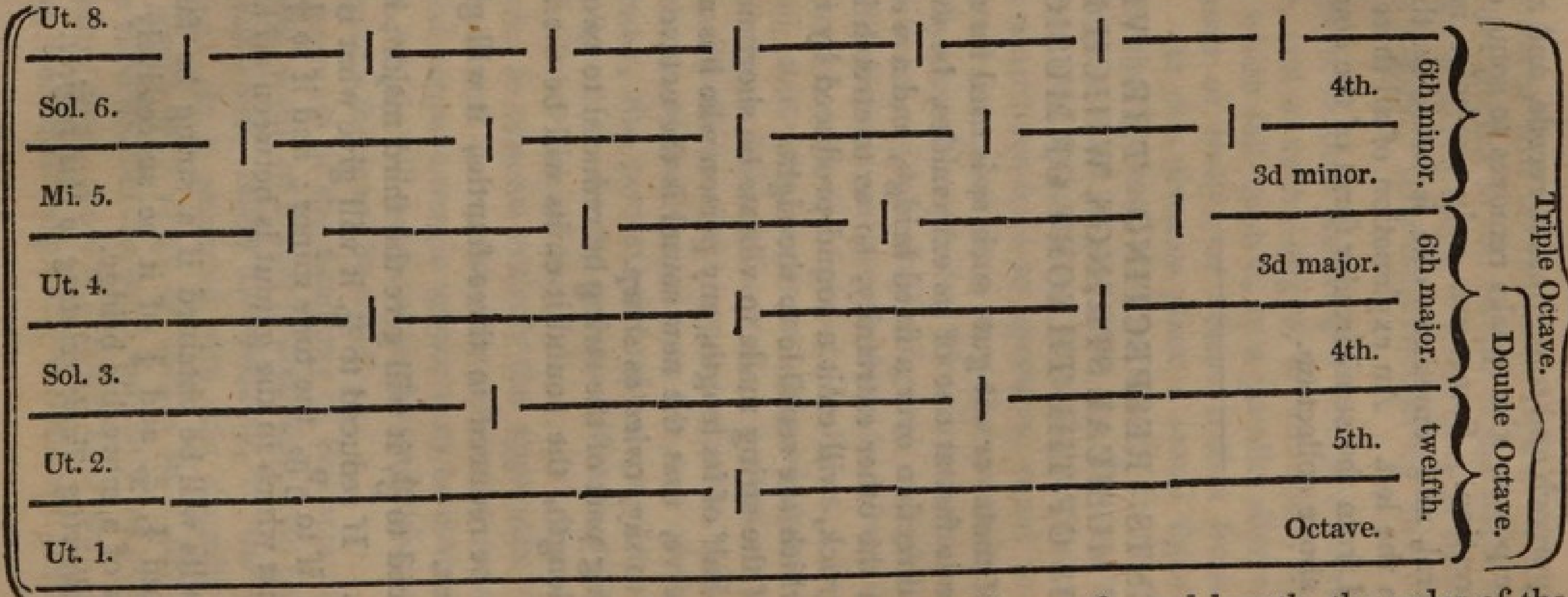
If the length be reduced to three-fourths, it will give the fourth of the first.

If it be reduced to $\frac{4}{5}$, it will give the third major; if to $\frac{5}{6}$, the third minor. If reduced to $\frac{8}{9}$, it will give what is called the tone major; if to $\frac{9}{10}$, the tone minor; and if to $\frac{15}{16}$, the semi-tone, or that which in the gamut is between *mi* and *fa*, or *si* and *sol*.

The same results will be obtained if a string be fastened at both ends, and $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of it be successively intercepted by means of a moveable bridge.

See the following Table relating to this subject.

Ingenious Manner in which Rameau *expresses the Relation of the Sounds in the Diatonic Progression.*



It may be here seen, that if these seven lines represent seven strings, of equal length, the order of the principal harmonic concords will be determined by the following numbers: Thus,

1 to 2	denotes the octave	5 to 6	denotes the third minor	5 to 8	denotes the sixth minor
2 to 3	the fifth	6 to 8	the fourth	1 to 4	the double octave
3 to 4	the fourth	1 to 3	the twelfth	1 to 8	the triple octave.
4 to 5	the third major	3 to 5	the sixth major		

Such is the result of a determinate degree of tension applied to a string, when the length of it has been made to vary. Let us now suppose that the length of the string is constantly the same, but that its degree of tension is varied. The following is what we are taught by experiment on this subject:—

If a weight be suspended at one end of a string of a determinate length, made fast by the other, and if the tone it emits be fixed, when another weight quadruple of the former is applied, the tone will be the octave of the former; if the weight be nine times as heavy, the tone will be the octave of the fifth; and so on: so that the tones will become acute in the ratio of the square roots of the weights.

The size of the strings has an effect in regard to the tones, as well as the different lengths of the string, and the weight by which it is stretched; for it is proved by experiment, that a string twice as small in diameter as another, every thing else being the same, emits a tone which is the octave of that of the other; and that if the diameter is only a third of that of the other, the tone is the octave of the fifth of that other string, following the order of the diatonic scale.

We may thence conclude, that the tones of the musical strings are in the direct ratio of the square root of the weights by which they are stretched, and in the inverse ratio of the lengths and diameters of these strings.

Consequently, to bring into unison strings which differ in length and diameter, and which are stretched by different weights, the compound ratio thence resulting must be exactly the same, in order that the frequency of the vibration in one may be compensated by the slowness of another. Thus, two strings of the same size, the lengths of which are as 2 to 1, and the stretching weights as 4 to 1, will have their vibrations isochronous, that is to say, they will be in unison: two strings, the diameters of which are as to 2 to 1, and the lengths as 1 to 2, stretched by equal weights, will be in unison also, as well as those which, being of equal lengths, have their diameters as 2 to 1, and the stretching weight as 4 to 1.

We may conclude, therefore, that two strings, the diameters of which are as 3 to 2, and the lengths as 1 to 3, cannot be in unison, unless the weights, by which they are stretched, be to each other in the same ratio as 1 to 4.

To determine the number of the vibrations made by a string of a given length and size, when stretched by a given weight.

A very ingenious method, invented by M. Sauveur, for finding the number of these vibrations, may be seen in the Memoirs of the Academy of Sciences for 1700. Having observed, when two organ-pipes, very low, and having tones very near to each other, were sounded at the same time, that a series of pulsations or beats were heard in the sounds; and by reflecting on the cause of this phenomenon, he found, that these beats arose from the periodical meeting of the coincident vibrations of the two pipes. Hence he concluded, that if the number of these pulsations which took place in a second, could be ascertained by a stop-watch, and if it were possible also to determine, by the nature of the consonance of the two pipes, the ratio of the vibrations which they made in the same time, he should be able to ascertain the real number of the vibrations made by each.

We shall here suppose, for example, that two organ-pipes are exactly tuned, the one to *mi* flat, and the other to *mi*: it is well known, that as the interval between these two tones is a semi-tone minor, expressed by the ratio of 24 to 25, the higher pipe will perform 25 vibrations while the lower performs only 24; so that at each 25th vibration of the former, or the 24th of the latter, there will be a pulsation: if 6 pulsations, therefore, are observed in the course of 1 second, we ought to conclude, that 24 vibrations of the one and 25 of the other take place in the tenth of a second; and consequently, that the one performs 240 vibrations, and the other 250, in the course of a second.

M. Sauveur made experiments according to this idea, and found that an open organ-pipe, 5 feet in length, makes 100 vibrations per second; consequently, one of 4 feet, which gives the lower triple octave, and the lowest sound perceptible to the ear, would make only $12\frac{1}{2}$; on the other hand, a pipe of one inch less $\frac{1}{16}$, being the shortest the sound of which can be distinguished, will give in a second 6400 vibrations. The limits, therefore, of the slowest and the quickest vibrations appreciable by the ear, are, according to M. Sauveur, $12\frac{1}{2}$ and 6400.

We shall not enlarge further on these details, but proceed to a very curious phenomenon respecting strings in a state of vibration.

Make fast a string by both its extremities, and by means of a bridge divide it into aliquot parts, for example, 3 on the one side, and 1 on the other, and put the larger part, that is to say, the $\frac{3}{4}$, in a state of vibration; if the bridge absolutely intercepts all communication from the one part to the other, these $\frac{3}{4}$ of the string, as is well known, will give the tone of the fourth of the whole string; if $\frac{4}{5}$ be intercepted, the tone will be the tierce major.

But if this bridge only prevents the whole of the string from vibrating, without intercepting the communication of motion from the one part to the other, the greater part will then emit only the same sound as the less, and the $\frac{3}{4}$ of the string, which in the former case gave the fourth of the whole string, will give only the double octave, which is the tone proper to the fourth of the string. The case is the same if this fourth be touched: its vibrations, by being communicated to the other three-fourths, will make them sound, but in such a manner as to give only the double octave.

The following reason, which may be rendered plain by an experiment, is assigned for this phenomenon: when the bridge absolutely intercepts all communication between the two parts of the string, the whole of the largest part vibrates together; and if it be $\frac{3}{4}$ of the whole string, it makes, agreeably to the general law, 4 vibrations in the time that the whole string would make 3: its sound therefore is the fourth of the whole string.

But, in the second case, the larger part of the string divides itself into 3 aliquot parts, each of which is equal to the less, and all these distinct portions perform their particular vibrations; for if bits of red paper, for example, be placed upon all the points of division, and bits of white paper in the middle of each division, the former will remain motionless, but the latter will drop off as soon as the string begins to vibrate.

If the part of the string immediately made to vibrate, instead of an aliquot part of the remainder, be only $\frac{2}{7}$ of it, the whole string will then divide itself into seventh parts, and will emit only that tone which belongs to $\frac{1}{7}$ of its length.

If the less part of the string be incommensurable to the greater, the sound will absolutely be discordant, and will almost immediately cease.

METHOD OF ADDING, SUBTRACTING, MULTIPLYING, AND DIVIDING CONCORDS.

It is necessary for those who wish to understand the theory of music, to know what concords result from two or more concords, either when added or subtracted, or when multiplied by each other. For this reason we shall give the following rules:

Problem I.—*To add one concord to another.*

Express the two concords by the fractions which represent them, and then multiply these two fractions together; that is to say, first the numerators and then the denominators: the number thence produced, will express the concord resulting from the sum of the two concords given.

Example I.—*Let it be required to add the fourth and fifth together.* The expression for the fifth is $\frac{2}{3}$, and that for the fourth $\frac{3}{4}$, the product of which is $\frac{6}{12} = \frac{1}{2}$, being the expression for the octave. It is indeed well known, that the octave is composed of a fifth and a fourth.

Example II.—*What is the concord arising from the addition of the third major and the third minor?* The expression of third major is $\frac{4}{5}$ and that of the third minor is $\frac{5}{6}$, the product of which is $\frac{20}{30}$ or $\frac{2}{3}$, which expresses the fifth; and this concord indeed is composed of a third major and a third minor.

EXAMPLE III.—*What is the concord produced by the addition of two tones major?* A tone major is expressed by $\frac{8}{9}$; consequently, to add two tones major, $\frac{8}{9}$ must be multiplied by $\frac{8}{9}$. The product $\frac{64}{81}$ is a fraction less than $\frac{64}{80}$, or $\frac{4}{5}$, which expresses the third major; hence it follows, that the concord expressed by $\frac{64}{81}$, is greater than the third major, and consequently, two tones major are greater than a third major, or form a third major false by excess.

On the other hand, by adding two tones minor, which are each expressed by $\frac{9}{10}$, it will be found that their sum $\frac{81}{100}$ is greater than $\frac{80}{100}$ or $\frac{4}{5}$, which denotes the third major: two tones minor therefore, added together, make more than a third major.

This third is, indeed, composed of a tone major and a tone minor, as may be proved by adding together the concords $\frac{8}{9}$ and $\frac{9}{10}$, which makes $\frac{72}{90} = \frac{8}{10}$ or $\frac{4}{5}$.

It might be proved, in like manner, that two semi-tones major make more than a tone major, and two semi-tones minor less even than a tone minor; and, in the last place, that a semi-tone major and a semi-tone minor make exactly a tone minor.

PROBLEM II.—*To subtract one concord from another.*

Instead of multiplying together the fractions which express the given concords, invert that which expresses the concord to be subtracted from the other, and then multiply them together as before: the product will give a fraction expressing the concord required.

EXAMPLE I.—*What is the concord which results from the fifth subtracted from the octave?* The expression of the octave is $\frac{1}{2}$; that of the fifth $\frac{2}{3}$, which inverted gives $\frac{3}{2}$; and if $\frac{1}{2}$ be multiplied by $\frac{3}{2}$ we shall have $\frac{3}{4}$, which expresses the fourth.

EXAMPLE II.—*What is the difference between the tone major and the tone minor?* The tone major is expressed by $\frac{8}{9}$, and the tone minor by $\frac{9}{10}$, which when inverted gives $\frac{10}{9}$:

the product of $\frac{8}{9}$ by $\frac{10}{9}$ is $\frac{80}{81}$, which expresses the difference between the tone major and the tone minor. This is what is called the *great comma.*

PROBLEM III.—*To double a concord, or to multiply it any number of times, at pleasure.*

In this case, nothing is necessary but to raise the terms of the fraction, which expresses the given concord, to the power denoted by the number of times it is to be multiplied; that is, to the square if it is to be doubled, to the cube if it be tripled, and so on.

Thus, the concord arising from the tone major tripled is $\frac{512}{729}$; for as the expression of the tone major is $\frac{8}{9}$, we shall have $8 \times 8 \times 8 = 512$ and $9 \times 9 \times 9 = 729$. This concord $\frac{512}{729}$ corresponds to the interval between *ut* and a *fa*, higher than *fa* sharp of the gamut.

PROBLEM IV.—*To divide one concord by any number at pleasure, or to find a concord which shall be the half, third, &c. of a given concord.*

To answer this problem, take the fraction which expresses the given concord, and extract that root of it which is denoted by the determinate divisor; that is to say, the square root, if the concord is to be divided into two; the cube root, if it is to be divided into three, &c.; and this root will express the concord required.

EXAMPLE.—As the octave is expressed by $\frac{1}{2}$, if the square root of it be extracted, it will give $\frac{7}{10}$ nearly; but $\frac{7}{10}$ is less than $\frac{1}{4}$, and greater than $\frac{2}{3}$, consequently the middle of the octave is between the fourth and the fifth, or very near *fa* sharp.

OF THE RESONANCE OF SONOROUS BODIES, THE FUNDAMENTAL PRINCIPLE OF HARMONY AND MELODY, WITH SOME OTHER HARMONICAL PHENOMENA.

EXPERIMENT I.—If you listen to the sound of a bell, especially when very grave, however indifferent your ear may be, you will easily distinguish, besides the principal sound, several other sounds more acute; but if you have an ear accustomed to appreciate the musical intervals, you will perceive that one of these sounds is the twelfth or fifth above the octave, and another the seventeenth major or the third major above the double octave. If your ear be exceedingly delicate, you will distinguish also its octave, its double and even its triple octave: the latter indeed are somewhat more difficult to be heard, because the octaves are almost confounded with the fundamental sound, in consequence of that natural sensation which makes us confound the octave with unison.

If the bow of a violoncello be strongly rubbed against one of its large strings, or the string of a trumpet marine, you will perceive the same effect. In a word, if you have an experienced ear, you will be able to distinguish these different sounds, either in the resonance of a string, or in that of any other sonorous body, and even in the voice.

Another method of making this experiment.—Suspend a pair of tongs by a woollen or cotton cord, or any other kind of small string, and twisting the extremities of it around the forefinger of each hand, put these two fingers into your ears. If the lower part of the tongs be then struck, you will first hear a loud and grave sound, like that of a large bell at a distance; and this tone will be accompanied by several others, more acute, among which, when they begin to die away, you will distinguish the twelfth and the seventeenth of the lowest tone. Rameau confirmed the truth of this phenomenon by the help of several organ-pipes.

This experiment respecting the resonance of sonorous bodies, is not new. It was known to Dr Wallis and to Mersesne, who speak of it in their works; but it appeared to them a simple phenomenon, with the consequences of which they were entirely unacquainted. Rameau first discovered the use of it, in deducing all the rules of musical composition, which before had been founded on mere sentiment, and on experience, incapable of serving as a guide in all cases, and of accounting for every effect. It forms the basis of his theory of thorough bass; a system which has been opposed with much declamation, but which, however, most musicians seem at present to have adopted.

All his harmony, therefore, is multiple, and composed of sounds which would give the aliquot parts of the sonorous body, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and we might add $\frac{1}{7}$, $\frac{1}{8}$, &c. But the weakness of these sounds, which go on always decreasing in strength, renders it difficult to distinguish them. Rameau, however, says, that he could distinguish very plainly the sound expressed by $\frac{1}{7}$, which is the double octave of a sound divided nearly into two equal parts, being the interval between *la* and *si* flat, below the first octave: he calls it a lost sound, and totally excludes it from harmony.

Experiment II.—If you adjust several strings to the octave, the twelfth, and the seventeenth, of the determinate sound emitted by another string, both ascending and descending; as often as you make that which gives the determinate sound to resound strongly, you will immediately see all the rest put themselves in a state of vibration: you will even hear those sounds which are tuned lower, if you take care to damp suddenly, by means of a soft body, the sound of the former.

Most people have heard the glasses on a table emit a sound when a person near them has been singing with a strong and a loud voice. The strings of an instrument, though not touched, are often heard to sound in consequence of the same cause, especially after swelling notes long continued.

In a manner somewhat similar, the diversity of tones agitates, in various ways, the fibres of our bodies, excites the passions, and produces in the soul sensations so different, *On the harmonical sounds heard with the principal sound: whether they have their source immediately in the sonorous body, or exist in the air or the organ?*

It is very probable that the principal sound is the only one that derives its origin immediately from the vibrations of the sonorous body. Philosophers of eminence have endeavoured to discover whether, independently of the total vibrations made by a body, there are also partial vibrations; but hitherto they have been able to observe only simple vibrations. Besides, how can it be conceived that the whole of a string should be in vibration, and that during its motion it should divide itself into two or three parts that perform also their distinct vibrations?

It must then be said, that these harmonical sounds of octave, twelfth, seventeenth, &c. are in the air or the organ: both suppositions are probable; for if such a determinate sound has the property of putting into a state of vibration bodies disposed to give its octave, its twelfth, &c. we must allow that this sound may put in motion the particles of the air, susceptible of vibrations of double, triple, quadruple, and quintuple velocity. What, however, appears most probable in this respect is, that these vibrations exist only in the ear: it seems indeed to be proved by the anatomy of this organ, that sound is transmitted to the soul only by the vibrations of those nervous fibres which cover the interior part of the ear; and as they are of different lengths, there are always some of them which perform their vibrations isochronous to those of a given sound. But, at the same time, and in consequence of the property above mentioned, this sound must put in motion those fibres susceptible of isochronous vibrations, and even those which can make vibrations of double, triple, quadruple, &c. velocity. Such, in our opinion, is the most probable explanation that can be given of this singular phenomenon.

Of the modern Music.

Every one knows that the gamut, or diatonic scale, is represented by the sounds *ut, re, mi, fa, sol, la, si, ut*, which complete the whole extent of the octave; and it appears, from the generation of it, as explained by Rameau, that from *ut* to *re* there is a tone major; from *re* to *mi* a tone minor; from *mi* to *fa* a semi-tone major; from *fa* to *sol* a tone major, as well as from *sol* to *la*; and in the last place, that from *la* to *si* there is a tone minor, and from *si* to *ut* a semi-tone major.

It is thence concluded, that in this scale there are three intervals which are not entirely just: these are,

1st, The third minor, from *re* to *fa*, which, being composed of a tone minor and a semi-tone major, is only in the ratio of 27 to 32; but this ratio is somewhat less than that of 5 to 6, which expresses exactly the third minor.

2d, The third major, from *la* to *fa*, is too high, being composed of two tones major; whereas, to be exactly in the ratio of 4 to 5, it ought to consist of a tone major and a tone minor.

3d, The third minor, from *la* to *ut*, is as far from being just as that of from *re* to *fa*, and for the same reason.

On the cause of the pleasure arising from Music.—The effects of Harmony on man and on animals.

It has often been asked, why two sounds, which form together the fifth and the third, excite pleasure, while the ear experiences a disagreeable sensation, by hearing sounds which are no more than a tone or a semi-tone distant from each other? Though it is difficult to answer this question, the following observations may tend to throw some light upon it.

Pleasure, we are told, arises from the perception of relations, as may be proved by various examples taken from the arts. The pleasure, therefore, derived from music, consists in the perception of the relations of sounds. But are these relations sufficiently simple for the soul to perceive and distinguish their order? Sounds will please when heard together in a certain order; but, on the other hand, they will displease if their relations are too complex, or if they are absolutely destitute of order.

This reasoning will be sufficiently proved by an enumeration of the known concords and discords. In unison, the vibrations of two sounds continually coincide, throughout the whole time of their duration; this is the simplest kind of relation. Unison also is the first concord in the octave: the two sounds of which it is composed perform their vibrations in such a manner, that two of the one are completed in the same time as one of the other: thus the unison is succeeded by the octave. It is so natural to man, that he who, through some defect in his voice, cannot reach a sound too grave or too acute, falls into the higher or lower octave.

When the vibrations of two sounds are performed in such a manner, that three of the one correspond to one of the other, these give the simplest relation next to those above mentioned. Who does not know, that the concord most agreeable to the ear is the twelfth, or the octave of the fifth? In that respect it even surpasses the fifth.

Next to the fifth, is the double octave of the fifth, or the seventeenth major, which is expressed by the ratio of 1 to 3. This concord, next to the twelfth, is the most agreeable.

The fourth, expressed by $\frac{3}{4}$, the third minor, expressed by $\frac{5}{6}$, and the sixth, both major and minor, expressed by $\frac{5}{8}$ and $\frac{3}{5}$, are concords, for the same reason.

But it appears that all the other sounds, after these relations, are too complex for the soul to perceive their order.

The following very strong objection, however, may be made to this reasoning. How can the pleasure arising from concords consist in the perception of them, since the soul often does not know whether such relations exist between the sounds? The most ignorant person is no less pleased with an harmonious concert than he who has calculated the relation of all its parts; what has hitherto been said, may therefore be more ingenious than solid.

We cannot help acknowledging, that we are rather inclined to think so; and it appears to us, that the celebrated experiment on the resonance of bodies may serve to account, in a still more plausible manner, for the pleasure arising from concords; because, as every sound degenerates into mere noise when not accompanied with its twelfth and its seventeenth major, besides its octaves, is it not evident, that when we combine any sound with its twelfth, or its seventeenth major, or with both at the same time, we only imitate the process of nature, by giving to that sound, in a fuller and more sensible manner, the accompaniment which nature itself gives it, and which cannot fail to please the ear, on account of the habit it has acquired of hearing them together? This is so agreeable to truth, that there are only two primitive concords, the twelfth and the seventeenth major; and that the rest, as the fifth, the third major, the fourth, and the sixth, are derived from them. We know also, that these two primitive concords are the most perfect of all, and that they form the most agreeable accompaniment that can be given to any sound; though on the harpsichord, for example, to facilitate the execution, the third major and the fifth itself, which, with the octave, form what is called perfect harmony, are substituted in their stead. But this harmony is perfect only by representation, and the most perfect of all would be that in which the twelfth and the seventeenth were combined with the fundamental sound and its octaves: Rameau, therefore, adopted it as often as he could in his choruses. We might enlarge farther on this idea; but what has been already said will be sufficient for every intelligent reader.

Some very extraordinary things are related in regard to the effects produced by the music of the ancients, which, on account of their singularity, we shall here mention. We shall then examine them more minutely, and shew that in this respect the modern music is not inferior to the ancient.

Agamemnon, it is said, when he set out on the expedition against Troy, being desirous to secure the fidelity of his wife, left with her a Dorian musician, who, by the effect of his airs, rendered fruitless for a long time the attempts of Ægisthus to obtain her affection; but that prince having discovered the cause of her resistance, got the musician put to death; after which he triumphed, without difficulty, over the virtue of Clytemnestra.

We are told also, that, at a later period, Pythagoras composed songs or airs capable of curing the most violent passions, and of recalling men to the paths of virtue and moderation. While the physician prescribes draughts for curing bodily diseases, an able musician might therefore prescribe an air for rooting out a vicious passion.

The story of Timotheus, the director of the music of Alexander the Great, is well known. One day, while the prince was at table, Timotheus performed an air in the Phrygian taste, which made such an impression on him, that being already heated with wine, he flew to his arms, and was going to attack his guests, had not Timotheus immediately changed the style of his performance to the Sub-Phrygian. This change calmed the impetuous fury of the monarch, who resumed his place at table. This was the same Timotheus, who at Sparta experienced the humiliation of seeing publicly suppressed four strings which he had added to his lyre. The severe Spartans thought that this innovation would tend to effeminate their manners, by introducing a more extensive and more variegated kind of music. This at any rate proves, that the Greeks were convinced that music had a peculiar influence on manners; and that it was the duty of government to keep a watchful eye over that art.

Who indeed can doubt that music is capable of producing such an effect? Let us only interrogate ourselves, and examine what have been our sensations on hearing a majestic or warlike piece of music, or a tender and pathetic air sung or played with expression. Who does not feel, that the latter tends as much to melt the soul and dispose it to pleasure, as the former to rouse and exalt it? Several facts in regard to the modern music, place it in this respect on a level with the ancient.

The modern music, indeed, has had also its Timotheus, who could excite or calm at his pleasure the most impetuous emotions. Henry III. king of France, having given a concert on occasion of the marriage of the Duke de Joyeuse, Claudin le Jeune, a celebrated musician of that period, executed certain airs, which had such an effect on a young nobleman, that he drew his sword, and challenged every one near him to combat; but Claudin, equally prudent as Timotheus, instantly changed to an air apparently Sub-Phrygian, which appeased the furious youth.

What shall we say of Stradella, the celebrated composer, whose music made the daggers drop from the hands of his assassins? Stradella having carried off the mistress of a Venetian musician, and retired with her to Rome, the Venetian hired three desperadoes to assassinate him; but fortunately for Stradella they had an ear sensible to harmony. These assassins, while waiting for a favourable opportunity to execute their purpose, entered the church of St John de Lateran during the performance of an oratorio, composed by the person whom they intended to destroy, and were so affected by the music that they abandoned their design, and even waited on the musician to forewarn him of his danger. Stradella, however, was not always so fortunate; other assassins, who apparently had no ear for music, stabbed him some time after at Genoa: this event took place about the year 1670.

Every body almost has heard, that music is a cure for the bite of the tarantula. This cure, which was formerly considered as certain, has by some been contested; but, however this may be, Father Schott in his works gives the tarantula air, which appears to be very dull, as well as that employed by the Sicilian fishermen to entice the thunny fish into their nets.—But it is probable, that fish are no great connoisseurs in music.

Various anecdotes are related respecting persons whose lives have been preserved by music effecting a sort of revolution in their constitutions. A woman being attacked for several months with the vapours, and confined to her apartment, had resolved to starve herself to death: she was, however, prevailed on, but not without great difficulty, to see a representation of the *Serva Padrona*, at the conclusion of which she found herself almost cured; and renouncing her melancholy resolution, was entirely restored to health by a few more representations of the like kind.

There is a celebrated air in Switzerland, called *Ranz des Vaches*, which had such an extraordinary effect on the Swiss troops in the French service, that they always fell into a deep melancholy when they heard it: Louis XIV. therefore, forbade it ever to be played in France, under the pain of a severe penalty. We are told of a Scotch air, which has a similar effect on the natives of Scotland.

Most animals, and even insects, are not insensible to the pleasure of music. There are few musicians, perhaps, who have not seen spiders suspend themselves by their threads in order to be near the instruments. We have several times had that satisfaction. We have seen a dog, who at the adagio of a sonata never failed to shew signs of attention, and some peculiar sensation, by howling.

The most singular fact, however, is that mentioned by Burney, in his History of Music. This author relates, that an officer being shut up in the Bastille, had permission to carry with him a lute, on which he was an excellent performer; but he had scarcely made use of it for three or four days, when the mice issuing from their holes, and the spiders suspending themselves from the cieling by their threads, assembled around him to participate in his melody. His aversion to these animals made their visit at first disagreeable, and induced him to lay aside this recreation; but he soon was so accustomed to them, that they became a source of amusement.

We have learned from persons worthy of credit, now in London, that, during their residence in the Levant, they have witnessed the influence of certain Greek songs on the oxen which the Greek farmers employ in agriculture.

Those who have seen at Bartholomew fair, in Smithfield, two elephants follow exactly the measure of the tunes played at the entrance of the place where they were kept, and humour all their variations by the motion of their head and trunk, will find no difficulty in believing what Buffon has said respecting the singular taste of these animals for harmony.

In a word, without deciding whether the fables of Amphion and Arion may not, in some measure, be founded on truth, we know that the noisy sound of trumpets, and the harmony of military instruments, excite the courage of soldiers and the ardour of horses; and the directors of caravans take care to be accompanied on their march by performers on different instruments, the music of which has such an effect on their camels, that they are better enabled to sustain the fatigue they must undergo in traversing the burning desarts of Arabia or Africa.

OF THE PROPERTIES OF CERTAIN INSTRUMENTS, AND PARTICULARLY WIND INSTRUMENTS.

WE are perfectly well acquainted with the manner in which stringed instruments emit their sounds; but erroneous ideas were long entertained in regard to wind instruments, such as the flute; for the sound was ascribed to the interior surface of the tube. The celebrated Euler first rectified this error, and it results from his researches,

1*st*, That the sound produced by a flute, is nothing else than that of the cylinder of air contained in it.

2*d*, That the weight of the atmosphere, which compresses it, acts the part of the stretching weight.

3*d*, That the sound of this cylinder of air, is exactly the same as that which would be produced by a string of the same mass and length, extended by a weight equal to that which compresses the base of the cylinder.

This fact is confirmed by experiment and calculation; for Euler found that a cylinder of air of 7½ Rhinlandish feet, at a time when the barometer is at a mean height, must give *c-sol-ut*; and such is nearly the length of the open pipe of an organ which emits that sound. The reason of its being generally made 8 feet, is because that length is required at those times when the weight of the atmosphere is greater.

Since the weight of the atmosphere produces, in regard to the sounding cylinder of air, the same effect as that produced by the weight which stretches a string, the more the weight is increased, the more will the sound be elevated; it is therefore observed, that during serene warm weather the tone of wind instruments is raised; and that during cold and stormy weather it is lowered. These instruments also become higher in proportion as they are heated; because the mass of the cylinder of heated air becoming less, while the weight of the atmosphere remains unchanged, the case is exactly the same as if a string should become less, and be still stretched by the same weight: every body knows that such a string would emit a higher tone.

But as stringed instruments must become lower, because the elasticity of the strings insensibly decreases, it thence follows, that wind and stringed instruments, however well tuned they may be to each other, soon become discordant.

A very singular phenomenon is observed in regard to wind instruments, such as the flute and huntsman's horn. With a flute, for example, when all the holes are stopped, if you blow faintly into the mouth aperture, a certain tone will be produced; if you blow a little stronger, the tone instantly rises to the octave, and by blowing successively with more force, you will produce the twelfth or fifth above the octave; then the double octave or seventeenth major.

Of some Musical Instruments or Machines, remarkable for their singularity or construction.

At the head of all these musical instruments, or machines, we ought doubtless to place the organ; the extent and variety of the tones of which would excite much more admiration, if it were not so common as it is in our churches; for, besides the artifice necessary to produce the tones by means of keys, what ingenuity must have been required to contrive mechanism for giving that variety of character to the tones obtained by means of the different stops, &c.? A complete description, therefore, of an organ, and of its construction, would be sufficient to occupy a large volume.

The ancients had hydraulic organs, that is to say, organs the sound of which was occasioned by air produced by the motion of water. These machines were invented by Ctesibus of Alexandria, and his scholar Hero. From the description of these hydraulic organs, given by Vitruvius in the tenth book of his architecture, Perrault constructed one which he deposited in the king's library, where the Royal Academy of Sciences held their sittings. This instrument, indeed, is not to be compared to the modern organs; but it is evident that the mechanism of it has served as a basis for that of ours. St Jerome speaks with enthusiasm of an organ which had twelve pair of bellows, and which could be heard at the distance of a mile. It thence appears, that the method employed by Ctesibus to produce air to fill the wind-box, was soon laid aside for one more simple; that is to say, for a pair of bellows.

The performer on the *tambour de basque*, and the automaton flute-player of Vaucanson, which were exhibited and seen with admiration in most parts of Europe, in the year 1749, may be classed among the most curious musical machines ever invented. We shall not, however, say any thing of the former of these machines, because the latter appears to have been far more complex.

The automaton flute-player performed several airs on the flute, with the precision and correctness of the most expert musician. It held the flute in the usual manner, and produced the tone by means of its mouth; while its fingers, applied on the holes, produced the different notes. It is well known, how the fingers might be raised by spikes fixed in a cylinder, so as to produce these sounds; but it is difficult to be conceived how that part could be executed which is performed by the tongue, and without which the music would be very defective. Vaucanson, indeed, confesses, that this motion in his machine was that which cost him the greatest trouble.

A very convenient instrument for composers, invented in Germany, consists of a harpsichord, which, by certain machinery added to it, notes down any air while a person is playing it. This is a great advantage to composers, as it enables them, when hurried away by the fervour of their imagination, to preserve what has successively received from their fingers a fleeting existence, and what otherwise it would often be impossible for them to remember. A description of this machine may be found in the Memoirs of the Academy of Berlin for the year 1773.

Of a new Instrument called the Harmonica.

This instrument was invented in America by Dr Franklin, who gave a description of it to father Beccaria, which the latter published in his works, printed in 1773.

It is well known, that when the finger, a little moistened, is rubbed against the edge of a drinking-glass, a sweet sound is produced; and that the tone varies according to the form, size, and thickness of the glass. The tone may be raised or lowered also by putting into the glass a greater or less quantity of water. Dr Franklin says, that an Irishman, named Puckeridge, first conceived the idea, about twenty years before, of constructing an instrument with several glasses of this kind, adjusted to the various tones, and fixed to a stand in such a manner, that different airs could be played upon them. Mr Puckeridge having afterwards been burnt in his house, along with this instrument, Mr Delaval constructed another of the same kind, with glasses better chosen, which he applied to the like purpose. About fourteen or fifteen years ago, an English lady at Paris performed, it is said, exceedingly well on this instrument, which, however, did not long continue in vogue: at present it is confined to cabinets and other musical curiosities.

A juggler, some years ago, to show his dexterity, placed on a table eight glasses of the same size, which had all the same tone, and boasted that he could tune them in an instant by pouring water into them, so as to play an air with the utmost precision. "Those who tune violins or organs, (said he), are not so dexterous as I; since they often labour for a quarter of an hour, and try the same pipe or string twenty times, before they can bring it to the proper tone." While he pronounced these words he poured water into the eight glasses; then striking them one after the other with a small rod, he immediately shewed that they emitted with great exactness the tones of the gamut, *ut, re, mi, fa, sol, la, si, ut*; and as he then amused the spectators by playing an air, which he accompanied with his voice, they overlooked the artifice he had employed in tuning his instrument so speedily.

Each of the glasses had a small hole at the proper height, so that when filled to the brim the water ran out, till there remained no more than the quantity requisite to give the glass the necessary tone. By these means, the instrument tuned itself in a moment; and the musician had no occasion to pour in or pour out water, at different times, to render the tone graver or more acute.

On what is called a False Voice.

A fine voice is certainly preferable to every instrument whatever. Unfortunately, many persons have only a false voice; but, in general, this does not arise from any defect in the organs of the voice, which are almost the same in all mankind: it originates from the ears, owing to an inequality of strength in these organs, or to some want of delicacy or tension, in consequence of which, as they receive unequal impressions, we necessarily hear false sounds, and the voice, which endeavours to imitate them, becomes itself false. On this subject Dr Vandermonde made a very simple experiment, which he relates in his Essay on Improving the Human Mind, and which may be repeated on children who pronounce with a false voice, in order that a remedy may be applied in that tender age, when the organs are still susceptible of modification.

The experiment, as he describes it, is as follows:—" I made choice (says he) of a clear day, and having fixed on a spacious apartment, I took up my station in a place judged most convenient for my experiments. I then stopped one of the ears of the child who was to be the subject of them, and made her recede from me, till she no longer heard the sound of a repeating watch which I held in my hand, or at least until the sound of the bell produced a very weak impression on her organs of hearing. I then desired her to remain in that place, and immediately going up to her unstopped her ear, and stopped the other, taking care to cause her to shut her mouth, lest the sound should be communicated to the ear through the eustachian tube. I then returned to my station, and making my watch again strike, the child was quite surprised to find that she heard tolerably well; upon which I made a sign to her to recede again till she could scarcely hear the sound."

It results from this experiment, that in the ears of persons who have a false voice, there is an inequality of strength, and the means of remedying this defect in children, is to ascertain by a similar mode which ear is the weakest.

" When this has been discovered, nothing better can be done, in my opinion, (says Dr Vandermonde), than to stop up the other as much as possible, and to take advantage of that valuable opportunity of frequently exercising the weak ear, but in such a manner as not to fatigue it. The one thus made to labour alone, will always retain the same force. The child's ear should from time to time be unstopped, in order to make it sing, and to discover whether both ears have the same degree of sensibility."

This natural defect may be then corrected, and any person may be made to acquire a true voice, provided the means pointed out by Dr Vandermonde be early employed.

Persons who have a false voice, in consequence of some inequality in the ears, may be compared to those who squint; that is to say, who, in order to see an object distinctly, do not turn equally towards it the axis of both eyes, because they have not the same visual powers. It is probable that the former, if they had early accustomed themselves to make use of only one ear, would hear distinctly different sounds which they would have imitated, and would not have contracted a false voice.

Of the Speaking Trumpet and Ear Trumpet.

As the sight is assisted by telescopes and microscopes, similar instruments have been devised also for assisting the faculty of hearing. One of these, called the speaking trumpet, is employed for conveying sound to a great distance: the other, called the ear trumpet, serves to magnify to the ear the least whisper.

Sir Thomas Morland, among the moderns, bestowed the most labour in endeavouring to improve this method of enlarging and conveying sound; and on this subject he published a treatise, entitled *De Tubâ Stentorophonica*, a name which alludes to the voice of Stentor, so celebrated among the Greeks for its great strength. The following observations on this subject are in part borrowed from that curious work.

The ancients, it would seem, were acquainted with the speaking trumpet, for we are told that Alexander had a horn, by means of which he could give orders to his whole army, however numerous. Kircher, on the authority of some passages in a manuscript preserved in the Vatican, makes the diameter of its greatest aperture to have been seven feet and a half. Of its length he says nothing, and only adds, that it could be heard at the distance of 500 stadia, or about 25 miles.

However this may be, the speaking trumpet is nothing else but a long tube, which at one end is only large enough to receive the mouth, and which goes on increasing in width to the other extremity, bending somewhat outwards. The aperture at the small end must be a little flattened to fit the mouth; and it ought to have two lateral projections to cover part of the cheeks.

Sir Thomas Morland says, that he caused several instruments of this kind to be constructed, of different sizes, viz. one of four feet and a half in length, by which the voice could be heard at the distance of 500 geometrical paces; another, 16 feet 8 inches, which conveyed sound 1800 paces; and a third, of 24 feet, which rendered the voice audible at the distance of 2500 paces.

The reason of this phenomenon is as follows:—As the air is an elastic fluid, so that every sound pronounced in it is transmitted spherically around the sonorous body, when a person speaks at the mouth of the trumpet, all the motion which would be communicated to a spherical mass of air, of four feet radius, for example, is communicated only to a cone, the base of which is the wider extremity of the trumpet. Consequently, if this cone is only the hundredth part of the whole sphere of the same radius, the effect will be the same as if the person should speak a hundred times as loud in the open air: the voice must therefore be heard at a distance a hundred times as great.

The ear trumpet, an instrument extremely useful to the deaf, is nearly the reverse of the speaking trumpet: it collects, in the auditory passage, all the sound contained within it; or it increases the sound produced at its extremity, in a ratio which may be said to be as that of the wide end to the narrow end. Thus, for example, if the wide end be six inches in diameter, and the aperture applied to the ear 6 lines, which in surfaces gives the ratio of 1 to 144, the sound will be increased 144 times, or nearly so; for we do not believe that this increase is exactly in the inverse ratio of the surfaces; and it must be allowed, that in this respect acoustics are not so far advanced as optics.

ELECTRICITY.

DEFINITIONS.

1. Electricity is that property in bodies which enables them, when excited by friction or heat, to attract other light bodies, and produce an effluvium that is sometimes luminous, attended with a snapping noise, and a faint phosphorical smell.

2. Electricity is called the second of the three species of attraction, gravity being the first, and magnetism the third.

3. Those bodies that produce electricity by friction or heat, are called electrics, and are said to be electric *per se*.

4. Those bodies that receive and communicate electricity are called conductors, and those that repel or will not suffer it to pass through them, are called non-conductors.

5. All bodies that are made to contain more than their natural quantity of electricity are said to be electrified positively, and those from whom part of their natural quantity is taken away, are said to be electrified negatively. These two electricities being first produced, one of them from glass, and the other from amber, wax, or rosin, the former was called vitreous, and the latter resinous electricity.

6. When a quantity of electricity is communicated to any body, it is said to be charged.

7. The effect of the explosion of a charged body, that is, the discharge of its electricity through any other body, is called the electric shock.

8. When any body is prevented from communicating with the earth, by the interposition of an electric body, it is said to be insulated.

9. The residuum of a charged body, as a jar or battery, is that part of the charge which remains in the body after the

first discharge, and by which it will give a second shock, though less than the first.

APHORISMS.

1. All substances are distinguished into electrics *per se*, and non-electrics: the latter of which are conductors, and the former non-conductors.

2. All kinds of metals, semi-metals, water, charcoal, and other bodies of a similar nature, are conductors; and all other bodies, whether mineral, vegetable, or animal, are non-conductors: many of the latter, however, may be made to conduct electricity by being heated to a certain degree.

3. Positive electricity is produced by the friction of uninsulated glass tubes or globes; and negative electricity is produced either from the rubber of those bodies, or from the friction of insulated glass bodies; or lastly, from the rubbing of globes or sticks of wax, sulphur, and other bodies of a similar nature.

4. It follows from the last aphorism, that the electricity of the excited body and the rubber are always opposite; that is, if that of the excited body be positive, that of the rubber will be negative; and the contrary. Those two bodies moreover will act on each other with greater force than any other body.

5. In charging any body, as a coated phial, if one side communicate with the excited body, and the other with the rubber, the electricity of the two sides of the charged body will be opposite.

6. There is a strong attraction between the two electricities on the opposite sides of a glass, so that when they are made to communicate by means of a conductor, they will be both discharged with a flash of light, and a snapping noise.

7. The substance of glass is impervious to electricity; but if the glass be thin, and the electricity on the opposite sides be very strong, that is, if the glass be overcharged, the opposite electricities will force a passage through the glass.

8. If an excited electric be in contact with an insulated conductor, the former will communicate its power to the latter, which will then attract light bodies, and give a spark, in the same manner as the excited electric.

9. The flash of light from a body to which electricity has been communicated, is more dense, and the sound louder, than from one that is excited; for the conductor parts with all its electricity at once, but the excited body with only so much as is at or near the part that is touched.

10. If insulated bodies have been attracted by, and have touched an excited body, they will soon after be repelled by that body, and will repel each other; nor will they return to the excited electric, till after they have touched some other body that communicates with the earth.

11. When an insulated conductor is brought within the sphere of action with an excited body, it acquires the electricity opposite to that of the body, and the nearer it is brought, the greater quantity it acquires, till the one receive a spark from the other, and then the electricity of both is discharged.

12. The electric explosion always takes the shortest course through the best conductors.

13. If the explosion between two bodies be interrupted by a non-conductor of a moderate density, the discharge will force a passage through it, in such a manner as to leave the appearance of a sudden expansion of the air about the centre of the explosion.

14. If an insulated conductor be pointed, or if an uninsulated conductor that is pointed, be brought very near the earth, there will be no other appearance of electricity during the time of excitation than a light, and a current of air, that may be perceived to come from those points.

15. The electric attraction acts *in vacuo*.

16. Electricity and lightning are in all respects of a similar nature.

17. All the effects of lightning may be imitated by electricity, and all the experiments in electricity may be performed by lightning, brought down from the clouds by means of an insulated pointed rod of metal, or by a kite.

Among the wonderful discoveries of human nature, there is hardly any that rank higher than electricity.

This phenomenon, like many others, was found out merely by accident; yet it has proved not only a source for various experiments, but likewise extremely beneficial to mankind.

The great Dr Franklin has improved more in this branch of knowledge than any other person. He even contrived to bring lightning from the clouds by means of conductors:—these conductors are of great service, when fixed to churches, and other public edifices, to preserve them from the dreadful effects of the rapidness of elemental fire.

When electricity is made use of physically, it is of great utility, and has been known to relieve, and sometimes entirely cure, various disorders. It is very serviceable in the rheumatism, and other chronic disorders.

"One circumstance," says Mr Gale, in his Recreations, "I shall mention, which I received from a gentleman who has been dead some years, but whose character as an artist and an ingenious person, will be a long time remembered; I mean Mr Benjamin Rackstrow, of Fleet-street.

"He told me, that having some company one day to see his museum and his electrical experiments, they were rather fearful of undergoing the shock; when a person who was much given to inebriety being in the room, and rather intoxicated, voluntarily offered to let the experiment be tried on him: this was agreed to, upon which he received it pretty smartly three or four times, and thought no more about it at that time. A few days afterwards he had occasion to go to Chichester, in Sussex, and being rather low in circumstances, was obliged to walk.

"This man had been affected for many years with a rupture, which was extremely troublesome; but on his journey he had not the least symptom of it: on which he wrote a letter to Mr Rackstrow, informing him of this agreeable circumstance, and imputing it entirely to his receiving the shock from his electrical apparatus. The man lived to confirm this by word of mouth; and what is really extraordinary, the rupture never returned: which is sufficient to establish its physical consequence. It is of farther service in palsies and contractions, and is performed by sparks, drawn by friction, from the electrical machine.

"Its real use being thus established, we may now, without offending, be a little merry with other circumstances which have and may happen again, by means of electricity.

"Some ladies and gentlemen, coming to Mr Rackstrow's, brought with them a negro servant who had not been long in England: after they had seen his natural and artificial curiosities, they desired to see some of his electrical experiments, and gave him a hint to play a trick or two upon poor Mungo. Mungo was not a little surprised at the shocks he received, but could not guess from whence they came; but when the room was darkened, and fire made to come out of his fingers' ends, he roared out like a mad bull, crying, the devil! the devil! And in endeavouring to get out of the room, overset the skeleton of a rhinoceros, run his head against a case of butterflies, and broke to pieces a fine bust of the Marquis of Granby; and having once more gained day-light, made a sudden spring into the street, and run immediately home, to the no small diversion of his master and family.

"Mrs Bulky being troubled with a tympany, was recommended to be electrified; she accordingly went to a professor in that way, who asked her if she could bear a pretty hard shock. O yes, sir, said she, as hard as you please, and as often as you please; I am very fond of being *shocked.* The man by this supposed she had before undergone the operation, and was not sparing to give her what she seemed so well to understand: but, alas! he wound up his instrument too high; so that he not only overset his patient, but actually conveyed her into a cellar where they sold ox-cheek and peas-soup:—down went the streaming pan full of savoury broth, and off flew her monument of a cap into the other boiling caldron.

"The cook reddened like a heated poker, the customers rose from their seats, and the greatest confusion took place in this subterraneous abode.

"All culinary business was at an end for the present; the electrical doctor came running to the assistance of his patient; but as soon as the cause of the disaster was explained, the occupier of the place declared the damages should be made good; her pan of leg of beef was entirely lost, her peas-soup spoiled by the powder and pomatum of the lady's head-dress,—the doctor was the cause of all, and he should pay for all; but he declared he would, sooner than pay a farthing, electrify the house till it fell about their ears.

"At last the lady, having adjusted herself in the best manner she could, gave the good woman a crown, and so compromised the matter; however, it cured her of her tympany, for she never went to the doctor afterwards.

"Many are the tricks played by means of an electrifying machine. A person in London had one in his shop, which was not seen by the passers-by; and he hung at the door an old steelyard, which, from its make, seemed to be very ancient: this attracted the attention and notice of many, who no sooner went to examine it, than they received the shock; those that knew what it was, only smiled and went on; others stared, and could not guess from whence it came.

"A drunken porter being called one day, and asked what he would have to carry the steelyard to a certain place, went to examine it; but he no sooner touched it than he felt a blow, and, turning round, with an oath declared, if he knew who it was, he would pay them well for their impudence. He then returned to speak about his job, and received another shock, and another after that; till, irritated by the supposed assaults, given by he could not tell whom, he stripped to the buff in order to fight all that came in his way, till he got a mob of boys and dogs at his heels, and was glad to get away at any rate.

"Such tricks are not recommended as proper to be practised, for they are really dangerous. A strange person might, on finding the truth, break the windows, or keep it in his mind, and do the electrifying gentleman some injury, which might make him repent of his experiments.

"Small electrical machines are often introduced in company, and create not only mirth, but produce real rational amusement; such can never be disagreeable, but must give satisfaction to all who have any idea of philosophical knowledge, and wish to improve their minds by mathematical experiments: to all such we may safely recommend the electrical apparatus, which will be both useful and profitable."

A description of all the machinery that has been used in electrical experiments would fill a volume. We therefore refer the reader to the numerous and laborious productions on the subject of electricity, where he may meet with ample descriptions of such apparatus, and hasten to detail some of the amusing experiments in this science.

We have divided the following amusements into such as are performed in the light, and such as require a dark chamber; beginning with the former.

The Animated Feather.

Electrify a smooth glass tube with a rubber, and hold a small feather (or piece of leaf-gold) at a short distance from it. The feather will immediately fly to the tube, and adhere to it for a short time, and then fly off, and the tube can never be brought close to the feather till it has touched the side of the room, or some other body that communicates with the ground. If, therefore, the operator take care to keep the tube constantly between the feather and the side of the room, he may drive it round to all parts without touching it; and what is very remarkable, the same side of the feather will be constantly opposite the tube.

While the feather is flying before the smooth tube, it will be immediately attracted by an excited rough tube, or a stick of wax, and fly continually from one tube to the other, till the electricity of both is discharged.

This was one of the first, and is one of the most common experiments in electricity; it is, however, very entertaining, and shows the nature of electric attraction and repulsion altogether as well as a more elaborate performance.

The Marvellous Fountain.

Suspend a vessel of water from the middle of the brass arch, and place in the vessel a capillary siphon. The water will at first issue by drops only, from the lower leg of the siphon; but when the vessel is put in motion, there will be one continued stream of water, and if the electrification be strong, a number of streams will issue in form of a cone, the top of which will be at the extremity of the tube. This experiment may be stopped and renewed, almost instantly, as if at the word of command.

The Magic Picture.

Have a large print, suppose of the king, with a frame and glass. Cut a panel out of the print at about two inches from the frame all round; with thin paste or gum fix the border that is cut off on the inside of the glass, pressing it smooth and close; then fill up the vacancy, by covering the glass well with leaf-gold, or thin tin-foil, so that it may lie close. Cover, likewise, the inner edge of the bottom part of the back of the frame with the same tin-foil, and make a communication between that and the tin-foil in the middle of the glass; then put in the board, and that side is finished. Turn up the glass, and cover the foreside with tin-foil, exactly over that on the backside, and when it is dry, paste over it the panel of the print that was cut out, observing to bring the corresponding parts of the border and panel together, so that the picture will appear as at first, only part of it behind the glass, and part before. Lastly, hold the print horizontally by the top, and place a little moveable gilt crown on the king's head.

Now if the tin-foil on both sides of the glass be moderately electrified, and another person take hold of the bottom of the frame with one hand, so that his fingers touch the tin-foil, and with the other hand endeavour to take off the crown, he will receive a very smart blow, and fail in the attempt. The operator who holds the frame by the upper end, where there is no tin-foil, feels nothing of the shock, and can touch the face of the king without danger, which he pretends to be a test of his loyalty. When a ring of persons take a shock among them, the experiment is called the conspirators.

The Tantalian Cup.

Place a cup or pot, of any sort of metal, on a stool of baked wood, or a cake of wax. Fill to the brim with any sort of liquor; let it communicate with the branch by a small chain, and when it is moderately electrified, desire a person to taste the liquor, without touching the cup with his hands, and he will immediately receive a shock at his lips; which, however, should not be very strong.

The motion of the wheel being stopped, you offer to taste the liquor yourself, and desire the rest of the company to taste it likewise, which they will do without any inconvenience. You then give the signal to the operator, and while you are amusing the company with discourse, the cup is again charged, and you desire the same person a second time to taste the liquor, when, to the no small diversion of the company, he will receive a second shock.

The Self-moving Wheel.

This wheel is formed of a thin round plate of window-glass, 17 inches diameter, well gilt on both sides, all but two inches next the edge. Two small hemispheres of wood are then fixed with cement to the middle of the upper and under sides, centrally opposite, and in each of them a thick strong wire, eight or ten inches long, which together make the axis of the wheel. It turns horizontally on a point at the lower end of its axis, which rests on a bit of brass cemented within a glass salt-cellar. The upper end of its axis passes through a hole in a thin brass plate, which keeps it six or eight inches distant from any non-electric, and has a small ball of wax or metal on the top, to keep in the fire.

In a circle on the table which supports the wheel are fixed twelve small pillars of glass, at about eleven inches distance, with a thimble on the top of each. On the edge of the wheel is a small leaden bullet, communicating by a wire with the gildings of the upper surface of the wheel; and about six inches from it is another bullet, communicating, in like manner, with the under surface. When the wheel is to be charged by the upper surface, a communication must be made from the under surface to the table.

When it is well charged it begins to move. The bullet nearest to a pillar moves towards the thimble on that pillar, and passing by, electrifies it, and then pushes itself from it. The succeeding bullet, which communicates with the other surface of the glass, more strongly attracts that thimble, on account of it being electrified by the other bullet; and thus the wheel increases its motion, till it is regulated by the resistance of the air. It will go half an hour, and make, one minute with another, 20 turns in a minute, which is 600 turns in the whole. The bullet of the upper surface gives in each turn 12 sparks to the thimbles, which makes 7200 sparks; and the bullet of the under surface receives as many from the thimbles, those bullets moving in the same time 2500 feet. The thimbles are well fixed, and in so exact a circle, that the bullets may pass within a very small distance of them.

If, instead of two bullets, you put eight, four communicating with the upper surface, and four with the under surface, placed alternately (which eight, at about six inches distance, complete the circumference), the force and celerity will be greatly increased, the wheel making 50 turns in a minute; but then it will not continue so long in motion.

The Magician's Chase.

On the top of a finely pointed wire, rising perpendicularly from the conductor, let another wire, sharpened at each end, be made to move freely, as on a centre. If it be well balanced, and the points be bent horizontally, in opposite directions, it will, when electrified, turn very swiftly round, by the reaction of the air against the current which flows from off the points. These points may be nearly concealed, and the figures of men and horses, with hounds and a hare or fox, may be placed upon the wires, so as to turn round with them, when they will look as if the one pursued the other. If the number of wires proceeding from the same centre be increased, and a still greater variety of figures be put upon them, the chase must be more diversified and entertaining. If the wire which supports the figures have another wire, finely pointed, rising from its centre, a second set of wires, furnished with another sort of figures, may be made to revolve above the former, and either in the same or the contrary direction, as the operator shall think fit.

If such a wire, pointed at each end, and the ends bent in opposite directions, be furnished, like a dipping-needle, with a small axis fixed in its middle, at right angles with the bending of the points, and the same be placed between two insulated wire strings, near and parallel to each other, so that it may turn on its axis freely upon and between them, it will, when electrified, have a progressive as well as circular motion, from one end of the wires that support it to the other; and this even up a considerable ascent.

The Planetarium.

From the branch suspend six concentric hoops of metal, at different distances from each other; and under them, on a stand, place a metal plate, at the distance of about half an inch. Then place upon the plate, within each hoop, and near to it, a round glass bubble, blown very light. These bubbles, and the distances between the hoops, should correspond to the different diameters of the planets, and those of their orbits: but as that cannot be, on account of the vast disproportion between them, it must suffice here to make a difference that bears some relation to them.

Now, the hoops being electrified, the bubbles placed upon the plate, near the hoops, will be immediately attracted by them; in consequence of which, that part of a bubble which touches a hoop will acquire some electric virtue, and be repelled. The electricity not being diffused over the whole surface of the glass, another part of the surface will be attracted, while the former goes to discharge its electricity upon the plate. This will produce a revolution of the bubble quite round the hoop, as long as the electrification is continued, and will be either way, just as the bubble happens to set out, or is driven by the operator. A ball hung over the centre of all the hoops, will serve to represent the sun in the centre of its system. If the room be darkened, the several glass balls will appear beautifully illuminated. This experiment affords a remarkable instance of electric attraction and repulsion.

The Incendiaries.

Let a person stand upon a stool made of baked wood, or upon a cake of wax, and hold a chain communicating with the branch: Upon turning the wheel, he will soon be electrified; his whole body, in reality, making a part of the prime conductor, and will exhibit the same appearances; emitting sparks wherever he is touched by any person standing on the floor. If the prime conductor be very large, the sparks may be rather painful than agreeable; but if it be small, the electrification moderate, and none of the company touch the eyes, or the more tender parts of the face, the experiment is diverting enough to all parties.

Many of the preceding experiments may be also performed to advantage by a person standing upon the stool as above, and holding in his hand what was directed to be fastened to the prime conductor. If he hold a large plumy feather in his hand, it is very pleasing to observe how it becomes turgid, its fibres extending themselves in all directions from the rib; and how it shrinks like the sensitive plants, when any unelectrified body touches it; when the point of a needle is presented to it, or to the prime conductor, with which he is connected.

If a dish, containing spirits of wine made warm, be brought to the electrified person, and he be directed to put his finger or a rod of iron into it, the spirit will be immediately in a blaze; and if there be a wick or thread in the spirit, that communicates with a train of gunpowder, he may be made to blow up a magazine, or set a city on fire with a piece of cold iron; and at the same time know nothing of what he is about.

An amusement of this sort may be performed by several persons, standing upon insulated stools, and many diverting circumstances may be added to those here mentioned. Care should be taken that the floor on which the stools stand be free from dust, but it is most eligible to have a large smooth board for that purpose.

The Inconceivable Shock.

Put into a person's hand a wire that is fixed on to the hook that comes from the chain which communicates with one side of the battery, and in his other hand put a wire with a hook at the end of it, which you direct him to fix on to the hook that comes from the other chain, which when he attempts, he will instantly receive a shock through his body, without being able to guess from whence it proceeds. The shock will be in proportion to the number of jars that are charged; but it is remarkable, that a small shock gives a much more pungent sensation in passing through the body, than one that is large.

This amusement may be diversified, and rendered still more entertaining, by concealing the chain that communicates with that which comes from the outside of the battery under a carpet, and placing the wire that communicates with the chain which comes from the inside, in such a manner that a person shall put his hand upon it without suspicion, at the same time that his feet are upon the other wire. Many other methods of giving a shock by surprise may be easily contrived; but great care should be taken, that these shocks be not too strong, and that they be not given to all persons indiscriminately.

When a single person receives a shock, the company is diverted at his sole expense; but all contribute their share to the entertainment, and all partake of it alike, when the whole company forms a circle, by joining their hands, and when the operator directs the person who is at one extremity of the circle to hold the chain which communicates with the coating, while he who is at the other extremity of the circle touches the other chain or wire. All the persons who form this circuit being struck at the same time, and with the same degree of force, it is often very pleasant to see them all start at the same moment, to hear them compare their sensations, and observe the very different accounts they give.

This experiment may be agreeably varied, if the operator, instead of making the company join hands, directs them to tread on each other's toes, or lay their hands on each other's heads. If, in the latter case, the whole company should be struck to the ground, as it once happened when Dr Franklin gave the shock to six very stout men, the inconvenience arising from it will be very little; the company that was struck immediately got up again, without knowing what had happened. This stroke was given with two jars, each of the measure of about six gallons, but not fully charged.

Magical Explosions.

We have shown in a preceding experiment how gunpowder may be fired by the intervention of spirits, but there is another method, more simple and expeditious, which we shall here describe. Make up gunpowder in the form of a small cartridge, in each end of which put a blunt wire, so that the ends within the cartridge may be about half an inch distant from each other; then joining the chain that comes from one side of the battery to one of the wires at the end of the cartridge, bring the chain that comes from the other side of the battery, to the wire at the other end, when the shock will instantly pass through the powder, and set it on fire.

By a similar method, fine brass or iron wire may be melted; for the explosion will pass from one chain to the other, through the wire, which will be first red-hot, and then melt into round drops. A battery of 35 jars has entirely destroyed fine brass wire of the 330th part of an inch in diameter, so that no particle of it could be found after the explosion. At the moment of the stroke, a great number of sparks, like those from a flint and steel, flew upward and laterally from the place where the wire was laid, and lost their light, in the day, at the distance of about two or three inches.

A stroke from a common jar will easily strike a hole through a thick cover of a book, or many folds of paper, leaving a remarkable bur or prominence on both sides, as if the fire had darted both ways from the centre.

The Prismatic Colour.

To the ends of each of the chains that come from the battery, fix an iron wire, and between those wires place a plate of tin, of about three inches square, and polished on one side, in a perpendicular direction.—The wire next the polished side should be finely pointed, and brought very near the surface of the plate.

By repeating the explosions of the battery, there will first appear a dusky red, about the edge of the central spot; presently after, generally after four or five strokes, there appears a circular space, visible only in an oblique position to the light, and looking like a shade on the plate: this expands very little during the whole course of the explosions. After a few more discharges, the second circular space is marked, by another shade beyond the first, of one-eighth or one-tenth of an inch in width, which never changes its appearance after any number of explosions. All the colours make their first appearance about the edge of the circular spot; more explosions make them expand toward the extremity of the space first marked out; while others succeed in their place, till, after 30 or 40 explosions, three distinct rings appear, each consisting of all the colours in the prism or rainbow.

It makes no difference whether the electricity issue from the pointed wire upon the plate, or from the plate upon the pointed wire, the surface opposite the point being marked exactly the same in both cases. The points themselves, from which the fire issues, or at which it enters, are coloured for about half an inch to a considerable degree; and the colours are repeated, as on the plate.

The innermost, that is, the last formed colours on the plate, are always the most vivid, and those rings are also closer to each other than the rest. Those colours may be brushed with a feather or the finger without injury, but they are easily peeled off by the nail, or any thing that is sharp.

The Artificial Spider.

Cut a piece of burnt cork, about the size of a pea, into the form of the body of a spider; make its legs of linen thread, and put a grain or two of lead into it to give it more weight. Suspend it by a fine line of silk between the electrified arch and an excited stick of wax, and it will, like a clapper between two bells, jump continually from one body to the other, moving its legs, at the same time, as if animated; to the no small surprise of those who are unacquainted with the electric influence.

The Artificial Earthquake.

In the middle of a large basin of water place a round wet board: this board represents the earth, and the water the sea.

On the board erect an edifice, composed of several separate pieces, which may represent a church, a castle, a palace, or, if you please, all of them.

Then placing a wire that communicates with the two chains of the battery, so that it may pass over the board and the surface of the water, upon making the explosion the water will become agitated, as in an earthquake, and the board, moving up and down, will overturn the structures it supports; at the same time that the cause of this commotion is totally concealed.

The Electrical Kite.

Take a large thin silk handkerchief, and extend it, by fastening the four corners to two slight strips of cedar. The handkerchief thus prepared and accommodated with a tail, loop, and string, will rise in the air like a common paper kite. To the top of the upright stick of the cross is to be fixed a pretty sharp pointed wire, rising a foot or more above the wood. To the end of the twine next the hand is to be tied a silk ribbon, and where the twine and silk join, a key or tin tube may be fastened.

This kite is to be raised when a thunder gust appears to be coming on, and as soon as the thunder clouds come over the kite, the pointed wire will draw the electricity from them, and the kite, with all the twine, will be electrified, the loose filaments of the twine will stand out every way, and be attracted by the finger. When the rain has wetted the kite and twine, so that it can conduct the electric fire freely, it will stream out plentifully from the key, on the approach of a man's knuckle. At this key a phial may be charged, and from the electric fire thus obtained, spirits may be kindled and all the other electric experiments performed, which are usually done by the help of a rubbed glass or tube, and thereby the identity of the electric matter with that of lightning completely demonstrated.

The Candle lighted by Electricity.

Charge a small coated phial, whose knob is bent outwards so as to hang a little over the body of the phial; then wrap some loose cotton over the extremity of a long brass pin or wire, so as to stick moderately fast to its substance: next roll this extremity of the pin which is wrapped up in cotton, in some fine powdered resin; then apply the extremity of the pin or wire to the external coating of the charged phial, and bring as quickly as possible the other extremity that is wrapped round with cotton to the knob: the powdered resin takes fire, and communicates its flame to the cotton, and both together burn long enough to light a candle. Dipping the cotton in oil of turpentine will do as well as if you use a larger sized jar.

Candle Bombs.

Procure some small glass bubbles, having a neck about an inch long, with very slender bores, by means of which a small quantity of water is to be introduced into them, and the orifice afterwards closed up. This stalk being put through the wick of a burning candle, the flame boils the water into a steam, and the glass is broken with a loud explosion.

Dancing Balls.

Take a common tumbler or glass jar, and having placed a brass ball in one of the holes of the prime conductor, set the machine in motion, and let the balls touch the inside of the tumbler; while the ball touches only one point, no more of the surface of the glass will be electrified, but by moving the tumblers about so as to make the ball touch many points successively, all these points will be electrified, as will appear by turning down the tumbler over a number of pith or cork balls placed on a table. These balls will immediately begin to fly about.

The Leyden Phial.

When a nail or a piece of thick brass wire, &c. is put into a small apothecary's phial and electrified, remarkable effects will follow; but the phial must be very dry or warm. Rub it once beforehand with your finger, on which put some pounded chalk. If a little mercury, or a few drops of spirit of wine, be put into it, the experiment succeeds the better.

As soon as this phial and nail are removed from the electrifying glass, or the prime conductor to which it has been exposed is taken away, it throws out a pencil of flame so long, that, with this burning machine in your hand, you may take about sixty steps in walking about your room. When it is electrified strongly, you may take it into another room, and there fire spirits of wine with it. If, while it is electrifying, you put your finger, or a piece of gold which you hold in your hand, to the nail, you receive a shock which stuns your arms and shoulders.

A tin tube, or a man placed upon electrics, is electrified much stronger by this means than in the common way. When you present this phial and nail to a tin tube, fifteen feet long, nothing but experience can make a person believe how strongly it is electrified. Two thin glasses have been broken by the shock of it. It appears extraordinary, that when this phial and nail are in contact with either conducting or non-conducting matter, the strong shock does not follow.

Rosin ignited by Electricity.

Wrap some cotton wool, containing as much powdered rosin as it will hold, about one of the knobs of a discharging rod. Then having charged a Leyden jar, apply the naked knob of the rod to the external coating, and the knob enveloped by the cotton to the ball of the wire. The act of discharging the jar will set fire to the rosin.

A piece of phosphorus or camphor wrapped in cotton wool, and used in the same way, will be much more easily inflamed.

Spirits ignited by Electricity.

Hang a small ball with a stem to the prime conductor, so that the ball may project below the conductor. Then warm a little ardent spirit, by holding it a short time over a candle in a metallic spoon; hold the spoon about an inch below the ball, and set the machine in motion. A spark will soon issue from the ball, and set fire to the spirits.

This experiment may be varied different ways, and may be rendered very agreeable to a company of spectators. A person, for instance, standing upon an electric stool, and communicating with the prime conductor, may hold the spoon with the spirits in his hand, and another person, standing upon the floor, may set the spirits on fire, by bringing his finger within a small distance of it. Instead of his finger, he may fire the spirits with a piece of ice; when the experiment will seem much more surprising. If the spoon be held by the person standing upon the floor, and the insulated person bring some conducting substance over the surface of the spirit, the experiment succeeds as well.

Electrified Air.

Fix two or three pointed needles into the prime conductor of an electrical machine, and set the glass in motion so as to keep the prime conductor electrified for several minutes. If, now, an electrometer be brought within the air that is contiguous to the prime conductor, it will exhibit signs of electricity, and this air will continue electrified for some time, even after the machine has been removed into another room. The air in this case is electrified positively; it may be negatively electrified by fixing the needles in the negative conductor while insulated, and making a communication between the prime conductor and the table, by means of a chain or other conducting substance.

The air of a room may be electrified in another way. Charge a large jar, and insulate it; then connect two or more sharp-pointed wires or needles with the knob of the jar, and connect the outside coating of the jar with the table. If the jar be charged positively, the air of the room will soon become positively electrified likewise; but if the jar be charged negatively, the electricity communicated by it to the air will become also negative. A charged jar being held in one hand, and the flame of an insulated candle held in the other being brought near the knob of the jar, will also produce the same effect.

To spin Sealing-wax into threads by Electricity.

Stick a small piece of sealing-wax on the end of a wire, and set fire to it. Then put an electrical machine in motion, and present the wax just blown at the distance of some inches from the prime conductor. A number of extremely fine filaments will immediately dart from the sealing-wax to the conductor, on which they will be condensed into a kind of net-work resembling wool.

If the wire with the sealing-wax be struck into one of the holes of the conductor, and a piece of paper be presented at a moderate distance from the wax, just after it has been ignited, on setting the machine in motion, a net-work of wax will be formed on the paper. The same effect, but in a slighter degree, will be produced, if the paper be briskly rubbed with a piece of elastic gum, and the melting sealing-wax be pretty near the paper immediately after rubbing.

If the paper, thus painted as it were with sealing-wax, be gently warmed by holding the back of it to the fire, the wax will adhere to it, and the result of the experiment will thus be rendered permanent.

The Electrified Camphor.

A beautiful experiment of the same nature is made with camphor. A spoon holding a piece of lighted camphor is made to communicate with an electrified body, as the prime conductor of a machine; while the conductor continues electrified by keeping the machine in motion, the camphor will throw out ramifications, and appear to shoot like a vegetable.

ELECTRICAL AMUSEMENTS IN THE DARK CHAMBER.

To exhibit a great number of pleasing and surprising amusements in the dark, as well as in the light, is the peculiar property of electricity: for though there are many beautiful experiments performed in the camera obscura, it is still by the aid of the sun's rays, or those of a candle or lamp; whereas, the electric apparatus contains within itself the particles of the fire by which these amusements are performed.

The Fiery Shower.

On the plate put a number of any kind of seeds, grains of sand, or brass dust. The conductor being strongly electrified, those light particles will be attracted and repelled by the plate suspended from the conductor, with amazing rapidity, so as to exhibit a perfect fiery shower.

Another way is, by a sponge that has been soaked in water. When this sponge is first hung to the conductor, the water will drop from it very slowly; but when it is electrified, the drops will fall very fast, and appear like small globes of fire, illuminating the basin into which they fall.

The Miraculous Luminaries.

To perform this amusement it is necessary to be provided with a quantity of the following phosphorus.—Calcine common oyster-shells by burning them in the fire about half an hour; then beat them into powder, of the clearest of which take three parts, and of flowers of sulphur one part, and put the mixture into a crucible about an inch and a half deep. Let it burn in a strong open fire for a full hour; when cool, turn it out, and break it into several pieces, and taking those pieces into a dark place, scrape off the brightest parts for use, which, when good, will be a white powder.

Then take a circular board of three or four feet diameter, on the centre of which draw the figure of the half-moon, of three or four inches diameter, and round it, at different distances, draw a number of stars of different magnitudes. On each of these figures fix the phosphorus just mentioned, to the thickness of about a quarter of an inch. The board being thus prepared, you must have ready a number of charged jars or phials, and by discharging one of them, at the distance of about an inch, over each figure, it will become illuminated. The light of the crescent will be so strong at first, that you may distinguish by it the figures on the dial of a watch. Round the board let there be placed a rim or hoop, and over that, at a sufficient distance from the figures, draw a curtain.

The board thus prepared is to be brought into the darkened room, and placed, by hooks, against the cieling. The curtain is then to be drawn back, and the moon and stars will then appear as emerging from behind a cloud, and will continue to shine for half an hour; the light, however, growing continually more faint.

The Globular Fires.

Let the room, and all the parts of the apparatus, be made very dry, and let the globe be strongly excited, so that the electricity may be very vigorous; the fire will then be seen to dart from the cushion toward the wire of the conductor. Sometimes these lucid rays (which are in part visible in daylight) will make the circuit of half the globe, and reach the wires; and they will frequently come in a considerable number, at the same time, from different parts of the cushion, and reach within an inch or two of the wires. The noise attending this beautiful phenomenon exactly resembles the crackling of bay-leaves in the fire. These lucid arches have frequently radiant points, often four or five in different parts of the same arch. These radiant points are intensely bright, and appear very beautiful. It is peculiarly pleasing to observe the circles of fire rise from those parts of the cushion where the amalgam or moisture has been put, or which have been lately scraped. Single points on the rubber will then appear intensely bright, and for a long time together will seem to pour out continual torrents of flame. If one part of the rubber be pressed closer than another, the circles will issue from that part more frequently than from any other.

When the conductor is taken quite away, circles of fire will appear on both sides the rubber, which will sometimes meet and completely encircle the globe. If, in that state, a finger be brought within half an inch of the globe, it is sure to be struck very smartly; and there will often be a complete arch of fire from it to the rubber, though it be almost quite round the globe.

If all the air be exhausted from the globe, the electricity will be found to act wholly within it, where it will appear in the form of a cloud or flame of reddish or purple-coloured light, filling the whole interior space of the globe.—When this amusement is finished, the globe and rubber must be taken away, that they may not incommode the apparatus of the following experiments.

The Illuminated Vacuum.

Take a tall receiver that is very dry, and through the top of it fix, with cement, a wire, not very acutely pointed. Then exhaust the receiver, and present the knob of the wire to the conductor, and every spark will pass through the vacuum, in a broad stream of light, visible through the whole length of the receiver, how tall soever it be. This stream often divides itself into a variety of beautiful rivulets, which are continually changing their course, uniting and dividing again in a most pleasing manner. If a jar be discharged through this vacuum, it gives the appearance of a very dense body of fire, darting directly through the centre of the vacuum, without ever touching the sides; whereas, when a single spark passes through, it generally goes more or less to the side, and a finger put outside of the glass will draw it wherever a person pleases. If the vessel be grasped by both hands, every spark is felt, like the pulsation of a large artery, and all the fire makes towards the hands. This pulsation is felt at some distance from the receiver, and a light is seen between the hands and the glass.

All this while the pointed wire is supposed to be electrified positively; if it be electrified negatively, the appearance is remarkably different. Instead of streams of fire, nothing is seen but one uniform luminous appearance, like a white cloud, or the milky way in a clear star-light night. It seldom reaches the whole length of the vessel, but generally appears only at the end of the wire, like a lucid ball.

If in the neck of a tall receiver a small phial be inserted, so that the external surface of the glass may be exposed to the vacuum, it will produce a very beautiful appearance. The phial must be coated on the inside, and while it is charging, at every spark taken from the conductor into the inside, a flash of light is seen to dart at the same time from every part of the external surface of the phial, so as to quite fill the receiver. Upon making the discharge, the light is seen to return in a much closer body, the whole coming out at once.

The Luminous Cylinder.

Provide a glass cylinder three feet long and three inches diameter: near the bottom of it fix a brass plate, and have another brass plate so contrived that you may let it down the cylinder, and bring it as near the first plate as you desire. Let this cylinder be exhausted and insulated, and when the upper part is electrified, the electric part will pass from one plate to the other, when they are at the greatest distance from each other the cylinder will admit. The brass plate at the bottom of the cylinder will moreover be as strongly electrified, as if it was connected by a wire with the prime conductor.

The electric matter in its passage through this vacuum is said to produce a delightful spectacle; not making, as in the open air, small brushes or pencils of rays, an inch or two in length, but coruscations of the whole length of the tube, and of a bright silver hue. These do not immediately diverge as in the open air, but frequently form a base that is apparently flat, dividing themselves into less and less ramifications, and very much resemble the most lively coruscations of the aurora borealis.

The Magical Constellations.

As the moon and stars in the zenith will become dull during the time of performing the preceding amusements, it will be proper to draw the curtain gently before them, that it may seem as if a cloud came slowly over them; and then the operator may, by his magical power, light up other constellations. In order to which, he must provide a large board, on which let him mark the stars that are in two or more constellations, which are contiguous and visible in the northern hemisphere, as Taurus, Gemini, &c.

To represent these stars, let there be a hole on each side of the spot that is marked for a star, about a quarter of an inch distant from each other, and let the extremities of two wires, neatly rounded, come through these holes, and be brought near together, exactly over the mark. These wires should be of different sizes, that they may the better represent the different magnitudes of the stars.

The other ends of the wires must be so disposed, that they may all receive a spark from the conductor at the same time, and the stars will then be all luminous at the same instant. These stars are not evanescent, like those made by the phosphorus, but will continue with equal splendour, as long as the motion of the wheel is continued. After the same manner any cipher, or the outlines of a drawing, may be exhibited.

The Aurora Borealis.

Make a Toricellian vacuum in a glass tube, about three feet long, and seal it hermetically:* it will then be always ready for use. Let one end of this tube be held in the hand, and the other be applied to the conductor, and immediately the whole tube will be illuminated from end to end; and when taken from the conductor will continue luminous, without interruption, for a considerable time, very often above a quarter of an hour. If, after this, it be drawn through the hand either way, the light will be uncommonly intense, and without the least interruption from one hand to the other, even to its whole length. After this operation, which discharges it in a great measure, it will still flash at intervals, though it be held only at one extremity, and quite still; but if it be grasped by the other hand at the same time, in a different place, strong flashes of light will hardly ever fail to dart from one end to the other; and this will continue 24 hours, and perhaps much longer, without fresh excitation. Small and long glass tubes exhausted of air, and bent in many irregular crooks and angles, will, when properly electrified, beautifully represent flashes of lightning.

The Circulating Lamps.

After keeping the company thus long in the dark, it will be proper to illuminate the room before you dismiss them.

* The Toricellian vacuum is made by filling a tube with pure mercury, and then inverting it, in the same manner as in making a barometer; for as the mercury runs out, all the space above will be a true vacuum. A glass is hermetically sealed by holding the end of it in the flame of a candle, till it is ready to melt, and then twisting it together with a pair of pincers.

In order to which, introduce the circulating wheel described in page 167. To the upper axis of which let there be fixed a number of radii, made of baked wood, at the end of each of which must hang a small globular lamp, filled with spirits; and let that of each lamp be tinged with a different colour. The wheel, having previously acquired its greatest velocity, is to be placed on the table, and a chain, depending from the branch, is to dip into each lamp as it passes by; so that all of them will become illuminated in a very short time. These lamps will not only enlighten the room, but by their variegated colours, and continual revolution, afford a very pleasing phenomenon.

MAGNETISM.

DEFINITIONS.

1. Magnetism is the science that explains the several properties of the attractive and repellent powers in the magnet or loadstone.

2. The magnet is a rich, heavy, iron ore, of a hard substance, a dusky grey colour, with some mixtures of a reddish-brown, and sparkling when broke.

3. The magnetic virtue is called the third species of attraction; gravity being the first, and electricity the second.

4. The two ends of a magnet, when it is properly formed, are called its poles; and when it is placed on a pivot, in just equilibrium, one end will turn toward the north, and is called its north pole, and the other end the south pole.

5. When the two poles of a magnet are surrounded with plates of steel, it is said to be armed.

6. If the end of a small iron bar be rubbed against one of the poles of a magnet, it is said to be touched, and is then called an artificial magnet.

7. If such a magnet be supported on a pivot, it is called a magnetic needle; one end of it turning toward the north, and the other toward the south.

8. The difference between the position of the needle, and the exact points of north and south, is called its declination.

9. A needle which is touched will incline toward the earth, and that is called its inclination or dipping.

APHORISMS.

1. The magnetic attraction is produced by effluvia emitted by the magnet, and passing from one pole to the other.

2. One pole of a magnet will attract iron, and the other repel it, but no other body.

3. The magnet attracts iron as well in vacuo as in the air.

4. The magnetic attraction will be continued through several pieces of iron placed contiguous to each other.

5. The magnetic effluvia pervades all bodies.

6. The magnetic attraction extends to a considerable distance.

7. The north pole of one magnet will attract the south pole of another; and the similar poles will repel each other.

8. The end of a needle touched by the north pole of a magnet will turn south, and that touched by the south pole will turn north.

9. The declination of the magnetic needle is different in different parts of the earth, and in the same part at different times.

10. The inclination of the needle is not always the same in different places, nor in the same place at different times.

11. The strength of natural magnets differs in those of different magnitudes, but not in proportion to their magnitudes.

12. The strength of a natural magnet is considerably increased by its being armed.

13. Iron acquires a magnetic power by being continually rubbed in the same direction.

14. Iron bars become magnetic by standing a long time nearly upright.

15. The magnetic virtue may be communicated by electricity.

16. A strong blow at one end of a magnetic bar will give it a magnetic power.

17. Fire totally destroys the power of magnets, as well natural as artificial.

The Magnetic Wand.

Bore a hole, three-tenths of an inch diameter, through a round stick of wood; or get a hollow cane about eight inches long and half an inch thick. Provide a small steel rod, and let it be very strongly impregnated with a good magnet: this rod is to be put in the hole you have bored through the wand, and closed at each end by two small ends of ivory that screw on, different in their shapes, that you may better distinguish the poles of the magnetic bar.

When you present the north pole of this wand to the south pole of a magnetic needle, suspended on a pivot, or to a light body swimming on the surface of the water (in which you have placed a magnetic bar), that body will approach the wand; and present that end which contains the south end of the wand, to the north or south end of the needle, it will recede from it.

The Mysterious Watch.

You desire any person to lend you his watch; and ask him if it will go when laid on the table. He will, no doubt, say it will; in which case you place it over the end of the magnet, and it will presently stop. You then mark the precise spot where you placed the watch, and, moving the point of the magnet, you give the watch to another person, and desire him to make the experiment; in which he not succeeding, you give it to a third (at the same time replacing the magnet), and he will immediately perform it.

This experiment cannot be effected, unless you use a very strongly impregnated magnetic bar, (which may be purchased at the optician's); and the balance of the watch must be of steel, which may be easily ascertained by previously opening it, and looking at the works.

The Magnetic Dial.

Procure a circle of wood or ivory, of about five or six inches diameter, which must turn quite free on a stand with a circular border; on the ivory or wood circle fix a pasteboard, on which you place, in proper divisions, the hours as on a dial. There must be a small groove in the circular frame to receive the pasteboard circle, and observe, that the dial must be made to turn so free, that it may go round without moving the circular border in which it is placed.

Between the pasteboard circle and the bottom of the frame, place a small artificial magnet, that has a hole in its middle. On the outside of the frame, place a small pin, which serves to shew when the magnetic needle is to stop. This needle must turn quite free on its pivot, and its two sides should be in exact equilibrio.

Then provide a small bag, with five or six divisions, like a lady's work-bag, but smaller. In one of these divisions put small square pieces of pasteboard, on which are written the numbers from 1 to 12. In each of the other divisions put twelve or more similar pieces, observing that all the pieces in each division must be marked with the same number. The needle being placed upon its pivot, and turned quickly about, it will necessarily stop at that point where the north end of the magnetic bar is placed, and which you previously know, by the situation of the small pin in the circular border.

You then present to any person that division of the bag which contains the several pieces on which is written the number opposite to the north end of the bar, and tell him to draw any one he pleases. Then placing the needle on the pivot, you turn it quickly about, and it must necessarily stop at that particular number.

The Magnetic Cards.

Draw a pasteboard circle; you then provide yourself with two needles, similar to those used in the foregoing experiment, (which you must distinguish by some private mark), with their opposite points touched with the magnet. When you place that needle, whose pointed end is touched, on the pivot described in the centre of the circle, it will stop on one of the four pips against which you have placed the pin in the frame; then take that needle off, and placing the other, it will stop at the opposite point.

Having matters thus arranged, desire a person to draw a card from a piquet pack, offering that card against which you have placed the pin of the dial, which you may easily do, by having a card a little longer than the rest. If he should not draw it the first time, as he probably may not, you must make some excuse for shuffling them again; such as letting the cards fall, as if by accident, or some other manœuvre, till he fixes on the card. You then tell him to keep it close, and not let it be seen. Then give him one of the two needles, and desire him to place it on the pivot, and turn it round, when it will stop at the colour of the card he chose; then taking that needle off, and exchanging it unperceived for the other, give it to a second person, telling him to do the same, and it will stop at the name of the identical card the first person chose.

The Communicative Crown.

Take a crown piece, and bore a hole in the side of it; in which place a piece of wire, or a large needle well polished, and strongly touched with a magnet.—Then close the hole with a small piece of pewter, that it may not be perceived. Now the needle in the Magnetic Perspective, when it is brought near to this piece of money, will fix itself in a direction correspondent to the wire or needle in that piece.

Desire any person to lend you a crown-piece, which you dexterously change for one that you have prepared as above. Then give the latter piece to another person, and leave him at liberty either to put it privately in a snuff-box or not. He is then to place the box on a table, and you are to tell him, by means of your glass, if the crown is, or is not in the box. Then bringing your perspective close to the box, you will know, by the motion of the needle, whether it be there or not; for as the needle in the perspective will always keep to the north of itself, if you do not perceive it has any motion, you conclude the crown is not in the box. It may happen, however, that the wire in the crown may be placed to the north, in which case you will be deceived. Therefore, to be sure of success, when you find the needle in the perspective remain stationary, you make some pretence to desire the person to move the box into another position, by which you will certainly know if the crown-piece be there or not.

You must remember that the needle in the perspective must here be very sensible, as the wire in the crown cannot possibly have any great attractive force.

The Magnetic Table.

Under the top of a common table place a magnet that turns on a pivot, and fix a board under it, that nothing may appear. There may also be a drawer under the table, which you pull out to show that there is nothing concealed. At one end of the table there must be a pin that communicates with the magnet, and by which it may be placed in different positions: this pin must be so placed as not to be visible by the spectators. Strew some steel filings, or very small nails, over that part of the table where the magnet is. Then ask any one to lend you a knife, or a key, which will then attract part of the nails or filngs, in the same manner as the iron attracts the needle. Then placing your hand in a careless manner on the pin at the end of the table, you alter the position of the magnet; and giving the key to any person, you desire him to make the experiment, which he will then not be able to perform. You thên give the key to another person, at the same time placing the magnet, by means of the pin, in the first position, when that person will immediately perform the experiment.

The Incomprehensible Card.

Insert in the middle of a card, and parallel to its two longest sides, part of a watch-spring, as thin as possible, and strongly impregnated: let it be so concealed as not to afford the least suspicion. This card should be a little longer than the others of the pack in which it is placed.

Offer any one to draw a card out of the pack, and present the long card dexterously to his hand. You then give him all the cards, and leave him to replace that card in the pack or not. He is then to lay the pack on the table, and by applying your magnetic perspective, you will discover whether the card be there or not.

If the person should not draw that card, you must be ready with some other experiment, to prevent suspicion of having failed in your design.

PNEUMATICS.

DEFINITIONS.

1. The atmosphere is that body of air which every-where surrounds the earth.

2. The air-pump is a machine contrived to produce a vacuum, by exhausting the air out of the vessel called a receiver.

3. The condenser is an instrument generally in form of a syringe, to force a greater quantity of air into any vessel than it naturally contains.

4. The animometer is an instrument that measures the strength of the wind.

5. The hygrometer is contrived to show the different degrees of moisture in the atmosphere at different times.

6. The thermometer measures the degrees of heat and cold of the air, and of other bodies.

7. The barometer shows the different weight of the air at different times.

APHORISMS.

1. The air is an elastic, ponderating, compressible, and expansible fluid, that is insensible only to the touch.

2. The elasticity of the air is increased by heat and decreased by cold.

3. The weight of the air is so small as not to be perceived but in large quantities.

4. The rarefaction and condensation of the air are indefinite.

5. Though air is greatly condensible by cold, it cannot be congealed.

6. Air is necessary to animal existence.

7. Adust air, that is, such as has passed through the fire or a heated tube, will not support animal life.

8. Air is contained in almost all bodies, and may be produced from them.

9. Sound is communicated by the air.

10. The atmosphere is of different densities at different heights, and is most dense near the earth.

11. The height of the atmosphere does not exceed 50 miles.

12. Wind is nothing but a current of air.

13. The velocity of the wind is from 1 to 60 miles in an hour.

To describe the numerous apparatus necessary for experimenting on air, (among which the air-pump, the animometer, and hygrometers, are the most conspicuously useful), would occupy more space than the limits of our small volume can allow. We shall therefore refer the reader to works professedly devoted to researches in Natural Philosophy, for descriptions and illustrations of these instruments, and proceed to enumerate a few amusing experiments in this branch of science.

The Bottle broke by Air.

Take a bottle that is square, not round or cylindrical; and if it be small, the glass must be thin. Put the mouth of this bottle over the hole in the place of the air-pump, and exhaust the air. By this means the bottle will be made to sustain the weight of the external air as long as it is able, but at last it will be suddenly burst into very small parts.

The same effect may be produced by the spring of the air, in the following manner. Seal the mouth of a bottle so close that not the least air can come out, and place it in the receiver; then as the air is drawn off from its surface, the spring of the included air will act against the sides of the bottle, and will continually increase as the air in the receiver becomes more rarefied, till at last it burst the bottle in pieces.

A similar effect is produced by laying a plate of glass on the top of an open receiver, and exhausting the air; for then the weight of the external air will press upon the glass and break it in pieces. In like manner, if a person lay his hand upon an open receiver, and the air be exhausted, his hand will be fixed to the receiver: for if the aperture of the receiver be four square inches, the weight on his hand will be equal to 60 pounds. This experiment will be attended with some pain in the person's hand.

The Brass Hemisphere.

Take two hemispheres of about four inches diameter, and whose circumferences exactly fit each other. Now, when they are placed together, and the air is exhausted from their cavities, the internal spring taken away, they will be pressed by a column of air equal to their surfaces, that is, twelve square inches and a half, which, multiplied by 15 pounds, the weight of the air on every inch, the sum will be 187 pounds and a half.

Therefore, give these hemispheres to any two persons, after they have seen them put together, and that they are not in any manner joined to each other, and desire them to pull the hemispheres asunder; to effect which, they must, between them, exert a force equal to the above number of pounds.

Water boiled by Air.

Take water that is made as warm as you can well bear to put your hand in it, but that has not boiled, and putting it under the receiver, exhaust the air.—Bubbles of air will soon be seen to rise, at first very small, but presently become larger, and will be at last so great, and rise with such rapidity, as to give the water all the appearance of a violent boiling. This agitation of the water will continue till the air is again let into the receiver, when it will immediately cease, and the water become quite motionless.

The Aerial Bubbles.

Take a piece of iron, brass, stone, or any other heavy substance, and putting it in a large glass with water, place it in the receiver. The air being exhausted, the spring of that which is in the pores of the solid body, by expanding the particles, will make them rise on its surface in numberless globules, which resembling the pearly drops of dew on the tops of the grass, afford a very pleasing appearance. On letting the air into the receiver, all these aerial forms immediately disappear.

The Floating Stone.

To a piece of cork tie a small stone, that will just sink it, and putting it in the vessel of water, place it under the receiver. Then exhausting the receiver, the bubbles of air which expand from its pores, and adhere to its surface, will render it, together with the stone, lighter than water, and consequently they will rise to the surface and float.

The Vegetable Air Bubbles.

Put a small branch of a tree with its leaves, or part of a small plant, in a vessel of water, and placing the vessel in the receiver, exhaust the air. When the pressure of the external air is taken off, the spring of that contained in the air vessels of the plant, by expanding the particles, will make them rise from the orifices of all the vessels for a long time together, and produce a beautiful appearance. This experiment shows how great a quantity of air is contained in every vegetable substance.

The Mercurial Rod.

Take a piece of stick, cut it even at each end with a penknife, and immerse it in a vessel of mercury. When the air is pumped out of the receiver, it will, at the same time, come out of the pores of the wood, through the mercury, as will be visible at each end of the stick. When the air is again let into the receiver, it falls on the surface of the mercury, and forces it into the pores of the wood, to possess the place of the air.

When the rod is taken out and weighed, it is found to be several times heavier than before, and has changed its colour, being now all over of a bluish hue. If this stick be cut transversely, the quicksilver will be seen to glitter in every part of it.

The Mystical Bell.

Fix a small bell to the wire that goes through the top of the receiver, and, shaking it by that wire, it will be distinctly heard while the air is in the receiver. As the air is exhausted, the ringing becomes gradually weaker, and at last, how much soever the bell be shook, the least sound cannot be heard. But when the air begins to enter again into the receiver, the sound becomes presently audible. This experiment proves that air is the medium of sound.

The self-moving Wheel.

Take a circle of tin, about ten inches diameter, or of any other dimension that will go into the receiver, and to its circumference fix a number of tin vanes, each about an inch square. Let this wheel be placed, between two upright pieces, on an axis whose extremities are quite small, so that the wheel may turn, in a vertical position, with the least force possible. Place the wheel and axis in the receiver, and exhaust the air. —Let there be a small pipe with a cock; one end of this pipe is to be on the outside of the top of the receiver, and the other end to come directly over the vanes of the wheel.

When the air is exhausted from the receiver, open the cock just mentioned. A current of air will rush against the vanes of the wheel, and put it in motion; and the velocity of its motion will increase till the receiver is again replete with air.

If the pump be kept continually working, after the air is exhausted, the motion of this wheel may be regarded not only as spontaneous, but perpetual.

The animated Figures.

Provide nine, twelve, or any number you please, of hollow cylinders, about nine inches long, and one and a half or two inches diameter. Let the bottom of each of these cylinders be closed, except a small hole; and in each of them place a piston, like that in a syringe. At the bottom of each piston let there be a worm spring, and over it the figure of a man, woman, or what else you please. These figures should be all different, and in different attitudes, and of such a size that they may completely enter the cylinders.

Place all the cylinders in a circular frame of wood, and having pushed each piston down to the bottom of the cylinder, and stopped the holes at bottom, draw it up again to what height you think proper, and there will then be a vacuum under each piston. Then place the frame in the receiver, and exhaust the air.

When the weight of the external air begins to be taken off, the force of the spring that is at the bottom of each piston being greater than its friction, and the weight of the figure placed over it, they will gradually rise up, and present themselves in their proper attitudes. When the air is again let into the receiver, they will in like manner retire to their separate apartments.

If the arms and legs of the figures be inflated with a due quantity of air, when the pressure of that in the receiver is taken off, they will be extended, and may be made to assume any attitude: and when the air is again let into the receiver, they will resume their former positions.

The Artificial Halo.

Place a candle on one side of a receiver, and let the spectator place himself at some distance from the other side. As soon as the air begins to be exhausted, and becomes attenuated and charged with vapours to a proper degree, the light of the candle will be refracted through that medium in circles of various colours, that livelily resemble those seen about the moon in a hazy night.

The Mercurial Shower.

Cement a piece of wood into the lower part of the neck of an open receiver, and pour mercury over it.—After a few strokes of the pump, the pressure of the air on the mercury will force it through the pores of the wood in form of a beautiful shower; which, if the receiver be clear and the weather be dry, will appear luminous in a dark chamber.

The Fountain in Vacuo.

Take a tall glass tube, hermetically sealed at the top and at bottom, by means of a brass cap screwed on to a stop-cock, and that to the plate of the pump. When all the air is exhausted, the cock is turned, the tube is taken off the plate and immersed in a basin of mercury or water; then the cock being again turned, the fluid, by the pressure of the air, will play up in the tube, in form of a fountain, and afford a very pleasing appearance.

The Cemented Bladder.

Tie the neck of a bladder to a stop-cock, which is to be screwed to the plate of the pump, and the air exhausted from the bladder; then turn the stop-cock to prevent the re-entrance of the air, and unscrew the whole from the pump. The bladder will be transformed into two flat skins, so closely applied together, that the strongest man cannot raise them half an inch from each other; for an ordinary sized bladder, of six inches across the widest part, will have one side pressed upon the other with a force equal to 396 pounds weight.

Cork heavier than Lead.

Let a large piece of cork be pendent from one end of a balance beam and a small piece of lead from the other; the lead should rather preponderate. If this apparatus is placed under a receiver on the pump, you will find that when the air is exhausted, the lead, which seemed the heaviest body, will ascend, and the cork outweigh the lead. Restore the air, and the effect will cease. This phenomenon is only on account of the difference of the size in the two objects. The lead, which owes its heaviness to the operation of the air, yields to a lighter, because a larger substance, when deprived of its assistance.

The animated Bacchus.

Construct a figure of Bacchus, seated on a cask; let his belly be formed by a bladder, and let a tube proceed from his mouth to the cask. Fill this tube with coloured water or wine, then place the whole under the receiver. Exhaust the air, and the liquor will be thrown up into his mouth. While he is drinking, his belly will expand.

The Artificial Balloon.

Take a bladder containing only a small quantity of air, add a piece of lead to it, sufficient to sink it if immersed in water.

Put this apparatus into a jar of water, and place the whole under a receiver. Then exhaust the air, and the bladder will expand, become a balloon lighter than the fluid in which it floats, and ascend, carrying the weight with it.

Experiment with a Viper.

Many natural philosophers, in their eagerness to display the powers of science, have overlooked one of the first duties of life—humanity; and, with this view, have tortured and killed many harmless animals, to exemplify the amazing effects of the air-pump. We will not stain the pages of this work by recommending any such species of cruelty, which, in many instances, can merely gratify curiosity; however, as many of our readers might like to read the effect on animals, we extract from the learned Boyle, an account of his experiment on a viper.

He took a newly caught viper, and shutting it up in a small receiver, extracted the air. At first, upon the air's being drawn away, it began to swell; a short time after it gaped and opened its jaws; it then resumed its former lankness, and began to move up and down within the receiver, as if to seek for air. After a while it foamed a little, leaving the foam sticking to the inside of the glass; soon after, the body and neck became prodigiously swelled, and a blister appeared on its back. Within an hour and a half from the time the receiver was exhausted, the distended viper moved, being yet alive, though its jaws remained quite stretched; its black tongue reached beyond the mouth, which had also become black in the inside. In this situation it continued for three hours; but on the air's being readmitted, the viper's mouth was presently closed, and soon after opened again; and these motions continued some time, as if there were still some remains of life.

It is thus with animals of every kind; even minute microscopical insects cannot live without air.

Experiments with Sparrows.

Count Morozzo placed successively several full grown sparrows under a glass receiver, inverted over water. It was filled with atmospheric air, and afterwards with vital air. He found,

First—That in *atmospheric* air,

	HOURS.	MIN.
The first sparrow lived	3	0
The second sparrow lived	0	3
The third sparrow lived	0	1

The water rose in the vessels eight lines during the life of the first; four during the life of the second; and the third produced no absorption.

Second—In *vital* air, or *oxygen*,

	HOURS.	MIN.
The first sparrow lived	5	23
The second	2	10
The third	1	30
The fourth	1	10
The fifth	0	30
The sixth	0	47
The seventh	0	27
The eighth	0	30
The ninth	0	22
The tenth	0	21

The above experiments elicit the following conclusions:—1. That an animal will live longer in vital than in atmospheric air. 2. That one animal can live in air in which another has died. 3. That, independent of air, some respect must be had to the constitution of the animal; for the sixth lived 47 minutes, the fifth only thirty. 4. That there is either an absorption of air, or the production of a new kind of air, which is absorbed by the water as it rises.

OPTICS.

DEFINITIONS.

1. WHATEVER grants a passage to light is called a medium.

2. By rays of light are understood its least parts, either successive in the same lines, or contemporary in several lines. —It is clear that light consists of parts both successive and

contemporary, because in the same place you may stop that which comes one moment, and let pass that which comes immediately after: the least sensible part which may be stopped, or suffered to proceed, is called a ray of light.

3. Refrangibility is that disposition of a ray of light to be refracted, or turned out of its course, when it passes out of one medium into another.—When a ray of light passes out of a rarer medium into a denser, Sir Isaac Newton supposes that it is refracted by the superior attraction of the denser medium, and by that means drawn out of its course.

4. Reflexibility is that disposition of a ray of light to be reflected or turned back into the same medium from any other medium upon whose surface it may fall.—Sir Isaac Newton supposes that light is reflected by impinging upon the solid parts of the body, but by some power of the body which is evenly diffused all over its surface, and by which it acts upon the ray, and impels it back without immediate contact.

5. Inflection is that disposition of a ray of light to be turned out of its course when it passes very near to the edges of bodies.

6. The angle of incidence is the angle which the line described by the incident ray makes with the perpendicular to the reflecting or refracting surface at the point of incidence.

7. The angle of reflection or refraction is the angle which the line described by the reflected or refracted ray makes with the perpendicular to the reflecting or refracting surface at the point of incidence.

8. Any parcel of rays diverging from a point, considered as separate from the rest, is called a pencil of rays.

9. A lens is a medium bounded by two spherical, or one plain and one spherical surface; and the line joining the centres, or which passes perpendicularly through each surface, is called the axis.—There are six lenses, a double convex, a double concave, a plano-convex, a plano-concave, a concave-convex, and a meniscus.

10. The focus of rays is that point from which they diverge, or to which they converge.—The focus of parallel rays is called the principal focus.

The sun's light consists of rays of different colours, and differently refrangible.

For if the sun's rays be admitted into a dark room through a small hole in a window-shutter, and be refracted through a prism, the image is not round, but a long figure with parallel sides and semicircular ends, the length of which is above five times its breadth; that end which has suffered the least refraction is red, and that which has suffered the greatest is violet: the whole image consists of seven distinct colours, lying in the following order—red, orange, yellow, green, blue, indigo, violet. The red is the least refrangible, and the others more in their order. These are called primary colours, all other colours being only different combinations of these. Each colour forms a distinct image of the sun, which images, in this experiment, running into each other, make a gradual change of colour in the image; but if a convex lens be placed before the prism, each image will be diminished, and by that means they will be separated, and each rendered distinct.

If two coloured images be formed with two prisms, and thrown one upon the other, then if that image be looked at through a prism, the images will be again separated.

The primary colours cannot be separated into other colours by any refraction.

For if in the last experiment all the colours but one be stopped, for instance, the red, and that be again refracted by a prism, it suffers no alteration in colour. By suffering the colours to pass in succession, from the red, each preserves its colour, but the quantity of refraction keeps increasing. The image of each colour is perfectly circular, which shews that the light of each colour is refracted regularly without any dilation of the rays; it is therefore incompounded, or homogeneal.

If the breadth of each colour in the spectrum formed by the prism be measured, it will appear that the breadth of the red, orange, yellow, green, blue, indigo, violet, are as the numbers 45, 27, 48, 60, 60, 40, 80, respectively.

If the circumference of a circle be divided into 45°, 27°, 48°, 60°, 60°, 40°, 80°, and the respective sectors be painted red, orange, yellow, green, blue, indigo, violet, and the circle be turned swiftly, it will appear nearly white; for the ideas we have from the impression of light remain for a short time, and thus the colours excite the same sensation as if they all entered the eye collected together.

If the direct image of the sun through a small hole be received upon a screen perpendicular to the rays, and the rays be then intercepted by a prism, and fall perpendicularly on the first side; if the distance from the place of the direct image to the nearest edge of the red and farthest of the violet be measured, they will be the tangents of the angles of deviation, and the radius of which is the distance from the point where the rays emerge to the place of the direct image.

The angles of incidence on the second side of the prism equal the refracting angle or the prism, to which add the deviations of the two extreme colours, and we get the two angles of refraction, the sines of which will be to the sines of incidence as 77 and 78 to 50: hence, if the difference between 77 and 78 be divided in the ratio of the breadth of each colour, it gives for the sines of refraction the common sine of incidence, being 50; that is, the sine of incidence. The sine of refraction of the red rays :: 50: not less than 77, nor greater than $77\frac{1}{8}$, the boundary of the red; and the same for the rest.

Candle-light is of the same nature as the light from the sun; for rays from a candle may be separated into all the different colours, and they lie in the same order as in the light from the sun.

The sun's light consists of rays which differ in flexibility, and those rays which are most refrangible are most reflexible.

For after forming a coloured image, as before, with a prism, by turning the prism about its axis, until the rays within it, which in going out into the air were refracted at its base, become so oblique to the base as to begin to be totally reflected thereby, those rays become first reflected which before, at equal incidences with the rest, had suffered the greatest refraction.

According to Sir Isaac Newton, the colours of natural bodies arise from hence, that some reflect one sort of rays, and others another sort more copiously than the rest.

For every body looks most splendid in the light of its own colour, and therefore it reflects that the most copiously: besides, by reflection you cannot change the colour of any sort of rays; and as bodies are seen by reflection, they must appear of the colour of those rays which they reflect. This is the opinion of Sir Isaac Newton; but Mr Delaval accounts for the colours of natural bodies in a manner different from this. See the Manchester Memoirs, Vol. II.

Thin transparent substances, as glass, water, air, &c. exhibit various colours according to their thickness.

For a very thin glass bubble, or a bubble of water, will appear to have concentric colours: the bubble blown with water, first made tenacious by dissolving a little soap in it, continually grows thinner at the top by the subsiding of the water, the rings of colours dilating slowly, and overspreading the whole bubble. A convex and concave lens of nearly the same curvature being pressed closely together, exhibit rings of colours about the point where they touch. Between the colours there are dark rings, and when the glasses are very much compressed, the central spot is dark. Sir Isaac Newton, to whom we owe all these discoveries, found the thickness of the air between the glasses where the colours appeared to be as 1, 3, 5, 7, 9, &c. and the thickness where the dark rings appeared to be as 0, 2, 4, 6, 8, &c. the coloured rings must have appeared from the reflection of the light, and the dark rings from the transmission of the light: the rays therefore were transmitted when the thickness of the air was 0, 2, 4, 6, 8, &c. and reflected at the thickness 1, 3, 5, 7, 9, &c. Sir Isaac Newton therefore supposes, that every ray of light, in its passage through any refracting surface, is put into a certain constitution or state, which in the progress of the ray returns at equal intervals, and disposes the ray at every return to be easily transmitted through the next refracting surface, and between the returns to be easily reflected by it. These he calls fits of easy transmission and reflection.

Three Objects discernible only with both Eyes.

If you fix three pieces of paper against the wall of a room at equal distances, at the height of your eye, placing yourself directly before them, at a few yards distance, and close your right eye, and look at them with your left, you will see only two of them, suppose the first and second; alter the position of your eye, and you will see the first and third; alter your position a second time, you will see the second and third, but never the whole three together: by which it appears, that a person who has only one eye can never see three objects placed in this position, nor all the parts of one object of the same extent, without altering the situation of his eye.

To construct the Camera Obscura.

Make a circular hole in the shutter of a window from whence there is a prospect of some distance; in this hole place a magnifying glass, either double or single, whose focus is at the distance of five or six feet; no light must enter the room but through this glass. At a distance from it, equal to its focus, place a very white pasteboard (what is called a Bristol board, if you can procure one large enough, will answer extremely well); this board must be two feet and a half long, and eighteen or twenty inches high, with a black border round it: bend the length of it inward to the form of part of a circle, whose diameter is equal to double the focal distance of the glass. Fix it on a frame of the same figure, and put it on a moveable foot, that it may be easily placed at that distance from the glass, where the objects appear to the greatest perfection. When it is thus placed, all the objects in front of the window will be painted on the paper in an inverted position, with the greatest regularity, and in the most natural colours. If you place a swing looking-glass outside the window, by turning it more or less, you will have on the paper all the objects on each side the window.

If, instead of placing the looking-glass outside the window, you place it in the room above the hole (which must then be made near the top of the shutter), you may have the representation on a paper placed horizontally on a table, and draw at your leisure all the objects reflected.

Observe, the best situation is directly north; and the best time of the day is noon.

The Magnifying Reflector.

Let the rays of light that pass through the magnifying glass in the shutter be thrown on a large concave mirror, properly fixed in a frame. Then take a thin strip of glass, and stick any small object on it; hold it in the intervening rays at a little more than the focal distance from the mirror, and you will see on the opposite wall, amidst the reflected rays, the image of that object, very large, and beautifully clear and bright.

Optical Augmentation.

Take a large drinking-glass of a conical figure, that is, small at bottom and wide at top; in which put a shilling, and fill the glass about half full of water; then place a plate on the top of it, and turn it quickly over, that the water may not get out. You will then see on the plate a piece of the size of a half-crown, and somewhat higher up, another piece of the size of a shilling.

This phenomenon arises from seeing the piece through the conical surface of the water at the side of the glass, and through the flat surface at the top of the water at the same time; for the conical surface dilates the rays, and makes the piece appear larger; but by the flat surface the rays are only refracted, by which the piece is seen higher up in the glass, but still of its natural size. That this is the cause will be further evident by filling the glass with water, for as the shilling cannot be then seen from the top, the large piece only will be visible.

After you have amused yourself with this remarkable phenomenon, you may give the glass to a servant, telling him to throw out the water, and take care of the two pieces of money; and if he has no suspicion of the deception, he will be not a little surprised to find one piece only.

To magnify small Objects by means of the Sun's rays let into a dark Chamber.

Let the rays of light that pass through the lens in the shutter be thrown on a large concave mirror, properly fixed in a frame; then take a slip, or thin plate of glass, and sticking any small object on it, hold it in the incident rays, at a little more than the focal distance from the mirror, and you will see on the opposite wall, amidst the reflected rays, the image of that object, very large, and extremely clear and bright.—This experiment never fails to give the spectator the highest satisfaction.

AMUSING SECRETS.

To make a Ring be suspended by a Thread after it has been burnt.

NOTHING is necessary for this purpose, but to employ a thread which has been soaked in a solution of common salt in river water. Though flame be applied to the thread, it will still have strength sufficient to sustain the ring.

To make People in a room have a hideous appearance.

Dissolve salt in an infusion of saffron in spirit of wine; then dip some tow in the solution, and having set fire to it, extinguish the other lights in the room.

To form Figures in relief on an Egg.

Delineate on the shell any figures at pleasure, with melted tallow, or any other fat oily substance proof against acids; then immerse the egg in strong vinegar, and let it remain till the acid has sufficiently corroded that part of the shell not covered with the tallow or oil.

To change a Colour from White to Blue.

Dissolve copper filings in a phial of volatile alkali: when the phial is stopped, the liquor will be bluish; but when unstopped, it will be white.

To make a Red Liquor, which, when poured into different glasses, shall become Yellow, Blue, Black, or Purple.

This phenomenon may be produced by the following process: Infuse a few shavings of logwood in common water, and when the liquor is sufficiently red, pour it into a bottle.

Then take three drinking-glasses and rinse one of them with strong vinegar; throw into the second a small quantity of pounded alum, which will not be observed if the glass has been newly washed, and leave the third without any preparation. If the red liquor in the bottle be poured into the first glass, it will assume a straw colour, somewhat similar to that of Madeira wine; if into the second, it will pass gradually from bluish-grey to black, provided it be stirred with a bit of iron, such as a key, for example, which has been privately immersed in good vinegar. In the third glass, the red colour will assume a violet tint.

To make Pomatum with Water and Wax, two substances which do not combine together.

Put into a new glazed earthen pot six ounces of river water, and two ounces of wax, in which, to render the process more marvellous, you must have concealed a strong dose of salt of tartar. If the whole be then exposed to a considerable degree of heat, it will assume the consistence of pomatum, and may be used for cleansing the skin.

How a Body of a combustible nature may be penetrated by fire without being consumed.

Put into an iron box a piece of charcoal, sufficient to fill it entirely, and solder on the lid. If the box be then thrown into the fire, it will become red, and it may even be left in it for several hours or days. When opened, after it has cooled, the charcoal will be found entire, though there can be no doubt of its having been penetrated by the matter of the fire, as well as the whole metal of the box which contains it.

Apparent transmutation of Iron into Copper or Silver.

Dissolve blue vitriol in water, till the latter is nearly saturated, and immerse into the solution small plates of iron or coarse filings of that metal. These small plates of iron or filings will be attacked and dissolved by the acid of the vitriol, while its copper will be precipitated and deposited in the place of the iron dissolved; it will be so completely covered with cupreous particles, that it will seem to be converted into copper. This is an experiment commonly shewn to those who visit copper mines. In Savoy, keys have been seen to become entirely of a copper colour, after being immersed some minutes in water collected at the bottom of a copper mine.

If you dissolve mercury in marine acid, and immerse in it a bit of iron; or if the solution be rubbed over the iron, it will assume a silver colour. Jugglers sometimes exhibit this chemical deception at the expense of the credulous and ignorant.

Remark.—In this case there is no real transmutation, but only the appearance of one. The iron is not changed into copper; the latter, held in solution by the liquor impregnated by the vitriolic acid, is only deposited in the place of the iron with which the acid becomes charged, while it abandons the copper. Every time, indeed, that a menstruum, holding any substance in solution, is presented to another substance which it can dissolve with more facility, it abandons the former, and becomes charged with the second. This is so certain, that when the liquor which has deposited the copper is evaporated, it produces crystals of green vitriol, which, as is well known, are formed by the combination of the acid with iron. This process is indeed practised, on a large scale, in the minès of Savoy. The liquor in question, which is nothing but a pretty strong solution of blue vitriol, is put into casks, or large square reservoirs; pieces of old iron being then immersed in it, are at the end of some time converted into some sort of sediment, from which copper is extracted. The liquor thus charged with iron is evaporated to a certain degree, and wooden rods are immersed in it, which become covered with crystals of green vitriol.

This experiment may be made also by dissolving copper in the vitriolic acid, and then diluting the solution with a little water. This is a new proof that the liquor only deposits the copper with which it is charged.

Different substances successively precipitated by adding another to the solution.

In the former experiment we have seen copper precipitated by iron; we shall now show iron itself precipitated. For this purpose, throw into a solution of iron a small bit of zinc, and, in proportion as the latter dissolves, the iron will fall to the bottom of the vessel: it may easily be known to be iron, because it will be susceptible of being attracted by the magnet.

If you wish to precipitate the zinc, nothing will be necessary but to throw into the solution a bit of calcareous stone, such as white marble, for example, or any other kind of stone capable of making lime. The vitriolic acid will attack this new substance, and suffer to be deposited at the bottom of the vessel a white powder, which is zinc.

To precipitate the lime or calcareous earth:—Pour into the solution liquid volatile alkali (spirit of hartshorn); the earth being abandoned by the acid, will deposit itself at the bottom of the vessel.

The calcareous earth may be precipitated also, and much better, by pouring into the liquor a solution of fixed alkali, such as fixed vegetable alkali; or by throwing into it fixed mineral alkali.

Remark.—It is by a similar effect that hard water decomposes soap, instead of dissolving it, and suffers to be deposited a greater or less quantity of calcareous earth. The manner in which this is done is as follows:

Water, in general, is hard only because it holds in solution selenite or gypsum (a combination of vitriolic acid with calcareous earth), which it has dissolved in its passage through the bowels of the earth, or which has been formed by the water first becoming impregnated with vitriolic salts, and afterwards in its course meeting with, and dissolving a portion of calcareous earth.

On the other hand, soap is an artificial combination of mixed alkali with oil, or with some other greasy substance, and which have no great affinity.

When soap, therefore, is dissolved in water impregnated with selenite, the vitriolic acid of the latter, having a greater tendency to unite with the fixed alkali, than with the calcareous earth which enters into the composition of the selenite, abandons that earth, and combines with the fixed alkali in such a manner that the soap is decomposed; and, as the oil is immiscible with water, it is diffused through it in the form of white flakes, while the calcareous earth of the selenite falls to the bottom.

By the mixture of two transparent Liquors, to produce a blackish Liquor.—Method of making good Ink.

Provide a solution of green or ferruginous vitriol, and an infusion of gall-nuts, or any other astringent vegetable substance, such as oak-leaves, well clarified and filtered; if you then pour the one liquor into the other, the compound will immediately become obscure, and at last black.

If the liquor be suffered to remain at rest, the black matter suspended in it will fall to the bottom, and leave it transparent.

Remark.—This experiment may serve to explain the formation of common ink; for the ink we use is nothing but a solution of green vitriol mixed with an infusion of gall-nuts, and a little gum. The blackness arises from the property which the gall-nuts have of precipitating, of a black or blue colour, the iron held in solution by the water impregnated with vitriolic acid; but as the iron would soon fall to the bottom, it is retained by the addition of gum, which gives to the water sufficient viscosity to prevent the iron from being precipitated.

The reader, perhaps, will not be displeased to find here the following recipe for making good ink:—

Take one pound of gall-nuts, six ounces of gum arabic, six ounces of green copperas, and one gallon of common water or beer; pound the gall-nuts, and infuse them in a gentle heat for twenty-four hours, without bringing the mixture to ebullition; then add the gum in powder. When the gum is dissolved, put in the green vitriol. And if you then strain the mixture, you will obtain very fine ink.

To produce inflammable and fulminating Vapours.

Put into a moderately sized bottle, with a short wide neck, three ounces of oil or spirit of vitriol, with twelve ounces of common water, and throw into it at different times an ounce or two of iron filings. A violent effervescence will then take place, and white vapours will arise from the mixture. If a taper be presented to the mouth of the bottle, these vapours will inflame and produce a violent detonation, which may be repeated several times, as long as the liquor continues to furnish similar vapours.

The Philosophical Candle.

Provide a bladder, into the orifice of which is inserted a metal tube, some inches in length, that can be adapted to the neck of a bottle, containing the same mixture as that used in the preceding experiment.

Having then suffered the atmospheric air to be expelled from the bottle by the elastic vapour produced by the solution, apply to the mouth of it the orifice of the bladder, after carefully expressing from it the common air (which you must not fail to do, or the bladder will explode). The bladder by these means will become filled with the inflammable air; which, if you force out against the flame of a taper, by pressing the sides of the bladder, will form a jet of beautiful green flame. This is what the chemists call a philosophical candle.

To make an Artificial Volcano.

For this curious experiment, which enables us to assign a very probable cause for volcanoes, we are indebted to Lemery.

Mix equal parts of pounded sulphur and iron filings, and having formed the whole into a paste with water, bury a certain quantity of it, forty or fifty pounds for example, at about the depth of a foot below the surface of the earth. In ten or twelve hours after, if the weather be warm, the earth will swell up and burst, and flames will issue out, which will enlarge the aperture, scattering around a yellow and blackish dust.

It is not impossible that what is here seen in miniature, takes place on a grand scale in volcanoes; as it is well known that they always furnish abundance of sulphur, and that the matters they throw up abound in metallic and probably ferruginous particles; for iron is the only metal which has the property of producing an effervescence with sulphur, when they are mixed together.

But it may easily be conceived, from the effect of a small quantity of the above mixture, what thousands or millions of pounds of it would produce: there is no doubt that the result would be phenomena as terrible as those of earthquakes, and of those volcanic eruptions with which they are generally accompanied.

To make Fulminating Powder.

Mix together three parts of nitre, two of well dried fixed alkali, and one of sulphur: if a little of this mixture be put into an iron spoon, over a gentle fire, capable however of melting the sulphur, when it acquires a certain degree of heat it will detonate with a loud noise, like the report of a small cannon.

This would not be the case if the mixture were exposed to a heat too violent: the parts only most exposed to the fire would detonate, and by these means the effect would be greatly lessened.

If thrown on the fire, it would not detonate, and would produce no other effect than pure nitre, which indeed detonates, but without any explosion.

To form a combination which when cold is liquid and transparent, but when warm becomes thick and opaque.

Put equal quantities of fixed alkali, either mineral or vegetable, and of well pulverized quicklime, into a sufficient quantity of water, and expose it to strong and speedy ebullition. Then filter the product, which at first will pass through with difficulty, but afterwards with more ease, and preserve it in a bottle well stopped. This liquor, when made to boil, either in the bottle or in any other vessel, will become turbid, and assume the consistence of very thick glue; but when cold, it will recover its fluidity and transparency.

To make a Flash like that of Lightning appear in a room, when any one enters it with a lighted candle.

Dissolve camphor in spirit of wine, and deposit the vessel containing the solution in a very close room, where the spirit of wine must be made to evaporate by speedy and strong ebullition. If any one then enters the room with a lighted candle, the air will inflame, while the combustion will be so sudden, and of so short a duration, as to occasion no danger.

It is not improbable that the same effect might be produced, by filling the air of an apartment with the dust of the seed of a certain kind of lycoperdon, which is inflammable.

Of Sympathetic Inks, and some Tricks which may be performed by means of them.

Sympathetic inks are certain liquors, which alone, and in their natural state, are colourless; but which, by being mixed with each other, or by some particular circumstance, assume a certain colour.

Chemistry presents us with a great many liquors of this kind, the most curious of which we shall here describe.

1st, If you write with a solution of green vitriol, to which a little acid has been added, the writing will be perfectly colourless and invisible. To render it visible, nothing will be necessary but to immerse the paper in an infusion of gall-nuts in water, or to draw a sponge moistened with the infusion over it.

2d, If you are desirous of having an ink that shall become blue, you must write with an acid solution of green vitriol, and moisten the writing with a liquor prepared in the following manner:

Make four ounces of tartar, mixed with the same quantity of nitre, to detonate on charcoal; then put this alkali into a crucible with four ounces of dried ox blood, and cover the crucible with a lid, having in it only one small aperture. Calcine the mixture over a moderate fire, till no more smoke issues from it, and then bring the whole to a moderate red heat; take the matter from the crucible, and immerse it, while still red, in two quarts of water, where it will dissolve by ebullition; and when the liquor is reduced to one-half, it will be ready for use. If you then moisten with it the writing above mentioned, it will immediately assume a beautiful blue colour. In this operation, instead of black ink, there is formed Prussian blue.

3d, If you dissolve bismuth in nitrous acid, and write with the solution, the letters will be invisible. To make them appear, you must employ the following liquor :

Boil a strong solution of fixed alkali with sulphur reduced to a very fine powder, until it dissolves as much of it as it can ; the result will be a liquor which exhales vapours of a very disagreeable odour, and to which, if the above writing be exposed, it will become black.

4th, Of all the different kinds of sympathetic ink, the most curious is that made from cobalt. It is a very singular phenomenon, that the characters or figures traced out with this ink, may be made to disappear and reappear at pleasure. This property is peculiar to ink made with cobalt ; for all the other kinds are at first invisible, until some substance has been applied to make them appear : when they have once appeared, they remain.

To prepare this ink, take zaffer, and dissolve it in aqua regia (nitro-muriatic acid) till the acid extracts from it every thing it can ; that is to say, the metallic part of the cobalt, which communicates to the zaffer a blue colour ; then dilute the solution, which is very acrid, with common water. If you write with this liquor on paper, the characters will be invisible ; but when exposed to a sufficient degree of heat, they will become green. When the paper has cooled, they will disappear.

It must, however, be observed, that if the paper be heated too much, they will not disappear at all.

Remark.—With this kind of ink some very ingenious and amusing tricks, such as the following, may be performed.

1*st, To make a drawing, which shall alternately represent Winter and Summer.*—Draw a landscape, and delineate the ground, and the trunks and branches of the trees, with the usual colours employed for that purpose, but the grass and leaves of the trees with the liquor above mentioned. By these means you will have a drawing, which, at the common temperature of the atmosphere, will represent a winter piece ; but if it be exposed to a proper degree of heat, not too strong, you will see the ground become covered with verdure, and the trees with leaves, so as to present a view in summer.

Screens painted in this manner, were formerly made at Paris. Those to whom they were presented, if unacquainted with the artifice, were astonished to find, when they made use of them, that the views they exhibited were totally changed.

2*d*, *The Magic Oracle.*—Write on several sheets of paper, with common ink, a certain number of questions, and below each question write the answer with the above kind of sympathetic ink. The same questions must be written on several pieces of paper, but with different answers, that the artifice may be better concealed.

Then provide a box, to which you may give the name of the Sibyl's cave, or any other at pleasure, and containing in the lid a plate of iron made very hot, in order that the inside of it may be heated to a certain degree.

Having selected some of the questions, take the bits of paper containing them, and tell the company that you are going to send them to the Sibyl, or Oracle, to obtain an answer; introduce them into the heated box, and when they have remained in it some minutes, take them out, and shew the answers which have been written.

You must, however, soon lay aside the bits of paper; for if they remain long in the hands of those to whom the trick is exhibited, they would see the answers gradually disappear, as the paper becomes cold.

Of Metallic Vegetations.

To see a kind of shrub rise up in a bottle, and even throw out branches, and sometimes a kind of fruit, is one of the most curious spectacles exhibited by chemistry. The operation by which this delusive image is produced, has been called chemical or metallic vegetation, because performed by means of metallic substances; and it is not improbable, that some respectable persons, who thought they saw a real palingenesy, have been deceived by a similar artifice. However this may be, the following are the most curious of these vegetations, which in fact are only a sort of crystallizations.

Arbor Martis, or Tree of Mars.

Dissolve iron filings in spirit of nitre (aquafortis), moderately concentrated, till the acid is saturated; then pour gradually into the solution a solution of fixed alkali, commonly called *oil of tartar per deliquium*. A strong effervescence will take place, and the iron, instead of falling to the bottom of the vessel, will afterwards rise, so as to cover its sides, forming a multitude of ramifications heaped one upon the other, which will sometimes pass over the edge of the vessel, and extend themselves on the outside, with all the appearance of a plant. If any of the liquor is spilt, it must be carefully collected, and be again put into the vessel, where it will form new ramifications, which will contribute to increase the mass of the vegetation.

Arbor Dianæ, or Tree of Diana.

This kind of vegetation is called the Tree of Diana, because it is formed by means of silver, as the former is called the Tree of Mars, because produced by iron.

Mix together two parts of very pure mercury, and four of fine silver, in filings or scales, by means of trituration with an ivory pestle in a porphyry mortar; then dissolve this amalgam in four ounces of very pure spirit of nitre, moderately strong, and dilute the solution with about a pound and a half of distilled water; shake the mixture, and preserve it in a bottle well stopped. Pour an ounce of this liquor into a glass, and throw into it a small bit, about the size of a pea, of an amalgam of mercury and silver, similar to the former, and of the consistence of butter. Soon after you will see rising from the ball of amalgam a multitude of small filaments, which will visibly increase in size, and, throwing out branches, will form a sort of shrubs.

The Lead Tree.

This is a more modern invention, and may be produced by the following very easy method:

To a piece of zinc, fasten a wire, crooked in the form of the worm of a still; let the other end of the wire be thrust through a cork. You then pour spring water into a phial or decanter, to which you add a small quantity of sugar of lead; thrust the zinc into the bottle, and with the cork at the end of the wire fasten it up. In a few days the tree will begin to grow, and produce a most beautiful effect.

Non-metallic Vegetation.

Cause to decrepitate, on burning charcoal, eight ounces of saltpetre, and place it in a cellar, in order that it may produce *oil of tartar per deliquium;* then gradually pour over it, to complete saturation, good spirit of vitriol, and evaporate all the moisture. The result will be a white, compact, and very acrid saline matter. Put this matter into an earthen dish, and having poured over it a gallon of cold water, leave it exposed to the open air. At the end of some days the water will evaporate, and there will be formed all around the vessel ramifications in the form of needles, variously interwoven with each other, and about fifteen lines in length. When the water is entirely evaporated, if more be added, the vegetation will continue.

It may be readily seen, that this is nothing but the mere crystallization of a neutral salt, formed by the vitriolic acid and the alkali of the nitre employed, that is to say, vitriolated tartar.

To produce Heat, and even Flame, by means of two cold Liquors.

Put oil of guaiacum into a basin, and provide some spirit of nitre, so much concentrated, that a small bottle, capable of holding an ounce of water, may contain nearly an ounce and a half of this acid. Make fast the bottle containing the acid to the end of a long stick; and, after taking this precaution, pour about two-thirds of the acid into the oil in the basin; the result will be a strong effervescence, which will be followed by a very large flame. If an inflammation does not take place in the course of a few seconds, you have nothing to do but to pour the remainder of the nitrous acid over the blackest part of the oil; a flame will then certainly be produced, and there will remain, after the combustion, a very large spongy kind of charcoal.

Oil of turpentine, oil of sassafras, and every other kind of essential oil, may be made to inflame in the like manner.

The same phenomenon may be produced with fat oils, such as olive oil, nut oil, and others extracted by expression, if an acid, formed by equal parts of the vitriolic and nitrous acids, well concentrated, be poured into them.

To fuse Iron in a moment, and make it run into Drops.

Bring a bar of iron to a white heat, and then apply to it a roll of sulphur; the iron will be immediately fused, and run down in drops. It will be most convenient to perform this experiment over a basin of water, in which the drops that fall down will be quenched.—On examination they will be found reduced into a kind of cast iron.

This process is employed for making shot used in hunting; as the drops, by falling in the water, naturally assume a round form.

Cement for mending broken China.

Calcine oyster-shells, and having pounded them, sift them through a silk sieve, and grind them on porphyry, till they are reduced to an impalpable powder. Then take the whites of several eggs, according to the quantity of the powder to be used, and form them with the powder into a kind of paste or glue. With this paste join the fragments of the porcelain, and press them together for the space of seven or eight minutes. No longer time is necessary to dry this mastic; which will stand both heat and water, and which will never give way, even if the article by any accident should have a fall.

Process for whitening Prints.

Paste a piece of paper to a very smooth table, that the boiling water used in the operation may not acquire a colour, which might lessen its success. When this precaution has been taken, spread out the print on the table, and sprinkle it with boiling water, taking care to moisten it thoroughly throughout, by means of a fine sponge. After this process with boiling water has been repeated three or four times, you will observe the stains or spots extend themselves; but this need excite no uneasiness, as it is only a proof that the dirt imbibed by the paper begins to be dissolved.

After this preparation, the prints must be put into a copper or wooden vessel, of such a size as to admit of their being freely stretched out in it; they are then to be covered with a boiling lye of potash, and care must be taken to keep it hot as long as possible. After the whole has cooled, take out the prints with care; spread them on stretched cords, and, when half dry, press them between leaves of paper, in order that they may not contract wrinkles.

By this process, spots and stains of every kind may be removed.

Method of taking Paintings from the Old Canvass, and transferring them to New.

Take the painting from its frame, and tack it down on a very smooth table, with the face upwards, and in such a manner that it may be well stretched, and free from wrinkles; then cover it with a stratum of strong glue, and lay over it some sheets of large white paper, of the strongest kind you can procure. When the whole has dried, draw the tacks, and having inverted the painting, that is, turned the back uppermost, without fixing it, dip a sponge in tepid water, and gradually moisten the canvass, trying it from time to time at the edges, to see whether it begins to detach itself from the painting.

When you find it sufficiently loose, detach it carefully along one of the edges, and fold back the part so detached: if you then roll it with both hands, the whole canvass may, by these means, be removed.—When this is done, wash well the back of the painting with a sponge dipped in water, until all the old size has been nearly removed; then cover the back of the painting with a new stratum of size, or the usual priming applied to new canvass, intended for pictures, and immediately spread over it a new piece of canvass, which must be somewhat larger than the painting, in order that it may be properly stretched and nailed down at the edges. In the last place, do over the canvass, portion by portion, with a stratum of glue, taking care to spread it with a painter's muller, so that it may pass through the pores of the cloth to the painting.

When the painting is dry, remove it from the table, and put it into its frame, after which you must thoroughly moisten the paper with a sponge dipped in warm water, that it may be taken off without leaving any traces behind it, and to wash out any stains that may still remain on the painting. Conclude the process by rubbing over the painting with pure nut-oil, and when dry, with the white of an egg properly beat up.

To fill a Glass with water, in such a manner that a person shall not be able to remove it without spilling it all.

Lay a bet with any one that you will fill a glass with water, and place it on a table in such a manner that it cannot be removed without spilling the whole water it contains. Then fill a glass with water, and placing over it a bit of paper, so as to cover the water, and the edge of the glass; clap the palm of your hand on the paper, and laying hold of the glass with the other, suddenly invert it on a very smooth table. If you then gently draw out the paper, the water will remain suspended in the glass, and it will not be possible to remove it, without spilling the water entirely.

To construct two Figures, one of which shall blow out a Candle, and the other light it again.

Prepare two figures, of any materials whatever, and insert into the mouth of each a tube of the size of a small quill. Put into one of these tubes a small piece of phosphorus, and into the other a few grains of gunpowder, taking care that each may be retained in the tube by a bit of paper. If the second figure be applied to the flame of a taper, it will extinguish it, and the first applied will light it again.

The same kind of phosphorus may be employed, on the point of a knife, to light a candle which has been newly extinguished.

Japan Vases.

The Japanese have the art of making a kind of vases with the shavings of paper, or with saw-dust, which, when covered with varnish, are capable of containing hot or cold liquors. These vases, which are exceedingly neat and light, are ornamented in an agreeable manner with flowers, birds, and animals, and with gilt borders.

This preparation is called *papier maché*, and is made of the shavings of white or brown paper, boiled in water, and beat in a mortar till they are reduced to a kind of paste. This paste is afterwards boiled with a solution of gum-arabic, to give it tenacity; and by being pressed into moulds, rubbed over with oil, it may be formed into toys of various kinds; which when dry are done over with a mixture of glue and lamp-black, and then varnished.

The black varnish used for these toys is prepared in the following manner:

Dissolve, in a glazed earthen pot, a little colophonium, or boiled turpentine, till it becomes black and friable, and gradually throw into the mixture three times as much amber finely pulverized; adding from time to time a little spirit or oil of turpentine. When the amber is dissolved, besprinkle the mixture with the same quantity of sarcocolla gum, continually stirring the whole, and add spirit of wine till the composition becomes fluid; then strain it through a piece of hair-cloth, pressing it between two boards. This varnish, when mixed with ivory-black, is applied in a warm place on the dried paste of the paper shavings; the articles are then put into a hot stove, next day removed into a hotter stove, and the third into one still hotter: each time they are left till the stove has cooled. The paste when thus varnished, is hard, brilliant, and durable, and capable of containing liquors either hot or cold.

To construct a Vessel from which water shall escape through the bottom, as soon as its mouth is unstopped.

Among the number of amusing tricks, founded on philosophical principles, we may class the following:

Provide a vessel of tin-plate, two or three inches in diameter, and five or six inches in height, having a mouth about three lines in width, and in the bottom several small holes, of such a size as to admit a small needle. Immerse this vessel in water, with its mouth open, and when full stop it very closely. If you are desirous of playing a trick to any person, give him this vessel, and desire him to unstop it; if he does so, placing it on his knees, the water will escape through the holes in the bottom, so that he will soon be all over wet.

Transparencies.

Those transparencies exhibited on the stage, and during public festivals, which are illuminated by a light placed behind them, are prepared in the following manner. A piece of strong linen or silk, stretched on a wooden frame, is done over with a solution of wax in oil of turpentine, and during the operation a chaffing-dish is placed below it, that the liquid may be every-where equally diffused. Any figures at pleasure are then delineated on the cloth with oil-colours, mixed up with spirit of turpentine.

Moveable transparencies, exceedingly amusing, may be formed in the following manner:

Affix the transparency to a very light circular frame, supported by an axis on which it can freely turn. The upper end of the cylinder must be closed by a circular piece of tin-plate, cut into inclined planes, like the ventilators constructed in windows to prevent smoke: if a lamp be then placed within the cylinder, it will illuminate the transparency, and at the same time make it turn round by the means of the current of air which falls on the tin-plate.

The figures exhibited by this transparency may be varied a thousand ways, according to the taste of the artist. They may be made to represent serpents twisting around a column, &c.

It is by the same mechanism that a spiral piece of card or paper, placed on a stove, turns round of itself, and serves as a thermometer to regulate the heat.

Method of fixing Crayons.

Crayon painting is superior to oil painting in brightness, freshness, splendour of colouring, and fidelity of likeness. It is attended with this advantage also, that it is not subject to that reflection of light which prevents the beauty of a painting from being seen except from a certain point of view. On account of these valuable qualities, it would certainly have been preferred to oil painting, had it been equally durable; but it has this inconvenience, that it is liable to be destroyed by the least friction. At the end of a few years, master-pieces of this kind perish, because the powder of the crayons detaches itself, or becomes mouldy, especially if great care be not taken to preserve these paintings from moisture, and from the heat of the sun. The following liquor, however, has been employed with success for fixing crayons: it is not expensive, and nothing is necessary but to immerse the painting in it for a few moments.

To prepare this liquor, dissolve Roman alum pulverized, in two glassfulls of very pure water, and when the water is saturated, decant it from off the alum which may have remained undissolved at the bottom of the vessel. This observation is of great importance; for if the alum which has not been dissolved were left in the liquor, by becoming dry it might tarnish the painting, and produce whitish spots in those places where the liquor accumulates itself in draining off.—Into this water, well impregnated with alum, put a small quantity of very transparent and pure fish-glue, leaving it to dissolve for twenty-four hours, and then boil the whole, that the glue may be dissolved completely. The liquor must afterwards be strained through a piece of linen, to free it from any impurities it may contain.

In the last place, pour the water thus impregnated with alum and glue, into a bottle containing three pints of brandy, not coloured, and mixed with a large glassfull of spirit of wine. A greater or less quantity of this liquor may be made according to the size of the paintings to be fixed, provided care be taken to increase the ingredients in the proper proportions. It is, however, to be observed, that it must not be used when too old, as in that case it would weaken the splendour of the painting.

Put the liquor, thus prepared, into a vessel of lead, or of any other substance, so large that the painting may be immersed in it, and heat it in a balneum mariæ, taking care that the fish-glue be well dissolved; for before the liquor is heated, especially if the weather be cold, it will deposit itself at the bottom. Place in each corner of the basin a bit of lead, in such a manner that the liquor may rise over it no more than a line at most, and then lay hold of the painting, keeping it in a horizontal position, and immerse it gently into the liquor. The pieces of lead, placed in the vessel, will prevent it from sinking too deep. The time employed in immersing and taking out the painting, ought not to exceed a second.

The painting must be taken out horizontally, and be deposited in the same position, in some place where it can rest on its two borders, which it will do if supported by two chairs.

If the above process be properly followed, it will be found that all the tints have retained their original freshness and primitive colour. Crayons fixed in this manner, will bear even to be covered with a varnish, which may supply the place of glass. To lay on this varnish, the following method may be employed:

When the painting is fixed and dry, apply over it, with a soft brush, a stratum or two of melted fish-glue, mixed with about a third of spirit of wine, and sufficiently strong that when cold it may form a sort of jelly. When this preparation is dry, apply that varnish used for varnishing prints, which will produce the same effect as on paintings in distemper.

Crayon paintings, fixed in the above manner, are attended with this advantage, that they may be retouched; for the crayons will make an impression as before; some strengthening touches may even be added with colours in distemper. This method employed for crayons, may be used also for fixing chalk drawings.

An Object being placed behind a Convex Glass, to make it appear before it.

Provide any object, such, for example, as a small arrow of wood, an inch and a half in length, and tie it perpendicularly to a piece of black card, which must be suspended from a wall at about the height of the eye. Throw a strong light on the card, and place before it a lenticular glass, two or three inches in diameter, in such a manner that it may be distant from the arrow about twice the length of its focus. If you then make a person stand at a proper distance, opposite to the glass, the arrow will appear to him to be suspended in the air before the glass.

It is evident, that this singular effect of dioptrics, with taste and a little ingenuity, may be applied to a variety of other amusements, which it is needless here to detail.

The Chinese Shadows, (Ombres Chinoises).

Make an aperture in a partition wall, of any size, for example, four feet in length and two in breadth, so that the lower edge may be about five feet from the floor, and cover it with white Italian gauze, varnished with gum-copal. Provide several frames of the same size as the aperture, covered with the same kind of gauze, and delineate upon the gauze different figures, such as landscapes and buildings, analogous to the scenes which you intend to exhibit by means of small figures representing men and animals.

These figures are formed of pasteboard, and their different parts are made moveable according to the effect intended to be produced by their shadows, when moved backwards and forwards behind the frames, and at a small distance from them. To make them act with more facility, small wires, fixed to their moveable parts, are bent backwards, and made to terminate in rings, through which the fingers of the hand are put, while the figure is supported by the left, by means of another iron wire. In this manner they may be made to advance or recede, and to gesticulate, without the spectators observing the mechanism by which they are moved; and, as the shadow of these figures is not observed on the paintings till they are opposite those parts which are not strongly shaded, they may thus be concealed, and made to appear at the proper moments, and others may be occasionally substituted in their stead.

It is necessary, when the figures are made to act, to keep up a sort of dialogue, suited to their gestures, and even to imitate the noise occasioned by different circumstances. The paintings must be illuminated from behind, by means of a reverberating lamp, placed opposite to the centre of the painting, and distant from it about four or five feet.

Various amusing scenes may be represented in this manner, by employing small figures of men and animals, and making them move in as natural a way as possible, which will depend on the address and practice of the person who exhibits them.

To direct a swarm of Bees at pleasure.

It is well known that the female bee is the queen of the hive, and that the fate of the whole swarm depends, in some measure, upon her alone. The distinguishing characters of this mother bee are, that she has very short wings. It is difficult for her to fly, and therefore she seldom goes abroad, except when she quits the hive for a new colony. On that occasion, the bees, like faithful subjects, follow her to whatever place she may have chosen; and for this reason, if a person can get possession of the queen bee, he is sure of being able to direct the swarm at his pleasure.

In that case, nothing is necessary but to confine her by means of a hair, or a very fine thread of silk, made gently fast around her corslet; the bees, attentive to all her actions, will surround her, go backwards and forwards, stop and seem obedient to the will of him who commands the mother bee, by merely following the movements of their queen.

This was the charm, or rather the secret, by which Mr Wildman, who had studied the instinct of bees, and who thus took advantage of their attachment for their queen, was able to make a swarm pass from one hive to another at pleasure. Having full confidence in the success of his experiments, he presented himself one day to the Society of Arts, with three swarms of bees which he brought along with him, partly on his face and shoulders, and partly in his pockets. He placed the hives to which these swarms belonged in an outer apartment, and on blowing a whistle they all immediately quitted him, and returned to their hives; but on blowing his whistle a second time, they returned to occupy their former place on the person, and in the pockets of their master. This exercise was repeated several times, to the great astonishment of the Society, and without any of the spectators being injured.

These astonishing experiments, the secret cause of which we have explained, were repeated some years ago, with equal success, before the Academy of Sciences at Paris, by Mr Wildman, who explained to the French Academicians the theory and practice of his wonderful art.

A Powder which inflames when exposed to the Air.

Put three ounces of rock alum, and one ounce of honey, or sugar, into a new glazed earthen dish, capable of standing a strong heat, and keep the mixture over the fire, stirring it continually, till it become very dry and hard. Then remove it from the fire, and pound it until it assume the form of a coarse powder.

Put this powder into a small matrass, or long-necked bottle, leaving part of the vessel empty, and having placed it in a crucible, fill up the crucible with fine sand, and surround it with burning coals.

When the matrass has been kept at a red heat for about seven or eight minutes, and no more vapour issues from it, remove it from the fire; then stop it with a piece of cork; and, having suffered it to cool, preserve the mixture in small bottles well closed.

If you uncork one of these bottles, and let fall on a bit of paper, or any other very dry substance, a few grains of this powder, it will first become bluish, then brown, and will be speedily converted into an ardent body, so as to burn the paper, or any other combustible substance on which it may have been exposed.

When a few grains of this powder catch fire, on being thus exposed to the air, they emit a light flame, which resembles that of common sulphur when it begins to burn; and they exhale, at the same time, an odour similar to that produced by the smoke of sulphur.

Fulminating Gold.

Put into a small matrass, resting on a little sand, one part of fine gold filings and three parts of aqua regia (nitro-muriatic acid). When the filings are completely dissolved, pour the solution into a glass, and add to it five or six times the quantity of common water.

Then take spirit of sal-ammoniac, or oil of tartar, and pour it drop by drop into this solution, until the gold is entirely precipitated to the bottom of the glass; decant the supernatant liquor, by inclining the glass, and having washed it several times in tepid water, dry it in a very moderate heat, placing it on paper capable of absorbing all the humidity.

If a grain of this powder, put into a metal spoon, be exposed to the flame of a taper, as soon as it becomes sufficiently heated, it will explode with a very loud report; but it sometimes happens that it pierces the spoon, and forces itself downwards with great violence.

To cut Glass by means of Heat.

Take a common drinking-glass, not very thick, and apply to the edge of it a lighted match, until the violence of the heat produces a crack in it; then move the match along the crack, following a spiral direction, and after five or six circumvolutions, the glass will form a sort of scroll, the parts of which separate when you invert it; but which will be rejoined when put again into its natural position.

This method may be employed to cut glass tubes; for if a small notch be made with a file in the place where the tube is to be divided, you may easily make it split in that place, by applying to it a piece of angular iron made red-hot.

To melt a piece of Money in a Walnut-shell, without injuring the shell.

Bend any very thin coin, and having put it into the half of a walnut-shell, place the shell on a little sand, in order that it may remain steady. Then fill the shell with a mixture made of three parts of very dry pounded nitre, one part of the flowers of sulphur, and a little saw-dust well sifted.

If you then inflame the mixture, as soon as it has melted you will see the metal completely fused in the bottom of the shell, under the form of a button, which will become hard when the burning matter around it is consumed. The shell employed for the operation will have sustained very little injury.

Phosphorus.

The name of phosphorus is given to certain bodies which shine or appear luminous in the dark. Some kinds of it are natural, and others artificial. The natural are those which shine without the assistance of art, such as certain kinds of rotten wood, glow-worms, and almost all fish when they begin to become putrid.

The artificial kinds of phosphorus are those prepared by art, such as Kunckel's phosphorus (the common phosphorus of the shops), the sulphuret or the sulphate of barytes calcined, called Bologna phosphorus, &c.

A Liquor which shines in the dark.

Take a bit of Kunckel's phosphorus, about the size of a pea, and having divided it into several portions, put them into half a glassfull of very pure water, and boil it in a small earthen vessel, over a very moderate fire. Have in readiness a long narrow bottle, with a well fitted glass stopper, and immerse it, with its mouth open, into boiling water. On taking it out, empty it of the water, and immediately pour into it the mixture, in a state of ebullition; then put in the stopper, and cover it with mastic, to prevent the external air from entering it.

This water will shine in the dark for several months, even without being touched; and if it be shaken during dry warm weather, a kind of brilliant flashes will be seen to rise through the middle of the water.

Various amusing tricks may be performed with this phosphorus, by covering the bottle which contains it with black paper, having words or figures cut out in it. As you may not only cause different words to appear, but may even conceal, with one of your fingers, some of the letters which compose them, so as to form other words, it will seem as if you had the power of making them appear at pleasure.

To make Luminous Characters appear on a piece of paper, or a wall, &c.

If any characters be traced out with a small bit of Kunckel's phosphorus, they will appear luminous in the dark. If this experiment be made during warm weather, the light will be more vivid, and will be the sooner dissipated, than if performed during cold or moist weather. By breathing on these characters they will disappear, but a moment after they will reappear of themselves.

A Liquor shut up in a bottle, which, when the bottle is unstopped, becomes luminous.

Put a little of Kunckel's phosphorus into essence of cloves, and fill with it a bottle, which must be kept closely shut: every time the bottle is unstopped, the whole liquor will appear luminous. This experiment, as well as the preceding, must be performed in the dark.

Kunckel's phosphorus may be preserved in a bottle filled with water; but it must be put back into the bottle as soon as it has been used, and care must be taken not to touch it with the naked fingers, because it would burn them, and occasion very acute pain; in short, it is impossible to be too careful in handling this dangerous substance.

Method of speedily delineating all sorts of Plants and Flowers.

Provide two balls and some printer's ink; then holding one of the balls in the left hand, place upon it the leaf or plant, the impression of which you are desirous of obtaining, and taking the other ball, which must be daubed over with ink, in the right hand, strike it gently once or twice against the plant, without deranging it. Then carefully remove the leaf or plant, and putting it between a sheet of paper folded double, lay it on a table covered with a woollen cloth, and press it two or three times with a wooden roller, covered with a handkerchief, or any thing else of the like kind. After this process, you will find on each leaf of the paper an impression of the upper and lower side of the leaf; which, besides being a perfect resemblance of nature, will even surpass the most beautiful engravings, especially if the operation has been performed with dexterity.

The changeable Rose.

Take a common full-blown red rose, and having thrown a little sulphur finely pounded into a chaffing-dish with coals, expose the rose to the vapour. By this process the rose will become whitish; but if it be afterwards immersed some time in water, it will resume its former colour.

The Magic Picture.

Provide a glass similar to those used for miniature paintings, that is to say, somewhat concave, and another piece of common glass of the same size, and exceedingly thin. Fill the concave side of the former with a mixture of hog's-lard and wax melted together; then apply the two pieces of glass to each other exactly, that the above composition may be enclosed between them; and, having wiped the edges very clean, cement upon them, with fish-glue, a small slip of swine's bladder. When it is thoroughly dry, clean the glasses, and apply to the flat side a portrait, or any other subject at pleasure, and enclose the whole in a frame, so as to conceal the edges.

If this portrait be exposed to heat, the composition between the two glasses will dissolve, and become transparent, and the portrait will be distinctly seen; but it will disappear when the substance cools. In this manner it may be made to re-appear as often as you choose.

The changeable Picture.

Paint upon thin paper, in a slight manner, and with very light colours, any subject at pleasure, but disposed in such a manner, that by painting the paper stronger on the other side, it may be entirely disguised. Then cover the last side with a piece of white paper, to conceal the second subject, and enclose the whole in a frame, and even between two pieces of glass.

If you hold this picture between you and the light, and look through it, a subject will be seen very different from that which it exhibits when looked at in the usual manner.

Golden Ink.

As writing, before the invention of printing, was the only method of transmitting to posterity the works and discoveries of celebrated men, it became, in the fourteenth and fifteenth centuries, an art much cultivated, and in which many persons excelled. The manuscripts of those periods contain writing, the neatness and regularity of which are astonishing. Transcribers were even acquainted with a method of ornamenting the initial letters with gold, which they applied in such a manner as to preserve all its splendour.

Writing, by the invention of printing, having become of less importance, soon degenerated, and the secret of applying gold to paper and parchment, like many other arts, was at length lost. The Benedictines, however, rediscovered this secret, and specimens of the process, and parchment containing writing in gold letters, as brilliant as those so much admired in the ancient manuscripts, have been seen at the Abbey Saint Germain des Pres, at Paris. This process may be exceedingly useful, and may furnish hints for improving some of the other arts, which are all connected, and mutually tend to promote each other.

The process translated from the German.

Take a certain quantity of gum-arabic, the whitest is the best; and, having reduced it to an impalpable powder in a brass mortar, dissolve it in strong brandy, and add to it a little common water, to render it more liquid. Provide some gold in a shell, which must be detached, in order to reduce it to a powder. When this is done, moisten it with the gummy solution, and stir the whole with your finger, or with a small hair-brush; then leave it at rest for a night, that all the gold may be better dissolved. If the composition becomes dry during the night, it must be diluted with more gum water, in which a little saffron has been infused; but care must be taken that the gold solution be sufficiently liquid to be employed with the pen. When the writing is dry, polish it with a dog's tooth.

Another Process.

Reduce gum-ammoniac to powder, and dissolve it in water in which gum-arabic has been previously dissolved, and to which a little garlic juice has been added. This water will not dissolve the gum so as to form a transparent fluid; for the result will be a milky liquor. With this liquor you must form your letters or ornaments, on paper or vellum, by means of a pen or hair-brush; then suffer them to dry, and afterwards breathe on them for some time, till they become somewhat moist, and immediately apply a few bits of gold-leaf cut to the size of the letters; press the gold-leaf gently with a ball of cotton, or bit of soft leather, and when the whole is dry, take a soft brush and draw it gently over the letters, to remove the superfluous gilding. The parts which you wish to polish and render brilliant, may then be burnished with a dog's tooth.

White Ink, to write on Black Paper.

Take egg-shells, and having carefully washed them, remove the internal pellicle, and grind them on a piece of porphyry. Then put the powder into a small vessel filled with pure water, and when it has settled at the bottom, decant the water, and dry the powder in the sun. This powder must be preserved in a bottle. When you are desirous of using it, put a small quantity of very pure gum-ammoniac into distilled vinegar, and leave it to dissolve during the night; next morning the solution will appear exceedingly white, and if you then strain it through a piece of linen cloth, and add to it the powder of egg-shells, in sufficient quantity, you will obtain a very white ink.

Red Ink.

Boil four ounces of Brazil wood in two pints of water, for a quarter of an hour, and having added a little alum, gum-arabic, and sugar-candy, suffer the whole to boil for a quarter of an hour longer. This ink may be preserved a long time; and the older it grows, it will still become redder.

Blue Ink.

Blue ink may be obtained by diluting indigo and ceruse in gum water.

Yellow Ink.

Take saffron and yellow berries *(graine d'Avignon,)* or gamboge, and dilute them as before, in gum water.

Green Ink.

This ink is made by boiling sap-green in water, in which a little rock alum has been dissolved.

Ink of different Colours, made from the juice of Violets.

Dip a camel's-hair brush in any acid, such as diluted spirit of vitriol, and draw it over a part of the paper. When the liquor is dry, write on it with a pen dipped in violet juice, and the writing will immediately appear of a beautiful red colour.

If a camel's-hair brush, dipped in an alkaline solution, such as that of salt of wormwood in water, be drawn over the other part of the paper, by writing on it when dry with juice of violets you will obtain characters of a beautiful green colour.

If you write with the juice of violets, and draw a brush dipped in spirit of hartshorn, or a solution of salt of wormwood dissolved in water, over another, you will have red and green writing.

By exposing this writing to the fire, it will become yellow.

If you write on paper with an acid, such as lemon-juice (which is as proper for this purpose as any other), and then suffer it to dry, the writing will be invisible till brought near the fire, when it will become as black as ink. The juice of onions produces the same effect.

The older writing of this kind is, the more beautiful the colour becomes; and, in like manner, the longer the spirit of vitriol, or solution of salt of wormwood, &c. has been left to dissolve, before they are used to write with, the brighter will be the colours.

Tracing Ink.

This name is given to a kind of ink employed for tracing out figures, and other subjects, intended to be engraved, as by means of pressure it may be transferred from paper, and fixed on the white wax with which engravers cover their plates.

To compose this ink, take gunpowder finely pounded, and add to it an equal quantity of printer's black; then put the whole into water with a little Roman vitriol, and stir the mixture, giving it such a consistence that it may be neither too thin nor too thick. Before the ink is used, shake it well, because the black is apt to deposit itself at the bottom of the vessel.

China, or Indian Ink.

China ink, which is employed for small drawings and plans, may easily be made by the following process. Take the kernels of the stones of apricots, and burn them in such a manner as to reduce them to powder, but without producing flame; which may be done by wrapping up a small packet of them in a cabbage leaf, and tying round it a bit of iron wire. Put this packet into an oven, heated to the same degree as that required for baking bread, and the kernels will be reduced to a sort of charcoal, with which an ink may be made similar to that brought from China.

Pound this charcoal in a mortar, and reduce it to an impalpable powder, which must be sifted through a fine sieve; then form a pretty thick solution of gum-arabic in water, and, having mixed it with the powder, grind the whole on a stone, in the same manner as colourmen grind their colours. Nothing is then necessary but to put the paste into some small moulds, formed of cards, and rubbed over with white wax, to prevent it adhering to them.

In regard to the smell of the China ink, it arises from a little musk which the Chinese add to the gum water, and may easily be imitated. The figures seen on the sticks of China ink, are the particular marks of the manufacturers, who, as in all other countries, are desirous of distinguishing whatever comes from their hands.

Dr Lewis thinks, from the information of Father du Halde, that China ink is composed of nothing but lamp-black and animal glue. Having boiled a stick of China ink in several portions of water, in order to extract all the soluble parts, and having filtered the different liquors which he evaporated in a stone vessel, he found that the liquors had the same odour as glue, and that they left, after evaporation, a pretty considerable quantity of a tenacious substance, which seemed to differ in nothing from common glue.

Ink Powder.

Common liquid ink, the method of making which we have already described, is not easily transported from one place to another; and, besides this inconvenience, it is apt to dry in the ink-holder. In bottles, unless well corked, it becomes decomposed and evaporates; and if the bottles happen to break, it may spoil clothes, or any other articles near it. For the convenience, therefore, of those who travel either by land or by sea, ink powder has been invented, which is nothing else than the substances employed in the composition of common ink, pounded and pulverized; so that it can be converted into ink in a moment, by mixing it up with a little water.

To Revive Old Writing.

It is often necessary to consult old charters, titles, deeds, and manuscripts, written many centuries ago, either to gratify curiosity, or to clear up some important point in law; but as the writing is sometimes so much effaced as to be scarcely legible, a Benedictine invented a liquor, which will make old manuscripts appear as fresh as if newly written. The process for preparing this liquor, which may be easily applied, is as follows:

Having provided a pot, capable of containing three quarts of water, take some white onions, freed from the exterior thick skin, and cut them into small morsels; put such a quantity of them into the pot as to occupy three-fourths of it: then fill up the remaining part with water, and add three gall-nuts well pounded. Boil the whole for an hour and a half, and throw into the mixture about the size of a nut of rock alum. Strain the mixture through a piece of cloth, squeezing the onions strongly to express the juice, and preserve the liquor, which, when cold, will have the appearance of orgeat.

When you intend to use this liquor, expose it to heat, which will render it clear; then dip in it a bit of rag, and apply it to the writing near the fire, that the liquor may make a stronger impression, you will have the pleasure of seeing the characters revived in their full lustre. If there be only a few words of the writing effaced, it will be sufficient to heat a little of the liquor in a silver spoon, and to apply it as above.

Another process, more simple, consists in putting three or four pounded gall-nuts into a certain quantity of spirit of wine; heating the mixture and exposing to the vapour of it the writing which you wish to revive.

Old papers or parchments, the writing of which cannot be read, or can be read only with difficulty, may be immersed also in water in which copperas has been dissolved; if they are then suffered to dry, the copperas will make the writing reappear with as much freshness as if it were new.

To take off the Impression of any Drawing.

The impression of any drawing may be taken off, by placing a piece of glass over the original, and then tracing out all the outlines with a bit of soft red chalk; but as red chalk makes no mark upon glass, it must be first rubbed over with gum water, to which a little vinegar has been added; when the gum is dry, it will be fit for drawing on. Without vinegar, red chalk would not mark on the gum; but if you rub the glass with the white of an egg, instead of gum, there will be no need of vinegar.

When the drawing has been traced on the glass, if you apply to it a piece of moistened paper, pressing it strongly down, and immediately remove it, lest it should adhere to the glass, you will find imprinted on it the drawing made with the red chalk. By these means, you will obtain an exact outline of any drawing or print you wish to copy. This resemblance, however, will be reversed; and for that reason, to give it the same appearance as the original, it must be recopied.

To take off the Impression of Old Prints.

Take Venice or Windsor soap, which must be cut into small pieces, a certain quantity of potash, with as much quicklime, and boil the whole in a pot. Wet the engraved side of the print gently with this liquor, then apply to it a sheet of white paper, and roll it several times with a roller, in order that the impression may be complete.

Method of teaching Drawing to Young Persons.

An artist proposes to teach young persons the elements of drawing, by making them first practise with a slate, because it may be soon cleaned with a wet cloth, or sponge. This method indeed would save the expense of paper, and afford the pupils an opportunity of easily correcting their faults, without being obliged to begin their drawing again entirely. But it is more advantageous to employ, instead of a slate, a piece of Bohemian glass, which might be made rough on one side, by rubbing it with a pumice-stone, or a flat bit of free-stone, or fine sand well moistened. Whatever figures have been drawn on this glass, may be effaced by a wet cloth, in the same manner as from a slate; and besides this advantage, as the glass is transparent, correct copies may be placed below it, which the scholars ought to follow till the hand is properly formed. What is here said of drawing, may be applied also to writing.

To construct a Lantern which will enable a Person to Read by night at a great Distance.

Make a lantern of a cylindric form, or shaped like a small cask placed lengthwise, so that its axis may be horizontal, and fix in one end of it a parabolic or spheric mirror, so that its focus may fall about the middle of the axis of the cylinder. If a small lamp or taper be placed in this focus, the light passing through the other end will be reflected to a great distance, and will be so bright that very small letters on a remote object may be read, by looking at them with a good telescope. Those who see this light, if they be in the direction of the axis of the lantern, will think they see a large fire.

To take off Impressions in Plaster of Paris or Sulphur.

As curious people, who cannot purchase the originals, are often desirous of obtaining impressions of medals, engraved stones, and other valuable articles preserved in cabinets, they may easily be procured, and at a very small expense. The whole process consists in a very simple operation, which will give a striking resemblance of the object, so as to exhibit all its parts with the greatest truth.

When you intend to take off an impression in plaster, that which has been pulverized and sifted through a piece of very fine silk must be employed. First rub over the medal, or engraved stone, very softly with oil, and having wiped it with cotton, surround the edge of it with a bit of thin lead. Mix up the sifted plaster with water, and stir it gently, to prevent it throwing up air-bubbles; then throw it over the medals, and suffer it to harden and dry. It may then easily be detached, and will form a mould, strongly marked, by means of which you may take off impressions in relief, either in plaster or sulphur.—Observe, before these moulds are used, they must be impregnated with oil.

The process for melted sulphur is the same as for plaster; but it is to be observed, that when the model is of marble, old lard ought to be employed in preference to oil, because the latter, by penetrating through the pores of the marble, would stain it.

Baits for Catching Fish.

In order to attract fish when angling, baits made of various kinds of grain, such as wheat, barley, oats, or boiled beans, mixed with aromatic herbs, and pounded with earth, may be employed. Fish are wonderfully attracted by strong-smelling substances, as camphor, assafœtida, &c. They seem to have a great fondness for a paste made of crusts of bread, honey, and assafœtida. It is said also, that they approach coloured objects through curiosity.

Some people tie a bit of scarlet rag to the hook, and rub it over with petroleum; and others highly extol heron oil. To obtain the latter, the flesh of the heron is cut small, and pounded in a mortar; it is then put into a long-necked bottle, closely corked, and preserved for two or three weeks in a warm temperature; the flesh, by putrefying, is converted into a substance that approaches near to oil, which is mixed up with honey, bread, and a little milk. Most fish, and particularly carp, are said to be very fond of this bait.

Artificial insects are much used also for catching fish, especially trout; they are made of different colours, according to the hours of the day, in order that they may imitate the natural objects which appear at these different periods.

Those who fish in fresh water, employ cheese sometimes as a bait, and prefer that which emits the strongest smell. The putrid livers and flesh of animals of every kind are likewise used.

Small, long, slender worms, of a white or pale yellow colour, with a red head, contained in small cells found in the roots of the water iris, are said to be excellent bait for trout, tench, carp, and various other kinds of fish.

Earth worms, as well as those engendered in meat, are of great service.

To procure the latter, and almost at every season, a dead cat, or bird of prey, must be exposed to the flies, and when the worms become very lively, it ought to be buried in moist earth, as much sheltered from the frost as possible. The worms may be taken out as they are wanted. As these worms are metamorphosed into flies towards the month of March, recourse must then be had to other animals of the like kind.

To produce variety in the colours of Flowers.

Variety is generally produced in flowers by sowing, in the same bed, seeds collected from different individuals; and there is reason to think that this variety in colour arises from the farina of the differently coloured flowers, which mutually fecundate each other.

This conjecture is supported by experience; for it is found, that if flowers of the same kind, but different in colour, that is, some red and others yellow, flower together, the seeds arising from them produce red, yellow, and orange flowers, and even some diversified with red and yellow. It is certain also, that the variegations of flowers are more singular, according as the variety of colours contrasted together in the same bed is greater; that by planting together in the same pot yellow and white ranunculuses, the seed resulting from them will produce sulphur-coloured ranunculuses; and that aurora-coloured ones may be obtained, in like manner, by a similar process, with yellow and red ranunculuses.

It may easily be proved by experiment, that this phenomenon arises only from the influence of the farina; because, when these flowers are planted separately, and at a distance from each other, they produce only the same colours.

To obtain double Flowers.

The more petals a flower has, it becomes the fuller and more beautiful. Flowers sometimes are converted into double ones by accident, but there are some which are only very little so, as may be observed among carnations. There is, however, an artificial method of making them become double, which is, to transplant them several times the first year, as in spring and autumn, without suffering them to flower. By following this method for two years consecutively, single carnations may sometimes be converted into double ones.

To obtain Flowers of different colours on the same stem.

Scoop out the pith from a small twig of elder, and having split it lengthwise, fill each of the parts with seeds that produce flowers of different colours. Surround them with earth, and then tying the two bits of wood, plant the whole in a pot filled with earth properly prepared.

The stems of the different flowers will thus be so incorporated, as to exhibit to the eye only one stem, throwing out branches covered with flowers analogous to the seed which produced them.

By selecting the seeds of plants which germinate at the same period, and which are nearly similar in regard to the texture of their stems, an intelligent florist may obtain artificial plants exceedingly curious.

THE END.

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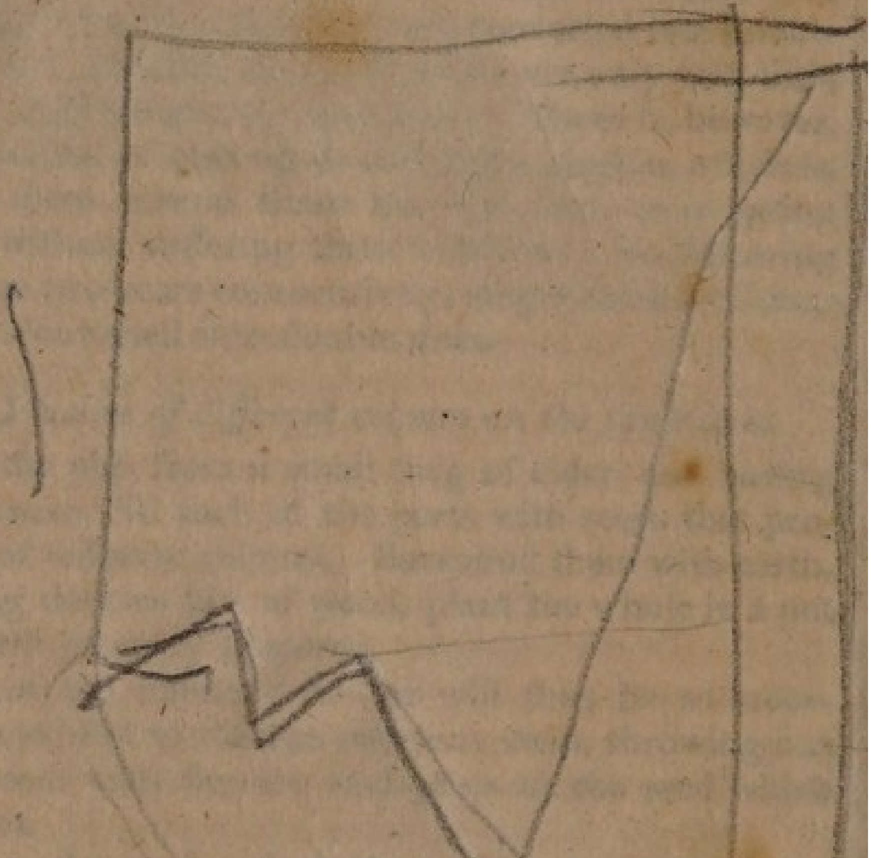

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