

Account of the life and writings of Robert Simson, M.D., late professor of mathematics in the University of Glasgow / By the Rev. William Trail.

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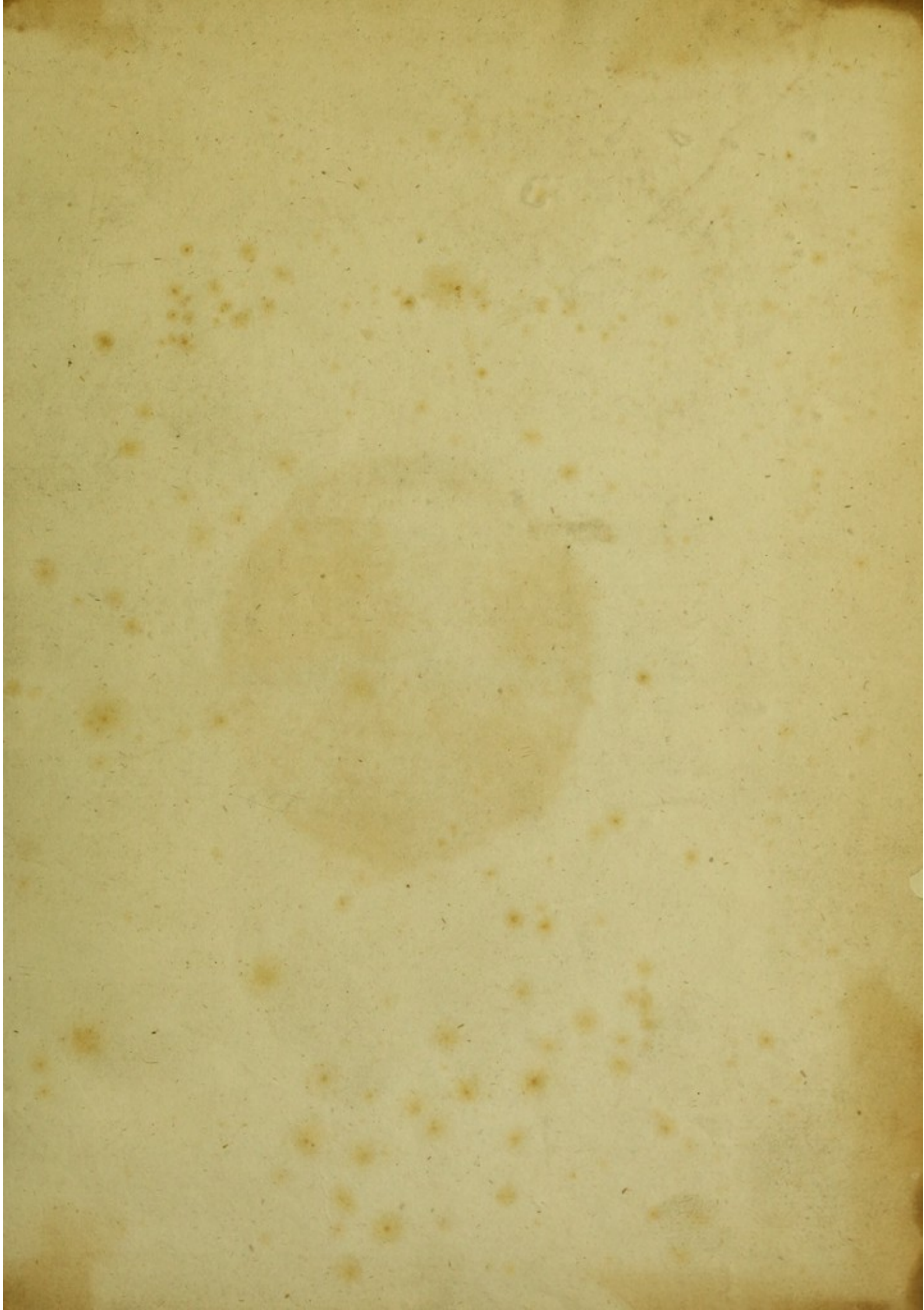
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LIFE AND WRITINGS
OF
ROBERT SIMSON, M.D.

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ACCOUNT OF THE
LIFE AND WRITINGS
OF
ROBERT SIMSON, M. D.

LATE PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

By the Rev. WILLIAM TRAIL, LL. D. F.R.S. Edin.

MEMBER OF THE ROYAL IRISH ACADEMY, AND CHANCELLOR OF ST. SAVIOUR'S, CONNOR.



Robertus Simson, M.D.

GEOMETRIAM SUB TYRANNO BARBARO SÆVA SERVITUTE DIU SQUALENTEM,
IN LIBERTATEM ET DECUS ANTIQUM VINDICAVIT UNUS.

PRINTED BY RICHARD CRUTTWELL, ST. JAMES'S-STREET, BATH;
FOR
G. AND W. NICOL, BOOKSELLERS TO HIS MAJESTY, PALL-MALL, LONDON.

1812.

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1812



TO
THE REVEREND THE PRINCIPAL,
AND TO
THE PROFESSORS,
OF THE
COLLEGE AND UNIVERSITY OF GLASGOW,
THIS ACCOUNT OF THE LIFE
AND WRITINGS
OF A DISTINGUISHED MEMBER
OF THEIR SOCIETY
IS MOST RESPECTFULLY INSCRIBED.

THE FRIENDS OF THE PHOENIX

AND TO

THE PHOENIX

OF THE

COLLEGE AND UNIVERSITY OF CALIFORNIA

THE SOCIETY OF THE

OF THE

A DISTINGUISHED MEMBER

OF THE SOCIETY

IS MOST RESPECTFULLY REQUESTED

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ADVERTISEMENT.

ABOVE thirty years ago the late EARL STANHOPE honoured me with a request to draw up an account of the Life and Writings of the late Dr. SIMSON, of Glasgow, which might be published in the new edition of the *Biographia Britannica*. The slow progress of that great work left me much at liberty as to the time of preparing an article which could appear only near the end of it; and for a number of years having been occupied by engagements of a different kind, I was in some measure compelled to postpone the execution of my undertaking, much longer than I wished to have done.

As there is not at present any near prospect of the completion of the *Biographia*, I could not properly, at my time of life, defer any longer embracing the opportunity afforded me of paying this small tribute of respect to the memory of that eminent man, by whose friendship and instruction I was honoured during some of the last years of his life. I thought it my duty therefore, though the undertaking was still

liable to interruptions from other concerns, to collect the necessary materials, and to arrange them for a separate publication.

With respect to the incidents of Dr. SIMSON's life, particularly those of the early part of it, I obtained satisfactory information from a short narrative drawn up by the Doctor's colleague and particular friend the late Mr. CLOW, which, from the intimacy subsisting between them for many years, may be considered as authentic and accurate. From several gentlemen who had been in the same College with Doctor SIMSON other circumstances have been communicated, and even from my own acquaintance with him, though only for a few years, some interesting particulars came within the reach of my own observation.

With regard to Doctor SIMSON's scientific pursuits, there are abundant materials of information in his Works which have been published, and particularly in his learned Prefaces and Notes which accompany them. Some circumstances, also, have been collected from his unpublished MS. papers, and from the small remains of his Correspondence. It is much to be regretted that the greater part of his Mathematical Correspondence, which appears to have been very extensive during a great part of his long life, had been either lost or destroyed in his own time. Some interesting fragments of it, however, were found among his papers; and a series of his letters to that eminent mathematician the late Earl STANHOPE, from 1750 to 1758, was in the most liberal and obliging manner communicated by the present Earl; and from them some curious notices of Dr. SIMSON's studies have been extracted.

Some history and explanation of the ancient geometrical Analysis, almost necessarily, forms a part of an account of the literary life of Dr. SIMSON, by whom that Analysis has been completely restored and illustrated. The *Mathematical Collections* of PAPPUS, in which is contained nearly every thing that is known from the Ancients of that celebrated instrument of investigation, which they employed so successfully in their geometrical inquiries, from the Doctor's early partiality for that branch of science, naturally became an object of his particular study. Some account therefore of that work, and of Dr. SIMSON's commentaries on it, is requisite for giving a just view of his geometrical labours, and for estimating their importance to, what is so much wanted, a corrected edition of that valuable Author. Even with the risk of several repetitions, it has appeared convenient to detach from the Memoir the particular account of PAPPUS, and some explanations of the ancient Analysis connected with it, and to annex them as an Appendix. These details, though uninteresting to many readers, may be acceptable to others; and my object in preparing them will be attained, if they can save some trouble to those who at any future time may undertake a new edition of the *Mathematical Collections*. Two short passages of PAPPUS, concerning the ancient classification of geometrical lines, are added in a second appendix, in the original language, with COMMANDINE's version of them.


To these is subjoined Dr. SIMSON's translation of the description of the Porisms of EUCLID, in the Preface to the seventh book of PAPPUS; in which are some material improvements of the former translations.

August, 1812.

Some history and explanation of the present geographical
 Analysis, which necessarily forms a part of an account of the
 history of the East Indies, by whom that Analysis has been
 completely revised and illustrated. The Mathematical Geog-
 raphy of Ptolemy, in which is contained nearly every thing
 that is known from the Ancients and is a valuable instrument
 of investigation, which they employed in their inquiries for
 geographical truths, from the stars, is very pertinent to
 that branch of Science, and is accordingly become an object of
 particular study. Some account therefore of that work, and
 of the Author's acquaintance with it, is requisite for giving
 a full view of his geographical system, and for clearing them
 up, in order to what is so much wanted, a corrected edition of
 the Author's Atlas. Even with the title of Geographical
 it has appeared convenient to detach from the Atlas, the
 particular account of Arabia, and some particulars of the
 ancient world, connected with it, and to insert them in an
 Appendix. The details, though interesting to many readers,
 may be necessary to others, and any object in preparing them
 will be attended, if they can find time to look at
 any thing more may be added, a new edition of the Atlas
 shall be published. The best papers of the press concerning
 the ancient constitution of geographical facts, are added in a
 second Appendix, in the original language, with Chinese
 translation of them.

To this is added, Mr. Stinson's translation of the
 description of the Kingdom of Siam, in the French and
 French book of Ptolemy; in which are some remarks and
 particulars of the same Kingdom.

London, 1712.



SECTION I.

General Account of DR. SIMSON's Life.

DOCTOR ROBERT SIMSON, late Professor of Mathematics in the University of Glasgow, was the eldest son of JOHN SIMSON, of Kirktonhill in Ayrshire, and was born on the fourteenth of October 1687, O. S. Being designed by his father for the Church, after having got the usual school education, he was sent to the University of Glasgow, where he was distinguished by his proficiency in classical learning, and in the sciences.

At this time, from temporary circumstances, it happened, that no Mathematical Lectures were given in the College; but young SIMSON's inquisitive mind, from some fortunate incident, having been directed to Geometry, he soon perceived the study of that science to be congenial to his taste and capacity. This taste however, from an apprehension that it might obstruct his application to subjects more connected with the study of theology, was anxiously discouraged by his father, though it would seem, with little effect.

Having procured a copy of EUCLID's Elements, with the aid only of a few preliminary explanations from some more

advanced students, he entered on the study of that oldest and best introduction to Mathematics. In a short time he read and understood the first six with the eleventh and twelfth books; and being delighted with the simplicity of language and accuracy of reasoning in EUCLID, notwithstanding the discouragements he met with, he persevered in his Mathematical pursuits;* and by his progress in the more difficult branches he laid the foundation of his future eminence. But though the bent of his inclination for these pursuits was strong, he did not neglect the other sciences then taught in the College; and in proceeding through the regular course of academical study he acquired the principles of that variety of knowledge, which he retained through life, and which contributed much to the estimation of his conversation and manners in society. His chief attention, however, was directed to his favourite science; and his reputation as a mathematician in a few years became so high, and his general character so much respected, that in 1710, when he was only twenty-two years of age, the Members of the College, without any solicitation on his part, made him an offer of the Mathematical Chair, in which a vacancy was in a short time expected to take place. From his natural modesty, however, he felt much reluctance, at so early an age, to advance abruptly from the situation of a Student to that of a Professor in the same College; and therefore he solicited permission to spend one year at least in London, where, besides other obvious advantages, he might have opportunities of becoming acquainted

* In some of his early MS. volumes still preserved, are notices of the books he had been reading, and of which he had been making abstracts. In one of these, dated in 1705, (then in his eighteenth year,) are mentioned OUGHTRED'S CLAVIS, RAPHSON'S TRACTS, JONES'S SYNOPSIS, KERSEY'S ALGEBRA, and several others.

with some of the eminent mathematicians of England, who were then the most distinguished in Europe. In this proper request he was readily indulged; and without delay he proceeded to London, where he remained about a year, diligently employed in the improvement of his mathematical knowledge.

This journey turned out very favourable to his views; and he had much satisfaction in the acquaintance of some respectable mathematicians, particularly of Mr. JONES, Mr. CASWELL, and Mr. DITTON. With the latter, indeed, who was then Mathematical Master of Christ's-Hospital, and well esteemed for his learning, he was more particularly connected; but from all of them he had opportunities of receiving information respecting the progress of the science, both in England and on the continent of Europe. When the vacancy in the Professorship of Mathematics at Glasgow did occur in the following year, the University, while Mr. SIMSON was still in London, appointed him to fill it; and the Minute of Election, which is dated March 11th, 1711, concluded with this very proper condition: "That they will admit the said Mr. ROBERT SIMSON, providing always, that he give satisfactory proof of his skill in mathematics previous to his admission." He returned to Glasgow before the ensuing session of the College, and having gone through the form of a trial, by resolving a geometrical problem proposed to him, and also by giving "a satisfactory specimen of his skill in Mathematics, and dexterity in teaching Geometry and Algebra;" having produced also respectable certificates of his knowledge of the science from Mr. CASWELL, and others, he was duly admitted Professor of Mathematics, on the 20th of November of that year.

Mr. SIMSON immediately after his admission entered on the duties of his office, and his first occupation necessarily was the arrangement of a proper course of instruction for the students who attended his lectures, in two distinct classes. He prepared elementary sketches of some branches in which there were not suitable treatises in general use. Among his papers one of these still remains, a translation of the three first books of L'HOSPITAL'S Conic Sections, in which geometrical demonstrations are substituted for the algebraical of the original, according to Mr. SIMSON'S early taste on this subject. There remain also some traces of his collections of Problems from HUYGHENS, GREGORY, and others, in optics and astronomy, for the use of his scholars; and it appears likewise that soon after his admission, he had given some public Lectures on the History of Mathematics, before the whole University, of which several happen to be preserved, and are proofs of the extent and accuracy of his learning at the early period of life when they were delivered.

Both from a sense of duty and from inclination, he now directed the whole of his attention to the study of mathematics; and though he had a decided preference for geometry, which continued through life, yet he did not devote himself to it, to the exclusion of the other branches of mathematical science; and in the progress of this Memoir it will appear that he was well acquainted with the modern analysis, particularly as it stood in the early part of his time. From 1711, he continued near fifty years to teach mathematics to two separate classes, at different hours, five days in the week, during a continued session (or term) of seven months; besides giving occasional

instruction which he was ever ready to communicate to those students, who wished for more particular explanations of his lectures, or to make further progress in the study of mathematics. Though the duties of a professor soon became familiar and easy to him, yet they occupied a considerable portion of his time, and divided it, so as often to interrupt the course of his private studies.

His manner of teaching was uncommonly clear, and engaging to young people; and most of his scholars retained through life an affection and reverence for the Professor. The College of Glasgow in his time was in great repute both at home and abroad, to which Dr. HUTCHESON, Dr. MOOR, Mr. ADAM SMITH, and himself, much contributed. The resort of students was great, and almost all of them attended Dr. SIMSON'S lectures. The knowledge of the elementary branches of mathematics, and of the most useful applications of them, were thence much diffused in the College, and some taste also for the study of the higher branches was excited; but the early age of the greater number of the students, their short residence in College, and the necessary appropriation of a considerable portion of their time to other sciences, seldom admitted of that long and nearly exclusive cultivation of one particular science, by which alone, especially in mathematics, eminence usually can be attained. Among Mr. SIMSON'S scholars, however, several rose to distinction as mathematicians. Dr. MATTHEW STEWART, who alone has applied the geometrical method of reasoning to the most complex physical investigations, by universal acknowledgement, is to be named the first. Mr.

WILLIAMSON,* a favourite pupil, from whom he had great expectations, died very young. Dr. JAMES MOOR, Greek Professor at Glasgow, and Professor ROBISON,† of Edinburgh, were all well known as mathematicians of superior abilities and attainments.

In the year 1758, Dr. SIMSON, being then seventy-one years of age, found it necessary to employ an assistant in teaching; and in 1761, on his recommendation, the Rev. Dr. WILLIAMSON was appointed his assistant and successor.

The resignation of Dr. SIMSON presented an opportunity to the Principal and Professors of recording in their minutes the affection which they felt for the Doctor, and their high admiration of his genius. A long paper for this purpose was drawn up by his colleagues, and it is expressed with all the warmth of attachment and respect, which it was natural for them to entertain for the father of the College, from whom the University had derived so much honour. Several of them had been his pupils, and all had lived with him in habits of friendship from the time of their becoming members of the Society. It is introduced in the following manner:

“The University Meeting do hereby gratefully, unanimously,
“and warmly express to Dr. SIMSON their most cordial thanks

* Mr. WILLIAMSON, afterwards the Rev. Dr. WILLIAMSON, Chaplain to the British Factory at Lisbon, in which station he died.

† In the third and fourth editions of the Encyclopedia Britannica (in the article SIMSON R.) is a sketch of Dr. SIMSON's life, which is known to be from the pen of Professor ROBISON, of Edinburgh; and though, from some accidents not now to be ascertained, there are some errors in dates, and in some small circumstances of the narrative, yet it bears the character of the distinguished ability and knowledge of the writer.

“ for his long, faithful, and eminent services to the University,
“ in the course of fifty years; during which very uncommon
“ period, he has with universal applause, been Professor of
“ Mathematics here; to the great honour of this University,
“ as well as to his own high reputation, which will last as long
“ as united elegance and science are admired among mankind.”

It proceeds to mention generally some of his most distinguished inventions, justly stating that he was the first, in modern times, who fully understood and explained the ancient analytical geometry; that he had already restored, with superior accuracy and elegance, some of the most valuable works of the ancient geometers, which for ages had been lost; and that he purposed to employ the remaining leisure of his life, in completing and publishing others, not less necessary for the full illustration of the nature and use of the ancient analysis: “ for the accomplishment of which valuable design, (the Minute concludes,) “ of so much importance to the advancement of true science, the University Meeting do wish and “ pray that he may long enjoy the blessing of a vigorous old “ age; and they intreat Dr. SIMSON to be entirely assured, “ that they will at all times heartily embrace every opportunity of testifying to him their gratitude, their affection, “ esteem, and veneration.”

During the remaining ten years of his life, he enjoyed a pretty equal state of good health; and continued to occupy himself in correcting and arranging some of his mathematical papers, and occasionally for amusement, in the solution of problems, and demonstration of theorems, which occurred from his own studies, or from the suggestions of others. His con-

versation on mathematical and other subjects continued to be clear and accurate, yet he had some strong impressions of the decline of his memory, of which he frequently complained; and this probably protracted, and finally prevented his undertaking the publication of some of his works, which were in so advanced a state, that with little trouble they might have been completed for the press.* His only publication, however, after the resignation of his office, was a new and improved edition of the Data of Euclid, which in 1762 was annexed to the second and corrected edition of the Elements. But from that period, though much solicited to bring forward some of his other works on the ancient geometry, though he knew well how much it was desired, and though he was fully apprized of the universal curiosity excited respecting his discovery and illustration of the Porisms of Euclid, he resisted every importunity on the subject.†

* The following extracts from Dr. SIMSON's Letters to the Writer of this Memoir mention the impression he had of the decline of his memory:

In a letter dated February 11th, 1766.—“As to the publishing any thing on the ancient geometry, give nobody any hopes of my doing it. You know my inability to do any thing that requires so much thought and application as that would do; and if it be not by your assistance in preparing them for the press, and in taking care of the printing, I have scarce any hopes of doing any thing that way, though I much desire it.” And in another letter of March 7th, 1767, he says, “I wish you had not mentioned any thing that might give expectation of the Porisms from me, as it is very doubtful whether I shall be able to publish any thing either about them, or any other geometrical subject; my memory, and consequently any other ability for such things, being so greatly decayed. However, your kind offer of assistance will make me give out some figures to be cut, though I believe they cannot be ready against the time you propose to be here.”

I shall have afterwards occasion to mention the impression of a decline of memory which Dr. SIMSON seems to have felt many years before.

† Sometimes he seems to have entertained serious thoughts of publishing the most important of his remaining works, the Sectio Determinata, and the Porisms; and in

A life like Dr. SIMSON's, purely academical and perfectly uniform, rarely contains occurrences, the recording of which could be either interesting or useful. But his mathematical labours and inventions form the important part of his life, and supply the best illustration of his character; and with respect to them, there are abundant materials of information in his printed works; and some circumstances also may be gathered from a number of MS. papers which he left; and which, by the direction of his executor, are deposited in the Library of the College of Glasgow.* It is to be regretted, that of the extensive correspondence which he carried on through life with many distinguished Mathematicians, a small portion only is preserved. The greater part of it had been lost or destroyed in his own time; so that in 1751, when he was desirous of reviewing some of his early letters to Dr. JURIN on a particular subject, he obtained a copy, probably from the archives of the Royal Society, of which Dr. JURIN at the time of the

one of his MS. books, of which the earliest date is of 1762, there is the following notice. "The title-page, if the book to which it belongs come to be printed, may be thus:

" APOLLONII PERGÆI
" De Sectione Determinata Libri duo,
" Restituti."

" Quibus adjecta sunt non pauca Porismata; inter quæ habentur quædam EUCLIDIS,
" quibus Doctrina Porismatum explicata et restituta est."

* By the liberal favour of the Principal and Professors, the Writer of this Memoir has had every accommodation for consulting these Papers, and also the very valuable Mathematical Library which Dr. SIMSON bequeathed to the University. He has received also from them some material communications respecting their distinguished Colleague; and he is particularly obliged to Professors JARDINE, YOUNG, and MILLAR, for procuring useful information, and aiding his researches.

correspondence (1723) was secretary. Through Dr. JURIN he had some intercourse with Dr. HALLEY; and both at that time, and afterwards, he had frequent correspondence with Mr. MACLAURIN, with Mr. JAMES STIRLING, Dr. JAMES MOOR, Mr. WILLIAMSON of Lisbon, and more particularly with the Rev. Dr. MATTHEW STEWART; of which there are some notices in his printed works, and of which also there are some remains among his unpublished papers. In the latter part of his life, his mathematical correspondence was chiefly with that eminent Geometer the late Earl STANHOPE,* and with GEORGE LEWIS SCOTT, esq; then a Commissioner of Excise, and well known for his scientific attainments; and from Dr. SIMSON'S letters to both these gentlemen, some illustration of his opinions and pursuits will be obtained, which shall be taken notice of in a subsequent part of this Memoir.

* The Doctor's affectionate respect for that venerable Nobleman is strongly marked by the concern which he expresses in a letter to himself on hearing a report of his being indisposed. " May it please the gracious God to preserve so valuable a life for many good purposes, both with respect to your Lordship's honourable family, and the public good; and particularly for promoting of real and useful knowledge of every kind." — Letter dated 7th March 1755.

SECTION II.

*Particular Account of Dr. SIMSON's Mathematical Studies,
and of the Works published by himself.*

IT has already been observed, that Dr. SIMSON directed his early attention to the ancient Geometry. Dr. HALLEY, by his edition of the Conics of APOLLONIUS, and more particularly by his publication of two Treatises of APOLLONIUS, accompanied with a corrected edition of the preface to the seventh Book of PAPPUS, gave considerable aid to those who embarked in that study. The *Mathematical Collections* of PAPPUS are indeed the chief repository of information respecting the Geometry of the Ancients, and especially respecting their analysis, which has been the subject of much discussion among the moderns. This most interesting work contains some curious mathematical history of former times, but is more particularly valuable, by the account, contained in the preface to the seventh Book, of the treatises of the analytical geometry of the ancients, which together obtained the name of *τόπος ἀναλυόμενος*. In this preface there is, first, a general exposition of the analysis employed by the ancients, both in the solution of problems, and in the demonstration of theorems; then follows a particular description of the nature and contents of a certain number of these treatises, which we may presume

were considered by PAPPUS as the most important; and an enumeration of the whole is added, consisting of thirty-three books. The seventh book of PAPPUS itself consists of a number of Lemmata, or subsidiary propositions, not contained in EUCLID, but assumed or employed in the several treatises which are so fully described in the preface.

It is through the medium of COMMANDINE's translation only, that the importance of this work of PAPPUS is yet known to the public. Though many manuscript copies of the original remain in various libraries of Europe, it has never been printed; and no attempt has been made to correct the translation of COMMANDINE, whose zeal, learning, and ability, in promoting the knowledge of the ancient Geometricians by useful translations, are very meritorious; though this posthumous version of PAPPUS, no doubt from its not having received his final corrections, remains in a less perfect state.*

Dr. SIMSON, however, seriously applied himself to the study of this translation; and notwithstanding its unfinished state, it was the means of enabling him to investigate, and fully to illustrate the principles, of the analytical geometry of the ancients. The generality of modern mathematicians, from not having fully considered the intimations of PAPPUS on this subject, fell into strange misapprehensions respecting it.† When they examined the works of the ancient geometers,

* For a particular account of PAPPUS, and of this translation, see the Appendix.

† VIETA must be excepted; who, though he did not use the ancient analysis in his *Apollonius Gallus*, yet he seems persuaded, that the ancients had a true geometrical analysis. For in his first Appendix to that Treatise (p. 339, Op. VIETÆ) he observes, "At Algebra quam tradidere, THEON, APOLLONIUS, PAPPUS, et alii veteres analystæ omnino geometrica est;" &c.

EUCLID, ARCHIMEDES, and APOLLONIUS, they admired the elegance and clearness of their demonstrations, but wondered by what contrivance they had invented and proved so many curious theorems, and obtained solutions of so many difficult problems. They asserted even that the ancients must have possessed an analysis equivalent to the algebraical; but that they had industriously concealed it, in order to excite the greater admiration of their inventions. This opinion, naturally improbable, is however unreservedly expressed by some eminent mathematicians; by FRANCIS SCHOOTEN, by his brother PETER SCHOOTEN, by PETER NONIUS or NUNEZ; and what is more remarkable, is avowed in later times by the profound Dr. WALLIS,* who was certainly acquainted with PAPPUS, both in the original, and in COMMANDINE's translation. But the important communications of that author respecting the geometrical analysis of the ancients seem not to have been duly considered by this very eminent Mathematician, though he published, with learned and valuable notes, a fragment of PAPPUS, not contained in COMMANDINE's translation, but which he discovered in one of the two Savilian MSS. of the *Mathematical Collections*.

Dr. BARROW also, in his Lectures,§ gives countenance to this erroneous judgment on the ancients, which has been

* WALLIS's Algebra, chap. ii.

§ In Dr. BARROW's four Lectures on the Discoveries of ARCHIMEDES, after stating his plan in p. 341, he adds, "unde patebit qualem analysin, et quam nostræ modernæ similem exercuerit." And in p. 376, after giving an algebraical analysis of a problem, (viz. Prop. v. Lib. 2. ARCHIMED. *de Sphæra et Cyliandro*,) producing a cubic equation, he derives from it the proportion given by ARCHIMEDES, in which a certain straight line is to be cut in order to resolve the problem; he adds, "Qui ipsissi-

pronounced by so many mathematicians of modern times; and even so late as the publication of Sir ISAAC NEWTON'S Fluxions in 1736, Mr. COLSON, the commentator on that work, states this to be the opinion of "many of our modern geometers."[†]

This charge, however, against the ancients, as is justly observed by Dr. SIMSON,[§] was altogether groundless. The only algebra known to the ancients was that of DIOPHANTUS,

"mus est analogismus iste, ad quem rem deduxit ARCHIMEDES; quod ipsum satis prodit et arguit, qualem is analysis usurparit. Nam huc eum devenisse, varias istas proportionum compositiones, divisiones, permutationes, ac inversiones, quales in discursu suo ostendat adhibendo, pene supra fidem est." But in this case the ancient analysis is properly applicable; and under the management of ARCHIMEDES, its power in this proposition need not excite surprize. He refers also to the elegant synthetic solution of this problem by HUYGHENS, by means of the trisection of an arch. HUYG. Illustr. Probl. prob. 1. Perhaps some of the discoveries of ARCHIMEDES may have been suggested to him by views and reasonings resembling those of CAVALLERIUS, as is conjectured by his Commentator TORELLI; but he established their truth by rigorous demonstrations, which might be naturally derived from the only analysis of ancient times, the geometrical.

[†] See COLSON'S Commentary on NEWTON'S Fluxions, p. 143: and it may be here remarked, that the observations of Sir ISAAC NEWTON, (page 1,) to which this part of the Commentary refers, seem to imply, that he considered the ancient geometry, which he greatly admired, as synthetical only, without having much considered the nature and merits of their analysis. Some of his observations in the *Arithmetica Universalis* naturally lead to the same inference. See sect. iv. cap. 1. art. 18: also, Appendix de *Æquationum Constructione Lineari*, art. 51. Dr. PEMBERTON'S account of Sir ISAAC NEWTON'S opinion of the ancient geometry, of his regret that he had not studied it more particularly, and also of his remarks on *De Omerique*, corresponds with these observations in Sir ISAAC'S works. It may be added, that opinions like that of Mr. COLSON are to be found in many respectable recent writers; and consistently with them, the ancient geometry is generally distinguished as *synthetical*, while the term *analysis* is almost exclusively applied to the modern system.

[§] See the Preface to Dr. SIMSON'S restoration of the *Loci Plani* of APOLLONIUS, where this opinion is particularly stated, with the Doctor's satisfactory refutation of it.

which was never applied to geometry. But they had an analysis of their own, of which they made no secret, and of which there are some short specimens, even in EUCLID'S Elements.* This analysis however is more fully described by PAPPUS, who also observes, that in order to facilitate the solution of geometrical problems by this method, the ancients composed no less than thirty-three books, which collected together got the title of *τόπος ἀγαλυμένος*; and of the chief of which, as has already been mentioned, he gives an interesting account in the preface to his seventh book.

What led probably to this prevailing mistake of modern mathematicians was, that in the most valuable ancient treatises still remaining, such as those of ARCHIMEDES, and even the Conics of APOLLONIUS, the analysis of their propositions (particularly of their theorems) is generally omitted; though there can be no doubt that this method was employed by them, both in resolving problems, and for ascertaining the truth of theorems, which now appear in their works only in the synthetic form, and which had been either proposed to them by others, or had occurred to them in their own studies. In copying their books, however, for general use, for the sake of shortness, (which, before the invention of printing, was an object of consequence,) and even for facility to the reader in acquiring the knowledge of their discoveries, the analysis was often omitted. In the fifth book of PAPPUS is an example of this omission, for which he assigns the reason just now mentioned, and which we may believe influenced the other more ancient geometers. He observes, that in the com-

* EUCLID'S Elem. book xiii. prop. 1, 2, 3, 4, 5.

parifon which he is about to give of the five regular solids having equal surfaces, for the sake of *brevity* and *perspicuity*, he is to employ only the synthetic method, and not the analytic, which some of the ancients had used in treating that subject.† But in the other analytical treatises of APOLLONIUS, containing solutions of general problems for facilitating the resolution of any particular geometrical problem which can be reduced to a case of them, a full analysis of every case is given, being essential to this important application of such problems. In the *Sectio Rationis*, recovered from the Arabic by Dr. HALLEY, is a specimen of this compleat manner of solution, which, we must presume, was followed in the other treatises, now unfortunately lost.

These books appear to have been all existing in the time of PAPPUS, but the greater part of them have since perished, or at least they are not known to remain, either in the original, or in any translation. The book of *Data* by EUCLID, in the Greek; two books of *Sectio Rationis*, by APOLLONIUS, in an Arabic version; and seven books of the *Conics* of APOLLONIUS, four in Greek and three in Arabic; are all that have been preserved. But fortunately the descriptions by PAPPUS of eleven more of these books are so particular and entire, that some eminent modern mathematicians have been able to restore them, with various success indeed, as shall be afterwards more particularly stated. Those by Dr. HALLEY, from his superior taste and knowledge of the subject, are in the purest style of geometry.

† His expression is, “Καὶ τὴν ἴφοδον τῶν ἀποδείξεων ἔχουσας, ἢ διὰ τῆς ἀναλυτικῆς λεγομένης θεωρίας δι’ ἧς ἔτι τῶν παλαιῶν ἐποιήθη τὰς ἀποδείξεις τῶν περιεσημένων σχημάτων, ἀλλὰ διὰ τῆς κατὰ συνθέσιν ἀγωγῆς ἐπὶ τὸ σαφές τε καὶ συντομώτερον ὕψ’ ἰμῶ, διασκευάσμενας.” &c. Ex Cod. BULLIALDI, ad fol. 99. a. COM.

The others, by VIETA, SNELLIUS, MARINUS GHETALDUS, FERMAT, SCHOOTEN, and WALLIS, of which some are algebraical, and some geometrical, though respectable works of ingenious men, are defective in that elegance and compleatness of solution which distinguish the analytical writings of the ancient Geometers. A field was thus still open, for a person of Dr. SIMSON's taste and genius, for attempting a more perfect restoration of these curious works of antiquity; and it appears, that soon after he was placed in the Mathematical Chair, he set about the investigation. From his papers still remaining, we learn that his endeavours were first directed to the improvement of the defective restorations of these books, by preceding geometers. From the same source also we know, that within a few years he turned his attention to the Porisms of EUCLID; and it was to be expected that the curious nature of these Propositions, and even the difficulty of the investigation, would attract the early notice of his ardent mind.

Unfortunately the description of the Porisms by PAPPUS, in all the MSS. of his *Collections* which have been examined, is so mutilated, that every attempt to restore them, before Dr. SIMSON's time, had failed. The first part of the description, which seems to be entire, is calculated only to excite curiosity; being too general for conveying any precise notion of these Propositions, or for giving any effectual assistance for the recovery of them: and the remainder, containing a detail of the contents of EUCLID's work, is through the whole so depraved by the injuries of time, that all endeavours to explain it were nugatory. Some Geometers indeed, of great name, flattered themselves that they had got

possession of the secret of this peculiar class of Propositions; but subsequent to them, Dr. HALLEY, with all his genius, his extensive knowledge, and his successful experience in unravelling some other pieces of ancient geometry, gives up the Porisms as a hopeless pursuit; and he admits that the description of PAPPUS, as it now stands, is unintelligible and useless.*

Dr. SIMSON, however, fully proved that those ingenious men before Dr. HALLEY, who supposed that they had discovered the Porisms, had certainly deceived themselves; and that the celebrated FERMAT alone, in modern times, had acquired some notion of the nature of Porisms, but without being able to unfold it completely; and without having restored any one Proposition, that could be supposed to belong to the Treatise of EUCLID.

The Doctor himself candidly informs us, how long and how seriously he had ineffectually laboured in search of this ænigma, as the nature of a Porism truly became, from the very mutilated state of the only existing description of it. An ardent curiosity was a prominent and well-known part of his character; and in this case, his curiosity was enlivened by his predilection for the study of the ancient geometry; but we may presume also, that a natural and laudable ambition for the distinction which would result from success, in a pursuit in which the greatest Mathematicians of his own and of the preceding age had failed, would animate his zeal and perseverance in the investigation. He had been occupied on this subject even in the year 1715,

* "Hactenus Porismatum Descriptio, nec mihi intellecta nec lectori profutura." See the Preface to the Seventh Book of PAPPUS, edited by Dr. HALLEY, octavo, 1706, page 87, note at the end of the Porisms.

and perhaps before that period; for he observes, that in that year he had demonstrated the first case of FERMAT's fourth Porism, before he had acquired the knowledge of the nature of that class of Propositions.*

The first direction of his research seems to have been, to endeavour to discover the Porisms, from the general description of them in the beginning of the account given by PAPPUS; and when this failed, he tried to recover some of the individual Porisms, from which he hoped to ascertain the distinctive qualities of these Propositions; but in this attempt he had no better success. He continued his researches, and devoted his whole attention to the subject. For a considerable time his imagination was compleatly occupied by it: his mind was harrassed by the constant, but unsuccessful exertion; he lost his sleep, and his health was injured: but all his endeavours were ineffectual; and therefore he finally determined to banish for ever the subject from his thoughts.

For some time he maintained this resolution, and applied himself to other pursuits;† but afterwards he happened to be walking with some friends on the banks of the river Clyde at

* SIMSON's Posthumous Works, p. 540, last line.

† It is not improbable, that Dr. SIMSON about this time turned his attention to Algebra, and that, on despairing of the Porisms, he had employed himself in the investigation of Serieses for the Circle, of which an account is given in this Memoir, (see Note H.) The Porisms, as I reckon, were first discovered in April 1722. For some time after, it is reasonable to suppose that he was almost entirely occupied in prosecuting his invention; and therefore, as he transmitted these Serieses to Dr. JURIN in February 1723, and from his letter also it appears that they had been found out some time before, it is very probable they were investigated before his invention of the Porisms. They may be considered as a respectable specimen of what Dr. SIMSON might have done in Algebra, had he devoted his attention to that branch of Mathematics.

Glasgow, and by accident being left behind his company, he inadvertently fell into a reverie respecting the Porisms. Some new ideas struck his mind, and with his chalk having drawn some lines on an adjoining tree, at that moment, for the first time, he acquired a just notion of one of EUCLID'S Porisms.* I have repeatedly heard him relate the occurrence, which he seemed to do with pleasure, and he mentioned even the scite of the tree on which he described the fortunate diagram.†

After this first discovery, however, it required time and much investigation, before he could restore, to his satisfaction, the general Proposition of EUCLID'S first book of Porisms; and it appears that his first communication of the discovery to the Mathematicians of London was through Mr. MACLAURIN. Just before Mr. MACLAURIN'S setting out from Scotland for France, by the way of London, in 1723, Dr. SIMSON communicated to him the Proposition he had recently restored, and which a short time after was printed in the Philosophical Transactions. It appears also that Dr. JURIN, then Secretary

* This incident is generally stated by Dr. SIMSON himself, in his Preface to the Porisms. Post. Works, pp. 319, 320.

† Though most of Dr. SIMSON'S notes and propositions are dated, yet there is no entry to be found among his papers of this particular incident. From the dates of several of his first investigations of the Porisms, both in his PAPPUS, and in other MSS. it had, with much probability, occurred in April 1722; for in that month are some of his first notices about the Porisms. At the end of one of them he adds, "Hodie hæc de Porismatis inveni, R. S. April 25, 1722." Also in a note on Prop. 131, lib. vii. PAPPUS, he says, "Postquam vero ipsa Porismata, ea velim quæ generali propositione in præfatione ad hunc librum septimum, complexus est PAPPUS, multo labore at fuc-
cessu præter quem sperare æquum fuit felici, tandem ex manca et contracta admodum PAPPUS descriptione investigavimus; facilius erit lemmata hisce inventia, ad pristinum nitorem restituere. April 27, 1722. ROB. SIMSON."

of the Royal Society, had mentioned in a letter to Dr. SIMSON this communication from Mr. MACLAURIN, for Dr. SIMSON, in a letter to Dr. JURIN, dated Feb. 1, 1723, in reply to one he had just received from him, says, "The Proposition of PAPPUS, which I shewed Mr. MACLAURIN when here,* is the general Proposition into which PAPPUS collected the Porisms of the first book of EUCLID's Three; which, together with the more general Proposition immediately subjoined, and which are both deficient, (I mean imperfect,) I have, with no little investigation, recovered and demonstrated, together with some few of the Porisms; the rest I have not had leisure enough to try. I mean those of the first book, for as to those of the two others, excepting what may be included in the second of the above-mentioned Propositions, I believe it will be extremely difficult for any body to restore them. If I understand you like to see them, I shall send them as soon as I can get the scribbles I have about them wrote over fair."†

It would appear that the result of this letter was an immediate communication of the Propositions to Dr. JURIN, which, as might be expected, were much approved by those eminent Geometers Dr. HALLEY, Mr. MACHIN, and Dr. JURIN himself. In the course of the same year the Paper was printed in the Philosophical Transactions,‡ and Dr. SIMSON, in a letter

* This is mentioned in the Preface to Dr. SIMSON's Conics, p. vi. 2d edit. See also Note A. at the end.

† The principal object of this letter was to communicate to Dr. JURIN some Serieses about the Circle, of which notice will be taken afterwards. See also Note H. at the end.

‡ Vol. XL. for 1723, p. 330. In the same volume of the Transactions, p. 248, is a Paper by Dr. PEMBERTON, containing some propositions about the Rainbow; at the

to Dr. JURIN, of January 10, 1724, expresses his satisfaction at the reception of his communication in the following manner: “ The honour you have done me in printing the Paper I sent up, in the Philosophical Transactions, is what I am very sensible of; and you may be sure the approbation any thing in it has had from such good judges as Dr. HALLEY, Mr. MACHIN, yourself, and the other learned gentlemen you were pleased to shew it to, cannot but give me a great deal of pleasure; and nothing could excite me more to endeavour to restore the other Porisms, of which there are the least data: but I find it a very difficult affair, especially to one who is so slow, as by much experience I find myself. However I shall at leisure hours, GOD willing, try what I can do. I desire you may give my humble respects to Dr. HALLEY and Mr. MACHIN; and be pleased to tell the Doctor, that as to the meaning of the words *Ἐὰν ὕπτια ἢ παραπλίε, ἢ παραλλήλα*, I never was solicitous about the meaning of the proposition, but at his desire I have considered the passage,” &c. Dr. SIMSON then enters into a long and learned discussion of the proper meaning of this passage, of which the result is shortly stated in a note, page 348, Post. Works.*

beginning of which he observes, “ For the greater brevity, I shall deliver them under the form of Porisms; as, in my opinion, the ancients called all Propositions treated by analysis only.” This intimation naturally gave some dissatisfaction to Dr. SIMSON, as assuming the appearance of being the first to announce to the world the nature of the ancient Porisms. In the copy of this volume of the Transactions belonging to the College Library of Glasgow, the Doctor wrote a short animadversion on the margin of the page, which concludes with these words: “ but he (Dr. PEMBERTON) has entirely mistaken the nature of a Porism, and his two Propositions have neither the form nor matter of Porisms. R. S.”

* The rest of the letter may be interesting to some readers, and is therefore placed in Note C, at the end.

For a few years after his discovery of the Porisms, his mind was much engaged in the further prosecution and illustration of it. It appears, however, from his papers, that at this very time he occasionally applied to other branches both of ancient and modern Mathematics; though, as was to be expected, the chief object of his study was the improvement and extension of his investigation of the Porisms, by recovering more of EUCLID'S Propositions, and adding also some of his own, and subsequently also the contributions of a few mathematical friends, to whom he communicated the interesting intelligence of his discovery. There are many indications of his intention of publishing the Porisms; but from various causes he postponed the execution of it, till, in the progress of life, he had acquired so very strong an impression of the decline of his faculties, that he reluctantly gave up the design. The *Treatise on Porisms*, however, with some other valuable tracts, were published a few years after the Doctor's death, at the sole expense of his highly respected and learned friend the late Earl STANHOPE; and by the full explanation of the Porisms contained in that volume, (of which I shall have occasion to give a more particular account,) this hitherto inexplicable portion of ancient geometry was, by the Doctor's perseverance and ingenuity, compleatly restored and explained. It may justly be admitted, that the satisfaction and the pride of invention, in the elucidation of this very difficult branch of ancient science, involved in such obscurity from the depraved state of the only remaining account of it, were properly high; and by liberal and scientific minds, the warmth with which he expresses these feelings will not be disapproved: " Descriptio
" autem quam tradit (PAPPUS) Porismatum adeo brevis est et

“ obscura, et injuria temporis aut aliter vitiata, ut nisi DEUS
 “ benigne animum et vires dederat in ea pertinaciter inquirere,
 “ in perpetuum forsan geometras latuissent.”*

It has been already observed, that before Dr. SIMSON obtained any just notion of the Porisms, he had begun to improve the restorations of other ancient geometrical treatises; which had been attempted by preceding geometers, particularly of the *Loci Plani*, and *Sectio Determinata*,† of APOLLONIUS. To those who have some knowledge of this subject, it is needless to explain the value of these two treatises; particularly of the former, as containing many elegant Theorems, and as being eminently useful in the resolution of Problems. The *Loci Plani* of APOLLONIUS had been restored in a certain manner, both by FERMAT, and by FRANCIS SCHOOTEN. In the posthumous works of the former very distinguished Mathematician,‡ is given a restoration, geometrical indeed, but synthetical only without analysis, and deficient also in other material points, particularly in the distinction of the cases, and in ascertaining the determinations; without which no geometrical resolution can be considered as complete.§

* Opera Reliqua, p. 513.

† The *Sectio Determinata* was not published in his own time, and I therefore defer any notice of it, till an account be given of the volume of Posthumous Works.

‡ FERMAT had restored this work of APOLLONIUS before 1629, as appears by a letter of his, p. 153 of his Works. But it was printed only in 1679, among his Posthumous Works. See FERMAT. Oper. Varia, tom. 11. p. 12. FERMAT had not seen the Preface to the 7th book of PAPPUS in Greek.

§ The use of *Loci* in the resolution of Problems is very obvious, even to those who are but little acquainted with ancient Geometry. The great importance of the complete distinction of cases and determinations in those treatises of the *τέρας ἀναλυ-*

The restoration by SCHOOTEN has similar defects; in a few only of the problems an analysis is given, but one purely algebraical; and he acknowledges in his preface* that his restoration of the *Loci Plani* was designed to be an illustration of the geometry of DES CARTES, by furnishing proper examples of his method. Though it appears that Dr. SIMSON had begun, at a very early period, to restore the *Loci Plani* after the ancient model, and though the work was almost completed before he published his *Conic Sections* in 1735, yet it was not printed till 1749. What reasons occurred for this delay cannot now be conclusively ascertained; but we learn from his papers, and from some remains of his correspondence, that at one time he had designed to add one or two books of *Loci* to those of APOLLONIUS. There are many detached Propositions, and even some Serieses of Propositions of that description, in which the two books of APOLLONIUS are quoted as part of the same work, which sufficiently ascertains his purpose, at the time of writing them.† It is indeed much to be regretted, that he did not pursue this

ομείνος, which consist of general Problems, to which other Problems may often be reduced, will be explained afterwards. The same attention to the cases and determination of *Loci*, and also of Porisms, is equally necessary, to render the application of them useful to the solution of Problems.

* F. SCHOOTENII *Exercitationes Mathematicæ*, Præf. 1657.

† In a letter to Earl STANHOPE, dated September 10, 1750, after mentioning that the *Loci Plani* were just printed, and about to be published, he adds, "that he once designed to have added several other *Loci Plani*, but thought it best now to give only those mentioned by PAPPUS, with a few to make some of them more complete."—There is also among his papers a sketch of a title for the *Loci Plani*, comprehending some additions to those of APOLLONIUS.

scheme; as no work could be more generally useful in geometrical investigations, than an extensive and well-arranged collection of *Loci*. The Doctor's plan seems also to have extended to solid *Loci*, of which indeed he gives a hint, in the preface to his *Conic Sections*; and among his papers are also some small Sets of such Propositions, which indicate the probability of the design which I suppose him to have entertained.* The hesitation about making additions to the Treatise of APOLLONIUS probably contributed to the delay in printing the work, which was not executed till 1749. He then met with some unexpected difficulties in treating with a bookseller for the sale of the whole impression, which alone prevented the publication at that time;† and except a few copies distributed among his friends in 1750, the book remained unpublished, till after his death. Such is the elegance of method, and the ingenious contrivance of demonstration in this work, that he has truly exhibited a copy, or at least so very nearly a copy, of the work of APOLLONIUS, that little regret need be had for the loss of the original. The preface also is well deserving the attention of those who wish to acquire just notions of the ancient books of analysis.

The first publication by Dr. SIMSON, except the Paper on Porisms in the Philosophical Transactions, was his Treatise *Sectionum Conicarum, libri v.* which appeared in 1735.

* In a great many of the Propositions of solid *Loci* the corresponding algebraical equations are added. From some observations in the preface to the seventh book of PAPPUS, it appears that the ancients had considered several varieties of *Loci*, particularly the plane, the solid, the linear, and those called *Loci ad medietates*, arising from mean proportionals; but of these last only a very few imperfect notices remain in the beginning of the third book.

† This account is given by Dr. SIMSON, in a letter to the late Earl STANHOPE, Sept 10th, 1750.

He had observed, in the first years of his study of Mathematics, that the Treatises on Conic Sections, then in most general use and estimation, were entirely algebraical; and the great merit of the work, written in that style by the Marquis DE L'HOSPITAL, contributed not a little to the popularity of this mode of treating geometrical subjects. It occurred therefore to Dr. SIMSON, that a treatise on Conic Sections, written on the purer model of antiquity, might have some influence in correcting the prevailing false taste, of introducing algebraical calculation into those branches of geometry where it was not necessary, and where it supplanted a more elegant form of analysis and demonstration. To exhibit, therefore, a just comparison of the two methods, he assumed the same definitions of the Conic Sections, as L'HOSPITAL and others before him had employed; and from them, with the true simplicity and accuracy of the ancient school, he deduced not only the properties of these curves as given by all preceding writers, but added many new and important Propositions of his own,* with the generalization and improvement of many, which had been previously discovered. It is unnecessary to enter into any particular discussion of

* The precise object which Dr. SIMSON had in view, was the occasion of his adopting the definitions of the curves given by L'HOSPITAL and others, in which the language is rather mechanical, than of a strictly scientific character. This I have frequently heard the Doctor express; and he observed at the same time, that he considered the derivation of the properties of these curves from the cone, after the original method of the ancients, as the best; and that if a definition of them, from a description in a plane, were to be assumed as most expedient, a more correct form of language than L'HOSPITAL's ought to be used; which indeed is generally done by those modern geometers who have relinquished the consideration of the cone, in defining this class of curves.

the merits of a work, which has been for so many years before the public, which was received with general approbation on its appearance, and which, notwithstanding more recent improvements, still maintains the reputation, of being the best example of the ancient method of demonstration, of having communicated large additions to the theory of the Conic Sections as it stood in his time, and of exhibiting a clear and just arrangement of the properties of these curves.

This Treatise became a part of Dr. SIMSON's Course of Lectures in the College; it was reprinted in 1750 with several additions, including some valuable communications by Dr. MATTHEW STEWART; and the three first books have been translated, and repeatedly printed, as an elementary introduction to this branch of science. In the preface, Dr. SIMSON gives a short but satisfactory sketch of the history of this portion of geometrical science from the age of MENÆCHMUS, reputed the first inventor, to his own time; in which some omissions have been remarked, but not of much importance in so short an abstract, as it was his purpose to communicate.*

To the second edition of the *Conics* is added an appendix, containing two Geometrical Problems, with a preliminary *Locus*; which Dr. SIMSON gives as examples of the superiority of the ancient analysis over that of the moderns, in the resolution of the same problems, by the authors there quoted. That superiority will be generally admitted; but it is proper to remark, that besides the solution of one of

* See Note A. at the end.

the problems by GUISNEE,† referred to by Dr. SIMSON, this author, in the same Treatise, gives another very elegant construction of the same problem, of trisecting the arch of a circle, by means of two local equations.* The Doctor's object, however, was to state a comparison between the pure geometrical analysis, and the usual algebraical method, of resolving an equation, and constructing that solution. The other method by the combination of *Loci*, though clothed in an algebraical dress, Dr. SIMSON would have considered, and with reason, as in effect geometrical; for the *Loci* in question might have been easily deduced geometrically, and thence the proper geometrical solution became obvious.‡

In the year 1752, Dr. SIMSON transmitted to the late Earl STANHOPE, in return for some valuable communications from his Lordship, an investigation of a rule of approximation to the roots of numbers which are not perfect squares, given by ALBERT GIRARD in 1629, without demonstration, in his

† GUISNEE *Appl. de Algebre à la Geometrice*, p. 191, 2d ed.

* The same Problem is resolved with great simplicity by BOSCOVICH, by means of an Hyperbola, and the circle to which the given arch belongs: see MAKO *de Arithm. et Geomet. Æquationum Resolutionibus*, p. 332. It may not be improper to mention here a remark by Dr. SIMSON, in a letter to the late Earl STANHOPE, respecting a Problem proposed by the Doctor's ingenious pupil Mr. WILLIAMSON of Lisbon. "As to the substituting a circle in place of one of the Hyperbolas, I never tried to do it in this Problem, because I observed the ancients were not solicitous about such solutions, and preferred the *Loci* which naturally arise from a Problem to any other, as affording the shortest composition. Thus in the 58th Prop. of the 5th Book of APOLL. *Conica*, though it is not difficult to solve the Problem by a circle and the given Parabola, yet APOLLONIUS takes the Hyperbola that arises from the Problem as giving the most natural and shortest solution." The letter is dated March 7, 1755.

‡ Dr. SIMSON's opinions on this subject are more fully stated in his correspondence with GEORGE LEWIS SCOTT, Esq; for which see Note K. at the end.

edition of the works of STEVINUS. That rule seems to have escaped the notice of Mathematicians, till Dr. SIMSON undertook the investigation of it; which, on the recommendation of Earl STANHOPE, was submitted to the Royal Society, and published in the Transactions of that year. The method is ingenious, and in some cases may be useful; though, from modern analytical improvements, such rules become of inferior consequence.

The next and only other work of Dr. SIMSON which was published in his life time, was his excellent edition of EUCLID, which appeared both in Latin and English in 1756, and was dedicated to his present MAJESTY when Prince of Wales. It contained the first six books of the *Elements*, with the eleventh and twelfth, being those usually taught in Universities. From his annually lecturing on these books, and his accurate knowledge of the ancient geometry, many corrections of the common text of EUCLID would naturally occur to him; but it was only after many and repeated solicitations from his mathematical friends, that he was induced to undertake the preparation of a new edition. Even after he had nearly completed it, a considerable delay occurred, from doubts which were then entertained respecting the construction and effect of the Statute of Queen ANNE, for the encouragement of learning. He was flattered by his friends with the hope that an edition corrected by him would have an extensive circulation, and would nearly supersede the common editions then in use; and he was therefore naturally anxious to secure the advantages of his work, at least for the term prescribed by the Statute. But from some of his letters to the Earl STANHOPE,

it would appear that such were the apprehensions entertained on this subject, particularly about the edition in Latin, that he had it in contemplation to solicit a security of his right by a private Act of Parliament; and the Act in favour of Mr. BUCKLEY, the Editor of *Thuanus*, was quoted as a precedent.* The Speaker ONSLOW was consulted; but on full consideration, this scheme was relinquished, and Dr. SIMSON relied on the protection of the Statute.

To judge with impartiality of the merits of this work, the state of the text in preceding editions must be attended to. Dr. SIMSON, from his veneration for the ancient Geometers, seems, with an excusable partiality, to have assumed, that the *Elements* of EUCLID, as they came from the author, were nearly without blemish; and he therefore ascribes all the errors and imperfections of the common editions, either to the carelessness of transcribers, or to the blunders of THEON, and other ancient Editors. His corrections are numerous, and many of them important; and even now, when most of them are adopted, it might be an useful exercise for the young mathematician to study the grounds of his emendations, which exhibit so clearly the precision of his ideas, and the logical accuracy of his understanding. Some animadversions were made on this edition, chiefly by those whose works had been criticised in the Doctor's notes; and to some of these, in a second edition, replies and explanations were made; but he had a great aversion to controversy, and his observations on what he had

* This Act was for preventing for fourteen years the importation of any Latin copies of that work; which was not provided for in the Statute of Queen Anne.

proved to be errors or defects in his predecessors, were never calculated to provoke it.†

Notwithstanding Dr. SIMSON's valuable corrections, there are still some difficulties in the *Elements*, which remain to be cleared up by some future Editor. The demonstration of the property of parallel lines (29 I. Elem.) is still theoretically defective, requiring the admission of some principle, not strictly belonging to the class of self-evident truths. It has by some been supposed, that the remedy for this difficulty must be sought for in a just definition of a straight line. No definition of a straight line has yet been found, and none perhaps can be found, from which all the properties assumed in the *Elements* to belong to it, can be rigidly demonstrated. There is manifestly also some defect in the definition of a solid angle, since what is given in Dr. SIMSON's, and in all other editions, does not discriminate the solid angle from a number of plane angles, formed at one point, which may exist according to the definition, but without forming the solid angle intended to be defined. The improvements and corrections of the Fifth Book are also important. His observation with respect to solid figures, in the note on Def. 10. XI. Elem. is curious, from remarking an error, which is so obvious when pointed out, but which had

† Soon after the publication of EUCLID, Sir ANDREW MITCHEL, then the British Minister at the Court of Berlin, asked Dr. SIMSON's permission to present, in the Doctor's name, a copy of the Latin EUCLID to FREDERIC III. The Doctor was gratified by the request, and transmitted to Sir ANDREW MITCHEL a copy of the book, on the page opposite to the title of which was written the following inscription:

“CESARI in belligerando, PTOLEMÆIS in promovendis artibus et scientiis, animique suam doctrina excolendo merito æquiparandus, Magnus Imperator
“FREDERICUS Tertius, BORUSSORUM Rex, ut librum hunc benigne accipiat
“summa veneratione exoptat. ROBERTUS SIMSON, Editor.”

escaped the notice of the many learned and acute Geometers, who had paid much attention to EUCLID'S *Elements*. An observation of a similar kind, and about the same time, was made by Mr. LE SAGE, which is recorded in the History of the Royal Academy of Sciences at Paris for 1756; and another important correction has been more recently made by LE GENDRE, of which a satisfactory history is given by Mr. PLAYFAIR, in the second edition of his *Elements of Geometry*.

The Book of EUCLID'S *Data* was annexed by Dr. SIMSON to a second edition of the *Elements* in octavo, (1762,) with many necessary corrections, and some valuable additional Propositions, both of his own and of his learned friends the late Earl STANHOPE, and Dr. STEWART. This book is one of the thirty-three, composed by the ancient Geometers, for facilitating the resolution of problems by their analysis; and is the most proper introduction to the study and practice of that analysis.* At the end, Dr. SIMSON has added two geometrical Problems, for illustrating the use of some Propositions of the *Data*, which would not be obvious to beginners; and with respect to one of them he observes, that he believes "it would be in vain to try to deduce the preceding construction from an algebraical solution of the Problem." The observation was perhaps hastily made, when he was seventy-five years of age; but he plainly had in view the most common method of resolving geometrical Problems by algebra, viz. by deducing a final equation with only one unknown quantity, resolving that equation, and constructing the solution of it; and with respect

* In a letter of Dr. SIMSON'S, in Note K. are some observations on the use of this book.

to this method, the remark would be just. An ingenious friend of the Doctor's, the late GEORGE LEWIS SCOTT, esq; intimated to him that a construction, similar to his, might be derived from the combination of two *Loci*, expressed by two equations arising in the algebraical solution of the problem. But this communication, with the Doctor's reply, is inserted in a note at the end, which will be more satisfactory than any abstract of it in this place.*

* See Note K.

SECTION III.

Of Dr. SIMSON's Posthumous Works.

THE strong impression which Dr. SIMSON felt of the failure of his memory, having prevented his publishing some important Geometrical Works; the copies of these works, with a large mass of miscellaneous papers, fell at his death, into the hands of his friend and executor Mr. CLOW, the Professor of Logic in the College of Glasgow.†

While Mr. CLOW was deliberating what was most expedient to be done with regard to these papers entrusted to his care, the late Earl STANHOPE, distinguished in his elevated rank by his ingenious cultivation and liberal patronage of the Mathematical Sciences, intimated his design of publishing those works of Dr. SIMSON which he had compleated, with any other pieces, which though unfinished, might without injury to his fame, be given to the public. The munificent proposal was most acceptable to Mr. CLOW; and after some correspondence respecting the selection, a large volume, in the year 1776, was, at his Lordship's sole expense, handsomely printed, under the care of Mr. CLOW, and liberally distributed.

† A considerable portion of these papers consists of various first sketches of his works, published by himself, or since his death.

This volume contains a restoration of the *Sectio Determinata* of APOLLONIUS, with two additional books by Dr. SIMSON; and a full explanation of the Porisms, with a restoration of a number of the Propositions of EUCLID's original work. To these two important works are added some smaller Tracts, which shall be taken notice of in the order in which they are placed in the volume.

The two books of *Sectio Determinata*, it was formerly observed, had engaged Dr. SIMSON's attention at an early period of his life. That treatise indeed, had been restored by SNELLIUS, but in a very imperfect manner, without the necessary distinction of the various situations of the Points, called (by APOLLONIUS) *Epitagnata*; and without a complete exposition of the Determinations, which, as is well known, are necessary to the perfect solution of any Problem; and are more particularly requisite, as has already been observed, in the books forming the *τόπος ἀναλυόμενος*, to render them useful for the purpose for which they were composed and collected.*

In a long and instructive preface Dr. SIMSON explains the many defects in the Work of SNELLIUS, which need not be here enumerated.† He takes notice also of the subsequent resolutions of some of the Problems, by ALEXANDER ANDERSONUS;‡ and of those likewise contained in the Treatise

* See the Preface to Dr. SIMSON's *Restoration of the Loci Plani*, pp. 7 and 8; where the use of the books of the *τόπος ἀναλυόμενος* in resolving Problems is briefly but very clearly stated.

† It is amusing to observe SNELLIUS blaming PAPPUS for placing the *Sectio Determinata* after the *Sectio Rationis*, in consequence of his own defective method of restoring the latter treatise by means of the former.

‡ In *Supplemento APOLLONII Redivivi*. Paris, 1612.

of Geometrical Analysis by HUGO DE OMERIQUE; and in the work itself he adopts some Propositions from these performances. In the Doctor's first attempts to restore the *Sectio Determinata*, the difficulty was not so much the giving proper and even elegant solutions of the various cases of the general Problem, as the finding the accurate distinction of the *Epitigmata*, the ascertaining the determinations; and chiefly the investigation of solutions, which required the use of the *Lemmata* assumed by APOLLONIUS, and demonstrated by PAPPUS,* by which the identity of the restoration with the original work might be recognized. In the preface he observes, that it was not till 1727,† and after many unsuccessful attempts, that he obtained such solutions of the fifth and sixth Problems of the first book. The remaining part of the work required much investigation, and amidst the various other studies in which he was engaged, it was not till above twenty years after, that he completed the restoration to his entire satisfaction, and in such a manner as to leave no doubt of its being truly the same as the original of APOLLONIUS; or so nearly the same, as to preclude the occasion of any further enquiry respecting it.‡

To the restoration of the work of APOLLONIUS, two other books, containing an extension of the general Problem, are

* For a more particular account of these *Lemmata*, see the Appendix.

† *Opera Reliqua*, Præfat. ad *Sec. Determ.* p. v.

‡ In a letter to Earl STANHOPE, Sept. 10, 1750, he mentions, that for more than twenty years he had been endeavouring to find the use and application of the *Lemmata* in PAPPUS, in the solution of the cases of the *Sectio Determinata*, but had not done so completely till last summer. And among his papers there is an intimation of his having just completed the restoration of the *Sectio Determinata*, by the use of the *Lemmata* of PAPPUS: to which he adds, "a Geometris intacta sunt a tempore PAPPI in hunc usque diem. 28 Jan. 1749. R. S."

added by Dr. SIMSON, of which it is necessary for me shortly to state the history. About two years before his death, when conversing with him on some geometrical subject, he mentioned that he had compleated the restoration of the *Sectio Determinata*. He informed me also, that he had written two additional books, a fair copy of which he took out of his repository, and gave me in confidence, with an injunction to publish them, or not, according to the reception which his restoration of the work of APOLLONIUS might meet with among mathematicians. At that time he was probably thinking of publishing the work of APOLLONIUS, but he did not enter into any particular explanation of his intentions.

During his last illness, and for some time before it, I happened to be at a great distance from him, and he died without my receiving any further communication on the subject. A short time after his death, therefore, I considered it to be my duty, after explaining these circumstances, to deliver the manuscript to Mr. CLOW, his particular friend, to whom he had assigned by his will the property and charge of all his papers. An account of this supplement being afterwards communicated to the late Earl STANHOPE, his Lordship was pleased to desire that it should be annexed to the Doctor's restoration of the work of APOLLONIUS.* From the nature of the subject, and

* At the end of a former copy of the restoration of this work of APOLLONIUS is the following. N. B. "Unicuique Problematum quæ in duobus de *Sectione Determinata* Libris continentur, addenda sunt tanquam corolloraria in proprijs locis, Problemata in quibus quadrata vel reſtangula, sunt quadratis vel reſtangulis majora vel minora, dato quam in ratione; vel quorum unum simul cum eo quod ad alterum datam habet rationum, datum est. Nam et hæc sæpius in usu veniunt." But instead of this, the Doctor had afterwards thought proper to expand the Problem in the two additional books just mentioned.

the constant references to the two former books, the additional books are rather uninteresting, especially when read without any particular application of them to the solution of Problems. But they have a similar utility to that of the former books; and when any geometrical Problems can be reduced to some particular cases of this supplement, it affords also an immediate construction and demonstration of such Problems.*

The next, and certainly the most important portion of the posthumous volume, contains Dr. SIMSON'S discovery and illustration of the Porisms of EUCLID; and as this volume is not in general circulation, it may be a gratification to some readers, besides the account already given, to add a more particular detail of the history of this curious piece of ancient science, and of the Doctor's successful investigation of it, after it had baffled the researches of all the ingenious men who attempted it before him.

The seventh book of the *Mathematical Collections*, it has been observed, is the only notice to be found in ancient authors of EUCLID'S Porisms; if we except the few short and not very satisfactory observations of PROCLUS, who lived, as is supposed, within a century after PAPPUS.† The transla-

* Since Dr. SIMSON'S death two other restorations of the *Seclio Determinata* have been published. One by W. WALES, in 1772; and another, by GIANNINI, for which see *Opuscula Mathematica, auctore PETRO GIANNINI*, Parmæ, 1773.—But with respect to them, it is sufficient to observe, that independently of the curious information in Dr. SIMSON'S valuable preface, the superiority of his work, as a restoration of APOLLONIUS, must be obvious to the geometrical reader.

† PAPPUS flourished as is generally reckoned about the year 400 of the Christian Æra, and PROCLUS towards the year 500, and the former is repeatedly quoted by the latter. See SUIDAS, and GERARD. VOSSIUS *de Universe Matheseos natura et Constitutione, et Chronologia Mathematicorum.*

tion by COMMANDINE of the six last books of PAPPUS, (which were all that were found in the MS. used by him) was published in 1588.—The attention of the Mathematicians of Europe was soon directed to that work; and it gave rise to the various attempts which were made to restore the lost works of the ancient Geometers, so particularly described in the preface to his seventh book. The Porisms of course were not neglected; though the very imperfect state of the description of them by PAPPUS, in all the manuscripts which have hitherto been examined, created peculiar, and apparently unconquerable, difficulties in the investigation.

ALBERT GIRARD, a Geometer of eminence in the early part of the 17th century, was the first who announced the restoration of the Porisms of EUCLID. In his *Trigonometry* (1629), and also in his edition of *Stevinus* (1634,) he states his having restored the Porisms; but in terms so general, that no precise opinion can thence be formed of his notions on the subject. But from the first of these intimations Dr. SIMSON reasonably infers, that GIRARD was not acquainted with the true nature of the ancient Porisms; and it appears also to be highly improbable, if he had had any success in recovering so curious a branch of ancient learning, that he would have concealed it from the world.*

* Dr. SIMSON in his preface justly observes, that the short notices of Porisms in C. RENALDINUS's work *De Resol. et Comp. Math.* (Bon. 1644) are of no use in explaining the ancient Porisms, and in truth have no relation to them. He seems to use the term Porism for expressing the general rule resulting from an algebraical investigation in general terms: see pp. 280, 1. They are general corollaries, in the common acceptance of that term. SCHOOTEN also, in his *Exercitationes Math.* (sect. 24.) uses the term Porism for an investigation, without any precise object, of a variety of relations among lines drawn in and about a circle, or other geometrical figure. He gives only an example of a circle.

BULLIALDUS is the next author who mentions them in one of his *Exercitationes Geometricæ*, (1657,) but in it he refers to FERMAT as the inventor, who had communicated the discovery in letters to some of his friends at Paris; BULLIALDUS, however, was unable to investigate them.* This very eminent and ingenious man FERMAT appears to have been the first in modern times, who made any near approach to the discovery of the Porisms. He supposed that he had ascertained the nature of these propositions, and that he had made such progress as to ensure a complete restoration of EUCLID's work. In his treatise on the subject, which was published only after his death in 1679, he gives five propositions (without demonstrations) as specimens of his invention. These are certainly Porisms, but none of them belonged to EUCLID's Treatise; and this circumstance, besides other arguments, justified Dr. SIMSON in stating that FERMAT had not acquired a correct notion of the nature of a Porism.† It is sufficient to remark that FERMAT's definition of Porisms

* Dr. SIMSON, on the margin of his copy of this tract of BULLIALDUS wrote the following remark: "Ex hoc Tractatu liquet BULLIALDUM nihil de naturâ Porismatum intellexisse. R. S. Mart. 29, 1739."

† An Eloge on Monsieur FERMAT appeared in the *Journal des Sçavans*, Feb. 9, 1665, eight years after the publication of BULLIALDUS's tract, in which the supposed discovery of FERMAT was announced to the world. This Eloge is also copied into the volume of posthumous works published in 1679 by his son; but in the enumeration of his works which it contained, no mention is made of the Porisms. Some readers may be gratified with FERMAT's definition or description of the Porisms. "Cum Locum investigamus Lineam rectam aut curvam inquirimus, nobis tantisper ignotum, donec Locum ipsum inveniendæ lineæ designaverimus, sed cum ex supposito Loco dato et cognito, alium Locum venamur, novus iste Locus Porisma vocatur ab EUCLIDE, qua ratione Locos ipsos unam speciem, et esse et vocari, verissime PAPPUS subjunxit."—FERMATII Opera, vol. ii. p. 118.

(which at the same time is not very clearly expressed,) is avowedly derived from the definition of them by the Mathematicians reckoned modern, (*νεωτέροι*), in the time of PAPPUS, and which he pointedly censures as inadequate. It may therefore be concluded from this authority, that FERMAT'S notion of a Porism was imperfect; more especially as by this definition a numerous class of Porisms, altogether unconnected with *Loci*, were necessarily excluded. The matter, however, is rendered perfectly clear by Dr. SIMSON'S full exposition of the nature of Porisms; of which he gives a new definition, very different from that of FERMAT, but fortified by accurate reasoning, and the illustration of many examples, both of EUCLID'S Porisms, and of others composed by himself and his friends. The Doctor also observes, that at an early period he had been studying FERMAT'S Tract on Porisms, and that in 1715 he had discovered the demonstration of cas. 1. prop. 4. of FERMAT'S Propositions, before he knew any thing of the nature of Porisms. So little could be derived from the labours of that distinguished Geometrician, by Dr. SIMSON, in his investigation of the common object of their pursuit.*

Dr. DAVID GREGORY also, an eminent Geometer in the latter part of the same century, seems, from having taken only a superficial view of the subject, to have deceived himself, when he observes, in the preface to his valuable edition of EUCLID, that it would not be difficult to restore the Porisms of EUCLID, if the Greek text of PAPPUS were published.

* Dr. HALLEY, in the preface to the *Sectio Rationis*, (last page) speaking of a work of FERMAT'S, adds, "qui (sc. FERMATVS) et Porismata EUCLIDIS opus longe "difficillimum, redintegrare pollicitus est, verum fidem non liberavit."

That text was within his reach, as there were two Greek manuscripts of PAPPUS in the Savilian Library, of which, as Professor of Astronomy, he had a particular charge; but there is no evidence of his having made any attempt, and he died while engaged in another important work, the publication of the *Conics* of APOLLONIUS, which was finished by his Colleague Dr. HALLEY. From the Savilian manuscripts, Dr. HALLEY was enabled to give an improved edition of the preface to the seventh book of PAPPUS, with a translation much superior to that of COMMANDINE. The general account of the Porisms is in some places corrected; but the detail of the contents of EUCLID'S work remained so very defective and unintelligible, that Dr. HALLEY, with his great abilities and learning, and with his successful experience in restoring and elucidating several works of APOLLONIUS, was compelled to give up the investigation of the Porisms as hopeless; and he was brought to this conclusion, both from the difficulty of the subject, and also from the mutilations which the injuries of time had occasioned in all the manuscripts which had been examined.*

From all these seemingly unfurmountable obstructions, it required an uncommon degree of zeal and fortitude in Dr. SIMSON to undertake the investigation; and from his own account, which has already been given, his perseverance was not less remarkable. From the time of his first discovery of the nature of a Porism, the Doctor, without neglecting his other scientific pursuits, occasionally applied his mind to this very curious subject; and notwithstanding the many difficul-

* See Note B. at the end.

ties which he encountered, he was at length completely successful; not only in explaining the distinctive character of these Propositions, but in the investigation of a number of the individual Porisms of EUCLID, and also of various other Porisms, which were useful, both in illustrating the nature of these Propositions, and in shewing the application of them in the resolution of difficult geometrical Problems.

After a certain progress in the prosecution of this subject, it became an important object to ascertain a just definition of the Porism. The definition given by the later Mathematicians, as stated by PAPPUS, but censured by him, "quod deficit hypothesi a Theoremate Locali,"* clearly implies that a Porism had an immediate reference to a *Locus*; though it is not less clear that PAPPUS considered *Loci* as only one class of Porisms, (a large one no doubt,) but that of course many Porisms have no connection whatever with *Loci*.

But the definition which PAPPUS quotes from the ancients, † as more characteristic of Porisms, is too general for any useful purpose; and though it does correspond to the nature of these Propositions, yet it is deficient in discrimination, and of itself neither conveys any precise notion of EUCLID's Porisms, nor gives assistance in the investigation of any individual Proposition.

* "Quod deest in hypothesi Theorematis Localis." HALL. versio.

† PAPPUS, in his description of the Porisms, (preface to 7th book,) states the following definitions of a Theorem, a Problem, and a Porism: "Differentias autem horum trium, melius intellexisse veteres, manifestum est ex definitionibus. Dixerunt enim *Theorema* esse quo aliquid proponitur demonstrandum; *Problema* quo proponitur aliquid construendum; *Porisma* vero esse quo aliquid proponitur investigandum." HALLII versio.

After much consideration of various forms of a Definition which had occurred to him, the Doctor finally settled the following: "Porisma est Propositio in qua proponitur demonstrare rem aliquam, vel plures datas esse, cui, vel quibus, ut et cuilibet ex rebus innumeris, non quidem datis, sed quæ ad ea quæ data sunt eandem habent relationem, convenire ostendendum est affectionem quandam communem in Propositione descriptam."*

The Doctor illustrates the propriety and accuracy of this definition by many examples; and shews particularly wherein the definition blamed by PAPPUS coincides with his, and wherein it is deficient, by excluding many genuine Porisms. The definition indeed, with much address, is so framed as to correspond with all the intimations of PAPPUS respecting Porisms, and also with the character of the individual Porisms of EUCLID, which Dr. SIMSON had discovered; and therefore may justly be considered as expressive of the notions on this subject entertained by the ancients. It is not pretended that

* From the necessary generality of expression for comprehending every class of Porisms, there is some obscurity in this definition; but the Latin language, which Dr. SIMSON used in all his mathematical writings, is in this case better fitted for giving precision, and for preventing ambiguity, than the English. In Mr. LAWSON'S translation of Dr. SIMSON'S Introduction to the Porisms, which is plainly meant to be strictly literal, this definition is expressed thus: "A Porism is a Proposition in which it is proposed to demonstrate that some one thing or more things are given, to which, as also to each of innumerable other things, not indeed given, but which have the same relation to those which are given, it is to be shewn that there belongs some common affection described in the Proposition." The following less literal translation is proposed by Mr. PLAYFAIR, for remedying part of the obscurity: "A Porism is a Proposition in which it is proposed to demonstrate that one or more things are given, between which and every one of innumerable other things, not given, but assumed according to a given law, a certain relation, described in the Proposition, is to be shewn to take place." Ed. Transact. vol. iii. p. 172, Note.

this was a definition of the ancients; for probably no precise definition was given by them, of either Theorem, Problem, or Porism. None appears in the works of the more early Geometers, which are still preserved in a considerable degree of purity, and where such definitions would naturally have had a place. And we may affirm with much probability, that if any useful and characteristic definition of a Porism had reached the times of PAPPUS, he would not have neglected so valuable a remnant of ancient mathematical science, in a work obviously designed for the preservation of the more curious portions of it. He does not omit a definition, which probably was only a traditional and pointed observation of some ancient Geometer; and though of no use in explaining the character of a Porism, yet it in some degree fortified his objection to the definition of the later Mathematicians, who, he states, from inability, could not accomplish the investigation of Porisms; but satisfied themselves with assuming the constructions as they found them in EUCLID, or other Geometers, and adding the demonstrations.

It is observed by PAPPUS, that a Porism is neither a problem nor a theorem, but something of an intermediate nature; and that it might be proposed either as a problem or as a theorem; some Geometers contending for the one, and some for the other. Dr. SIMSON has given a form to the enunciation of a Porism, implying this intermediate character between a problem and a theorem. In his enunciation it is affirmed that certain things *may* be found, which shall have the relations or properties therein described. Perhaps this form resembles more that of a theorem than of a problem; but at the same

time, the things, of which it is said that they may be found, must be actually investigated by analysis, as if the Proposition were a problem. Were it simply proposed to investigate certain things which would have the properties expressed in the Porism, it may be regarded as a problem; but if these things are found by a construction described in the enunciation, the Proposition becomes a theorem, affirming the truth of the properties asserted; and then a demonstration only is required, without any investigation; in the manner which appears to have been practised by the later Mathematicians, alluded to by PAPPUS.*

The most satisfactory illustration of these definitions is by examples, of which Dr. SIMSON gives a great variety;

* The enunciation of a Porism as a problem, is not consistent with the usual character of such Propositions. Problems commonly (whatever difficulty attend the actual resolution of them) are almost immediately recognized by those having some knowledge of geometry, as either possible in certain circumstances of the data, or as altogether impossible; and it is unusual to propose as a problem, "to find things with certain properties, respecting the possibility of which no judgment can be formed without an analysis, or such consideration as is equivalent to an analysis." For example, if it had been proposed as a problem in the time of APOLLONIUS, to find in a given parabola a point having the property of the focus, that point being then unknown; such a Proposition would not have been considered as a proper problem, but in reality would have been a genuine Porism.—PROCLUS, in his description of Porisms, mentions the 1 Prop. 3 Elem. "to find the center of a circle," as a Porism, being in some measure between a problem and a theorem. But in that case, as a circle, from EUCLID's definition of it, must have a center, the Proposition "to find that center," seems to be a proper Problem. Had the circle been defined from another of its properties, as for instance, from its being produced by the extremity of a straight line moving at right angles to another straight line given in magnitude and position, and in the same plane, so that the square of the moving line be equal to the rectangle by the segments into which it divides the given line; then the finding of the center would be a proper Porism, and might be enunciated thus: "within a given circle (defined in the manner now mentioned) a point may be found, from which all straight lines drawn to the circumference will be equal."

and he distinguishes clearly the Locus and Local Theorem from the Porism, though all the three, in a large class of the latter, are convertible into each other. He proceeds to give examples of Porisms altogether unconnected with *Loci*, and adds likewise an algebraical or arithmetical Porism; all of which, however, are clearly comprehended in his own definition, and display the defect of the definition censured by PAPPUS.

After an ample exposition of the nature of Porisms, the Doctor proceeds to the restoration of some of EUCLID'S Porisms, beginning with the general Proposition contained in PAPPUS, though in an imperfect state, and distributed into the ten cases also remarked by him; adding the second general Proposition, which is an extension of the first. He then investigates several other Porisms ascertained to belong to EUCLID, from the remaining fragments of his description; and by employing some of the *Lemmata* assumed by EUCLID, and preserved by PAPPUS, he proves the identity of his investigation with that of the original author. The Doctor's investigation of Porisms, of course, is in the ancient analytic method; though from the general nature of these Propositions, some variation in the form, from what is used in a common problem, is requisite. The things to be investigated, as in the common analysis, are supposed to be found; and the relation of them to the innumerable other things must also be assumed, and often, as existing in different situations, to express effectually the general nature of the Proposition.—It is unnecessary here, however, to enter more particularly into this discussion, as the propriety and use of the form adopted by Dr. SIMSON can

can be fully explained only by examples, and will be intelligible to those who have duly considered what he has written on the subject.*

FERMAT'S five Propositions, though stated by him in the form of theorems, are undoubtedly Porisms, and as no investigation or demonstration was given of them so far as Dr. SIMSON could learn, they are properly added. He gives them the appropriate form of Porisms, renders them more general, and adds the necessary analysis and composition. The second Prop. of FERMAT belongs to the parabola, and is demonstrated in the Doctor's *Conics*, Prop. 19. Lib. v. 2d edit.

The Porisms of EUCLID, so far as may be conjectured from the description of PAPPUS in its present imperfect state, seem all to have had reference to the circle and straight lines, and therefore may be called plane; but it is obvious that the Conic Sections may supply many solid Porisms, and superior lines in like manner may be the source of superior classes of Porisms, which the ancients would have called linear. Dr. SIMSON concludes this work with some elegant Porisms of his own, and some by Dr. STEWART, one of the very few to whom he had, in the earlier part of his life, freely communicated his invention.‡ He takes occasion also to shew the

* I shall quote here an important observation of Dr. SIMSON'S on this subject. *Opera Reliq.* p. 337. "Semel igitur ad minimum, punctum, recta, angulus, vel quodcunque fuerit de quo in Porismate aliquid generaliter affirmatur, sumendum est utcunque, intra tamen limites, si qui sint in Porismate præscriptos, ut legitime fiat analysis. Sæpe autem brevius et facilius ea quæ investiganda sunt inveniuntur, si tum quævis ex prædictis rebus sumatur utcunque, tum denuo modo quodam particulari sumatur, quando hoc fieri potest."

‡ See Note, 2 C. at end.

utility of Porisms in producing easy resolutions of geometrical problems, which, without that assistance, might be complex and difficult.

This application of Porisms by the ancients is stated by PAPPUS; who insinuates also, that little had been done respecting them by the geometers subsequent to EUCLID; but as EUCLID's treatise is part of the *τόπος ἀναλυόμενος*, we may presume that this use of the Porisms was in contemplation of the author,† and was probably regarded by subsequent geometers as the most important.

I cannot omit adverting in this place to a very ingenious theory of Porisms proposed by Mr. Professor PLAYFAIR, of Edinburgh, first briefly in his account of the life of Dr. STEWART,* and afterwards more fully explained in a memoir on that subject in the third volume of the Transactions of the Royal Society of Edinburgh. The result of his investigation is, that a Porism is the case of a problem which becomes indeterminate; or more particularly, “a Porism is a Proposition affirming the possibility of finding such conditions as will render a certain problem indeterminate, or capable of innumerable solutions.”§ But though I admire the ingenuity, and fully admit the soundness, of this definition,

† It is proper to observe that Dr. SIMSON gives a corrected edition of all the *Lemmata* belonging to the Porisms, which must be very useful to those who may attempt to recover more of EUCLID's Porisms.

* This life of Dr. STEWART, read before the Edinburgh Society in July 1784, was published in the first volume of the Transactions.

§ In this paper is given a comprehensive and very ingenious theory of the nature of Porisms founded on this definition; to which the reader who is desirous of satisfactory information on the subject is referred.

and also the utility of the principle on which it is founded, in the discovery of Porisms, I must acknowledge my doubt of that particular notion of a Porism having ever been adopted, or even proposed, among the ancient geometricians. The circumstance of its being so satisfactory as a definition is to me a proof that it was never generally known or embraced: for had it ever been approved and established, it seems scarce possible that it should afterwards have been neglected and lost. That among the ancients the consideration of the relations subsisting among the data, in some problems, might have occasionally suggested the particular case in which these problems would become indeterminate, is very probable. It might also have often occurred to them, that this indeterminate case involved an important general Proposition, which might be separately stated as such, and preserved. Many Porisms of EUCLID may possibly have been invented in that way; but still I entertain a doubt, if ever the ancients were in possession of this notion as a principle, and as the proper ground of the definition of a Porism. PAPPUS mentions the definition of the ancients, and apparently as the only one which they were known to possess, though, as has been remarked, it be of no particular use. He mentions also a definition of the later mathematicians, which he censures as erroneous: but if such a complete and satisfactory definition, which not only accurately distinguishes that class of Propositions, but points out an obvious source of the discovery of them, had ever been generally understood among the ancients, it is difficult to suppose that it could ever have been lost; and had it reached the time

of PAPPUS, it is most improbable that he should neglect the recording of it in his detailed account of EUCLID's treatise on this subject. With these strong internal probabilities, and the total want of external evidence, I must (with deference, however, to the opinion of those who may think differently) adhere to the judgment which I have already expressed concerning the recent origin of this excellent definition proposed by Mr. PLAYFAIR.*

The very compleat account of the Porisms given by Dr. SIMSON in this posthumous treatise may be considered as so satisfactory an illustration of the nature and character of EUCLID's work, that the justice of the concluding remark of his preface will readily be admitted: "His igitur doctrinam Porismatum satis explicatam, et in posterum ab oblivione tutam fore, sperare liceat." I may take the liberty of adding, that though Dr. SIMSON's fame, as an accurate, elegant, and ingenious Geometrician, be established from his other works; yet the restoration of the Porisms of EUCLID will be regarded by posterity as the most important production of those powers of investigation and genius, with which he was so eminently endowed.

There remain some small tracts in the posthumous volume, which shall be very briefly mentioned.

The treatise *De Logarithmis* is formed on the model of the fifth book of EUCLID; and notwithstanding the great variety

* See Note D. at the end. I may here observe that the two definitions are perfectly consistent: but though Dr. SIMSON's might easily be derived from Mr. PLAYFAIR's, yet the deduction of Mr. PLAYFAIR's from the other is by no means so obvious, and does not appear to have ever occurred to Dr. SIMSON.

of explanations which have been given of that celebrated and most useful invention, yet a deduction of the properties of *Logarithms*, with the same strict manner of demonstration which is used by EUCLID in treating of proportion, remained a desideratum in the science. It was undertaken at the request of the late Earl STANHOPE, who suggested to him some valuable hints on the subject; and the tract, though short, is compleat, and appears to be the last work which he finished. His correspondence with the Earl STANHOPE about the plan of it was in the year 1752; and the fair copy from which the tract was printed, is dated Feb. 19, 1762. But in this some corrections were made with the sanction of the Earl STANHOPE.

The next tract, *De Limitibus Quantitatum et Rationum*, is only a fragment; but it contains a rigorous demonstration of the principles of fluxions, and of prime and ultimate ratios. It is well known that from the time of the invention of fluxions, some inaccurate expressions respecting infinitely little quantities, and some loose reasonings founded on such expressions, had crept into that branch of science. Much discussion arose, and a serious controversy was maintained for some time; which, in a science professing to be founded on self-evident principles, and conducted with strict demonstration, was of itself a matter of some reproach. Mathematicians of considerable name took a part in the business; and many treatises were written, with a view to remove the objections which had been started, and also to give a rigid demonstration of the principles and rules of this new and most important analysis. The doctrine of limits was certainly implied (though not in language

calculated to prevent every objection) in Sir ISAAC NEWTON'S Propositions of prime and ultimate ratios. The principle, however, came afterwards to be stated in more accurate language; and is now by universal consent admitted to be the found foundation on which this branch of the science ought to rest. This certainly has been properly explained in other treatises; yet it may be hoped that this fragment of Dr. SIMSON'S will be considered by the admirers of the simplicity and accuracy of ancient geometrical reasoning, not only as free from even the pretence of objection, but as a demonstration, with elementary perspicuity, of the principles of fluxions.*

At the end of the volume is an Appendix, containing a few geometrical problems resolved in the ancient analytical method. This addition was made at the request of the late Earl STANHOPE; but it is proper to mention, that it does not appear that Dr. SIMSON ever intended to make any selection of such problems, with a view to publication. There are, no doubt, among his papers, a great number of geometrical problems; some of which are valuable, but almost all of them appear to have been taken up only from the suggestion of the moment, either from his reading, his correspondence, or from his own particular studies; but generally without any decisive marks of his having given a particular consideration of the best possible solutions. The few added in this place were selected just when the volume was nearly printed, and without any parti-

* It appears from Dr. SIMSON'S papers, that he had been thinking of the method of limits as far back as 1736, and then proposed to extend this method to the first and some other Propositions of the *Principia*. No trace however remains of his having completed this design.

cular object in view, except that examples might be given of the accurate determination of problems, about which the ancient Geometers were so very curious, and which has often been neglected by the moderns. I must observe, however, that the last problem, being a case of the *Tactions* of APOLLONIUS, will be mentioned more particularly afterwards.

SECTION IV.

Of Dr. SIMSON's Unpublished Papers and Correspondence.

I SHALL conclude the account of Dr. SIMSON's labours and inventions with a few observations on his unpublished papers, and his correspondence.* From these papers it does not appear that he ever seriously embarked in the restoration of any other of the treatises in the *τόμος ἀναλυόμενος*, except those already mentioned. In an early MS. volume, indeed, he gives an arrangement of the cases of the Problem of the Tactions, and in other papers the solutions of most of them.†

* Wherever in this Memoir I have made use of Dr. SIMSON's unpublished papers, the reference to them is particularly remarked, that any inadvertence or mistake of mine might not be attributed to him.

† The Tactions of APOLLONIUS were first restored by VIETA, in a treatise, which he calls APOLLONIUS *Gallus*. Afterwards they were restored by various mathematicians, both geometrically and algebraically. A Treatise of the Tactions by J. GUGL. CAMERER was published at Gotha and Amsterdam in 1795, and is mentioned with commendation by MONTUCLA, (tom. iii. p. 14,) but it contains only an edition of VIETA's treatise, with notes and additions, and a curious history of the Problem. The history is interesting, from the accounts which it contains of the labours of some foreign mathematicians upon this problem, which are little known in this country. He gives the preface and *Lemmata* of the Tactions in Greek, with some various readings of several manuscripts of PAPPUS. Though VIETA's solutions are elegant, yet they are in several respects deficient. There is not a full distinction either of the cases, or of the necessary determinations. No analysis is given, and no attempt to restore the APOLLONIAN solutions by the use of the *Lemmata* in PAPPUS, which had been assumed in the work of APOLLONIUS. See CAMERER *de Tactionibus*, p. 4, and p. 12.

It would seem, however, that as there was little difficulty in this problem, it had not been sufficiently interesting to engage him in a complete restoration of it. The use of one of the *Lemmata*, (Prop. 117, Lib. vii. PAPPUS,)[†] in resolving a case of the Tactions, did not for some time occur to him; but having discovered it, he deduced from it the elegant solution of that case which is printed in the posthumous volume. I found among his papers the following date of this solution: "Feb. 9, 1734, mane, post horam I^{ma} ante meridiem." So minute was he in some of these notices; and in this case perhaps, at the moment, he felt a little satisfaction from having overcome the only difficulty in restoring the Tactions in the APOLLONIAN method.

PAPPUS, in his description of the Tactions, observes that there are other problems respecting Tactions, which were generally neglected by the editors; of whom, however, some prefixed one of these problems to the first of the two books of APOLLONIUS, as an easy and proper introduction to the doctrine of Tactions.* PAPPUS gives the general description of this problem, from which Dr. SIMSON stated the arrangement and solution of the several cases; and he also proposed and resolved another problem of Tactions, viz. "of points, lines, and circles, any two being given to describe a circle through the points, or touching the straight lines or circles, of which

[†] See Note E. at the end.

* This Problem is "of points, straight lines, and circles, given in position any two, to describe a circle given in magnitude, which may pass through the given point or points, and touch also (if possible) the given lines or line." This problem was also resolved by MARINUS GHETALDUS.

“ the centre shall be in a straight line given in position.” But all these problems, though very easy, may be occasionally useful, when a proposed problem can be reduced to a case of any of them.

With respect to the *Inclinations* of APOLLONIUS, he seems to have bestowed little attention; only it may be observed, that, after repeated unsuccessful trials, he found (February 20, 1723) the solution of the problem about the Rhombus, by the Lemma of PAPPUS, (Prop. 70, lib. vii.) which APOLLONIUS no doubt had employed in his solution. The figure of this Lemma in COMMANDINE'S PAPPUS, which is erroneous, was at the same time amended.* The *Inclinations* had been restored by MARINUS GHETALDUS; and subsequently to Dr. SIMSON'S time, by Dr. HORSLEY, the late Bishop of St. Asaph, in 1772; and also by Mr. WALES, in 1779. I may further remark, that Dr. SIMSON made some useful corrections of another Lemma of the *Inclinations*, viz. Prop. 85, lib. vii. PAPPUS. And in this Proposition is an example of the deduction of a problem to a case of the *Sectio Determinata*, which is thence considered by PAPPUS as completely resolved; and in the demonstration a reference is made to that case, being *Epitagma 2. Prob. 3. lib. i. Sect. Determ.* In COMMANDINE'S time the lost treatise *De Sect. Determ.* had not been restored; and therefore, in his commentary on this Proposition, he gives a solution of that case, as necessary to the solution of this lemmatic problem.

* See Note F. at the end, in which is given Dr. SIMSON'S solution, and the amended figure of the Lemma. It may be observed, that an ingenious correction of the Lemma, and of its figure, as it is delineated in the Savil. MS. No. 3, is given by Dr. HORSLEY, in his *Refutation of the Inclinations*.

Among the Doctor's papers there remain also some fragments respecting *Loci ad superficiem*, of which EUCLID composed two books, making a part of the *τόπος ἀναλυόμενος*. PAPPUS, in his preface to his seventh book, gives no description of these two books, nor any account of their contents, from which, as in other cases, geometers might have been enabled to explain or restore them. There are, indeed, a few *Lemmata* at the end of his seventh book, said there to belong to the *Loci ad superficiem*, but the first, as it is enunciated, and the other four,*

* These five *Lemmata* are in COMMANDINE's translation, and some of the MSS. in a very imperfect state, as will appear from the following short account of them.

The enunciation of Lemma 1, (Prop. 235.) is a simple Conic *Locus*, but in a sort of supplementary enunciation is an allusion to a *Locus ad superficiem*, which in its present state is unintelligible; but a conjecture about the meaning of it is proposed by Dr. WALLIS, in the Savilian copy of COMMANDINE.

In fol. 300. a. COMMAND. 1588, the four last lines, though in the type of the commentary, is a part of the text, and the Greek, of which it is a version, is in the Savil. MS. No. 3. It is the enunciation of a well-known Conic *Locus*, but without any demonstration; and at the end it is said, "that it will be shewn by premising the following *Locus*." This probably was Lem. 2, as the next Proposition 236 (viz. the *Locus* referred to) is Lem. 3.

Prop. 236. Lem. 3. is also a well-known Conic *Locus*, consisting of three cases, belong to the three sections; but the first case of the parabola is alone demonstrated in this Proposition.

Prop. 237. Lem. 4. This is the enunciation of the two cases omitted in the preceding Prop. 236; viz. those of the ellipse and hyperbola, and these are analyzed and demonstrated.

Prop. 238. Lem. 5. This proposition is only a repetition of what I suppose to have been Lem. 2, and is the same *Locus* to the three sections as was there stated. PAPPUS demonstrates only the first case of the parabola, by a reference to Prop. 236; but the other two cases of the ellipse and hyperbola are easily demonstrated by a similar reference to Prop. 237.

I may further remark that the *Locus ad hyperbolem*, in Prop. 237, is assumed by PAPPUS in the solution of a solid problem, Prop. 34, b. iv. without however quoting either Prop. 237 of b. vii. or APOLLONIUS. And no doubt it was at the time a well-known solid *Locus*.

are all solid *Loci*; and truly contain only two *Loci*, of which the one is preliminary to the other. What follows the enunciation of the first (viz. Prop. 235) in its present mutilated state is unintelligible; it has, however, some allusion to a *Locus ad superficiem*, but the other four have none whatever. It is indeed somewhat singular, that these Propositions are introduced in this place. PAPPUS, in his preface, professes to give an account of the treatises of the *τόπος ἀναλυόμενος*, only so far as the *Conics* of APOLLONIUS, in the arrangement assumed by him, which is not the order of time, excluding thereby (from design, we may fairly suppose) the *Loci ad superficiem*, and several others. Perhaps these *Lemmata* may be a fragment of some other work, as probably is the Proposition, called *λήμμα τῆ ἀναλυόμενε τόπε*, subjoined to them in several MSS. which has no obvious connection with any other Propositions of that seventh book. It is possible, however, that these five Propositions may have been *Lemmata in Locos ad superficiem*; but being solid *Loci*, might have been considered as a proper addition to the "*Lemmata in Conica APOLLONII*," to which they are subjoined.

In the description of the *Loci Plani*, however, a short account is given of the *Loci ad superficiem*, and they are mentioned also in Prop. 28 and 29, lib. iv. PAPPUS; which are problems resolved by means of *Loci ad superficiem*, and which Dr. SIMSON has corrected and rendered intelligible. These are all the materials remaining for the investigation of this difficult and curious, though perhaps not very important, piece of ancient geometry.

Dr. SIMSON had begun very early to consider these *Loci*; for in 1721 I find some equations for expressing the relation of the co-ordinates, by means of three indeterminate quantities. It appears also that he had afterwards designed to give a regular explanation of these *Loci*; for there remains a preface for such an essay, in which he proposes to explain the method of investigating such *Loci* by examples. From these he hopes that it would be easy for one well acquainted with solid *Loci* to investigate and describe any *Loci ad superficiem* which can arise from Conic Sections variously situated; (“*vario situ dispositis.*”) In the beginning of the preface, where he remarks the very little known of these *Loci* among the moderns, he has a marginal correction, intimating that he had not, at the time of writing this preface, seen the treatise *Des Courbes a double Corbure*, which he ascribes (surely from inadvertence) to D. N.* But as there is no treatise of that period of this title, or indeed on this subject, but that of M. CLAIRAUT, published in 1730, there can be little doubt of that being the work alluded to by Dr. SIMSON.†

* M. NICOLE, in a Memoir of the Academy of Sciences, (1734) but subsequent to the publication of CLAIRAUT's work, makes an allusion to the curves of double curvature. M. NICOLE was one of the censors of CLAIRAUT's treatise.

† The very slight notice of the ancient *Loci ad superficiem*, by DES CARTES, in his Geometry, (book ii.) was of no use for Dr. SIMSON's investigation. “At vero duabus conditionibus deficientibus ad hujus puncti determinationem, Locus in quo illud reperitur superficies est,” &c. This, to be sure, corresponds to an equation with three unknown quantities, but it is not pursued by DES CARTES. His commentator SCHOOTEN gives two examples, which may perhaps be included in the general description of the *Loci ad superficiem* by PAPPUS, in his account of *Loci Plani*; but the Data and the Locus are all in the same plane, and they furnish no illustration of what Dr. SIMSON considers as the object of the ancient treatise of EUCLID.—See CARTESII *Geomet.* Amst. 1683, p. 34; and SCHOOT. *Comment.* p. 228. See also SCHOOTENII *Sectiones Miscellaneæ.* Lugd. Bat. 1657, p. 494.

All these are comprehended in the following equation:

$$xx + \frac{a}{b}xy + ax + by + \frac{a}{c}yy + \frac{a}{d}yz + \frac{a}{e}xz + gz + \frac{b}{k}zz = \text{Dato.}$$

The signs being any how changed.*

In proceeding however to examples, he treats the subject geometrically, and without any reference to the equation. There are indeed several examples, but all of them, except a few of the simplest forms, are unfinished; and though in the first there is a considerable progress, and an investigation of the common section of the *Locus ad superficiem* sought, when cut by several planes in different positions, yet he mentions several others to be ascertained, but which are not further prosecuted. The Doctor was not easily diverted from an investigation, merely from the difficulty of it; and we may therefore reasonably infer, that he abandoned the subject, partly from its having been previously taken up by CLAIRAUT, and partly from considering it as a branch of ancient geometry, of inferior importance and utility.† The omission of any description of this and some other treatises of the *τύπος ἀναλυόμενος*, in the preface to the seventh book, is a presumption of a similar estimation of the *Loci ad superficiem* being entertained by PAPPUS.‡

* See Note G.

† See Note G. at the end, some of the easiest examples.

‡ In vol. ii. p. 151, of FERMAT'S works, there is a letter of his to ROBERVAL, in which, after some observations about *Loci*, and particularly respecting one of the second book of APOLLONIUS, he adds, "*de Locis solidis et ad superficiem multa quoque sunt detecta.*" He slightly alludes to the same subject in a letter to Sir KENELM DIGBY. WALLISII *Opera*, tom. ii. p. 859. Also in page 116 of second vol. of FERMAT'S works, in the preface to his *Porisms*, after mentioning the restoration of ancient treatises of geometry by SNELLIUS, VIETA, and MARINUS GHETALDUS, he adds, "*sequebantur Loci plani, Loci solidi, et Loci ad superficiem, et huic quoque*

Besides these fragments of geometrical speculations, there are a great variety of Problems, Theorems, Loci, Data, and Porisms, dispersed through the Doctor's papers, and particularly in the MS. volumes, which he called *Adversaria*, of which eighteen remain. These in general, as has already been observed, appear to have been analyzed or demonstrated, as the circumstances of his reading, conversation, or study, happened to suggest; yet they may be considered as a valuable repository of such Propositions, from which useful selections might be made, for promoting the study of geometry in the ancient method. They are deposited in the College library of Glasgow; and it is much to be wished, that some gentleman acquainted with the subject, and who has leisure and inclination for the task, would take the trouble of arranging a proper collection.*

It has been repeatedly remarked, that notwithstanding Dr. SIMSON'S early and continued partiality for geometry as

“parti non ignoti nominis geometræ, succurrerant, eorumque opera manuscripta, licet et adhuc inedita, latere non potuerint.” And in another letter to Mr. ROBERVAL (p. 153) he observes, “J'en ay plus de cens Propositions tres belles et particulièrement des *Lieux solides et ad superficiem*, mais le loisir me manque, &c.” No further account however of these restorations of *Loci ad superficiem* is to be found.

* There are in these volumes, also, a number of geometrical demonstrations, by the Doctor, of theorems, in the works of various eminent analysts, where they are treated either algebraically, or if geometrically, with inferior skill.—I may remark that most of the Propositions alluded to are dated; and on some particular occasions not only the day, but even the hour is noted. Sometimes also, when he had accomplished a solution or a demonstration, which pleased at the moment, some expression of his satisfaction is added. He began this practice very early, even before he was elected Professor of Mathematics, and it was continued till about fourteen months before his death. In the book of *Adversaria Q.* is the latest date now to be found, 11th August 1767; he being then in his eightieth year.

cultivated by the ancients, he did not entirely neglect the modern analysis, but became well acquainted with algebra, and with the common applications of it which were pursued in the early part of the last century.* He certainly did not approve of the prevailing use of algebra in geometrical investigations, which could be accomplished with more elegance, and not seldom with more ease, by the ancient method; but he made no scruple of acknowledging the superior power of algebra in complex geometrical inquiries, and more especially in the application of it to such as are physical.—It is probable however, from all the information which can now be obtained respecting his studies, that though he understood the modern analysis as delivered by Sir ISAAC NEWTON, yet he did not continue to devote much of his time to the study of it, and thence did not enter much into the profound analytical speculations of succeeding mathematicians.

It must be admitted, also, that his rigid notions respecting algebraical reasoning, particularly in geometry, had a tendency to limit investigations, by that method; and to discourage him from pursuing the methods of modern analysts, who felt no such difficulties as had occurred to him. As an example, however, of Dr. SIMSON's knowledge of algebra, I may mention his invention of some very simple and excellent serieses for finding the circumference of a circle, of which he appears to have given an intimation in 1722 to Dr. JURIN, then secretary of the Royal Society.—Dr. JURIN undertook to obtain the sentiments

* Dr. SIMSON often amused himself, especially in the latter part of his life, in resolving any geometrical problems which attracted his notice, by the algebraical method, though he rarely made any entry of these solutions in his *Adversaria*.

of his learned friends on these serieses; and therefore Dr. SIMSON, in a letter of Feb. 1, 1723, transmitted to him his paper on the subject,* containing the serieses, with two elementary Propositions from which they were derived. Though Dr. JURIN was not apprized that any thing of the kind had been done by others, yet by inquiry he found that Mr. MACHIN, near twenty years before, had communicated to the Royal Society some serieses of the same form, but had afterwards withdrawn them. Mr. MACHIN readily gave a copy of his serieses to be communicated to Dr. SIMSON, which Dr. JURIN sent to him immediately after; (March

* I annex a copy of the portion of this letter which relates to the serieses. The remainder of it respects the Porisms, and is taken notice of in another place. For the paper enclosed, see Note H. at the end.

“ SIR,

“ Friday last I had the favour of your very civil answer to the letter I lately gave you the trouble of: some private affairs have kept me from transcribing the enclosed trifle, else you should have had it sooner.—The series for finding the circumference of the circle which are in Mr. SHERWIN'S Book of Tables, being deduced from tangents of arches which have a given ratio to the circumference, I was desirous to try if the like might not in a general way be drawn from tangents, whose arches are not in a given ratio to it; and found a great many easily flow'd from the two Propositions and their corollaries, which I have enclosed. The Propositions, though obvious enough, I do not find in Mr. BRIGGS, or any of the authors I have searched for them: the corollaries are long since known, if I mistake not. Among the series I have set down, there is one which Mr. JONES, in his Synopsis, says he had from Mr. MACHIN, and which is certainly one of the best that can be found by this or any other method. Whether any or all of the rest have been taken notice of by any body, I am wholly ignorant; but you may soon discover.

“ I thankfully acknowledge the favour of your kind offer to obtain the sentiments of some of your learned friends upon the enclosed paper, but I entreat you may give yourself no more trouble about it, since I am ashamed to have given you already so much, when the thing is scarce worth taking notice of, and which you will easily see is not to be shewn to any but as such.”

“ Glasgow, Feb. 1, 1723.”

1723). The extreme modesty with which Dr. SIMSON mentions this ingenious investigation to Dr. JURIN, is remarkable; for though from the present state of science, those series may have lost their former importance, yet near a century ago they had the merit of novelty and utility, and had then occupied the attention of Mr. MACHIN and Dr. SIMSON. It may thence be satisfactory to some readers to see a fuller account of them, which is given in a note at the end.*

The Mathematicians of the end of the sixteenth and beginning of the seventeenth centuries began to apply algebra to geometrical inquiries; and this method was much promoted and extended by DES CARTES. The use of negative and impossible roots of equations was also introduced into this analysis; though by preceding algebraists they were at first not observed, and afterwards when observed were reckoned useless. While algebra, in its early and humble state, was employed almost entirely about numbers, these roots were naturally neglected, and the language of the first writers on the science was thence not encumbered with the metaphysical difficulties which arose from the processes and reasonings of the followers of DES CARTES. Some of these were reasonably considered by Dr. SIMSON as defective in that precision of definition, and strictness of argument, which have ever been the boast of pure geometry, and to which he had been accustomed from his almost constant study of that branch of the science. The Doctor, from this cause perhaps, conceived a prejudice against an application of algebra, which

* See Note H.

was accompanied with such difficulties; and was thence led to treat of that science after the manner of the early writers on it, with the limited definition and use of the negative sign. Many detached observations on this subject remain among his papers;* and some short essays also on cubic equations, in which he endeavours to explain them without admitting negative or impossible roots. In the present state of that science, it is unnecessary to mention the detail of these speculations; and it may be sufficient to remark, that though the method of investigation was different, the results were similar to those contained in the Essay on the Negative Sign by that learned promoter and patron of mathematical science, Mr. BARON MASERES; of which, from the correspondence of opinions, Dr. SIMSON spoke with much approbation. Dr. SIMSON's notions on this subject probably varied a little in the progress of life, by his confirmed partiality for the ancient analysis, and an increasing prejudice against the algebraical. The elegance and satisfactory clearness of every step in the geometrical method must, where it can be employed, be universally preferred to the almost mechanical process of an algebraical calculation. His animadversions, however, on the application of algebra to geometry, chiefly referred to those cases where it was not necessary, and in which the more excellent method of the ancients could be successfully employed. In Plane Geometry, and in the Conic Sections, the ancient method is still generally used, and by good judges is allowed to be the best for forming the

* See in Note K, at the end, some observations by Dr. SIMSON on this subject, in his letter to Mr. GEO. LEWIS SCOTT.

taste, and exercising the reasoning powers, of young persons; and Dr. SIMSON has done essential service by his geometrical works, in exciting curiosity respecting these branches, and facilitating the study of them.

Dr. SIMSON has no where in his writings given any specific opinion of the proper and necessary use of algebra in geometrical enquiries, and therefore we cannot judge with precision of his views respecting it. It may be observed, however, that as the indefinite number of classes of geometrical curves, in the present state of both the ancient and modern analysis, can be conveniently treated of only by algebra, this application of algebra must be admitted as necessary; and the definition of the Conic Sections by equations may be regarded as an useful introduction to the definition of superior curves on that principle. Dr. SIMSON seems, in the early part of his life at least, in some measure to have admitted this; for in a great number of solid *Loci* among his papers, though demonstrated geometrically, the suitable equation is added. Probably, however, he never set about taking an accurate and comprehensive view of the subject in all its bearings; and having made his early choice from taste, the ancient analysis continued through life to be the favourite object of his study.

Considering the unbounded extent of mathematical science, and the variety of branches into which it may, and has been usually distributed; it may be for the advantage of the most successful progress of the whole, that individual mathematicians should apply themselves chiefly to particular divisions of it. Though there be a close connection among

these several branches, yet a moderate knowledge of the general system may be sufficient for the successful cultivation of any one portion of it, to which the attention of a mathematician may happen to be directed. Had Dr. SIMSON applied his ardent mind to the modern analysis, most probably the results of his investigations would have been important; but his not having devoted much of his attention to the study of it, was certainly favourable to his progress in the laborious researches which he undertook respecting the ancient geometry. It will be readily admitted also, that if some of the great modern analysts happen not to have studied the ancient geometry, as may be presumed to be the case, they were not thereby materially obstructed in their profound investigations, by which human knowledge has been so much enlarged; though, perhaps, by such previous study, their works might in some instances have been improved in simplicity and perspicuity.

It appears from the Doctor's letters to the late Earl STANHOPE,* that his Lordship had recommended to him to attempt the demonstration of some of the more important modern geometrical discoveries by the strict and more elegant method of the ancients. This was in 1751, and though he states it to be very much his own wish to make the trial, he considers it to be beyond his power, from the decline of his faculties. The tract on *Logarithms*, and the fragment *De Limitibus*, seem to be his only attempts of that kind; but he proposes to recommend it to his scholars, Dr. STEWART, Dr. MOOR, and Mr. WILLIAMSON. There are indeed among his papers a few imperfect sketches (apparently of an old date) of attempts to

* See Note I. at the end.

demonstrate geometrically the celebrated theorem of the Circle invented by Mr. COTES. He does not seem to have bestowed much time to this subject; but I have heard him mention that his plan was to demonstrate first the more easy cases, and then to attempt to frame a general method of deducing any case of this Proposition from the next preceding and more simple case, from which a compleat demonstration of the theorem in its full extent might be derived. But in this he was unsuccessful, and he regretted the disappointment. But as neither Dr. SIMSON, nor any other geometer, so far as is known, have ever accomplished a geometrical demonstration of this theorem, we may consider the investigation of Mr. COTES as an important example of the superior power of the modern analysis, in the discovery and demonstration of a Proposition, very general and difficult indeed, but strictly geometrical; and though it be commonly expressed algebraically, yet may, by means of compound ratios, be enunciated in the pure language of geometry.

Long before any symptom of decline in his health appeared, he seems to have felt some remarkable decay of his memory and attention. This was not observed by his friends, either in conversation or in correspondence; but his complaints of it were so particular and serious, that no doubt can be entertained of the fact, nor of the influence which it had in diverting his mind from some of the more severe investigations which he had formerly wished to pursue.*

In the preface to the Doctor's *Conic Sections*, when recommending the geometrical method of treating the subject, in

* See Note K. at the end.

preference to the algebraical, he has this remarkable sentence :
“ In quibus autem differat analysis geometrica ab ea quæ
“ calculo instituitur algebraica, atque ubi hæc aut illa sunt
“ usurpanda, atque quæ sint in mathematicis utriusque partes
“ propriæ, alias differendum.” From this observation it must
be acknowledged, there naturally arises a presumption, which
has been generally entertained, that the Doctor had considered
the subject, and probably written something concerning it:
and the opinion of so distinguished a mathematician on a point
of some difficulty, and of which he was well qualified to judge,
would have been particularly interesting. But among his
papers no trace of such an attempt is to be found. Professor
ROBISON, in his Life of Dr. SIMSON already mentioned, seems
to think that a dissertation on *Loci*, which he had seen when
attending the Doctor's Lectures in the College of Glasgow,
might have been the tract to which the preceding quotation
referred.* Of this, however, notwithstanding the weight of
Mr. ROBISON's opinion, a reasonable doubt may be enter-
tained, considering the extent of the discussion implied in that
quotation, which must have gone far beyond what such a dis-
sertation as Mr. ROBISON mentions, could be supposed to
contain. But Dr. SIMSON's own testimony, in his correspon-
dence with Mr. SCOTT respecting this very sentence, is con-
clusive, and puts an end to all speculation regarding it, as he
therein explicitly declares, that on the subject of it “ he had no
“ papers, and never wrote any thing on that matter.” Fortu-
nately, a considerable part of this correspondence is preserved;
and the most important portion of it is placed in a note at the

* No trace of this dissertation on *Loci* remained among his papers.

end.* The letters both of Mr. SCOTT and Dr. SIMSON will be read with interest: they contain some valuable observations on the relative merits of the ancient analysis and of the algebraical, in the solution of geometrical problems. The subject indeed is not exhausted; and as the correspondence took place from occasional circumstances, the discussion, though entertaining and instructive, is of course limited very much to the points which thus accidentally arose. It touches little on the peculiar powers of the modern analysis, and is confined chiefly to the comparison of the two methods in cases where either may be employed.†

* See Note K.

† For some remarks on this correspondence, see the latter part of Note K.

SECTION V.

Sketch of Dr. SIMSON's Character.

IT was formerly observed that Dr. SIMSON's mathematical labours and inventions, of which some account has been given in this memoir, were the most interesting occurrences of his life, and most illustrative of his capacity and genius.

In attempting, however, to give a brief outline of the Doctor's character, besides the talents displayed in his geometrical works, which claim the first consideration, it may be expected that some observation also should be made of the well-known discriminative features of his mind, which, though not important in themselves, appear to have had an influence on its greater qualities. These peculiar features are still affectionately remembered by his pupils, friends, and acquaintances who survive; and, from the eminence of his genius, they may be regarded by others as objects of a natural and liberal curiosity.

Dr. SIMSON was originally possessed of great intellectual powers, an accurate and distinguishing understanding, an inventive genius, and a retentive memory: and these powers, being excited by an ardent curiosity, produced a singular capacity for investigating the truths of mathematical science. By such talents, with a correct taste, formed by the study of

the Greek Geometers, he was also peculiarly qualified for communicating his knowledge, both in his lectures and in his writings, with perspicuity and elegance. He was at the same time modest and unassuming; and though not indifferent to literary fame, he was cautious and even reserved in bringing forward his own discoveries, but always ready to do justice to the merits and inventions of others. Though his powers of investigation in the early part of life were admirable, yet before there appeared any decline of his health, he felt strong impressions of the decay both of his memory and other faculties; occasioned probably by the continued exertion of his mind in those severe studies, which for a number of years he pursued with unremitting ardour.

Besides his mathematical attainments, from his liberal education he acquired a considerable knowledge of other sciences, which he preserved through life, by occasional reading, and in some degree also, by his constant intercourse with so many literary men in his College.† He was esteemed a good classical scholar, and though the simplicity of geometrical demonstration does not admit of much variety

† Dr. SIMSON in the early part of his life studied Botany, in which he made great progress, and it became a source of amusement to him in his walks. In the year 1746, when the University of St. Andrew's wished to confer on him the degree of Doctor, as he was a layman, a degree of medicine was proposed from the circumstance just mentioned, though he had no other pretension to distinction in that science.

It may be mentioned also in this place, that a few years after he became a Professor, he composed a tract on the School Logic, with rigorous demonstrations of its rules. Some sketches of it remain, which strongly mark the accuracy of his reasoning powers to whatever subject they were directed.

of style, yet in his works a good taste in that point may be distinguished. In his Latin prefaces also, in which there is some history and discussion, the purity of language has been generally approved. It is to be regretted indeed that he had not had an opportunity of employing, in early life, his mathematical and Greek learning, in giving an edition of PAPPUS in the original language.‡

Dr. SIMSON never was married; and the uniform regularity of a long life, spent within the walls of his College, naturally produced fixed and peculiar habits, which however, with the sincerity of his manners, were unoffending, and became even interesting to those with whom he lived. The strictness of these habits, which indeed pervaded all his occupations, probably had an influence also on the direction and success of some of his scientific pursuits. His hours of study, of amusement, and of exercise, were all regulated with uniform precision. The walks even in the squares or garden of the College were all measured by his steps, and he took his exercise by the hundreds of paces, according to his time or inclination.

‡ Dr. SIMSON's learned discussion concerning some Greek terms in the account of the Porisms by PAPPUS has already been mentioned, and a copy of it is inserted in Note C. I may mention here also another example of his critical knowledge. In the dialogue of PLATO, MENO, there is a passage, of which, even in the best editions, some sentences are somewhat obscure, and probably corrupted. It relates to an ingenious speculation, of all knowledge being only reminiscence; and as an illustration, an uninstructed youth is led, merely by an examination, to assent to the truth of a geometrical proposition, (an easy case of the 47. 1. ELEM. EUCL.) This had attracted Dr. SIMSON's notice; and he found that some natural and easy emendations, with the addition of a diagram, which, in the dialogue, is plainly supposed to be exhibited to the youth, rendered the argument intelligible and correct. He had drawn up a detailed statement of the passage, and the necessary corrections which, in his own time I saw in the hands of one of his colleagues, who has been dead many years: but no trace of it is now to be found among the Doctor's papers, or any where else.

It has been repeatedly mentioned, that an ardent curiosity was an important feature of his character. It contributed essentially to his success in his mathematical investigations, and it displayed itself in the small and even trifling occurrences of common life. Almost every object and occurrence excited it, and suggested some problem which he was impatient to resolve. This disposition, when opposed, as it often necessarily was, to his natural modesty, and to the formal civility of his manners, occasionally produced an embarrassment, which was amusing to his friends, and sometimes a little distressing to himself.

The Doctor, in his disposition, was both cheerful and social; and his conversation, when he was at ease among his friends, was animated and various, enriched with much anecdote, especially of the literary kind, but always unaffected. It was enlivened also by a certain degree of natural humour; and even the slight fits of absence to which in company he was occasionally liable, contributed to the entertainment of his friends, without diminishing their affection and respect, which his excellent qualities were calculated to inspire. One evening in the week he devoted to a club, chiefly of his own selection, which met in a tavern near the College. The first part of the evening was employed in playing the game of whist, of which he was particularly fond; but though he took no small trouble in estimating chances, it was remarked that he was often unsuccessful. The rest of the evening was spent in cheerful conversation, and as he had some taste for music, he did not scruple to amuse his party with a song; and it is said that he was rather fond of singing some Greek odes, to which modern music had been adapted. On Saturdays he usually dined in the village of

Anderston, then about a mile distant from Glasgow, with some of the members of his regular club, and with a variety of other respectable visitors, who wished to cultivate the acquaintance, and enjoy the society, of so eminent a person. In the progress of time, from his age and character, it became the wish of his company that every thing in these meetings should be directed by him; and though his authority, growing with his years, was somewhat absolute, yet the good-humour with which it was administered, rendered it pleasing to every body. He had his own chair and place at table; he gave instructions about the entertainment, regulated the time of breaking up, and adjusted the expense. These parties, in the years of his severe study, were a desirable and useful relaxation to his mind, and they continued to amuse him till within a few months of his death.

Strict integrity and private worth, with corresponding purity of morals, gave the highest value to a character, which, from other qualities and attainments, was much respected and esteemed. Upon all occasions, even in the gayest hours of social intercourse, the Doctor maintained a constant attention to propriety. He had serious and just impressions of religion, which appear occasionally in his papers, and may be traced even in the midst of some of his mathematical investigations;* but he was uniformly reserved in expressing particular opinions on the subject of religion; and from his sentiments of decorum,

* As an example, the following date of the solution of a problem, which happened to be his birth-day, may be quoted, as it stands in one of his MS. volumes:

“ 14 Octobris 1764

“ 14 Octobris 1687

“ 77

“ DEO Optimo Maximo, Benignissimo Servatori, fit laus et gloria.”

he never introduced religion as a subject of conversation in mixed society, and all attempts to do so in his clubs were checked with gravity and decision.

Dr. SIMSON in his person was tall and erect, and his countenance, which was handsome, conveyed a pleasing expression of the superior character of his mind.* His manner had always somewhat of the fashion which prevailed in the early part of his life, but was uncommonly graceful. He was seriously indisposed only for a few weeks before his death, and through a very long life had enjoyed an uninterrupted state of good health. He died on the first of October 1768, when his eighty-first year was almost compleated; having bequeathed his small paternal estate in Ayrshire to the eldest son of his next brother, in whose family it still remains.†

* In the College hall of Glasgow is a portrait of Dr. SIMSON, which, though painted when he was in the vigour of life, yet expresses a likeness to his countenance and figure as they appeared in advanced age. On the title-page of this Memoir is placed a small engraving from this picture, with an inscription written by the late Dr. MOOR, Greek Professor at Glasgow, happily expressing Dr. SIMSON's disapprobation of the modern use of algebra in geometry, and his singular merit and success in restoring the ancient analysis. See the Edinburgh Transactions, vol. i. p. 75. History.

† Dr. SIMSON, from his constant residence in the College, and his engagement in study, gave little attention to the improvement of his estate, which descended to him encumbered with debts, and which became much more productive to his heir than it had been to himself. He was at the same time well acquainted with business, having been for a great number of years Clerk to the Faculty, and in that character he had the chief management of the property and other affairs of the College, which he conducted with his habitual regularity and accuracy. In his manner of living he was simple and unexpensive, except in collecting a valuable mathematical library, which he bequeathed to his College. But from an income, ample with respect to his habits, he made no accumulation; having enjoyed the satisfaction of advancing young relations, in his own time, by many acts of generosity.

NOTES.

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NOTE A. p. 28.

FROM APOLLONIUS and his commentators to MYDORGIUS and others of the seventeenth century, Dr. SIMSON considers this branch of geometry as stationary for that long period. It has been properly observed however by different authors, that the points, in subsequent times called *Foci*, were ascertained by APOLLONIUS only in the ellipse and hyperbola, but not in the parabola. The Rev. Dr. ROBERTSON, Savilian Professor of Astronomy at Oxford, in his learned History of Conic Sections, annexed to his Treatise on that subject,* after mentioning this observation, adds, that in two small tracts, now very rare, edited by GOGAVA, Louv. 1548, the first mention is made, so far as he knew, of the focus of the parabola.† GOGAVA considers the second tract (*De Speculo Ustorio*) as of Arabic origin, and very ancient; and also that VITELLO had borrowed from it. VITELLO flourished about 1470,

* *Sectionum Conicarum, Libri VII.* &c. Oxon. 1792, pp. 340, 362.

† This publication appears to be a first edition, though other dates are given to it by writers who quote it; and it has a Preface by GEMMA FRISIUS. The second of these two small tracts seems to have been generally known before the edition by GOGAVA. ROGER BACON quotes it; and VERNERUS, in his tract *De Cubo Duplic.* Nurem. 1522, append. 12ma. plainly alludes to it. MAUROLICUS also, in a miscellaneous volume, (Messanzæ, 1558,) states

near a century before GOGAVA; and in his *Optics*, prop. 42, 43, book ix. (page 400, edit. Refneri,) the focus of the parabola, and the application of it in forming a concave parabolic mirror are distinctly described, but without stating the source from which he derived them.

It is plain, however, that the point, afterwards called the focus of the parabola, was used by PAPPUS, though not formally distinguished or named, in prop. 238, lib. vii. (fol. 303. b. edit. 1583,) which is a *Locus* to the parabola. But he does not mention the property, easily flowing from that which he assumes, on which is founded the construction of the parabolic reflecting mirror; and it is not improbable that this last property had not been observed in the time of PAPPUS, and that it was discovered long after by the Arabian opticians. This Prop. of PAPPUS must have been well known to Dr. SIMSON, though he appears not to have considered it as requiring notice in the short history of Conics in his preface.

It may be convenient in this place to make an observation on another portion of the preface to Dr. SIMSON'S *Conics*, viz. the paragraph beginning, "*Propositionum jamdudum, &c.*" (p. vi. Præfat. prim. et sec. edit.) and ending "*contulerint.*"

This paragraph was added in the first edition, (1735,) in consequence of an objection made by Mr. MAC LAURIN to a Proposition of the Doctor's, as not containing a sufficient acknowledgment of a communication by Mr. M. on the subject of it. Dr. SIMSON, in a letter to a common friend not named, which remains among his papers, states this circumstance. He says, "I have not time to write about the Proposition Mr. MAC LAURIN objects against. To take away all controversy in this matter, I send you an addition to the preface;" viz. the above-mentioned paragraph. After some farther explanation of his anxiety to do justice to Mr. MAC LAURIN he says, "you may shew him this addition to the preface, before printing it."—The explanation

the nature of this tract, observing, that it had been erroneously attributed to ARCHIMEDES, and it is in an abstract of the works of ARCHIMEDES that he mentions it. He considers it, however, to be the work of PTOLEMY, without giving any authority. See the above mentioned volume of MAUROLYCUS, fol. 72. It was printed indeed some years after GOGAVA'S publication; but probably MAUROLYCUS had not seen these small tracts, as he makes no mention of them, or of the editor GOGAVA. The first tract, *De Parabola*, is supposed to be of the time of ROGER BACON, about 1270; but the second, *De Speculo Ufforio*, is much more ancient, being quoted by ROGER BACON.

seems to have been quite satisfactory, for in the following year, 1736, the mathematical correspondence between Dr. SIMSON and Mr. MAC LAURIN continued on its usual friendly footing.

Among Dr. SIMSON's papers there is an account of his *Conics*, drawn up soon after the publication, in the form of a letter, and in the name of some friend of the Doctor's, apparently with the design of being transmitted to a distance, and perhaps for publication.* It is a sort of enlargement of the preface, giving a more particular detail of the author's views in composing the treatise, and of the improvements which he had made in it. The following passage in it gives a more particular account of the communication to Mr. MAC LAURIN. "The Scholium in p. 198, (first edit.) and the Prop. it flows from, contain a very general and useful property of the Sections, which the author discovered upon this occasion. When Mr. MAC LAURIN, the learned professor of mathematics in the university of Edinburgh, was going to France in the year 1723, Mr. SIMSON communicated to him a *Locus* of PAPPUS Alexand. which he had restored, viz. *Si a tribus punctis datis in recta linea ducantur tres rectæ lineæ, et ipsarum intersectiones duæ tangant rectas positione datas, tanget reliqua intersectio rectam lineam positione datam*: which was afterwards printed in the Philosophical Transactions. When Mr. MAC LAURIN had come home from France, he told Mr. SIMSON that he found, when the three given points were not in a straight line, all the rest remaining as in the former case, that the *Locus* described would be a conic section, and that by help of this *Locus* he could easily describe a conic section through five given points: This *Locus*, and method of description, he gave Mr. SIMSON without any demonstration, which he said he had made by help of a *Lemma* in Sir ISAAC NEWTON's *Principles*; and it was in searching for the demonstration of them that Mr. SIMSON found out the Propositions in the Scholium, and those they flow from, which he did in the same method

* Since writing this account, Mr. FRYER of Bristol, who is engaged in a history of mathematics, pointed out to me a letter from the celebrated Professor HUTCHESON of Glasgow, on Dr. SIMSON's *Conic Sections*, which he had found in the *Bibliothèque Raisonnée* for April, May, and June 1735; and which, on comparison, I ascertained to be the very letter here alluded to. In that journal it is translated into French, and Professor HUTCHESON is mentioned as an old friend of M. SMITH, one of the publishers, at Amsterdam. The letter is dated Glasgow, 28 Feb. 1734.

“they are now printed in his book.” The copy of this letter, found among Dr. SIMSON’s papers, is without any signature, but it is in the Doctor’s handwriting.*

In the note * page 29, the proper reference is to BOSCOVICH *Sect. Con. Elementa.* art. 275.

NOTE B. p. 43.

To the examples mentioned by Dr. SIMSON, of misapprehensions entertained by eminent mathematicians respecting Porisms, may be added that of CASTILLON. In his commentary on the *Arithmetica Universalis*, (Amst. 1761,) p. 216, vol. 1, after quoting a passage from MARINUS’s preface to the *Data*, he adds, “Hæc observatio mihi viam aperuit ad restituenda EUCLIDIS Porismata;” and afterwards, page 268, when speaking of the ancient problem of the Tactions, he adds, “omnia soluta habeo, et me editurum spero propediem cum Porismatis EUCLIDIS restititit.” The same author, in his commentary on the appendix, vol. ii. page 264, shews more particularly his misapprehension of the nature of Porisms, by supposing them to be the constructions of EUCLID’s data; and then he adds, “ad hunc (sc. librum Porismatum) non pauca colligere cæperam, sed cum audivissem virum doctissimum ROBERTUM SIMSONUM, rem perfecisse, et sua scripta reliquisse, ab incepto destiti, sperans et rogans ut tanti viri cogitationes in publicam lucem emittantur.” Dr. SIMSON had got this book a few years before his death, and was much amused with these observations of the author. It is very surprising that so many respectable mathematicians deceived themselves on the subject of Porisms, especially those who were apprized of the failure of Dr. HALLEY, who candidly acknowledges it, but who, of all who attempted the investigation of them before Dr. SIMSON, seemed to be, from his talents and his knowledge, the most likely to succeed.

* In this letter also is given a solution of the problem, “To describe a Conic Section which shall pass through two given points, and touch three straight lines given in position.”—This he easily deduces from Prop. 12. b. v. (1st ed.) by means of the before-mentioned *Locus*, published in the *Phil. Transactions* for 1723, p. 330, without the transformations employed in Prop. 25 and 26 of NEWTON’s *Princip.* lib. 1.

NOTE C. p. 22.

“ I subjoin the remainder of Dr. SIMSON's letter to Dr. JURIN, of which the first part is in the text at page 22. “ At his (Dr. HALLEY's) desire I have “ considered that passage as narrowly as I could, and shall be very glad to have “ his opinion concerning the following conjectures; since, both with respect “ to the language and the matter, I esteem him the most skilful judge.

“ The first thing to be considered is, that the figures PAPPUS speaks of in “ this Proposition (for I suppose σχῆματός, or some such word, ought to be under- “ stood to be joined with ὑπὲρ &c.) are such as are made by four straight lines, “ either all of them intersecting one another, or two of them, at the most, “ parallels: (the case of the parallelogram, because of its plainness, being “ omitted.) I mean, PAPPUS speaks of the complex figures made by the “ actual intersections of all the lines, and not of simple Trapezia, whose sides, “ if produced, would meet: this is plain from his supposing three points of “ intersection in their sides; ‘ Si dentur, in earum una, puncta tria.’

“ Of the three kinds of these figures he mentions, that which he calls “ παραλλήλων is easily, and I think certainly, determined, from some places in “ his Lemmas, to be the figure made by two parallel lines meeting two other^s “ which are not; so that either of these two figures A or B is the παραλλήλων.*

“ The places which verify this are first in Prop. cxxxvi. at the letter C in “ page 247† (of the edition at Ven. 1569,) COMMANDINE has it, ‘ ita KH “ ad HL hoc est in lineis parallelis GH ad HM.’ where the Greek has “ τὴν ἴστιν ἐν παραλλήλων &c. and the figure GHMLK‡ is plainly such as has been “ described above. Again, in Prop. cxxxv. fol. 246, there occurs, “ ut FM “ ad DH, ita in lineis parallelis FK ad KH;’ which I doubt not will be in “ the Greek MSS. ἐν παραλλήλων, and the figure is the same as in the last.

“ The same way of speaking is twice used in the first demonstration of the “ first Lemma, fol. 238.§ And all the figures to which these two passages refer “ are of the other kind, (viz. fig. B.) except one. And from this it appears “ that παραλλήλων is promiscuously applied to both..

* Fig. A and B.

† This should be fol. 246. b.

‡ Fig. 1.

§ It appears that the MS. used by COMMANDINE had ἐν παραλλήλων in this prop. (127.) See his Note D.

“ It is also plain, that the passage in fol. 242, at letter B, is not corrupted, “ as COMMANDINE suspects; it is *καὶ γὰρ ἡ θκ πρὸς τὴν θε ἐν παραλλήλω*, which he “ should have rendered into Latin thus, ‘ ut autem AF ad FG, ita LH ad “ HM, etenim in eadem ratione (sc. cum utrisque illis rationibus) est HK ad “ HG ἐν παραλλήλω, viz. est AF ad FG ut HK ad HG ἐν παραλλήλω AFGHK,” “ and LH ad HM ut HK ad HG, ἐν παραλλήλω LHKGM.’

“ Having determined the figure which PAPPUS calls *παραλλήλον*, the rest, viz. “ those where none of the lines are parallel, must belong to other two kinds, “ *ὕπτιον* and *παρυπτιόν*. The first of which words usually signifies *supinus, re-* “ *supinus, retrorsum vergens*, and is applied by ARISTOTLE to the recurved “ horns of Oxen. Now all the figures whatsoever that can be made by the “ intersecting of four straight lines none of which are parallels, are with “ respect to their form, of one kind; and have every one a reversed or retroflexed “ angle, which will agree very well with the signification of *ὕπτιος retrorsum* “ *vergens*. But, then, if every one of these figures be *ὕπτιον*, what comes of “ the remaining branch of the division *παρυπτιόν*? This is a difficulty cannot be “ taken away, if we suppose PAPPUS to speak of the form of these figures “ absolutely considered; but may be removed, if it be found he has regard “ to the twofold manner in which they are formed, by two lines drawn “ intersecting two other lines, and so as to give a just reason of using these “ terms *ὕπτιον* and *παρυπτιόν* respectively.

“ I shall first explain the two ways after which these figures are formed, so “ as to give rise to the names PAPPUS uses, and then shew what ground there “ is to think he considered them thus, and not absolutely with respect to their “ form.

“ FIG. 2. Let two lines AB, AC, be drawn, intersecting two others DB, DC; “ then if the angles BAC, BDC, made by these lines, lie one over another, *i. e.* “ *ὕπτιως*; or which is the same thing, if one of them, BDC, be the reversed “ angle of the figure; then such lines may be said to make the *ὕπτιον σχῆμα*.

“ FIG. 3. But when the two lines AB, AC, are drawn intersecting two “ others DB, DE, so that the two angles BAC, BDE, made by these lines, “ do not lie below one another, but one of them is upon one side and the “ other upon the other hand of the *ὕπτια γωνία* AFD, which they form by their “ two nearest sides AC, DE, and so lie beside (*παρὰ* juxta) the reversed angle, “ as also at one another’s sides; then these pairs of lines may be said, with “ respect to the angles they make, to form the *παρυπτιόν σχῆμα*.

“ And that PAPPUS had regard to these two different ways in which one pair
 “ of lines meet another pair, I think will appear plain enough by comparing
 “ Prop. cxxxvi. with Prop. cxlii.; in each of which he considers the figures we
 “ are now speaking of, as formed by two lines intersecting two others; and these
 “ figures are not only, absolutely considered, exactly the same, but the things
 “ demonstrated are also the same, viz. the converse of Prop. cxxix.; nor is there
 “ any thing to give occasion to the splitting it into two cases, other than the
 “ consideration of the different ways the one pair of lines meets the other pair;
 “ viz. in Prop. cxxxvi. the lines DH, HE, meet the lines BAE, DAG, *ὀπίσσω*,
 “ or so as to form the *ὀπίσσω σχῆμα*. And in Prop. cxlii. the lines BD, DE, meet
 “ the lines AB, AC, *παρῶπιός*, or so as to form the *παρῶπιόν σχῆμα* with respect
 “ to their angles.”

“ Some, perhaps, may incline rather to give *παρὰ* the negative or contrary
 “ force, when joined with *ὀπίσσω*; because the angles made by the pairs of
 “ lines which form the figure, are the internal opposites of two triangles, of
 “ which the reversed angle is the common external. I shall leave this to others
 “ to determine; and shall only say, I do not remember *παρὰ* is used in this sense
 “ in any mathematical term; for in EUCLID'S Data I think it is quite amiss
 “ to translate it *contra*.”

He adds a Porism without demonstration, which it is unnecessary to insert,
 and concludes, with an apology for writing in haste,

“ I am, Sir, your much obliged and

“ affectionate Servant,

“ GLASGOW, Jan. 10, 172 $\frac{3}{4}$.”

ROBERT SIMSON.”

NOTE [2 C.] p. 49.

On the subject of Porisms, Dr. SIMSON was certainly reserved; and this probably in part arose from a natural desire of publishing his discovery of them in a compleat form, without the risk of his being anticipated, which from partial communications was not unreasonably to be apprehended. Candour was a striking point of his own character; and we may observe in his works the precision with which he remarks the inventions, the im-

provements, and even the smallest hints contributed by his friends;* and perhaps he felt some dissatisfaction from not having always met with the same liberality. Some instances were well known, and there were reports of others; but though they may account for his reserve, it is neither of importance to his reputation, nor in any respect expedient, to investigate or to record them. His reserve on the subject of Porisms I had frequent opportunities of remarking, though favoured with much familiar conversation with him on mathematical subjects. From PAPPUS, and other writers, by whom Porisms were mentioned, and also from some very distant allusions from Dr. SIMSON, I certainly had got a general, but as I afterwards learnt, an imperfect notion of Porisms. I conceived them to be entirely geometrical, and that they were a class of Propositions in which certain points or lines were to be found, which might have a *general* property expressed in the enunciation. I occasionally submitted to Dr. SIMSON some Propositions which I considered to be of that class; but without admitting or denying them to be Porisms, with some pleasantries he said they were Propositions. It appears from his *Posthumous Works*, p. 459, that, with his usual accuracy, a simple Proposition of that kind (which I had stated to him as a problem) had been entered in the fair MS. of the Porisms from which the treatise was printed.

NOTE D. p. 52.

As every notice of the Porisms in former times is interesting, I shall insert some extracts from PROCLUS respecting them, in his Commentary on the first book of EUCLID's *Elements*. I quote from HERVAGIUS's edition, the only printed one, though very erroneous, as is observed by BAROCIUS in his translation, who says it is "dilaniatum potius quam impressum." But as BAROCIUS had access to several MSS. from which he chose the best readings, his translation, where it differs from the printed Greek, is a better authority; I therefore annex the translation, both it and the original being scarce.

The first mention of the Porisms is in page 58 of HERVAGIUS's edition, and in page 121 of BAROCIUS, viz. in the Commentary on the first Prop. of the *Elements*, where he professes to give only a brief account of several mathe-

* See SIMSON's *Conics*, 2d edit. præfat. p. vi. and Prop. 19, lib. 5, at the end. See also *Loci Plani*, p. 223.

mathematical terms, such as Lemma, Porism, Case, &c.* The notice of the Porism is as follows:

“Τὸ δὲ πόρισμα λεγέται μὲν καὶ ἐπὶ προ-
 “βλημάτων τινῶν οἴον* τὰ Ἐυκλείδει γεγραμ-
 “μένα, πορίσματα λεγέται δὲ ἰδίως ὅταν ἐκ τῶν
 “ἀποδεικνυμένων ἄλλο τι συναφανῆ θεώρημα
 “μὴ προθεμένων ἡμῶν ὁ καὶ δια τῆς πόρισμα
 “κεκλήκασι ὡς περ τι κέρδος ὅν τῆς ἐπιση-
 “μοικῆς ἀποδείξεως πάρεργον.”

* ὁ α.

This very short description is unsatisfactory, but a subsequent passage (PROCLUS, HERVAGII, p. 80. BAROC. p. 173.) is more particular and intelligible:

“Ἐν τῶν γεωμετρικῶν ἐστὶν ὀνομάτων το
 “πόρισμα. τῆτο δὲ σημαίνει διπλὸν. καλεῖσι
 “γὰρ πορίσματα καὶ ὅσα θεωρήματα συνα-
 “λασκεινάζονται ταῖς ἄλλων ἀποδείξεσιν οἷον
 “ἔργματα καὶ κέρδη τῶν ζητήτων ὑπάρχοντα,
 “καὶ ὅσα ζητεῖται μὲν εὐρέσεως δὲ χερίζει
 “καὶ οὐτε γενέσεως μόνης οὐτε θεωρίας ἀπλῆς.
 “ὅτι μὲν γὰρ τῶν ἰσοσκελῶν αἱ πρὸς τῆ βασι
 “ἴσαι, θεωρησάσαι οἶσι, καὶ ὄντων τῶν παραγμάτων
 “ἐστὶν ἡ τοιαύτη γνάσις. τὴν δὲ γωνίαν δίχα

“Corollarium vero, dicitur quidem
 “et de quibusdam problematibus ut
 “corollaria, quæ EUCLIDI ascripta
 “sunt. Dicitur autem proprie corol-
 “larium, cum ex iis quæ demonstrata
 “sunt quoddam aliud theorema ap-
 “paruerit, nobis minime proponen-
 “tibus, quod etiam propterea corol-
 “larium vocarunt, tanquam lucrum
 “quoddam, quod sit præter gignentis
 “scientiam demonstrationis propo-
 “situm.”

“Unum quid geometricorum no-
 “minum corollarium est; hoc autem
 “duplex quidpiam significat; vocant
 “enim corollaria quæcunque etiam
 “theoremata, una cum aliorum de-
 “monstrationibus probantur, veluti
 “lucra inexpectata atque emolu-
 “menta quærentium existentia: Et
 “quæcunque quærentur quidem, in-
 “ventionem autem indigent, et neque
 “generationis solæ causa quærentur
 “neque simplicis contemplationis;
 “nam quod quidem ‘Æquicrurium
 “qui ad basim sunt anguli æquales
 “sunt,’ contemplari oportet, existen-
 “tiumque rerum hujuscemodi cog-

* “Age de iis etiam quæ his annexa sunt breviter differamus, quid Sumptio (λήμμα)
 “quid Casus, quid Corollarium (πόρισμα), quid Instantia, quid Inductio.”——BAROC.
 Versio, p. 120.

“ τεμῆν ἢ τρίγωνον συστήσασθαι, ἢ ἀφελῆν ἢ
 “ θέσθαι, ταῦτα πάντα ποίησιν τίνος ἀποδείξει.
 “ τε δοθέντος κύκλου τὸ κέντρον εὑρεῖν ἢ δυο
 “ δοθέντων συμμετρῶν μεγεθῶν τὸ μέγιστον καὶ
 “ κοινὸν μέτρον εὑρεῖν ἢ ὅσα τοιαῦτα μέλαξὺ
 “ πως ἐστὶ προβλημάτων καὶ θεωρημάτων. οὕτε
 “ γὰρ γεόμετροι εἰσὶν ἐν τήτοις τῶν ζητούμενων
 “ ἀλλ’ εὐρέσεις, οὕτε θεωρία ψιλῆ. δεῖ γὰρ
 “ εἶναι ὅψιν ἀγαγεῖν, καὶ πρὸ ὀμμάτων ποιή-
 “ σασθαι τὸ ζητούμενον. τοιαῦτα ἄρα ἐστὶ καὶ
 “ ὅσα πορίσματα Ἐυκλείδους γέγραφε, βιβλία
 “ προβλημάτων* συνάξας. ἀλλὰ περὶ μὲν τῶν
 “ τοιούτων πορισμάτων παρέρσω λαγεῖν. τὰ δὲ
 “ ἐν τῇ στοιχειώσει πορίσματα συναναφαινέσθαι
 “ μὲν ταῖς ἄλλων ἀποδείξεσιν, ἀντὶ δὲ προση-
 “ γεμένης τυγχάνει ζητήσεως. οἷον καὶ τὸ νῦν
 “ προκείμενον.” &c.

“ nitio est. ‘ Angulum autem bifa-
 “ riam fecare,’ vel ‘ triangulum con-
 “ stituere,’ vel ‘ rectam lineam æqua-
 “ lem abscindere, vel ponere,’ hæc
 “ omnia ut aliquid fiat postulant.
 “ ‘Dati vero circuli centrum reperire,’
 “ vel ‘ duabus magnitudinibus com-
 “ mensurabilibus datis, maximam ip-
 “ sarum communem mensuram in-
 “ venire,’ vel quæcunque id genus
 “ alia, quodammodo inter proble-
 “ mata atque theoremata sunt; neque
 “ enim quæstorum ortus in his,
 “ neque sola contemplatio sed inventio
 “ est. Opus est siquidem quæsitum
 “ in conspectu et præ oculis ponere.
 “ Talia igitur sunt quæcunque etiam
 “ corollaria EUCLIDES scripsit, quippe
 “ qui libros corollariorum construxit.
 “ Verum de hujuscemodi quidem co-
 “ rollariis dicere prætermittatur. Quæ
 “ autem in elementari institutione sunt
 “ corollaria, simul quidem cum alio-
 “ rum demonstrationibus apparent,
 “ ipsa vero non præcipue quærentur,
 “ veluti id quod in præsentia propo-
 “ nitur;” viz. Cor. 15. 1 Elem.

† From the sense, this word ought to be πορίσματα, and not προβλήματα; for whatever be the origin of that class of Propositions, there can be no doubt of their being always known and distinguished by the term πορίσματα. BAROCIUS accordingly translates it in this place by “ corollariorum;” most probably from his having found that reading in a MS. which he relied upon. In the MS. of PROCLUS, in the Bodleian Library, which is esteemed valuable, προβλήματα is in the text, but on the margin πορίσματα is written in a character much resembling the text. In this sentence γ probably is omitted after βιβλία.

In this extract is a very explicit statement of the two very different species of *Porisms*, (or *Corollaries*, as they are named by BAROCIUS,) viz. the *Porisms* composed by EUCLID, a curious and difficult class of Propositions, requiring investigation, as well as construction and demonstration; and the *Porisms* or *corollaries* of EUCLID's Elements, which result from the demonstration of other Propositions, and which often present themselves unexpectedly. PROCLUS proceeds to illustrate this latter class of *Porisms* by the example of a corollary annexed to 15 Prop. 1 Elem. the preceding extract being part of his commentary on that Proposition. He then gives a more diffuse description of this corollary of the elements, of which a portion is annexed.

(HERVAGII. p. 80. BAROC. p. 174.)

“Ἔστιν ἐν τῷ πόρισμα θεώρημα διὰ τῶ ἄλλῃ
 “ προβλήματος ἢ θεωρήματος ἀποδείξεως,
 “ ἀπεργαματεύως ἀναφαιρόμενον. οἷον γὰρ
 “ κατὰ τύχην περιπίπτειν εἰκόταμεν τοῖς πο-
 “ ρίσμασιν, ἢ γὰρ προβεμένοις, ἢ δὲ ζητήσασιν
 “ ἀπαλήθῃ, ὅθεν αὐτὰ καὶ τοῖς, ἐξμάσιας
 “ εἰκόταμεν, καὶ ἴσως οἱ δεινοὶ τὰ μαθημα-
 “ τικὰ, κατὰ τύχην αὐτοῖς ἔθεντο τὴν ἑπιτομήσαν,
 “ ἐνδοκίμοι τοῖς πολλοῖς καὶ ἐπὶ τὸ φαί-
 “ νόμνον κίεθος ἐπισημένους* ὅτι ἄρα τὰ
 “ ἀληθῆ θεῶ δῶρα καὶ τὰ ἐξμασια τὰῦτα ἐστὶν
 “ ἕχ οἷα ἐκείνοις δοκῆ.”

* Forſan ἠδοκίμοις.

“ Corollarium est theorema quod
 “ ex alius problematis vel theorematis
 “ demonstratione ex improvifo emer-
 “ git: nam veluti casu quodam in
 “ corollaria incidere videmur, nec
 “ proponentibus enim nobis, neque
 “ etiam quærentibus obviam sese
 “ offerunt. Unde hæc quoque lucris
 “ assimilavimus: et fortasse mathe-
 “ maticarum rerum periti hoc ipsis
 “ imposuere nomen, ostendentes vul-
 “ go, quippe quod apparenti gaudet
 “ lucro, quod utique vera DEI mu-
 “ nera, veraque lucra hæc sunt, non
 “ autem quæ illi videntur,” &c.

Then follow several distinctions of corollaries, into arithmetical and geometrical, into those arising from theorems and those from problems, with some others of no particular importance. These extracts, it must be allowed, are not expressed with uniform clearness, but they discriminate sufficiently the *Porisms* of EUCLID from the *corollaries* of the Elements; and they correspond with the more general expressions of PAPPUS on this subject, which PROCLUS in these passages plainly had in view. For illustrating the

the distinction, he premises that the 5th Prop. 1 Elem. is a theorem, and requires demonstration: also, that the 1st, 3d, and 9th Propositions are problems, each requiring something to be done or constructed; he then adds, that to find the centre of a given circle, (1. 3. Elem.) or to find the greatest common measure of two magnitudes, (2. 7. Elem.) or such like things, are in some sort of an intermediate character between problems and theorems. For the construction of these things sought is not given in the enunciation, but invention is requisite for finding that construction, and also for discovering the demonstration; for it is necessary to exhibit to the eye the construction of the things sought. Such, says he, is the nature of the second kind of Porisms, of which EUCLID composed books.* Of these observations, not altogether clear, I have given, in a sort of paraphrase, the meaning as I understand it to be. It is plain he was acquainted with the observation of PAPPUS, that Porisms are of a middle nature between problems and theorems, though some doubt may be entertained, whether the examples which he quotes be properly stated as Porisms, as was observed in a former note p. 47. It may be remarked here, that PROCLUS, in referring to the 2d. 7th Elem. as a Porism, supposes that there may be arithmetical Porisms as well as geometrical, and this Proposition is altogether unconnected with *Loci*, or even with geometrical position of any kind.

I may observe further in this place, that *Loci*, which by PAPPUS are reckoned a class of Porisms, have also somewhat of the intermediate character between problems and theorems; though they are generally reckoned to belong to the latter class. In the *Locus* the construction must be investigated, as in the *Porism*; and every *Locus* is easily convertible into a Porism.

The etymology of the Porism, or corollary of the Elements, from *πρόρισμα* signifying gain, may be right; and the other meaning of the Greek term *πρόρισμα*, implying investigation, may be the ground of its being applied to the Porisms of EUCLID. In the first sense it is the common corollary, which is an acquisition (or gain) from another Proposition, from the demonstration of which it results often unexpectedly. † *Πρόρισμα* also, from the other signification of the word, properly denotes

* See Note, p. 90.

† Corollarium, assumed by BAROCIUS as the proper translation of *πρόρισμα*, may express the first meaning of it, but has no connection with the other.

any thing to be investigated, which corresponds with the character of the Porisms of EUCLID.* And thus, without any connection between these two classes of Propositions, they may incidentally, from the two unconnected meanings of a Greek word, have obtained the same name.

It is necessary, however, to remark, that the first extract from PROCLUS, (p. 58, HERVAG.) containing the very short description of Porisms, does not, as it stands, accurately correspond with the detailed and more intelligible description in the second. If the word *προβλήματων* in the former passage be the true reading, it might thence certainly be urged, that Porisms have a reference to problems; and an argument might be derived in favour of the opinion, that the ancient notion of Porisms was similar to that proposed by Mr. PLAYFAIR. But this preliminary description of Porisms by PROCLUS is surely too short and general to be the foundation of any such inference, without other more precise authorities; and the subsequent passage, which is much more particular and intelligible, contains no such reference to problems. Besides, it is well known, that the only printed edition of PROCLUS is very erroneous.† If it may be supposed that *προβλήματων* is printed for *προσίματων*,‡ as it most certainly is in another place in the second extract, the two passages will be more consistent; and though the expression in the first be short and inexplicit, it

* Notwithstanding these notices of EUCLID's Porisms, PROCLUS does not speak of their curiosity and importance in the manner that PAPPUS does. And it is somewhat remarkable, that when PROCLUS gives a catalogue of the works of EUCLID, (p. 20, HERVAG. edit.) he makes no mention whatever of his Porisms. Dr. SIMSON, most probably, had considered these passages in his first enquiries on this subject; but as they could not give the smallest aid towards his discovery of the true nature of Porisms, he does not seem afterwards to have attended to the remarks of PROCLUS, as there is no allusion to them among his papers.

† BAROCIUS, in the dedication of his translation of PROCLUS, says, “*quamvis nefcius non esse quod impressi fuerant BASILIE quatuor PROCLI DIADOCHI libri commentariorum in primum Elementorum EUCLIDIS; quos adeo laceros et corruptos vidi, ut nihil boni ex iis elicere potuerim: editi namque erant perinde ac si editi nunquam fuissent.*” Afterwards, having mentioned the different MSS. to which he had access, he adds, “*ubi ex iis omnibus exemplaribus quoad fieri potuit unum integrum feci quod postremo e Græca lingua in Latinam converti, &c.*”——PROCLUS BAROCII, Patav. 1569.

‡ I must observe, that in the translation by BAROCIUS, which, though it includes many corrections of the printed Greek, yet contains many errors, the MS. which he relied on must have had *προβλήματων* in this place, as in the edition by HERVAGIUS.

being only an introductory notice of Porisms, yet it may be understood. I conceive, therefore, that in both extracts his object is the same, viz. to distinguish the Porisms composed by EUCLID from the Porism which is the common corollary; and he does it on the same principle, though more briefly in the first than in the second: and if *πορίσματα* be assumed as the true reading in the first passage, it may be freely translated thus: "The term Porism is applied to that class of Propositions of which are the Porisms composed by EUCLID; but more properly the term Porism is applied to Propositions not proposed by us, but arising from the demonstration of other Propositions; and being thus an unexpected acquisition- they obtained that name."

It may be further observed, that in the Commentary of PROCLUS on the 1st, 1. Elem. there are various distinctions of problems stated; one of which, viz. the *πλεονέξον* and the *ἐλλείπεις* may here be properly mentioned. By the former term (exceeding) is meant any problem which has more conditions or data than are necessary to the solution. As for instance, if it be proposed to describe an equilateral triangle, of which each of the angles shall be two thirds of a right angle. This last condition is implied in the former, and is therefore superfluous. And if an unnecessary condition be added, the probability is that it will be inconsistent with the others, and render the problem so stated impossible. The other term (deficient) expresses the want of a necessary condition, by which the problem becomes, according to modern language, indeterminate.* *Ἐλλείπεις δὲ*

* In PAPPUS and PROCLUS the term *ἀδιορίστος*, commonly translated *indeterminatum*, or indeterminate, is applied to problems which do not require or admit of any determination, a meaning altogether different from the modern use of the word: *ἀορίστος* is indeed used by PROCLUS in the modern sense of indeterminate. The following sentence may be quoted, HERVAGIUS, p. 61, and BAROC. 127, at the close of the commentary on the first proposition of the Elements: "φανερὸν οὖν ἐκ τούτων ὅτι τὰ κυρίως λεγόμενα προβλήματα, βυλεῖται τῇ ἀορίστῳ διαφεύγειν, καὶ μὴ εἶναι τῶν ἀπειραχῶς γιγνομένων. λεγέται δὲ ὅλως κακῆνα προβλήματα διὰ τὴν ἁμυνομίαν τὰ προβλήματα." &c. "Ex his itaque manifestum est, quod ea quæ proprie problemata appellantur, indeterminationem effugere debent, et non esse ex eorum numero quæ infinitis modis fiunt. Problemata tamen et illa dicuntur per problematis æquivocationem, &c." BAROC.

In this passage it is intimated that indeterminate Propositions are also called *problems*, from a double meaning of the word; and among such *problems Loci* must of course have a place, and therefore *Porisms* also; but it is not easy to reconcile the different observations of PROCLUS on this subject.

ἔστι πρόβλημα, το προσθήκης ἄλλης δέομενον ἵνα ἐκ τῆς ἀοριστίας, εἰς ἰσότητα καὶ ἔσονται ἰσχυριστικὸν ἀχθῆν.† HERVAG, edit. p. 61.

If any such notion had existed in the times of PAPPUS or PROCLUS, as the ingenious definition of Mr. PLAYFAIR involves, we might have expected some reference or allusion to it in a discussion like this concerning problems. PROCLUS, in the present state of the text, even with all the skill of BAROCIUS, is not always intelligible, nor consistent in his different observations on the same subject. See his comment on the 1st Prop. EUCL.; also on the 22d Prop. compared with his comment on the corollary to the 15th Prop.

NOTE E. p. 57.

The 117th Prop. of the seventh book of PAPPUS has become remarkable, from the consideration which has been given to it by many distinguished mathematicians of modern times, more than from any particular excellence belonging to it, as it stands in that author. In 1742, an extension of it was proposed by M. CRAMER of Geneva to M. DE CASTILLON, viz. by supposing the three points not to be in a straight line, as is assumed in the Proposition of PAPPUS; and then the problem was thus expressed, “a circle and three
“ points being given in position, to inscribe a triangle in the circle of which
“ the three sides shall respectively pass through the three given points.” It was not however till 1776, that CASTILLON published a solution, in the Berlin Memoirs of that year. The solution is geometrical, and distinguished into many cases. In the same volume is an algebraical solution by LA GRANGE, by means of some trigonometrical formulas. In the *Petersburgh Acts* for 1780, are solutions of the same problem by Messrs. EULER, LEXEL, and FUSS. In the *Memorie di Matematica e Fisica della Societa Italiana*, tom. iv. Verona, 1788, are two memoirs respecting this problem.* The first is by GIOE-

† HUGO DE OMERIQUE, in the fourth book of his *Analysis Geometrica*, seems to have had some of the distinctions of PROCLUS in view. The *Deficiens Problema* of the latter is called *diminutum* by the former, who gives some examples of such problems, in which the solution turns out to be a *Locus*.

* The following solutions and extensions of this problem were pointed out to me by Mr. PLAYFAIR.

DANO DI OTTAJANO, in which is a sketch of the history of the problem; and though his solutions are geometrical, yet he appears to have had but an imperfect notion of the ancient geometrical analysis. He resolves the problem before mentioned, adding a variation of it, viz. that the three lines from the angular points of the inscribed triangle to the three given points may make angles with the sides of the triangle equal to given angles; and concludes with an important extension to the case of a polygon, of any number of sides, which he inscribes in a given circle, so that the sides respectively shall pass through the same number of given points. The other memoir in the same volume is by Signor GIAN. FRANCESCO MALFATTI, and contains a good solution of the very general problem last mentioned; from reducing the case of polygons to that of the triangle, by the use of two very easy theorems. I should have added to the account of the first memoir, that the author acknowledges his inability to resolve the problem algebraically, and expresses a wish that some mathematician would attempt a solution purely analytical. Hence also this problem may be considered as an example of what Dr. SIMSON observes, that many geometrical problems are resolved more easily by the ancient analysis than by the modern.

In the Berlin Memoirs for 1796, is a memoir by LHUILIER, containing an algebraical solution of the most general case of the polygon. He premises two trigonometrical theorems, one of which is the basis of LA GRANGE's solution of the case of the triangle, and by means of them proceeds successively from the simple to the more complex cases of polygons; and in all the cases deduces a quadratic equation, from the solution of which the problem is resolved. The expressions of course become very complex; and a geometrical construction derived from them would be not less so. LHUILIER extends the problem also to Conic Sections, and adds a similar one respecting the sphere.

I shall subjoin Dr. SIMSON's solution of the case, when the three points are not in a straight line; dated August 30, 1731, many years before this problem acquired celebrity from having employed the skill of so many eminent mathematicians. The Doctor wrote all his mathematical notes in Latin, and I give this Proposition in his own words.*

† In this and some other Propositions of Dr. SIMSON's copied into these notes, the Data are quoted as numbered in the old edition. I may remark also, that there are some differences in the phraseology of Dr. SIMSON's early writings from what he used in more advanced life, but they are not important.

(“ Est Prop. 117, lib. vii. generalior facta.”) Fig. 4.

“ Datis tribus punctis A, B, C, et circulo DEF positione dato,
 “ a duobis ex punctis A, B, inflectere ad circumferentiam AE, EB
 “ occurrentes circulo in D, G, quæ faciant DGC, rectam lineam.”

“ Factum puta ; et ducatur DF parallela ipsi AB junctæ, occurrens circulo
 “ in F, et juncta FG occurrat AB in H. Quoniam igitur propter parallelas
 “ est angulus BHF æqualis angulo DFG, hoc est, propter circumulum, angulo
 “ DEG, erunt triangula AEB, BHG æquiangula, et igitur rectangulum ABH
 “ æquale erit dato rectangulo (92 Dat.) EBG, et datur AB, quare et BH et
 “ punctum H, et igitur juncta HC positione dabitur ; occurrat hæc ipsi DF
 “ in K, et erit angulus DKH æqualis dato angulo BHK ; quare eo deventum
 “ est, ut a duobus punctis datis C, H, inflectantur ad circumferentiam CG GH,
 “ ita ut DF datum faciat angulum DKH cum ipsa HC ; puta factum, et
 “ ducatur DL parallela ipsi CH, et juncta LF occurrat CH in M, ergo dabitur
 “ punctum M, ut prius ostensum de puncto H ; [sc. quoniam angulus CMF
 “ æqualis est angulo FLD, hoc est angulo FGC, igitur æquales sunt anguli
 “ HMF HGC, et triangula HMF, HGC, similia, quare et CHM rectangulum
 “ dato FHG est æquale, et datur GH, ergo et MH et punctum M,] et propter
 “ datum angulum DKH dabitur angulus KDL, et igitur recta LF magnitudine
 “ dabitur [88 Dat. [quoniam igitur a dato puncto M ad circumulum positione
 “ datum ducta est MFL, faciens FL quæ circulo interceptur datam, dabitur
 “ ML positione, et igitur LD et AD, et BE. Q. E. I.

“ Compositio.

“ Fiat rectangulum ABH æquale rectangulo contento segmentis cujusvis
 “ rectæ quæ a puncto B ad circumulum producitur, et jungatur HC, fiat vero
 “ rectangulum CHM æquale rectangulo contento segmentis cujusvis rectæ
 “ quæ a puncto H ad circumulum producitur, et a puncto M ducatur MFL quæ
 “ abscindat segmentum LEDF capiens angulum angulo CHB æqualem, junctaque
 “ HF occurrat circumferentiæ in G, juncta vero BG eidem occurrat in E, et
 “ juncta AE eidem occurrat in D. Erunt puncta D, G, C, in recta linea. Jungatur
 “ DF, et connectentur DL, DG, CG: et quoniam rectangulum ABH æquale
 “ est ex constructione, rectangulo EBG, erit angulus BHG æqualis angulo
 “ E hoc est angulo DFG, igitur parallelæ sunt AB, DF, ergo angulus DKH

“ æqualis est ipsi BHK, hoc est ex constructione angulo LDF quare parallelæ
 “ sunt DL HK; et propter rectangulum CHM æquale rectangulo FHG in
 “ circulo sunt puncta C, M, G, F; ergo angulus CGF æqualis est [angulo
 “ CMF, hoc est] ipsi DLF, et propter circulum anguli DLF et DGF
 “ simul æquales sunt duobus rectis, ergo anguli CGF, DGF, simul æquales sunt
 “ duobus rectis, et igitur recta est linea DGC. Q. E. D.”—“ Transcriptum
 “ Aug. 30, 1731.”

It may also be mentioned that Dr. SIMSON at the same time gave two variations of the problem, viz. the same things being given to make the line DG parallel to a given straight line; and also the same things being supposed, to inflect AEB, so that the line DG may make an angle with the line drawn from D to the third given point, equal to a given angle.

NOTE F. p. 58.

The case of inclinations respecting the Rhombus has always been considered as an elegant geometrical Problem, and it attracted Dr. SIMSON's attention in his early study of PAPPUS. His amendment of the figure of a Lemma in COMMANDINE's PAPPUS, (Prop. 70. lib. vii. PAPPUS,) which no doubt was used by APOLLONIUS in the solution of this problem, and the Doctor's corresponding solution of it, are dated Feb. 20, 1723. The following transcript was taken from a later copy without date, but is substantially the same with the original paper, with some slight amendments.

It is proper, however, to mention, that the Problem of the trisection of an arch of a circle is treated of in the iv. b. of PAPPUS; and in Prop. 31st it is proposed to be done by an *inclination* which is a solid Problem, there resolved by an hyperbola. The case of *inclinations* employed is that of a rectangular parallelogram, though the solution be the same in any other parallelogram. Of course, a Rhombus being a parallelogram, it is also comprehended in the general solution. But this case of the Rhombus is a Problem in the treatise by APOLLONIUS *De Inclinationibus*, and is plane. Dr. SIMSON, however, in his notes on this Prop. of PAPPUS shews, that when the Hyperbola is employed in the case of the Rhombus, the requisite circle is described about the vertex of the Hyperbola, and then the intersections of the Hyperbola

and circle may be determined by plane geometry, and without the description of the Hyperbola. Thus the Problem of the Rhombus, though a case of a solid Problem, as often happens, becomes plane; and is resolved in this manner by Dr. SIMSON. But the following solution of the Doctor's is better, being no doubt the APOLLONIAN, as the lemma, Prop. 70. vii. PAPPI, is employed.

“*Prop. de Rhombo cui inservit Prop: 70. lib. vii. PAPPI.*”

[“*Schemate hujus sc. Prop. 70, emendato ut in Fig 6.*”]

“Rhombus existente, ABCD, et producta AC ad E, facere EF datam, et quæ ad punctum B vergat. Fig. 5.”

“Factum puta et producta BC ad G, angulo FCG æqualis fiat BFG, et GE jungatur. Ergo angulus GFE, æqualis est ipsi [FCB hoc est, ipsi BCA seu] GCE. Sunt igitur F, C, G, E, in circulo; quare angulus FGE, æqualis est [ipsi FCE vel ei quæ ipsi deinceps est, hoc est] angulo CDB: et in triangulis FGE, BDC, ostensus est GFE æqualis angulo DCB æquiangula propterea sunt triangula. Ergo ut BC ad BD, ita EF ad EG, et data ex hypothesi est FE, quare data est EG, five ipsi æqualis FG. Est autem propter æquiangula triangula BGF, FGC, BG ad GF ut GF ad GC; igitur rectangulum BGC æquale est quadrato ex GF: datum proinde est BGC rectangulum, et data est BG ergo [84 Dat.] dabitur CG, et punctum G propterea datum erit; et data est GF magnitudine, et CD positione ergo (31 Dat.) positione data est GF et punctum F, sed et punctum B, ergo recta BFE positione data est.

“Componetur ita: Sit BH data recta, et ut BC ad BD, ita fiat BH ad quartam BK: ad datam vero rectam BC applicetur rectangulum BGC æquale quadrato ex BK, excedens quadrato; factaque GL ipsi BK æquali, centro G intervallo GL describatur circulus qui rectis AC, CD occurrat in E, F, punctis: Erit juncta EF æqualis datæ BH vergetque ad punctum B.

“Jungantur enim GE et GF; et quoniam GL media est proportionalis inter BG, GC, erunt puncta E, F, B, in recta linea per Prop. 70. lib. vii. PAPPI. Et quoniam est BG ad GL seu GF ut GF ad GC, æquiangula erunt triangula BGF, FGC; quare angulus BFG æqualis est ipsi FCG, et angulus

“ GFE ipsi [FCB hoc est, ipsi BCA seu] GCE. Ergo in circulo sunt puncta F,
 “ C, G, E, angulusque FGE æqualis erit [ipsi FCE vel ei qui ipsi deinceps est
 “ hoc est] angulo CDB: æquiaugula igitur sunt CBD, EFG triangula, et ut
 “ CB ad BD, hoc est ut BH ad BK, ita erit EF ad EG vel FG; æqualis autem
 “ est BK ipsi FG seu GL, ergo æqualis est BH ipsi EF. Q. E. D.

“ Potuisset compositio æque ac analysis facta fuisse sine PAPPI Lem-
 “ mate, sc. jungendo BF et producendo eam ad AC, in E; ita enim non opus
 “ fuisset Lemmate ad ostendendum puncta E, F, B, in recta linea esse. Hæc
 “ autem compositio stricte loquendo respondisset problemati quo requiritur a
 “ puncto B ducere rectam BFE, et facere EF datam; et non huic quo re-
 “ quiritur producere AC ad E, et facere EF datam quæ ad punctum B vergit;
 “ hoc est in angulo exteriori Rhombi ECF, seu interiore ACF, aptare rectam
 “ EF magnitudine datam quæ vergat ad punctum B; et ut compositio huic
 “ enunciationi ad amissim respondeat videtur Lemma PAPPI utile, et non
 “ aliam ob causam. R. S.”

“ In casu anguli interioris Rhombi, addatur determinatio ope, Prop. 73, 74,
 “ lib. vii. PAPPI, et fiant figuræ respondententes pro casu, [Prop. 70. PAPPI,
 “ et inde in problemate] quando ponenda EF in angulo interiore ACD, et
 “ etiam per B punctum; [vel ad B vergens.]

“ Lemma in linea ultima, fol. 205, a. PAPPI, (COMMAND. 1589) assump-
 “ tum ab eo, est ut sequitur. Vid. fig. Prop. 70. ‘ Si a puncto C in diametro
 “ DF ducantur ad circumferentiam rectæ CL CK, quæ faciant cum diametro
 “ æquales angulos LCF, KCF, erunt ductæ inter se æquales;’ cujus demon-
 “ stratio facilis est. Vide fig. 6. [Schema emendatum, Prop. 70. lib. vii. PAPPI.]”

NOTE G. p. 62.

As Dr. SIMSON had at one time intended to compose a treatise on *Loci ad superficiem*, it may be satisfactory to some readers to see his preface, in which he expresses this intention; and this, with some excerpts from the remaining fragments on this subject, may give a general notion of the Doctor's views of this subject.

“ De *Loci ad superficiem*, quæ ab EUCLIDE aliisque geometris antiquis
 “ tradita fuere, injuria temporis deperdita sunt, exceptis paucis iis quæ apud

“PAPPUM habentur. Recentiores* autem nihil de *Locis* hisce cognoscere videntur, exceptis *Locis* quæ sunt ad superficiem rotatione alicujus ex sectionibus conicis circa diametros vel alias rectas genitam, cum tamen magnus sit numerus aliorum qui hoc modo minime formari possunt. Operæ igitur pretium facturum ab eruditis videri spero; si viam qua *Locis* hi investigari possunt exemplis quibusdam patefaciam, quibus rite intellectis, non difficile erit ei qui doctrina *Locorum solidorum* probe instructus fuerit, *Locos* ad quamvis superficiem, quæ quovis modo ex sectionibus conicis vario situ dispositis oriri potest, investigare et describere.

“Hi autem omnes [phrasi algebraica] sequente æquatione continentur, viz.

$$“xx + \frac{a}{b}xy + ax + by + \frac{a}{c}yy + \frac{a}{d}yz + \frac{a}{e}xz + gz + \frac{b}{k}zz = \text{Dato.}”$$

“Signis utcunque mutatis.”

The first example, as was observed in the Memoir, is unfinished. The Doctor had proceeded so far as to ascertain the common sections of the superficies required with several planes; some others were proposed, but not investigated, and there is no trace of his having resumed the inquiry. The portion of this example which is written becomes not a little complex, requiring also subsidiary Propositions; and therefore the enunciation only of the *Locus* is here stated; which may be referred to the diagram of the subsequent Proposition. [Fig. 8.]

“Data sint in plano quovis, puta in plano chartæ in quo schema est, duo puncta A, B, et juncta BA, in ea sumatur quodvis punctum D, et ipsi AD ad rectos angulos, ducatur in eodem plano recta DE, et a termino ejus E ad planum erigatur ad rectos angulos EF, sitque rectangulum BDA simul cum rectangulo DEF, et quadrato quod fit ex EF æquale spatio dato, puta (simplicitatis gratia) quadrato ex ipsa AB: quæritur superficies in qua omnia puncta F versantur.”

To this no equation is annexed.

* “N. B. scripta fuerunt hæc priusquam librum, D. N. *des Courbes a double Courbure* vidissem, itaque mutanda est hæc introductio.”

The following is a short example of a *Locus ad superficiem*, found among several unfinished sketches of such Propositions which were mentioned in the Memoir. It is given as it stands in the Doctor's own words.

“*Locus ad Superficiem.* $xx + \frac{ay^2}{b} + \frac{az^2}{c} = d^2$ [FIG. 8.]”

“Data positione recta linea AB, punctoque in ipsa A, in dato positione plano ABC; si a puncto D in recta AB ipsi ad rectos angulos ducatur recta DE, et a termino ipsius E, erigatur ad planum ABC perpendicularis recta EF; fueritque quadratum ex AD, simul cum spatio quod ad quadratum ex DE, et eo quod ad quadratum ex EF, datas habent rationes, æquale dato spatio, scil. quadrato ex data recta AB. Tanget punctum F, superficiem positione datam.”

“Sit enim quadratum ex DG, id quod ad quadratum ex DE datam habet rationem, et juncta AG occurrat circulo centro A semidiametro AB descripto in H, idem vero circulus occurrat DE in K, L, punctis. Quoniam igitur quadrata ex AD, DG, hoc est quadratum ex AG, una cum eo quod ad quadratum ex EF datam habet rationem æquale est quadrato ex AB, seu AH, auferatur commune quadratum ex AG, et reliquum rectangulum KGL æquali erit spatio quod ad quadratum ex EF datam habet rationem. Fiat autem ut DG ad DE, ita DK ad DM; quadrata igitur ex hisce rectis proportionalia erunt, et [per. 19. 5.] quoniam quadratum ex DK est ad quadratum ex DM ut quadratum ex DG ad quadratum ex DE, erit in eadem ratione data reliquum rectangulum KGL ad excessum quadratorum ex DM, DE, hoc est, facta DN æquali ipsi DM, ad rectangulum MEN. Datur igitur ratio rectanguli MEN ad rectangulum KGL, hujus autem ratio ad quadratum ex EF datur, quare [per 8. Dat.] datur ratio MEN ad quadratum ex EF. Quoniam vero datur ratio DK ad DM, tangit autem punctum K circumferentiam positione datam, semidiametro, scil. AB descriptam, tanget punctum M ellipsin positione datam, cujus semiaxis major est ipsa AB; occurrat vero EF sphaeroidi oblongæ hac ellipsi descriptæ in O. Et quoniam data ostensa fuit ratio rectanguli MEN, hoc est quadrati ex EO ad quadratum ex EF, dabitur ratio EO ad EF. Si igitur altitudo EO omnium punctorum O hujus sphaeroidis supra planum ABC diminuatur, vel augeatur, in ratione

“ data EO ad EF superficies solidi, diminutione hac vel additione producti erit
 “ Locus punctorum F. Q. E. I.”

Another short Propofition exhibits a method of describing the Cassinian curve by means of *Loci ad superficiem*.

PROB. [Fig. 9.]

“ Describere curvam Cassinianam.”

“ Sint A, B, Foci, C centrum, et D junctum in curva; igitur junctis AD
 “ BD, erit rectangulum ADB æquale dato spatio.”

“ Plano in quo est curva ducatur ad rectos angulos recta DE ipsi DA
 “ æqualis, et tanget punctum E conum rectangulum cujus vertex est punctum
 “ A, axis vero plano ABD ad rectos angulos; et quoniam rectus est angulus
 “ EDB, et rectangulum EDB æquale dato spatio, tanget idem punctum E
 “ hyperbolam æquilateram cujus una assymptotos est DB, altera vero recta
 “ BF quæ ad planum ADB est perpendicularis. Revolvatur igitur hyperbola
 “ hæc circa BF tanquam axem, et describetur solidum hyperbolicum acutum;
 “ communis vero sectio superficiem hujus cum superficie cono prædicti, erit linea a
 “ cujus punctis si ducantur perpendiculares ad planum ADB, ipsi occurrent
 “ in punctis quæ sunt in curva Cassiniana, ut patet.”

Another example may be the 28 Prop. lib. iv. PAPPI, as corrected by Dr. SIMSON.

PROB. V. PROP. 28. Lib. iv. PAPPI. [Fig. 7.]

“ Hic igitur lineæ ortus magis mechanicus est, ut dictum
 “ fuit, geometrice vero per Locos qui ad superficies “ dicuntur”
 “ resolvi potest hoc modo.”

“ Sit circuli quadrans positione datus ABC, et ducatur, ut contingit, recta
 “ linea BD, et ad BC perpendicularis EF quæ ad circumferentiam DC pro-
 “ portionem datam habeat. Dico punctum E ad lineam esse.

* Deest hoc verbum in MS. BULL.

“Intelligatur enim a circumferentia ADC recti cylindri superficies, et
 “in ipsa linea spiralis [CGH] descripta, [incipiens a puncto C, ita ut
 “velocitas puncti describentis in recta CM quæ perpendicularis est ad
 “planum ACB, fit ad velocitatem ipsius CM in quadrante circuli CDA in
 “data ratione quam habet EF ad circumferentiam DC, quæ propterea
 “spiralis] positione data erit. Sitque HD latus cylindri, et ad planum
 “circuli perpendiculares ducantur EI, BL, et per H ipsi BD parallela
 “ducatur HL, [ipsi EI occurrens in I.] Itaque quoniam proportio rectæ lineæ
 “EF ad circumferentiam DC eadem est quæ proportio EI, hoc est DH
 “ad eandem DC propter spiralem; erit recta EF ipsi EI æqualis. Suntque
 “FE, EI, positione, ergo et juncta FI positione erit, [N. B. Non dicit FE, EI,
 “positione datas, datum non est punctum E, sed intelligit tantum angulum
 “EFB datum esse, et EI perpendicularem esse ad planum circuli, unde
 “quoniam rectus est angulus EFB, erit:] et FI ad BC perpendicularis. Est
 “igitur FI in plano [sc. quod per BC transit, et per dimidium anguli recti
 “inclinatum est ad planum subjectum ABC, angulus enim EFI dimidium
 “est recti] quare et punctum I est in eodem; atque idem punctum est in
 “superficie, fertur enim HL et per lineam spiralem CGH, et per rectam
 “lineam BL, et ipsam positione datam, semper existens [sc. ipsa HL] parallela
 “subjecto plano; ad lineam igitur est punctum I, [sc. ad communem sectionem
 “superficiæ a recta HL descriptæ, cum plano prædicto quod per BC transit]
 “ergo est punctum E ad lineam [quæ sc. formatur ducendo perpendiculares
 “a linea in qua versatur punctum I, ad subjectum planum.] Hoc quidem
 “univerſe resolutum est, [sc. quæcunque fuerit data ratio EF ad DC.] Si
 “autem hæc ratio æqualis fuerit rationi quam habet BA ad circumferentiam
 “quadrantis ADC, prædicta linea [in qua est punctum E] quadratrix efficitur.”

The 29th Prop. next following is, “Potest etiam illud per lineam spiralem
 in plano descriptam resolvere simili ratione.”—It is unnecessary in this place
 to give the investigation, but I shall remark some of the most material
 corrections of it by Dr. SIMSON.

In line 6. fol. 59. a. COMMAND. “in superficie cylindrica, in qua est linea
 “spiralis.”—Dr. SIMSON supposes from Note F on this Proposition, that in
 the MS. used by COMMANDINE the reading was *ἐν κυλινδροειδὲι ἄρα ἐπιφανείᾳ*,
 which is more correct, and is also confirmed by the MS. BULL. It should
 therefore have been translated “in superficie cylindroide.”—Also, instead

of "in qua est linea spirali," it should be "quæ formata est a linea spirali:" for in MS. BULL it is "τῆ ἀπὸ τῆς ἐλίκου," which is plainly the true reading.

Lin. 8. "ad lineam est ipsum K," add, "sc. ad eam lineam quæ est communis sectio superficiei cylindroides et superficiei conicæ." Lin. 11. for "ergo K et I sunt in superficie," read, "ergo est punctum I in superficie, sc. in ea quam vocavit πλῆκτοειδου." Lin. 13. for, "ad rectam igitur lineam est punctum I;" read, "ad lineam igitur est punctum I,* " sc. eam quæ communis est sectio superficiei πλῆκτοειδου† cum plano quod est super rectam BC, " et ad subjectum planum est inclinatum per dimidium recti."

NOTE H. p. 65.

For the satisfaction of some readers, I shall insert Dr. SIMSON's Paper on Series, enclosed in his letter to Dr. JURIN, of Feb. 1, 1723, (for which see note p. 65.) The two Preliminary propositions are very easy, and are now to be found in all the compleat treatises of trigonometry; but as they are short, they are inserted as they stood in Dr. SIMSON's original paper.

Dr. SIMSON about a month after received Dr. JURIN's answer, (of March 5, 1723,) communicating Mr. MACHIN's serieses; and to his paper he has added a note, stating the correspondence of five of his own serieses with those of Mr. MACHIN's. Mr. MACHIN communicated seven; and two of them, which had not before occurred to Dr. SIMSON, were readily investigated by him.

"PROP I. Fig. 11. Sint in circulo cujus centrum est C duo quilibet arcus AB, AD, qui simul sumpti sunt quadrante BL minores; erit excessus quadrati quod fit a semidiametro supra rectangulum sub tangentibus arcuum AB, AD ad quadratum a semidiametro, ut tangentes arcuum AB, AD simul sumpti, ad tangentem summæ arcuum.

"Sint enim AE, AF tangentes arcuum AB, AD, et fit BG, tangens summæ ipsarum BD, et juncta CL, occurrat ipsi AF in H, ducatur vero FK parallela ipsi AC, et occurrans EC in K, et jungatur KH. Igitur quoniam anguli HCK, HFK recti sunt, erunt puncta H, F, C, K, in circulo; ergo angulus FHK equalis est ipsi FCE, et proinde similia sunt triangula rectangula

† For some observations on πλῆκτοειδου, see Appen. II.

* This is confirmed by MS. BULL, which has πρὸς γεαμμῆς ἀρα τὸ ι.

“ HFK, CBG. Est autem ratio EF ad BG composita ex rationibus EF ad FK
 “ et FK ad BG; *i. e.* ex rationibus EA ad AC, et FH ad CB seu AC; ergo EF
 “ est ad BG ut rectangulum EA in FH, ad quadratum ex AC; sed est EA in FH
 “ excessus rectanguli EAH, *i. e.* quadrati ex AC, supra rectangulum contentum
 “ tangentibus EA, AF, et est hic excessus ad quadratum ex AC ut EF ad BG.
 “ 2. E. D.

“ COR. 1. Hinc si arcus AB, AD, æquales fuerint, erit excessus quadrati a
 “ femidiametro supra quadratum tangentis arcus cujusvis octante circumferentiæ
 “ minoris ad quadratum a femidiametro, ut tangens duplicatus ad tangentem
 “ arcus duplicati.

“ COR. 2. Hinc etiam inveniri potest tangens arcus qui multiplex est
 “ arcus alterius, cujus tangens datur. Si a tangens arcus A, et erit tangens
 “ arcus $n \times A$.

$$\frac{na - \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} a^3 + \frac{n \cdot n - 1 \cdot n - 2 \cdot n - 3 \cdot n - 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} a^5 \&c.}{1 - \frac{n \cdot n - 1}{1 \cdot 2} a^2 + \frac{n \cdot n - 1 \cdot n - 2 \cdot n - 3}{1 \cdot 2 \cdot 3 \cdot 4} a^4 \&c.}$$

“ Numeratoris hujus seriei si n fit numerus par, sumendi sunt tot termini
 “ quot sunt unitates in $\frac{1}{2} n$; denominatoris vero tot quot sunt unitates in $\frac{1}{2} n + 1$.
 “ Si vero fuerit n numerus impar, tam numeratoris quam denominatoris
 “ sumendi sunt termini $\frac{n+1}{2}$. Hanc seriem video apud CHRIST. WOLFIUM,
 “ ex finibus derivatum.

“ PROP. 2. fig. 12. Sint duo quilibet arcus AB, AD, quorum major non
 “ excedit quadrantem; erit quadratum a femidiametro una cum rectangulo
 “ sub tangentibus arcuum ad quadratum ex femidiametro, ut differentia tangen-
 “ tium ad tangentem differentię arcuum.

“ Simili prorsus modo quo precedens demonstratur.

“ COR. 1. Hinc si arcus minor AB, æqualis fuerit dimidio quadrantis, erit
 “ radius una cum tangente majoris arcus ad ipsorum differentiam, ut radius ad
 “ tangentem arcus quo AD superat dimidium quadrantis. Si vero arcus major
 “ AD æqualis fuerit dimidio quadrantis; erit radius una cum tangente minoris
 “ arcus, ad ipsorum differentiam, ut radius ad tangentem arcus quo dimidium
 “ quadrantis superat arcum AB, *i. e.* in utroque casu erit summa tangentium ad
 “ ipsorum differentiam ut radius ad tangentem differentię arcuum. Corolla-
 “ rium vero hoc, ni fallor, jamdudum innotuit.

“ Præcedentium ope innumeræ series exhiberi possunt ad longitudinem cir-
 “ cumferentiæ inveniendam. Plurimæ vero, ad usum paratiores, in sequente
 “ serie continentur. Sc. sint tangentes $\frac{1}{a}, \frac{1}{2^n a}$ et fit A arcus cujus tangens est $\frac{1}{a}$
 “ et erit, existente radio = 1,

$$\left. \begin{aligned} & \frac{1}{a} - 1 \times \frac{1}{\beta + 2\gamma + 4\delta + 8\epsilon} \&c. \\ & - \frac{1}{3} \times \frac{2^n}{2^n a^3} + \frac{1}{3} \times \frac{1}{\beta^3 + 2\gamma^3 + 4\delta^3 + 8\epsilon^3} \&c. \\ & + \frac{1}{5} \times \frac{2^n}{2^n a^5} - \frac{1}{5} \times \frac{1}{\beta^5 + 2\gamma^5 + 4\delta^5 + 8\epsilon^5} \&c. \\ & - \frac{1}{7} \&c. \end{aligned} \right\} = \text{Arcui A.}$$

$$\left. \begin{aligned} \text{Est vero } \beta &= \frac{1}{4aa + 3} \times a \\ \gamma &= \frac{1}{16aa + 3} \times 2a \\ \delta &= \frac{1}{64aa + 3} \times 4a \\ \epsilon &= \frac{1}{256aa + 3} \times 8a \\ &\&c. \end{aligned} \right\} \text{Terminorum autem } \beta, \gamma, \delta, \&c. \text{ tot sumendi sunt, quot sunt unitates in numero integro } n.$$

“ Facile deducitur hæc series ex Cor. 1 et 2. Prop. 1. et ex Prop. 2. Sit jam
 “ arcus D datum habens tangentem, datamque rationem ad circumferentiam;
 “ Sc. fit $2rD = \text{circumf.} = c$; fitque mA is arcus qui vel proxime excedit arcum
 “ D, vel ab ipso proxime deficit existente m numero integro; et inveniatur
 “ tangens arcus mA ope Cor. 2. Prop. 1. deinde inveniatur tangens differentiæ
 “ arcuum mA & D ope Cor. 1. Prop. 2. Sit tangens hic t , et erit arcus cujus
 “ est tangens, $t - \frac{1}{3}t^3 + \frac{1}{5}t^5 \&c.$ ut notum est; vocetur hic arcus B, et erit
 “ $m\bar{A} + B = D$, unde $rm\bar{A} + rB = rD = \text{circumferentiæ circuli cujus diameter}$
 “ = I.

Ex. 1. Sit $a=4$, $n=0$, $r=4$, $m=3$, eritque $t=\frac{5}{99}$.

$$\begin{aligned} \text{Et } rmA + rB &= \frac{12}{4} + \frac{20}{99} \\ &= \frac{1}{3} \times \frac{12}{4^1} + \frac{20^1}{99^1} \\ &+ \frac{1}{5} \times \frac{12}{4^5} + \frac{20^5}{99^5} \\ &\&c.* \end{aligned}$$

N. B. Quoniam $\frac{1}{4}$ est tangens arcus A erit tangens arcus m A. i. e. $3A = \frac{47}{52}$ unde tangens arcus $45^\circ - 3A$ erit $\frac{5}{99}$.

Ex. 2. Sit $a=5$, $n=1$, $r=4$, $m=4$; eritque $\beta = \frac{1}{515}$; $t = 239$.

$$\begin{aligned} \text{Et } rmA - rB &= \frac{16}{5} - \frac{16}{515} - \frac{4}{239} \\ &= \frac{1}{3} \times \frac{32}{10^3} - \frac{16}{515^3} - \frac{4}{239^3} \\ &+ \frac{1}{5} \times \frac{32}{10^5} - \frac{16}{515^5} - \frac{4}{239^5} \\ &\&c. \&c. \&c. \end{aligned}$$

Ex. 3. Sit $n=0$ manentibus cæteris.

$$\begin{aligned} \text{Tunc } rmA - rB &= \frac{16}{5} - \frac{4}{239} \\ &= \frac{1}{3} \times \frac{16}{5^3} - \frac{4}{239^3} \\ &+ \frac{1}{5} \times \frac{16}{5^5} - \frac{4}{239^5} \dagger \end{aligned}$$

“ Et manifestum est series has celerius convergere quo major est n , terminos vero ipsarum, propter eandem causam, complexiores evadere, et proinde plus laboris ad ipsos computandos requiri, ut ex duabus ultimo præcedentibus seriebus liquet.

* “ This is Mr. MACHIN’s 4th, in the paper he sent me down in Dr. JURIN’s Letter of the 5th of March 1723.”

† “ Quæ est series optima a doctissimo Dom. MACHIN inventa.” Et est ejus 6^{ta}.

Ex. 4. Sit $a=2, n=0, r=4, m=2$. Eritque $t=\beta=\frac{1}{7}$

$$\begin{aligned} \text{Et } rmA - rB &= \frac{8}{2} - \frac{4}{7} \\ &= \frac{1}{3} \times \frac{8}{2^3} - \frac{4}{7^3} \\ &\quad \times \frac{1}{5} \times \&c. \end{aligned}$$

This is Mr. MACHIN's 2d.

Ex 5. Sit $a=3$. Cæteris manentibus.

$$\begin{aligned} \text{Eritque } rmA + rB &= \frac{8}{3} + \frac{4}{7} \\ &= \frac{1}{3} \times \frac{8}{2^3} + \frac{4}{7^3} \\ &\quad + \frac{1}{5} \times \&c. \end{aligned}$$

This is Mr. MACHIN's 3d.

Ex. 6. Denique, fit $a=2, n=0, r=4, m=1$, eritque $t=\frac{1}{3}$.

$$\text{Et } rA + rB = \frac{4}{2} + \frac{4}{3} - \frac{1}{3} \times \frac{4}{2^3} + \frac{4}{3^3} + \frac{1}{5} \times \frac{4}{2^5} + \frac{4}{3^5} \&c.$$

Quæ omnium est simplicissima. Est 1^{ma} Dom. MACHIN.

Similiter posito $r=6$, derivantur aliæ series, ut si fit

$$a=\sqrt{3}, n=1, m=2. \text{ erit } t = \frac{1}{15\sqrt{3}}.$$

$$\text{Et } rmA - rB = 2\sqrt{3} - \frac{2\sqrt{3}}{5 \times 3}$$

$$- \frac{1}{3} \times \frac{\sqrt{3}}{2 \times 3} - \frac{2\sqrt{3}}{5^3 \times 3^4}$$

$$+ \frac{1}{5} \times \frac{\sqrt{3}}{2^3 \times 3^2} - \frac{2\sqrt{3}}{5^5 \times 3^7}$$

&c.

“Convergit hoc duplo celerius ferie simplici ex tangente $\frac{1}{\sqrt{3}}$ derivata, sed plus quam duplo labore.”*

These few notes were added by Dr. SIMSON to his original paper, after he received Mr. MACHIN's serieses.

Above thirty years after this correspondence, Dr. SIMSON gives some account of it to the late Earl STANHOPE, in a letter dated Jan. 9, 1758, of which the following extract will not be unacceptable.

After acknowledging some ingenious communications of series from Lord STANHOPE, he adds: “In the beginning of the year 1723 I sent up to Dr. JURIN, Secretary to the Royal Society, a method of deriving series for finding the circumference of the circle from the series $1 - \frac{1}{3} + \frac{1}{5}$ &c. which by itself is of no use. I had before this desired him to inform me if he knew if any application had been of this series for finding of others that converged quickly; he wrote to me that he knew of none, and desired me to send up the paper, which I did, and gave seven series as examples of the method. He wrote me in answer, that on communicating it, he found that Mr. MACHIN had in the year 1705 or 1706, given in a paper to the Royal Society upon this affair, which he afterwards took back again; but that he had prevailed on Mr.

* “In the paper wrote and signed by Mr. MACHIN, which Dr. JURIN sent me inclosed in his letter of the 5th of March 1723, there are seven serieses set down for finding the arch of 45° , the

“1st of which is the same with the 6th in this letter; and the

“2d is the same with the 4th of this; and the

“3d is the same with the 5th of this; and the

“4th is the same with the 1st of this; and the

“6th the same that is here the 3d.

“Mr. MACHIN's 5th is for the arch of 45° . as all his others are, when radius is 1.

$$\frac{3}{4} + \frac{1}{20} + \frac{1}{1985} - \frac{1}{3} \times \frac{3}{4^3} + \frac{1}{20^3} + \frac{1}{1985^3} + \frac{1}{5} \times \&c.$$

“His 7th is $\frac{8}{10} - \frac{1}{100} - \frac{11}{5637} + \frac{10893}{41480222056636}$

$$- \frac{1}{3} \times \frac{8}{10^3} - \frac{1}{100^3} - \frac{11^3}{5637^3} + \frac{10893^3}{41480222056636^3} + \&c.$$

“Vide the investigation of the 5th and 7th series (of Mr. MACHIN) in a MS. book.”

“MACHIN to send me a short account of it, which he did in a paper signed by him, which Dr. JURIN inclosed in his letter to me. It contained seven series, two of which were different from those I had sent, the rest were the same. In case your Lordship has not seen these series, I shall set down two of them.”

Dr. SIMSON then states the series of Mr. MACHIN, No. 5, in the preceding note; and also one which he had sent to Dr. JURIN, and was not returned by Mr. MACHIN, viz. Ex. 2d. in the preceding paper. Of this last he observes, “This series is one of mine, and converges very quickly; and because one part of it consists of powers of $\frac{1}{10}$ is raised with little more trouble than Mr.

“MACHIN’s, by the powers of $\frac{1}{5}$ and $\frac{1}{239}$, which was published in Mr. JONES’s *Synopsis*. It gives the circumference to the diameter 1.” He adds, “The series which Mr. EULER gives in page 107, vol. i. of his *Introductio in Analysin Infinitorum*, (Lausan. 1748,) is one of those which I sent up, and which Mr. MACHIN sent down, [being Ex. 6. before mentioned] but it does not converge so quickly as the foregoing two.” &c.

In the remainder of this long letter are some other observations on this and some other matters in the work of EULER referred to. But the insertion of them is unnecessary, since of these methods of squaring the circle by Mr. EULER, there is a full and explanatory detail, with many judicious remarks and improvements by Mr. BARON MASERES, in his great repository of curious mathematical tracts, the *Scriptores Logarithmici*, vol. iii. p. 169, &c.†

† It may be proper to refer the reader to several notices of these serieses of Mr. MACHIN’s, in Baron MASERES’s *Essay on the Negative Sign*; also in his *Scriptores Logarithmici*, vol. iii. p. 157; and in Dr. HUTTON’s *Mensuration*. Dr. HUTTON’s explanation of Mr. MACHIN’s series, published by Mr. JONES, is reprinted in the same volume of *Script. Log.* with another tract also of the Doctor’s on the same subject, p. 207, which had been published in the *Philosophical Transactions*, and contains some curious discussions respecting such serieses, and some important improvements.

NOTE I. p. 69.

In a letter to Earl STANHOPE, of 22d of March 1751, he mentions very particularly the decline of his memory, and his inability to attempt some things recommended to him by his Lordship. “Persons of my age (now past sixty-
 “three) generally lose the ability they had when younger, of a quick and ready
 “imagination; and their memory, (which, in my opinion, is either the imagi-
 “nation of sensations past, or the recalling imaginations we had formerly)
 “manifestly decays; and so far with me, that I have oftentimes difficulty to
 “recall those I had the last hour, or even a few minutes before. And in long
 “investigations, where it is necessary to look back a good way, this inability is
 “most easily observed, especially when most of the steps are not wrote down; for I
 “remember since I could go through a longer series of steps without writing,
 “than I can now well do with the help of it.* This, my Lord, makes me
 “afraid that I shall not be able to engage in the undertaking you are pleased to
 “recommend to me, and which, indeed, would be very agreeable to me; the
 “applying the method of the ancients to the modern inventions, so as they
 “might be demonstrated in such a way as would (to use your Lordship’s just
 “and elegant description of accuracy and strictness) convince an EUCLID, an
 “ARCHIMEDES, or an APOLLONIUS, risen from the grave, to examine
 “them. My scholars, Mr. MOOR, Mr. WILLIAMSON, and particularly
 “Mr. STEWART of Edinburgh, I hope may be able to do something this
 “way; and I shall not fail to recommend it to them, and direct them as far
 “as I can.”——“I hope, if GOD grant me health, I may do something
 “towards restoring some pieces of the ancients; the first six, and the eleventh
 “and twelfth books of EUCLID’s *Elements* are near ready, excepting the
 “figures. Next to this, I have in most forwardness APOLLONIUS *de Sectione*
 “*Determinata*; but PAPPUS, though I have done a good deal towards restoring
 “him, is so large, that it frightens me to meddle with him.”——Towards
 the end of this letter he adds, “And I hope to do EUCLID justice, if it please
 “GOD to give me opportunity to publish the eight books of his *Elements*,
 “which want of security from being pirated does yet delay.”

* I shall mention in this place an observation of Dr. SIMSON on mathematical enquiry, which, though not immediately connected with the subject of this letter, may gratify the

NOTE K. p. 72.

Among the small remains of Dr. SIMSON's mathematical correspondence which have been preserved, are two letters from the late GEORGE LEWIS SCOTT, esq; Commissioner of Excise, with copies of two from Dr. SIMSON in reply. The general object of these letters was mentioned in the memoir, and for the satisfaction of mathematical readers, the portions of them respecting the comparative merits of the ancient and modern analysis are subjoined. Mr. SCOTT's first letter is dated London, 12th April 1764, and after some observations on Dr. SIMSON's edition of the *Data*, which the Doctor had solicited from him, and in which he discovers some prejudice against that book, thinking its use to be rather logical than mathematical, he adds:

“To be more particular, I observed that the 27 first Propositions relating to magnitudes and their ratios merely, might be much more concisely treated, and receive an additional evidence from the algebraic method. I shall give but one instance from Prop. xvii. which is the xith in BARROW, and is by him left in darkness; you have given it light; but would it not be shorter and clearer to state it and its demonstration in symbols?

“The first hypothesis, that $x - a : y :: m : n$ where a , m , and n , are data. The thesis that $x - a : x + y$, in ratione data. The hypothesis may be changed into

$$“x - a : y :: a : b \text{ therefore } x - a + y : x - a :: a + b : a :: a : \frac{a^2}{a + b}.”$$

$$“\text{Therefore by 12. 5th Elem. } x - a + y : x - a :: x + y : x - a + \frac{aa}{a + b}”$$

$$“\text{and } x - a + y : x - a :: a + b : a”$$

$$“\text{Therefore } x - a + \frac{a^2}{a + b} : x + y :: a : a + b. \text{ q. d. e.}”$$

“The second hypothesis is that $x - a : x + y :: a : b$. which may be shewn in like manner.

reader. It is also in a letter to Earl STANHOPE, April 9, 1753. “What your Lordship says of the usefulness, and often the necessity, of using the method of induction, I have frequently had the experience of; and though it be a proof of the weakness of the human mind, it is at the same time a good help to the finding out whether a Proposition be true or not, and most powerfully excites us to search after a strict demonstration.”

“ I have followed your reasoning; mine is essentially the same; but is not
 “ the distinction by symbols of the indeterminate from the determinate or
 “ given quantities or magnitudes concerned in the question of use? and is not
 “ the conciseness of the expression of great use also? Many of the Propositions
 “ of the data might be thereby reduced to intuitive evidence. In this particular
 “ demonstration, it is shewn at once, what magnitude is to be taken from x
 “ (viz. $a - \frac{a^2}{a+b}$) that the remainder may be in a given ratio to $x+y$. I men-
 “ tion these particulars only to introduce a more general observation, which is,
 “ that wherever magnitudes and their ratios are considered, the method of the
 “ ancients seems inferior to that of the moderns, with respect to facility, though
 “ not at all so in point of accuracy. When I look into APOLLONIUS, I
 “ admire the great man; and I may say the same of GREGORY of St. Vincent
 “ and of HUYGENS; but who can help complaining of the tediousness of their
 “ demonstrations? They seem to me in many cases like a man who would
 “ reject the Indian symbols of number, and perform all the arithmetical
 “ operations in words at length. This might be done; but to what good
 “ purpose? When I say to what purpose, I mean geometrical purpose. Other-
 “ wise I much approve of the elements being demonstrated in words at
 “ length, because of their logical use. For in the affairs of life we have no
 “ symbols, but common words; and it is of service to habituate the mind, to
 “ accurate and distinct expressions, and even to formal syllogisms.”

In the same letter after, mentioning the Propositions of the *Data* from
 the 27th to the 62d, as very easy, he adds: “ In these Propositions I see
 “ little or no room for algebra; and indeed, in questions of position, it often
 “ comes in awkwardly enough. DR. WALLIS, one of the great patrons of
 “ this method, owns it. It is therefore a proper enquiry to determine, if
 “ possible, when one method is to be used, and when the other. You seem to
 “ promise something of this kind in the preface to your *Conics*. I wish you
 “ could find resolution to execute what I am persuaded is almost ready in your
 “ manuscripts. THOMAS SIMPSON, in his *Exercises*, says, that sometimes
 “ the algebraical, sometimes the geometric method is preferable. I believe it.
 “ But it seems to me that notwithstanding the praises bestowed by Sir ISAAC
 “ NEWTON and his followers on the method of the ancients, his own *Principia*
 “ are but algebra disguised, as MACHIN used to say. Though at the same

“ time he owned that he thought Sir ISAAC’s taste of demonstration preferable
 “ to HUYGENS’s. I own myself of his opinion; as the only difficulty attending
 “ Sir ISAAC’s arguments, are the steps suppressed for the sake of brevity.”

I shall add an extract from another letter of Mr. SCOTT’s, dated 1st May 1764, as it is connected with the subject of the preceding.

When mentioning Dr. SIMSON’s edition of the *Data*, he adds, “ I think
 “ that such as would form themselves upon the model of the ancients, will
 “ do well to study the *Data*. I even wish that I had been initiated in that way.
 “ But having once begun in the modern analysis, I have not had leisure to
 “ strike into another road, especially, as, to confess the truth, it seems to me
 “ much more tedious and more doubtful with respect to success. I mean
 “ not that the method of the ancients leaves any doubt in the mind; but
 “ that the success of attempting the solution of a difficult problem is more
 “ doubtful in the way of the ancients, than in that of the moderns; although,
 “ when the solution is once found, it be often true that demonstrations formed
 “ according to the ancients are more agreeable, and more satisfactory to
 “ the mind, than those derived from algebra. I say often, for it is far from
 “ being always so. To me a demonstration derived from the consideration
 “ of a multitude of ratios, and their compositions, and expressed in the
 “ manner of the ancients, is so far from being clearer than the algebraical
 “ method, that it seems vastly more difficult and obscure. And I would
 “ appeal to any beginner, and ask him, whether the general algebraic solution
 “ of the problem for determining the *foci* of Lenses, or HUYGENS’s method,
 “ be most clear. I might bring many other instances, where Propositions
 “ can be demonstrated in the same manner as EUCLID has done in his
 “ first four books, which often reduce the truth of a Proposition to almost
 “ intuitive evidence. I admit that it would be in vain to look out for any
 “ assistance from algebra; and yet I think with Dr. BARROW, that most
 “ of the Propositions of the second book are rather clearer when treated
 “ algebraically, than in EUCLID’s method. The perspicuity of the fourth
 “ of that book has often been insisted on. It is very clear, undoubtedly;
 “ but still our friend Dr. MATTHEW STEWART thinks that the original
 “ demonstration was from the preceding Propositions, without a figure,
 “ which is entirely analogous to the algebraic method. The fifth book is
 “ not peculiar to geometry, but is equally applicable to quantity as it is to

“ magnitude; nor does the doctrine of proportion receive any evidence from
 “ its being expounded by lines. The symbols of algebra would, I think, be
 “ of use here. In the sixth book there are many Propositions which may
 “ be treated either geometrically or algebraically, without any preference of
 “ advantage on either side: for wherever an analogy is used, it may as justly
 “ be said to be algebraical, as it can be called geometrical. I may demonstrate
 “ geometrically the 47. El. 1. from the doctrine of similar triangles, and I
 “ may do the same algebraically. The two demonstrations differ only in stile;
 “ and GUISSNEE is mistaken, when he says this theorem cannot be demonstrated
 “ algebraically. I readily admit that the theorems about the congruity and
 “ similitude of triangles cannot be demonstrated by algebra, and that these
 “ are the true geometrical elements; which being established, the other Pro-
 “ positions in the *Elements* may be derived, either by the method of the
 “ ancients, or by that of the moderns, equally well; at least, if the ancient
 “ method has the advantage in easy questions, that of the moderns seems
 “ of almost indispensable necessity in more difficult enquiries. I admit, however,
 “ that many parts of your writings, and of Dr. MATTHEW STEWART's, might
 “ furnish strong objections to what I advance, nor do I pretend to answer
 “ such objections: all I say at present is, that I know a person very well versed
 “ in both ancient and modern geometry, who seems diffident of his own
 “ abilities to comprehend Dr. STEWART's determination of the Sun's distance.
 “ He and I would, I believe, find the same difficulties in Dr. STEWART,
 “ that BULLIALDUS did in ARCHIMEDES. And to conclude the subject,
 “ I must confess that I cannot find that superiority of evidence in the method
 “ of the ancients, that you and many other able geometers, in this island,
 “ contend for. Their great genius and accuracy I do see, and I see sometimes
 “ the precipitation and inaccuracy of modern geometers. But these are the
 “ faults of individuals, and not of the art.”

In the same letter, after some remarks on the data not connected with the
 particular subject of the preceding extract, he adds, “ In your notes you
 “ bring two instances of the use of the data, and think that the construction
 “ of the last problem could not be derived from an algebraical solution.
 “ I find, indeed, that the algebraic solution is more complicated than one
 “ would at first suspect, on reading the problem. Even in very small numbers,
 “ the solution brings you to large ones. Thus, if $x : y :: 6 : 7$. also $y - 1 : z :: 3 : 4$

“ and $xx + zz = 100$. I find $x = \frac{168}{277} + \frac{\sqrt{28224 + 2203812}}{277}$ and

“ $\frac{\sqrt{2232036}}{277} = \frac{1494}{277}$, therefore $x = \frac{168}{277} + \frac{1494}{277} = \frac{1662}{277} = 6$.

“ and $x = -\frac{1326}{277}$, therefore $y = 7$, and $-\frac{1547}{277}$

Also $z = 8$, and $-\frac{2432}{277}$, and both roots answer the

“ question.

“ In the general solution, supposing $x : y :: a : b$

$$y - a : z :: a : c$$

$$xx + z^2 = d^2$$

“ I find $\frac{bc}{a^2}x - c = z$, then taking $a : b :: c : f = \frac{bc}{a}$ and

“ $a : \sqrt{aa + ff} :: \sqrt{aa + ff} : b = \frac{aa + ff}{a}$. Also $b : c :: f : \frac{cf}{b} = p$. I have

“ $xx - 2px = \frac{a}{b}(dd - cc)$ and the construction derived by this method is not

“ so simple as yours. But the equation $xx + zz = dd$ being obviously a locus

“ ad circulum, and the equation $\frac{bc}{aa}x - c = z$, a locus ad rectam, I find an

“ easy construction by combining them. For taking $a : b :: c : f = \frac{bc}{a}$ I have

“ $\frac{f}{a}x - c = z$, which is very easily constructed; and then describing a circle

“ from the origin of the abscissa x with the Radius d , I have the other Locus,

“ the interfection of which with the former determines the value of x . This

“ is not less simple than yours; and you have also taken the advantage of the

“ Locus ad circulum.—Hitherto there appears not to me any advantage in

“ the ancient method. Nor do I see any from the first of the instances given

“ at the end of your *Conics*. It is true your solution is more simple than

“ NEWTON's, but your investigation is not peculiar to ancient geometry. For

“ what should hinder an algebraist from supposing the angle BCD (in your figure)

“ equal to BAC, and then calling $BD = x$, $DC = y$, and $AB = a$, we have by

“ the similar triangles, and by multiplying the extremes and means of the

“ analogy, $ax + x^2 = y^2$, which is a Locus ad hyperbolam æquilateram, very

“ easily constructed. If Sir ISAAC’s investigation be more complex, it is not
 “ because he made use of algebra, but because he did not make a convenient
 “ election of terms. Now this election cannot be taught precisely, though
 “ Sir ISAAC himself has given many excellent observations relating thereto.
 “ It is a sagacity and habit to be acquired by the study of good models; and in
 “ this respect there can be no doubt but the study of the ancients may be of
 “ great use, even to an algebraist. I believe Mr. D’ALAMBERT speaks truly,
 “ when he says, that the study of geometry is perhaps too much neglected.”

“ After some other observations, he adds. “ To conclude, I long much to
 “ see a treatise of which you gave us hopes in the preface to your *Conics*, when
 “ you say *in quibus autem differat analysis geometrica ab ea quæ calculo*
 “ *instituitur algebraico, atque ubi hæc aut illa sit usurpanda alias disserendum.*
 “ Such a treatise is much wanted; and I am persuaded that you must have it
 “ almost ready for the press among your papers, so that the labour of publishing
 “ would not be great.”

These letters, it is obvious, were written by Mr. SCOTT without any intermediate letter from Dr. SIMSON. I shall subjoin the material parts of the Doctor’s two answers, which he dispatched separately, though he mentions that he had received the second before he had finished his answer to the first.

Dr. SIMSON’s first letter, dispatched July 27, 1764.

“ I am now, by the weakness of my memory and of my eyes, in no good
 “ condition to write letters of any length; but yet I shall try to answer the
 “ several particulars you mention, to the best of my ability. I am glad the
 “ Problems have come to your hands. You perhaps have not yet leisure to
 “ read them, else I would have expected your opinion about them. Your
 “ censures and corrections I value more than commendations. In the preface
 “ to the *Data* it is said, that they are of the most general and necessary use
 “ in the solution of problems of every kind; and whoever tries to investigate
 “ the solutions of problems geometrically, will soon find this to be true; for
 “ the analysis of a problem requires that consequences be drawn from the
 “ things that are given, until the thing that is sought be shewn to be given.
 “ Now supposing that the book of the *Data* were not extant, these consequences
 “ must be found out and demonstrated from the things given; which, in most

“ cases, would take no little time and pains. But now having that book, the
 “ Propositions of it need only be cited, or supposed to be known; for though
 “ the ancients did not cite the particular Propositions either of EUCLID’s
 “ *Elements*, or of his *Data*, yet they supposed them to be known, as is evident
 “ from the solutions of problems given by PAPPUS, in which he always makes
 “ use of the *Data* as being known to every geometer. The *Elements* are
 “ necessary in every geometric reasoning, whether in demonstrating theorems
 “ or in resolving problems: and in the analysis of problems the *Data* are of
 “ as general and necessary use.”

“ The demonstration of the 17th *Data*, which you gave in symbols, is very
 “ right; the shortness of the expression by symbols is no doubt of use, but
 “ then there is greater hazard of committing mistakes by using them, than
 “ when the words at length are made use of; and I believe that Dr. BARROW’s
 “ mistake in this Proposition has been owing to this short-hand writing, which
 “ cost more time to understand, than if he had used the ordinary way. As A
 “ is with him the given magnitude, and $\frac{B}{C}$ the given ratio, it is plain that
 “ $A + B$ is the first of the magnitudes mentioned in the enunciation; from
 “ which taking A , the remainder B has to C , the second magnitude, a given
 “ ratio; and the thing he should have demonstrated is, that $A + B$ is major
 “ $A + B + C$ data quam in ratione, which he has not done, but only shewed
 “ that $A + B$ is major $B + C$ *d. q. in r.* Many such mistakes are observable
 “ in problems done algebraically; and in the 31 Prop. lib. i. of the *Loci plani*
 “ p. 96, I have taken notice of a remarkable one in SCHOOTEN’s *Exer-*
 “ *citationes*, p. 249. The given magnitude AE (in your notation $a - \frac{a^2}{a+b}$.) which
 “ is to be taken from AB (or x) is easily seen to be the excess of AD (or a)
 “ above ED , which is the 4th proportional to DC , DB and AD the given
 “ magnitude; and if the given ratio of DB to BC be a to b , as you make it,
 “ then the ratio of DC to DB will be that of $a + b$ to a .

“ You seem to think, that wherever magnitudes and their ratios are con-
 “ sidered, that the algebraical method is better than that of the ancients in
 “ respect of facility, though not of accuracy. But what if the construction
 “ deduced from the first method, be nothing so good, either in respect of
 “ shortness or elegance, as that from the other? which is often the case.

“ You may at leisure try the algebraic solution of Prob. 2. in pages 465 and
 “ 466 of the English EUCLID, which will furnish you with an instance of this.

“ The length of the demonstrations in the method of the ancients arises,
 “ for most part, from the strict regard they had to accuracy; yet I do not
 “ say that none of them can be made shorter; for sometimes, by making
 “ use of a more proper medium of the demonstration, they can be shortened:
 “ an instance of which you have in the 20th Prop. b. ii. of APOLLONIUS’S
 “ *Conics*; which, by making use of the properties of the Asymptotes instead of
 “ those of the Diameters, is made shorter in the 40th Prop. of b. iii. of the
 “ book which I published sometime ago. I expressed myself indistinctly
 “ in the preface to that book, which has made some think I designed to
 “ publish something, to shew when the ancient and when the modern method
 “ ought to be used; but I meant to say nothing more than that it was not
 “ proper to be done in that preface.”

“ I cannot guess on what account Mr. MACHIN, to whom you assent, thought
 “ Sir ISAAC’S taste of demonstration preferable to that of Mr. HUYGENS.
 “ Though I have the greatest veneration for Sir ISAAC’S profound and penetrating
 “ genius, and his most valuable and useful discoveries, yet I cannot but prefer
 “ HUYGENS’S elegance and perspicuity, especially in his *Horologium oscilla-*
 “ *torium*, to Sir ISAAC’S; nor do I think the first has fallen into so many
 “ mistakes as the other, in things purely mathematical.”*

I shall subjoin some extracts from Dr. SIMSON’S letter in answer to Mr.
 SCOTT’S 2d letter, of May 1, 1764.

“ DEAR SIR,

“ Your approbation of the edition of the *Data*, with which you begin your
 “ second letter of May 1st, gives me great satisfaction. You observe that the
 “ demonstrations made by a multitude of ratios and their compositions, ex-
 “ pressed in the manner of the ancients, are more difficult and obscure than
 “ the algebraic method. I confess a multitude of them makes a demonstra-
 “ tion difficult in any of the methods, and I found it was not only difficult to
 “ my scholars to understand the 20th Prop. of HUYGENS’S *Dioptrics*, but
 “ uneasy to me to present it to them, by reason of the great number of ratios
 “ that are considered in the demonstration, where there is only a straight

* Sir ISAAC NEWTON himself entertained a high opinion of the elegance of HUYGENS’S taste in geometry. See Pemberton’s View, Preface.

“ line and its parts to aid the memory: and on this account I contrived a
 “ demonstration by help of a figure, by which the composition of ratios is
 “ avoided, and nothing but the properties of parallel lines, and similar
 “ triangles, enter into; it which made it much easier both to my scholars and me.”

“ No doubt the fourth of the second book of the *Elements* might have been
 “ shewn from the third and the second of it, and EUCLID, or even a far less able
 “ geometer, could not but see it; but he preferred the way by figures in all the
 “ demonstrations of the second book, as shewing the equality of the spaces
 “ compared to the eye, which is more clear than when it is perceived by
 “ the imagination. As to the fifth book, I do not see that the demonstrations
 “ in it can receive any help from algebra; and the straight lines made use of
 “ in it make the demonstrations clearer and easier, than they would be without
 “ them. I do not understand your meaning, when you say any analogy
 “ may be called algebraical as well as geometrical. The expressing lines by
 “ a single letter does not make analogy or any thing else algebraical, any
 “ more than when they are expressed by the two letters at their ends, nor do I
 “ think any thing can be called algebraical, where no operation peculiar to
 “ algebra is made use of. I should be glad to see an algebraical demonstration
 “ of some proposition in the 5th book, *ex. gr.* of the 17th, by which perhaps
 “ I might understand your meaning. The 47th Prop. of 1st B. cannot be done
 “ by similar triangles in the place where it now stands; and when it is done
 “ by them, I do not see that the demonstration has the least dependence
 “ on any thing in algebra. You think algebra almost indispensibly necessary
 “ in difficult enquiries: I have, on the contrary, been obliged to make use of the
 “ ancient method in many problems, in which I could not find that algebra was
 “ of any use to me; such were the two problems, the solutions of which I sent
 “ some time ago; and so are many others, of which the following is one. *Three*
 “ *points and three straight lines being given in position, it is required to place*
 “ *a triangle so that the three angular points may respectively be on the three*
 “ *given straight lines, and the three sides pass through the three given points.**”

“ The solution which you give of the last problem at the end of the *Data*,
 “ by help of two *Loci*, is very good; but I am not yet persuaded, that the con-

* The enunciation in this letter has reference to a figure which is here omitted. It is also to be remarked that in the *Opera Reliqua*, p. 388, is a solution of a case of this problem, viz. when the three given points are in one straight line.

“struction arising from it, or the demonstration of it, will be quite so simple
 “as those given in that place: if you had leisure to write them out, without
 “making use of any algebraical notation or operation, they might then be
 “better compared. You think I have taken advantage of the *Locus ad Cir-*
 “*culum*, but I assure you it never came into my mind: the 34th Prop. in
 “the *Data*, as you will see by looking to it, does indeed require the description
 “of a circle, by which the position of the straight line AD (in Prob. p. 465,
 “is found.

“It would take up too much time, and writing too much for my eyes, to
 “shew the advantage of the ancient method above the algebraic, and how the
 “precepts given in this last method, particularly in the *Arithmetica Universalis*,
 “lead those who observe them, from the right solution of geometrical
 “problems, into such as are quite out of the natural method; many instances
 “of this occur in that book, among which is that problem which is the
 “first at the end of the second edition of the *Conics*. You say the solution
 “there given is not peculiar to the ancient geometry, and you add, ‘what should
 “hinder an algebraist to suppose the angle BCD in my figure equal to BAC
 “&c.?’ True, an algebraist may give a geometrical solution of a problem, but it is
 “not therefore an algebraic solution. The solution you give in the algebraic
 “notation depends nothing on algebra, for the *Locus* $ax + x^2 = y^2$ flows from
 “one of the first properties of the hyperbola demonstrated in the *Conics*,
 “and is the very same which is made use of in the 1st Prob. of the appendix.
 “It may be said that algebraists are blameable, not so much in making a wrong
 “choice of a medium, as for hindering themselves from making any choice
 “not pointed out by their rules; one of which almost constantly made use
 “of by them is, to let fall a perpendicular, that by help of the 47. 1st Elem.
 “they may get an equation, which otherwise they could not so easily obtain.*

* An observation of DES CARTES tends to confirm some of Dr. SIMSON’s remarks in
 this letter; for which see BRANKER’s Algebra, p. 65, (1668.) DES CARTES, in a letter
 “not yet printed, writes thus: *In searching the solution of geometrical questions, I always*
 “*make use of lines parallel and perpendicular as much as is possible; and I consider no other*
 “*theorems but these two.* (The sides of like triangles have like proportions,) and (in
 “rectangle triangles the square of the greatest side is equal to the square of the other two
 “sides,) and I am not afraid to suppose many unknown quantities, that I may reduce the
 “proposed question to such terms, as that it depends on no other theorems but these two.” This
 letter of DES CARTES, however, was afterwards printed, Amst. 1683, Epist. 72, part iii.

“ I forgot, when mentioning the last problem, at the end of the *Data*, to
 “ observe that in the equation you first bring it to, you give the value of
 “ what is called the negative root, viz. $x = -\frac{1326}{277}$.† It is one of the absur-
 “ dities introduced into algebra in the last age, to suppose every equation has
 “ as many roots as there are units in the index of its highest power, and
 “ consequently that every quadratic equation has two: but the contrary, in
 “ the present case, can easily be shewn. Your equation is of the same form
 “ with this $x^2 - 4x = 12$, the root of which you know is $x = \sqrt{12+4} + 2$ or 6.
 “ And that this equation can have no other root, *i. e.* that there can be
 “ no number but 6, from the square of which taking the quadruple of the
 “ number, the remainder will be 12, appears thus. Suppose it possible, that
 “ y is also the root of the equation, or that $yy - 4y = 12$. Then y must be
 “ either greater or less than 6 or x ; (if any deny this axiom, I will reason
 “ no further with him on this matter;) suppose that it is greater, $y - 4$ is greater
 “ than $x - 4$, and therefore the product of y by $y - 4$ is greater than the product
 “ of x by $x - 4$; that is, $yy - 4y$ is greater than $xx - 4x$, and consequently than
 “ 12. Therefore $yy - 4y$ is not equal to 12; that is, y is not the root of the
 “ equation. The same thing follows, if y be said to be less than 6.

“ *The passage you have set down from the preface to the Conics, might,*
 “ *I confess, make any one think that I designed to have published something*
 “ *on the subject there mentioned; but indeed, as I wrote in my last, I blun-*
 “ *dered in the expression, and have no papers, and never wrote any about*
 “ *that matter. R. S.**

page 296. The sentences succeeding the foregoing extract give additional explanations
 of his opinion, to which the reader is referred. I may also remark, that in this letter, (Ep.
 72.) and in the following, (Epist. 73.) some difficulties are mentioned in the algebraical
 solution of the case of the *tactions*, in which three circles are given.

† Dr. SIMSON's notions about negative quantities in algebra have been frequently men-
 tioned. His comment on the negative root of a quadratic equation, mentioned in Mr.
 SCOR's letter, though not connected with the other matters in this correspondence, is retained
 as a characteristic specimen, in his own words, of the Doctor's manner of treating that subject.
 He made no secret of his opinions respecting it: they were well known to all his
 contemporaries; and whatever objections may be made against them, he remained firm
 in maintaining them.

* This letter was dispatched to Mr. SCOR, 13th August 1764; and no trace of corres-
 pondence between these gentlemen, of a subsequent date, remains.

With respect to this correspondence mathematical readers will form their own opinion; but it is proper to remark, that Dr. SIMSON, at the time of writing his letters, was in his 77th year. I venture to add a few remarks, which, to those who have not considered the subject particularly, may be acceptable.

In all those geometrical investigations, in which the chain of argument can be rendered by the ancient analysis both intelligible to the understanding, and pleasing to the imagination, Dr. SIMSON's reasoning in its favour will not be objected to. It must be admitted also, as he states, that unless operations peculiar to algebra be introduced, an investigation, though clothed in the characters of algebra, may yet be properly geometrical. He gives an example of the solution of a problem by the combination of two local equations; and such may always be regarded as purely geometrical, when the *Loci* employed have been demonstrated in known geometrical works, and are easily deducible from the *Data* in the problem without any algebraical operation. But even when an algebraical process is requisite for investigating the *Local Equations*; or when, in the investigation of a geometrical problem by algebra, some inferences may be more immediately drawn by means of known geometrical propositions than by the operations of that art; the algebraical solutions may by such aids be much abridged, and often rendered much more elegant.

It is further to be remarked, that when geometry comes to be employed in practical mensuration, or in physical enquiries, the most obvious and most expeditious methods of investigation, whether geometrical or algebraical, will usually be preferred. It is well known that in such applications, in general it is useful, and often absolutely necessary, to express conclusions (in whatever way they may be obtained) arithmetically; and hence it is frequently expedient to investigate by algebra such conclusions, though they might without difficulty be obtained by geometrical reasoning, as they must ultimately assume the arithmetical form in the practical uses of them. I must also observe, that Mr. SCOTT's remark respecting the combination of ratios is perhaps not fully answered by Dr. SIMSON; and that the remark may also in some cases be extended to any long series of steps, by the combination and separation of squares and rectangles arising from the segments of a straight line, by the use of the Propositions of the iid b. of EUCLID, or of others founded on that book. Though a demonstration may be regarded as strictly

geometrical, when such ratios, and the combinations of them, are set down either in words at length, or in a kind of short-hand, by using some algebraical characters; yet when the mind loses the distinct perception of the particular geometrical magnitudes compared, the evidence is similar in its impression to that of algebraical reasoning, in which the previous demonstration of the rules employed is the ground of our assent to the truth of the conclusions, and not the immediate perception of the geometrical magnitudes and their relations.—An analysis or demonstration in which many combinations of ratios are employed, may generally indeed be much shortened by admitting multiplication, division, or other operations peculiar to arithmetic or algebra; but the investigation or demonstration then becomes truly algebraical. It must however be admitted, in the particular case here supposed, that the evidence of the algebraical process is not materially different, or rather that the impression of that evidence on the mind, is not very different, from that of the geometrical method. The same observation may be extended also to the case, when many combinations of squares and rectangles from the segments of a straight line necessarily make a principal part of a geometrical analysis or composition.

Dr. SIMSON acknowledges the difficulty of pursuing such long trains of reasoning, by combining the ratios in the Proposition of HUYGHENS particularly alluded to. He found it even expedient to contrive a new demonstration by figures, which might facilitate his own labour in lecturing, and more effectually keep alive the attention of his scholars, by presenting objects better calculated for that purpose, and also for aiding the memory in retaining the long series of steps in that Proposition. But in such Propositions an algebraical deduction would in many cases be not less satisfactory, and far more expeditious, than any other method.

It must be conceded, however, by the patrons of the algebraical analysis, that the ancient method, where it can be applied, with very few exceptions, is more elegant, and gives a more immediate and more pleasing conviction of the truth of a geometrical proposition, than the other. It must be admitted, also, that there are many geometrical problems, even in the class denominated *plane*, of which the algebraical solution is far from being obvious, and is often more difficult than the geometrical; even from the complexity of the calculation, without regarding the inferiority of the constructions which

generally result from the solution of equations. Dr. SIMSON, in one of his letters to Mr. SCOTT, mentions an example; and others equally illustrative of the point might easily be pointed out.*

That many geometrical problems are more easily resolved by algebra than by the ancient analysis, there is no doubt; but when the object is to obtain a solution, in which rigid accuracy is combined with perspicuity in every step of both the analysis and demonstration, the ancient method is indisputably to be preferred. But the distinguishing value of the modern analysis, though not particularly considered in this correspondence, is the means which it affords of resolving geometrical problems, inaccessible to the ancient analysis in its present state; and of applying geometry to physical investigations, equally beyond the reach of that more elegant, though less powerful instrument. Such, indeed, has been the success of the modern analysis, in the vast enlargement of some important branches of science, and of the arts depending on them, that all reasonings on any theoretical imperfection in its principles, which does not affect the truth of the conclusions, is superseded by the wonderful extent and utility of its power and application.†

In a miscellaneous MS. volume of the year 1722 were found, annexed to the solution of a geometrical problem, some short observations by Dr. SIMSON on the ancient analysis; which, though apparently suggested merely by that solution, and written without any particular view, yet mark the accuracy of his notions respecting that analysis at this early period, before he had made much progress in the restoration of the lost analytical works of the ancient geometers. They may however, notwithstanding these circumstances, be considered as a suitable addition to the Doctor's letters on the subject in his old age, which have been

* The problem, (117 lib. vii. PAPPUS) and the extensions of it, of which a short history is given in note E, may also be mentioned as examples.

† Such is the power of the modern analysis in many complex investigations, that an observation of WOLFIIUS (tom. v. Math. Univ. p. 210) will not be considered as altogether extravagant, even by the rational admirers of ancient geometry. "Sane si ARCHIMEDES et APOLLONIUS nostro ævo reviviscerent, in stuporem raperentur visis inventis recentiorum, quæ per algebram fuerunt in apicem producta; neque unquam sibi persuassissent patere ad talia mortalibus aditum." The original and powerful genius of ARCHIMEDES however, had his life been prolonged, might have anticipated many of the great discoveries of modern times; and have adorned them also with his elegant and accurate manner of demonstration.

given in this note. To prevent unnecessary trouble, the particular references to the problem are omitted.

“Elegans est hoc exemplum veræ et genuinæ tam analysis quam constructionis et demonstrationis, methodo veterum institutarum, quasque algebra recentiorum nunquam præbuisse.”

“In genere notandum est, analysim eo puriorem et elegantiore habendum esse, quo minus artificii in constructione analysi præmissa invenitur; *i. e.* quo pauciores, æque non nisi ob rationes obvias ducendæ lineæ, constructionem istam ingrediuntur.

“Analyses enim hac methodo institutæ præferendæ sunt iis quæ, quamvis breviores esse poterint, tamen ope constructionis minus obviæ perficiuntur, *i. e.* hæ ipsæ analyses alterâ analysi indigent, ut ad istas constructiones perveniatur, quæ propterea minus perfectæ sunt censendæ. Perfectissimæ enim sunt analyses quæ a datis procedunt ad ista quæ immediate cum iis connectuntur, atque ita donec ad quæsitum perveniatur. Verum quidem est, eos qui sagaci et acri pollent ingenio, posse per pauciores gradus ad quæsitum, quasi saltando, pervenire, sed hoc nullo modo potest fieri, nisi vel animo perceperint grados omisso, quod redit ad methodum descriptam, vel tentando inciderint in constructionem quandam, a datis satis remotam, id quod omni cura ab analysta est vitandum. Cum nihil verâ et genuinâ analysi magis distat, nihil magis abhorreat, quam tentandi methodus; hanc enim amovere et certissima via ad quæsitum perducere, præcipuus est analyseos finis; quamvis fatendum, tentando et in tenebris quasi palpitando, quæsitum posse inveniri, sed hac methodo nunquam, nisi cogente demum, ob deficientem analysim, necessitate procedendum. De compositionis et demonstrationis natura, cum eæ ex analysi semper efficiuntur, eodem modo judicandum.

“Porro animadvertendum est nihil in analysi de quæsito vel ejus quavis affectione supponendum quod ex datis non clare fluat. *Ex gr.* In investigatione loci alicujus non supponendum est locum esse *rectam* vel *circulum*, quoniam hoc est quod quæritur. Et hoc vitio laborat analysis ex qua constructionem suam Prob. in Prop. 7, p. 25, lib. i. APOLLONII *de Locis Planis*, hausisse videtur FERMATIUS, nam inveniendos duo puncta in quibus locus quæsitus secat datas positione rectas, et affirmando *rectam*, hæc duo jungentem, esse locum quæsitum, videtur eum etiam in analysi supposuisse, locum quæsitum esse *rectam*.

“ Notandum denique in difficilioribus problematis tractandis multum ad solutionem conducere, si a casu aliquo simpliciore inciperimus.”

He proceeds to illustrate this last observation by a particular case, which is too long for insertion.

APPENDIX.

APPENDIX

APPENDIX I.

*Account of the Mathematical Collections
of PAPPUS.†*

§. I. *Two first Books of PAPPUS.*

PAPPUS, an eminent Mathematician of Alexandria, flourished about the end of the fourth century.‡ In the very brief accounts which remain respecting him, he is mentioned as the author of several treatises all of which; except his *Mathematical Collections*, probably the most valuable of his writings, appear to have perished. This work, as the

† In Dr. HUTTON's learned work, the *Mathematical Dictionary*, there is an excellent abstract of the contents of the *Mathematical Collections*, to which I at first intended to have referred. But on further consideration, I found that, for the purpose of giving a satisfactory account of Dr. SIMSON's commentaries and corrections of that work, a more detailed statement was necessary, as will readily appear to the intelligent reader.

‡ See SUIDAS; (Πάππος,) also GER. VOSSIUS *de Chronologia Mathematicorum*; and MONTUCLA tom. i. In SUIDAS several works of PAPPUS are mentioned; but neither his *Collections*, nor a treatise alluded to by himself in b. iv. of the *Collections*, (COMMAND. fol. 56, 1588.)

name imports is miscellaneous, and besides a variety of Propositions, both problems and theorems, contains some curious notices, not to be found elsewhere, of the history of mathematics and of mathematicians in his own and in preceding times. Its chief importance however is, as has already been mentioned, that from it alone we derive satisfactory information respecting the ancient geometrical analysis. In the preface to the seventh book, particularly, is given a description of the most valuable treatises in the collection, which in the celebrated school of Alexandria* got the title of *τύπος ἀναλυόμενος*, and which was composed by some of the most eminent geometers of antiquity, for promoting the use of that analysis.

Of the eight books of the *Mathematical Collections*, the first and about one half of the second are presumed to be lost; the rest have reached the present time, though with many imperfections, and in some passages so mutilated, that the meaning cannot be certainly ascertained. The original Greek (except some short extracts) has never been printed; and the only translation of it, by COMMANDINE, was first published at Pisaurum in 1588; and another edition, with little variation or improvement, was printed in 1660 at Bologna.† This translation is

* It is remarked by VOSSIUS, in the before-mentioned work, “ ab EUCLIDIS tempore usque ad tempora Saracenica, vix ullum invenire sit nobile mathematicum, quin vel patria fuit ALEXANDRINUS, vel saltem Alexandriae dederit operam MATHESI.”

† Though the second edition is said to be corrected, yet this appears to be only in some press errors of the former edition, while new errors of the same kind are introduced: but there is no attempt to amend the obscure and deficient passages in that translation. From some convenience of the bookseller, the original edition succes-

accompanied with a commentary, often tedious, and in some places defective; but at the same time extremely valuable, from the explanation which it contains of some difficulties, and the correction of many errors in the MS. used by COMMANDINE, and which pervade all the MSS. of PAPPUS that have hitherto been examined. From COMMANDINE's manner of referring to the Greek, it appears that he had only one MS. for his guide. He died before the work had received his last corrections; and no account is given of the history or character of the MS. which he followed.† From a family dispute between two sons-in-law, the publication was suspended for some time after his death; and at length, by the munificence of his patron the DUKE of URBINO, the translation was printed, but confessedly without any correction whatever of the errors or omissions in the unfinished work of COMMANDINE.‡

In this state, however, it was a very interesting communication to the mathematicians of that age, and excited much curiosity respecting some branches of the ancient geometry, which, before this publication, were little considered or known in modern Europe. Accordingly, not long after its appearance,

it successively got three title-pages and dates; the first, Pisauri 1588; the second, Venetiis 1589; and the third, Pisauri 1602.

† COMMANDINE in many hundred notes quotes the CODEX GRÆCUS. In a very few places indeed he does quote the GRÆCI CODICES; but this last expression seems to have arisen merely from inadvertence, as in no instance is there any comparison of readings of different MSS. though, in a great many cases, from the sense, he corrects the blunders of the one manuscript to which he usually refers.

‡ See the Dedication, and also the address to the reader, in which these circumstances are mentioned; and it is also observed that COMMANDINE had made diligent search for the two first books of PAPPUS, but without success.

SNELLIUS, VIETA, MARINUS GHETALDUS, and FERMAT, attempted, from the description of the *τίπος ἀναλυόμενος* in the seventh book, to restore several of the lost books of APOLLONIUS, which made a principal part of that collection. Afterwards SCHOOTEN, Dr. WALLIS, and subsequently Dr. HALLEY, pursued the same track; and thus a number of these ancient treatises were restored, with various success indeed, and with different characters of elegance and accuracy, as has already been observed in the foregoing Memoir.

The two first books of PAPPUS are not in COMMANDINE'S translation, from their not being found in any of the MSS. to which he had access; and they are wanting also in several other MSS. remaining in various libraries of Europe.* In a MS. of PAPPUS, in the Savilian Library at Oxford, (No. 9,) there is preserved about one half of the second book, which was published by Dr. WALLIS in 1688, with a Latin translation, and valuable notes, explanatory of the Greek arithmetic.

From this remaining fragment, it is reasonably conjectured by Dr. WALLIS, that these two books related solely to that arithmetic; and thence he infers that the loss of them is not greatly to be lamented. We learn from the fragment, indeed, that APOLLONIUS had composed a treatise on this subject, to which PAPPUS frequently refers; and Dr. WALLIS derives from it some information respecting the notation and algorithm of the ancient arithmetic. From the defective mode of notation

* See at the end of this Appendix a short notice of the MSS. of PAPPUS, which are generally known to be preserved in the libraries of Europe. It may be observed in this place, that some of them, besides the Savilian MS., have the fragment of the second book published by Dr. WALLIS.

among the Greeks, as compared with that of modern Europe, though there be much ingenuity in some of their methods, we need not be surpris'd at the great inferiority of their system. Dr. WALLIS remarks, that the business of the second book of PAPPUS appears to be nearly equivalent to what is now considered as a very simple proposition, viz. that the multiplication of any numbers, all or any of which have cyphers annexed, may be performed by multiplying these numbers without the cyphers, and then adding all the cyphers to the product. The first book he with much probability conjectures to have been employed about the simple operations of the addition and subtraction of numbers.* And I need only further remark, that there is no appearance among Dr. SIMSON's papers of his having directed his attention to this fragment.

The succeeding five books of PAPPUS are geometrical, and the last is almost entirely mechanical. They are all of a miscellaneous character; though in some of them there is a certain degree of method, and titles have been prefixed to them, alluding to the principal matters contained in the several books.† These titles do not appear to have been in the MS. followed by COMMANDINE, as he does not mention them in his translation; and probably, indeed, they did not belong to the original work, but were added by some commentator, perhaps of a much later age. Though I purpose to give a particular account of the contents of the several books, it may be proper

* See WALSH *Fragmentum Secundi Libri PAPPUS*, and the *Epilogus*.

† The MS. BOLL. has those titles, from which they are copied in this Appendix. The two Savilian MSS. at Oxford have them, and likewise the Parisian MSS.

first to mention the distinctions of geometrical propositions (especially of problems) assumed by the ancients, as they are stated by PAPPUS in two passages of his work, expressed nearly in the same words.‡

COMMANDINE's translation of these passages is given in Appendix II. and the Greek not having been printed, is added from the two Savilian MSS. § and the MS. BULL. for the satisfaction of those who may wish to consider them accurately. From these extracts it appears that it was the difficulty, or rather the impossibility, of resolving some problems by the circle and straight line, which suggested the investigation of other curve lines, by the description of which the solution of such problems might be accomplished. The doubling of the cube,* and the trisection of an arch of a circle, were two celebrated problems, which exercised the ingenuity of the more ancient geometers, but which were found not to be resolvable by plane geometry. From the very brief accounts which remain of these speculations, it appears that the first attempt of producing new curves, which might be employed in geometrical science, was from the section of a solid by a plane; and the only solids considered in the early state of the science, which, by such a

‡ The first passage is in the third book, fol. 4. b. in COMMANDINE'S PAPPUS, (1588,) and the other in the fourth book, fol. 60.

§ Accurate copies of these passages from the two Savilian MSS. were procured for me, in the most obliging manner, by the Rev. Dr. ROBERTSON, Savilian Professor of Geometry; who also gave me every facility for consulting these MSS. and other books in the very curious collection placed under his care. I met with the same obliging attention, when consulting some MSS. and books in the Bodleian Library.

* Usually called the Deliacal Problem, from the well-known tradition about doubling the cubical altar at Delos.

section could produce curves different from the circle, were the cylinder and cone. But as the sections of the latter comprehended the curves resulting from the sections of the former, the three new curves, arising from the different possible sections of the cone by a plane, obtained the name of Conic Sections. By these curves the two before-mentioned problems were easily resolved; and from this origin, all problems requiring for their solution the description of one or more of them, were called solid, though they had no other relation to solid figures.

Some other curves were also invented by ingenious men of those times for the same purpose; but the Ancients did not pursue this branch of geometry, and considered only a small number of such lines, without having had any notion of the unbounded number which modern speculations have brought into notice; and therefore, without proposing any principle of systematic arrangement. All curves, besides the Conic Sections, according to the account of PAPPUS, in the passages alluded to, were by the ancients denominated *Lines*, and problems resolvable only by such curves were called *Linear Problems*. All geometrical problems therefore, among them, were divided into three classes, *plane*, *solid*, and *linear*; and theorems were sometimes distinguished in the same manner, as they had reference to those three classes of lines. The superior lines treated of by PAPPUS, and other ancient writers, were the *conchoid*, the *cissoïd*, the *spiral*, and the *quadratrix*; and a few others are slightly alluded to.

In the solution of the two celebrated problems, which are supposed to have given rise to the investigation of new curves, some of these last-mentioned curves have been employed, as

well as the Conic Sections. And we are informed by PAPPUS, that the difficulty of describing the Conic Sections with mechanical accuracy led some of the ancient geometers to employ those higher curves, the description of which was found to be more easy. The conchoid in particular was used for finding between two given straight lines two mean proportionals, from which the doubling the cube was an obvious inference; and the trisection of an arch of the circle was accomplished also by the same curve, and likewise by the spiral and quadratrix.* From PAPPUS it appears, however, that the early Mathematicians had at first some reluctance in admitting either the Conic Sections or superior curves in the solution of problems, considering them as not strictly geometrical;† but afterwards these lines became objects of much curious investigation, even among the ancients; and in modern times ultimately were of the most extensive utility, both in abstract and in physical science.

The above-mentioned passages of PAPPUS naturally suggest an observation which may be stated in this place; that expressions occur on this subject, in the writings both of ancient and modern geometers, in which there seems to be some want of that precision and consistency which properly belong to the language of mathematical enquiry. The description of any geometrical line from the data by which it is defined, must always be assumed as possible, and is admitted as the legitimate

* These problems are resolved in that manner by PAPPUS, in b. iii. and iv.

† Even some of the more early of the Mathematicians, reckoned modern, entertained a similar notion; particularly VIETA.

means of a geometrical construction: it is therefore properly regarded as a *postulate*. Thus the description of a straight line and of a circle are the postulates of plane geometry, assumed by EUCLID. The description of the three *Conic Sections*, according to the definitions of them, must also be regarded as postulates; and though not formally stated like those of EUCLID, are in truth admitted as such by APOLLONIUS,† and all other writers on this branch of geometry. The same principle must be extended to all superior lines.

It is true, however, that the properties of such superior lines may be treated of, and the description of them may be assumed in the solution of problems, without an actual delineation of them. A plausible though imperfect representation of geometrical lines is indeed extremely useful in assisting the imagination, when we are employed in the investigation of their affections; but the degree of accuracy with which they are exhibited is of no importance to the truth of the reasoning, or to the satisfaction with which it is perceived. For it must be observed, that no lines whatever, not even the straight line or circle, can be truly represented to the senses according to the strict mathematical definitions;‡ but this by no means affects the theoretical conclusions

† See Prop. 52, 53, 54, APOLLON. b. i. in which the description of the three sections is assumed precisely according to the definitions given of them by APOLLONIUS.

‡ CARTESII *Geometria*, lib. ii. at the beginning. It may also be remarked, that even in the *Elements* of EUCLID, particularly in the xith and xiith books, certain constructions are assumed, which, though perfectly consistent with the rigour of geometrical demonstration, would in mechanical practice be extremely difficult. But they must be considered as postulates, though not stated to be such by EUCLID.

which are logically deduced from such definitions. It is only when geometry is applied to practice, either in mensuration, or in the arts connected with geometrical principles, that accuracy of delineation becomes important.

Among the ancients, the description of curve lines with a certain degree of ease and accuracy was important; as we have reason to believe that such descriptions were often used by them in practical applications of geometry to mensuration and mechanics. This the limited nature of their system of computation rendered particularly expedient; and influenced by this consideration, we may presume, the Greek geometers, in the solution of problems resolvable by the *Conic Sections*, sometimes employed superior curves, on account of the greater facility of description; of which there are examples in the *Collections* of PAPPUS.* In recent times, however, the power of the modern analysis, especially with the aid of trigonometry, renders the accurate delineation of curves seldom necessary; and in the few cases where it may be usefully employed, those curves ought no doubt to be assumed, the description of which, in the existing state of mechanical arts, most conveniently ensures the required degree of exactness.

But in a scientific consideration of the subject, it is a principle admitted ultimately, both by the ancient and modern geometers, that the proper solutions of problems

* PAPPUS enumerates several solutions of the Deliacal problem, some of them purely mechanical, and one of them by the conchoid; a curve of a class superior to the *Conic Sections*. SIR ISAAC NEWTON even admits, that in certain cases the conchoid may be preferred to the *Conic Sections*, on account of the simplicity of description. *Arith. Universalis, Appendix, constructio Æquationum linearis.*

must be effected by the description of the lowest class of lines by which they are practicable; and that though constructions of problems by curves superior to those by which they are resolvable, may be demonstrably true, and in particular cases, may be practically useful, yet in theory are to be rejected as irregular.

§. II. *The Third Book of PAPPUS.*

THIS book, as the title imports, contains geometrical problems, both plane and solid.*

The problems considered in this book are four: first, the duplication of the cube, or what is equivalent, the finding two mean proportionals between two given straight lines, which is solid: second, a problem respecting the mediatates: third, a sort of paradoxical problem, to place two straight lines, drawn from two points in one side of a triangle, to a point within it which may be greater than the other two sides: fourth, to describe the five regular solids in a sphere. These three are considered as plane.

The first problem, which was an object of much consideration among the ancients, is resolved by PAPPUS in several of the methods then known; but he begins with a refutation of an unscientific attempt to resolve it by plane geometry, which seems, in his time, to have excited some attention. The

* The title in MS. BULL. is Πάππος Ἀλεξανδρείως συναγωγῶν Γ, περιέχει δε περιβλημικά γεωμετρικά ἐπίπεδα καὶ στερεά.

† The transcript from the Parisian MS. in the Savilian library has συναγωγῶν μαθηματικῶν.

examination of it, which had been solicited from PAPPUS by the author and his friends, requires a long train of reasoning, from the complex construction which had been proposed by this inaccurate geometer; and some things, assumed by PAPPUS in this argument, are demonstrated in four propositions annexed to it. Both the text and the diagram have been injured by transcribers; but COMMANDINE's translation is much amended by Dr. SIMSON, and a few of his corrections, indeed, are confirmed by the MS. BULL. and several in the figure are from the Parisian MSS. communicated by Dr. MOOR. Before giving the particular solutions of the Deliacal problem, PAPPUS premises a statement of the ancient distinction of lines, already explained. In this account he mentions the solution of the Deliacal problem by means of the *Conic Sections*, without however giving a detail of that solution; but he explains several others, viz. three mechanical, by ERATOSTHENES, HERO, and PAPPUS himself; and a fourth of NICOMEDES, by the description of another curve, supposed to be invented by him, and called the *Conchoid*. EUTOCIUS, in his commentary on ARCHIMEDES (Prop. 2. lib. ii. *de Sphæra et Cyliandro*) gives several other solutions, (ten in all,) and particularly two by MENECHMUS by the intersection of two *Conic Sections*.—It may be observed also, that in some of the MSS. of PAPPUS, at the end of the third book are placed some variations of the mechanical solutions of this problem, not taken notice of in COMMANDINE's edition.†

† In Prop. 24. lib. iv. The solution of this problem by the conchoid is repeated; and the solution in PAPPUS's method in this book is also repeated in lib. viii. Prop. 11.

The second problem respecting the *medietates* is branched out into twenty-one Propositions, with several definitions and explanations. He distinguishes three species, and when three lines (or three numbers) are either arithmetical, geometrical, or harmonical proportionals, according to the common definitions of these terms, they were anciently called *medietates*: while the term *analogy* was usually applied only to the geometrical proportionals; so that strictly an *analogy* is a *medietas*, but not conversely.

The principal problem‡ which he proposes, is to place the three medietates in a semicircle. The resolution of this by PAPPUS is in Prop. 16th, but he begins with animadverting on a geometer of his time for his manner of treating the problem, though unjustly, as Dr. SIMSON observes in a note on this passage.§

Before this 16th Prop. however are ten problems according to COMMANDINE's enumeration, respecting the three medietates. But after that Proposition he observes, that NICOMACHUS the Pythagorean, and others, had treated not only of these original medietates, but also of three* others which were added

‡ Called the second. (The Deliacal problem being the first.) It is proposed in fol. 7 b. COMMAND. PAPPI. And in the same page is the definition here referred to.

§ Dr. SIMSON quotes VINCENTIO VIVIANI *de Locis solidis*, lib. iii. page 90, where there are some Propositions concerning the medietates, and also a remark of this mistake of PAPPUS. I may add, that BLONDEL, in his *Resolution des 4 Problemes d'Architecture*, makes the same corrections of PAPPUS's censure; and adds, that the solution by PAPPUS (Prop. 16.) is defective. But his objection to that Prop. is rather verbal than geometrical. See BLONDEL, p. 36. 37.

* PROCLUS, in his *Commentary on EUCLID*, (p. 19. HERVAGII,) states that EUDOXUS of Cnidus, about the time of PLATO, added three analogies (τρεις αναλογιας) to the three then known, and the former probably were the medietates alluded to in this place by PAPPUS.

among the ancients, and four more among the moderns, making ten altogether, which he defines and illustrates by a number of Propositions to the 27th inclusive. These, particularly the ancient medietates, were much considered in the Platonic School, in which probably they originated, and to which PAPPUS belonged. ERATOSTHENES composed two books on the medietates, (probably the *Loci ad medietates*) and though they were part of the *τόπος ἀναλυόμενος*, PAPPUS gives no description of them, and has preserved no Lemmata connected with them.† This may be considered as a presumption of his opinion that in his time they were not regarded as important; and he further remarks of the more ancient medietates, that they were useful, chiefly in explaining allusions to them in ancient writings; and probably this was the principal object of the details which he has given respecting them.

The general nature of the third problem has been mentioned, and it is expanded into sixteen Propositions. It is a sort of paradox, and he refers to a treatise of ERYCEMUS on Paradoxes, which, in the time of PAPPUS, was generally circulated, and probably in some estimation. Some of the Propositions are curious, but in the present state of the science they cannot be considered important, and are interesting only as examples of the geometrical speculations of former times, (Prop 28—43.)

† From the observation of PAPPUS at the end of his description of the *Inclinations* of APOLLONIUS, it would seem that these *Loci ad medietates* were *plane*, and, in the order of the *τόπος ἀναλυόμενος*, placed last.

The fourth and last general problem of this book is to inscribe in a sphere the five regular solids; and as these solids were objects of much speculation in the Platonic school, they were naturally attended to by PAPPUS, who belonged to it. This problem occupies the remainder of this book in the last sixteen Propositions, of which eleven are preliminary to the other five; and these last contain the solution of the problem respecting the before-mentioned five regular solids.

§. III. *Of the Fourth Book.*

THE fourth book is miscellaneous, containing theorems of the three ancient classes of Propositions formerly mentioned; plane, solid, and linear.* The first may be remarked as an extension of the 47. 1 Elem.; and the 4th is a curious theorem, with an analysis and composition, which was rendered more general by Dr. M. STEWART, in a paper published in the first volume of the *Edinburgh Physical Essays*; † and some Propositions also are added by Dr. SIMSON, in his notes.

* The title in MS. BULL. is as follows: Πάσης συναγωγῆς δ' ὅτις ἔστιν ἀθροῦν ‡ θεωρημάτων ἐπιπέδων, καὶ στερεῶν καὶ γραμμικῶν."

‡ Probably it ought to be ἀθροῦν, as it is in the title of the eighth book.

§ In Dr. STEWART'S Paper, in that volume, are several Propositions connected with this proposition of PAPPUS, and which are analysed and demonstrated in the ancient manner. A scholium, containing some conic theorems without demonstrations, is added; and at the end of it is a theorem, also without demonstration, which is truly a Porism, and is analysed and demonstrated by Dr. SIMSON, Prop. 91 of his treatise. Dr. STEWART, from delicacy no doubt to Dr. SIMSON, who wished to keep his expo-

The 10th is the case of one of the problems of the *Tactions* of APOLLONIUS, in which three circles touching each other are given, to which the three preceding Propositions are preliminary; all however requiring the corrections remarked by Dr. SIMSON.†

Before the 13th Prop. a property of the figure called *Arbelon*‡ is stated, as an ancient Proposition generally known, and of which the demonstration is in the 16th, the intermediate Propositions being necessary to it.

At the 19th Prop. he gives the definition of the Spiral of CONON, and demonstrates some of its principal properties.|| This is the curve fully treated of by ARCHIMEDES, and in a subsequent part of the book it is employed in the solution of some problems. PAPPUS (Prop. 24) repeats the definition of the

† The Porisms secret till he should publish his treatise, states the Proposition as a theorem, though he was well acquainted with Dr. SIMSON's discovery, and had investigated many curious Porisms.—See *Edinburgh Physical Essays*, vol. i. p. 141—172.

‡ There are some amendments on the 12th, necessary to make the demonstration complete.

§ The Arbelon (*ἀρβηλον*) is mentioned in the *Lemmata* of ARCHIMEDES, (Prop. 4, 5, 6;) and it is remarkable, that though PAPPUS in his *Collections*, and particularly in this book, often quotes ARCHIMEDES, there is no allusion to the *Lemmata* in his long and curious discussion of the properties of the Arbelon. There is also a lemma of COMMANDINE's, for demonstrating a Proposition of PAPPUS on this subject, (Prop. 14,) which is the first Proposition of the *Lemmata* of ARCHIMEDES, and which COMMANDINE afterwards (fol. 52, b.) asserts to be composed by himself. From this it may be inferred, that COMMANDINE certainly had never seen the *Lemmata* of ARCHIMEDES, and most probably neither had PAPPUS. The *Lemmata* have never been found in Greek, and have by several learned men been supposed not to be the work of ARCHIMEDES; and the circumstance now mentioned favours that supposition.

|| In Prop. 21, respecting the Spiral, the demonstration is conducted on principles like the method of Indivisibles of CAVALLERIUS.

conchoid of NICOMEDES, and the solution of the Deliacal Problem by it, as in the preceding book: he afterwards employs this curve for resolving another celebrated problem of antiquity, the trisection of an arch of a circle, which was found impracticable by plane geometry; and which, with the Deliacal Problem, roused the efforts of mathematicians to investigate new curves for resolving them.

He proceeds to explain the origin and properties of the quadratrix, (*τετραγωνίζουσα*.) assumed by DEMONSTRATUS and NICOMEDES, subsequent geometers, for the quadrature of the circle, from which it obtained its name. But he observes that another geometer, SPORUS, was not satisfied with this application of that curve, and he gives some detail of the objections proposed by SPORUS; and particularly, that in the construction of this line the problem to be resolved by it is in some measure assumed. PAPPUS however, besides the common description of its origin and of its properties, gives what he considers as a more strictly geometrical description, by *Locis ad Superficiem*.*

After several Propositions on this subject, there is a repetition of the ancient classification of lines, and nearly in the same words† in which it was stated in the third book. It seems to be introduced here for illustrating the problem of the trisection of an arch of a circle, as in the former book it had a more particular reference to the Deliacal problem.

* This is Prop. 28, which is so much corrupted in COMMANDINE as to be scarcely intelligible; and in Note G. at the end of the preceding Memoir is placed Dr. SIMSON's corrected edition of it. Another description of it is given in Prop. 29.

† The Greek of this passage is added in Appendix II.

This problem is resolved by PAPPUS in various ways; first, by a solid inclination, to which Dr. SIMSON adds an improvement; then by an hyperbola without the inclination;* and he mentions that in another treatise† he had resolved it by means of the conchoid, of which the method is obvious.

He subjoins some linear problems, such as dividing an angle or arch of a circle in a given ratio; the construction of an isosceles triangle, of which each of the angles at the base shall have a given ratio to the angle at the vertex; which are easily resolved, by assuming the quadratrix or spiral, as proper means of geometrical solutions.

Besides the very important emendation of Prop. 28th and 29th of this book by Dr. SIMSON, there are various other necessary corrections in other Propositions, and some useful readings adopted by him from MS. BULL. which it is unnecessary to specify. At the end of this book, there is in MS. BULL. a very imperfect sketch of a problem said to have been used by ARCHIMEDES, in his attempt to find a straight line equal to the periphery of a circle; and also that this use of it had been animadverted on. It is not in COMMANDINE's translation, and in its present state is altogether unintelligible.‡

* This Inclination is alluded to in Note F. p. 99.

† From the very brief notice of this work, it seems to have been a comment on the Analemma of DIODORUS: “ και ημεις εν τω εις το αναλήμμα Διοδορου τριχα τμησιν την γωνίαν βλησμένοι κηχρήμεθα τη προειρημένη γραμμη.” MS. BULL. Vide COMMAND. PAPP. fol. 56, a. l. 6.

‡ This Proposition, in the same unintelligible state, is found in Savil. MS. No. 3, and at the end of the fourth book.

§. IV. *The Fifth Book.*

To the fifth Book there is a preface, in which PAPPUS makes some philosophical observations on the curious instincts of animals, which in many cases supply the place of reason. He mentions more particularly the instinct of bees, by which they construct the receptacle for their provision on geometrical principles, by employing the hexagon, which of the three equilateral and equiangular figures that occupy the space round a point, supplies, with the smallest labour, the most convenient accommodation. This observation seems to be introductory to the purpose of the book, which, at the end of the preface, he states to be; to prove that of plane figures which are equilateral and equiangular, and have equal perimeters, the greater space is contained by the figure with the greater number of sides; and that of all plane figures, of equal perimeters, the circle is the greatest.*

This subject of isoperimetrical figures is largely treated of in this book, containing fifty-seven Propositions.‡ Besides

* The title of the fifth book in MS. BULL. is the following, Πάππος Ἰλλυξοῦδος συναγωγῆς ἑ, περιέχει δὲ συγκριθεῖς τῶν ἴσων περιμέτρων ἔχοντων ἐπιπέδων σχημάτων πρὸς ἀλλήλα τε καὶ τὸν κύκλον, || καὶ συγκριθεῖς τῶν ἴσων ἐπιφανειῶν ἔχοντων στερεῶν σχημάτων πρὸς ἀλλήλα καὶ τὴν σφαίραν.

|| Note in MS. BULL. it is erroneously τῶν κυκλῶν, but in the Savil. MS. No. 3. it is τὸν κύκλον.

‡ In Prop. 1. an easy Proposition is assumed, which is demonstrated in a Lemma (Note B. on that Prop.) by COMMANDINE. It is remarkable, that a Proposition of the same import in Greek is found on the margin of MS. BULL. And in the text of MS. SAVIL. No. 3, is a similar Proposition. It would require however the examination of other MSS. to ascertain the history of this Lemma.

the object mentioned in the preface, several other things are considered. It is proved, that of plane figures with equal perimeters, the greatest is that which is equilateral and equiangular. This principle is extended to solids; the regular solids (as they are called) are compared, and it is proved, that, of those with equal surfaces, the greatest is that with the greater number of sides. These Propositions are meant no doubt to be introductory to a demonstration of the Proposition, that of all solids with equal surfaces the greatest is the sphere. There are many references to ARCHIMEDES, and several of the Propositions in his treatise of the Sphere and Cylinder are demonstrated in this book by PAPPUS,* and, as he mentions, in a different manner.

It was formerly observed, that a number of Propositions in it, respecting the comparison of the five regular bodies having equal surfaces, were demonstrated only synthetically from the brevity and the facility of communication in that method; and that this explains the omission of the analysis in many ancient Propositions, though there can be no doubt of that analysis having been used by the authors in the investigation of them. He introduces the Propositions respecting these regular solids by a general dissertation on them before Prop. 18. in which he professes great reverence for the doctrines of the divine PLATO, in whose school it is supposed that these bodies were first defined, and their relations fully treated of. He

* In Prop. 35. (fol. 96. COMMAND.) is a very material improvement of the demonstration by Dr. SIMSON, by which the long note (Z) of COMMANDINE becomes unnecessary. This may be remarked in many of the Doctor's corrections, which, even when short, often supersede the use of long and sometimes tedious comments by COMMANDINE.

remarks, that it is assumed by philosophers, without proof, that the sphere is the greatest of solids, having equal superficies: he proceeds to give some popular illustrations of that truth; and in Prop. 18. he proposes a comparison of the sphere with the regular solids. In the end of the book he states, what is easily ascertained, that there can be only five regular solids, which are comprehended by equal, similar, and equilateral polygons, viz. the tetraedron, cube, octaedron, dodecaedron, and icosaedron; in some of the Propositions Dr. SIMSON has remarked several proper and necessary corrections.

§. V. *Sixth Book.*

THE sixth Book of PAPPUS, as the title intimates,† is employed chiefly in explaining and correcting some Propositions of THEODOSIUS, and some other ancient writers, in treatises containing chiefly, what is popularly called, the doctrine of the sphere. In a short preface the object of this book is stated, with a reference to three examples of Propositions criticised in it, viz. the 6th, iii. THEODOSIUS *on the Sphere*; the second Prop. of EUCLID'S *Phænomena*; and the fourth Prop. of THEODOSIUS *on Days and Nights*. This collection of treatises, obtained, in the Alexandrian school, the name of μικρὸς ἀστρονόμος, or, as it is in PAPPUS μικρὸς ἀστρονομήμενος. Among the Arabians they were called *Libri intermedii inter σοιχωιωτήν et magnum constructorem*, that is, between EUCLID and PTOLEMY:

§ The title of this book in MS. BULL. is Πάππου Ἀλεξανδρείᾳ συναγωγῆς ἔχοντι, περιέχει δὲ τῶν ἐν τῷ μικρῷ ἀστρονομίᾳ θεωρημάτων ἀποδείξεων λύσεις.

and in contrast to the greater work of the latter (the *Almagest*, sometimes called μέγας ἀστρονόμος) these smaller tracts got the name of μικρὸς ἀστρονόμος.

According to VOSSIUS, this collection contained the nine following works, viz. THEODOSII *Sphærica*, EUCLIDIS *Optica*, ejusdem *Phænomena*, THEODOSII *de Habitationibus*, ejusdem *de Noctibus et Diebus*, AUTOLYCUS *de Sphæra*, ejusdem *de Ortu et Occasu*, ARISTARCHI *de magnitudinibus et distantis Solis et Lunæ*, HYPsicLIS ἀναφορικόν, sive *de ascensionibus*.† FABRICIUS, when stating this collection, includes in it also MENELAI *Sphærica*, and EUCLIDIS *Data et Catoptrica* :‡ and he adds, that these treatises, or most of them, are often found together in Greek MS. in the libraries of Italy and France.

It may be also observed, that all these treatises, except that of HYPsicLES, are quoted or alluded to in this sixth book, in which PAPPUS examines several ἐνστασεις (*Instantiæ*), to be found in them. By this term he appears to understand the objections to certain Propositions in these writers, or the limitations and exceptions necessary to be made respecting them, but which had been omitted by the respective authors. PROCLUS gives an explanation of this term;§ and from his diffuse and not very satisfactory account, this word ἐνστασις (*Instantia*) seems to have been applied to any objection made to a Proposition, or even to the denial of it, as preparatory to an indirect demonstration of it, by proving the absurdity of

† GER. VOSSIUS *de Scientiis Mathematicis, et Chronologia Mathematicorum*, cap. xxxiii. §. 18.

‡ FABRICII *Biblioth. Græc.* Harles, Hamb. 1795. tom. iv. p. 16, and also p. 212. SAVILIUS in *Prælect.* II. in EUCLIDEM.

§ PROCLUS, p. 58. HERVAGII; p. 121. BAROCII.

that denial. In PAPPUS, however, the application seems to be confined to the former meaning, with which the examples which he gives, from THEODOSIUS and others, correspond.†

Prefixed to Prop. 39 of this book, is a long discussion concerning the positions in the before mentioned tract of ARISTARCHUS SAMIUS and this, with the 39th, 40th, and 41st Propositions on the same subject, is inserted by Dr. WALLIS in his edition of *Aristarchus* published at Oxford in 1688. He adds the Greek of this extract from PAPPUS according to the two Savilian manuscripts, which he takes an opportunity of characterizing.*

In the explanation and correction of several passages in these tracts by PAPPUS are introduced some curious Propositions, by which the doctrine of the sphere of that period was improved; and there are some Propositions relating to a sort of perspective or projection of the sphere, which are connected with Propositions of EUCLID'S *Phænomena*. It may be also observed, that in four Propositions, viz. 31, 32, 33, 34, which are preliminary to some following Propositions respecting the sphere, some distinctions of magnitudes are stated as examples, which may be either increased or diminished without limit; or may be increased indefinitely,

‡ Examples of *instantiæ* may be found in Prop. 21, of this sixth book, to which the preceding ten Propositions refer. Also in Prop. 51, (fol. 145. a. at the bottom, COMMAND.) Other criticisms on EUCLID'S *Phænomena*, and also on HIPPARCHUS, are mentioned in subsequent Propositions of this book. There are also many references to PTOLEMY.

* In his *præfatio ad Lectorem* he observes, "quorum (sc. MSSorum) qui "elegantius scribitur (sc. No. 3.) est mendosior; quique festinantius et minus "elegantius scribitur (scil. No. 9.) est emendatior."

while there is a limit to the decrease, and conversely. And I mention them here, merely to remark the inaccuracy of COMMANDINE's translation, by introducing the term *infinite* without authority from the original. Dr. SIMSON observes that he had not considered this book so particularly as he purposed afterwards to do, but which he appears not to have accomplished: at the same time he had made some necessary and useful corrections in several Propositions as they stand in COMMANDINE's translation.*

§. VI. *Seventh Book.*

THIS Book, as the title imports, is wholly employed on that curious subject, the ancient analysis.†

THE collection of treatises known by the name of *τέπος ἀναλυόμενος*, next to the *Elements* and *Data* of EUCLID, were important for facilitating the resolution of geometrical problems. They were composed, by the elder ARISTÆUS, EUCLID, and APOLLONIUS;‡ but at what time the collection

* It may be observed, that in the MS. BULL. and in the Savil. MS. No. 3, are some marginal notices and at the end of Prop. 59, a scheme respecting climates which may deserve consideration; in that Proposition also are references to PTOLEMY.

† The title of the 7th book, from one of the Parisian MSS. No. 2368, is as follows; Πάσις Ἀλεξανδρείᾳ συναγωγῆς ζ' ὃ περιέχει τὴν τάξιν, καὶ τὴν περιοχὴν καὶ τὰ λήμματα τῶ ἀναλυόμενου τέπου.

‡ In this first statement of the authors in the preface to this viith book these three only are mentioned. In a subsequent more particular account (also in this preface) PAPPUS mentions two books on the *Medietates* by ERATOSTHENES. But neither this work of ERATOSTHENES, nor the *Loci* of ARISTÆUS, nor the *Loci ad Superficiem* by EUCLID, are described by PAPPUS.

obtained the name by which they are distinguished in the preface to this book, is unknown. They must have been composed at different periods, from the known interval of the times of the respective authors, who all lived above 500 years before the age of PAPPUS. The expression of PAPPUS is not definite; yet it seems to imply that the collection was made, and the name imposed, before his own time, for his expression seems to suppose the order of the books as existing.‡

The preface to this book, one of the most valuable remnants of ancient geometry, contains a particular description of the nature and contents of a certain number of these treatises, which, both from the subjects of them, and from their being thus distinguished by PAPPUS, may be considered as by far the most important part of the collection; and the book itself contains a number of Lemmata or elementary Propositions assumed (probably without proof) in the treatises described in the preface, but of which the demonstrations are added by PAPPUS.

PAPPUS, however, premises a short but accurate account of the ancient method of analysis and synthesis; of which a free translation will be more satisfactory than any abridgement, for conveying a correct notion of this curious branch of science, so much valued among the ancients, and, till the

‡ Dr. HALLEY, whose authority in such points is very great, at the end of his preface to the *Sectio Rationis*, intimates an opinion that the collection was made by PAPPUS. But the circumstances now mentioned render this inference from the words of PAPPUS at least doubtful; and Dr. SIMSON has remarked another mistake of Dr. HALLEY's in the before mentioned preface respecting the principal object in forming this collection.

time of Dr. HALLEY and Dr. SIMSON, so little understood among the moderns.

It is employed both for the resolution of problems, and also for investigating the truth of theorems either asserted or conjectured to be true; though it is in the former class of Propositions that the use and importance of this method is chiefly known.

In the analysis of a problem, what is proposed to be done, or to be found, is supposed to be obtained; and the consequences of such an assumption are successively deduced by means of all previously known practical Propositions, which appear to be connected with it, till at length we arrive at some conclusion, which, from the state of geometrical science at the time, we know to be practicable; and thence the solution of the problem is deduced. Synthesis, or composition is the reverse of analysis; and by assuming the last practicable consequence of the analysis, we proceed in a contrary order through the several steps of that analysis till we necessarily reach the construction of the thing required, and then the problem is resolved. In this analysis however, if we arrive at a consequence which we know, from previous Propositions, to be impossible, then the problem proposed must itself be impossible; and further, if we find, from the progress of the composition, that in certain relations of the given magnitudes the construction is practicable, while in others it becomes impossible; the ascertainment of these relations becomes a necessary part of the solution, and is called the determination of the problem.

In the analysis of a theorem,* the assertion in the enunciation is assumed as true; and by reasoning from it, by the application of known geometrical Propositions which appear to be connected with it, we trace the successive consequences of the first assumption, till we arrive at some one which we know to be true, or to be false. If we arrive in this manner at a true Proposition, by proceeding from it, in a contrary order, through the several steps of the analysis, we shall necessarily arrive at the proposed assertion; and this last proceeding will be a demonstration or composition of the theorem. In like manner, if in this investigation we arrive at a conclusion which we know to be false, from the same necessary concatenation of Propositions, we infer the falsehood of the assumed assertion; and if it were requisite, we might also demonstrate that inference. But it not being my design, in this general account of the seventh book of PAPPUS, to attempt a full exposition of the ancient analysis, which, for the use of those who have not considered the subject, would require also the illustration of examples, I can only refer to those treatises where such explanatory examples of the practice of analysis may be found.† I shall

* The analysis of theorems is often useful. A theorem may be proposed to a geometer, that he may investigate the demonstration; it may be only suspected to be true from analogy, from the apparent relations of magnitudes in geometrical diagrams, and even from accident; but in all such cases an analysis will ascertain the truth or falsehood of the assumed Proposition.

† In the remaining works of APOLLONIUS, as published by Dr. HALLEY, and also in PAPPUS, many problems and theorems are treated analytically. In Dr. SIMSON's different works are various examples in the pure stile of the ancient geometry. The analysis of theorems is well illustrated in Dr. STEWART's *Propositiones Geometricæ more veterum demonstratæ*. For problems, I may refer to Dr. HORSLEY's (Bishop of St. Asaph) *Delectus Problematum*; and to Mr. Professor LESLIE's *Geometrical Analysis* in his *Elements of Geometry*. In that treatise the general problems of APOLLONIUS, mentioned in this book of PAPPUS, are introduced as examples.

therefore only further observe, that, in this practice, some of the principles and rules commonly laid down by writers on the application of the algebraical analysis to the solution of geometrical problems, may here also be usefully employed; though several of them, no doubt, are more peculiarly fitted for the modern system.†

After this general account of the ancient analysis, PAPPUS, in his Preface, proceeds to enumerate the books contained in the *τύπος ἀναλυόμενος*, and afterwards to give a particular account of the nature and contents of a certain number of them, viz. of twenty-four; the whole number being thirty-three.‡

These twenty-four books are, I. *Data* of EUCLID; III books *De Porismatis*, also by EUCLID; and the following twenty by APOLLONIUS: II. *De Sectione Rationis*; II. *De Sectione Spatii*; II. *De Sectione Determinata*; II. *De Tactionibus*; II. *De Inclinationibus*; II. *De Locis Planis*; and VIII. *De Conicis*.

These books, though all useful in facilitating the solution of geometrical problems, yet are of different characters, and promote the object of the whole collection in different ways; as may appear from the many references to them in this Memoir, of which the following short recapitulation is added.

† See NEWT. *Arith. Universalis*, cap. 23.—See also some observations of Dr. SIMSON on the ancient analysis, in Note K. at the end of the Memoir, p. 121—128.

‡ In this preface PAPPUS mentions his purpose of giving a description of these books as far as the *Conics* of APOLLONIUS; and therefore, according to his arrangement, which is not in the order of time, he appears to omit by design the nine following books: V. of ARISTÆUS *de Locis Solidis*, (which were no doubt superseded by the *Conics* of APOLLONIUS;) II. of ERATOSTHENES *de Medietatibus*; and II. by EUCLID, *de Locis ad Superficiem*.

I. *The Data of EUCLID.* This book, one of the few which have been preserved in the original language, may be considered as a collection of elementary problems; and the demonstrations as given by EUCLID, would become the analyses of these Propositions, had they been enunciated as problems. The book was constantly used by the ancients in their resolutions of problems, though it was not their practice to quote the particular Propositions; and this application of the Data was continued among such of the moderns as follow strictly the ancient method of analysis, and is now so well known, that it is unnecessary to give any detail of it.* The *Sectio Rationis*, *Sectio Spatii*, *Sectio Determinata*, and the Treatises *De Tactionibus*, and *De Inclinationibus*, are all general problems of frequent recurrence in geometrical investigations, and were resolved by APOLLONIUS in the most complete manner, all the possible cases being distinguished; and of each case a separate analysis and composition are given, with the respective determinations in all those cases which required them.†

The use of these general problems, as has been repeatedly mentioned, was for the more immediate resolution of any proposed geometrical problems which could be easily reduced to a particular case of any one of them.‡ By such a

* See in Note K. at the end of the Memoir, some remarks on the Data, in the correspondence between Dr. SIMSON and Mr. SCOTT.

† PAPPUS, in this preface, alluding to a problem connected with the Treatise of Tactions, states with brevity, but with precision, what is requisite in a perfect solution of such general problems: “καὶ ταῦτ’ ἀναλῦσαι, καὶ συνθεῖναι, καὶ διορίζεσθαι κατὰ πτώσιν.”

‡ When a problem can be resolved with equal facility by the use of known elementary Propositions, a reference in such a case to any of these general problems becomes unnecessary.

reduction the proposed problem was considered as fully resolved; because it was then necessary only to apply the analysis, composition, and determination of that case of the general problem to this particular Proposition, which was shewn to be comprehended in it. The apparatus of separate solutions, with the determinations of every possible case which is essential to the use of these general problems, may appear forbidding; and if regularly perused without examples of the application, may sometimes appear tedious and uninteresting; and this perhaps, may have created some prejudice against the study of them. Dr. HALLEY, indeed, seems to think that the books of the *τόπος ἀναλυόμενος*, were intended merely for the instruction of beginners in the study of the geometrical analysis;§ but it is justly observed by Dr. SIMSON, that though they may be most advantageously employed for that purpose, yet that it is manifest from the contents of those books, of which he gives some detail, and also from the manner in which they are referred to in the small remains which we possess of the ancient geometry, the chief object of them was what has now been stated. Even in PAPPUS there are some examples of problems being resolved by a reduction of them to cases of these general problems. In the 85th Prop. of this seventh book, a problem, useful in the *Treatise of Inclinations*, is by analysis brought to a case of the *Sectio Determinata*, as was formerly mentioned in the Memoir. Another example is Prop. 164, of the same book, the last lemma belonging to the Porisms; which is a problem resolved by PAPPUS, by reducing it to a case of the *Sectio Spatii*, and the composition of the problem is considered as completed, merely

§ HALLEII *Sect. Rationis*, Præfat. See also *Loci Plani*, SIMSON's Præfat. p. vii.

by a reference to the construction of that case. This tract of APOLLONIUS being lost, and no restoration of it having been made in the time of COMMANDINE, the case referred to is resolved by him, though in a tedious manner, which may be compared with the solution of it in Dr. HALLEY'S restoration of that work.*

The other books, described in this preface by PAPPUS, are useful for the same purpose of facilitating the solution of geometrical problems, though in a manner different from what has just been mentioned of the preceding treatises. The treatises of *Loci* in particular are very important, and in a method well understood by those even slightly acquainted with either the ancient or modern geometry. The *Loci Plani* of APOLLONIUS are useful for the solution of plane problems; and often also, with *Loci* of a higher order, may be required in the solution of superior problems. Though the original work of APOLLONIUS be lost, the elegant and compleat restoration of it by Dr. SIMSON leaves nothing to be regretted on that subject. The *Loci Solidi* of ARISTÆUS might have been very serviceable in the solution of solid problems, but it has perished; and probably, even in the time of PAPPUS, had been superseded by the great work of APOLLONIUS on *Conic Sections*, from which not less important aid might have been derived for resolving that class of problems.†

* SCHOOTEN also resolves this problem, *Exercit. Mathem.* p. 104, 105, and seems to blame PAPPUS for not resolving it directly, but by a reference to a case of the *Sectio Spatii*; from which it appears, that SCHOOTEN was not apprised of the true use of these ancient books. His Proposition also is not so general as it is stated in PAPPUS. See Dr. SIMSON'S edition of this Prop. *Opera Reliqua*, p. 529.

† As seven books of the *Conics* of APOLLONIUS have been recovered, and the eighth restored in a satisfactory manner by Dr. HALLEY, it is unnecessary to enter

The porisms of EUCLID were a peculiar class of Propositions, and used among the ancients in the solution of some of the most difficult geometrical problems. The long account of them in this preface has been particularly unfortunate in suffering from the injuries of time; so that till Dr. SIMSON's persevering industry and ingenuity were employed on them, no satisfactory explanation of the nature of these Propositions could be discovered, nor could any single Proposition of EUCLID's treatise be restored. But in the preceding Memoir so full an account of Dr. SIMSON's restoration of them has been given, that it is unnecessary in this place to make any further observations respecting them.

With respect to the *τόπος ἀναλυόμενος*, I shall only further observe, that on the revival of mathematical learning in Europe, if the study and application of the ancient analysis had continued, without being nearly superseded by the use of the modern algebra; it is highly probable that later geometricians would, from time to time, have made additions to the ancient collection; and would have investigated various other general problems, with compleat solutions of their several cases, for the same general purpose for which these ancient books were composed by EUCLID and APOLLONIUS.

At the end of the description of the *Conics* of APOLLONIUS, in this preface to the seventh book of PAPPUS, there are some interesting observations of a more general nature, which have

into any discussion respecting the account of this work by PAPPUS. It is, however, both curious and interesting; and Dr. HALLEY, in his preface to the corrected edition of this preface to the seventh book of PAPPUS, makes some valuable observations on it. Dr. SIMSON adds some useful notes on the *Lemmata* of this work.

been particularly remarked by Dr. HALLEY. I allude to the account of the celebrated ancient problem of the "*Locus ad tres et quatuor rectas;*" and the extension of it mentioned by PAPPUS to the cases where there are a greater number of straight lines than four, to which lines are drawn in given angles from the point, of which the *Locus* is to be investigated. The original *Locus* is solid, but these other cases produce *linear Loci*, that is, curves of superior orders; which, however, had not been investigated in the time of PAPPUS. In some general observations on the subject by PAPPUS, it may be remarked that he mentions the method of expressing dimensions above the cubic, by means of compound ratios.

This last article of the preface is concluded by a distinct enunciation of the celebrated theorem of GULDINUS, which, twelve centuries after the age of PAPPUS, excited much curiosity and admiration. Whether this theorem was invented by PAPPUS, or by some other geometer, is not stated. It is proper to remark, however, that no imputation is conveyed on the originality of the discovery by GULDINUS; as in COMMANDINE's translation, which was the only account then in print, the Proposition is altogether unintelligible.* But as this portion of the preface has not been considered in Dr. SIMSON's notes,

* The improved translation of this preface of PAPPUS, by Dr. HALLEY, appeared only in 1706, long after the time of GULDINUS; and in it this theorem from PAPPUS was first intelligibly published. MONTUCLA indeed remarks, (tom. i. p. 325,) that the work of GULDINUS was published before the second edition of COMMANDINE's PAPPUS in 1660. But that edition could not have given him any aid, for it contained no valuable improvement of the former; and in this particular point, it is an exact transcript of the passage in the first edition.

it becomes unnecessary in this place to enter into any further discussion of it.

Having given so particular an account of the preface of the seventh book, (certainly one of the most curious remains of ancient geometry,) a very short statement of the contents of the book itself will be sufficient. It consists of a great number of Propositions, (238,) some of which are problems, but the greater part theorems. They are called *Lemmata*, and divided into classes according to the several treatises described in the preface, to which they respectively belong. They appear in general to be elementary Propositions assumed, but we may suppose not demonstrated, in these ancient books, from which they were collected probably by PAPPUS; and the collector has added the demonstrations. Some of them indeed are curious and important Propositions; but the greater number of them are chiefly valuable for the aid which they give in the attempts of modern geometers in restoring accurately the lost books of the *τύπος ἀναλυόμενος*. In such attempts, it was an interesting object, not merely to resolve the general problems contained in that collection, with their cases and determinations, but also, where it was possible, to follow the particular mode of solution employed by the original authors. These *Lemmata*‡ preserved by PAPPUS have

‡ According to PROCLUS, in his comment on Prop. 1. I. Elem. a Lemma (sumptio BAROC.) is an assumption of a geometrical truth, in the demonstration of a Proposition; which truth, however, has not been demonstrated by known geometrical writers. In this sense is the term used by PAPPUS, and the *Lemmata* of the seventh book seem to be of that description. It appears from the books of APOLLONIUS'S *Conics* still preserved, that the *Lemmata* of PAPPUS belonging to

supplied the means of doing this; and when proper analyses and compositions were discovered, which required also the use of the particular Lemmata mentioned by PAPPUS, there was a high degree of probability, that such solutions were truly those given by EUCLID and APOLLONIUS.

Many necessary corrections of these Lemmata, as they appeared in COMMANDINE's translation, were made by Dr. SIMSON. Some of the corrections were supplied by the Parisian MS. of PAPPUS, of which Dr. MOOR procured a copy, as has formerly been mentioned; and they are remarked in the Doctor's restorations of the *Loci Plani*, and *Sectio Determinata*. In the Lemmata belonging to the Porisms, which are all given in his posthumous work on that subject, and in an improved state, are many important emendations; but it does not appear

that work were assumed by him, in the manner now mentioned; and Dr. HALLEY, in his edition of APOLLONIUS, gives these Lemmata from the Savilian MSS. of PAPPUS. In the demonstrations of the Propositions of APOLLONIUS, Dr. HALLEY refers occasionally to those Lemmata, as the want of them appeared in the text for rendering the demonstrations complete. It appears likewise from the same edition, (p. 97 of the translation of vith, viith, and viiith, books) that ABDOLMELEC SCHIRAZITA, an Arabian epitomizer of the *Conics*, had collected some other Lemmata, (8.) besides those in PAPPUS, which were assumed without proof in the demonstrations of the seventh book, and which therefore he prefixes to that seventh book; and they are placed by Dr. HALLEY immediately after the Lemmata of PAPPUS belonging to it.

In modern times, however, a Lemma is understood to be an easy preliminary Proposition, which might have been incorporated with the more important one to which it is prefixed, but is more conveniently detached from it, and sometimes is itself a Proposition deserving notice. In the treatise of ARCHIMEDES on the Sphere and Cylinder (before Prop. 17. b i.) is an easy Lemma demonstrated by himself; and at the end of Prop. 17th are several Lemmata, which he states to have been demonstrated by those who preceded him; and are Propositions of the xith b. of EUCLID. They are in subsequent Propositions assumed as known, but a formal quotation of preceding works was not the practice of the ancient geometers.

that any of these last were derived from that MS. as he never quotes it; and most of them, indeed, were made before Dr. MOOR obtained that copy of the seventh book, †. A considerable number of the Lemmata of the seventh book (about 90) have been published, as corrected by Dr. SIMSON; and the attention and accuracy with which he has examined these Propositions, will be obvious to the intelligent reader, by comparing them with the edition of them in COMMANDINE'S translation. And though the remaining Lemmata in this book do not appear to have been so particularly considered by the Doctor, yet there are in his notes on them many necessary corrections, with useful explanations, some of which have been mentioned in the notes added to this memoir.

The Lemmata in this seventh book belong to the several treatises described in the preface; according to the annexed arrangement from COMMANDINE'S translation; in which enumeration, however, are some irregularities, as is observed by Dr. SIMSON.

† In Dr. SIMSON'S PAPPUS is a memorandum for enquiry if the copy of the *Codex Regius* contained any of the necessary emendations which he had made on a Lemma (Prop. 130.) of the Porisms, but no mention is made of the result; which is a further presumption that the Doctor, though he had had the use of the MS. for correcting the Lemmata of the two treatises above mentioned, yet that he had not had an opportunity of consulting it respecting the Porisms.

Treatises.		No. of Lemmata.
No. 1.	APOLLONIUS, <i>De Sectione Rationis et Spatii,</i> } from beginning to Prop. 21 incl. }	21
No. 2.	————— <i>De Sectione Determinata,</i> to Prop. 64.	43
No. 3.	————— <i>De Inclinationibus,</i> to — 95.	31
No. 4.	————— <i>De Tactionibus,</i> to — 118.	23
No. 5.	————— <i>De Locis Planis,</i> to — 126.	8
No. 6.	EUCLIDES, <i>De Porismatis,</i> to — 164.	38
No. 7.	APOLLONIUS, <i>De Conicis,</i> to — 234.	70
No. 8.	EUCLIDES, <i>De Locis ad Superficiem,</i> to — 238.	4*
		238

Of these Lemmata, since COMMANDINE's translation, have been published:

By Dr. HALLEY, Greek and Latin, No. 1, and 7.	- - 91
By Dr. SIMSON, in Latin, and corrected, Nos. 2, 5, and 6.	89
By CAMERER, Greek and Latin, No. 4.	- - - - 23
	203

A few others have been mentioned in this Memoir as corrected by Dr. SIMSON and others.

It is proper also to mention in this place, that a Proposition of some curiosity, though not in COMMANDINE's translation, is found at the end of this book in the two Parisian MSS. and also in those belonging to the Savilian Library at Oxford. It is entitled *λήμμα τῆ ἀναλυομένης τόσε*; and an accurate copy of it, from the Parisian MSS. is placed in Dr. SIMSON's PAPPUS, from which a translation by the Doctor is added.

* Four only in COMMANDINE's translation are numbered as Propositions, though it seems that five Lemmata were intended. And it is also to be remarked, that the four last lines of fol. 300. a. COMMANDINE, though printed in the character of the Commentary, are part of the text, of which the Greek is to be found in Savil. MS. No. 3. Dr. SIMSON, however, does not appear to have considered these Lemmata; at least he has left no notes respecting them. See the Memoir, p. 48.

“ Propositio PAPPI ALEXANDRINI quam ex Cod. MSS.†
 “ Bibliothecæ Parisiensis paucis abhinc Septimanis exscripsit
 “ D^{nus} JACOBUS MOOR, collega meus, 7^{mo} Nov^{ris} 1748.

“ Si latera circa angulum rectum trianguli BAC, (Fig. 10.)
 “ viz. BA AC, secentur in K, L, ita ut tam BK ad KA quam AL
 “ ad LC sit ut BA ad AC, et jungantur BL CK, sibi mutuo
 “ occurrentes in F; et ad basim BC ducatur AFG, erit AG ad
 “ BC perpendicularis.

“ Ducatur enim CE parallela ipsi BA occurratque ipsis BF,
 “ AG, (productis) in E, H; et puta verum esse theorema, sc.
 “ angulum AGB rectum esse: æquiangula igitur sunt BAC
 “ ACH triangula, quia ABC angulus æqualis est ipsi GAC seu
 “ HAC (8. 6.) et recti sunt anguli BAC, ACH; ut igitur BA ad
 “ AC, ita AC ad CH. Est autem propter parallelas EC ad CH
 “ ut (BK ad KA hoc est ex hypothesi, ut BA ad AC hoc est ut)
 “ AC ad CH: æquales igitur sunt EC, CA. Et propter parallelas
 “ est AL ad LC ut BA ad CE, hoc est ad AC, quod quidem
 “ verum est ex hypothesi.

“ Componetur vero ita. Eadem manente constructione,
 “ quoniam ex hypothesi est BA ad AC ut AL ad LC, hoc est
 “ ut BA ad CE; erunt AC, CE, æquales. Est itidem ex hypo-
 “ thesi BA ad AC ut (BK ad KA hoc est ut) CE seu AC ad
 “ CH: ergo (6. 6.) æquiangula sunt triangula BAC, ACH, et
 “ angulus ABC æqualis ipsi HAC seu GAC, et in triangulis ABC,
 “ GAC communis est angulus ACB, ergo reliquus AGC æqualis
 “ est reliquo BAC, sc. angulo recto. Q. E. D.

“ N. B. In Prop. PAPPI facile ostenditur AK, AL, æquales esse.”

† “ Viz. No. 2368, et No. 2440.

In the same MS. volume from which the preceding Proposition is copied, there is the following different statement of it :

[The same figure.]

“ Si fit triangulum ABC rectum habens angulum BAC,
 “ ducatur autem BD parallela ipsi AC æqualis autem rectæ
 “ BA; et CE parallela ipsi BA æqualis autem ipsi AC; et CD
 “ BE jungantur sibi mutuo occurrentes in F, et juncta AF
 “ occurrat basi BC in G, erit AG perpendicularis ad BC.”

This Proposition, which is equivalent to the preceding, was suggested by the figure of the 47 Prop. 1. EUCL. and had been demonstrated by Dr. SIMSON, before he met with the other. There is no intimation in the manuscripts hitherto examined of the purpose for which this Proposition was placed at the end of this book; and though it is called a lemma of the *τόπος ἀναλυόμενος*, there is no apparent connection between it and any of the treatises described in this book, or with any of the *Lemmata* belonging to them; but from its being found in so many manuscripts of PAPPUS, there is a presumption of its having been placed in the *Mathematical Collections* by him.

§. VII. *The Eighth Book.*

THE object of this last book* is to give some account of the ancient mechanics; and though a curious document of the state of that branch of science in the time of PAPPUS, yet from

* The title from MS. BULL. is “ Πάππου Ἀλεξανδρείας συναγωγῆς ἧν περιέχει δὲ
 “ μηχανικὰ προβλήματα σύμμικτα δ' ἀθηναῖα.”

§ σύμμικτα is wanting in MS. Savil. No. 3.

the great improvement both in the theory and practice of mechanics in modern times, it is comparatively of little value.

The original genius of ARCHIMEDES was distinguished in this department, as appears in part even from the portions of his numerous writings which have been preserved, and also from the many references to his mechanical inventions in this book of PAPPUS, and in many other ancient writers. A considerable part of this book is employed in describing what were then, and still are, called the Mechanic Powers, and the most obvious combinations of them for the common purposes of life; especially for raising up and for drawing very great weights.* PAPPUS avows its being chiefly borrowed from HERO, a distinguished geometer and mechanic, whose works he frequently quotes, and some of which still remain.† There is a long preface, which from some statements of the mechanical notions, and of the arts of that period, becomes curious. It contains also some observations on the utility of mechanics, and of the connection of mechanics with geometry; in it also are distinguished several branches of that science, with notices of treatises on it which are lost. Besides many references to HERO, there are very ample testimonies in this preface of the same of ARCHIMEDES, from his mechanical writings

* Towards the end of the book are descriptions of the ancient machines for these purposes; and in the MS. copies are designs of such machines made from these descriptions. It may be observed, however, that those drawings are different from each other, and from the engravings in COMMANDINE, which no doubt had been copied from sketches in the MS. which he followed.

† This is the elder HERO, of Alexandria, who is supposed to have lived about fifty years after ARCHIMEDES.—GER. VOSSIUS.

and inventions, † in many of which his geometrical and arithmetical science was employed. But as has already been remarked, the chief value of this book is from the information which it affords of the state of mechanical science in the age of PAPPUS. I must observe also, that though Dr. SIMSON makes a few corrections on this book, he seems never to have considered it particularly, and therefore it is unnecessary to give a more particular account of its contents. §

§. VIII. Conclusion.

FROM the short account which has been given of the several books of PAPPUS, the miscellaneous nature of the Collection is sufficiently apparent. Several ancient mathematicians are mentioned by him, of whom no other notice remains; and

† In Prop. 10th, he mentions the well-known observation of ARCHIMEDES, that with a fixed station he could move the earth. And the resolution of the general mechanical problem, “to move a given weight with a given power,” he calls the 40th invention of ARCHIMEDES.

§ It is proper, however, to mention an observation of Dr. SIMSON on Prop. 14 of this book: “Addatur hic (post verbum *quidem* in linea quarta hujus paginæ, viz. fol. 320. b.) “*Geometricè* ut dudum emendavit GREGORIUS a ST^o. VICENTIO, pag. 291.” *Quad. Circuli*.—In the same place is also the following note, referring to the *Codex BULL.*; he says, “In quo post verba *ἀρχαίως εὐρεῖν*, legitur *μεθοδεύεται* δε τὸν τζόπων τέλει. “Verbo vero *μεθοδεύεται* significatur constructio geometrica.” GREGORY also makes another remark on COMMANDINE’s Note B. in the same page of PAPPUS, (1588.) The last sentence of the enunciation of Prop. 12, (fol. 317. a.) in COMMANDINE’s translation is “*Invenitur autem methodo investigata hoc pacto;*” but it ought to have been “*investigabitur autem geometricè ita;*” the Greek (MS. BULL.) being “*εὐρισκίται μεθοδεύθεν ἕτως.*” Dr. SIMSON also remarks, respecting the last sentence of COMMANDINE’s Note B. on Prop. 14, viz. ‘*Mirum est PAPPUM,*’ &c. “*Non mirum quoniam ex elementis conicis satis hujus demonstratio patet.*”

many curious particulars are detailed respecting the ancient state of mathematics, and of the problems and theorems which engaged the attention of the geometers of those times. It is written with only a general attention to method; there is little uniformity in the style, and the work has probably been composed at different times. Some propositions are demonstrated minutely, and rather diffusely, while in others many important steps are omitted,* as is often remarked by COMMANDINE, and these omissions he has generally supplied. As a further proof of this character of the *Collections*, the very frequent repetitions in different parts of it, of the same Propositions, sometimes with the same, and sometimes with different demonstrations, may be mentioned.‡ It is, however, a most interesting work, and more especially from the account of the analytical geometry of the ancients in the seventh book; an edition of the Greek, from a collation of the many manuscripts which are known still to exist in the libraries of Europe, would be most acceptable to all the admirers of the elegance and accuracy of ancient geometry; and for such an undertaking Dr. SIMSON's notes

* Dr. SIMSON makes a remark to this purpose, *Opera Reliqua*, p. 526, at the end of Prop. 76. Many of the imperfections of PAPPUS, as they appear in the existing MSS., must be attributed to the transcribers.

‡ The following examples may be remarked. The two statements of the ancient classification of problems, in b. iii. fol. 4. b. and b. iv. fol. 60. The two accounts of the Conchoid in these two books. The two solutions of the Deliacal Problem, b. iii. fol. 7, and b. viii. Prop. 2, nearly in the same words. In Dr. SIMSON's edition of the *Sectio Determinata*, several duplicates are mentioned. Other repetitions also might be pointed out, such as Props. 31, 57, 178, 194, book vii. Also Props. 87, 153, 206, in the same book. Also Props. 23, 58, 193, b. vii. Several of these are remarked by Dr. SIMSON, in his notes, both printed and unpublished. See *Opera Reliqua*, pp. 409, 410.

would be highly valuable.‡ He does not seem to have taken a regular and accurate survey of the whole, with any view to the immediate completion of such a work; nor has he considered every passage which would require correction or explanation; yet his emendations and illustrations, several of which have been mentioned in this Memoir, will be an important repository of materials for the assistance of any future editor.

It is proper likewise to observe, that among Dr. SIMSON'S notes are several generalizations of Propositions in PAPPUS, and also some connected Propositions, which, though valuable in themselves, may probably not be all considered as properly belonging to a new edition of that author. Several also of the Doctor's notes contain corrections of COMMANDINE'S commentary, most of which might be omitted in conducting a new edition; of which the proper object must be, to give the text of PAPPUS free from errors, and to introduce such comments only as are necessary for explaining real difficulties, and for filling up material deficiencies in the demonstrations. It may be also observed, that from what is known respecting several of the remaining MSS. and the similarity of their defects, it is much to be apprehended that few things of material importance, beyond what Dr. SIMSON has already remarked, are likely to be discovered; though an accurate examination of these MSS. would certainly be most desirable.

‡ By comparing Dr. SIMSON'S MS. notes on the Lemmata of the seventh book with his corrected edition of these Lemmata in his restorations of the *Loci Plani Sectio Determinata*, and *Porisms*, his taste and judgment may be remarked in selecting only those notes which were necessary, and omitting the others which he had written on his first review of these Propositions.

After every aid is obtained from remaining MSS. the ability and intelligence of an editor must be depended on for the best use of existing materials; and it is to be considered, that, in geometry, the nature of the subject, and the train of argument, may often enable an editor to supply the defects and correct the errors of manuscripts, to which, indeed, those on mathematical subjects appear to be more liable than any others.

The copy of COMMANDINE'S PAPPUS which Dr. SIMSON used, and in which he wrote his notes, was the first edition PISAURI, 1588, and VENETIIS, 1589, and to this all quotations in this memoir refer.

It is necessary also to mention, that a few years after the Doctor's death, an application was made to his executor Mr. CLOW, on the part of the Delegates of the Clarendon Press, in the University of Oxford, for a transcript of all the MS. notes in this volume, as they were left by the Doctor. This was readily complied with, on the reasonable condition, that whatever notes and corrections of Dr SIMSON's might be adopted in a new edition of PAPPUS, they should be particularly distinguished and acknowledged as his. Some time after, Mr. CLOW deposited this valuable *legacy* from his deceased friend, in the library of the College of Glasgow; and having had, by the favour of the Principal and Professors, every convenience for consulting it, I have been enabled to give from it some interesting particulars of Dr. SIMSON's geometrical studies.

§. IX. *Of the MSS. of PAPPUS.*

A Considerable number of manuscripts of the collections of PAPPUS are to be found in various libraries of Europe, but all of them which have been examined, are mutilated; and contain many errors, from the ignorance or carelessness of the transcribers.†

COMMANDINE appears to have had the use of only one manuscript, of which the history and fate are unknown; but from his numerous corrections of it in his commentary, in which he generally inserts the erroneous passage of the MS. a judgement may be formed of its very deficient state. In it, as in several other MSS.‡ the two first books are entirely wanting.

In several MSS. however, a portion of the second book (about one half) is preserved. This was found in one of the Savilian MSS. viz. No. 9, and published by Dr. WALLIS, in 1688, with learned notes. It is understood also, that one.

† Dr. SIMSON, in a note on a Proposition, of the seventh book of PAPPUS, remarks, “non pauca autem in hac Propositione, in eo codice (Parisiensi Regio sc.) vitiata sunt, ut in omnibus fere PAPPI Propositionibus, et, ut videtur, in omnibus manuscriptis.”

‡ The same is to be remarked of the Savil. MS. No. 3; also of a MS. of PAPPUS in the Neapolitan Royal Library, mentioned by FABRICIUS *Bibl. Græc.* tom. v. page 790, Hamburg, 1798, &c. In the MS. BULLIALDI, afterwards to be mentioned, the two first books also are wanting. GER. VOSSIUS, *de Scientiis Math.* cap. xvi. 7, when stating the books of the *Mathematical Collectionss* adds, “sed duo primi videntur deperisse.”

of the Parisian MSS. No. 2368, has this portion of the second book, beginning at the same words as the fragments in the Savilian MS.‡ From a MS. of PAPPUS, which belonged to JOSEPH SCALIGER, the portions of the preface to the viith book, describing the *Sectio Rationis* and *Sectio Spatii*, were published in Greek by SNELLIUS, in his restoration of these books, in 1607; and his edition of the Greek preface to the *Sectio Determinata*, 1608, was probably from the same MS.

The MS. of PAPPUS which was in the library of ISAAC VOSSIUS, and was carried from England to the University of Leyden, had also about one half of the second book*; and probably, the precise portion which is in the other MSS. now mentioned.

In the edition of the *Bibliotheca Græca*, published by HARLES, vol. ix. p. 170, is a catalogue of the MSS. of PAPPUS known to that writer, including those now mentioned and several others. I shall in this place therefore mention only two more, not taken notice of by him. In the year 1795, J. GUGL. CAMERER published an edition of the APOLLONIUS GALLUS by VIETA; and prefixes to it the preface and Lemmata belonging to the Tactions, in Greek. For this purpose he made use of three MSS. the two in the Parisian library No. 1440, and No. 2368; and another in the Strasburgh library, not mentioned in Fabricius; but from the very short references

‡ A MS. copy of this fragment of the second book, (with the third book also,) taken from this Parisian MS. is in the Savilian Library at Oxford; and from a note at the end of this copy, it appears that the Parisian MS. was written in 1562.

* See the account of ISAAC VOSSIUS in the *Biog. Brit.* This MS. is mentioned in the Oxford Catalogue of Manuscripts, No. 2126.

to that MS. nothing can be pronounced respecting its history and value.

There is also at Edinburgh an elegant manuscript of five books of PAPPUS, viz. the third, fourth, fifth, sixth, and eighth; but unfortunately the seventh, the most valuable portion of the work, is wanting.

This MS. was purchased at Paris, in 1748, by Dr. JAMES MOOR, then Greek professor at Glasgow, from Mr. DE MAIRAN, of the Academy of Sciences; and on the first page of it is written *D'Ortous de Mairan*, probably by his own hand. DE MAIRAN had informed Dr. MOOR that it had belonged to BULLIALDUS; and on that account Dr. SIMSON, who had it for some time in his possession, and took many notes from it, calls it *Codex BULLIALDI*.† Like the other MSS. of PAPPUS it abounds with errors; but Doctor SIMSON obtained from it several improved readings and corrections, which he has remarked in his copy of COMMANDINE'S translation. Some

† Dr. SIMSON, in his copy of COMMANDINE'S PAPPUS, has the following notices of this MS. which, without doubt, he had from Dr. MOOR; and which may therefore be considered as authentic. In fol. 320. b. (PAPPI, edit. Pis. 1588.) “3^{tio} Nov^{ris} A. D. 1748. Predictam GREGORII (a Sanct. Vinc.) emendationem veram esse, ostendit “codex manuscriptus elegantissimus, quem tribus abhinc diebus Parisiis huc attulit “D. JACOBUS MOOR, collega meus doctissimus, &c.”

Also in fol. 1. b. PAPPI, “Hisce litteris adscriptis emendavimus tum schema “COMMANDINI tum MSⁱ GRÆCI quem Dom. *D'Ortous de Mairan* dicit fuisse “BULLIALDI. R. S.”

In a detached paper, referring to fol. 77. b. PAPPI, is a note in Dr. SIMSON'S hand, “25 Martii 1750, inveni hæc ita esse in codice GRÆCO qui BULLIALDI fuerat, “quemque collega meus D. JACOBUS MOOR, a domino Mairan emptum Parisiis huc “(Glasguam) attulit.”

of them are curious and important, and all merit the attention of any future editor of that work.

Some peculiarities in this MS. may be mentioned, for the satisfaction of those who may be inclined to consult it.

At the end of the third book are five pages of MS. not in COMMANDINE's edition, which contain some variations of the solution of the Deliacal problem, in addition to those contained in the beginning of that book. §

At the end of the fourth book is a very imperfect sketch of a Proposition, not in COMMANDINE, which has already been mentioned in the preceding account of this fourth book.

In this MS. are many corrections on the margin, and some even in the text, by a later hand; and among them are a great number of the emendations proposed by COMMANDINE of the very same errors, in the MS. used by him, from which a connection between these two MSS. may be inferred. Many of these emendations are servilely copied, retaining even the mistakes into which COMMANDINE, in a few of them, had fallen.

This MS. though elegantly written, has been copied by a person totally ignorant of the subject; of which the number of gross errors in its first state is a sufficient proof. It may be remarked, also, that at the beginning of a Proposition, or of a paragraph, there is usually a red letter, and a new line; but

§ To these additional pages the following title is prefixed, “*ἄλλως.—το δέκαλον θεωρήμα ἐν τῷ τρίτῳ τῆς τοῦ Πάππου συλλογῆς καὶ τὴν ἀποδείξιν περιέχον, καὶ τὴν ὀργάνικην κατασκευὴν τῆς διπλασιασμῆς τοῦ κύβου καὶ τῶν δύο μεσῶν ἀνάλογον.*”—It may be observed that the SAVIL. MS. No. 3, has a similar addition at the end of the 3d book.

frequently also this distinction is made in the middle of a sentence, while the beginning of Propositions and subjects in other places is not distinguished in that or in any other manner. The enumeration of the Propositions is often irregular, and many of the diagrams, though neatly drawn, are altogether erroneous. The MS. was sold at Dr. MOOR's death, and was afterwards purchased for the library of the Faculty of Advocates at Edinburgh, in which the writer of this Memoir had every facility for consulting that MS. and other books in that great collection. From the well-known liberality of the Curators of that establishment, the aid of this MS. will, without doubt, be most readily given, when it can be useful for preparing an edition of that interesting work in the original language.

This MS. not having the seventh book, Dr. MOOR procured a copy of it to be taken from the MS. No. 2368, in the Parisian library. This also for some time was in Dr. SIMSON's possession, and he adopts from it some corrections of COMMANDINE's translation in his restoration of the *Loci plani*, and *Sectio determinata*; but, as was already mentioned, there is no reference to it in his exposition of the Porisms†. I add with regret, that this copy, in the dispersion of Dr. MOOR's library, seems to have been lost; and these particulars are stated, as they may perhaps facilitate the recovery of it from some obscure situation into which it may have accidentally fallen.

† In a note on COMMANDINE's PAPPUS, the Doctor gives the following account of this transcript, which he must have got from Dr. MOOR "Hunc autem librum 7^{mum} PAPPI ex eo codice (scil. No. 2368, Reg. Bibl. Par.) descripsit Dom. CAPERONIER, linguæ Græcæ in Academia Parisiensi Professor; sumptibus D. JAC. MOOR, collegæ mei doctissimi. Schemata vero ejus depinxit D. JOA. BRISBANE, M. D."

The two Savilian MSS. No. 3, and No. 9, have been repeatedly referred to. There is also in that library a transcript from the Parisian MS. No. 2368, of a portion of the second book of PAPPUS, (the same as the fragment in MS. SAVIL. No. 9, published by Dr. WALLIS,) and the whole of the third book. This transcript is not mentioned by Dr. WALLIS; and in the following Appendix II. it is quoted as MS. SAVIL. B. From a note at the end, it appears that the Parisian MS. No. 2368, was written in 1562, by the direction of P. RAMUS.

APPENDIX II.

TWO passages of PAPPUS, on the ancient division of geometrical lines into classes, have, in the preceding Memoir, been repeatedly alluded to. For the satisfaction of those who may wish to consider them accurately, the Greek, from the SAVILIAN MSS. and the MS. BULL. is printed in this Appendix, along with COMMANDINE's translation. Some of the various readings in these MSS. are remarked, but without taking notice of either the smaller differences, which do not affect the sense, or of some gross errors, manifestly arising from the carelessness of the transcribers.

Some of COMMANDINE's notes are also mentioned; but as Dr. SIMSON has not left any observations on these passages, it is not my purpose to enter into any examination or explanation of them, except only by pointing out the references to some Propositions, not explicitly stated either in PAPPUS, or in the commentary of his translator.

PAPPI, MS. SAVIL. No. 3. Fol. 10, b.

“ Ἄ μὲν ἔδει με προαιτεῖν ἔστι ταῦτα. Παοίς
 “ δὲ κρίνειν σοί τε καὶ ἄλλοις τοῖς ἐν γεωμετρίᾳ
 “ γεωμετρουμένοις τὰ ὑπ’ ἐκείνου προσγραφέντα ||
 “ περὶ τῆς κατασκευῆς, καὶ τὰ ὑφ’ ἡμῶν ἐπι-
 “ λεχθέντα καλῶς ἔχειν ἠγνοῦμαι, καὶ τὰ δόξαντα
 “ τοῖς ἀρχαίοις περὶ τοῦ προειρημένου προβλή-
 “ ματος ἐκθίσθαι. Καὶ πρῶτον εἰπεῖν ὀλίγη
 “ περὶ τῶν ἐν γεωμετρίᾳ προβλημάτων ἀρχὴν
 “ λαβὼν ἐντεῦθεν. Τῶν ἐν γεωμετρίας προ-
 “ βλημάτων οἱ παλαιοὶ τρία γένη φασὶν εἶναι.
 “ Καὶ τὰ μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ
 “ δὲ στερεὰ, τὰ δὲ γραμμικά. Τὰ μὲν οὖν δι-
 “ εὐθείας καὶ κύκλου περιφέρειας δυνάμενα
 “ λύεσθαι λέγοιτο ἂν εὐκότως ἐπίπεδα. Καὶ
 “ γὰρ αἱ γραμμαὶ δι’ ἂν λύεται τὰ τοιαῦτα
 “ προβλήματα, τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδῳ.
 “ ὅσα δὲ προβλήματα λύεται παραλαμβανο-
 “ μένης εἰς τὴν γένεσιν μιᾶς τῶν τοῦ κώνου
 “ τομῶν, ἢ πλειόνων, ταῦτα στερεὰ κέκληται.
 “ πρὸς γὰρ τὴν κατασκευὴν ἀναγκασίον ἔστι
 “ χρῆσθαι στερεῶν σωμάτων† ἐπιφανείαις.
 “ Λέγω δὲ τὰς κωνικάς. Τρίτον δ’ ἔτι κα-
 “ ταλέπιται γένος, ὃ καλεῖται γραμμικόν.

“ Quæ igitur me præmississe oportebat, hæc sunt. Itaque omittens
 “ explicare et tibi, et iis, qui in geometria exercitati sunt, ea, quæ ille
 “ scripsit de constructione, et quæ nos objecimus; optimum fore iudicavi, si exponerem quid antiqui
 “ de dicto problemate senserint: et
 “ primum nonnulla dicerem de problematibus, quæ in geometria considerantur, inde sumpto initio.

“ Problematum geometricorum antiqui tria genera esse statuerunt, et eorum alia quidem *plana* appellari, alia *solida*, alia *linearia*. Quæ igitur per rectas lineas, et circuli circumferentiam solvi possunt, merito *plana* dicantur; etenim *linearæ*, per quas ejusmodi problemata solvuntur, in plano ortum habent. Problemata vero quæcunque solvuntur, assumpta in constructionem aliqua conisectione, vel pluribus, *solida* appellantur; namque ad constructionem necesse est solidarum figurarum superficiesibus, nimirum conicis, uti. Restat tertium genus, quod *lineare* appellatur. *Linearæ* enim aliæ præter jam

|| προσγραφέντα, MS. B. SAVIL.

† σχημάτων, idem.

“ γραμμᾶί γὰρ ἕτεραι παρὰ τὰς εἰρημίνας εἰς
 “ τὴν κατασκευὴν λαμβάνονται πικιλωτέραν
 “ καὶ μεταπλασμένην ἔχουσαι τὴν γένεσιν.
 “ ὅπῃαι τυγχάν* (οὐσι αἱ ἕλικες) καὶ τετρα-
 “ γωνίζουσαι καὶ κογχλοειδῆς, † καὶ κισσοει-
 “ δῆς, πολλὰ καὶ παραδόξα περὶ αὐτὰς ἔχουσαι
 “ συμπτώματα. Τοιαύτης δὲ τῆς διαφορᾶς
 “ τῶν προβλημάτων οὗσης, οἱ παλαιοὶ γεωμέτραι
 “ τὸ προειρημένον ἐπὶ τῶν δύο εὐθειῶν πρόβλημα
 “ τῆ οὐσι στερεὸν ὑπάρχον, οὐχ οἰοίτε ἦσαν
 “ κατασκευάζειν τῷ γεωμετρικῷ λόγῳ κατα-
 “ κολουθοῦντες. ἐπεὶ μὴδε τὰς τοῦ κώνου τομὰς
 “ ῥᾶδιον ἐν ἐπιπέδῳ γράφειν ἦν.” [ὡς δὲ δύο
 “ δοθεισῶν εὐθειῶν ἀνίσταν, δύο μέσας ἀνάλογον
 “ λαβὴν ἐν συνεχῇ ἀνάλογίᾳ.] Τῶς δὲ ὀργάνοις
 “ μεταλαβόντες αὐτὸ θαυμασίως εἰς χειρουργίαν
 “ καὶ κατασκευὴν ἐπιτίθειον ἔγαγον. ὡς ἐστὶν
 “ ἰδεῖν ἀπὸ τῶν φερομένων αὐτοῖς συντηγμάτων.
 “ λίγῳ δ’ ἐν τῷ ἐρατοσθένους μεσολάβῳ, καὶ
 “ τῶς φίλωνος καὶ ἥρωνος μηχανικοῖς, ἢ κατὰ
 “ πικλιχοῖς. οὗτοι γὰρ ὁμολογοῦντες στερεὸν
 “ εἶναι τὸ πρόβλημα, τὴν κατασκευὴν αὐτοῦ
 “ μόνον ὀργανικῶς πεποιήθηται, συμφώνως ἀπολ-
 “ λωτίῳ τῷ περιγίῳ. ὅς καὶ τὴν ἀνάλυσιν
 “ αὐτοῦ πεποιήθηται διὰ τῶν τοῦ κώνου τομῶν.
 “ Καὶ ἄλλοι διὰ τῶν ἀριστάϊου τόπων στερεῶν,

“ dictas in constructionem affumun-
 “ tur, varium, et transmutabilem or-
 “ tum habentes, quales sunt helices,
 “ et [quas Græci τετραγωνίζουσας appel-
 “ lant, nos] quadrantes [dicere possu-
 “ mus,] conchoides, et cissoïdes, qui-
 “ bus quidem multa, et admirabilia
 “ accidunt. Cum igitur tales sint pro-
 “ blematum differentiarum, antiqui geo-
 “ metrae problema ante dictum in
 “ duabus rectis lineis, quod natura
 “ solidum est, geometrica ratione
 “ innixi construere non potuerunt;
 “ quoniam neque conic sectiones facile
 “ est in plano designare: instrumentis
 “ autem ipsum in operationem manu-
 “ alem, et commodam, aptamque
 “ constructionem mirabiliter tradux-
 “ erunt, quod videre licet in eorum
 “ voluminibus, quæ circumferuntur,
 “ ut in ERATOSTHENIS mesolabo,
 “ in PHILONIS, et HERONIS me-
 “ chanicis et catapulticis. Hi enim
 “ afferentes problema solidum esse,
 “ ipsius constructionem instrumentis-
 “ tantum perfecerunt, congruentur
 “ APOLLONIO Pergæo, qui et resolu-
 “ tionem ejus fecit per conic sectiones:
 “ alii per locos solidos ARISTÆI:
 “ nullus autem per ea, quæ proprie

§ βεβιασμένη, idem.

* In MS. Savil. No. 3, quæ inter uncus, rubro liquore exarata a secunda manu.—In altero codice Savil. “ αἱ ἕλικες” absunt. In MS. BULL. τυγχάνουσι αἱ . . . καὶ.

† κογχλοειδῆς, MS. B. SAVIL.

|| Quæ inter uncus, in COMMANDINI versione omittuntur; sed quamvis supervacanea, forsitan non erronea.

“ οὐδείς δὲ διὰ τῶν ἰδίων ἐπιπέδων καλομένων.
 “ Νικομήδης δὲ καὶ λόγῳ διὰ κογχλαιοῦ*
 “ γραμμικῆς, δι’ ἧς καὶ τὴν γωνίαν ἐτριχότο-
 “ μισεν. ἐκθησόμεθα οὖν τέσσαρας αὐτοῦ κατα-
 “ σκυάς, μετὰ τινος ἐμῆς ἐπιξεργασίας. ὡς
 “ πρῶτην μὲν ἐρατοσθένειον, δευτέραν δὲ τὴν
 “ τῶν περὶ νικομήδην, τρίτην δὲ τὴν τῶν περὶ
 “ ἥρωνα μάλιστα πρὸς τὰς χειουργίας ἀρμό-
 “ ζουσαν, τοῖς ἀρχιτεκτονεῖν βουλομένοις. Καὶ
 “ τελευταίαν τὴν ἐφ’ ἡμῶν ἀνευρημένην. στερεοῦ†
 “ γὰρ παντός, ἕτερον στερεον ὅμοιον τῷ δοθέντι
 “ κατασκευάζεται πρὸς τὸν δοθέντα λόγον.
 “ εἰάν δύο τῶν δοθεισῶν εὐθειῶν δύο καὶ ‡ μίσαι
 “ κατὰ τὸ συνεχὲς ἀνάλογον ληφῶσιν, ὡς
 “ ἥρων ἐν μηχανικῆς καὶ καταπαλτικῆς.

“ plana appellantur. At NICOME-
 “ DES, et ratione illud fecit per lineam
 “ conchoidem, per quam et angulum
 “ tripartito dividit. Exponemus igitur
 “ quatuor ejus constructiones una cum
 “ quadam nostra tractatione. Qua-
 “ rum prima quidem est ERATOS-
 “ THENIS, secunda NICOMEDIS,
 “ tertia HERONIS, maxime ad ma-
 “ nuum operationem accommodata,
 “ iis qui architecti esse volunt. Ultima
 “ autem est a nobis inventa: solido
 “ enim quocunque dato, alterum so-
 “ lidum dato simile construitur ad
 “ datam proportionem, si duabus
 “ datis rectis lineis, duæ mediæ in
 “ continua analogia affumantur, ut
 “ inquit HERO in mechanicis et
 “ catapulticis.

* κογχοειδης, MS. SAVIL. B.

† στερεῦ δεδομένα. MS. BULL. et ut videtur MS. quo utebatur COMMANDINUS.

‡ Deest καὶ MS. B. SAVIL. et MS. BULL.

PAPPI, MS. SAVIL. No. 3. Fol. 86, b.

“ Τὴν διθεῖσαν γωνίαν εὐθύγραμμον εἰς τρία
 “ ἴσα τεμεῖν οἱ παλαιοὶ γεωμέτραι θελήσαντες,
 “ ἠπόρησαν, δι’ αἰτίαν τοιαύτην. τρία γένη φασὲν
 “ εἶναι τῶν ἐν γεωμετρίας προβλημάτων, καὶ τὰ
 “ μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ δὲ στερεὰ,
 “ τὰ δὲ γραμμικὰ. τὰ μὲν οὖν δι’ εὐθείας καὶ
 “ κύκλου περιφέρειας δυαίμενα λύεσθαι, λέγοιτ’
 “ ἂν εὐκότως ἐπίπεδα. καὶ γὰρ αἱ γραμμαῖ
 “ δι’ ὧν εὐρίσκεται τὰ τοιαῦτα προβλήματα
 “ τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδῳ ὅσα δὲ
 “ λύεσθαι* προβλήματα, παραλαμβανομένης
 “ εἰς τὴν γένεσιν μιᾶς τῶν τοῦ κώνου τομῶν, ἢ
 “ καὶ πλειόνων, στερεῶ τὰυτα κέκληται. πρὸς
 “ γὰρ τὴν κατασκευὴν χρῆσασθαι στερεῶν σχη-
 “ μάτων ἐπιφανείαις, λέγω δὲ τὰς κωνικαῖς,
 “ ἀναγκαῖον. τρίτον δὲ τι προβλημάτων† ὑπο-
 “ λήπεται γένος, τὸ καλούμενον γραμμικόν,
 “ γραμμαῖ γὰρ ἕτεραι παρὰ‡ τὰς εἰρημέας εἰς
 “ τὴν κατασκευὴν λαμβάνονται, ποικιλωτέραν
 “ ἔχουσαι τὴν γένεσιν, καὶ βεβιασμένην μάλλον,
 “ ἐξ ἀτακτοτέρων ἐπιφανειῶν, καὶ κινήσει ἐπι-
 “ πλεγμένων γενόμεναι, τοιαῦται δὲ εἰσιν αἱ τε
 “ ἐν τοῖς πρὸς ἐπιφανείαις καλουμένοις τόποις
 “ εὐρισκόμεναι γραμμαῖ, ἕτεραι τε τούτων

“ Antiqui geometræ datum angulum
 “ rectilineum tripartito secare volentes
 “ ob hanc causam hæsitaverunt. Pro-
 “ blematum, quæ in geometria confi-
 “ rantur, tria esse genera dicimus; et
 “ eorum alia quidem *plana*, alia *solida*,
 “ alia vero *linearia* appellari. Quæ
 “ igitur per rectas lineas, et circuli
 “ circumferentiam solvi possunt, me-
 “ rito dicuntur *plana*: *linearæ* enim per
 “ quas talia problemata inveniuntur,
 “ in plano ortum habent. Quæcun-
 “ que vero solvuntur, assumpta in
 “ constructionem aliqua conicæ sectione,
 “ vel etiam pluribus, *solida* appellata
 “ sunt, quoniam ad constructionem
 “ solidarum figurarum superficiebus,
 “ videlicet conicis, uti necessarium est.
 “ Relinquitur tertium genus proble-
 “ matum, quod *lineare* appellatur;
 “ lineæ enim aliæ, præter jam dictas
 “ in constructionem assumuntur, quæ
 “ varium et difficilem ortum habent,
 “ ex inordinatis superficiebus, et mo-
 “ tibus implicatis factæ. Ejusmodi
 “ vero sunt etiam lineæ, quæ in locis
 “ ad superficiem dictis inveniuntur,

* λέγει, MS. SAVIL. No. 9.

† τῶν προβλημάτων, idem.

‡ ὡς, idem.

“ ποικιλότεραι, καὶ πολλὰ τὸ πλῆθος ὑπὸ
 “ δημοτρίου τοῦ ἀλεξανδρέως ἐν ταῖς γραμμικαῖς
 “ ἐπιστάσεσι, καὶ φίλωνος τοῦ πύανου, ἐξ ἑπιπ-
 “ λουῆς πλεκτοειδῶν ἢ τε καὶ ἐτέρων παντοίων
 “ ἐπιφανειῶν εὐρισκόμεναι, πολλὰ καὶ θαυμαστὰ
 “ συμπτώματα περὶ αὐτὰς ἔχουσαι. καὶ τινες
 “ αὐτῶν ὑπὸ τῶν νεωτέρων ἠξιώθησαν λόγου
 “ πλείους. μία δὲ τις ἐξ αὐτῶν ἔστιν, ἢ καὶ
 “ παράδοξος ὑπὸ τοῦ μενελάου κληθεῖσα γραμμὴ.
 “ τοῦ δὲ αὐτοῦ γένους ἔτιραι ἑλικεῖς εἰσι τετρα-
 “ γωνίζουσαι τε, καὶ κοχλοειδεῖς καὶ κισσοειδεῖς.
 “ δοκῆ δὲ πως ἀμάρτημα τὸ τοιοῦτον οὐ μικρὸν
 “ εἶναι τοῖς γεωμέτραις, ὅταν ἐπίπεδον προβλήμα
 “ διὰ τῶν κωνικῶν ἢ τῶν γραμμικῶν ὑπὸ τινος
 “ εὐρίσκεται. καὶ τὸ σύνολον ὅταν ἐξ ἀνοικείου

“ et aliæ quædam magis variæ, et
 “ multæ a DEMETRIO ALEXAN-
 “ DRINO [ἐν ταῖς γεωμετρικαῖς ἐπιστάσεσι,
 “ hoc est] in linearibus aggressioni-
 “ bus; et a PHILONE TYANEO ex
 “ implicatione [πλεκτοειδῶν,*] et aliarum
 “ varii generis superficierum inventæ,
 “ quæ multa et admirabilia sympto-
 “ mata continent: et nonnullæ ipfa-
 “ rum a junioribus dignæ existimatæ
 “ sunt, de quibus longus sermo habe-
 “ retur. Una autem aliqua ex ipfis
 “ est, quæ et admirabilis a MENELAO
 “ appellatur.

“ Ex hoc genere sunt lineæ helices,
 “ et quadrantes, et conchoides, et cif-
 “ foides: videtur autem quodam-
 “ modo peccatum non parvum esse
 “ apud geometras, cum problema
 “ planum per conica, vel linearia ab
 “ aliquo invenitur, et ut summatim

ἢ πλεκτοειδῶν. MS. BULL. et ut videtur MS. COMMAND.

* Πλεκτοειδῶν, or πλεκτοειδῶν. This word, as appears from the application of it in Prop. 29. lib. iv. denotes a class of geometrical surfaces, produced by some precise rule, on which mathematical reasoning respecting such surfaces may be founded. By COMMANDINE, and also by VOSSIUS, they are supposed to have got this name from the complex motions by which they are produced. The *Plethoïdes*, in Prop. 29, (see fig. of that Prop. in PAPPUS) is formed by the motion of the straight line LKI passing through the straight line BLN, to which it is always perpendicular, and also through the line produced by the common section of the conic superficies, and of the superficies called *Cylindroïdes*, both mentioned in that Proposition. See note G. p. 104 and 105.

Among the ancients, geometrical curves and figures generally got names from some resemblance which they were supposed to have to common objects. Such are the *Conchoid* and *Cissoïd*, and also the *Lunule*, or *Meniscus*, the *Pelecoïdes*, the *Hippopeda*, and several others mentioned by PROCLUS. (See PROCL. on Def. 4. 8. 1 Elem.)—It is possible that πλεκτοειδης may have had a similar origin, not now to be traced; but the etymology implied in COMMANDINE's translation of it may be right.

“ γένους λύεται, || δὴν ἔστι τὸ ἐν τῷ ἑ τῶν
 “ ἀπολλωνίου κωνικῶν, ἐπὶ τῆς παραβολῆς πρό-
 “ βλημα, καὶ ἐν τῷ περὶ τῆς ἕλικος ὑπὸ ἀρχι-
 “ μέδους λαμβανομένη στερεῶν ἔνυσις ἐπὶ κύκλου,
 “ μηδενὶ γὰρ προσχωόμενον στερεῶν, δυνατὸν εὑρεῖν
 “ τὸ ὑπ’ αὐτοῦ γραφόμενον βεβήημα. λέγω δὴ
 “ τὸ τῆν περιφέρειαν τῶν ἐν τῇ πρώτῃ περιφορᾷ
 “ κύκλου, ἴσην ἀποδείξαι τῇ πρὸς ὀρθᾶς ἀγομένη
 “ εὐθείᾳ τῇ ἐκ τῆς γενέσεως τῆς ἐφαπτομένης
 “ τῆς ἕλικος. τοιαύτης δὲ τῆς διαφορᾶς τῶν
 “ προβλημάτων ὑπαρχούσης, οἱ πρότεροι γεω-

“ dicam, cum ex improprio solvitur
 “ genere, quale est in quinto libro
 “ conicorum APOLLONII † problema
 “ in parabola: et in libro de lineis
 “ spiralibus ARCHIMEDIS: assumpta
 “ solida inclinatio in circulo: fieri enim
 “ potest, ut nullo utentes solido, pro-
 “ blema ab ipso descriptum invenia-
 “ mus. † Dico autem circumferentiam
 “ circuli in prima circulatione descrip-
 “ tam demonstrare æqualem rectæ
 “ lineæ, quæ a principio lineæ spiralis
 “ ad rectos angulos ducitur ei quæ est
 “ circulationis principium, et a recta
 “ linea spiralem contingente termi-
 “ natur. Itaque cum hujusmodi fit
 “ problematum differentia, antiqui

|| λύεται γενέσει, MS. SAVIL. No. 9.

† In COMMANDINE's time, the first four books only of APOLLONIUS's *Conics* had been discovered; and he thence makes no observation on this criticism of PAPPUS, respecting a Proposition in the fifth book. ALEXANDER ANDERSON, however, in his *Exercitationum Mathematicarum Decas Prima*, (Paris 1619,) long also before the fifth, sixth, and seventh books of APOLLONIUS were recovered from the Arabic, inferred from investigations founded on the general description of that fifth book by PAPPUS, that the problem about the Parabola, here alluded to by him, was truly a solid problem; and in *Exerc. V.* (page 24,) he states it, and resolves it. In the fifth book of APOLLONIUS we now find the first case of Prop. 58. is equivalent to the problem of ANDERSON.

† The Proposition of ARCHIMEDES mentioned in this place is the 18th in his treatise *De Helicibus*. In that Proposition the 7th is necessary, and in that 7th Proposition (*De Helicibus*) the inclination alluded to is assumed.—COMMANDINE, in Note C. respecting this inclination, refers to Prop. 128. b. i. of VITELLO's *Optics*. That Proposition is indeed a plane inclination, and is assumed in Prop. 5th of ARCHIMEDES's Treatise. But in Prop. 7. of the same treatise, which (as has been observed) is used in Prop. 18th, a solid inclination is assumed, the same as Prop. 130. b. i. of VITELLO, in which an hyperbola is required. See also ALHAZEN, b. v. Propp. 32, 33. In Prop. 42 of b. iv. of PAPPUS, is a *Locus ad Hyperbolam*, which he states to have been assumed by ARCHIMEDES, in the solid inclination employed by him in his treatise *De Helicibus*.

“ μετραι, τὸ προειρημένον ἐπὶ τῆς γωνίας πρό-
 “ βλημα, τῇ φύσει στερεὸν ὑπάρχον διὰ τῶν
 “ ἐπιπέδων ζητούντες, οὐχ οἷοι ἦσαν εὐρίσκειν,
 “ οὐδέ πω || γὰρ* τοῦ κώνου τομαὶ συνήθως ἦσαν
 “ αὐτοῖς. καὶ διὰ τοῦτο ἠπόρησαν. ὕστερον
 “ μέντοι διὰ τῶν κωνικῶν ἐτριχοτόμησαν τὴν
 “ γωνίαν εἰς τὴν εὐρεσιν χηισάμενοι τῇ ὑπο-
 “ γηγραμμένην βεύσει.”

“ geometræ problema jam dictum in
 “ angulo, quod natura solidum est,
 “ per plana inquirentes invenire non
 “ potuerunt : nondum enim ipsis cog-
 “ nitæ erant conicæ sectiones, et ob eam
 “ causam hæsitaverunt. Postea vero
 “ angulum tripartito dividerunt ex
 “ conicis, ad inventionem infra scripta
 “ inclinatione utentes. †

|| ε δὲ πω, MS. SAVIL. No. 9.

* γὰρ αἰ, MS. BULL. αἰ τῆ κωνῆ τομαὶ συνεχῆς εἰσιν. MS. SAVIL. No. 9.

† Prop. 31, lib. iv. PAPPI.

APPENDIX III.

AS the volume of Dr. SIMSON's posthumous works is not in general circulation, it may be agreeable to the Mathematical reader, to see the Doctor's very improved translation of the general description of EUCLID's Porisms, in the preface to the seventh book of PAPPUS, annexed to the account of his labours. The mutilated detail of the contents of the three books of Porisms, not being materially altered by Dr. SIMSON, except in a few Propositions restored by him, is not added.

“ POST Tactiones in tribus libris habentur Porismata EUCLIDIS, collectio
“ artificiosissima multarum rerum quæ spectant ad analyfin difficiliorum et
“ generalium problematum, quorum quidem ingentem copiam præbet natura.
“ Nihil vero additum est iis quæ EUCLIDES primum scripserat, præterquam
“ quod imperiti quidam qui nos præcefferunt secundas descriptiones paucis
“ ipsorum [sc. Porismatum] apposuerunt. Cum vero unumquodque definitum
“ habeat demonstrationum numerum,† ut ostendimus,‡ EUCLIDES unam
“ eamque maxime evidentem in singulis posuit. Habent autem subtilem et
“ naturalem contemplationem, necessariamque et maxime universalem, atque
“ iis quæ singula perspicere et investigare valent admodum jucundam Specie

† “ Forfan intelligit demonstrationes diversorum casuum ejusdem Porismatis; vel propositionum quæ sunt ejus conversæ.

‡ “ In quibusdam ex lemmatibus, ni fallor, ad Porismata.

“ autem hæc omnia neque theoremata sunt, neque problemata, sed mediæ
 “ quodammodo inter hæc naturæ, ita ut eorum Propositiones possunt vel ut
 “ theoremata, vel ut problemata formari. Unde factum est, ut inter multos
 “ Geometras alii hæc genere theoremata existiment, alii vero problemata,
 “ respicientes ad formam tantum propositionis. Differentias autem horum
 “ trium melius intellexisse veteres manifestum est ex definitionibus. Dixerunt
 “ enim theorema esse quo aliquid propositum est demonstrandum; problema
 “ vero quo aliquid propositum est construendum; Porisma vero esse quo aliquid
 “ propositum est investigandum. A Neotericis autem immutata est hæc
 “ Porismatis definitio, qui hæc omnia investigare haud potuerunt, sed
 “ Elementis hisce adhibitis, ostenderunt tantum quid sit quod quæritur, non
 “ autem illud investigaverunt. Et quamvis a definitione et ab ipsis rebus quæ
 “ traditæ sunt redarguerentur, hoc tamen modo, ab accidente, definierunt.
 “ Porisma est quod deficit hypothesi a Theoremate Locali [hoc est, Porisma est
 “ Theorema Locale deficiens five diminuta in hypothesi ejus.] Hujus autem
 “ generis Porismatum Loca Geometrica sunt species, quorum magna est copia
 “ in Libris de Analyti; ac seorsim a Porismatibus collecta, sub propriis titulis
 “ traduntur, eo quod magis diffusa et copiosa sit hæc præ cæteris speciebus. E
 “ Locis enim quædam plana sunt quædam solida, quædam linearia, et præter
 “ hæc sunt Loca ad medietates [five a mediis proportionalibus orta.] Accidit
 “ hoc etiam Porismatibus, Propositiones habere concisas propter difficultatem
 “ multarum rerum quæ subintelligi solent; unde evenit Geometras non paucos
 “ ex parte tantum rem perspicere, dum ea quæ inter ostensa magis necessaria
 “ sunt haud capiunt. Multa autem ex iis in una Propositione minime com-
 “ prehendi possunt, quia ipse EUCLIDES non multa in unaquaque specie
 “ posuerit, sed ut specimen daret multæ copię, pauca ad principium primi
 “ libri posuit ejusdem omnino speciei cum uberrima illa [quæ in primo libro
 “ habetur] Locorum specie, ut decem sint numero. Quare has [Propositiones
 “ scilicet hujus speciei] una Propositione comprehendi posse animadvertentes,
 “ eam ita describimus.

“ Si quadrilateri cujus anguli oppositi vel ex adverfo, vel ad eandem partes
 “ sunt positi,* [lateribus productis] data sint in uno ipsorum tria puncta

* “ Primum horum PAPPUS vocat ἑπίπλευρον, alterum παραπλευρον, de quibus vide Not. C.
 “ pp. 85, 86, 87.

‘ [interfectionum scilicet ;] vel si in quadrilatero cujus duo latera sunt inter se
 ‘ parallela [data sint duo puncta interfectionum in altera parallelarum ;] cætera
 ‘ vero puncta præter unum tangant rectam positione datam ; etiam hoc tanget
 ‘ rectam Positione datam.’ “ Hoc autem de quatuor tantum rectis dicitur
 “ quarum non plures quam duæ per idem punctum transeunt. In quolibet
 “ vero proposito rectarum numero ignoratur, quamvis vera sit hujusmodi
 “ Propositio, viz.

‘ Si quotcunque rectæ occurrant inter se, nec plures quam duæ per idem
 ‘ punctum ; data vero sint puncta omnia in earum una, unumquodque autem
 ‘ punctum in alia tangat rectam, positione|| datam.’ Vel generalius sic. ‘ Si
 ‘ quotcunque rectæ occurrant inter se, neque sint plures quam duæ per idem
 ‘ punctum, omnia vero puncta [interfectionem scilicet] in earum unâ data sint ;
 ‘ reliquorum numerus erit numerus triangularis, cujus latus exhibet numerum
 ‘ punctorum rectam positione datam tangentium ; quarum interfectionum si
 ‘ nullæ tres existant ad angulos trianguli spatii [nullæ quatuor ad angulos
 ‘ quadrilateri, nullæ quinque ad angulos quinquelateri, &c. i. e. Universim, si
 ‘ nullæ harum interfectionum in orbem redeant] unaquæque interfectio reliqua
 ‘ tanget rectam positione datam.’

“ EUCLIDEM autem hoc nescivisse haud verisimile est, sed principia sola
 “ respexisse : nam per omnia Porismata non nisi prima principia, et semina
 “ tantum multarum et magnarum rerum sparsisse videtur. Hæc autem juxta
 “ hypothesium differentias minime distinguenda sunt ; sed secundum diffe-
 “ rentias accidentium et quæditorum. Hypotheses quidem omnes inter se
 “ differunt, cum specialissimæ sint : accidentium vero et quæditorum unum-
 “ quodque, cum sit unum idemque multis diversisque hypothesebus contingit. §

“ Talia itaque inquirenda offeruntur in primi libri Propositionibus ; (in
 “ principio septimi habetur* diagramma huc spectans) ‘ Si a duobus punctis
 ‘ datis inflectantur duæ rectæ ad rectam positione datam, abscindat autem
 ‘ earum una à rectâ positione data segmentum dato in ea puncto adjacens,
 ‘ auferet etiam altera ab aliâ rectâ segmentum datum habens rationem.’ i. e.
 “ *Quod ad alterum segmentum habebit rationem eandem rationi quæ ex*
 “ *hypothese data est.*” Deinde in subsequentiibus ; &c.”

|| “ Unum tangat unam, aliud tangat aliam rectam positione datam, et sic deinceps.”

§ “ Ex gr. Multa sunt Porismata quæ diversas hypotheses habent, sed quæ omnia
 “ concludunt punctum aliquod tangere rectam positione datam ; vel rectam aliquam vergere
 “ ad punctum datum, &c.”

* “ Non jam habetur.”

ERRATA.

Page 18, line last of note, for 87 read 37

38, l. 6, note, for *rationum* read *rationem*

40, l. 5, note, for 280 read 270

89, l. 5, after μέγα place a period.

90, l. last of note, for γ read $\bar{\gamma}$ (3)

91, l. dele note, "*for san ἠδομαι*"

94, l. 1 note, for *indeterminatum* r. *indeterminatus*

114, l. 22, for *letter after*, read *letter, after*

121, l. 4, for *into*; it read *into it*;

Page 131, l. 3, for *treatises all of which*; read
treatises, all of which

132, l. 18, for *ascertained* read *determined*

136, l. 5, note, for *Geometry*; read *Astronomy*;

145, l. 1, note, for δ' ὀπίθω read δ', ὀπίθω

149, l. 12, for *plane*, read *plane*

170, l. last, for *same* read *same*

175, l. last note, for *deperisse* read *deperiisse*

176, the first note, from page 180, unnecessary.

