# GEOMETRICAL OPTICS.

MACKINNEY & TAYLOR.

# Geometrical Optics

By

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Published by

J. & H. TAYLOR, 54, Tenby Street North,

BIRMINGHAM.

1908.

# Preface.

THE growing demand for a book on Geometrical Optics based upon the Curvature System has led to the production of this small volume, which is mainly a reprint of certain chapters from "The Key to Sight Testing."

The lists of Optical Experiments and Books, with reference numbers furnishing the source of the necessary information, we trust will prove an incentive to the further study of Optics.

V. H. M.
H. L. T.

*January, 1908.*

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# Section I.

## Geometrical and Physical Optics.

### Chapter I.

### CURVATURE SYSTEM AND OPTICAL NOTATION.

WHEN a stone is thrown into water we notice that concentric circles of ripples emanate from the point of disturbance, and that any straight line from that point to any circle measures the distance of the ripple from the point of origin. Without pressing the analogy too closely we may illustrate the progress of light waves from a luminous point by this illustration, and as light travels in straight lines a *ray* corresponds to any straight line from the centre, so that it becomes a *radius* for any given ripple or wave, and is thus the *path of a wave.*

The curvature method of treating geometrical optics so simplifies the teaching of this branch of the science, that one wonders why so many writers have given it such slight recognition. This method deals with the subject strictly from the physical standpoint, and in accordance with the generally accepted wave theory of light. It lends itself to the satisfactory explanation of the phenomena of physical optics, and thus in many instances cumbersome mathematical proofs may be replaced by simple reasoning—in conjunction with a graphical representation. Furthermore, it offers an especial advantage because it harmonizes with the spectacle-lens notation now used by opticians. The introduction of the curvature method into the present volume will enable the reader to grasp the subject much more readily and to retain the knowledge so gained.

It is evident that we need some unit by means of which we can get a definite idea of the value of certain measurements, the simplest of which is that of length. We may express this in the standards known as feet, inches, metres, etc., but in addition we must know the point of origin and the direction of measurement. Optical measurements of distance are now universally made according to the metric system, and specified as so many metres (m), centimetres (cm), or millimetres (mm).

The unit of curvature is defined as that curve which corresponds to a radius of one metre (39·37 or approximately 40 inches) and is called a *dioptre.** It may be used to express the curvature of a beam of light, mirror, or lens surface, as well as the power of a lens, as will be seen presently.

Similarly, the unit of angular measurement is defined as that angle which, at the centre of a circle, stands on an arc equal in length to the radius of the circle, and is called a *radian*.

Let any circle ABC be taken, Fig. 1 (we may with advantage think of this as a sphere), this circle has a definite fixed radius $r$. The curvature may be represented by R and defined as follows :—

*The curvature of a circle is the angle through which a curve turns per unit length.*

If, therefore, we denote the angle by $\theta$, and its corresponding length of arc by $l$, we have by definition—

$$R = \frac{\theta}{l} \qquad (1)$$

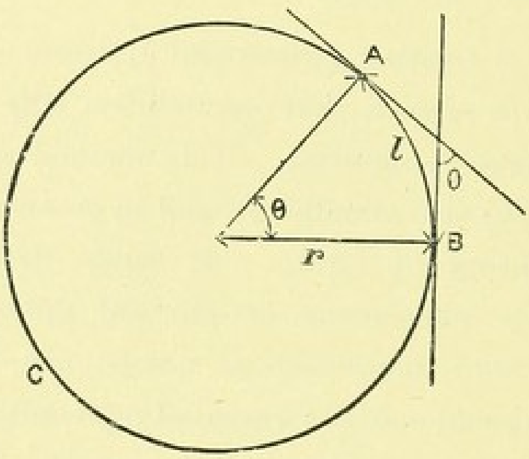


Fig. 1.

Also, from the definition given above of the unit of angular measure—the radian—it is seen that :

$$\theta \text{ (in radians)} = \frac{\text{arc}}{\text{radius}} = \frac{l}{r}$$

$$i.e.: \ l = \theta r.$$

Substituting this value of $l$ in (*1*)

$$R = \frac{\theta}{\theta r} = \frac{1}{r}$$

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* The word *dioptrie* was proposed by Monoyer in 1872 and adopted by the Brussels International Congress in 1875.

Expressing this in words we have : *The curvature of a circle (or surface) is the reciprocal of its radius.*

From the definition of unit curvature, when $r$ = 1 metre, 100cm, or 1000mm.
= 40 inches (approx.)
then R = 1 Dioptre.

It is therefore seen that R, the curvature of the light wave, or any other surface, real or imaginary, is given in dioptres when the radius of curvature is known, no matter in what units the radius may be given.

$$R \text{ (The curvature of the surface in dioptres)} = \frac{1}{r \text{ (in metres)}}$$

$$= \frac{40}{r \text{ (in inches)}} \quad \text{since 40 inches} = 1 \text{ metre (approx.)}$$

$$= \frac{100}{r \text{ (in centimetres)}} \text{ or } \frac{1000}{r \text{ (in millimetres)}}$$

The following table shows that as R increases so $r$ decreases.

| Radii of Curvature ($r$) in metres and inches | | | | Curvatures (R) in Dioptres |
|---|---|---|---|---|
| 1 | metre | | 40 inches | 1 D |
| 2 | ,, | | 80 ,, | $\frac{1}{2}$ D |
| 4 | ,, | | 160 ,, | $\frac{1}{4}$ D |
| $\frac{1}{2}$ | ,, | | 20 ,, | 2 D |
| $\frac{1}{5}$ | ,, | | 8 ,, | 5 D |
| $\frac{1}{10}$ | ,, | (10cm) | 4 ,, | 10 D |

The dioptre as a unit is one very useful to opticians in general ; when, however, we have to deal with microscope and other lenses, whose surfaces often have curves with radii of the order of a few millimetres, or telescope lenses for the great "refractors," the radii of whose surfaces may measure several metres, then it becomes advisable to adopt multiples and sub-multiples of this unit. Dr. Drysdale* has suggested the following :—

| RADIUS OF CURVATURE. | CURVATURE. |
|---|---|
| Kilometre. | Millidioptre. |
| Metre | Dioptre. |
| Centimetre. | Hectodioptre. |
| Millimetre. | Kilodioptre. |

These are admirable, for the range of curvatures may be very great.

*Proc. Optical Society, December, 1902.

Light which impinges upon any surface is known as *incident*, while that which is returned from the surface is *reflected;* and when any is transmitted, after being bent from its course at the surface, it is termed *refracted light.*

As light travels in straight lines there will always be one particular path of a wave perpendicular to the mirror or lens surface. Such a path (*ray*) is said to have *normal* incidence, and all angles of incidence are reckoned from this, and not from the surface itself.

For convenience, a notation* has been adopted in this work, which is now very generally used, by which distances of the point of origin of paths of light, and also points of assemblance of paths, after encountering simple reflecting or refracting surfaces, are denoted by small letters, while capitals are used for the corresponding curvatures of the waves of light at those distances.

In some cases the assemblance of paths is not real, and then by prolonging them backwards a *virtual* point is obtained.

Thus, while $u$ and $v$ symbolize the two distances referred to, U and V represent corresponding curvatures. Similarly, $r$ is used for the radius of curvature of a mirror, and $r_1$, $r_3$ (odd numbers) for the radii of curvatures of the surfaces of a lens.

Refractive Indices (see later) are denoted by *even* numbers, for instance $n_0$, $n_2$, $n_4$, etc.

It may further be mentioned that in the use of + and − signs the *product* (multiplication) of like signs is always positive, while that of unlike signs is always negative. So that $-(-U)$ equals $+U$, etc.

This notation may be brought to a close by asking the reader *always* to consider light as passing from left to right, also to treat divergent light and surfaces curved in the direction of divergent light as *negative*, (Fig. 2);

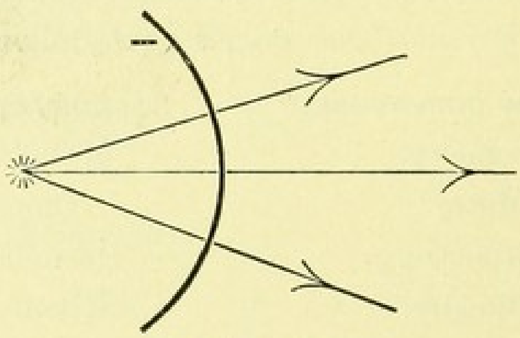



Fig. 2.

* This notation, with slight variations, has been advocated by Herschel, Prof. S. P. Thompson, Dr. C. V. Drysdale and other writers.

but convergent light, and surfaces curved in the direction of convergent light, as *positive*, (Fig. 3).

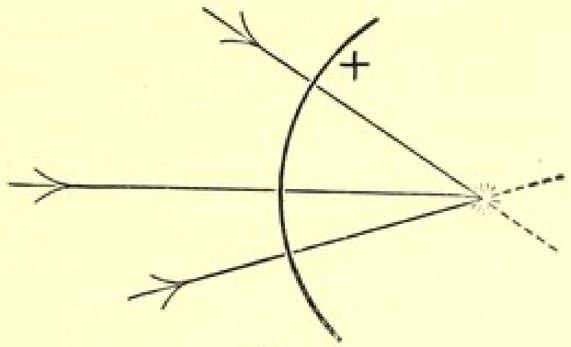



Fig. 3.

By adhering to the above rules, the solution of all ordinary mirror and lens problems becomes an easy matter.

**Summary.**

(*a*) The radian is the unit of angular measurement.

(*b*) The curvature of a circle is the reciprocal of its radius.

(*c*) Capital letters indicate curvatures, and small letters the corresponding distances.

(*d*) Odd numbers are used for radii, even numbers for refractive indices.

(*e*) Light is considered as passing from L to R.

(*f*) Convergent light and similarly curved surfaces are +, and divergent light and similarly curved surfaces —.

## Chapter II.

### NATURE AND PROPERTIES OF LIGHT.

ACCORDING to the wave theory, the sensation of light is a direct consequence of a disturbance in the *ether*, a substance which it is necessary to conceive, although no eye has seen nor instrument weighed it. Ether is the key to the satisfactory explanation of *all* physical phenomena. As Lewis Wright says in his book on " Light ":—

> " Heat, Light, Colour, Electricity, Chemical Actinism,—all alike are " simple disturbances in, or propagations of disturbances through, that " Something which we call Ether. Invisible themselves, these " wonderful motions make all things visible to us, and reveal to us " such things as are."

Light, then, as we understand it, is a transverse (since the vibration is at right-angles to the direction of propagation) undulatory disturbance in this unknown substance. These waves or undulations, once set up, travel at the enormous rate of about 300 million metres per second, and when vibrating at the rate of between 375 and 857 billion times per second, they cause the sensation of light. The rate of vibration also determines our colour sensation. Supposing the range of frequency to be small, and something of the order of 400 billion times per second, then the sensation produced is that of red light ; if the frequency is of the order of 500 billion times per second the sensation is that of yellow light ; and, finally, if of the order of 700-800 billion times per second the sensation is that of blue or violet light. Should the range of frequency be at all great then the resultant colour sensation will be a compound one, and if it embraces the entire visual range (between 375 and 857 billion times per second) then the resultant sensation will be that of white light.

In Fig. 4, let us suppose that light has travelled from the point A to the point B in one second. Then the distance AB represents 300 million metres, or 300,000 million millimetres. The light vibrates in a transverse direction, that is, at right-angles to the line of propagation AB.

CD represents one such complete vibration and is *one wave-length.* If the light proceeding from A consists of all vibrations between 375 and 857 billions per second, then the wave-lengths of the wave motions proceeding from A to B will differ one from another. We have reason for believing that the velocity for any coloured light is practically the same in air and free space ; in other media, however, the bending of the path of the light due to refraction is a consequence of a change in velocity, and experiments show that the change in velocity is dependent upon the colour of the light—in other words, upon the frequency of vibration, or wave-length, of the incident light.

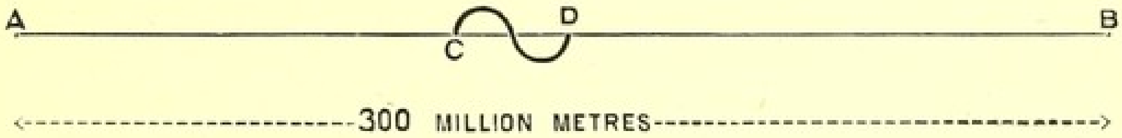



Fig. 4.

It is not a difficult matter to realize therefore, that under certain circumstances a composite beam of light may be split up into its component parts, and such breaking up is termed *dispersion.* Light of one particular rate of vibration, and therefore of one particular wave-length, is called *monochromatic light.* Sunlight may be split up into a number of coloured lights, viz.:—red, orange, yellow, green, blue, indigo, and violet. One way of breaking up the light is by passing it through an optical prism (Chapter VI.), the resultant band of different coloured lights being termed a *spectrum,* and in the case of sunlight—the solar spectrum.

The connection between velocity, frequency of vibration, and wave-length of light may be represented by $f = \frac{V}{\lambda}$; where $f$ denotes the frequency of vibration per second, V the distance traversed per second, and $\lambda$ the wave-length.

From which we see that $\lambda = \frac{V}{f}$

Now the frequency (375 billions per second) which produces the sensation of red light represents one limit of the visual spectrum, so that frequencies below this do not produce the sensation of light at all in the human eye. Also the frequency (857 billions per second) which produces the sensation of violet light represents the other limit of the visual spectrum.

As the velocity in air or free space is the same for all colours, viz.: 300,000 million millimetres per second, we have

*For Red Light* the limiting wave-length

$$\lambda_r = \frac{300{,}000 \text{ million}}{375 \text{ billion}} \text{ mm.} = \cdot 00080 \text{ mm.}$$

*For Violet Light* the limiting wave-length

$$\lambda_v = \frac{300{,}000 \text{ million}}{857 \text{ billion}} \text{ mm.} = \cdot 00035 \text{ mm.}$$

The Greek letter $\mu$ (pronounced "mu") is employed by many to denote 1 micron, that is, $\frac{1}{1000}$th mm. (·001 mm.), while others employ the same symbol to denote "refractive index." In this work it has the former signification. We may say, therefore, that the visual spectrum ranges from 0·35 $\mu$ (violet limiting wave-length) to 0·80 $\mu$ (red limiting wave-length).

*The Laws of Light.*

1. In a homogeneous medium light from any source of illumination is propagated from every point of it in the form of concentric spherical waves which spread outwards in the direction of the radii of their spheres.

From this law follows the *rectilinear propagation of light*, in other words light travels away from its source in every direction in straight lines.

2. The intensity of illumination at a given point varies inversely as the square of its distance from the source of illumination. This is known as *the law of inverse squares*.

3. The intensity of illumination which is received obliquely is proportional to the cosine* of the angle of turning of the surface.

The intensity of illumination of any surface is the amount of light received on unit area of that surface. The unit of illumination is that given by a light of unit brightness at unit distance. In England the practical unit may be taken as that illumination given by a standard candle at a distance of one metre, and called one *candle-metre*.

The intensity of a source of light is the amount of light emitted as compared with that from a standard source of illumination. The British standard is a particular candle burning 120 grains of pure spermaceti wax per hour. The practical standard, however, is the Vernon-Harcourt 10 c.p. Pentane Lamp, having a light intensity extremely constant with careful use, which cannot be said of the standard candle.

If I denotes the illumination, K the candle-power, and *d* the distance

*For explanation see Tables at end.

of the receiving screen from the source ; then, from the law of inverse squares we have

(1) $I = \frac{K}{d^2}$ when the receiving screen faces the source normally.

(2) $I = \frac{K}{d^2} \cos \alpha$ where $\alpha$ is the angle of turning of the surface from its position when it faced the source normally.

The quantity of light Q received by any surface is given by Lambert's equation

(3) $Q = B \frac{A_1 A_2}{d^2} \cos \alpha_1, \cos \alpha_2,$ where $A_1$ and $A_2$ are the areas of the luminous source and screen respectively, $d$ the distance between the centres of the two surfaces, and $\alpha_1$ and $\alpha_2$ the angles made by the imaginary line joining the two surface centres with their respective normals. (Fig. 5.)

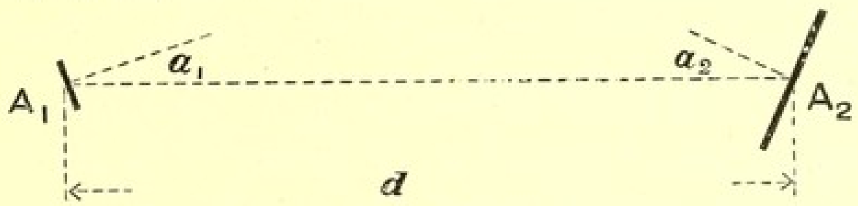



Fig. 5.

All these equations (1) (2) and (3) are only correct when the dimensions of the luminous source are small as compared with the distance $d$.

In practice equation (1) is of great use, for by means of it we are able, with a photometer, to compare the intrinsic brilliancy of two light sources, by reducing to equal illumination the light given out by them.

The essentials of the photometric process are first, the production of two identical surfaces (or one surface), illuminated separately from each source ; and secondly, some means of obtaining equality of illumination on the said identical surfaces (or one surface).

*The Aberration of Light.* Owing to the facts that light travels with a finite velocity, and that the earth has a certain period of revolution round the sun, light emitted by the heavenly bodies will, more often than not, appear to have proceeded along a different path from what it really has, and we see them in a false position ; hence the term "aberration of light."

*The Laws of Reflection.*

1. The incident and reflected paths of the waves of light lie in the same plane with the normal to the surface at the point of incidence, and are on opposite sides of the normal.

2. The angles which the incident and reflected paths of the waves of light make with the normal to the surface are equal to one another.

From these two laws we see that in Fig. 6, first, the incident path B, reflected

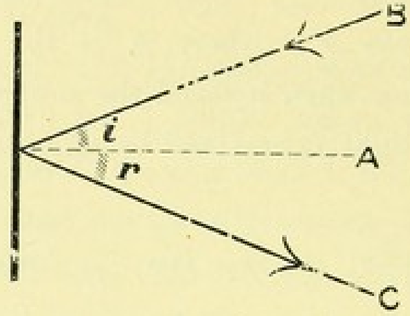


Fig. 6.

path C, and normal A, all lie in the plane of the paper; secondly, the reflected path is on the opposite side of the normal to the incident path: and thirdly, the angles $i$ and $r$ are equal to one another.

*The Laws of Refraction.*

1. The incident and refracted paths of the waves of light lie in the same plane with the normal to the surface at the point of incidence, and are on opposite sides of the normal.

2. For light of any one colour the sines of the angles of incidence and refraction are in a constant ratio for the two media. (Snell's law).

From these two laws we see that in Fig. 7, first, the incident path B, refracted

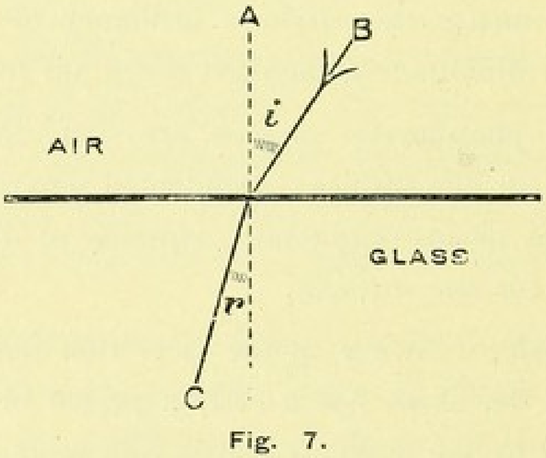


Fig. 7.

path C, and normal A, all lie in the plane of the paper; secondly, the refracted path is on the opposite side of the normal to the incident path; and thirdly, *$\frac{\sin i}{\sin r}$ = K, a constant for the same two media and for light of a particular colour.

*For explanation see Tables at end.

*Refractive Index.*

The *absolute* refractive index of a substance is simply the ratio between the velocity of light in free space, or air for all practical purposes, to its velocity in the substance under consideration. If, therefore, we denote the velocity of light in free space or air by $V_0$, the velocity in the substance by $V_2$, and the refractive index by the letter $n$, then we have

$$n = \frac{V_0}{V_2}$$

$V_0$ for air we have seen is equal to 300 million metres per second. Now the velocity in a certain kind of crown glass is about 200 million metres per second, therefore the refractive index $n = \frac{V_0}{V_2} = \frac{3}{2} = 1.5$.

In transparent media other than gases, the change of velocity is dependent upon the colour of the light, that is, upon the frequency of vibration or wave, length ; so that the above value will only be correct for a particular coloured light.

In practice it is usual to determine the refractive indices for red, yellow and blue light. The yellow chosen is that given out by luminous sodium vapour, readily produced by burning common salt in a Bunsen flame. The spectrum given by this is simply a narrow band of yellow light—in reality double—so that, for all practical purposes it is monochromatic, and when a refractive index for any substance is stated without reference to any colour it may be assumed that it refers to this yellow sodium light.

Since the band is quite narrow we may assume that it has but one frequency, and therefore one particular wave-length, given as $\lambda = 0{\cdot}5890\ \mu$.

The spectrum of luminous hydrogen vapour consists of four very narrow bands or lines of light. These are red, green, blue and violet in colour, and the refractive index of a piece of glass is usually determined for the red and blue lines of hydrogen, as well as for the yellow line of sodium, the reason for which will be apparent when we have studied Chapter VI.

It is usual to denote the refractive index for sodium light by the symbol $n_D$, and the refractive indices for the red and blue lines of hydrogen by $n_C$ and $n_F$ respectively. $n_D$ represents the *mean refractive index*. The difference between $n_F$ and $n_C$ $(n_F - n_C)$ is a measure of the dispersion of the glass, and may be termed the *mean dispersion*.

The *dispersive power* $\omega$ is given by the equation $\omega = \frac{n_F - n_C}{n_D - 1}$, where 1 is the refractive index for air. (The actual figures being 1.0002922.)

The *dispersive reciprocal* or *efficiency* N is therefore given by

$$N = \frac{n_D - 1}{n_F - n_C} .$$

All the above constants are given in their lists by makers of glass so as to enable purchasers to get some idea of the properties peculiar to the various glasses. Partial dispersions in three portions of the visual spectrum are given too.

The refractive index $n$, which by definition is equal to $\frac{V_0}{V_2}$ is also equal to $\frac{\sin i}{\sin r}$, where $i$ is the angle of incidence, and $r$ the angle of refraction.

We may see that $n^* = \frac{V_0}{V_2} = \frac{\sin i}{\sin r} = \frac{n_2}{n_0}$, where $n_2$ is the refractive index for the second medium and $n_0$ the refractive index for the first medium.

Let BD in Fig. 8 represent the surface of separation between two media whose refractive indices are $n_0$ and $n_2$ respectively, $n_2$ being the greater. Let CD represent the path of the incident plane wave front BC, the normals to the surface being shown as dotted lines. Had the light not met the new medium $n_2$, in a given time the point B would have travelled a distance equal to CD, but because it meets the new medium the wave front is retarded and bent out of its course, and the point B will only reach E in a given time.

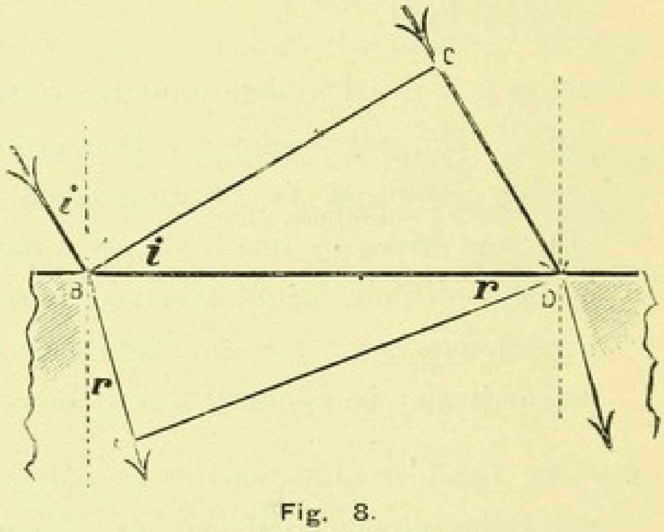



Fig. 8.

BCD and BED are right angled triangles because the path of a plane wave is at right angles to the wave front. Therefore the angle which the path makes with the normal, and the angle which the wave front makes with the surface are both angles of incidence *(i)*. Similarly those marked $(r)$ are the angles of refraction.

* When $V_0$ represents the velocity in any medium but air, $n$ is termed the *relative* refractive index ; $n_0$ for air = 1, therefore $n = n_2$ usually.

Now $\sin i = \frac{CD}{BD}$ and $\sin r = \frac{BE}{BD}$

And $\frac{\sin i}{\sin r} = \frac{\frac{CD}{BD}}{\frac{BE}{BD}} = \frac{CD}{BE}$

But CD and BE represent the velocities $V_0$ and $V_2$

$$\therefore n = \frac{V_0}{V_2} = \frac{\sin i}{\sin r} = \frac{n_2}{n_0}$$

The above holds good for all angles of incidence. We know also, from our second law of refraction, that for two given media, and a particular colour of light, $\frac{\sin i}{\sin r}$ is a constant.

*Diffraction.* Although we may consider light as a grand wave motion and represent it diagramatically by concentric circles, experiment shows that the motion set up in the ether is a very complex one indeed.

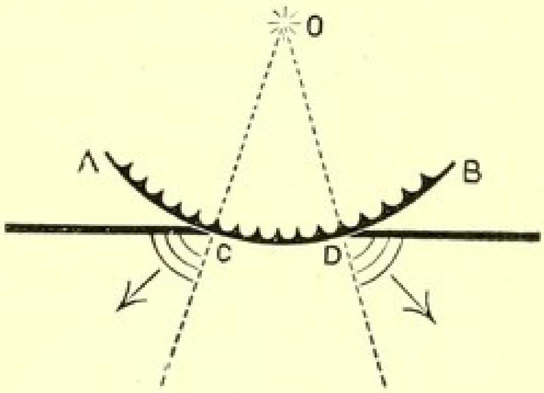


Fig. 9

In Fig. 9 let O be a luminous source, then if A B is the first main or *grand* wave, it will be formed by the mutual interference of the little *secondary* waves, which all assist to that end. If C D is an aperture, then only a portion of the main wave can pass on, and at each edge of the aperture some of the secondary waves will become detached from the main beam and proceed in the direction shown, very diagramatically, by the arrows. Some of these will meet in opposition and destroy one another, others will meet in such a manner as to assist one another, the final result being that to an eye placed on either side of the transmitted portion of the main wave, alternate dark and light bands of light will appear. If the source O is white, or other compound light, then alternate dark and spectral coloured bands will be seen. This is one method of producing diffraction spectra, and the further we move the eye from the main beam the fainter will become the luminous diffaction bands or spectra.

*Interference.* Just as secondary waves assist or annul one another, so do main waves assist or annul one another. But here at least two sources, or two apparent sources, of illumination are necessary. Phenomena due to the interference of the main waves of light are called *interference phenomena.*

*Polarization.* Experiment shows that however light is vibrating when proceeding from its source, oblique reflection and unequal refraction (as seen in doubly refracting crystals) have the property of splitting the light up into two paths, a particular percentage of the light along each of which has a purely transverse vibration ; that is, it vibrates in one plane only. Experiment also proves that the light following each of these paths is vibrating in a direction at right-angles to the other. Often it will be found that the sorting out of the vibrations has not been complete, and that a certain amount of residual ordinary white light is still present. Light vibrating in one plane only is said to be completely plane polarized. Phenomena due to the polarization of light are studied under the title of "Polarization."

*Fluorescence.* Certain substances, such as a solution of sulphate of quinine, or a piece of Uranium glass, possess the property of converting some of the shorter wave-lengths of light into longer ones, and transmitting them as such. This is termed Fluorescence, and by means of it we are able, among other things, to convert invisible light into visible light, in other words, to give to invisible waves such a wave-length as will enable our eyes to detect their presence.

*Phosphorescence.* Some substances possess the property of giving out luminous waves after having been withdrawn from the presence of any luminous source, and this phenomenon is called Phosphorescence. It is simply a case of energy being stored at a greater rate than it is emitted. Fluor spar glows for a short period after exposure to a strong light, while certain sulphides, and the compound sold as Balmain's luminous paint, glow for many hours after exposure to a strong light.

*Calorescence.* Certain substances possess the property of converting the longer wave-lengths into shorter ones ; it is thus the converse of fluorescence.

**Summary.**

(*a*) The visible spectrum ranges between 0.35 $\mu$ and 0.8 $\mu$.

(*b*) Monochromatic light is of one particular wave-length.

(*c*) The intensity of illumination varies inversely as the square of the distance.

(*d*) The angle of incidence of light is equal to the angle of reflection.

(*e*) $\frac{V_0}{V_2} = \frac{\sin i}{\sin r} = \frac{n_2}{n_0}$

# Chapter III.

## REFLECTION AT PLANE AND CURVED SURFACES.

*Experiment.* Hold some small source of illumination in front of a *thin* piece of plane mirror, and note that the image apparently occupies a similar position behind the mirror to what the source does in front.

Now replace the thin piece of plane mirror by a *thick* piece, and, looking obliquely, notice that there are *several* images, only one of which occupies a symmetrical position with the object with regard to the silvered surface.

The image seen in the thin plane mirror is a virtual or imaginary one, the object being apparently seen in a position it does not occupy, because the mirror surface alters the course of the actual light waves. In Fig. 10 the actual, but invisible, divergent wave of light AB from the luminous source O, occupies a momentous position. At A it is about to be reflected, while at B it has a distance B $B_1$ to travel before reflection, and it is obvious that by the time B reaches $B_1$, A will have reached $A_1$. Had the mirror not been present, A would have occupied the position $A_2$. In the

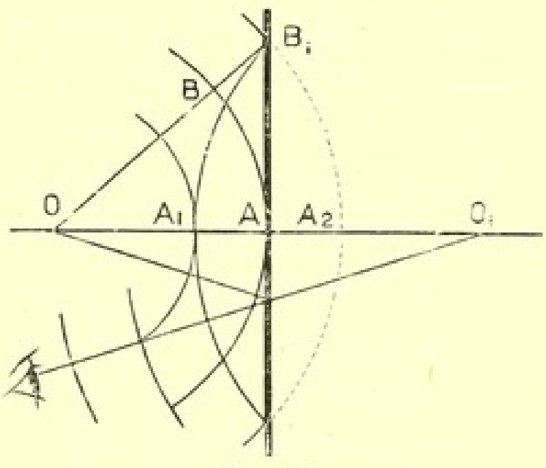


Fig. 10.

figure the distances A $A_1$, A $A_2$, B $B_1$ must be equal, on the assumption that light travels away from its source in spherical waves, and that reflection is instantaneous, or equal in retardation effect at all angles of incidence. Hence the curvature of the wave at the particular moment when it occupies the reflected position at $A_1$ must be equal to that which it would have at $A_2$ if the mirror were not present, for the curves stand on the same chord, and have equal sags, under the assumptions made. Therefore, to an eye placed as shown, the object will appear to be at $O_1$. This apparent position of the object is called the image position—and because the curves above referred to are similar in all respects this apparent object

$O_1$ will appear to occupy a similar position to that of the actual object, only on the opposite side of the mirror. If the position of the luminous source be such that the distance represented by $O\,A_2$ or $O\,B_1$ equals one metre, then, since $A_1\,O_1 = O\,A_2$, the reflected wave of light will have a curvature of one Dioptre.

Or replacing the thin mirror by a thick piece, several images of the object will be seen by looking obliquely, a phenomenon of very great importance in optical work. By careful observation we may also note that it is the second image which is the brightest, and that beyond this there seems to be a great number of images, all equidistant, but getting less intense as the apparent distance from the mirror surface increases.

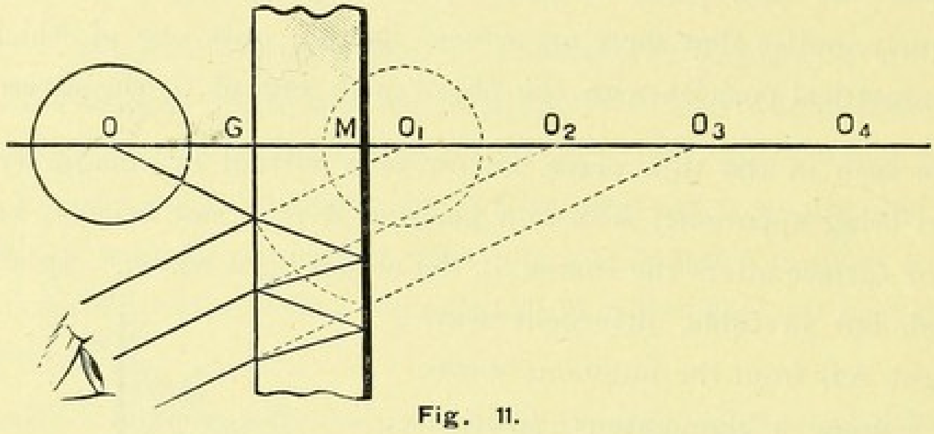



Fig. 11.

In Fig. 11 let O represent the position of the luminous source, then $O_2$ will represent the position of the image formed by the highly reflecting mirror surface M (neglecting for the moment the effect of refraction at surface G) and O M will be equal to M $O_2$. Before the light passes through the thickness of glass G M to the mirror surface M it has lost some of its original intensity, because the surface G, although transparent, possesses the property of reflecting a definite amount. The light reflected by a very perfect transparent glass surface is of sufficient intensity, when the object is a luminous source, to enable an image of the object to be seen by oblique observation. Hence another image $O_1$, of less intensity than $O_2$, is seen in front of this bright one $O_2$, the distance depending upon the thickness of glass G M. The distance O G will be equal to the distance G $O_1$.

Just as a certain amount of light is lost by reflection at the glass surface when the light passes from air into glass, so also is another definite amount lost by reflection on the light passing from the glass into air on its return path. This, as the diagram shows, gives rise to a third image $O_3$. The diagram might be extended to show $O_4$, $O_5$ &c., in fact theoretically there is an infinite number of these images, but the formation of each causes a loss in the light which passes on, until finally the intensity of the light is not sufficient to produce another visible image.

When the thickness of glass in front of the reflecting surface does not exceed a few millimetres all the visible secondary images are formed so close to the chief (bright) one $O_2$ as to become scarcely distinguishable at all moderate angles—say up to 45° from the normal. They may, however, cause the bright image to appear fuzzy, and these multiple images are often the cause of much annoyance in optical instruments, ruining the definition of the bright image. Metallic mirrors, or glass mirrors silvered on the front surface, are sometimes employed, but they become costly if at all perfect in other respects, and are very apt to tarnish or get marked during use.

If a plane mirror is rotated through a given angle, say $\theta$, the light reflected by the mirror will be rotated through just twice the angle, viz., $2\theta$. This is the principle made use of in instruments like the sextant.

The truth of this will be evident if we consider the movement of the normal to the surface, which rotates exactly to the same extent as the mirror, and as the angle between it and the incident light increases, so will the angle between it and the reflected beam increase, to just the same extent. Thus the reflected beam moves at twice the rate of the normal to the surface, and consequently of the mirror also.

*Practical Use of Plane Mirrors.*

(*a*) To measure small deflections.

(*b*) In instruments designed for the measurement of angles, as the sextant, range finder, etc.

(*c*) In retinoscopy.

(*d*) In the kaleidoscope.

(*e*) For producing the effect of doubling a distance, as in the use of reversed type in testing.

(*f*) For deflecting a beam of light for illuminating purposes.

*Experiment.* Hold a pin vertically in front of a concave mirror of not more than 20 cm. radius (5 D curvature),—provided the experiment is to be performed within arm's length,—and notice that when the pin is quite close to the mirror an image, *virtual* and *erect*, and very similar to that seen in a plane reflecting surface, is seen. As the pin is withdrawn from the mirror the image rapidly becomes larger and indistinct, finally fading away. Withdrawing the pin still further the image reappears, but it is *inverted*, and the size rapidly varies with the position of the pin. Keeping the pin stationary, in such a position that the inverted image is visible, place a light (a candle flame) to one side of the pin, thus forming a second object, and a piece of white paper, or other small screen, on the other side of the pin. Notice that a *real* and *inverted* image of the light is formed upon the screen, and if this were absent it would actually be formed in the air, although apparently seen in the mirror. It is an *aerial* image, and the reason why the aerial image of the pin cannot be received upon a screen is that the light from the pin is merely reflected light, and far too feeble to produce a real visible image, the surrounding light preventing the production of sufficient contrast.

There are certain terms which are used in connection with mirrors of this description. The distance apart of the edges used is called the *aperture* of the mirror, and the centre of the curved portion is known as the *pole* or *apex*. Further, an imaginary straight line joining the centre of curvature of the mirror with the geometrical centre of the curved part (pole or apex) is termed the *principal* or *optic axis*.

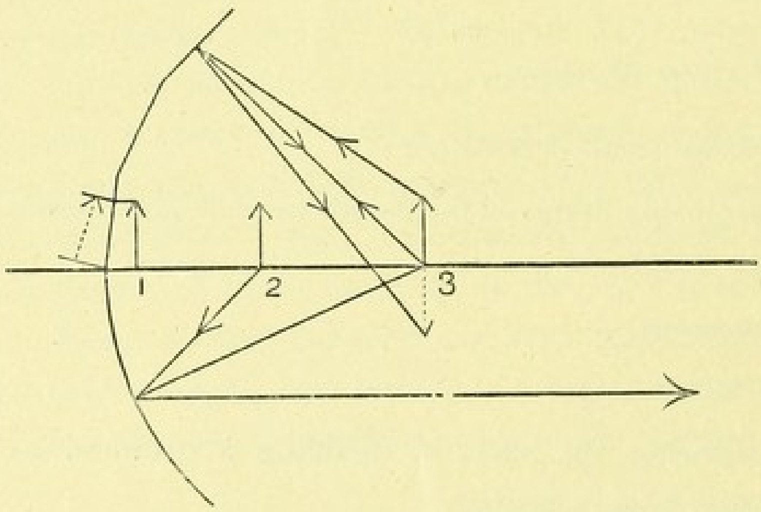



Fig 12

To explain the formation of the virtual erect image we may imagine the mirror to consist of an infinite number of plane surfaces, Fig. 12 showing three only for the upper part. Each of these is capable of producing a virtual and erect image (as explained in Chapter II.) identical in position behind the mirror with the object in front. As the object is moved from the mirror more of these supposed innumerable facets take up the reflection for the eye to view, and so the image increases rapidly in size until the object arrives at No. 2 position in the diagram, midway between the mirror and its centre of curvature. At this point the paths (rays) of light emanating from every point of the object are reflected along nearly parallel lines and so completely fill the view, the image seeming to fade away.

Let us now pass to the position marked No. 3, the centre of curvature of the mirror. The paths from this point on the axis will be reflected upon themselves from every part of the mirror, but as a part of the object is above the axis, it is evident that by making the angle of reflection equal to that of incidence for all of the supposed innumerable facets, the reflection of that part of the object will appear below the axis, the convergence of all the paths to that point forming a real image, and from the position, inverted.

As the object passes from No. 2 to No. 3 position the image decreases from an infinite size to that of the object, gradually taking form in an inverted position. No 3, it will be noticed, has a point exactly at the centre of curvature, so that all paths from that point are reflected upon themselves, forming normals to the supposed facets. The position of the image of the top of the object is thus readily understood by the construction shown.

Waves of light from an infinite distance undergo a change upon meeting the mirror such that after reflection they are twice the curvature of the mirror itself, and thus converge to point No. 2 which is termed the *principal focus*, so that the convergence or focal power of a mirror is twice its curvature, and our experimental mirror of 5 dioptres curvature has a convergence or focal power of 10 dioptres.

Consideration of the path from the point of the object at No. 3 in Fig 12, and also of that reflected to the point of the image, show that they are radii of the incident and reflected wave fronts respectively, and that they mark the direction in which these travel, just as in the lower part of the diagram the path parallel to the axis and that from the principal focus indicate the same.

These points of object and image are termed *conjugate* points, because light diverging from one converges to the other, and in the particular instance taken (No. 3), it will be noticed that object and image are at the same distance from the mirror, are alike in size, but are reversed in position, the image being inverted. If instead of the object being above the axis it were bisected by it, object and image would be coincident but reversed, and this position, viz., the centre of curvature, is the only one where such coincidence could occur. As the object approaches the mirror, so the image recedes from it, and becomes larger and larger, until the object gets to the principal focus, when the image is at an infinite distance. Movement within this point produces an image in a position which we may describe as beyond infinity, that is on the other side of the mirror, upright and virtual in character, which gradually lessens until object and image coincide at the surface of the mirror itself.

For any given *position* of the object upon the principal axis there is a given *position* for the image, and these points are known as *conjugate foci*, the conjugate point for the principal focus being at infinity. These points are interchangeable for object and image, although the object cannot in practice be placed where the virtual image is, as it would not be reflected if behind the mirror. This relation is known as the *Law of Conjugate Foci*, and applies also to direct *refraction* through a lens.

Unfortunately, when dealing with spherical mirrors, the expressions "principal focus" and "focal length" are not quite so definite as the above explanation would imply, and a glance at Fig. 13 shows why this is so.

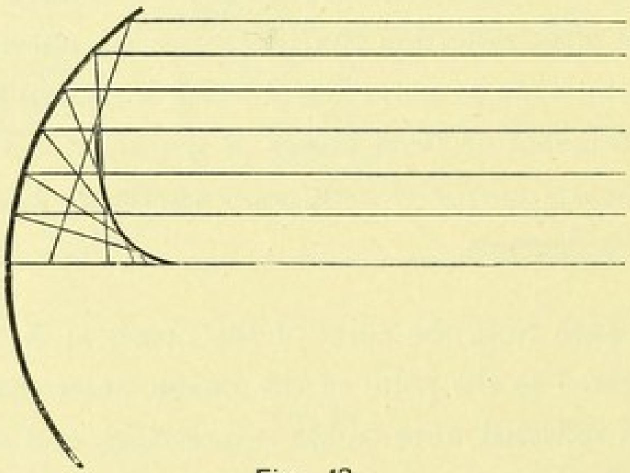


Fig. 13.

The parallel waves of light (the direction of which is represented by straight lines), coming from an infinite distance, strike each facet of which we supposed the mirror to be formed, and, following the law of reflection, cut the axis at very different points, in fact they envelop a surface called *the caustic*, and the curve indicated is therefore a caustic curve. It must suffice here to mention that the excess in bending, or curvature, of the peripheral parts of the waves of light gives rise to *astigmatism by reflection*, a condition in which the focus does not consist of a point or circle, but of two lines, really forming two line foci. This causes such distortion as to prevent the use of these mirrors in many cases. It is worthy of note that no distortion of image occurs when the object is at the centre of curvature.

Parabolic mirrors are employed when parallel light is to be brought to a focus, or when a parallel beam is required from a point of light. The parabola is obtained by cutting a cone by a section parallel to one of its sides, and is really an ellipse with one of its foci at infinity. The geometrical property of the parabolic curve is that it always satisfies the second law of reflection, and at the same time converges all portions of incident plane waves to the same point on its axis (Fig. 14). Mirrors of this type are of great service in practice.

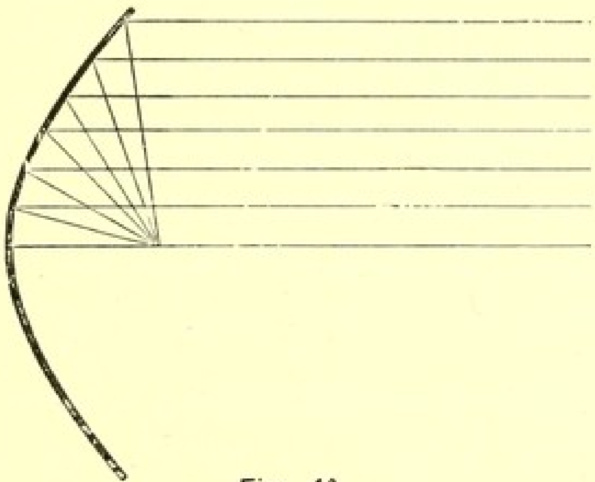


Fig. 14.

Convex mirrors do not lend themselves very extensively to practical use, and unless light is artificially made convergent before it reaches the mirror, they cannot produce a real image. Under ordinary conditions some form of virtual erect image of reduced size will be seen, and the paths of the waves of light may be traced in a similar manner to what has been done in a concave mirror, the reflected paths being produced backwards to form the virtual image.

To graphically demonstrate the formation of any image, real or virtual, by a curved surface, it is only necessary to consider that cone of light the limits of which are given by a line from the top of the object, passing through the centre of curvature of the surface, and a line from the same point of the object, running parallel to the axis until it meets the curved surface. (Fig. 15.)

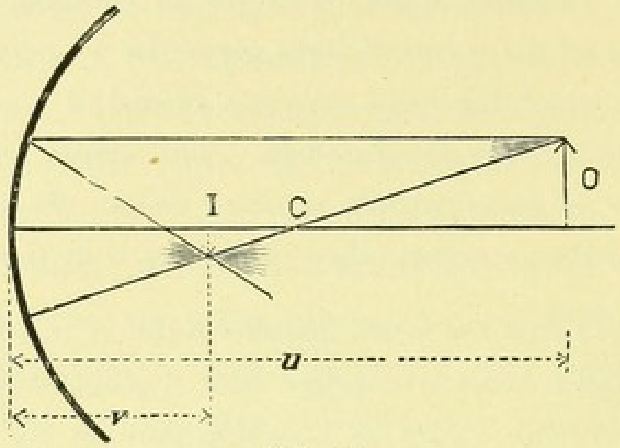



Fig 15.

It will be seen that for size of image and object the concave mirror may be likened to a pinhole camera with the pinhole at the centre of curvature, and Fig. 15 shows that the relation in size will also be equal to the ratio of the convergences of the incident and reflected light with respect to the mirror, although it must be noted that incident light is *actually* divergent.

*Practical Use of Curved Mirrors.*

(*a*) In the Retinoscope, Ophthalmoscope, &c.

(*b*) In Reflecting Telescopes.

(*c*) For Microscopic Illumination.

(*d*) As Reflectors.

(*e*) As Search Lights, &c. (Parabolic).

# Chapter IV.

## REFRACTION AT PLANE SURFACES.

WHEN light is incident upon a transparent medium, besides that amount which is reflected, according to the laws laid down in Chapter III, a certain portion enters the new medium, and should it be incident obliquely to the normal at the point of incidence, it is refracted, or bent out of its original path.

*Experiments.*

(1) Hold a pencil obliquely, and immerse half of it in water; note that the half under water appears bent with respect to the part above. (Refraction.)

(2) Hold a piece of thick plane glass in front of one eye, look through and over it at the same time, and note that on tilting it objects are apparently moved. (Deflection).

(3) Place a coin in a thin glass, and fill the glass with water to the brim. Put the glass on a table with another coin by the side of it, looking down at both coins. Note that the one seen through the water appears much nearer to the eye than that lying upon the table. (Displacement).

(4) Take a prism with a large angle (a right-angled one), and note that it is possible to view an object by *internal* reflection at one surface. (Total Reflection).

It has been mentioned that light is a motion in an unknown substance termed *ether*, the particles of which vibrate in a transverse direction to that of propagation, and that the many secondary waves set up destroy each other by mutual interference, so that the *grand* or main waves of light alone are what we need consider. Call a portion of such a main wave B C (Fig. 16), and let it meet a transparent piece of glass obliquely with respect to the plane surface. We wish to determine graphically in what manner and to what extent the course of the light is altered. In the first place we must think of monochromatic light, for then there will be but one change of velocity. Light of any colour moves at the same rate in ether or air, but so soon as light enters a medium such as a transparent piece of glass the rates of propagation of the different coloured lights vary one with another.

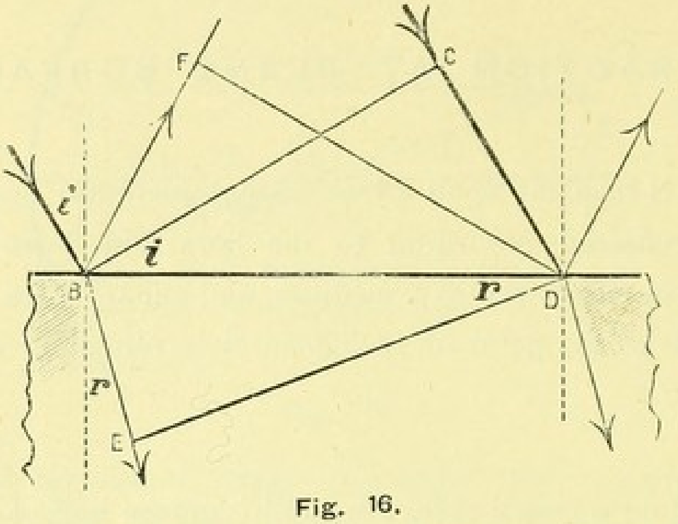



Fig. 16.

In the figure let the angle of incidence $i = 30^\circ$, and the refractive index $n_2$ of the glass for certain monochromatic light $= 1 \cdot 5$. Taking the refractive index of air as unity, this means that the light will travel only $0 \cdot 6\dot{6}$ times, or two thirds its rate in air. Thus its speed is retarded upon entering the glass, for as was shown in Chapter II $\frac{n_2}{n_0} = \frac{V_0}{V_2}$. It follows, therefore, that to obtain graphically the refracted path of the waves of light, we have simply to strike an arc from B such that its radius is equal in value to two thirds C D, drawing a tangent to this arc through D, which will give us the position of the wave of light E D in the glass.

Since we know from our second law of refraction that—

$$\frac{n_2}{n_0} = \frac{\sin i}{\sin r} \text{ or that } \sin r = \sin i \ \frac{n_0}{n_2}$$

$$\therefore \ \sin r = \frac{0 \cdot 5000 \times 1}{1 \cdot 5} = 0 \cdot 33\dot{3}$$ (see table of sines at end of book.)

$\therefore \ r = 19^\circ \ 30'$. Which acts as a check upon the above construction.

Besides the light entering and refracted by the new medium, a certain percentage is reflected at the surface of separation of the two media according to the laws of reflection, and will therefore occupy a position F D, when the refracted wave occupies the position E D. It will be noticed that the effect of retardation, or slowing down of the light in the glass, is to give it an entirely new direction of propagation. The explanation of the experiment, in which the pencil held obliquely in water appears bent, is that light coming from the pencil and entering the eye is bent out of its original course at the surface of separation, hence we see that part of the pencil under water in an incorrect position.

With reference to experiments 2 and 3, we may note that when the apparent movement of an object, due to refraction, is (for all practical purposes) a lateral one, it may come under the heading "Deflection," and when the apparent movement is a compound one, or a purely vertical one, it may come under the heading of "Displacement." This nomenclature will be found convenient in practice.

It is well to note that although waves of light are merely deflected by passing through a plate of glass with parallel surfaces, the deflection depends upon the thickness of the plate, being proportionate to it.

Before considering experiment 4, it is necessary to study the refraction of light by a prism, which may be defined as a transparent solid, two of whose faces at least are plane surfaces intersecting in a line.

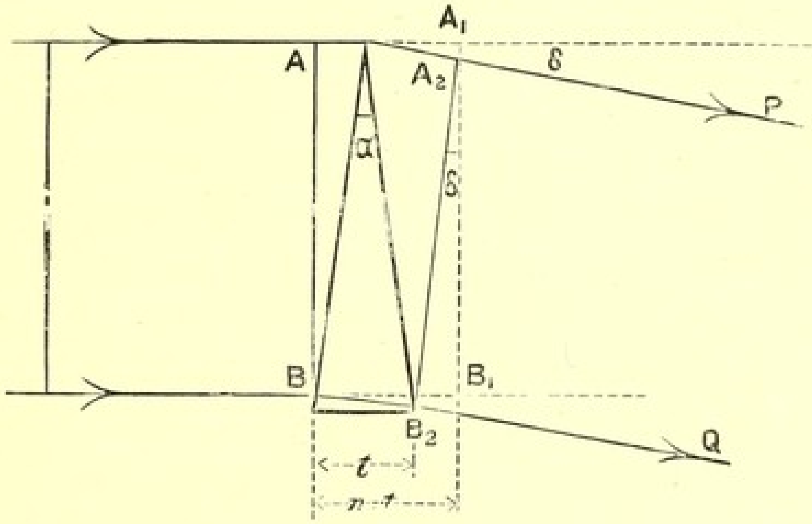



Fig 17.

In the case of a thin prism Fig. 17, with a zero thickness at its apex, and a thickness *t* at its base, we see that while the point A of the wave of light is only retarded for a very short period, the point B is retarded considerably, and therefore by the time A reaches $A_2$, B will only just be emerging from the prism at the point $B_2$. Had the prism not been in its path the wave would have occupied the position $A_1$ $B_1$. The amount of bending does *not* depend upon the thickness of material traversed, for $A_2$ P is parallel to $B_2$ Q, but upon the *change in the velocity*, or rate of propagation of the wave. The angle $\alpha$, made by the intersection of the two faces through which the light passes, is termed the *refracting angle* ; while the angle $\delta$, made by the incident and refracted waves (that is, the old and new paths) is termed the *angle of deviation*. Now the amount by which the wave is retarded depends upon the refractive index $n_2$ of the prism. If $n_2 = n_0$ then the point $B_2$ would be coincident with $B_1$ and no refraction would take place. It follows therefore that :—$BB_1 = n_2 t$

$$\therefore\ B_2B_1 = n_2 t - t = (n_2 - 1)\,t$$

But angles are proportional to their arcs standing on the same radius. Therefore, for thin prisms (whose refracting angles do not exceed 10°), we may take the chords $B\,B_2$ and $B\,B_1$ as being equal in length to their corresponding arcs, with a sufficient degree of accuracy, and may therefore substitute $\delta$ for $B_2B_1$, and $\alpha$ for $t$. By so doing we obtain the important relation, which holds good for *thin* prisms only, $\delta^\circ = (n_2 - 1)\,\alpha^\circ$.

*Refraction by a Thick Prism.*

The refraction of light, and particularly of plane waves, through a thick prism may be put to practical use in many ways, so that it is advisable to study the relations existing between the angles of incidence, refraction and deviation, the refracting angle, and the refractive index of the transparent substance under consideration.

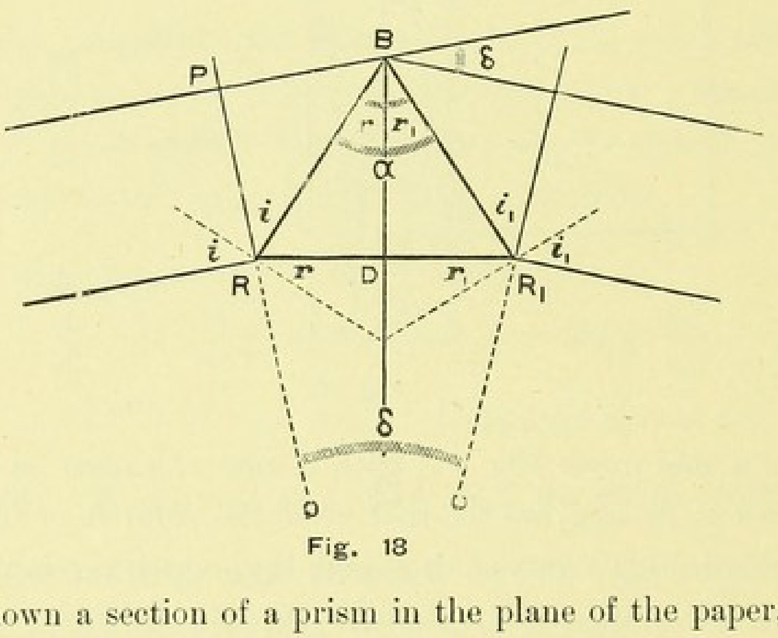



Fig. 18

In Fig. 18 is shown a section of a prism in the plane of the paper, to which the faces are supposed to be perpendicular. If we consider the portion P R of a plane wave incident upon the face R B at an angle $i$, then as the whole of this portion of the wave comes under the influence of the prism. by the time the point P has arrived at B, the point R will only have reached D, as its speed has been reduced, and the wave P R occupies the position B D in the prism. Upon emerging the wave is again bent away from its original path, for it enters a medium of less refracting power, and therefore its path makes a greater angle ($i_1$), with the normal to the second surface than what it did ($r_1$) upon meeting that surface. The angle through which the light is deviated is therefore $\delta$, known as the *angle of deviation*. The angle $a$, made by the intersection of the two faces of the prism, is the *refracting angle*. The angle $r_1$ is the angle of refraction for light retracing its path, and we see from the figure that the refracting angle is equal to the sum of the angles of refraction. That is :

$$r + r_1 = a \qquad (1)$$

Also $i$ and $i_1$ are the exterior angles of the triangles R B Q and $R_1$ B Q respectively, therefore

$$i + i_1 = \delta + a$$
$$= \delta + (r + r_1)$$
$$\therefore \quad \delta = i + i_1 - (r + r_1)$$
$$= i + i_1 - a \cdot \qquad (2)$$

Experiment shows that the angle of deviation $\delta$ varies with the angle of incidence, that is, the angle at which the wave front strikes the surface of the prism ; and further, that the condition for a minimum value for $\delta$ (minimum deviation) is the same as that for symmetrical refraction through the prism. Therefore, when the angle of emergence equals the angle of incidence (and $r = r_1$), the angle $\delta$ has its minimum value. The condition for minimum deviation being symmetrical refraction, the accurate measurement of this angle of deviation becomes of considerable practical importance, for when $\delta$ is a minimum, and it is easy to tell when this is the case with an instrument for measuring angles, such as a goniometer, the wave B D produced, Fig. 18, will exactly bisect the angle $\delta$.

Therefore $i = i_1 = \frac{a + \delta}{2}$, also

$$r = r_1 = \frac{a}{2}.$$

But the refractive index of the substance,

$$n_2 = \frac{\sin i}{\sin r} = \frac{\sin i_1}{\sin r_1}$$

$$\therefore \quad n_2 = \frac{\sin\left(\frac{a + \delta}{2}\right)}{\sin\left(\frac{a}{2}\right)} \qquad (3)$$

The goniometer enables the angles $\alpha$ and $\delta$ to be accurately measured. Here, therefore, is an easy and accurate method of determining the refractive index of transparent substances, but we must remember that this equation (3) only holds good when the deviation is a minimum. Liquids may be made to take the required form by putting them in prism-shaped glass troughs.

A goniometer consists essentially of a collimator and a telescope mounted upon a horizontal circle, and facing a small table for holding the prism, about which they can rotate.

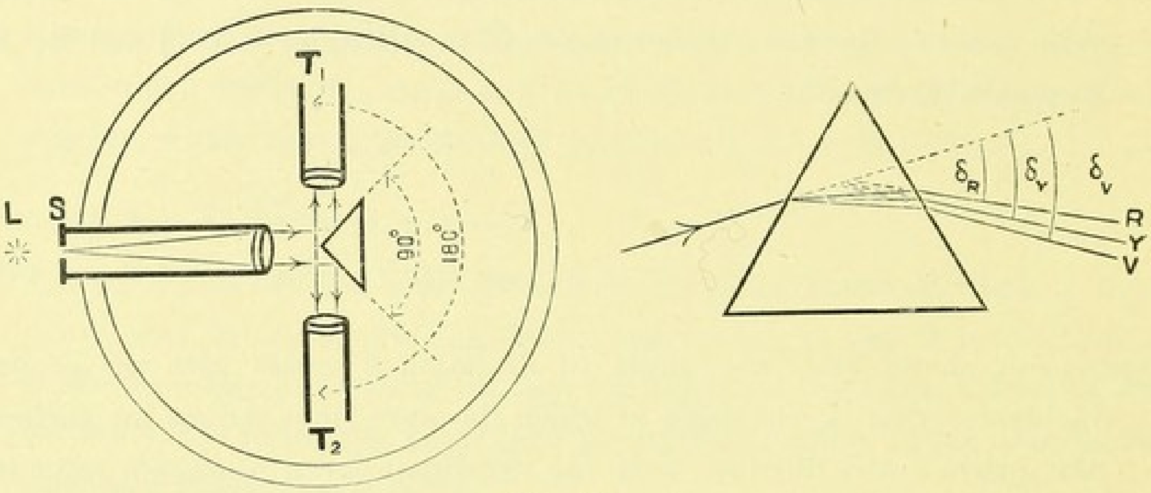



Fig. 18a.

The collimator is an instrument for producing parallel light, and consists simply of a vertical slit and a convex lens, placed at a distance from the slit equal to its focal length. The telescope is a simple astronomical one, fitted with cross-wires. To measure the refracting angle $\alpha$, the apex of the prism is faced down the collimator, and the telescope first moved into position (1) and then to position (2). The angle made by the two positions of the telescope may be read off upon the graduated circle, and this angle will be just *twice* the angle $\alpha$. To measure the angle $\delta$ a direct reading is taken, without the prism in position, and then, with the prism replaced as shown by the sketch annexed to Fig. 18a, a reading of the position of the telescope is taken for minimum deviation. That is when the angle $\delta$ is as small as it possibly can be for the particular coloured light. The prism itself should be rotated about a vertical axis at the same time as the telescope is moved, the slit of light in the field of view of the telescope being always kept in sight. $\alpha$ and $\delta$ in formula (3) should then be replaced by their respective values and the refractive index $n$ worked out. A good instrument will give it correct to the fourth decimal place, with care

From the diagram it will be noticed too that the angle $\delta$ increases as the *time period* for the light decreases, so that the angle $\delta$ is greater for violet than red light. Hence the refractive index will be greater for violet than for red light for the same piece of glass.

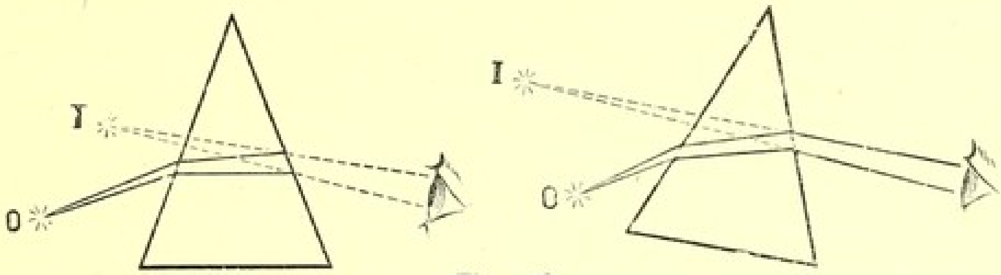



Fig. 19.

Fig. 19 illustrates vision of an object through a prism, and the effect of movement of the prism upon the position of the image. All waves emanating from O and meeting the prism surface will be spherical ones, and in each case the section of a small cone of light is shown, the first figure giving the position for minimum deviation, and the dotted lines indicating the direction in which the image is seen.

In the second figure the object and eye are in precisely the same places, but the prism has been so tilted that the base is nearer the object. Not only is the deviation greater, but the image has suffered displacement, being further away, and the visual angle is smaller, which always causes the image to appear smaller. These effects are more pronounced as the angle of the prism increases in size, and as the position of minimum deviation is departed from in either direction.

We may thus see, that as the base of a prism is turned towards an object and the apex away, so the object appears to decrease in size, and as the base is turned away, so it appears to increase in size. Other phenomena, due to dispersion, observed when looking through a prism, are dealt with in Chap. VI.

*Refraction of a Spherical Wave at a Plane Surface.* In Fig. 20, let a divergent spherical wave of light, proceeding from the point O, meet the plane surface of a block of more highly refracting transparent substance. That portion B, which meets the surface of separation first, will come under the influence of the more highly refracting medium first, and therefore be retarded by a certain amount before any other portion. Hence, by the time the whole wave A B C is in the second medium, its shape will be altered. If the new medium had not been encountered the wave would at some instant have occupied the position shown by the dotted lines of the curve.

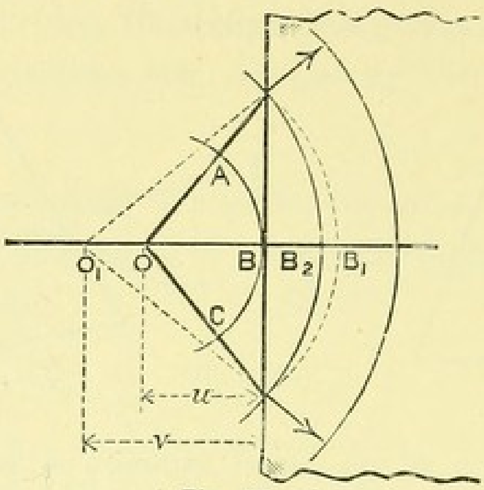



Fig. 20.

As, however, the new medium retards the wave, it becomes flattened, and appears to be proceeding from the new centre $O_1$. If, then, $B B_1$ represents the velocity of the wave in the first medium, $B B_2$ represents the velocity in the second.

But $\frac{n_2}{n_0} = \frac{V_0}{V_2}$ and $n_2 = \frac{V_0}{V_2}$ when $n_0$ equals unity, as for air.

Therefore $n_2 = \frac{B B_1}{B B_2}$, or $B B_1 = n_2 B B_2$ ; an important relation.

The path of the wave which meets the surface normally is not refracted. The refracted wave will really form a hyperbola, but we may consider it as the portion of a sphere. If $r_1$ and $r_3$ denote the radii of the incident and refracted waves, then :—$n_0 r_3 = n_2 r_1$ . (but $n_0 = 1$)

$$\therefore r_3 = n_2 r_1. \qquad (4)$$

Thus the radius of the refracted wave is $n_2$ times the radius of the incident wave. Or, denoting O B by $u$ and $O_1$ B by $v$, we have :—

$$v = n_2 u.$$

Suppose that the incident wave had been plane, or, secondly, that the surface had been a spherical one, with centre at O. In neither case would any refraction take place, for in both cases all points of the incident wave would come into contact with the new medium simultaneusly (and therefore normally). The rate of propagation would be altered of course, and so the wave would be retarded ; but since all points of the wave would be retarded to the same degree, no alteration in the shape of the wave would take place, and consequently there would be no refraction.

We have seen how curvature can be expressed in dioptres (Chap. I.) and curvature was shown to be the reciprocal of its radius, in metres. We may follow the effect of refraction, therefore, through any number of media, with any number of surfaces—curved or otherwise—and all we need do is to add and subtract, noting the change undergone in the shape of the wave at each successive refraction, in order to determine the curvature of the final emergent wave, and therefore the position at which the wave is brought to a focus.

*Total Reflection.* Experiment 4, described previously, illustrates a very important phenomenon—that of total reflection. This only takes place when light impinges upon a medium of less refractive index. In Fig. 21

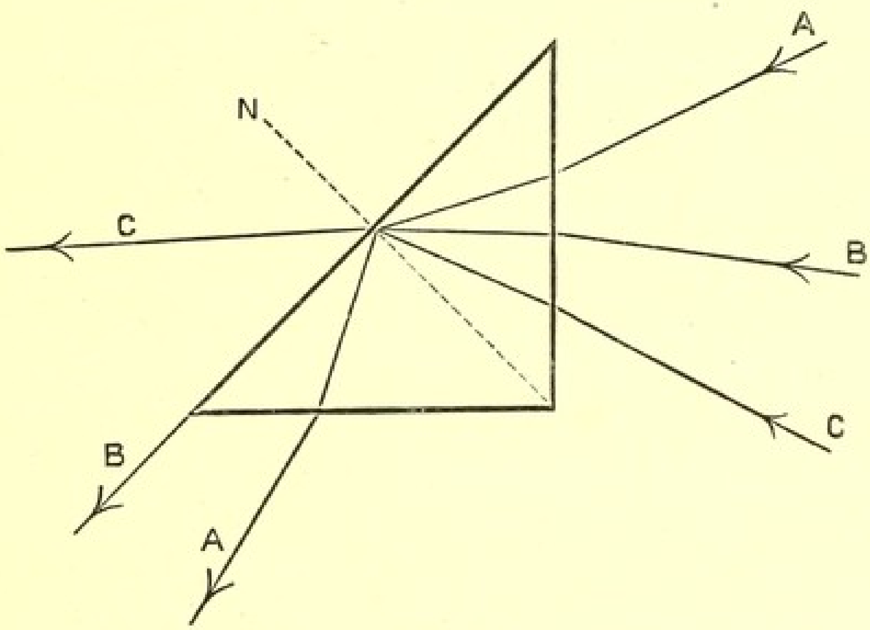



Fig 21.

three paths of light are shown passing into a prism, the paths indicating the direction of march of the wave fronts. When they arrive at the long side of the prism, their various paths become quite different, for the one is refracted into air, the second is deflected along the boundary, while the third is totally reflected, in accordance with the laws of reflection. In the case of the second path, the angle which this makes with the normal is known as the *critical angle*. If we reverse the path, then the angle of incidence in air is 90°, and sin 90° = 1. But when $n_0 = 1$, $n_2 = \frac{\sin i}{\sin r}$, therefore when $i = 90°$ in air, $n_2 = \frac{1}{\sin c}$, which is unity divided by the sine of the critical angle. Conversely, if we know the $n$ of a substance we can calculate the value of its critical angle.

Right angled prisms are used in various instruments to obtain total internal reflection, and it will be observed that the images so obtained will be inverted.

Herewith are given the refractive indices and critical angles of three substances, which should be verified by use of the above formulæ.

| | | |
|---|---|---|
| Water | $n = 1{\cdot}33$ | Critical angle $48^\circ{\cdot}34'$. |
| Crown glass | $n = 1{\cdot}51$ | ,, $41^\circ{\cdot}14'$. |
| Flint glass | $n = 1{\cdot}53$ | ,, $40^\circ{\cdot}46'$. |

**Summary.**

(*a*) In refraction by a thin prism $\delta^\circ = (n_2 - 1)\ a^\circ$

(*b*) With a thick prism $n_2 = \dfrac{\sin\left(\dfrac{a^\circ + \delta^\circ}{2}\right)}{\sin\dfrac{a^\circ}{2}}$

(*c*) The condition for minimum $\delta$ is the same as for symmetrical refraction.

(*d*) Tilting a prism before the eye alters the size and position of the image seen.

(*e*) $n_2 = \dfrac{1}{\sin c}$ (critical angle).

# CHAPTER V.

## MIRROR AND THIN LENS PROBLEMS.

WE have seen that light consists of a wave motion in the ether, and we may represent it diagrammatically, in section, by concentric circles, as shown in the diagrams; but what is more important, we can definitely state the *dioptric power of the light* at any particular moment, provided we know the position of the point from which it diverges, or to which it is converging. The dioptric power of the light at any particular instant is the wave curvature expressed in dioptres. For example, in Fig. 22, A is a source

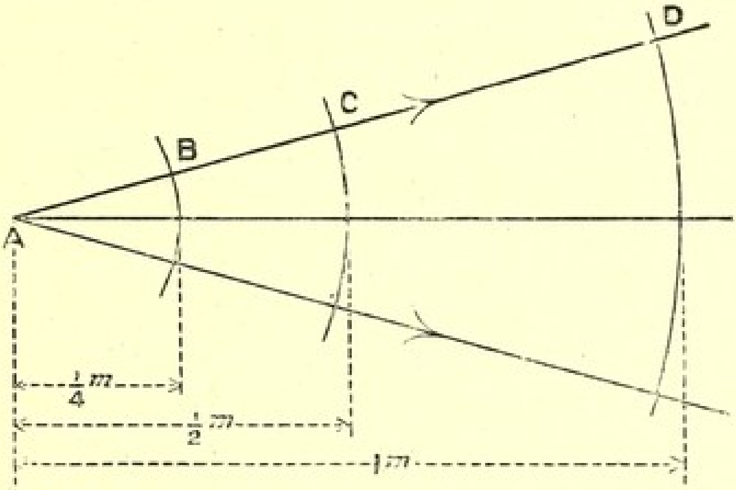



Fig. 22.

of light, and since light is a wave motion, at $\frac{1}{4}$ metre distance we may represent the light by the wave B, at $\frac{1}{2}$ metre by C, etc. The radius of the wave in linear measurement does not immediately convey to our minds its curvature, hence we have resource to the unit of curvature, the dioptre. The imaginary wave with a radius of one metre we say has a *power* of one dioptre, and as the radius decreases, so the curvature of the wave correspondingly increases, and vice versâ. When, therefore, the radius is $\frac{1}{4}$ metre (25 c.m., 10 inches) the curvature of the light wave is given by *4 dioptres.* Thus the power in dioptres is the reciprocal of the radius in metres, and in a similar manner all spherical surfaces may have their curvature defined. A lens, for instance, has two curved surfaces, $R_1$ and $R_3$; if $R_1$ and $R_3$ have equal radii ($r_1$ and $r_3$), each $\frac{1}{5}$ metre (20 c.m.)

in length, the curvature of each surface is equal to 5 dioptres. Again, the change in curvature (alteration in the shape of the wave of light) due to reflection or refraction may obviously be expressed by another curve, as has already been shown. This curve is termed F, and when expressed in dioptres represents the reflecting or refracting power, as the case may be, of the system under consideration. F is therefore known as the *focal-power* or *convergence* of the system. The reciprocal of F, in dioptres, gives the focal length, *f*, of the system, in metres. This F is the algebraic sum of the incident and reflected, or emergent, curvatures of the wave, and denoting the incident curvature by U and the reflected or emergent one by V we see that F is equal to the sum of U and V when U and V are curved in opposite directions, and the difference of U and V when U and V are curved in the same direction. F therefore is equal to $V + U$, or $V - U$, or $U - V$, according to circumstances. If, however, we decide always to treat divergent curves as negative and convergent curves as positive, we may write once for all

$$F = V - U,$$

and re-arrange this equation to suit the particular problem we wish to solve.

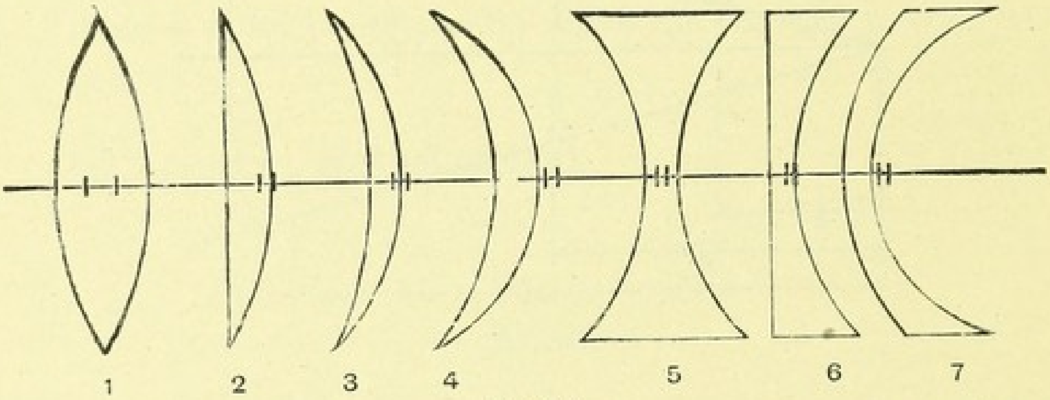



Fig. 23.

1. Double convex. 2. Plano convex. 3. Periscopic convex. 4. Deep periscopic convex. 5. Double concave. 6. Plano concave. 7. Deep periscopic concave.

In Fig. 23 names are given to the various types of lenses shown, which represent the chief forms in use. The focal power or convergence does not depend upon the shape of a lens, but upon its total refracting power, and two lenses may vary widely, so far as shape is concerned, and yet have the same focal power—producing the same change in curvature upon the wave of light.

*The Principal or Optic Axis of a Mirror* is the imaginary line, produced if necessary, joining the centre of curvature of the mirror and the geometrical centre of that portion of the mirror under consideration.

*The Principal Axis of a Lens* is the imaginary line, produced if necessary, joining the centres of curvature of the two surfaces.

*Optical Centre.* The optical centre is that point upon the principal axis through which pass all waves which have their paths parallel before and after reflection or refraction.

*The Optical Centre of a Mirror.* From the above definition of an optical centre it follows that the optical centre in this case is coincident with the centre of curvature of the surface.

*The Optical Centre of a Lens.* In practice one requires to deal with this imaginary point from two separate positions: first, directly from the front; and secondly when the lens is seen in section—edgeways. For convenience we may term the first, the *direct optical centre;* and the second, the *sectional optical centre.* Then we have—

*The Direct Optical Centre* is that point at which the principal axis cuts the lens.

*The Sectional Optical Centre* is that point on the principal axis at which the imaginary line joining two parallel radii at the points of contact with their respective surfaces cuts it. (This satisfies the definition of *the* optical centre as above defined.)

The method for determination of the position of the direct optical centre of a lens in practice is to hold it in front of the eye so that two cross lines, such as are formed by the bars of window panes, are in perfect alignment outside and within the lens when viewed through it. The point of intersection of the lines as seen through the lens is the direct optical centre. The cross lines, whether upon a white card placed horizontally, or as window bars in a vertical plane, must always be in a plane parallel to that in which the lens is held.

The graphical determination of the position of the sectional optical centre is seen in Fig. 24 (I. and II.). C and $C_1$ are the centres of curvature of the surfaces, and any two radii C B and $C_1$ A are drawn parallel to each other. B and A are joined, and in (I.) where this line cuts the principal axis is the position of the optical centre. In (II.) it is necessary to produce B A along the path of the wave outside the lens, according to the law of refraction, $C_1$ A being the normal to the first surface. Similarly in (I.) we may trace this, and also the path of emergence B E, and in all cases D A and B E must be parallel.

The deviation of the path will be represented by *d*, and is exactly similar to that caused by an equivalent rectangular block of glass P Q R S, of thickness *t*.

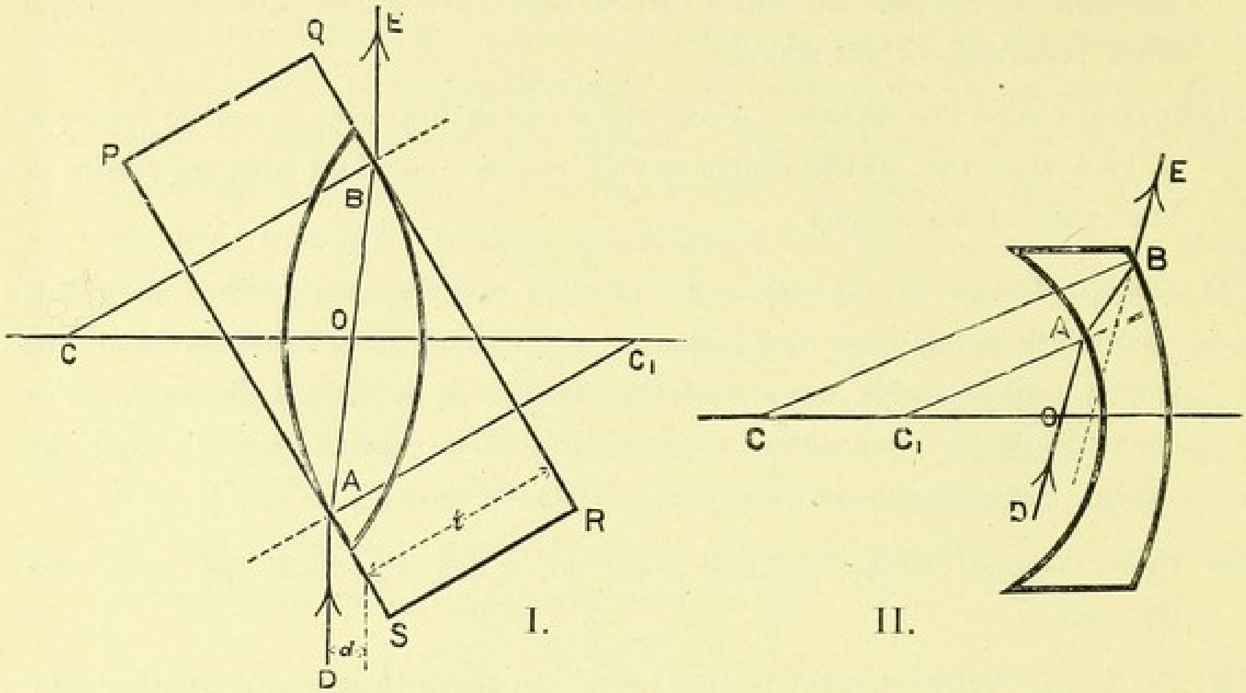



Fig. 24.

We are now in a position to investigate two important fundamental formulæ applicable to both spherically curved mirrors and thin spherical lenses, remembering the notation adopted in Chapter I.

In Fig. 25 (I., II., III., IV.), in each case let U represent the curvature of the incident wave (incident curvature), that is, the curvature of the wave of light just as it comes into contact with the reflecting or refracting surface; then *u*, the radius of curvature of the incident wave is called the *first conjugate distance.* By reflection or refraction, as the case may be, the curvature of the wave is altered, and the new form will be complete when the last portion of the wave has just been reflected or refracted. At this moment the curvature of the wave V is called the emergent curvature; then *v*, the radius of this curve, will represent the *second conjugate distance.* These are the true conjugate distances, if we define them as being the radii of the incident and reflected, or emergent, waves. In practice, however, it is convenient to measure both *u* and *v* from the geometrical centre of the mirror surface, or from the optical centre of the lens (not shown in III. and IV.), for these are points whose positions may be determined with great accuracy. Moreover, by denoting the curvature which the incident

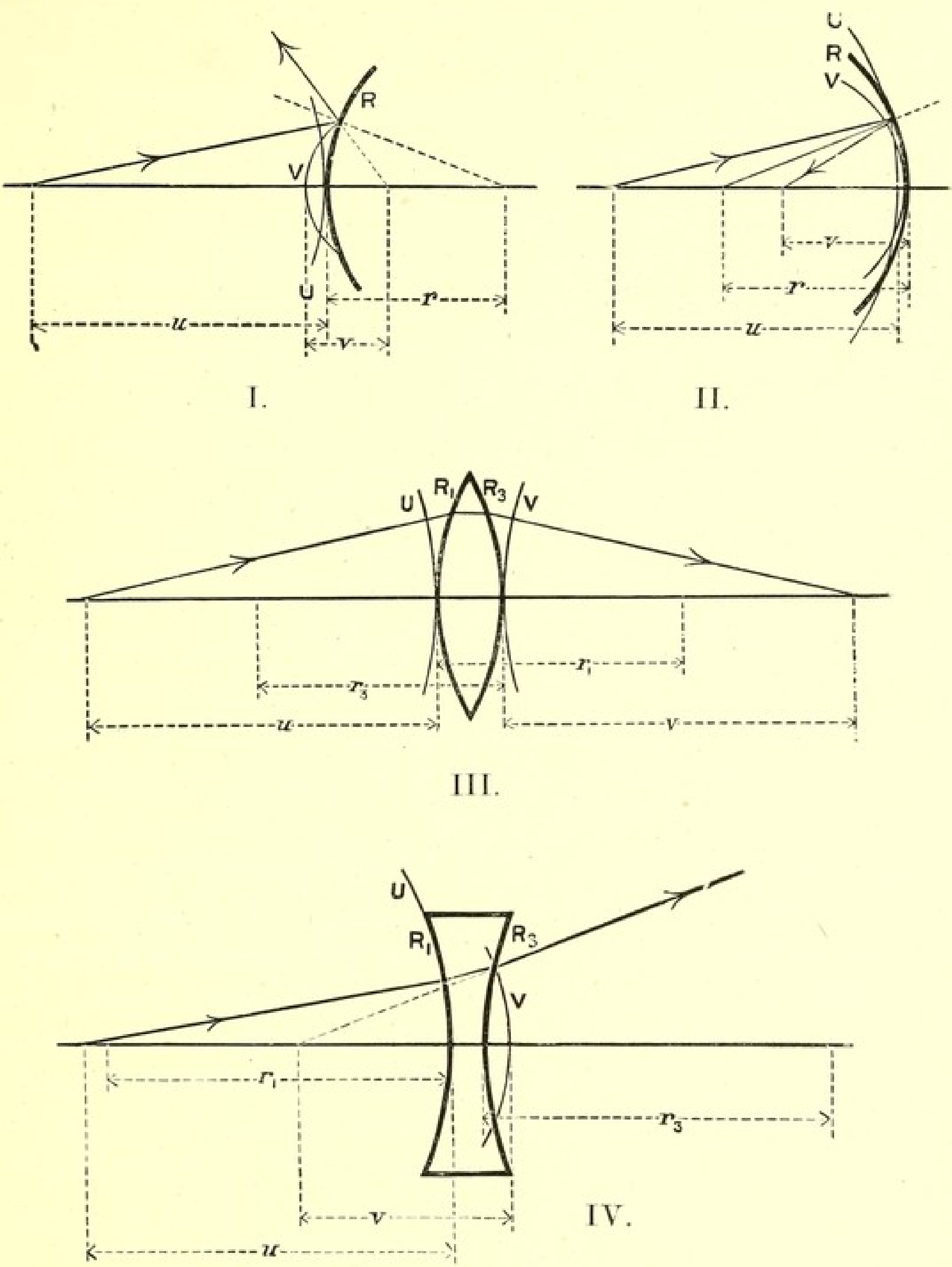



Fig. 25.

wave would have, at the mirror surface, or optical centre of a lens, as the incident curvature, and the curvature that the emergent wave would have at the mirror surface, or optical centre of a lens, the emergent curvature, the relations which exist between these curves and the other factors in mirror and lens problems can be expressed, mathematically, in a simple manner.

*Mirror Formulæ :*

(1) $$R = \frac{V-U}{2} = \frac{1}{2} F$$

$$\text{or } 2R = V - U = F$$

A relation expressing mathematically the law of conjugate foci.

(2) $$m = \frac{v}{u} = \frac{U}{V}$$

Where $m$ represents the *magnification*, or the ratio of the linear dimensions of image and object.

In Fig. 25a a plane wave of light A B, falling upon the concave mirror of curvature R, receives a curvature V upon reflection.

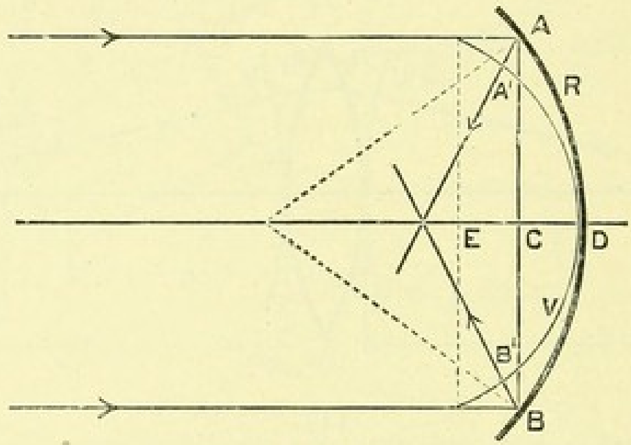



Fig. 25a.

Since the incident wave A B is plane its curvature $U = O$, therefore the curvature V of the reflected wave is equal to F. Upon incidence the centre portion of the wave A B has a distance C D still to travel before being reflected, so that by the time the point C reaches D the points A and B of the wave will have been reflected a distance equal to C D and will be in the positions A′ and B′. Therefore C E = C D, and since curvatures are proportional to their sags on equal chords we have

$$R = \frac{1}{2} F \text{ or } \mathbf{F = 2R}$$

but according to our definition of F, as well as from the figure, treating divergent light as negative and convergent as positive,

$$F = V - U$$

$$\therefore 2R = V - U$$

Whatever the curvature of the incident wave, that of the reflecting surface must be the mean between the curvatures of the incident and reflected waves, or :

$$R = \frac{V-U}{2} \quad \text{and therefore}$$

$$2R = V - U = F$$

Concave mirrors produce convergent waves, or less divergent ones than the original; whereas convex mirrors produce divergent, or less convergent ones. We have therefore with light from L to R

*a.* Concave Mirrors $-R$ and $+F$

*b.* Convex Mirrors $+R$ and $-F$

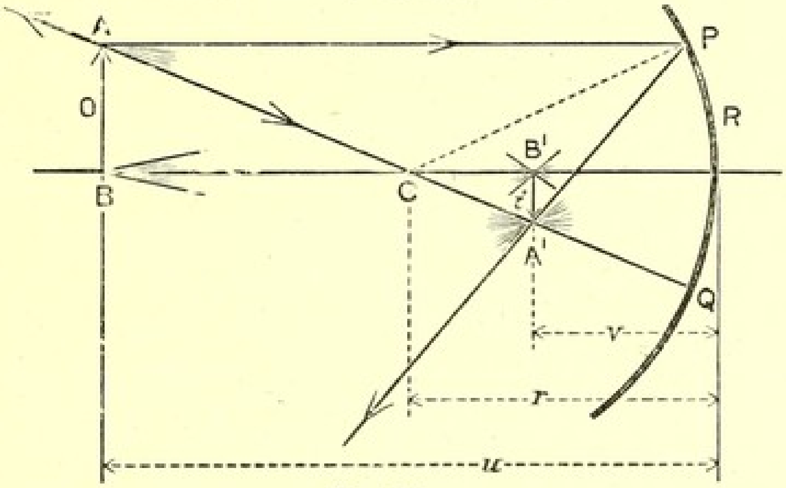



Fig. 26.

The magnification formula may be deduced from Fig. 26. A divergent cone of light from the point A of an object A B falls upon a mirror of curvature R, and is transformed into a cone converging to the point $A^1$. Similarly a divergent cone from B is transformed into a cone converging to $B^1$. In the plane $A^1 B^1$, therefore, a perfect representation, the *image* of the object A B will be formed.

In order to graphically determine the position and size (magnification) of the image only two lines need be drawn from the extremities of the object, A P parallel to the axis, and A Q through the centre of curvature (optical centre) C of the reflecting surface. The line A P, representing a path, will be reflected along $A^1$ P by the mirror, A P and $A^1$ P, making equal angles with C P. The line A Q will retrace its path, since it strikes the mirror normally. Therefore—

$$AB : A^1B^1 :: BC : B^1C$$

$$\text{but } BC = u - r \text{ and } B^1C = r - v$$

$$\therefore BC : B^1C :: u : v$$

Calling A B, $o$; and $A^1 B^1$, $i$, we have :—

$$o : i :: u : v;$$

$$\text{but by definition } m = \frac{i}{o}$$

$$\therefore m = \frac{v}{u}, \text{ and since } v = \frac{1}{V} \text{ and } u = \frac{1}{U}$$

$$\therefore m = \frac{U}{V}.$$

Therefore we see that the magnification is given by the ratio of the curvatures (in dioptres) of the incident and reflected waves.

From the equations

(1) $$F = 2\,R = V - U$$

$$\text{or } \frac{1}{f} = \frac{2}{r} = \frac{1}{v} - \frac{1}{u}$$

(2) $$m = \frac{v}{u} = \frac{U}{V}$$

the whole of the properties of mirrors may be deduced.

The equations

(1) $$F = V - U$$

and (2) $$m = \frac{v}{u} = \frac{U}{V},$$

are applicable to the solution of thin lens problems, but the alteration in the shape of the wave by refraction at a thin lens depends upon the refractive index of the glass from which the lens is made. For lenses therefore we must not write

$$F = V - U = [2\,R].$$

Calling the curvatures of the two surfaces of the lens $R_1$ and $R_3$, the refractive index $n_2$, and the refractive index of the surrounding medium $n_0$, the relation existing between these quantities and the focal power F is given by the equation—

$$F = (n_2 - n_0)\ (R_1 - R_3).$$

If the surrounding medium is air $n_0 = 1$, and we may therefore write :—

$$F = (n_2 - 1)\ (R_1 - R_3) \qquad (3)$$

$$\therefore\ V - U = (n_2 - 1)\ (R_1 - R_3) \qquad (4)$$

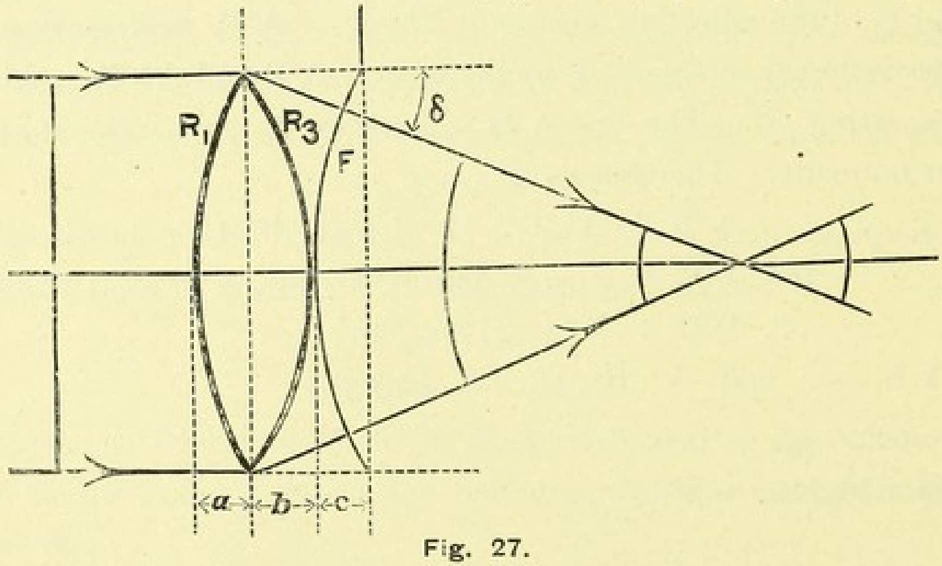



Fig. 27.

In this figure $t = a + b$

and $n_2\,t = a + b + c$

$\therefore\ (n_2 - 1)\,t = c$

In Fig. 27 let $t$ denote the thickness of the convex lens L, $a$ the sag corresponding to the curvature of its first surface, $b$ the sag corresponding to the curvature of its second surface, and $c$ the sag corresponding to the curve F, which represents the change undergone in the curvature of the wave, that is, the focal power of the lens. Then $t$ represents the sag corresponding to the sum of the curvatures $R_1$ and $R_3$. Also, as in the case of refraction by a thin prism (Chap. IV.), we see that the retardation, represented by the sag $c$, is given by $(n_2 - n_0)\,t$. If $n_0 = 1$ this may be written $(n_2 - 1)\,t$.

$$\text{But} \quad t = a + b$$

$$\therefore \quad c = (n_2 - 1)(a + b), \quad \text{and consequently}$$

$$\mathbf{F} = (n_2 - 1)(R_1 + R_3).$$

Remembering the rule of treating surfaces curved like divergent waves as negative, and surfaces curved like convergent waves as positive, we must write :—

$$F = (n_2 - 1)(R_1 - R_3)$$

having $+ R_1$ and $- R_3$ according to our convention of signs.

From the three equations

$$F = V - U \qquad (1)$$

$$m = \frac{v}{u} = \frac{U}{V} \qquad (2)$$

$$F = (n_2 - 1)(R_1 - R_3) \qquad (3)$$

the whole of the properties of thin lenses may be deduced.

Equation (3) may be written

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{r_1} - \frac{1}{r_3}\right).$$

Herewith are given a few examples of mirror and thin lens problems. In each case sketches should be made by the reader for illustration.

*Example (1). How far from a concave mirror, whose radius of curvature, r, is equal to 1 metre, would you place an object, to obtain a real image of three times the size?*

We have
$$m = \frac{v}{u} = 3$$

$$\therefore \quad v = 3u.$$

Also, for a concave mirror producing a real image $\frac{2}{r} = \frac{1}{v} - \frac{1}{u}$ becomes $\frac{2}{r} = \frac{1}{v} + \frac{1}{u}$ since the incident light is divergent, and therefore

$u$ is negative. Strictly speaking we should write $-\frac{2}{r} = -\frac{1}{v} - \frac{1}{u}$ (See Fig. 25 II.)

$$\therefore \quad \frac{2}{r} = \frac{u + v}{v\,u} = \frac{4\,u}{3\,u^2}\,, \text{ since } v = 3\,u.$$

Putting in the value $r$, we have :—

$$\frac{2}{1} = \frac{4}{3\,u}\,, \text{ or } 3\,u = \frac{4}{2}$$

$$\therefore \quad u = \frac{2}{3} \text{ metres} = 66.\dot{6} \text{ c.m.}$$

$$\text{and } v = 3\,u = 2 \text{ metres} = 200 \text{ c.m.}$$

The object would therefore be placed at $\frac{2}{3}$ of a metre from the concave mirror. The image formed must be a *real* one, for the light waves after reflection are convergent, and the object is at a point on the axis between the centre of curvature and the focal point.

*Example (2.) Determine the size and position of the image of an object 5 c.m. long placed 25 c.m. in front of a convex mirror, whose radius of curvature is 50 c.m.*

Here $u = 25$ c.m. $\therefore$ U $= 4$ dioptres

But $\frac{2}{r} = \frac{1}{f}$ (the focal length is half the radius of curvature).

$\therefore \; f = 25$ c.m.

Also F $= \frac{1}{f}$ (metres)

$= 4$ dioptres.

Since the mirror is convex, both incident and reflected light is divergent, but the reflected divergent light is curved in the direction of convergent light passing from L to R. So that in this case we have $-$ U, $+$ V, $+$ R, and also $-$ F, according to our notation. (Fig. 25 I).

Instead of F $=$ V $-$ U, we have $-$ F $=$ V $-$ ($-$ U).

$\therefore$ V $= -$ F $-$ U in this case.

$= -$ 8 dioptres. (The negative sign indicates a *virtual* image.)

$\therefore \; v = \frac{1}{8}$ metres $= 12{\cdot}5$ c.m.

$$\text{Also } m = \frac{v}{u} \text{ or } \frac{\text{U}}{\text{V}}$$

$$= \frac{12{\cdot}5}{25} \text{ or } \frac{4}{8} = \frac{1}{2}$$

The virtual image is therefore apparently situated 12·5 c.m. behind the mirror, and is only one half the size of the object.

*Example (3). An object is placed 20 c.m. in front of a convex lens which produces a real image of the object upon the other side, at a distance of 1 metre. What is the focal power of the lens, and the magnification?*

In this case, since

$$m = \frac{v}{u} = \frac{100 \text{ c.m.}}{20 \text{ c.m.}} = 5. \quad \text{(magnification)}$$

Also we have $-$ U and $+$ V.

Therefore F = V − U becomes

$$F = V - (-U) = V + U$$

$$u = 20 \text{ c.m.} \qquad \therefore \quad V = 5 \text{ dioptres.}$$

$$v = 100 \text{ c.m.} \qquad \therefore \quad V = 1 \text{ dioptre.}$$

$$\therefore \quad F = 1 + 5 = 6 \text{ dioptres (focal power).}$$

The focal length, $f = \frac{1}{F} = \frac{1}{6}$ metres $= 16{\cdot}\dot{6}$ c.m.

Which means that parallel light falling upon the lens will be brought to a focus, on the other side, at $16{\cdot}\dot{6}$ c.m., from the optical centre of the lens.

*Example (4). The refractive index of a double-convex lens is stated to be 1·53, and the curvatures of its surfaces in dioptres 3 and 5 respectively. What is the focal power of the lens?*

Here $n_2 = 1{\cdot}53$, $R_1 = 3{\cdot}0$, and $R_3 = 5{\cdot}0$.

Now $F = (n_2 - 1)(R_1 - R_3)$

But since the lens is a double-convex one, we have $-R_3$ because this surface is curved like divergent light passing from left to right.

$$\text{Therefore} \quad F = (n_2 - 1)(R_1 + R_3)$$
$$= 0.53 \times 8 = 4.24 \text{ dioptres.}$$

*Measurement of Curvature in Practice.* We often require in practice to determine the curvature in dioptres of a mirror or lens surface. This is most easily accomplished by applying to the surface an instrument, called a *spherometer*, consisting essentially of a tripod or ring, in the middle of which a vertical rod, capable of being raised or lowered by mechanical means, is fixed.

The principle of the instrument is based upon a theorem of Euclid (Book III., Prop. 35):—

"*If two chords of a circle cut one another at a point within the circle, the product of the segments of one chord is equal to the product of the segments of the other chord.*"

Therefore in Fig. 28, $c \times c_1 = h \times m$, and $m = 2r - h$.

Therefore if $c = c_1$ we have

$$c^2 = h(2r - h)$$
$$= 2rh - h^2$$

From which we see that

$$\frac{c^2 + h^2}{2h} = r,$$

and therefore that, since $R = \frac{1}{r}$,

$$(5) \qquad \frac{2h}{c^2 + h^2} = R \quad \text{(the curvature of the surface).}$$

The spherometer is so constructed that it fulfils the conditions of the above equation. The distance from its vertical rod to the edge of the ring, or one of the tripod legs, corresponds to the distance $c$ in Fig. 28, while a movement of the vertical rod enables the elevation (or depression), corresponding to the distance $h$ in the figure, to be measured.

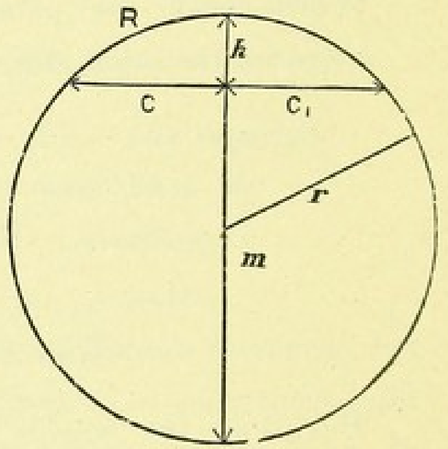


Fig 28.

The spherometer therefore gives the exact values for $c$ and $h$ for the surface under test. These values are then substituted for $c$ and $h$ in equation (5) and R, the curvature of the surface in dioptres, determined. To obtain R in dioptres the distances $c$ and $h$ must of course be expressed in metres.

The instrument affords an easy method of determining experimentally the focal power of a mirror, since F = 2R, and as, for lenses,

$$F = (n_2 - 1)(R_1 - R_3),$$

if we know the refractive index of the substance of which a lens is composed F is easily determined. Moreover, should F, $R_1$ and $R_3$, be known $n_2$ can be determined.

The "lens measure" of the optical trade has a dial attached, with the readings so arranged that the actual curvatures measured have been multiplied by ·51, *i.e.* ($n_2 - 1$). For this purpose it is assumed that crown glass of 1·51 refractive index will be used, so than an *optically* denser material would give an incorrect reading, as in the case of flint glass or "pebble." The instrument is more convenient than accurate, because it measures the dioptric power of each surface directly.

## Summary.

(*a*) F denotes focal power or convergence of a mirror, lens, or lens system.

(*b*) For Mirrors $R = \frac{V - U}{2} = \frac{1}{2} F$

$$\text{or } \frac{1}{f} = \frac{2}{r} = \frac{1}{v} - \frac{1}{u}$$

$$m = \frac{v}{u} = \frac{U}{V}$$

(*c*) For thin Lenses.

$$F = V - U = (n_2 - 1)(R_1 - R_3)$$

# Chapter VI.

## CHROMATIC AND SPHERICAL ABERRATION.

A SINGLE lens, producing a real image of an object, is by no means satisfactory for the purpose, and if the lens is at all powerful, that is, of short focal length, then a glance at the image on a screen will show that it is not an exact representation of the object, nor will it be possible to obtain an outline so sharply defined as in the actual object, no matter where the screen is placed. Moreover, if it is held rather inside what is considered to be the best focus, and the luminous object is giving out composite light (white light), then the image is found to be surrounded by a red haze; likewise, if the screen is held a little outside the best focus, then the image is found to be surrounded by a blue haze.

Single lenses are liable to two great classes of errors, the first only concerning us here.

Class (1) Axial.
,, (2) Oblique.

The effective aperture of the refracting system of the eye, that is, the area employed for producing the macular image, is so small that the oblique aberrations due to corrective lenses may be neglected, but it is quite different with a photographic objective of considerable aperture, whose purpose is to produce an image with good definition throughout its area. All the human eye requires is that the macular image shall have perfect definition, but the retinal area outside the macula may have quite a blurred one.

Axial aberrations are *chromatic and spherical*.

*Experiment.* Take a prism of about 10° refracting angle, view some such object as a horizontal window bar through it, and note that, besides appearing deflected and bent, its edges are tinged with a number of colours.

Owing to the fact that different coloured lights do not travel all at the same speed through transparent substances, such as glass or water, whenever refraction takes place there results a *dispersion* or breaking up of a composite beam of white light into its various coloured constituents. Even when light falls normally upon the separating surface the different coloured lights, that is the different wave motions in the ether, are retarded in varying degrees, and it is not difficult to conceive that light whose time period is short (green, blue, violet), should be retarded most, and consequently bent most, when refraction takes place, for this light has to vibrate so much more frequently.

Phenomena due to dispersion are many and varied. Let us examine a few. In the first place, instead of the simple refracted wave front shown in Fig. 18, when the incident light is white the refracted wave front must be represented by a fan shape, because the rates of propagation differ for the colours in the order V.I.B.G.Y.O.R. Red is retarded least, and violet most, and as the refractive index, $n_2$, of the glass is by definition $= \frac{V_0}{V_2}$ (since $n_0 = 1$), it follows that the refractive index varies with the colour of the light *for the same piece of glass*. In practice it is usual therefore to state the $n$ for a particular position in the solar spectrum, that is for a particular coloured light, that chosen being yellow, and the line in the spectrum is known as the " D " line. This colour, with its characteristic line, may be produced artificially by burning sodium, and so producing luminous sodium vapour.

If these refracted waves proceed and finally emerge from another surface of the glass *parallel to the first*, they will do so parallel one with another. Therefore their various new paths will also be parallel, and the amount by which the waves are *deflected* from their original path also depends upon the colour of the light—in other words upon the refractive index, or change in velocity. Colours so produced by refraction will not be seen by the eye, because they will overlap one another almost instantaneously upon the retina, and therefore act as a composite wave of white light. If, however, one surface of the glass is tilted with respect to the other to form a prism, the light becomes separated out quite clearly, the incident beam being split up into the component beams upon emerging from the glass. The phenomenon witnessed in the experiment can now be explained. When the prism is held thin edge (apex) up the bar appears tinged with colours—the top edge red and the lower edge blue and violet, other colours will not be visible. We might, at first sight, expect that red would appear below and blue at the top, but in the case of the window bar we have really *two* beams, or portions of composite waves of light, one from the upper and one

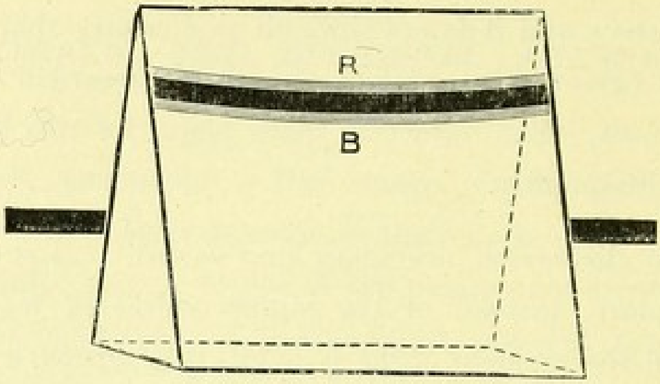



Fig. 29.

from the lower white pane, so that two spectra are formed, and where their ends overlap the dark bar there the particular colours will be visible. On looking through the prism the image of the bar is also bent, because the light reaching the eye from the ends of the bar has to pass obliquely through the prism, and therefore through a greater thickness of glass, hence the retardation will be greater, and consequently also the angle of deviation, i.e., the angle which the emergent path of the wave of light makes with the original path (angle $\delta$ in Fig. 17). The angle of deviation increasing gradually, as we look from the centre outwards the bar will appear curved, with its ends up when the prism is held apex up (Fig. 29), and vice versâ. Light passing obliquely through the prism acts just as if it were passed through a prism of greater refracting angle.

The effect of dispersion, and consequently the difference in the refractive indices for different coloured lights with the same piece of glass, gives rise to a phenomenon termed *chromatic aberration*. This is in practice termed a *defect*, because it produces a want of definition in the image. Let us consider the formation of an image by a convex lens. In Fig. 30 A B is a luminous object giving out waves of white light, differing in time period, or rate of vibration, for its constituent colours. All these different coloured waves, forming white light, travel with practically the same velocity in air, and so will meet the lens together, but they will not travel with equal velocities through the lens, violet waves being retarded most, and therefore have the greatest curvature. This being the case they come to a focus first, as the diagram shows. Moreover, the violet image, so formed, is smaller than the others, the red being the largest. The distance V R is termed the *chromatic difference*, and is a measure

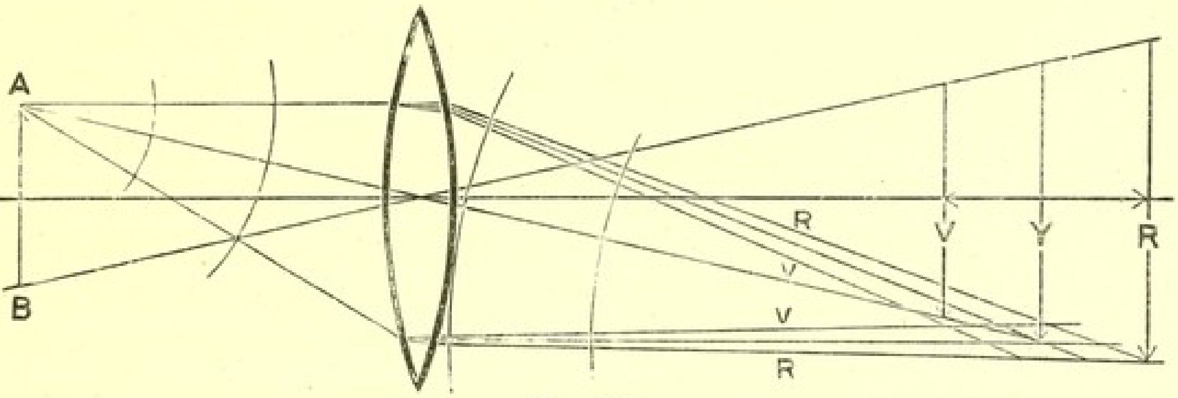



Fig. 30.

of the aberration or defect. In practice this defect is corrected by neutralizing the chromatic difference with another lens of opposite power, without destroying the deviating or focussing power. This can be accomplished by making the second lens from another kind of glass. . For instance, if we take a glass list* we may find, among others, constant, such as the following :—

| | $n_D$ | $\Delta = (n_F - n_C)$ | $N = \left(\frac{n_D - 1}{n_F - n_C}\right)$ |
|---|---|---|---|
| Crown. | 1·5175 | ·00856 | 60·5 |
| Flint. | 1·6214 | ·01722 | 36·1 |

These constants tell us that the flint glass retards the waves of light to a greater extent than the crown, and therefore that, other things being equal, greater refraction (and deviation) will occur when light passes through it. Furthermore, they tell us that between two given positions in the solar spectrum, the F and C lines, the *difference* in refractive index for the flint glass is about double that for the crown, and consequently but a comparatively small amount of deviating power is required in the flint glass to neutralize the dispersive effect of the crown. Thus a residual deviating (focussing) power will remain. A combination of two or more prisms or lenses, which in this way unite all the coloured waves again, is termed an *achromatic* system, the exact manner in which the waves are united in each case being seen in Fig. 31.

* For an explanation of the symbols see list at end of book.

Although the two spectra formed by the crown and flint prisms C and F respectively, at a given distance, may be equal in total length, it is more than probable that there will be irregularities in the spreading out of the

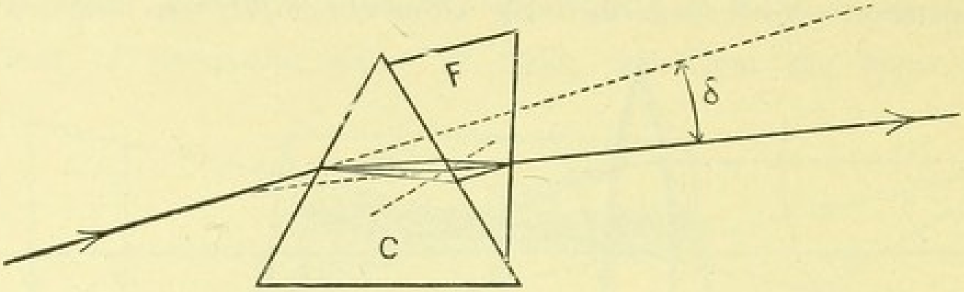



Fig. 31.

individual colours. This fact is known as the *irrationality of dispersion*. In all probability the various coloured waves will *not* be perfectly united, and when this is so what is known as a *secondary spectrum* is formed.

We find that a lens with surfaces ground truly spherical refracts light to a greater extent as it strikes the lens nearer its periphery, so that this part will have quite a different power from the central portion. We may satisfy ourselves that such is actually the case in the following manner :—Taking a rather large double convex lens let us imagine two paths of a plane incident wave, and assume that the path near the axis makes an angle of incidence of 5° at the first surface, and that the path near the periphery makes an angle of incidence of 25°. Call these angles $i_1$ and $i_2$ respectively. Then the ratio $\frac{i_2}{i_1} = 5$.

Now we have seen that $n_0 \sin i = n_2 \sin r$

Let $n_0 = 1$ and $n_2 = 1{\cdot}5$, then for Case I. we have—

$$\sin r_1 = \frac{\sin i_1}{n_2}$$

$$= \frac{{\cdot}0871557}{1{\cdot}5} = {\cdot}058104$$

$$\therefore \; r_1 = 3^\circ \; 20'$$

And for Case II.—

$$\sin r_2 = \frac{\sin i_2}{n_2}$$

$$= \frac{{\cdot}4226183}{1{\cdot}5} = {\cdot}281745$$

$$\therefore \; r_2 = 16^\circ \; 22'.$$

Therefore the ratio $\frac{r_2}{r_1} = 4{\cdot}91$. But the ratio $\frac{i_2}{i_1} = 5$. Therefore the path near the periphery of the lens must be bent proportionately more, and consequently cut the axis at a point *nearer* to the refracting surface than the point to which the central path is bent. It is clear, too, that refraction at the second surface of the lens will increase this effect, or error of spherical aberration, as it is termed. Moreover, if the angle of emergence of the light from the second surface is equal to the original angle of incidence at the first surface, a condition known as symmetrical refraction, then the error of spherical aberration will be at a minimum for the particular lens and aperture under consideration. For this reason a plano-convex lens should have its convex surface faced towards parallel incident light, a feature noticeable in the objectives of opera glasses and telescopes. It is rarely a matter of any difficulty to reduce this aberration to a minimum, but it is one of considerable trouble to satisfy the condition for its elimination. The distance between the axial focus and the peripheral focus for the given aperture and lens may be termed the *aplanatic difference*, and is a measure of the aberration or defect. A combination of lenses which satisfies the condition for the elimination of spherical aberration is termed an *aplanatic* system, and if a lens system is corrected both for chromatic and spherical aberration the whole combination may be termed a *collinear lens*, and the emergent waves of light will be perfect spherical ones.

The defect of chromatic aberration is quite marked in the human eye, and we need only look out of the corner of the eye at a window bar against a white background to prove the defect. Since, however, we depend almost entirely upon direct or axial vision the presence of this error causes us no inconvenience. The human eye possesses the defect of spherical aberration, but not to any great extent, owing to the peculiar formation of its crystalline lens, whose refractive index gradually increases from the periphery inwards, thereby counteracting the effect of the gradual excess over the necessary refracting power from the centre outwards due to the sphericity of its surfaces.

**Summary :—**

(*a*) Axial aberrations are chromatic and spherical.

(*b*) $n$ varies with the colour of the light for the same refractive medium.

(*c*) The order of colours in ordinary dispersion is expressed by the coined word VIBGYOR.

(*d*) The refraction of spherical lenses is greater as the incident path of light moves from the axis to the periphery.

# Chapter VII.

## OCULAR REFRACTION.

THE human eye possesses a refracting system, diaphragm and screen, in fact all the essentials for producing and receiving a well defined real inverted image, and in order that we may see clearly the screen has to be precisely at the focus of the refracting system.

Now it often happens that the *retina* (the receiving screen of the eye) does not occupy this ideal position, and when such is the case the eye is said to have a refractive error, and it is termed an *ametropic* eye.

Because we can alter (accommodate) the focal power of the visual refracting system by the action of the ciliary muscle, it is possible for us to see both near and distant objects in rapid succession.

*An Emmetropic or Normal Eye* is one which brings parallel incident light to an exact focus upon the surface of the fovea centralis (point of most distinct vision) of the retina, and which possesses the normal (average) accommodative power for the particular age.

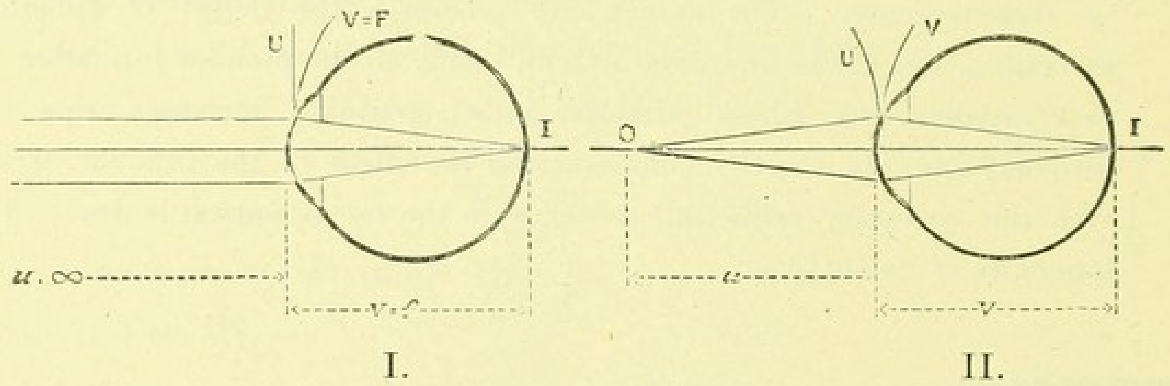



Fig. 32.

Fig. 32 represents the two instances of a young emmetropic eye, having ample accommodative power, focussed (I.) for a distant object, and (II.) for a near object. Because the eye is young, even the very divergent waves proceeding from O (in II.) are brought to a focus at I. upon the retina, and it is evident that the refracting power must be greater than in I., where F, the focal power, equals V, since U = 0 (plane wave), while in II. we have F = V + U, but V is the same in both instances so that the focal power in the second must be greater than in the first.

We have represented the refracting system by a single surface for simplicity, in reality the eye possesses an adjustable lens in addition to the transparent anterior covering surface, the *cornea*, whose position corresponds with the single surface shown in the diagrams.

A *Presbyopic or Old-age Eye*, because of its age, has lost more or less of its accommodative power; and, therefore, is unable to make its refracting system powerful enough to bring to a focus upon the retina light waves diverging from near points, and additional refractive power, in the form of a convex lens, becomes a necessity for near work in order to *maintain* clear vision.

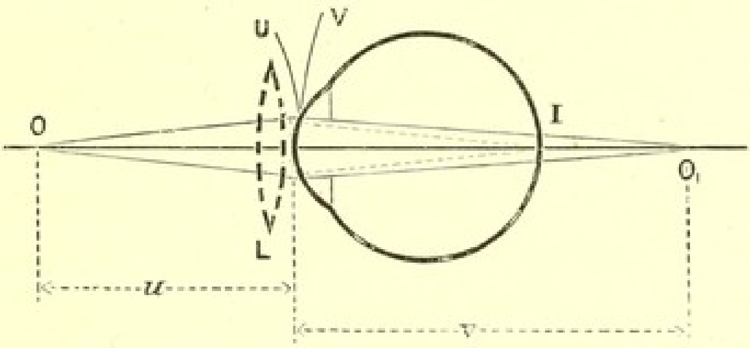



Fig. 33.

Fig. 33 represents an old emmetropic eye, in other words a normal presbyopic eye. Plane waves of light are brought to a focus just as in Fig. 32 I., but the eye is now unable, *unaided*, to refract waves diverging from O sufficiently to focus them upon the retina, and they would meet at $O_1$, a disc of light instead of a point being formed at I. Thus, waves diverging from so near a point as O cannot be focussed upon the retina at all, because of the loss of accommodation through age, and a suitable convex lens L must be placed in front of the eye to supply the deficiency.

In an *Hyperopic or Long-sighted Eye* the focal power of the refracting system is too weak for the existing length of eyeball, and parallel light would only be brought to a focus beyond the retina. Near objects will only accentuate the defect if accommodative power is not available.

In Fig. 34 a young hyperopic eye is indicated, which, by reason of a deficiency in its refracting system, cannot focus parallel light upon the retina, the point focus being at $O_1$. By employing accommodation this may be remedied,

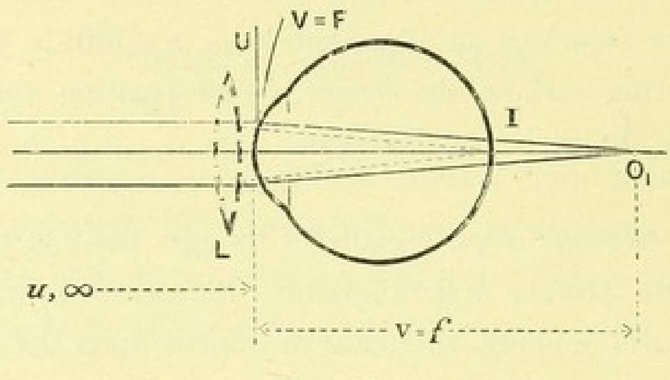
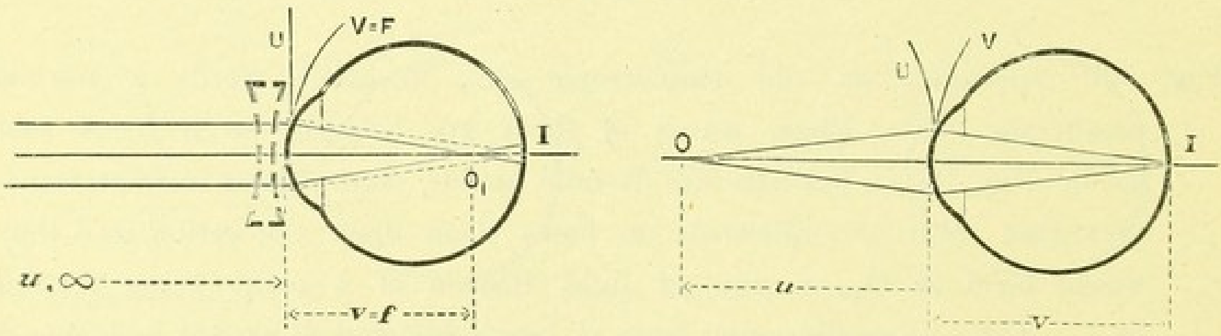



Fig. 34.

but it may then have insufficient remaining for viewing near objects, and it is better to correct the error by placing a lens L in front of the eye, thus removing the latent defect.

A *Myopic or Short-sighted Eye* has the focal power of the refracting system too strong for the existing length of eyeball, so that parallel light is brought to a focus in front of the retina. This error must be corrected by a concave lens L, and this without exception, for the eye has no means of *reducing* the focal power of its refracting system.




I. Fig. 35. II.

Fig. 35 illustrates a myopic eye, in which the refracting system is too strong to focus parallel light upon the retina, the point focus being at $O_1$, and a negative or concave lens must be placed in front of the eye in order to weaken the power. But we have seen that light diverging from a near object requires a stronger refracting system to focus it upon the retina, and consequently there will be some particular point (O in II.) from which divergent light will focus exactly upon the retina, and this is known as the *far point* of such an eye, accommodation becoming necessary for any position inside O which can be viewed by its aid.

*An Astigmatic Eye* may be defined as one which possesses a toroidal surface or its equivalent. This may be defined as one having varying curvatures in different meridians, with a maximum and minimum curvature at right angles, *toroid* being the scientific name for a surface like that of a bicycle tyre. In Fig. 36 such a surface is represented, AB and CD being the respective radii for the maximum and minimum curvatures.

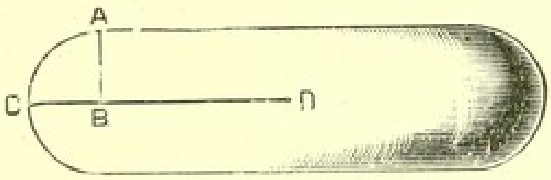



Fig. 36.

With such a surface separating two media of different refractive indices light will be brought to two line foci, separated one from another, instead of to a single point. The distance between the foci is known as the *interval of Stürm* (H to V in Fig. 37), and between these two foci a disc of light will be formed, known as the *circle of least confusion*. For convenience the meridian of least power may be termed the axis of the surface or lens.

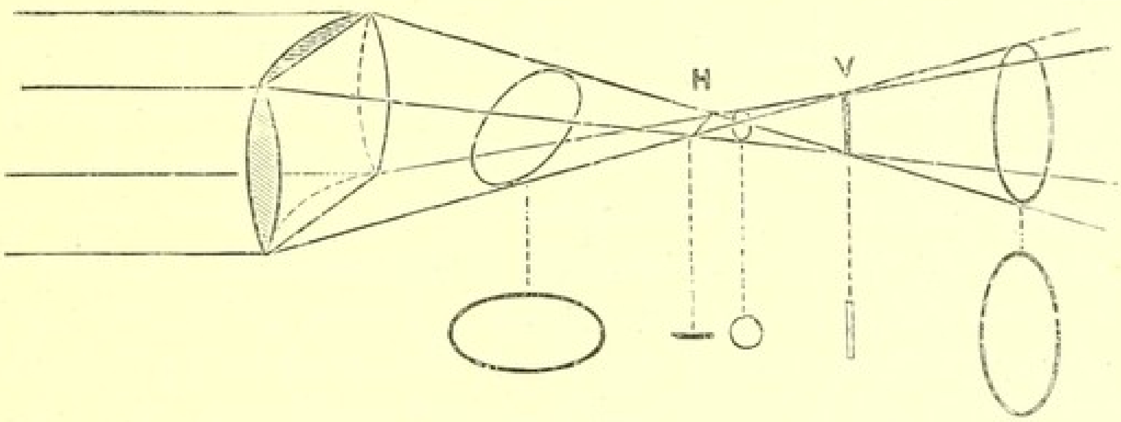



Fig. 37.

The equivalent *in effect* of a toroidal surface is a sphero-cylindrical lens (Fig. 37), a crossed cylindrical lens of unequal powers, or a tilted spherical lens. The error of refraction known as astigmatism is, therefore, capable of being corrected by one of these types of lenses, or by one having a toroidal surface, suitably placed before the eye.

Although the human eye is a compound system, it may with advantage be studied as a simple refracting system, having one surface separating its assumed single medium from air, when it is known as a "reduced" eye, so that we may liken it to a block of glass with a curved end, and may investigate the effect of refraction at a single curved surface by reference to Figs. 38 and 39, the first of which shows a surface of separation between two media of refractive indices $n_0$ and $n_2$, the former being to the left and the latter to the right.

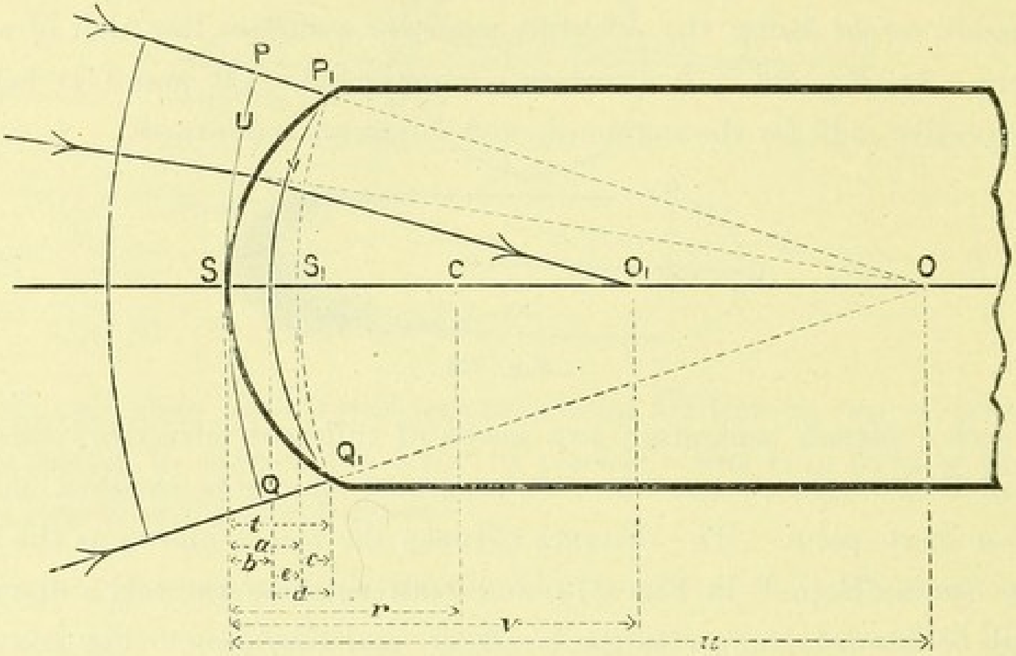



Fig. 38.

An incident wave is converging towards O, and, passing from a medium of low into one of high refractive index, the paths of this wave will be bent towards the normal. One path is shown as proceeding to $O_1$, and OS is then the first conjugate distance $u$, $O_1S$ being the second conjugate distance $v$. By the time the limits of the incident wave U reach P and Q the centre portion S is just coming under the influence of the more highly refracting medium, and by the time P reaches the surface at $P_1$, and Q at $Q_1$, S, instead of reaching $S_1$ will only have travelled a distance equal to $b$, owing to the retardation due to the medium of higher refractive index. We may say, therefore, that light would have travelled a distance $a$, but only travels over $b$ in the given time.

$$\text{So that } \frac{a}{b} = \frac{V_0}{V_2} \left(\text{but } \frac{V_0}{V_2} = \frac{n_2}{n_0}\right)$$

$$\therefore \quad \frac{a}{b} = \frac{n_2}{n_0}$$

$$\text{or} \quad n_0\, a = n_2\, b$$

$$\text{or} \quad n_0\,(t - c) = n_2\,(t - d)$$

But curvatures are proportional to their sags standing upon the same chord.*

$$\therefore \quad n_0 (R - U) = n_2 (R - V)$$
$$\text{or} \quad n_2 V - n_0 U = (n_2 - n_0) R \qquad (1)$$

This equation holds under all conditions if the rules laid down in Chapter I. are adhered to (light from left to right, etc.), provided $n_0$ *always* represents the incident medium and $n_2$ the emergent medium.

If the incident light had been *divergent*, and still came to a focus in the optically denser medium, the equation would have been :—

$$n_0 (R + U) = n_2 (R - V)$$
$$\text{and then} \quad n_2 V + n_0 U = (n_2 - n_0) R.$$

which harmonizes with $F = V + U$, for a convex lens producing a real image. We need only remember equation (1), however, and apply the rules.

We have throughout defined the focal power (F) of a system as the change in curvature of the wave, or as the algebraic sum of the curvatures U and V, so that if, as in Fig. 38, $e$ represents the change undergone :

$$e = d - c \quad \text{or} \quad F = V - U,$$

which again is in accordance with our notation, for this is the relation for *convergent* incident light upon a convex lens.

When the incident wave is plane (Fig. 39), we have $F = V$, because $U = 0$, and calling the incident medium $n_0$, and the emergent medium $n_2$, we may write down once for all the simple relation :—

$$n_2 F = (n_2 - n_0) R$$
$$\text{or} \quad F = \left(\frac{n_2 - n_0}{n_2}\right) R \qquad (2)$$

It will be obvious from this equation that the value of F depends upon which way light is passing, for since $n_2$ always represents the emergent medium it may stand either for the higher or lower refractive index.

Or again, in Fig. 39, let us consider a plane wave U incident (in air) upon R, and refracted so as to converge to $O_1$, then $SO_1$ will be the focal length measured *in the glass*. Similarly, if $U_1$ is a plane wave incident in glass (think of the diagram turned round, so that light still passes from L to R), then $SO_2$ is the focal length *in the air*, and $\frac{SO_1}{SO_2}$ must equal $\frac{n_0}{n_2}$, because

* This is not absolutely correct, because the curvature is only proportional to the sag when either the curvature or aperture is small.

from *glass to air*, when the incident wave meets the surface at P, this portion will have its speed accelerated, and the gain over the central portion Q is the time taken by Q to reach S, plus the rate of increase in speed, because during that time it will be travelling faster. The rate of increase in speed is determined by $\frac{V_2}{V_0}$ or $\frac{n_0}{n_2}$, $V_2$ being the velocity in the *emergent* medium.

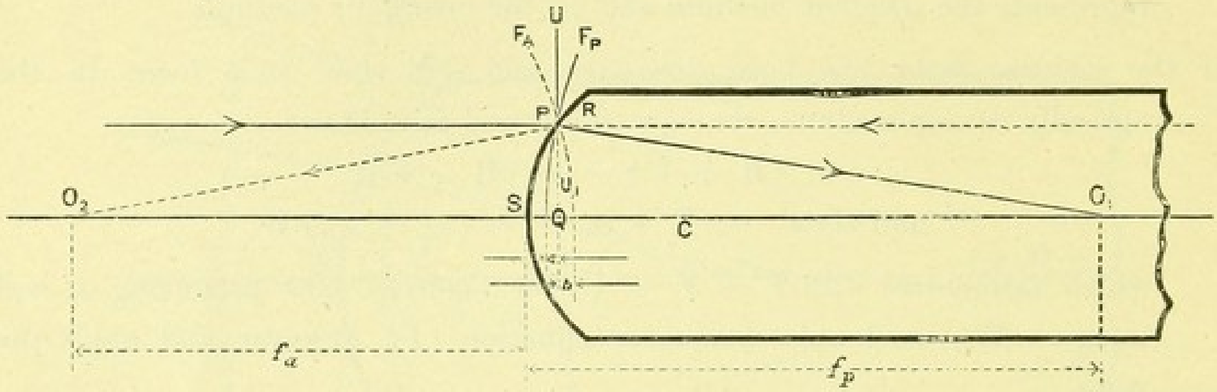



Fig. 39.

It will be found convenient to term $SO_1$ (the focal length in the optically denser medium) the *posterior focal length* ($f_P$), and $SO_2$, the focal length in the optically rarer medium, the *anterior focal length* ($f_A$). Hence we may write,

$$\frac{b}{a} = \frac{F_A}{F_P} = \frac{f_P}{f_A} = \frac{n_2}{n_0} \text{ (optically rare to dense medium).} \quad (3)$$

$$\frac{b}{a} = \frac{F_A}{F_P} = \frac{f_P}{f_A} = \frac{n_0}{n_2} \text{ (optically dense to rare medium).} \quad (4)$$

which, from air to glass, means that the posterior focal length is equal to the anterior focal length multiplied by the index of refraction of the glass, because air is unity.

From equation (2) it follows that

$$\frac{F_A\ F_P}{F_A - F_P} = R \quad (5)$$

$$\text{or } f_P - f_A = r.$$

So that the difference between the posterior and anterior focal lengths gives the radius of curvature of the surface.

Also from equations (1) and (2) we have

$$n_2\ F = n_2\ V - n_0\ U$$

$$\text{or } F = \frac{n_2\ V - n_0\ U}{n_2}. \quad (6)$$

Which tells us that from air to glass ($n_2 = 1{\cdot}5$) the focal power determined will be $F_P$, whereas from glass to air ($n_2 = 1{\cdot}0$) the focal power determined will be $F_A$, $n_2$ *always* being the emergent medium.

From our previous definition of the optical centre it follows that for a single surface it is always coincident with the centre of curvature, C, in the figures. And since an object point and its corresponding image are joined by a secondary axis through this point, it follows that if $u$ and $v$ are measured from the point C (optical centre) the magnification of the image is given by $m = \frac{v}{u}$, as for a single thin lens. However, it is usual to measure $u$ and $v$ from the surface, so that for a real image formed in glass we have :—

$$m = \frac{v - r}{u + r} \qquad (7)$$

From equation (1) it may be shown that for this case

$$r = \frac{(n_2 - n_0)\, u\, v}{n_2\, u + n_0\, v},$$

and by substituting these values in the above equation we find that

$$m = \frac{n_0\, v}{n_2\, u} = \frac{n_0\, U}{n_2\, V} \qquad (8)$$

This equation holds under all circumstances providing $u$ and $v$ are measured from the vertex of the surface.

The following examples will illustrate the practical application of the preceding equations.

*Example (1).* To determine the anterior and posterior focal lengths of Donders' *reduced* eye. We have given :

| | |
|---|---|
| Radius of curvature ... ... ... | = 5 mm. |
| Refractive index of air ... ... ... | = 1·0 |
| Refractive index of the medium of the eye | = 1·333 |

Since $r = 5$ mm.

$$R = \frac{1000}{5} = 200 \text{ dioptres.}$$

To determine $f_P$ (air to eye) we have from equation (2)

$$F_P = \left(\frac{1{\cdot}333 - 1{\cdot}0}{1{\cdot}333}\right) 200$$

$$= 50 \text{ dioptres}$$

$$\therefore \quad f_P = \frac{1000}{50} = \mathit{20\ mm.}$$

To determine $f_A$ (eye to air, the eye being on the left)

$$F_A = \left(\frac{1\cdot0 - 1\cdot333}{1\cdot0}\right)(-200)$$

$$= 66\cdot6 \text{ dioptres}$$

$$\therefore\ f_A = \frac{1000}{66\cdot6} = 15\ mm.$$

The result is positive in both cases, indicating a measurement to the right, and this is in strict agreement with our notation, for we have always to treat light as passing from L to R. Also $f_P - f_A$ equals $r$, and $f_P = f_A \times 1\cdot333$.

*Example* (*2*). Given that an object (in air) is 10 cm. high, and situated 100 cm. from the surface of the glass block, which has a curvature of 10 dioptres, determine the size and position of the image.

We have given : Curvature of surface = 10D
Refractive index of air = 1·0
Refractive index of glass = 1·5
Size of object = 10 cm.
First conjugate distance $u$ = 100 cm.
∴ U = 1 dioptre.

We know that $n_2\ V - n_0\ U = (n_2 - n_0)\ R$ (Equation (1) )

And also light is divergent ( − U)

Hence (1) becomes :—$n_2\ V - (-\ n_0\ U) = (n_2 - n_0)\ R$

$$\text{or}\quad n_2\ V + n_0\ U = (n_2 - n_0)\ R$$

$$\text{or}\quad V = \frac{(n_2 - n_0)\ R - n_0\ U}{n_2}.$$

And it is air to glass, therefore $n_0 = 1$, and $n_2 = 1\cdot5$.

$$\therefore\ V = \frac{(1\cdot5 - 1\cdot0)\ 10 - 1\ (1\cdot0)}{1\cdot5}$$

$$= 2\cdot66 \text{ dioptres.}$$

$$\therefore\ v = \frac{100}{2\cdot66} = 37\cdot5 \text{ cm.}$$

The image, therefore, is situated 37·5 cm. from the surface, and is a real inverted one formed in the glass block itself, since the *sign* is positive.

The size of the image, obtained with the aid of equation (8) is given by

$$10\ \left(\frac{1\cdot0 \times 37\cdot5}{1\cdot5 \times 100}\right)$$

$$= 2\cdot5 \text{ cm.}$$

We may represent the refracting system of the eye by a single surface separating two media, air and water respectively, if we give to the radius of curvature of the surface the value of 5 mm. We have for air $n = 1\cdot0$ and for water $n = 1\cdot333$ (Donders' "reduced" eye). With these data we can obtain sufficiently accurate solutions to all practical problems relating to vision, some of which are dealt with in a subsequent chapter.

The difference between an *emmetropic* and an *ametropic* eye has already been referred to, and some idea has been given of the conditions known as hyperopia, myopia, astigmatism and presbyopia. The fact of the object seen and the retinal image being conjugate foci has also been specified as necessary for distinct vision, but it does not follow that an eye having distinct vision is emmetropic, for we have seen that the hyperope may in many cases overcome his defect by using accommodation for distant vision, the amount used being a measure of the defect, which is described as *latent*, or hidden, in opposition to a *manifest* error, such as would be seen were the eye under atropine. The normal or emmetropic eye, therefore, should have the average accommodation available for its particular age, in other words its near point should be normal in position.

**Summary :—**

(*a*) The human eye may be studied as a single refracting system, being then known as a "reduced" eye.

(*b*) $n_2 V - n_0 U = (n_2 - n_0) R$, when $n_0$ is the incident medium and $n_2$ the emergent medium.

(*c*) Consequently $F = \left(\frac{n_2 - n_0}{n_2}\right) R = \frac{n_2 V - n_0 U}{n_2}$

(*d*) A toroid, a sphero-cylinder, a crossed cylinder of unequal powers, and a tilted spherical lens may be equivalents.

# CHAPTER VIII.

## THICK LENSES AND LENS SYSTEMS.

ALTHOUGH we can in practice treat the refracting system of the eye as consisting of a single spherical surface separating two media of different refractive indices, we must know the meaning and properties of what are called the *cardinal points*, because of the combined effect when corrective lenses are placed before the eye.

The optical centre of a single lens has been defined as that point upon the principal axis through which every path of light must pass whose emergent direction is parallel to its original direction of incidence, and it is still the same imaginary point in the case of a thick lens or lens-system, its position depending upon the relative curvatures of the surfaces.

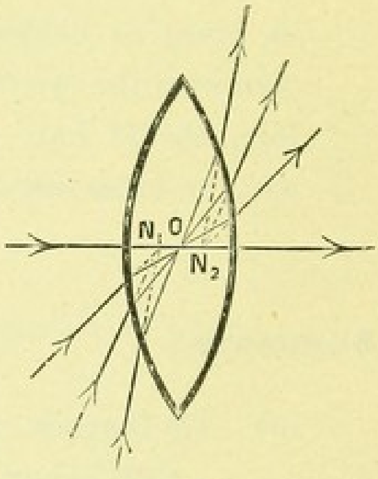


Fig 40.

In the equi-curved convex lens shown in section in Fig. 40 the optical centre O will occupy a position midway between the two surfaces, for the emergent paths are all parallel to their respective incident paths, and therefore the conditions of our definiton are satisfied. Although in each case the emergent path is parallel to the incident path it is considerably *deflected* to one side, because of the extreme thickness of the lens, and consequently we cannot assume, as we did with thin lenses, that the images of the optical centre are coincident with it. Suppose a small air bubble to occupy the position of the imaginary optical centre, then an eye placed anywhere to the left of the lens would, apparently, see it at $N_1$, and if to the right of the lens at $N_2$. To an observer the points $N_1$ and $N_2$ are the collecting points for all paths of light emerging in a direction parallel to that of incidence. They are, apparently, the knots or *nodes* formed by such paths (rays) of light, and are therefore termed *nodal points*, being simply the images of the imaginary optical centre viewed through each surface of the lens respectively. But we must remember that the *secondary axes*, as paths of light having parallel emergence are termed, never actually pass through the nodal points, but through the optical centre.

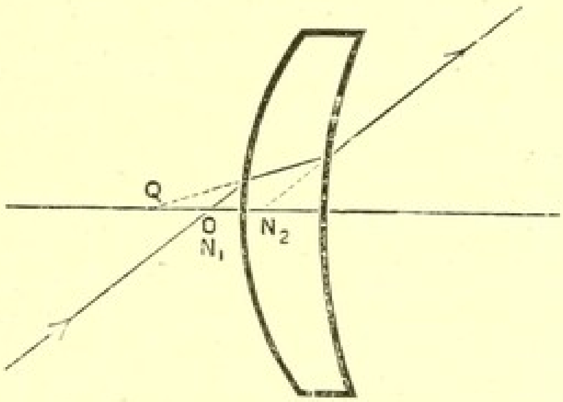



Fig. 41.

Fig. 41 represents a meniscus or periscopic lens, a single secondary axis being shown, the paths of incidence and emergence being parallel, and where the uniting path cuts the optic axis must lie the optical centre O. But as the nodal points are images of the optical centre, and this is outside the lens, it is evident that if it were a tangible object it would itself be seen by an eye placed to the left, so that there cannot be an image $N_1$ because no refraction takes place. We may however determine a position Q which will represent the position of the image (that is $N_1$) as seen from inside the lens, by the graphical construction of Fig. 24. In fact this may be called the optical centre for thin lenses, being considered coincident with O, and various writers still so describe it, but as light must pass through the optical centre, and all secondary axes pass through O, which we *may* term $N_1$, it is clear that it is an error. The position of Q is easily located, and it indicates the approximate positions of O ($N_1$) and $N_2$. This point does not vary in position with a change in refractive index, while O ($N_1$) and $N_2$ vary with the refractive index and curvatures of the two surfaces, and when the optical centre is on one surface or inside the lens Q is coincident with it. In future, when O is outside the lens or on one surface it will be referred to as the 1st or 2nd nodal point as the case may be.

Fig. 42 shows why all measurements should be made from the nodal points. A B being an object very remote from the lens, and the image *a b* therefore being in the focal plane, *b* coinciding with the focal point F.

The position of $a$ is determined by a secondary axis A $N_1$ $N_2$ $a$ and a path from A parallel with the optic axis, refracted by the lens at C, and on emergence again at G. If these incident and emergent paths are produced so as to intersect, they determine the position of one nodal plane, and consequently one nodal point, in this case $N_2$. The position of $N_1$ can also be determined by turning the lens round and passing a similar path through in an opposite direction.

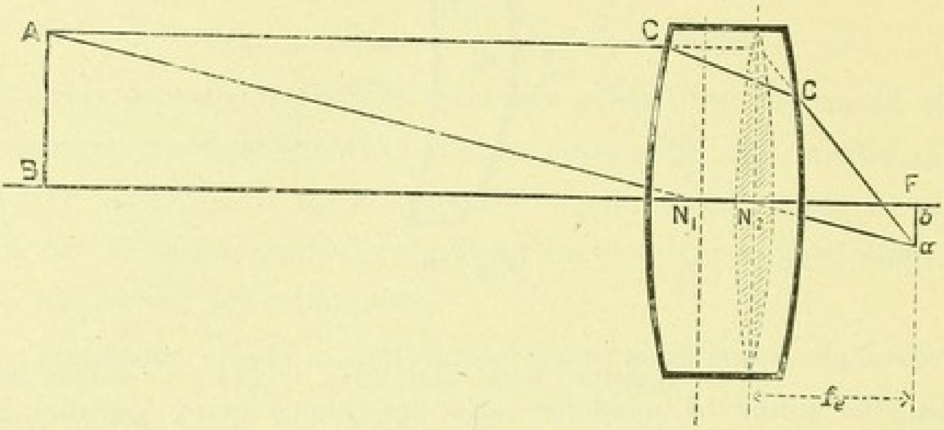



Fig. 42.

However much A C is refracted by the thick lens this work could be done by a single thin lens placed in the nodal plane of emergence, and shown in dotted outline. It is the equivalent lens, and its optical centre coincides with $N_2$. The same holds true for a compound system of lenses, even to determining $N_1$ by turning the system round, and in every case where the incident and emergent media have the same refractive index the anterior and posterior focal lengths $f_A$ and $f_P$ are equal (measured of course from the nodal points) since, if the lens were reversed, for light from A to be focused at $a$ the point $N_1$ would have to occupy the present position of $N_2$. Thus if the lens had light passed through from right to left the equivalent thin lens would lie in the nodal plane $N_1$, but as $N_1$ and $N_2$ are not coincident the equivalent lens is not the exact equivalent. For it to be so we must imagine it by some means to suddenly move from one nodal plane to the other before the light has time to emerge from it.

As the triangles A B $N_1$ and $a$ $b$ $N_2$ are similar in all respects, the corresponding sides being parallel, we can measure conjugate distances $u$ and $v$ from $N_1$ and $N_2$ respectively, and so apply our simple lens equations for solving thick lens problems. Hence, for thick lenses or lens-systems in air,

$F = V - U$ and $m = \frac{U}{V}$ as with thin lenses.

In practice the experimental determination of the positions of the nodal points is made by supporting the lens or lens-system in a V-shaped trough, which itself can be rotated on its support about a vertical axis. The image of an object should be received on a screen with cross-lines upon it. Experiment will show that by moving the lens or lens-system along the trough, keeping the image in focus, one position can be found in which no amount of rotation of the trough and lens system, about a vertical axis, will cause any movement of the image on the screen with respect to the cross-wires upon it. Fig. 43 shows that with the lens in such a position the axis of rotation, which is the centre line of the support, must be coincident with the nodal plane of emergence. The position of the other nodal plane may be determined by turning the lens round, and making the present nodal point of incidence the nodal point of emergence. The distances $N_1$ and $N_2$ from the first and last lens surfaces respectively can be determined with considerable accuracy, for $p$, $q$, $u$, and $v$ are all easily obtained, and the positions of the nodal planes upon the lens mount may be marked. If the object is at a considerable distance, $v$ will equal $f$, the true equivalent focal length.

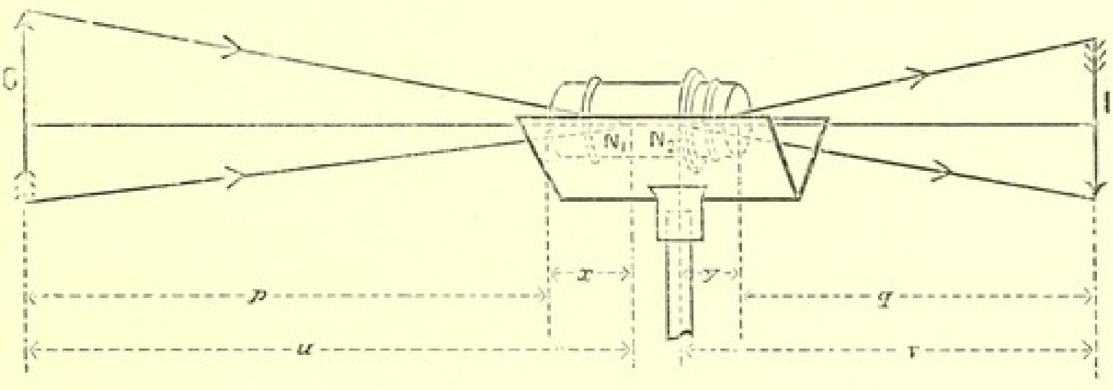



Fig. 43.

Gauss discovered that every optical system, however complex, acts as a simple thick lens; and, therefore, as a simple thin lens placed in one nodal plane to receive the light and then rapidly shifted to the other nodal plane to emit it. He also deduced equations which enable us to determine the properties of any lens-system, however complex, dealing with it *as a whole*, instead of as a series of separate parts, and so that they may be applicable to all cases he laid down the condition that all conjugate distances must be measured from *planes of unit magnification*, called *principal planes*, which are not necessarily the nodal planes in all cases, as we shall see.

Fig. 44 illustrates *planes of unit magnification*, irrespective of what we shall afterwards call *symmetrical planes*. O is an object in the plane of the optical centre of a rectangular block of glass, and to an eye on the left it appears of the same size, but in the first nodal plane $N_1$. Similarly, from the right it appears at $N_2$, and the incident and emergent paths of light, being parallel to each other, show the position of $N_1$ and $N_2$. If now we curve the surfaces of the block it becomes a lens, and although, if the object is large, the images may be distorted, unit magnification is preserved, so far as the secondary axes are concerned.

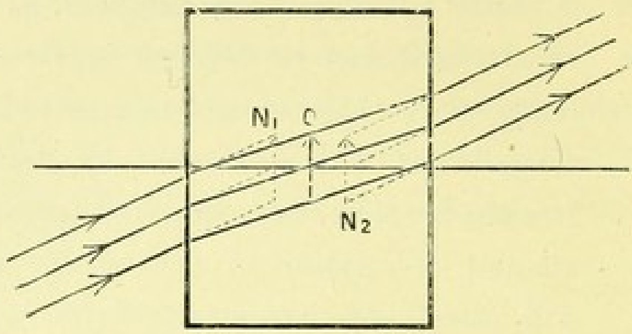


Fig. 44.

For any thick lens in air therefore, the nodal planes are identical with the principal planes of Gauss, and this applies also to all complex systems with the same surrounding medium. When single lenses are separated the nodal points of the system may overlap, so that in Fig. 43, to an eye on the right the optical centre *might* appear at $N_1$, and from the left at $N_2$, so that $u$ might be measured from the position of $N_2$ in the figure and $v$ from the position of $N_1$.

The planes drawn perpendicular to the optic axis through the principal focal points $F_1$ and $F_2$ are known as *focal planes*; while if an object be so placed upon one side of a lens system as to produce a real inverted image of the same size upon the opposite side (unit magnification for real image) then the positions of object and image are known as *symmetrical planes*, and the points where they cut the axis are symmetrical points.

There is one important thing to remember when we are dealing with thick lenses or separated thin lenses *in air*, viz., that the principal points of Gauss, called sometimes by others *equivalent points*, are coincident with the nodal points defined above.

If now we imagine a thick lens with air on one side and water on the other the effect upon light emergent in water will be to reduce the amount of bending, so that we shall have now an anterior and posterior focal length differing in value in the ratio of the refractive indices of air and water respectively, just as in the case of a single surface. And if $f_A$ and $f_P$ are the corresponding focal lengths in air and water, and $n_0$, $n_4$, the refractive indices for the incident and emergent media respectively, as in Chapter VII.

$$\text{Then } \frac{F_A}{F_P} = \frac{f_P}{f_A} = \frac{n_4}{n_0} \quad \text{(optically rare to dense medium)}$$

$$\text{and } \frac{F_A}{F_P} = \frac{f_P}{f_A} = \frac{n_0}{n_4} \quad \text{(optically dense to rare medium)}$$

We have already stated that the focal lengths are measured from the principal planes, which must also be planes of unit magnification, and the effect of having water upon one side, with increase of equivalent focal length on that side, is to separate the principal planes from the nodal planes as seen

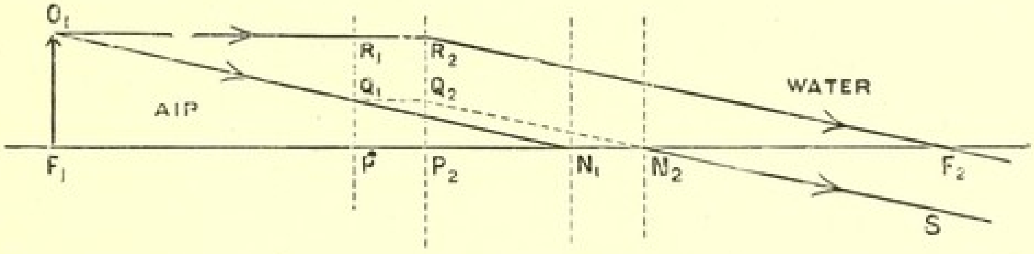



Fig. 45.

diagrammatically in Fig. 45, where $P_1$ $P_2$ are the principal points and $N_1$ $N_2$ the nodal points of a system having water to the right, in which is the posterior focus $F_2$ and air to the left, in which is the anterior focus $F_1$. As $P_1$ and $P_2$ are in planes of unit magnification an object in $P_1$ viewed from air would appear of the same size as if it were in $P_2$ and viewed from the water, so that an equivalent thin lens would have to occupy the plane of $P_1$ and be suddenly moved to $P_2$ in order to do the work of the system, the water included.

What effect does the water have upon the optical centre ? Let us imagine the system of lenses reduced to a single curve with water to the right. Then the optical centre will be at the centre of curvature, with the single nodal point, and the principal points will be united at the intersection of the principal axis with the curve. Thus as we increase the refractive index of the medium to the right so the optical centre, of which the nodal points are still images, is displaced to the right.

In Fig. 45 light from $O_1$ in the first focal plane emerges from the system as a parallel beam, and since $O_1$ $N_1$ and $N_2$ S are parallel (forming a secondary axis) then $P_1 P_2 = N_1 N_2$. Also triangles $N_1$ $O_1$ $F_1$ and $F_2$ $R_2$ $P_2$ are equal in every respect so that $F_1 N_1 = P_2 F_2$, and as $P_2$ $F_2$ is the posterior focal length, $F_1$ $N_1$ is also. $F_1$ $P_1$ is the anterior focal length, so that the equal distances $P_1$ $N_1$ and $P_2$ $N_2$ are each given by the difference between the posterior and anterior focal lengths. As we have already stated, with air on both sides $f_A = f_P$, $N_1$ coincides with $P_1$ and $N_2$ with $P_2$.

Such a system as that represented is similar to what we find in the human eye, and we require to know the positions of $P_1$ $P_2$ in order to calculate conjugate distances, whereas when dealing with the size of the image (magnification) we have to take measurements from $N_1$ and $N_2$ so as to employ the relations existing between the "quantities" of similar triangles.

The equivalent thin lens, to do the work of the system, including the water, has to be placed in the plane $P_1$ to receive the light, and rapidly shifted to the plane $P_2$ to emit it. These planes are planes of unit magnification insomuch as $P_1$ viewed from air will be magnified to a similar extent as $P_2$ viewed from inside the water. Therefore for the thin lens equation $F = V - U$ (representing the law of conjugate foci) to still hold true we *must* measure the conjugate distances from $P_1$ and $P_2$, which, with such a system as represented, do not coincide with the nodal points. The nodal points, however, are still of some use, for when dealing with magnification we often find it convenient to measure $v$ and $u$ from the nodal points $N_1$ and $N_2$, so as to employ, as stated above, the relations existing between the "quantities" of similar triangles. So that even with such a complicated system we may still write

$$F = V - U \quad (u \text{ and } v \text{ from } P_1 \text{ and } P_2)$$

$$m = \frac{U}{V} \quad (u \text{ and } v \text{ from } N_1 \text{ and } N_2) \text{ or } m = \frac{n_0 U}{n_2 V} \quad \text{(See page 59)}$$

employing the same notation as has been adopted throughout the book.

**Summary**:—

(*a*) The nodal points are images of the optical centre.

(*b*) An equivalent thin lens may be found for any thick lens or lens system, if we assume it to move from one definite position to another with infinite rapidity.

(*c*) The principal planes are those of unit magnification, and from them $u$ and $v$ are measured.

(*d*) For lenses or lens systems in air $N_1$ and $N_2$ coincide with $P_1$ and $P_2$ respectively.

(*e*) These are separated when the media are different on the two sides of the lens or system.

(*f*) For "magnification" it is convenient and usual to measure $u$ and $v$ from $N_1$ and $N_2$.

# CHAPTER IX.

## SCHEMATIC AND REDUCED EYES.

THE theory of Gauss, explained in the last chapter, affords us great help when applied to the eye, for the difficulty of calculating the exact course of light through so many media, with different refraction at the various surfaces, becomes apparent when we realize the complexity of the organ of vision. Working with the theory of Gauss we can find for the entire ocular refracting system an anterior focus, a posterior focus, two principal points and two nodal points. The question arises as to what eye we are going to take as a standard; evidently it must be emmetropic, neither too short nor too long in the bulb, nor may we have a cornea whose curvature is in any way abnormal. A model eye is needed, and many scientists at different times have given careful attention to measuring the various curvatures, and calculating the refractive indices of the media. The resulting figures give what are known as *the optical constants of the human eye*, and the model constructed to conform to these measurements is called a *schematic* (diagrammatic) *eye*. Listing was the first to construct such, and the schematic eye of Listing was named accordingly. He, too, introduced the idea of a *reduced* (simplified) *eye* for the purpose of obtaining results sufficiently accurate with a minimum amount of calculation. Helmholtz gave us figures which have been generally accepted as very accurate; and later, Tscherning, with the most recent appliances, has made more exact measurements in several respects. These we may tabulate as follows :—

| | LISTING. | HELMHOLTZ. | TSCHERNING. |
|---|---|---|---|
| Refractive index ($n$) of cornea ... | — | 1·3507 | 1·377 |
| ,, ,, of aqueous ⎫ ,, ,, of vitreous ⎭ | 1·3376 | 1·3365 | 1·3365 |
| Mean refractive index of lens ... | 1·4545 | 1·4371 | 1·42 |
| Radius of curvature of anterior surface of cornea ... ... | 8 mm. | 7·829 mm. | 7·98 mm. |
| Ditto, posterior surface ... ... | — | — | 6·22 mm. |
| Radius of anterior surface of lens (at rest) ... ... ... ... | 10 mm. | 10 mm. | 10·2 mm. |
| Ditto, posterior surface ... ... | 6 mm. | 6 mm. | 6·17 mm. |
| Distance from anterior of cornea to anterior of lens ... ... | 4 mm. | 3·6 mm. | 3·54 mm. |
| Thickness of lens ... ... ... | 4 mm. | 3·6 mm. | 4·06 mm. |
| Thickness of cornea ... ... | — | — | 1·15 mm. |

Here we have a complete set of measurements, from which we can calculate the value of the total ocular refraction. By the theory of Gauss the entire refracting system of the human eye may be represented by a single thin lens, having the property of moving with infinite rapidity from the principal plane of incidence to the principal plane of emergence. If then we know the positions of the various cardinal points we shall be in a position to draw to scale our schematic eye, from which we may deduce data for the reduced eye on the lines suggested by Listing. Below are the figures of Helmholtz and Tscherning for the various cardinal points.

| | HELMHOLTZ. | TSCHERNING. | |
|---|---|---|---|
| Position of 1st. p.p. ... | 1·75 mm. | 1·54 mm. | measured from the anterior surface of the cornea. |
| ,, ,, 2nd. p.p. ... | 2·11 mm. | 1·86 mm. | |
| ,, ,, 1st. n.p. ... | 6·95 mm. | 7·30 mm. | |
| ,, ,, 2nd. n.p. ... | 7·31 mm. | 7·62 mm. | |
| ,, ,, anterior focus ... | 13·73 mm. | 15·59 mm. | In front of cornea. |
| ,, ,, posterior focus | 22·79 mm. | 24·75 mm. | Back of anterior cornea. |

The positions of the nodal planes are given by the difference between the anterior and posterior focal lengths, thus, taking Helmholtz's figures :—

$$f_P - f_A = (22{\cdot}79 - 2{\cdot}11) - (13{\cdot}73 + 1{\cdot}75)$$
$$= 5{\cdot}20 \text{ mm.}$$

So that the first nodal point is 0·25 mm. to the anterior side of the posterior surface of the lens, and the second is 0·11 mm. to the posterior side of the same surface, the distance between them being the same as between $P_1$ and $P_2$. (0·36 mm.)

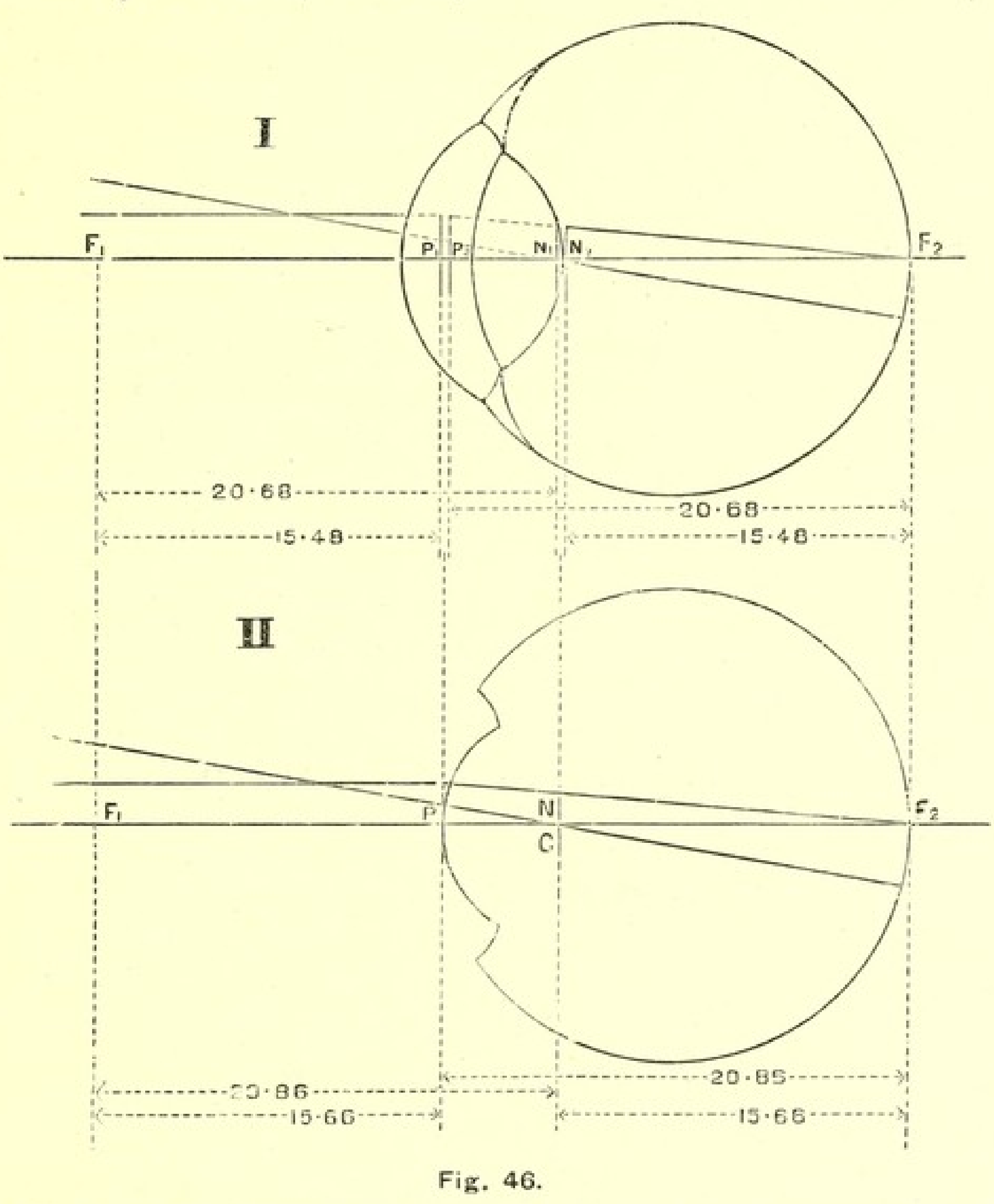



Fig. 46.

Fig. 46, I, gives us the schematic eye (distances twice "actual") after data supplied by Helmholtz. In this the two principal planes and two nodal planes almost coincide, and the principal planes are not very far from the cornea. A glance is sufficient to show that the entire refracting system of the human eye can, to a first approximation, be represented by a single surface separating two media having different refractive indices. In such a case the principal planes are coincident with the surface of separation, and nodal planes are coincident with the centre of curvature of the separating surface.

Listing proposed that the separation between $F_1$ and $F_2$, the anterior and posterior focal points respectively, should remain as for the schematic eye, and that refraction should be assumed to occur only at one surface, whose position should be taken as being midway between $P_1$ and $P_2$. Working on the schematic eye data (after Helmholtz) we have, therefore, for the reduced eye, Fig. 46, II,

$$\text{Anterior focal length } (f_A) = 15{\cdot}66 \text{ mm.}$$
$$\text{Posterior focal length } (f_P) = 20{\cdot}86 \text{ mm.}$$

Now, for refraction at a single surface separating two media

$$n_2 = n_0 \frac{f_P}{f_A} \text{ (Chap. VII.)}$$

and as $n_0 = 1 \therefore n_2 = \frac{20{\cdot}86}{15{\cdot}66} = 1{\cdot}332$ (the refractive index of the reduced eye.)

We also know that the reciprocal of $F_A = \frac{1}{f_A}$. Therefore, for light parallel within the eye, and passing from an optically denser to an optically rarer medium

$$\frac{1}{f_A} = \frac{n_2 - n_0}{n_2 r} \text{ (Equation (2) Chap. VII.)}$$

Therefore $\frac{1}{15{\cdot}66} = \frac{0{\cdot}332}{r} \quad \therefore r = 5{\cdot}20$ mm.

The position of the principal planes is represented by the point P on the adapted cornea, and the nodal planes by the point N coincident with the new centre of curvature C.

Donders proposed a *reduced eye* with the following data :—

$$f_A = 15 \text{ mm.}$$
$$f_P = 20 \text{ mm.}$$
$$n_2 = 1{\cdot}3\dot{3} \text{ (that of water.)}$$
$$r = 5{\cdot}0 \text{ mm.}$$

For rough calculations, and for making comparisons respecting the size of retinal images, etc., these data answer very well.

It is usual to reckon the positions of the various cardinal points from the anterior surface of the cornea, and this method of procedure is adopted in the following table which gives the results previously obtained in this chapter, together with results calculated from Tscherning's data in a like manner. All measurements are in mms.

| | HELMHOLTZ (*Schematic*). | HELMHOLTZ (*Reduced*). | TSCHERNING (*Schematic*). | DONDERS (*Reduced*). |
|---|---|---|---|---|
| Position of anterior focus ... | 13·73 | 15·66 | 15·59 | 15·0 |
| ,, posterior ,, ... | 22·79 | 20·86 | 24·75 | 20·0 |
| ,, 1st P.P. ... | 1·75 | On cornea | 1·54 | On cornea |
| ,, 2nd P.P. ... | 2·11 | | 1·86 | |
| ,, 1st N.P. ... | 6·95 | 5·2 | 7·30 | 5·0 |
| ,, 2nd N.P. ... | 7·31 | | 7·62 | |

The anterior focus must not be confused with what is known as the "near point" in testing vision ; it has quite a different signification, and, as previously explained, is the focus for light which, being parallel within the eye, converges after leaving the cornea and meets at this point, so that when the pinhole is placed at the anterior focus for the observation of entoptic phenomena light is actually parallel in the eye, diverging from the pinhole, and being parallel, casts shadows of obstructions within the media upon the retina.

The results obtained with the *reduced* eye, equally with those from the *schematic* eye, hold good for the actual eye. This being so, it is evident how much easier it is to study the formation of images on the retina by aid of the simplest optical system than with the very complex one of the normal human eye.

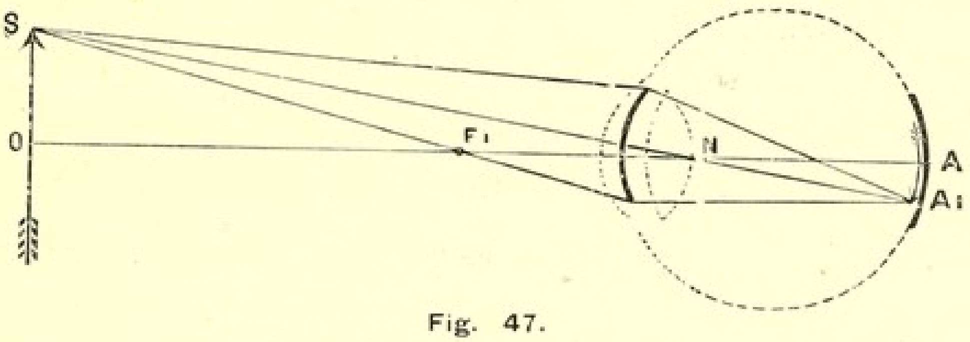



Fig. 47.

The normal eye is shown in the dotted outline, the "simplified" or "reduced schematic" eye being in dark lines. N is the single nodal point, O A the optic axis, $F_1$ the anterior principal focus.

In Fig. 47 is shown the formation of an image by the reduced eye, light from one extremity of an object passes along a secondary axis S $A_1$, through the nodal point, undergoing no refraction. Two other paths are shown, which are refracted at the "reduced" cornea and meet at $A_1$, which is the image of S, on the secondary axis. Every point in the object could be made to furnish an image in the same way, each having a path traversing an individual secondary axis, which must pass through the nodal point N, suffering no refraction, every other path being refracted at the imaginary cornea. Evidently, as the object approaches the anterior focus, the image moves further away from the nodal point, until with the object at the anterior focus light is parallel within the eye. A reference to Fig. 39 will make this evident.

It is instructive to note how the point $F_1$ might become a "near point" for testing. If we could increase $n$ sufficiently to make the light parallel within the eye convergent, so as to meet at the retina, we should get an image. The crystalline lens would do this if the power of accommodation were great enough, but as we should require about 58 dioptres of available accommodation this is impossible; in fact we should need a crystalline lens which could accommodate almost exactly the same number of dioptres as the total power of the eye.

We may summarize by saying that the refraction of the eye for parallel light incident in air and meeting the cornea is approximately 43D, but for light from the anterior focus it would require 58D of accommodation *extra* to form an image.

The magnitudes of image and object are proportionate to their distance from the nodal point of the reduced eye. Taking Donders' data the image is formed 15 mm. from the nodal point. If, therefore, we assume the object to be 15 metres away from the nodal point, the image will be $\left(\frac{15 \text{ metres}}{15 \text{ mm.}}\right)$, or 1000 times smaller than the object.

The results of some calculations based upon Tscherning's figures will be found instructive. He has given a certain value to the posterior surface of the cornea, and, so far as is known, no figures have previously been determined for this surface, a fact greatly adding to the interest of the following table.

| | | |
|---|---|---|
| Anterior surface of cornea ... ... ... | 47·24 D | |
| Posterior „ „ ... ... ... | −4·73 D | |
| Anterior surface of crystalline ... ... ... | 6·13 D | Eye at rest. |
| Posterior „ „ ... ... ... | 9·53 D | |

Adding these we obtain 58·17 D, which is, for all intents and purposes, the total refractive power of the eye. It is instructive to compare these figures with those obtained by the Helmholtz schematic and Donders reduced eyes. The approximation is very close.

From Tscherning's table we notice the remarkable power of the anterior corneal surface. The reason is obvious, when we remember that light entering the cornea comes from air, with $n = 1$, into a refracting substance whose $n = 1·377$. How different, when light passes from the aqueous humour, whose $n = 1·3365$, to the lens whose total $n$ is only 1·42! In the former case the difference is nearly 0·4 while in the latter it is less than 0·1. Again, the actual curvature of the cornea is deeper than that of the anterior surface of the lens. The comparatively low power of the anterior surface of the lens is also worthy of notice. In fact, it will be observed that it is nearly neutralized by the inner concave (posterior) surface of the cornea.

**Summary :—**

(*a*) A schematic eye is merely a model for comparison.

(*b*) A reduced eye is one simplified for the purpose of calculation.

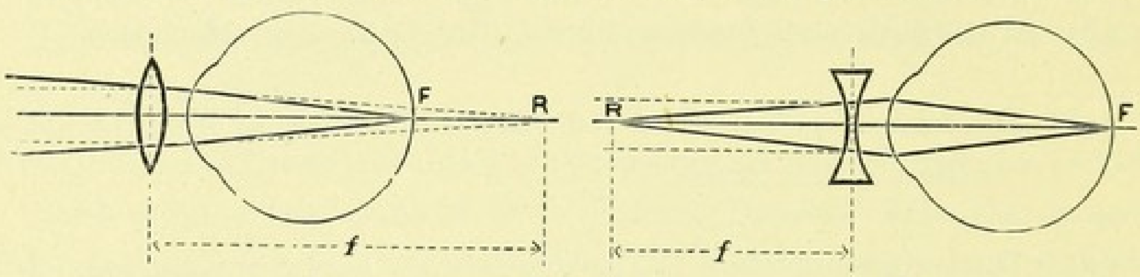
(*c*) The cardinal points of the reduced eye are reckoned from the anterior surface of the cornea.

(*d*) The anterior surface of the cornea is the principal refracting surface in the actual eye.

# CHAPTER X.

## POSITION OF CORRECTIVE LENSES.

WE have already seen that the far point (P.R.) of an eye is conjugate to the retina, and, moreover, that the focal point of the lens correcting the ametropia coincides with the far point. It is clear that such must be the case from Fig. 48, I and II. In each figure the position of the P.R. is




I. Fig. 48 II.

indicated by the letter R, and therefore, the eye is capable, without accommodating, of focussing this point R upon the retina at F. If, however, a distant object is to be seen clearly, a positive lens (I), or negative (II), of focal length $f$, must be placed in front of the eye. The distant object and far point (P.R.) are thus seen to be conjugate foci with respect to the corrective lens, and any movement of the convex lens away from the eye necessitates a *decrease* in power to retain the same positions for conjugate foci, while any movement of the concave lens from the eye necessitates an increase in power to retain the same positions for conjugate foci.

The first of these statements requires some qualification, for if light proceeding from any point $o$ is to be focussed at a point $o_1$, and $u$ and $v$, as usual, represent the first and second conjugate distances respectively, then the weakest possible convex lens that will focus light proceeding from $o$ at the point $o_1$ must be situated midway between $o$ and $o_1$. In other words $v$ must equal $u$. For we have, for a convex lens producing a real image (convergent light),

$$F = V + U = \frac{1}{v} + \frac{1}{u} = \frac{u+v}{v\,u}$$

Now it is a well known rule of proportion that the *result* of the sum of two quantities divided by the product of the same two quantities has its minimum value when the two quantities are equal, providing the sum of the two quantities (in this case $u + v$) always remains constant. This can be verified by assigning to $u$ and $v$ actual values and keeping $u + v$ constant. Therefore when a *convex* corrective lens occupies a position midway between the far point and the distant object it will have a minimum value, the lens being the weakest one possible for correcting the ametropia present. For such a position of the corrective lens each conjugate distance would be just twice its focal length—the distant object and far point occupying symmetrical planes with respect to the corrective lens.

The above conclusions do not hold good for near vision and reading because the object viewed may be at a distance *less* than twice the focal length of the corrective lens when qualification is needed:—"Any movement of position of corrective lens necessitates a *decrease* in power to retain the same positions for conjugate foci, *providing the object is distant from the lens more than twice its focal length.*" To be quite precise we may say that if a positive corrective lens is of such a power as to give a focus upon the retina when near to the eye, then when it is withdrawn from the eye it will produce a focus in front of the retina, in other words, it will be *too strong in effect.* But if the convex lens passes the midway position, and therefore, is situated nearer to the object viewed than to the far point of the eye, the reverse will be the case, because a weaker lens placed nearer the eye would do the same work.

These qualifying remarks do not apply to the case of withdrawing a negative (concave) lens from the eye, for such cannot possibly produce a real inverted image of the same size as the object, and, therefore, we cannot speak of symmetrical planes when referring to concave lenses, which can only produce a real image when they receive light of sufficient convergence. This must be artificially produced, for in nature all light is divergent, even that which, coming from a great distance, we term *parallel light.* Any movement in position of a corrective concave lens always necessitates an *increase* in power to retain the same positions of conjugate foci, evident from Fig. 48 II, for if we consider first the distant object (indicated by dotted lines) viewed through the concave lens, then as this is withdrawn from the eye $f$ must of necessity decrease to retain the same position for conjugate foci. Consequently a lens at some distance from the eye must be equivalent to a weaker one placed closer to the eye. In other words when a concave lens is withdrawn from the eye it will produce a focus in front of the retina (providing it corrected the ametropia in the first instance) and, therefore will be *too weak in effect.* If we suppose the object to be a near one a similar effect is produced, because, although the distance between the object and the lens appreciably diminishes, this only assists in producing the same effect. That such must be the case will be realized if we imagine the corrective lens to be coincident with the near object viewed, for then no refraction can take place, so that the equivalent power of the lens is zero and an "infinitely great" increase in power is necessary to retain the same positions for conjugate foci, and this infinitely strong lens only does the work of a comparatively weak one placed close to the eye.

There are two other important features connected with the position of corrective lenses. Firstly, we should make it a rule wherever possible to place the corrective lens at 15 mm. distance from the cornea, that is, approximately coincident with the anterior focal point of the eye. In order to correct the ametropia its power must differ from that of one placed in contact with the cornea, which would be a *contact* lens, and although not possible for ordinary wear, is convenient in investigation, being known as the *equivalent contact* lens. Secondly, for varying distances of the object, the difference between the power of the contact lens and one at 15 mm. distance (each to correct the ametropia present) will not remain constant. This latter effect is of great importance, for the corrective lens which we place at 15 mm., perhaps with a statement emphasizing that it is for *constant* use, is kept in this one position, has but one power, and is employed in conjunction with the eye to view objects at varying distances.

For example, consider a + 4 D lens placed at 15 mm. from the cornea. Let us determine :—

(1) The power of the equivalent lens at the cornea for a distant object.

(2) The power of the equivalent lens at the cornea for a near object, which we will suppose is 250 mm. from the corrective lens 15 mm. in front of the cornea.

In (1) the + 4 D lens would focus light at a distance of 250 — 15 = 235 mm. behind the cornea. The equivalent power of the contact lens is, therefore $\frac{1000}{235} = 4{\cdot}26$ dioptres for a distant object. In (2) where the object is 250 mm. from the + 4 D corrective lens, the light would emerge from the + 4 D lens parallel, (since $f = 250$ mm. for + 4 D). Hence the contact lens must have a focal length of 250 + 15 = 265 mm., and an equivalent power, therefore, of $\frac{1000}{265} = 3{\cdot}77$ dioptres.

Now let us take the case of a + 8 D under similar conditions. In (1) this + 8 D lens would focus light at a distance of 125 – 15 = 110 mm. behind the cornea. The equivalent power of the contact lens is, therefore, $\frac{1000}{110}$ = 9·09 dioptres for a distant object. In (2), where the object is 250 mm. from the + 8 D corrective lens, the light would be brought to a focus at 250 — 15 = 235 mm. behind the cornea. For the contact lens we have therefore, as conjugate distances

$$u = 250 + 15 = 265$$
$$v = 250 - 15 = 235$$

The equivalent power, therefore, is given by

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$= \frac{1000}{235} + \frac{1000}{265}$$
$$= 4{\cdot}26 + 3{\cdot}77$$

= 8·03 dioptres for the near object, which is 250 mm. from the + 8 D lens at 15 mm. from cornea.

Working on these lines Mr. S. D. Chalmers* has drawn up the following table :—

| Power of Lens. | Equivalent Lens at Cornea. | | Power of Lens. | Equivalent Lens at Cornea. | |
|---|---|---|---|---|---|
| | Object at Infinity. | Object at 250 mm. from Lens. | | Object at Infinity. | Object at 250 mm. from Lens. |
| + 1 | + 1·02 | + ·90 | – 1 | – ·98 | – ·88 |
| + 2 | + 2·06 | + 1·83 | – 2 | – 1·94 | – 1·73 |
| + 3 | + 3·14 | + 2·79 | – 3 | – 2·87 | – 2·57 |
| + 4 | + 4·26 | + 3·77 | – 4 | – 3·77 | – 3·37 |
| + 5 | + 5·40 | + 4·79 | – 5 | – 4·65 | – 4·16 |
| + 6 | + 6·60 | + 5·83 | – 6 | – 5·50 | – 4·93 |
| + 7 | + 7·82 | + 6·91 | – 7 | – 6·34 | |
| + 8 | + 9·09 | + 8·03 | – 8 | – 7·14 | |
| + 9 | + 10·40 | + 9·17 | – 9 | – 7·93 | |
| + 10 | + 11·77 | + 10·37 | – 10 | – 8·70 | |

There are many points of interest in these tabulated results. First to be noticed is the change in the equivalent power for an object at 250 mm. between + 7 and + 8 D.

* Optical Society Transactions, 1907.

We are given—

| Power of Lens. | *Twice Focal Length of Lens.* | Equivalent Lens at Cornea with *Object at 250 mm.* |
|---|---|---|
| + 7 | 285·6 | + 6·91 |
| + 8 | 250·0 | + 8·03 |
| + 9 | 222·2 | + 9·17 |

With + 7 the equivalent lens is less, for + 8 it is almost identical, while for + 9 the power of the equivalent lens is more. This merely emphasize the midway position effect referred to earlier in the chapter.

For spherical corrections the effects indicated by these results are not probably of great importance, as accommodation would automatically counteract them. But for cylindrical and sphero-cylindrical corrections the effect, as Mr. Chalmers has pointed out, is very important. For example, suppose a corrective lens at 15 mm. from the cornea has a power of + 4 D Sph. ◯ + 4 D Cyl., so that one chief meridian has + 4 D, and the other has + 8 D power. For a *distant object* this lens is equivalent to + 4·26 in one chief meridian, and + 9·09 in the other chief meridian, or to + 4·26 Sph. ◯ + 4·83 Cyl. at the cornea, but for a *near object* the same lens is equivalent to + 3·77 in one chief meridian and + 8·03 in the other chief meridian, or to + 3·77 Sph. ◯ + 4·26 Cyl. at the cornea.

Here again the spherical difference is unimportant, but the cylindrical difference of 0·57 dioptres is much too great to be neglected.

So far we have referred to the position of corrective lenses rather as parts of a whole, in combination with the eye, than as influencing the optical properties of a system, such as they form with the refracting media of the organ and the air space between. It becomes necessary to consider them in this latter sense when we investigate the influence of their position upon the size of retinal images, and upon the total refractive power of the system, for we may have a case where, by moving a corrective lens from the eye its apparent power is increased, but the power of the system as a whole is thereby lessened. In Chapters VIII. and IX. it was laid down that conjugate distances were measured from $P_1$ and $P_2$, the principal points, whereas the relative sizes of object and image were governed by their distances from $N_1$ and $N_2$, the nodal planes. Further, we know that for distinct vision the object seen and the retinal image must be at conjugate points, and that the anterior focal distance is in air, while the posterior focal distance is measured entirely within the eyeball, except in some few types where $P_2$ is actually in front of the eye. Finally, it is evident that a displacement of any portion of the refracting system, by removal of a lens, or any addition to any portion, as in the exercise of accommodation, causes a displacement of the principal planes, and consequently also of the nodal planes.

For the purpose of illustration two extreme instances will be taken, the one a case of high myopia, and the other a case of aphakia from lens extraction.*

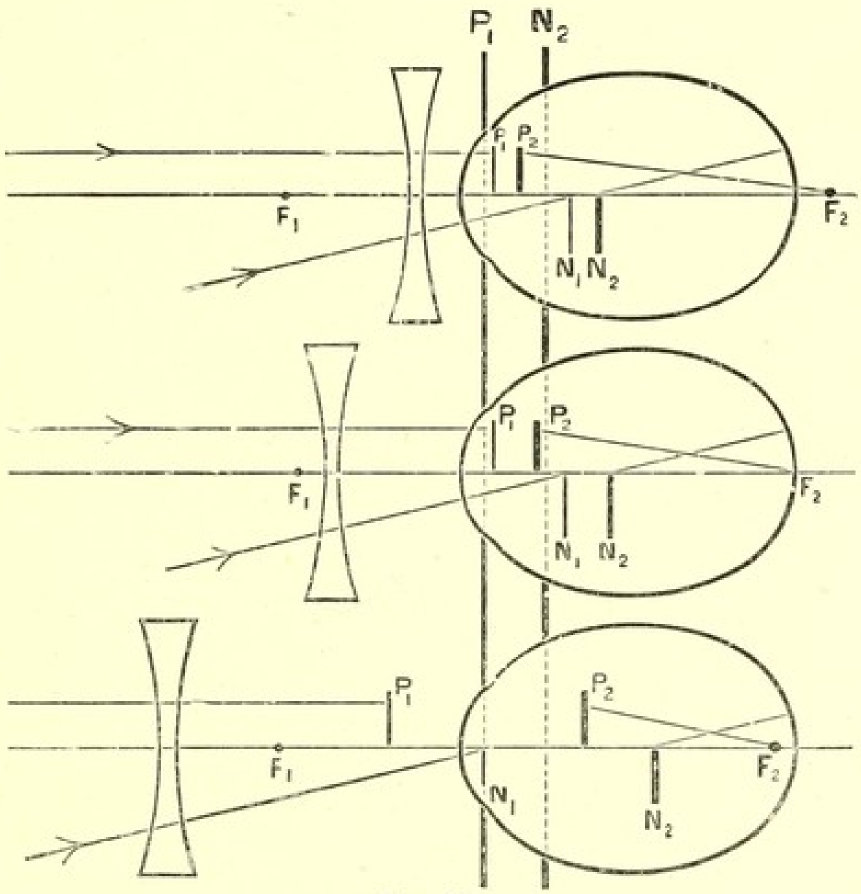



Fig. 49.

In Fig. 49 a high myopic eye is shown, with the corrective lens in three different positions with regard to $F_1$, the anterior focal point. The vertical lines $P_1$ and $N_2$ represent the first principal and second nodal planes respectively of a *normal* eye, the influence of position of the lens on $F_2$, which should be upon the retina, being shown. The secondary axis in each case is the same, so that tracing it back from the second nodal point the size of the retinal image can be estimated, and it is noticeable that, although the distance of the lens in the third position brings $F_2$ within the eye, yet the retinal image is actually lessened because $N_2$ has been also

* Proc. of the Optical Convention, 1905.

moved nearer to the retina in the opposite direction. The parallel light is shown incident to $P_1$ and refracted at $P_2$, and a striking feature is the widening out of the principal and nodal planes, to exactly the same extent in each case, so that from $P_1$ to $P_2$ is identical with the distance apart of $N_1$ and $N_2$. In this case, quite contrary to what we find in the next, the posterior focal length gradually lessens, and so the power as a whole increases as the influence of the lens decreases, as we saw in dealing with the removal of concave lenses from the eye.

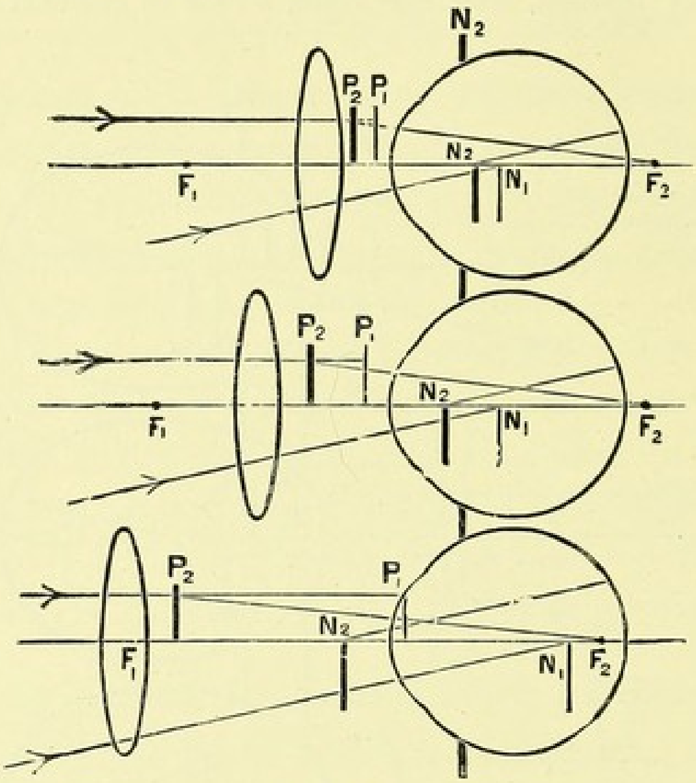



Fig. 50.

The eyes represented in Fig. 50 are, in consequence of the loss of the crystalline lens, highly hyperopic, and entirely deprived of accommodation. The first feature noticeable is the reversal in position of the nodal planes and principal planes, in each case the *second* plane being towards incident light, and as, with withdrawal of the lens the focal length of the system increases $P_2$ moves rapidly outward, and consequently $N_2$ does the same. This has a very marked effect upon the size of the retinal image, which is regulated by the position of the second, or *back* nodal point, as it is sometimes called, although in this case it is actually in front. In both this figure and the last the course of the plane wave entering the system, shown by the line parallel to the axis, and also the position of the secondary axes, should be carefully studied, noting the transference in all cases from the first to the second plane, and if in any case we wished to place a single lens in position to represent the whole refracting system of eye and corrective lens it would be put at $P_2$, the distance between that and $F_2$ representing the posterior focal length. (Chap. VIII.)

These diagrams show the necessity of placing the corrective lens as nearly as possible to the anterior focal point of the eye, so that the retinal image may be as nearly as possible the normal size, and comparing Figs. 49 and 50 the most striking facts are the differences created by removal of the two lenses upon the posterior focal distances of the two systems, and the greater differences in the size of the retinal images caused by such removal in high hyperopia than in high myopia. Although extreme types have been taken the same arguments hold good in ordinary cases, but to a less degree, provided accommodation is not used, and we see the necessity of always considering the position of nodal points when dealing with questions involving vision with corrective lenses.

Considering the effect of accommodation, it is clear that without a lens in front of the eye the posterior focal length is shortened and $P_2$ moves towards the retina, so that $N_2$ does the same, and images are reduced. If, instead of accommodating, the eye has a convex lens placed in front to enable objects to be seen without accommodation then $P_2$ and $N_2$ move away from the retina and images are enlarged, as in presbyopic corrections.

The above remarks *do not imply* that a single individual would perceive an object to be of one particular size with lenses varying in power, even if each lens were placed precisely at the anterior focal point. Dr. Lindsay Johnson has laid stress upon the point that if a certain lens corrects the ametropia present then a stronger or weaker one must give rise to circles of confusion which would stimulate the neighbouring cones, so that the object viewed, even if seen fairly clearly, will appear larger. Should the error so introduced be corrected by accommodation then this again will alter the size of the retinal image because the nodal point $N_2$ will shift, whereas the focal point $F_2$ will not. On the other hand all persons having a normal length of eyeball, and who wear their corrective lenses at the anterior focal point, have an image formed upon their retina of an identical size when the same object at a given distance is viewed.

In astigmatism, corrected by a lens in front of the eye, there is frequently trouble, because the retinal images vary in size in the different meridians. This is readily seen if we notice that there are always two principal meridians, one of highest and the other of least refraction, so that, from what has preceded, there should be two cylindrical lenses at different distances from the eye to produce equal proportion for the image. When the one meridian is hyperopic and the other myopic, as in mixed astigmatism, the defect is very noticeable, and wearers of the corrective lenses can rarely be relied upon to estimate exact proportion, because circles appear to them as oval in shape, and when the meridians are oblique, squares seem to be diamond shaped. In all cases of high degree, only a lens in actual contact with the cornea will give an approximate constancy in proportion.

Mr. Conrad Beck has pointed out a source of trouble which is likely to occur with toroidal (toric) lenses, and which experience has shown to exist. Their position in front of the eye has a greater effect upon the displacement of the nodal planes than ordinary sphero-cylindrical combinations, and so the retinal images vary more, especially when the error of refraction is large.

**Summary**:—

(*a*) When a + lens occupies a position midway between the *p.r.* and a *distant* object it will have a minimum value.

(*b*) Any movement of a + lens away from the eye necessitates a decrease in power to retain the same positions for conjugate foci, provided the object is distant from the lens more than twice its focal length.

(*c*) Any movement away of a corrective — lens *always* necessitates an increase in power to retain the same positions for conjugate foci.

(*d*) The position of the object viewed affects the power of a corrective lens.

(*e*) In astigmatism no lens, except actually in contact with the cornea, gives images proportionately correct.

# CHAPTER XI.

## AIDS TO NORMAL VISION.

A DISTINCTION must be made between the effect of lenses placed in front of the eye in order to correct any ametropia present, and the effect of lenses either single, as in the reading glass and hand magnifier (simple microscope), or combined, as in the telescope, field-glass and microscope. In the former case the correction of the ametropia does not imply a difference in the apparent size of the object, for except in a few cases the retinal image is normal in size. But in the latter case the apparent size of the object is usually increased, and on this account the increment is an aid to normal vision, the apparent size of the *virtual* image projected by the eye being greater than that of the object seen by direct vision.

This apparent enlargement of the object is termed magnification, and we must note the difference between magnification and magnification with resolving power. This is clearly illustrated in the case of a bright star seen by the naked eye, and afterwards viewed through a telescope of moderate power, because the peculiar stellate appearance in the first case causes the actual object to appear larger than the well defined telescopic image. Again, in many cases of myopia the retinal image of an object, as seen within the far point of the uncorrected eye, is much larger than that of the emmetrope, the pathological stretching of the myopic retina, however, causes adjacent points to be further removed, without any filling in, as it were, of the interspaces, because the retinal elements are not so closely packed as in emmetropia so that resolution may easily be subnormal.

We may give an example of actual magnification to illustrate this, and also to show that magnification as here understood is always *linear*, and not measured by area or cubical capacity. Suppose that a scale is divided into millimetres and also tenths of a millimetre. With a certain lens it would be just possible to recognise the finer divisions, and it is conceivable that in some forms of ametropia we might get the same enlargement of the millimetre division without the recognition of the separation into tenths.

It has been stated that the images seen by means of aids to normal vision are virtual ones, and so they cannot be readily measured. The difficulty is increased by the fact that we cannot decide at what distance from the eye they are formed, in other words to what distance they are projected. If a magnifying glass of about + 12 D be taken, and a small piece of ruled foolscap be viewed lying upon a dull black background, the lines may be focussed and magnified, but, providing the paper does not overlap the field of view through the lens, the lines appear to be further from the eye than the dull background ; whereas, if the small piece of paper be similarly viewed when resting upon a whole sheet of the same, the lines appear to be closer to the eye, the magnification suggesting nearness in the latter case, whereas in the former there is nothing for comparison.

It is necessary, therefore, to arbitrarily fix a distance to which virtual images shall be assumed to be projected, and this is taken by universal consent at 10 inches (25 cm.) from the eye (plane of the pupil—eyepoint of the instrument), a distance suitable for distinct near vision in all cases where near objects are concerned, but not applicable to vision of distant objects as seen through the telescope, etc.

In the case of instruments employed to view near objects magnification is defined as the ratio of the linear dimension of the virtual image at 25 cm. distance, to that of the object when placed at the same distance and viewed direct. This is a conventional magnification and does not represent the true value which is given by the ratio of the sizes of the retinal images, or $\frac{\theta_2}{\theta_1}$ for any instrument, where $\theta_2$ represents the angle subtended by the virtual image, and $\theta_1$ that subtended by the object. The conventional method of expressing magnification is very important for purposes of comparison, and it *may* represent the apparent magnification, especially in the use of the reading glass. The *apparent* differs from the conventional magnification, and may be defined as the ratio of the linear dimension of the virtual image, when projected to the actual object plane, to that of the object in its actual position. It is also estimated by the double vision method where one eye views the virtual image through the lens and the other views the object direct, comparing the relative sizes.

Assuming a virtual image to be projected, in which case the object must be just within the focal point of the lens system, and occupying a position less than 25 cm. distance from the eye, the conventional value must exceed the apparent, whereas with the object at a greater distance than 25 cm., the reverse will be the case, while, with the object at 25 cm. the two values will agree.

This is illustrated in Fig. 51, where A B is the object and $A_1$ $B_1$ the virtual image at 25 cm. distance. Supposing A B to be outside the near point of distinct vision, so that it may be clearly seen, A B would subtend a larger angle at P than would $A_3$ $B_3$, of the same size at the conventional distance, and, consequently, the conventional *m* will always be greater than the apparent *m*.

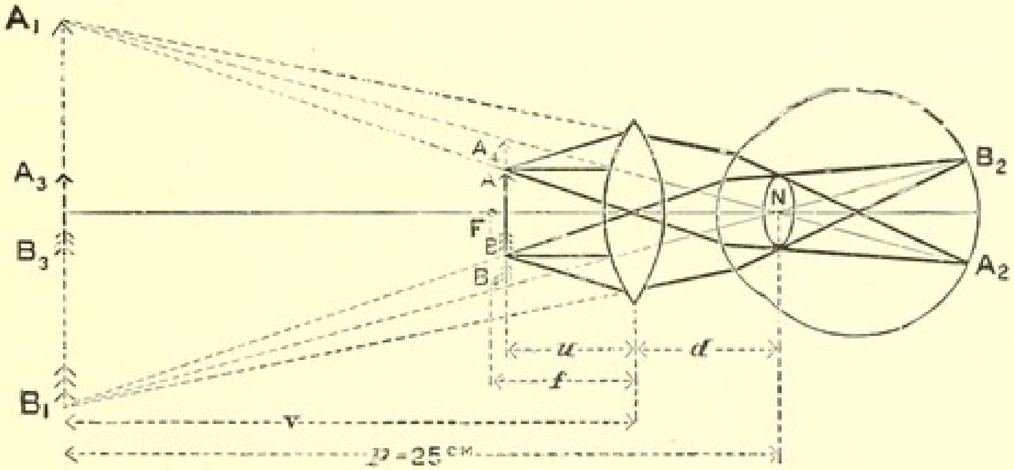



Fig. 51.

To put the matter clearly, suppose we place a millimetre rule at 10 inches from the eyes of an emmetropic youth of 16 years, who will be able to accommodate easily up to within about 4 inches from the eye. He now views an object of 1 mm. length with a lens of + 8 D placed ½ inch from one eye. The object then will be just under 5 inches from the lens, and if with the free eye he views the object it will be projected upon the scale, while the virtual image will also be seen upon the scale and will be only a trifle larger (1·1 times approximately). This is the apparent magnification. The conventional magnification will be obtained by the same double vision method, except that the free eye views the scale and notes how many mms. the virtual image of the 1 mm. object covers ; the number will be nearly 3, each mm. of the scale representing the size of the object as it appears when at 10 inches (25 cm.) from the eye. If, as often happens with the reading glass, the object is actually at 25 cm. from the eye then conventional and apparent magnification correspond.

The conventional magnification is given by :—

$$m = \frac{\mathrm{I}}{\mathrm{O}} = \frac{r}{u} \quad \text{but } u = \frac{f\,r}{f + r}$$

$$= \frac{r\,(f + r)}{f\,r}$$

$$= 1 + \frac{r}{f}\ \left(\text{or } 1 + \frac{\mathrm{F}}{\mathrm{V}}\right)$$

If we represent the distance of the lens from the eye by $d$, and the distance of normal distinct vision by $p$, then :—

$$m = 1 + \frac{p - d}{f}$$

from which the table at the end of the book is calculated.

From this it follows that the conventional magnification is decreased as the distance of the lens from the eye is increased. The apparent magnification, on the other hand is increased as the lens is withdrawn from the eye, until a midway position is reached, after which it decreases.

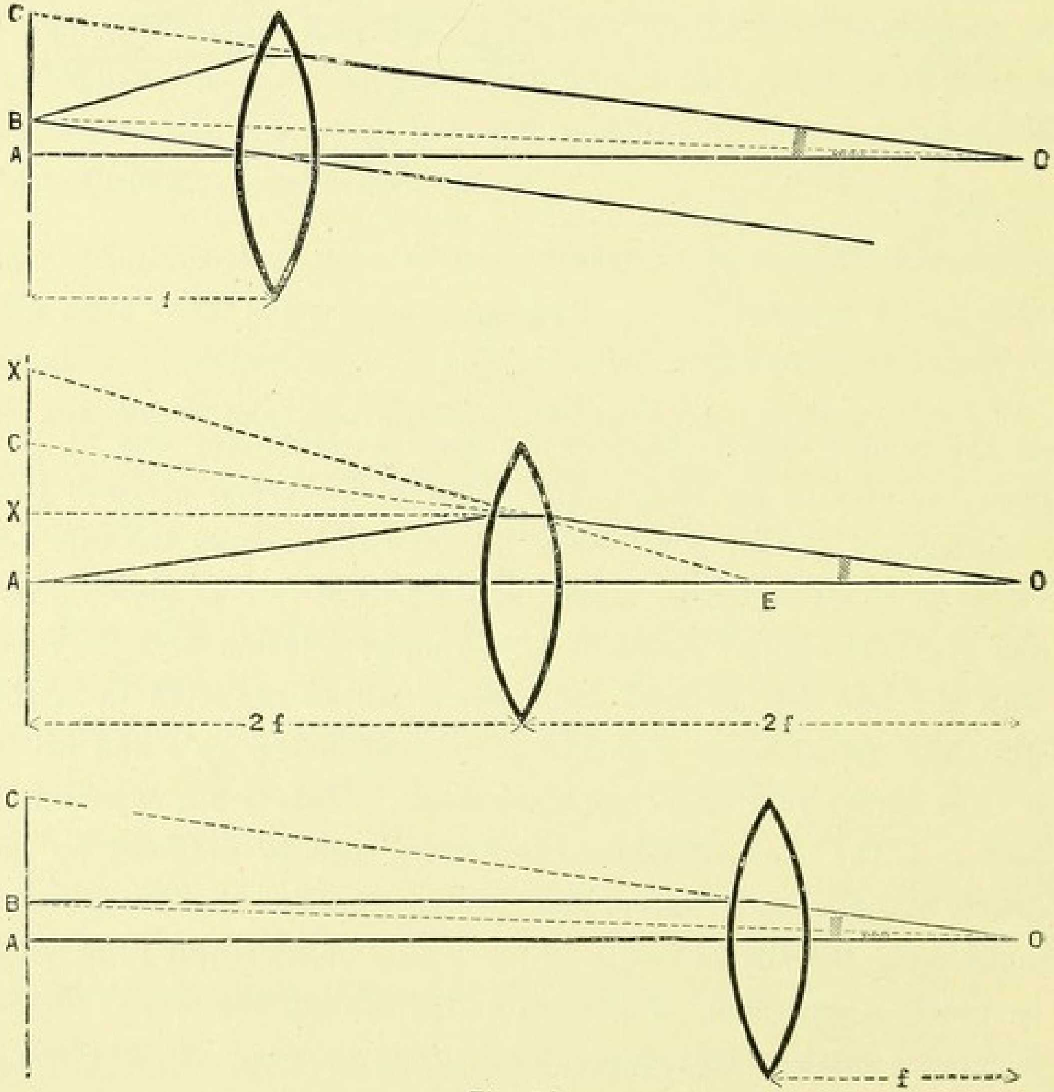



Fig. 52.

The angles are shaded, $\theta_2$ being the larger and $\theta_1$ the smaller.

It is of considerable importance to consider the greatest effect produced by a convex lens placed between an object and the eye. In Fig. 52 A C represents an object such as a page of print, and O the optical centre (back nodal point) of the refracting system of the eye, and let A O equal four times the focal length of the lens shown, for the particular case we are considering. Then the print and the eye are in symmetrical planes when the lens is midway between them, as in the second figure, while in the first and third the print and eye respectively are in the focal planes of the lens.

It is proposed to show that, for a single convex lens, the value of $m$ depends upon :—

(*a*) The distance of lens from object.

(*b*) The distance of eye from object.

(*c*) The focal power of the lens.

If we suppose the lens to be thin and in contact with the object, light apparently, as well as actually, diverges from the object itself, and $\theta_2 = \theta_1$, giving unit magnification, the image being in the object plane.

With the lens at a distance of its focal length from the object light emerges parallel from the lens, apparently coming from an infinite distance, and a smaller object A B now subtends the same angle as A C, as the first diagram shows.

But $f = \frac{1}{4}$ AO, therefore AB $= \frac{1}{4}$ AC and $m = \frac{\theta_2}{\theta_1} = \frac{\text{AC}}{\text{AB}} = 4.$

Therefore the magnification is four times, and as the emergent light is parallel the object is distinctly seen.

When the lens occupies the midway position (twice its focal length) AO $= 4f$, and light converges to O in the eye, crosses over and floods the retina with light, so that an infinitely small point A on the axis will subtend the angle $\theta_2$, hence nothing definite will be visible, the magnification being infinite and useless.

In the third illustration we have the converse of the first, light diverging from the object emerging convergent from the lens, but as the eye cannot adapt itself to focus convergent light the object will not be seen clearly. As AB apparently subtends the angle $\theta_2$ and $f = \frac{1}{4}$ AO $\therefore m = \frac{\theta_2}{\theta_1} = \frac{\text{AC}}{\text{AB}} = 4,$ as in the first case. Finally, with the lens just in front of the eye (15 mm.), its optical centre being coincident with the anterior focal point, the *real* inverted image on the retina will be normal in size, because, as explained in

Chapter X, the nodal point $N_2$ of the system (lens and eye) will move forward by an amount identical with the movement of $F_2$, the posterior focal point. If the eye is emmetropic the real image will be formed in front of the retina, and the object will appear too large and blurred, but if the lens happens to correct the ametropia there is perfect unit magnification. It is therefore evident that when the distance of object to eye is equal to $4f$ then $m$ increases from unity to infinity, and decreases from infinity to unity, as the lens is moved from the object to the eye.

Let us suppose now that AO is less than $4f$; then, for any equivalent position of the lens the magnifying power will be less, although it will still produce the maximum value when in the midway position. Thus, if in the second diagram the eye is moved forward to E, a distance from the lens equal to its focal length, then an object AX is seen as AX′, the magnification being obviously three times. If now the lens is placed midway between A and E, $m$ will have its greatest value, but it will not be infinitely great, as when the eye was at O, because the object seen under the new angle $\theta_2$ would be of some considerable size.

When AO is greater than $4f$, the lens, when in certain positions will cause light diverging from the object to *converge* to a point in front of the eye, thus forming a *real inverted* aerial image, and light diverging from this will enter the eye, the magnification being given by the ratio of the conjugate distances from the lens $\left(m = \frac{v}{u}\right)$, so that the magnification may be negative, the image being less than the object. When the lens is in the midway position for conjugate foci the object might just as well be inverted, placed in the image plane, and viewed direct.

With any given position of the lens which enables a virtual image to be projected, $m$ increases with the distance between object and eye. In the first diagram, where the lens is at its focal length from the object, and the eye at O, $m = 4$, but if the eye recedes from O, it is clear that light still emerges parallel from the lens, and therefore from any point of the object, as B for instance, every path makes the same angle with the axis and so $\theta_2$ remains constant. As the eye recedes it is clear that $\theta_1$ diminishes and, therefore, $m$ $\left(\frac{\theta_2}{\theta_1}\right)$ increases; the field of view gets less and less and becomes infinitely small when $m$ becomes infinitely great.

From (1) Fig. 52 it must be obvious that $m$ increases with an increase in the focal power of the lens, providing a virtual image is viewed. For as $f$ decreases any oblique path from some point in the object such as B will be bent to a greater extent, consequently any given object will be seen under a greater angle than previously.

We are now in a position to study, with regard to magnification, etc., the effect of placing in front of the eye various optical aids to normal vision. Although it has been shown that the instrument and the eye as a whole can be readily dealt with on the Gauss system, the subject requires some further investigation. The simplest aids to vision are the hand-magnifier and reading-glass, the first always held close to the eye and the other at some distance away. The hand-magnifier or simple microscope has already been dealt with somewhat fully, magnification being expressed conventionally by the equation $m = 1 + \frac{F}{V}$. The reading-glass is identical in action, Fig. 51 illustrating the formation of the real retinal image $A_2$ $B_2$ and the projected virtual image $A_1$ $B_1$. It is important to note that if we take any point of the object nearer the axis that A or B the path of the emergent beams will not make so great an angle with it, and a point actually upon the axis will give a path parallel with it. This indicates why the field of view lessens as the eye is withdrawn from the lens, for clearly such beams as shown in the diagram could not enter the pupil, but only those whose path was nearer the axis of the entire system. Consequently a reading-glass must have a large aperture. This, owing to aberration, limits its power, which in practice is usually of the order of 4D. The reading-glass should be employed in the midway position (2) Fig. 52, so that a maximum value of $m$ may be obtained with a minimum of power. The crossing point of the greatest number of rays emerging from the system is termed the eye-point, N in the accompanying figure, and this is the position of greatest advantage for field of view and illumination; an important matter in the microscope, opera-glass, etc.

*The Binocular Magnifier* is an arrangement by which a lens is placed before each eye and decentered inwards, so as to equalize the functions of accommodation and convergence.

In Fig. 53 a convenient form of such instrument is shown. This is made by Messrs. C. W. Dixey and Son, and consists of two decentered convex lenses, so placed that their planes are at right angles to the ocular visual axes when the object is viewed. A suitable bar allows of a movement towards or away from the eyes, and is attached to a frame which fits upon the face. A small vertical folding diaphragm in the median plane prevents confusion during use of the lenses.

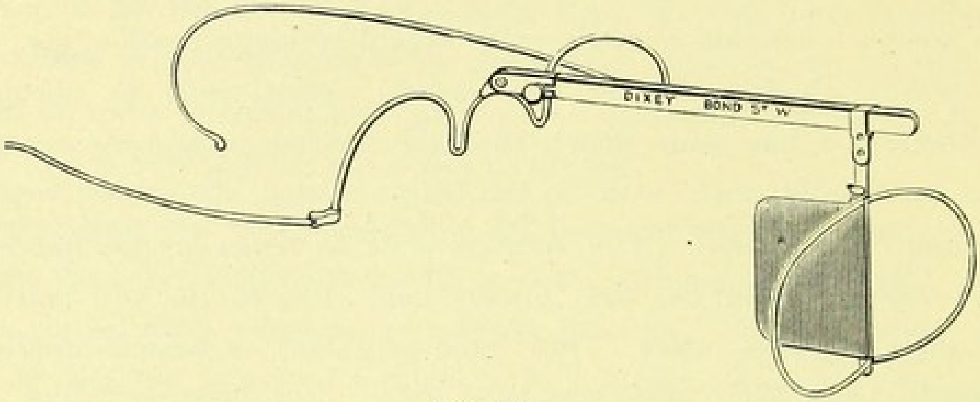



Fig. 53.

*The Astronomical Telescope*, employed to view distant objects, and giving an inverted image, may have its magnification defined as the ratio of the apparent size of the object seen through the instrument to the apparent size seen direct. The tube length equals the sum of the focal lengths of eyepiece and objective, which condition ($t = f_0 + f_2$) gives parallel emergent light. The instrument should be adjusted by *racking in* and not racking out, otherwise accommodation will be brought into play ; if $t$ exceeds $f_0 + f_2$ then emergent light will be convergent and only suitable for hyperopic eyes, while if $t$ is less than $f_0 + f_2$ the divergent light will be suitable for myopes, the difference of tube length of any particular instrument from its normal indicating approximately the amount of ametropia present. Thus, with a telescope whose objective is + 2D and ocular + 40D, $t$ will be $(50 + 2{\cdot}5) = 52{\cdot}5$ cm. Suppose it is racked in 5 mm. then parallel incident light focusses at 50 cm. from the objective and at the eyepiece will have a divergence of $\left(\frac{100}{52 - 50}\right) = 50$D. As the eyepiece is + 40D, clearly the emergent light would suit a myope of 10D provided the eye is placed near the instrument.

*The Terrestrial Telescope* is essentially like the foregoing except than a lens or lens system is placed between the real image formed by the objective and the eyepiece, the tube length being extended. This is done in order to give an erect image.

*The Galilean Telescope or Opera Glass* has a negative eyepiece which is placed a distance equal to its focal length inside the focal point of the objective in order to obtain parallel emergent light. The tube length is given by $f_O - f_E$, and the image is erect.

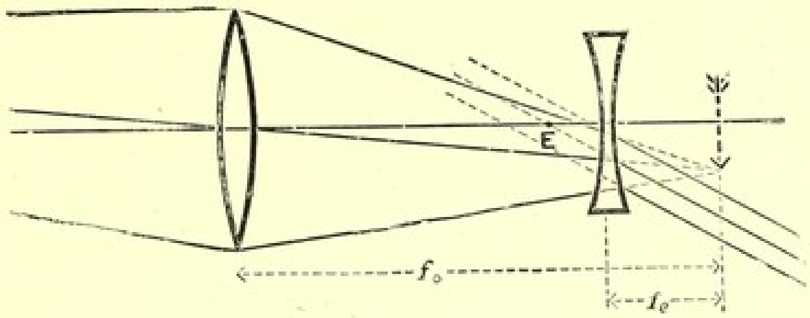



Fig 54.

Fig. 54 shows the real inverted image of a distant object which would be formed if the convergent light had not met the concave lens which makes it parallel, and an emergent parallel pencil is illustrated, from which it is seen that the true eyepoint E is consistently virtual, which prevents the eye being placed in its true position, and causes a limited field of view, the only defect of an excellent instrument. Its construction is simplified by the fact that the negative eyepiece has the opposite chromatic effect to the objective.

Suppose that such an instrument has an objective of + 5D and an eyepiece of — 40D, then the tube length will be (20 — 2·5) = 17·5 cm. for parallel emergent light, and if it be then screwed *in* 5 mm. we shall have at the eyepiece $\left(\frac{100}{20-17}\right) = 33\dot{3}D$, this being the convergence of the light from the objective. But as the eyepiece is — 40D, the emergent light will have a divergence of $6\dot{6}D$.

If the tube be screwed *out* 5 mm. from the position for parallel emergent light then a convergence of $\left(\frac{100}{20-18}\right) = 50D$ is only partly neutralized by the – 40D of the eyepiece, and the emergent light is convergent 10D, and, just as with the astronomical telescope, myopia and hyperopia may be in effect corrected by the amount of *draw* which gives the necessary divergence or convergence to emergent light, provided no accommodation is employed. For this reason an opera glass should be opened to its full extent and gently racked in. Optometers have been constructed on this principle, but they are not reliable because of the increase in magnifying power as the convex lens is withdrawn from the eye.

*The Prism Binocular* is exactly like an astronomical telescope the real image of which is erected by means of two totally reflecting prisms, causing a loss of light but giving a large field of view. In these, as in all other cases, a correcting lens, spherical or cylindrical, may be inserted in the eyepiece.

*Separated thin lenses in air.* It is necessary to give some attention to the focal lengths and magnification of two (or more) separated thin lenses in air, whether they are both positive, both negative, or one positive and the other negative. The curvature system can readily be applied here, and as an illustration let us take the case of two convex lenses $L_1$ and $L_2$ in

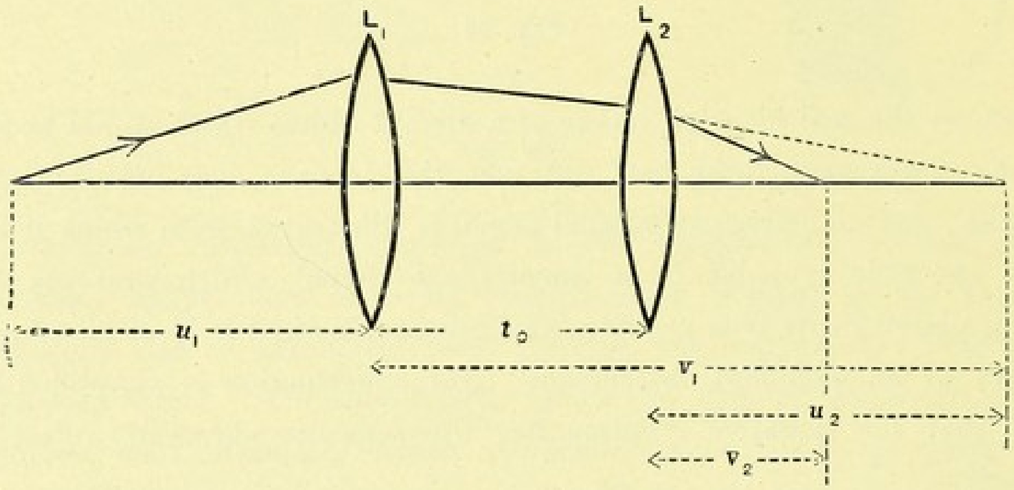



Fig. 55.

Fig. 55, the first being + 10 D, the second + 12 D, and the interval between them 25 mm. If we imagine parallel light passing from left to right it receives a convergence of 10 D at $L_1$ and would focus at 100 mm. upon the other side, therefore when it arrives at L it actually has a convergence of $\left(\frac{1000}{100-25}\right) = 13\cdot\dot{3}$ D. As it emerges from $L_2$ it will have received the extra convergence due to this lens and now is $(13\cdot\dot{3} + 12)$ $= 25\cdot\dot{3}$ D, coming to a focus at $\left(\frac{1000}{25\cdot\dot{3}}\right) =$ approx. 39·5 mm. from $L_2$. If we suppose the parallel light to pass from right to left we should find the focus at about 36·8 mm. from $L_1$, by a similar calculation, and as these two measurements depend upon the relative powers of the lenses, *only being equal when the lenses are of equal curvatures, powers and signs,* they are denoted as the *first back focal length*, measured from $L_1$, and the *second back focal length* measured from $L_2$.

It is evident, however, that such a combination must have an equivalent focus, the same upon both sides, measured from the principal points, which in such a case coincide with the nodal points (Chapter VIII), so that we must carefully distinguish between $f_E$, $f_{B1}$ and $f_{B2}$, and also between $F_E$, $F_{B1}$ and $F_{B2}$.

Below are given the equations which show these differences, the details being deduced from a consideration of Fig. 55 supposing the incident light to be parallel :—

Equiv. focal length $f_E = \dfrac{f_1 f_2}{f_1 + f_2 - t}$ $\therefore$ $F_E = F_1 + F_2 - F_1 F_2 t$. 1.

1st back ,, $f_{B1} = \dfrac{f_2 (f_1 - t)}{f_1 + f_2 - t}$ $\therefore$ $F_{B1} = \dfrac{F_1 + F_2 - F_1 F_2 t}{1 - t F_1}$. 2.

2nd back ,, $f_{B2} = \dfrac{f_1 (f_2 - t)}{f_1 + f_2 - t}$ $\therefore$ $F_{B2} = \dfrac{F_1 + F_2 - F_1 F_2 t}{1 - t F_2}$. 3.

From these it is clear that we can obtain the position of the principal points by subtracting 2 from 1 and 3 from 1, the differences expressing the distances from the surfaces of the lenses inwards.

If we represent the magnification due to $L_1$ by $m_1$ and that due to $L_2$ by $m_2$ then $m = m_1 m_2 = \dfrac{v_1}{u_1} \times \dfrac{v_2}{u_2} = \dfrac{U_1 U_2}{V_1 V_2}$.

**Summary** :—

(*a*) The magnification caused by high ametropia may not have the resolving power of true magnification.

(*b*) Magnification is given by $m = \dfrac{\theta_2}{\theta_1}$.

(*c*) Conventional magnification assumes that the virtual image is formed at a distance of 25 mm. from the eye, for the purposes of comparison. It is given by $m = 1 + \dfrac{(p - d)}{f}$, and is sometimes the apparent value.

(*d*) The maximum magnification with a minimum of power is obtained when the lens is midway between object and eye.

(*e*) For an astronomical telescope $t = f_o + f_E$.

(*f*) For the opera glass $t = f_o - f_E$.

(*g*) The equivalent focal length of a lens system must be carefully distinguished from the first and second back focal lengths.

## Symbols and Abbreviations with their Alternatives and Meanings employed by various authors.

| | |
|---|---|
| A. Ac. Acc. ... | Accommodation. |
| Ah. ... ... | Hyperopic Astigmatism. |
| Ahm. Amh. ... | Mixed Astigmatism. |
| Am. ... ... | Ametropia. Ametropic. |
| A.m. ... ... | Myopic Astigmatism. |
| As. ... ... | Astigmatism. Astigmatic. |
| As. H. ... ... | Hyperopic Astigmatism. |
| As. M. ... ... | Myopic Astigmatism. |
| Ax. ... ... | Axis. |
| c. ... ... | Cum. With. |
| Cc. ... ... | Concave. (Minus, or —). |
| cm. ... ... | Centimetre. |
| Cve. ... ... | Concave. Minus. |
| Cvx. ... ... | Convex. Plus. |
| Cx. ... ... | Convex. (Plus, or +). |
| Cyl. ... ... | Cylinder. Cylindrical. |
| D. ... ... | Dioptre. |
| Dcc. ... ... | Double Concave. |
| Dcx. ... ... | Double Convex. |
| E. Em. ... ... | Emmetropia. Emmetropic. |
| f ... ... | Focal length. |
| H ... ... | Hyperopia. Hyperopic. Horizontal. |
| Hy. ... ... | Hyperopia. Hyperopic. |
| H. As ... ... | Hyperopic Astigmatism. |
| In. ... ... | Inch. ″ |
| M ... ... | Myopia. Myopic. |
| m ... ... | Metre. |
| Mer. ... ... | Meridian. |
| M As ... ... | Myopic Astigmatism. |
| mm. ... ... | Millimetre. |
| My. ... ... | Myopia. Myopic. |
| NA ... ... | Numerical Aperture. |
| OD ... ... | Right eye. |
| OS ... ... | Left eye. |

| | | |
|---|---|---|
| OV | ... ... | Both eyes. |
| P. Pb. Pr. | ... | Presbyopia. Presbyopic. |
| Pc | ... ... | Periscopic. |
| Pcc | ... ... | Periscopic concave. |
| Pcx | ... ... | Periscopic convex. |
| Pr | ... ... | Prism. |
| P.D. | ... ... | Prism dioptre. |
| R | ... ... | Right. Curvature of a surface. |
| r | ... ... | Radius of curvature. Punctum remotum |
| R E | ... ... | Right eye (Oculus dexter). |
| S. Sph. | ... ... | Spherical. Spherical lens. |
| U | ... ... | Curvature of incident wave. |
| V | ... ... | Curvature of reflected or refracted wave. |
| F | ... ... | Focal power. Change undergone in curvature |
| V | ... ... | Velocity. |
| w | ... ... | With. |
| $\alpha$ | ... ... | Refracting angle. |
| $\delta$ | ... ... | Angle of deviation. |
| $\lambda$ | ... ... | Wave length. |
| $n$ $\mu$ | ... ... | Refractive Index. |
| $\mu$ | ... ... | 1 micron $\left(\frac{1}{1000}\text{ mm.}\right)$ |
| $^\circ$ | ... ... | Degree. |
| $'$ | ... ... | Minute. Foot. |
| $''$ | ... ... | Second. Inch. |
| $'''$ | ... ... | Line. |
| $\triangle$ | ... ... | Prism Dioptre. |
| $\Delta$ | ... ... | Mean dispersion $(n_F - n_C)$ |
| $\omega$ (Omega) | ... | Dispersive power. Index of dispersion. |
| N (Nu) | ... | Efficiency. Dispersive reciprocal. |
| $+$ | ... ... | Plus. Convex. Positive. |
| $-$ | ... ... | Minus. Concave. Negative. |
| $=$ | ... ... | Equal to. |
| $>$ | ... ... | Greater than. |
| $<$ | ... ... | Less than. |
| L | ... ... | At right angles to. |
| $\perp$ | ... ... | Perpendicular to. |
| $\parallel$ | ... ... | Parallel to. |
| $\bigcirc$ | ... ... | Combined with. |
| $\infty$ | ... ... | Infinity (practically 20 ft. or 6 m.) |
| ℞ | ... ... | Recipe. Prescription. |

# Useful Tables and Data.

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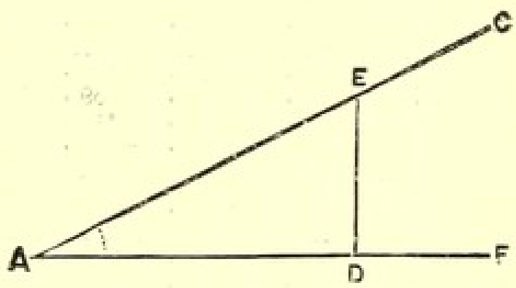
## TABLE I.—The Trigonometrical Ratios.




Let F A C be any angle. From any point in either F A or C A a line can be drawn perpendicular to the other. Let E be chosen in A C, and a perpendicular drawn to F A. Then the angle A D E is a right angle and A D is called the *base*, D E the *perpendicular*, and A E (the side opposite the right angle) the *hypotenuse*. We will denote these by B, P, and H, respectively.

Then $\frac{P}{H}$ is called the *sine* of the angle at A.

,, $\frac{B}{H}$ ,, *cosine* ,,

,, $\frac{P}{B}$ ,, *tangent* ,,

,, $\frac{B}{P}$ ,, *cotangent* ,,

,, $\frac{H}{B}$ ,, *secant* ,,

,, $\frac{H}{P}$ ,, *cosecant* ,,

## TABLE II.—Trigonometrical Values.

| Angle | | Chord. | Sine. | Tangent. | Co-tangent. | Cosine. | | | |
|---|---|---|---|---|---|---|---|---|---|
| Degrees | Radians | | | | | | | | |
| 0° | 0 | 0 | 0 | 0 | ∞ | 1 | 1·414 | 1·5708 | 90° |
| 1 | ·0175 | ·017 | ·0175 | ·0175 | 57·2900 | ·9998 | 1·402 | 1·5533 | 89 |
| 2 | ·0349 | ·035 | ·0349 | ·0349 | 28·6363 | ·9994 | 1·389 | 1·5359 | 88 |
| 3 | ·0524 | ·052 | ·0523 | ·0524 | 19·0811 | ·9986 | 1·377 | 1·5184 | 87 |
| 4 | ·0698 | ·070 | ·0698 | ·0699 | 14·3007 | ·9976 | 1·364 | 1·5010 | 86 |
| 5 | ·0873 | ·087 | ·0872 | ·0875 | 11·4301 | ·9962 | 1·351 | 1·4835 | 85 |
| 6 | ·1047 | ·105 | ·1045 | ·1051 | 9·5144 | ·9945 | 1·338 | 1·4661 | 84 |
| 7 | ·1222 | ·122 | ·1219 | ·1228 | 8·1443 | ·9925 | 1·325 | 1·4486 | 83 |
| 8 | ·1396 | ·140 | ·1392 | ·1405 | 7·1154 | ·9903 | 1·312 | 1·4312 | 82 |
| 9 | ·1571 | ·157 | ·1564 | ·1584 | 6·3138 | ·9877 | 1·299 | 1·4137 | 81 |
| 10 | ·1745 | ·174 | ·1736 | ·1763 | 5·6713 | ·9848 | 1·286 | 1·3963 | 80 |
| 11 | ·1920 | ·192 | ·1908 | ·1944 | 5·1446 | ·9816 | 1·272 | 1·3788 | 79 |
| 12 | ·2094 | ·209 | ·2079 | ·2126 | 4·7046 | ·9781 | 1·259 | 1·3614 | 78 |
| 13 | ·2269 | ·226 | ·2250 | ·2309 | 4·3315 | ·9744 | 1·245 | 1·3439 | 77 |
| 14 | ·2443 | ·244 | ·2419 | ·2493 | 4·0108 | ·9703 | 1·231 | 1·3265 | 76 |
| 15 | ·2618 | ·261 | ·2588 | ·2679 | 3·7321 | ·9659 | 1·218 | 1·3090 | 75 |
| 16 | ·2793 | ·278 | ·2756 | ·2867 | 3·4874 | ·9613 | 1·204 | 1·2915 | 74 |
| 17 | ·2967 | ·296 | ·2924 | ·3057 | 3·2709 | ·9563 | 1·190 | 1·2741 | 73 |
| 18 | ·3142 | ·313 | ·3090 | ·3249 | 3·0777 | ·9511 | 1·176 | 1·2566 | 72 |
| 19 | ·3316 | ·330 | ·3256 | ·3443 | 2·9042 | ·9455 | 1·161 | 1·2392 | 71 |
| 20 | ·3491 | ·347 | ·3420 | ·3640 | 2·7475 | ·9397 | 1·147 | 1·2217 | 70 |
| 21 | ·3665 | ·364 | ·3584 | ·3839 | 2·6051 | ·9336 | 1·133 | 1·2043 | 69 |
| 22 | ·3840 | ·382 | ·3746 | ·4040 | 2·4751 | ·9272 | 1·118 | 1·1868 | 68 |
| 23 | ·4014 | ·399 | ·3907 | ·4245 | 2·3559 | ·9205 | 1·104 | 1·1694 | 67 |
| 24 | ·4189 | ·416 | ·4067 | ·4452 | 2·2460 | ·9135 | 1·089 | 1·1519 | 66 |
| 25 | ·4363 | ·433 | ·4226 | ·4663 | 2·1445 | ·9063 | 1·075 | 1·1345 | 65 |
| 26 | ·4538 | ·450 | ·4384 | ·4877 | 2·0503 | ·8988 | 1·060 | 1·1170 | 64 |
| 27 | ·4712 | ·467 | ·4540 | ·5095 | 1·9626 | ·8910 | 1·045 | 1·0996 | 63 |
| 28 | ·4887 | ·484 | ·4695 | ·5317 | 1·8807 | ·8829 | 1·030 | 1·0821 | 62 |
| 29 | ·5061 | ·501 | ·4848 | ·5543 | 1·8040 | ·8746 | 1·015 | 1·0647 | 61 |
| 30 | ·5236 | ·518 | ·5000 | ·5774 | 1·7321 | ·8660 | 1·000 | 1·0472 | 60 |
| 31 | ·5411 | ·534 | ·5150 | ·6009 | 1·6643 | ·8572 | ·985 | 1·0297 | 59 |
| 32 | ·5585 | ·551 | ·5299 | ·6249 | 1·6003 | ·8480 | ·970 | 1·0123 | 58 |
| 33 | ·5760 | ·568 | ·5446 | ·6494 | 1·5399 | ·8387 | ·954 | ·9948 | 57 |
| 34 | ·5934 | ·585 | ·5592 | ·6745 | 1·4826 | ·8290 | ·939 | ·9774 | 56 |
| 35 | ·6109 | ·601 | ·5736 | ·7002 | 1·4281 | ·8192 | ·923 | ·9599 | 55 |
| 36 | ·6283 | ·618 | ·5878 | ·7265 | 1·3764 | ·8090 | ·908 | ·9425 | 54 |
| 37 | ·6458 | ·635 | ·6018 | ·7536 | 1·3270 | ·7986 | ·892 | ·9250 | 53 |
| 38 | ·6632 | ·651 | ·6157 | ·7813 | 1·2799 | ·7880 | ·877 | ·9076 | 52 |
| 39 | ·6807 | ·668 | ·6293 | ·8098 | 1·2349 | ·7771 | ·861 | ·8901 | 51 |
| 40 | ·6981 | ·684 | ·6428 | ·8391 | 1·1918 | ·7660 | ·845 | ·8727 | 50 |
| 41 | ·7156 | ·700 | ·6561 | ·8693 | 1·1504 | ·7547 | ·829 | ·8552 | 49 |
| 42 | ·7330 | ·717 | ·6691 | ·9004 | 1·1106 | ·7431 | ·813 | ·8378 | 48 |
| 43 | ·7505 | ·733 | ·6820 | ·9325 | 1·0724 | ·7314 | ·797 | ·8203 | 47 |
| 44 | ·7679 | ·749 | ·6947 | ·9657 | 1·0355 | ·7193 | ·781 | ·8029 | 46 |
| 45° | ·7854 | ·765 | ·7071 | 1·0000 | 1·0000 | ·7071 | ·765 | ·7854 | 45° |
| | | | Cosine | Co-tangent | Tangent | Sine | Chord | Radians | Degrees |
| | | | | | | | | Angle | |

TABLE III.—REFRACTIVE INDICES, &c., OF OPTICAL MATERIALS FOR D LINE.

| | Soft Crown Glass. | Hard Crown Glass. | Light Flint Glass. | Densest Flint Glass. | Canada Balsam. | Water. | Quartz (ordinary ray). | Quartz (extra-ordinary ray). |
|---|---|---|---|---|---|---|---|---|
| Refractive Index ($n$) | 1·515 | 1·517 | 1·547 | 1·713 | 1·526 | 1·333 | 1·544 | 1·553 |
| Mean Dispersion | 0·009 | 0·008 | 0·012 | 0·024 | 0·023 | 0·006 | 0·007 | 0·008 |
| Efficiency, Dispersive Reciprocal | 56·6 | 60·5 | 45·8 | 29·9 | 41·5 | 55·6 | 69·9 | 69·0 |

TABLE IV.—CONVENTIONAL MAGNIFICATION. The distance of distinct vision is 25 cms., the formula of Chapter XI. being used $\left(M = 1 + \frac{p-d}{f}\right)$

| Power of Lens in Dioptres. | Distance of Lens from the Eye in cms. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1·5 | 2 | 4 | 6 | 8 | 10 | 12 |
| 80 D | 19·8 | 19·4 | 17·8 | 16·2 | 14·6 | 13 | 11·4 |
| 40 D | 10·4 | 10·2 | 9·4 | 8·6 | 7·8 | 7 | 6·2 |
| 20 D | 5·7 | 5·6 | 5·2 | 4·8 | 4·4 | 4 | 3·6 |
| 16 D | 4·7 | 4·6 | 4·3 | 4 | 3·7 | 3·4 | 3·1 |
| 12 D | 3·76 | 3·70 | 3·47 | 3·23 | 3·00 | 2·76 | 2·52 |
| 10 D | 3·35 | 3·30 | 3·10 | 2·90 | 2·70 | 2·5 | 2·30 |
| 8 D | 2·88 | 2·84 | 2·68 | 2·52 | 2·36 | 2·2 | 2·04 |
| 6 D | 2·38 | 2·35 | 2·23 | 2·11 | 2·00 | 1·88 | 1·76 |
| 4 D | 1 94 | 1·92 | 1·84 | 1·76 | 1·68 | 1·60 | 1·52 |
| 3 D | 1·69 | 1·67 | 1·61 | 1·55 | 1·50 | 1·44 | 1·38 |
| 2·5 D | 1·58 | 1·57 | 1·52 | 1·48 | 1·42 | 1·37 | 1·32 |
| 2 D | 1·47 | 1·46 | 1·42 | 1·38 | 1·34 | 1·30 | 1·26 |

## TABLE V.—Deviation Constants.

Prisms are measured in three ways :—

1. By the *angle at the apex* (or refracting angle) contained by the sides of the glass, and written in degrees, viz., 1°, $1\frac{1}{2}$° or 1° 30′. The size of this angle varies with *n* of the glass for the same deviation. (See below.)

2. By the *angular deviation* produced by the prism, which will be nearly half the apical angle. This is also written in degrees, and should have " d " added to it to distinguish it from the first, viz., 1°d, 2°d.

3. By *prism dioptres*, which express linear deviation, the unit being a prism which deviates a ray of light 1cm. from its path at a distance of 1 metre. Written 1$^{\Delta}$, 2$^{\Delta}$, etc. (The *centrad* is practically the same measurement.)

| | | | |
|---|---|---|---|
| 1$^{\Delta}$ | = 1·00 cm. | 1° ($n$ = 1·57) | = 1·00 cm. (10·0 mm.) |
| 1° ($n$ = 1·5 ) | = 0·87 ,, | 1° ($n$ = 1·60) | = 1·06 ,, (10·6 ,, ) |
| 1° ($n$ = 1·54) | = 0·94 ,, | 1°d | = 1·745 ,, (17·45 ,, ) |

| Prisms ($n$ = 1·54). | | |
|---|---|---|
| Angle of Dev. | Angle of Ref. | Linear Dev. (At 1m.) |
| 0·5 | 0·93 | 8·6 mm. |
| 1·0 | 1·85 | 17·4 ,, |
| 1·5 | 2·78 | 26·2 ,, |
| 2·0 | 3·70 | 34·8 ,, |
| 2·5 | 4·63 | 43·6 ,, |
| 3·0 | 5·55 | 52·4 ,, |
| 3·5 | 6·48 | 61·2 ,, |
| 4·0 | 7·40 | 69·8 ,, |
| 5·0 | 9·23 | 87·4 ,, |
| 6·0 | 11·5 | 105·0 ,, |
| 7·0 | 12·58 | 122·8 ,, |
| 8·0 | 14·63 | 140·2 ,, |

| Prisms ($n$ = 1·54). | |
|---|---|
| Prism Diop. | Ref. Ang. |
| 1·00$^{\Delta}$ | 1·06° |
| 2·00$^{\Delta}$ | 2·16° |
| 3·00$^{\Delta}$ | 3·24° |
| 4·00$^{\Delta}$ | 4·32° |
| 5·00$^{\Delta}$ | 5·40° |
| 6·00$^{\Delta}$ | 6·47° |
| 7·00$^{\Delta}$ | 7·54° |
| 8·00$^{\Delta}$ | 8·62° |

## TABLE VI.—Metric and English Linear Equivalents.

| | | | | |
|---|---|---|---|---|
| To convert | inches to millimetres | multiply by | 25·4 | (Roughly—multiply by 100 and divide by 4) |
| ,, | mm. to inches | ,, | 0·03937 | (Roughly—multiply by 4 and divide by 100) |
| ,, | metres to feet | ,, | 3·2808 | (Roughly—multiply by 10 and divide by 3) |
| ,, | feet to metres | ,, | 0·3048 | (Roughly—multiply by 3 and divide by 10) |

To reduce French inches to English × 39·37 and ÷ 37.

135·3 French lines = 1 foot.

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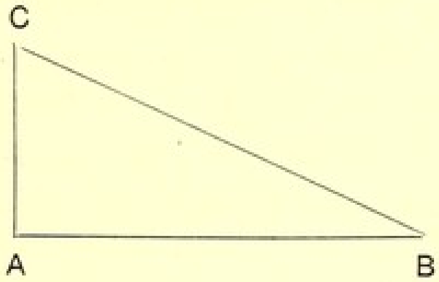
## CALCULATION for Resultant Prisms :—




Let A B = in cm. the °d. of the horizontal prism.

,, A C = in cm. the °d. of the vertical prism.

Then C B = in cm. the °d. of the resultant prism.

And A B C = angle of the "base apex line" with the horizontal (or by calculation).

(1) Square the vertical prism.

(2) Square the horizontal prism.

(3) Add the two, and extract the square root, this is the *power* of the resultant prism.

Then :—

$$\frac{\text{Horizontal effect required}}{\text{Power of resultant prism}} = \left\{ \begin{array}{l} \textit{Cosine of} \text{ the angle of rotation} \\ \text{of the base apex line from the} \\ \text{horizontal.} \end{array} \right.$$

To find the effects (V. and H.), of an oblique prism :—

Sine of angle of rotation from the horizontal × °d. of the oblique prism = the vertical effect.

Similarly :—

Cosine of the *same* angle × °d. of the oblique prism = the horizontal effect.

## The Slide Rule.

Parts.—Rule, slide and cursor.

1st Scale of Rule (top part from left 1 to middle 1)
1st Scale of Slide ( ditto ditto )

Representations.—The 1st scale is taken as a unit of measure, and the spaces 1—2, 1—3, 1—4, etc., are proportional to the logs of 2, 3, 4, etc., to 10, which last has unity for its logarithm. The spaces 1—2, 2—3, etc., are also divided in the same manner, giving, for example, between 2 and 3 the logs of such numbers as 2·1, 2·3, etc. The lower scale of rule and of slide are alike, and just double the length of the top scales.

Multiplication.—Use the two lower scales of the rule and slide. Make a 1 of the slide coincide with one of the factors which must be read off on the rule scale ; and the product will be found on the scale of the rule opposite to the other factor read on the slide.

*Number* of figures in Product may be determined either by

(1) The characteristic of the log.

(2) By adding the two factors if the right hand 1 is used, or if the left hand 1 is used by subtracting 1 from the sum of the two factors.

Division.—Use the two lower scales of rule and slide. Place the divisor read on the slide above the dividend read on the rule ; the quotient will be found on the rule below a 1 of the slide.

*Number* of figures in quotient may be determined by

(1) Deducting the figures in the divisor from those in the dividend if the right hand 1 of the slide points out the answer, and

(2) By deducting the figures in the divisor from those in the dividend and adding one if the left hand 1 of the slide points out the answer.

Proportion.—Perform the necessary operation for finding the *quotient*, and, without reading it, look for the product of this quotient by reading off the third factor on the slide and noting the answer on the rule.

Or use the following rule :—Put the proportion into the form of 2 equal ratios, for instance let $\frac{2}{3} = \frac{4}{x}$ then place 2 over 3 and under 4 on slide read $x$ on rule.

Squares and Square Roots.—The numbers on the upper scales of the rule are the squares of the numbers on the lower scale. Hence, to obtain the square or the square root of a number it is merely requisite to read the numbers opposite to each other, either by the help of the cursor, or by the end marks 1 on the slide.

*Note.*—In taking the square root of a number the *left* hand top scale must be used if the number has an *odd* number of figures, and the *right* hand top scale if the number has an *even* number of figures.

Cubes of Numbers.—This may be done either by

(1) Setting the 1 on the lower scale of the slide against the number to be cubed on the lower scale of the rule, when over the number on the first scale of the slide will be found the required cube ; or by

(2) Inverting the slide, keeping the same face upwards, and now (using the lower scale of rule and what was formerly the *1st scale* of slide) setting opposite each other the marks which indicate on each scale the number of which the cube is required ; this cube will now be found on the upper scale of the rule opposite to the 1 of the slide (now on the right hand).

Cube Roots.—To find the cube root of a number place (having inverted the slide, keeping the same face up) the now right hand 1 of the slide against the number and seek for a number on the lower scale of the rule which is opposite to the same number on the (now) lower scale of the slide. This number is the cube root required.

*Note.*—The number of figures in the answer is of course 1 for every 3 (and 1 for the remainder over the even threes, if there is one) in the number whose cube root is to be extracted.

Sines and Tangents.—On the reverse side of the slide will be found the sine and tangent scales marked *S.* and *T.* respectively. Both scales are divided so that the spaces, reckoned from the left of the scales, to 1, 2, 3, etc., represent the logs of the natural sines and tangents of angles of 1°, 2°, 3°, etc., in a circumference whose radius is 100.

The scale required for use is placed into contact with the upper scale of the rule, the sines and tangents can then be read off this scale direct for the various angles.

*Note.*—Values of the tangents of angles greater than 45° may be obtained by dividing 1 by the tangent of the complementary angle.

To multiply say 38 by sin 15° *direct*, place one extremity of the sine scale against 38, the product will then be found on the scale of the rule opposite to the mark corresponding to sin 15°. (Likewise Tangents).

LOGARITHMS.—On the reverse side of the slide and in the centre is a scale divided into equal parts, and adapted to measure the spaces on the lower scale of the rule which represent the decimal parts of the logarithms of numbers.

To find the log of a number the slide is left in its usual position and the left hand (1) put against the number (on the lower scale of the rule) for which the log is required ; the log of the number will now be found on the scale of equal parts opposite to the mark in the opening at the back of the *right* extremity of the rule. Example, for log 2 we find 301, which must be read as 0·301.

| — | Aberration. | Manifestation. | Remedy. |
|---|---|---|---|
| Class I. | DUE TO THE MATERIAL. | | |
| A ... ... | Chromatic aberration— (a) Of focal plane (b) Of Magnification | Colour tinging edges of images and of visible field ... ... ... ... ... ... | Use achromatic lens made of two kinds of glass. Partial remedy: stop down with diaphragm. |
| B ... ... | Secondary Spectrum ... ... | Faint colour tinge in images of minute bright objects... ... ... ... ... ... | Use apochromatic lenses made of three kinds of glass; or special achromatics of glass chosen to annul secondary spectrum. |
| C ... ... | Streakiness ... ... ... | Streaks in glass. Images of bright points confused ... ... ... ... ... | Reject. Glass is bad. |
| D ... ... | Double refraction ... ... | Images doubled ... ... ... ... ... | Reject. Cause: crystalline structure or bad annealing. |
| Class II. | DUE TO THE FORM. | | |
| E ... ... | Cylindricity ... ... ... | Vertical and horizontal lines not in focus at same time ... ... ... ... ... | Reject. Lens has got astigmatism, being badly ground. But sometimes due to oblique mounting. |
| F ... ... | Spherical Aberrations – | | |
| | (a) Central ... ... | Images of bright points blurred, even at centre of field ... ... ... ... | Partial remedy: stop down. Curves badly calculated. In telescopes, correct by retouching to correct for error arising from surface being truly spherical. Other remedy: use separated component lenses, or recalculate curves, so that work of refraction shall be shared between various surfaces. |
| | (b) Coma ... ... ... | Image of points not at centre of field blurred into pear shape ... ... ... ... | |
| | (c) Radial astigmatism ... | Image of points not at centre distorted into two focal lines at different distances from lens ... ... ... ... ... ... | |
| G ... ... | Chromatic differences of the Spherical Aberration ... | Faint colour tinge of blurred image ... ... | Remedies and corrections as preceding. |
| H ... ... | Curvature of Focal plane ... | Images at centre and at margin are in focus at different distances ... ... ... | Different combinations of lenses needed. |
| I ... ... | Distortion of image in its plane | Barrel distortion or pin-cushion distortion of image ... ... ... ... ... ... | Combination badly designed. Try shifting stop. |
| Class III. | DUE TO APERTURE. | | |
| K ... ... | Diffractional aberration ... | Spurious disks of stars; sometimes faint symptoms of rings round star ... ... | |
| Class IV. | DUE TO BAD MOUNTING. | | |
| L ... ... | Obliquity of mounting ... | Vertical and horizontal lines not in focus at same time ... ... ... ... ... | Set straight in mounting. |
| M ... ... | Decentring ... ... ... | Image shifted out of direct line ... ... | Lens badly cut or mounted. Thickest part must be in centre. |
| Class V. | DUE TO STRAY REFLEXION. | | |
| N ... ... | Flare Spot ... ... ... | Flare intruding in middle of field ... ... | Combination not well adjusted. Alter distance between lenses. |

After S. P. Thompson.

# List of Optical Works.

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| | | |
|---|---|---|
| 1 | Abney, | Colour Vision |
| 2 | ,, | Photography |
| 3 | Baly | Spectroscopy |
| 4 | Bayley, | Instruction in Photography |
| 5 | Beck and Andrews, | Photographic Lenses |
| 6 | Bell, | The Art of Illumination |
| 7 | Cole, | A Treatise on Photographic Optics |
| 8 | Cross & Cole, | Modern Microscopy |
| 9 | Czapski, | Theorie d. Optischen Instrumente nach Abbe |
| 10 | Dallmeyer, | Tele-Photography |
| 11 | Drude, | The Theory of Optics |
| 12 | Edser, | Light |
| 13 | Ferraris, | Dioptrische Instrumente |
| 14 | Forster, | This World of Ours |
| 15 | Gage, | The Microscope |
| 16 | Glazebrook, | Light |
| 17 | ,, | Physical Optics |
| 18 | Gleichen. | Lehrbuch der Geometrischen Optik |
| 19 | Green, | Colour Blindness |
| 20 | Heath, | Elementary Geometrical Optics |
| 21 | ,, | Geometrical Optics |
| 22 | Hovestadt, | Jena Glass |
| 23 | Jones (Chapman), | Science and Practice of Photography |
| 24 | Kayser, | Handbuch der Spectroscopie, (2 vols.) |
| 25 | Kohlrausch, | Physical Measurements |

| | | |
|---|---|---|
| 26 | Lloyd, | The Wave Theory of Light |
| 27 | Lummer, | Photographic Optics |
| 28 | Lockyer, | Spectrum Analysis |
| 29 | Mann, | Manual of Advanced Optics |
| 30 | Palaz, | Photometry |
| 31 | Pendlebury, | Lenses and Systems of Lenses |
| 32 | Percival, | Optics |
| 33 | Preston, | Theory of Light |
| 34 | Pringle, | Practical Photomicrography |
| 35 | Raymond, | Plane Surveying |
| 36 | Roscoe and Schuster, | Spectrum Analysis |
| 37 | Schellen, | Spectrum Analysis |
| 38 | Schuster, | Theory of Optics |
| 39 | Schuster & Lee, | Physical Measurements |
| 40 | Stanley, | Drawing Instruments |
| 41 | ,, | Surveying Instruments |
| 42 | Stein, | Photometric Measurements |
| 43 | Steinheil & Voit, | Handbuch der Angewandten Optik |
| 44 | Stewart, | Light |
| 45 | Suter, | Handbook of Optics |
| 46 | Tait, | Light |
| 47 | Taylor & Baxter, | The Key to Sight-Testing, (2nd Edition, Taylor & Mackinney) |
| 48 | Taylor (Dennis), | A System of Applied Optics |
| 49 | Taylor (Traill), | The Optics of Photography |
| 50 | Thompson (S.P.), | Optical Tables and Data |
| 51 | Thompson, (Lord Kelvin) and Tait | Elements of Natural Philosophy |
| 52 | Tscherning, | Physiologic Optics |
| 53 | Verdet, | Optique Physique (2 vols.) |
| 54 | Watts, | Introduction to the Study of Spectrum Analysis |
| 55 | Wood, | Physical Optics |
| 56 | Wright, | Light |
| 57 | ,, | Optical Projection |

58 Proceedings of the Optical Convention (No. 1, 1905).

*a* "Present Position of Photometric Measurements." A. C. Jolley

*b.* "Measurement of Refractive Index." S. D. Chalmers

*c.* "Polishing of Glass Surfaces." Lord Rayleigh.

*d.* "Discussion on Aberrations." Dr. C. V. Drysdale and S. D. Chalmers

*e.* "The Consideration of the Equivalent Planes of Optical Instruments." Conrad Beck

*d.* "Small Telescopes and Binoculars." Dr. C. V. Drysdale

59 Transactions of the Optical Society.

*a.* "Stroboscopy." Dr. C. V. Drysdale, 1905-6

*b.* "A New Spectrometer—its uses and advantages." Val. H. Mackinney, 1905-6

*c.* "A Simple Direct Method of Testing Spherical and Cylindrical Lenses." Dr C. V. Drysdale, 1899-1900

*d.* "A Simple Direct Method of Determining the Curvatures of Small Lenses." Dr. C. V. Drysdale, 1900-1901

*e.* "Measurement of Aberrations of Photogıaphic Lenses." S. D. Chalmers, 1905-6

*f.* "Lens Aberrations." Prof. S. P. Thompson, 1899-1900

*g.* "The Testing of Photographic Lenses." 1903-4

*h.* "Some Properties of Glass." W. Rosenhain, 1902-3

*i.* "Testing of Optical Instruments." Dr. C. V. Drysdale, 1901-2

# Experimental Optics.

Experiment.

1. **The Wave Theory of Light.** Wave motion and the stroboscope. Refer 59a.

2. **The Rectilinear Propagation of Light.** The Pin-hole Camera. Refer 12, page 5.

3. **The Formation of Shadows.**
   Umbra, penumbra. Eclipses.
   Proof of Law of Inverse Squares.
   Refer 12, page 3, and 16, page 8.

4. **Photometry.**
   Inverse Square Law.
   Light Standards, Photometers.
   Comparison of Light Sources.
   Percentage Light Transmitted
   (Absorption). Refer 30, 42 and 58a.

5. **Verification of the Laws of Reflection.** Refer 12, page 21, or 16, page 31.

6. **Curvatures of Concave and Convex Mirrors.**
   Focal length $= \frac{1}{2}$ radius of curvature.
   Focal power $=$ 2 (curvature of surface.)
   With Optical Bench. Refer 16, page 102.
   With Spherometer. Refer Chapter V.
   Magnification, $m = \frac{r}{u}$. For verification refer 16, page 103. Also refer 59 d., experiment 10.

7. **Verification of the Laws of Refraction.** $n_0 \sin i = n_2 \sin r$.
   Refer 16, page 58, or 12, page 48.
   Both laws of reflection and refraction may be verified with the aid of the instrument called a Goniometer. Refer Chap. IV.

8. **Determination of Refractive Indices.** (Solids and Liquids).

*a.* In Rectangular form — Microscope Method. Refer 16, page 79.
Graphical ,, ,, ,, 67.

*b.* In Prism form — Graphical Method.
Minimum Deviation Method, with Goniometer. Refer Chap. IV.
Normal Incidence Method, Critical ,, ,, Return Path (Abbe) ,, — Refer 59 b.

*c.* In Lens form — Microscopic Method.
Deduction Method. [F=($n_2 - n_o$) ($R_1 - R_3$)]. Refer Chap. V.
Chalmers' Method. Refer 58 b.

*d.* By Total Reflection. For special methods refer 25 and 39.

9. **Focal Power of Lenses.** $F$ (focal power) in dioptres. $= \frac{1}{f}$ (focal length) in metres.

*Convex and Concave.* (Thin lenses).

Graphical Method, employing law of conjugate foci. Refer 12 page, 111.

Drysdale's Method. Refer 59 c.

Deduction Method. [F=($n_2 - n_o$) ($R_1 - R_3$) ]

Neutralization Method. (Contact method).

Symmetrical Planes Method. (Unit magnification, convex lenses only.)

*Thick Convex Lenses.*

Equivalent Focal Power and Nodal Points. Refer Chap. VIII.

Sir Howard Grubb's Method, Moëssard's Method — Refer 7, page 208.

Drysdale's Method ($Cx$ and $Cc$). Refer 59 c.

Magnification $m = \frac{v}{u}$, measurements from optical centre (thin lenses) and respective nodal points (thick lenses in air). For verification arrange conjugate foci method upon optical bench.

10. **Curvature of Small Lenses.** (Short focal length). Refer 59 d.

Spherometer. Abbe's design, made by Zeiss of Jena.

Note that diameter of lens is limited by power.

11. **Use of the Vernier.** Refer 41, page 182.

12. **Testing of Optical Surfaces.**
Reflection Method (direct observation).
Telescope Method.
Contact Method (adhesion).
Newton Ring Method (contact). Refer 12, page 408. Also 58 c. and 59 h.

13. **Lens Testing.** Refer 5, Appendix II., and 7, Chap. V. Also refer 58 d, 59 e, 59 f and 59 g.

14. **Test for Striæ and Strain in Glass.** Refer 5, Appendix II. Also make use of pair of Nicol Prisms to detect strain.

15. **The Consideration of the Nodal and Principal Planes of Lens Systems.** Refer Chap. VIII. Also 58 e.

16. **Eye-Piece Systems of Lenses.** Refer 20, or 12, Chap. X., and 58 f.

17. **The Camera Obscura.** Refer 16, page 140.

18. **Construction of Simple Microscopes.** Refer Chap. IX., 20 and 21.

19. **Construction of Compound Microscopes.** Refer Chap. IX. and 15.
For methods of determining magnification and size of minute objects refer also 15.

20. **The Camera Lucida.** 15 or 16, page 166.

21. **Construction and Magnification of Prism Binoculars.** Refer Chap. IX., also 58 f.

22. **Construction and Magnification of Refracting Telescopes.** Refer Chap. IX., also 58 f.

23. **Construction of Reflecting Telescopes.** Refer 16, page 163, and 20.

24. **The Photographic Camera.**

Refer Lens Testing, Experiment 13.

Determine Size and Value of Stops.

State Principle and Use of Telephoto Lens.

Give Advantages and Disadvantages of Pinhole Photography.

Refer 5, 7, and 59 i.

25. **The Projection Lantern.** Refer 57.

26. **The Spectrometer and Spectrum Analysis.**

Purity, Brightness, and Resolution. Refer 38 and 29.

Determination of the Optical Constants of Glass. Refer 22 and 59 b.

For numerous other experiments refer 29.

To produce a pure spectrum on a screen, refer 16, page 187.

27. **The Polariscope.** For numerous experiments on polarised light, refer 29, 33, and 56.

28. **The Sextant.** Refer 14 and 41.

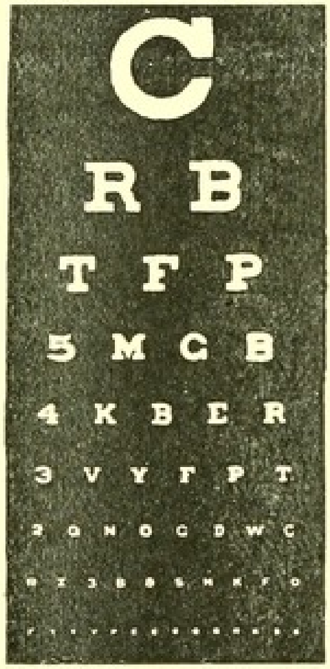
29. **The Theodolite.** Refer 41.

30. **The Eye and Vision.** Refer Chap. VII. Also 16, page 145, and 47 and 52.

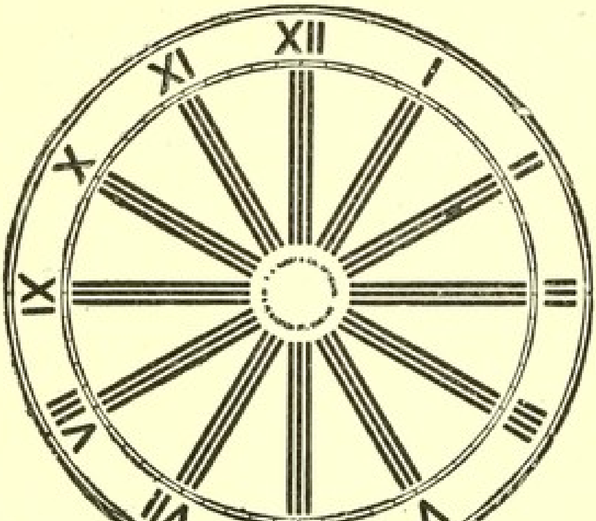
# Index.

*The following list gives the prices at which many useful articles may be purchased :—*

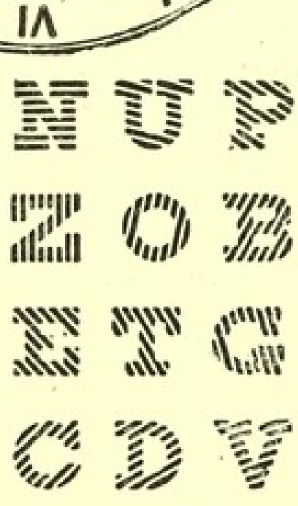
| | | £ | s. | d. |
|---|---|---|---|---|
| 1. | SNELLEN'S TEST TYPE, four different sheets, various letters and arrangements, paper 21 inches by 9 ... ... ... per sheet | 0 | 0 | 6 |
| 2. | SNELLEN'S TEST TYPE, four different sheets, on cards with metal edge to top ... ... ... ... ... per card, 1/- ; per set | 0 | 4 | 0 |
| 3. | SNELLEN'S TEST TYPE, reversed for use with Mirror ... per card | 0 | 1 | 0 |



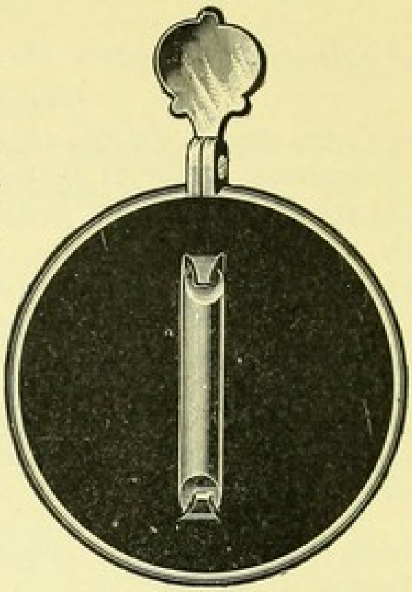

| | | | | |
|---|---|---|---|---|
| 4. | TEST TYPE, white letters on black ground (as illustrated) ... ... ... per card | 0 | 1 | 3 |
| 5. | SNELLEN'S READING TYPE, 8 × 5½, distances in feet ... ... ... ... | 0 | 0 | 6 |
| 6. | JAEGER'S TEST TYPE, folding 14 × 11, distances in centimetres ... ... ... | 0 | 0 | 9 |
| 7. | THE "ORTHOPS" CHART— | | | |
| | Ordinary Type, 40 × 25 ... ... | 0 | 9 | 6 |
| | Reversed Type ... ... ... ... | 0 | 10 | 6 |
| | Portable Chart, 1½ × 12½ ... ... | 0 | 7 | 6 |
| | 2 extra cards for changing type | 0 | 2 | 0 |

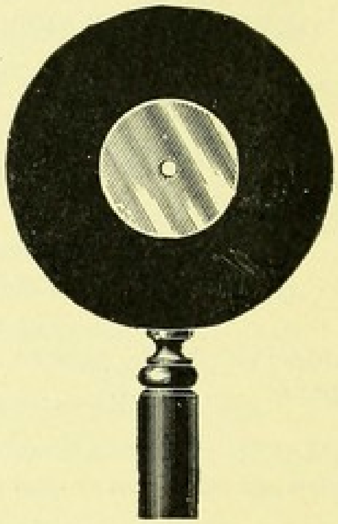



| | | | | |
|---|---|---|---|---|
| 8. | ASTIGMATIC FAN, Large size paper ... ... ... | 0 | 1 | 6 |
| 9. | ASTIGMATIC FAN, Linen, mounted on rollers... ... | 0 | 3 | 0 |
| 9A. | ASTIGMATIC FAN-CARD... | 0 | 1 | 6 |
| 10. | ASTIGMATIC CLOCK FACE (as illustrated), 18 × 18 ... | 0 | 1 | 0 |
| 10A. | ASTIGMATIC STAR ... | 0 | 0 | 6 |
| 11. | TEST TYPE (after Dr. Pray) for Astigmatism, circles ... | 0 | 0 | 6 |
| 12. | TEST TYPE (after Dr. Pray) for Astigmatism, letters (as illustrated) ... ... ... | 0 | 1 | 6 |



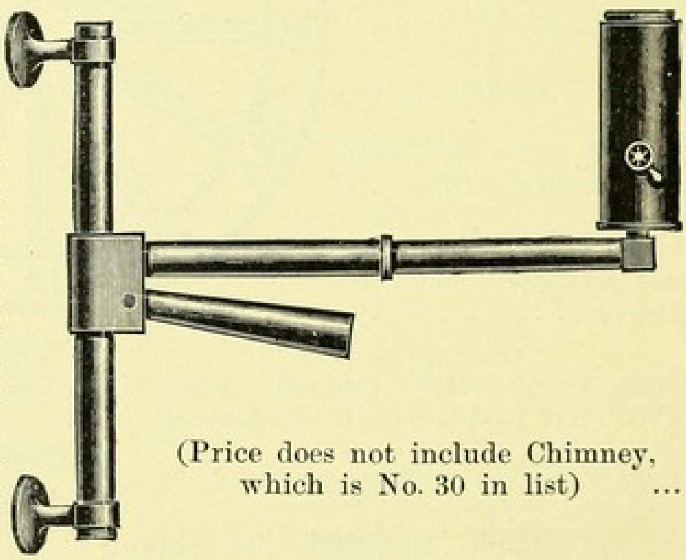
| | | | | |
|---|---|---|---|---|
| 13. | PINHOLE DISC, black bronzed metal, in nickelled steel frame, with ebony handle ... | 0 | 2 | 0 |
| 14. | STENOPAIC SLIT, mounted as above ... | 0 | 2 | 0 |
| 15. | STENOPAIC SLIT, adjustable for various widths ... ... ... ... ... ... | 0 | 3 | 6 |
| 16. | REVOLVING SLITS & PINHOLES of various sizes, with spring clip and groove for holding lens ... ... ... ... .. ... | 0 | 6 | 0 |





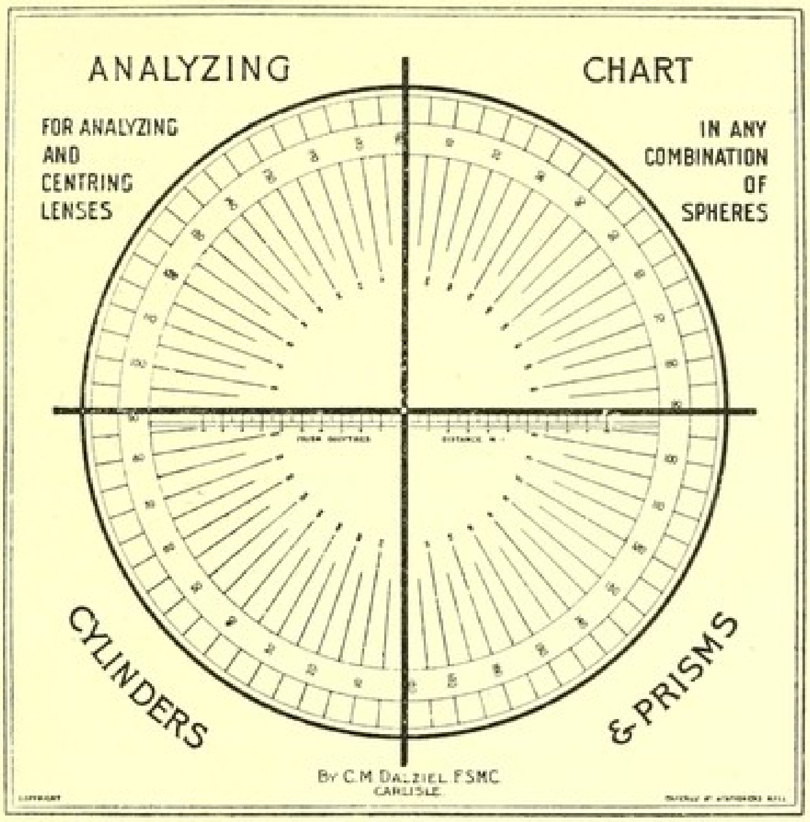
| | | £ | s. | d. |
|---|---|---|---|---|
| 17. | DISC OR LENS HOLDER, nickelled steel, ebony handle ... ... ... ... | 0 | 1 | 0 |
| 18. | TEST RING, with vivid blue, green, or opaque glass ... ... ... ... ... | 0 | 1 | 0 |
| 19. | MADDOX ROD for testing muscular insufficiency (*as illustrated*) ... ... | 0 | 1 | 6 |
| 20. | MADDOX MULTIPLE ROD for testing muscular insufficiency ... ... ... | 0 | 2 | 6 |
| 21. | MADDOX PRISM mounted in Test Ring... | 0 | 2 | 0 |
| 22. | CHROMATIC TEST in nickelled ring, with handle ... ... ... ... | 0 | 1 | 9 |
| 23. | JACKSON'S CROSSED CYLINDER in nickelled ring, with handle ... ... | 0 | 2 | 6 |
| 24. | NEAR POINT MEASURE, boxwood ... | 0 | 5 | 0 |
| 25. | TEST CARDS for Near Point Measure ... | 0 | 0 | 6 |
| 26. | RETINOSCOPE, plain or concave mirror, with ebony handle... ... ... ... | 0 | 2 | 6 |
| 27. | THORINGTON'S RETINOSCOPE ... (*as illustrated.*) | 0 | 4 | 6 |
| 28. | QUEEN'S PRACTISE EYE for practising Retinoscopy ... ... ... ... ... | 0 | 2 | 6 |
| 29. | ASBESTOS RETINOSCOPIC CHIMNEY | 0 | 6 | 0 |
| 30. | ASBESTOS RETINOSCOPIC CHIMNEY, with Iris Diaphragm ... | 0 | 10 | 0 |

30A. MACKINNEY'S OPHTHALMIC BRACKET, with electric or gas fittings. This is very simple in construction, and is self-fixing in position by merely releasing the handle which is attached to an eccentric cam. The bars are telescopic, and the whole is highly finished in dead black

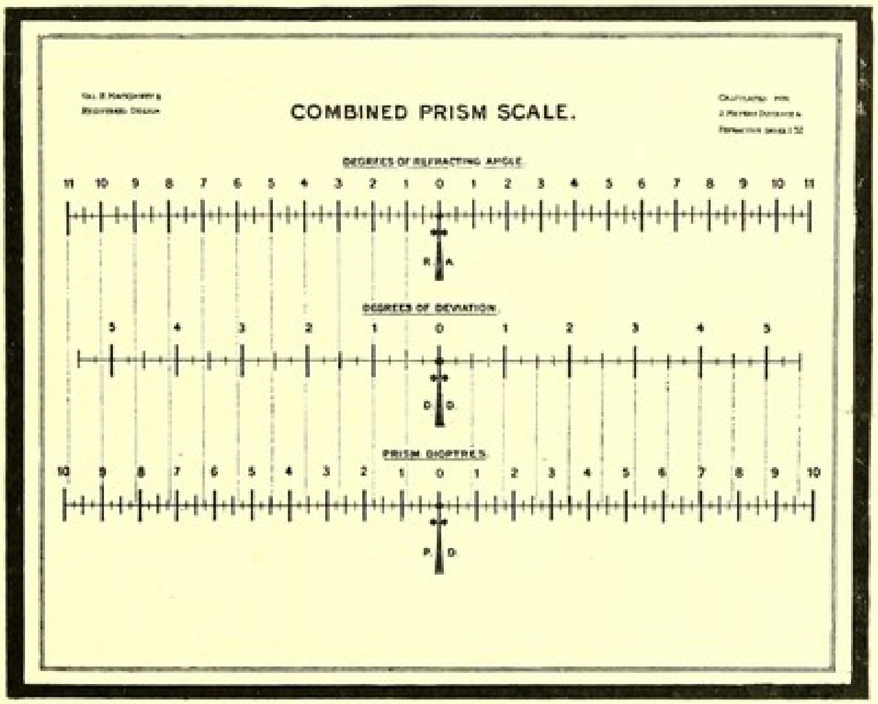


(Price does not include Chimney, which is No. 30 in list) ... 1 12 6

| | | | | |
|---|---|---|---|---|
| 31 | MORTON'S OPHTHALMOSCOPE, best English make ... ... | 2 | 12 | 6 |
| 32. | MORTON'S OPHTHALMOSCOPE, foreign make (to order only) ... | 2 | 7 | 6 |
| 33. | MANIFOLD PRESCRIPTION BOOKS, of 100 pages ... ... ... | 0 | 1 | 6 |

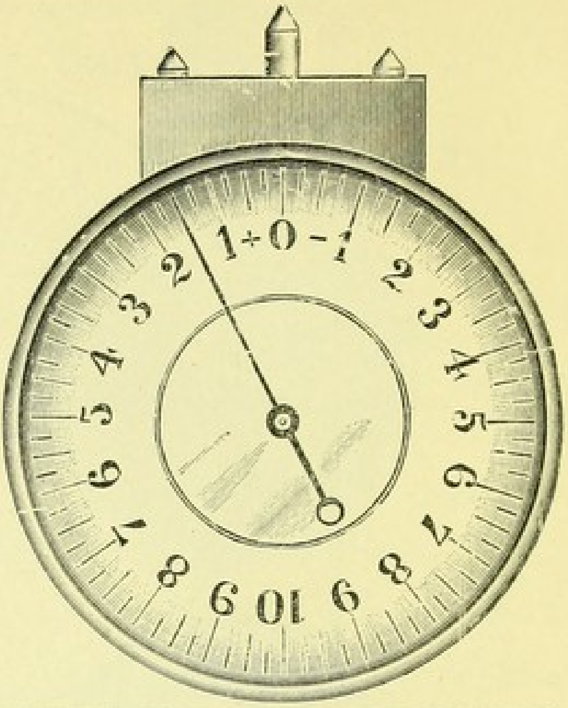



| | | £ | s. | d. |
|---|---|---|---|---|
| 34. | **DALZIEL'S CHART** for analyzing and centering lenses (*as illustrated*) | 0 | 3 | 6 |



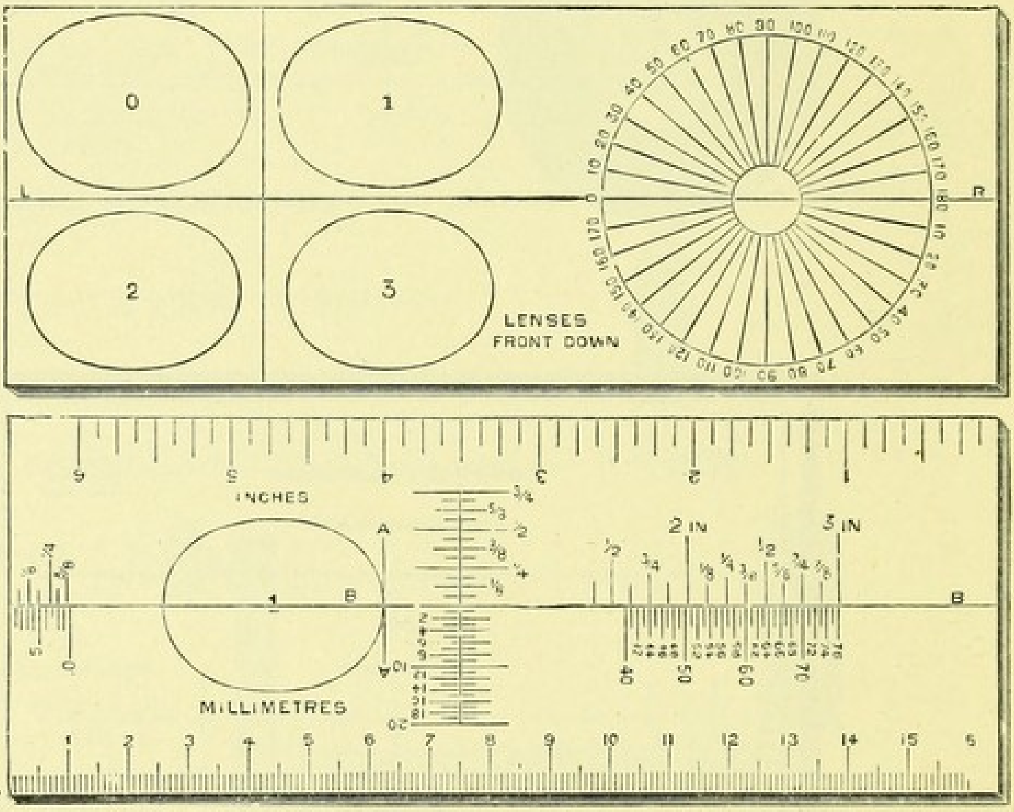

34a. **MACKINNEY'S PRISM CHART**, for testing the prismatic power of a lens in D of R.A, D of D and P.D. Also useful for testing muscular imbalance at a near distance ... ... ... ... ... 0 2 0

| | | s. | d. |
|---|---|---|---|
| 35. | CONDENSING LENS, diameter 2½-in., 2½ or 3 inch focus ... | 1 | 6 |
| 36. | CONDENSING LENS, diameter 2½-in., in strong nickel mount... | 3 | 6 |
| 37. | LENS MEASURE (*as illustrated*), for testing cylindrical and spherical lenses ... ... | 17 | 6 |
| 38. | The SPHEROMETER, a cheaper form of measure for testing spherical lenses... ... ... | 14 | 6 |
| 39. | BOXWOOD SCALE, with centering cross, and reversed protractor for testing axes of cylinders (*as illustrated*) ... | 1 | 6 |



40. SPECTACLE FRONT MEASURE, for testing heights, projection, and centres of spectacles with analyzing scale on reverse. (*as illustrated*)

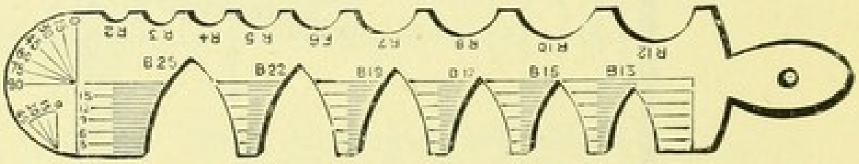
s. d.
2 6



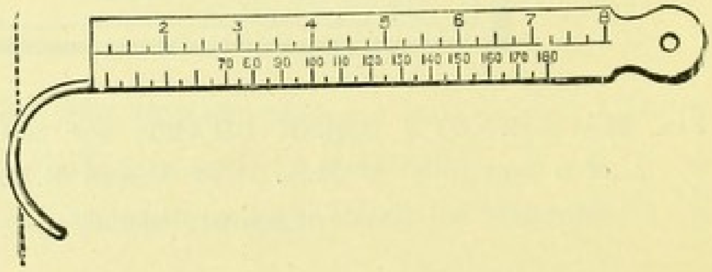

41. NOSE CONTOUR MEASURE.

(*Regd.*) A device by which the contour of any nose can be expressed in three measurements, from which a spectacle bridge can be made exactly to fit. (*As illustrated.*)

| | s. | d. |
|---|---|---|
| Boxwood ... ... | 3 | 6 |
| Celluloid ... ... | 4 | 0 |



42. CURL SIDE MEASURE (*As illustrated*) ... 4 0



£ s. d.

43. FOLDING BOXWOOD METRE RULE, folds into $5\frac{1}{2}$ inches, marked in millimeters and inches ... ... ... ... ... ... 0 1 0

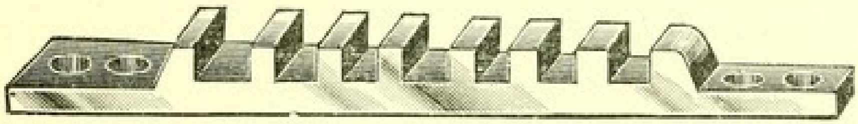
44. SET OF SIX TRIAL FRAMES, centres $2\frac{1}{8}$ to $2\frac{5}{8}$ inches, with six different bridges, ticketted with details on each. Empty. ...The set 0 9 0

45. SET OF SIX TRIAL FRAMES, centres $2\frac{1}{8}$ to $2\frac{1}{2}$ inches, with six different bridges, measurements on ends in gilt letters. Empty. The set 0 12 0

46. SET OF SIX TRIAL FRAMES, glazed, measurements on ends in gilt letters ... ... ... ... ... ... ... ... ... 0 18 0

47. SET OF TWELVE TRIAL FRAMES, centres 2 inch to $2\frac{5}{8}$, in polished walnut case, with separate division for each centre, measurements marked on frame in gilt letters ... ...The set 1 7 0

48. JOINT HOLDER *(as illustrated)* for holding spectacle joints while screw is turned, preventing injury to frame ... ... 0 0 9

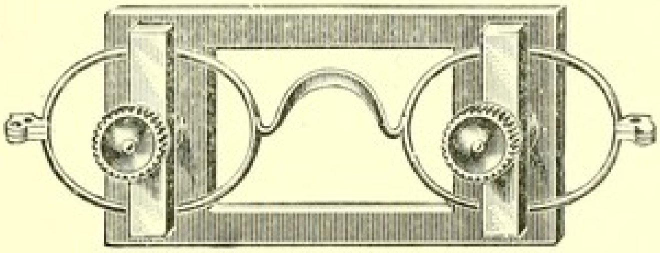


49. SCREW DRIVERS (OPTICIANS'), two sizes, for ordinary and fine joints, with strong handle ... ... ... ... ... each 0 0 6

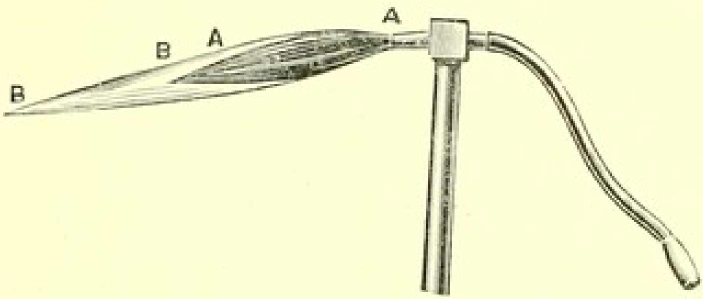
50. SCREW DRIVERS, high quality, two sizes, one for Gold work, recommended... ... ... ... ... ... ... ... each 0 1 3

50A. SCREW SLOTTER, for cutting new slots in screw heads... ... 0 0 8

51. BRASS SOLDERING CLIP (*as illustrated*) for holding spectacle frame during repair. 0 1 6

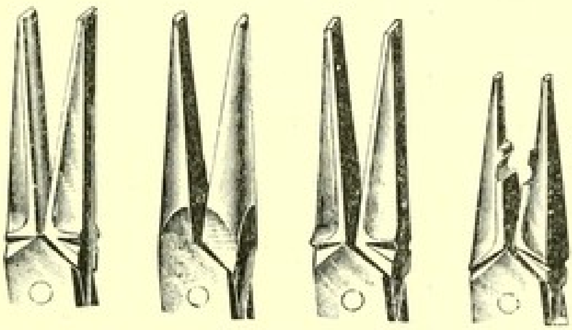


52. BLOW PIPE, flexible *(as illustrated)* very useful for repairs. 0 2 0




53. SET OF OPTICIANS' PLIERS *(as illustrated)*, consisting of one pair each, flat nose, half-round nose, sharp nose, and shaped and slotted nose, for holding spectacle or eyeglass joints, well nickel-plated. The set. 0 6 0

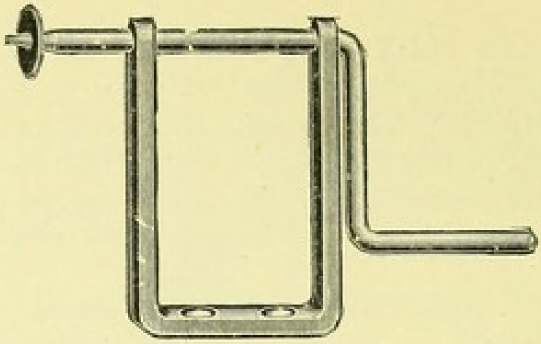
s. d.
Per pair ... 1 6



54. The same SET OF PLIERS, in polished case, complete with flat file, three-square file, and two screw drivers (one broad and one narrow), forming a very useful set of tools for an optician ... per case 0 13 6

55. Or with best quality drivers ... ... ... ... ... ... ... 0 15 0

£ s. d.

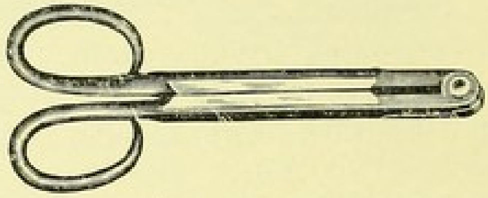


55A. MILLED WHEEL, for clearing eyewires of solder or rough parts. Made to screw on work bench or fit in vice. ... 0 2 6

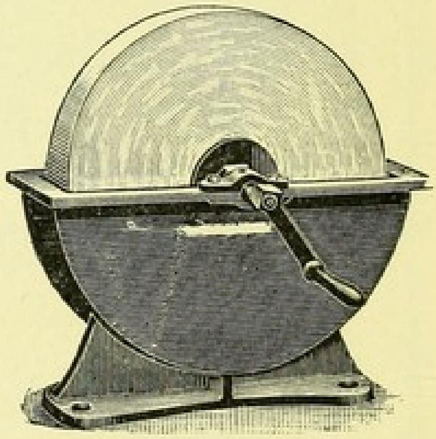
56. GAUGE RINGS (American make, guaranteed accurate) for sizing interchangeable lenses ... ... ... ... ... ... ... 0 1 6

57. GAUGE PLATES (American make, guaranteed accurate) for testing frames ... ... ... ... ... ... ... ... ... 0 3 6

58. TOURMALINE PINCETTE for testing pebbles ... ... ... ... 0 5 0



59. SHANKS OR NIPPERS for edging Spectacle lenses, previous to grinding ... ... 0 1 6



60. BILSTON OR ADAMANT GRINDSTONE, (*as illustrated*) iron trough—

| 12 x 2 | 14 x 2½ | 16 x 3 | 18 x 3 | 20 x 3 |
|---|---|---|---|---|
| 12/- | 14/6 | 17/- | 26/- | 31/- |

Larger sizes quoted for.

61. CRAIGLIETH GRINDSTONE, in iron trough—

| 12 x 1 | 14 x 1 | 16 x 1 | 18 x 1 | 20 x 1 |
|---|---|---|---|---|
| 19/- | 23/- | 28/6 | 40/- | 48/6 |

## TRIAL CASES.

POLISHED WALNUT CASE, with 18 convex and 18 concave cylinders, ·25 to 5·50 ... ... ... ... ... ... ... ... ... £ s. d. 1 1 0

POLISHED MAHOGANY CASE, detachable lid, lined with satin.

Contents:
32 pairs spherical c/x ·25 to 20 D.
32 ,, ,, c/c ·25 to 20 D.
6 plano tinted glasses
2 discs (pinhole and slit)
1 single cell trial frame ... ... ... ... 2 10 0

POLISHED MAHOGANY CASE, detachable lid, lined with satin, good lock.

Contents:
30 pairs spherical c/x ·25 to 20 D.
30 ,, ,, c/c ·25 to 20 D.
18 single cylinders c/x ·25 to 7 D.
18 ,, ,, c/c ·25 to 7 D.
8 plano tinted glasses
4 discs stenopaic slit, pin-hole, etc.)
1 single cell trial frame
1 two cell graduated trial frame ... ... ... ... 3 18 0

POLISHED WALNUT CASE, fixed lid, contents as above ... ... ... 4 4 0

£ s. d.

**POLISHED MAHOGANY CASE**, detachable lid, lined satin, brass fittings and good lock, all lenses mounted in test rings with pierced handles.

Contents:
- 30 pairs spherical c/x ·25 to 20 D.
- 30 „ „ c/c ·25 to 20 D.
- 18 single cylinders c/x ·25 to 7 D.
- 18 „ „ c/c ·25 to 7 D.
- 12 prisms $1^{\Delta}$ to $12^{\Delta}$
- 12 plano blue and smoke glasses
- 1 each plano red, white, green and yellow glasses
- 4 discs (stenopaic slit, pin-hole, etc.)
- 1 single cell trial frame
- 1 two cell graduated trial frame ... ... 7 0 0

**POLISHED MAHOGANY CASE**, detachable lid, lined satin, brass fittings and good lock.

Contents:
- 32 pairs spherical c/x ·25 to 20 D.
- 32 „ „ c/c ·25 to 20 D.
- 18 pairs cylinders c/x ·25 to 7 D.
- 18 „ „ c/c ·25 to 7 D.
- 12 prisms $1^{\Delta}$ to $12^{\Delta}$
- 12 plano blue and smoke glasses
- 1 each plano red, green, amber, and white
- 4 discs (stenopaic slit, pin-hole, etc.)
- 1 chromatic test
- 1 single cell trial frame
- 1 two cell graduated trial frame

All lenses mounted in fine test rings, with pierced handles ... ... 8 8 0

**POLISHED MAHOGANY CASE**, highly finished with bevelled plate glass top, removable lid, tray for lenses to lift out, good fittings and lock.

Contents:
- 30 pairs spherical c/x ·25 to 20 D.
- 30 „ „ c/c ·25 to 20 D.
- 18 single cylinders c/x ·25 to 7 D.
- 18 „ „ c/c ·25 to 7 D.
- 12 prisms $1^{\Delta}$ to $12^{\Delta}$
- 12 blue and smoke glasses
- 1 each plano ruby, amber, green, and white
- 4 discs
- 1 single cell trial frame
- 1 double cell graduated trial frame

All lenses in fine test rings, with pierced handles ... ... ... 8 0 0

**POLISHED MAHOGANY CASE**, highly finished, with bevelled plate glass top, removable lid, tray for lenses to lift out, good fittings and lock.

Contents:
- 32 pairs spherical c/x ·25 to 20 D.
- 32 „ „ c/c ·25 to 20 D.
- 18 pairs cylinders c/x ·25 to 7 D.
- 18 „ „ c/c ·25 to 7 D.
- 12 prisms $1^{\Delta}$ to $12^{\Delta}$
- 12 plano blue and smoke glasses
- 1 each plano red, amber, green, and white
- 5 discs
- 1 chromatic test
- 1 Maddox rod
- 1 single cell trial frame
- 1 double cell graduated trial frame ... ... ... 9 16 0

**LEATHER COVERED**, finely finished case, with velvet and leather fittings, all lenses mounted in tempered spun rims, and nickel plated.

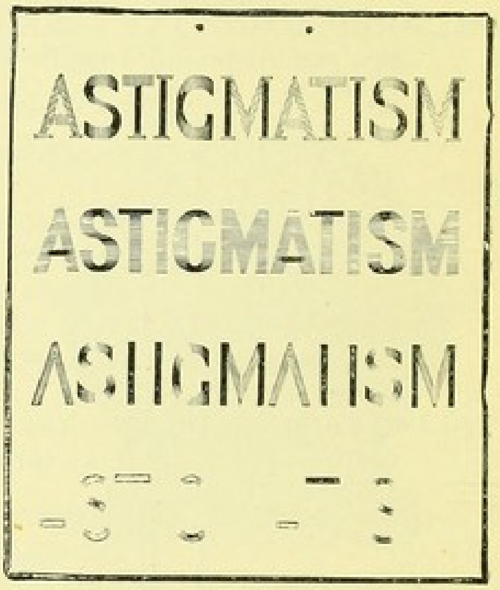
Contents:
- 32 pairs spherical c/x ·12 to 20 D.
- 32 „ „ c/c ·12 to 20 D.
- 20 pairs cylinders c/x ·12 to 6 D.
- 20 „ „ c/c ·12 to 6 D.
- 10 prisms $1^{\Delta}$ to $10^{\Delta}$
- 3 coloured glasses
- 1 Maddox rod
- 6 stenopaic and occluding discs
- 1 double cell graduated trial frame
- 1 single cell trial frame ... ... ...... ... ... 9 0 0

or with best standard trial frame, adjustable sides ... 10 0 0

# WORKS ON OPTICS.

| | | £ | s. | d. |
|---|---|---|---|---|
| HARTRIDGE—"Refraction of the Eye" | ... ... ... ... | 0 | 5 | 0 |
| "The Ophthalmoscope" | ... ... ... ... | 0 | 4 | 0 |
| KEYSTONE—"The Optician's Manual" | ... ... ... ... | 0 | 6 | 3 |
| "The Optician's Manual Supplement" | ... ... ... | 0 | 3 | 4 |
| "Skiascopy" | ... ... ... ... ... ... | 0 | 4 | 2 |
| TAYLOR—"The Manipulation and Fitting of Ophthalmic Frames" | ... | 0 | 4 | 0 |
| THOMPSON—"Optical Tables and Data" | ... ... ... ... | 0 | 6 | 0 |
| LAURANCE—"The Eye" (Physiological and Anatomical) | ... ... | 0 | 3 | 6 |
| "General and Practical Optics" | ... ... ... ... | 0 | 10 | 6 |
| TAYLOR & MACKINNEY—"The Key to Sight Testing" (2nd Edition) | ... | 0 | 10 | 6 |

# CHARTS FOR ADVERTISING PURPOSES.

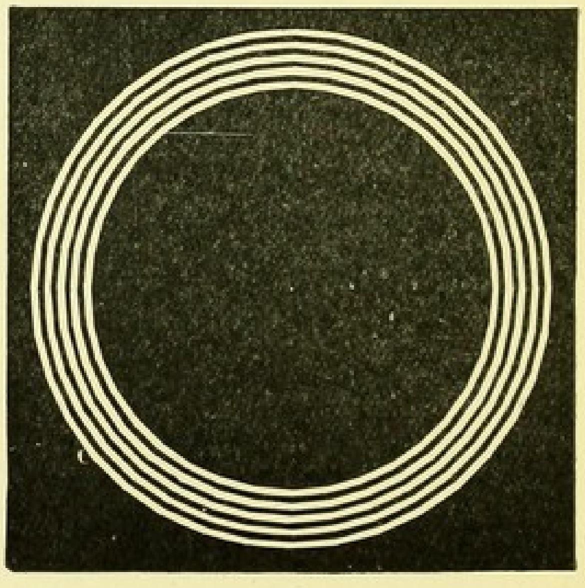



This Chart illustrates the relative value of vertical and horizontal lines in deciphering letters, and provides a good means of explaining to clients the significance of astigmatism.

The first and second lines illustrate the difference between a word having the horizontal components of the letters blurred, and one in which the vertical positions are indistinct.

In the third only vertical lines are shown, and in the fourth horizontal.

1/- each.



A set of three charts, consisting each of a series of tangents (at 10° intervals) to concentric circles. These charts are printed in black, red, and blue, and are delicate tests for irregular astigmatism, besides illustrating (by the different extent of the blurring) the chromatic aberration of the eye.

1/- each ; 2/6 the set of three.