THE RELATION BETWEEN THE LENGTH OF APHAKIC EYES AND THEIR APPROPRIATE CORRECTING GLASSES.

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MANY of the formulæ employed in ophthalmic practice are misapplied from a want of knowledge of the conditions on which they are based. In this paper I wish to draw attention to one formula the improper application of which has given rise to serious error. In the estimation of axial ametropia, of swelling of the fundus, or of the depth of a posterior staphyloma, we are all in the habit of using the convenient formula that a difference of 3 D corresponds to a difference of level of 1 *mm* in the fundus. If, for instance, when the surgeon is emmetropic, and the curvature of the patient's cornea is normal, the optic disc is seen distinctly with + 1 D behind the sight-hole of the ophthalmoscope, and a certain part of a retinal vessel is seen distinctly with + 12 D behind the sight-hole, the surgeon is justified in drawing the conclusion that the optic disc lies $\frac{1}{3}$ *mm* in front of its normal position and that the retinal vessel lies 4 *mm* in front of its normal position, or $3\frac{2}{3}$ *mm* in front of the level of the optic disc. If, however, the crystalline lens has been removed this conclusion is erroneous, as the dioptric system of the eye is in that case entirely different.

It will be convenient to consider the conditions on which this rule-of-thumb formula is based, and we shall be at once led to find the proper relation between the the length of an aphakic eye and its appropriate correcting glass.

If we take as our standard the simple reduced eye of Donders, in which the complex dioptric system of the eye

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is considered to be practically represented by a single spherical surface, whose radius of curvature is 5 *mm*, whose first principal focal distance f_1 is 15 *mm* in front of the cornea, the second principal focal distance f_2 being behind the cornea -20 *mm*, we get from the familiar formula

$$(p - f_1)(q - f_2) = f_1 f_2$$

the relation between the conjugate focal distances p and q. Here p represents the distance of the punctum remotum from the cornea, and q denotes the distance of the retina from the cornea, or the antero-posterior diameter in the reduced eye. Since q is measured behind the cornea it is of course a negative quantity.

Also if the eye be axially ametropic, $p - f_1$ denotes the focal distance of the glass necessary to correct the ametropia when it is placed at the first principal focus of the eye.

$$p - f_1 = \frac{f_1 f_2}{q - f_2} = \frac{(15)(-20)}{-l} = \frac{300}{l}$$

where l represents the excess or defect in length of the eye in millimetres.

Thus, suppose the eye be 3 *mm* too long, $l = +3$ *mm*

$$\therefore p - f_1 = \frac{300}{3} = +100 \; mm$$

A lens, therefore, the focal distance of which is $+100$ *mm* (or -10 D lens) will be required to correct this eye for distant objects.

If x denote the power in dioptres of the correcting lens placed at the first principal focus of the eye

$$\frac{1000}{-x} = p - f_1 = \frac{300}{l}$$

$$\therefore l = -\cdot 3\, x \text{ and } x = \frac{-l}{\cdot 3} \; *$$

Now in an aphakic eye if the corneal curvature is normal ($r = -7.829$ *mm*) $f_2 = -31.095$ *mm* $f_1 = 23.266$ *mm*.

$$\therefore p - f_1 = \frac{f_1 f_2}{q - f_2} = \frac{(23.266)(-31.095)}{-l'} = \frac{723.455}{l'}$$

* If the complex dioptric system of the eye be taken into account, a more accurate expression for the power (x) of the correcting lens 13·7451 *mm* in front of the cornea is given

$$\text{where } x = \frac{-l}{\cdot 321} = -3\cdot 115\, l, \text{ and } l = -\cdot 321\, x$$

where l' represents the excess or defect in length of the eye above or below 31.095 *mm*, which is the antero-posterior diameter of the emmetropic aphakic eye.

And $p - f_1$ denotes the focal distance of the correcting glass placed at the first principal focus of the eye required to correct this excess or defect. If X denote the power in dioptres of the correcting glass placed at the first principal focus of this aphakic eye

$$\frac{1000}{-X} = p - f_1 = \frac{723.455}{l'}$$

$$\therefore l' = - \cdot 723455 \text{ X, and } X = \frac{-l'}{\cdot 723455} = - 1\cdot 38226\, l'$$

If an eye of normal length (22.8237 *mm*) be rendered aphakic by the extraction of its crystalline,

$$l' = 22.8237 - 31.095 = - 8.2713 \; mm$$

The correcting glass for distance after the operation will be

$$X \text{ or } (-1.38226)\,(-8.2713) = + 11.5 \text{ D}$$

if placed about 23 *mm*, in front of the cornea. A neuritis which in an aphakic eye causes + 3 D of swelling, would represent a displacement forwards of about 2.17 *mm*, whereas in a normal eye + 3 D of swelling would indicate a displacement of barely 1 *mm*.

Again suppose a myopic eye be 31.1 *mm* long,

$$l = 31.1 - 22.8 = + 8.3 \; mm \; ; \; x = - 3.115\, l = - 25.85 \text{ D}$$

Thus a myope who has been using — 25 D or — 26 D glasses may be expected to see without glasses if his crystalline lens is removed. Surprise has often been expressed at the very great change in refraction after extracting the lens in myopic cases. It will be noticed that the change in refraction due to the operation increases with the previous degree of axial myopia.

Ant.-Post. Dimension.	x	l	l'	X	Change of Refraction.
23 *mm*	— ·5 D	+ ·1763	— 8·095	+ 11·2 D	11·7 D
26 *mm*	— 9·9 D	+ 3·1763	— 5·095	+ 7· D	16·9 D
29 *mm*	—19·2 D	+ 6·1763	— 2·095	+ 2·9 D	22·1 D
32 *mm*	—28·5 D	+ 9·1763	+ ·905	— 1·25 D	27·3 D

This point is of considerable importance to those who treat high degrees of myopia by the extraction of the transparent lens. If the result of the operation does not correspond with this table, it is certain that the myopia was not purely axial in character. Incidentally I may point out that in aphakic eyes provided with the necessary correction 23 *mm* in front of the cornea, the retinal images are 1.5 times larger than in normal eyes. This is distinctly advantageous to the patient but should warn us against forming too favorable a view of his visual acuity when examining him with test types.

I lay no claim to any originality in the formulæ here given. They follow immediately from the data which may be found in the standard books on physiological optics, such as those of Helmholtz, Donders, and of Landolt. The numerical values of the constants used in this paper are those given by Landolt. The frequency of error arising from an improper application of the rule about normal eyes to the length of aphakic eyes justifies, I trust, this brief reference to the essential difference between the two conditions.