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ELEMENTARY OPHTHALMIC OPTICS

J. HERBERT PARSONS

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ELEMENTARY OPHTHALMIC OPTICS

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ELEMENTARY OPHTHALMIC OPTICS

INCLUDING

OPHTHALMOSCOPY & RETINOSCOPY

BY

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1901

PREFACE

THIS book is intended to supply the student of ophthalmology with all the optics which is necessary for an intelligent knowledge of his subject. The proofs are in some cases a little confusing at first reading, but they never demand more than a very elementary acquaintance with mathematics. Even where a trigonometrical nomenclature has been inevitable the meaning of the signs is usually self-evident. Special care has been taken to emphasise all important propositions by italics, so that those who do not care to grapple with the proofs are provided with a readily accessible synopsis of results. A short bibliography of the best books upon the subject has been added, so that the advanced student may know where to look for further information.

It is hoped that the book will prove a useful addendum to such practical works as Morton's *Refraction of the Eye*, Hartridge's *Refraction*, etc.

CONTENTS

BIBLIOGRAPHY

The following are the works to which I am most indebted in the preparation of this book, and which are likely to prove most useful to the student:—

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CHAPTER I

LIGHT

LIGHT may be defined as the specific force which, acting upon the retina, produces the sensation called sight. The subjective sensation of sight may be elicited by other changes in the retina, as in the case of phosphenes, etc.; hence the qualification "specific."

Incandescent bodies, such as a flame or the sun, are called self-luminous; and they emit certain vibrations which, passing into the eye, affect the retina and cause objects to be seen. They have also the property of rendering other objects, which are not self-luminous, visible.

Light travels through a homogeneous medium in straight lines. A luminous body emits light in all directions, and it is often convenient to consider the portion of light which travels along some particular line in space apart from all the rest: such a portion of light is called a *ray*. A collection of such rays, which during their course do not deviate far from some fixed central ray, is called a *pencil*, and the central ray is called the *axis* of the pencil. If the rays of a pencil meet in a point, this point is called the *focus* of the pencil.

If an opaque body lies in the area of illumination it will cut off certain of the rays of light.

If B be a luminous point, and A an opaque body, then a *shadow*, EF, will be cast upon the screen, S. As, however, most luminous bodies are of appreciable size, if CD be a luminous body, then A will cause a

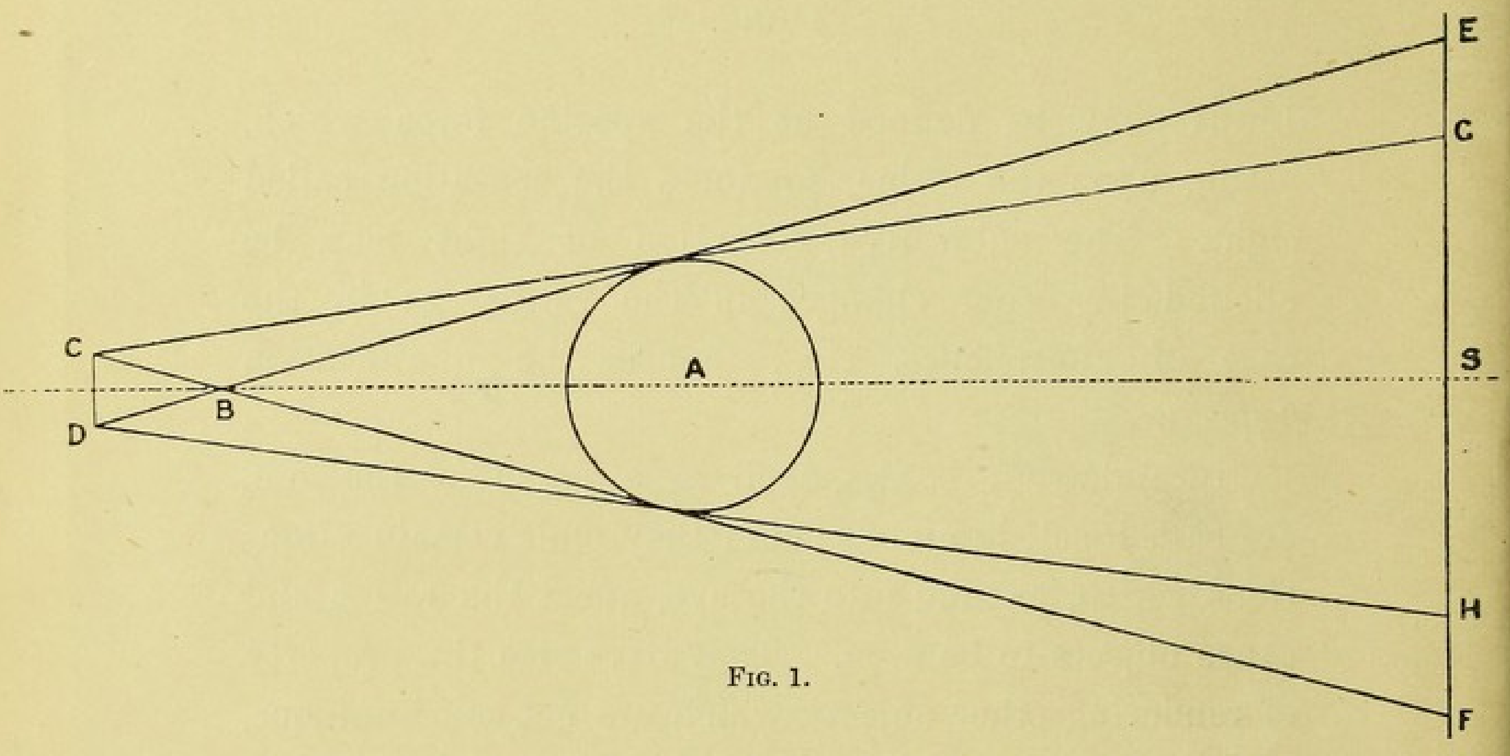

FIG. 1.

shadow of the light coming from C at GF, and a shadow of the light coming from D at EH, and similarly for all intermediate points. The areas EG, HF will continue to receive rays of light from CD, and will therefore still be partially illuminated. GH is called the *umbra* or *total shadow*, and EG, HF, the *penumbra* or *partial shadow*.

For the explanation of the phenomena of light, physicists have introduced the conception of an hypothetical medium, *ether*, the peculiar undulations of which constitute the phenomena. This is the Undulation or Wave Theory, which explains the phenomena more accurately than the Corpuscular or Emission Theory, which previously held the field.

Ether is an imponderable entity pervading space, possessed of hypothetical physical properties. It is of importance in speaking of "particles" of ether to remember that we are dealing with a useful imaginary conception, and that such particles are not to be confounded with the atoms and molecules with which we are familiar in chemistry.

A simple comparison will enable us to obtain some idea of the movements of the particles of ether along a ray of light. Imagine a thread hanging vertically, and held at the upper end by the hand. If the hand is moved to and fro the thread will assume a wavy form. In the waves which traverse the thread from above downwards each individual particle of the thread always remains at the same height above the ground, but at this level it will move in straight lines or in circular or elliptical paths around its centre of gravity, according as the hand is moved in straight lines or curves. The movements of the particles of ether along a ray of light are exactly similar. The particles remain in the immediate neighbourhood of their original place of rest, and move in straight or curved lines around this point, the direction of movement being transverse to the direction of the propagation of the light. What actually manifests itself as light is not the ether particles themselves, but only the wave-forms which they assume during their movement, with their varying *phases* of deviation and velocity.

As stated above, the planes of movement of the ether particles are perpendicular to the direction of propagation of the waves. It is especially in this respect that the waves of light differ from the waves of elastic fluids, *e.g.* waves of sound, in which the particles of fluid oscillate parallel to the direction of propagation.

If the path of the particles is straight, the light is said to be *plane polarised;* if circular or elliptical, *circularly or elliptically polarised.* Ordinary light, which is a combination of all kinds of polarised light, is said to be *unpolarised.*

If every particle of ether repeatedly traverses exactly the same path in the same time, the light is said to be *simple, monochromatic, or homogeneous,* and the time in which it traverses its path once is called its *period of vibration.* The *wave-length* is clearly a function of this period of vibration. The chief peculiarity distinguishing light of different wave-lengths is colour. Ordinary light from the sun and luminous bodies, which is a combination of all kinds of wave-length within certain limits, is called *mixed* light. The limits of wave-frequency are from 400 to 700 billions a second, according to the colour of the light.

The velocity of propagation of light is extremely great. For space it has been estimated by astronomical observations at about 300,000 kilometres or 186,000 miles a second. In transparent substances it is less according to their density, and is not quite the same for light of different wave-lengths. This affords a means of splitting up mixed light into its coloured constituents, a process known as *dispersion*, the resultant band of colours being known as a *spectrum*.

When a ray of light, traversing one medium, meets the surface of another medium, it usually breaks up into three parts:—

1. Part is *reflected* back into the first medium.
2. Part is *refracted* into the second medium.
3. Part is *scattered or diffused* by the surface bounding the two media, so that this surface becomes illuminated, and acts itself as a source of light.

If the new medium is opaque none of the light is refracted. Scattering is due to unevenness of the surface, so that different rays are reflected and refracted in innumerable directions, some of which enter the eye, so that the surface appears luminous.

With non-metallic bodies the amount of light reflected increases with the obliquity of the incident light. This is not the case with metallic bodies; from which fact it is probable that the velocity of propagation of light through a thin metallic lamina is extremely slow.

The amount of light which is reflected also depends upon the difference in density of the two media. This sometimes leads to a wrong diagnosis of cataract in old people. In old age the refraction of the cortical layers of the lens increases, so that more light is reflected from the surface. As the lens substance is non-metallic, the amount of reflected light is still further increased by increasing the obliquity of the incident light. Consequently, by oblique illumination these lenses often appear to be opaque, or to have opaque striæ in them. A correct diagnosis is ensured by noticing that light is readily transmitted through them by illumination with the ophthalmoscope.

We have now to consider the laws which govern the reflection and refraction of light.

CHAPTER II

REFLECTION

ANY ray meeting the surface between two media is called an incident ray. The plane containing the incident ray and the normal (or perpendicular) to the surface is called the plane of incidence, and the acute angle between the incident ray and the normal is called the *angle of incidence;* and the acute angle between the reflected ray and the normal is called the *angle of reflection.* If the direction of the incident ray is changed, the angle through which this ray produced must be turned in order to assume the new position is called the *deviation* of the ray.

For all surfaces the angle of incidence is equal to the angle of reflection, and is in the same plane with it.

These laws can readily be proved experimentally by a method which forms the basis of the "artificial horizon" used in astronomy. The consistency of the results obtained thus is a very severe test of the accuracy of the laws.

The following is a simple explanation founded upon the wave-theory.

Let AB, PQ be parallel rays incident to the surface at B and Q. QD is the wave-front, which will be perpendicular to the direction of propagation of the light. As the successive points of *QD* reach the reflecting surface, hemispherical waves will diverge in all directions from the points of incidence. By the time *D* has reached *B*, the wave from *Q* will have diverged in all directions to a distance equal to *DB*. If a semicircle be described in the plane of incidence, with centre *Q*, and radius equal to *DB*, the

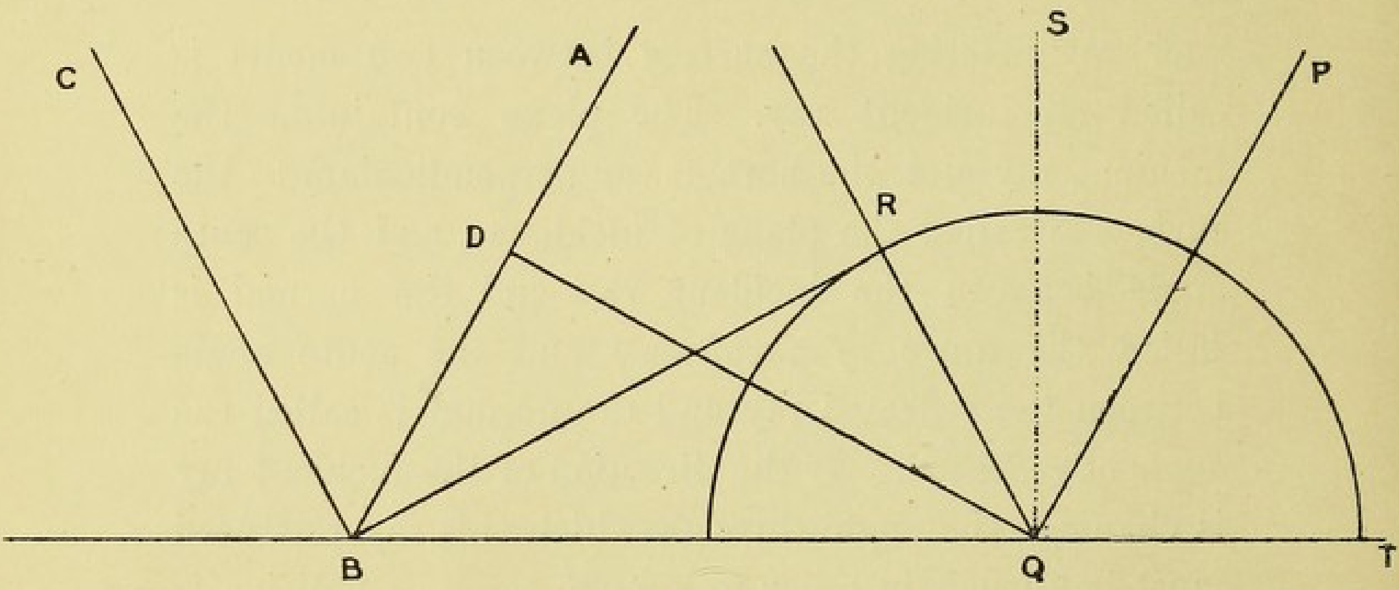

FIG. 2.

tangent *BR* will be the wave-front of the reflected light, and *QR* will be the reflected ray of the incident ray *PQ*.

From the equality of the right-angled triangles *QDB*, *BRQ*, the angle *RQB* is equal to the angle *DBQ*.

But since *PQ* is parallel to *AB*, the angle *DBQ* is equal to the angle *PQT*.

Therefore the angle *RQB* is equal to the angle *PQT*.

And if *SQ* is perpendicular to *BT*, the angle *RQS* must be equal to the angle *PQS*. That is, the angle of reflection is equal to the angle of incidence.

The following is a useful geometrical method for finding the reflected ray.

Let P be any point on a ray PQ incident at Q to the surface AB.

Draw PN perpendicular to AB, and produce it to P', so that PN is equal to $P'N$.

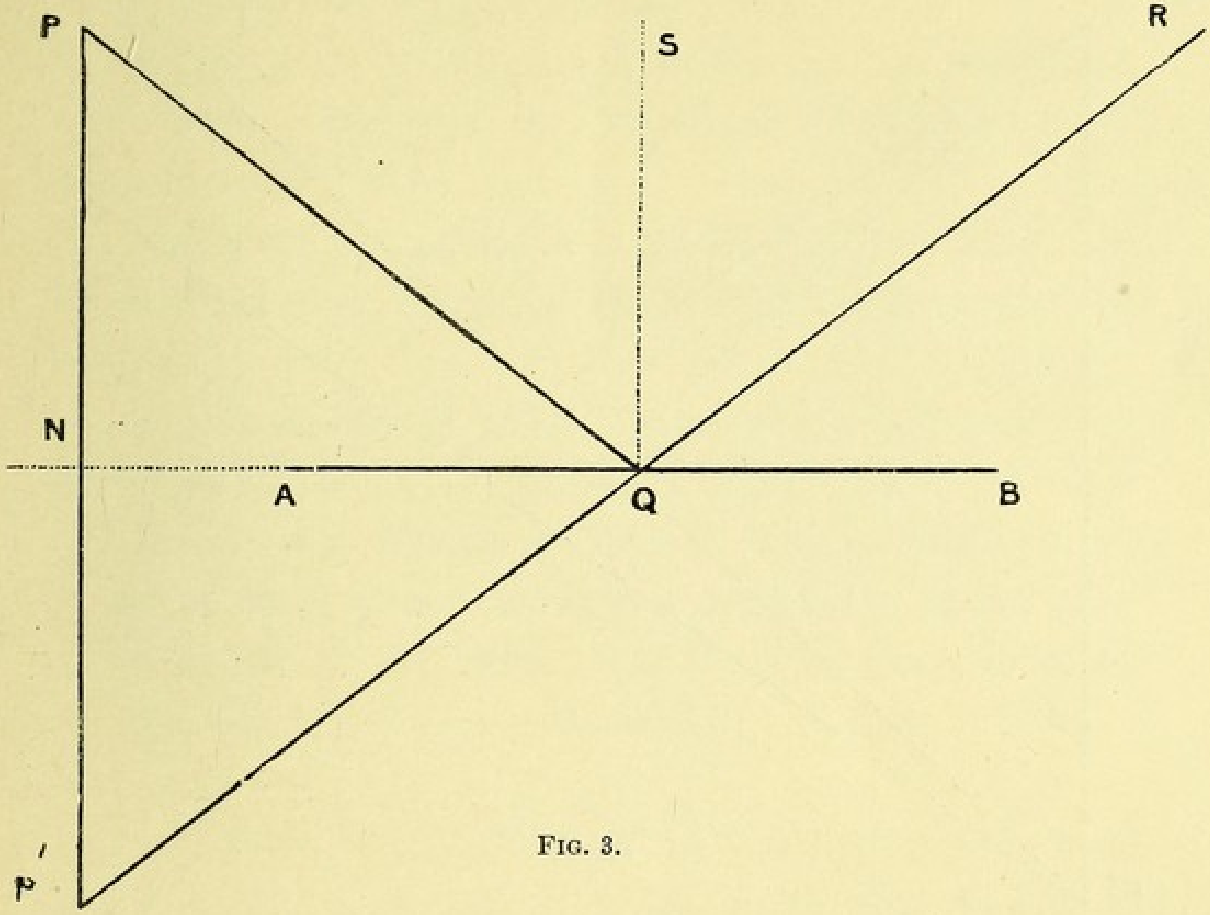

Fig. 3.

Join $P'Q$ and produce it to R.

Then, in the equal right-angled triangles PNQ, $P'NQ$, the angle PQN is equal to the angle $P'QN$.

But the angle $P'QN$ is equal to the angle RQB.

Therefore, PQ and RQ make equal angles with the plane AB, and therefore with the normal to it, SQ.

Therefore QR is the reflected ray.

The same construction applies for any curved surface, if *AB* represents the tangent plane at the point *Q* of the curve.

It is clear that if *RQ* be considered the incident ray, *QP* will be the direction of the reflected ray. It is indeed a universal rule that *the path of a ray of light is reversible.*

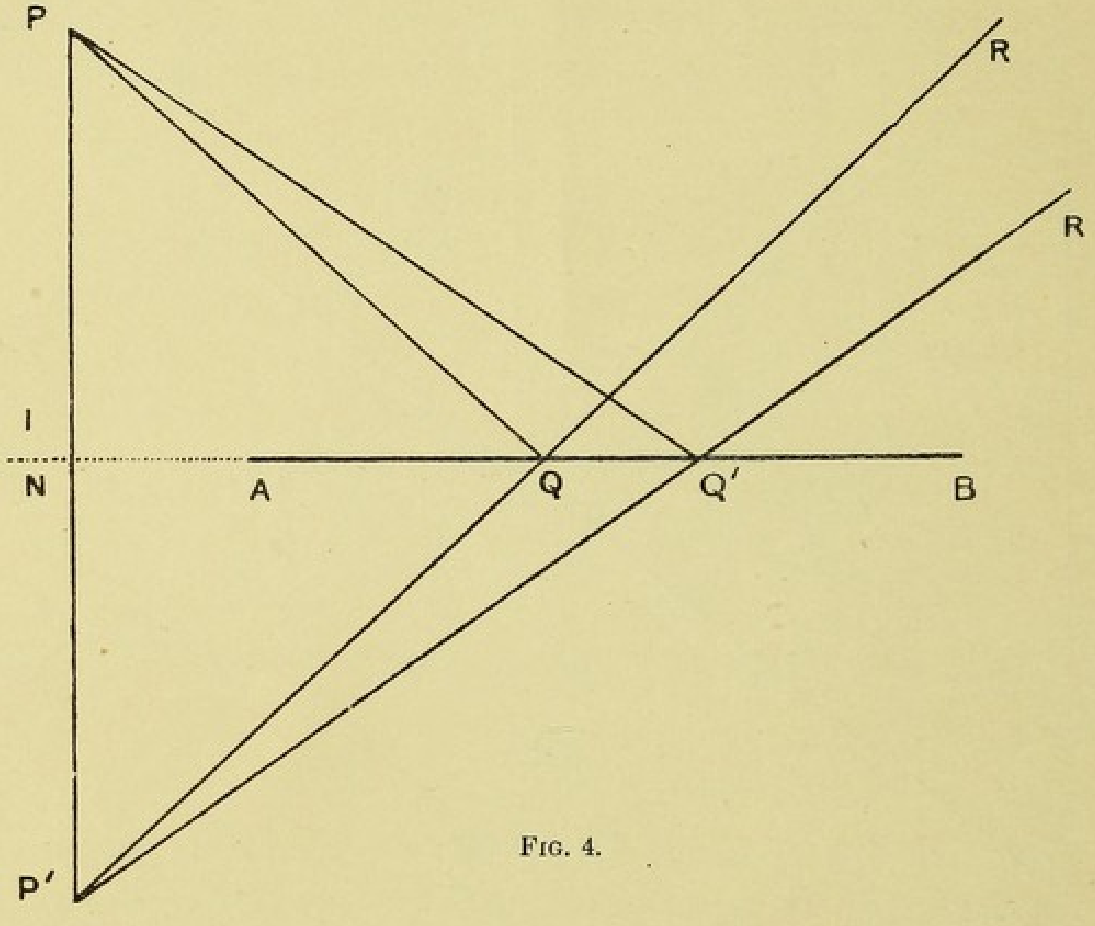

Fig. 4.

Plane Mirrors

The foci of the incident and reflected pencils lie on the same perpendicular to the mirror, at equal distances from it, on opposite sides.

Let *P* be a luminous point, situated in front of a plane mirror *AB*.

Let *PQ* be any incident ray from *P*, and *QR* its course after reflection.

Draw PN perpendicular to the mirror and produce it to meet RQ produced, as at P'.

Then the angle RQB is equal to the angle PQN.

But the angle RQB is equal to the angle $P'QN$.

Therefore the angle $P'QN$ is equal to the angle PQN; and the triangles PQN, $P'QN$ are equal in all respects, so that $P'N$ is equal to PN.

The position of P' is independent of the particular ray chosen, and will be the same for PQ' and its reflected ray $Q'R'$.

Therefore the pencil of rays from P will have the same form after reflection as it would if it originated in P'. [*Q.E.D.*]

It follows that P' is the image of P; and similarly for all other points of an object situated in front of a plane mirror. Hence it follows that *the image of an object in front of a plane mirror is situated at the same distance behind the mirror that the object is in front of it, and is equal in size to the object.*

If a plane mirror is rotated in the plane of incidence, the direction of the reflected ray is changed by double the angle through which the mirror is turned.

Let AB be a plane mirror perpendicular to the plane of the paper, and capable of rotation round an axis through Q, also perpendicular to the plane of the paper. Let PQ represent an incident ray.

If PQ is perpendicular to AB, the ray will be reflected along its own course.

If the mirror is rotated through an angle AQA' ($=\theta$), and NQ is the normal in the new position, then the direction of the reflected ray QR will be such that the angle PQN is equal to the angle NQR.

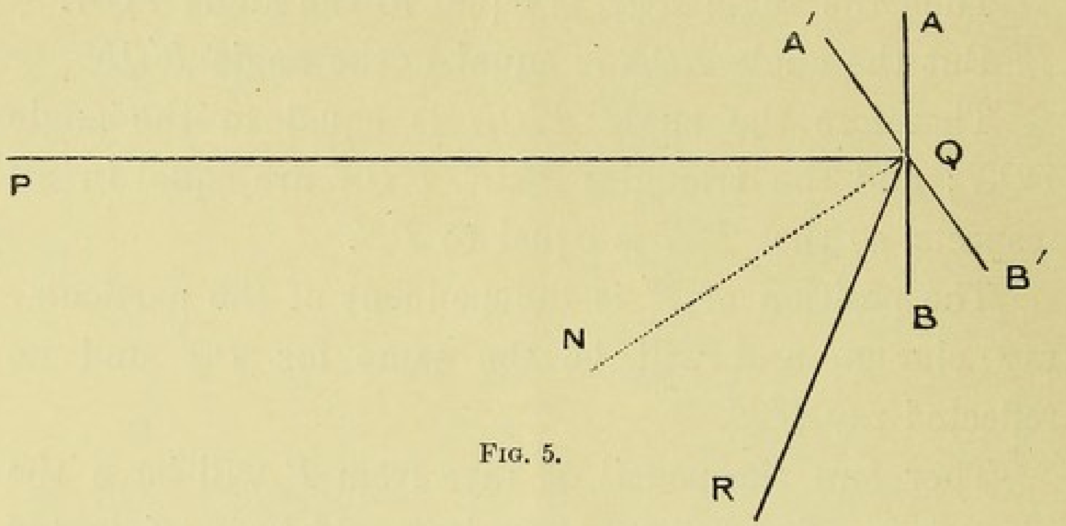

FIG. 5.

But the angle PQN is equal to the angle $AQA' = \theta$.

Therefore the deviation of the reflected ray is equal to 2θ.

SPHERICAL MIRRORS

Reflection of a Small Pencil at a Spherical Surface

Let P be a luminous point, the focus of a small pencil incident upon AH, a concave spherical mirror,

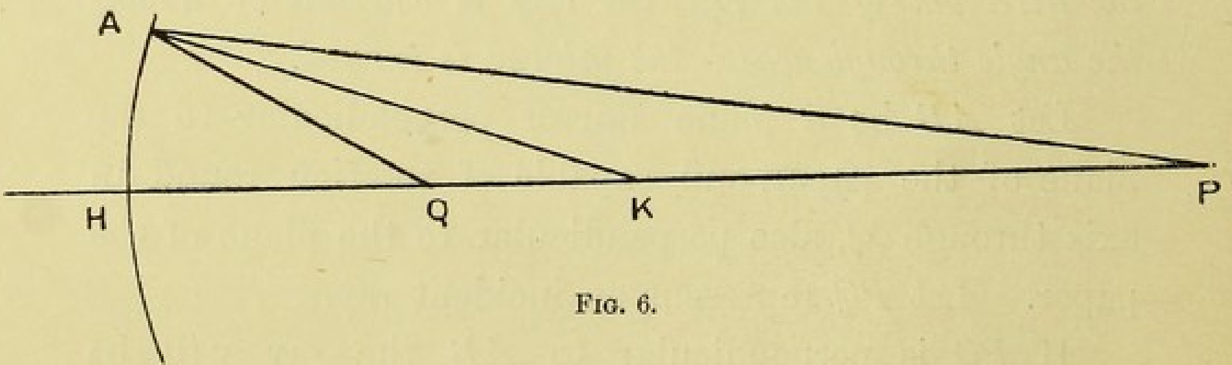

FIG. 6.

of which K is the centre and H the vertex. Then PKH is the axis of the pencil. Let PA be any incident ray, which after reflection cuts the axis in Q.

Let $PH = f_1$, $QH = f_2$, and the radius of the sphere, $KH = r$.

By the law of reflection, the angle PAK is equal to the angle KAQ.

Therefore $PA : AQ = PK : QK$ [Euc. vi. 3.

Since, by hypothesis, the pencil is small, AH is small; and PA will be practically equal to PH, and QA equal to QH.

Therefore, for this approximation,

$$PH : QH = PK : QK.$$

That is, $$f_1 : f_2 = f_1 - r : r - f_2,$$

or $$f_1 (r - f_2) = f_2 (f_1 - r)$$

$$f_1 r - f_1 f_2 = f_1 f_2 - f_2 r$$

$$f_2 r + f_1 r = 2 f_1 f_2.$$

Divide by $f_1 f_2 r$.

Therefore $$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{r} \quad . \quad . \quad . \quad (1).$$

This formula determines the position of the point Q, in which the reflected ray cuts the axis; and shows that for small pencils all the rays from P are reflected to Q. These points are called *conjugate foci*, and *the line joining them passes through the centre of the mirror.*

If the incident rays are parallel to the axis, so that P is at infinity, then $f_1 = \infty$.

Therefore $$\frac{1}{\infty} + \frac{1}{f_2} = \frac{2}{r}.$$

Therefore $$f_2 = \frac{r}{2} = F.$$

That is, parallel rays are brought to a focus at the middle point of *KH*, which is called the *principal focus* (ϕ) of the mirror. In other words, *the principal focal distance* (*F*) *is half the radius of curvature.*

Since *F* depends only upon the curvature of the mirror, we may write (1).

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F} \quad . \quad . \quad . \quad . \quad (2).$$

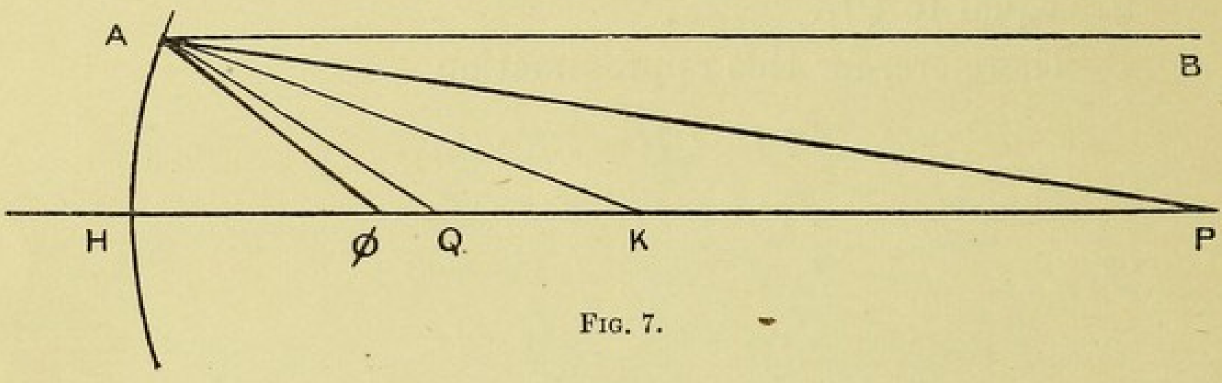

FIG. 7.

If l_1, l_2 are the distances of a pair of conjugate points measured from the principal focus, ϕ, so that $l_1 = P\phi$ and $l_2 = Q\phi$ in Fig. 7, then

$$l_1 = f_1 - F \quad \text{and} \quad l_2 = f_2 - F.$$

But, by (2)

$$f_1 f_2 = F(f_1 + f_2).$$

Therefore $\quad (f_1 - F)(f_2 - F) = F^2.$

That is, $\quad l_1 l_2 = F^2 \quad . \quad . \quad (3).$

It must be noticed that all these quantities, f_1, f_2, r, etc., are algebraical symbols, which in addition to signifying numbers also denote by their signs the directions in which the quantities are measured. On this understanding the formulæ are of universal application, and apply equally to convex mirrors.

From formula (1), if *P* moves towards the mirror

f_1 diminishes and f_2 increases until $f_1 = f_2 = r$, and the foci coincide at the centre of curvature. This is clearly correct, for the rays are now normal to the mirror, and are therefore reflected along their own course. If f_1 still diminishes, f_2 still increases, and P simply takes the place of Q, and *vice versâ*. If f_1 becomes less than F or $\frac{r}{2}$, then $\frac{1}{f_1}$ is $> \frac{1}{F}$ or $\frac{r}{2}$, and therefore $\frac{1}{f_2}$, and hence also f_2, must be negative. In this case the conjugate focus is behind the mirror, and the reflected waves diverge as if they had come from this point. This is called a *virtual* focus, in contradistinction to a *real* one, in which the rays actually meet.

If the mirror is convex, then the radius is measured in the opposite direction from H, and is negative. Therefore either f_1 or f_2 must always be negative, that is, the conjugate focus must always be behind the mirror and virtual. This can be deduced geometrically by a similar construction to that used for a concave mirror.

The importance of formula (3) is shown by the fact that the whole theory of reflection of a pencil of convergent or divergent rays from a spherical surface, whether concave or convex, can be stated by the help of the principal focus: thus, a *pair of conjugate foci always lie on the same side of the principal focus, and at distances* l_1, l_2 *from it, such that* $l_1 l_2 = F^2$, *where* $F = \frac{r}{2}$.

If the axis be turned through a small angle about the centre of curvature, the points P and Q will describe lines which are approximately straight. If the whole system be rotated round the axis these lines will describe planes. Hence a small plane object at P will have a plane image at Q. The planes described by P and Q are called *conjugate planes;* that described by H is called the *principal plane;* and that described by ϕ, the *focal plane.*

To construct the Image formed by a Spherical Mirror

From what has already been proved we know the paths of three rays originating in any point P:—

1. The ray PA, parallel to the axis, passes after reflection through the principal focus, ϕ.

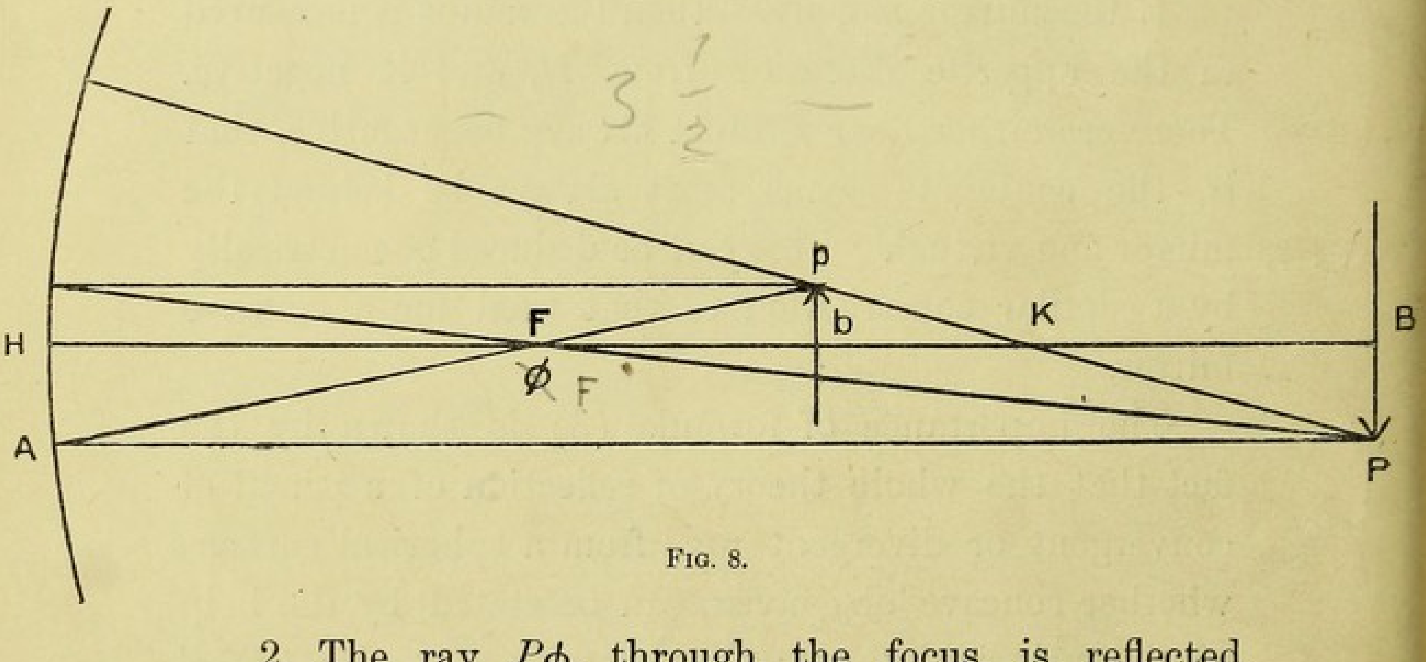

Fig. 8.

2. The ray $P\phi$, through the focus, is reflected parallel to the axis.

3. The ray PK, through the centre of curvature, is reflected upon itself.

The intersection of any two of these three will define the image, p, of the point P; and similarly for all other points in the object.

The Size of the Image

To simplify the figure, we will consider only that part of the object upon one side of the axis. It is clear that the same proof holds good for the part upon the other side, and therefore for the whole object.

Let o be the size of the object PB, and i the size of the image pb.

Then, from the similar triangles KBP, Kbp,

$$PB : pb = BK : bK.$$

Therefore $$\frac{o}{i} = \frac{f_1 - r}{r - f_2} \quad . \quad . \quad (1).$$

or, *the sizes of object and image are directly as their distances from the centre of curvature.*

Again, in Fig. 6,

$$PK : QK = PA : AQ$$
$$= PH : QH \text{ (approximately).}$$

Therefore $$\frac{o}{i} = -\frac{f_1}{f_2} . \quad . \quad . \quad (2).$$

The negative sign is introduced to show that the o and i are measured upon opposite sides of the axis. Therefore, *the sizes of object and image are directly as their distances from the mirror.*

Since o and f_1 are always positive, i can only be positive when f_2 is negative, or *vice versâ:* that is, all erect images are virtual, and all inverted images are real.

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From formula (2), *p.* 14.

$$\frac{1}{f_2}=\frac{1}{F}-\frac{1}{f_1}=\frac{f_1-F}{f_1F}.$$

Therefore $$\frac{f_1}{f_2}=\frac{f_1-F}{F}=\frac{l_1}{F}.$$

Therefore, from (2), *p.* 17.

$$\frac{o}{i}=\frac{F-f_1}{F}=-\frac{l_1}{F} \quad . \quad . \quad (3),$$

$$=-\frac{2l_1}{r} \quad . \quad . \quad (4).$$

Similarly, $$\frac{o}{i}=\frac{F}{F-f_2}=-\frac{F}{l_2} \quad . \quad . \quad (5).$$

From (3) or (5), if the mirror is concave so that F is positive, the image is erect and virtual when f_1 is less than F, and the image is then always larger than the object. If the mirror is convex, so that F is negative, the image is always erect and always smaller than the object.

Formula (4) is used in ophthalmometry.

The relative sizes of object and image can also be stated in terms of the divergence of the rays, that is, of the angle (a) between any two rays from a given point of the object and of the angle (a') between the same two rays after reflection.

Let PA be a ray incident at the point A upon a convex spherical mirror, the centre of which is K. Then, in the figure, θ is the angle of incidence and of reflection, a is the original divergence of the incident ray from the axis, and $-a'$ the angle through which the ray diverges from the axis after reflection.

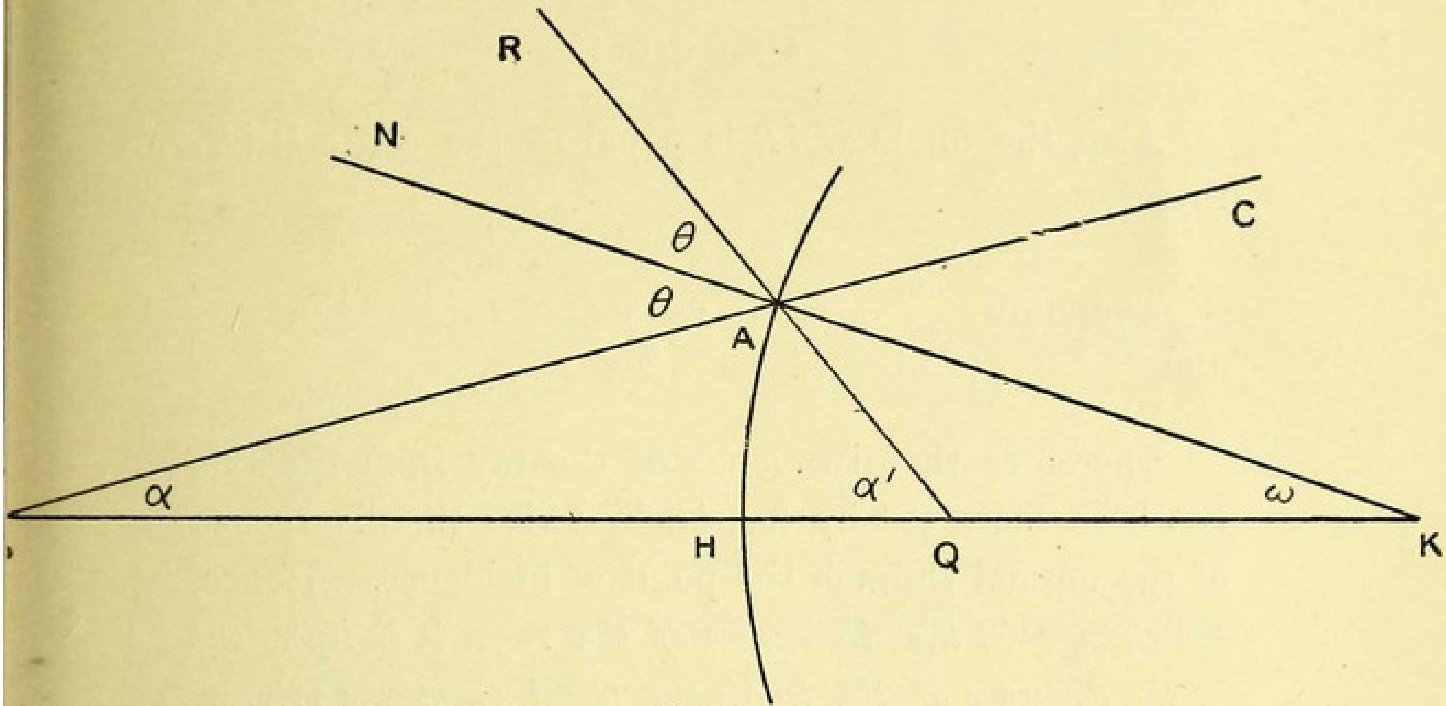

FIG. 9.

Then, since AH is approximately straight and perpendicular to the axis,

$$\frac{AH}{PH} = \tan a.$$

Therefore $\qquad AH = f_1 \cdot \tan a.$

Similarly, $\qquad AH = -f_2 \cdot \tan a'.$

Therefore $\qquad \dfrac{f_1}{f_2} = -\dfrac{\tan a'}{\tan a},$

or, for a concave mirror,

$$\frac{f_1}{f_2} = \frac{\tan a'}{\tan a}.$$

Therefore, from (2) p. 17.

$$\frac{o}{i} = -\frac{\tan a'}{\tan a} \qquad . \qquad . \qquad . \quad (6).$$

It will be seen from Fig. 9 that since the angle *NAP* is the exterior angle of the triangle *APK*, therefore

$$\theta = a + \omega.$$

Also, the angle *NAR* is equal to the angle *QAK*, therefore

$$a' = \theta + \omega.$$

Therefore $a' = a + 2\omega,$

and $a' - a = 2\omega.$

Therefore the divergence of the ray from the axis has been increased by 2ω, *i.e.* by twice the divergence of the normal through the point of incidence. Hence, *the change in the divergence of the rays is independent of the distance of the luminous object from the mirror.* It is the same for every incident ray through *A*.

If the rays were reversed, so that *RA* was incident, the divergence becomes a convergence, and the diminution is equal to 2ω.

If the mirror is concave, and *QA* represents the incident ray, reflected to *C*, a' becomes the angle of divergence before, and a after, reflection; and the divergence is diminished by 2ω. If *CA* is the incident ray, the convergence is increased by 2ω.

If the mirror is plane, $\omega = O$, and the divergence of the rays is not altered by reflection.

These facts were first pointed out by von Helmholtz.

We may summarise the chief results which can be deduced algebraically from the formulæ or geometrically by construction.

For Concave Mirrors

1. The image of an object beyond the centre of curvature is situated between the centre and the principal focus. It is real, inverted, and diminished.

2. The image of an object between the centre and the principal focus is situated beyond the centre. It is real, inverted, and magnified.

3. An object situated between the principal focus and the mirror has its image behind the mirror. It is virtual, erect, and magnified.

4. An object at the centre of curvature, or at the mirror, has its image identical with itself.

5. An object at the principal focus has an infinitely large image situated at infinity.

For Convex Mirrors

The image is always virtual, erect, and diminished.

It must be well understood that the formulæ for spherical mirrors are only approximately correct, and only apply to small pencils of light. Only parallel rays near the axis are at all accurately reflected to the principal focus. In a parabola, which is the section of a cone by a plane parallel to one of the sides, any line parallel to the axis of the curve meets the line joining the focus to the point of contact at an angle which is bisected by the normal. That is, if the curve in Fig. 7 were a parabola, the angle BAK would always be equal to $KA\phi$, irrespective of the distance of BA from the axis. Hence it is only when the curve of the mirror is a paraboloid of revolution that all incident parallel rays are reflected to one point, which is also the mathematical focus of the curve. In the case of a sphere, the reflected rays cut the axis at gradually increasing distances from the focus as the incident parallel rays are more distant from the axis. This is called *spherical aberration*, and is the cause of blurring of the outer parts of the image of a large object.

In order that all the rays from a given luminous point may be brought to a focus at a *real* conjugate point, the curve of the mirror must be an ellipsoid of revolution around the major axis, the points being the foci of the ellipse. From this it will be seen that the curve is a different ellipse for every distance between the foci.

In order that all the rays from a given luminous point may be brought to a focus without aberration at a *virtual* conjugate point, the curve of the mirror must be an hyperboloid of revolution about the axis.

Such reflecting surfaces, with no aberration, are called *aplanatic*.

CHAPTER III

REFRACTION

We have next to consider the behaviour of the portion of light which, travelling in one medium, meets the surface of another medium and passes into it. We are here dealing with an *incident ray* and the corresponding *refracted ray*. The acute angles which they make with the normal at the point of incidence are called respectively the *angles of incidence and refraction.*

It can be proved experimentally that *the angles of incidence and refraction always lie in the same plane, and their sines are to one another in a constant ratio.*

The following is a simple explanation of this proposition upon the wave-theory.

Let *PQ*, *AB* be parallel rays incident to the surface at *Q* and *B*. *QD* is the wave-front. As the successive points of *QD* reach the refracting surface, hemispherical waves will diverge in all directions from the points of incidence. By the time *D* has reached *B*, the wave from *Q* will have diverged in all directions to a distance less than *DB*, because we are considering the passage of the ray from a less dense into a denser medium. If a semicircle be described in the plane of incidence, with centre *Q*, and radius equal to the distance traversed whilst D travels to B, the tangent BR will be the wave-front of the refracted light, and QR will be the refracted ray of the incident ray PQ.

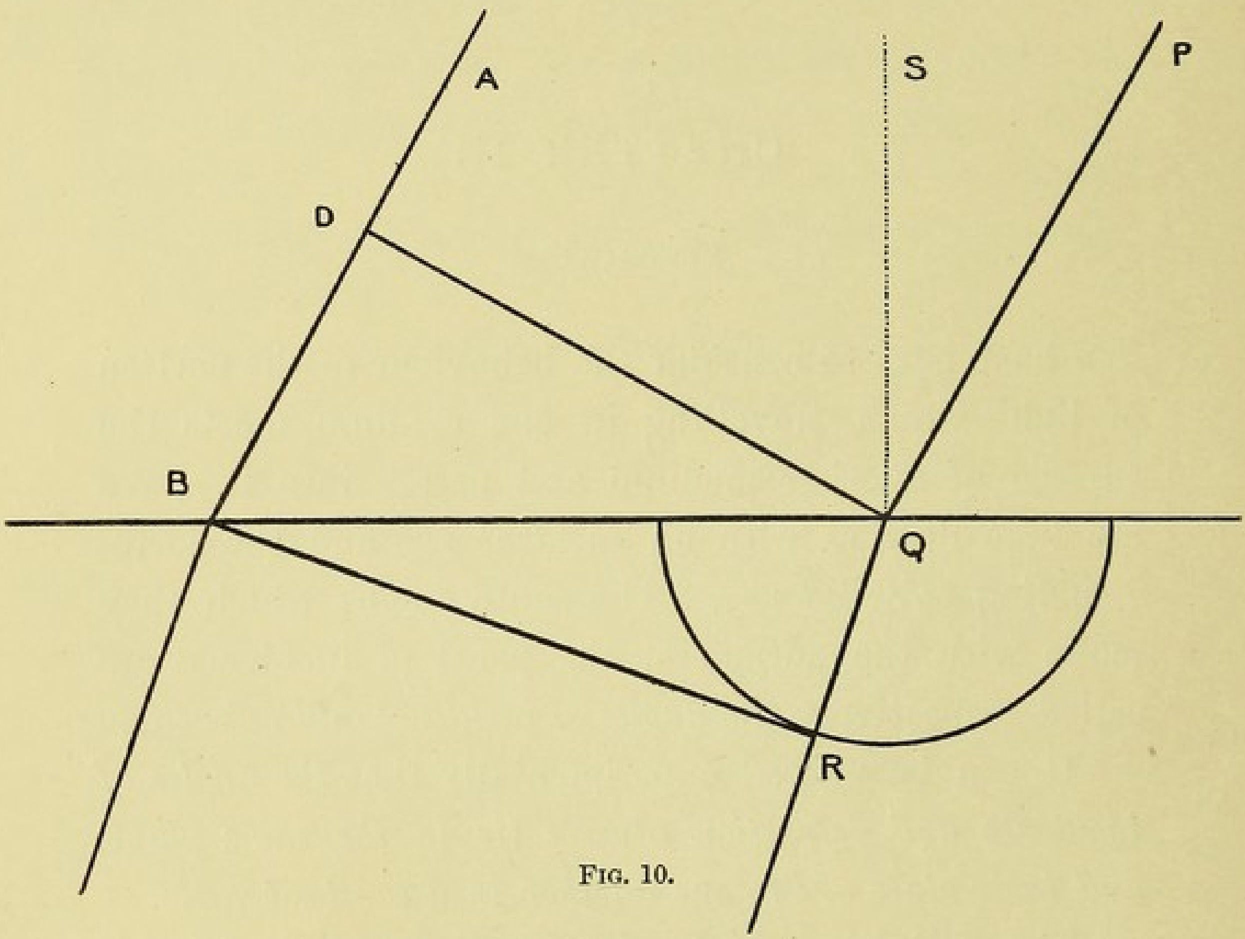

Fig. 10.

Since QDB and QRB are right-angled triangles, the angle DQB is equal to the angle PQS, *i.e.* the angle of incidence (ι); and the angle QBR is equal to the angle of refraction (ρ).

The sine of the former is $\frac{DB}{BQ}$, and the sine of the latter is $\frac{QR}{BQ}$.

Therefore $$\frac{\sin \iota}{\sin \rho} = \frac{DB}{QR} = N \quad . \quad . \quad (1),$$

and N is a constant for the given media. It is called the *index of refraction.* If the first medium is a vacuum it is called the *absolute index of refraction.* In other cases it is the *relative* refractive index for the given media.

The equation (1) may be stated in various forms, thus :—

(*a*) The sines of the angles of incidence and refraction are directly as the velocities of propagation of the incident and refracted light.

(*b*) The relative index of refraction is the ratio of the velocity of light in the first medium to its velocity in the second.

(*c*) The absolute index of refraction in any medium is inversely as the velocity of light in that medium.

Plane Laminæ

It can be shown by experiment, or deduced from the wave-theory, that *if a ray passes through any number of media, bounded by parallel planes, into its original medium, the initial and final directions of the ray are parallel to each other.*

Let $A, B, C \ldots\ldots$ be the media, $n_1, n_2, n_3 \ldots\ldots$ their relative indices of refraction, and $\theta_1, \theta_2, \theta_3 \ldots\ldots$ the angles of incidence at each refraction.

Then
$$\frac{\sin\theta_1}{\sin\theta_2} = n_1,$$
$$\frac{\sin\theta_2}{\sin\theta_3} = n_2,$$
$$\cdots\cdots$$
$$\frac{\sin\theta_m}{\sin\theta_1} = n_m.$$

By multiplication, therefore

$$n_1, n_2, n_3 \ldots\ldots n_m = 1 \quad . \quad . \quad (a).$$

If n, n' be the absolute refractive indices of two

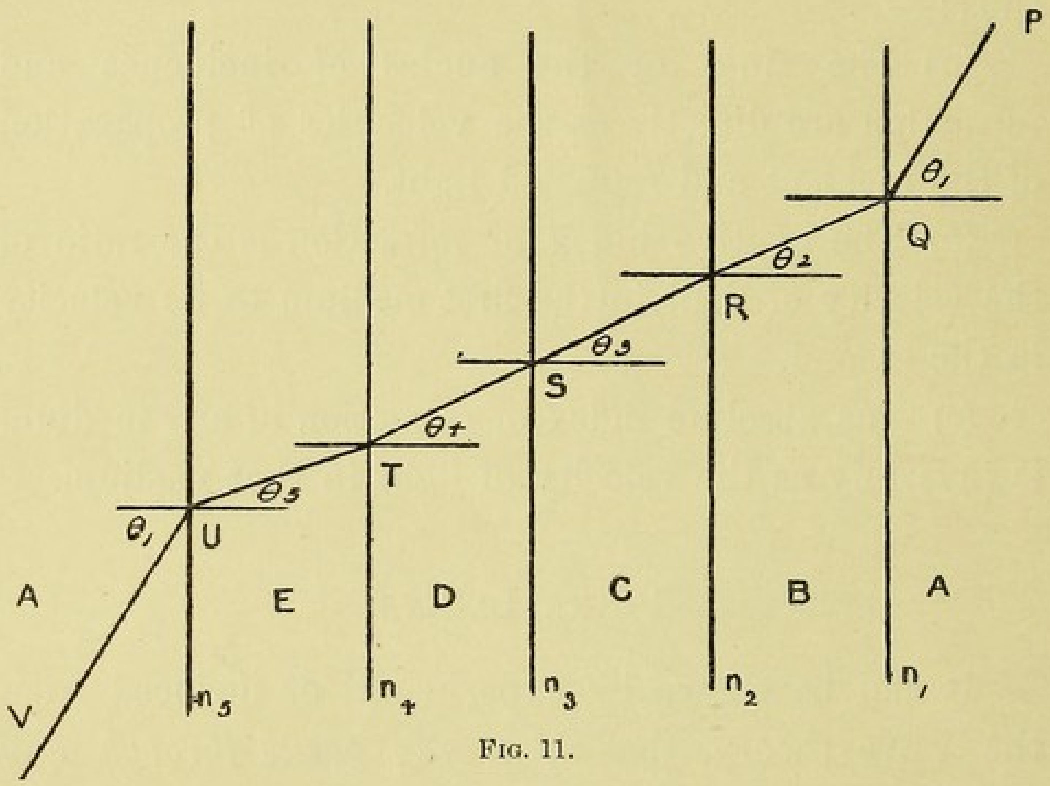

FIG. 11.

media, and N its relative refractive index, we have from (a)

$$n \,.\, N \,.\, \frac{1}{n'} = 1$$

Therefore $$N = \frac{n'}{n} \quad . \quad . \quad . \quad (2).$$

That is, *the relative refractive index between any two media is found by dividing the absolute refractive index of the second by that of the first.*

Hence, from (1),

$$\frac{\sin \iota}{\sin \rho} = N = \frac{n'}{n}.$$

Therefore $n \,.\, \sin \iota = n' \,.\, \sin \rho$ . . (3).

Several interesting results can be deduced from this formula. Thus, if $n' = -n$, then $\iota = \rho$, and the refraction becomes a reflection. Consequently, *all proofs derived from refraction can be applied to reflection by making* $n' = -n$.

Equation (3) may be written

$$\sin \iota = \frac{n'}{n} \,.\, \sin \rho.$$

If $\sin \rho$ is greater than $\frac{n}{n'}$, $\sin \iota$ is greater than 1, which is an impossible angle, and there is no real direction for the refracted ray. The angle whose sine is $\frac{n}{n'}$ (*i.e.* the angle $\sin^{-1} \frac{n}{n'}$) is the greatest angle at which a ray travelling in a denser medium can be refracted into the rarer medium. It is called the *critical angle* for the media. When the incident angle is greater than $\sin^{-1} \frac{n}{n'}$, the whole of the light is reflected. This is called *total internal reflection*, and since none of the light is lost by refraction the reflected light is very brilliant. This property is made use of in some optical instruments.

If R in Fig. 10 represents a luminous point, an eye situated at P will be able to see it. Since the eye will not take into account the bending of the ray, the point will appear to be somewhere upon PQ produced. It will consequently appear nearer the surface than it really is. This is the case when the iris is seen through the cornea. Owing to the curvature of the cornea the iris is also magnified, the exact position and distortion being dependent upon the position of the observer's eye.

Prisms

We shall only consider prisms which are denser than the surrounding medium, since we have only to deal with these in ophthalmic practice.

If a ray passes through a prism which is more highly refractive than the surrounding medium, the deviation is always towards the base of the prism.

This proposition can be simply deduced from a plane lamina.

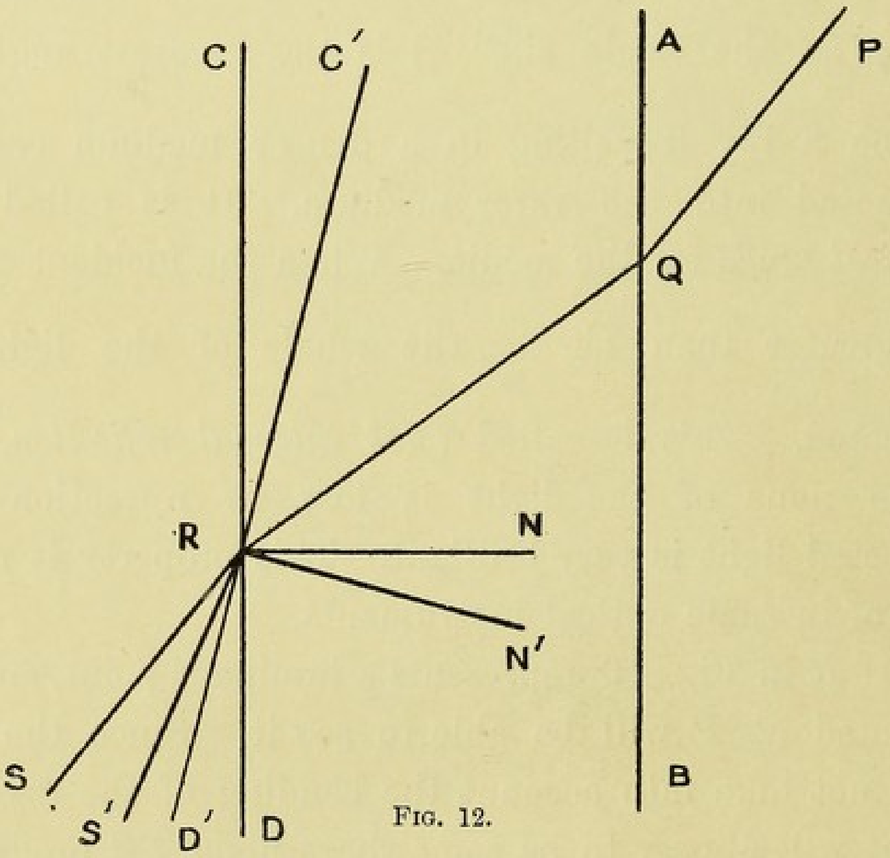

Fig. 12.

If *AB*, *CD* be the parallel faces of a plane lamina, and *PQRS* be the course of the ray, then *PQ* is parallel to *RS*. If the face *CD* be rotated round *R*, so that *C* approaches *A*, then the angle of incidence at *R* is increased, and consequently the deviation is also increased. Hence, in a prism the ray is deviated towards the base of the prism.

Since objects seen through a prism will still be regarded as being in the direction of the line of vision, *i.e.* the direction of the incident ray, they always appear to be displaced towards the apex of the prism.

It follows also from the above proof that the greater the angle between *BA* and *CD*, the more will the ray be deviated, *i.e. as the refracting angle of the prism increases the deviation also increases.*

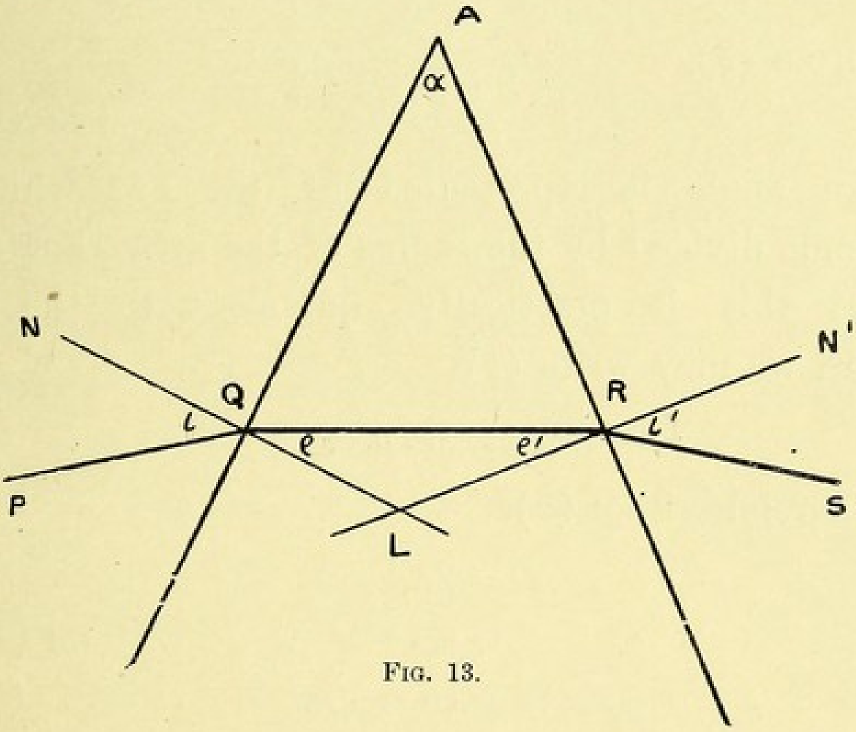

Fig. 13.

Let *PQRS* be the course of a ray through a prism whose refracting angle is a. Draw the normals *NL*, *N'L* at *Q* and *R* respectively.

Then $$\left.\begin{aligned} \sin \iota &= n \,.\, \sin \rho \\ \sin \iota' &= n \,.\, \sin \rho' \end{aligned}\right\} \quad . \quad . \quad (1).$$

The angle $AQR = 90° - \rho$,
and the angle $ARQ = 90° - \rho'$.

All the angles of the triangle $AQR = 180°$.

Therefore

$$a + (90° - \rho) + (90° - \rho') = 180°.$$

Therefore $\rho + \rho' = a$ . . (2).

If D be the deviation of the ray, then

$$D = (\iota - \rho) + (\iota' - \rho')$$
$$= \iota + \iota' - a \quad . \quad . \quad (3).$$

The deviation is minimum if $\iota = \iota'$; i.e. when the ray passes symmetrically through the prism.

Under these circumstances, from (2)

$$a = 2\rho \quad . \quad . \quad . \quad (4),$$

and from (3), $D = 2\iota - 2\rho$

$$= 2\iota - a \quad . \quad . \quad . \quad (5).$$

An angle is represented by the arc which it subtends divided by the radius of the arc. For small angles this is practically equivalent to the sine. Hence, we may write (1)

$$\iota = n \,.\, \rho.$$

Therefore, from (5)

$$D = 2n\rho - a$$
$$= na - a \quad . \quad . \quad [\text{from } (4)]$$
$$= (n-1)a \quad . \quad . \quad (6).$$

If the prism is made of glass, $n = \frac{3}{2}$, so that

$$D = \frac{a}{2}.$$

That is, *the deviation produced by a feeble glass prism* *is equal to half its refracting angle.* It must be carefully noted that this is only true (*a*) when the refracting angle is small, (*b*) when the ray passes symmetrically through the prism. Error has arisen, especially from neglect of the second point.

Spherical Surfaces

Refraction of a Small Pencil at a Spherical Surface

Let P be the focus of a small pencil, and PA a ray incident to the spherical surface at A, PHK being the

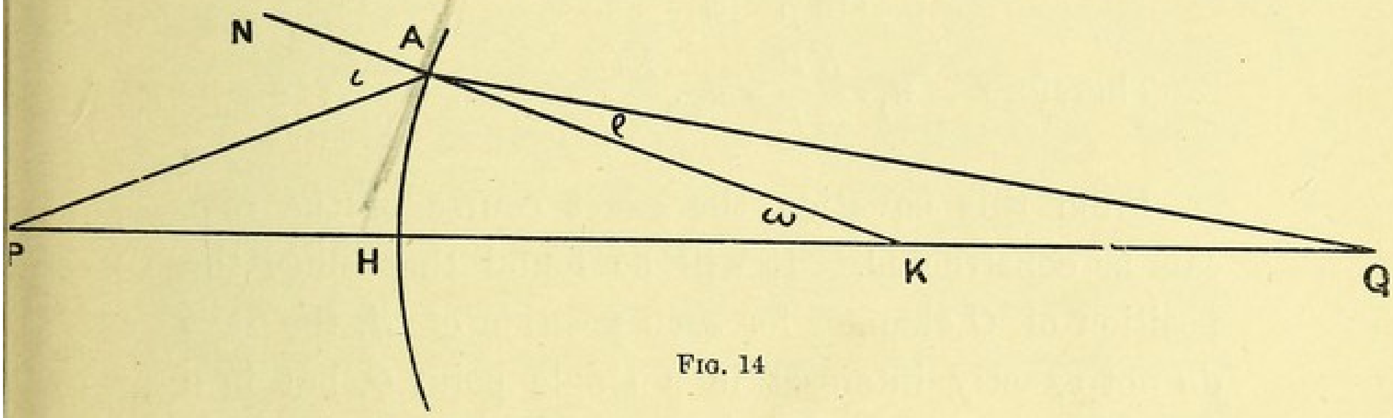

Fig. 14

axis of the pencil. PA is refracted to Q. Let K be the centre of the sphere: draw KAN normal to the surface at A.

Let the angle $HKA = \omega$; the angle $PAN = \iota$, and the angle $KAQ = \rho$.

Then in the triangle KAP

$$\frac{PA}{\sin PKA} = \frac{KP}{\sin PAK}$$

$$= \frac{KP}{\sin PAN},$$

i.e.

$$\frac{PA}{\sin \omega} = \frac{KP}{\sin \iota},$$

therefore $$\frac{\sin \iota}{\sin \omega} = \frac{KP}{PA} \quad . \quad . \quad . \quad (a).$$

Similarly, in the triangle AKQ,

$$\frac{\sin \rho}{\sin \omega} = \frac{KQ}{QA} \quad . \quad . \quad . \quad (b).$$

Therefore, from (a) and (b)

$$\frac{\sin \iota}{\sin \rho} = \frac{KP}{AP} \times \frac{AQ}{KQ}.$$

But $$\frac{\sin \iota}{\sin \rho} = \frac{n_2}{n_1}.$$

Therefore $$n_1 \cdot \frac{KP}{AP} = n_2 \cdot \frac{KQ}{AQ} \quad . \quad . \quad . \quad (c).$$

From this equation the exact course of the rays can be constructed. It will be found that since the position of Q changes for each position of A, the rays do not exactly intersect in a single point Q, but in a curved line, which is called the *caustic curve.* This property of spherical surfaces is called *spherical aberration.* In order that there may be no aberration, the curve must be a Cartesian oval.

If the pencil is small, we may neglect the aberration, and PA will be approximately equal to PH, and QA to QH, so that we may write (c)

$$n_1 \cdot \frac{KP}{PH} = n_2 \cdot \frac{KQ}{QH} \quad . \quad . \quad (d).$$

The relationship between P and Q is then reciprocal, and they are conjugate foci.

Let $PH = f_1$, $QH = f_2$, and $KH = r$.

Then $KP = f_1 + r,\ KQ = f_2 - r.$

Therefore, from (d)

$$n_1 \cdot \frac{f_1 + r}{f_1} = n_2 \cdot \frac{f_2 - r}{f_2} \qquad . \qquad . \qquad (e),$$

which may be written

$$\frac{n_1}{f_1} + \frac{n_2}{f_2} = \frac{n_2 - n_1}{r} \qquad . \qquad . \qquad (1).$$

The distances f_1, f_2 are called *conjugate focal distances.* As f_1 increases, f_2 must diminish, and *vice versâ*; hence *the conjugate foci always move in the same direction.*

In this equation we have considered f_1 positive when P lies in front of H, and f_2 positive when Q lies behind H. This convention is most convenient for lenses, but where there are several refracting surfaces it is best to consider all quantities measured from left to right positive, and quantities measured from right to left negative. In this convention f_1 will be negative, and (1) becomes

$$\frac{n_1}{f_1} - \frac{n_2}{f_2} = \frac{n_1 - n_2}{r}.$$

Retaining for the present the convention for lenses, if f_1 is infinite, so that the incident rays are parallel, then

$$f_2 = \frac{n_2 r}{n_2 - n_1} = F_2 \qquad . \qquad . \qquad (2).$$

Similarly, if f_2 is infinite, so that the refracted rays are parallel, then

$$f_1 = \frac{n_1 r}{n_2 - n_1} = F_1 \qquad . \qquad . \qquad (2a)$$

These positions of P and Q are called the *principal foci*, and F_1, F_2 are the *anterior and posterior principal focal distances* respectively.

Equation (1) may therefore be written

$$\frac{F_1}{f_1}+\frac{F_2}{f_2}=1 \quad . \quad . \quad (3).$$

The path of the rays being reversible, this formula applies, without change of sign, to both convex and concave spherical surfaces. It follows from (3) that the focal lengths are always both positive or both negative, and therefore the product F_1F_2 is always positive. They are both positive if the medium containing the centre of the spherical surface is the more highly refractive, and both negative if this medium is the less refractive.

From (2) and (2a) it follows that

$$\frac{F_1}{F_2}=\frac{n_1}{n_2} \quad . \quad . \quad . \quad (4).$$

Therefore, *the principal focal distances are to one another directly as the refractive indices of the corresponding media.*

Also from (2) and (2a)

$$F_2-F_1=\frac{n_2r}{n_2-n_1}-\frac{n_1r}{n_2-n_1}$$
$$=r \quad . \quad . \quad . \quad (5).$$

Therefore, *the difference between the principal focal distances is equal to the radius of curvature.*

From (3) we have

$$f_1F_2+F_1f_2=f_1f_2.$$

Therefore

$$f_1 f_2 - f_1 F_2 - F_1 f_2 + F_1 F_2 = F_1 F_2,$$

or

$$(f_1 - F_1)(f_2 - F_2) = F_1 F_2.$$

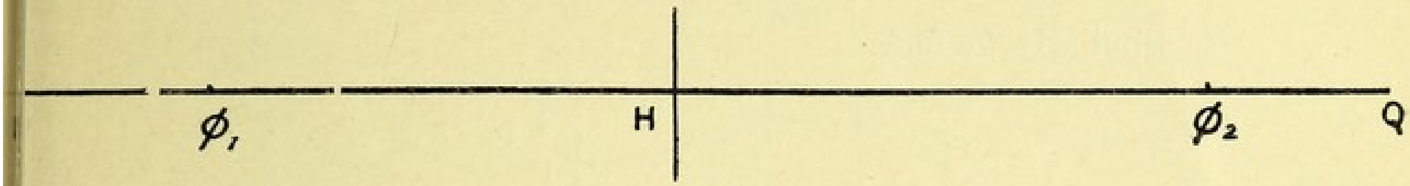

FIG. 15.

If l_1, l_2 be the distances $P\phi_1$, $Q\phi_2$ (Fig. 15), of a pair of conjugate foci, P, Q, measured from the focal points, ϕ_1, ϕ_2, then

$$l_1 = f_1 - F_1, \text{ and } l_2 = f_2 - F_2.$$

Therefore

$$l_1 l_2 = F_1 F_2 \qquad . \qquad . \quad (6).$$

The relative positions of conjugate foci can easily be traced by this equation.

If the axis be turned through a small angle about the centre of curvature, the points P and Q will describe lines which are approximately straight. If the whole system be rotated round the axis these lines will describe planes. Hence a small plane object at P will have a plane image at Q. The planes described at ϕ_1, ϕ_2 are called the *focal planes*, and the plane at H is called the *principal plane* of the surface.

We can obtain symmetrical expressions in terms of distances from the centre of curvature.

If in Fig. 14, $KP = g_1$ and $KQ = g_2$, then

$$\left.\begin{aligned} f_1 + r &= g_1 \\ f_2 &= g_2 + r \end{aligned}\right\}$$

and formula (1)

$$\frac{n_1}{f_1}+\frac{n_2}{f_2}=\frac{n_2-n_1}{r} \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

becomes

$$\frac{n_2}{g_1}+\frac{n_1}{g_2}=\frac{n_2-n_1}{r} \quad \cdot \quad \cdot \quad \cdot \quad (1a).$$

Similarly we find

$$F_1=\frac{n_1 r}{n_2-n_1}=G_2 \quad \cdot \quad \cdot \quad \cdot \quad (2a)$$

$$F_2=\frac{n_2 r}{n_2-n_1}=G_1 \quad \cdot \quad \cdot \quad \cdot \quad (2).$$

Also,

$$\frac{F_1}{f_1}+\frac{F_2}{f_2}=1 \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

and

$$\frac{G_1}{g_1}+\frac{G_2}{g_2}=1 \quad \cdot \quad \cdot \quad \cdot \quad (3a).$$

We can also obtain symmetrical expressions in terms of distances from another pair of conjugate foci.

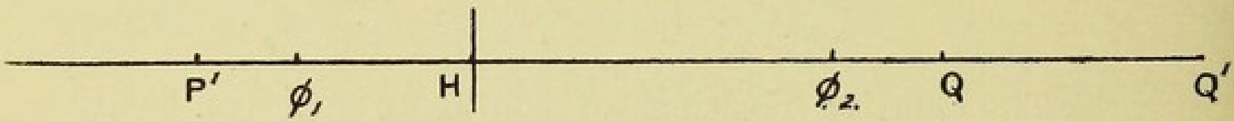

FIG. 16.

Let Q be the conjugate focus of P, and Q' the conjugate focus of another point P'; and let

$$\left.\begin{array}{ll} PH = f'_1 & : QH = f'_2 \\ P'H = f_1 & : Q'H = f_2 \end{array}\right\}$$

$$\left.\begin{array}{ll} PP' = h_1 & : QQ' = -h_2 \\ \phi_1 P' = -H_1 & : \phi_2 Q' = -H_2 \end{array}\right\}$$

Then

$$f'_1 - f_1 = h_1 \quad \cdot \quad \cdot \quad \cdot \quad (a).$$

$$f'_2 - f_2 = h_2 \quad \cdot \quad \cdot \quad \cdot \quad (b).$$

$$F_1 - f_1 = H_1 \quad \cdot \quad \cdot \quad \cdot \quad (c).$$

$$F_2 - f_2 = H_2 \quad \cdot \quad \cdot \quad \cdot \quad (d).$$

Further, from formula (3)

$$\frac{F_1}{f_1}+\frac{F_2}{f_2}=1 \quad \cdot \quad \cdot \quad \cdot \quad (e),$$

and

$$\frac{F_1}{f'_1}+\frac{F_2}{f'_2}=1 \quad \cdot \quad \cdot \quad \cdot \quad (f).$$

Substituting the values of f'_1, f'_2 of (a) and (b) in (f) we have

$$\frac{F_1}{h_1+f_1}+\frac{F_2}{h_2+f_2}=1,$$

or
$$F_1h_2+F_1f_2+F_2h_1+F_2f_1=h_1h_2+h_1f_2+h_2f_1+f_1f_2.$$

But by (e)
$$F_1f_2+F_2f_1=f_1f_2.$$

Therefore
$$F_1h_2+F_2h_1=h_1h_2+h_1f_2+h_2f_1,$$

or
$$(F_1-f_1)\,h_2+(F_2-f_2)\,h_1=h_1h_2.$$

Therefore, from (c) and (d)

$$H_1h_2+H_2h_1=h_1h_2,$$

or
$$\frac{H_1}{h_1}+\frac{H_2}{h_2}=1 \qquad . \quad . \quad . \quad . \ (3b).$$

To construct the Image formed by Refraction at a Spherical Surface

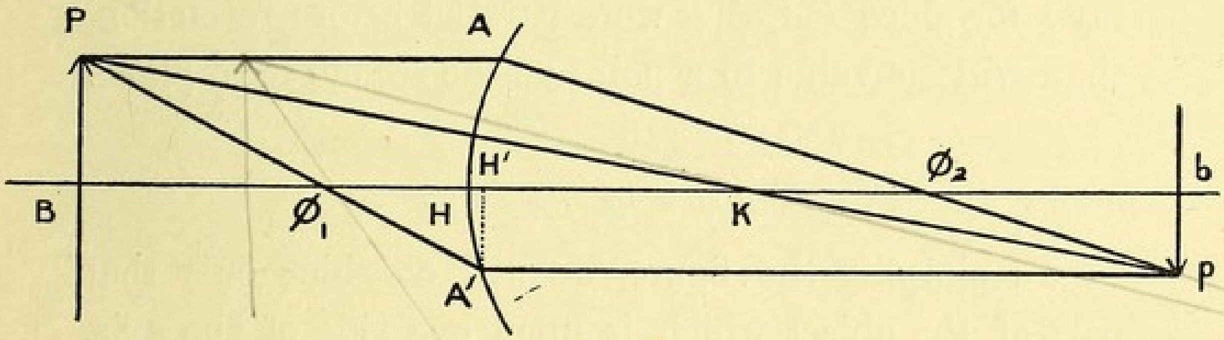

FIG. 17.

From what has already been proved we know the paths of three rays originating in any point P:—

(1) The ray PA, parallel to the axis, passes after refraction through the principal focus, ϕ_2.

(2) The ray $P\phi_1$, through the principal focus ϕ_1, is refracted parallel to the axis.

(3) The ray PK, through the centre of curvature, does not deviate.

The intersection of any two of these three will define the image, p, of the point P: and similarly for all other points in the object.

The refracted ray may also be constructed thus:

Let PA meet the principal plane in A. Draw $\phi_1 B$ parallel to PA, meeting the principal plane in B.

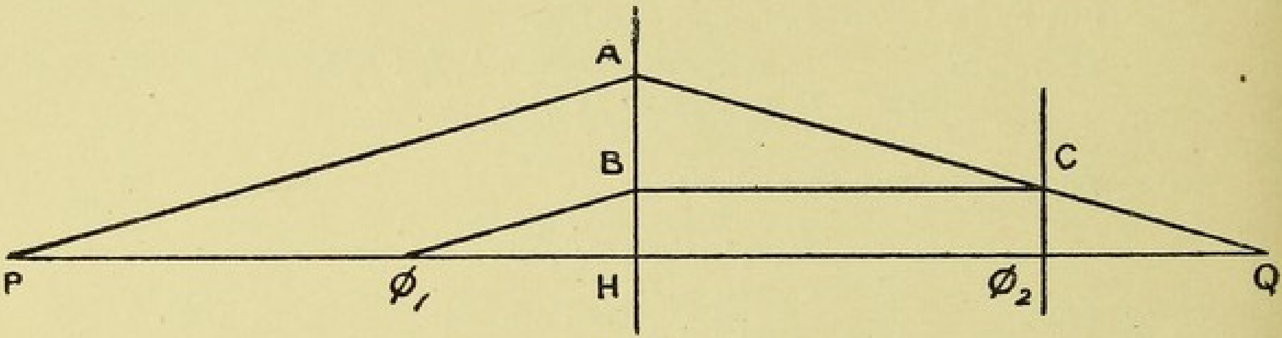

FIG. 18.

$\phi_1 B$ will be parallel to the axis after refraction: let it meet the focal plane in C. Then AC is the refracted ray; for since PA, $\phi_1 B$ were parallel before refraction, they will converge to a point on the second focal plane.

The Size of the Image

To simplify the figure, we will consider only that part of the object which is upon one side of the axis, the part above the axis being considered positive, and that below negative.

Let o be the size of the object PB, and i the size of the image pb (Fig. 17).

Then, in the similar triangles KPB, Kpb,

$$PB : pb = BK : bK.$$

Therefore $$\frac{o}{i} = -\frac{f_1 + r}{f_2 - r} \quad . \quad . \quad . \quad (1).$$

From (e), p. 33,

$$\frac{f_1 + r}{f_2 - r} = \frac{n_2}{n_1} \cdot \frac{f_1}{f_2}.$$

Therefore $$\frac{o}{i} = -\frac{n_2}{n_1} \cdot \frac{f_1}{f_2} \quad . \quad . \quad (2).$$

And from the similar triangles $PB\phi_1$, $A'H'\phi_1$,

$$PB : B\phi_1 = H'A' : H'\phi_1.$$

Therefore, for a small object,

$$o : f_1 - F_1 = i : F_1,$$

or $$\frac{o}{i} = -\frac{f_1 - F_1}{F_1} = -\frac{l_1}{F_1}$$

and similarly $$\frac{o}{i} = -\frac{F_2}{f_2 - F_2} = -\frac{F_2}{l_2} \quad . \quad (3),$$

Hence, for a *convex* surface,

(1) The image of an object beyond the anterior principal focus is real and inverted. It is smaller than the object if the object is greater than $2F_1$ from the surface, and greater if less than $2F_1$.

(2) The image of an object between the anterior principal focus and the surface is virtual, erect, and magnified, being situated behind the object.

If the refractive surface is plane, the focal distance is infinite, and equation (3) becomes $\frac{o}{i} = 1$.

Similarly, from Fig. 17,

$$PB : pb = BK : bk$$
$$= g_1 : g_2$$

Therefore $$\frac{o}{i} = -\frac{g_1}{g_2} \quad . \quad . \quad . \quad (4).$$

But $$\frac{o}{i} = \frac{F_1 - f_1}{F_1} = \frac{F_2}{F_2 - f_2} \quad . \quad . \quad (3).$$

Hence, from (2), (2*a*), (3), (3*a*) p. 36,

$$\frac{o}{i}=\frac{G_1-g_1}{G_2}=\frac{G_1}{G_2-g_2} \quad . \quad . \quad (3a).$$

The relative sizes of object and image can also be stated in terms of the divergence of the rays.

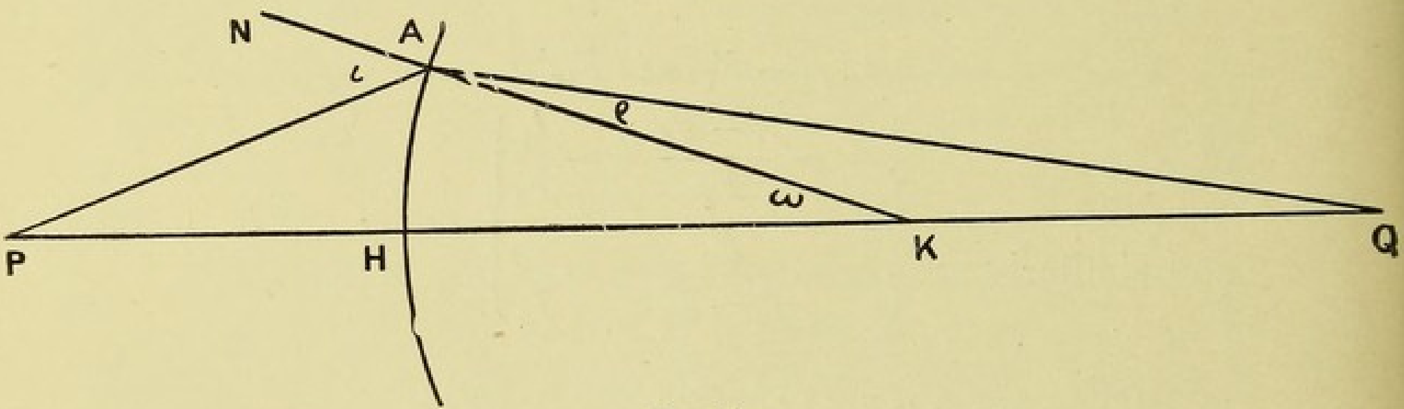

FIG. 19.

In Fig. 19 the angle APH $(=a)$ is the angle of divergence of the incident ray from the axis, and the angle AQH $(=-a')$ is the angle of divergence of the refracted ray.

Then $$\frac{AH}{PH}=\tan a.$$

Therefore $$AH=f_1 \, . \tan a.$$

Similarly, $$AH=-f_2 \, . \tan a'.$$

Therefore $$f_1 \, . \tan a=-f_2 \, . \tan a',$$

or $$\frac{\tan a}{\tan a'}=-\frac{f_2}{f_1}.$$

But $$\frac{o}{i}=\frac{n_2}{n_1} \cdot \frac{f_1}{f_2} \quad . \quad [\text{by } (2)].$$

Therefore $$\left.\begin{aligned} \frac{o}{i}&=-\frac{n_2}{n_1} \cdot \frac{\tan a'}{\tan a} \\ o \, . \, n_1 \, . \tan a&=-i \, . \, n_2 \, . \tan a' \end{aligned}\right\} \quad (5).$$

or

This is von Helmholtz's formula.

The ratio of the linear dimensions of object and image is sometimes called the *linear magnification*, or simply *magnification* (m). The change in the angle of divergence is called the *angular magnification* (γ). Since for small angles the tangents are practically equal to the angles, we have

$$\frac{\tan \alpha'}{\tan \alpha} = \gamma.$$

Hence $$m\gamma = \frac{n_1}{n_2} \quad . \quad . \quad . \quad (5a).$$

Similarly, replacing the tangents by the angles in (5), we get in general (*i.e.* omitting the special sign)

$$o \, . \, n_1 \, . \, \alpha = i \, . \, n_2 \, . \, \alpha' \quad . \quad . \quad . \quad (5b).$$

The product of the angle of divergence and the refractive index of the medium is called the *optical divergence.* Hence, *in refraction at a spherical surface, the product of the optical divergence of a ray with the linear magnification of the image in the same medium, remains constant for a small angle of incidence.* (Lagrange's Law.)

In Fig. 19 PAN is the angle of incidence (ι), KAQ is the angle of refraction (ρ), and PKA is ω.

Then $$\iota = \alpha + \omega,$$
and $$\rho = \omega - (-\alpha').$$
$$= \omega + \alpha'.$$

Hence $$n_1 \, . \sin(\alpha + \omega) = n_2 \, . \sin(\alpha' + \omega).$$

Therefore

$$n_1 (\sin \alpha \cos \omega + \cos \alpha \sin \omega) = n_2 (\sin \alpha' \cos \omega + \cos \alpha' \sin \omega),$$

or $$\cos \omega \, (n_1 \sin \alpha - n_2 \sin \alpha') = \sin \omega \, (n_2 \cos \alpha' - n_1 \cos \alpha).$$

If these angles are very small their cosines are approximately equal to 1, and their sines are equal to the angles: therefore

$$n_1 \alpha - n_2 \alpha' = (n_2 - n_1)\omega \quad . \quad . \quad . \quad (6).$$

That is, *the optical divergence changes on refraction to* $(n_2 - n_1)\omega$, *and is consequently independent of the distance of the luminous point.*

The optical divergence is diminished if the more refractive medium is on the concave side of the spherical surface, and *vice versâ*.

For a plane surface $\omega = 0$, and the optical divergence of the rays is not altered by refraction.

We are now in possession of all the data required to determine the position of the principal foci of an eye which has no lens (aphakia), given the radius of the cornea = 8 mm., and the refractive index of the aqueous and vitreous (which is the same) = 1·3365 (air being = 1).

$$F_1 = \frac{n_1 r}{n_2 - n_1}$$

$$= \frac{8}{1{\cdot}3365 - 1} = 23{\cdot}774 \text{ mm.}$$

$$F_2 = \frac{n_2 r}{n_2 - n_1}$$

$$= \frac{1{\cdot}3365 \times 8}{1{\cdot}3365 - 1} = 31{\cdot}774 \text{ mm.}$$

Therefore the anterior focus is 23·774 mm. in front of the cornea and the posterior focus 31·774 mm. behind it. Since the length of the normal eye along the axis is only about 23 mm., the aphakic eye is strongly hypermetropic.

CHAPTER IV

REFRACTION (*continued*)

LENSES

A LENS is a portion of a refracting medium formed by two surfaces of revolution having a common axis, the *axis* of the lens. The surfaces are usually either plane or spherical. The terms double convex (or bi-convex), double concave (or bi-concave), plano-convex, plano-concave, convexo-concave, scarcely require explanation. A convexo-concave lens is also called a meniscus or periscopic lens.

We shall first deal with thin lenses, or those in which the thickness is negligible.

The Optical Centre of a Lens

Let C, C' be the centres of the two spherical surfaces of a lens, and let two parallel radii CQ, $C'R$ be drawn to the surface. Let QR represent the path of a ray through the lens.

Then, since CQ, $C'R$ are parallel, the angle CQR is equal to the angle QRC', and the incident and emergent rays, PQ, RS, are also parallel. In fact the ray acts exactly as if it passed through a plane lamina, bounded by the parallel tangent planes at Q and R, as indicated by the dotted lines.

Let O be the point where QR cuts the axis, and r_1, r_2 the radii of curvature.

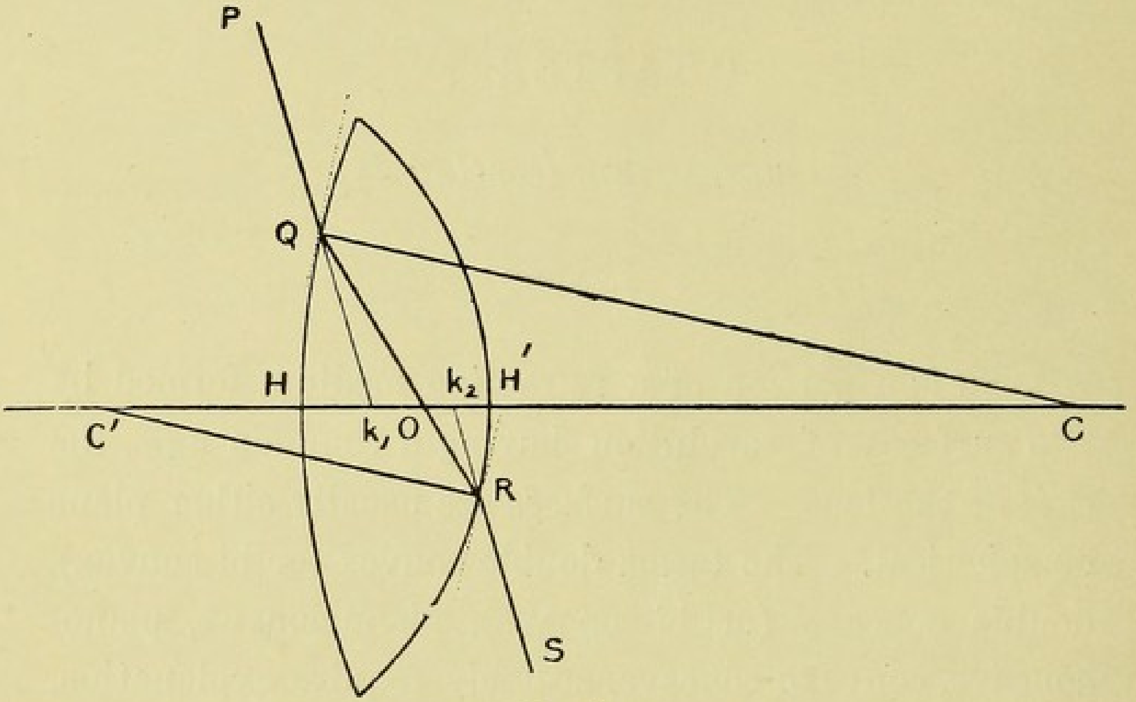

Fig. 20.

Then, from the similar triangles COQ, $C'OR$,

$$\frac{CO}{C'O}=\frac{r_1}{r_2} \quad . \quad . \quad . \quad (a).$$

Therefore O is a fixed point, such that every ray which, on emergence, is parallel to its original direction must pass through it, and conversely. O is called the *optical centre*, or briefly, the *centre* of the lens.

From (a) we have

$$\frac{r_1}{r_2}=\frac{CO}{C'O}=\frac{r_1-CO}{r_2-C'O}=\frac{OH}{OH'} \quad . \quad (b).$$

Hence, in a bi-convex or bi-concave lens the optical centre lies within the lens, *its distances from the two* *surfaces being directly as their radii.* In plano-convex and plano-concave lenses it lies upon the curved surface: in a meniscus it lies outside the lens.

The thickness of the lens may be neglected in most of the trial-lenses used in ophthalmology, and in this case *PQRS* may be considered a straight line. To this approximation, *the rays which pass through the centre of a lens do not deviate.*

Any straight line, other than the principal axis, through the centre, is called a *secondary axis.*

The deviation of a ray through a lens is the same as that through a prism formed by tangent planes at the points of incidence and emergence.

The expression for minimum deviation is

$$D = (n - 1)\, a \quad . \quad (c)\ [(6)\ \text{p. } 30].$$

If we compare Fig. 21 with Fig. 13, the angle *CBD*

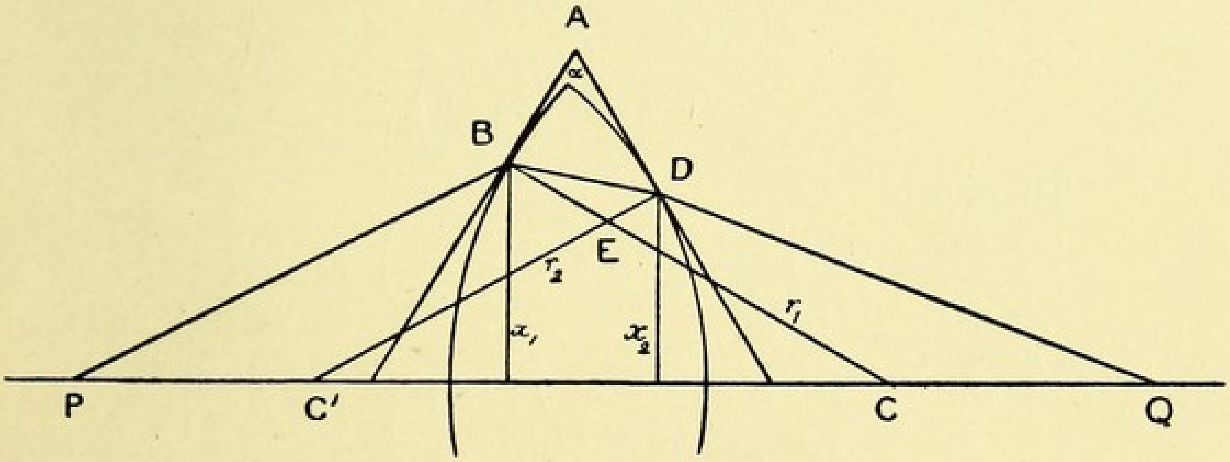

FIG. 21.

is ρ and the angle $C'DB$ is ρ'. But in the triangles *BED*, *CEC′*, the angle *BED* is equal to the angle *CEC′*. Therefore

$$\begin{aligned}(ECC' + EC'C) &= (EBD + EDB)\\ &= \rho + \rho'\\ &= a \qquad\qquad [(2)\text{ p. } 30].\end{aligned}$$

Since the angles are small, we may replace them by their sines. Therefore

$$a = \frac{x_1}{r_1} + \frac{x_2}{r_2} \quad . \quad . \quad . \quad (d).$$

Now $\frac{x_1}{PB}$ and $\frac{x_2}{DQ}$ are the sines of the angles which the rays make with the axis, and they are, to the usual approximation, equal to $\frac{x_1}{f_1}$ and $\frac{x_2}{f_2}$. The deviation is the sum of these two angles, and identifying the angles with their sines we have

$$D = \frac{x_1}{f_1} + \frac{x_2}{f_2}.$$

But from (c) and (d) we have

$$\begin{aligned}D &= (n-1)a\\ &= (n-1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right).\end{aligned}$$

Therefore

$$\frac{x_1}{f_1} + \frac{x_2}{f_2} = (n-1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right).$$

Therefore

$$\frac{1}{f_1} + \frac{1}{f_2} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad (1).$$

If f_1 is infinite, the incident rays are parallel, and f_2 is the posterior focal distance, F_2; whence

$$\frac{1}{F_2} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

Making f_2 infinite gives the same value for the anterior focal distance, F_1, so that *for thin lenses, with the same medium on each side, the anterior and posterior focal distances are equal.* Hence we may neglect the subscripts and write

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F} \quad . \quad . \quad (2).$$

In ordinary bi-convex lenses where $r_1=r_2$, we get

$$F=\frac{r}{2(n-1)},$$

and since $n=\frac{3}{2}$ for glass, we get $F=r$, so that *the focal distance of an ordinary glass lens is equal to the radius of curvature of its surfaces.*

To construct the Image formed by a Thin Lens

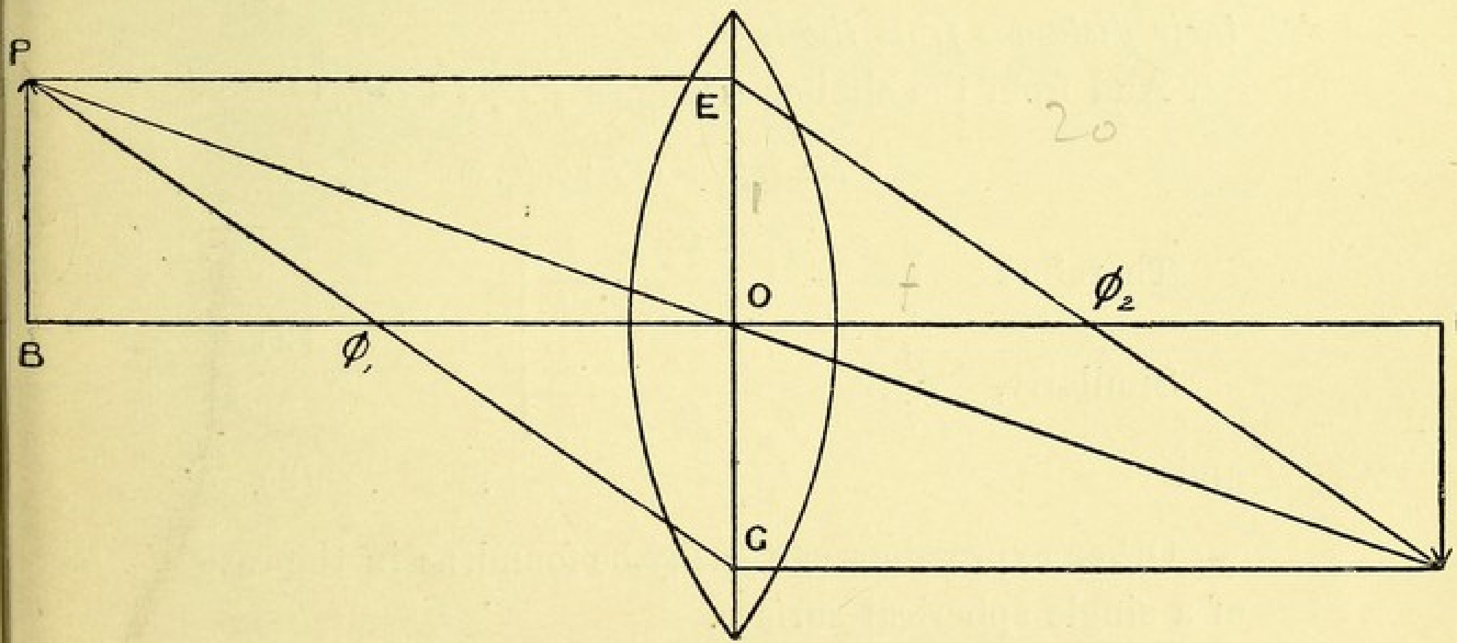

FIG. 22.

We know the paths of three rays originating in any point P:—

1. The ray *PO*, passing through the centre of the lens, does not deviate.

2. The ray *PE*, parallel to the axis, passes after refraction through the focus ϕ_2.

3. The ray *PG*, passing through the focus ϕ_1, is refracted parallel to the axis.

The intersection of any two of these three will define the image, *p*, of the point *P*; and similarly for all other points in the object.

The Size of the Image

From the similar triangles *PBO*, *pbO*,

$$PB : pb = BO : bO.$$

Therefore $$\frac{o}{i} = -\frac{f_1}{f_2} \quad . \quad . \quad . \quad (1).$$

That is, *the sizes of object and image are directly as their distances from the lens.*

And from the similar triangles $PB\phi_1$, $GO\phi_1$,

$$PB : B\phi_1 = OG : O\phi_1.$$

Therefore $$\left.\begin{aligned} \frac{o}{i} &= -\frac{f_1 - F}{F} = -\frac{l_1}{F} \\ \text{Similarly,}\quad \frac{o}{i} &= -\frac{F}{f_2 - F} = -\frac{F}{l_2} \end{aligned}\right\} \quad . \quad (2),$$

and $$l_1 l_2 = F^2 \quad . \quad . \quad (3).$$

Other expressions can easily be found, as in the case of a single spherical surface.

From these formulæ, or by construction, it follows that *for convex lenses:—*

1. The image of an object beyond the principal

focus is real, inverted, and situated upon the opposite side of the lens. It is magnified if the object is less than $2F$ from the lens, equal if the distance is equal to $2F$, and diminished if the distance is more than $2F$.

2. The image of an object between the principal focus and the lens is virtual, erect, and magnified, and is situated behind the object and upon the same side of the lens.

For *concave* lenses the image is virtual, erect, and diminished.

The *refractive force* (R) of a surface or lens is expressed in dioptres by the inverse of the anterior focal distance, measured in metres.

$$R = \frac{1}{F_1} = \frac{n-1}{r}.$$

Thus, we have calculated the anterior focal distance of the cornea as rather less than 24 mm. Its refractive power is therefore equivalent to $\frac{1}{0{\cdot}024}D$, *i.e.* about $42D$.

Similarly, for thin lenses,

$$R = \frac{1}{F} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

That is, it is the sum of the refractive forces of the two surfaces.

The focal distance of the crystalline lens is 62·46 mm., consequently, if we neglect its thickness, its refractive force is about $16D$.

Cardinal Points

We have hitherto neglected the thickness of the lens. For the comprehension of the optical properties of the eye it is necessary to take this factor into account. It is further necessary to consider the effect of several different refractive media, with spherical surfaces of contact, when centred upon the same axis. In each case the image of an object formed by the first surface might be taken as the object whose rays are refracted by the second surface, and so on; but this would be tedious, and the investigation has been much simplified by the theory of Gauss.

According to this theory, as modified by Listing, all such systems possess three pairs of *cardinal points*, viz., two principal foci, two principal points, and two nodal points.

Rays which pass through a *principal focus* (ϕ_1, ϕ_2) are, after refraction, parallel to the axis.

Rays which pass through the first *principal point* (h_1) after refraction pass through the second (h_2). Planes perpendicular to the axis through the principal points are called the principal planes, and each is the exact image of the other and, indeed, the only image of the same size and direction.

Rays which pass through the first *nodal* point (k_1) after refraction pass through the second (k_2), and the direction of the refracted ray is parallel to the direction of the incident ray. The nodal points were added by Listing.

It will be convenient to summarise here the leading facts about the cardinal points, and then to proceed to the discussion of thick lenses and centred systems.

1. The distance of the first nodal point from the first principal focus is equal to the second focal distance; and the distance of the second nodal point

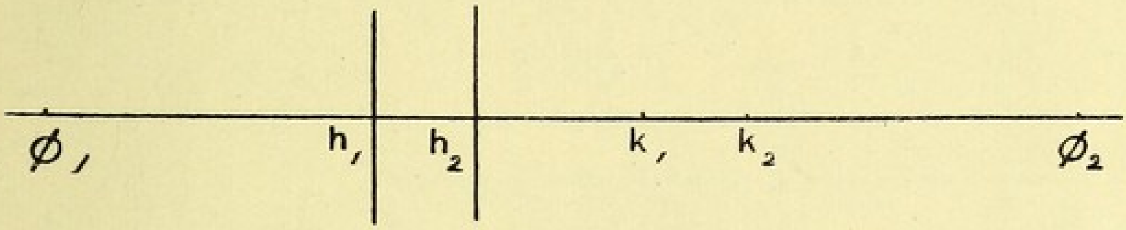

FIG. 23.

from the second principal focus is equal to the first focal distance. That is, in Fig. 23,

$$\left.\begin{array}{l}\phi_1 k_1 = \phi_2 h_2 \text{ or } G_1 = F_2 \\ \phi_1 h_1 = \phi_2 k_2 \text{ or } F_1 = G_2\end{array}\right\} \quad . \quad (1).$$

2. The distances of the corresponding principal and nodal points from each other is equal to the difference of the two focal distances. That is,

$$\left.\begin{array}{l}k_1 h_1 = k_2 h_2 = \phi_2 h_2 - \phi_1 h_1, \\ \text{or} \quad G_1 - F_1 = F_2 - G_2 = F_2 - F_1\end{array}\right\} \quad . \quad (2).$$

3. The distance between the two principal points is equal to the distance between the two nodal points. That is,

$$h_1 h_2 = k_1 k_2 \; . \quad . \quad . \quad (3).$$

4. The two principal focal distances are in the ratio of the indices of refraction of the first and last media. That is,

$$\frac{\phi_1 h_1}{\phi_2 h_2} \text{ or } \frac{F_1}{F_2} = \frac{n_1}{n_{m+1}} \quad . \quad . \quad (4).$$

5. If the first and last media are the same, and $n_1 = n_{m+1}$, as in most optical instruments, but not in the eye, then the two principal focal distances are equal, and the corresponding principal and nodal points coincide.

To construct the Image formed by a Homocentric System

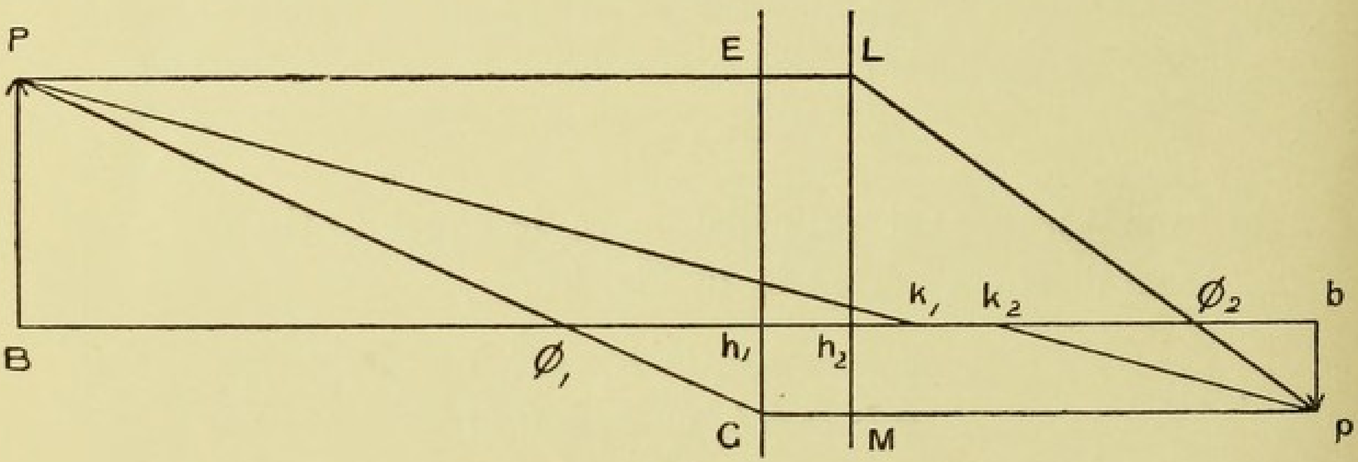

FIG. 24.

The intersection of any two of the following three rays will give the position of the image, p, of the point P:—

(1) The ray PE, parallel to the axis, will cut the second principal plane in L, so that $Lh_2 = Eh_1$, and it will pass through ϕ_2. Hence its direction is Lp.

(2) The ray PG, through ϕ_1, will, after refraction, be parallel to the axis. Hence its direction is Mp.

(3) The ray Pk_1, meeting the first nodal point, will pass through k_2, and will then pass parallel to its former course. Hence its direction is k_2p, parallel to Pk_1.

The Size of the Image

From the similar triangles $PB\phi_1$, $Gh_1\phi_1$, and the similar triangles $Lh_2\phi_2$, $pb\phi_2$, we obtain, exactly as in the case of thin lenses,

$$\frac{o}{i} = -\frac{l_1}{F_1} = -\frac{F_2}{l_2} \quad . \quad . \quad (5).$$

Whence $$l_1 l_2 = F_1 F_2 \quad . \quad . \quad (6),$$

and $$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1 \quad . \quad . \quad (7).$$

Thick Lenses

In refraction at a single spherical surface the nodal points coalesce in the centre of curvature. All rays passing through it are normal to the surface, and therefore do not deviate. We have regarded the optical centre of a lens as a similar nodal point, but this is only strictly accurate in the case of a meniscus with parallel sides, such as the cornea (approximately), the centre of both curvatures being then the same. In all others every ray is refracted except the axis, and there are two nodal points, *i.e.* two conjugate foci such that any incident ray through one will emerge in a parallel direction through the other.

In Fig. 25 the position of the nodal points is found by producing the rays *PQ*, *RS* to meet the axis. Then k_1, k_2 are the nodal points.

For we have shown that any ray through *O* will emerge parallel to its former course. Hence *PQ*, *RS* are rays which fulfil the conditions of the definition.

Further, any ray which would pass through k_1 will pass through O as far as the first surface is concerned. Therefore they are conjugate points with regard to the

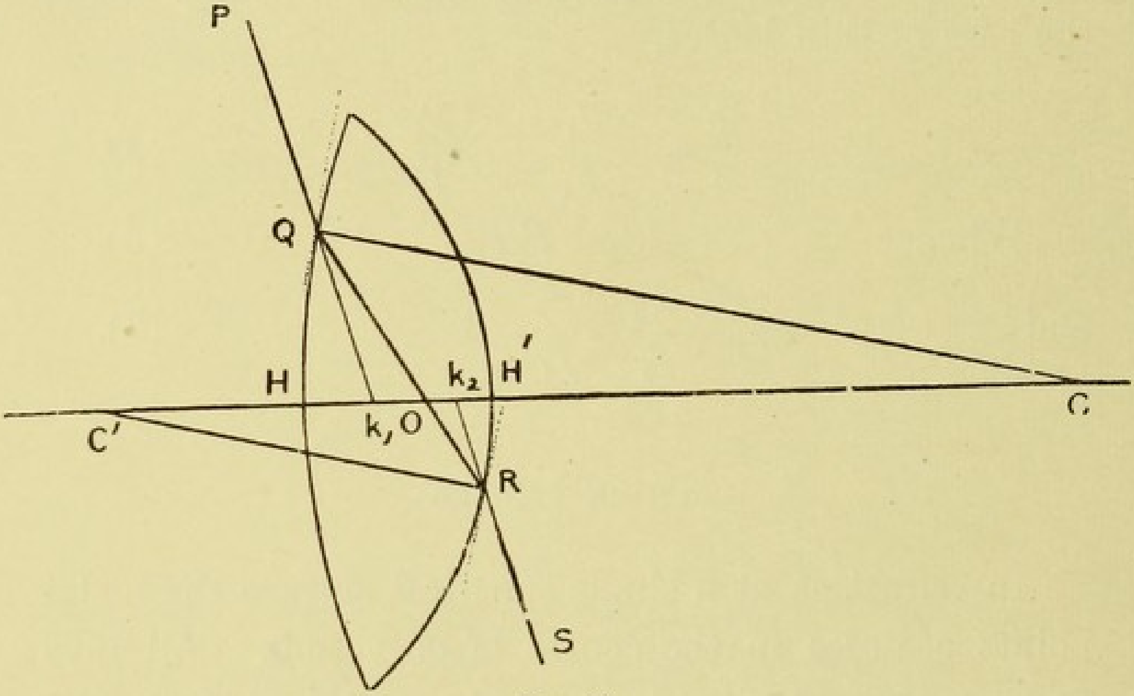

FIG. 25.

first surface, and k_2 and O are conjugate points with regard to the second.

Let the lens, of refractive index n, and thickness nc, be situated in air (refractive index = 1). Then we may write

$$\frac{r_1}{n-1}=F_1, \quad \frac{r_2}{n-1}=F_2 \quad [(2a) \text{ and } (2) \text{ p. } 33].$$

Then $$CC'=r_1+r_2-nc.$$

And $$\frac{CO}{C'O}=\frac{r_1}{r_2}. \qquad [(a) \text{ p. } 44].$$

Now $$\frac{CO}{CQ}=\frac{C'O}{C'R}=\frac{C'O+CO}{C'R+CQ}=\frac{CC'}{C'R+CQ}.$$

Therefore $$CO=\frac{r_1}{r_1+r_2}(r_1+r_2-nc)$$

$$= r_1 - \frac{nc \,.\, r_1}{r_1 + r_2}$$

$$= r_1 - OH.$$

Therefore $\qquad OH = \dfrac{nc \,.\, r_1}{r_1 + r_2} = \dfrac{nc \,.\, F_1}{F_1 + F_2}$

And similarly, $OH' = \dfrac{nc \,.\, r_2}{r_1 + r_2} = \dfrac{nc \,.\, F_2}{F_1 + F_2}$ $\qquad$. (1).

Since k_1 and O are conjugate foci at the first refraction,

$$\frac{1}{k_1H} + \frac{n}{OH} = \frac{n-1}{r_1} = \frac{1}{F_1} \quad [(1) \text{ p. } 33].$$

If $k_1H = h_1$, and $k_2H' = h_2$, we have

$$\frac{1}{h_1} = -\frac{n(F_1 + F_2)}{nc \,.\, F_1} + \frac{1}{F_1}.$$

Therefore, suppressing the negative sign, which is peculiar to the particular case, we have in general

$$h_1 = \frac{cF_1}{F_1 + F_2 - c}$$

and similarly $\qquad h_2 = \dfrac{cF_2}{F_1 + F_2 - c}$ $\qquad$ . . (2).

A lens not only has two nodal points, it also has two principal points. In the case of a single refracting surface, the incident and refracted rays are directed to the same point upon the surface. This is not true of more than one surface; but we can find two surfaces, perpendicular to the axis, which are the exact images of each other, so that a refracted ray is directed to the corresponding point on the second surface to that to which the incident ray is directed upon the first surface, the line joining these corresponding points being parallel to the axis.

To determine the principal planes we must find the position of an object which will form identical images on each side. For a bi-convex lens the position is obviously inside the lens.

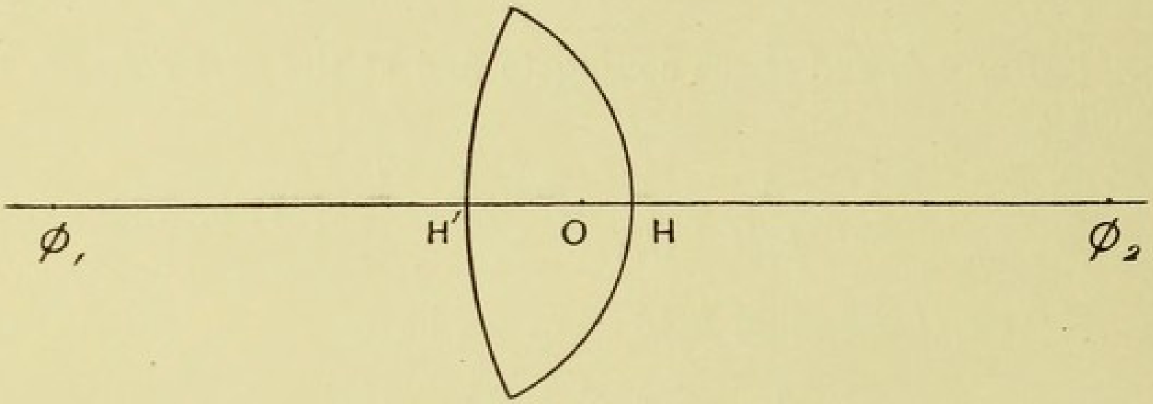

FIG. 26.

In Fig. 26 let O be the position. Then, considering each surface separately, as if the other were away, we have from (3) p. 39,

$$\frac{o}{i_1}=\frac{F}{F-f},$$

and
$$\frac{o}{i_2}=\frac{F'}{F'-f'}.$$

Therefore, since $i_1=i_2$,

$$f:f'=F:F'.$$

Therefore, when the lens has the same medium on each side, O coincides with the optical centre, and the principal points coincide with the nodal points.

The algebraical proof is as follows. Let o be the size of the object, i_1 its image after one refraction, and

i_2 the image of i_1 after the second refraction. Then, from (2), p. 39,

$$\frac{o}{i_1}=\frac{nf_1}{f_2} \text{ and } \frac{i_2}{i_1}=\frac{nf'_1}{f'_2}.$$

Therefore $$\frac{o}{i_2}=\frac{f_1f'_2}{f_2f'_1} \quad . \quad . \quad . \quad (a).$$

But by (*b*), p. 44,

$$\frac{OH}{OH'}=\frac{r_1}{r_2}.$$

In the first refraction O is conjugate to k_1, therefore $Hk_1=f_1$, and $OH=f_2$. In the second refraction k_2 is conjugate to O, therefore $OH'=f'_2$, and $H'k_2=f'_1$.

Therefore $$\frac{f_2}{f'_2}=\frac{r_1}{r_2}.$$

In the first refraction

$$\frac{1}{f_1}+\frac{n}{f_2}=\frac{n-1}{r} \quad \text{[by (1), p. 33]},$$

or $$\frac{f_2+nf_1}{f_1f_2}=\frac{n-1}{r_1} \quad . \quad . \quad (b).$$

In the second refraction

$$\frac{f'_2+nf'_1}{f'_1f'_2}=\frac{n-1}{r_2} \quad . \quad . \quad (c).$$

Dividing (*c*) by (*b*)

$$\frac{f_1f_2(f'_2+nf'_1)}{f'_1f'_2(f_2+nf_1)}=\frac{r_1}{r_2}=\frac{f_2}{f'_2},$$

which reduces to

$$f_1f'_2=f'_1f_2.$$

Therefore, from (*a*) $\quad o=i_2$.

Hence the object and image are equal and in the same direction when they lie at k_1 and k_2, *i.e.* in the nodal planes. Therefore the principal and nodal planes coincide.

We have now determined the nodal points and the principal points. It remains to determine the foci.

Let P be a point in the anterior focal plane, and let PE be a ray parallel to the axis; then the refracted ray will cut the axis in the posterior focus. Also, since the nodal points coincide with the principal points, h_2p will be parallel to Ph_1; and since the

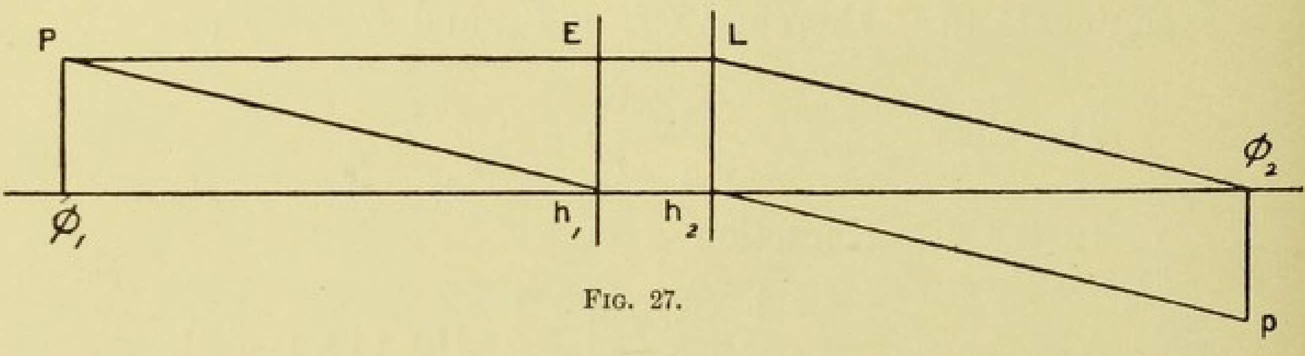

FIG. 27.

parallel rays EL, h_1h_2 are equally refracted at the surface Lh_2, h_2p will also be parallel to $L\phi_2$.

Therefore, in the right-angled triangles $P\phi_1h_1$, $Lh_2\phi_2$, Ph_1 is parallel to $L\phi_2$, and $P\phi_1$ is equal to Lh_2, therefore ϕ_1h_1 is equal to ϕ_2h_2. That is, when the medium is the same on each side of the lens, the focal distances are equal, and this distance is called the *focal length* of the lens.

Let f_1, f'_1 be the distances of conjugate foci in front of and behind H (in Fig. 25) respectively; and f_2, f'_2 the distances of conjugate foci behind and in front of H' respectively: refraction at each surface being considered separately. Then

$$\left.\begin{aligned} \frac{1}{f_1}+\frac{n}{f'_1}&=\frac{n-1}{r_1} \\ \frac{1}{f_2}+\frac{n}{f'_2}&=\frac{n-1}{r_2} \\ \text{and} \qquad f'_1+f'_2&=nc \end{aligned}\right\}$$

Eliminate f'_1, f'_2 from these equations, and we obtain

$$c=\frac{1}{\frac{n-1}{r_1}-\frac{1}{f_1}}+\frac{1}{\frac{n-1}{r_2}-\frac{1}{f_2}}$$

$$=\frac{1}{\frac{1}{F_1}-\frac{1}{f_1}}+\frac{1}{\frac{1}{F_2}-\frac{1}{f_2}}$$

$$=\frac{F_1 f_1}{f_1-F_1}+\frac{F_2 f_2}{f_2-F_2}.$$

Therefore

$$f_1 f_2(F_1+F_2-c)-F_1 f_2(F_2-c)-F_2 f_1(F_1-c)=cF_1F_2 \qquad (a).$$

Divide by f_2, then

$$f_1(F_1+F_2-c)-F_1(F_2-c)=\frac{cF_1F_2}{f_2}+\frac{F_2 f_1(F_1-c)}{f_2}.$$

Now make $f_2=\infty$, in which case f_1 becomes the distance of the principal focus from the lens ($=\psi_1$).

Then $$\psi_1=\frac{F_1(F_2-c)}{F_1+F_2-c}.$$

Similarly, $$\psi_2=\frac{F_2(F_1-c)}{F_1+F_2-c}.$$

But the principal focal distance (Φ_1) is the distance of the focus from the principal point $=\psi_1+h_1$.

Therefore

$$\Phi_1 = \frac{F_1(F_2 - c)}{F_1 + F_2 - c} + \frac{cF_1}{F_1 + F_2 - c} \quad [(2) \text{ p. } 55],$$

and

$$\Phi_2 = \frac{F_2(F_1 - c)}{F_1 + F_2 - c} + \frac{cF_2}{F_1 + F_2 - c}.$$

But $\Phi_1 = \Phi_2$, and we may suppress these subscripts, and write F instead, whence

$$F = \frac{F_1 F_2}{F_1 + F_2 - c} \quad . \quad . \quad (3),$$

or the power of the lens $\left(\frac{1}{F}\right)$,

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} - \frac{c}{F_1 F_2}.$$

If the values of ψ_1, ψ_2 and F are substituted in the equation (a), it can be reduced to the form

$$(f_1 - \psi_1)(f_2 - \psi_2) = F^2,$$

or

$$l_1 l_2 = F^2 \quad . \quad . \quad . \quad (4),$$

where l_1, l_2 are the distances of a pair of conjugate foci measured respectively in front of and behind the focal points.

If H_1, H_2 are the distances of these points from the principal points, then $l_1 = H_1 - F$, and $l_2 = H_2 - F$, and

$$(H_1 - F)(H_2 - F) = F^2.$$

Therefore

$$\frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{F} \quad . \quad . \quad (5).$$

If Fig. 24 represents a lens, we have

$$PB : B\phi_1 = Gh_1 : h_1\phi_1,$$

$$\left.\begin{array}{ll}\text{or} & \dfrac{o}{i} = -\dfrac{l_1}{F} \\ \text{Similarly,} & \dfrac{o}{i} = -\dfrac{F}{l_2}\end{array}\right\} \quad . \quad . \quad . \quad (6).$$

The expressions in formulæ (2) and (3) can easily be found in terms of n and r. They are as follows :—

$$\left.\begin{array}{l} h_1 = \dfrac{cr_1}{(r_1 + r_2) - (n-1)c} \\ h_2 = \dfrac{cr_2}{(r_1 + r_2) - (n-1)c} \end{array}\right\} \quad . \quad (2),$$

$$F = \frac{r_1 r_2}{(n-1)[r_1 + r_2 - (n-1)c]} \quad . \quad (3).$$

CHAPTER V

REFRACTION (*continued*)

THE HOMOCENTRIC SYSTEM

IN the previous chapter we have considered the formulæ relating to lenses, both without and with consideration of their thickness. We have not yet proved all the data necessary to determine the refractive conditions of the eye, in which, in addition to the crystalline lens, we have also the cornea to account for. We therefore proceed to discuss the general case of a series of centred spherical refracting surfaces, in which we have m surfaces, with $m + 1$ refracting media, of refractive indices $n_1, n_2, n_3 \ldots\ldots n_{m+1}$. This will include all possible cases, and is the only satisfactory proof for the conditions found in the eye, where the lens is itself a homocentric system of many laminæ.

The best demonstration of this problem, and the one which is simplest for the mathematician, is by means of convergents. The following exposition of the subject does not make any demand upon special mathematical knowledge, and is essentially that given by von Helmholtz in his *Physiologische Optik*.[1]

[1] 2nd ed. pp. 72-82.

It is clear that the same approximation with regard to rays having only a small deviation from the axis must be adhered to; and since each refraction is but a repetition of the first, rays which were homocentric to begin with will remain so for an indefinite number of refractions. In order to prevent confusion, we must adopt the convention whereby all quantities measured from left to right are positive, and those measured from right to left are negative.

We will first prove the general form of equation (3*b*) p. 37.

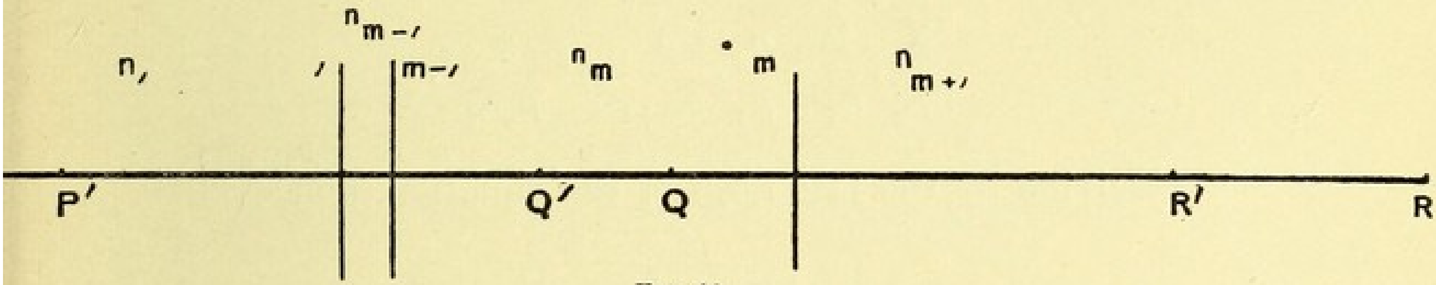

FIG. 28.

Let 1 be the first refracting surface, $m-1$ the last but one, and m the last. Let Q, Q' be the conjugate foci of P, P' respectively after all but the last refraction, and R, R' the corresponding conjugate foci after the last refraction. Then, in the usual notation, $PP' = h_1$, $QQ' = h_m$, and $RR' = h_{m+1}$, since there are $m+1$ media; and $H_1 =$ the distance of P' from the first principal focus, and H_2 the distance of R' from the second. Let λ_1, λ_2 be the distances of the principal foci of the first $(m-1)$ surfaces from P' and Q', and μ_1, μ_2 the distances of the principal foci of the m^{th} surface from Q' and R'.

Then $$\frac{\lambda_1}{h_1} + \frac{\lambda_2}{h_m} = 1. \qquad . \quad . \quad (a).$$

And, if the m^{th} surface is convex,

$$-\frac{\mu_1}{h_m}+\frac{\mu_2}{h_{m+1}}=1 \qquad . \qquad . \quad (b).$$

Divide (a) by λ_2, and (b) by μ_1; then

$$\frac{\lambda_1}{\lambda_2}\cdot\frac{1}{h_1}+\frac{1}{h_m}=\frac{1}{\lambda_2},$$

and

$$-\frac{1}{h_m}+\frac{\mu_2}{\mu_1}\cdot\frac{1}{h_{m+1}}=\frac{1}{\mu_1}.$$

Add these equations together; then

$$\frac{\lambda_1}{\lambda_2}\cdot\frac{1}{h_1}+\frac{\mu_2}{\mu_1}\cdot\frac{1}{h_{m+1}}=\frac{\mu_1+\lambda_2}{\mu_1\lambda_2}.$$

Divide by the last term of the equation, and

$$\frac{\mu_1\lambda_1}{\mu_1+\lambda_2}\cdot\frac{1}{h_1}+\frac{\mu_2\lambda_2}{\mu_1+\lambda_2}\cdot\frac{1}{h_{m+1}}=1.$$

If $h_1=\infty$, then $h_{m+1}=H_2$; therefore

$$H_2=\frac{\mu_2\lambda_2}{\mu_1+\lambda_2}.$$

Similarly,

$$H_1=\frac{\mu_1\lambda_1}{\mu_1+\lambda_2}.$$

Therefore

$$\frac{H_1}{h_1}+\frac{H_2}{h_{m+1}}=1 \qquad . \qquad . \quad (1).$$

If PB is an object, and pb its image after refraction through a homocentric system, then the ray PE parallel to the axis must pass through ϕ_2, and must therefore be Lp. The point where PE and pL intersect determines the position of h_2, and similarly if pM be parallel to the axis, and GP its refracted ray through ϕ_1, the point of intersection of pM and PG determines the position of h_1. Since there are no other points

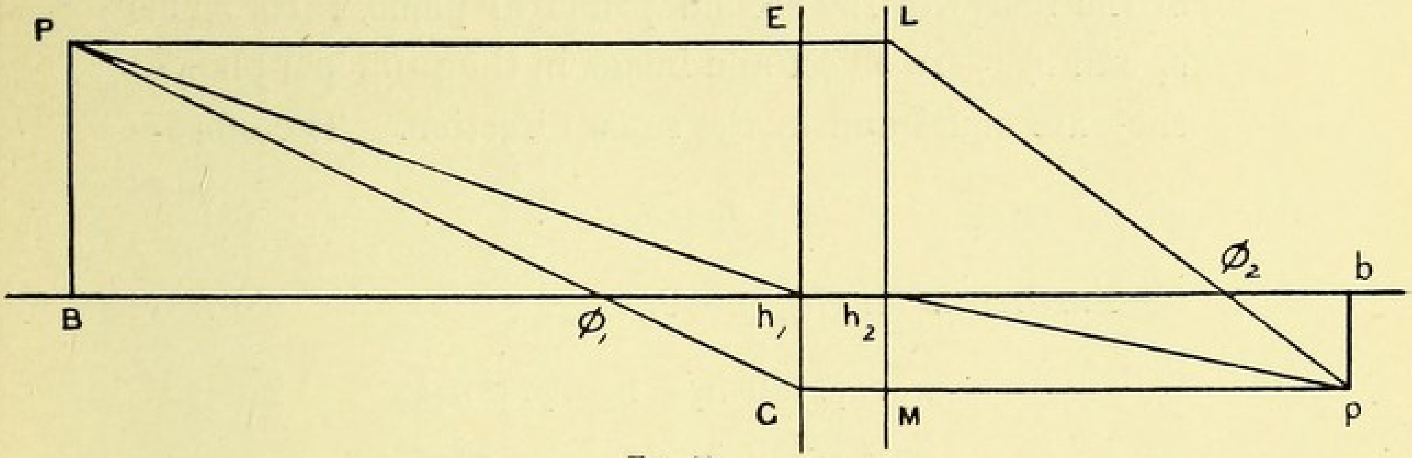

FIG. 29.

where these lines can intersect, there are only two principal points in such a refracting system.

In the right-angled triangles $Lh_2\phi_2$, $pb\phi_2$

$$Lh_2 : pb = h_2\phi_2 : \phi_2 b.$$

Therefore $$o : i = F_2 : f_2 - F_2.$$

And, since the image is inverted in the figure,

$$-\frac{o}{i} = \frac{F_2}{f_2 - F_2}.$$

From (1) we have

$$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1 \quad . \quad . \quad (2).$$

Therefore $$\frac{o}{i} = \frac{F_1 - f_1}{F_1} = \frac{F_2}{F_2 - f_2} \quad . \quad . \quad (3).$$

If $B\phi_1 = l_1 = f_1 - F_1$, and $b\phi_2 = l_2 = f_2 - F_2$, then from (2)

$$l_1 l_2 = F_1 F_2 \quad . \quad . \quad . \quad (4),$$

and $$\frac{o}{i} = -\frac{l_1}{F_1} = -\frac{F_2}{l_2} \quad . \quad . \quad (3).$$

If o_1 is the size of an object in the first principal plane, and o_2, o_3...... the sizes of the images formed by the respective refracting surfaces, o_{m+1} being the size of the image in the second principal plane, then, since o_1 and o_{m+1} are object and image in the principal planes, they are equal and in the same direction. Therefore

$$o_1 = o_{m+1} \quad . \quad . \quad . \quad (a).$$

Further, from (5), p. 40,

$$n_1 . o_1 . \tan a_1 = n_2 . o_2 . \tan a_2$$
$$n_2 . o_2 . \tan a_2 = n_3 . o_3 . \tan a_3$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$n_m . o_m . \tan a_m = n_{m+1} . o_{m+1} . \tan a_{m+1}.$$

Therefore, subtracting

$$n_1 . o_1 . \tan a_1 = n_{m+1} . o_{m+1} . \tan a_{m+1} \qquad (b).$$

Therefore, by (a)

$$n_1 . \tan a_1 = n_{m+1} . \tan a_{m+1}.$$

But $$o = -f_1 . \tan a_1.$$

[*e.g.* in Fig. 29, $PB = Bh_1 . \tan Ph_1B$],

and $$-i = -f_2 . \tan a_{m+1}.$$

Therefore $$\frac{n_1 . o}{f_1} = -\frac{n_{m+1} . i}{f_2}.$$

But from (2)

$$f_2 = \frac{f_1 F_2}{f_1 - F_2}.$$

Therefore

$$\frac{n_1 . o}{f_1 - F_1} = -\frac{n_{m+1} . i}{F_2}$$

And from (3)

$$\frac{o}{i} = -\frac{f_1 - F_1}{F_1}.$$

Therefore $$\frac{F_1}{F_2} = \frac{n_1}{n_{m+1}} \quad . \quad . \quad (5).$$

This proves (4), p. 51, for all cases.

If $a_1 = a_{m+1}$, the refracted ray is parallel to the incident ray, and they pass through the nodal points.

But $n_1 . o_1 . \tan a_1 = n_{m+1} . o_{m+1} . \tan a_{m+1}$.

Hence in this case

$$n_1 . o_1 = n_{m+1} . o_{m+1}.$$

Therefore $$o_{m+1} = \frac{n_1}{n_{m+1}} . o_1 \quad . \quad . \quad (c).$$

The images of the same object o_1 vary as their distances from the second principal focus (Fig. 29). If the image of o_1 is on the second principal plane, its size is equal to o_1 and its distance from ϕ_2 is equal to F_2. If it is on the second nodal plane its size is o_{m+1} (c), and its distance from ϕ_2 is equal to G_2.

Therefore $$\frac{o_1}{o_{m+1}} = \frac{F_2}{G_2}.$$

Therefore, from (c),

$$G_2 = \frac{n_1}{n_{m+1}} = F_1 \quad . \quad . \quad (6).$$

The distance between the second principal and nodal planes $= F_2 - G_2 = F_2 - F_1 \quad . \quad . \quad (7).$

Similarly, the distance between the first principal and nodal planes

$$= G_1 - F_1 = F_2 - F_1 \quad . \quad . \quad (7a),$$

and

$$G_1 = F_2 . \quad . \quad . \quad (6a)$$

These results prove the rules (1), (2), and (3), p. 51, for all cases.

Lastly, we have to consider the method of finding the cardinal points of a system consisting of two other homocentric systems. This is of great importance in ophthalmology, seeing that it is the condition which arises when spectacles are placed in front of the eyes.

Let A and B be the two systems, with their

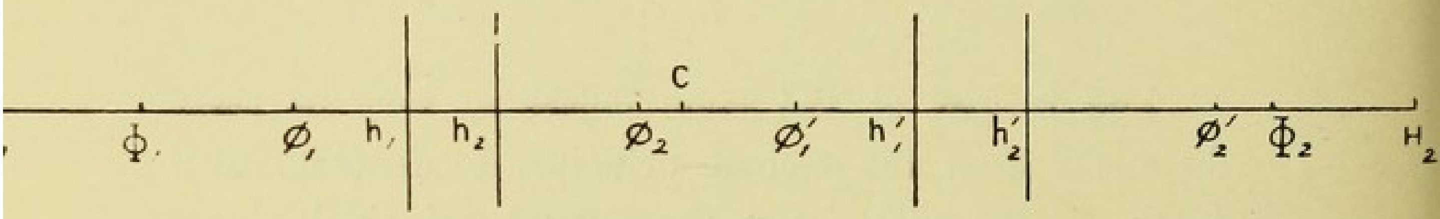

FIG. 30.

cardinal points according to the usual notation. The distances will be represented by letters with a single dash for A and a double dash for B.

Let $$h_2 h'_1 = d.$$

Any ray parallel to the axis after passing through A and B must pass through the first principal focus of the combined system Φ_1. But any ray parallel to the axis after passing through B must also pass through ϕ'_1, the first principal focus of B.

Now $h_2\phi'_1 = d - F''_1$, therefore by (2)

$$\left.\begin{aligned} h_1\Phi_1 &= \frac{(d - F''_1)F'_1}{d - F''_1 - F'_2} \\ \text{Similarly,} \quad h'_2\Phi_2 &= \frac{(d - F'_2)F''_2}{d - F''_2 - F'_2} \end{aligned}\right\} \quad . \quad . \quad (a).$$

The two principal points of the combined system (H_1, H_2) will be the points where an object, *e.g.* at C in the middle medium forms equal images upon the same side of the axis. If o is the size of the object at C, i_1 the size of its image at H_1 formed by A, i_2 the size of its image at H_2 formed by B, then by (3), p. 65,

$$\frac{i_1}{o} = \frac{F'_2}{F'_2 - x},$$

$$\frac{i_2}{o} = \frac{F''_1}{F''_1 - y},$$

where $x = h_2C$, and $y = Ch'_1$.

But $i_1 = i_2$, therefore

$$\frac{F'_2}{F'_2 - x} = \frac{F''_1}{F''_1 - y}.$$

Therefore $$\frac{x}{F'_2} = \frac{y}{F''_1} \quad . \quad . \quad . \quad (b).$$

But $$x + y = d.$$

Therefore $$\frac{x}{F'_2} = \frac{d - x}{F''_1},$$

and $$\frac{d - y}{F'_2} = \frac{y}{F''_1}.$$

$$\left.\begin{aligned} \text{Therefore} \quad x &= \frac{dF'_2}{F''_1 + F'_2} \\ \text{and} \quad y &= \frac{dF''_1}{F''_1 + F'_2} \end{aligned}\right\} \quad . \quad . \quad (c).$$

If we represent H_1h_1 by h_1, and h'_2H_2 by h_2, we have

$$h_1 = \frac{x \cdot F'_1}{x - F'_2},$$

and

$$h_2 = \frac{y \cdot F''_2}{y - F''_1}.$$

Therefore, from (c),

$$\left.\begin{aligned} h_1 &= \frac{dF'_1}{d - F''_1 - F'_2} \\ h_2 &= \frac{dF''_2}{d - F''_1 - F'_2} \end{aligned}\right\} \quad . \quad . \quad (8).$$

And since $F_1 = h_1\Phi_1 - h_1H_1$, and $F_2 = h'_2\Phi_2 - h'_2H_2$, we have

$$\left.\begin{aligned} F_1 &= \frac{F'_1F''_1}{F''_1 + F'_2 - d} \\ F_2 &= \frac{F'_2F''_2}{F''_1 + F'_2 - d} \end{aligned}\right\} \quad . \quad . \quad (9).$$

The nodal points are easily found from (1), p. 51.

In the case of spectacles, the medium is the same on each side of the first system, and formulæ (8) and (9) are simplified by putting $F'_1 = F'_2$. If the lens is placed in such a case, so that $d = F''_1$, *i.e.* at the anterior focus of the second system, we have

$$h_1 = -F''_1$$

$$h_2 = -\frac{F''_1F''_2}{F'_1}$$

$$F_1 = F''_1$$

$$F_2 = F''_2$$

That is, the first principal plane (and consequently also the first nodal plane) remains unaltered, and the change in the second principal plane (and second nodal plane) depends only upon the focal length of the lens.

Spherical and Chromatic Aberration

All the formulæ deduced are approximate, and apply with near approach to accuracy only to small pencils of light. With large pencils the inaccuracy of focusing of spherical surfaces manifests itself as spherical aberration.

We have hitherto confined our attention to the action of rays of uniform wave-length. Sunlight, however, consists of mixed light, a fact first proved by Newton by means of refraction through a strong prism, whereby the white mixed light was *dispersed* into a many-coloured band, the *spectrum*. Colour is therefore a function of refrangibility, which is dependent upon wave-length. The red end of the spectrum is least refracted, and the violet end most.

For prisms with small refracting angles and spherical lenses with large radii, the amount of dispersion which takes place is too small to be appreciated. For stronger prisms or lenses some method has to be adopted to retain their refrangibility whilst eliminating their dispersion. Newton thought this was impossible, but Chester Moor Hall succeeded in making an achromatic telescope. The discovery fell into oblivion, until it was re-discovered by Dollond, a London optician.

At first the inequality of dispersion made it impossible to unite all the different rays, so that secondary, tertiary, etc., spectra were successively formed. Blair, however, succeeded by counteracting the dispersion of crown-glass by fluid lenses containing antimony and mercury chlorides mixed with ammonium chloride or hydrochloric acid. Fluid lenses are very inconvenient, and it was left for Abbé and Schott in 1883 to satisfactorily solve the problem. They succeeded in preparing a great number of various kinds of glass of the most different refractive and dispersive powers, so that high-power lenses can now be made which are practically achromatic.

The dispersion which occurs in non-achromatised lenses causes a blurred, coloured outline in the image, which is said to be due to *chromatic aberration.*

CHAPTER VI

THE DIOPTRIC SYSTEM OF THE EYE

THE reflection of light is sometimes spoken of as *catoptrics,* as opposed to refraction, or *dioptrics.* We have now to investigate the transparent, refractive, or dioptric media of the eye.

The refracting media of the eye are the cornea, the aqueous, the lens, and the vitreous. Each of these has an anterior and a posterior refracting surface, but it will be shown that the surfaces of refraction can be reduced in number without any wide divergence from accuracy.

The anterior surface of the normal cornea is generally considered to be a segment of an ellipsoid of revolution, the axis of revolution being the major axis. For practical purposes, however, the central part, which is most used in vision, is spherical, whilst the outer parts are flatter, thus approaching the curve known as a Cartesian oval, though the flattening differs in the various quadrants. It is excessively difficult to measure the curvature of the posterior surface, but it may be considered practically parallel to the anterior. Moreover, the cornea has only a slightly higher refractive index than the aqueous, and refraction at this surface may be neglected, the cornea being regarded simply as a spherical surface separating air and aqueous.

The aqueous has almost identically the same refractive index as the vitreous, and we may consider these fluids as identical for optical purposes.

The anterior surface of the lens is part of the surface of an oblate spheroid, and the posterior is part of the surface of a paraboloid of revolution. It must be remembered that these estimates of curvature are themselves open to the errors of experiment, so that we shall not transgress if we regard both surfaces as spherical, the anterior being the less curved. The lens is of greater refractive index than the aqueous or vitreous, but it has the further complication that the refractive indices of the successive layers increases from the periphery towards the nucleus. The effect of this is twofold. First, it tends to correct aberration by increasing the convergence of the central rays. Second, the total refractive index of the whole lens is increased, being greater than the refractive index of the nucleus. For it may be looked upon as the combination of a central bi-convex lens, encapsuled in two menisci. The menisci will, of course, tend to counteract the effect of the central lens, but not so much as if their refractive index was the same. In old age the index of the peripheral layers increases, so that the total refractive index of the lens becomes less, and the eye becomes hypermetropic. According to Donders, this amounts to 0·33 D at 60, 1 D at 68, and 2·25 D at 80. This has nothing to do with the presbyopia due to progressive diminution in the power of accommodation. Its effect in producing a false appearance of cataract has already been referred to (p. 6).

We have, then, to deal with three spherical refracting surfaces, separating three media, viz., the anterior surface of the cornea, separating air and aqueous, the anterior surface of the lens, separating aqueous and lens-substance, and the posterior surface of the lens, separating lens-substance and vitreous, which has the same refractive index as aqueous. These are all, to very near approximation, centred along the *optic axis.* The optic axis meets the retina at the fovea centralis, though some authorities say a minute distance above and to the nasal side of that point.

Given then, (1) the radius of curvature of each surface, (2) the refractive index of each medium, and (3) the distances between the refracting surfaces, measured along the optic axis, the system will obey the theory of Gauss, and we have all the data required for a complete solution of its dioptric properties.

It is then found that the system can be further simplified to a hypothetical one possessing only two surfaces. The principal points of this *schematic or diagrammatic eye* of Listing lie close together in the front part of the aqueous, and the nodal points lie close together at the back part of the lens.

Further, these surfaces are so close together that but little error is introduced if we consider them fused into one, occupying a position midway between the two, the nodal points being similarly fused. We thus arrive at the simplest expression of the dioptric system of the eye, which is called the *reduced eye.*

The following are the measurements of the normal eye as determined by Listing and von Helmholtz.[1] The methods of obtaining them will be described in chap. viii.

Index of refraction of aqueous and vitreous	$\frac{103}{77}$	1·3365
Total index of refraction of the lens . .	$\frac{16}{11}$	1·4371
Radius of cornea	8 mm.	7·829 mm.
,, anterior surface of lens . .	10 ,,	10· ,,
,, posterior ,, ,, . .	6 ,,	6· ,,
Distance from anterior surface of cornea to anterior surface of lens . . .	4 ,,	3·6 ,,
Thickness of lens	4 ,,	3·6 ,,

In accommodation the radius of the anterior surface of the lens changes from 10 mm. to 6 mm., and that of the posterior from 6 mm. to 5·5 mm. (von Helmholtz).

From these data we can determine the cardinal points of the cornea and of the lens, and finally of the eye as a whole. Thus the principal points of the cornea coincide at the intersection of the anterior surface with the optic axis. The nodal points coincide at the centre of curvature, and the foci are deduced from (2) and (2*a*), p. 33. The principal points and nodal points of the lens coincide with each other. The focal distances are equal, since the refractive index of the vitreous is the same as that of the aqueous. These points are determined by the formulæ for thick lenses. The cardinal points of the eye as a whole are then deduced, from the combination of the two systems, by the formulæ (8) and (9), p. 70.

The following are the results of these calculations, and give the measurements of Listing's schematic eye.

[1] *Loc. cit.* pp. 89, 140.

	mm.
Anterior focal length of cornea	23·266
Posterior " " "	31·095
Focal length of lens	50·617
Distance of anterior principal point of lens from its anterior surface	2·126
Distance of posterior principal point of lens from its posterior surface	– 1·276
Distance of principal points of lens from one another .	0·198
Posterior focal length of eye	20·713
Anterior " " "	15·498
Position of first principal point	1·753
" second " "	2·106
" first nodal point	6·968
" second " "	7·321
" anterior focal "	– 13·745
" posterior " "	22·819

(To avoid negative signs, the distances behind the cornea are considered positive.)

If now the principal points be merged into one, midway between the two, we obtain the measurements of *Listing's reduced eye*, as follows :—

	mm.	
Principal point . .	2·3448	. behind the anterior surface of the cornea.
Nodal point . .	0·4764	. in front of the posterior surface of the lens.
Posterior principal focus	22·819	. behind the anterior surface of the cornea, *i.e.* on the retina.
Anterior principal focus	12·8326	. in front of the anterior surface of the cornea.
Radius of curvature of surface . .	5·1248	...
Index of refraction of medium . .	1·3365	. *i.e.* the same as the aqueous.

For purposes of rapid calculation Donders suggested a less accurate reduced eye. *Donders' reduced eye*, like Listing's, consists of a single spherical surface and a single refracting medium. The radius of curvature is 5 mm., and the index of refraction of the medium 1·333, which is the index of water. The anterior focal distance is 15 mm., and the posterior focal distance 20 mm.

The enormous advantage in the economy of space of the arrangement of the dioptric media of the eye is very striking. Thus the posterior focal distance of the cornea alone is 31 mm., and that of the lens 50·6 mm., whilst that of the combined system is less than 23 mm.

Construction of the Retinal Image

This is very easily accomplished for the reduced eye. All that has to be done is to draw lines from points of the object through the nodal point to the retina. For the schematic eye it is exactly as in Fig. 24. The image is real, inverted, and diminished. It is reinverted psychologically, so that objects appear erect.

The Size of the Retinal Image

This is given by the formula

$$\frac{o}{i} = \frac{F_1 - f_1}{F_1} = \frac{F_2}{F_2 - f_2}.$$

The Distance of the Retinal Image

This is given by the formula

$$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1,$$

or

$$l_1 l_2 = F_1 F_2.$$

The Minimum Visual Angle

This is the angle subtended at the posterior nodal point by a single foveal cone. If θ is the required angle and c the diameter of a foveal cone, then

$$\tan\frac{\theta}{2} = \frac{\frac{c}{2}}{k_2\phi_2} = \frac{\frac{c}{2}}{\phi_1 h_1} = \frac{\cdot 001}{15\cdot 498308} = \cdot 000064523.$$

This value is found from the mathematical tables to be the tangent of an angle of $13''\cdot 308$. Therefore $\theta = 26''\cdot 616$.

In addition to the real image, there are also several *false images* formed in the normal eye; for it must be remembered that the refracting surfaces are also reflecting surfaces. Most of the reflected rays are in such a direction that they do not impinge upon the retina, and hence no image is perceived; but there are certain rays from the posterior surface of the lens to the back of the cornea which are again reflected so as to reach the retina. The image formed thus may in some cases give rise to an entoptic image and become the basis of a monocular diplopia.

The position of the *centre of rotation* of the eyeball is important. It is situated 13·54 mm. behind the summit of the cornea (Donders), and varies from 14·52 mm. to 13·22 mm. in myopic and hypermetropic eyes.

Strictly speaking, the *optic axis* is the line through the nodal point and the centre of rotation, and the refractive surfaces are *not* accurately centred upon it. The *line of vision* is the line passing from the fovea through the nodal point to the point of fixation in the

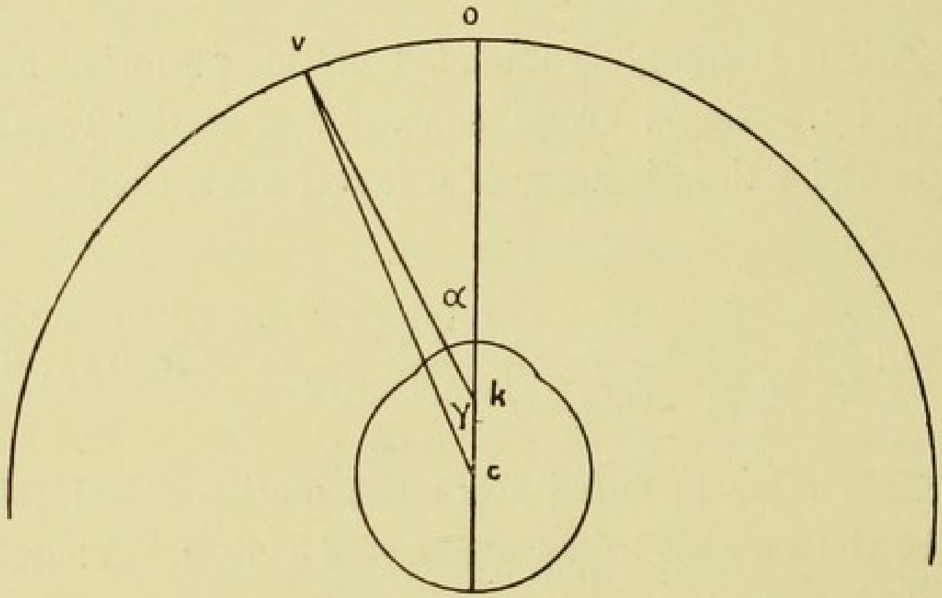

FIG. 31.

field of vision. The *line of fixation* is the line passing from the centre of rotation to the fixation point.

The angle between the line of vision and the major axis of the corneal ellipse is called the *angle* α.[1] Donders considered the major axis of the corneal ellipse to coincide with the optic axis, as in Fig. 31. In this case the angle α varies with the *angle* γ, which is the angle between the optic axis and the line of fixation. The angle α is about 5° (Donders); hence

[1] According to Sulzer (*La forme de la cornée humaine*) the curve of the cornea is not an ellipsoid of revolution, and it has no axis of symmetry. If this is true, the angle α has now no definite meaning.

when the lines of vision of the two eyes are parallel, the optic axes diverge about 10°. The deviation is greater in hypermetropia, and may be negative in myopia.

The *angle* κ is the angle which the fixation line makes with the normal to the cornea through the centre of the pupil. The centre of the pupil is usually slightly to the inner side of the cornea, so that the angle κ is not equal to the angle γ. It is very much easier to measure, and is therefore the one generally used in cases of strabismus.

CHAPTER VII

AMETROPIA

We have to deal in practice with two principal types of ametropia or error of refraction. These are known as *axial* and *curvature ametropia*. *Simple hypermetropia and myopia* are forms of axial ametropia, and are due to abnormality in the length of the eye, whereby the eye is either too short, so that the posterior principal focus lies behind the retina, or too long, so that this focus is in front of the retina. Curvature ametropia is due to abnormal curvature of one or more of the refracting surfaces. It may rarely give rise to simple ametropia, but is most common as *astigmatism*. A normal alteration of refraction due to change of curvature is that which occurs in accommodation. It is clear from the preceding chapters that there is a third possible cause of ametropia, viz., a change in the index of refraction of one or more of the media. We have already mentioned an instance of this form in the acquired hypermetropia of old age.

In simple hypermetropia the eye is too short, and parallel rays are not brought to a focus without some degree of accommodation. In simple myopia the eye is too long, and parallel rays are brought to a focus in the vitreous. Tracing the rays in the reverse direction, those coming from any point upon the retina are convergent when they pass out of the myopic eye at rest, so that a real image of the point is formed at a finite distance in front of the eye. This position is called the *remote point* of the eye. In emmetropia, in which the emergent rays are parallel, the *punctum remotum* is at infinity. In hypermetropia, the emergent rays are divergent when the eye is at accommodative rest, consequently the remote point, found by producing these divergent rays backwards until they meet, is virtual and situated at a finite distance behind the eye. When there is no lens, as in aphakia, the posterior principal focus is behind the eye, and the condition is one of extreme hypermetropia.

There are two chief forms of astigmatism, viz., regular and irregular. Irregular astigmatism is due to irregularity of one of the refracting surfaces, usually the cornea, and is most frequently the result of previous injury whereby a scar or nebula is formed. Regular astigmatism may be due to the cornea or to the lens, but by far the more often to the cornea. In this condition the curvature is greater in one meridian than in the meridian perpendicular to it. Hence it is impossible for all the rays to be brought to a focus at the same point. Usually the vertical meridian has a greater curvature than the horizontal, so that the rays passing through the vertical meridian are brought to a focus first. The distance between the two foci is known as the *focal interval of Sturm.*

It is impossible to represent the course of the rays satisfactorily in a diagram upon a plane surface, but it may be reproduced easily by passing parallel rays through the combination of a convex lens with a cylindrical one. As its name implies the curvature of a cylindrical lens is a portion of a cylinder, and its "axis" is that diameter which is parallel to the axis of the generating cylinder. This use of the word "axis" must be carefully distinguished from its use in speaking of spherical lenses. In all planes parallel to the axis the rays will pass through a plate with parallel sides, so that if the lens is thin the deviation of the rays is negligible. In all planes at right angles to the axis the lens will act exactly like a spherical lens—the rays will be refracted at the spherical surface, and brought to a focus at a distance dependent upon the strength of the lens. At intermediate angles the rays will undergo refraction at an elliptical surface, the curvature of which will vary *pari passu* with the angle.

Fig. 32 may be taken to represent transverse sections of the cone of light transmitted through a

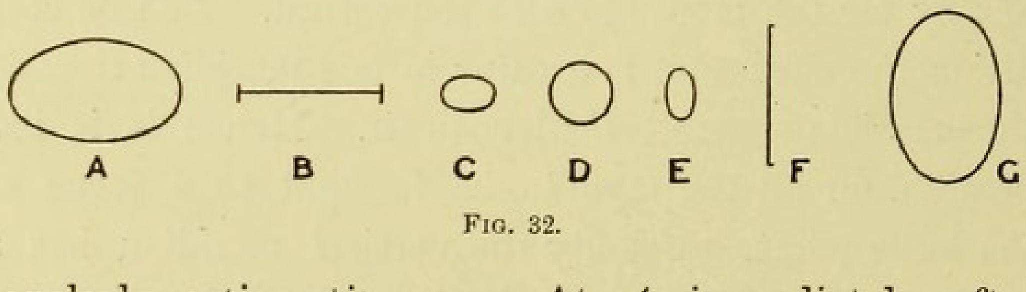

Fig. 32.

regularly astigmatic eye. At *A*, immediately after refraction, the rays form an oblate ellipse; at *B* the vertical rays have come to a focus, and the horizontal still form a horizontal plane of light; at *C* the vertical rays are beginning to diverge, the horizontal still converging; at D the conditions of divergence and convergence are equal, and the section is a circle; at E the vertical divergence is greater than the horizontal convergence, and the section is a prolate ellipse; at F the horizontal convergence is complete, these rays having come to a focus, whilst the vertical continue to diverge; and at G both sets of rays are diverging in unequal degrees. Hence there can be no common focal point for all the rays, and a retina situated at A, B, C, etc., will receive corresponding diffusion images. Of these, that at D will give the clearest possible general image of the whole of an object, and this is called the *circle of least diffusion.*

The actual variety of regular astigmatism depends both upon the conditions of curvature of the cornea and the length of the eye. If Fig. 32 represents the refraction of parallel incident rays, and B, C, D, etc., represent the diffusion images upon the retina, then in B the vertical meridian will be emmetropic and the horizontal hypermetropic, and the condition is one of simple hypermetropic astigmatism. A will represent compound hypermetropic, and F compound myopic astigmatism, whilst C, D, E, all represent forms of mixed astigmatism.

A rarer form of ametropia is found in the condition known as *keratoconus* or *conical cornea.* Here the curvature of the cornea approaches an hyperboloid.

The Correction of Ametropia

The first essential in the correction of errors of refraction is to bring the rays to a focus upon the retina. The rays from a distant object (*i.e.* practically anything over 6 metres) are approximately parallel. Such rays will be brought to a focus behind the retina in the hypermetropic eye, and in front of it in the myopic. It is easy to see that we can find a convex lens which will give the exact degree of extra convergence in the first case, and a concave lens which will give the exact degree of divergence in the second case, to enable the eye to now focus the rays upon the retina. It is also clear that the strength of this lens will be different according to its distance from the eye. These facts are shown either geometrically or algebraically by the formulæ in chap. v. p. 70. We there proved that if the lens was placed in the anterior focal plane of the eye (*i.e.* 13·745 mm. or about half an inch in front of the cornea) the first principal and nodal planes of the combined system are the same as those of the eye alone, and that the focal distances of the combined system are the same as those of the second. If the error of refraction is an axial one the focal distances of the eye are clearly the same as those of an emmetropic eye; hence we may also state that the focal distances of the combined system are the same as those of an emmetropic eye.

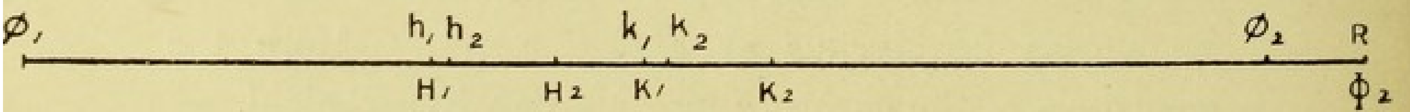

FIG. 33.

In Fig 33 let R be the retina of a myopic eye, $\phi_1\phi_2$, etc., the cardinal points of the eye, and let H_1, H_2, and K_1, K_2, be the principal and nodal points of the combined system. Let the eye be x mm. too long, *i.e.* $\phi_2 R = x$. Let F' be the focal length of a thin concave lens placed at ϕ_1, so as to bring the parallel incident rays to a focus at R. And let F''_1 F''_2, be the focal lengths of the eye. Further, let $h_2 R = p$, and let the distance of the remote point of the eye from $h_1 = p_r$.

Then $$p - F''_2 = -x,$$
and $$F' = p_r - F''_1.$$

But, by the formula $l_1 l_2 = F_1 F_2$,
$$(p_r - F''_1)(p - F''_2) = F''_1 F''_2.$$

Therefore $$p - F''_2 = \frac{F''_1 F''_2}{p_r - F''_1}.$$

Therefore $$-x = \frac{F''_1 F''_2}{F'} \quad . \quad . \quad (1)$$
$$= \frac{15{\cdot}498 \times -20{\cdot}713}{F'}$$
$$= -\frac{321}{F'}.$$

And $$F' = \frac{321}{x} \quad . \quad . \quad . \quad (2).$$

This formula gives the focal length of the correcting lens when it is placed in the anterior focal plane of the eye.

We showed, on p. 70, that

$$h_2 = -\frac{F''_1 F''_2}{F'}.$$

Therefore $$h_2 H_2 = -x \quad . \quad . \quad (3).$$

Similarly, by (3) p. 51,

$$h_2H_2 = k_2K_2 = -x \quad . \quad . \quad (4).$$

Also
$$\left.\begin{array}{l} F_1 = F''_1 \\ F_2 = F''_2 \end{array}\right\} \quad . \quad . \quad . \quad (5).$$

But
$$\frac{o}{i} = \frac{F''_1}{F''_1 - F'}$$

$$= \frac{F_1}{F_1 - F'} \quad . \quad . \quad (6).$$

Therefore, *the size of the image formed by an axially ametropic eye combined with the correcting lens placed in the anterior focal plane of the eye is the same as the size of the image formed by an emmetropic eye.*

This fact is of enormous importance, since only under these conditions are the tests of visual acuity comparable with those of emmetropia.

We have now to consider the conditions if the position of the correcting lens is altered.

For the eye alone, we have from (3), p. 65,

$$i = o \, . \, \frac{F''_1}{l_1}$$

(omitting the negative sign, which merely indicates the inversion of the image).

For the eye *plus* the lens, we have

$$i' = o \, . \, \frac{F_1}{l'_1}.$$

Therefore
$$\frac{i'}{i} = \frac{l_1 F_1}{l'_1 F''_1} \quad . \quad . \quad . \quad a)$$

But by (9), p. 70,

$$F_1 = \frac{F' F''_1}{F' + F''_1 - d}.$$

Therefore $$\frac{i'}{i} = \frac{l_1 F'}{l'_1(F' + F''_1 - d)} \quad . \quad . \quad (b).$$

If, in Fig. 34, P is the position of the object, and h_1 that of the lens; and ϕ_1 is the anterior focus of the

FIG. 34.

lens, ϕ'_1 that of the eye, Φ_1 that of the combined system, and h'_1 the principal point of the eye, then

$$l_1 = P\phi'_1$$

and $$\begin{aligned} l'_1 = P\Phi_1 &= P\phi'_1 + \phi'_1\Phi_1 \\ &= P\phi'_1 + (h_1\Phi_1 - h_1\phi'_1) \\ &= P\phi'_1 + h_1\Phi_1 - (h_1h'_1 - \phi'_1h'_1) \\ &= l_1 + h_1\Phi_1 - (d - F_1''). \end{aligned}$$

But by (a), p. 69,

$$h_1\Phi_1 = -\frac{F'(F''_1 - d)}{F' + F''_1 - d}.$$

Therefore $$\begin{aligned} l'_1 &= l_1 + \frac{(F''_1 - d)^2}{F' + F''_1 - d} \\ &= \frac{l_1F' + l_1(F''_1 - d) + (F''_1 - d)^2}{F' + F'' - d} \quad (c). \end{aligned}$$

Therefore, from (b)

$$\frac{i'}{i} = \frac{F'}{F' + (F''_1 - d) + \frac{1}{l_1}(F''_1 - d)^2} \quad (7).$$

(1) If, in this equation $F''_1 = d$, we find that $i' = i$, the same result we arrived at before.

(2) For a *convex lens*:—

If the object is so far away that $\frac{1}{l_1}(F''_1 - d)^2$ may be neglected, the equation becomes

$$\frac{i'}{i} = \frac{F'}{F' + F''_1 - d} \quad . \quad . \quad (d).$$

The ratio $i' : i$ under these circumstances is increased by increasing the value of d, and *vice versâ*. Therefore, for distant objects, *the image increases as the convex lens is carried away from the anterior focus of the eye, and diminishes as the lens is brought nearer the eye.*

Equation (d) may be written

$$\frac{i'}{i} = \frac{F'}{F' - (d - F''_1)} \quad . \quad . \quad (e).$$

If F''_1 is less than d, and if $d - F''_1$ is greater than F' the ratio $i' : i$ becomes negative, and an inverted image is formed by the lens in front of the eye.

When the object is nearer the lens, we must take into account the expression $\frac{1}{l_1}(F''_1 - d)^2$. If F''_1 is still less than d, when

$$d - F''_1 > F_1 + \frac{1}{l_1}(F''_1 - d)^2 \quad . \quad (f),$$

the ratio $i' : i$ becomes negative, and an inverted image is also formed under these conditions.

If
$$d - F''_1 = F_1 + \frac{1}{l_1}(F''_1 - d)^2 \quad . \quad (g),$$

we get
$$\frac{i'}{i} = \frac{1}{0} = \infty .$$

From equation (g) we find that

$$d - F''_1 = \frac{l_1}{2}\left(1 \pm \sqrt{1 - \frac{4F'}{l_1}}\right),$$

i.e. when $l_1 = 4F'$,

$$\frac{i'}{i} = \infty \quad \text{and} \quad d - F''_1 = \frac{l_1}{2} \qquad . \quad (h).$$

If $l_1 < 4F'$, there is no real value for $d - F''_1$. Hence l_1 must be > or $= 4F'$.

If $l_1 = 4F'$, *we see that the image is infinite when the convex lens is midway between the object and the anterior focus of the eye, and diminishes in size on each side of this point.* It will be seen that the law is the same as that for a simple convex lens (see (1), p. 48).

If F''_1 is greater than d, $(F''_1 - d)$ will be positive, and from (7) i' will always be less than i. That is, *the image is always diminished if the convex lens is closer to the eye than the anterior focus of the eye.*

From (7) the images are also equal if $l_1 = d - F''_1$, *i.e.* when the object and the lens are in contact.

(3) For a *concave lens*:—

Here F' is negative, and from (7), if $F''_1 > d$ then $i' > i$; and if $F''_1 < d$, then $i' < i$, except when $l_1 = d - F''_1$, and then $i' = i$, as with a convex lens. Hence, *a concave lens nearer than the anterior focus of the eye magnifies the image, whilst beyond this point it diminishes it.*

It follows from the above deductions that *the convex lens required to correct simple hypermetropia* *must be stronger the nearer it is placed to the eye; and the concave lens required to correct simple myopia must be weaker the nearer it is placed to the eye.*

Spectacle glasses are, for mechanical reasons, usually rather farther from the eye than its anterior focus, consequently *the retinal image is larger in corrected hypermetropia, and smaller in corrected myopia, than in the emmetropic eye.* The practical rule follows, that *the lenses should be as near the eyes as possible in high myopia.* As we shall see later, there are other reasons also in favour of this rule.

A glance at Fig. 24 will show that the size of the retinal image depends upon two factors: (1) the visual angle subtended by the object; (2) the distance between the second nodal point and the retina (G_2). If the visual angle be kept constant and minimal, the retinal image, and therefore the *visual acuity,* will vary directly as the distance between the second nodal point and the retina.

If we replace d in equations (9), p. 70, by $F''_1 + y$, where y is the variable distance of the correcting lens from the anterior focal plane of the eye, we have (remembering that $F'_1 = F'_2 = F'$, say)

$$F_1 = \frac{F'F''_1}{F' - y} = G_2 \qquad . \quad . \quad (8).$$

By (1), p. 87, the distance of the retina from the posterior focus of the emmetropic eye $= \pm x = \pm \frac{F''_1 F''_2}{F'}$; the *plus* sign representing myopia, and the *minus* sign hypermetropia. Therefore, omitting the diffusion circles, the emmetropic retinal image (r) and the ametropic retinal image (r') have the following relationship,

$$r : r' = F_1 : F_1 \pm \frac{F''_1 F''_2}{F'} \quad . \quad . \quad (9).$$

That is, *the uncorrected hypermetropic eye has smaller, and the uncorrected myopic eye has larger, retinal images than the emmetropic eye.*

We must now consider the effect of accommodation upon the images. The optical effect of accommodation is to increase the distance between the second nodal point and the retina. Hence, *accommodation is accompanied by increase in the size of the retinal image.* The *relative visual acuity,* or visual acuity for near objects, is therefore greater than the *absolute visual acuity*, or acuity for distant objects. This accounts in large measure for the difficulty in relaxing the accommodation in ophthalmoscopic examination by the direct method.

The uncorrected *hypermetropic* eye has smaller retinal images than the emmetropic eye; these are increased beyond the size of the emmetropic images by a correcting lens outside the anterior focal plane of the eye. Similarly the correction of presbyopia also results in the production of larger images.

The uncorrected *myopic eye* has larger retinal images than the emmetropic eye; these are made smaller than the emmetropic images by a correcting lens outside the anterior focal plane of the eye. Bringing the lens inside the anterior focal plane magnifies the images beyond those of the emmetropic eye. Further, near objects also form larger retinal images than in the emmetropic eye, the increase in length of the eye being usually greater than the slight increase in the distance between the second nodal point and the retina, due to accommodation, in the emmetropic eye.

We have still to consider the effect of accommodation upon the visual acuity of ametropic eyes which are corrected for distance. It might at first sight be thought that, with the correcting lens in the anterior focal plane, the eyes were in a condition of artificial emmetropia; but this is only true for distance. The correcting lens must necessarily be placed some distance in front of the eye, and the effect of this is to alter the amount of the range of accommodation as well as its position; so that convex glasses diminish it, and concave glasses increase it. Hence, in *hypermetropia* a greater amount of accommodation is necessary to produce the same effect as in emmetropia; and there is therefore a greater relative increase in the size of the images. In *myopia* the reverse occurs: there is a wider range of accommodation than in emmetropia, and the size of the image is increased relatively less.

These results are of peculiar importance in the correction of *anisometropia*, where the ametropia is different in the two eyes. Here, if the difference is great (*e.g.* $4D$) much confusion results from the inequality of the two retinal images, any variation in accommodation causing greater inequality, to which must also be added an unnatural relationship between the amount of accommodation and that of convergence. It is questionable whether any general rule can be formulated for these cases, and the subject requires further clinical investigation.

We have seen that in *presbyopia*, correction for distance, with the lens farther than the anterior focal plane, increases the size of the retinal image. The same applies if the object is brought closer to the lens to any distance greater than twice the focal length of the lens. If it is brought closer still, however, the size of the image is diminished by carrying the lens farther away from the anterior focal plane of the eye. It is to be noticed that it may still remain larger than the emmetropic image, but the movement of the lens causes a progressive diminution. If there is high hypermetropia as well as presbyopia, the image may increase in size. These facts are best seen by applying appropriate figures to equation (7), p. 89. Hence it is not generally true to suppose that presbyopes can increase the power of their reading-glasses by wearing them on the tip of the nose, nor is it generally true that this increases the size of the retinal image, as often stated.

In *aphakia* we have found that the anterior focal plane of the eye is 23·2659 mm. in front of the cornea, hence the correcting lens will only vary directly as the length of the eye if it is placed at this distance (nearly 1 inch) in front of it. The posterior principal focus is 31·0949 mm. behind the cornea, so that if the eye was emmetropic, and therefore 22·8231 mm. long, before removal of the lens, the image is far behind the retina, and the diffuse retinal image is much larger than in the normal eye. If the correcting glass is worn in about the usual position, *e.g.* 15 mm. in front of the eye, it will be well inside the anterior focal plane, and the image will be diminished. The remote point of the eye is found by the formula

$$\frac{n}{f_1} - \frac{1}{f_2} = \frac{n-1}{r},$$

which gives $f_2 = -64{\cdot}1941$ mm. The focal length of the correcting lens, 15 mm. from the eye, is therefore $-64{\cdot}1941 - 15 = -79{\cdot}1941$ mm., and the strength of the lens $= \dfrac{1000}{79{\cdot}1941} =$ about $12{\cdot}5D$.

We have seen that the size of the retinal image varies as $G_2(=F_1)$, the distance of the second nodal point from the retina. In the aphakic, uncorrected eye we have $F''_1 = 23{\cdot}2659$ mm., and the focal length of the correcting lens, $F' = -79{\cdot}1941$ mm., when placed 15 mm. from the eye. Therefore

$$\begin{aligned} F_1 &= \frac{F'F''_1}{F''_1 + F' - d} \\ &= \frac{-79{\cdot}1941 \times 23{\cdot}2659}{-23{\cdot}2659 - 79{\cdot}1941 + 15} \\ &= 20{\cdot}0670 \text{mm.} \end{aligned}$$

In the normal eye $F_1 = 15{\cdot}4983$.

Therefore

$$\frac{i'}{i} = \frac{20{\cdot}067}{15{\cdot}4983} = 1{\cdot}35.$$

That is, *the retinal image in the corrected aphakic* *eye is about a third larger than in the emmetropic eye.* Hence a visual acuity of $\frac{6}{9}$ is only equivalent to about the normal $\frac{6}{12}$.

Since the posterior focal length of the aphakic eye is 31·0949 mm., it is 8·2718 mm. more than the length of the normal eye. By formula (1), p. 87, this is equivalent to a lens of $-\frac{8271 \cdot 8}{321} = -25 \cdot 76 D$. Hence, if the eye had this degree of myopia before the lens was removed, it would require no correcting lens for seeing distant objects afterwards. The size of the retinal image will be greater than the size of the emmetropic image in the ratio of the corresponding anterior focal distances.

$$\frac{i'}{i} = \frac{23 \cdot 2659}{15 \cdot 498} = 1 \cdot 43.$$

That is, the image is nearly half as large again as in the emmetropic eye. Hence the visual acuity is greatly increased in the successful cases of removal of the lens for high myopia.

A slight change in the position of the crystalline lens will produce a large effect in the dioptric condition of the eye. If the lens moves towards the cornea the eye will become myopic; if towards the retina, hypermetropic. The former is the usual method of accommodation in most snakes and amphibia, whilst the latter occurs in teleostean fishes (Beer).

CHAPTER VIII

THE OPHTHALMOMETER

We have found that certain measurements of the eye are necessary in order that the cardinal points may be calculated. These might be determined upon the dead eye, but the *post-mortem* diminution of intra-ocular pressure renders this method inaccurate. In the living eye the difficulty is increased owing to the unavoidable movements, but this can be overcome by the ophthalmometer, the earliest form of which was devised by von Helmholtz.

The basis of ophthalmometry is the formula

$$\frac{o}{i}=\frac{l}{F}=\frac{2l}{r},$$

or

$$r=\frac{2li}{o} \quad . \quad . \quad . \quad (1)$$

where l is the distance of the object from the refracting surface, o the size of the object, i the size of the image, and r the radius of curvature. The formula is only exact if l is large, so that the rays are approximately parallel. More accurately it is

$$r=\frac{2li}{o-i}.$$

The only difficulty is the measurement of i.

The object generally used is the distance between two lights, or two white objects, called *mires*—the image being the distance between their images. Owing to the movements of the eye, the latter cannot be accurately measured by the usual method employed by physicists of looking at the images through a telescope which has a micrometer at the focus of the objective. The difficulty is overcome by doubling the image.

Suppose we wish to measure the distance D between two points, A and B, and that we have some means of doubling the images of A and B, so that we now have four points, A_1, A_2, corresponding to A, and B_1, B_2, corresponding to B, separated by the distance d, so that $A_1A_2 = B_1B_2 = d$. Now, if we can vary the distance d, there will be a point at which A_2 coincides with B_1, and at that point d will be equal to D. We can thus determine D if the amount of doubling, d, is known.

The same result can be attained by keeping the distance d constant, and varying the distance between A and B until contact between A_2 and B_1 occurs.

Various methods have been employed to effect the doubling of the images. We will first consider the earliest form of ophthalmometer of von Helmholtz.

In this instrument the doubling is brought about by two plane-glass plates, set at a variable angle to one another.

If an object O (Fig. 35) is seen through a glass plate, aa, by an observer at l, it will apparently occupy the position O'; if seen through bb, O''.

Let *aa* (Fig. 36) be one of the plates, *AB* the

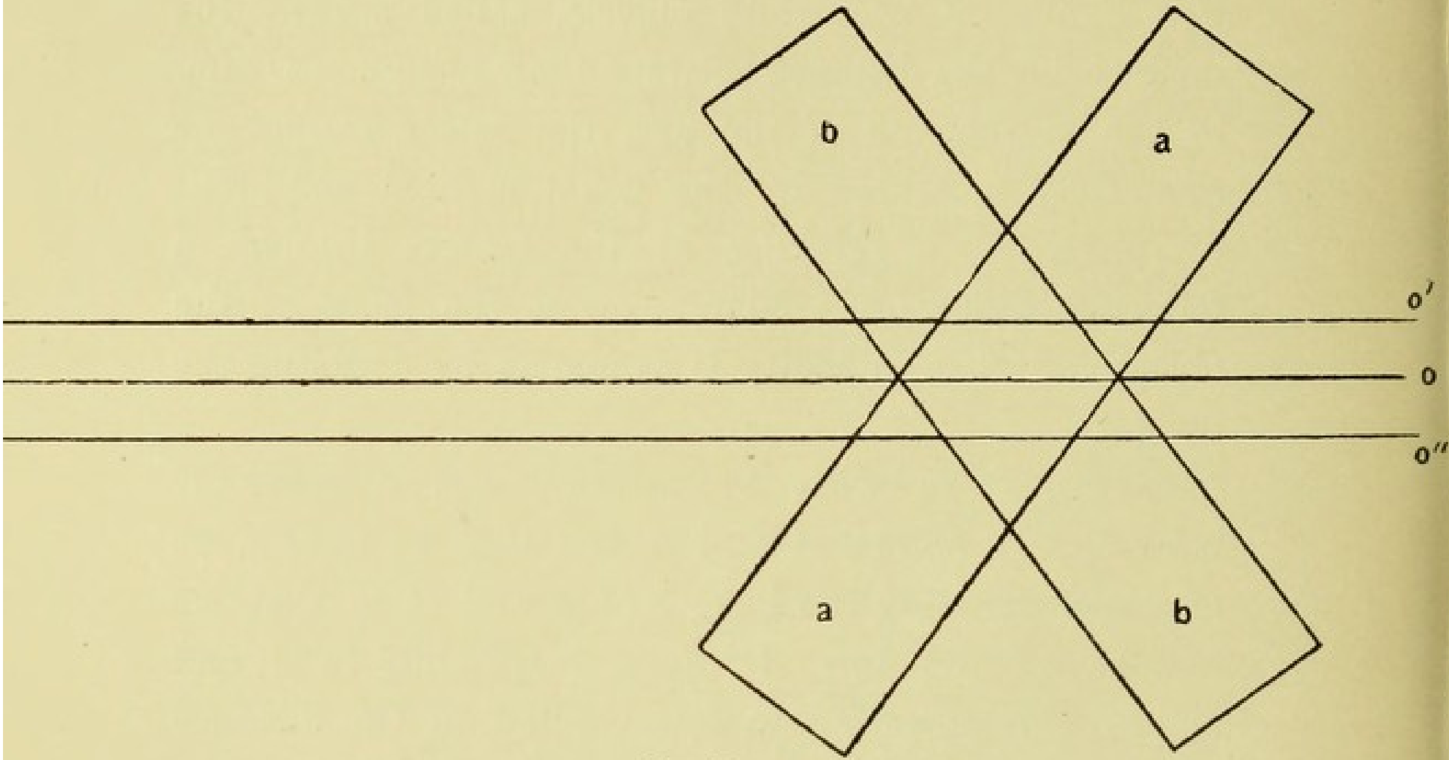

FIG. 35.

incident, *CD* the refracted ray. Then, since the re-

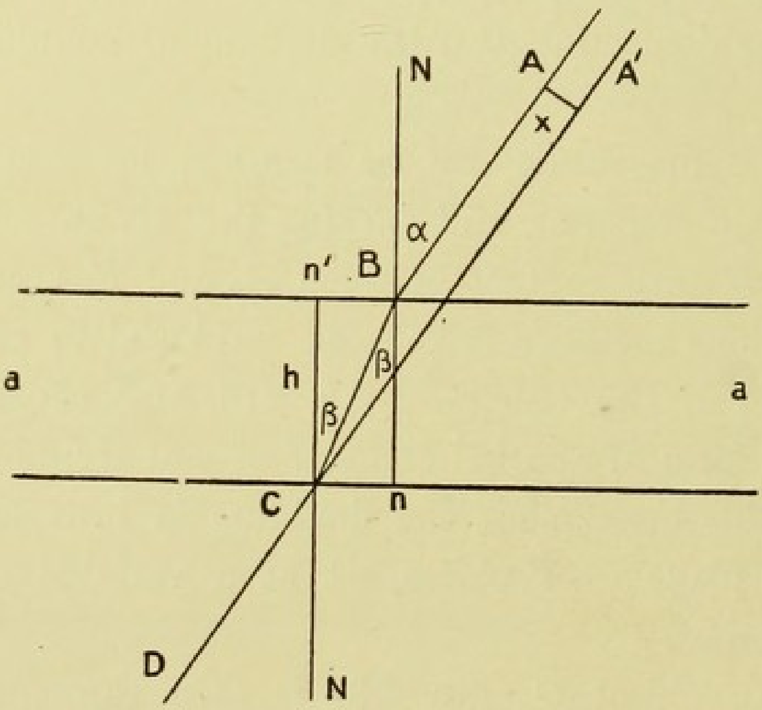

FIG. 36.

fracted ray is parallel to the incident ray, the angle

ABN is equal to the angle $DCN'(=a)$. Similarly the angle of refraction CBn is equal to the angle $BCn'(=\beta)$. Let h be the thickness of the glass plate. Produce DC backwards to A'. It is required to find the perpendicular distance between A and $A'(=x)$.

Now $$\frac{x}{BC} = \sin BCA'$$
$$= \sin(n'CA' - n'CB)$$
$$= \sin(a - \beta).$$

And $$\frac{h}{BC} = \cos\beta.$$

Therefore $$x = h \,.\, \frac{\sin(a-\beta)}{\cos\beta}.$$

If there are two such plates, arranged as in Fig. 35, then

$$O'O'' = 2x$$
$$= 2h \,.\, \frac{\sin(a-\beta)}{\cos\beta} \quad . \quad . \quad (2).$$

The angle a is measured by the instrument; the angle β is calculated from the formula for refraction, $\sin a = n \,.\, \sin\beta$; and the thickness of the plates, h, is known. Therefore the distance between the images can be calculated. In actual practice these calculations are already worked out and classified in tables supplied with the instrument.

The Curvature of the Cornea

In order to measure the curvature of the cornea a dark room is necessary, and the instrument must be about 2 metres from the eye, so that slight movements of the eye can be neglected, and the formula is more exact.

Fig. 37 represents the arrangement of the instrument. There is a screen in front of the ophthalmometer carrying two lights, *a* and *b*, about half a metre apart. Four points are seen reflected on the cornea, and the plates are adjusted until the two middle points coincide. The size of the image is then calculated,

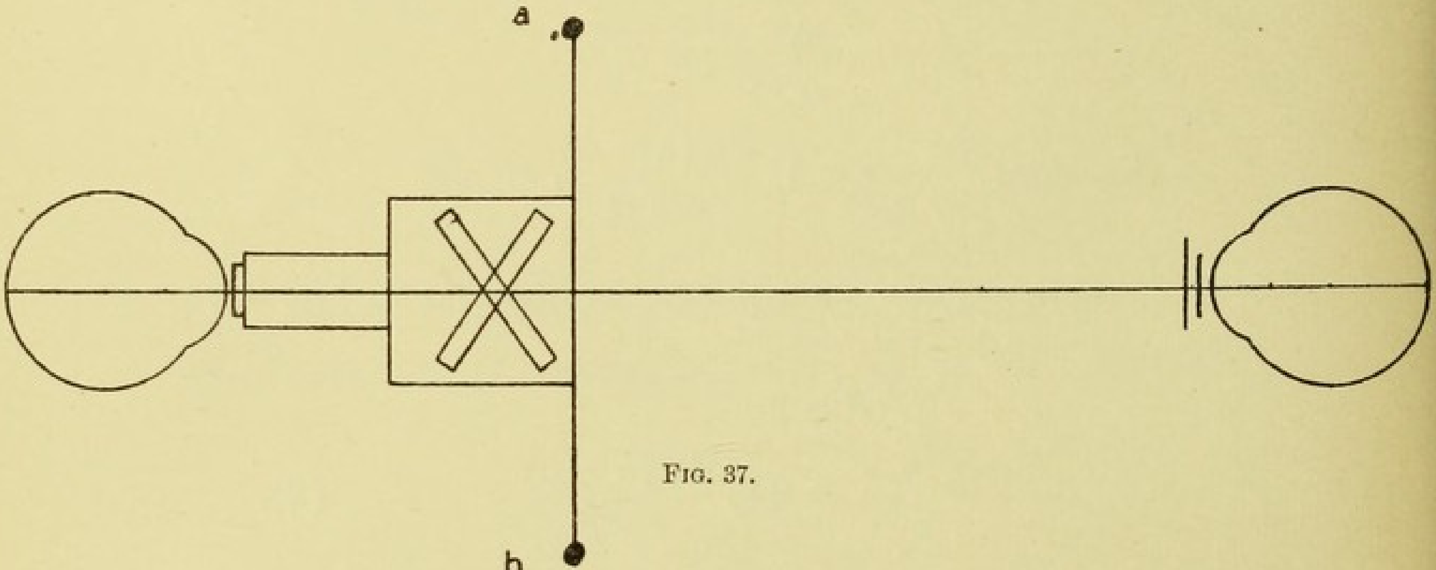

FIG. 37.

or found from the table, and the radius of curvature calculated from formula (1).

The complete measurement of a cornea by this method involves at least thirty-two determinations, which are made more difficult by the necessary distance of the instrument from the eye.

Many other methods of doubling the image have been devised, such as bisecting the ocular of the telescope, or the use of prisms (Landolt's Ophthalmometer). Coccius used Iceland spar, and Javal and

Schiötz a Wollaston's prism. The latter is made of two rectangular prisms of quartz, cemented together so as to form a thick plate. If the two prisms are cut in the opposite axes of the quartz crystal, the combination is doubly refracting, and symmetrical as regards the incident ray. The method has the advantage of producing two complete cones of light, instead of dividing a single cone into halves. The images are brought into contact by altering the distance between the two lights. Mandelstamm and Schöler attempt to measure the image directly by means of a microscope and micrometer.

Tscherning has introduced a valuable modification of the ophthalmometer, which he calls the ophthalmophakometer. It is specially designed to obtain brighter reflections from the surfaces of the lens and the posterior surface of the cornea. The eye is placed at the centre of an arc, which carries a telescope (without any doubling mechanism) at its mid-point, and three carriers. One carrier has a six-volt lamp; another a vertical bar carrying two weaker lamps, one above and the other below the arc; and the third carrier has a bright ball upon it for use as a fixation point.

The Distance between the Anterior Surface of the Cornea and the Anterior Surface of the Lens

This was originally measured by Donders with a corneal microscope. Von Helmholtz used the ophthalmometer, taking advantage of the fact that the iris is in contact with the anterior surface of the lens. He measured the distance of the apparent situation of the corneal images from the edge of the pupil. For objects situated at a considerable distance, the reflected

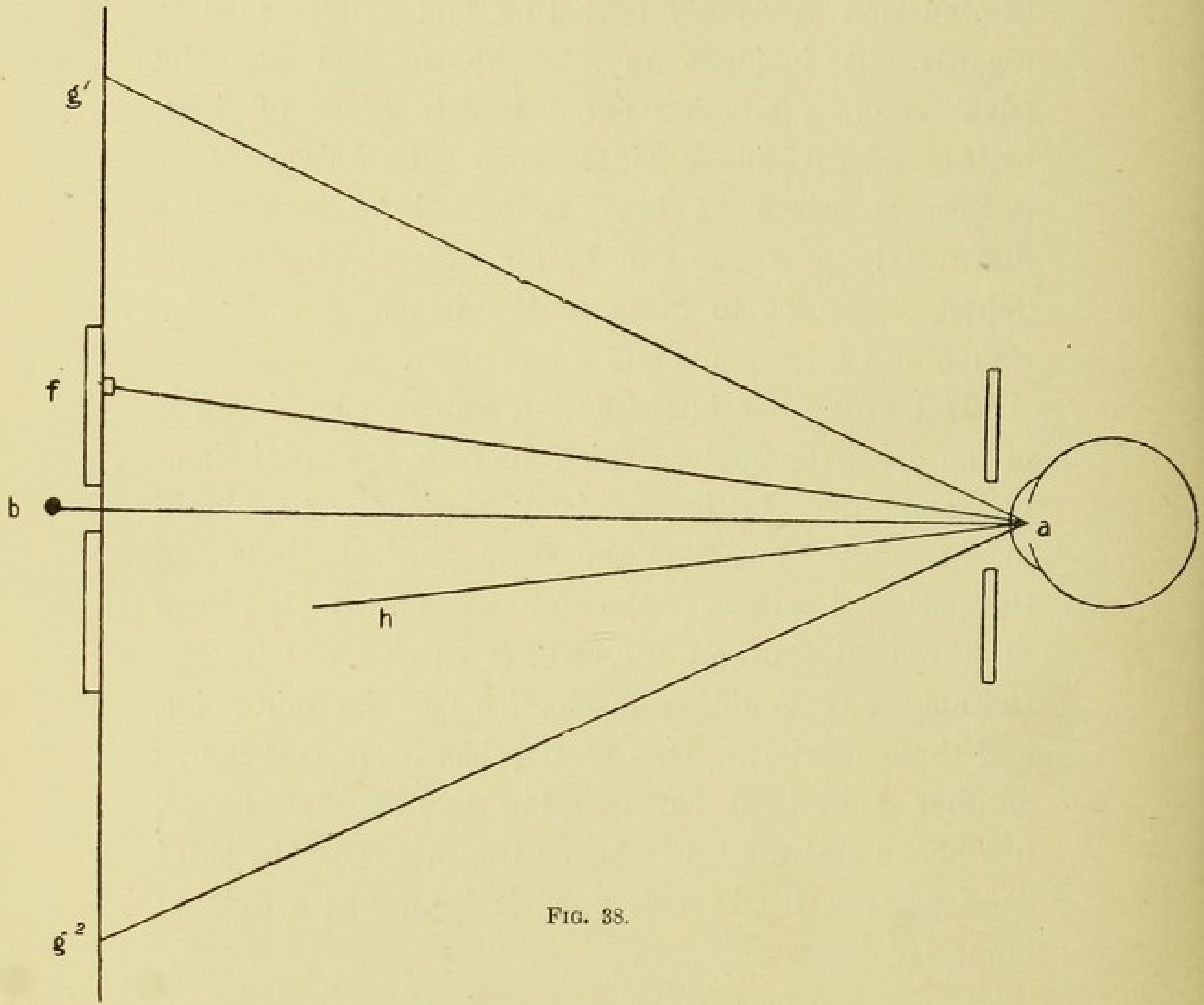

Fig. 38.

image in the corneal axis lies behind the cornea at a distance equal to half its radius of curvature.

In Fig. 38, if there is an object at *b*, its image, seen through the ophthalmometer at g^1, will be doubled, and the pupil will also be doubled. The fixation point, *f*, of the eye is then moved until the doubled corneal images correspond with the edges of the doubled pupil, as in Fig. 39.

The ophthalmometer is then moved to an equal distance on the other side of b, viz. g^2, and the operation is repeated. The lines g^1a, g^2a, will intersect in the plane of the pupil. The angle fah is the angle a. By taking this into account, and also allowing for the difference between the real and apparent positions of the images, the distance between

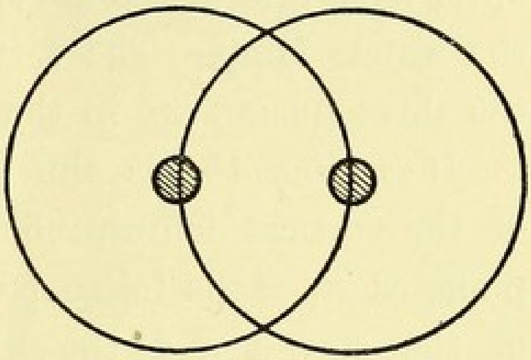

FIG. 39.

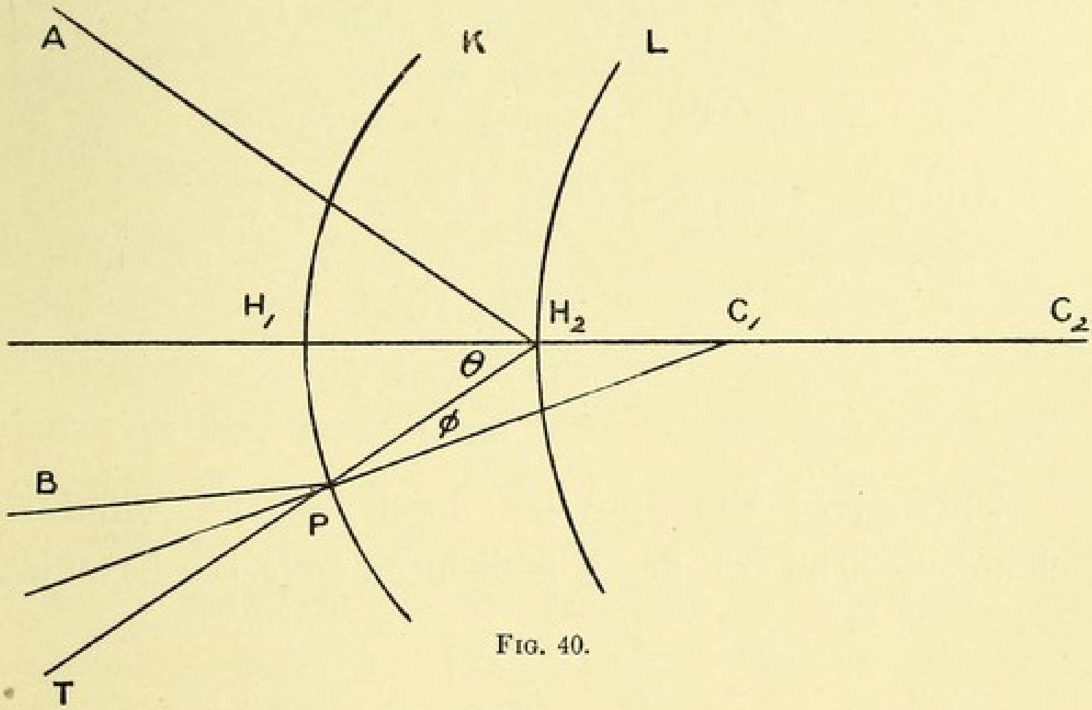

FIG. 40.

the anterior surface of the cornea and that of the lens can be calculated.

Tscherning's method with his instrument is as follows:—The two vertical lamps are made so feeble as only to give clear corneal images, and the strong lamp is placed at the end of the arc and made as bright as possible, so as to give the best possible image from the lens. The fixation point is placed so that the optic axis bisects the angle between the telescope and the strong lamp. The vertical lamps are moved until the three images are in the same vertical line.

If A (Fig. 40) is the strong lamp, B the position of the vertical lamps, referred to the same horizontal plane as A, T the telescope, K, L the anterior surfaces of the cornea and the lens, C_1, C_2, their respective centres of curvature, then $\theta = \frac{1}{2} TH_2A$, and $\phi = \frac{1}{2}\ TPB$. If r_1 is the radius of curvature of the cornea, then from the triangle H_2C_1P we have

$$H_2C_1 : H_1C_1 = \sin\phi : \sin\theta,$$

or

$$H_2C_1 = \frac{r_1 \,.\, \sin\phi}{\sin\theta}.$$

And

$$\begin{aligned} H_1H_2 &= H_1C_1 - H_2C_1 \\ &= r_1 - \frac{r_1 \sin\phi}{\sin\theta} \\ &= r_1 \frac{\sin\theta - \sin\phi}{\sin\theta}. \end{aligned}$$

This gives the *apparent* depth of the anterior chamber. The *real* depth is deduced from the formula

$$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1.$$

The Angle a

When the angle a exists, its measurement is of importance. It has therefore been investigated with much care by several methods.

Using the ophthalmophakometer, the eye is directed towards the telescope, and the two vertical lamps are lit, being placed in the same vertical plane as the telescope. If there is no angle *a*, all the reflections will be placed in the same vertical line, as in Fig. 41.

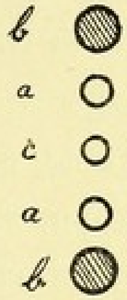

Fig. 41.

The brightest, *a,a*, are from the anterior surface of the cornea; *c*, small and bright, is from the posterior surface of the lens; *b, b*, large and much less distinct, are from the anterior surface of the lens.

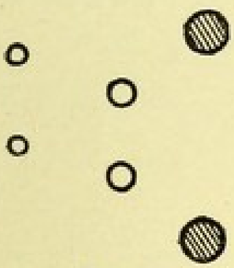
Fig. 42.

More frequently they will be disposed as in Fig. 42.

Under these circumstances the eye is to be directed towards the fixation point upon the other carrier, and this is moved until the images are brought into line. The angle is then read off upon the arc, and will be found to vary from 4° to 7°. The vertical deviation can be measured in the same manner by turning the arc vertically, so that the lights are horizontal. It is usually 2° or 3° below the line of vision.

Sometimes it is impossible to get the images in line. This is due to the refracting surfaces not being accurately centred. The axis of the lens is generally about 0·25 mm. above the centre of curvature of the cornea.

The Curvature of the Anterior Surface of the Lens

This was measured by von Helmholtz by a method resembling that used to determine the depth of the anterior chamber. As the reflection from the anterior surface of the lens is too large and too diffuse to give exact measurements by doubling, two objects were taken such that the corneal image was of the same size as the one from the lens. Since the size of the image varies as the focal length of the refracting surface, and the radius of curvature of the cornea is known, the radius of the anterior surface of the lens can be calculated.

Tscherning's method with the ophthalmophakometer is as follows:—The strong lamp is placed above the telescope, and the fixation point as far away as

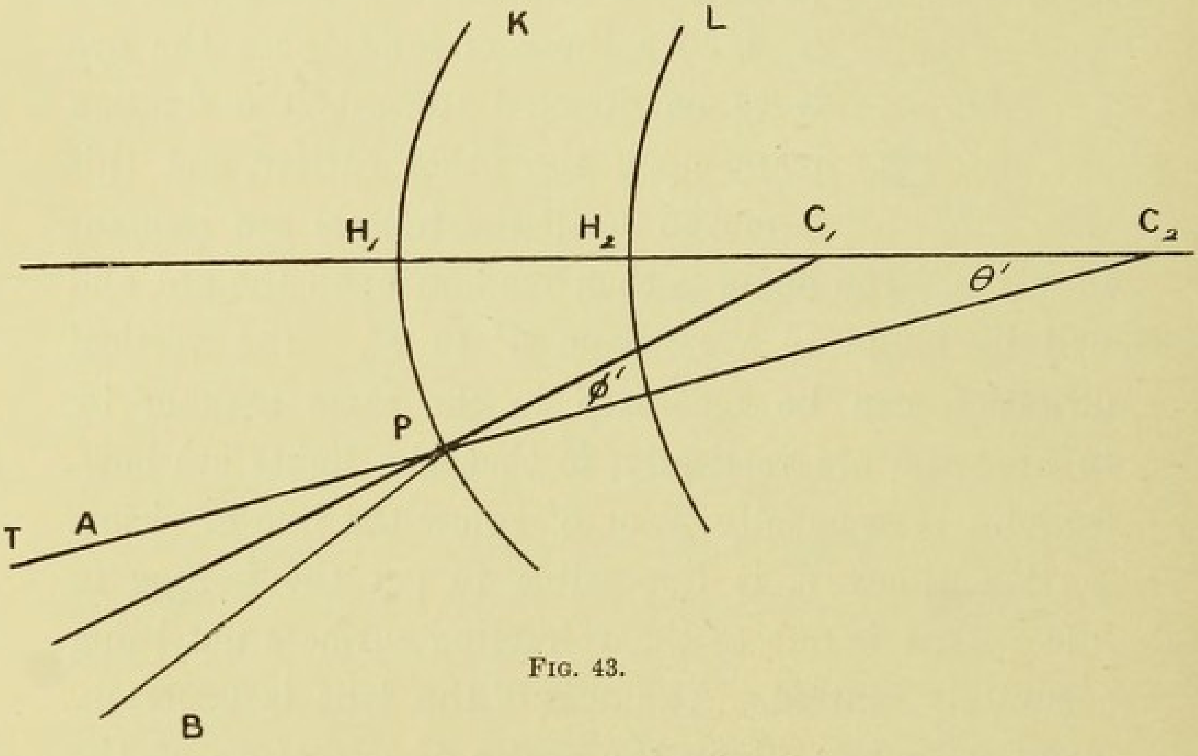

FIG. 43.

possible without losing its image behind the iris. The corneal images from the vertical lamps are then brought into the same line as the image from the lens.

In Fig. 43, using the same notation as in Fig. 40, we have

$$C_2C_1 = r_1\frac{\sin\phi'}{\sin\theta'},$$

and

$$C_2H_1 = C_1H_1 + C_2C_1$$

$$= r_1 + r_1\frac{\sin\phi'}{\sin\theta'}$$

$$= r_1\frac{\sin\theta' + \sin\phi'}{\sin\theta'}.$$

This gives the apparent position of the centre, C_2. The real position is calculated from this, and on subtracting the depth of the anterior chamber, the radius of curvature of the anterior surface of the lens is determined.

The Position and Curvature of the Posterior Surface of the Lens

There is no guide to the position of the posterior surface of the lens such as the edge of the iris affords to the anterior surface. It is measured by producing a reflection, and then reversing the positions of the object and the observing eye. If the lens image is covered in each case by a corneal image, the position of the lens image will be at the point of intersection of the observer's visual lines passing through the two corneal images, and the position of the posterior surface of the lens can be calculated.

The radius of curvature can be calculated in the same manner as for the anterior surface, or by the method used for the cornea.

The Indices of Refraction

Von Helmholtz measured the indices of refraction by enclosing a quantity of the fluid between a plano-concave lens and a plate of glass. He then measured with the ophthalmometer the images obtained from known objects, and deduced the focal distances of the plano-convex lens formed by the fluid. He also measured the radius of curvature of the concave surface of the lens. Knowing then the radius of curvature of the surface of the fluid and the focal distances of the lens formed out of it, the index of refraction could easily be calculated.

Another very accurate method makes use of the critical angle. A drop of the fluid is placed in the centre of the lower face of a right-angled prism over a marked hollow in the table. The light entering the hypotenuse face of the prism is reflected from the spot, and viewed through a telescope directed towards the third face of the prism. At a certain position of the telescope the mark can no longer be seen through the drop. The incident light has then been so refracted in passing through the glass as to be totally reflected at the surface of the drop. This gives the critical angle for the fluid, and consequently the index of refraction.

The Centre of Rotation of the Eye

The ophthalmometer is placed in the centre of a graduated horizontal arc, along which a fixation point can be moved. A hair is suspended vertically in front of the observed eye, which is rotated until the hair covers first one and then the other corneo-scleral margin. The angle ($= yxy'$, Fig. 44) is read off from the scale, and is found to be about 56°. The diameter of the cornea is then measured by doubling its image with the ophthalmometer, and by making doubled images from the centre of the cornea coincide with the edges of the doubled cornea, as in Fig. 39; yz and the angle yxz are then known, hence xz can be calculated. Adding the height of the cornea ($= 2{\cdot}6$ mm.) we have the distance of the centre of rotation behind the anterior surface of the cornea. Donders found this distance to average 13·54 mm. in emmetropic, 14·52 mm. in myopic, and 13·22 mm. in hypermetropic eyes.

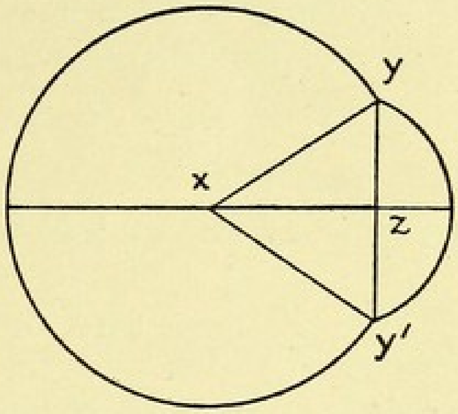

FIG. 44.

J. J. Müller determined the path of a definite point of the cornea during movements of the eyeball, and Berlin arranged a number of fine objects, so that they appeared to be in the same line in different positions of the eyeball. These observers state that the centre of rotation is not a fixed point, but recedes from the cornea on rotation upwards.

CHAPTER IX

THE OPHTHALMOSCOPE

UNDER ordinary circumstances the pupil of the eye looks black, and no red reflex, much less a clear image, is obtained from the fundus. The reason of this follows very simply from the conditions of refraction.

If, as in Fig. 45, there is a source of light, *L*, in

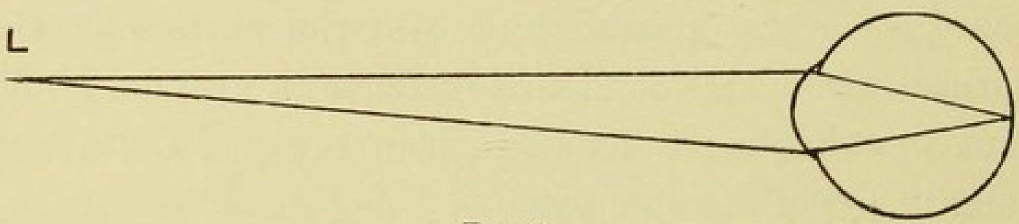

FIG. 45.

front of the eye, and the eye is focused upon it or accommodated for it, the light and a spot upon the retina are conjugate foci; consequently, reversing the direction of the rays, all rays from the illuminated spot of the retina are brought to a focus at the source of light. Therefore no rays will enter an observing eye unless it is situated actually at the source of light; and the problem solved by von Helmholtz in the construction of the ophthalmoscope was practically this one of making the observing eye at the same time the source of illumination of the observed fundus.

If the eye is not focused for the source of light the conditions are different, and some slight luminosity of the pupil may be seen. This is one cause of luminosity in the pupils of the hypermetropic eyes of young children and most carnivora. In the latter it is much intensified by the tapetum, an iridescent and strongly reflecting layer of the choroid, which is quite vestigial in the human eye.

Extreme hypermetropia is also the cause of the so-called *amaurotic cat's eye*, due to glioma of the retina, detached retina, etc. Here the retina is pushed forwards and the fundus at this spot becomes highly hypermetropic, the luminosity of the pupil being often the first symptom noticed. The same applies to the reflex from the eye in aphakia, the luminosity being increased by the enlargement of the pupil if an iridectomy has been done in the operation for the removal of the lens.

In hypermetropia the conjugate focus of the source of light L (Fig. 46) is a point, l, behind the retina; hence the emergent rays from the illuminated area of

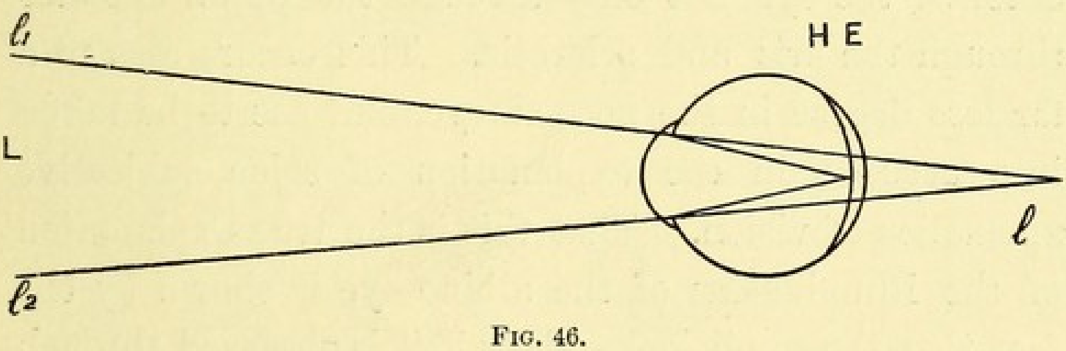

FIG. 46.

the fundus are divergent, as if coming from l. Therefore an observing eye, situated anywhere within the area, $l_1 l_2$, of the cone of emergent rays will catch some of them, and the pupil of the observed eye will appear feebly illuminated. Under these circumstances it is not necessary for the observing eye to occupy the exact position of the source of light, but only a spot in its immediate neighbourhood. The pupil may be obliquely illuminated by focusing the rays of light from a lamp beside the observer's head by means of a convex lens, and in this manner intraocular tumours and detachment of the retina may be seen.

In high myopia the emergent rays are strongly convergent, and become divergent after coming to a focus as the remote point. Beyond this point some of the divergent rays may enter an observing eye suitably situated, and the observed pupil appears illuminated.

If the eye is not naturally ametropic it can easily be made so by a strong lens in front of it, or by putting it under water, so that the eye is seen through the plane surface of the water, the refraction at the surface of the cornea being reduced to a minimum. Belarminoff used a lamina of glass in contact with the cornea.

The luminosity of albinos' eyes is due to light entering the eye, not only through the pupil, but also through the iris and sclerotic. This occurs also to a far less degree in the normal eye, and has to be taken into account in the explanation of some subjective anomalies of vision. That this is the true explanation of the illumination of the albino eye is shown by the fact that the pupil looks black if it is observed through a small hole in an opaque screen.

The original ophthalmoscope of von Helmholtz (1851) was merely a plane plate of glass (Fig. 47). A source of light was placed beside the observed eye, and the glass plate obliquely in front of it, so that a portion of the light was reflected from the surface of the plate into the eye. On looking through the transparent plate, an observer could now receive some of the rays from the fundus into his own eye, and thus obtain an image of the illuminated fundus. Since but a small proportion of the light received upon the plate is reflected at its surface the illumina-

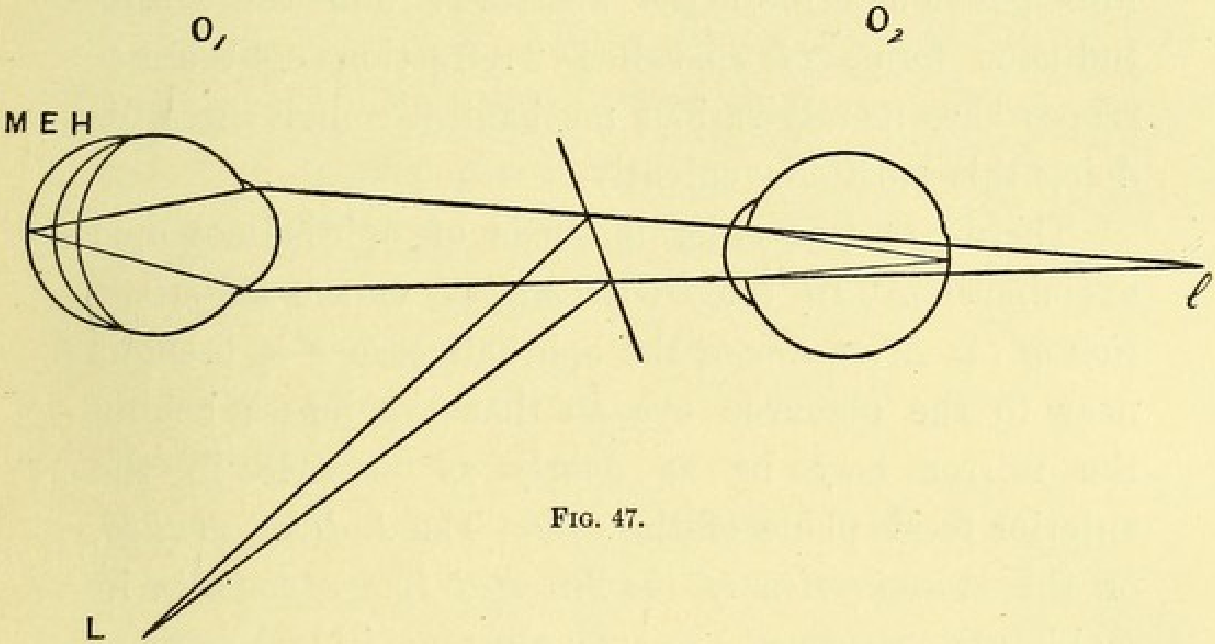

Fig. 47.

tion is feeble. Nevertheless, the principle is worth bearing in mind as a ready means of getting a view of a fundus in the absence of a more satisfactory ophthalmoscope. Moreover, as recently pointed out, an error of refraction in the observed eye may be obviated by using the corresponding spectacle glass of the patient as the ophthalmoscopic mirror.

Von Helmholtz next increased the amount of light reflected by superposing three plane plates, thus obtaining better illumination. The back of the glass was next converted into a more powerful mirror by silvering it, leaving a small portion unsilvered, or leaving a hole in the mirror, through which the observer might look. The illumination was still feeble, since the rays reflected by a plane mirror are divergent, so Ruete (1852) introduced the perforated concave mirror which still holds the field. The final modification was the addition of a battery of small lenses of various strengths, which might be brought into position behind the aperture; and the multitudinous forms of so-called "refraction ophthalmoscopes" are merely various mechanical contrivances for doing this most conveniently.

There are two chief methods of ophthalmoscopic examination. In the *Direct Method*, or the *Examination of the Erect Image*, the ophthalmoscope is brought near to the observed eye, so that the lenses behind the mirror may be as nearly as possible in the anterior focal plane of the eye. The *Indirect Method*, or the *Examination of the Inverted Image*, consists in making the observed eye strongly myopic by means of a convex lens placed in front of it, so that an inverted image of the fundus is formed in front of the lens, and this is examined by the observing eye placed at a convenient distance for distinct vision.

The Field of Illumination

The field of illumination of the observed fundus will vary with (1) the condition of refraction or accommodation of the observed eye, (2) the position and nature of the source of light, (3) the position and nature of the ophthalmoscopic mirror. As regards (1), we will suppose that the accommodation is completely at rest or paralysed. The effect of an effort of accommodation will be to produce a curvature myopia of corresponding degree. As regards (2), we will suppose that the source of light is a luminous point situated beside the patient's head. The effect of a luminous circular area is exactly the same as that of a luminous point situated at a greater distance, on the condition that the area is equally illuminated all over. Under these circumstances the variants will be the condition of refraction of the eye, whether emmetropia, myopia, or hypermetropia, and the position and nature—whether plane, concave, or convex—of the mirror. We shall not consider the effect of convex mirrors as they are not of much practical importance.

(*a*) *The Plane Mirror.*—Fig. 47 represents the field of illumination with the plane mirror. The rays reflected from the mirror are divergent, as if they came from *l*, the image of the actual source of light, *L*. We may, for convenience of description, call the image *l* the *immediate* source of light, in contradistinction to *L*, the *actual* source of light. It will be seen that the field of illumination is greatest in the hypermetropic eye, and least in the myopic eye, being reduced to a point when the myopia is such that *l* is the remote point of the eye. In emmetropia the field is intermediate in size. As the mirror is carried farther away from the eye, the divergence of the rays becomes less, and at a great distance approaches parallelism. In this case the rays are practically brought to a focus on the retina in emmetropia, whereas there is an area of diffusion in hypermetropia, the rays not having yet come to a focus, and also in myopia, the rays having come to a focus in the vitreous and crossed. The size of the area of diffusion is obviously dependent upon the size of the aperture of entry, *i.e.* the size of the pupil.

(*b*) *The Concave Mirror.*—The focal length of the concave ophthalmoscope mirror is usually about 20

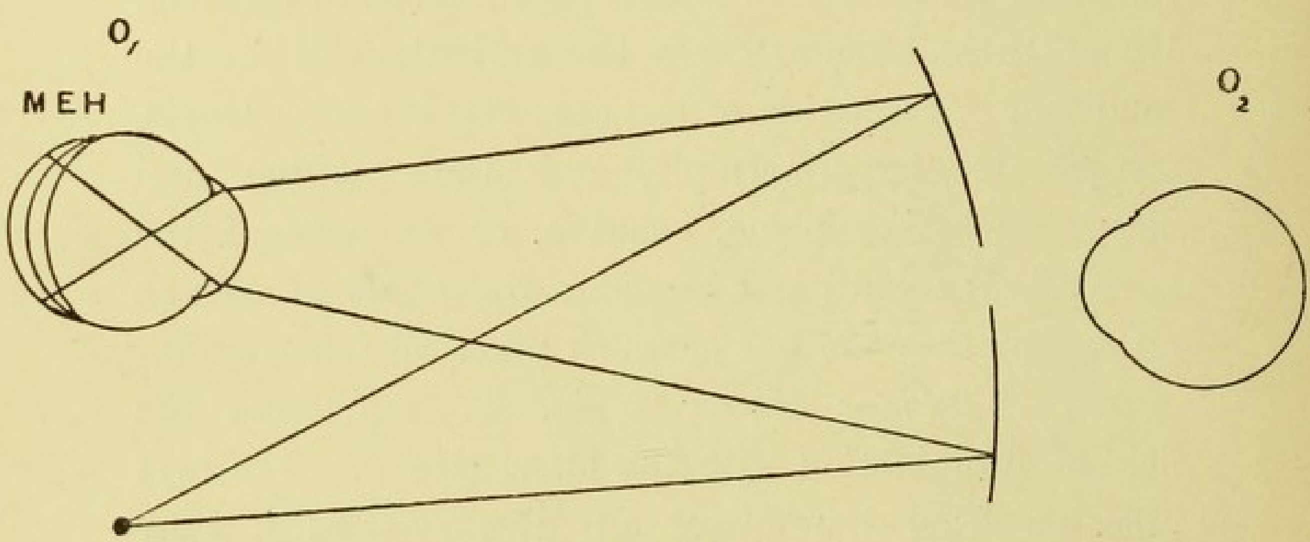

FIG. 48.

cm. (8 inches). The nature of the rays entering the eye will vary according to the distance of the mirror.

If this is more than 20 cm. (Fig. 48), say 30 cm., from the light, the rays will be convergent, as if coming to a focus 60 cm. in front of the mirror. They will be rendered more convergent by the dioptric mechanism of the eye, will cross in the vitreous and form an area of diffusion upon the retina, which will be least in hypermetropia, greatest in myopia, and intermediate in emmetropia. If the mirror is 20 cm. from the light the reflected rays will be parallel, and will come to a focus upon the retina in emmetropia, behind it in hypermetropia, and in the vitreous in myopia. If the mirror is less than 20 cm. from the light the reflected rays are divergent, as if coming from a virtual focus behind the mirror, and the mirror acts like a plane mirror. If the mirror is more than 40 cm., say 1 m., from the light the reflected rays will come to a focus in front of the eye, in the given case 25 cm. from the

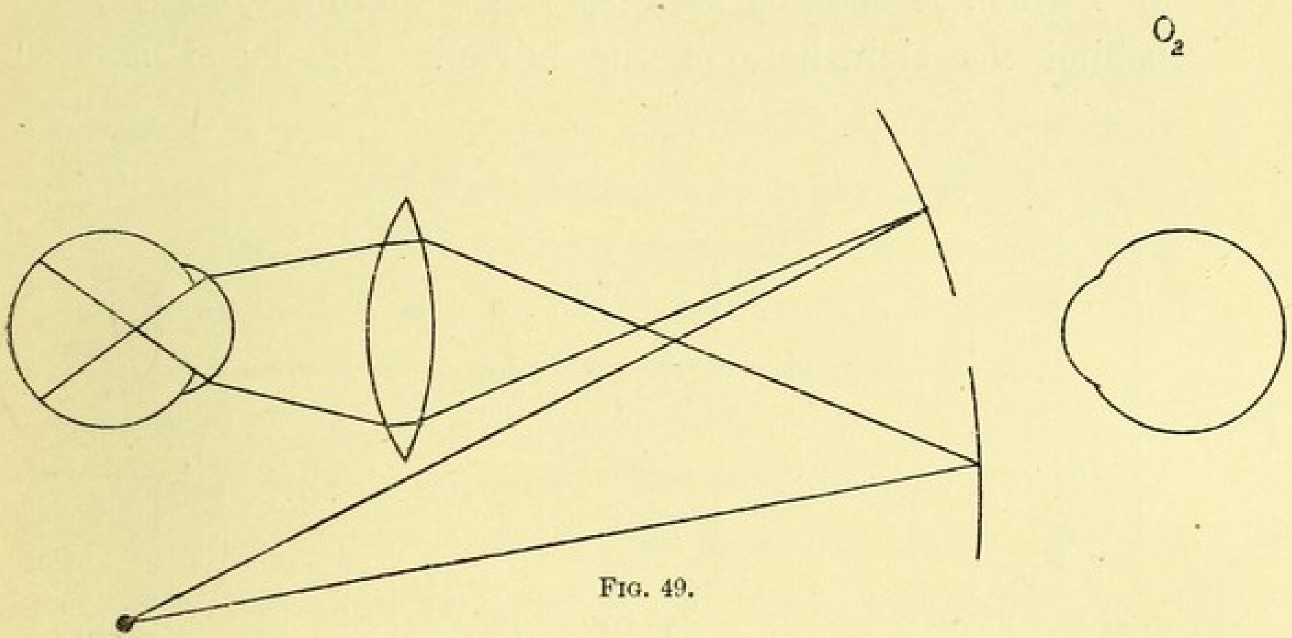

Fig. 49.

mirror, and divergent rays from this image as the immediate source of light will enter the eye, acting exactly like a plane mirror 37·5 cm. from the eye.

In the indirect method (Fig. 49) a convex lens is placed a short distance in front of the eye. The divergent rays from the immediate source of light are rendered convergent by this lens, and still more convergent by the dioptric media of the eye; hence they cross in the vitreous in all cases, and the field of illumination is least in hypermetropia, greatest in myopia, and intermediate in emmetropia.

The Ophthalmoscopic Field of Vision

It will be seen from the above that whenever the light is not brought accurately to a focus upon the retina the field of illumination is dependent upon the size of the aperture of entry, *i.e.* upon the pupil of the observed eye (which we will call O_1). This area is not necessarily, or even generally, the same as the area which is actually seen by the observing eye (O_2) behind the sight-hole of the mirror. The latter is

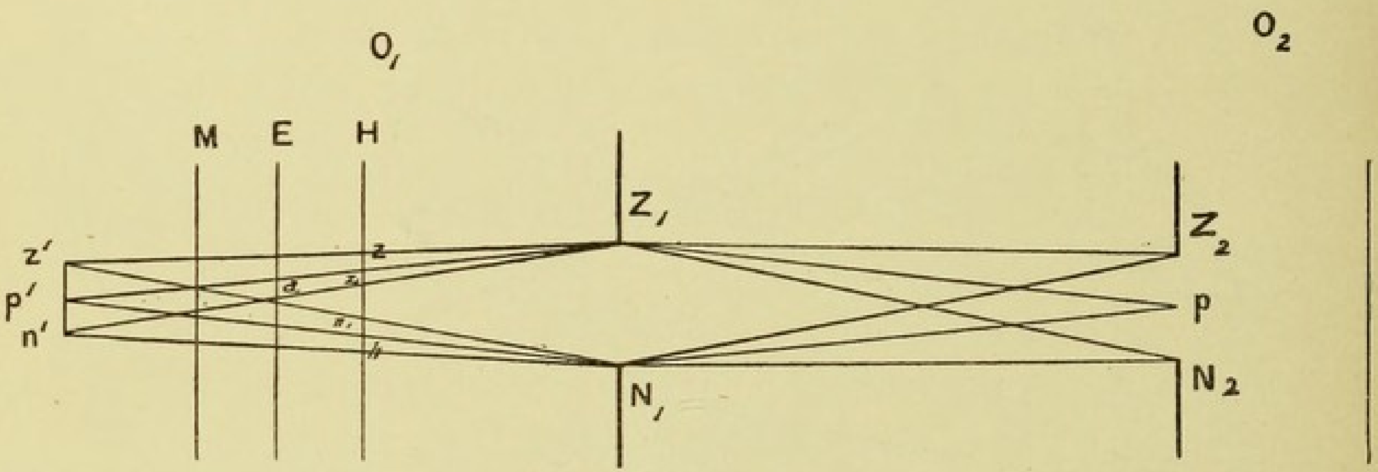

FIG. 50.

called the ophthalmoscopic field of vision, and is dependent, not only upon the pupil of O_1, but also upon the pupil of O_2 or the sight-hole of the mirror, whichever is the smaller.

Von Helmholtz utilised the reversibility of rays to delimit the ophthalmoscopic field.

If we regard a point in the centre of the observer's pupil as a luminous point, and find out what area of the observed fundus it could illuminate, we shall also have found the ophthalmoscopic field when the observer's pupil is reduced to a point.

Let Z_1N_1, Z_2N_2 be the pupils of O_1, O_2 respectively; and *M, E, H* the positions of the retina if O_1 is myopic, emmetropic, or hypermetropic. p' is the conjugate focus of *p*, the centre of the pupil of O_2, in the refraction by O_1. The ophthalmoscopic field is clearly the area cut off by the lines $p'Z_1$, $p'N_1$ on the retina. It is therefore greatest in hypermetropia, least in myopia, and intermediate in emmetropia. In reality the observer's pupil is not a point, but the area Z_2N_2, and all that is necessary is to find the image of Z_2N_2, formed at p'. The extreme rays $z'Z_1$, $n'N_1$ will then cut off a larger area, *zn*, which is the true ophthalmoscopic field.

From the laws of refraction, $z'n'$ will be the same size as Z_2N_2 when the latter is twice the focal distance of O_1 (about 30 mm.) from the anterior surface of the cornea of O_1. If O_2 is farther off, $z'n'$ is smaller; hence *the ophthalmoscopic field in the direct method diminishes as the distance between the eyes increases.* It is further obvious that *dilation of either pupil increases the ophthalmoscopic field in the direct method.*

In the indirect method, if the convex lens (*AB*, Fig. 51) is held at its own focal distance from the pupil, Z_1N_1, of O_1, Z_2N_2, which is a considerable distance from the lens, will form a very small image, $z'n'$, approximately at the level of Z_1N_1. The ophthalmoscopic field is therefore the area *zn*, cut off upon the retina by the lines $z'A$, $n'B$.

Hence, under ordinary conditions, *by the indirect method,*

(1) *the field of illumination is smaller than the ophthalmoscopic field;*

(2) *the ophthalmoscopic field is greatest when the lens is at its principal focal distance from the pupil of the observed eye ;*

(3) *the ophthalmoscopic field is greatest, within certain limits, when the lens is large,* and

(4) *when it is of short focal distance ;*

(5) *the ophthalmoscopic field is independent of the size of the observed pupil, so long as this is greater than the small image of the observer's pupil formed by the lens.*

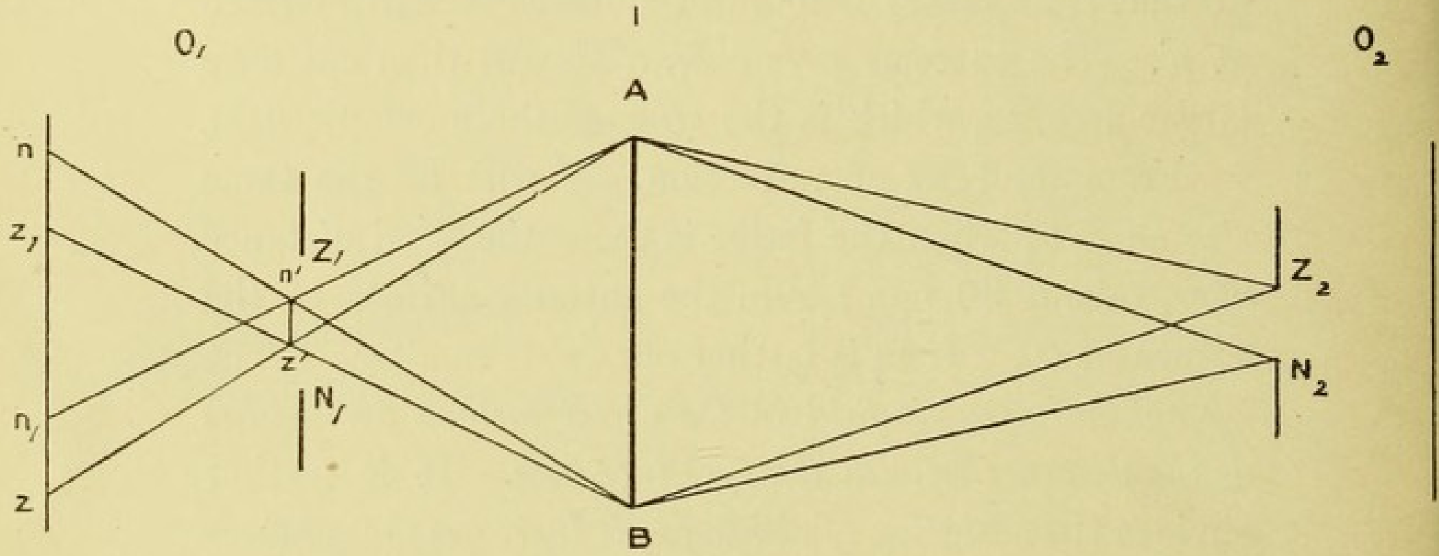

FIG. 51.

The field of illumination can be increased by using a plane mirror, but the brightness is then diminished ; or the light may be brought close to the concave mirror, but this is unpleasant for the observer.

The Brightness of the Image

The apparent brightness of a surface is independent both of its size and distance, and as long as the size of the pupil remains constant, is simply proportional to the intrinsic brightness of the surface. As long as an object subtends an angle greater than the minimum visual angle, it is impossible by any optical arrangement to obtain an image whose brightest part exceeds the brightest part of the object. If the cone of light from any point of the object, viewed through any optical arrangement, does not completely cover the pupil, the brightness of the retinal image will be to that of the object, seen by the naked eye, as the area of pupil covered to the total area of the pupil. This is only true when a clear image is formed upon the retina. If the retinal image is not clear there will be a peripheral zone which is not so bright, and the width of this zone will be equal to the diameter of a circle of diffusion.

If in the ophthalmoscopic examination, the patient accommodates for the immediate source of light, the brightness of the retinal image is the same whether the mirror is plane, concave, or convex. If the eye is not accommodated for the immediate source of light, the peripheral zone of the width of a circle of diffusion diminishes in brightness from within out; the central portion is equally bright throughout, and if the source of light is uniformly luminous, the brightness is independent of the kind of mirror. When the actual source of light is not uniform the diffusion circles tend to equalise the brightness of the retinal image. This is one reason why a strong concave mirror gives "better illumination" by the direct method than a plane mirror. It also gives better illumination by the indirect method. Here, with a plane mirror, the immediate source of light is a long distance from the lens, which therefore forms an image near the nodal point of the eye. Hence the diffusion circles are very large.

We have hitherto considered the brightness of the field of illumination; we have now to consider the apparent brightness of the ophthalmoscopic field.

From Fig. 51 it is clear that any point between z_1n_1 will emit a cone of rays which will entirely cover the pupil Z_2N_2, whereas rays from zn_1, z_1n will not cover the whole of the pupil Z_2N_2; hence this peripheral zone will appear less bright.

Similarly, in Fig. 50, the rays from a will include the rays corresponding to every point of $z'n'$, and will cover the whole of the pupil Z_2N_2; and in the hypermetropic eye, every point between z_1n_1 will also send a cone of rays covering the whole of Z_2N_2; whereas in the myopic eye, no point on the retina will send a cone of rays which will cover the whole of Z_2N_2. Hence, *by the direct method the image is brightest in hypermetropia, less bright in emmetropia, and still less bright in myopia; and the apparent brightness is also influenced by the size of the pupils and the distance between them. In the indirect method nearly the whole area of illumination is of maximum brightness.*

Preliminary Examination

We will suppose that the observer is emmetropic, or that his error of refraction has been corrected, and that the accommodation of the observed eye is at rest or paralysed. If a portion of the fundus, preferably that part which includes the optic disc, is illuminated by the concave mirror from a distance of about 1 metre, the appearances will vary according to the condition of refraction of the observed eye. If this is *emmetropic or very slightly myopic* only a blurred image of the disc will be seen. Consider two points upon the fundus a little distance apart, *e.g.* upon opposite edges of the disc. The rays passing out of the eye from these points will be parallel or very slightly convergent, and their direction will be that of their axes, which is the continuation of the lines joining the points and the nodal point of the eye. As these axes constantly diverge from one another, the observer at a distance of 1 metre cannot receive portions of both pencils of rays upon his own pupil, consequently he cannot obtain a clear image of the whole intermediate region between the spots. He may get a clear image from two spots very close together, but only if his accommodation is almost completely suspended, so that nearly parallel rays are brought to a focus upon his retina.

If the observed eye is *hypermetropic*, the emitted rays from the two points will be divergent in each case, coming apparently from their virtual conjugate foci behind the eye. The greater the distance from the eye, the greater will be the area over which these divergent rays spread, so that at 1 metre distance some of the peripheral rays of each pencil will enter the observed eye, and by slight accommodation the observer will obtain a clear image upon his own retina. The actual image which he sees will be exactly as if the two points were situated at their conjugate foci behind the observed eye, and the eye itself were taken away. Hence it will appear to be erect. If the observer now shifts a little to one side, more rays will enter his eye from the neighbourhood of the opposite point, and less from the neighbourhood of the point on the same side as his movement. Although the points actually remain stationary, they will appear to move in the same direction as his own movement, when compared with the well-marked edge of the observed pupil. The observer mentally regards this very distinct circular frame as a fixed object of comparison, and as more of the fundus upon the opposite side comes into view, whilst a corresponding amount upon the same side disappears, this is mentally interpreted as a movement of the image in the same direction as his own movement.

If the observed eye is *highly myopic*, the emitted rays from the two points will be strongly convergent in each case, and a real inverted image of the ophthalmoscopic field will be formed at the remote point of the eye, *i.e.* between the observer and the observed eye. The rays will diverge from this image, and the effect will be exactly the same as if there were an actual inverted object in this position. If the myopia is sufficiently high, the image will be beyond the observer's near point, and he will be able to accommodate for it. If he moves to one side he will see more of the observed fundus upon the same side, and less upon the opposite side, so that the fundus will appear to have moved in the opposite direction.

This apparent movement of the image affords a ready means of diagnosing the higher degrees of ametropia.

If the observer now approaches nearer, *e.g.* to about 20 cm. from the eye, he will be at the most suitable position for distinct vision, and will be able to examine the superficial parts of the eye more accurately. By assisting his accommodation with convex glasses behind the mirror, he can approach still nearer, whilst he at the same time magnifies the parts examined. If he is hypermetropic or presbyopic the use of a convex lens becomes imperative. In this manner various pathological conditions are recognised. Irregularity of the pupil or synechiæ are at once evident; whilst a wound of the iris may point to the presence of an intraocular foreign body. The edge of a displaced lens may be seen, if the pupil is dilated, as a black arc, due to the total refraction of the emergent rays. In the same way notches in the edge of the lens are diagnosed. The method is, however, of most value in the diagnosis of opacities in the dioptric media. The normal pupil shows a uniform red reflex, due almost entirely to the rich choroidal blood-supply. If the observed eye is directed slightly to the nasal side the reflex is paler, owing to the anæmic optic papilla. A reflection either of the immediate source of light or of the mirror, whichever is smaller, is formed by the anterior surface of the cornea. This corneal reflex may be large and bright, so as to greatly obscure or even entirely stop the red reflex. If there is any opaque body in the course of the rays reflected from the fundus, no light can pass at this spot and it appears black. The whole field may be black, as when the lens is entirely opaque or when the vitreous is filled with blood. In the latter case oblique illumination will show the red blood behind the transparent lens. It is very important for fine opacities not to have the light too bright, as they may be partially transparent to strong illumination. They can often only be diagnosed by using a plane mirror, or by reducing the strength of the source of light or removing it to a greater distance from the mirror. Indeed it is too often forgotten that, in the examination with the undilated pupil, more is gained by the larger field due to physiological dilation with weak illumination than by the smaller field, perhaps nearly obscured by a bright corneal reflex, with strong illumination.

An opacity in any of the media will not necessarily prevent the light from being focused upon the retina immediately behind it. Thus a central opacity of the cornea will not prevent the focusing of an object upon the macular region, for the rays passing through the clear peripheral parts of the cornea will be refracted towards the macula, only those rays being cut off which are incident to the corneal surface at the opaque region. There is thus a loss of brightness rather than of definition, though definition will also be impaired by the superposition of a diffuse entoptic image upon the clear image of the external object. So, too, for opacities in the lens or vitreous, only those rays are cut off which are refracted so as to fall upon the opacity. On the other hand, the black appearance by transmitted light is due to certain rays reflected from the retina being stopped, and is independent of the real colour of the opacity by reflected light. If the opacity is sufficiently large and so situated as to reflect a portion of the light into the observer's pupil, its true colour will become manifest.

The first point to determine about an opacity is whether it is movable. This is done by telling the patient to move the eye in different directions, and then to look straight forward. A floating opacity will then continue to move after the eye is brought to rest. It must therefore be either in the aqueous or the vitreous, the former affording no difficulty in diagnosis. If the opacity only moves with the eye it may be in the cornea, lens, or vitreous.

The next point is to determine its exact position. This is effected in the preliminary examination with the mirror alone by *parallactic displacement*.

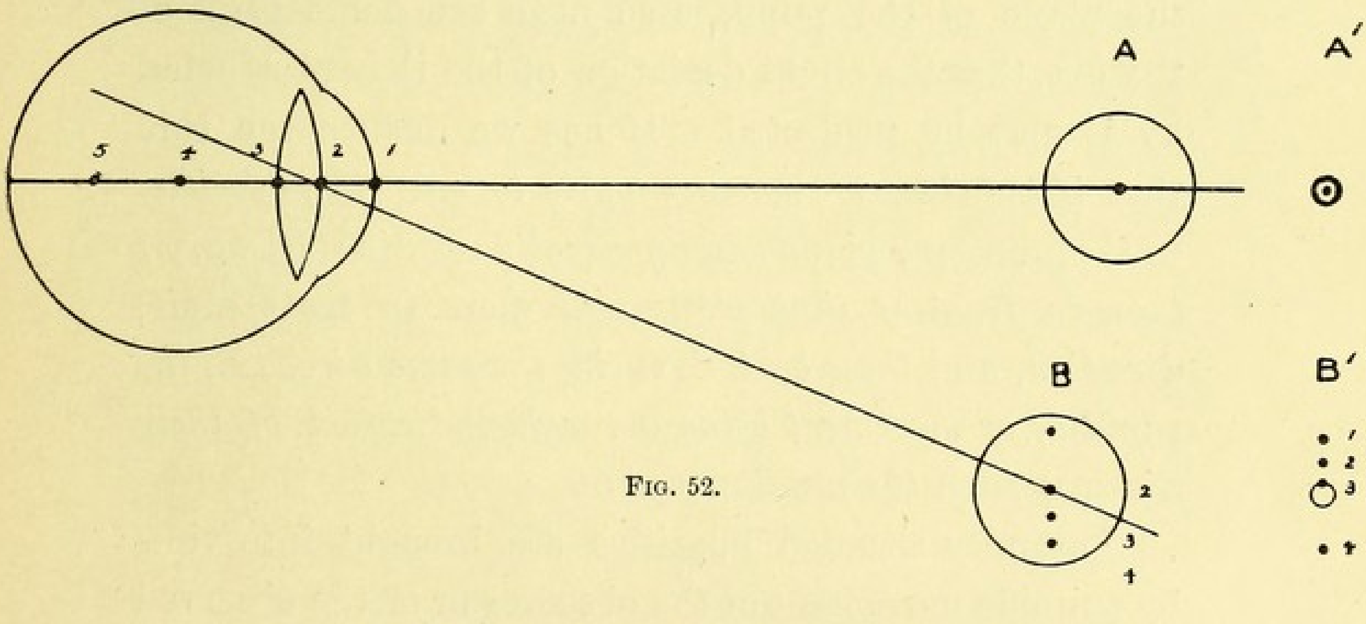

FIG. 52.

In Fig. 52, if R is the centre of rotation of the eye, and if there are opacities at 1, 2, 3, 4, 5, then, when the eye is rotated a small amount, the opacities 1, 2, and 3, in front of the centre of rotation will move in the direction of rotation, and 5, behind the centre, will move in the opposite direction, whilst 4, at the centre, will not move. It is obvious that the amount of movement will be greater the farther the opacity is from the centre of rotation. Now, we have no means of defining the centre of rotation in the ophthalmoscopic examination, but all the movements will be referred to the edge of the pupil for comparison. If the observer is situated at *A*, all the opacities will appear as a single spot in the centre of the pupillary area. If he shifts his position to *B*, or if the observed eye is rotated a corresponding amount in the opposite direction, the opacity 2 will remain in the centre of the pupil, whilst 1 will move towards one edge of the pupil, and 3, 4, and 5 towards the opposite edge, 5 being lost entirely behind the iris. The opacity 2, in the plane of the pupil, is so near the nodal point of the eye, that the slight deviation of the rays obstructed by it may be neglected. Hence we deduce the rule that *if the observer moves slightly to one side, opacities in the pupillary plane will apparently remain stationary; those in front of that plane will move in the opposite direction, and those behind it in the same direction, the rapidity of movement being a rough indication of their distance from the pupillary plane.*

Laterally situated opacities are brought into view by suitable movement of the observer or of the observed eye.

The corneal reflex may be used as a valuable guide to the position of opacities which are situated near the optic axis. With a concave mirror of 20 cm. focal distance, situated 20 cm. from the light, the image of the light will be infinite in size and situated at infinity, *i.e.* the rays from the mirror will be parallel and will be refracted by the cornea to its principal focus, 4 mm. (*i.e.* half its radius of curvature) behind its anterior surface. At this spot an image of the mirror will be formed. If a plane mirror, or concave one with shorter focal distance is used, the image of the light formed by the mirror may be smaller than the diameter of the mirror, in which case the corneal reflex will be an image of this immediate source of light and not of the mirror.

The corneal reflex then is a virtual image situated about 4 mm. behind the cornea, *i.e.* immediately behind the anterior surface of the lens (behind 2 in Fig. 52). The centre of curvature of the cornea is situated 8 mm. behind its anterior surface, *i.e.* less than 1 mm. behind 3. The corneal reflex will always cover this spot, and it follows that *an opacity at the centre of curvature of the cornea will always be covered by the corneal reflex: opacities in front of this point move in the same sense with regard to the reflex as the eye moves; and opacities behind it move in the opposite direction to the movement of the eye.* Therefore, in Fig. 52, in the first position of the eye, the opacities 1, 2, 3, 4, 5 will all appear in the centre of the corneal reflex (A'); in the second position they will appear as in B'; so that a posterior polar opacity will scarcely leave the edge of the reflex, whereas an anterior polar opacity will move much farther from it.

The minute examination of superficial opacities may be effected by two methods. By the first method the parts are illuminated by the mirror, and a strong convex lens (*e.g.* + 20*D*) is used behind it. This acts as a magnifying glass, and as the observer gradually approaches nearer to the eye, deeper and deeper parts become successively focused upon his retina. In this manner, by first focusing the edge of the iris, the relative position, anterior or posterior, of a corneal or lental opacity may be determined, and its minute structure examined. To examine the deeper parts of the vitreous the strength of the lens is gradually diminished. In this manner the position of opacities is demonstrated by the direct method. The amount of light reflected from the opacity is increased, and therefore the appearance which it would have to the naked eye made more evident by increasing the angle of incidence of the illuminating rays. This is effected by the second method, or that of *lateral or focal illumination.* The light is placed to the side and slightly in front of the patient, and is focused by a strong convex lens (+ 15 to + 20*D*) upon it. The position of the apex of the cone of light can be varied at will, both laterally and antero-posteriorly, and if the examination is carried out in a darkened room, the illuminated area is very bright in comparison with the surrounding parts. By holding the illuminating lens in the left hand, and looking through a strong convex lens or corneal loup, held in the right, any superficial opacity can be minutely investigated.

CHAPTER X

THE OPHTHALMOSCOPE (*continued*)

The Indirect Method

We have seen that the ophthalmoscopic field is large by the indirect method. It is therefore best to pass on from the preliminary examination with the mirror alone to the examination by the indirect method, which has much the same relationship to the direct method that microscopic examination by a low power, with coarse adjustment, has to examination by a high power, with fine adjustment. We have also seen that the ophthalmoscopic field is largest when the lens is situated at its own focal distance from the pupillary (or, more accurately, the anterior nodal) plane of the eye, and that it is independent of the size of the pupil. We therefore get a good general view of the fundus with an undilated pupil. This is somewhat obscured, however, by three reflexes. One of these is the ordinary corneal reflex; the other two are due to the lens, one, a small virtual image formed behind the lens by reflection from its anterior surface, and the other, a real inverted image formed in front of the lens by reflection from its posterior surface. These reflexes travel in opposite directions, away from the line of vision, if the lens is slightly tilted; but it must not be tilted more than is necessary, otherwise the eccentric incidence of the rays falling upon it will cause a serious deformation of the image. Thus the image of a circular disc seen through a much-tilted lens will be elliptical, and, as will be seen later, this appearance might be attributed to astigmatism in the eye, instead of an artificial astigmatism produced by the lens.

The corneal reflex is magnified by the lens, and if it is situated at the posterior focus it will occupy the whole area of the lens. This constitutes a difficulty in obtaining the maximum field; hence for practical purposes it is best to hold the lens either nearer or farther off, and preferably the latter, and at such a distance that the lens is its own focal distance from the anterior focal plane of the eye. Slight tilting will then cause the corneal reflex and the image of the fundus to move in opposite directions, and a clear view will be obtained.

We have already considered the rays entering the eye in the indirect method.

The course of the emergent rays will be evident from Fig. 53. If the eye is *emmetropic* they will be parallel, and impinging upon the lens, will be brought to a focus at its principal focus. The image of the fundus will therefore be real and inverted, and situated at the principal focus of the lens. If the eye is *myopic* the emergent rays will be convergent; they will be made more convergent by the lens, and will be brought to a focus at less than its focal distance. The image will again be real and inverted. If the eye is *hypermetropic* the emergent rays will be divergent, and they

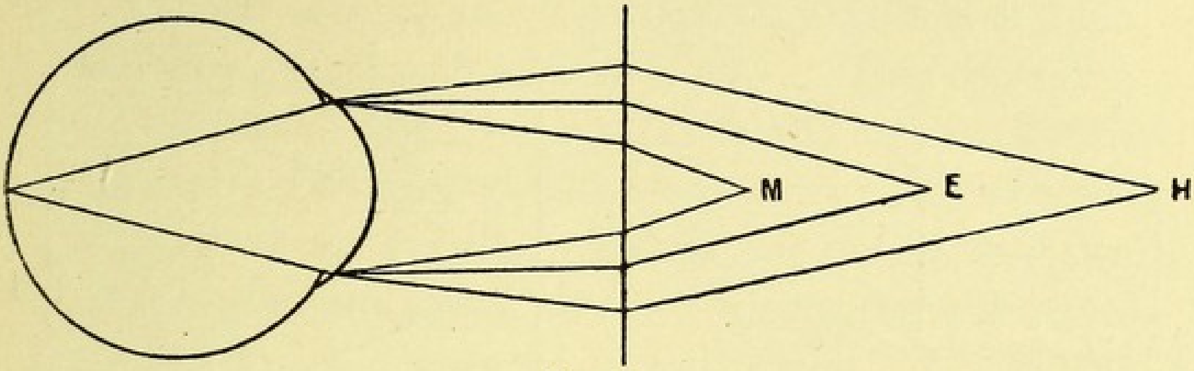

FIG. 53.

will be refracted by the lens to a focus beyond its principal focus. The image will still be real and inverted.

The effect in all cases is to produce an artificial myopia, the degree of which, and therefore the position of the image, will depend upon, (1) the refraction of the eye, (2) the strength of the lens, and (3) its distance from the eye. By keeping (2) and (3) constant, the refraction of the eye may be determined by this method (Schmidt-Rimpler).

In all cases the image is magnified, the amount of magnification depending upon the same three factors. In the following investigation we shall use the reduced eye for the sake of simplicity.

If the eye is emmetropic (Fig. 54) the rays from any two points, *a* and *b*, upon the retina, which are parallel to the axis, will pass through ϕ_1, the anterior focus of the eye, and will then diverge. All the rays from *a* and *b*, after refraction, will be parallel to these two since the eye is emmetropic; and of these, two, *OA*, *OB*, will pass through the optical centre of the lens. Since the rays incident to the lens are parallel, the image, *AB*, will be formed at ϕ', the focus of the

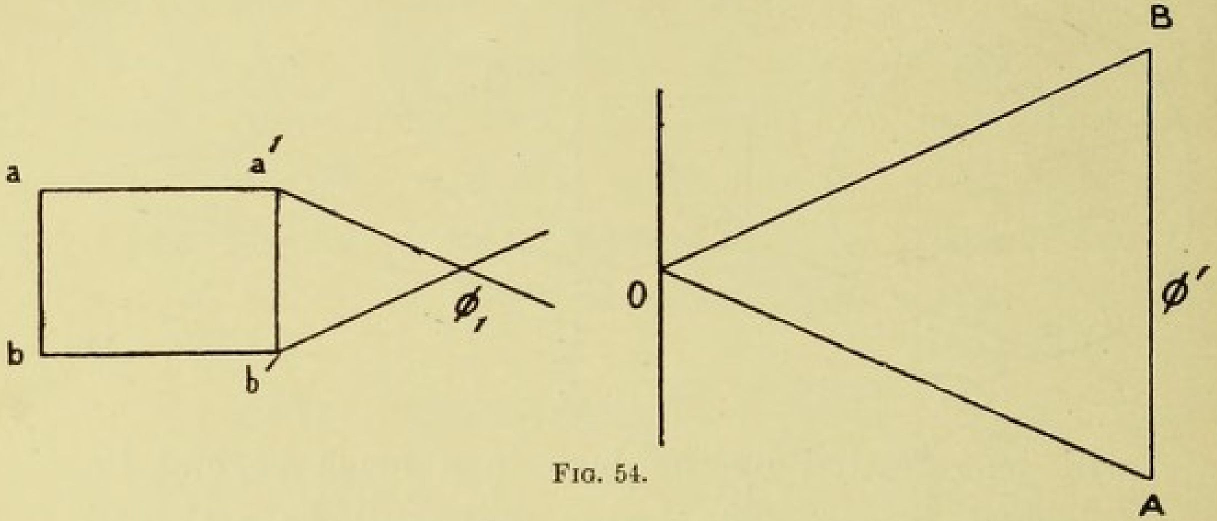

FIG. 54.

lens, and *AB* will represent the size of the image. If o is the size of ab, and i that of AB, then from the similar triangles $a'b'\phi_1$, OAB,

$$\frac{o}{i} = \frac{F_1}{F'}.$$

If the strength of the lens is $+13D$, $F' = 77$ mm.; therefore

$$\frac{i}{o} = \frac{77}{15} = 5{\cdot}1.$$

Hence the magnification is about 5 times for a lens of 75 mm. focal distance; similarly, it is 4 times for 60 mm., 3 times for 45 mm., and twice for 30 mm. Hence, *the stronger the lens the smaller and brighter is the image.* Conversely, the weaker the lens the greater the magnification; but the greater also the focal distance, and, therefore, the necessary distance from the patient. Since the rays incident to the lens are parallel its distance from the eye is immaterial; and hence, *in emmetropia the magnification is independent of the distance of the lens from the eye.*

If the lens is placed so that it is at its own focal distance from anterior focal plane of the eye (Fig. 55), since the rays from *a* and *b* which are parallel to the axis pass through its own focus, they will be parallel to the axis after refraction. Hence, *when the lens is at its own focal distance from the anterior focus*

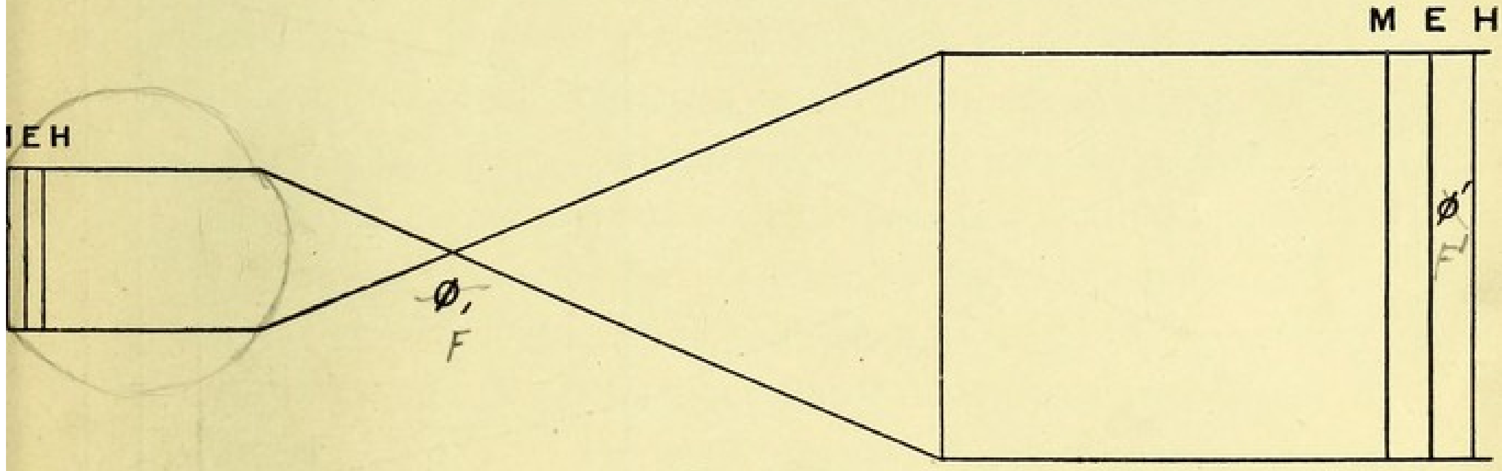

FIG. 55.

of the eye the magnification is the same in myopia, emmetropia, and hypermetropia.

If the lens is nearer the eye than the above distance the rays under consideration will diverge after refraction. Hence, *if the lens is less than its own focal distance from the anterior focus of the eye the magnification is greatest in hypermetropia, least in myopia, and intermediate in emmetropia.*

Conversely, if the lens is farther from the eye the rays under consideration will converge after refraction. Hence, *if the lens is more than its own focal distance from the anterior focus of the eye the magnification is* *greatest in myopia, least in hypermetropia, and intermediate in emmetropia.*

It follows from these laws, that *as the lens is moved from the anterior focus of the eye towards the observer the image remains the same size in emmetropia, diminishes in size in hypermetropia, and increases in myopia.* This affords a ready means of determining any axial error of refraction.

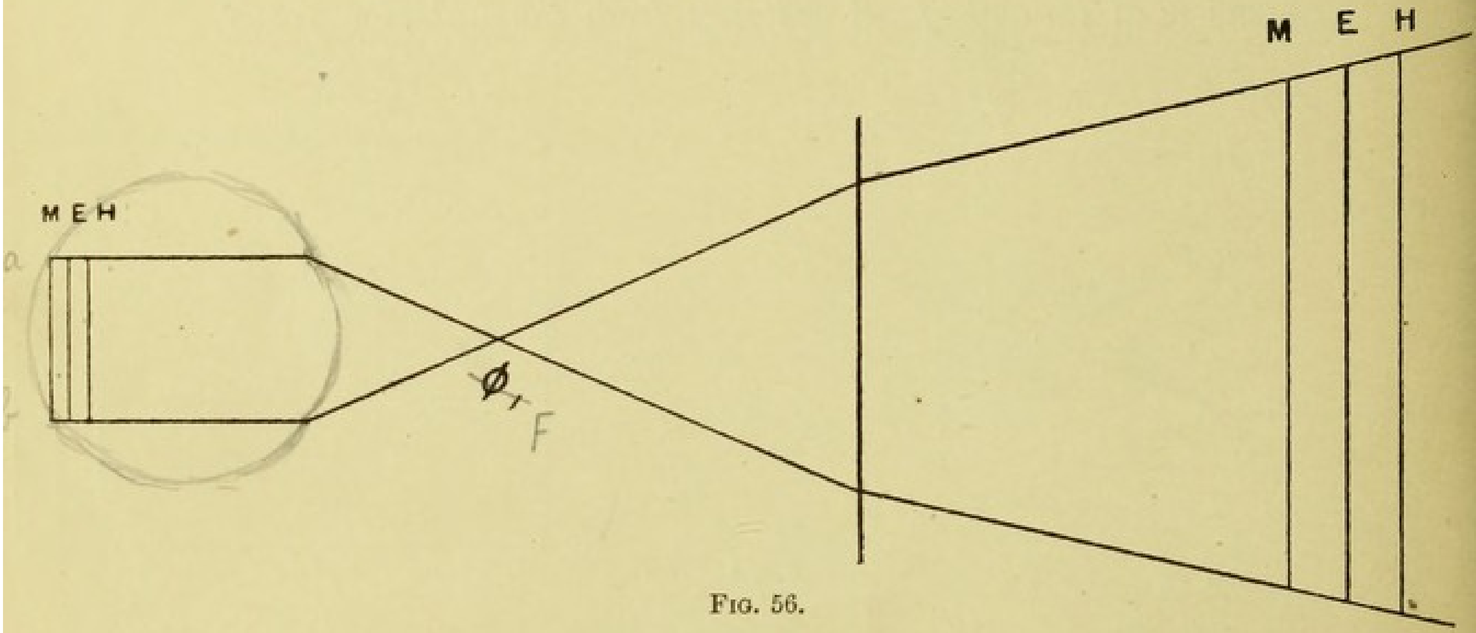

FIG. 56.

The actual size of the image in any given case is very easily found, viz., first find the image at the remote point of the eye (*i.e.* the real inverted image in myopia, and the virtual erect image in hypermetropia), and then consider this image as being viewed through the convex lens.

In curvature ametropia, such as we meet with in astigmatism, the results are easily deduced if we remember that there are now two anterior foci to the eye, one for each meridian. There are also two nodal points to the (reduced) eye, but only one principal point at the anterior pole of the (reduced) eye. When the lens is at its focal distance from the principal point of the eye, all the rays passing through that point will be refracted by the lens parallel to the axis. Hence, *when the lens is at its focal distance from the cornea the magnification is the same in emmetropia and any ametropia of curvature.* Under these circumstances the disc appears circular. If the lens is *nearer* the eye the image is elliptical, with its long axis in the less refractive meridian (*i.e.* generally *horizontal*). If the lens is *farther* from the eye the long axis of the ellipse is in the more refractive meridian (*i.e.* generally *vertical*). As mentioned above, it is essential here that the lens should be held vertically, as any inclination makes it itself astigmatic. If the disc is really oval, *e.g.* in high myopia, the axis of the ellipse will of course remain unaltered.

Ametropia of index of refraction occurs in old age. Aphakia may be considered an extreme form of it. Here it is the position of the nodal point which remains invariable; and if the lens is at its focal distance from this point, the image is the same size in emmetropia and ametropia of refractive index. If the lens is moved nearer the eye the image increases in hypermetropia and diminishes in myopia, whilst it remains the same size in emmetropia. Since the image is formed a considerable distance beyond the focus of the lens in aphakia after cataract extraction, owing to the high hypermetropia induced, it is convenient to use a stronger lens, *e.g.* + 18D.

Differences of level are made very evident by *parallactic displacement* in the indirect method.

Thus, in Fig. 57, if there are two spots, a and b, at different levels in the fundus, when the lens is shifted slightly so that its optical centre moves from

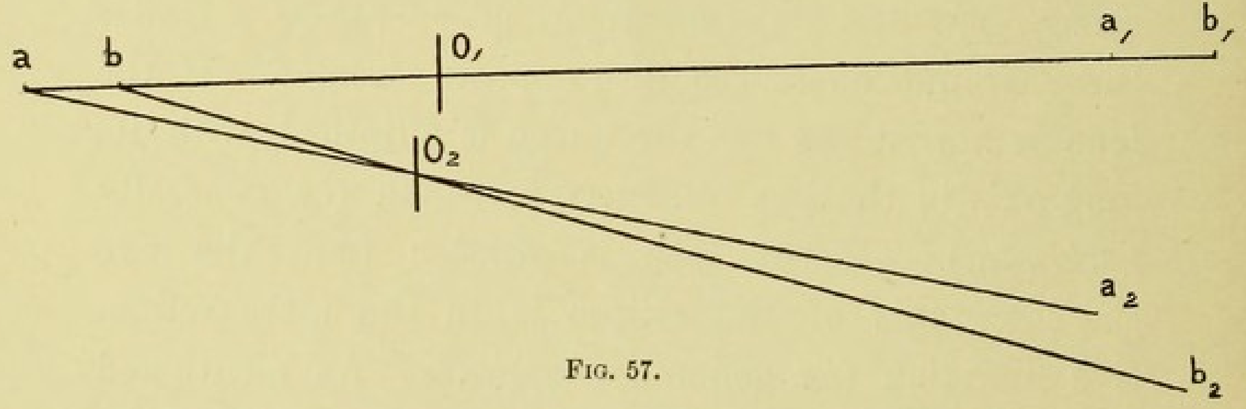

FIG. 57.

O_1 to O_2 the images of a and b will move from a_1 to a_2 and b_1 to b_2. It is of historical interest that this displacement was at one time wrongly interpreted, so that a glaucomatous cupping of the disc was diagnosed as a swelling.

The Direct Method

In the direct method of examination the observer approaches as near as possible to the eye to be observed. If the eye is *hypermetropic* the emergent rays will be divergent, as if coming from the virtual conjugate focus behind the retina. Owing to the short distance between the eyes a large pencil will fall upon the observer's pupil, and may be brought to a focus upon his retina if he makes a suitable effort of accommodation. If his accommodation is relaxed he will only obtain a clear image by placing a convex lens behind the sight-hole of the mirror. The object of this lens is to render parallel the rays diverging from the virtual focus behind the eye; hence its focal length must be equal to its distance from that point. If it is situated in the anterior focal plane of the eye, its strength will exactly represent the strength of the correcting lens when worn at this spot, *i.e.* it will be an accurate measure of the hypermetropia. In practice it is difficult to approach so close that the lens behind the mirror will be in the anterior focal plane of the eye; hence the focal distance of the correcting lens is too long by its distance beyond the anterior focal plane, *i.e. the correcting lens found by the direct method is usually weaker than the true amount of hypermetropia.*

If the observed eye is *emmetropic* the emergent rays will be parallel, and consequently can only form an image upon the observer's retina if his accommodation is absolutely relaxed—unless, indeed, he counteracts the amount of his accommodation by a corresponding concave lens in front of his eye. The closer the two eyes are together the larger will be the number of points upon the observed fundus which can send pencils of rays into the observer's pupil. As we have already seen, at a metre's distance the field is reduced to a point, or at most to two points very close together, so that the effect is that of a blurred image.

If the observed eye is *myopic* the emitted rays are convergent. If the myopia is moderate the real image of the fundus at the far point of the eye will be behind the observer's eye, *i.e.* he will catch the convergent rays before they have come to a focus. These convergent rays, entering his emmetropic eye, are brought to a focus in his vitreous; hence he cannot possibly obtain a clear image unless he counteracts the convergence by an equivalent concave lens behind the mirror. Since in practice this lens is beyond the anterior focal plane of the observed eye, its focal length is too short by its distance from that plane, *i.e.* the *correcting lens found by the direct method is usually stronger than the true amount of myopia.* There is another important reason why the lens is usually very markedly stronger. It is difficult to relax the accommodation entirely when the eye is apparently close to the object looked at; hence, the lens will really represent the myopia *plus* the amount of accommodation still exerted. The latter factor will vary with different observers, diminishing as skill is acquired in the art of relaxing the accommodation.

If the observed eye is very highly myopic its *punctum remotum* will be situated somewhere in the space between the eye itself and the observer's retina, and it may be in such a position that it will be impossible to obtain a clear image with any correction. For example, the remote point may be just behind the sight-hole of the mirror. Here it is too close to the observer's eye to be accommodated for, and no correcting glass situated at the same position will have any effect upon the rays.

In hypermetropia it is as if an erect image were situated at the remote point behind the eye, and in emmetropia there is an erect image at the remote point, *i.e.* at infinity. In all cases of myopia in which a clear image is obtained the convergent rays are caught before they have come to a focus. Hence in all cases an inverted image is formed upon the observer's retina, and this is, of course, pyschologically interpreted as due to an erect object. *The image by the direct method is, therefore, always erect.*

It is also always magnified. The apparent magnification is found by comparing the observer's retinal image of a portion of the observed fundus, *e.g.* the

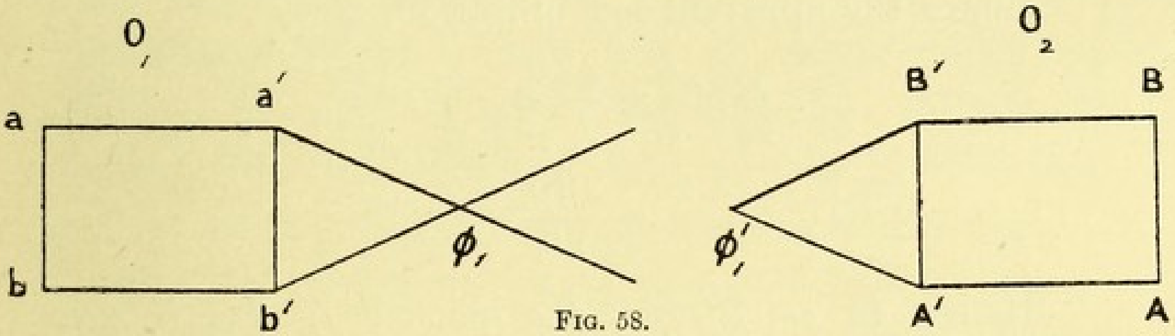

FIG. 58.

optic disc, with the retinal image which he would have of the same object if it were seen in air at the distance of distinct vision. This distance is usually reckoned at about 22 cm., but as this is more or less arbitrary so also is the apparent magnification.

If the eye is emmetropic (Fig. 58) the rays from any two points, *a* and *b*, upon the retina, which are parallel to the axis, will pass through ϕ_1, the anterior focus of the eye, and will then diverge. All the rays from *a* and *b*, after refraction, will be parallel to these two since the eye is emmetropic; and of these two, ϕ'_1B', ϕ'_1A' will pass through the anterior focus of the observer's eye, and will be parallel after refraction, and will determine the size of the retinal image, AB. If o is the size of ab, and i that of AB, then from the similar triangles, $a'b'\phi_1$, $B'A'\phi'_1$

$$\frac{o}{i}=\frac{F_1}{F'_1},$$

where F_1, F'_1 are the anterior focal distances of the observed and the observer's eyes respectively.

If the object were seen in air at the distance of 22 cm., we should have

$$\frac{o}{i'}=\frac{220}{F'_1}.$$

Hence the apparent magnification,

$$M=\frac{i}{i'}=\frac{220}{F'_1}$$

$$=\frac{220}{15}=14{\cdot}6.$$

Hence, *in emmetropia the observed fundus is seen magnified about* $14\frac{1}{2}$ *times by the direct method.*

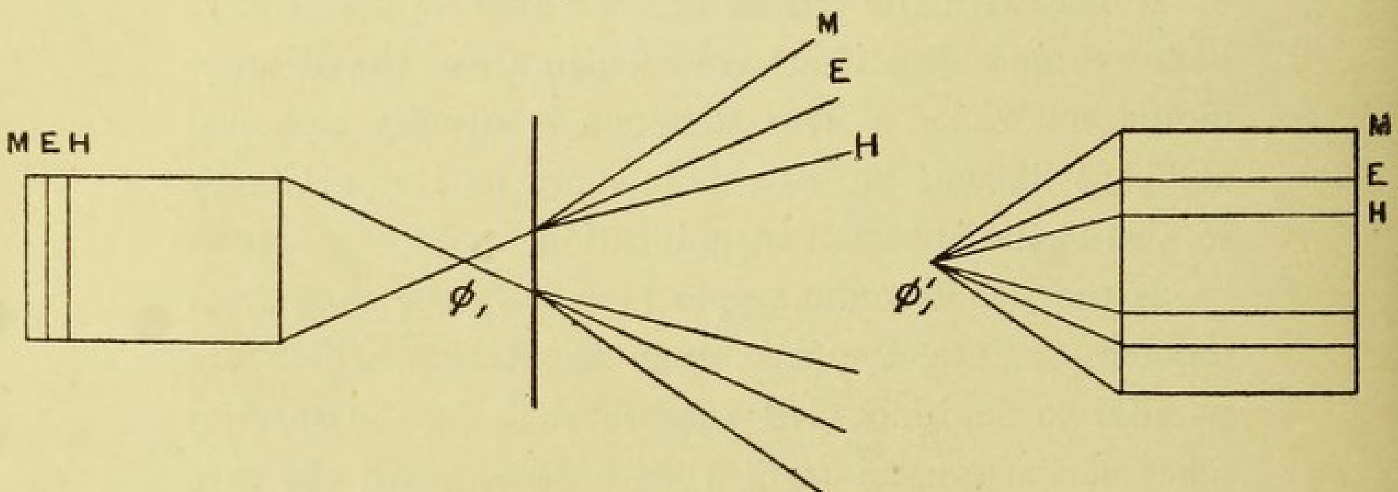

FIG. 59.

If in axial ametropia the correcting lens is placed so that its optical centre is at ϕ_1, it will have no effect upon the director rays under consideration, hence the magnification will be the same in hypermetropia, emmetropia, and myopia. If, however, it is placed beyond ϕ_1, as is usually the case in practice, it will cause these rays to diverge less in hypermetropia (+ lens) and more in myopia (− lens). Hence *the apparent magnification of the image by the direct method is generally greatest in myopia, least in hypermetropia, and intermediate in emmetropia.*

In curvature ametropia, such as occurs in astigmatism, there are two anterior foci to the eye, and the results are easily deduced from Fig. 58. Thus, in the more refractive meridian, ϕ_1 will be nearer the eye, and the rays will diverge from ϕ_1 more than in the figure. In the less refractive meridian ϕ_1 will be farther from the eye, and the rays will diverge less. Hence, *in curvature ametropia (astigmatism), the magnification is greatest in myopia, least in hypermetropia, and intermediate in emmetropia by the direct method.* Hence, in the usual form of regular astigmatism, the image of the disc is an ellipse with the axis *vertical* (the opposite of the usual image by the indirect method, with the lens near the eye). It is obvious that there can be no clear image of the whole field, by the direct method, in astigmatism. *Only lines perpendicular to the meridian which is corrected are seen clearly.* Lines in any meridian other than the two principal ones cannot be seen clearly by any spherical correcting glass, but only by a cylindrical one.

The high magnification by the direct method enables the observer to examine the details of the fundus better than by any other method.

Differences of level may be demonstrated by *parallactic displacement.* If there are two spots on the fundus, one slightly in front of the other, a slight movement of the observer will show an apparent movement of the anterior spot in the opposite direction. In this manner a stereoscopic effect of depth is produced.

Of far more importance is the accurate estimation of the position of opacities or differences of level by the direct method. An opacity in the vitreous of an emmetropic eye is under the same optical conditions as the fundus of a hypermetropic eye. Similarly the bottom of a cupped disc in an emmetropic eye is under the same conditions as the fundus of a myopic eye. Hence, by finding the correcting lens which is necessary to accurately focus the opacity or the bottom of the cup we can determine its exact position. For this purpose it is of great importance that the lens should be as nearly as possible in the anterior focal plane of the eye, *i.e.* about half an inch from the cornea. The strength of the lens will then give the position of the point without further correction, and applying the formula $x = \frac{321}{F'}$ (2) p. 87, we find that a difference of $1D$ is equivalent to an elevation or a depression of $\frac{1}{3}$ mm., according to whether a convex or a concave lens is required. In the aphakic eye the anterior focal plane is nearly an inch in front of the eye, and the ophthalmoscope should be held so that the lenses are in this position. Owing to the strong hypermetropia of refractive index, a difference of $3D$ is then equivalent to 2 mm., as compared with 1 mm. in the emmetropic eye.

CHAPTER XI

RETINOSCOPY

The condition of refraction of an eye can be very accurately determined by means of the movements of the shadow which is seen when light is thrown into the eye by the ophthalmoscopic mirror from a distance of about 1 metre. On account of the importance of

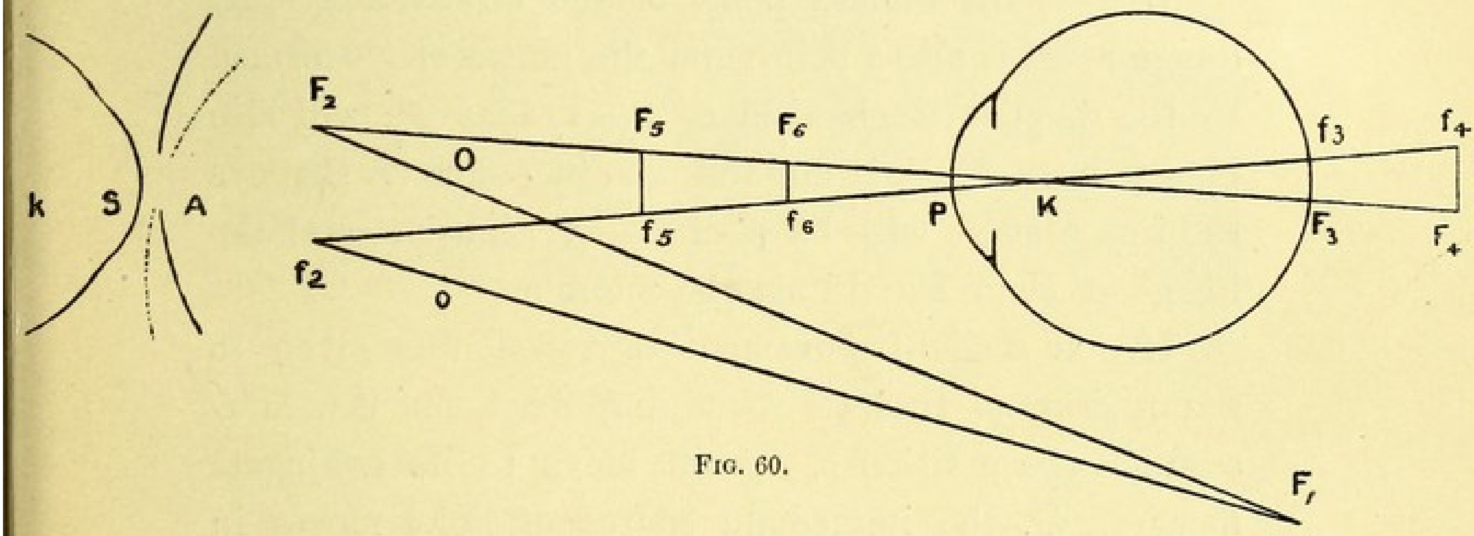

FIG. 60.

understanding the theory of Retinoscopy, the subject will be considered *ab initio*, although this will involve the repetition of some points already explained.

Let F_1 (Fig. 60) be a luminous point (the actual source of light), A a concave mirror held in front of the observer's eye, O the centre of the mirror, KP the observed eye (reduced), K being its nodal point. An image of F_1 is formed by the mirror at F_2, the conjugate focus of F_1. This is the immediate source of light. Since F_1 is a considerable distance from the mirror, F_2 will be near its principal focus.

The rays of light diverging from F_2 will illuminate an area of the observed retina, the centre of which will be F_3, the point where the line F_2K cuts the retina. The dioptric apparatus of the eye will form an image of the point F_3 upon the line F_2KF_3, its position depending upon the refractive condition of the eye. In hypermetropia it will lie behind the retina, *e.g.* at F_4; in myopia it will lie in front of the eye, *e.g.* at F_5; and in emmetropia it will be at infinity. This image may be called the remote image.

If F_2 is the remote point of the observed eye, its image at F_3 is also a point, and the image of F_3, formed by the dioptric mechanism of the eye at F_2, will also be a point. If F_2 is not the remote point of the eye the image at F_3 will be a circle of diffusion, and the image at F_4 or F_5 will also be a circle.

If now a slight movement of rotation is given to the mirror, so that, A remaining fixed, the centre O assumes the position o, F_2 will move to its conjugate focus f_2, *i.e.* the immediate source of light moves in the direction of rotation of the mirror, or, as it is commonly called, *with* the mirror. On the other hand F_3 will move to f_3, *i.e. against* the mirror. F_4 and F_5 will move to f_4 and f_5 respectively, *i.e.* against the mirror in hypermetropia, and with it in myopia.

If, instead of a concave mirror, a plane one is used, F_2 will be as far behind the mirror as F_1 is in front of it; and when the mirror is rotated the immediate source of light will move against the mirror and the retinal image with it. Hence in hypermetropia the remote image will move with the mirror, and in myopia against it.

If the observer is attempting to see the fundus of the observed eye, he really accommodates for the remote image, and if F_2 is the remote point of the eye he will see a point of light at F_2. This is not the case in retinoscopy. The observer is now accommodating for the pupil of the observed eye, with the object of watching the movement of the shadow across it. Hence under all circumstances he sees a circle of diffusion in the observed pupil. This circle of diffusion is of twofold origin. First, the object he is looking for is generally a circle of diffusion upon the observed retina, except in the one instance where the myopia is such that F_2 is the far point of the eye. This circle of diffusion depends upon the aperture of the dioptric system, *i.e.* the observed pupil. Second, the observer is never accommodating for F_4 or F_5, or even F_2, hence a diffusion circle is formed upon his own retina, which is due to the aperture of his own dioptric system, *i.e.* to his own pupil. Hence in every case the observer sees a circle of diffusion which is nearly always of twofold origin, and is referred to the observed pupil.

It has been assumed that the actual source of light is a point. In practice the light must be considerably larger than the sight-hole of the mirror; for if it were not, the reflection would at times disappear entirely as the mirror was rotated. This does not, however, affect the theory; for if the source of light is a uniformly illuminated circular area, it can be replaced by a luminous point farther off which will cause a retinal diffusion image of the same size and nature.

We will assume then, as practical conditions, that the observer's eye is situated at rather more than 1 metre from the observed eye, their optic axes coinciding. The accommodation of the observed eye is at rest or paralysed, and the observing eye is accommodated for the observed pupil. If we consider only the immediate source of light it will be unnecessary to refer to the curvature of the mirror.

Clearly the first thing to be determined is the portion of the observed fundus which, when illuminated, can send rays into the observer's eye. This is the ophthalmoscopic field of vision, and it is determined by the method described in chap. ix. Precisely the same construction gives the retinal image, the point *p* (Fig. 50) being now, however, outside the observer's eye. The remote image can be determined without previously determining the retinal image, and by this means the explanation of retinoscopy is much simplified.

If the remote point of the eye is known, the size of the image is at once found by the length of the line perpendicular to the optic axis, and cut by the limiting rays which enter the pupil from the source of light. Thus, in Fig. 61, if ZN is the observed pupil, L the source of light, and L_1 the conjugate focus of L, H will represent the position of the retina if the eye is hypermetropic, M_2 and M_1 the positions of the retina if the eye is slightly or more strongly myopic respectively.

Then the remote images of the retinal images Z_3N_3, Z_2N_2, Z_1N_1 will be z_3n_3, z_2n_2, and z_1n_1 respectively. If the remote point is not known, all that need be done is to draw a line from the extremities of the retinal

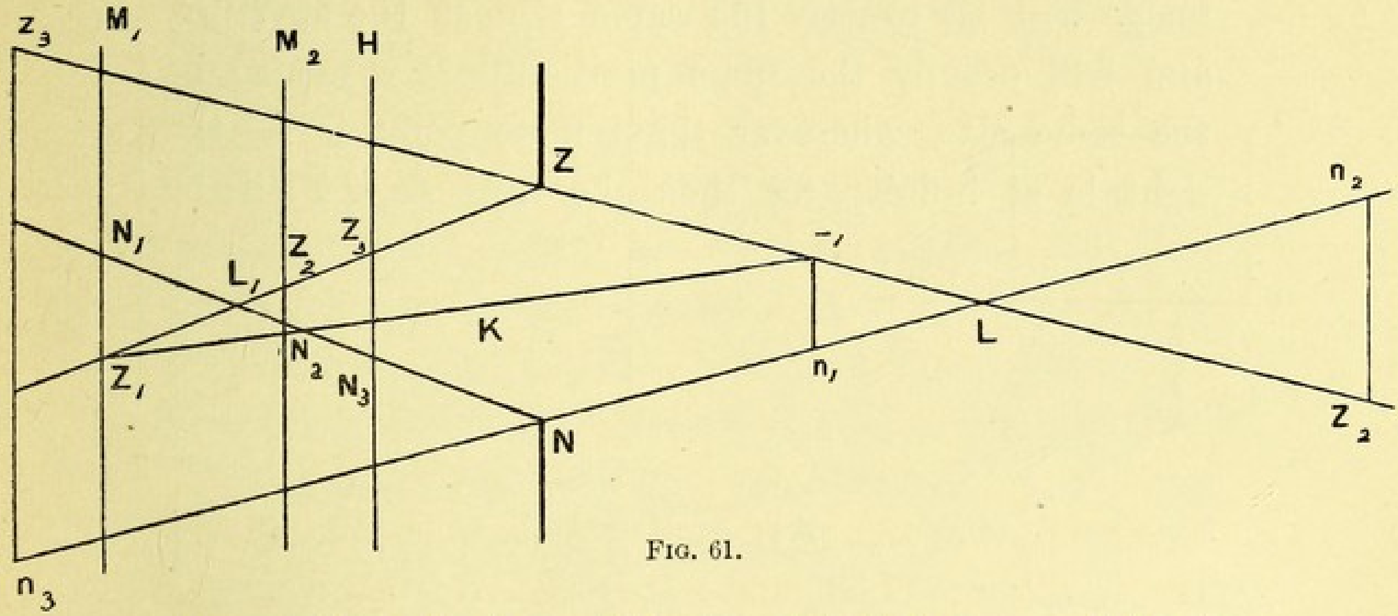

FIG. 61.

images through the nodal point, *K*, of the eye. Where these lines cut the limiting rays *ZL, NL* will be the corresponding points of the remote image.

The distribution of light in the remote image will be an exact image of that of the retinal image.

We have hitherto considered that the only obstacle to rays entering the observed eye from the source of light is the pupil. The results are not quite the same if a second diaphragm is interposed.

If the diaphragm, *AB* (Fig. 62), is situated between the light *L* and the pupil *ZN*, it will have no effect if the aperture is greater than the distance between the limiting rays *ZL, NL*. If, however, the aperture is smaller some of the outer rays will be cut off, and only a portion, *ab*, of the pupil will be illuminated. The size of the remote image will now be determined by *ab* instead of *ZN*, and will be $a_1\beta_1$ instead of z_1n_1, or $a_2\beta_2$ instead of z_2n_2, for the given remote points, *L* being upon the optic axis. If the remote point corresponds with the plane of the diaphragm the remote image will be exactly the same size as the aperture, and will occupy the same position. It will also be the same size, wherever the remote point is, if the light is at infinity, for then the rays *La*, *Lb* will be

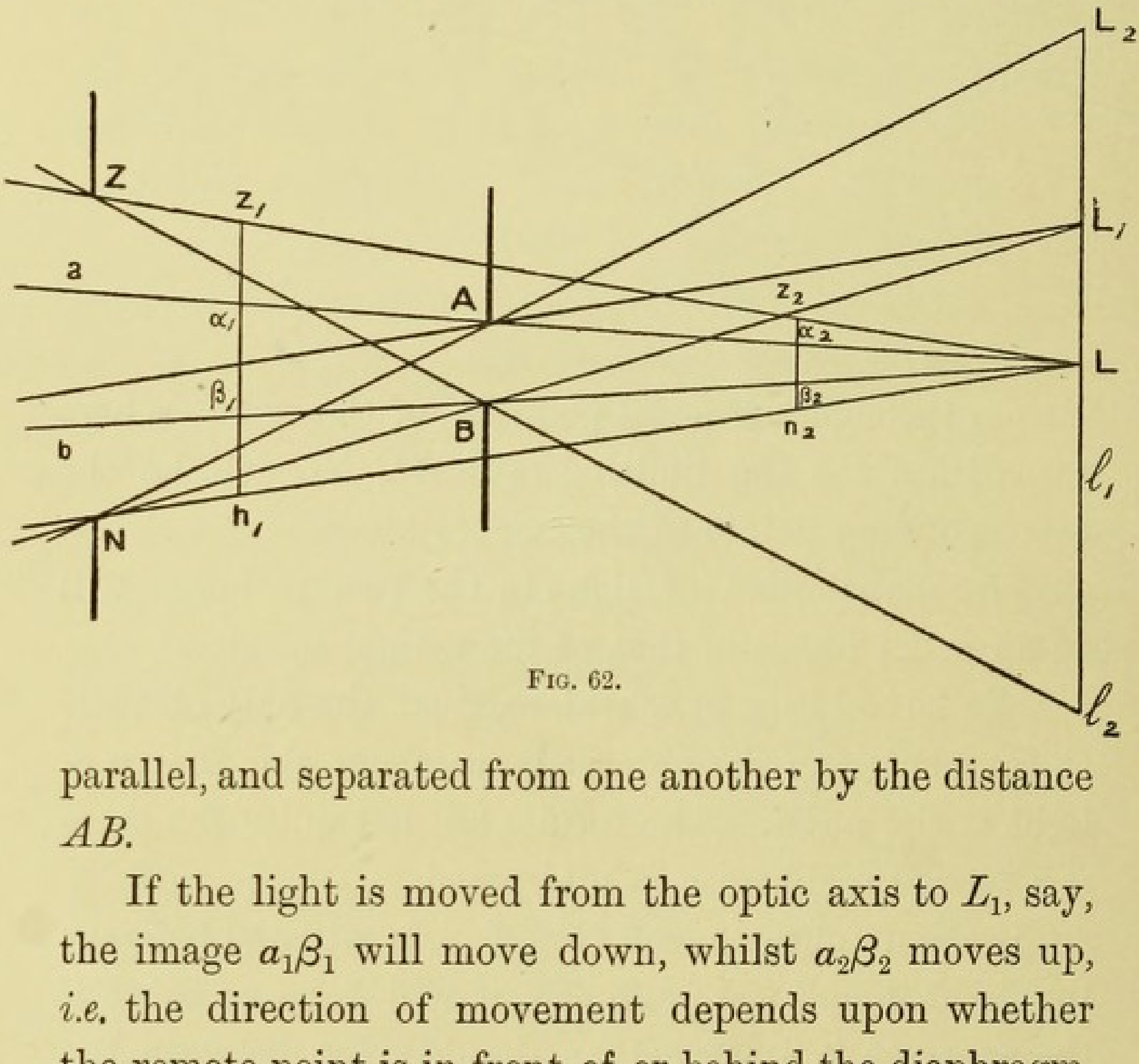

FIG. 62.

parallel, and separated from one another by the distance *AB*.

If the light is moved from the optic axis to L_1, say, the image $a_1\beta_1$ will move down, whilst $a_2\beta_2$ moves up, *i.e.* the direction of movement depends upon whether the remote point is in front of or behind the diaphragm. If it coincides with the diaphragm the image will not move at all. The movement of the image depends, therefore, upon the condition of refraction of the eye.

If the light is moved to L_2, only a single ray, *LN*, can enter the pupil. The same reasoning applies to l_1, l_2. Hence, as the light passes from L_2 to l_1, the limiting ray passes across the whole pupil from N to Z; whilst as it passes from l_1 to l_2, the limiting ray swings round Z, and passes across the whole diameter of the diaphragm from A to B.

For the distance $L_1 l_1$, the illuminated area of pupil, ab, will not be altered, consequently the remote image remains the same size; but for the distances $L_1 L_2$, $l_1 l_2$, the image continually diminishes in size until at L_2 or l_2 it is reduced to a point. Beyond these points the fundus ceases to be illuminated and no image is formed.

The effect of placing the light between the eye and the diaphragm can be determined easily by a similar construction, as also that of the virtual image of a light behind the pupil.

The remote image is not necessarily an exact reproduction of the varying brightness of the parts of the retinal image when there is a second diaphragm.

The tangents AZ, BN (Fig. 62) will include all the rays which can enter the eye, representing thus the total visual field. The effect of the diaphragm upon the diffuse peripheral rays will depend upon the position of the image with regard to the intersection of these tangents and of the internal tangents AN, BZ. Under some circumstances the peripheral area of diminishing illumination will be suppressed.

Since, under the conditions of retinoscopy which we have assumed, the observing eye is accommodated for the observed pupil, this position will for the time being represent its remote point, and any retinal image which may be formed will form a remote image in this plane. This image may be distinguished from the (objective) remote image, formed from the observed retina, as the subjective remote image. It is clear that if this subjective remote image is accurately determined, we can easily determine the corresponding subjective retinal image; and we have then only to reverse this image (as is always done psychologically for every image formed upon the retina) to have an exact picture of what the observer really sees.

Now, since the objective remote image is dependent upon the condition of refraction of the observed eye, we must consider three separate cases in order to determine the subjective remote image under all circumstances. The three cases are:—(1) when the objective remote image is behind the observed eye, *i.e.* the condition of hypermetropia; (2) when it is between the two eyes, *i.e.* high myopia; (3) when it is behind the observing eye, *i.e.* slight myopia and emmetropia.

(1) *Hypermetropia*

In Fig. 63, let Z_1N_1 be the pupil of the observed eye, O_1, and Z_2N_2 the pupil of the observer's eye, O_2; and let L_1 be the immediate source of light. Let F_4 be the (virtual) *punctum remotum* of O_1; then the objective remote image will lie in the plane F_4G, which is drawn through F_4 perpendicular to the optic axis. The total visual field will lie between the points f_2 and f'_2, which are found by drawing the internal tangents to the two pupils, viz. N_2Z_1, Z_2N_1.

Now, the actual objective remote image formed by L_1 will be f_1G, found by joining L_1Z_1, L_1N_1, and producing these tangents to meet the plane of the remote point. But of this area only f_1f_2 lies within the visual field, the remainder, f_2G, being invisible to the eye O_2.

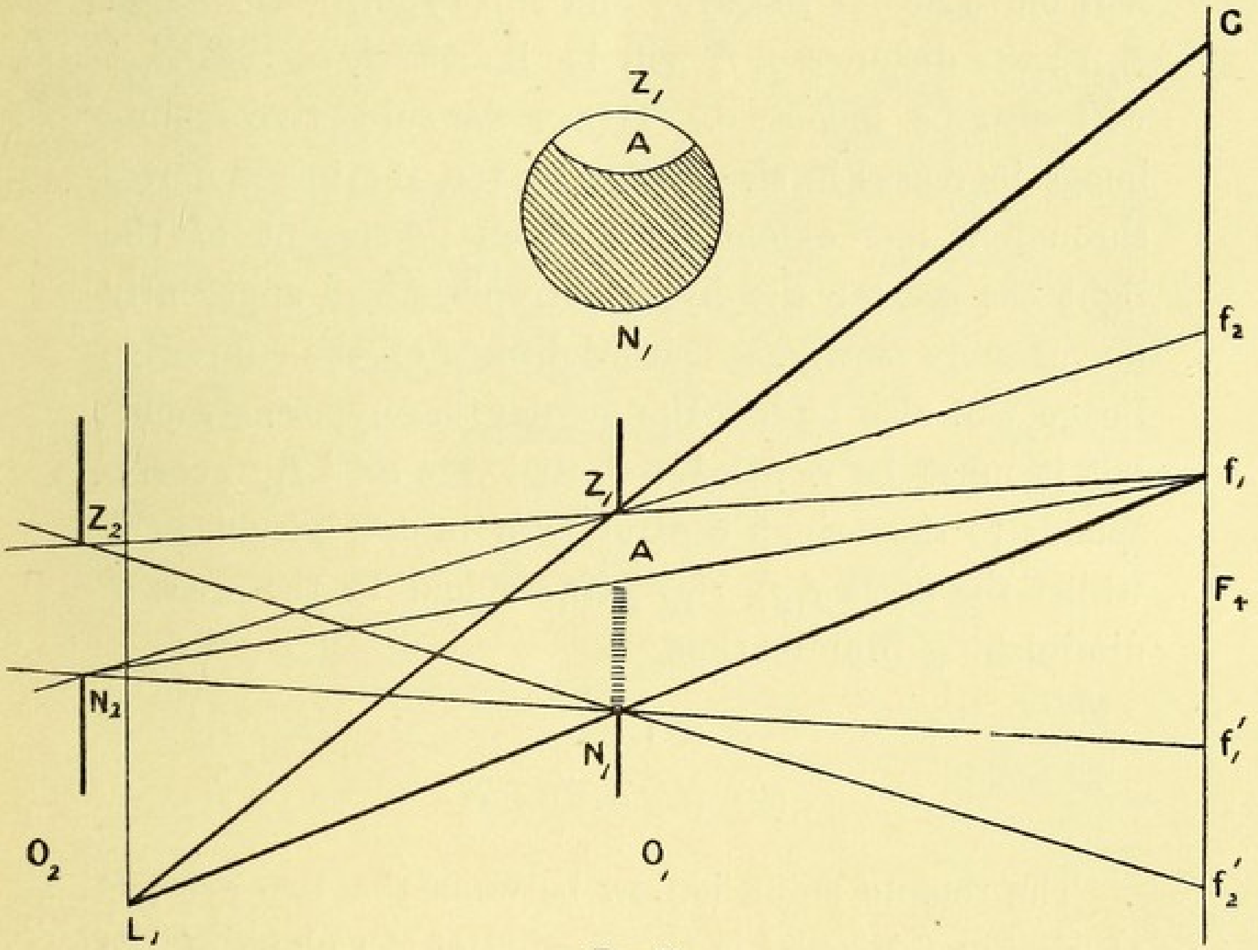

FIG. 63.

Further, the subjective remote image lies in the plane Z_1N_1, for which the eye O_2 is accommodated. Therefore the part of the objective remote image which is visible to O_2 is Z_1A, found by joining f_1N_2, f_2N_2. The illuminated portion of the visual field will therefore be bi-convex, as shown in the small figure. Also, in hypermetropia the subjective image is on the same side of the optic axis as the objective, the immediate source of light being upon the opposite side.

As the light is moved upwards f_1 will go downwards, and hence A, the edge of the illuminated area, will also move in the opposite direction. The effect of an increase in the hypermetropia is easily seen. F_4 will move nearer the eye; and since f_1 will still lie on L_1N_1, the distance f_1f_2 will be increased, so that Z_1A will also be increased. Hence the subjective remote image increases in size. Clearly, too, the image moves through a less extent, for a given movement of the light the greater the hypermetropia, *i.e.* it apparently moves more slowly. The brightness of the subjective image will also be less the greater the hypermetropia; but it must be remembered that the area $f_1f'_1$ corresponds to the bright central area of the visual field, whilst the areas f_1f_2, $f'_1f'_2$ correspond to the areas of diminishing illumination.

(2) *High Myopia*

The remote point is now between the two eyes, as at F_6, Fig. 64. f_1G is the portion of the plane of the objective remote image which is illuminated, but only the portion f_1f_2 will be visible to the eye O_2, f_2G being outside the visual field. The portion AN_1 of the subjective image will be illuminated, and will be bi-convex, as shown in the small figure.

Hence in high myopia the objective and the subjective images are still both upon the same side of the optic axis, but they are also upon the same side as the immediate source of light. The dark crescent is therefore upon the opposite side. As the light is moved the edge of the shadow will move in the same direction.

If the myopia is less, so that the remote point is at

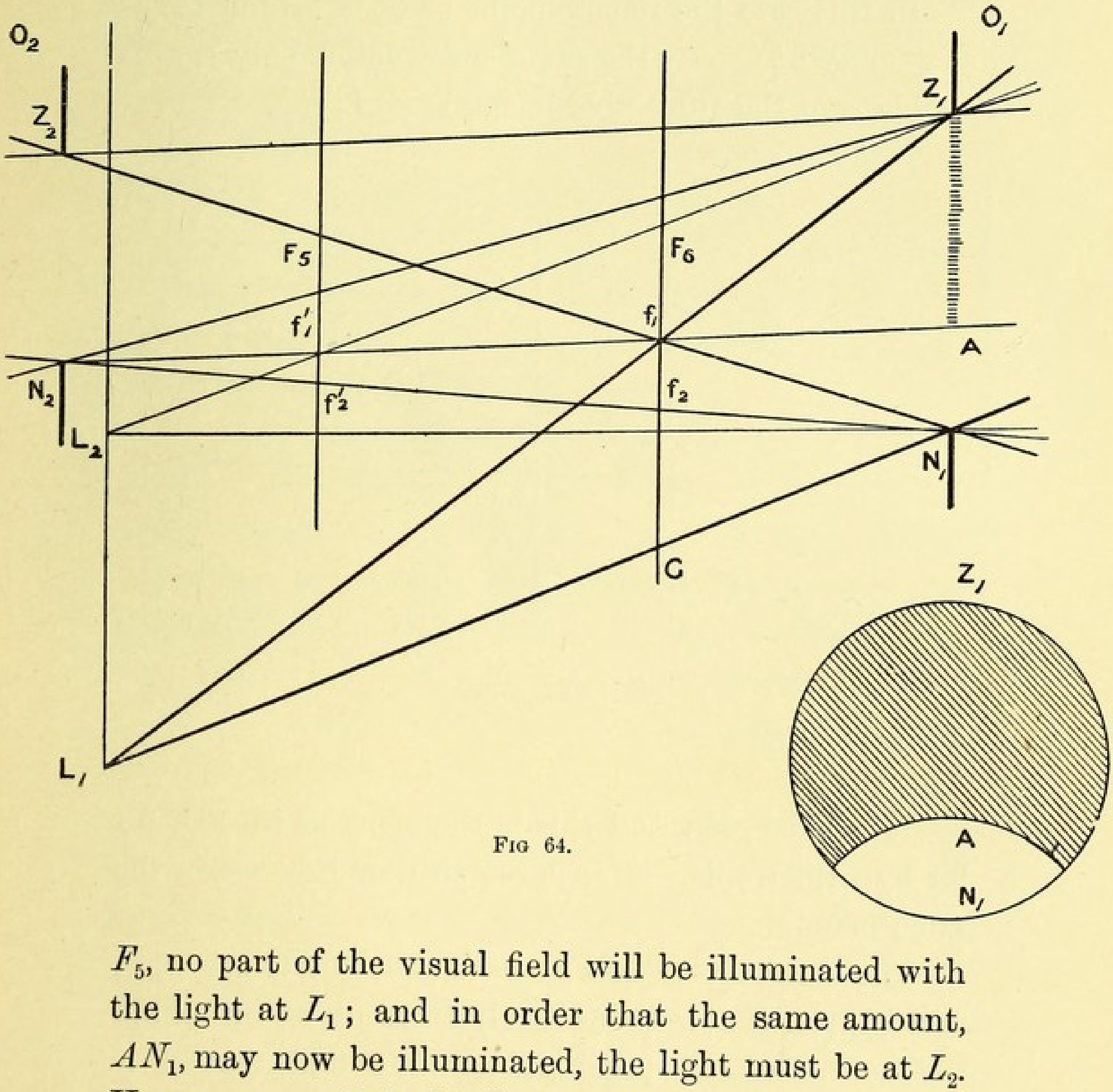

FIG 64.

F_5, no part of the visual field will be illuminated with the light at L_1; and in order that the same amount, AN_1, may now be illuminated, the light must be at L_2. Hence a very slight displacement of the light will now cause a very large displacement of the subjective image. In fact, as soon as the light lies on the tangent N_2N_1

the whole of the pupil will be illuminated, and will remain so until the light reaches the tangent $Z_2 Z_1$.

(3) *Low Myopia and Emmetropia*

In this case the remote point of O_1 is behind O_2, as at F (Fig. 65). By the same construction only $f_1 f_2$ is visible, and the subjective image is $Z_1 A$.

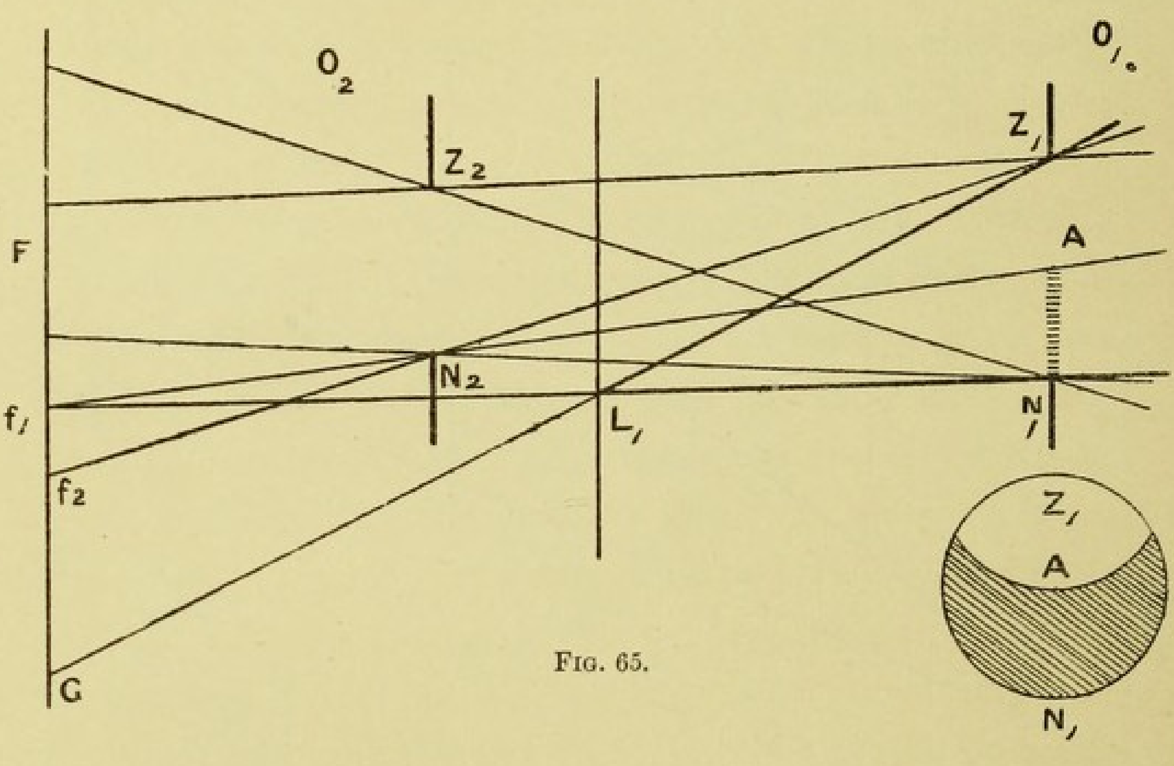

FIG. 65.

It will be seen that this is the same as in Fig. 63 for hypermetropia. Hence the shadow behaves in the same manner.

The construction of Fig. 65 enables us to see what will happen if F is immediately in front of or behind the centre of the pupil $Z_2 N_2$. We find that if F actually coincides with the centre, it is impossible to see the edge of the image—the slightest movement of the light illuminates the whole of the observed pupil or plunges it in shadow. This is the *point of reversal* of the shadow.

A glance at Figs. 63 and 65 will show the effect of altering the sizes of the two pupils. In Fig. 63, reducing the size of Z_2N_2 will have but little effect upon the edge of the shadow, whereas in Fig. 65 it will have a relatively enormous effect.

We arrive then at the following conclusions:—

(1) *With a plane mirror, the shadow moves with the mirror in hypermetropia, emmetropia, and low myopia, and against it in high myopia. The opposite is the case with a concave mirror.*

(2) *The slower the shadow moves and the feebler the illumination, the higher is the ametropia.*

(3) *The shadow depends upon both pupils, but more upon the observed pupil, except near the point of reversal, when it depends more upon the observer's pupil or the sight-hole of the mirror, whichever is smaller.*

The direction of movement of the shadow is the same whether the ametropia is axial, curvature, or of refractive index. We have yet to consider astigmatism.

Here the behaviour of the shadow will be in accordance with the refraction of the principal meridians, when the light is moved in the directions of those meridians. If the light is moved in any other meridian, the shadow still appears to move in the direction of the nearest principal meridian. This is due to an optical illusion, which is, however, very useful for determining the axes. The shadow really moves in the direction of rotation of the mirror, but since its edge is determined by the oval diffusion images whose axes are in the directions of the principal meridians, the apparent movement is always perpendicular to the edge. The illusion is exactly the same as when a ruler is moved behind a circular aperture. It always apparently moves perpendicular to the edge, whatever may be its real movement (movement in the same direction as the edge being of course excepted).

In measuring refraction by retinoscopy correcting lenses are placed in front of the eye until the point of reversal is found. This can only be determined approximately, as the shadow becomes indistinct before it is actually arrived at. For practical details the reader is referred to other works upon the subject.

A word must be said in conclusion as to Leroy's theory of retinoscopy, which is adopted by most German ophthalmologists. According to this theory, the cause of the shadow is the observer's pupil projected upon the pupil of the observed eye. It may be stated at once that the theory is quite correct, but it has nevertheless led to much ambiguity.

Let O_1 be the observed, and O_2 the observer's eye; and let Z_1N_1, Z_2N_2 be their respective pupils, F_5 being the far point of O_1. F_5 will be the conjugate focus of a point F_3 (Fig. 60) upon the retina of O_1; *i.e.* if F_3 be illuminated it will form a real image at F_5. This image will be formed by all the rays which can pass through the pupil Z_1N_1; *i.e.* the rays will diverge from F_3 to Z_1N_1, and will then converge to the point F_5. Beyond F_5 the rays will again be divergent.

But all of these rays, for the position of F_5 given in Fig. 66, cannot enter the observer's pupil Z_2N_2, but only those having the base n_1N_2. The remainder of the cone falls upon the iris at z_1N_2. "Since the rays constituting this part of the cone are not seen by the observer, the portion of the pupil which is opposite to them, and from which they come (represented by shading in the figure) appears unilluminated; the only portion of the pupil that does appear illuminated being that which is here shown as unshaded, and from

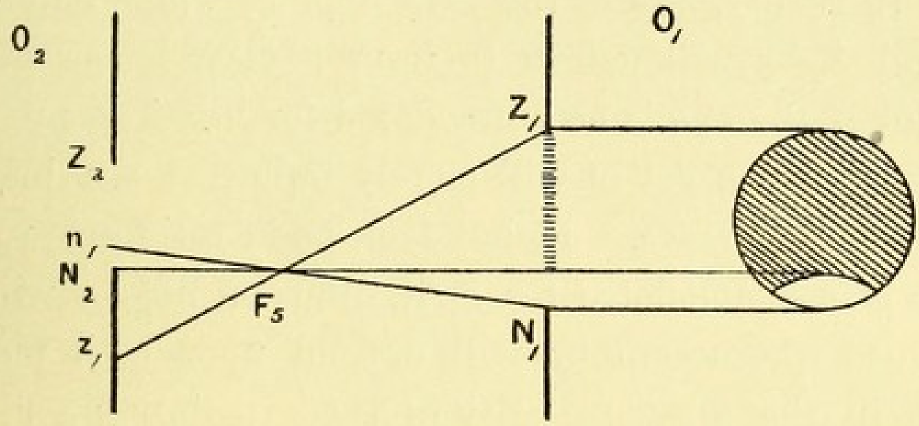

FIG. 66.

which the observer receives rays that enter his own pupil."[1] A suitable construction will give an analogous result for the hypermetropic eye.

It is natural to come to the conclusion from this reasoning that the observer's pupil is the main cause of the shadow. This is only true in the immediate neighbourhood of the point of reversal, or what Parent calls the "zone ou parcours de mauvaise observation et point neutre"—in fact, where observation is most difficult.

A simple experiment proves that it is not generally

[1] Fuchs' *Text-Book of Ophthalmology*, 2nd American ed. p. 21.

true. If the sight-hole of the mirror be made smaller than the observer's pupil, it will replace the latter as the optical aperture of his dioptric system. If, instead of being round, it is made square or triangular, any shadow caused by it will be similarly altered in contour. It will be found to have no effect upon the shadow seen in the observed pupil, except just as the point of reversal is approached, and then only in the most favourable circumstances as regards the disposition of the light, etc.

In reality, any of the rays from F_5 which enter the pupil Z_2N_2 will suffice to form a clear image of the point if the eye is accommodated for it. The number of rays which get in will merely influence the brightness of the image. If, however, as in retinoscopy, the eye is accommodated for Z_1N_1, diffusion images will be formed whose contour will depend upon both pupils, but in the great majority of cases mainly upon the observed pupil.

THE END

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