THE PRINCIPLES OF REFRACTION IN THE HUMAN EYE

BASED ON THE LAWS OF CONJUGATE FOCI

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IN THE HUMAN EYE

BASED ON THE LAWS OF CONJUGATE FOCI

BY

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AUTHOR'S PREFACE

This elemetary treatise on the refractive function of the eye is an elaboration, with much added matter, of an article written by the author and first published in the *American Journal of Ophthalmology*, in 1903. It is now offered in a permanent form under the conviction that the undergraduate in medicine and the beginner in ophthalmology would find their studies along these lines much facilitated by some such compact statement of fundamental principles, presented in a simple way and devoid of the complicated mathematics usually found in treatises claiming to deal fully with the subject.

It might be objected that even the amount of optics here given is by no means necessary to the undergraduate, who will not need such knowledge unless he proposes to follow ophthalmology as a special work in practice. To this the pertinent reply is that every graduate going out from a medical school should have an intelligent understanding of all the functions of the human economy, with a fair knowledge of the methods of investigating into their normal and commoner pathological states. Of the function of such an important organ as the eye he should certainly have, as a part of his general medical culture, such knowledge as shall enable him to comprehend the laws which underlie its primal and essential purpose as an optical instrument and the means usually employed in testing this function. Furthermore if this knowledge does not come to him during the collegiate course it is not likely to be acquired after the responsibilities of general practice have been entered upon.

It may be contended further that such primary teaching should not be necessary in our medical schools, with the advanced knowledge now required of our matriculants.

It is a matter of regret, however, that few undergraduates come to their studies in ophthalmology with such sufficient groundwork in physics as shall allow the professor to assume that their knowledge will enable them to comprehend readily even the simple laws involved in the study of the eye as a collecting refractive system for the formation of images upon the retina. Even those who have gone through the ordinary university or college training have, for the most part, to be taught what optics is necessary from the beginning. It is hoped, and indeed expected, that this will be remedied in the near future ; but under any circumstances it will be necessary for the instructor to go over some of this ground in the application of general optical laws to the special conditions found in the eye, in order to make his teaching consecutive and thorough.

Since all the phenomena pertaining to the object and its image formed by the refracting system of the eye rest upon, and are only to be explained by, the laws of conjugate foci, it would seen that if we could first make our students thoroughly familiar with these simple primary laws by diagrams and models, together with the elementary equations involved, it would much facilitate their understanding of all the practical problems which arise in the study of the eye as an optical instrument. This, I am fully aware, may be considered as repeating much that has been said over and over again in our text-books ; but we must not forget that what has become a very familiar way of looking at the subject to us is still a very unfamiliar way to the average student, who has so many other things to engross his attention ; and while it is not possible to make a royal road to knowledge, we may still be able to render the path comparatively smooth and easy to the comprehension of fundamental principles.

The conjugate focus is, of course, mentioned often in our treatises and text-books dealing with the optics of ophthalmology, but it is always in a subsidiary way and incidentally, and not as the bed rock upon which the whole fabric of refraction as it pertains to the formation of images rests. It has been borne in upon me, from a long experience in the teaching of medical students, that if we are able to seize and impress upon them some central fundamental law back to which all the phenomena involved can and must be referred for substantiation, we will have rendered their understanding much more logical and therefore easier. Such a fundamental law in refraction I consider the conjugate focus to be.

Applied in this way we have found these laws to be of the greatest assistance in solving all the problems involved in the determination of the optical properties of the eye; but we believe they have a special value in unraveling the intricacies inherent in the complex optical system that lies at the basis of that method of examination known as skiascopy. Much of the confusion that prevails regarding the optical theory of that method is due to the neglect of a proper application of these laws to the requirements of this special case.

An application of these primary principles to all of the details of individual conditions cannot, of course, be attempted in an elemetary treatise such as this claims to be.

For valued assistance in the construction of the diagrams, all of which are original, which illustrate the text of this work, I wish to acknowledge my indebtedness to the well-known scientific and technical skill of Mr. C. F. Prentice, M. E., of New York.

Washington, D. C., October 1, 1904.

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THE PRINCIPLES OF REFRACTION IN THE HUMAN EYE

BASED ON THE LAWS OF CONJUGATE FOCI

CHAPTER I.

The Essential Functions of Optical Systems.

a. The prime function of all optical apparatus is to change the course of rays of light from their original to some other definite direction.

b. In ophthalmic optics we have principally to deal with those apparatus which act upon rays of light in such a manner that those emanating from each single point of an illuminated *object* shall be directed, after being acted upon, towards another corresponding point, or assume a direction as if they had come from such a corresponding point. This corresponding point is called the *image* of the point in the object.

The inherent properties of these optical apparatus are such that rays of light follow the same path going and coming, so that if the object should occupy the position of the image, the image would occupy the position of the object.

c. The points to be considered in any study of the activities of such apparatus are: the essential properties of the apparatus itself, including the curvature of the surfaces and, in the case of refracting apparatus, the index of refraction of the medium, and the positions of the object and image in relation to the apparatus—which positions are called the *foci* of the apparatus.

d. When any one of the essential properties of the apparatus undergoes a modification, the relation of one or both foci to the apparatus is changed. So long, however, as the essential properties of the apparatus remain fixed, any change in the position of one focus is accompanied by a corresponding change in the position of the other focus, and in accordance with certain definite laws. On account of this intimate, inseparable connection, they are called *conjugate foci* (from *conjugare*, to join together).

e. Rays of light emanating from luminous objects are divergent, and never become mathematically parallel. The apparent divergence in small bundles of rays, such as are utilized in ordinary optical systems, however, becomes less and less as the distance from the luminous body increases, until a point is reached where the rays may, for all practical purposes, be considered as parallel, and they so continue indefinitely.

The point at which parallelism of the rays, or an infinite distance, as it is called, begins, varies with the power of the optical apparatus. It has been found that when one conjugate focal distance has become 200 times greater than the other, a state of parallelism of rays for that particular apparatus sets in, and a farther removal of this conjugate focus has no perceptible effect on the position of the other. Thus, when a removal of one conjugate focus (the object) beyond a distance of 200 feet from the apparatus ceases to influence the position of the other conjugate focus (the image), we know that 200 feet is the point at which parallelism for the rays, or infinity, begins for that special apparatus, and that the other conjugate focus is at one foot.

This conjugate focus for infinity, or parallel rays, has been denominated the *principal focus*, to distinguish it from the conjugate foci for divergent and convergent rays.

For the refracting apparatus of the standard or schematic *human eye*, the principal focus for parallel rays, in accordance with this law, is accepted to be 20.7 mm., or about one inch, with infinity at 200 inches, 16.66 feet or 5 meters.

CHAPTER II.

The Laws of Conjugate Foci in Refracting Systems.

The laws of conjugate foci, in all refracting systems, so far as they pertain to the relative positions of the image and the object, are :

1. The rays of light follow the same path going and coming, thus making it possible for the object and image to replace each other.

2. The two foci, representing the object and the image, always move in the *same* direction. If one moves to the right the other moves to the right also, and *vice versa.**

3. When the object (or image) is situated outside of the principal focus (for parallel rays), the image (or object) is found on the opposite side of the refracting system, and is positive, real and inverted.

4. When the object (or image) is situated within the principal focus (for parallel rays), the image (or object) is found on the same side of the refracting system and is virtual and erect ; that is, the rays proceed as if coming from a real object situated at that point.

The operation of these laws for a single refracting surface is shown diagrammatically in Figs. 1 and 2, where we will follow the movements of the object emitting the rays, beginning at infinity on the left and going to infinity on the right.

The object in Fig. 1 being at infinity on the left and sending out parallel rays a a, has its conjugate focus at the *posterior principal focus* a_1, on the opposite side of the refracting system *O.* Infinity and the principal focus are therefore conjugate foci, and both are positive and real.

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* In *reflecting systems*—convex and concave mirrors—the two foci move in *opposite* directions.

When the object is advanced to the right and is found at a finite distance *b*, the conjugate focus, as represented by the image, recedes in the same direction to b_1, and being still on the opposite side of the refracting system is positive, real and inverted.

When the object is still farther advanced till it reaches

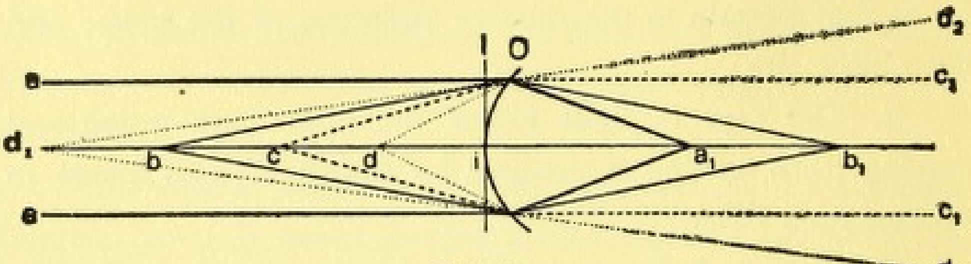

Fig. 1

c, the conjugate focus recedes to infinity on the right, as represented by the parallel rays c_1 c_1, *c* becoming the *anterior principal focus.* The image is now infinitely large, but being on the opposite of the refracting surface *O*, is still inverted, positive and real.

When the object is advanced to *d*, falling inside the anterior principal focus *c* (for parallel rays), the conjugate focus with its image passes beyond infinity, the rays becoming divergent, d_2 d_2, as if they came from a real object d_1, situated on the same side of the refracting medium as the object *d*. The conjugate focus d_1 is, therefore, negative and the image is erect and virtual.

As the object *d* is still farther advanced to the right towards the refracting surface *O*, the conjugate focus and image d_1 also moves in the same direction, but at a more rapid rate, until the image finally overtakes the object, and both are merged into one at *i* on the surface, which is the principal plane of the system. The object still proceeding in the same direction, passes to the *right* of the refracting surface *O* and finds itself, say, at *d* (Fig. 2) and within the principal posterior focus c, for parallel rays c_1 c_1, coming from the left. The conjugate focus will then also have moved to the right, according to rule 4, and be found on the same side of the refracting surface O at d_1; the conjugate focus is, therefore, negative and the image is erect and virtual, the rays d_2 d_2 proceeding with a divergence as if they came from a real object situated at d_1.

The object still receding to the right from the refracting system, its conjugate focus and image likewise recede in the same direction, but at a more rapid pace, until the object arrives at c, the posterior principal focus (for parallel rays c_1 c_1), when the conjugate focus will again find itself at infinity on the left, following the parallel rays c_1 c_1 and become real, positive and inverted.

Proceeding with the object then to b, the conjugate focus advances from infinity on the left to the finite distance b_1, the image being real and inverted. A farther advancement of the object along the axis to the right is accompanied with a corresponding advance of the conjugate focus and image

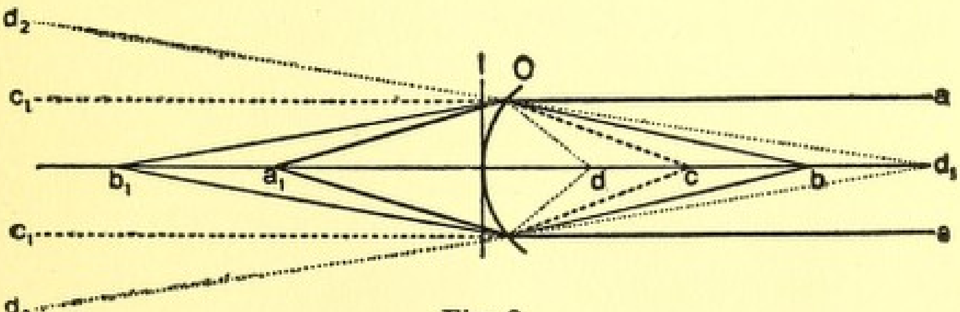

Fig. 2

until the object reaches infinity (parallel rays a a), when the image is found at a_1, the anterior principal focus of the system.

Comparing these two diagrams, it will be seen that one is just the reverse of the other, and that the object could, with the same results, have been started from the right and moved towards the left; demonstrating rule 1 of the laws of conjugate foci, that the image and the object can replace each other.

In studying the phenomena of the dioptric apparatus of the eye, we shall find that the laws of conjugate foci admirably lend themselves to a satisfactory elucidation of all the problems which it is necessary for us to solve in practice.

CHAPTER III.

Application of the Laws of Conjugate Foci to the Human Eye.

In making an application of these laws to the human eye we shall, for the sake of uniformity and simplicity, consider *that one conjugate focus is permanently fixed on the retina; for it is there, for the purposes of best vision, that a distinct image must always be formed.* The other conjugate focus, which is the varying one, will then be found at the point where the object is situated, which emits the rays going to form the distinct retinal image. This position of the object has been called the "far point" of the eye.

The far point and the retina are, therefore, always at conjugate foci.

Whenever the retina changes its relations to the principal focus of the refractive system, either by an increase or decrease in the refractive power of the eye itself or an alteration in the distance between the retina and the refractive system; in other words, whenever the conjugate focus, represented by the retina, alters its position relative to the posterior principal focus for parallel rays, so also must the other conjugate focus change its position relative to the refractive system, and always in keeping with the laws of conjugate foci, as above explained.

It has been agreed by convention, to adopt one single position of the retina in respect to the refracting system of the eye as a standard with which every other position shall be compared. This accepted place is the posterior principal focus of its refracting system, the retina then lying at the focus for parallel rays coming from infinity.

In this standard eye, the retina and infinity (from which parallel rays proceed) are at conjugate foci, and the optical condition is called Emmetropia (E.).

As compared with this, the only other possible positions of the retina are two: First, that in which the retina lies *outside the principal focus* for parallel rays, which is called *Myopia* (*M.*), and, second, that in which it lies *in front of the principal focus*, called *Hypermetropia or Hyperopia* (*H.*).

Every eye, in all possible optical states, static or dynamic, must fall in one of these three categories.

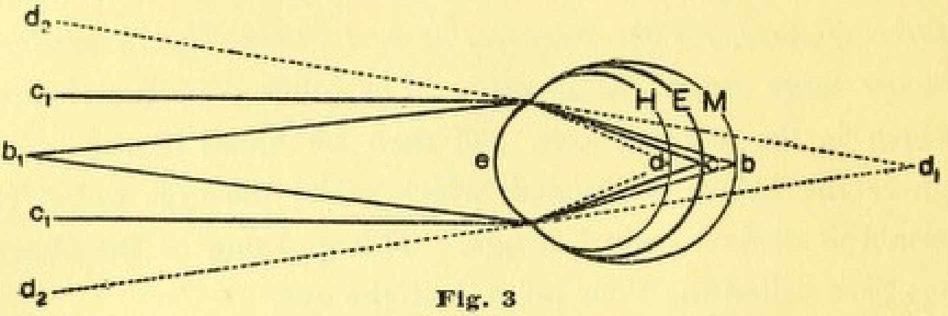

Fig. 3

It must be borne in mind, in these studies of what is called the *static* refraction, that is, a state of optical repose in which there is nothing added to the refraction by any effort of the eye itself, that the absolute refracting power of the eye is not, as might readily be supposed, the matter at issue at all. We have solely to do with the relative positions of the retina and its conjugate focus. It is a fact, which has been demonstrated innumerable times, that the actual refracting power in myopic conditions, where the retina is behind the posterior principal focus, may be lower than in emmetropia, while in hypermetropia, in which the opposite condition prevails, it may be higher.

Let us now apply the laws of conjugate foci as above stated to an explanation of the phenomena manifest in these refractive conditions of the eye.

In Fig. 3 is shown the position of the retina in relation to the refracting system *e* in each of the categories of *H.*, *E.* and *M.*, as they are ordinarily represented, in which the retina *c* lies at the posterior principal focus for parallel rays (emmetropia, *E.*), while *b* gives the position of the retina, behind the posterior principal focus (myopia, *M.*), and *d* its position in front of it (hypermetropia, *H.*).

As, however, according to our manner of considering the subject, *the retina is regarded as occupying a fixed position, we can very properly assume that the differences in distance between it and the refracting system in the different categories is attained by a variation in the position of the refracting system itself, as shown in Fig. 3a, in which H, E and M represent the positions of the refracting system in relation to the retina, d, in each of these categories respectively.*

Emmetropia (*E.*). This proposition is simple, since the standard optical eye finds one conjugate focus at the retina (*c*, Fig. 3, and *d*, Fig. 3a), and the other, the far point, at infinity with parallel rays c_1 c_1. As the object and the image are on opposite sides of the refracting system, the image is real and inverted.

Myopia (*M.*). When the retina is located beyond the principal focus of the refracting system, *c*, Fig. 3, and finds

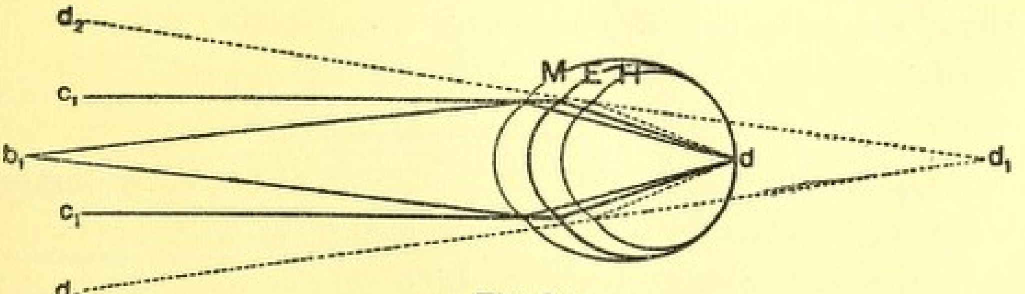

Fig. 3 a

itself at *b*, or when *M*, Fig. 3a, is separated by more than the principal focal distance from *d*, its conjugate focus, representing the far point, will be found at b_1. These conjugate foci, at the retina and b_1, being on opposite sides of the refracting system, are positive. In accordance with law 2 of conjugate foci, in proportion as *b*, Fig. 3, recedes to the right or *M*, Fig. 3a, advances to the left, the conjugate focus b_1

advances toward *e*, and if such a thing were physically possible and the retina *b*, Fig. 3, could remove to infinity on the right, or *d*, Fig. 3a, be separated from *M* by an infinite distance, b_1 would be found at the anterior principal focus of the refracting system, as at *c*, Fig. 1.

The *degree or amount* of *M.*, that is, the difference between *E.* and *M.*, is expressed by the difference between the parallelism of the rays, c_1 c_1, Figs. 3 and 4, in *E.*, and the divergence of the rays from b_1 in *M.* This difference is represented by the power of the convex lens *l*, as shown in section in Fig. 4, which gives parallel rays *a* coming from the right a positive focus at a finite distance b_1 on the left. For example, if the far point be at b_1 10 inches (25 cm.) in front of the refracting system, it means that a lens of 4 D. positive refracting power, with a focal distance of 25 cm., is necessary to bring parallel rays *a* from infinity on the right to a focus at b_1, the far point of *M*, on the left. It is commonly considered that the myopic eye *exceeds* the emmetropic eye in its refraction by that amount. What, however, is really expressed is the difference in the positions of their two conjugate foci or far points.

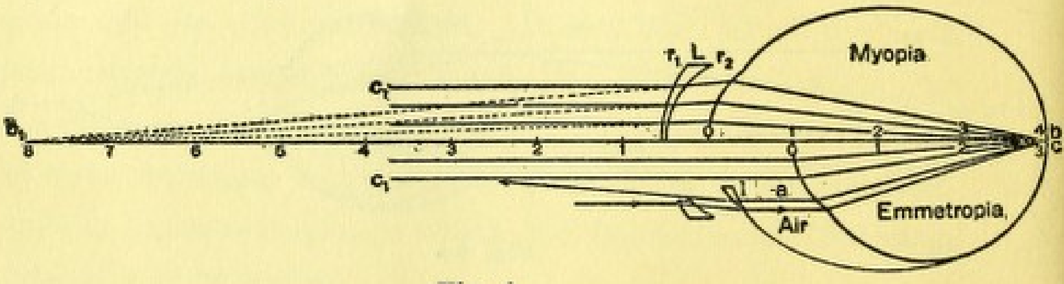

Fig. 4

When it is desired *to convert M. into E.*, that is, to bring infinity on the left in front, c_1 c_1, into conjunction with the retina *b*, it is necessary to render the parallel rays c_1 c_1 artificially divergent, as if they came from the conjugate focus b_1. This is accomplished by the concave lens *L* placed in the path of these parallel rays, whose negative focus is at b_1. In the example taken, with the far point at 10 inches (25 cm.), a dispersive (concave) lens of 4 D. power placed close to the eye would give the parallel rays c_1 c_1 coming from infinity a divergence as if they came from its negative focus b_1, 25 cm., in front of it. The focus conjugate to the retina will then have been removed from 10 inches

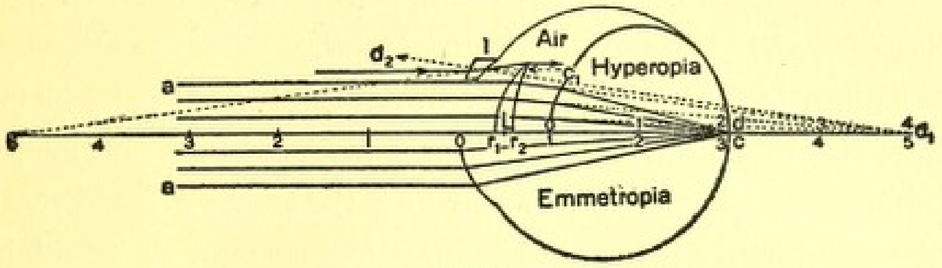

Fig. 5

in front of the eye back to infinity, the myopia expressed and measured by + 4 D. will have been neutralized by the — 4 D. lens, and emmetropia will prevail.

Hypermetropia (*H.*). In this optical state the one conjugate focus, represented by the retina, lies at d, in front of the posterior principal focus of the refracting system, the other being at infinity (Fig. 3); or H and d, Fig. 3 a, are separated by less than the posterior principal focal distance. Both conjugate foci d and d_1 consequently fall on the same side, to the right of the refracting system, in accordance with rule 4 of conjugate foci, and the image d_1 is negative.

Since, in accordance with rule 1 of conjugate foci, it is immaterial in which direction we follow the rays, we will assume, for a simpler demonstration, that the rays start from the conjugate focus on the retina at d (Figs. 3 a and 5).

The rays, then, coming from the retina at d do not, after refraction by the dioptric apparatus, become convergent as in *M.*, nor parallel as in *E.*, but are divergent, d_2 d_2, Figs. 3 a and 5, as if they came from the conjugate focus at d_1, which is the conjugate focus of d and the far point for that particular optical state.

As d recedes from the refracting system *H.* and moves towards the posterior principal focus c, as shown in Fig. 2, the conjugate focus d_1 also recedes in the same direction, but more rapidly, so that by the time d has reached the posterior principal focus of the system, d_1 has gone to infinity, and emmetropia prevails, since the retina and infinity are once more at conjugate foci. The hypermetropia has therefore disappeared. When, on the other hand, d moves away from c and towards the refracting system *H,* d_1 also advances in the same direction to the left, but at a more rapid rate, and if such a thing were physically possible and the retina d could reach *H,* the principal place of the refracting system, d_1 would be found there also, and they would be superposed.

The *degree or amount of H.*, that is, the difference between the far point of *E.* and the far point of *H.* in this case is expressed by the power of the concave lens l, Fig. 5, which would render parallel rays c_1 coming from infinity on the right, divergent, d_2, as if they came from d_1. The far point of the system and the focus of the lens would then coincide at d_1 and both be negative. If, for example, d_1 be 20 inches (50 cm.) behind the refracting system, the difference is expressed by a concave (dispersing) lens l, which, placed close to the cornea, will have a negative focus of 20 inches behind the refracting system.

As commonly stated, the hypermetropia has 2 D. of refraction *less* than *E.* but what is really expressed is the difference in the position of the conjugate foci or far points in the *H.* and *E.*

When, however, it is desired to *correct the H.* and bring the far point d_1 back to infinity, it is necessary to render the divergent rays d_2 d_2 coming from the conjugate focus d_1, parallel. This is accomplished by a convex lens *L*, of 2 D., which, placed close to the cornea, has its focus for parallel rays a a at d_1. When this is done rays divergent from d_1 passing through this lens, will be made parallel and the conjugate focus or far point will be removed from d_1, to infinity, *a a*, on the left, and emmetropia will prevail. The hypermetropia expressed and measured by — 2 D. has been neutralized by the + 2 D. lens and is said to be corrected.

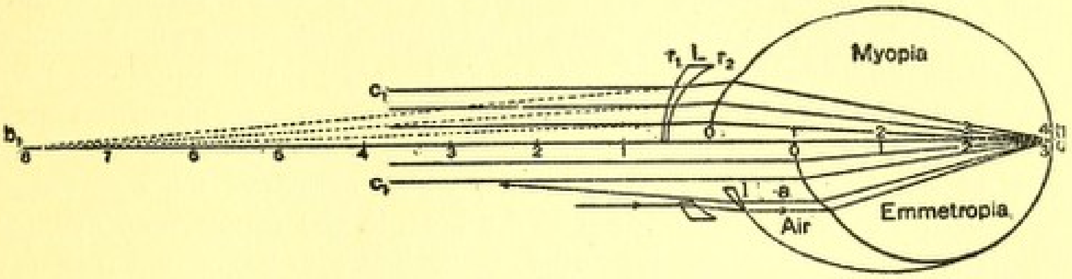

Fig. 4

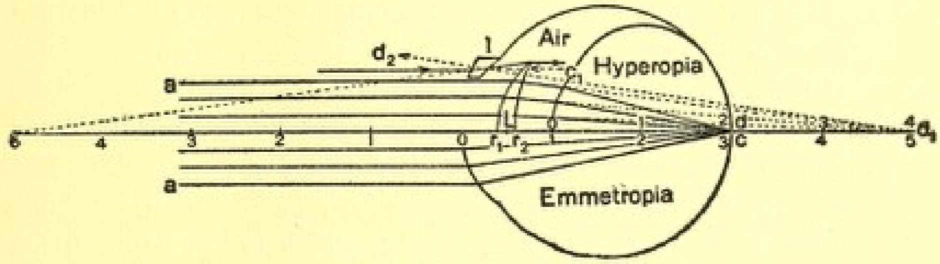

Fig. 5

Explanation of Figures 4 and 5.

These figures showing the relative dioptric values of the emmetropic, myopic and hypermetropic eye have been constructed by C. F. Prentice, M. E., of New York, who describes them as follows:

"The emmetropic, myopic and hypermetropic eyes are represented as solid bodies of glass with an index of refraction, $n = 1.5$, and anterior refracting spherical surfaces of a uniform radius = 1. Introducing these values into the formula $\frac{1}{f} = \frac{n-1}{n\ r}$, which expresses the refraction for parallel incident rays, our schematic emmetropic eye is found to have a depth of focus on the optical axis = 3 (numbers below the optical axis in Figs. 4 and 5).

"Fig. 4 shows the *emmetropic eye* (lower portion of the diagram) *compared with the myopic eye* (the upper half of the diagram), with an optical axis longer by a radius of 1; that is to say, its depth of focus from the retina, which is common to both, is made arbitrarily = 4 (figures above the optical axis). The conjugate focal distance anterior to the cornea of the myopic eye is found by the formula $\frac{n}{e} - \frac{1}{a} = \frac{n-1}{r}$, wherein $e = 4$ (within the eye), and a (in the air), are conjugate distances upon the optic axis. The anterior conjugate focus is found by this formula to be — 8, which, being counted from the cornea in the opposite direction to the incident rays, is established at b_1, which is thereby the far point of the myopic eye. The correcting concave meniscus L is arbitrarily placed one-half radius in front of the myopic cornea, so that the distance of the center of this lens is 7½ units from its focus, which thus coincides with the far point of the eye at b_1. By making the radius r_2 of the posterior surface of this lens = 1 unit, we obtain, through the formula for lenses whose thickness can be neglected, $-\frac{1}{F} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$; the value of the radius of curvature of the anterior surface, r_1, = 1.36, as carried out in the diagram. The convex meniscus l (a section of which is shown at the lower part of the diagram), which expresses the difference between the refracting power of the myopic and emmetropic eye, is shown to have its anterior surface coincident with the myopic cornea, and hence the curvature of this anterior surface = 1, and its focus is = 8. The curvature of its posterior surface, r_2, is obtained from the formula $\frac{1}{F} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ and becomes 1.33, as in the diagram.

"Fig. 5 gives, in a similar manner, the comparative dioptric values of the *emmetropic and hypertropic eye.* The hypermetropic eye (upper part of the diagram), being shorter by one unit of radius, its depth of focus is therefore only = 2 (figures above the optical axis). Applying the formula $n - \frac{1}{a} = \frac{n-1}{r}$, in which $e = 2$, we find the conjugate focal distance, $a = 4$, which being counted from the cornea in the direction of incidence is located *behind* the eye at d_1, and is therefore the hypermetropic far point. The convex correcting meniscus L is placed one-half unit of radius in front of the cornea of the hypermetropic eye, hence its focus will be 4½ units from d_1, the far point behind the eye. Since the radius of curvature of the anterior surface $r_1 = 1$, the radius of the posterior surface r_2 will be, according to the foregoing formula 1.8. The concave meniscus l, expressing the difference between the emmetropic and the hypermetropic refraction, has the curvature of its posterior surface coincident with the emmetropic cornea with its radius = 1, and its focus at $d_1 = 5$ (figures below the optical axis). Introducing these values into the equation $-\frac{1}{F} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, we find r_1 the curvature of the anterior surface = 1.66."

CHAPTER IV.

The Conjugate Focus in Accommodation (*A.*)

In the *static* refractive conditions which we have been considering, that is, the refractive state uninfluenced by any voluntary muscular effort on the part of the eye itself, the differentiation of the three categories of *E.*, *M.* and *H.* consists in a determination of the relative positions of the far point and the retina as conjugate foci, the matter of the actual amount of refracting power not entering into consideration.

There is, however, another condition of what is called *dynamic refraction*, in which the eye has actually added to its refracting power by means of an increased curvature of the crystalline lens, brought about through a contraction of the ciliary muscle. This ability to increase its refractive power, serving, as it does, to adapt the eye to distinct vision at distances within the far point, is called the *accommodation* power (*A.*). We shall find that the laws of conjugate foci apply to the elucidation of the phenomena of this refractive state as pertinently as we have found them to do in the conditions of static refraction.

When an eye has added, by means of its accommodation, something to its refracting power, the position of the retina, representing one conjugate focus, remaining unchanged, it is only the position of the other conjugate focus that is altered.

As the position of the external focus conjugate to the retina in a static refractive condition has been called the the *far point* (*punctum remotum*), another name must be used to designate the conjugate focus under accommodation. This properly should be termed the *accommodation point* (*punctum accommodatum*), but the term *near point* (*punctum* *proximum*) has been so long employed that it may still be retained as sufficiently descriptive and accurate for most practical uses.

Let us now examine into the effect produced upon the position of the remote conjugate focus or far point by the accommodation in the three categories of static refraction.

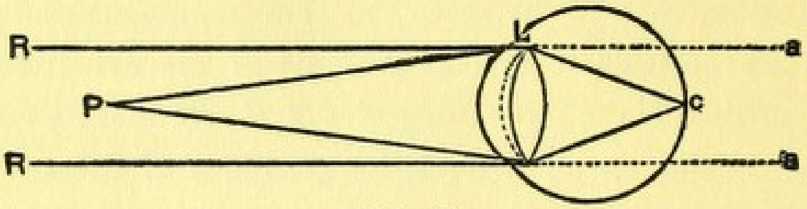

Fig. 6

A. in Emmetropia. In this case the effect of an increase in refraction, the retina remaining fixed, is to advance the far point from infinity to a finite distance, that is, to convert *E.* into *M.* The amount of accommodation then, that is, the refractive power added, as indicated by the dotted line *L*, Fig. 6, would be expressed by the power of the additional lens which placed at *L* would bring parallel rays *a a* to a focus at the point of accommodation, *P.* If, for example, the point *P* is 8 inches (20 cm) from *L*, the added refraction would be expressed by the value of *M.* with its far point at *P*, that is, by $+ 5$ D. As this, at the same time, gives the difference between the far point of the emmetropic eye, *R*, and the accommodation point *P*, the general formula for the amount of accommodation is $A = P - R$; therefore, in this special case, $A = 5 - \infty = 5$ D.

On the other hand, if we know the amount of *A.* and the position of *R*, the position of *P* is found by the formula ; $P = A + R$. In this case, therefore, $P = 5 + \infty = 5$ D. $= 20$ cm. $= 8$ inches.

The *total amount* of accommodation possessed by an eye would be expressed when *P* gives the *nearest* point of distinct vision ; that is, the position of the external conjugate focus under the full force of the accommodation power. Owing to the fact, however, that the accommodation power diminishes with age, this nearest point recedes towards the far point as we grow older. When this recession has reached a point beyond that at which reading, writing, etc., must be done, *presbyopia* (*Pr.*), (πρεσβυς, an old man, and οψ, eye) is said to have set in. In emmetropia this occurs at about the forty-fifth year. This deficiency in the power of *A.* must then be artificially supplied by a convex lens placed in front of the eye, which, by adding to its refraction will bring back the receded nearest point within the distance at which the work is to be done. If, for example, *P* has receded to 20 inches, 50 cm., leaving only 2 D. of *A.*, and it is necessary that the near point should be at 10 inches, 25 cm., representing 4 D. of *A.*, a lens of $4 - 2 = 2$ D. must be added.

A. in Myopia. In myopia the conjugate focus or far point is already at a finite distance. The act of accommodation can then only bring the conjugate focus nearer to the eye and thus increase the degree of *M.*; the amount of advancement

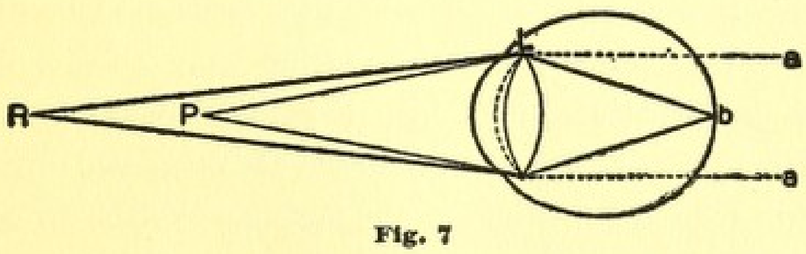

Fig. 7

being measured by the difference in the positions of the foci in the two degrees of *M.* Let *R*, Fig. 7, be the far point in a static condition, say, at 50 cm. (20 inches) from *L*, representing 2 D. of *M.*, and *P* an accommodation point at 10 inches = 4 D. of *M.*; then, since $A = P - R$, $4 - 2 =$ 2 D. of *A.* On the other hand, the amount of *A.* and the far point *R* being known, the accommodation point is obtained by the general formula $P = A + R$. A being 2 D, and R being 2 D., $P = 2 + 2 = 4$ D. = 10 inches = 25 cm.

In myopia *Pr.* sets in at a later period than in *E.*, for the obvious reason that with the same amount of accommodation power P lies much nearer the eye. If, for example, there is $M. = 2$ D. with the far point R at 20 inches, and an A. of 3 D., the near point P would be 2 D. + 3 D. = 5 D. = 8 inches = 20 cm. This eye will not need assistance to supply a defect of *A.* for reading as the emmetropic eye with the same amount of *A.* would. In *E.* this *A.* would bring the conjugate focus or near point only to 13.3 inches, 33.3 cm.,

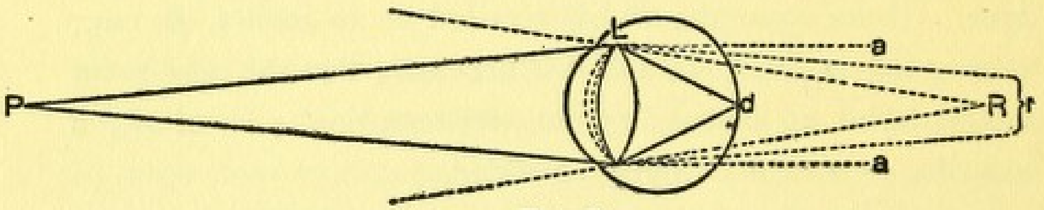

Fig. 8

which is outside the conventional limits of 10 inches. When $M. = 4$ D. with its far point at 10 inches, there will be no *Pr.*, even when the *A.* is totally abolished, as it usually is about the seventieth year, since the conjugate focus yet falls without the conventional near point of 10 inches.

A. in Hypermetropia. In hypermetropia the case is somewhat different from that found in the other two categories of static refraction, owing to the fact that R has a negative value, falling, as it does, behind the refracting system. In Fig. 8, R represents the far point of an eye with *H.* of 4 D., its conjugate focus being 10 inches (25 cm.) behind L. If, by an act of accommodation, a refracting power is added, the conjugate focus will be moved back from R towards infinity to the right, let us say, to r, 20 inches behind L. The amount of *A.* will then be measured by the difference in the positions of the conjugate foci R and r in accordance with the formula $A = P - R$. R in this case is represented by 4 D., and r by 2 D. Since both R and r are negative, the amount of A. is still expressed by the usual formula and $4 - 2 = 2$ D. The accommodation point r has a negative value, being behind the refracting system. If r be at 40 inches (1 m.) $A = 4 - 1 = 3$ D., the accommodation point yet remaining negative.

The usual formula $P = A + R$ also holds here for the determination of the position of the point of accommodation. In the last example, $P = +3 + (-4) = -1$ D., and the accommodation point is one meter behind the eye, since R is negative and greater than A. When r reaches infinity, $A = 4 - 0 = 4$ D. the *H.* is abolished and emmetropia prevails.

If r be now advanced by a further increase in refraction from infinity on the right to a finite distance on the *left*, say, P, the conjugate focus representing the accommodation point becomes positive and a condition of myopia prevails. If this finite distance is at 20 inches (50 cm.) in front of L, the difference between R and P (R being negative and P positive) is expressed by $A = P - (-R) = R + P$. If P be 2 D., $A = 4 + 2 = 6$ D., because it has required 4 D. of A. to bring r to infinity and 2 D. more to bring the conjugate focus to P, 20 inches in front of L. P, the position of the accommodation point, is obtained by the usual formula, $P = A + R$. Since R is negative, $P = A + (-R) = A - R = 6 - 4 = +2$ D. $=$ 20 inches in front of the eye.

In *H. Pr.* sets in earlier than in *E.*, owing to the fact that a certain amount of *A.*, equal to the degree of *H.*, is consumed in converting the *H.* into *E.* by bringing the conjugate focus from a negative position behind the eye to an infinite distance in front of it. Should there be, for example, *H.* $=$ 3 D. in a person of thirty-five years, having an accommodation power of 5.5 D., three diopters of *A.* would be taken up in overcoming the *H.*, leaving only 2.5 D. with which to bring the nearest point to 40 cm. This being still without the conventional nearest point of 25 cm., a + 1.5 D. lens, such as is required for an emmetrope of fifty years, will be necessary to bring the accommodation point to the required 10 inches.

When such a hypermetrope shall have reached the age of forty-seven years, the total amount of *A.* will have been exhausted in converting the *H.* into *E.*, and *P* will have receded to infinity, with no accommodation power available for work at a finite distance. He will then require a + 4 D. to bring his conjugate focus to the working distance of 10 inches, such as would be required by an emmetrope of about seventy years.

It is estimated that the loss of *A.* is about 1 D. for every five years from the thirty-fifth year, when it is 5.5 D. to the seventieth, when it is practically abolished. From this rule, however, the variations are most frequent.

CHAPTER V.

The Conjugate Focus in Ophthalmoscopy

In ophthalmoscopy the interior of the eye is illuminated by the light thrown into it through the pupil by a proper mirror. The objects on the fundus of the eye then become in themselves luminous bodies giving off rays of light, some of which pass out through the pupil, and in so passing out are subjected to refraction by the dioptric apparatus of the eye. It is on account of this refraction that, under ordinary conditions, these rays cannot be so utilized by an observing eye placed in front of the eye thus illuminated as to have a clear image of these objects made on its retina.

It is the aim of the science of ophthalmoscopy to arrange it so that these rays coming from the bottom of the eye may be acted upon in such manner that they will be focused upon the retina of the observing eye, giving a clear image of the details of the fundus of the observed eye. The problems to be solved all have their basis in the laws of conjugate foci. The fundus and the far point being at conjugate foci, any illuminated object on the fundus must have its image at that far point, and if the object or this image is to be seen by the observing eye, its dioptric apparatus must also be adapted to that far point.

It was through a recognition of the laws of conjugate foci as exemplified in the refracting power of the eye that the genius of Helmholtz was enabled to give us the ophthalmoscope. It had, seemingly, never occurred to anyone before him to take account of the fact that the eye was an optical instrument of the ordinary kind, and that the rays of light coming out of it were acted upon in the same manner as when going into it. The neglect to recognize this fundamental fact accounts for the failure of all previous efforts to observe the details of the fundus of an illuminated living eye. If the fundus of an eye, through an illumination, became itself the source of emitted rays, he reasoned that the rays should be refracted by the optical system of the eye in passing out through it, and would, in accordance with the well-known laws of optics, then proceed toward the conjugate focus of the object from which they came ; at which focus an image of that object would be formed. If, then, the eye of an observer could be so placed in the path of those emergent rays that they could be focused on its retina, the object from which those rays proceeded would be seen in all its details. The theory of ophthalmoscopy, as he unfolded it for the first time, consists simply in bringing the illuminated fundus of the observed eye and the retina of the observing eye into the positions of conjugate foci.

As the position of the far point or conjugate focus of the observed eye necessarily differs in each of the three categories of *E.*, *M.* and *H.*, the means by which this far point is to be brought to the retina of the observer must also be different for each. Let us now examine into the conditions and requirements of each one of these categories separately, it being assumed always that the observing eye is emmetropic and in a state of static refraction with its far point at infinity.

Ophthalmoscopy by the Direct Method

Ophthalmoscopy in E. In emmetropia the rays emerge from the observed eye parallel, c_1 c_1 (Figs. 3 and 3a), coming, as they do, from the fundus of the emmetropic eye *E.* The conjugate focus is, therefore, at infinity. As the emmetropic observing eye placed in the path of these rays is adapted for parallel rays and also has its far point at infinity, the two retinæ will be at conjugate foci and a clear image of the fundus of the observed eye will be formed on the retina of the observer. It will be noticed as a necessary condition that the retina of each eye has its conjugate focus or far point at the same place, namely, infinity. An emmetropic eye, therefore, can see clearly the details of the fundus of another emmetropic eye without the intervention of any auxiliary lens when both eyes are in a condition of static refraction.

Ophthalmoscopy in M. Here, the far point being at a finite distance, the rays from the fundus emerge convergently towards the conjugate focus at b_1 (Figs. 3 and 3a). The emmetropic observing eye when placed in the path of these rays cannot focus them on the retina, since it is adapted only for parallel rays. In order that it may so focus them, the rays must be made parallel. This can be done by the interposition of a concave lens *L* (Fig. 4) of such strength as shall give the rays emerging convergently towards b_1 a parallel direction, c_1 c_1. As b_1 is the negative focus of this lens, this also marks the position of the far point and the degree of *M.* of the observed eye. By this it will be seen that the ophthalmoscope becomes also an *optometer* by which one may measure the degree or amount of *M.* Example: If the retinæ of the two eyes are brought into the positions of conjugate foci, as indicated by the formation of a clear image of the fundus of the observed eye on the retina of the observing, by a — 4 D. lens placed behind the ophthalmoscope, we know that this lens has rendered parallel the rays converging towards b_1, which is the far point of the observed eye. This point b_1 lies then at the negative focus of the lens, namely, 10 inches, which represents a power of 4 D. This lens of — 4 D., through which the fundus is seen clearly, is, therefore, the correcting lens of the *M.*

Ophthalmoscopy in H. In this category, where the far point lies behind the refracting system at d_1 (Figs. 3 and 3a), the rays from the fundus emerge divergently, d_2 d_2. The emmetropic observing eye placed in the path of these divergent rays can focus them on its retina only after they have been rendered parallel. This, however, may be accomplished by the interposition of a convex lens *L* (Fig. 5), whose focus for parallel rays, *a a*, is at d_1. The two retinæ are then placed at the same conjugate focus (infinity) and the details of the fundus of the observed eye are distinctly pictured on the retina of the observing eye. Example: The details of the fundus are clearly seen through a + 2 D. lens placed behind the ophthalmoscope. This shows that the principal focus of the lens, which is 20 inches, falls at d_1 and coincides with the far point of the observed eye situated at that distance behind its refracting system. Neglecting any difference between the eye and the position of the lens, the far point of the eye will fall at this focus and the amount of *H.* will be 2 D.

The Observing Eye is not Emmetropic. The conditions here are somewhat changed, but the law still holds, and it is yet possible under certain specific contingencies for an ametropic eye, that is, one not adapted for parallel rays, to see the fundus of another ametropic eye distinctly while both are still in their static refractive state. This can only occur, however, when the far points of the two eyes happen to fall at the same conjugate focus. Example: Suppose that the far point of the myopic eye, b_1, Fig. 3, is 60 cm. in front of *e*. If, now, a hypermetropic eye with its far point at 50 cm. behind its refracting system, d_1, Fig. 3a, were placed 10 cm. in front of this eye, the far point of both would fall at the same place, that is, 60 cm. in front of *e*, the two retinæ would then be at conjugate foci and the image of the one would be clearly pictured on the other. The fundus of a myopic eye is the only object in nature that a hypermetropic eye can see without the exercise of its accommodation power, since nowhere else have we rays that are otherwise than artificially convergent.

On the other hand, and similarly, a myopic eye with its far point at b_1, 100 cm. from e, Fig. 3, placed 10 cm. in front of a hypermetropic eye with its rays coming from the retina divergently, d_2 d_2, as if they came from its far point, d_1, Fig. 3a, 90 cm. behind its refracting system, will have a distinct view of the fundus d, since the two far points b_1 and d_1 will then fall together and the two retinæ be at conjugate foci.

Should the observing eye be possessed of accommodation power it is still possible for it to bring its far point to the same position as that of the observed eye, provided the latter has a conjugate focus farther from the eye than the former. Thus, an emmetropic observing eye can, by advancing its far point from infinity to a finite distance in front of it, bring its conjugate focus to the negative far point of a hypermetropic observed eye; and a myopic observing eye, by increasing its myopia through its accommodation and thus advancing its far point, can bring its conjugate focus to fall at the negative far point of a hypermetropic eye of a higher degree than its own *M.*; and the hypermetropic observing eye can, through its accommodation, bring its far point from a negative position back to infinity, where it will be at conjugate focus with the emmetropic eye, and by a still further increase of *A.* can bring its far point to a finite distance in front of it that shall correspond to the negative far point of another, though less, degree of hypermetropia. In practice, however, it is always better to render the observing eye emmetropic by means of suitable correcting glasses placed behind the ophthalmoscopic mirror.

This method of ophthalmoscopy, in which the two retinæ are brought into the positions of conjugate foci and the one eye looks directly into the other, is called the *direct method*, and as the objects at the fundus are seen in their natural positions the image is said to be *erect*.

Ophthalmoscopy by the Indirect Method

In accordance with the laws of conjugate foci, there must be formed in the air at the far point of a myopic eye, after it has been illuminated, an inverted image of the fundus of that eye in all its details. Should an observer place himself at a sufficient distance from this ærial image so that his own far point, either static or by accommodation, shall coincide with this image, an image of this image will be formed on his retina and he will be able to see it distinctly.

This principle is used in another method of ophthalmoscopy, which is called the *indirect* method, since it is the *image* of the fundus of the eye under observation and not the fundus itself which is observed ; and since the image is formed and seen to be *inverted*, it is so called, in contradistinction to the erect image just described. This is only practically possible, of course, in myopia of high degrees, with the far point very near the eye under observation. We can, however, render, artificially, any eye myopic and bring its far point very close by the interposition of a strong convex lens in the path of the emergent rays. This should be placed in front of the ophthalmoscope and close to the observed eye. It is customary to use one of about 20 D. This will place the conjugate focus of an emmetropic eye, with an inverted image of the fundus, at two inches in front of the lens. An emmetropic observing eye at a distance of 18 inches will then be able, by means of its accommodation, to bring its far point to the position of this image, and an image of this image will thus be pictured on its retina. The position of the aerial image will be closer to the auxiliary lens when the rays come from the eye under observation convergently, as in *M.*, and farther from it when they come divergently, as in *H.*; the exact position in any case depending, of course, on the degree of ametropia of the eye that is being examined.

Should the emmetropic observing eye not use its *A.*, a lens placed behind the ophthalmoscopic mirror with its focal distance coincident with the aerial inverted image, will render the rays coming from it parallel and adapt them to the static emmetropic refraction. Should the observing eye be myopic, or hypermetropic, a lens of such power will be required as shall bring the inverted image into conjugate focus with the retina of the observing eye. In that degree of *M.* in which the far point coincides with the position of the aerial image no lens will be required. In *H.*, lenses stronger than in *E.* will be necessary. It is also possible to accommodate the observing eye to varying positions of the aerial image by a variation in the distance of the ophthalmoscope, while using the same lens behind the mirror.

CHAPTER VI.

The Conjugate Focus in Skiascopy (Shadow Test).

The laws of conjugate foci as we have found them to apply in a consideration of all the foregoing methods for determining the optical condition of the eye, hold equally good in what is called the "shadow test," though we shall encounter some necessary modifications in their application to these special conditions due to the complexity of the optical system on which the method rests ; for, simple and easy as it is in practice, the optical laws involved are quite complicated. While in the direct method of ophthalmoscopy, for example, the observing eye seeks to have an image of the fundus of the eye under observation pictured distinctly on its retina, in the shadow test, the details of the fundus are not desired ; the object of observation being the shadowy edge of an aerial image of a bright spot formed from the light thrown by an ophthalmoscopic mirror on the fundus of the observed eye.*

This aerial image, real or virtual, being formed by the refracting system of the observed eye, lies necessarily at the conjugate focus of the fundus. That to which the observer chiefly directs his attention is the movement of the shadowy edge of this image as it appears to pass across the pupillary field of the observed eye as compared with a rotation movement of the mirror giving the illumination. Simple as the method is in practice, its optical principles may seem at first sight somewhat complicated, but they are easily resolvable by the laws already exposed.

*For this reason the incorrectness and absurdity of the term "retinoscopy" must be apparent.

The phenomena of movement differ according as a concave or plane ophthalmoscopic mirror is employed. We will consider those with the plane mirror first.

Skiascopy with a Plane Mirror. When a plane mirror m, Fig. 9, is used, the real *source of illumination* is an image L_1 of the flame L situated as far behind the mirror m as the

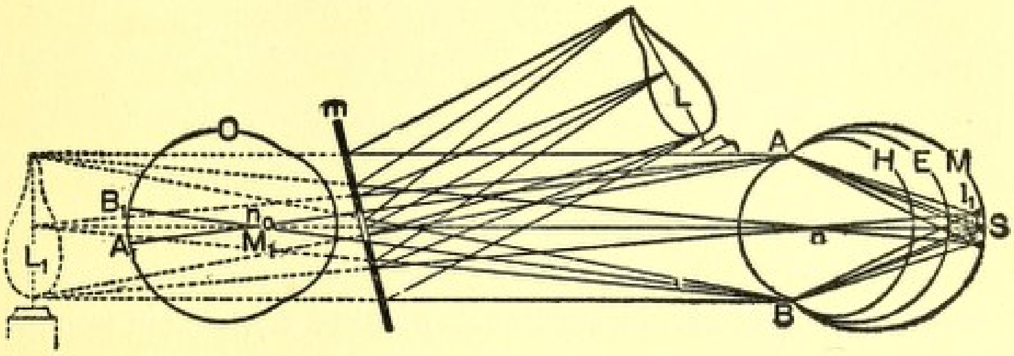

Fig. 9

flame L itself is in front of it. This *flame image* L_1, in keeping with the well-known laws of reflection, *always moves in a direction opposite to that of the mirror rotation;* when the mirror is rotated to the right, the flame image moves to the left, and *vice versa.* The diffuse *spot of illumination* S, made by this flame image on the fundus of the eye under observation by its refracting system, is positive and inverted, being formed by the same rays that constitute l_1, which is the intraocular image of L_1 and therefore must *move in a direction opposite of its object, the flame image* L_1; when the flame image moves to the left, the bright spot moves across the fundus to the right, and *vice versa*, that is, *in the same direction as the mirror rotation.*

When, in its turn, this *bright spot becomes the object* and a source of illumination on the fundus, sending out rays which are acted upon by the refracting system of the eye, *an image of it, real or virtual, is formed at the conjugate focus of the fundus.*

When the eye is myopic, *M*, this image of the bright spot *S* is formed at the conjugate focus which is at a finite distance, the far point of the eye, and, being in front of the eye, is positive and inverted, M_1, Fig. 10. When the eye is

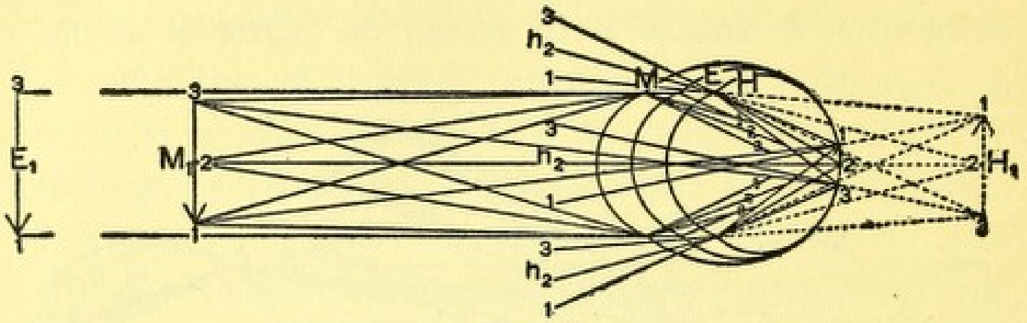

Fig. 10

emmetropic, *E*, this image lies at infinity, E_1, also in front of the refracting system and is likewise positive and inverted. When the eye is hypermetropic, *H*, with its conjugate focus and far point behind the retina and on the same side of the refracting system as the bright spot on the fundus, the image is virtual and erect, the rays h_2 h_2 h_2 coming from the eye divergently as if from a real object, *1 2 3* at its negative far point H_1.

The *movements of this image* in respect to the movements of the object *S* will be as follows: In *E.* and *M.* the movements of E_1 and M_1 will be *against* the movements of *S*, being on opposite sides of the refracting system from *S* and therefore *against the mirror rotation.* In *H.*, on the contrary, the movement of H_1, it being on the same side of the refracting system as *S*, will be in the *same* direction as *S* and also in the *same direction as the mirror rotation.*

The direction of movement apparent to an observing eye, placed in the path of the rays emerging from the observed eye, however, will depend on the position of this eye relative to the image formed at the conjugate focus of the fundus of the observed eye—that is, on whether the observing eye receives the rays which come from a real aerial image of the bright spot formed in front of it, or the rays which proceed from the bright spot itself to form an image behind its nodal point by its own refracting system.

We will first consider the latter condition, where *the conjugate focus falls behind the nodal point of the observing eye.* In Fig. 11 the image M_1 of S is formed behind the nodal point n_1, by the refracting system of the observing eye O_1. This image is real and inverted. Now this image may be anywhere between n_1 and infinity behind O_1, depending upon the refractive condition of M or the position of O_1 in respect to M. But within these limits, no matter where it is formed, whether on the retina, in front of or behind it, its movements, as perceived by the retina of the eye O_1, are always in the same direction, and just as it would perceive the motion of any object in space in front of it. Though this is called the *erect* image, it is really formed inverted on the retina, just as the images of all objects in front of the eye are; but, in accordance with the law of projection, they are perceived as erect, and all movements of S to the right are perceived by O_1

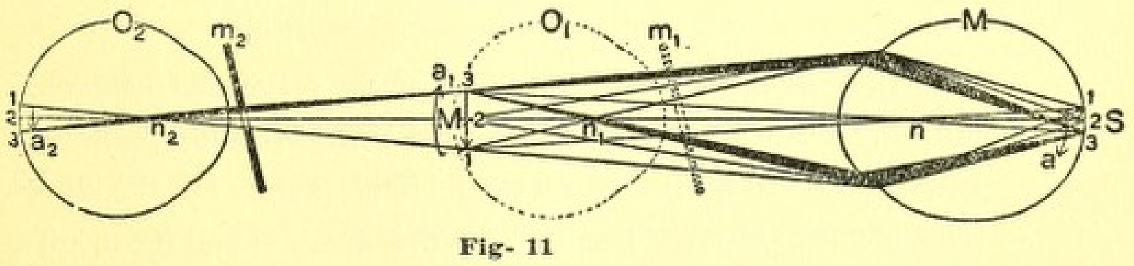

Fig. 11

as to the right, and *vice versa*, and of course *with the mirror rotation.* The observing eye, therefore, will bring parallel rays forming E_1, Fig. 10, divergent rays h_2 h_2 h_2, coming from H_1, and rays convergent to all points *beyond the position of its nodal point*, n_1, Fig. 11, to a focus somewhere behind its own nodal point; and in all these conditions the apparent movement across the pupil will be *with* the *mirror rotation.*

When the *conjugate focus falls in front of the nodal point* n_2 *of the observing eye* O_2, Fig. 11, there is formed there a real inverted image, M_1, of the bright spot S. This image being on the opposite side of the refracting system of M, moves always in a direction the *opposite of* S, *and likewise of the mirror rotation.* When S moves down in the direction of the arrow a, the image M_1 moves upward in the direction of the arrow a_1. The observing eye O_2 will perceive this movement exactly as it would the movement of any other object moving in the same direction in front of it. Hence, in O_2 the image with its shadowy edge moves *against*, whereas in O_1 it moves *with* the mirror rotation.

When, however, the observing eye O, Fig. 12, is so placed that the *conjugate focus of the observed eye M falls*

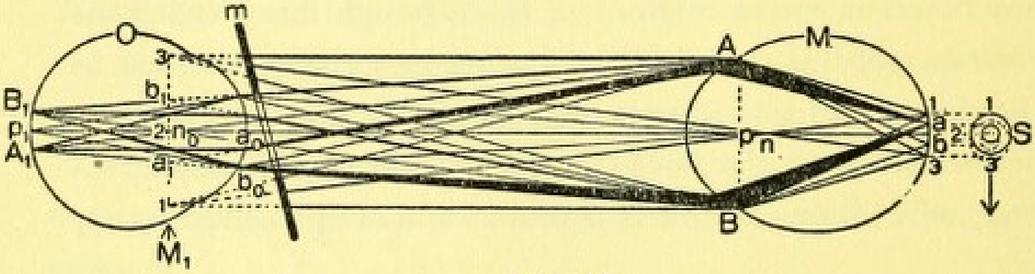

Fig. 12

exactly at its nodal point n_0, *there is no movement observed*, but only a diffuse, steady illumination with no outline, however extensively the mirror is rotated. The reason for this is as follows: As we have seen, the flame image L_1, Fig. 9, of a certain magnitude, has always an image l_1 of a definite and proportionate size, made by the refracting system of the observed eye. The pencils forming this image make the bright spot S on the fundus, which is always blurred in outline except when l_1 happens to fall exactly on the fundus. So long as the pupil of the observed eye is fully illuminated during a rotation of the mirror, there is formed, by the refracting system of this eye, an image *1 2 3*, M_1, Fig. 12, of the bright spot S at the nodal point n_0 of the observing eye. This image is always larger than the hole in the mirror (the latter has been purposely exaggerated in the drawing). Now, in order that any specific direction of movement be noted, it is necessary for the shadowy edge of the image M_1 of the bright spot S, to appear to pass across the pupil of the observed eye during a mirror rotation. Hence, the appropriateness of the term *skiascopy* (from σκια, a shadow, and σκοπειν, to examine).

In this case, where the image M_1 falls at the nodal point n_0, the image of the upper edge *1* of S is found at *1* of M_1. The pencils of light which go to form the image of *1* at M_1, however, are cut off by the mirror surrounding the sight-hole and do not enter the eye O at all, and therefore no image of this edge of the bright spot S is perceived. It is only the pencils coming from a in S, much within the limits of *1*, that pass along the lower edge of the sight-hole and enter the eye O to form an image at a_1. When a rotation of the mirror causes the bright spot S to move, say, downward in the direction of the arrow, a and *1* move downwards also; but, synchronously, another point between a and *1* takes the place of a, which will still form its image at a_1. But no movement downward of S, when there is still a full illumination of the pupil, is sufficient to bring *1* down to the position occupied by a, and have its image at a_1 within the eye O. The same holds good also in the upward movement of the points b and *3* of the bright spot S, *3* never, on any rotation of the mirror, reaching up to the place occupied by b. Consequently, the shadowy edge, *1–3*, of the bright spot S can never have its image within the eye of the observer O, and therefore it can never appear to pass across the pupil, to whatever extent the mirror may be rotated. As some point of S, within *1* and *3*, always has its image at M_1, there is always a steady illumination perceived by O during all mirror rotations.

Those pencils of light, however, from a of the bright spot S, projected to A and B, which mark the outline of the pupil of the eye M, are directed, after their refraction, to a_0 and b_0. After entering O these rays are brought by its refracting power not to a_1, but, after crossing each other to the points A_1 and B_1, on the secondary axes $A_1\ n_0\ A$ and $B_1\ n_0\ B$, which pass through the nodal point n_0 to corresponding points A and B of the pupil of M. These converging pencils, refracted at a_0 and b_0, cross each other near a_1 on the plane passing through n_0, and do not come together again anywhere to form an image of a which is an image of the bright spot

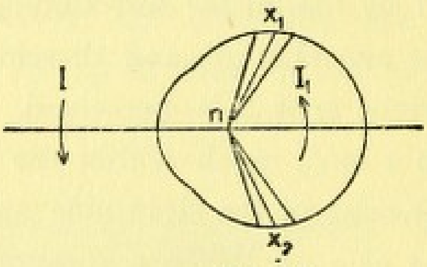

Fig. 13

S. Becoming more and more widely separated, they pass on to opposite sides of $A_1\ B_1$. The same is true of the rays coming from any other point of S, each pair of converging pencils crossing in the plane n_0 to opposite sides of $A_1\ B_1$. *In other words, the image of S formed at the nodal point* n_0 *becomes itself an illuminated object which sends out divergent rays in all directions, which are not acted upon by any optical apparatus to form an image anywhere, and so long as there is a single point of S which has its image at* n_0 *the fundus of O will be fully illuminated.* This is shown in Fig. 13, where a single illuminated point at n gives off rays $x_1\ x_2$, filling the whole of the background of the eye.

As, however, in this method of examination the observing eye is usually accommodated for the pupil of the observed eye, $A_1\ B_1$ on the retina of O becomes the image of $A\ B$ the pupil of *M*, all points of *A p B* of *M* having their conjugate foci in $A_1\ p_1\ B_1$ of *O*.

On comparing diagrams 11 and 12 it will be observed that the pencils from *1* and *3*, corresponding to the shadowy edge of the bright spot *S*, enter the observing eyes O_1 and O_2, Fig. 11, through the nodal points n_1 and n_2, to form their images *1* and *3* in each eye respectively; whereas, in Fig. 12 these pencils from the shadowy edge of *S* do not enter the eye at all, being cut off by the mirror so that no image whatever of them is formed, which the observing eye can take note of during any rotation of the mirror.

Broadly stated, then, since in order to have a movement *against* the mirror rotation an inverted image of *S* must be formed *in front* of the nodal point of the observer's eye at *I*, Fig. 13, while, in order that there be a movement *with* the mirror rotation an erect image must be formed *behind* the nodal point at I_1, the point where the one changes to the other must necessarily be the nodal point itself, *n*.

It will be seen from this that the nodal point of the observer's eye, *the position of no motion* and full illumination under all mirror rotations, *is the* point of reversal *from the movement* with *to the movement* against *the mirror rotation; making the position of the observer's nodal point in relation to the conjugate focus of the observed eye the key of the situation.**

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*In most treatises on skiascopy the exact position of the point of reversal in its relation to the observing eye is left indefinite, obscure or is incorrectly stated. Some European writers of distinction have placed it at the pupil of the observing eye, which, from the foregoing exposition, must be seen to be wholly wrong. The diaphragms of the ophthalmoscopic mirror and of the iris play but a subsidiary part in the phenomena, as we have shown, and as neither represents any cardinal point in the refracting system, and as the phenomena of the shadowy edge would be the same if they were eliminated, the part they play can very properly be neglected in any academic discussion of the questions involved.

The term *koroscopy* (or pupilloscopy, as it was formerly called), introduced as a name for the method and as indicating the importance of the pupil in the phenomena of the test, is, therefore, without meaning or significance.

It should be stated in this connection that while, theoretically, the point of *no movement* is the one which marks the position of the conjugate focus, it is much easier, in practice, to take the point where one movement succeeds the other as indicating with sufficient accuracy this position. This space between the two movements, when the ametropia is very high and the examination is made near the observed eye, may be sufficient to be taken into consideration in an exact estimation of the optical condition.

In determining the refraction of the eye by skiascopy, we employ the same principles as in all the other objective methods, namely, by finding the far point of the eye, or the conjugate focus of the fundus. This is not done, however, by determining directly the point of reversal, the place where the nodal point of the observing eye and the conjugate focus of the fundus of the observed eye fall together. Since this point of reversal is at infinity in *E.* an inconvenient distance in low degrees of *M.* and negative in *H.*, it would not be practically possible. Nor is it necessary, since we can artificially bring the far point of any eye to any desired finite distance by the interposition of a lens in the path of the emerging rays, as is done in the indirect ophthalmoscopic method. In other words, we can create an artificial myopia at whose far point we can easily place the nodal point of the observing eye at which to obtain the phenomena of reversal.

Let us assume, for example, the fixed far point to be at 1 *meter* (*M.* = 1 D.) at which point the observing eye finds itself. The far point of any eye can be brought, by means of a proper lens placed just in front of it, to this point of reversal. The difference between the actual far point of this eye and 1 D. of *M.* is then expressed by the strength of the auxiliary lens which it has been found necessary to place in the path of the rays in order to bring that far point to 1 m. The far point of the observed eye is, therefore, expressed by the difference between the number (x) of the lens employed and 1 D., or $x - 1$ dioptries of refraction, from which the position of the far point is easily found.

Examples: The observing eye being at 1 m. and a lens $x = +1$ D. being necessary to bring about a reversal of movement, there is emmetropia, since $x - 1 = +1 - 1 = 0$ dioptries of refraction. Hence, the far point $= \frac{1 \text{ meter}}{0} = \infty$ (infinity). If a lens of $+ 0.5$ D. is required, then there is $+ 0.5 - 1 = - 0.5$ D. of refraction with a far point $\frac{1 \text{ M.}}{0.5}$

= 2 meters, and there is *M.* of 0.5 D. If a + 4 D. lens is found necessary to bring about a reversal, there is + 4 — 1 = + 3 D. of *H.*, with a negative far point $\frac{1 M.}{3}$ = 33 cm. behind the observed eye. Should a — 3 D. be required to bring about a reversal, there is — 3 — 1 = — 4 D. of *M.* with the far point at $\frac{1 M.}{4}$ = 25 cm. in front of the observed eye, a — 3 D. lens being necessary to extend the far point from 25 cm. (4 D.) to 1 m. (1 D.). Should the nodal point of the observing eye be placed at 2 m. in front of observed eye, we would have to substitute 0.5 D. for 1 D. in the above calculations ($x = 0.5$). If it be at one-half a meter, 2 D. ($x = 2$) will have to be substituted, and so on.

In this method the *accommodation of the observed eye* can, by its action, add some dynamic to the static refraction and thus change the position of its far point by bringing it nearer. On the contrary, an *accommodation on the part of the observing eye can have but little effect*, since the amount of accommodation used to fix the observed eye does not displace the nodal point of the observer's eye to any material extent from the position it occupies in a condition of static refraction and from the actual position of the image of the bright spot.

Skiascopy with the Concave Mirror. The difference between this method and that with the plane mirror rests on the difference in the positions of the flame image giving the illumination. With the plane mirror, as we have seen, the flame image is as far behind the mirror as the flame is in front of it, and moves always against the mirror rotation. With the *concave mirror the flame image is at the focus of the mirror in front of the observer and moves always with the mirror rotation.* This is shown in Fig. 14, where an image of the flame *L* is formed by the concave mirror *m* with its center of curvature at *c*, at L_1. When the mirror is rotated upward in the direction of the arrow, the image L_1 moves also in the same direction. As a result of this, the direction of movement of the bright spot across the fundus of the observed eye is the *opposite of that of the flame image and of the mirror rotation, and necessarily the contrary of that with the plane mirror.* The relative direction of the movement of the image of the bright spot on the fundus made at the conjugate focus of the fundus, however, remains the same; that is, *contrary* to that of the bright spot itself, in the case of *E.* and *M.* where the movement is *with* the mirror rotation, and in the *same* direction in *H.* with the movement *against*

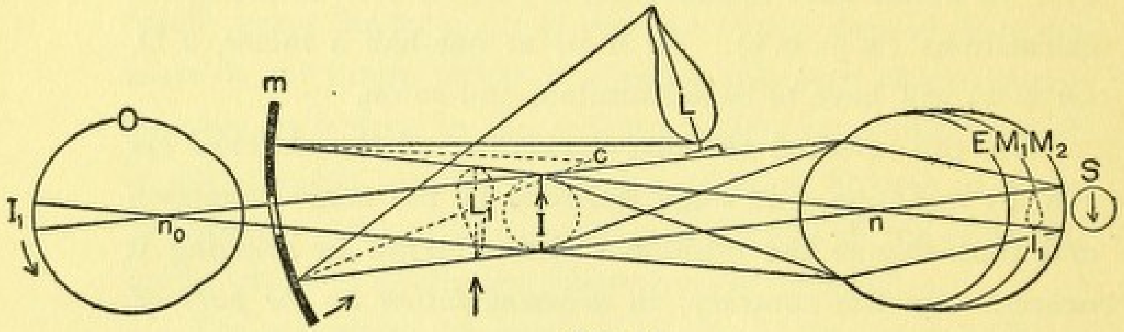

Fig. 14

the mirror rotation. When the observer, then, is at a sufficient distance from the eye under observation to allow the flame image to be formed somewhere in front of the observed eye, the phenomena of movements apparent to the observer will be the *reverse of those with a plane mirror;* that is, in *E.* and *H.* and in *M.* with a far point behind the nodal point n_0 of the observer's eye *O*, as at I_1, the movement will be *against* the mirror rotation, while in *M.*, with the far point in front of the nodal point of the observer's eye, as at *I*, the movement will be *with* the mirror rotation.

With the exception of this change in the direction of the movements across the pupil, the rules for the estimation of ametropia are the same as with the plane mirror given above. When, however, the concave mirror is approached sufficiently near to allow the flame image L_1 to fall behind the nodal point *n* of the observed eye, the bright spot *S* on the fundus and the flame image will move in the *same* direction, and the phenomena will be the same as with the plane mirror.

For a perfect working out of the law, as laid down in the preceding sections, it is necessary that all points of the image, during every rotation of the mirror, lie in the plane passing through the nodal point. If, for any cause, some points of the image fall before this plane and some behind, it is evident that there will be conflicting shadows and a lack of steady, uniform illumination. These shadows I have called "internal shadows," * and they are caused by any irregularity in the refracting media which causes the focus of the different points of the image to fall in different planes and at different distances from the nodal point. These "internal shadows" are most apparent under a wide pupil, which unveils the irregular refraction at the periphery of the refracting media, particularly that of the cornea and in corneal and lenticular opacities.

This irregular astigmia—which it really is—often vitiates the findings by skiascopy and renders it more or less uncertain. For an accurate determination, skiascopy demands a freedom from any irregularity in the area of the pupillary opening.

The apparent *rate of motion of the image of the bright spot* relative to that of the mirror rotation, depends upon the position of this image in respect to the nodal point of the observed eye, or, to be exact, upon the relative positions of the two conjugate foci. The distance of the one focus on the fundus from the nodal point represents the short arm of the lever, and the distance from the nodal point to the position of the image at the other conjugate focus represents the long arm, the fulcrum or center of rotation being the nodal point *n*, Figs. 15 and 16. The longer the long arm, therefore, the greater the extent of its excursions, with a relatively *constant* extent of excursion of the short arm over the fundus. Thus, in *M.*, Fig. 15, when the far point, with its inverted, real

* Trans. Amer. Oph. Soc., 1892.

image, lies at M_2, the excursion will be shorter, with a definite amount of excursion at S_2, than it will be when the far point is at M_1, with a corresponding fundus excursion at S_1. As M_1 recedes still further from n the excursions increase in extent for the same fundus excursions of the bright spot, until

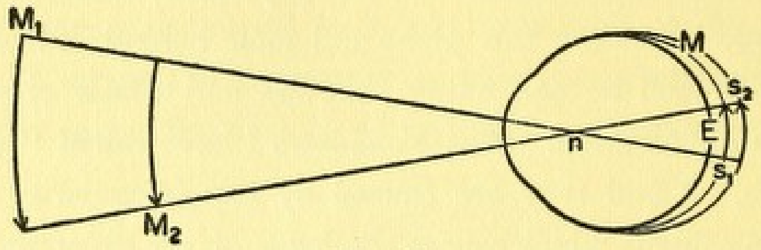

Fig. 15

the image M_1 attains to infinity, when it reaches its maximum, and emmetropia prevails.

Likewise in *H.*, Fig. 16, when the virtual erect image lies at H_1, at the negative far point behind the eye, its excursions, corresponding to the fundus excursions S_1, will be less than when the far point is at H_2, corresponding to the con-

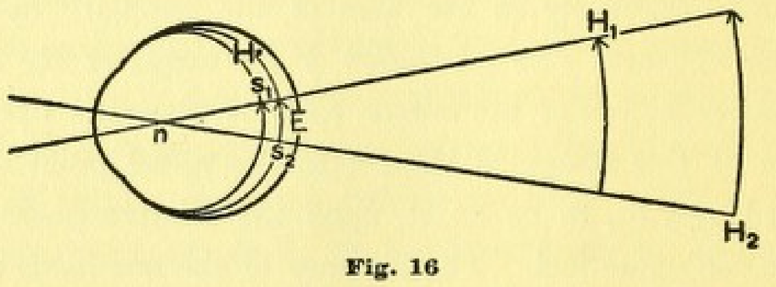

Fig. 16

jugate focus on the fundus at S_2; increasing as H_2 recedes from n, reaching the maximum when H_2 lies at infinity when emmetropia prevails; diminishing again as it becomes positive and approaches a point of reversal fixed at a finite distance in front of the observed eye, as in the myopic conditions, Fig. 15.

The same law regulates the *size of the image of the bright spot* formed at the far point. The closer this is to the refracting system—in accordance with the well-known law governing the relative size of the image to the object—the smaller it is, increasing *pari passu* as it is removed towards infinity, where it reaches its maximum, diminishing again as it is brought to a finite distance at the artificially-established point of reversal.

Thus, in Fig. 15, the inverted image formed at M_1 is larger than that formed at M_2 and the dimensions increase as it is removed towards infinity on the left, where it attains its largest size, being infinitely large. Likewise in *H.*, Fig. 16, the size of the virtual image at H_2 is larger than at H_1, and increases as it is removed to infinity on the right, where it becomes infinitely large, diminishing in size as it becomes positive and inverted at a finite distance at the point of reversal in front of the observed eye, Fig. 15.

It can be said then, in a general way, that both the *rate of motion and size of the image are smaller in the higher degrees of ametropia and increase until the condition of emmetropia with the far point at infinity is attained—diminishing again as the point of reversal at a finite distance is approached.*

The *edge of the image is always shadowy* and indistinct in outline, except when one special relation prevails, and that is when the fundus of the observed eye and the source of illumination are at conjugate foci and, at the same time, the retina of the observing eye is at conjugate focus with the fundus of the observed eye. Under this rare concatenation of conditions the image is seen to be sharp in outline and at its brightest. As these conditions are departed from, and proportionately, the outline becomes more shadowy and indistinct and the image duller.

CHAPTER VII.

The Conjugate Foci in Astigmia (Astigmatism).*

In the optical state of the eye known as regular† astigmia (or, more commonly, as astigmatism), where a point can no longer have its conjugate focus in another single point, the two principal refracting meridians at right angles to each other are of unequal power and each has conjugate foci‡ of its own, which can be studied separately from those of the other. These two foci conjugate to the retina, are on planes perpendicular to the visual axis, and are separated the one from the other by what is called the *focal interval of Sturm*, the boundary of the interval being its anterior and posterior focal lines. It has been customary, hitherto, to consider the focal interval of Sturm as applied solely to the two posterior conjugate foci in their relations to the retina, the anterior conjugate focus being fixed at infinity.

Astigmia has been divided, in accordance with this view, into: 1. *Simple astigmia*, in which one focus is on the retina E, Fig. 17, and the other (a) either in front of the retina at M_2 with a focal interval $E\ M_2$ (*myopic astigmia*), or (b) behind the retina at H_1 with a focal interval $E\ H_1$ (*hypermetropic astigmia*). 2. *Compound myopic astigmia*, where both foc. $M_1\ M_2$, are in front of the retina with the focal interval $M_1\ M_2$. 3. *Compound hypermetropic astigmia*,

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*The name commonly used to indicate this form of refraction was, in the first place, erroneously, derived from στιγμα–στιγματος, which means a *spot* or *blemish*. The proper derivation is from στιγμη–ης, which means *a mathematical point*. The correct word is, therefore, *astigmia*. A complete history of the introduction of the term can be found in an article by the author, in the *Amer. Jour. of Oph.*, Dec., 1903.

† Where the refraction is such as to give more than two foci in the planes at right angles to each other, the astigmia is called *irregular*. This is not corrigible by any of the ordinary optical apparatus and will, therefore, not be considered here.

‡ It must be understood, however, that these foci conjugate to the retina are not focal *points*, but focal *lines*

where both foci H_1 H_2 are behind the retina with a focal interval H_1 H_2. 4. *Mixed astigmia*, where one focus, M_2, is in front of, and the other, H_1, is behind the retina with a focal interval M_2 H_1.

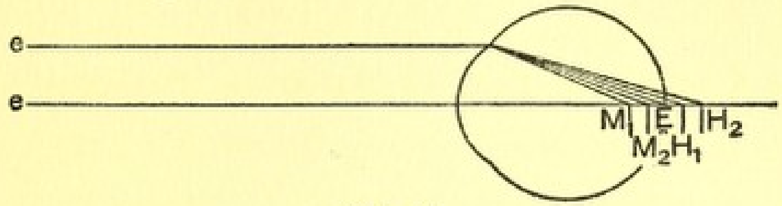

Fig. 17

In keeping with our general plan, however, we can with equal propriety regard *the anterior and posterior focal lines of the interval of Sturm as separating the other*, *external*, *conjugate foci*, *the position of the focus on the retina remaining fixed*. The dimensions and position of the interval of Sturm on the visual axis, outside the eye, must vary, therefore, with the varying positions of the conjugate focus of each of the two meridians, in respect to the principal plane of the eye, as in ordinary spherical refraction ; in other words, we may regard each meridian as representing in its refraction a single eye.

Studying them, then, from this standpoint, we have : 1. The conjugate focus of one meridian at infinity *e e*, Fig. 18, the other at a finite distance M_1, focal interval *e* M_1, *simple M. astigmia.* 2. One conjugate focus at infinity *e e*, the other behind the refracting system at H_1, focal interval *e* H_1, *simple H. astigmia.* 3. Both conjugate foci at a finite distance, but separated by a focal interval M_1 (Fig. 19) corresponding to the meridian m_1, and M_2 corresponding to the meridian m_2, focal interval M_1 M_2 *compound M. astigmia.**

*It is interesting to note that both Young and Airy, who were the first to study with any degree of accuracy this condition, in the determination of their astigmia, used the anterior conjugate foci in the same manner as we have indicated, this being to them the obviously simple and direct way, as they were both myopic. Young's description of his method is as follows :[1] "The eye, in a state of relaxation, collects to a focus on the retina those rays which diverge vertically from an object at a distance of 10 inches from the cornea, and the rays which diverge horizontally from an object at 7 inches distance." This would make his astigmia $\frac{1}{7} - \frac{1}{10} = \frac{1}{23\frac{1}{3}} = 1.71 +$ D.
Airy, in his second communication to the Cambridge Philosoph. Soc., 1846,

[1] Philosoph. Trans. for 1801 ; On the mechanism of the eye.

4. Both conjugate foci behind the refracting system, but with a focal interval H_1 corresponding to the meridian h_1, and H_2 corresponding to h_2, *compound H. astigmia.* 5. One conjugate focus in front of the eye at M_1 (Fig. 18), the other at H_1 behind it, the focal interval being M_1 H_1, *mixed astigmia.*

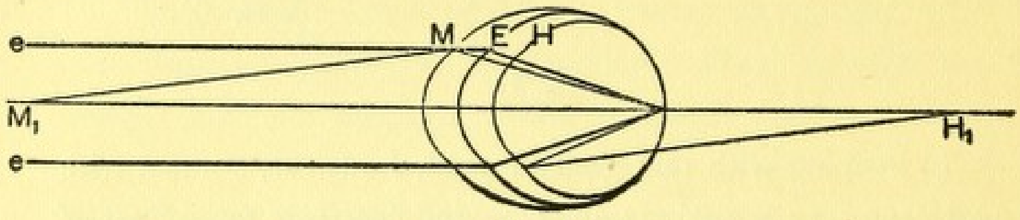

Fig. 18

When the two conjugate foci are, by suitable optical means, brought together, the focal interval is abolished, and the *astigmia is corrected.* The optical agent used for this purpose is a *cylindrical lens,*† which refracts the light only in the meridian of its curvature, the light falling in the direction of its axis being unaffected. Such a lens is to be placed

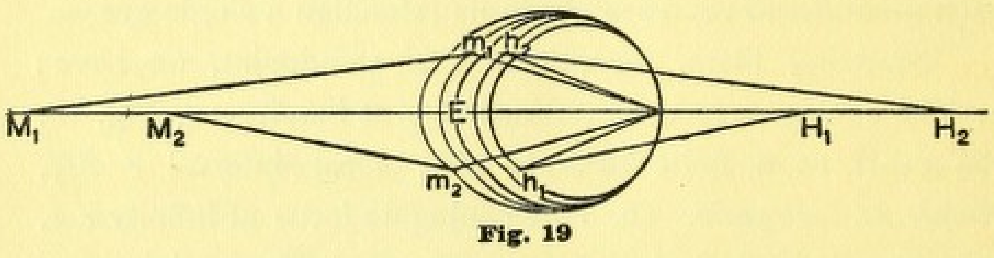

Fig. 19

before the eye with its curvature corresponding to the meridian whose refraction it is desired to effect, and it should be

describes his method as follows: "The evidence of this astigmatism and the measure of it are given by the simple observation of bringing a luminous point nearer and nearer to the eye; the lines of focal convergence, according to the usual rules of focal position, move in the same direction in the interior of the eye, and then one line and the other line are successively brought upon the retina; or the image of the point becomes successively a line in one direction or in the other direction, these directions being at right angles to each other. It was found in 1825" (the date of the first discovery of this condition in his eye) "that the distances at which the luminous point must be placed to give linear images, were 3.5 inches (11.43 D.) and 6 inches (6.66 D.), and the difference in the reciprocals for these numbers, or 0.119 (4.77 D.), is the proper measure of the astigmatism." This had changed in 1846 to 4.7 inches (8.51 D.) for one far point on conjugate focus, and 8.9 inches (4.49 D.) for the other focus, with a difference of 0.100 reciprocal = 4.02 D., showing that though the myopia had diminished, the astigmia had remained nearly the same.

† For an exhaustive study of th· properties of cylindrical lenses, fully illustrated by diagrams, together with the mathematical formulæ thoroughly worked out, the student is referred to a work on "Ophthalmic Lenses," by C. F. Prentice, M. E., published by The Keystone, Philadelphia, Pa.

of such quality and power as to bring the conjugate focus of that meridian to the same point as that of the other meridian.

In Fig. 20, for example, of the rays coming from the

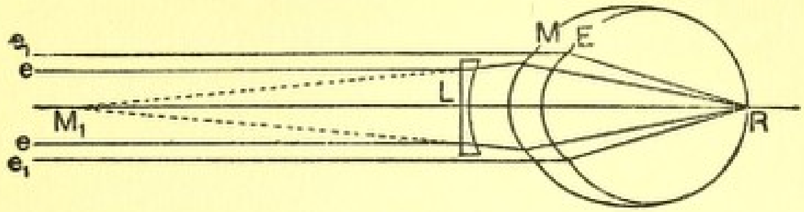

Fig. 20

retina *R* those passing out through the emmetropic meridian *E* will be parallel, $e_1\ e_1$, with their conjugate focus at infinity; while those passing out through the myopic meridian *M* at right angles to it will have their focus at M_1. If, now, a concave cylindrical lens *L* be interposed, with its curvature corresponding to the faulty meridian *M*, it will affect only those rays that have passed out through that meridian, leaving those passing out through the emmetropic meridian, lying in the plane of its axis, unaffected. Should the negative focus of the cylinder lie at M_1 the conjugate focus of the faulty meridian, the rays convergent towards M_1 will be rendered parallel, *e e*, and the far point of this meridian will be removed to infinity and coincide with that of the emmetropic

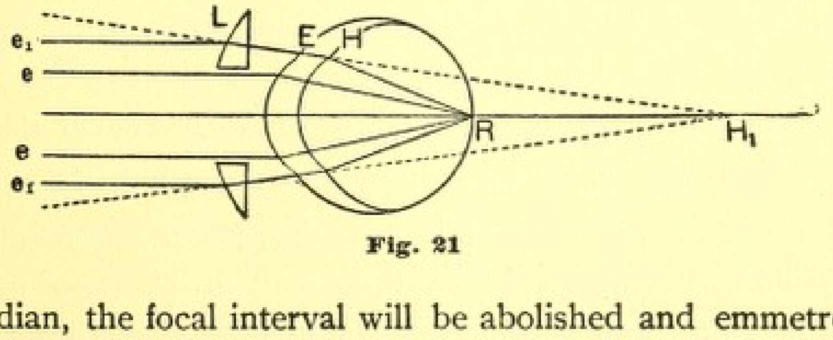

Fig. 21

meridian, the focal interval will be abolished and emmetropia will be established for both meridians.

Likewise in the case of *H.*, as shown in Fig. 21, the rays coming from the retina *R*, which pass out through the faulty meridian *H*, will be divergent, as though they came from H_1, the negative conjugate focus of R for this meridian. A convex cylindrical lens L, with its convex surface corresponding to the faulty meridian and its axis in the plane of the emmetropic meridian E, will render these divergent rays parallel, e_1 e_1, when its focus falls at the far point H_1. The foci of both meridians, E and H, will then fall at the same place, infinity, the focal interval will be abolished and emmetropia will have been established for both meridians.

Examples : There is simple myopic astigmia, with one conjugate focus at infinity (e_1 e_1, Fig. 20), the other at M_1, 20 cm. in front of the principal plane. A concave cylinder with the (negative) focus of its refracting meridian at 20 cm. (— 5 D.) placed close to the eye with its curvature corresponding to the faulty meridian of the eye (or, what is the same thing, with the axis of the cylinder corresponding to the axis of this meridian) will render parallel rays divergent, as if they came from its focus. This focus is, at the same time, the conjugate focus of the faulty meridian, which is thus set back to infinity, where the conjugate focus of the other meridian is to be found. The astigmia has now been corrected and E. prevails.

One meridian, h_2 (Fig. 19), with its axis at 180° (horizontal), has its far point at 50 cm. ($H_2 = 2$ D.) and the other, h_1, with its axis at 90° (vertical) at 20 cm. ($H_1 = 5$ D.), both behind the refracting system. The focal interval H_1 H_2 is then represented by the difference in the power of the two correcting lenses whose focal distances correspond to these two points : $+ 5 - + 2 = + 3$ D., which expresses the amount of the astigmia. A + 3 cylinder, then, with its axis at 90° corresponding to the axis of the meridian of least refraction h_1 will remove the focus of this meridian from 20 cm., H_1 (+ 5), back to 50 cm., H_2 (+ 2), and the focal interval will be abolished and the astigmia corrected. The common conjugate focus, however, still remains at 50 cm., H_2, behind the eye, and there will be yet a *H.* of 2 D. present, which can be overcome and the eye rendered emmetropic by a spherical lens which acts upon both meridians equally, and of such power as to shift the conjugate focus from 50 cm. back to infinity. The strength of this spherical lens is + 2 D., which must be added to the cylinder (+ 3) in order to correct the total ametropia; the formula for the glass being + 2 sph. ◯ + 3 cyl. axis 90°.

One meridian has its far point at M_1, 50 cm. in front of *M*, Fig. 18 ($M = 2$ D.) and the other at H_1, 33.3 cm. behind *H.* ($H = 3$ D.). The astigmia represented by the focal interval M_1 H_1 is therefore $= + 3 - (- 2) = 3 + 2 = 5$ D. As H_1 is negative, it will require a + 3 D. cylinder to bring it to infinity for this meridian, while it will require a — 2 D. to set back the conjugate focus of M_1 to infinity in the meridian at right angles to it. If the axis of H_1 is at 90° and that of M_1 at 180°, the formula for the correcting lens would be + 3 axis 90° ◯ — 2 axis 180°. It is not customary, however, to use this formula for crossed cylinders, as they are called, in the correction of mixed astigmia. It is more convenient to bring one meridian to a state of emmetropia by the proper spherical lens and then correct the astigmia by adding a cylinder, as in the case of ordinary compound astigmia. In the example here given, a + 3 spherical lens would remove H_1 back to infinity on the right and render meridian *H* emmetropic; but it would, at the same time, render the myopic meridian *M* more myopic by exactly that amount by advancing M_1 towards the eye; that is to say, all the astigmia, $3 + 2 = 5$ D., would be myopic, and the formula would be + 3 sph. ◯ — 5 cyl. axis 180°. By using a — 2 spherical all the astigmia would be rendered hypermetropic, and the formula would be — 2 sph. ◯ + 5 cyl. axis 90°.

Determination of Astigmia by Skiascopy.

The movement of the bright spot on the fundus being neccssarily in the same plane as the mirror movement (at right angles to its rotation axis), it is only the refraction in a single meridian corresponding to that plane that is determined in skiascopy. As it is possible, however, to make the axis of mirror rotation correspond to any meridian of the refracting system it is desired to examine, it is easy to determine separately the refraction in any two meridians at right angles to each other as we have it in astigmia. To this end it is only necessary to find the points of reversal for the meridians of least and greatest refraction separately (always at right angles to each other), to know the value of the focal interval of Sturm. Knowing, then, the far points of the two meridians, the strength of the cylindrical lens required to bring them together marks the degree or amount of astigmia.

Example : Meridian with its axis at 90° requires + 3 to bring about a reversal at 1 meter; that with its axis at 180° requires a + 2. There is, therefore, 1 D. of *H.*, with its axis at 180° and 2 D. of *H.* with its axis at 90°. The astigmia is, therefore, $+ 2 - 1 = + 1$ D. axis 90°, with a general *H.* of 1 D. common to both meridians (compound hypermetropic astigmia).

Determination of Astigmia by the Ophthalmoscope.

In the determination of general ametropia by means of the ophthalmoscope (direct method), we saw that when the two retinæ were brought into the positions of conjugate foci, and the details of the fundus of the observed eye were pictured distinctly upon the retina of the observing eye, the ametropia was abolished, and the focus of the lens through which this was effected marked the far point of the observed eye (the observing eye being emmetropic). In astigmia, there being two foci, corresponding to the meridians of least and greatest refraction at right angles to each other, it is necessary to determine the conjugate focus or far point of each of these meridians separately. This is done by taking as the objects of observation, the retinal vessels running in various directions across the background of the eye. In astigmia the vessels whose general course corresponds to one direction will be seen more distinctly than those running in a direction at right angles to it. These directions will correspond to the *axes* of the two principal meridians, respectively. It now remains to find the lens through which the vessels corresponding to the axis of each of these meridians are seen most distinctly, separately, and the difference in their power will express the amount of astigmia.

Example : The vessels running in a vertical direction are seen most distinctly with a + 3 D., while those running in a horizontal direction are seen distinctly with a + 1 D. There is then *H.* = 3 D. with the axis at 90° (vertical), and *H.* = 1 D. with the axis at 180° (horizontal). There is then a general *H.* = 1 D. and an astigmia of 3 — 1 = 2 D. axis 90°; compound hypermetropic astigmia corrected by + 1 sph. ⊂⊃ + 2 cyl. axis 90°.

CHAPTER VIII.

Visual Acuteness and the Testing Thereof.

In some of the methods for determining the refractive state of the eye described in the foregoing chapters, such as by the ophthalmoscope and skiascopy, the person under examination remains passive, and for a successful result the eye may be entirely blind or the individual incapable, for any reason, of giving any information as to his visual sensations.

These methods are called *objective* in contradistinction to the *subjective* method, in which we must depend upon the expression of the sense of the person being examined for our knowledge and guidance.

The sense of vision is the manifestation of a judgment as to impressions made on the retina, and objects are recognized or *seen* by a mental interpretation of the images formed there by the dioptric apparatus of the eye. If these images are blurred, the visual sensation is indistinct. Since the far point of an eye and its retina are at conjugate foci, an object can have its image distinctly formed on the retina only when it is situated at that far point, or, expressed conversely, when the image is clear and distinct we know that the object is at the far point. When every point of an object is represented by another point in its image clearly separated from every other point, the image as a whole will be distinctly and sharply defined in all its parts. As this condition is departed from, and in proportion, does the image become blurred, from an overlapping of the diffusion circles of adjacent points.

There is, however, a natural limitation to the ability of the visual judgment to separate impressions which are made on the retina close to each other, and there is a distance between adjacent points within which they cease to be interpreted as two and are merged into a single sensation.

It has been found from an examination of a large number of normal eyes that this limit is reached when the distance separating the two impressions falls below that which subtends an angle of one minute. This is shown in Fig. 22, where *a* and *b* are the extreme points of the image of an object *A B*. If *a* and *b* be more closely approximated than this there will be only one impression and *A B* will not be recognized as an object with dimensions, but only as a single point.

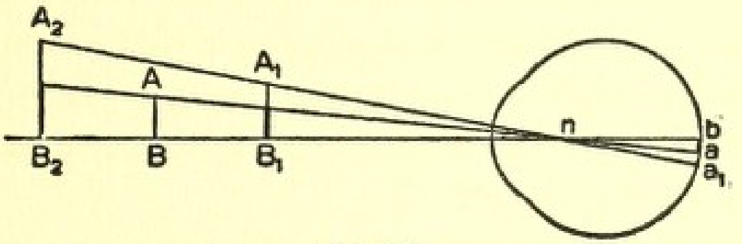

Fig. 22

The distance separating *a* and *b* is measured by the angle *a n b* (*n* being the nodal point of the eye). When this angle falls below 1′ *a* and *b* are no longer distinguished as two impressions and, therefore, *A B* is not recognized as an object, but as a single point. This angle is called the *visual angle*, and one minute, marking, as it does, the limit of perception of an object, is said to be the *smallest* visual angle. It is impossible to measure *a b* and its angle within the eye; but the angle *A n B* being the opposite angle, is equal to *a n b*, and this is easily measured by the size of the object *A B* and its distance from *n*.

It will be seen that the size of the visual angle is dependent not alone upon the size of the object, but also on its distance from *n*. Thus, when the object *A B* is brought closer to the eye and is found at A_1 B_1, the visual angle is increased to a_1 *n b* and is of the same size as that of an object

A_2 B_2 much larger, but farther removed. Two factors, then, enter into an estimation of visual acuteness : the size of the object and its distance from the eye.

Objects used for testing visual acuteness are constructed in accordance with one or the other of these demands as a basis ; either the object remains of the same size and its distance from the eye is altered, or a certain definite distance is chosen and the size of the test object is varied.

The latter has been found to be the most convenient in practice, and the *distance chosen is infinity*. The main reason for this choice is that this is the far point of the emmetropic eye and it is generally our object, in making their correction, to bring the far point of all forms of ametropia to this same place, and so the testing of visual acuteness and the reduction to emmetropia can be accomplished at the same time. A series of test objects, usually in the form of letters, has been formed on the basis of this method. These vary in size, the letters as a whole subtending an angle of 5 minutes, and each line and space entering into its composition an angle of 1 minute at a certain specific distance.

The letters are designated by the distances at which they subtend this angle. The one so subtending it at 5 meters is called No. 5, that so subtending it at XL feet No. XL, etc., according as the metric or foot system is used.

The degree of visual acuteness is most conveniently expressed in fractions, the formula being $V = \frac{d}{D}$, in which the numerator d gives the distance at which the examination is made, and the denominator D the size of the test object as measured by the distance at which it subtends an angle of 5 minutes. For example, if No. 10 is the smallest line of letters distinguished at 5 meters distance, $V = \frac{5}{10}$, since the eye sees at a distance of 5 meters what a normal eye should see at 10 meters. If No. XL is seen at XX feet, $V = \frac{XX}{XL}$. If No. 5 should be seen at 6 meters V would be expressed by $\frac{6}{5}$. These fractions are not usually reduced to their lowest terms, though it may be done, thus:

$$\frac{5}{5} = \frac{1}{1} = 1; \quad \frac{XX}{XL} = \frac{1}{2} = 0.5; \quad \frac{XX}{X} = \frac{2}{1} = 2.$$

CHAPTER IX.

Formulae for Conjugate Foci.

In order that the action of the laws governing the refraction of the eye, in the various manners described in the foregoing chapters, may be brought together under some general mathematical formulæ, applicable to all possible conditions, we give a construction and demonstration based on the simple exposition for conjugate foci according to Gavarret (*Les images par reflexion et refraction*, Paris, 1866).

Let F, Fig. 23, be the anterior principal focal point and F_1

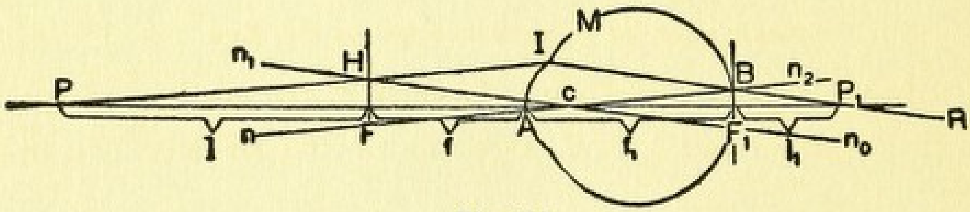

Fig. 23

the posterior focal point for parallel rays of the refracting surface M. Let P be any point situated on the principal axis passing through A the apex of the refracting surface M with its center of curvature at C. Any incident ray from P cutting the anterior focal plane at H and falling on the refracting surface at I, will be refracted in the direction $I\ R$ parallel to the secondary axis $n_1\ n_0$ passing through H. This will cut the posterior principal plane drawn through F_1 at B, where the secondary axis $n\ n_2$, parallel to the incident ray $P\ I$, meets the posterior focal plane. The refracted ray $I\ R$ meets the principal axis at P_1, and P_1 then becomes the image of P.

Since the incident ray $P\ I$ meets the anterior principal plane drawn through F at H at the same point as the secondary axis $n_1\ n_0$ parallel to the refracted ray $I\ R$, we have the similar triangles: PHF and CBF_1, which give $\frac{PF}{CF_1} = \frac{HF}{BF_1}$, The similar triangles CHF and P_1BF_1 gives $\frac{CF}{P_1F_1} = \frac{HF}{BF_1}$. As the second members of these two equations are the same, $\frac{PF}{CF_1} = \frac{CF}{P_1F_1}$, from which $PF \times P_1F_1 = CF_1 \times CF$.

But since the difference between the focal distances AF_1 and AF is equal to the radius AC, the distance of the center C from the posterior focus F_1 is equal to the anterior focal distance AF, and the distance of the center from the anterior focus F is equal to the posterior focal distance AF_1; hence, $CF_1 = AF = f$, and $CF = AF_1 = f_1$. Therefore, since $PF \times P_1F_1 = CF_1 \times CF$, by substituting values we have $PF \times P_1F_1 = ff_1$. If we designate PF by l and P_1F_1 by l_1, we have $ff_1 = ll_1$, which becomes the common formula for all conjugate foci.

Application of these Formulae to the Eye.

In the application of these formulæ to the eye, we accept as fixed values the optical constants of the reduced eye of

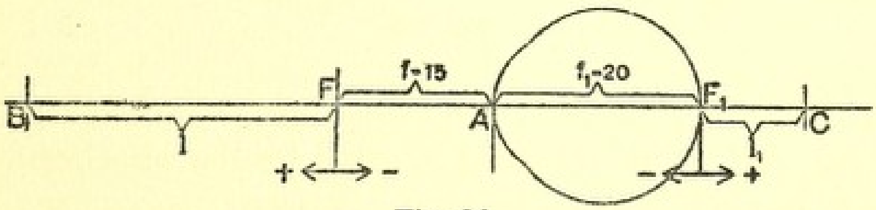

Fig. 24

Donders, where the posterior principal focal distance for parallel rays AF_1 (Fig. 24), $= f_1 = 20$ mm., and the anterior principal focal distance $AF = f = 15$ mm. B and C indicate the positions of other conjugate foci, and the relation of these to the principal foci F and F_1 have the values l and l_1. The varying values of l and l_1 are counted from F and F_1 in such manner that all values of l to the left of F are considered as positive (+), while all values to the right of F are negative (—). All values of l_1 to the left of F_1 are counted as negative (—), while all values to the right of F_1 are counted as positive (+). It will be seen that F_1 marks the position the retina occupies in emmetropia, being at the posterior principal focus for parallel rays.

The general formula for determining the values of l and l_1 being $l\,l_1 = f\,f_1$ (1) and the values of f and f_1 being fixed, any change in the value of l must be associated with a change in the value of l_1, and *vice versa*, and in accordance with the following formulæ, deduced from (1):

$$l = \frac{f\,f_1}{l_1} \text{ (2)}, \quad l_1 = \frac{f\,f_1}{l} \text{ (3)}.$$

Examples: $l = o$. Then $l_1 = \frac{f f_1}{l} = \frac{15 \times 20}{o} = \frac{300}{o} = \infty$; that is, when B is at the anterior principal focus F, the conjugate focus C is at infinity on the right. Similarly, when $l_1 = o$, C being at F_1, by formula (3) B or the other conjugate focus is found at infinity on the left.

When $l = \infty$, $l_1 = \frac{300}{\infty} = o$. B then recedes to infinity and C advances to F_1.

When l is positive, to the left of F, l_1 is also positive and to the right of F_1, and *vice versa*. When, however, l_1 is negative and to the left of F_1, l is also negative and to the right of F, and *vice versa*, and both are on the same side of the principal plane A.

Examples: $l = 200$ mm., $l_1 = \frac{300}{200} = 1.5$ mm., C is then positive and 1.5 mm. to the right of F_1 and $20 + 1.5 = 21.5$ mm. behind A. Conversely, when C is 21.5 mm. behind A, B or the far point will be 200 mm. in front of F, or 215 mm. in front of the principal plane A. This will correspond to the far point of a myopia at 4.5 D.

When $l_1 = -1$, C being 1 mm. to the left of F_1, $l = \frac{300}{-1} = -300$ mm.; B, the far point, will then be 300 mm. to the right of F, that is, $-300 + 15 = -285$ mm. behind A, which represents the position of the far point of 3.5 D. of H.

When $l_1 = -20$ mm., C will find itself at A, the principal plane, and l will be $\frac{300}{-20} = -15$ mm. Since $F\ A$ is equal to 15, B will find itself also at A, and the image and object will be superposed.

It will be seen, from the examples given, that a variation of 1 mm. in the position of C in its relation to F_1, the position of the retina in E. brings an alteration in the position of the other conjugate focus or far point of about 3.5 D. When l_1 is positive, this far point will be in front of the eye, a condition of myopia; when it is negative, the far point will be behind the eye, the condition being hypermetropic.