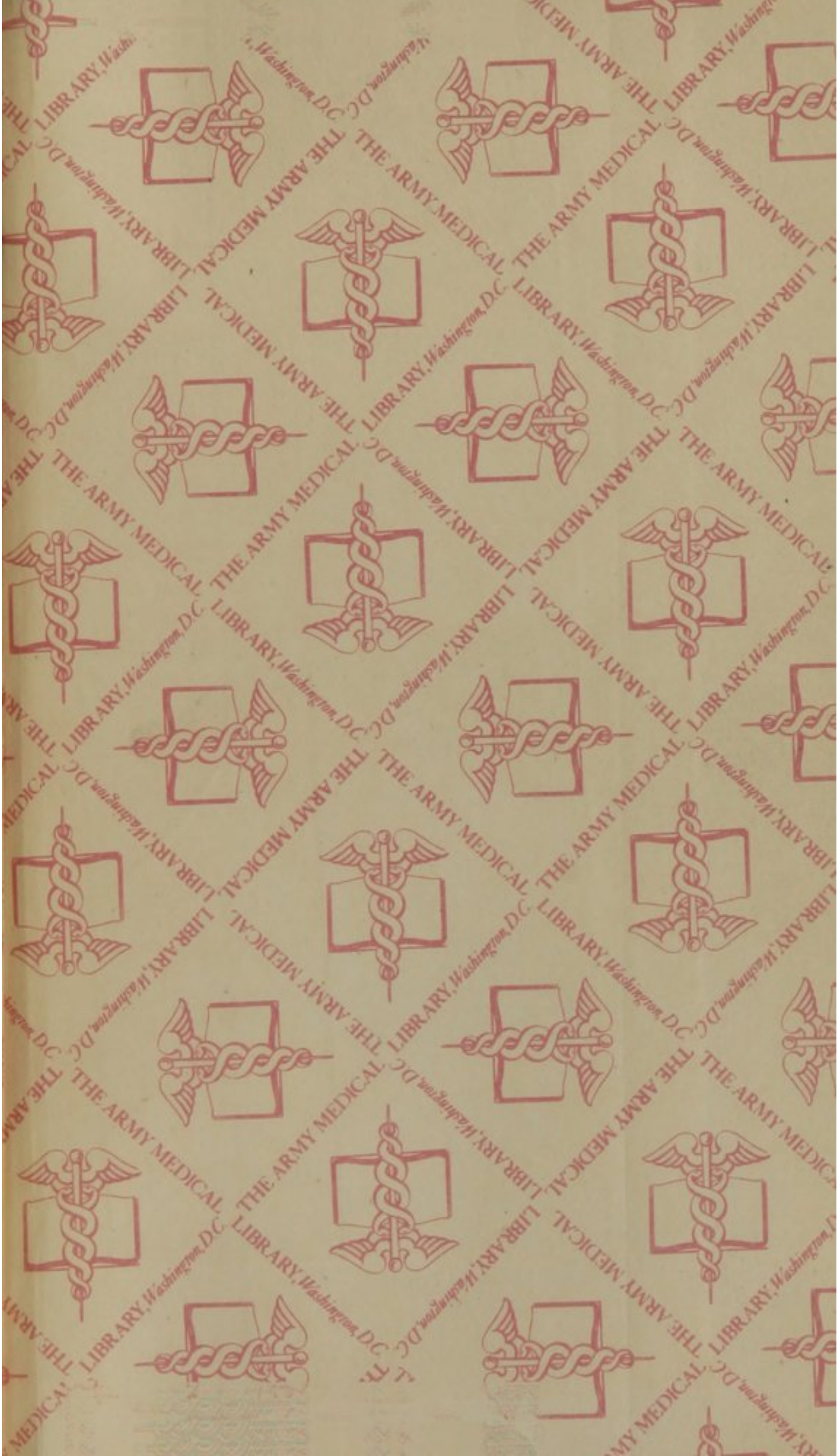
The scholar's assistant, or, A plain, comprehensive, and practical system of arithmetic : to which is prefixed an introduction, containing a practical illustration of the primary rules, and of the tables of money, weights, and measures : designed for the use of schools in the United States / by the Rev. J.G. Cooper.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THE

SCHOLAR'S ASSISTANT;

OR,

A PLAIN, COMPREHENSIVE, AND PRACTICAL

SYSTEM OF ARITHMETIC:

TO WHICH IS PREFIXED

AN INTRODUCTION,

CONTAINING

A PRACTICAL ILLUSTRATION OF THE PRIMARY RULES, AND OF THE TABLES OF MONEY, WEIGHTS, AND MEASURES.

DESIGNED

FOR THE USE OF SCHOOLS IN THE UNITED STATES.

BY THE

REV. J. G. COOPER, A. M.

EDITOR OF THE WORKS OF VIRGIL; AUTHOR OF A PLAIN, PRACTICAL LATIN GRAMMAR; ALSO, OF AN ABRIDGMENT OF MURRAY'S ENGLISH GRAMMAR, WITH IMPROVEMENTS.

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1830.

CONTENTS.

TO TEACHERS, AND INSTRUCTERS OF YOUTH.

Gentlemen,—The favourable reception which many of you have given to my EDITION of the WORKS OF VIRGIL, and to my IMPROVED LATIN AND ENGLISH GRAMMARS, leads me to hope, that the SCHOLAR'S ASSISTANT will also meet your approbation.

My experience in the business of instruction led me to a knowledge of the wants of the pupil, in the department of ARITHMETIC, and made me sensible of the defects of the books in common use. And, as far as my acquaintance extends, I believe many of you are persuaded, that important improvements might be made upon them. Whether I have fully supplied those defects, and made the desired improvements, must remain with you, and the public, to judge.

Arithmetic is of every day use in the business of life. Hence arises its importance to the farmer, the mechanic, the merchant, the gentleman, the scholar; and, I may add, to every member of the community. Its operations are founded upon mathematical principles; and those principles should be constantly kept in view, fully explained, elucidated, and rendered intelligible to the pupil, in every stage of his progress. To effect this has been my chief aim. Through the whole work, I kept the pupil in my view, and gave such explanations of the subject before him, and such instructions, as I supposed he might require.

THE SCHOLAR'S ASSISTANT may justly claim the following improvements: 1. It is a full and comprehensive system of practical arithmetic. 2. The arrangement of its several parts is calculated to facilitate the progress of the learner. 3. The principles of arithmetic are fully explained

and elucidated. 4. The examples are not mere obsolete and useless questions; but are interesting, instructive, and of a practical nature; a large portion of which are entirely new and original. 5. It explains itself; and, therefore, will save much labour on the part of the teacher, and will render the study pleasant to the pupil.

For the correctness of these positions, I refer you to the work itself. You will find Vulgar and Decimal Fractions, the Reduction of Foreign and Domestic Currencies, the Rules of Proportion, both Simple and Compound, the Extraction of the Square and Cube Roots, Arithmetical and Geometrical Progression, Simple and Compound Interest, Annuities, &c., more comprehensive, and their principles better explained, and rendered more intelligible, than they are in other works of the kind. In addition to these, the introduction contains more than eight hundred familiar, easy, and practical exercises, designed to elucidate the primary rules, to explain the several tables, and to show their use and application.

Other works, that I have seen, are deficient in arrangement, and in suitable introductory matter, calculated to engage the attention of the child, to interest his mind, and to develope his reasoning faculties. He is hurried through the primary rules, before his mind is sufficiently disciplined in the property of numbers.

The plan of the present work entirely remedies these defects. It is somewhat novel, and might seem to require an apology; but, by adopting it, I had an opportunity of explaining more fully the elementary principles, of dwelling upon them longer, and of giving that particular and minute instruction, which the child stands in need of, in his first essays in numbers. The questions upon the tables are designed chiefly as a mental exercise; and, in this view, they will be found very useful. The instructer may add others, as he shall deem expedient.

The division of the work necessarily led to some repetition. But this can be no objection. It is the very method of useful instruction. Children must have "line upon line, precept upon precept, here a little, and there a little," in order to induce thought and reflection, and to develope the powers of his mind,

In other books, a large proportion, and, in some, even the greater part, of the examples, are calculated in pounds, shillings, pence, and farthings; a currency obsolete and useless, and even *worse* than useless. This is a material defect. As well might the pupil be taught to perform the ordinary calculations of life, in the copecs and rubles of Russia, the candareens and tales of China, or in any other foreign currency. But such a course of instruction certainly could find no advocates. Here all the questions and examples are calculated in the easy and simple currency of the United States. Pounds, shillings, and pence are introduced only to show their nature and properties, and determine their values in the currency of our country.

The practical utility of the SCHOLAR'S ASSISTANT I have fully tested. The manuscript was used by my pupils for some months. Their progress, in every respect, fulfilled my expectations; and I can say, with confidence, that the saving of time, on their part, was full fifty per cent.; and the saving of labour, on the part of instruction, was in a greater proportion.

That a work of this magnitude, considerably larger than the common books, and indeed larger than I contemplated, should be free from errors, I cannot suppose. The circumstance of its being prepared during the daily occupations of a school; a great part of the examples being original, and requiring calculation; and the whole printed from manuscript, precludes the expectation. For the errors of the author, as well as for those of the press, I solicit your kind indulgence. If the present work shall meet your approbation, I have it in contemplation to furnish you with an "Introduction to Algebra," upon a similar plan.

That the SCHOLAR'S ASSISTANT may be found to answer the purposes for which it was designed, namely, to facilitate the progress of the pupil, and lighten the labour of the instructer, is the humble hope of, Gentlemen,

Your obedient friend and servant,

J. G. COOPER.

Philadelphia, June 20, 1830.

EXPLANATION OF CHARACTERS.

Signs.	*Signification.*
$=$	equal, as, 100 cents=1 dollar.
$+$	Addition, as, $4+6=10$.
$-$	Subtraction, as, $10-4=6$.
$\times$	Multiplication, as, $4\times6=24$.
$\div$	Divison, as, $10\div2=5$.
$:\ ::\ :$	Proportion as, $2 : 4 :: 6 : 12$.
$\surd$	the Square Root as, $\surd9=3$.
$\sqrt[3]{}$	the Cube Root, as, $\sqrt[3]{27}=3$.
$\overline{\qquad}$	a vinculum. It shows that the numbers, over which it is drawn, are to be considered as a simple quantity.

INTRODUCTION.

ALL the operations of Arithmetic are performed by the help of the following characters or figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first figure, when it is alone, represents a unit, or one of any thing; as one apple—one man—one dollar. The figure 2 represents two units, or ones, of any thing; as two apples—two dollars—two hats. The figure 3 represents three units, or ones. The figure 4 represents four units, or ones. The figure 5 represents five units, or ones. The figure 6 represents six units, or ones. The figure 7 represents seven units, or ones. The figure 8 represents eight units, or ones; and the figure 9 represents nine units, or ones. These are the simple values of the figures. The cypher or nought has in itself no value. In connection with the preceding figures, it causes them to assume different values, by removing them to higher places. Thus, in the combination 10, the nought or cypher removes the one or unit to the ten's place, thereby causing it to assume ten times the value which it would have, if it stood in the unit's place, itself having, at the same time, no value. When any of the figures stand together, they have a local value, which varies according to their situation. The first figure on the right hand always represents so many units or ones. It is, therefore, said to stand in unit's place. The next, on the left hand, is said to stand in ten's place, because a figure standing there represents ten times as much as it would do, if it stood in unit's place. Thus, in the combination 21, the figure 2 stands for two tens or twenty; but, if it stood in the place of the 1, it would represent two units only.

The simple value of any figure is increased ten times by removing it one place from the units; one hundred times, by removing it two places, a thousand times, by removing it three places, from the units; and so on, to any number of places: thus, in the combination 4321, the figure 4 stands for four thousand; the figure 3 stands for three hundred; the figure 2 stands for

two tens or twenty; and the figure 1 stands for a simple unit. The whole combination represents four thousand three hundred and twenty-one; and so for any other combination of figures. This is called Numeration; which means the expressing of numbers by figures, and the giving to each its proper value.

Question 1. How are the operations of arithmetic performed?

2. What do you mean by a unit?

3. How many units does the figure 2 represent?—how many does the figure 3 represent?—how many does the figure 4 represent?—how many does the figure 5 represent?—how many does the figure 6 represent?—how many does the figure 7 represent?—how many does the figure 8 represent?—how many does the figure 9 represent?

4. When are the figures said to have their simple value?

5. When are they said to have a local value?

6. Has the cypher or nought any value in itself?

7. What then is its use?

8. When several figures are combined, in what proportion do they increase in value from the unit's place?

9. Does the figure in the unit's place always retain its simple value?

10. How would you set down, in figures, four thousand three hundred and twenty-one?

11. Will you explain the reason of this, and assign to each figure its local value?

12. What is the expressing of numbers by figures called?

ADDITION.

Addition is the operation of finding the sum of two or more numbers of the same kind or denomination.

ADDITION TABLE.

2	and	1	are	3	3	and	1	are	4	4	and	1	are	5
2	"	2	"	4	3	"	2	"	5	4	"	2	"	6
2	"	3	"	5	3	"	3	"	6	4	"	3	"	7
2	"	4	"	6	3	"	4	"	7	4	"	4	"	8
2	"	5	"	7	3	"	5	"	8	4	"	5	"	9
2	"	6	"	8	3	"	6	"	9	4	"	6	"	10
2	"	7	"	9	3	"	7	"	10	4	"	7	"	11
2	"	8	"	10	3	"	8	"	11	4	"	8	"	12
2	"	9	"	11	3	"	9	"	12	4	"	9	"	13
2	"	10	"	12	3	"	10	"	13	4	"	10	"	14

5	and	1	are	6	6	and	1	are	7	7	and	1	are	8
5	"	2	"	7	6	"	2	"	8	7	"	2	"	9
5	"	3	"	8	6	"	3	"	9	7	"	3	"	10
5	"	4	"	9	6	"	4	"	10	7	"	4	"	11
5	"	5	"	10	6	"	5	"	11	7	"	5	"	12
5	"	6	"	11	6	"	6	"	12	7	"	6	"	13
5	"	7	"	12	6	"	7	"	13	7	"	7	"	14
5	"	8	"	13	6	"	8	"	14	7	"	8	"	15
5	"	9	"	14	6	"	9	"	15	7	"	9	"	16
5	"	10	"	15	6	"	10	"	16	7	"	10	"	17

8	and	1	are	9	9	and	1	are	10	10	and	1	are	11
8	"	2	"	10	9	"	2	"	11	10	"	2	"	12
8	"	3	"	11	9	"	3	"	12	10	"	3	"	13
8	"	4	"	12	9	"	4	"	13	10	"	4	"	14
8	"	5	"	13	9	"	5	"	14	10	"	5	"	15
8	"	6	"	14	9	"	6	"	15	10	"	6	"	16
8	"	7	"	15	9	"	7	"	16	10	"	7	"	17
8	"	8	"	16	9	"	8	"	17	10	"	8	"	18
8	"	9	"	17	9	"	9	"	18	10	"	9	"	19
8	"	10	"	18	9	"	10	"	19	10	"	10	"	20

Questions on the Table to be answered by the learner.

1. What is the sum of 2 and 4? Ans. 6. Why is it 6?
2. What is the sum of 2 and 7? Ans. 9. Why is it 9?
3. What is the sum of 3 and 5? Ans. 8. Why is it 8?
4. What is the sum of 4 and 6? Ans. 10. Why is it 10?
5. What is the sum of 5 and 7? Ans. 12. Why is it 12?
6. What is the sum of 6 and 8? Ans. 14. Why is it 14?
7. What is the sum of 7 and 9? Ans. 16. Why is it 16?
8. What is the sum of 8 and 8? Ans. 16. Why is it 16?
9. What is the sum of 9 and 5? Ans. 14. Why is it 14?
10. What is the sum of 10 and 4? Ans. 14. Why is it 14?
11. What is the sum of 9 and 9? Ans. 18. Why is it 18?
12. What is the sum of 8 and 5? Ans. 13. Why is it 13?
13. What is the sum of 7 and 10? Ans. 17. Why is it 17?
14. What is the sum of 2, 3 and 4? Ans. 9. Why is it 9?
15. What is the sum of 3, 5 and 4? Ans. 12. Why is it 12?
16. What is the sum of 4, 6 and 7? Ans. 17. Why is it 17?
17. What is the sum of 5, 6 and 4? Ans. 15. Why is it 15?
18. What is the sum of 6, 8 and 5? Ans. 19. Why is it 19?
19. What is the sum of 7, 4 and 7? Ans. 18. Why is it 18?
20. What is the sum of 9, 3 and 6? Ans. 18. Why is it 18?
21. What is the sum of 10, 4 and 6? Ans. 20. Why is it 20?

22. What is the sum of 9, 5 and 3? Ans. 17. Why is it 17?
23. What is the sum of 5, 6 and 7? Ans. 18. Why is it 18?
24. What is the sum of 3, 4, 5 and 6? Ans. 18. Why is it 18?
25. What is the sum of 4, 5, 6 and 4? Ans. 19. Why is it 19?
26. What is the sum of 3, 5, 2 and 1? Ans. 11. Why is it 11?
27. What is the sum of 8, 4, 3 and 2? Ans. 17. Why is it 17?
28. What is the sum of 9, 2, 3 and 4? Ans. 18. Why is it 18?
29. What is the sum of 2, 6, 10 and 2? Ans. 20. Why is it 20?
30. What is the sum of 4, 7, 5 and 3? Ans. 19. Why is it 19?

After this manner, the teacher may require the learner to add together any other numbers that may be in the table, demanding at the same time a reason for the answer. This will be found a very useful exercise.

PRACTICAL EXAMPLES.

31. Jane has 4 apples, and her two sisters have 3 apples each; how many have all three? Ans. 10. Why have they 10?
32. Susan bought a slate for 10 cents, quills and a pencil for 4 cents, and paper for 6 cents; how much did she spend in all? Ans. 20 cents. Can you prove that?
33. William spends 2 cents a day for cakes, and John spends 5 for nuts, and 3 for apples; how much do they both spend in a day? Ans. 10 cents. Can you prove that?
34. If you buy a knife for 10 cents, and apples for 5 cents; how much do you spend? Ans. 15 cents. Can you prove that?
35. A fine little boy asked his father for 12 cents to buy a toy, 2 cents to buy nuts, 4 cents to buy apples, and 1 cent for candy; how much did he spend in all? Ans. 19 cents. Will you prove that?
36. Jane, setting out for school, asked her sister to give her 2 cents for apples, 3 for nuts, 4 for cakes, and 8 cents for paper; how much did her sister give her in all? Ans. 17 cents. Will you prove that?
37. If you buy a book for 15 cents, and paper for 5 cents, how much do the book and paper come to? Ans. 20 cents. Will you prove that?

38. John is 8 years old, and William is 11; what is the sum of their ages? Ans. 19 years. How will you prove that?

39. Susan is 9 years old, and Jane 14 years; what is the sum of their ages? Ans. 23 years. How will you prove that?

40. In a school of young ladies, there are three classes; in one class there are 8, in another 6, and in a third 9; how many are there in the whole? Ans. 23. How can you prove that?

41. A teacher has 5 boys reading Virgil, 4 boys reading Cæsar, and 8 reading Sallust; how many has he in all? Ans. 17. How will you prove it?

42. In a school 9 boys write, 4 cypher, and 5 study grammar; how many are there in the school? Ans. 18. How will you prove it?

43. If a hat costs 3 dollars, a pair of shoes 2 dollars, and a coat 9 dollars; how much do they all cost? Ans. 14 dollars. How do you prove it?

44. Mary has 4 tickets for good behaviour, 5 for good reading, and 6 for grammar; how many has she in all? Ans. 15. How will you prove it?

45. James has 7 credits for good deportment, 5 for reading, and 8 for writing; how many has he in all? Ans. 20. How will you prove it?

46. William says 5 lessons in the morning, and 4 in the afternoon, beside 2 tasks: how many lessons does he say altogether? Ans. 11. Will you prove that?

47. Three poor persons asked charity; and a good boy gave to the first one 5 cents, to the second 8, and to the third one 6 cents, which was all the money he had; how much did he give in all? Ans. 19 cents. How will you prove it?

48. Jane, on her way to school, meeting some poor children, who asked charity, gave to the first 6 cents, to the second 7 cents, and to the third 8 cents, which was all she had; how much did she give in all? Ans. 21 cents. How will you prove it?

49. A farmer sold a load of hay for 8 dollars, five bushels of corn for 3 dollars, and ten bushels of oats for 6 dollars; how much money did he receive in all? Ans. 17 dollars. How will you prove that?

50. A merchant sold cloth for 12 dollars, tea for 3 dollars, and sugar for 5 dollars; how much did the whole amount to? Ans. 20 dollars. How will you prove that?

When the numbers to be added consist of two or more places, you must set them one under the other, observing to set the units of each number under one another; the tens under the tens, the hundreds under the hundreds, &c. as in the following examples:

51. What is the sum of 425, 146 and 620? Ans. 1191.

Here there are three places of figures; the first, or right hand, place is the unit's, the second is the ten's, and the third is the hundred's place.

Put them thus:

425
146
620
1191 the sum of the numbers.

After you have placed them under each other, you must add together the first column, or the figures in the unit's place, and observe the sum. If it is 10, or more; then set the units in the sum or amount, under the column of units, and add the tens into the column of tens; and proceed in the same way with the other columns; but, in the last column, you must set down the whole sum. In this example, the sum of the first column is 11; which is one ten, and one unit, or 1; set the 1 under the line of units, and add the 1, which is 1 ten, into the line or column of tens. This line amounts to 9 only; which you must place under the column of tens; and because it does not amount to ten, there is none to be added to the next line. The sum of the third column, or line of hundreds, is 11; the whole of which you must set down, as in the example. In the same way, you must proceed in the following questions.

52. What is the sum of 621, 542 and 134? Ans. 1297
53. What is the sum of 46, 74, and 92? Ans. 212
54. What is the sum of 942, 356 and 372? Ans. 1670
55. What is the sum of 876, 543 and 321? Ans. 1740
56. What is the sum of 1334, 2467, and 4321? Ans. 8122
57. What is the sum of 2468, 1357 and 7892? Ans. 11715
58. What is the sum of 1976, 2543 and 9045? Ans. 13564
59. What is the sum of 987, 578 and 679? Ans. 2244
60. What is the sum of 104, 560 and 809? Ans. 1473
61. What is the sum of 859, 789 and 999? Ans. 2647
62. What is the sum of 333, 645 and 777? Ans. 1755
63. What is the sum of 444, 555 and 888? Ans. 1887
64. What is the sum of 540, 372 and 421? Ans. 1233
65. What is the sum of 842, 573 and 375? Ans. 1780

66. James bought a grammar for 45 cents, a slate for 18 cents, and an English Reader for 65 cents; how much did they all cost? Ans. 128 cents.

67. A farmer has 34 acres of land in one field, 42 acres in another, and 26 acres in another; how many acres has he in all? Ans. 102.

68. A gentleman bought a horse for 145 dollars, a carriage for 275, and a harness for 45 dollars; how much did they all cost him? Ans. 465 dollars.

69. In the year 1820 there were in the city of New York 123,700 inhabitants, in the city of Philadelphia 108,100 inhabitants, and in the city of Baltimore 62,700; how many were there in the three cities? Ans. 294,500.

70. At the same time there were in the city of Boston 43,300 inhabitants, in New Orleans 27,200 inhabitants, and in Charleston 24,800 inhabitants; how many were there in the three cities? Ans. 95,300.

71. A merchant bought four pieces of cloth; the first measured 45 yards, the second 36 yards, the third 29 yards, and the fourth 56 yards; how many yards were there in all? Ans. 166.

72. A farmer has in one field 46 sheep, in another 75 sheep, in a third 61 sheep, and in a fourth 37; how many has he in all? Ans. 219.

73. A mechanic owes to one person 2,45 cents, to another 3,25 cents, to a third 1,87 cents, and to a fourth 96 cents; how much does he owe in all? Ans. 8,63 cents.

74. A merchant sent his clerk to collect money. Of one person he received 147 dollars, of a second 59 dollars, and of a third he received 226 dollars; how much did he collect in all? Ans. 432.

75. Jane received, as a new-year's present, from her father 1,37 cents, from her mother 87 cents, from her aunt 56 cents, from her uncle 75 cents, and from her brother 25 cents; how much did she receive in all? Ans. 3,80 cents.

76. A person bought a coat for 27 dollars, a vest for 4 dollars, a hat for 7 dollars, and a pair of boots for 6 dollars; how much did they all cost him? Ans. 44 dollars.

77. A person travels the first day 28 miles, the second day 39 miles, the third day 45 miles, and the fourth day 52 miles; how many miles has he travelled in all? Ans. 164 miles.

78. A young lady bought a Latin grammar for 68 cents, a copy of Virgil for 1,85 cents, and a treatise on algebra for 1,56 cents; how much did they all cost? Ans. 4,09 cents.

79. A lady has three daughters at school; for the first she pays 16,50 cents a quarter, for the second 12,50 cents, and for the third 8,75 cents; how much does she pay in all? Ans. 37,75 cents.

80. A gentleman has three sons at a boarding school, and one at college; for the one at college he pays 168 dollars a year;

for the oldest at the boarding school he pays 156 dollars; for the second 135, and for the youngest he pays 124 dollars; how much does he pay for all of them? Ans. 583 dollars.

81. A person deposited in the Bank of the United States the following sums—345 dollars, 765 dollars, 1142 dollars, and 127 dollars; how much has he in the bank altogether? Ans. 2379.

82. The income of a lady is derived from real estate and public funds; from the rent of a farm she receives 246 dollars, from house rent 574 dollars, and from stock in the funds 341 dollars; how much is her income altogether? Ans. 1161 dollars.

83. The expenses of a gentleman's family for three years were as follow; the first year he spent 542 dollars, the second year 679 dollars, and the third year 746 dollars; how much did he spend altogether? Ans. 1967 dollars.

84. A gentleman built a house, which cost him as follows: The brick-maker's bill was 364 dollars, the mason's bill was 476, the carpenter's bill was 625 dollars; and the painter and glazier's bills were 254 dollars; what did the house cost him? Ans. 1719 dollars.

85. A servant went to market, and bought a turkey for 87 cents, a pair of chickens for 52 cents, vegetables for 44 cents, and beef for 1,49 cents; how much did he spend? Ans. 3,32 cents.

In placing the following numbers for addition, the learner must observe to set units under units, tens under tens, hundreds under hundreds, &c., as above directed.

86. Add together 4621, 3746, 2154, 1405, 3246. Ans. 15172.

87. Add together 74521, 2351, 52432, 20461, 13456. Ans. 163221.

88. Add together 87654, 43210, 14532, 10246, 32145. Ans. 187787.

89. Add together 32465, 31524, 39982, 99210, 21031. Ans. 224212.

90. Add together 54216, 39495, 21746, 21543, 21467. Ans. 158467.

91. Add together 21462, 35261, 42574, 31798, 12345. Ans. 143440.

92. Add together 74521, 14752, 31462, 54671, 34576. Ans. 209982.

93. A miller ground in the month of September 5246 bushels of wheat, in October 6425, in November 3674, and in December 5725; how many bushels did he grind in the whole? Ans. 21070 bushels.

94. A merchant sold in the month of March 7461 barrels of flour, in April 3246 barrels, and in May he sold 10741 barrels; how many did he sell in the whole? Ans. 21448 barrels.

95. The amount of goods sold by a merchant, in the month of October was 4746 dollars, in the month of November 5796 dollars, and in the month of December 7894 dollars; how much did he sell in all? Ans. 18436 dollars.

96. How many days are in the six first months of the year, allowing 28 days for February, January having 31, March 31, April 30, May 31, and June 30? Ans. 181 days.

97. How many days are in the six last months of the year, July having 31 days, August 31, September 30, October 31, November 30, and December 31? Ans. 184.

98. Suppose, in the four principal streets of Philadelphia, there are 827 houses in the first, 685 in the second, 597 in the third, and 546 in the fourth; how many houses are there in all? Ans. 2655 houses.

QUESTIONS. 1. What is addition?—2. How do you place the numbers before you add them together?—3. If the numbers consist of one place only, how do you set them?—4. If there are two places, how do you set them?—5. If the numbers consist of three or more places, how do you set them?

6. What is the first place called?—7. Why is it so called?

8. What is the second place called?—9. Why is it so called?

10. What is the third place called?—11. Why is it so called?

12. What is the fourth place called?—13. Why is it so called?

14. What is the fifth place called?—15. Why is it so called?

16. Do the other places, to the left hand, increase in the same proportion?

17. How do you proceed in adding up the columns?

18. What do you set down under the first column?

19. What do you do with the tens that remain, after setting down the units?

20. What do you do with the second column, or column of tens?

21. Do you set down the right hand figure of the sum under the column, and add the remaining figure or figures into the next column or hundred's place?

22. Do you proceed in this way to the last column?

23. What do you do with it?

24. Will you explain this process by an example?

SUBTRACTION.

SUBTRACTION is the operation of taking a less number from a greater, to find the difference or remainder.

The less number must be placed under the greater, so that units may stand under units, tens under tens, hundreds under hundreds, as in addition.

The upper number is called the minuend or number to be lessened; the less number is called the subtrahend or number to be subtracted; and the difference of the two numbers is called the remainder.

The remainder added to the lower number will equal the upper number: this proves the operation.

SUBTRACTION TABLE.

1 from 1 and 0 remains			2 from 2 and 0 remains			3 from 3 and 0 remains		
1	2	1	2	3	1	3	4	1
1	3	2 remain	2	4	2 remain	3	5	2 remain
1	4	3	2	5	3	3	6	3
1	5	4	2	6	4	3	7	4
1	6	5	2	7	5	3	8	5
1	7	6	2	8	6	3	9	6
1	8	7	2	9	7	3	10	7
1	9	8	2	10	8	3	11	8
1	10	9	2	12	10	3	12	9

4 from 4 and 0 remains			5 from 5 and 0 remains			6 from 6 and 0 remains		
4	5	1	5	6	1	6	7	1
4	6	2 remain	5	7	2 remain	6	8	2 remain
4	7	3	5	8	3	6	9	3
4	8	4	5	9	4	6	10	4
4	9	5	5	10	5	6	11	5
4	10	6	5	11	6	6	12	6
4	11	7	5	12	7	6	13	7
4	12	8	5	13	8	6	14	8
4	13	9	5	14	9	6	15	9

7 from 7 and 0 remains			8 from 8 and 0 remains			9 from 9 and 0 remains		
7	8	1	8	9	1	9	10	1
7	9	2 remain	8	10	2 remain	9	11	2 remain
7	10	3	8	11	3	9	12	3
7	11	4	8	12	4	9	13	4
7	12	5	8	13	5	9	14	5
7	13	6	8	14	6	9	15	6
7	14	7	8	15	7	9	16	7
7	15	8	8	16	8	9	17	8
7	16	9	8	17	9	9	18	9

Questions upon the Table to be answered by the learner.

1. Take 2 from 5, and how many will remain?
2. Take 2 from 7, and how many will remain?
3. Take 3 from 7, and how many will remain?
4. Take 3 from 9, and how many will remain?
5. Take 3 from 11, and how many will remain?
6. Take 4 from 7, and how many will remain?
7. Take 4 from 9, and how many will remain?
8. Take 5 from 9, and how many will remain?
9. Take 5 from 11, and how many will remain?
10. Take 5 from 13, and how many will remain?
11. Take 6 from 9, and how many will remain?
12. Take 6 from 13, and how many will remain?
13. Take 6 from 15, and how many will remain?
14. Take 7 from 11, and how many will remain?
15. Take 7 from 16, and how many will remain?
16. Take 7 from 13, and how many will remain?
17. Take 8 from 11, and how many will remain?
18. Take 8 from 17, and how many will remain?
19. Take 8 from 15, and how many will remain?
20. Take 9 from 15, and how many will remain?
21. Take 9 from 17, and how many will remain?
22. Take 8 from 15, and how many will remain?
23. Take 8 from 13, and how many will remain?
24. Take 7 from 15, and how many will remain?
25. Take 6 from 16, and how many will remain?

After this manner, the teacher may require the pupil to subtract any other numbers, that may be in the table. This will be found a very useful exercise.

PRACTICAL EXERCISES.

1. If you have 5 apples, and give your brother 2; how many will you have remaining? Ans. 3. Why so?

2. If you have 8 cents, and give 3 of them for marbles; how many will you have remaining? Ans. 5. Why so?

3. Jane has 11 nuts, and gives her sister 4; how many has she left? Ans. 7. Why so?

4. James has 16 cents, and buys a book for 12 cents; how many cents has he left? Ans. 4. Why so?

5. Mary had 10 cents, but on her way to school she spent 4 for cakes; how many has she left? Ans. 6. Why so?

6. A person borrowed 15 dollars, and paid 9; how many dollars remain unpaid? Ans. 6. Why so?

7. A little boy on his way to school, having 14 cents in his pocket, gave 6 cents to a poor woman asking charity; how many had he left? Ans. 8. Why so?

8. Susan gave 25 cents for a book, the price of which was only 14 cents; how much change did she receive from the bookseller? Ans. 11. Why so?

9. A merchant bought a lot of goods for 20 dollars, and sold them again for 27 dollars; how much did he gain by the bargain? Ans. 7. Why so?

10. If you have a 10 dollar bill, and you buy books and paper to the amount of 6 dollars; how much will you have left? Ans. 4 dollars. Why so?

11. A person bought a barrel of apples for 2 dollars, and gave the seller a five dollar bill; how much change must he receive? Ans. 3. Why so?

12. A person is travelling to a place 27 miles distant, and after he has travelled 16 miles; how many miles further has he to go? Ans. 11. Why so?

13. James has 25 cents, and buys an inkstand for 15 cents; how much has he left? Ans. 10. Why so?

14. Eliza has 22 cents, and buys a toy for 11 cents; how much has she remaining? Ans. 11. Why so?

15. A farmer sold a cow for 20 dollars, and received in part pay 8 dollars; how much remains unpaid? Ans. 12. Why so?

16. Jane is 12 years old, and her sister is 7; what is the difference of their ages? Ans. 5. Why so?

17. John has been going to school 7 years, and his brother 2 years; how much longer has John been at school than his brother? Ans. 5. Why so?

18. A lady has two daughters at school; for one she pays a quarter 15 dollars, for the other 8; how much more does she pay for one than for the other? Ans. 7. Why so?

19. Subtract 25 from 56, how many remain? —. Why so?
20. Subtract 16 from 37, how many remain? —. Why so?
21. Subtract 24 from 48, how many remain? —. Why so?
22. Subtract 64 from 79, how many remain? —. Why so?
23. Subtract 125 from 146, how many remain? —. Why so?
24. Subtract 84 from 99, how many remain? —. Why so?
25. Subtract 144 from 256, how many remain? —. Why so?
26. Subtract 241 from 472, how many remain? —. Why so?
27. Subtract 365 from 676, how many remain? —. Why so?
28. Subtract 526 from 839, how many remain? —. Why so?
29. Subtract 641 from 794, how many remain? —. Why so?
30. Subtract 402 from 578, how many remain? —. Why so?

31. Subtract 528 from 649, how many remain? — Why so?

32. Subtract 1462 from 2762, how many remain? — Why so?

33. Subtract 569 from 899, how many remain? — Why so?

34. Subtract 4723 from 7924, how many remain? — Why so?

35. Subtract 3465 from 8496, how many remain? — Why so?

36. Subtract 14762 from 65862, how many remain? — Why so?

37. Subtract 8450 from 9765, how many remain? — Why so?

When the figures in the several places of the upper number, or minuend, are greater than the correspondent figures of the lower number or subtrahend, the operation is easy. All you have to do is, to take the lower figure from the upper one, beginning at the right hand figure, or unit's place, and set the difference under its proper column: and do the same with all the figures, and the work is done, as in the preceding examples.

But if the lower figure is greater than the upper figure, you must add 10 to the upper figure, and from the sum take the lower figure, and set down the difference in its proper place; and, because you added 10 to the upper figure, you must add 1 to the next lower figure on the left hand, and take their sum from the upper figure, and set down the difference in its proper place. But, if it should so happen that this sum is greater than the upper figure, you must add 10 to the upper figure as you did before, and from this sum take the subtrahend, and set down the difference in its proper place; and so proceed till the work is done.

The reason for adding 10 to the upper figure, and only 1 to the under one or subtrahend, is, that 1 in the ten's place is equal to 10 in the unit's place; and 1 in the hundred's place is equal to 10 in the ten's place; and 1 in the thousand's place is equal to 10 in the hundred's place; and so on, increasing in a tenfold proportion in every place toward the left hand.

This operation does not affect the difference of the numbers or remainder; for it is evident, that if you add ten to the upper number, and 10 to the lower number, the difference will be the same as if nothing had been added to either.

38. Subtract 1764 from 3456, how many will remain?

Ans. 1692.

Place the numbers under each other thus: 3456 the minuend.
1764 the subtrahend.
1692 the remainder.

Begin with the right hand figure; and because 4 is less than 6, you set down the 2, the difference, in the unit's place under the line. In the ten's place, the lower figure is greater than the upper one, you must therefore add 10 to the upper figure, which will make the sum 15; from this take 6, the lower figure, and set the difference or remainder to the left hand of the 2 already found; and because you added 10 to the upper figure, to preserve the equality of the numbers, you must add 1 to the next place, on the left hand of the lower number, which, with the figure 7, will make 8. This you cannot take from the upper figure because it is less than 8; you must therefore add 10 to it, which will make the sum 14; from this you can take 8, and the remainder will be 6; which set down to the left hand of the 9 which you previously found. Again, because you added 10 to the upper figure in hundred's place, you must add 1 to the next lower figure in thousand's place, and take this sum from the upper figure, and set the difference to the left hand of the 6 already found; and the operation is done. In like manner proceed to find the difference of any other numbers, when any one of the lower figures is greater than the upper one.

39. Subtract 4625 from 8467; how many remain?
Ans. 3842.

40. Subtract 3479 from 5627; how many remain?
Ans. 2148.

41. Subtract 17845 from 34762; how many remain?
Ans. 16917.

42. Subtract 35276 from 70425; how many remain?
Ans. 35149.

43. Subtract 2576 from 9405; how many remain?
Ans. 6829.

44. Subtract 1748 from 3740; how many remain?
Ans. 1992.

45. Subtract 9547 from 10456; how many remain?
Ans. 909.

46. Subtract 8205 from 9128; how many remain? Ans. 923.

47. Subtract 30581 from 45760; how many remain?
Ans. 15179.

48. Subtract 2469 from 5972; how many remain?
Ans. 3503.

49. Subtract 3699 from 7422; how many remain?
Ans. 3723.

50. Subtract 7426 from 9104; how many remain?
Ans. 1678.

51. Subtract 13254 from 25203; how many remain?
Ans. 11949.

52. Subtract 72064 from 112531; how many remain?
Ans. 40467.

53. Subtract 474681 from 742610; how many remain?
Ans. 267929.

54. Subtract 142651 from 572103; how many remain?
Ans. 429452.

55. Subtract 94721 from 123214; how many remain?
Ans. 28493.

56. The distance from Philadelphia to New York is 96 miles, and to Trenton, on the same road, is 38 miles; how far is the distance from Trenton to New York? Ans. 58 miles.

57. The distance from Philadelphia to Pittsburg is 310 miles, and to Lancaster, on the same road, is 66 miles; how far is the distance from Lancaster to Pittsburg? Ans. 244 miles.

58. A lady has an income of 575 dollars a year, and spends out of it 426 dollars; how much does she save a year?
Ans. 149 dollars.

59. A gentleman has an income of 1200 dollars a year, and his family expenses are 846 dollars; how much does he save of his income? Ans. 354 dollars.

60. The income of a school is 3240 dollars, but the teacher pays for house rent and assistance 1416 dollars; how much does he save for himself? Ans. 1824 dollars.

61. A merchant bought a quantity of goods for 4216 dollars, and sold to the amount of 2542 dollars; what amount has he remaining on hand? Ans. 1674 dollars.

62. A grocer bought a quantity of tea for 426 dollars, and sold the same for 512 dollars; how much did he gain by the bargain? Ans. 86 dollars.

63. A gentleman has a tract of land containing 2427 acres, of which he sold 749 acres; how many acres has he remaining?
Ans. 1678 acres.

64. A merchant deposited in the Bank of the United States 1260 dollars, and afterward drew out 746 dollars; how much has he remaining? Ans. 514 dollars.

65. Baltimore contains 74625 inhabitants, and the District of Columbia 31472; how many more inhabitants are in Baltimore than in the District of Columbia? Ans. 43153.

66. A mason built two houses of brick; in one he put 47642, in the other 27875; how many more bricks did he put in the one than in the other? Ans. 19767.

67. There are 365 days in a year, and the month of December

contains 31 days; how many days do all the other months of the year contain? Ans. 334 days.

68. The present year is 1830, and America was discovered by Columbus in the year 1492; how many years have passed since it was discovered? Ans. 338 years.

69. The declaration of American Independence was signed on the 4th day of July, 1776; how many years will have passed by the 4th day of July, 1832? Ans. 56 years.

70. The war of our independence commenced in the year 1775, and a general peace was made in 1783; how many years did it continue? Ans. 8 years.

71. General Washington was born in the year 1732, and died in the year 1799; how old was he at his death? Ans. 67 years.

72. Constantine the Great was converted to Christianity in the year 312 of the Christian era; how many years have passed to the present year 1830? Ans. 1518 years.

73. The victory at New Orleans was gained on the 8th day of January 1815; how many years will have passed on the 8th of January 1831? Ans. 16 years.

74. The first settlers of New England landed at Plymouth in December 1620; how many years will have passed by December 1831? Ans. 211 years.

75. Twelve lunations make 354 days; how many days less are 12 lunations than 12 months? Ans. 11 days.

QUESTIONS. 1. What do you understand by subtraction?

2. How do you place the numbers to be subtracted?

3. What is the upper number called?

4. What is the lower number called?

5. What is the difference of the two numbers called?

6. How do you prove the operation?

7. When you have placed the numbers in proper order for subtraction, how do you proceed, if the lower figure is less than the upper one?

8. When the lower figure is greater than the upper one, what do you do?

9. Why do you add 10 to the upper figure, and only 1 to the lower one?

10. How does the value of the figures increase from the right hand or unit's place?

11. Is 1 added to any figure in a higher place, equal to 10 added to a figure in the next lower place?

12. Is this the principle on which the process is founded?

13. Is the difference of the two numbers any way changed by adding 10 to the upper number in an inferior place, and only 1 in the next superior place of the lower number?

14. Can you prove this position, or make it evident by an example?

MULTIPLICATION.

The operation of finding the sum of any given number, taken as often as there are units in any other given number, is called Multiplication.

The number to be multiplied is called the multiplicand, and is commonly the larger number.

The number by which you multiply, is called the multiplier, and is commonly the less number.

The result of the operation is called the product of the two numbers.

The multiplicand and multiplier are sometimes called factors, because by them the operation is carried on, and the result produced.

MULTIPLICATION TABLE.

twice 1 make 2	3 times 1 make 3	4 times 1 make 4	5 times 1 make 5
2 4	3 2 6	4 2 8	5 2 10
3 6	3 3 9	4 3 12	5 3 15
4 8	3 4 12	4 4 16	5 4 20
5 10	3 5 15	4 5 20	5 5 25
6 12	3 6 18	4 6 24	5 6 30
7 14	3 7 21	4 7 28	5 7 35
8 16	3 8 24	4 8 32	5 8 40
9 18	3 9 27	4 9 36	5 9 45
10 20	3 10 30	4 10 40	5 10 50
11 22	3 11 33	4 11 44	5 11 55
12 24	3 12 36	4 12 48	5 12 60

6 times 1 make 6	7 times 1 make 7	8 times 1 make 8	9 times 1 make 9
6 2 12	7 2 14	8 2 16	9 2 18
6 3 18	7 3 21	8 3 24	9 3 27
6 4 24	7 4 28	8 4 32	9 4 36
6 5 30	7 5 35	8 5 40	9 5 45
6 6 36	7 6 42	8 6 48	9 6 54
6 7 42	7 7 49	8 7 56	9 7 63
6 8 48	7 8 56	8 8 64	9 8 72
6 9 54	7 9 63	8 9 72	9 9 81
6 10 60	7 10 70	8 10 80	9 10 90
6 11 66	7 11 77	8 11 88	9 11 99
6 12 72	7 12 84	8 12 96	9 12 108

10 times	1 make	10	11 times	1 make	11	12 times	1 make	12
10	2	20	11	2	22	12	2	24
10	3	30	11	3	33	12	3	36
10	4	40	11	4	44	12	4	48
10	5	50	11	5	55	12	5	60
10	6	60	11	6	66	12	6	72
10	7	70	11	7	77	12	7	84
10	8	80	11	8	88	12	8	96
10	9	90	11	9	99	12	9	108
10	10	100	11	10	110	12	10	120
10	11	110	11	11	121	12	11	132
10	12	120	11	12	132	12	12	144

Questions upon the Table to be answered by the learner.

1. How many are 3 times 4?
2. How many are 4 times 5?
3. How many are 4 times 7?
4. How many are 5 times 6?
5. How many are 5 times 8?
6. How many are 5 times 9?
7. How many are 6 times 4?
8. How many are 6 times 7?
9. How many are 6 times 9?
10. How many are 7 times 4?
11. How many are 7 times 7?
12. How many are 7 times 9?
13. How many are 8 times 4?
14. How many are 8 times 5?
15. How many are 8 times 6?
16. How many are 8 times 8?
17. How many are 8 times 11?
18. How many are 8 times 12?
19. How many are 9 times 3?
20. How many are 9 times 5?
21. How many are 9 times 8?
22. How many are 9 times 11?
23. How many are 9 times 12?
24. How many are 10 times 5?
25. How many are 10 times 8?
26. How many are 11 times 5?
27. How many are 11 times 8?
28. How many are 11 times 11?
29. How many are 12 times 4?
30. How many are 12 times 6?
31. How many are 12 times 8?
32. How many are 12 times 11?
33. How many are 12 times 9?
34. How many are 12 times 12?

After this manner, the teacher may require the learner to multiply any other numbers in the table. This will be found a very useful exercise.

PRACTICAL EXERCISES.

1. If you buy one orange for 5 cents, what will 3 oranges cost? Ans. 15 cents. Why so?

2. James bought a penknife, and gave 22 cents for it; how much would 4 cost at the same price? Ans. 88 cents. Can you prove that?

3. Jane bought 4 oranges, and gave 6 cents a piece for them; how much did they all cost? Ans. 24 cents. How will you prove that?

4. If one yard of ribbon cost 3 cents, what will 12 yards cost? Ans. 36 cents. How will you prove that?

5. A lady has 4 daughters, and on a Christmas morning, gave each of them 9 cents; how many cents did she give in all? Ans. 36 cents. How will you prove that?

6. A person can earn 4 dollars a week; how many dollars can he earn in 5 weeks? Ans. 20 dollars. Will you prove it?

7. Jane spends 6 cents a day for cakes and nuts; how many cents will she spend, at that rate, in 6 days? Ans. 36 cents. Why so?

8. A cord of wood sells for 5 dollars; how much will 12 cords come to? Ans. 60 dollars. How will you prove it?

9. A pound of butter sells for 21 cents; how much will 8 pounds come to at that rate? Ans. 1,68 cents. Will you prove it?

10. A spelling book sells for 12 cents; how much will 11 come to? Ans. 1,32 cents. Why so?

11. An English grammar sells for 32 cents; how much will 3 come to at that rate? Ans. 96 cents. Why so?

12. A pair of fowls sells for 42 cents; what will 4 pair sell for at that rate? Ans. 1,68 cents. How will you prove it?

13. A barrel of flour may be bought for 5 dollars; how much will 9 barrels come to at that rate? Ans. 45 dollars. Why so?

14. A yard of cloth will cost 4 dollars; what will 9 yards come to at that rate? Ans. 36 dollars. Will you prove that?

15. A person can travel 5 miles in an hour; how many miles will he travel at the same rate in 11 hours? Ans. 55 miles. Why so?

16. There are 24 hours in a day; how many hours are there in two days? Ans. 48 hour. Will you prove it?

17. If your quarter bill is 12 dollars, how much will your quarter bills amount to in a year? Ans. 48 dollars. Will you prove it?

18. If you pay 2 dollars a barrel for apples, how much will 14 barrels amount to? Ans. 28 dollars. Why so?

19. A lady pays 3 dollars a week for her daughter at a boarding school; how much will she have to pay for 12 weeks? Ans. 36 dollars. Will you prove it?

20. There are 3 feet in a yard; how many feet are there in 11 yards? Ans. 33 feet. How will you prove it?

21. There are 12 inches in one foot; how many inches are there in 9 feet? Ans. 108 inches. Why so?

22. There are 4 pecks in one bushel; how many pecks are there in 12 bushels? Ans. 48 pecks. Why so?

23. Ten cents make one dime; how many cents will 8 dimes make?—How many will 12 make?

24. Ten dollars make one eagle; how many dollars will 10 eagles make?—How many will 9 make?

25. There are 8 furlongs in one mile; how many furlongs are there in 9 miles?—How many in 12 miles?

26. There are 4 roods in an acre; how many roods are there in 12 acres?—How many in 10 acres?

27. If you divide an apple into 4 equal parts, how many of the like parts will 8 apples contain?—How many will 6 apples contain?—How many will 12 apples contain?

28. If you multiply 4321 by 3, what will the product be?

29. If you multiply 632 by 2, what will the product be?

30. If you multiply 822 by 4, what will the product be?

31. If you multiply 9120 by 4, what will the product be?

When the product of the multiplier, and any figure of the multiplicand does not exceed 9, as in the above examples, (except the last place, where you always set down the whole product) the operation is easy. All you have to do, is, to set the product down in its proper place. But when it exceeds 9, you must set down the units in the product under the figure multiplied, and add the tens to the product of the next figure and the multiplier: and so proceed till you have multiplied every figure in the multiplicand.

32. Multiply 4536 by 4: what will be the product?

Ans. 18144.

Set the multiplicand thus, 4536
The multiplier thus, 4

18144 the product.

Here the product of 4 and 6 is 24. You set down the 4 in the unit's place, and add the 2, which are two tens, to the product of 4 and the figure 3 in the ten's place, which make 14. Here again you set down the 4, which are four tens or forty, and the 1 which is a hundred, you add to the product of 4 and 5 in the hundred's place, which make 21. You set down the 1, and add the 2 to the product of 4 and 4 in the thousand's place, which make 18. And because it is the last figure, you set down the whole 18. In like manner you must proceed in any other example, whatever may be the number of figures, or places in the multiplicand.

33. What will be the product of 64354, multiplied by 5?

Ans. 321770.

34. What will be the product of 74216, multiplied by 6?

Ans. 445296.

35. What will be the product of 37992, multiplied by 4?
Ans. 151968.

36. What will be the product of 546201, multiplied by 5?
Ans. 2731005.

37. What will be the product of 62174, multiplied by 6?
Ans. 373044.

38. What will be the product of 54327, multiplied by 7?
Ans. 380289.

39. What will be the product of 46284, multiplied by 8?
Ans. 370272.

40. What will be the product of 57405, multiplied by 8?
Ans. 459240.

41. What will be the product of 64217, multiplied by 7?
Ans. 449519.

42. What will be the product of 7846, multiplied by 9?
Ans. 70614.

43. What will be the product of 8476, multiplied by 8?
Ans. 67808.

44. What will be the product of 54764, multiplied by 9?
Ans. 492876.

45. What will be the product of 32146, multiplied by 10?
Ans. 321460.

46. What will be the product of 54721, multiplied by 11?
Ans. 601931.

47. What will be the product of 721401, multiplied by 11?
Ans. 7935411.

48. What will be the product of 47426, multiplied by 12?
Ans. 569112.

49. What will be the product of 532106, multiplied by 12?
Ans. 6385272.

50. What will be the product of 42176, multiplied by 8?
Ans. 337408.

51. What is the product of 74861, multiplied by 9?
Ans. 673749.

52. What will be the product of 72161, multiplied by 11?
Ans. 793771.

53. What will be the product of 8421, multiplied by 12?
Ans. 101052.

54. If one barrel of flour sells for 5,55 cents, what will 6 barrels come to?
Ans. 33,30 cents.

55. If one load of hay costs 8,75 cents, what will 7 loads come to?
Ans. 61,25 cents.

56. If one cord of wood costs 4,86 cents, what will 8 cords cost?
Ans. 38,88 cents.

57. If one yard of broad-cloth costs 7,65 cents, what will 9 yards come to? Ans. 68,85 cents.

58. A lady purchased 54 yards of muslin at 12 cents a yard; what was the cost of the whole? Ans. 6,48 cents.

59. A farmer sold 49 acres of land at 11 dollars an acre; what did the whole amount to? Ans. 539 dollars.

60. A carpenter worked 10 days, at 1,46 cents a day; how much did his wages amount to? Ans. 14,60 cents.

61. What will 764 pounds of pork come to, at 8 cents a pound? Ans. 61,12 cents.

62. A person spends 9 dollars a week; how much will he spend in 46 weeks? Ans. 414 dollars.

63. If you save 12 cents a week, how much will you save in 52 weeks, or one year? Ans. 6,24 cents.

64. James spends for nuts and cakes 8 cents a day; how much will he spend in a year or 365 days? Ans. 29,20 cents.

65. Jane spends for cakes, nuts, and apples, 6 cents a day; how much will she spend in 216 days? Ans. 12,96 cents.

66. Susan pays 87 cents for a pair of shoes, and wears out 7 pair in a year; how much do they all come to? Ans. 6,09 cents.

67. How much will 11 pair of hose come to at 75 cents a pair? Ans. 8,25 cents.

68. If you give 9 cents for one orange; what will 248 come to at that rate? Ans. 22,32 cents.

69. What will 675 pounds of cheese come to at 11 cents a pound? Ans. 74,25 cents.

70. How much will a keg of raisins, containing 246 pounds, come to, at 12 cents a pound? Ans. 29,52 cents.

When the multiplier is more than 12, you must multiply by each figure separately, and add together the several products for the products required. But you must observe to place the product of the first figure of the multiplicand, and any figure of the multiplier under its own proper multiplier; as in the following example.

71. What is the product of 456, multiplied by 24?

Ans. 10944.

456 the multiplicand.
24 the multiplier.

1824 product of the units of the multiplier.
912 product of the tens of the multiplier.

10944 the product required.

When the multiplier consists of two or more places, you must put it under the multiplicand, so that units may stand under units, tens under tens, &c. as in this example. Then multiply by the first figure or units, and place the product as before instructed, or as in this example. Then multiply by the figure in the next or ten's place, and set the product under the former product; but observe to set it one place higher, or under its own multiplier. Proceed in the same manner with each figure of the multiplier; and add together the several products; the sum will be the product required.

72. What will be the product of 475, multiplied by 34?
Ans. 16150.

73. What will be the product of 5462, multiplied by 42?
Ans. 229404.

74. What will be the product of 64215, multiplied by 214?
Ans. 13742010.

75. What will be the product of 7421, multiplied by 36?
Ans. 267156.

76. What will be the product of 5204, multiplied by 37?
Ans. 192548.

77. What will be the product of 874, multiplied by 52?
Ans. 45448.

78. What will be the product of 1476, multiplied by 324?
Ans. 478224.

79. What will be the product of 2456, multiplied by 721?
Ans. 1770776.

80. What will be the product of 5723, multiplied by 314?
Ans. 1797022.

81. What will be the product of 7430, multiplied by 452?
Ans. 3358360.

82. What will be the product of 57921, multiplied by 623?
Ans. 36084783.

83. What will be the product of 8432, multiplied by 635?
Ans. 5354320.

84. What will be the product of 999, multiplied by 777?
Ans. 776223.

85. What will be the product of 9040, multiplied by 832?
Ans. 7521280.

86. What will be the product of 20546, multiplied by 369?
Ans. 7581474.

87. What will 342 yards of cloth come to at 3,64 cents a yard?
Ans. 880,88 cents.

88. What will 376 barrels of flour come to at 5,26 cents a barrel?
Ans. 1977,76 cents.

89. What will 365 acres of land come to at 16,46 cents an acre? Ans. 6007,90 cents.

90. A gentleman bought a farm containing 317 acres of land at 9,75 cents an acre; what did the whole amount to?
Ans. 3090,75 cents.

91. A merchant bought 476 barrels of flour, and gave 5,36 cents a barrel; what did the whole come to?
Ans. 2551,36 cents.

92. A farmer sold 1246 bushels of wheat at 1,35 cents a bushel; what did the whole come to? Ans. 1682,10 cents.

93. What will 128 barrels of pork come to at 12,25 cents a barrel? Ans. 1568,50.

94. There are 60 minutes in an hour; how many minutes are in 24 hours? Ans. 1440.

95. There are 24 hours in one day; how many hours are in 184 days? Ans. 5416 hours.

96. How many hours are in 365 days? Ans. 8760 hours.

97. How many hours are in the month of February, when it has only 28 days? Ans. 672 hours.

98. In an acre of land are 160 perches; how many perches are in 46 acres? Ans. 7360 perches.

99. A hogshead contains 63 gallons; how many gallons are in 25 hogsheads? Ans. 1575 gallons.

100. A bushel contains 32 quarts; how many quarts are in 346 bushels? Ans. 11072 quarts.

Questions. 1. What is multiplication?—2. What is the number to be multiplied called?—3. What is the number, by which you multiply, called?—4. What is the result of the operation called?

5. What are the multiplicand and multiplier, taken together, sometimes called?

6. Why are they so called?—7. How do you place the numbers for the operation?

8. Having placed them properly, how do you proceed?

9. If the product of any figure of the multiplicand and multiplier is less than 10, what do you set down?

10. But if it is more than 9, what do you set down?

11. What do you do with the tens?—12. What do you do with the product of the last figure and the multiplier?

13. When the multiplier consists of two or more places, how do you place the figures for the operation?—14. How do you proceed?

15. How do you place the product of the multiplicand and

the second figure of the multiplier, or the one in the ten's place?

16. Do you proceed in the same manner with all the figures of the multiplier?

17. What is the reason for placing the product of the multiplicand and any figure of the multiplier after the units, one place higher than the preceding one?

18. What do you do with the several products?

DIVISION.

Division is the operation of finding how often one number is contained in another. It is the reverse of multiplication.

The number to be divided is called the dividend.

The number by which you divide is called the divisor.

The number of times the divisor is contained in the dividend is called the quotient.

The remainder, if there is any, must be less than the divisor, and always of the same denomination with the dividend.

When the divisor is less than 12, it is called Short Division: when more than 12, it is called Long Division.

DIVISION TABLE.

Note.—The first column contains the dividend; the second the divisor; and the third the quotient, or the number of times the divisor is contained in the dividend.

4	divided by	2 = 2	6	divided by	3 = 2	8	divided by	4 = 2
6	"	2 = 3	9	"	3 = 3	12	"	4 = 3
8	"	2 = 4	12	"	3 = 4	16	"	4 = 4
10	"	2 = 5	15	"	3 = 5	20	"	4 = 5
12	"	2 = 6	18	"	3 = 6	24	"	4 = 6
14	"	2 = 7	21	"	3 = 7	28	"	4 = 7
16	"	2 = 8	24	"	3 = 8	32	"	4 = 8
18	"	2 = 9	27	"	3 = 9	36	"	4 = 9
20	"	2=10	30	"	3=10	40	"	4=10
22	"	2=11	33	"	3=11	44	"	4=11
24	"	2=12	36	"	3=12	48	"	4=12

10	divided by	5 = 2	12	divided by	6 = 2	14	divided by	7 = 2
15	"	5 = 3	18	"	6 = 3	21	"	7 = 3
20	"	5 = 4	24	"	6 = 4	28	"	7 = 4
25	"	5 = 5	30	"	6 = 5	35	"	7 = 5
30	"	5 = 6	36	"	6 = 6	42	"	7 = 6
35	"	5 = 7	42	"	6 = 7	49	"	7 = 7
40	"	5 = 8	48	"	6 = 8	56	"	7 = 8
45	"	5 = 9	54	"	6 = 9	63	"	7 = 9
50	"	5=10	60	"	6=10	70	"	7=10
55	"	5=11	66	"	6=11	77	"	7=11
60	"	5=12	72	"	6=12	84	"	7=12

16	divided by	8 = 2	18	divided by	9=2	20	divided by	10=2
24	"	8 = 3	27	"	9=3	30	"	10=3
32	"	8 = 4	36	"	9=4	40	"	10=4
40	"	8 = 5	45	"	9=5	50	"	10=5
48	"	8 = 6	54	"	9=6	60	"	10=6
56	"	8 = 7	63	"	9=7	70	"	10=7
64	"	8 = 8	72	"	9=8	80	"	10=8
72	"	8 = 9	81	"	9=9	90	"	10=9
80	"	8=10	90	"	9=10	100	"	10=10
88	"	8=11	99	"	9=11	110	"	10=11
96	"	8=12	108	"	9=12	120	"	10=12

22	divided by	11 = 2	24	divided by	12 = 2			
33	"	11 = 3	36	"	12 = 3			
44	"	11 = 4	48	"	12 = 4			
55	"	11 = 5	60	"	12 = 5			
66	"	11 = 6	72	"	12 = 6			
77	"	11 = 7	84	"	12 = 7			
88	"	11 = 8	96	"	12 = 8			
99	"	11 = 9	108	"	12 = 9			
110	"	11 = 10	120	"	12 = 10			
121	"	11 = 11	132	"	12 = 11			
132	"	11 = 12	144	"	12 = 12			

The divisor and quotient multiplied together will always equal the dividend, if there is no remainder; and if there is a remainder, add it to the product, and the sum will equal the dividend.

Parallel lines (thus =) express equality, and imply that the numbers between which they placed are equal. In the table, for example, 4 divided by 2 = 2. They imply that the quotient of 4 divided by 2 is equal to 2, that is, it is 2, and so in all other cases.

Questions upon the Table to be answered by the learner.

1. If you divide 6 by 3, what is the quotient? —. Why so?
2. If you divide 12 by 4, what is the quotient? —. Why so?
3. If you divide 15 by 5, what is the quotient? —. Why so?
4. If you divide 24 by 6, what is the quotient? —. Why so?
5. If you divide 35 by 7, what is the quotient? —. Why so?
6. If you divide 48 by 6, what is the quotient? —. Why so?
7. If you divide 56 by 8, what is the quotient? —. Why so?
8. If you divide 63 by 9, what is the quotient? —. Why so?
9. If you divide 72 by 8, what is the quotient? —. Why so?
10. If you divide 81 by 9, what is the quotient? —. Why so?
11. If you divide 96 by 8, what will the quotient be? —. Why so?
12. If you divide 99 by 11, what will the quotient be? —. Why so?
13. If you divide 108 by 9, what will the quotient be? —. Why so?
14. If you divide 121 by 11, what will the quotient be? —. Why so?
15. If you divide 132 by 11, what will the quotient be? —. Why so?
16. If you divide 144 by 12, what will the quotient be? —. Why so?
17. If you divide 77 by 7, what will the quotient be? —. Why so?
18. If you divide 84 by 7, what will the quotient be? —. Why so?
19. If you divide 55 by 11, what will the quotient be? —. Why so?
20. If you divide 36 by 12, what will the quotient be? —. Why so?
21. If you divide 48 by 12, what will the quotient be? —. Why so?
22. If you divide 66 by 11, what will the quotient be? —. Why so?
23. If you divide 72 by 12, what will the quotient be? —. Why so?
24. If you divide 96 by 12, what will the quotient be? —. Why so?

In like manner, the teacher may require the pupil to divide any other numbers in the table, by the given divisors. It is important that the scholar should perfectly understand this, and

the foregoing tables, and be able to repeat them all without hesitation; otherwise his future progress will be slow.

PRACTICAL EXERCISES.

1. If you have 25 cents and divide them into 5 parts, how many will each part contain? Ans. 5. Why so?

2 If 6 yards of cloth cost 30 dollars, what will one yard cost? Ans. 5 dollars. Why so?

3. If 8 barrels of flour cost 40 dollars, what will one barrel cost? Ans. 5 dollars. Why so?

4. If 9 pounds of butter cost 2,70 cents, what will one pound cost? Ans. 30 cents. Why so?

5. If 12 yards of muslin cost 1,44 cents, what will one yard cost? Ans. 12 cents. Why so?

6. If 8 barrels of apples cost 24 dollars, what will one barrel cost? Ans. 3 dollars. Why so?

7. If you buy a spelling book for 11 cents, how many can you buy for 132 cents? Ans. 12. Why so?

8. If you buy one cord of wood for 6 dollars, how many can you buy for 54 dollars? Ans. 9. Why so?

9. If you buy one ton of coal for 7 dollars, how many tons can you buy for 84 dollars? Ans. 12 tons. Why so?

10. A farmer sold 11 fat cattle for 121 dollars; how much was the price of one? Ans. 11 dollars. Why so?

11. A merchant sold 7 yards of broad-cloth for 49 dollars; how much was the price of one yard? Ans. 7 dollars. Why so?

12. If you buy a quire of paper for 12 cents, how many quires can you buy for 108 cents? Ans. 9. Why so?

13. If you buy a slate for 9 cents, how many can you buy for 99 cents? Ans. 11. Why so?

14. If you buy an inkstand for 8 cents, how many can you buy for 72 cents? Ans. 9. Why so?

15. If you buy 9 oranges for 63 cents, how much is the price of one? Ans. 7 cents. Why so?

16. A person bought 8 pounds of sugar for 80 cents; how much was it a pound? Ans. 10 cents. Why so?

17. The stage fare is commonly 6 cents a mile; how many miles may a person travel for 66 cents? Ans. 11 miles. Why so?

18. A gentleman and his wife pay for boarding, 7 dollars a week; how many weeks can they board for 56 dollars? Ans. 8 weeks. Why so?

19. A horse travels at the rate of 6 miles an hour; in how many hours will he travel 42 miles? Ans. 7 hours. Why so?

20. In 5 days a man travels 100 miles; how many miles does he travel in one day? Ans. 20 miles. Why so?

When the dividend consists of two or more places of figures, and each figure contains the divisor one or more times without a remainder, you must find the quotient (or the number of times the dividend contains the divisor) and set it under the figure divided, beginning at the left hand; as in the following examples.

21. If you divide 246 by 2, what will the quotient be?

Ans. 123.

divisor 2)246 the dividend.
123 the quotient.

23. If you divide 3696 by 3, what will the quotient be?

Ans. 1232.

divisor 3)3696 the dividend.
1232 the quotient.

24. If you divide 4804 by 4, what will the quotient be?

Ans. 1201.

25. If you divide 39630 by 3, what will the quotient be?

Ans. 13210.

26. A merchant bought a quantity of pork for 484 dollars, and paid 4 dollars a hundred pounds; how many hundreds were there? Ans. 121.

27. A farmer bought a flock of sheep, for which he paid 246 dollars; how many were there in the whole, provided he paid 2 dollars a piece? Ans. 123.

But if there is a remainder after division, except in the last figure, or unit's place, such remainder is to be considered as so many tens, (because 1 in a higher place is equal to ten in the next lower place) and added to the next figure for a dividend: the sum you must divide by the divisor, and place the quotient, as above directed, and so proceed till the operation is finished.

28. What will be the quotient of 2568, divided by 2?

Ans. 1284.

divisor 2)2568 dividend.
1284 quotient.

Here the first figure contains the divisor once without a remainder; but 5 divided by 2 leaves 1, which is one ten (or ten times as much, as if it stood in the next lower place). To this add the 6 in the next lower place; and the sum 16 divide by 2, and place the quotient under the 6. There being no remainder, divide the 8 by 2, and the operation is finished.

29. What will be the quotient of 1455, divided by 3?

Ans. 485.

divisor 3)1455 dividend.
485 quotient.

Here the first figure in the dividend will not contain the divisor; you must consider it 10, and add it to the 4 in the next place, making 14, which contains the divisor 4 times, leaving 2 a remainder. This 2 is two tens or 20, and, added to the next lower figure, becomes 25. This contains the divisor 8 times, and 1 remains; which is 1 ten, and added to the 5 in the last place, makes 15. This sum contains the divisor 5 times, and no remainder. In this manner proceed in the following examples. When there is a remainder after dividing the last figure, set it to the right of the quotient with this mark or character + before it, which signifies that so many are over or remain.

30. What will be the quotient of 4568, divided by 3?

Ans. 1522+2.

31. What will be the quotient of 54631, divided by 4?

Ans. 13657+3.

32. What will be the quotient of 62074, divided by 4?

Ans. 15518+2.

33. What will be the quotient of 5374, divided by 5?

Ans. 1074+4.

34. What will be the quotient of 7856, divided by 6?

Ans. 1309+2.

35. What will be the quotient of 85362, divided by 7?

Ans. 12194+4.

36. What will be the quotient of 94621, divided by 7?

Ans. 13517+2.

37. What will be the quotient of 85643, divided by 8?

Ans. 10705+3.

38. What will be the quotient of 76543, divided by 9?

Ans. 8504+7.

39. What will be the quotient of 68954, divided by 10?

Ans. 6895+4.

40. What will be the quotient of 54985, divided by 11?
Ans. 4998+7.

41. What will be the quotient of 47652, divided by 12?
Ans. 3971.

42. What will be the quotient of 8261, divided by 7?
Ans. 1180+1.

43. What will be the quotient of 87546, divided by 8?
Ans. 10943+2.

44. What will be the quotient of 15431, divided by 9?
Ans. 1714+5.

45. What will be the quotient of 21452, divided by 11?
Ans. 1950+2.

46. What will be the quotient of 65241, divided by 12?
Ans. 5436+9.

47. Divide 67458 dollars among 8 persons, how much will each one have? Ans. 8432+2.

48. Divide 7642 acres of land among 9 persons, how many acres will each one have? Ans. 849+1.

49. Divide 1496 bushels of wheat into 7 parts, how many bushels will there be in each part? Ans. 213+5.

50. Divide 1243 cords of wood among 6 persons, how much will each one have? Ans. 207+1.

51. Divide 2236 barrels of flour among 11 persons, how many barrels will each one have? Ans. 203+3.

52. Divide 16482 cents among 12 persons, how many will each one have? Ans. 1373+6.

QUESTIONS. 1. What do you understand by Division?—2. Of what is it the reverse?

3. What is the number to be divided called?—4. What is the number divided by, called?—5. What is the number of times the divisor is contained in the dividend, called?

6. When any number is over after division, what is it called?

7. Must it always be less than the divisor?—8. Must it always be of the same denomination with the dividend?

9. When the divisor is less than 12, what is it called?—10. When the divisor is more than 12, what is it called?

11. How do you prove the operation?—12. If there is a remainder, what do you do with it?—13. What do parallel lines signify?—Will you give an example to elucidate this?

15. When each figure of the dividend contains the divisor without a remainder, how do you proceed?—16. Will you give an example?

17. When, after dividing any figure, there shall be a remainder, what do you do with it?—18. Will you give an example?

19. Why do you call each one of the remainder 10?—20. What do you do then?—21. Where do you place the quotient figure?

22. If there is a remainder after dividing the last figure of the dividend, how do you place it?—23. Will you give an example to explain your meaning?

When the divisor is more than 12, you must find by inspection or trial, how often it will be contained in as many figures of the dividend, as there are in the divisor; and put that to the right of the dividend, but separate from it, as the first quotient figure. If, however, the first figure on the left hand in the dividend is less than the first figure in the divisor, you must take one place more from the dividend, and find how often the divisor will be contained in that number, and place it as before. Then multiply the divisor by the quotient figure, and set the product under that part of the dividend, which you had designated. Subtract this from the dividend, and to the right hand of the remainder, add, or bring down, the next figure in the dividend. This then becomes the dividend for the next quotient figure, which must be placed to the right of the quotient figures already found; and so proceed till the operation is finished. But if, after you have brought down the figure from the dividend, it will not contain the divisor, you must put a cypher in the quotient, and bring down one or more additional figures, till it will contain the divisor: As in the following examples.

1. Divide 4524 by 21, what will the quotient be?

Ans. 215+9.

```
divisor 21)4524(215 quotient.
          42··
          ——
           32
           21
           ——
           114
           105
           ———
             9 remainder.
```

```
215+9 rem.
 21   divisor.
 ———
  224
 430
 ———
4524  proof.
```

2. Divide 157462 by 34, what will the quotient be?

Ans. 4631+8.

```
divisor 34)157462(4631 quotient.
          136···
          ---
           214          quotient 4631+8 rem.
           204                34      divisor.
           ---              -------
            106             18532
            102             13893
            ---             -------
              42  dividend 157462 proof.
              34
              --
               8 remainder.
```

Here the first two figures of the dividend will not contain the divisor; you must therefore take three figures, that is, 157, as a dividend for the first quotient figure, which is 4. This place in the quotient, and multiply the divisor by it, placing the product under 157, the portion of the dividend set apart: Substract the product from the dividend, and the remainder is found to be 21. To this, bring down the next figure of the dividend, and place it to the right hand of the remainder, making 214. This now becomes a dividend for the second quotient figure; find by inspection, or trial, how often the divisor is contained in this number, and place that in the quotient to the right hand of the quotient figure already found: and so proceed till the operation is finished.

3. Divide 64251 by 42, what will the quotient be?
Ans. 1529+33.
4. Divide 10462 by 57, what will the quotient be?
Ans. 183+31.
5. Divide 7231 by 62, what will the quotient be?
Ans. 116+39.
6. Divide 84031 by 74, what will the quotient be?
Ans. 1135+41.
7. Divide 92054 by 84, what will the quotient be?
Ans. 1095+74.
8. Divide 104701 by 92, what will the quotient be?
Ans. 1138+5.
9. Divide 12474 by 99, what will the quotient be? Ans. 126.
10. Divide 46215 by 124, what will the quotient be?
Ans. 372+87.
11. Divide 34762 by 216, what will the quotient be?
Ans. 160+202.

12. Divide 24374 by 136, what will the quotient be?
Ans. 179+30.

13. Divide 45261 by 144, what will the quotient be?
Ans. 314+45.

14. Divide 52135 by 223, what will the quotient be?
Ans. 233+176.

15. Divide 3467 by 57, what will the quotient be?
Ans. 60+47.

16. If you divide 4746 dollars among 26 men, how much will each one have? Ans. $182,38,4+

17. Forty-eight men engage to perform a piece of work for the sum of 6472 dollars; how much will each man have?
Ans. 134,83+ cents.

18. The President of the United States has a salary of 25,000 dollars a year; how much has he a day?
Ans. 68,49+ cents.

19. If a person has a salary of 7500 dollars a year, how much has he a day? Ans. 20,54+ cents.

20. If a person spends 1240 dollars a year, how much does he spend one day with another? Ans. $3,39,7+

21. A company of 48 persons built a bridge which cost 9765 dollars; how much did each one pay? Ans. 203,41+ cents.

22. A farmer planted 4320 trees in 48 rows; how many trees were there in each row? Ans. 90 trees.

23. If you divide 478,97 cents among 56 men, how many will be the share of each? Ans. 855,3+ mills.

24. There is a tract of land containing 42475 acres; how many sections of 640 acres each does it contain, and what number of acres remain? Ans. 66 sections, 235 acres remain.

25. The sum of 18756 dollars is to be paid by a company of men, in 24 equal payments; how much is each payment?
Ans. 781,50 cents.

26. A lady has an annual income of 1210,50 cents; how much is that in a day? Ans. $3,31,7,8+

27. A farmer sold 36 barrels of pork, and received for the same 456 dollars; what was it a barrel? Ans. 12,66⅔ cents.

28. Seventy-five persons associated for the purpose of establishing a bank, and they contributed altogether the sum of 142675 dollars; how much was each one's share, supposing they all contributed equally? Ans. $1902,33⅓.

29. A company of 32 persons purchased several tickets in a lottery, and they drew altogether the sum of 25462 dollars, after all deductions were made; how much was the share of each one?
Ans. $795,68,7+

30. A merchant purchased 244 yards of cloth for 874,50 cents; how much was the price of a yard? Ans. $3,58,4.

31. A person travelled in 44 days 1045 miles; how many miles did he travel a day on an average? Ans. 53 m. 5 fu.

QUESTIONS. 1. What is the difference between long and short division?

2. Where do you place the quotient in long division?—3. Where do you place the divisor?

4. How do you proceed to find the first quotient figure?

5. What do you do after that?—6. Where do you place the product of the divisor and quotient figure?

7. What do you add to the remainder?—8. How do you proceed after you have brought down the next figure of the dividend?

9. But, if it will not then contain the divisor, what do you do?

10. After you have put a cypher in the quotient to the right hand of the figure last found, what do you do?

11. Do you proceed in this way till the operation is finished?

12. Will you illustrate this by an example?

13. How do you prove the work?—14. If there is a remainder after the division, what do you do with it?

FEDERAL CURRENCY.

This currency was established by the congress of the United States, in the year 1786; and therefore it is called Federal Currency. It increases in a tenfold proportion, from a mill, the lowest denomination, to an eagle, which is the highest. It is more simple than any currency in the world.

TABLE.

10 mills make 1 cent.	1000 mills=10 dimes=1 dollar.
10 cents make 1 dime.	100 mills= 1 dime $=\frac{1}{10}$ doll.
10 dimes make 1 dollar.	7.5 mills=$\frac{3}{4}$ a cent.
10 dollars make 1 eagle.	5 mills=$\frac{1}{2}$ a cent.
	2.5 mills=$\frac{1}{4}$ a cent.

Questions on the Table.

1. How many mills are in 5 cents?—2. How many in 8 cents?—3. How many in 9 cents?—4. How many in 6 cents?

5. How many cents in 4 dimes?—6. How many in 8 dimes?—7. How many in 9 dimes?—8. How many in 7 dimes?

9. How many dimes in 3 dollars?—10. How many in 6 dollars?—11. How many in 8 dollars?

12. How many dollars in 4 eagles?—13. How many in 6 eagles?—14. How many in 9 eagles?—15. How many in 10 eagles?

16. How many mills in 5 dollars?—17. How many in 8 dollars?—18. How many in 9 dollars?

19. How many mills in 7 dimes?—20. How many in 4 dimes?

21. How many in 8 dimes?—22. How many in 9 dimes?

23. How many mills make 1 dollar?—24. How many make 1 dime?—25. How many make $\frac{1}{2}$ a cent.

Any sum of money in eagles, dollars, dimes, cents and mills, may be written as so many mills, thus: 7 eagles, 4 dollars, 5 dimes, 6 cents and 4 mills, may be written, 74564 mills, making seventy-four thousand, five hundred and sixty-four mills. The 4 in unit's place, represents 4 mills, the 6 in ten's place, represents 60 mills or 6 cents; the 5 in hundred's place, represents 500 mills or 5 dimes; the 4 in thousand's place, represents 4000 mills or 4 dollars; and the 7 in ten-thousand's place, represents 70,000 mills or 70 dollars: the whole therefore is 74 dollars, 56 cents and 4 mills. But the usual way of setting down any sum of money in dollars, cents, and mills, is, to separate the first place by a comma, or decimal point for mills: the two next for cents, and the remainder will be dollars, whatever may be the number of places, thus: 74, 56, 4, which are 74 dollars, 56 cents, and 4 mills, as before.

1. How many dollars, cents and mills are in 36452 mills?
2. How many dollars, cents and mills are in 732651 mills?
3. How many dollars, cents and mills are in 125046 mills?
4. How many dollars, cents and mills are in 847526 mills?
5. How many dollars, cents and mills are in 3452601 mills?
6. How many dollars, cents and mills are in 140754 mills?

PRACTICAL EXERCISES.

1. If you buy a book for one dollar, how many dimes must you give for it? Ans. 10. Why so?

2. If you buy a hat for 4 dollars, how many dimes or ten cent pieces must you give for it? Ans. 40. Why so?

3. If you buy an arithmetic for 50 cents, how many dimes or ten cent pieces must you give for it? Ans. 5. Why so?

4. If you purchase a grammar for 30 cents, how many mills will there be? Ans. 300. Why so?

5. A bushel of apples costs 25 cents; how many mills will there be? Ans. 250. Why so?

6. If you give 12,5 mills for a slate, how many mills will 5 come to? Ans. 62,5 mills. Why so?

7. If a cord of wood costs 4,25,5 mills, what will 5 cords cost? Ans. 21,27,5 mills. Why so?

Dollars, cents, and mills are multiplied exactly as whole numbers are; only observe, in the product, to place the proper decimal points. They are divided also as whole numbers are, only observe to place the proper decimal points, in order to divide the quotient into dollars, cents, and mills; or into cents and mills, as the case may be.

They are added as whole numbers are, only observe to place the proper decimal points, and to place dollars under dollars, cents under cents, and mills under mills.

8. If your quarter bill is 6,25 cents, how much will your bills amount to in four quarters? Ans. 25,00 cents.

9. If you multiply 62,5 mills by 7 mills, what will the product be? Ans. 4,37,5 mills, or 4 dols. 37½ cts.

10. If you purchase 24 peaches at 5 mills each, what will they amount to? Ans. 12,0 mills.

11. If you buy 246 quills at 5 mills apiece, what will the whole come to? Ans. 1,23,0 mills.

12. If one yard of muslin costs 12,5 mills, what will 9 yards come to? Ans. 1,12,5 mills.

13. If one yard of broad-cloth costs 3,37,5 mills, what will 5 yards come to? Ans. 16,87,5 mills.

14. If 6 persons give in charity 1,37,5 mills each, how much do they all give? Ans. 8,25 cents.

15. There are 57,46,4 mills to be divided among 8 persons; how much will each one have? Ans. 7,18,3 mills.

16. A sum of money amounting to 954,62,5 mills is to be divided among 5 persons; how much will each one have?
Ans. 190,92,5 mills.

17. A lady bought 8 yards of silk for 10,37,6; how much was the silk a yard? Ans. 1,29,7 mills.

18. A gentleman bought 12 yards of cloth for 27,62,4 mills; how much was the cloth a yard?
Ans. 2,30,2 mills; or, 2 dols. 30 cts. 2 ms.

19. If a little boy spends for apples and cakes 6,2,5 tenths of a mill a day; how much will he spend in 12 days?
Ans. 75,0 mills.

20. If you save 12,5 mills a day, how much will you save in 30 days, or one month? Ans. 3,75 cents.

21. If you save 37,5 mills a week, how much will you save in 52 weeks, or one year? Ans. 19,50 cents.

22. If a person spends 18,7,5 tenths of a mill a day, how much will he spend in a year, or 365 days?

Ans. 68,43,7,5 tenths of a mill.

23. Add together the following sums, 5,46,7 mills, 8,65,5 mills, and 9,37,5 mills; what is the sum? Ans. 23,49,7 mills.

24. Add together 14,25,9 mills, 42,75,6 mills, and 75,31,7 mills; what is the sum? Ans. 132,33,2 mills.

25. Add together 216,12,5 mills, 312,18,7 mills, 165,43,8 mills, and 374,87,5 mills; what will be the sum?

Ans. 1068,62,5 mills.

26. Add together 374,90,4 mills, 104,61,8 mills, 225,05,7 mills, 524,17,6 mills, and 720,21,8 mills; what is the sum?

Ans. 1948,97,3 mills.

27. How much will 49 pounds of butter come to, at 16,5 mills a pound? Ans. 8,08,5 mills.

28. How much will 26 pounds of beef come to, at 6,5 mills a pound? Ans. 1,69 cents.

29. How much will 12 pair of shoes come to, at 1,37,5 mills a pair? Ans. 16,50 cents.

30. If 5 tons of coal come to 36,75,5 mills, how much is it a ton? Ans. 7,35,1 mills.

31. If 8 cords of wood cost 43,87 cents, how much is it a cord? Ans. 5,48,3+ mills.

32. How much will 16 reams of paper come to, at 2,37,5 mills a ream? Ans. 38,00 cents.

Questions. 1. Why is this currency called Federal Currency?—2. Is it more simple than any other?—3. Upon what principle does it proceed?

4. What are its denominations?—5. May any sum in dollars, cents, and mills be set down as so many mills?

6. Will you give an example?—7. How do you divide or distinguish any given sum in mills, into dollars, cents, and mills?—8. Will you give an example?

9. How are dollars, cents, and mills multiplied?—10. Will you give an example?

11. How are they divided?—12. Will you give an example?

13. How are they added together?—14. Will you give an example?—15. What must you observe in all these cases?

Before Congress established the Federal currency, the currency of the United States was *pound*, *shilling*, *penny*, and *farthing;* and in some of the States accounts are kept, and money reckoned, in that currency, at the present time.

TABLE.

4 farthings make 1 penny, *d.*
12 pence make 1 shilling, *s.*
20 shillings make 1 pound, £.
960 *qrs.*=240 *d.*=20 *s.*=1£.

The parts of a penny are thus represented:
$\frac{1}{4}$ of a penny=1 farthing, marked *qr.*
$\frac{1}{2}$ of a penny=2 farthings.
$\frac{3}{4}$ of a penny=3 farthings.

Questions on the Table.

1. How many farthings are in 3 pence? —. Why so?
2. How many farthings are in 5 pence? —. Why so?
3. How many farthings are in 9 pence? —. Why so?
4. How many farthings are in 11 pence? —. Why so?
5. How many pence are in 2 shillings? —. Why so?
6. How many pence are in 4 shillings? —. Why so?
7. How many pence are in 7 shillings? —. Why so?
8. How many pence are in 9 shillings? —. Why so?
9. How many shillings are in 2 pounds? —. Why so?
10. How many are in 5 pounds? —. Why so?—11. How many are in 6 pounds? —. Why so?—12. How many are in 9 pounds? —. Why so?
13. How many pence are in 10 farthings, and how many farthings over? —. Why so?
14. How many pence are in 13 farthings, and how many over? —. Why so?
15. How many pence are in 15 farthings, and how many over? —. Why so?
16. How many pence are in 19 farthings, and how many over? —. Why so?
17. How many pence are in 20 farthings, and how many over? —. Why so?
18. How many shillings are in 19 pence, and how many pence over? —. Why so?
19. How many shillings in 27 pence, and how many pence over? —. Why so?
20. How many shillings in 30 pence, and how many pence over? —. Why so?

21. How many shillings in 37 pence, and how many pence over? —. Why so?
22. How many shillings in 45 pence, and how many pence over? —. Why so?
23. How many shillings in 55 pence, and how many pence over? —. Why so?
24. How many pounds are in 24 shillings, and how many shillings over? —. Why so?
25. How many pounds in 30 shillings, and how many shillings over? —. Why so?
26. How many pounds in 45 shillings, and how many shillings over? —. Why so?
27. How many pounds in 50 shillings, and how many shillings over? —. Why so?
28. If you divide a penny into four parts, what will 1 part be called?—How will you represent it?
29. If you divide it into two parts, what will 1 part be called? —How will you represent it?
30. If you divide it into four parts, what will 3 parts be called?—How will you represent them?

The teacher may ask the pupil any other question of the like nature, that he may deem expedient.

PRACTICAL EXERCISES.

1. In 12 pounds, how many shillings? Ans. 240 shillings.
2. In 124 pounds, how many shillings? Ans. 2480 shillings.
3. In 54 shillings, how many pence? Ans. 648 pence.
4. In 124 shillings, how many pence? Ans. 1488 pence.
5. In 75 shillings, how many pence? Ans. 900 pence.
6. In 21 pence, how many farthings? Ans. 84 farthings.
7. In 52 pence, how many farthings? Ans. 208 farthings.
8. In 17 pence, how many farthings? Ans. 68 farthings.
9. In 144 shillings, how many pounds? Ans. £7, and 4*s.* over.
10. In 75 shillings, how many pounds? Ans. £3, and 15*s.* over.
11. In 150 shillings, how many pounds? Ans. £7, and 10*s.* over.
12. In 66 pence, how many shillings? Ans. 5*s.*, and 6*d.* over.
13. In 116 pence, how many shillings? Ans. 9*s.*, and 8*d.* over.
14. In 136 pence, how many shillings? Ans. 11*s.*, and 4*d.* over.
15. In 90 pence, how many shillings? Ans. 7*s.*, and 6*d.* over.

16. In 17 farthings, how many pence?
Ans. 4*d.*, and 1 *qr.* over.

17. In 27 farthings, how many pence?
Ans. 6*d.*, and 3 *qrs.* over.

18. In 12 farthings, how many pence? Ans. 3*d.*

If you would bring pounds to shillings, you must multiply the given pounds by 20, because 20 shillings make a pound. If you would bring shillings to pence, you must multiply by 12; because 12 pence make a shilling; and if you would bring pence to farthings, you multiply by 4; because 4 farthings make a penny.

If you would bring pounds to pence, you must multiply first by 20, and that product by 12. If you would bring shillings to farthings, multiply the given shillings by 12, and that product by 4.

On the other hand, if you would bring shillings to pounds, you must divide the given shillings by 20; because 20 shillings make a pound.

If you would bring pence to shillings, you must divide by 12; because 12 pence make a shilling.

If you would bring farthings to pence, you must divide by 4; because 4 farthings make a penny.

Questions. 1. What currency was used in the United States before Congress established the Federal currency?

2. Is this currency sometimes used at the present, in some of the states?

3. Can you repeat the table?—4. How do you represent one-fourth of a penny?—5. How one-half of a penny?—6. How three-fourths of a penny?

7. How do you bring pounds to shillings?—8. How do you bring shillings to pence?—9. How do you bring pence to farthings?—What is the reason for your doing so?

10. How do you bring pounds to pence, and farthings?

11. How do you bring shillings to farthings?—What reason can you give for the operation?

12. How do you bring shillings to pounds?—13. How do you bring pence to shillings?—14. How do you bring farthings to pence?—What is the reason for doing so?

TROY WEIGHT.

The denominations are *pound, ounce, pennyweight*, and *grain*. By this weight jewels, and gold and silver are weighed.

TABLE.

24 grains, (*gr.*) . make 1 pennyweight, *dwt.*
20 pennyweights, make 1 ounce, *oz.*
12 ounces, - make 1 pound, *lb.*
5760 *grs.* = 240 *dwts.* = 12 *oz.* = 1 *lb.*

Questions on the Table.

1. How many ounces in 2 pounds? —. 2. How many in 4 pounds? —. 3. How many in 7 pounds? —. 4. How many in 10 pounds? Why so?

5. How many ounces in 1 and a half pound? —. Why so?

6. How many ounces in 3 and a half pounds? —. Why so?

7. How many ounces in 7 pounds and a quarter? —. Why so?

8. How many pennyweights in 3 ounces? —. Why so?

9. How many pennyweights in 4 and a half ounces? —. Why so?

10. How many pennyweights in 5 ounces? —. Why so?

11. How many pennyweights in 3 and a half ounces? —. Why so?

12. How many grains in 2 pennyweights? —. Why so?

13. How many grains in 3 and a half pennyweights? —. Why so?

14. How many grains in 4 and a half pennyweights? —. Why so?

PRACTICAL EXERCISES.

1. In 11 pounds of silver plate, how many ounces? Ans. 132.
2. In 5 pounds of gold bullion, how many ounces? Ans. 60.
3. In 6 ounces of silver, how many pennyweights? Ans. 120.
4. In 9 ounces of silver plate, how many pennyweights? Ans. 180.
5. In 8 pennyweights of silver, how many grains? Ans. 192.
6. In 12 pennyweights of gold, how many grains? Ans. 288.
7. The standard weight of a Johannes is 18 pennyweights; how much is its weight in grains? Ans. 432.
8. If an Eagle weighs 17 *dwts.* and 4 *grains*, what is its weight in grains? Ans. 412.

Here you multiply the pennyweights by 24, because 24 grains make a pennyweight, and to the product add the 4 grains, and the sum will be the number required: and so in all cases, in which there are two or more denominations.

9. A French crown weighs 19 *dwts.*, what is its weight in grains? Ans. 456.
10. A doubloon weighs 16 *dwts.* and 21 *gr.*; what is its weight in grains? Ans. 405.

11. A Spanish dollar weighs 17 *dwts.* and 6 *grains;* what is its weight in grains? Ans. 314.

12. An English guinea weighs 5 *dwts.* and 6 *grains;* what is its weight in grains? Ans. 126.

13. In 3 pounds of of silver, how many grains? Ans. 17280.

14. In 4 pounds 5 ounces of silver, how many grains?
Ans. 25440.

Here you multiply the pounds by 12, to bring them to ounces, and to the product add the 5 ounces, and multiply the sum by 20, to bring them to pennyweights; and this product by 24, to bring them to grains. If there had been pennyweights also, as in the following examples, these must have been added in the proper place, and the sum multiplied by 24; and if there had been grains also, these must have been added to the last product, and the sum would have been the number required.

15. How many grains are in 6 pounds, 3 ounces, 4 pennyweights of silver plate? Ans. 36096.

16. How many grains in 2 pounds, 7 ounces, 10 pennyweights, and 12 grains of silver plate? Ans. 15132.

To bring ounces to pounds, you must divide by 12, because 12 ounces make a pound; if you would bring pennyweights to pounds, you must divide, in the first place, by 20; because 20 pennyweights make an ounce; and then divide these ounces by 12, to bring them to pounds. If grains are given to be brought to pounds, divide them by 24, in the first place, because 24 grains make a pennyweight; then bring these to ounces, and these again to pounds, as above directed.

17. In 256 ounces, how many pounds?
Ans. 21 *lbs.*, and 4 *oz.* over.

18. In 364 ounces, how many pounds?
Ans. 30 *lbs.*, and 4 *oz.* over.

19. In 180 pennyweights how many ounces? Ans. 9 *oz.*

20. In 396 pennyweights, how many ounces?
Ans. 19 *oz.*, and 16. *dwts.* over.

21. In 405 grains, how many pennyweights?
Ans. 16 *dwts.*, and 21 *grs.* over.

22. In 1142 grains, how many pennyweights?
Ans. 47 *dwts.*, and 14 *gr.* over.

23. In 17280 grains, how many pounds? Ans. 3 *lbs.*

24. In 36096 grains, how many pounds?
Ans. 6 *lbs.*, and 3 *oz.* 4 *dwts.* over.

25. In 25440 grains, how many pounds?
Ans. 4 *lbs.*, and 5 *oz.* over.

QUESTIONS. 1. What articles are weighed by troy weight?—2. What are its denominations?—3. Will you repeat the table?

4. How do you bring, or reduce pounds to ounces?—Why so?

5. How do you bring, or reduce ounces to pennyweights?—Why so?

6. How do you bring, or reduce pennyweights to grains?—Why so?

7. When ounces are given along with pounds, what do you do?

8. When pennyweights are given along with ounces, what do you do?

9. When grains are given along with pennyweights, what do you do?

10. When pounds, ounces, pennyweights, and grains, are given to be brought or reduced to grains, how do you proceed?

11. How do you bring ounces to pounds?—Why so?—12. How do you bring pennyweights to ounces?—Why so?—13. How do you bring grains to pennyweights?—Why so?

14. How do you bring pennyweights to pounds?—Why so?

15. How do you bring grains to ounces?—Why so?

16. How do you bring grains to pounds?—What reason can you give for the operation?

AVOIRDUPOIS WEIGHT.

BY this weight such commodities as are coarse and subject to waste, and all metals, except gold and silver, are weighed.

The denominations are, *ton*, *hundredweight*, *quarter*, *pound*, *ounce*, and *dram*.

TABLE.

16 drams,	(*dr*) make	1 ounce, *oz.*
16 ounces	-	1 pound, *lb.*
28 pounds,	-	1 quarter, *qr.*
4 quarters, or 112 *lbs.*		1 hundred weight, *Cwt.*
20 hundred weight,		1 ton, *T.*

573440 *dr.* = 35840 *oz.* = 2240 *lbs.* = 20 *Cwt.* = 1 *T.*

Questions on the Table.

1. How many hundred weight are in 4 tons? —. Why so? —2. How many in 6 tons? —. Why so?—3. How many in 8 tons? —. Why so?—4. How many in 12 tons? —. Why so?

6. How many quarters are in 6 hundred weight? —. Why

so?—7. How many in 12? —. Why so?—8. How many in 16? —. Why so?—9. How many in 22? —. Why so?

10. How many pounds are in 2 quarters? —. Why so?—11. How many are in 3 quarters? —. Why so?—12. How many are in 4 quarters? —. Why so?

13. How many ounces are in 3 pounds? —. Why so?—14. How many in 6 pounds? —. Why so?—15. How many in 8 pounds? —. Why so?—16. How many in 12 pounds? —. Why so?—17. How many in 11 pounds? —. Why so?

18. How many drams are in 4 ounces? —. Why so?—19. How many in 6 ounces? —. Why so?—20. How many in 8 ounces? —. Why so?—21. How many in 10 ounces? —. Why so?—22. How many in 12 ounces? —. Why so?

PRACTICAL EXERCISES.

1. In 6 tons of coal, how many hundred weight? Ans. 120.
2. In 11 tons of hay, how many hundred weight? Ans. 220.
3. In 5 hundred weight, how many quarters? Ans. 20.
4. In 13 hundred weight, how many quarters? Ans. 52.
5. In 3 quarters, how many pounds? Ans. 84.
6. In 8 quarters, how many pounds? Ans. 224.
7. In 11 quarters, how many pounds? Ans. 308.
8. In 9 pounds, how many ounces? Ans. 144.
9. In 12 pounds, how many ounces? Ans. 192.
10. In 17 pounds, how many ounces? Ans. 282.
11. In 15 ounces, how many drams? Ans. 240.
12. In 9 ounces, how many drams? Ans. 144.
13. In 4 tons, 16 cwt. how many cwt.? Ans. 96.
14. In 10 tons, 5 cwt. 3 qrs. how many qrs.? Ans. 823.

Here you multiply the tons by 20, because 20 cwt. make 1 ton; and to the product add the 5 cwt., and multiply this sum by 4, because 4 quarters make 1 cwt., and to the product add the 3 quarters, and the sum is the number required. If there had been pounds and ounces also given, you must have multiplied the quarters by 28, because 28 pounds make one quarter, and to the product, have added the given pounds: these pounds you must have multiplied by 16, because 16 ounces make 1 pound, and have added to the product the given ounces; the sum would have been the number required.

15. In 6 cwt., 3 quarters, and 16 pounds; how many pounds? Ans. 762.
16. In 14 quarters, 12 pounds, 10 ounces; how many ounces? Ans. 6474.

17. In 21 pounds, 13 ounces, 5 drams; how many drams?
Ans. 5590.

18. In 2 tons, 8 cwt., 3 qrs., 17 pounds, 11 oz., 12 dr.; how many drams? Ans. 1402300.

19. In 42 cwt., how many tons? Ans. 2 tons, and 2 cwt. over.

20. In 19 quarters, how many cwt.?
Ans. 4 cwt., and 3 qrs. over.

21. In 346 pounds, how many quarters?
Ans. 12 qrs., and 10 lbs. over.

22. In 1476 ounces, how many pounds?
Ans. 92 lbs., and 4 oz. over.

To bring hundredweight to tons, you must divide by 20, because 20 cwt. make one ton: to bring quarters to cwt., you must divide by 4, because 4 qrs. make one cwt.: to bring pounds to quarters, you must divide by 28, because 28 lbs. make one qr.: to bring ounces to pounds, you must divide by 16, because 16 oz. make one lb., &c.

If you would bring, for instance, drams to cwt., you must first divide them by 16, which will bring them to ounces: then again, you must divide by 16, which will bring them to pounds; then again, you must divide by 28, which will bring them to quarters: and these you must divide by 4 to bring them to cwt., as in the following examples.

23. In 47654 drams, how many cwt.?
Ans. 1 cwt., 2 qrs., 18 lbs., 2 oz., 6 dr.

24. In 65427 oz., how many tons?
Ans. 1 ton, 16 cwt., 2 qr., 1 lb. 3 oz.

25. In 54621 lbs., how many tons?
Ans. 24 tons, 7 cwt., 2 qr., 21 lb.

26. In 476 qrs., how many tons? Ans. 5 tons, 19 cwt.

Questions. 1. What commodities are weighed by avoirdupois weight?

2. What are the denominations?—3. Will you repeat the table?

4. How do you bring tons to cwt.? —. Why so?—5. How do you bring cwt. to quarters? —. Why so?—6. How do you bring quarters to lbs.? —. Why so?

7. How do you bring pounds to oz.? —. Why so?—8. How do you bring ounces to drams? —. Why so?

9. How do you bring cwt., qrs., and lbs. to pounds? —. Why so?

10. How do you bring lbs., ozs., and drs. to drams? —. Why so?

11. How do you bring cwt. to tons? —. Why so?—12. How do you bring qrs. to cwt.? —. Why so?—13. How do you bring lbs. to cwt.? —. Why so?—14. How do you bring drs. to lbs.? —. Why so?—15. How do you bring lbs. to cwt.? —. Why so?—16. How do you bring drs. to grs.? —. Why so?—17. How do you bring ozs. to tons? —. Why so?

APOTHECARIES WEIGHT.

By this weight, apothecaries mix their medicines, but buy and sell by avoirdupois weight. The pound is the same with the pound troy, but the subdivisions are different.

The denominations are *pound*, *ounce*, *dram*, *scruple*, and *grain*.

TABLE.

20 grains (*gr.*)	make 1 scruple,	....	℈
3 scruples	= 1 dram,	.....	ʒ
8 drams	= 1 ounce,	.....	℥
12 ounces	= 1 pound,	.....	℔

5760 gr.=288 scruples=96 dr.=12 oz.=1 lb.

Questions on the Table.

1. How many ounces are in 3 pounds? —. Why so?

2. How many drams are in 7 oz.? —. Why so?—3. How many in 11 oz.? —. Why so?—4. How many in 13 oz.? —. Why so?—5. How many in 24 oz.? —. Why so?

6. How many scruples are in 5 drams? —. Why so?—7. How many in 12 drams? —. Why so?—8. How many in 17 drams? —. Why so?

9. How many grains are in 5 scruples? —. Why so?—10. How many grains in 8 scruples? —. Why so?—11. How many in 11? —. Why so?—12. How many in 15? —. Why so?

13. How many grains in 1 scruple and a half? —. Why so?

14. How many scruples in 1 dram and a half? —. Why so?

15. How many drams in 1 oz. and a half? —. Why so?

16. How many ounces in 1 pound and a half? —. Why so?

PRACTICAL EXERCISES.

1. In 7 pounds, how many ounces? Ans. 84 oz.
2. In 34 ounces, how many drams? Ans. 272 drs.
3. In 14 drams, how many scruples? Ans. 42 ℈.
4. In 13 scruples, how many grains? Ans. 260 grs.
5. In 3 pounds, 5 ounces; how many ounces? Ans. 41 oz.
6. In 11 ounces, 7 drams; how many drams? Ans. 95 drs.
7. In 7 drams, 2 scruples; how many scruples? Ans. 23 ℈.
8. In 2 scruples, 17 grains; how many grains? Ans. 57 grs.
9. In 3 lbs., 5 ozs., 6 drs.; how many drams? Ans. 334 drs.
10. In 11 ozs., 7 drs., 2 scruples, 15 grains; how many grains? Ans. 5755 grs.
11. In 137 ounces, how many pounds?
Ans. 11 lbs. and 7 ozs. over.
12. In 579 drams, how many ounces?
Ans. 72 ozs. and 1 dr. over.
13. In 462 grs., how many scruples?
Ans. 23 scruples, and 2 grs. over.
14. In 642 drams, how many lbs.?
Ans. 6 lbs. and 8 ozs., 2 drs. over.
15. In 7624 grains, how many ounces?
Ans. 15 ozs. and 7 drs., 0 scruples, 4 grs. over.

When pounds, ounces, drams, scruples, and grains are given to be brought to grains; you must multiply the pounds by 12, and to the product add the ounces; these multiply by 8, to bring them to drams, and to the product add the given drams: this sum multiply by 3, to bring them to scruples; and to the product add the given scruples: this sum multiply by 20, and to the product add the given grains: the sum will be the number required.

To bring grains to pounds, the operation is just the reverse. You must divide the given sum by 20, which will bring the grains to scruples; these divide by 3, to bring them to drams: these again by 8, to bring them to ounces; and these again by 12, to bring those ounces to pounds.

Questions. 1. By whom is this weight used?—2. For what purpose do they use it?—3. Do they buy and sell by this weight?

4. What are the denominations?—5. Will you repeat the table?

6. How do you bring, or reduce pounds to ounces? — Why so?

7. How do you bring ounces to drams? —. Why so?
8. How do you bring scruples to grains? —. Why so?
9. How do you bring drams to scruples? —. Why so?
10. How do you bring pounds and ounces to ounces? —. Why so?
11. How do you bring drams, scruples and grains, to grains? —. Why so?
12. How do you bring ounces to pounds? —. Why so?
13. How do you bring drams to pounds? —. Why so?
14. How do you bring grains to drams? —. Why so?
15. How do you bring scruples to pounds? —. Why so?
16. How do you bring grains to ounces? —. Why so?

LONG MEASURE.

This measure is used for lengths and distances.

The denominations are, *degree*, *league*, *mile*, *furlong*, *pole* or *rod*, *yard*, *foot*, *inch*, and *barley-corn*.

TABLE.

3 barley-corns (*b. c.*)		make 1 inch, *in.*
12 inches	- -	make 1 foot, *ft.*
3 feet	- -	make 1 yard, *yd.*
5½ yards, or 16½ feet		make 1 rod, pole or perch, *P.*
40 rods, or 220 yards		make 1 furlong, *fur.*
8 furlongs, or 1760 yards		" 1 mile, *M.*
3 miles	- -	make 1 league, *L.*
60 geographic miles		make 1 degree, *deg.*
69½ statute miles	-	make 1 degree, *deg.*

190180 *b. c.*=63360 *in.*=5280 *ft.*=1760 *yds.*=320 *rods*=8 *fur.*=1 *mile.*

A hand is 4 inches, and is used for measuring the height of horses.

A fathom is 6 feet, and is used for measuring the depth of water.

Questions on the Table.

1. What is the difference between a geographic, and a statute mile?
2. Is a geographic mile, therefore, longer than a statute mile

3. How many geographic miles in 4 degrees? —. Why so? —4. How many in 6 degrees? —. Why so?—5. How many in 5 degrees and a half? —. Why so?

6. How many furlongs in 3 miles? —. Why so?—7. How many in 5 miles? —. Why so?—8. How many in 8 miles? —. Why so?

9. How many rods in 3 furlongs? —. Why so?—10. How many in 5 furlongs? —. Why so?

11. How many yards in two rods? —. Why so?—12. How many in 3 rods? —. Why so?—13. How many in 5 rods? —. Why so?

14. How many feet in 5 yards? —. Why so?—15. How many in 8 yards? —. Why so?—16. How many in 12 yards? —. Why so?

17. How many inches in 4 feet? —. Why so?—18. How many in 8 feet? —. Why so?—19. How many in 11 feet? —. Why so?—20. How many in 12 feet? —. Why so?

21. How many barley-corns in 8 inches? —. Why so?—22. How many in 11 inches? —. Why so?

23. How many furlongs are in 4 miles and a half? —. Why so?—24. How many in 6 miles and a half? —. Why so?

25. How many inches in 5 feet and a half? —. Why so? —26. How many in 8 feet and a half? —. Why so?—

27. How many feet in 4 rods? —. Why so?—28. How many in 6 rods? —. Why so?

29. How many rods in 2 miles? —. Why so?—30. How many in 4 miles? —. Why so?

PRACTICAL EXERCISES.

1. In 6 degrees, how many statute miles? Ans. 417 miles.
2. In 8 degrees, how many geographic miles? Ans. 480 m.
3. In 30 degrees and a half, how many geographic miles? Ans. 1830 miles.
4. In 16 degrees and one fourth, how many geographic miles? Ans. 975 miles.
5. In 5 leagues and one third, how many miles? Ans. 16 m.
6. In 7 miles, how many furlongs? Ans. 56 fur.
7. In 6 miles, how many yards? Ans. 10560 yds.
8. In 10 miles and a half, how many furlongs? Ans. 84 fur.
9. In 56 rods, how many yards? Ans. 308 yds.
10. How many yards in 16 furlongs? Ans. 3520 yds.
11. How many feet in 19 yards and a half? Ans. 58 ft. 6 in.
12. In 21 feet, how many inches? Ans. 252 in.

13. In 12 yards and a third, how many feet? Ans. 37 feet.
14. In 11 furlongs and a half, how many rods? Ans. 460 rods.
15. In 15 miles and a half, how many yards? Ans. 27280 yds.
16. In 12 feet, 5 inches, and 2 barley-corns; how many barley-corns? Ans. 449 b.c.
17. In 3 miles, 5 furlongs, and 31 rods; how many rods? Ans. 1191 rods.

When it is required to bring higher denominations to lower, you must multiply, in all cases, by the number which it takes o the lower denomination, to make one of the higher: as in the last example, you multiply the miles by 8, because it takes 8 of the lower denomination (to wit, furlongs) to make one mile. Again, these are to be reduced, or brought to rods; you must therefore multiply by 40, because it takes 40 of the lower denomination, (to wit, rods) to make one furlong.

One denomination is said to be higher than another, when it is of more value, or greater in number or magnitude. Thus a dollar is of a higher denomination than a dime, because it is of more value; and a mile of a higher denomination than a rod, because it is greater in length; and so in all other cases.

When it is required to bring lower denominations to higher, the operation is just the reverse; you must divide the lower denomination by the number which it takes of that denomination, to make one of the higher: thus, if inches are to be brought to feet, you must divide by 12, because it takes 12 inches to make one foot; and so in all other cases.

18. In 47625 barley-corns, how many feet?
Ans. 1322 ft. 11 inches over.
19. In 12651 inches, how many yards?
Ans. 351 yds. 1 ft. 3 in. over.
20. In 5460 yards, how many furlongs?
Ans. 24 fur. 180 yds. over.
21. In 74210 feet, how many furlongs?
Ans. 112 fur. 96 yds. 2 ft. over.
22. In 8462 rods, how many miles?
Ans. 26 miles, 142 rods over.
23. In 532 furlongs, how many leagues?
Ans. 22 L. and 4 fur. over.
24. In 1462 miles, how many degrees?
Ans. 24 deg. and 22 M over.
25. In 47246 yards, how many miles?
Ans. 26 M. and 1486 yds. over.

QUESTIONS. 1. For what purpose is long measure used?
2. What are its denominations?—3. Will you repeat the table?

4. How do you bring degrees to miles? —. Why so?—5. How do you bring miles to rods? —. Why so?—6. How do you bring miles to yards? —. Why so?—7. How do you bring leagues to rods? —. Why so?—8. How do you bring yards to inches? —. Why so?—9. How do you bring feet to barley-corns? —. Why so?—10. How do you bring miles to inches? —. Why so?—11. How do you bring furlongs to feet? —. Why so?.

12. How do you bring barley-corns to feet? —. Why so?
13. How do you bring inches to yards? —. Why so?
14. How do you bring yards to miles? —. Why so?
15. How do you bring furlongs to leagues? —. Why so?
16. How do you bring miles to degrees? —. Why so?
17. How do you bring feet to degrees? —. Why so?
18. How do you bring barley-corns to leagues? —. Why so?
19. When higher denominations are to be brought to lower, what do you do?
20. When is one denomination said to be higher than another?
21. When lower denominations are to be brought to higher, how do you proceed?

CLOTH MEASURE.

THIS is used by merchants and others, to measure cloth, tapes, &c.

The denominations are, *inch*, *nail*, *quarter*, *yard*, and *ell*.

TABLE.

$2\frac{1}{4}$ inches (*in.*)	make 1 nail, *na.*
4 nails	make 1 quarter of a yard, *qr.*
4 quarters	make 1 yard, *yd.*
3 quarters	make 1 Ell Flemish, *E. Fl.*
5 quarters	make 1 Ell English, *E. E.*
5 quarters	make 1 Ell French, *E. Fr.*
$2\frac{1}{2}$ quarters	make 1 Ell Hamburg, *E. H.*

36 inches=16 nails=4 quarters=1 yard.

Questions on the Table.

1. How many quarters in 5 yards? —. Why so?—2. How many in 8 yards? —. Why so?—3. How many in 13 yards? —. Why so?

4. How many quarters in 6 ells English? —. Why so?

5. How many quarters in 7 ells French? —. Why so?—6. How many in 12? —. Why so?—7. How many in 15? —. Why so?—8. How many in 21? —. Why so?

9. How many quarters in 7 ells of Hamburg? —. Why so?—10. How many in 10? —. Why so?—11. How many in 16? —. Why so?

12. How many nails in 5 quarters of a yard? —. Why so?—13. How many in 8? —. Why so?—14. How many in 12? —. Why so?—15. How many in 22? —. Why so?

16. How many nails in 4 yards? —. Why so?—17. How many in 7 yards? —. Why so?—18. How many in 12 yards? —. Why so?

19. How many inches in 6 nails? —. Why so?—20. How many in 8? —. Why so?—21. How many in 12? —. Why so?

PRACTICAL EXERCISES.

1. In 8 Ells Flemish, how many quarters? Ans. 24 qrs.
2. In 19 Ells Flemish; how many quarters? Ans. 57 qrs.
3. In 11 Ells English, how many quarters? Ans. 55 qrs.
4. In 16 Ells English, how many quarters? Ans. 80 qrs.
5. In 12 Ells Hamburg, how many quarters? Ans. 30 qrs.
6. In 15 quarters, how many nails? Ans. 60 na.
7. In 19 quarters, how many nails? Ans. 76 na.
8. In 5 yards, 2 quarters, how many quarters? Ans. 22 qrs.
9. In 6 yards, 3 quarters, how many nails? Ans. 108 na.
10. In 12 nails, how many inches? Ans. 27 in.
11. In 15 quarters, 3 nails; how many nails? Ans. 63 na.
12. In 462 nails, how many quarters?
Ans. 115 qrs. and 2 na. over.
13. In 1746 quarters, how many yards?
Ans. 436 yds. and 2 qrs. over.
14. In 1664 nails, how many yards? Ans. 104 yds.
15. In 742 quarters, how many Ells English?
Ans. 148 E. and 2 qrs. over.
16. In 524 quarters, how many Ells French?
Ans. 104 E. and 4 qrs. over.

17. In 375 qrs., how many Ells Flemish? Ans. 125 Ells.
18. In 31415 nails, how many yards?
Ans. 1963 yds. and 7 na. over.
19. In 7421 nails, how many Ells English?
Ans. 371 Ells, and 1 na. over.

Questions. 1. For what is this measure used?—2. What are its denominations?—3. Will you repeat the table?
4. How do you bring yards to quarters? —. Why so?—
5. How do you bring Ells English to quarters? —. Why so?
6. How do you bring Ells Flemish to quarters? —. Why so?
7. How do you bring Ells French to quarters? —. Why so?
8. How do you bring yards to nails? —. Why so?
9. How do you bring Ells English to nails? —. Why so?
10. How do you bring Ells Flemish to nails? —. Why so?
11. How do you bring quarters to inches? —. Why so?
12. How do you bring yards to inches? —. Why so?
13. How do you bring inches to yards? —. Why so?
14. How do you bring nails to Ells English? —. Why so?
15. How do you bring nails to Ells Flemish? —. Why so?
16. How do you bring inches to Ells English? —. Why so?

LAND MEASURE.

This is used in measuring land, and has respect to length and breadth.

The denominations are, *yard*, *perch*, *rood*, and *acre*.

TABLE.

9 square feet make 1 square yard, *yd.*
30¼ sq. yards make 1 sq. perch, *P.*
40 sq. perches make 1 rood, *R.*
4 roods make 1 acre, *A.*
43560 feet=4840 yds.=160 perches=4 roods=1 acre.

Questions on the Table.

1. How many roods are in 6 acres? —. Why so?—2. How many roods in 12 acres? —. Why so?—3. How many in 15 acres? —. Why so?

4. How many perches are in 6 roods? —. Why so?—
5. How many in 7 roods? —. Why so?

6. How many perches are in 3 roods and a half? —. Why so?—7. How many in 11 roods and a quarter? —. Why so?

8. How many square feet are in 4 sq. yards? —. Why so?

9. How many perches are in 1 acre and 2 roods? —. Why so?

10. How many roods are in 120 perches? —. Why so?

11. How many roods in 175 perches, and how many over?

12. How many in 212 perches, and how many over?

13. How many acres are in 340 perches, and how many over?

PRACTICAL EXERCISES.

1. In 4 acres, 2 roods; how many roods? Ans. 18.
2. In 7 acres, 3 roods; how many roods? Ans. 31.
3. In 12 acres, 1 rood; how many roods? Ans. 49.
4. In 12 roods, how many perches? Ans. 480.
5. In 15 roods, 20 perches; how many perches? Ans. 620.
6. In 7 roods, 35 perches; how many perches? Ans. 315.
7. In 7 square yards, how many square feet? Ans. 63.
3. In 13 square yards, how many square feet? Ans. 117.
9. In 14 roods, how many acres?
Ans. 3 acres, and 2 R. over.
10. In 21 roods, how many acres? Ans. 5 A. and 1 R. over.
11. In 547 perches, how many roods?
Ans. 13 R. and 27 P. over.
12. In 617 perches, how many roods?
Ans. 15 R. and 17 P. over.
13. In 1476 perches, how many acres?
Ans. 9 A. and 36 P. over.
14. In 43560 square feet, how many roods? Ans. 4.
15. In 4 square perches, how many yards? Ans. 121.
16. In 12 square perches, how many yards? Ans. 363.
17. In 28 square perches, how many sq. yards? Ans. 847.

QUESTIONS. 1. For what purpose is land measure used?
2. What are its denominations?
3. Will you repeat the table?—4. How do you bring roods to acres?
5. How do you bring perches to roods?—6. How do you bring perches to acres?
7. How do you bring square yards to perches?

8. How do you bring acres to roods?—9. How do you bring roods to perches?—10. How do you bring perches to square yards?

11. How do you bring perches to acres?—12. How do you bring square feet to roods?

13. How do you bring square feet to acres?—What is the reason for the operation?

LIQUID MEASURE.

This is used for measuring wine, brandy, spirits, beer, cider, &c.

The denominations are, *gill*, *pint*, *quart*, and *gallon*.

TABLE.

4 gills (*g.*)	- -	make 1 pint, *pt.*
2 pints	- -	make 1 quart, *qt.*
4 quarts	- -	make 1 gallon, *gal.*
31½ gallons	- -	make 1 barrel, *bar.*
63 gallons	- -	make 1 hogshead, *hhd.*
2 hogsheads, or 126 gal.		make 1 pipe or butt, *P.* or *B.*
4 hogsheads, or 252 gal.		make 1 tun, *T.*

2016 gills=504 pints=252 quarts=63 gal.=1 hhd.

Questions on the Table.

1. How many hogsheads are in 5 tuns of wine? —. Why so?—2. How many hogsheads in 11 pipes of brandy? —. Why so?

3. How many gallons are in 3 hogsheads? —. Why so?—4. How many gallons in 5 hogsheads? —. Why so?

5. How many gallons are in 4 barrels? —. Why so?—6. How many gallons in 6 barrels? —. Why so?

7. How many quarts are in 5 gallons? —. Why so?—8. How many in 12 gallons? —. Why so?—9. How many in 15 gallons? —. Why so?

10. How many pints are in 5 quarts? —. Why so?—11. How many in 16 quarts? —. Why so?

12. How many gills are in 3 quarts? —. Why so?—13. How many in 13 quarts? —. Why so?

14. How many gills are in 5 pints? — Why so?—15. How

many in 12 pints? —. Why so?—16. How many in 18 pints? —. Why so?

17. How many tuns are in 24 hogsheads? —. Why so?—18. How many in 40 hogsheads? —. Why so?

19. How many pipes are in 16 hogsheads? —. Why so?—20. How many in 24 hogsheads? —. Why so?

21. How many hogsheads are in 252 gallons? —. Why so?

22. How many quarts are in 11 gallons? —. Why so?—23. How many in 15 gallons? —. Why so?—24. How many in 22 gallons? —. Why so?

25. How many pints are in 7 quarts? —. Why so?—26. How many in 16 quarts? —. Why so?

27. How many gills are in 7 pints? —. Why so?—28. How many in 12 pints? —. Why so?—29. How many in 15 pints? —. Why so?

PRACTICAL EXERCISES.

1. In 5 hhds. and 10 gals. how many gallons? Ans. 325.
2. In 7 tuns and 3 hhds., how many hhds.? Ans. 31.
3. In 21 hhds. and 46 gals., how many gals.? Ans. 1369.
4. In 29 hhds. how many pipes?
   Ans. 14 P. and 1 hhds. over.
5. In 37 gallons, how many quarts? Ans. 148.
6. In 174 gallons, how many quarts? Ans. 696.
7. In 47 quarts, how many pints? Ans. 94 pints.
8. In 16 pints, how many gills? Ans. 64.
9. In 8 gallons and 3 quarts, how many pints? Ans. 70.
10. In 10 quarts and 1 pint, how many gills? Ans. 84.
11. In 15 pints, how many gills? Ans. 60.
12. In 12 pints and 3 gills, how many gills? Ans. 51.
13. In 23 hhds., how many tuns?
    Ans. 5 tuns and 3 hhds. over.
14. In 246 gallons, how many hhds.?
    Ans. 3 hhds. 57 gal. over.
15. In 6 barrels, how many gallons? Ans. 189.
16. In 10 barrels and 12 gallons, how many gallons?
    Ans. 327.
17. In 627 gallons, how many hhds?
    Ans. 9 hhds. 60 gals. over.
18. In 124 quarts, how many gallons? Ans. 31.
19. In 546 quarts, how many gallons?
    Ans. 136 gals. and 2 qts. over.
20. In 742 pints, how many gallons?
    Ans. 92 gals. and 6 pts. over.

21. In 476 gills, how many pints? Ans. 119.
22. In 1244 gills, how many gallons?
Ans. 38 gals. and 3 qt. 4 gills, over.
23. In 17456 pints, how many hhds.?
Ans. 34 hhds. and 40 gals. over.
24. In 7465 quarts, how many tuns?
Ans. 7 T., 1 hhd., 39 gals., 1 qt. over.

Questions. 1. For what is this measure used?—What are its denominations?—3. Will you repeat the table?
4. How do you bring tuns to hhds.? —. Why so?—5. How do you bring hhds. to gallons? —. Why so?
6. How do you bring barrels to gallons? —. Why so?
7. How do you bring gallons to quarts? —. Why so?
8. How do you bring gallons to pints? —. Why so?
9. How do you bring quarts to gills? —. Why so?
10. How do you bring gallons to gills? —. Why so?
11. How do you bring hhds. to tuns? —. Why so?
12. How do you bring gallons to hhds.? —. Why so?
13. How do you bring quarts to gallons? —. Why so?
14. How do you bring pints to gallons? —. Why so?
15. How do you bring gills to pints? —. Why so?
16. How do you bring gills to gallons? —. Why so?
17. How do you bring quarts to hhds.? —. Why so?
18. How do you bring gills to hhds.? —. Why so?

DRY MEASURE.

By this, are measured grain, fruits, salt, &c.
The denominations are, *pint*, *quart*, *peck*, and *bushel*.

TABLE.

2 pints (*pt.*) make 1 quart, *qt.*
8 quarts - make 1 peck, *P.*
4 pecks - make 1 bushel, *bu.*
64 pints=32 quarts=4 pecks=1 bushel.

A gallon dry measure, contains $268\frac{4}{5}$ cubic inches, and is larger than a gallon wine measure, which contains only 231 cubic inches.

Questions on the Table.

1. How many pecks are in 3 bushels? —. Why so?—2. How many in 6 bushels? —. Why so?—3. How many in 11 bushels? —. Why so?

4. How many quarts are in 6 pecks? —. Why so?—5. How many in 8 pecks? —. Why so?

6. How many pints are in 8 quarts? —. Why so?—7. How many in 14 quarts? —. Why so?

8. How many bushels are in 12 pecks? —. Why so?—9. How many in 20 pecks?

10. How many pecks in 40 quarts? —. Why so?—11. How many in 64 quarts? —. Why so?

12. How many quarts are in 16 pints? —. Why so?—13. How many in 24 pints? —. Why so?

14. How many bushels are in 128 quarts? —. Why so?

15. How many pecks are in 72 pints? —. Why so?

PRACTICAL EXERCISES.

1. In 15 bushels, how many pecks? Ans. 60.
2. In 20 bushels, 3 pecks, how many pecks? Ans. 83.
3. In 17 pecks, how many quarts? Ans. 136.
4. In 11 pecks, 7 quarts, how many quarts? Ans. 95.
5. In 25 pecks, 5 quarts, how many quarts? Ans. 205.
6. In 36 pints, how many quarts? Ans. 18.
7. In 42 quarts, how many pints? Ans. 84.
8. In 13 pecks, 4 quarts, 1 pint, how many pints? Ans. 217.
9. In 32 pecks, how many bushels? Ans. 8.
10. In 56 pecks, how many bushels? Ans. 14.
11. In 94 pecks, how many bushels?

Ans. 23 *bu.*, and 2 *p.* over.

12. In 49 quarts, how many pecks? Ans. 6 *p.*, and 1 *qt.* over.
13. In 174 quarts, how many pecks?

Ans. 21 *p.*, and 6 *qts.* over.

14. In 250 quarts, how many bushels?

Ans. 7 *bu.*, and 3 *p.* 2 *qt* over.

15. In 156 pints, how many pecks? Ans. 9 *p.*, and 6 *qts.* over.
16. In 346 quarts, how many bushels?

Ans. 10 *bu.*, and 3 *p.* 6 *qts.* over.

QUESTIONS. 1. For what purpose is this measure used?

2. What are its denominations?—3. Will you repeat the table?

4. How do you bring bushels to pecks?—Why so?—5. How do you bring pecks to quarts?—Why so?—6. How do you bring quarts to pints?

7. How do you bring bushels and pecks to pecks?—Why so?
8. How do you bring pecks and quarts to quarts?—Why so?
9. How do you bring quarts and pints to pints?—Why so?
10. How do you bring bushels, pecks, and quarts, to quarts?—Why so?
11. How do you bring pecks to bushels?—Why so?
12. How do you bring quarts to bushels?—Why so?
13. How do you bring pints to pecks?—Why so?
14. How do you bring pints to bushels?—Why so?
15. How many pints make a bushel?—Why so?
16. How many cubic inches are in a gallon, dry measure?
17. What number is in a gallon, wine measure?

TIME.

THE denominations are, *second*, *minute*, *hour*, *day*, *week*, *month*, and *year*. The year is the greatest denomination. The others are used to mark the divisions and subdivisions of a year.

TABLE.

60 seconds (*sc.*),	make	1 minute, *min.*
60 minutes	-	1 hour, *h.*
24 hours,	-	1 day, *d.*
7 days,	-	1 week, *w.*
52 weeks,	-	1 year, *y.*
30 days,	-	1 month, *m.*
12 months, or 365 days,		1 year, *y.*

The names of the months are, January, 31 days; February, 28 days, (except in every fourth year it has 29); March, 31 days; April, 30 days; May, 31 days; June, 30 days; July, 31 days; August, 31 days; September, 30 days; October, 31 days; November, 30 days; and December, 31 days.

But the names of the months, and the number of days each one contains, can be more easily remembered from this verse:

Thirty days have September,
April, June, and November;
All the rest have thirty-one,
Except February alone;
Which has but twenty-eight days clear,
And twenty-nine every leap year.

The solar year contains 365 days, 5 hours, 48 minutes, and

55 seconds, and is the exact time in which the earth makes one revolution around the sun. This would always keep the seasons the same.

The common year consists of 365 days; but being shorter than the true year, the seasons shift, or fall back 5 hours, 48 minutes, and 55 seconds every year. To remedy this inconvenience, Julius Cæsar, some time before the Christian era, directed that one day, every fourth year, should be added to the month of February, which would make the civil year nearly of the same length with the solar year; hence the year thus corrected is called the Julian year. Accordingly the 24th day of February was reckoned twice (*bis*), which being the sixth of the calends of March, called in Latin *sextilis*, the day was called from this circumstance *bissextilis*, and the year in which it took place was called bissextile. The Julian year, for three successive ones, contains 365 days, and every fourth one, 366 days. This is the year now in common use.

The year contains 52 weeks, 1 day, and 6 hours; but when the year is divided into weeks, the day and a quarter is rejected.

Thirty days are not the twelfth part of a year, but in ordinary calculations, and in the transactions of life, thirty days are considered a month: the 5 days and six hours being rejected.

The addition of 6 hours every year, or 1 day in four years, is too much; and, in a series of years, would affect the seasons, and occasion a change in the equinoxes; to remedy this, nearly one day in every 100 years is to be omitted.

Questions on the Table.

1. How many months are in 5 years? —. Why so?—2. How many in 11 years? —. Why so?

3. How many days in 4 months? —. Why so?—4. How many in 7 months? —. Why so?—5. How many in 10 months? —. Why so?

6. How many weeks in 2 years? —. Why so?—7. How many in 4 years? —. Why so?

8. How many days in 7 weeks? —. Why so?—9. How many in 12 weeks? —. Why so?—10. How many in 16 weeks? —. Why so?

11. How many hours in 3 days? —. Why so?—12. How many in 5 days? —. Why so?

13. How many minutes in 5 hours? —. Why so?—14. How many in 9 hours? —. Why so?

15. How many seconds in 3 minutes? —. Why so?—16. How many in 7 minutes? —. Why so?

17. How many months in 180 days? —. Why so?—18. How many in 240 days? —. Why so?—How many in 330 days?—. Why so?

20. How many weeks in 210 days? —. Why so?—21. How many in 112 days? —. Why so?—22. How many in 140 days? —. Why so? —.

23. How many days in 96 hours? —. Why so?—24. How many in 144 hours? —. Why so?

25. How many minutes in 3 hours? —. Why so?—26. How many in 5 hours? —. Why so?—27. How many in 12 hours? —. Why so?

28. How many seconds in 4 minutes? —. Why so?—29. How many in 7 minutes? —. Why so?—30. How many in 12 minutes? —. Why so?

PRACTICAL EXERCISES.

1. In 5 years and 4 months, how many months? Ans. 64.
2. In 7 months and 21 days, how many days? Ans. 231.
3. In 3 years and a half, how many weeks? Ans. 182.
4. In 8 weeks and 4 days, how many days? Ans. 60.
5. In 15 weeks and 6 days, how many days? Ans. 111.
6. In 11 days, 4 hours; how many hours? Ans. 268.
7. In 17 days, 6 hours; how many hours? Ans. 414.
8. In 12 hours, how many minutes? Ans. 720.
9. In 13 hours, 20 minutes; how many minutes? Ans. 800.
10. In 14 minutes, how many seconds? Ans. 840.
11. In 8 minutes, 30 seconds; how many seconds? Ans. 510.
12. In 5 months, 15 days; how many days? Ans. 165.
13. In 31 weeks, 4 days; how many days? Ans. 221.
14. In 8 days, 7 hours; how many hours? Ans. 199.
15. In 16 hours, 18 minutes; how many minutes? Ans. 978.
16. In 17 min. 14 sec.; how many seconds? Ans. 1034.
17. In 75 months, how many years?
Ans. 6 years, and 3 m. over.
18. In 175 days, how many months?
Ans. 5 m. and 25 days over.
19. In 148 days, how many weeks?
Ans. 21 weeks, and 1 day over.
20. In 247 min., how many hours? Ans. 4 h. and 7 m. over.
21. In 374 seconds, how many minutes?
Ans. 6 min. and 14 sec. over.

22. In 4521 seconds, how many hours?
Ans. 1 h. 15 m. 21 sec. over.

23. In 17461 minutes, how many days?
Ans. 12 d. 3 h. and 1 m. over.

24. In 3471 days, how many months?
Ans. 115 m. and 21 d. over.

25. In 54761 days, how many years of 365 days?
Ans. 150 years, and 11 d. over.

26. In 3 m., 5 d., 8 h., 25 m.; how many minutes?
Ans. 137305 m.

QUESTIONS. 1. What are the denominations of time?
2. For what purpose are they used?—3. Will you repeat the table?
4. What are the names of the months, and the number of days each one contains?
5. What is the length of the solar year?—6. Is this the true year?—7. Would it always keep the same seasons to the same days of the year?
8. How many days does the common year contain?
7. Did this difference between the solar and common years cause any inconvenience?
8. What was that inconvenience?—9. Who corrected it in a great degree?—10. What did Julius Cæsar order to be done?
11. What was the year called, in which February had 29 days, or the 24th day was reckoned twice?
12. From what circumstance was it so called?
13. What is the year thus corrected, called?—14. Do we use the same?
15. Is it exactly correct?—16. Is the correction, that is, the addition of a day in 4 years, two much or two little?
17. What is necessary to be done in order to keep the same seasons to the same days of the year?
18. Are 52 weeks exactly a year?—19. How much do they fall short of it?—20. Are 30 days exactly a month, or the 12th part of a year?—21. How much do 12 such months fall short of a true year?
22. How do you bring years to months? —. Why so?
23. How do you bring months to days? —. Why so?
24. How do you bring weeks to hours? —. Why so?
25. How do you bring hours to seconds? —. Why so?
26. How do you bring days to minutes? —. Why so?
27. How do you bring months to years? —. Why so?
28. How do you bring days to years? —. Why so?

29. How do you bring weeks to years? —. Why so?
30. How do you bring days to months? —. Why so?
31. How do you bring hours to days? —. Why so?
32. How do you bring minutes to hours? —. Why so?
33. How do you bring seconds to hours? —. Why so?
34. How do you bring hours to months? —. Why so?
35. How do you bring seconds to days? —. Why so?
36. How do you bring minutes to years? —. Why so?

MOTION, OR CIRCLE MEASURE.

This measure is used by astronomers and navigators. Its denominations are *second*, *minute*, *degree*, and *sign*.

TABLE.

60 seconds (″) make 1 minute, ′
60 minutes - make 1 degree, °
30 degrees - make 1 sign, *sig*.
12 signs, or 360 degrees make 1 great circle.

Questions on the Table.

1. How many degrees are in 6 signs? —. Why so?—2. How many in 9 signs? —. Why so?—3. How many in 11 signs? —. Why so?

4. How many minutes are in 4 degrees? —. Why so?—5. How many in 7 degrees? —. Why so?—6. How many in 12 degrees? —. Why so?

7. How many seconds are in 5 minutes? —. Why so?—8. How many in 8 minutes? —. Why so?—9. How many in 12 minutes? —. Why so?

10. How many signs are in 90 degrees? —. Why so?—11. How many in 120 degrees? —. Why so?—12. How many in 240 degrees? —. Why so?

13. How many degrees are in 180 minutes? —. Why so?—14. How many degrees in 240 minutes? —. Why so?
15. How many in 300 minutes? —. Why so?

16. How many minutes are in 240 seconds? —. Why so?

17. How many in 360 seconds? —. Why so?—18. How many in 480 seconds? —. Why so?—19. How many in 960 seconds? —. Why so?

PRACTICAL EXERCISES.

1. In 5 signs and 7 degrees, how many degrees? Ans. 157.
2. In 11 signs, 21 degrees; how many degrees? Ans. 351.
3. In 15 degrees, 6 minutes; how many minutes? Ans. 906.
4. In 22 degrees, 12 minutes; how many minutes? Ans. 1332.
5. In 45 degrees, how many seconds? Ans. 162000.
6. In 115 degrees, how many minutes? Ans. 6900.
7. In 72 minutes, how many seconds? Ans. 4320.
8. In 45 degrees, 8 minutes; how many seconds? Ans. 162480.
9. In 17 minutes, 12 seconds; how many seconds? Ans. 1032.
10. In 125 degrees, how many signs? Ans. 4 sig. and 5 deg. over.
11. In 247 degrees, how many signs? Ans. 8 sig. and 7 deg. over.
12. In 462 minutes, how many degrees? Ans. 7 deg. and 42′ over.
13. In 7941 minutes, how many degrees? Ans. 132 deg., 21′ over.
14. In 1246 seconds, how many minutes? Ans. 20′, and 46″ over.
15. In 5472 seconds, how many minutes? Ans. 91′, and 12″ over.
16. In 42161 seconds, how many degrees? Ans. 11 deg., 42′, 41″ over.
17. In 3145 minutes, how many signs? Ans. 1 sig., 22 deg., 25′ over.
18. In 4721 minutes, how many signs? Ans. 2 sig., 18 deg., 41′ over.

Questions. 1. By whom is circle measure used?—2. What are its denominations?—3. Will you repeat the table?
4. How do you bring signs to degrees? —. Why so?
5. How do you bring degrees to minutes? —. Why so?
6. How do you bring minutes to seconds? —. Why so?
7. How do you bring signs to minutes? —. Why so?

8. How do you bring degrees to seconds? —. Why so?
9. How do you bring minutes to seconds? —. Why so?
10. How do you bring degrees to signs? —. Why so?
11. How do you bring minutes to signs? —. Why so?
12. How do you bring seconds to minutes? —. Why so?
13. How do you bring seconds to degrees? —. Why so?
14. How do you bring minutes to degrees? —. Why so?

END OF THE INTRODUCTION.

ARITHMETIC.

ARITHMETIC is that part of Mathematics, which treats of numbers, explains their properties, and shows how to apply them to the business of life.

All the operations of arithmetic are performed by the help of these characters or figures; 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, cypher or nought.

Numeration, Addition, Subtraction, Multiplication, and Division, are its primary rules.

NUMERATION.

NUMERATION is that part of Arithmetic, which treats of the proper arrangement of figures, to express any given number, and the giving to each figure its proper value.

Every figure has an absolute and relative value. When it stands alone, it represents so many units or ones of any thing. This is its absolute or simple value. When it is connected with one or more figures, its value is increased. This is called its relative or local value. In every place from the right hand toward the left, it increases in a tenfold proportion; that is, any figure represents ten times as many units in the second place, as if it stood in the first place; a hundred times as many in the third place, as if it stood in the first; and ten times as many, as if it stood in the ten's place; and so in any other places, as may be seen in the following Table.

TABLE.

100 Million's place	10 Million's place	Million's place	100 Thousand's place	10 Thousand's place	Thousand's place	Hundred's place	Ten's place	Unit's place	
							1	1	=One.
							2	1	=Twenty-one.
						3	2	1	=Three hundred and twenty one.
					4	3	2	1	=Four thousand 321.
				5	4	3	2	1	=Fifty-four thousand 321.
			6	5	4	3	2	1	=Six hundred and 54 thousand 321.
		7	6	5	4	3	2	1	=Seven millions 654 thousand 321.
	8	7	6	5	4	3	2	1	=87 millions 654 thousand 321.
9	8	7	6	5	4	3	2	1	=987 millions 654 thousand 321.

Nine figures or places of figures are sufficient for common purposes; but sometimes it may be necessary to express a greater number, as in the following table, which extends to thirty places of figures.

100000 quadrillion's place	10000 quadrillion's place	1000 quadrillion's place	100 quadrillion's place	10 quadrillion's place	Quadrillion's place	100000 trillion's place	10000 trillion's place	1000 trillion's place	100 trillion's place	10 trillion's place	Trillion's place	100000 billion's place	10000 billion's place	1000 billion's place	100 billion's place	10 billion's place	Billion's place	100000 million's place	10000 million's place	1000 million's place	100 million's place	10 million's place	Million's place	100 thousand's place	10 thousand's place	Thousand's place	Hundred's place	Ten's place	Unit's place
5	4	3	1	9	8,	7	6	5	4	3	2,	1	9	8	7	6	5,	4	3	2	9	8	7,	6	5	4	3	2	1

Numbers are also expressed by letters; and hence are called numeral letters, or Roman numbers.

1	2	3	4	5	6	7	8	9	10	20	30	40
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XX.	XXX.	XL.

50	60	70	80	90	100	500	1000	1830
L.	LX.	LXX.	LXXX.	XC.	C.	D.	M.	MDCCCXXX.

A letter of less value standing before one of greater value, is to be subtracted from it; but placed after it, is to be added to it. Hence, by combining these letters, various numbers may be expressed.

Questions on the Table.

1. How many would 4, in the second, or ten's place represent? —. Why so?
2. How many would 6 in the third, or hundred's place represent? —. Why so?
3. How many would 3 in the fourth, or thousand's place represent? —. Why so?
4. How many would 2 in the fifth, or ten thousand's place represent? —. Why so?
5. How many would 8 in the sixth place represent? —. Why so?
6. How many would 9 in the seventh place represent? —. Why so?
7. How many would 5 in the eighth place represent? —. Why so?
8. How many would 6 in in the ninth place represent? —. Why so?
9. How many would 5 in the tenth place represent? —. Why so?
10. How many would 9 in the eleventh place represent? —. Why so?
11. How many would 7 in the twelfth place represent? —. Why so?
12. How many would 1 in the thirteenth place represent? —. Why so?
13. How many would 9 in the first place represent? —. Why so?

PRACTICAL EXERCISES.

1. Numerate 4365. Why is it four thousand, three hundred and sixty-five?
2. Numerate 75400. Why is it seventy-five thousand, four hundred?
3. Numerate 642100. Why is it six hundred forty-two thousand, one hundred?
4. Numerate 8467521. Why is it so much?
5. Numerate 97654304. Why is it so much?
6. Numerate 985406242. Why is it so much?
7. Write down in figures four hundred and seventy-five.
8. Write down in figures seven thousand and four.
9. Write down in figures twelve thousand eight hundred and ninety six.

10. Write down in figures one hundred, fifty-four thousand, six hundred, and twenty-four.

11. Write down in figures one million, one hundred thousand.

12. Write down seven millions, four hundred and forty-five thousand, three hundred and thirty-nine.

13. Write down in figures forty-four millions, seven hundred, thirty-six thousand, eight hundred, and ten.

14. Write down in figures five hundred seventy-nine millions, four hundred twenty-seven thousand, seven hundred, and twenty-seven.

The teacher may require the learner to write down any other numbers, that he may think proper. After being written on the slate, let the pupil numerate them, and give the reason for assigning to the figures their respective places. In this way he will soon become expert in this part of arithmetic.

QUESTIONS. 1. Is arithmetic a part of mathematics?

2. What part is it?—3. What do you use in performing its operations?

4. What are the primary rules of arithmetic?—5. Of what does numeration treat?

6. Have figures a two-fold value?—7. What are those values called?

8. When is a figure said to have a simple, or absolute value?

9. When has it a local, or relative value?—10. In what proportion does this increase from the right hand toward the left?—17. Will you explain this more fully by an example?

12. Will you repeat the first table, and give the values of the numbers?

13. Are numbers sometimes expressed by letters?

14. What are these letters then called?—15. What is meant, when a letter of less value or signification, is put before one of greater value?

16. What is meant, when it comes after it?—17. Will you repeat the several combinations of letters, and give to each one its proper value?

18. Is this a very inconvenient way of expressing numbers?

ADDITION.

ADDITION is the operation of finding the sum of two or more numbers of the same kind; and is both simple and compound.

Simple addition is the operation of finding the sum of two or more numbers of the same denomination and kind.

Compound addition is the operation of finding the sum of two or more numbers of the same kind, but of different denominations.

This character + placed between two figures or numbers implies that they are to be added together, thus 4+2 are 6; that is, 4 added to 2 are 6.

NOTE.—For directions for placing numbers to be added, and for performing the operation, see *Introduction*, under Addition.

EXAMPLES.

yards.	*acres.*	*inches.*	*rods.*
14567	84236	72154	942315
35823	27143	12673	231423
46254	34555	83426	654272
21024	46623	30237	342734
52433	21352	77434	542735

minutes.	*seconds.*	*days.*	*gallons.*
846217	274216	572461	2164534
300304	132123	982705	1254317
299177	725462	212471	3613512
411311	402703	240613	4000251
245124	275461	352417	2546503
431542	123125	641243	1310742

Addition may be proved by beginning at the top, and adding downward; and, if the result is the same as when you added upward, the operation is correctly performed. Or, separate the upper line of figures, and add together the remainder. Then add the upper line to the sum of the remainder; and if they equal the sum of the whole, then the operation is correctly performed. For the whole of any sum or number must be equal to all its parts.

PRACTICAL EXERCISES.

1. What will be the sum of 54765, 3214, 721, 8970, and 92115? Ans. 159785.

2. What will be the sum of 147462, 90042, 7475, 3210, 72143 and 9214? Ans. 329546.

3. What will be the sum of 80462, 37472, 3164, 98762, 21040, and 376421? Ans. 617321.

4. What will be the sum of 1468, 457, 321, 58469, 30000, and 312? Ans. 91027.

5. A merchant made deposits in the bank of the United States, of the following sums, to wit: 4765 dollars, 2146 dollars, 1041 dollars, and 347 dollars; how much has he in the bank? Ans. 8299 dol.

6. A merchant bought 476 barrels of flour for 2474 dollars, 226 barrels for 1116 dollars, 540 barrels for 2526 dollars: how many barrels of flour has he in all; and how much did it cost him? Ans. 1242 barrels in all.—$6116 cost of the whole.

7. A merchant retiring from business has the several items of property, to wit:—In cash 14740 dols. 25 cts., in real estate 27460 dols. 50 cts., in bank stock 10474 dols., and in notes and bonds 20475, dols. 75 cts.; how much property has he in the whole? Ans. 73150 dols. 50 cts.

8. A merchant bought at public sale, the following quantities of dry goods, to wit: 746 yards of broad-cloth, 1749 yards of muslin, 542 yards of linen, 2164 yards of flannel, and 175 yards of silks; how many yards did he purchase in the whole? Ans. 5376 yds.

9. A farmer in a favourable season raised 2146 bushels of wheat, 4746 bushels of indian corn, 785 bushels of rye, and 940 bushels of oats; how many bushels did he raise in the whole? Ans. 8617.

QUESTIONS. 1. What is addition?—2. How is it divided? 3. What is the difference between them?—4. How do you place the numbers in order to add them together?

5. At which place do you begin to add?—6. When the sum of the units is above 9, what do you do?

7. What do you do with the tens?—8. Why do you add only 1 for every ten?—9. Is 1 in a superior place, equal to 10 in the next lower or inferior place?

10. How do you prove addition?

SUBTRACTION.

SUBTRACTION is the operation of taking a less number from a greater, to find the difference between them.

Subtraction is both simple and compound. When the numbers are of the same denomination and kind, it is called simple subtraction; when they are of the same kind, but of different denominations, it is called compound subtraction.

The greater number is called the minuend, and the less or lower number is called the subtrahend. The difference of the two is called the remainder.

NOTE.—For directions for placing the numbers, and for performing the operation, see *Introduction*, under Subtraction.

A strait line — placed between two figures or numbers implies, that the latter is to be subtracted from the former: thus, 4 — 2, and 2 remain; that is, 2 subtracted from 4, and 2 remain.

EXAMPLES.

	miles.	*rods.*	*yards.*
From	7468912	46215424	896543204
Take	4321641	26854515	749875165
Rem.	3147271		
Proof	7468912		

	men.	*bushels.*	*acres.*
From	8642004	54678954	146548910
Take	5368123	12389768	74895654
Rem.			

Subtraction may be proved by adding the remainder to the less number, and if the sum is equal to the greater number or minuend, the operation is correctly performed. For the difference of two numbers, added to the less, must be equal to the greater.

PRACTICAL EXERCISES.

1. What is the difference between 47698 dollars, and 3241 dollars? Ans. 44457.

2. A merchant has in goods the value of 27460, and owes 2976 dollars upon them; how much has he paid? Ans. 24484 dolls.

3. A gentleman deposited in the bank of the United States, 2504 dollars, and drew out afterward by a check 1605 dollars; how much has he remaining in the bank? Ans. 899 dolls.

4. What is the difference between 10000 and 40? Ans. 9960.

5. What is the difference between 12000 and 101?
Ans. 11899.

6. What is the difference between 150040 and 100101?
Ans. 49939.

7. A lady has an annual income of 1647 dollars; of which she spends 1010 dollars; how much does she save?
Ans. 637 dolls.

8. William, Duke of Normandy, afterward called the Conqueror, overcame Harold, the reigning king of England, in 1066; how many years have passed, reckoning to the year 1830? Ans. 764.

9. A farmer has a tract of land containing 10140 acres, of which he sold 526 acres; how many acres has he remaining?
Ans. 9614 acres.

QUESTIONS. 1. What is subtraction?—2. How is it divided?—3. What is the difference between them?—4. What is the greater number called?—5. What is the less called?—6. What is the difference between them called?

7. How do you place the numbers?—8. How do you perform the operation?—9. When the upper figure is less than the lower one, what do you do?

10. How do you prove subtraction?—11. What is the principle on which this is founded?

MULTIPLICATION.

MULTIPLICATION is the operation of finding the sum of any given number, taken as often as there are units in another given number. It is both simple and compound.

It is simple, when both the numbers are of the same denomination and kind. It is compound, when both the numbers are of the same kind, but of different denominations.

The number to be multiplied is called the multiplicand: the number multiplied by is called the multiplier. The result of the operation is called the product.

The two numbers, taken together, are sometimes called factors, because by them, the operation is carried on, and the result produced.

This character $\times$, placed between two figures or numbers, implies that they are to be multiplied together: thus, 4×2 make 8; that is, 4 multiplied by 2 make 8. It is commonly called St. Andrew's cross.

For the *Table*, and directions for placing the numbers and performing the operation, see *Introduction*, under Multiplication.

EXAMPLES.

Multiplicand	746745	546325	8563045
Multiplier	6	8	9
Product	4480470	4370600	77067405

Multiplicand	567456	346754	4865472
Multiplier	11	12	12
Product			

7. Multiply 5231 by 145. Product 758495
8. Multiply 14063 by 382. Prod. 5372066
9. Multiply 671612 by 114. Prod. 76563768
10. Multiply 7034652 by 144. Prod. 1012989888
11. Multiply 814263 by 75. Prod. 61069725
12. Multiply 7063115 by 96. Prod. 678059040

When there are cyphers at the right hand of the multiplier, or multiplicand, or both, they may be omitted in the operation, and added to the right hand of the product.

When there is one, or more cyphers between the significant figures of the multiplier, they are to be omitted, and the product of the next figure of the multiplier and multiplicand, put as many places higher, as there are cyphers omitted in the operation; as in the following examples.

13. Multiply 74621 by 500. Prod. 37310500

Multiplicand	74621
Multiplier	500
Product	37310500

14. Multiply 65472000 by 65. Prod. 4255680000

Multiplicand 65472000
Multiplier 65
327360
392832
Product 4255680000

15. Multiply 546300 by 4200. Prod. 2294460000

Multiplicand 546300
Multiplier 4200
10926
21852
Product 2294460000

16. Multiply 346754 by 3005. Prod. 1041995770

Multiplicand 346754
Multiplier 3005
1733770
1040262
1041995770

17. Multiply 39580 by 2410. Prod. 95387800
18. Multiply 190046 by 161500. 30692429000
19. Multiply 3800920 by 80750. 306924290000
20. Multiply 3760410 by 4840. 18200384400
21. Multiply 47001881 by 1140090. 53586374509290
22. Multiply 62123000 by 130000. 8075990000000

When the multiplier is 10, 100, 1000, 10000, &c. annex the number of cyphers to the multiplicand, and the operation is performed.

23. Multiply 47654 by 10. Product 476540
24. Multiply 75476 by 100. 7547600
25. Multiply 86427 by 1000. 86427000

When the multiplier is equal to the product of any two numbers, the operation may sometimes be shortened by multiplying first by one of those numbers, and the product thence arising by the other; as in the following examples.

26. Multiply 54632 by 24. Product 1311168

Multiplicand 54632

6

327792=product by 6.

4

1311168=product of 6×4

Here 24 equals the product of 6 multiplied by 4. You may therefore multiply by 6, and then multiply that product by 4; or you may multiply by 4, in the first place, and that product by 6. The product will be the same either way. It will produce the same result, as if you multiplied by 24; and so in all other cases.

27. Multiply 5740632, by 32.	Product	183700224
28. Multiply 7063115, by 96.	Prod.	678059040
29. Multiply 370731, by 36.	Prod.	13346316
30. Multiply 43102, by 64.	Prod.	2758528
31. Multiply 12071, by 99.	Prod.	1195029

Multiplication may be proved in several ways:

First. Double the multiplicand, and multiply that sum by half the multiplier.

Secondly. Take unit from the multiplier, and multiply by the remainder; to the product add the multiplicand.

Thirdly. Divide the product by the multiplier; the quotient will be equal to the multiplicand, if the operation is correct.

Multiplication is the most useful rule in arithmetic. It performs, in a short way, what would require many additions. It brings numbers of higher denominations to lower; and, by knowing the value of one thing, we may determine the value of any number of things of the same kind.

PRACTICAL EXERCISES.

1. One hundred cents make 1 dollar; how many cents are in 125 dollars? Ans. 12500.

2. One thousand mills make 1 dollar; how many mills make 146 dollars? Ans. 146000.

3. Ten dimes make 1 dollar; how many dimes make 546 dollars? Ans. 5460.

4. One hundred mills make 1 dime; how many mills make 244 dimes? Ans. 24400.

5. A person purchased 147 acres of land, at 16 dollars an acre; what did the whole come to? Ans. 2352.

6. A merchant bought 144 pieces of cloth, each containing 29 yards; how many yards were in the whole? Ans. 4176.

7. The floor of a room is 24 feet long and 22 feet wide; how many square feet does it contain? Ans. 328.

8. In a square foot there are 144 inches; how many inches are in 25 square feet? Ans. 3600.

9. How many square inches are in 144 square feet? Ans. 20736.

10. A house contains 26 windows, each containing 24 panes of glass; how many panes are in the whole? Ans. 624.

11. There are 20 shillings in 1 pound; how many are there in 456 pounds? Ans. 9120*s*.

12. There are 12 pence in 1 shilling; how many pence are in 254 shillings? Ans. 3048.

13. There are 24 hours in 1 day; how many hours are in 151 days? Ans. 3624.

14. There are 28 pounds in 1 quarter of a *Cwt.*; how many pounds are in 124 quarters? Ans. 3472.

QUESTIONS. 1. What is multiplication?—2. How is it divided?—3. What is the difference between them?

4. What is the number to be multiplied, called?—5. What is the number by which you multiply, called?—6. What is the result of the operation, called?—7. What are the two given numbers, taken together, sometimes called?—8. Why are they called factors?

9. What does St. Andrew's cross, placed between two numbers, imply?

10. How do you place the numbers for multiplication?

11. At what place do you begin to multiply?

12. When the product of the multiplier, and any figure of the multiplicand is greater than 9, what do you do?

13. Why do you add only the number of tens to the next higher place?

14. When there are cyphers at the right hand of the multiplier, what do you do with them?

15. If there are cyphers at the right hand of the multiplier and multiplicand, what do you do with them?

16. When there is one, or more cyphers between the significant figures of the multiplier, what do you do?

17. When you multiply by 10, 100, 1000, &c., how is the operation performed?

18. When the multiplier is the exact product of any two numbers, how may the operation be performed?

19. Is multiplication a very useful rule?—20. Why is it so useful?

DIVISION.

Division is the operation of finding how often one number is contained in another. It is the reverse of multiplication.

Division is both simple and compound. It is simple, when the dividend is a whole number, or integer: It is compound, when the dividend consists of two or more denominations.

The number to be divided, is called the dividend.

The number by which you divide, is called the divisor.

The number of times the divisor is contained in the dividend, is called the quotient.

The remainder, if there is any, must always be less than the divisor, and of the same denomination with the dividend.

Division is also divided into short and long. It is short, when the divisor does not exceed 12: It is long, when it is greater than 12.

This character ÷ placed between two numbers implies that the former number is to be divided by the latter, thus: 4÷2 signifies that the 4 is to be divided by the 2.

For direction in placing the terms, and for proceeding in the operation, see *Introduction*, under Division.

EXAMPLES.

	miles.	*rods.*	*cents.*
Divisor	4)654324	6)8574684	8)146237456
Quotient	163581	1429114	18279682

	yards.	*gallons.*	*mills.*
Divisor	10)17869400	9)7468542	11)564521705
Quotient	1786940		

minutes.	*seconds.*	*days.*
12)9476521764	11)546215747	12)462174684

The operation may be proved by multiplying the quotient by the divisor, and adding the remainder, if there is any, to the product. If the work is correct, it will equal the dividend.

When the divisor is more than 12, and not greater than 144; and, at the same time, is the exact product of 2 numbers, the operation may be performed by dividing, first, by one of those numbers, and then dividing this quotient by the other. But observe, if there is a remainder after the last division, it must be multiplied by the first divisor, and to the product add the first remainder, if there is any; the sum will be the true remainder. But, if there is a remainder after the first division, and none after the last, that is the true remainder; as in the following examples. :

10. Divide 7463521, by 18. Quotient 414640, and 1 rem.

6)7463521
3)1243920+1 Here 6 and 3 are the factors,
414640 for $6\times3=18$.

11. Divide 734071, by 72. Quotient 10195, and 31 rem.

9)734071
8) 81563+4 1st. rem.
10195+3×9=27+4=31 true rem.

Here 9 and 8 are the factors, for $9\times8=72$.

Here after the last division, 3 remain, which you multiply by 9 the first divisor, and to the product add 4, the remainder after the first division; the sum is 31, the true remainder. In this manner, the operation, in some cases, may be considerably abbreviated.

12. Divide 20208, by 48. Quotient, 421.
13. Divide 57384659, by 144. Quotient, 398504, 83 rem.
14. Divide 43737, by 21. Quotient, 2082, 15 rem.
15. Divide 5704392, by 108. Quotient, 52818, 48 rem.

When there are cyphers at the right hand of the divisor, the operation may be shortened by separating or cutting them off; but, observe to separate or cut off as many figures from the right hand of the dividend. These must be annexed to the remainder after division, if there is any; but if there is none, the figures so cut off are the true remainder.

When the divisor is 10, 100, 1000, &c., the operation may be performed, simply by separating the cyphers, and cutting off from the dividend as many figures, as there are cyphers so separated; the figures so cut off, will be the remainder, and the others will be the quotient; as in the following examples:

16. Divide 746524, by 20. Quotient, 37326, and 4 rem.

2,0)74652,4 rem.
37326 quotient.

17. Divide 6543201, by 400. Quotient, 16358, and 01 rem.

4,00)65432,01
16358+01 rem.

18. Divide 4560700 by 2140. Quotient, 2131, and 360 rem.
19. Divide 783567 by 2100. Quotient, 373, and 267 rem.
20. Divide 137000 by 1600. Quotient, 85, and 1000 rem.
21. Divide 4765431 by 5100. Quotient, 934, and 2031 rem.
22. Divide 47652 by 10. Quotient, 4765, and 2 rem.
23. Divide 24675 by 100. Quotient, 246, and 75 rem.
24. Divide 5798642 by 1000. Quotient, 5798, and 642 rem.
25. Divide 14765216 by 3000. Quotient, 4921, and 2216 remain.
26. Divide 3456745 by 4625. Quotient, 747, and 1870 rem.

4625)3456745(747 quotient.
32375 ··
21924
18500
34245
32375
1870 rem.

Having placed your terms for operation, find how often the divisor is contained in an equal number of figures of the dividend, taken from the left hand, and put it in the quotient; but if they are less than the divisor, as in this example, you must take another figure from the dividend, and put in the quotient a figure, expressing the number of times the divisor is contained in that portion of the dividend. Multiply the divisor by this figure, and place the product under that part of the dividend so set apart. Subtract this from the dividend, and to the remainder bring down the next figure of the dividend. This now becomes the dividend for the next quotient figure. Find how often the divisor is contained in it, and put the figure to the right hand of the one already in the quotient. Multiply the divisor by the last quotient figure, and place the product under the dividend: find the remainder, and to this, bring down the

following figure of the dividend; and so proceed till the operation is finished.

But, if the remainder should be so small, that, when the figure from the dividend is annexed to it, it will not contain the divisor, you must put a cypher in the quotient, and bring down the following figure of the dividend; and, if with this annexed, it will not contain the divisor, as may sometimes be the case, you must put another cypher in the quotient, and so continue to do, until it will contain the divisor, if there are so many figures remaining in the dividend; and, if there are not, this sum will be the remainder.

27. Divide 63125 by 123.	Quot. 513, rem. 26
28. Divide 5374602 by 671.	Quot. 8009, rem. 569
29. Divide 9736205 by 2507.	Quot. 3883, rem. 1524
30. Divide 23470525 by 6425.	Quot. 3653
31. Divide 36737660 by 158694.	Quot. 2315
32. Divide 101442075 by 4025.	Quot. 25203

PRACTICAL EXERCISES.

1. The sum of 5500 dollars is to be divided among 125 persons; how much will each one have? Ans. 44 dollars.

2. How many shares of stock of the Bank of the United States, can be purchased with 6552 dollars, at 117 dollars a share? Ans. 56.

3. If 45980 pounds of bread be distributed among 2420 persons; how much will be the share of each? Ans. 19 lb.

4. A farmer bought a plantation containing 364 acres for 8736 dollars; what was the price by the acre? Ans. 24 dollars.

5. The earth revolves on its axis once in 24 hours; how far will any point of the equator be carried in an hour; the circumference being estimated at 25000 miles? Ans. $1041\frac{2}{3}$ miles.

6. How many generations have past since the birth of Christ to 1830, allowing 30 years to a generation? Ans. 61.

7. What number being multiplied by 7969, the product will be 1864746? Ans. 234.

8. A Virginia planter sold 56 hogsheads of tobacco for 6496 dollars; what did it bring him by the hogshead on an average? Ans. 116 dollars.

9. A merchant bought 4104 yards of cloth; how many pieces would there be of 36 yards each? Ans. 114.

10. Suppose the distance to Liverpool in England to be 3496 miles, and a ship leaving Philadelphia arrives in Liver-

pool in 23 days; how many miles must she sail one day with another? Ans. 152 miles.

QUESTIONS. 1. What is division?—2. Is it the reverse of multiplication?—3. How is it divided?—4. Has it any other division?—5. When is it said to be short?—6. When is it said to be long?—7. When is it said to be simple?—8. When is it said to be compound?

9. What is the number to be divided, called?—What is the number by which you divide, called?—11. What is the number of times the divisor is contained in the dividend, called?—12. If there is a remainder after division, is it of the same denomination with the dividend?—13. Is it also less than the divisor?

14. What character is that, placed between two numbers, implies that the former is to be divided by the latter?

15. How is the operation proved?

16. How do you proceed when the divisor is less than 12?

17. Will you explain the operation by an example?

18. When the divisor is more than 12, but less than 144, and is, at the same time, equal to the product of 2 numbers, how may the operation be performed?

19. Will you explain this by an example?

20. When there are cyphers on the right hand of the divisor, how may the work be abridged?

21. Will you explain this by an example?

22. When the divisor is 10, 100, 100, &c. how may the work be shortened?—23. Will you explain this by an example?

24. What are the figures so cut off from the dividend?

25. When the divisor is more than 12, and is not the product of 2 numbers, nor has cyphers on its right hand, how do you proceed in the operation?—26. Will you explain this by an example?

COMPOUND ADDITION.

COMPOUND ADDITION is the operation of finding the sum of two or more numbers of the same kind, but of different denominations.

Place the numbers so that the same denominations may stand under each other. First add together the right hand column, or column of the lowest denomination, and divide the sum by such number, as it takes of that denomination to make one of

the next higher; set down the remainder, if there is any, under the said column, and add the quotient of such division to the following column, or column of the next higher denomination; and so proceed till the operation is finished.

The work may be proved, in all cases, as in addition of whole numbers, or integers.

FEDERAL CURRENCY.

The addition of this currency is, in every respect, the same as addition of whole numbers or integers. The denominations increase from the lowest to the highest, in a tenfold proportion, and decrease from the highest to the lowest in the same proportion; that is, ten of every lower denomination make one of the next higher, and consequently one of every higher makes ten of the next lower.

For the table, and its application, see *Introduction*, under Federal Money.

EXAMPLES.

D.	*c.*	*m.*
46	25	4
54	37	5
62	50	2
17	46	3
24	16	7
204	76	1

E.	*D.*	*d.*	*c.*
12	4	5	3
16	3	2	4
20	2	7	5
31	4	5	3
51	9	6	7
132	4	7	2

D.	*d.*	*c.*	*m.*
1472	7	3	7
2141	9	8	4
2374	6	3	8
1027	3	5	2
9423	8	7	4
16440	5	8	5

If mills are given also, you add them together, and because 10 mills make 1 cent, you set down for mills all over even tens, and add these to the column of cents: you find the sum of this column, and set down for cents all over even tens, and add them to the column of dimes, because 10 cents make 1 dime. You add up the column of dimes, and set down for dimes all over even tens, because 10 dimes make 1 dollar, and add these to the column of dollars. If eagles are also giving, you must add up the column of dollars, and set down for dollars all over even tens, and add them to the column of eagles, because 10 dollars make 1 eagle. But in business, it is usual to consider eagles and dollars, all as dollars, and dimes and cents, all as cents; as in the first example.

PRACTICAL EXERCISES.

1. A merchant bought, September 14th, goods amounting to 1246 dollars, 50 cents; Oct. 12th, he bought to the amount of 1602 dollars, 75 cents; Nov. 2nd, he bought goods to the amount of 742 dollars, 87½ cents, and Dec. 1st, to the amount of 1799 dollars, 37½ cents; how much was the amount of his several purchases? Ans. $5391,50 cents.

2. A merchant sold 4 lots of goods; the 1st, for 1012 dollars, 27 cents, and 5 mills; the 2d, for 1250 dollars, 75 cents; the 3d, for 741 dollars, 87 cents, and 5 mills; and the 4th, for 549 dollars, 62 cents, and 5 mills; what did the whole amount to? Ans. $3554,52,5.

3. Four several lots of ground were sold in Philadelphia for the following sums: No. 1, for 1574 dollars, 87½ cents; No. 2, for 1241 dollars, 12½ cents, No. 3, for 1165 dollars, 62½ cents, and No. 4, for 965 dollars, 37½ cents; for how much did they all sell? Ans. $4947,00.

4. A lady of fortune made the following deposits in the bank of the United States: At one time, 742 dollars, 50 cents; at another time, 1112 dollars, 75 cents; at a third time, 549 dollars, 37½ cents; at a fourth, 826 dollars, 37½ cents; how much has she in bank? Ans. $3231,00.

5. A gentleman has the following property: a house valued at 5216 dollars, 87½ cents; a farm valued at 1650 dollars, 37½ cents; bank stock to the amount of 7466 dollars, and 25 cents; and cash 5216 dollars, 62½ cents; what is the sum of all his estate? Ans. $19550,12½.

6. A person deceased left to his 4 daughters, the following sums: to the oldest, 2425 dollars, 25 cents; to the next, 2174 dollars, 37½ cents; to the third he left a house valued at 2725 dollars, 50 cents; and to the youngest, whose education was not finished, the sum of 3216 dollars, 50 cents; how much did the whole amount to? Ans. $10541,62½.

7. A person has due to him the following sums of money, (viz.) 574 dollars, 06 cents, 2 mills, and 5 tenths of a mill; 425 dollars, 25 cents, and 4 mills; 175 dollars, 76 cents, 4 mills; 341 dollars, 87 cents, 5 mills; 1021 dollars, 12 cents, 5 mills; how much is due to him in all? Ans. $2538,08,0,5.

SUBTRACTION OF FEDERAL CURRENCY.

The operation is the same as subtraction of integers, or whole numbers.

EXAMPLES.

	D.	c.	m.	D.	c.	m.	D.	c.	m.
From	1746	46	5	1216	37	5	746	62	5
Take	1278	52	4	718	94	1	379	83	2
Rem.									

	D.	c.	m.	D.	c.	m.	D.	c.	m.
From	1946	50	0	8476	12	5	5476	75	0
Take	984	62	5	3792	16	8	2593	48	2
Rem.									

PRACTICAL EXERCISES.

1. A person borrowed the sum of 1647 dollars, 25 cents, and paid 1141 dollars, 12,5 cents; how much does he still owe?
Ans. $506,12,5.

2. A merchant purchased goods amounting to 2416 dollars, 87,5 cents, and paid in hand 748 dollars, 62,5 cents; how much does he owe?
Ans. $1668,25.

3. A gentleman purchased a farm for 7416 dollars, 50 cents, and paid in hand 2746 dollars, 18,7,5 cents; how much remains unpaid?
Ans. $4670,31,2,5.

4. A lady has an annual income of 1226 dollars, 80 cents, and her expenses are 941 dollars, 50 cents; how much less are her expenses than her income?
Ans. $285,30.

5. A young lady had left her by her father the sum of 6427 dollars, 50 cents, but her education is estimated to cost 1189 dollars, 37 cents, 5 mills; how much will she have to receive, when she arrives at the age of 18 years?
Ans. $5238,12,5.

6. A gamester began to play cards with the sum of 1749 dollars, 62,5 cents, and after playing 15 games found that he had only 916 dollars, 25 cents remaining; how much had he lost?
Ans. $833,37,5.

7. A lady went a shopping with the sum of 55 dollars, 55,5 cents; and her purchases amounted to 49 dollars, 56,2,5 cents; how much had she remaining?
Ans. $5,99,2,5.

8. Add together the following sums: 149 dollars, 01 cent; 75 dollars, 75,5 cents; 114 dollars, 25 cents; 137 dollars 37,5 cents; 87,5 cents, and 1144 dollars, 50 cents; and from the amount subtract 1047 dollars, 75 cents; what will remain?
Ans. $574,01,5.

9. A merchant purchased a lot of goods for 1894 dollars, 62,5 cents, and sold the same without delay for 1961 dollars, 55 cents; how much did he gain by the bargain?

Ans. $66,92,5.

10. A gentleman's income is 969 dollars, 80,5 cents a year, and his expenses are 627 dollars, 87,5 cents; how much does he save of his income? Ans. $341,93.

11. What is the difference between ten dollars, ten dimes, and ten cents; and nine dimes and one cent? Ans. $10,19.

MULTIPLICATION OF FEDERAL CURRENCY.

The operation is the same as multiplication of whole numbers.

When a sum of money is multiplied by an integer; separate the product into dollars, cents, and mills, or into dollars and cents, (as the case may be,) by pointing off the first place on the right hand for mills (if any) and the two next places for cents: the remainder will be dollars. Or, when any given number is multiplied by dollars, cents, and mills; by cents only; by cents and mills; or by mills only; observe the same rule.

EXAMPLES.

	D.	c.	m.	
Mult.	78546	37	5,	by 9.
			9	
	706917	37	5	product.

	D.	c.	
Mult.	4764	50,	by 11.
		11	

	D.	c.	m.	
Mult.	17645	86	5,	by 12.
			12	

	D.	c.	
Mult.	9865	36,	by 8.
		8	

	D.	c.	
Mult.	7461,75,		by 24.
		24	
	29847,00		
	149235,0		
	179082,00		Product.

	D.	c.	
Mult.	987,46,		by 54.
		54	
	3949,84		
	49373,0		
	53322,84		Product.

In the first example, dollars, cents and mills are given; in the product, therefore, separate, as above directed, the right

hand figure for mills, the two next for cents: the remainder will be dollars.

In the two last examples, only dollars and cents are given; separate the two places on the right hand for cents; the remainder will be dollars. And so in all other examples.

PRACTICAL EXERCISES.

1. What will 48 peaches come to at 15 mills, or 1 cent, 5 mills? Ans 72 cents.

2. What will 25 pounds of beef come to at 65 mills, or 6 cents, 5 mills? Ans. $1,62,5.

3. What will 12 pounds of butter come to at 125 mills, or 12 cents, 5 mills? Ans. $1,50.

4. What will 5 bushels of apples come to at 375 mills, or 37 cents, 5 mills? Ans. $1,87,5.

5. What will 117 bushels of wheat come to at 875 mills, or 87 cents, 5 mills? Ans. $102,37,5

6. What will 25 barrels of flour come to at 4 dollars, 75 cents a barrel? Ans. $118,75.

7. What will 45 yards of cloth come to at 125 cents, or 1 dollar, 25 cents a yard? Ans. $56,25.

8. What will 127 acres of land come to at 15 dollars, 50 cents an acre? Ans. $1968,50.

9. What will 48 cords of wood come to at 4 dollars, 87 cents, 5 mills a cord? Ans. $234,00.

10. What will 18 pair of shoes come to, at 94 cents a pair? Ans. $16,92.

11. What will 24 pounds of tea come to, at 1,75 cents, or 1 dollar, 75 cents a pound? Ans. $42,00.

12. If a mechanic earns 125 cents, or 1 doll. 25 cents a day, how much will he earn in 28 days? Ans. $35.

13. A gentleman bought a piece of cloth containing 28 yards, at 4375 mills, or 4 dolls. 37 cents, 5 mills a yard; what did the whole cost him? Ans. $122,50.

14. A merchant bought 246 pounds of coffee at 155 mills, or 15 cents, 5 mills, a pound; what did the whole come to? Ans. $38,13.

15 A lady bought, for the use of her family, a barrel of sugar, weighing 312 pounds, at 115 mills, or 11 cents, 5 mills a pound; what did the whole come to? Ans. $35,88.

16. A gentleman and his wife on a journey, spend 240 cents, or 2 dollars, 40 cents, a day; how much will their expenses be for 24 days? Ans. $57,60.

17. If a person spends 145 cents, or 1 dollar, 45 cents a day; how much will he spend in 56 days at that rate? Ans. $81,20.

18. How much will a hogshead of wine containing 63 gallons come to, at 224 cents, or 2 dollars, 24 cents a gallon?
Ans. $141,12.

19. How much will 51 yards of linen come to at 42 cents a yard? Ans. $21,42.

20. If a ton of hay sells for 7 dollars, 50 cents, or 750 cents, what will 13 tons amount to? Ans. $97,50.

21. If a person pays for board 350 cents, or 3 dollars, 50 cents a week, how much will he pay in 26 weeks? Ans. $91.

22. If a ton of coals cost 625 cents, or 6 dollars, 25 cents, how much will 11 tons come to? Ans. $68,75.

When there are mills and tenths of a mill given, you must separate in the product the right hand figure for tenths of a mill, the next figure for mills, and the cents, as above directed.

23. How much will 126 pounds of coffee come to, at 18 cents, 7,5 mills a pound? Ans. $23,62,5.

24. What will 132 pounds of beef come to, at 6 cents, 2,5 mills a pound? Ans. $8,25.

25. What will 174 yards of silk come to at 93 cents, 7,5 mills a yard? Ans. $163,12,5.

26. What will 346 bushels of corn come to, at 56 cents 2,5 mills a bushel? Ans. $194,62,5.

27. What will 2164 pounds of pork come to, at 7 cents, 5 mills a pound? Ans. $162,30.

28. What will 127 yards of muslin come to, at 21 cents, a yard? Ans. $26,67.

29. What will 526 barrels of flour come to, at 5 dollars, 06 cents a barrel? Ans. $2661,56.

DIVISION OF FEDERAL CURRENCY.

The operation is the same as division of integers or whole numbers. But observe to separate the quotient into dollars and cents, or into dollars, cents and mills, as the case may be.

EXAMPLES.

Divide 647 dols. 75 cts. by 5.

5)675,75

135,15 quotient.

Divide 7684 dols. 87 cts. 5 m. by 6.

6)7684,87,5

1280,81,2,5 quotient

		Dolls. cts. m.		*Dolls. cts. m.*
3.	Divide	14765 62 5	by 11. Quotient	1342 32 9,5+
4.	"	1574 25 0	by 12. "	131 18 7,5
5.	"	9876 76 5	by 9. "	1097 41 8,3⅓
6.	"	108 25 2	by 36. "	3 00 7
7.	"	436 56 0	by 214. "	2 04 0
8.	"	673 62 5	by 317. "	2 12 5
9.	"	328 42 5	by 755. "	0 43 5
10.	"	385 02 0	by 186. "	2 07 0
11.	"	10476 31 5	by 144. "	72 75 2+
12.	"	11216 81 2,5	by 75. "	149 55 7,5

PRACTICAL EXERCISES.

1. A lady bought 7 yards of silk for 11,37,5 ; how much was it a yard ? Ans. 1 doll. 62 cts. 5 m.

2. A miller bought 378 bushels of wheat for 380 dollars, 26 cents, 8 mills ; how much was it a bushel ?

Ans. 1 doll. 00 cts. 6 m.

3. A gentleman bought 15 cords of wood for 70 dollars, 75 cents ; how much was it a cord ? Ans. $4,71,6,¢

4. A person purchased a box of tea, weighing 14 pounds, for 17 dollars, 50 cents ; how much was it a pound ? Ans. $1,25.

5. A grocer bought 350 pounds of coffee for 41 dollars, 62 cents, 5 mills ; how much was it a pound ?

Ans. 11 cts. 8,9 m.+a pound.

6. A gentleman bought 8 yards of broad-cloth for 51 dolls. 62 cents, 5 mills ; how much was it a yard ? Ans. $6,45,3,125.

7. A mechanic performed a piece of work in 16 days, for which he received 25 dollars, 76 cents ; how much did he earn a day ? Ans. $1,61.

8. A lady bought for her own use a barrel of sugar, weighing 367 pounds, for 41 dollars, 87 cents, 5 mills ; how much was it a pound ? Ans. 11 cts. 4,1+

9. A farmer sold 14 tons of hay for 137 dollars, 50 cents ; how much was it a ton ? Ans. $9,82,1,4+

10. If 496 pounds of rice cost 15 dollars, 37 cents, 6 mills ; how much is it a pound ? Ans. 3 cts. 1 m.

11. If 24 pair of shoes cost 35 dollars, 25 cents ; how much is the price of one pair ? Ans. $1,46,8+

12. A person spent in 22 days the sum of 37 dollars, 50 cts. ; how much did he spend a day ? Ans. $1,70,4,5+

13. A person spends in a year 741 dollars, 62 cents, 5 mills ; how much is it a day, 365 days being a year ? Ans. $2,03,1,8+

14. The rent of a plantation is 466 dollars, 50 cents a year; what is it a day? Ans. $1,27, 8+

15. A gentleman rents a house for 650 dollars a year; how much is it a month? Ans. $54,16,6,¢.

QUESTIONS. 1. What is compound addition?—2. How do you place the numbers for the operation?

3. How do you proceed in adding up the several columns?

4. How may the work be proved?

5. What are the denominations of Federal currency?

6. By whom was it established?—7. In what year was it established?

8. How is addition performed?—9. What is the reason that it is so performed?—10. In what proportion or ratio, do the several denominations increase?

11. In the transactions of business, how are eagles and dollars considered?—12. How are dimes and cents considered?

13. How is subtraction of Federal currency performed?

14. Why is it so performed?—15. How do you place the terms?—16. How is multiplication of this currency performed?

17. What rule do you observe in dividing the product into dollars and cents; or into dollars, cents, and mills?

18. When mills and tenths of a mill are given, how do you point off the product?

19. How is division of this currency performed?—20. What do you observe in regard to the quotient?

21. How is division proved?

ADDITION

OF POUNDS, SHILLINGS, PENCE, AND FARTHINGS.

For the table, and its application, see *Introduction*.

When the several denominations do not increase in the same ratio, or proportion as in Federal currency, you must add up each column separately, beginning at the right hand, or lowest denomination, and note the sum: divide this sum by as many of that denomination as make one, or unit, of the next greater: set down the remainder, if there is any, under the said column, and add the quotient to the next higher denomination; and so proceed till you come to the last denomination, which add up exactly, as in addition of whole numbers.

This is a general rule, applicable to all cases, whatever may be the denominations.

The work may be proved, as in addition of whole numbers, which see.

EXAMPLES.

£.	s.	d.
487	13	8
512	6	4
671	11	3
764	18	10
2436	10	1

£.	s.	d.
8764	12	8
5421	11	9
2132	6	4
4578	16	7

£.	s.	d.
1245	15	5
3142	12	11
1452	13	2
2785	17	8

£.	s.	d.
746	10	$4\frac{1}{2}$
124	16	2
234	12	$5\frac{1}{4}$
342	6	6
542	7	$11\frac{3}{4}$
125	14	6
421	17	$8\frac{1}{2}$

£.	s.	d.
55	10	$6\frac{3}{4}$
42	7	2
75	12	$4\frac{1}{4}$
12	13	7
17	17	$6\frac{1}{2}$
17	11	9
28	14	$7\frac{1}{4}$

£.	s.	d.
254	16	$7\frac{1}{4}$
147	15	3
241	11	$7\frac{3}{4}$
525	4	8
317	2	$6\frac{1}{2}$
742	11	8
942	17	$9\frac{3}{4}$

In the first example, you add up the column of pence, and find the sum to be 25; which you divide by 12, because 12 pence make 1 shilling, or 1 of the next higher denomination. You set 1, the remainder, under the column of pence, and add or carry 2, the quotient, to the column of shillings; this you also add up, which, with the 2 from the last column, makes 50 shillings. These you must divide by 20, because 20 shillings make 1 pound, or 1 of the next higher denomination; set down the remainder or 10, and add or carry the quotient 2 to the column of pounds; which you add up as whole numbers. If farthings had been given also, as in the last examples, you must have added them up, and divided the sum by 4, because 4 farthings make 1 penny, or one of the next higher denomination; set down the remainder, and added the quotient to the column of pence.

SUBTRACTION

OF POUNDS, SHILLINGS, PENCE, AND FARTHINGS.

When any denomination of the subtrahend is less than that of the minuend, the operation is performed simply by taking their difference, and setting it down under its proper column. That difference will be the remainder required. But, if any term of the subtrahend is greater than the same term of the minuend, you must add to the upper number so many of the said denomination, as will make 1 or unit of the next greater denomination, and from this sum subtract the subtrahend, and set the remainder down under its proper column; add or carry 1 or unit to the subtrahend of the next higher denomination; and so continue to do, when the lower number is greater than the upper one, till you come to the last denomination; which subtract as whole numbers. This is a general rule, applicable to all cases, whatever may be the denominations.

The work may be proved as subtraction of whole numbers, which see.

EXAMPLES.

	£.	s.	d.	£.	s.	d.	£.	s.	d.
Minuend	450	16	8	365	14	5	549	12	6
Subtrahend	230	12	6	247	15	3	427	16	8
Remainder	220	4	2						

	£.	s.	d.	£.	s.	d.	£.	s.	d.
Minuend	756	11	$7\frac{1}{2}$	1100	7	$6\frac{1}{4}$	476	15	$4\frac{1}{2}$
Subtrah.	527	8	$9\frac{1}{4}$	746	12	$3\frac{1}{2}$	219	11	$8\frac{3}{4}$
Rem.									

MULTIPLICATION

OF POUNDS, SHILLINGS, PENCE, AND FARTHINGS.

When the product of the multiplier and any term of the multiplicand is less than the number required of that denomination to make 1 or unit of the next higher, you set down the product under its own proper column, and proceed to the next one.

But, if the product is greater, you must divide it by the number required of that denomination to make 1 or unit of the next

higher; set down the remainder, if there is any, under its own proper column, and add the quotient to the product of the multiplier and the next term of the multiplicand. If this sum is greater than the number required of that denomination to make 1 or unit of the next higher, you must divide, as before, set down the remainder, if there is any, and add the quotient to the next higher denomination; but, if it should be less, you set down the product in its proper place. In like manner you proceed, till you come to the last denomination, which you multiply, and set down as whole numbers.

This is a general rule, applicable to all cases, whatever the denominations may be.

EXAMPLES.

	£	s.	d.		£	s.	d.	
Multiplicand	25	12	6,	by 5.	154	15	4,	by 8.
Multiplier			5				8	multiplier
Product	128	2	6		1238	2	8	product.

	£	s.	d.	£	s.	d.	£	s.	d.
Multiplicand	46	5	$6\frac{1}{2}$	65	15	$3\frac{1}{4}$	56	10	$7\frac{3}{4}$
Multiplier			9			11			12
Product									

DIVISION

OF POUNDS, SHILLINGS, PENCE, AND FARTHINGS.

When the divisor is contained exactly in any term of the dividend, you have only to divide it, and set down the quotient under its own proper column, beginning at the left hand, or highest denomination. But, when there is a remainder, you must multiply it by the number required of the next lower denomination to make 1 or unit of the higher, that is, you must reduce it to the next lower denomination, and add the numbers standing in that lower place to it. This sum divide, and set down the quotient. The remainder, if there is any, you must reduce to the next lower denomination, and add to it the number in the next lower place: this sum divide as before; and so proceed till the work is done.

This rule is general, applicable to all cases, whatever may be the denominations.

EXAMPLES.

Divide £147 16*s*. 6*d*., by 4.

	£.	*s*.	*d*.
4)	147	16	6
Quotient	36	19	1½

Divide £465 12*s*. 8*d*., by 6.

	£.	*s*.	*d*.
6)	465	12	8
Quotient	77	12	1¼+

Divide £547 15*s*. 7½*d*. by 4.
£79 17*s*. 4*d*. by 7.
£1461 18*s*. 10*d*. by 11

Divide £1142 17*s*. 9½*d*. by 9.
£124 11*s*. 4*d*. by 10.
£7640 11*s*. 4¾*d*. by 12.

In the first example, after dividing the pound by 4, there are 3 remaining: which multiply by 20, because 20 shillings make a pound; to the product add the 16 shilling in the next lower place; the sum is 76 shillings. This divides by 4 without a remainder: the quotient 19 set down in shilling's place; and divide 6 the next lower denomination by 4; the quotient is 1, which set down in pence place: the remainder 2, you multiply by 4, because 4 farthings make 1 penny: divide the product 8, by the divisor, and nothing remains: the quotient is 2, or ½ a penny, that is, 2 farthings; and the work is done.

In like manner proceed in the other examples. On the same principle are all compound numbers divided, whatever may be their denominations.

QUESTIONS. 1. What is componnd addition?—2. How many shillings make a pound?—3. How many pence make a shilling?—4. How many farthings make a penny?—5. How do you proceed in the addition of this money?—6. How do you consider the last column, or highest denomination?

7. Do you proceed in a similar way in the addition of any other denomination?

8. What is compound substraction?—9. How do you proceed in compound subtraction?—10. What do you mean by the subtrahend?—11. What by the minuend?—12. Is this rule applicable to numbers, whatever may be their denominations?

13. What is compound multiplication?—14. How do you proceed in compound multiplication?—15. How do you consider the last term or highest denomination?

16. Is this rule applicable to numbers consisting of other denominations?

17. What is compound division?—18. With what denomination do you begin?—19. How do you proceed in the

operation?—20. Will this method apply to numbers consisting of other denominations?

21. How do you prove the operation of compound addition?

22. Do you prove that of subtraction, multiplication, and division in the same way?

ADDITION OF TROY WEIGHT.

For the table, and the denominations, see INTRODUCTION, under troy weight.

For directions to perform the operation, see ADDITION of pounds, shillings and pence.

EXAMPLES.

lb.	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*
8	5	10	12	12	7	14	16	48	10	19	23
6	4	11	15	11	9	10	12	56	9	7	00
9	2	4	6	7	8	17	19	26	11	12	20
10	7	6	16	14	7	8	22	19	7	6	5
11	5	14	7	17	8	12	21	12	8	11	15
Sum											

SUBTRACTION OF TROY WEIGHT.

For directions to perform the operation, see SUBTRACTION of pounds, shillings and pence.

EXAMPLES.

	lb.	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*
From	48	6	10	4	17	10	12	16	25	7	17	16
Take	25	7	6	8	4	11	7	20	12	8	16	19
Rem.												

MULTIPLICATION OF TROY WEIGHT.

For directions to perform the operation, see MULTIPLICATION of pounds, shillings, and pence.

EXAMPLES.

	lb.	*oz.*	*dwt.*	*gr.*
Multiply	22	5	10	16
by				4
Prod.				

	lb.	*oz.*	*dwt.*	*gr.*
Multiply	46	10	14	6
by				7

	lb.	*oz.*	*dwt.*	*gr.*
Multiply	16	8	4	6
by				8
Prod.				

	lb.	*oz.*	*dwt.*	*gr.*
Multiply	116	3	2	7
by				9

DIVISION OF TROY WEIGHT.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

	lb.	*oz.*	*dwt.*	*gr.*	
Divide	36	10	11	12	by 4.

	lb.	*oz.*	*dwt.*	*gr.*	
Divide	47	11	16	17	by 6

	lb.	*oz.*	*dwt.*	*gr.*
4)	36	10	11	12
Quot.				

	lb.	*oz.*	*dwt.*	*gr.*
6)	47	11	16	17
Quot.				

Divide 18 *lb.* 7 *oz.* 12 *dwt.* 15 *gr.* by 8.
Divide 12 *lb.* 8 *oz.* 16 *dwt.* 4 *gr.* by 7.
Divide 55 *lb.* 10 *oz.* 14 *dwt.* 22 *gr.* by 9.
Divide 116 *lb.* 9 *oz.* 17 *dwt.* 4 *gr.* by 12.

PRACTICAL EXERCISES.

1. A silversmith purchased 4 ingots of silver; the first weighed 2 lb. 6 oz. 10 dwt. 4 gr.; the second, 3 lb. 8 oz. 0 dwt. 16 gr.; the third, 3 lb. 11 oz. 16 dwt. 22 grains; and the fourth, 4 lb. 3 oz. 7 dwt. 16 gr.; what was the weight of the whole?

Ans. 14 lb. 5 oz. 15 dwt. 10 gr.

2. A gentleman just married purchased the following articles of a silversmith, (viz.) bowls weighing 4 lb. 6 oz. 11 dwt. 20 gr.; plates weighing 5 lb. 10 oz. 16 dwt. 8 gr.; spoons weighing 3 lb. 4 oz. 6 dwt. 5 gr.; and salts weighing 10 oz. 4 dwt. 12 gr.; what was the weight of the whole? Ans. 14 lb. 7 oz. 18 dwt. 21 gr.

3. A gentleman presented his daughter, on her marriage day, with the following articles of plate, (viz.) spoons weighing 3 lb.

11 oz. 6 dwt.; a tea-set weighing 4 lb. 8 oz. 0 dwt. 20 gr.; a tankard weighing 5 lb. 3 oz. 18 dwt.; and salts weighing 8 oz. 16 dwt. 7 gr.; what was the weight of the whole?

Ans. 14 lb. 8 oz. 1 dwt. 3 gr.

4. A silversmith purchased 16 lb. 8 oz. 12 dwt. 20 gr. of silver, and manufactured 5 lb. 11 oz. 16 dwt. 4 gr. of it; how much has he remaining? Ans. 10 lb. 8 oz. 16 dwt. 16 gr.

5. A silversmith wrought 24 lb. 5 oz. 16 dwt. of silver bullion into spoons, plates, salts, and bowls; and afterward weighed the articles, and found that their weight was only 23 lb. 11 oz. 18 dwt. and 22 gr.; how much was the loss in the whole?

Ans. 5 oz. 17 dwt. 2 gr.

6. A silversmith has 6 ingots of silver, each weighing 6 lb. 4 oz. 14 dwt. and 6 grains; how much is the weight of the whole? Ans. 38 lb. 4 oz. 5 dwt. 12 gr.

7. A person has a quantity of silver, in weight 16 lb. 7 oz. 0 dwt. 20 gr., and wishes to divide it into 5 equal parts; how much will there be in each part? Ans. 3 lb. 3 oz. 16 dwt. 4 gr.

8. A gentleman deceased ordered his plate to be equally divided between his three daughters. Its weight was found to be 17 lb. 2 oz. 17 dwt. and 16 gr.; what was the share of each one? Ans. 5 lb. 8 oz. 19 dwt. $5\frac{1}{3}$ gr.

ADDITION OF AVOIRDUPOIS WEIGHT.

For the table, and denominations, see *Introduction* under avoirdupois weight.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

T.	*C.*	*qr.*	*lb.*
16	5	2	16
12	14	3	22
24	6	1	16
54	17	0	22
31	5	3	11
44	16	1	25
Sum			

C.	*qr.*	*lb.*	*oz.*
5	2	21	12
21	1	16	4
32	2	24	8
41	3	12	5
54	1	11	14
61	0	22	0

Qr.	*lb.*	*oz.*	*dr.*
2	16	4	5
1	21	10	10
3	4	6	11
0	26	12	9
3	14	14	14
1	11	15	5

SUBTRACTION OF AVOIRDUPOIS WEIGHT.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	T.	*C.*	*qr.*	*lb.*
From	44	16	1	21
Take	29	17	0	14
Rem.				

	C.	*qr.*	*lb.*	*oz.*
From	56	2	12	8
Take	41	3	13	4
Rem.				

	qr.	*lb.*	*oz.*	*dr.*
From	3	11	12	6
Take	1	16	8	12
Rem.				

MULTIPLICATION OF AVOIRDUPOIS WEIGHT.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	T.	*C.*	*qr.*	*lb.*
Mult.	7	14	2	10
by				5
Prod.				

	C.	*qr.*	*lb.*	*oz.*
Mult.	16	2	21	6
by				7
Prod.				

	T.	*C.*	*qr.*	*lb.*
Mult.	16	7	1	16
by				8
Prod.				

	T.	*C.*	*qr.*	*lb.*	*oz.*
Mult.	5	1	24	6	10
by					9
Prod.					

DIVISION OF AVOIRDUPOIS WEIGHT.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 22 *T.* 16 *C.* 2 *qr.* 21 *lb.*, by 5.

	T.	*C.*	*qr.*	*lb.*
5)	22	16	2	21
Quot.				

Divide 24 *C.* 3 *qr.* 17 *lb.* 10 *oz.* 6 *dr.*, by 7

	C.	*qr.*	*lb.*	*oz.*	*dr.*
7)	24	3	17	10	6
Quot.					

Divide 56 *T.* 12 *C.* 1 *qr.* 16 *lb.*, by 8.
Divide 25 *T.* 7 *C.* 3 *qr.* 24 *lb.*, by 10.
Divide 14 *C.* 1 *qr.* 22 *lb.* 7 *oz.* 10 *dr.*, by 3.
Divide 37 *C.* 2 *qr.* 17 *lb.* 11 *oz.* 13 *dr.*, by 11.

PRACTICAL EXERCISES.

1. A grocer bought 3 hogsheads of sugar; the weight of the 1st, was 12 C. 3 qr. 16 lb.; of the 2d, was 14 C. 2 qr. 22 lb.; and of the 3d, was 15 C. 3 qr. 16 lb.; how much was there in the whole? Ans. 43 C. 1 qr. 26 lb.

2. A planter in Virginia sold 4 hogsheads of tobacco; No. 1, weighed 8 C. 2 qr. 16 lb.; No. 2, weighed 9 C. 3 qr. 20 lb.; No. 3, weighed 10 C. 1 qr. 17 lb.; and No. 4, weighed 12 C. 0 qr. 17 lb.; what was the weight of the whole?

Ans. 41 C. 0 qr. 14 lb.

3. A merchant purchased 4 bags of cotton, weighing as follow: No. 1, 4 C. 1 qr. 16 lb.; No. 2, 5 C. 2 qr. 14 lb.; No. 3, 3 C. 3 qr. 22 lb.; and No. 4, 5 C. 1 qr. 20 lb.; what was the weight of the whole? Ans. 19 C. 1 qr. 16 lb.

4. A merchant bought 24 C. 3 qr. 16 lb. of rice, and sold 15 C. 1 qr. 22 lb.; how much remains on hand?

Ans. 9 C. 1 qr. 22 lb.

5. A gentleman bought 21 T. 17 C. 2 qr. of coal, and sold 9 T. 19 C. 3 qrs.; how much has he on hand?

Ans. 11 T. 17 C. 3 qr.

6. A merchant purchased 6 hogsheads of sugar, each weighing 16 C. 2 qr. 12 lb.; how much was the weight of the whole?

Ans. 99 C. 2 qr. 16 lb.

7. A grocer bought 12 boxes of figs, each weighing 3 qr. 14 lb. 12 oz.; what was the weight of the whole? Ans. 42. qr. 9 lb.

8. A merchant has 4 parcels of iron; in each of which there are 5 C. 3 qr. and 20 lb.; how much is there of the whole?

Ans. 23 C. 2 qr. 24 lb.

9. A farmer cut 25 T. 17 C. 3 qrs. of hay; which he wished to put into 3 stacks for the better convenience of his stock; how much was in each stack? Ans. 8 T. 12 C. 2 qr. $9\frac{1}{3}$ lb.

10. In a certain heap there are 16 T. 15 C. 3 qrs. of coal, which it is required to divide into 5 equal parts; how much will be in each part? Ans. 3 T. 7 C. 0 qr. $16\frac{4}{5}$ lb.

11. A merchant has 10 T. 17 C. 2 qr. of iron, which he wishes to divide into 8 equal parts; how much will there be in each? Ans. 1 T. 7 C. 0 qr. 21 lb.

ADDITION OF APOTHECARIES' WEIGHT.

For the table, and the denominations, see *Introduction* under apothecaries' weight.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

	lb	℥	ʒ	℈	gr.
	6	7	5	1	10
	10	11	3	2	8
	12	4	1	0	11
	14	8	6	2	4
	7	6	5	1	16
	4	10	7	0	14
Sum					

	lb	℥	ʒ	℈	gr.
	14	7	6	2	19
	11	11	7	1	14
	12	7	6	2	5
	7	5	2	0	8
	16	9	1	2	16
	21	8	5	1	12

SUBTRACTION OF APOTHECARIES' WEIGHT.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	lb	℥	ʒ	℈	gr.
From	12	7	6	1	10
Take	9	11	5	2	16
Rem.					

	lb	℥	ʒ	℈	gr.
	18	6	5	1	14
	11	7	6	2	10

MULTIPLICATION OF APOTHECARIES' WEIGHT.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	lb	℥	ʒ	℈	gr.
Mult.	5	10	6	1	12
By					5
Prod.					

	lb	℥	ʒ	℈	gr.
	7	6	7	2	16
					8

DIVISION OF APOTHECARIES' WEIGHT.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

	℔	℥	ʒ	℈	gr.		℔	℥	ʒ	℈	
Divide	14	7	6	1	10, by 6.	Divide	18	10	7	2, by 8.

	℔	℥	ʒ	℈	gr.		℔	℥	ʒ	℈
6)	14	7	6	1	10	8)	18	10	7	2
Quot.										

Divide 6 ℔, 8 ℥, 5 ʒ, 2 ℈, 10 gr., by 6.
Divide 11 ℥, 6 ʒ, 1 ℈, 19 gr., by 9.
Divide 11 ℔, 10 ℥, 6 ʒ, 1 ℈, 19 gr., by 7.
Divide 24 ℥, 7 ʒ, 2 ℈, 16 gr., by 10.

PRACTICAL EXERCISES.

1. A druggist would mix 4 sorts of drugs; the 1st sort weighing 7 ℔, 4 ℥, 6 ʒ, 2 ℈, 16 gr.; the 2d sort, 16 ℔, 11 ℥, 5 ʒ, 0 ℈, 19 gr.; the 3d, 9 ℔, 9 ℥, 5 ʒ, 2 ℈, 16 gr.; the 4th, 8 ℔, 11 ℥, 7 ʒ, 0 ℈, 12 gr.; what is the weight of the whole?

Ans. 43 ℔, 2 ℥, 1 ʒ, 1 ℈, 3 gr.

2. An apothecary having mixed 12 ℔, 10 ℥, 5 ʒ, 2 ℈, 16 gr. of drugs, would take from the mass 4 ℔, 5 ℥, 6 ʒ, 1 ℈, 12 gr.; how much will remain? Ans. 8 ℔, 4 ℥, 7 ʒ, 1 ℈, 4 gr.

3. A druggist would compound 4 several sorts of drugs in equal proportions of 5 ℔, 10 ℥, 6 ʒ, 2 ℈, 15 gr., each; what will be the weight of the whole? Ans. 23 ℔, 7 ℥, 3 ʒ, 2 ℈.

4. A druggist compounded a quantity of drugs, weighing in the whole, 16 ℔, 10 ℥, 7 ʒ, 2 ℈, 16 gr.; which he would divide into 6 parcels; how much would there be in each?

Ans. 2 ℔, 9 ℥, 6 ʒ, 1 ℈, $19\frac{2}{6}$ gr.

ADDITION OF LONG MEASURE.

For the table, and denominations, see *Introduction* under long measure.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

Deg.	*M.*	*fur.*	*P.*
12	42	5	30
16	36	7	27
39	21	4	21
46	25	3	17
21	17	6	33
26	15	2	16
Sum.			

Yds.	*ft.*	*in.*	*b.*	*c.*
246	2	10	1	
341	1	11	2	
410	2	7	1	
114	0	6	0	
211	1	5	2	
416	2	8	1	

SUBTRACTION OF LONG MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	Deg.	*m.*	*fur.*	*P.*
From	24	36	5	16
Take	17	41	3	18
Rem.				

Yds.	*ft.*	*in.*
176	2	10
127	1	11

Ft.	*in.*	*b.c.*
2	7	2
1	9	2

MULTIPLICATION OF LONG MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	Deg.	*m.*	*fur.*	*P.*
Multiply	10	16	4	21
by				6
Prod.				

	Yds.	*ft.*	*in.*	*b.c.*
Multiply	125	2	10	2
by				7

	Deg.	*m.*	*fur.*	*P.*
Multiply	16	4	7	30
by				8
Prod.				

	Yds.	*ft.*	*in.*	*b.c.*
Multiply	241	2	11	1
by				9

DIVISION OF LONG MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 46 deg. 35 m. 5 fur. and 26 P. by 5.
Divide 246 yds. 2 ft. 10 in. and 2 b.c. by 7.

Deg. m. fur. P.	*Yds. ft. in. b.c.*
5)46 35 5 26	7)246 2 10 2

Divide 160 deg. 52 m. 4 fur. 16 P. by 9.
Divide 48 deg. 54 m. 7 fur. 24 P. by 11.
Divide 45 m. 6 fur. 120 yds. 2 ft. by 12.
Divide 134 m. 7 fur. 175 yds. 1 ft. by 10.

PRACTICAL EXERCISES.

1. What is the sum of 5 deg. 16 m. 5 fur. 36 P.; 10 deg. 54 m. 5 fur. 26 P.; 26 deg. 36 m. 4 fur. 16 P.; 35 deg. 40 m. 7 fur. 36 P.; and 12 deg. 16 m. 2 fur. 25 P.?

Ans. 90 deg. 45 m. 2 fur. 19 poles.

2. A person set out on a journey; the first day he went 25 m. 4 fur. 16 P.; the second, 27 m. 7 fur. 36 P.; the third, 30 miles, 5 fur. 35 P.; the fourth, 35 miles, 5 fur. 39 P.; how far did he travel in the whole? Ans. 120 m. 0 fur. 6 poles.

3. What is the difference between 140 degrees, 40 m. 7 fur. 16 P. and 74 deg. 50 m. 4 fur. 26 P.?

Ans. 65 deg. 50 m. 2 fur. 30 poles.

4. Suppose the distance from Philadelphia to Trenton, to be 38 m. 5 fur. 36 P., and to New York, 95 m. 7 fur. 16 P.; how far is the distance from Trenton to New York?

Ans. 57 m. 1 fur. 20 poles.

5. Suppose the distance from Philadelphia to Lancaster to be 65 m. 6 fur. 36 P.; how many miles must a person travel to go the distance 6 times? Ans. 395 m. 1 fur. 16 poles.

6. A gentleman has a field exactly square, and one side measures 175 yds. 2 ft. 9 in.; can you tell how many yards will be in all the sides? Ans. 703 yds. 2 feet.

7. A person in 6 days travelled 154 miles, 6 fur. 22 P.; how far did he travel one day with another?

Ans. 25 m. 6 fur. 17 poles.

8. A farmer has a square field, whose sides altogether measure 3 m. 3 fur. 24 P.; what is the length of one side?

Ans. 6 fur. 36 poles.

ADDITION OF CLOTH MEASURE.

For the table, and the denominations, see *Introduction* under cloth measure.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

	Yds.	*qr.*	*na.*	*E. E.*	*qr.*	*na.*	*E. F.*	*qr.*	*na.*
	125	2	1	54	4	3	41	2	3
	216	3	2	62	3	1	62	1	2
	171	3	1	41	3	2	34	2	3
	55	2	3	25	4	3	21	1	0
	16	1	2	16	1	2	49	2	1
	142	2	1	17	4	2	52	0	2
Sum									

SUBTRACTION OF CLOTH MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	Yds.	*qrs.*	*na.*	*E.E.*	*qr.*	*na.*	*E.F.*	*qr.*	*na.*
From	75	2	3	66	3	2	124	2	2
Take	46	3	1	24	4	3	85	2	3
Rem.									

MULTIPLICATION OF CLOTH MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	Yds.	*qr.*	*na.*	*E.E.*	*qr.*	*na.*	*E.F.*	*qr.*	*na.*
Multiply	116	3	2	74	4	1	56	2	1
by			6			8			9
Prod.									

DIVISION OF CLOTH MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 175 yds. 3 qrs. and 2 na. by 6.
Divide 216 yds. 2 qrs. and 1 na. by 8.

	Yds.	*qr.*	*na.*		*Yds.*	*qr.*	*na.*
	6)175	3	2		8)216	2	1
Quotient							

Divide 76 E. Eng. 4 qrs. and 3 na. by 9.
Divide 57 E. Eng. 3 qrs. and 2 na. by 7.
Divide 216 E. Flem. 2 qrs. and 3 na. by 11.
Divide 124 E. Flem. 1 qr. and 3 na. by 12.

PRACTICAL EXERCISES.

1. A merchant bought 4 pieces of cloth; No. 1, containing 25 yds. 3 qrs. 1 na.—No. 2, 30 yds. 2 qr. 2 na.—No. 3, 32 yds. 1 qr. 2 na.—No. 4, 36 yds. 3 qrs. 3 na.; what is the number of yards in the whole? Ans. 125 yds. 3 qrs.

2. A merchant sold the following pieces of cloth, viz: No. 1, containing 12 yds. 3 qrs. 3 na.—No. 2, 16 yds. 2 qrs. 1 na.—No. 3, 14 yds. 2 qrs. 3 na.—No. 4, 18 yds. 1 qr. 3 na., and No. 5, 20 yds. 2 qrs. 3 na.; how much did he sell in all?
Ans. 83 yds. 1 qr. 1 na.

3. A person bought a piece of broad-cloth, containing 24 yds. 2 qrs. 3 na., and sold of it 8 yds. 3 qrs.; how much has he on hand? Ans. 15 yds. 3 qrs. 3 na.

4. A merchant tailor purchased a piece of cloth, containing 42 yds. 3 qrs. 2 na., and cut from it 5 yds. 1 qr. 3 na.; how much remains? Ans. 37 yds. 1 qr. 3 na.

5. A merchant bought 6 pieces of cloth, each containing 26 yds. 2 qrs. 1 na.; how many yards were in the whole?
Ans. 159 yds. 1 qr. 2 na.

6. If you buy 18 yds. 2 qrs. 1 na. of cloth, and make use of 8 yds. 2 qrs. 2 na.; how many yards have you remaining?
Ans. 9 yds. 3 qrs. 3 na.

7. If you have 42 yds. 1 qr. 3 na. of cloth, and divide them nto 5 parts; how many yards will there be in each?
Ans. 8 yds. 1 qr. $3\frac{4}{5}$ na.

8. A merchant bought 8 pieces of broad-cloth, containing in the whole 164 yds. 2 qrs. 3 na.; how many yards were in each piece, supposing they were all of equal length?
Ans. 20 yds. 2 qrs. $1\frac{3}{8}$ na.

ADDITION OF LAND MEASURE.

For the table, and the denominations, see *Introduction* under land measure.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

	A.	*R.*	*P.*	*A.*	*R.*	*P.*	*A.*	*R.*	*P.*
	120	2	30	27	3	36	196	3	30
	219	3	25	19	2	21	247	2	36
	112	1	26	41	1	00	516	2	11
	74	2	16	35	3	11	172	3	17
	27	2	27	48	2	17	217	1	00
	71	3	36	55	0	36	415	3	33
Sum									

SUBTRACTION OF LAND MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	A.	*R.*	*P.*	*A.*	*R.*	*P.*	*A.*	*R.*	*P.*
From	74	2	16	54	2	17	144	1	36
Take	27	3	32	27	3	12	79	3	27
Rem.									

MULTIPLICATION OF LAND MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	A.	*R.*	*P.*		*A.*	*R.*	*P.*
Multiply	47	3	27	Multiply	175	3	21
by			6	by			8
Prod.				Prod.			

	A.	*R.*	*P.*		*A.*	*R.*	*P.*
Multiply	214	2	25	Multiply	96	3	36
by			11	by			12
Prod.				Prod.			

DIVISION OF LAND MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 172 *A.* 3 *R.* 34 *P.* by 6.

	A.	*R.*	*P.*
6)	172	3	34
Quot.			

Divide 68 *A.* 3 *R.* 39 *P.* by 9.

	A.	*R.*	*P.*
9)	68	3	39
Quot.			

Div. 240 *A.* 2 *R.* 26 *P.* by 8.

Div. 500 *A.* 2 *R.* 29 *P.* by 12.

Div. 360 *A.* 3 *R.* 39 *P.* by 7.

Div. 211 *A.* 3 *R.* 21 *P.* by 11.

PRACTICAL EXERCISES.

1. A farmer has several tracts of land; No. 1. containing 47 acres, 3 roods, 26 perches; No. 2, 27 acres, 2 roods, 26 perches; No. 3, 36 acres, 3 roods, 29 perches; No. 4, 17 acres, 2 roods, 27 perches; how much has he in all? Ans. 130 A. 0 R. 28 P.

2. A planter of Virginia has the following fields under cultivation; No. 1, containing 46 acres, 2 roods, 27 perches; No. 2, containing 59 acres, 3 roods, 16 perches; No. 3 containing 65 acres, 2 roods, 36 perches; and No. 4, containing 74 acres, 1 rood, 16 perches; how many acres has he in all?

Ans. 246 A. 2 R. 15 P.

3. A gentleman having a tract of land containing 527 acres, 3 roods, 36 perches, sold 129 acres, 3 roods, 39 perches; how much has he remaining? Ans. 397 A. 3 R. 37 P.

4. A farmer having 456 acres, 2 roods, 34 perches of land, set off to his son on his marriage day, 175 acres, 2 roods, 36 perches, as his portion of his estate; how much had he remaining? Ans. 280 acres, 3 roods, 38 perches.

5. A person has 4 tracts of land, each containing 24 acres, 1 rood, and 22 perches; how much has he in all?

Ans. 97 acres, 2 roods, 8 perches.

6. A gentleman has 6 fields exactly of the same size, each containing 12 acres, 2 roods, and 20 perches; how many acres do they all contain? Ans. 75 acres, 3 roods.

7. A landholder divided 1326 acres, 3 roods, and 36 perches of land equally among his 5 sons; how many acres had each one? Ans. 265 acres, 1 rood, $23\frac{1}{5}$ perches.

8. A farmer having a tract of land containing 216 acres, 2 roods, and 16 perches, would divide it into 12 equal lots; how many acres must each one contain? Ans. 18 A. 0 R. 8 P.

ADDITION OF LIQUID MEASURE.

For the table, and the denominations, see *Introduction*, under Liquid Measure.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

	T.	*hhd.*	*gal.*	*Hhd.*	*gal.*	*qt.*	*Gal.*	*qt.*	*pt.*
	12	3	36	18	21	3	56	3	1
	10	2	45	25	48	1	47	2	0
	9	1	21	36	12	2	75	3	1
	16	2	31	75	52	3	27	0	1
	22	3	48	81	24	2	36	3	1
Sum									

SUBTRACTION OF LIQUID MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	T.	*hhd.*	*gal.*	*Hhd.*	*gal.*	*qt.*	*Gal.*	*qt.*	*pt.*
From	27	2	26	127	42	1	57	2	0
Take	18	3	45	59	61	2	49	3	1
Rem.									

MULTIPLICATION OF LIQUID MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	T.	*hhd.*	*gal.*		*Hhd.*	*gal.*	*qt.*	*pt.*
Multiply	12	3	25	Multiply	12	42	3	1
By			7	By				8
Prod.				Prod.				

	Gal.	*qt.*	*pt.*		*Hhd.*	*gal.*	*qt.*	*pt.*
Multiply	54	2	1	Multiply	24	16	2	1
By			11	By				12
Prod.				Prod.				

DIVISION OF LIQUID MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

T. Hhd. gal. qt.
Div. 35 3 47 2, by 8.

T. hhd. gal. qt.
8)35 3 47 2
Quot.

Hhd. gal. qt. pt.
Div. 74 36 2 1, by 9.

T. hhd. gal. qt.
9)74 36 2 1

Hhd. gal. qt.
Divide 64 48 3, by 11.
Divide 121 36 2, by 10.

Gal. qt. pt.
Divide 146 3 0, by 12.
Divide 97 3 1, by 6.

PRACTICAL EXERCISES.

1. A merchant has 4 sorts of wine in as many casks, which measure as follow: No. 1, 124 gals. 2 qts. 1 pt.—No. 2, 100 gals. 3 qts.—No. 3, 87 gals. 2 qts. 1 pt.—No. 4, 96 gal. 0 qt. 1 pt.; how many gallons has he in all? Ans. 409 gal. 0 qt. 1 pt.

2. A farmer filled the following casks with cider: No. 1, containing 75 gals. 2 qts.—No. 2, 67 gals. 1 qt. 1 pt.—No. 3, 96 gals. 2 qts.—No. 4, 110 gals. 3 qts.; how much cider do they all contain? Ans. 350 gals. 0 qts. 1 pt.

3. A farmer made 627 gals. 3 qts. 1 pt. of cider, and sold 249 gals. 1 qt., how much remains? Ans. 378 gals. 2 qts. 1 pt.

4. A grocer purchased 5 hhds. 46 gals. 2 qts. of wine, and sold 2 hhds. 54 gals. and 3 qts.; how much remains unsold? Ans. 2 hhds. 54 gals. 3 qts.

5. A farmer, at a cider-making, filled 6 casks, each containing 85 gals. 3 qts. 1 pt.; how much cider did he make in all? Ans. 515 gals. 1 qt. 0 pt.

6. A brewer filled 8 casks with beer, each containing 45 gals. 3 qts.; how many gallons were there in the whole? Ans. 366.

7. A grocer purchased 25 bbls. 12 gals. and 3 qts. of cider, which he would divide into 8 equal parts; how many barrels allowing 32 gallons to the barrel, would there be in each part? Ans. 3 bbls. 5 gals. $2\frac{3}{8}$ qts.

8. A wine merchant imported 14 hhds. 47 gals. and 3 qts. of wine, which he would put into 12 equal casks; how many gallons were there in each? Ans. 1 hhd. 14 gals. 1qt. 1pt. $\frac{10}{12}$ over.

ADDITION OF DRY MEASURE.

For the table, and the denominations, see *Introduction*, under Dry Measure.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

	Bu.	*p.*	*qt.*	*Bu.*	*p.*	*qt.*	*Bu.*	*p.*	*qt.*
	75	3	1	125	2	4	226	0	6
	84	2	0	264	3	1	420	3	5
	46	3	7	136	2	0	731	2	4
	26	2	6	45	0	7	129	0	7
	16	3	5	84	3	2	314	5	2
	10	0	6	275	0	6	114	2	1
Sum									

SUBTRACTION OF DRY MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	Bu.	*p.*	*qt.*	*Bu.*	*p.*	*qt.*	*Bu.*	*p.*	*qt.*
From	219	2	4	74	1	6	274	1	5
Take	126	8	7	36	2	7	95	2	6
Rem.									

MULTIPLICATION OF DRY MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	Bu.	*p.*	*qt.*	*pt.*		*Bu.*	*p.*	*qt.*
Mult.	117	3	6	1	Mult.	246	2	6
by				7	by			9
Prod.								

	B.	*p.*	*qt.*	*pt.*		*B.*	*p.*	*qt.*
Mult.	74	3	5	1	Mult.	56	2	6
by				11	by			12
Prod.								

DIVISION OF DRY MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 112 *bu.* 3 *p.* 4 *qt.* by 7.

	Bu.	*p.*	*qt.*
7)	112	3	4
Quot.			

Divide 227 *bu.* 3*p.* 7*qt.* 1 *pt.* by 8.

	Bu.	*p.*	*qt.*	*pt.*
8)	227	3	7	1

Div. 255 *bu.* 2 *p.* 7 *qt.* 1 *pt.* by 9.

Div. 736 *bu.* 2 *p.* 5 *qt.* by 12.

Div. 274 *bu.* 4 *p.* 5 *qt.* 1 *pt.* by 10.

Div. 327 *bu.* 5 *p.* 6 *qt.* by 11.

PRACTICAL EXERCISES.

1. A farmer raised of Indian corn 526 *bu.* 3 *p.* 7 *qt.*; of wheat 375 *bu.* 2 *p.* 6 *qt.*; of oats 216 *bu.* 0 *p.* 6 *qt.*; of barley 146 *bu.* 3 *p.* 5 *qt.*; of rye 312 *bu.* 2 *p.*; how many bushels did he raise in all? Ans. 1578 *bu.* 1 *p.* 0 *qt.*

2. A miller purchased of one farmer 246 *bu.* 3 *p.* of wheat; of another 327 *bu.* 2 *p.*; of a third 504 *bu.* 0 *p.* 6 *qt.*; of a fourth 462 *bu.* 3 *p.* 5 *qt.*; how many bushels did he purchase in all? Ans. 1541 *bu.* 1 *p.* 3 *qt.*

3. A farmer raised 749 *bu.* 2 *p.* 7 *qt.* of wheat; and sold 247 *bu.* 3 *p.* 5 *qt.*; how many bushels has he remaining? Ans. 501 *bu.* 3 *p.* 2 *qt.*

4. A farmer raised 562 *bu.* 2 *p.* 6 *qt.* of the several kinds of grain; and estimating the consumption of his family and stock, to be 215 *bu.* 3 *p.* 7 *qt.*; how much will he have for sale. Ans. 346 *bu.* 2 *p.* 7 *qt.*

5. A miller has in 5 several bins, 215 *bu.* 2 *p.* 6 *qt.* of wheat each; how many bushels has he in all? Ans. 1078 *bu.* 1 *p.* 6 *qt.*

6. If a farmer raises 816 *bu.* 2 *p.* 6 *qt.* of wheat yearly; how many bushels would he raise in 8 years? Ans. 6533 *bu.* 2 *p.*

7. A miller has 2150 *bu.* 3 *p.* of wheat; and can grind the whole in 7 days; how many bushels must he grind one day with another, to finish the whole? Ans. 307 *bu.* 1 *p.*

8. If you divide 765 *bu.* 1 *p.* 6 *qt.* into 9 equal parts; what will one part be? Ans. 85 *bu.* 0 *p.* $1\frac{5}{9}$ *qt.*

ADDITION OF MOTION, OR CIRCLE MEASURE.

For the table, and the denominations, see *Introduction* under Motion, &c.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

Sig.	°	′	″	°	′	″	°	′	″
2	14	46	25	21	34	25	16	27	33
7	27	54	56	32	16	24	12	24	43
8	17	27	25	17	42	37	11	17	16
10	11	12	14	35	16	21	10	42	24
14	21	42	24	15	13	12	9	16	15
11	9	29	30	11	11	17	8	21	31
Sum.									

SUBTRACTION OF MOTION, OR CIRCLE MEASURE.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

Sig.	°	′	″	*Sig.*	°	′	″	°	′	″
From 11	16	10	12	9	29	21	16	75	41	48
Take 5	24	35	4	6	22	29	46	36	56	55
Rem.										

MULTIPLICATION OF MOTION, OR CIRCLE MEASURE.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

Sig.	°	′	″	*Sig.*	°	′	″
Mult. 3	25	27	31	Mult. 5	12	16	29
by			6	by			8
Prod.				Prod.			

	°	′	″		°	′	″
Mult.	47	37	27	Mult.	54	26	56
by			11	by			12
Prod.				Prod.			

DIVISION OF MOTION, OR CIRCLE MEASURE.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

Divide 8 *sig.* 25° 31′ 46″, by 6.

	sig.	°	′	″
6)	8	25	31	46
Quot.				

Divide 3 *sig.* 46° 56′ 44″, by 7.

	sig.	°	′	″
7)	3	46	56	44

Divide 11 *sig.* 16° 45′ 55″ by 8.

Divide 9 *sig.* 17° 45′ 55″ by 5.

Divide 154° 56′ 46″ by 9.

Divide 241° 35′ 27″ by 11.

PRACTICAL EXERCISES.

1. There are 3 places on the same parallel of latitude; the first is 34°, 26′, 35″, in east longitude; the second is 45°, 27′, 35″, east of the first; and the third is 28°, 39′, 46″, east of the second; what is the longitude of the last place?

Ans. 108 deg. 34′, 7″.

2. There are 2 places in the same latitude; one situated in 56°, 27′, 45″, east longitude; and the other in 75°, 46′ 25″ west longitude; how far are they from each other?

Ans. 132 deg. 14′, 10″

3. There are 2 places in the same latitude, one in 96°, 27′, 32″ east longitude, and the other in 37°, 46′ 29″ east longitude; what in the difference of their longitude?

Ans. 58 deg. 40′, 53″.

4. When the earth has moved through 5 sig. 26 deg. 27 min. 36 sec., how far has she to go, to complete one revolution around the sun? Ans. 6 sig. 3 deg. 32′, 24″.

ADDITION OF TIME.

For the table, and the denominations, see *Introduction*, under Time.

For directions to perform the operation, see *Addition* of pounds, shillings, and pence.

EXAMPLES.

Y.	*m.*	*d.*
12	6	20
16	7	16
42	11	24
19	9	21
24	3	22
36	7	25
Sum		

D.	*h.*	*m.*	*sec.*
35	16	24	20
27	21	46	55
24	16	30	32
17	12	18	00
41	15	10	12
14	9	9	9

M.	*d.*	*h.*
8	20	16
10	21	18
9	17	9
11	23	23
22	9	10
31	11	11

SUBTRACTION OF TIME.

For directions to perform the operation, see *Subtraction* of pounds, shillings, and pence.

EXAMPLES.

	Y.	*m.*	*d.*	*Y.*	*m.*	*d.*	*D.*	*h.*	*m.*	*sec.*
From	25	7	21	55	8	20	216	12	20	30
Take	16	8	24	31	9	22	174	16	31	45
Rem.										

MULTIPLICATION OF TIME.

For directions to perform the operation, see *Multiplication* of pounds, shillings, and pence.

EXAMPLES.

	Y.	*m.*	*d.*	*h.*		*D.*	*h.*	*m.*	*sec.*
Multiply	5	7	24	8	Multiply	162	12	17	31
By				6	By				8
Prod.					Prod.				

	Y.	*m.*	*d.*		*D.*	*h.*	*m.*	*sec.*
Multiply	24	5	10	Multiply	217	15	12	45
By			9	By				12
Prod.				Prod.				

DIVISION OF TIME.

For directions to perform the operation, see *Division* of pounds, shillings, and pence.

EXAMPLES.

	Y.	*M.*	*d.*	*h.*		*D.*	*h.*	*min.*	*sec.*
Div.	116	10	26	6, by 7.	Div.	241	16	20	25, by 8.
7)	116	10	16	6	8)	241	16	20	25
Quot.									

	Y.	*m.*	*d.*	*h.*		*d.*	*h.*	*min.*	*sec.*
Div.	17	9	24	8, by 6.	Div.	324	17	34	44, by 8.
Div.	23	10	16	10, by 9.	Div.	235	19	36	55, by 11.

PRACTICAL EXERCISES.

1. A lady has 3 daughters; the age of the eldest is 15 years, 5 months, and 16 days—of the next, 12 years, 10 months, and 22 days—of the youngest, 10 years, 11 months, and 12 days; what is the sum of their ages? Ans. 39 yrs. 3 m. 20 d.

2. A person resided in Philadelphia from his birth 24 years, 9 months, and 22 days—afterward in New York, 6 years, 8 months, and 10 days: from thence he removed to Boston, where he resided 10 years, 3 months, and 12 days; what is his age? Ans. 41 yrs. 9 m. 14 d.

3. A gentleman has two sons; the elder is 17 years, 5 months, and 19 days, and the difference of their ages is 4 years, 7 months, and 24 days; what is the age of the younger?

Ans. 12 yrs. 9 m. 25 d.

4. A young lady finished her education when she was 16 years, 4 months, and 15 days old; she had been at school 7 years, 9 months, and 24 days; at what age did she begin?

Ans. 8 yrs. 6 m. 21 d.

5. A gentleman took a lease of a farm for 7 years, and after he had held it 4 years, 3 months, and 16 days, how much time remained? Ans. 2 yrs. 8 m. 14 ds.

6. What will be the product of 5 years, 10 months, 16 days, 7 hours, and 20 minutes, multiplied by 6?

Ans. 35 yrs. 3 m. 7 d. 20 h. 0 min.

7. A gentleman resided 7 years, 5 months, and 12 days in Philadelphia, which was exactly one seventh part of his age; how old was he? Ans. 51 yrs. 1 m. 24 d.

8. A married lady is 36 years, 10 months, and 24 days old, and her eldest daughter is 14 years, 11 months, and 19 days; how old was she when her daughter was born?

Ans. 21 yrs. 11 m. 5 d.

9. What will be the quotient of 64 years, 10 months, 17 days, 5 hours, 24 minutes, and 36 seconds, divided by 6?

Ans. 10 yrs. 9 m. 22 d. 20 h. 54 min. 26 sec.

VULGAR FRACTIONS.

A vulgar fraction is a part or parts of an integer, or whole number, and is expressed thus, $\frac{2}{3}$, $\frac{1}{4}$, which mean two divided by three, or two-thirds, and one divided by four, or one-fourth.

The upper number is called the numerator, and expresses the integer.

The number below the line is called the denominator, and expresses the parts into which the integer is divided.

Vulgar fractions are *proper*, *improper*, *compound*, and *mixed*.

A proper fraction has the numerator less than the denominator, thus, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$.

An improper fraction has the numerator equal to, or greater, than the denominator, thus, $\frac{2}{2}$, $\frac{5}{2}$, $\frac{7}{6}$.

A compound fraction is a fraction of a fraction, thus, $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{1}{2}$.

A mixed number is a whole number and a fraction, thus, $7\frac{2}{3}$, $5\frac{1}{2}$.

A whole number, or integer, is expressed in the form of a fraction thus, $\frac{6}{1}$, $\frac{8}{1}$.

Let the pupil express by figures, in the form of a fraction, the following numbers: one divided by two, or one-half; two divided by three, or two-thirds; three-fourths; five-sixths; seven-eighths; nine-tenths; eleven-twelfths; thirteen-fifteenths; twenty divided by twenty-four; twenty-five divided by one hundred; forty divided by twenty-two; eleven divided by five; eight divided by four; five divided by three; two divided by one, &c. What kind of fractions are they?

PRACTICAL EXERCISES.

1. How do you set down the $\frac{1}{2}$ of 6 in the form of a vulgar fraction?—A. I set down $\frac{1}{2}$ and $\frac{6}{1}$, and multiply the numerators of the fractions together, for a new numerator, and the denominators together for a new denominator; and I have this fraction $\frac{6}{2}$, or 6 divided by 2. This gives 3, the half of 6; for 6 contains 2, three times.

2. How do you set down the $\frac{1}{3}$ of 12 in the form of a vulgar fraction?—A. I set down $\frac{1}{3}$ and $\frac{12}{1}$, and I multiply the numerators together for a new numerator, and the denominators together for a new denominator; and then I have this fraction, $\frac{12}{3}$. This gives 4, the third of 12; for 12 contains 3, four times.

3. How do you set down the $\frac{1}{5}$ of 20 in the form of a vulgar fraction?—A. I set down $\frac{1}{5}$ and $\frac{20}{1}$ and multiply the numerators together for a new numerator, and the denominators together for a new denominator, and I have $\frac{20}{5}$. This gives 4, the fifth of 20; for 20 contains 5, four times.

4. How do you set down $\frac{1}{3}$ of 7 in the form of a vulgar fraction?—A. I set down $\frac{1}{3}$ and $\frac{7}{1}$, and multiply as in the preceding examples, and I have $\frac{7}{3}$. This being divided gives 2, and $\frac{1}{3}$ over; for 7 contains 3 twice, and one-third, or one divided by three, over.

5. Will you set down 2 divided by 3 in the form of a vulgar fraction?—How much is it?—What kind of a fraction is it?

6. Will you set down 11 divided by 12?—How much is it?—What kind of a fraction is it?

7. Will you set down 9 divided by 16?—How much is it?—What kind of a fraction is it?

8. Will you set down 7 divided by 5?—What kind of a fraction is it?—How much is it?

9. Will you set down 46 divided by 16?—What kind of a fraction is it?—How much is it?

10. What is the $\frac{1}{3}$ of 63? Ans. 21.—How do you find it?

11. What is the $\frac{2}{3}$ of 33? Ans. 22.—How do you find it?

12. What is the $\frac{2}{3}$ of 75? Ans. 50.—How do you find it?

13. What is the $\frac{4}{5}$ of 30? Ans. 24.—How do you find it?

14. What is the $\frac{5}{6}$ of 120? Ans. 100.—How do you find it?

15. What is the $\frac{7}{8}$ of 16? Ans. 14.—How do you find it?

16. What is the $\frac{11}{12}$ of 60? Ans. 55.—How do you find it?

17. What is the $\frac{1}{20}$ of 100? Ans. 5.—How do you find it?

18. If you have $\frac{5}{6}$ of a farm valued at 2500 dollars, what is your part worth? Ans. 2083 dolls. $33\frac{1}{3}$.—How do you find it?

19. If you own $\frac{3}{4}$ of a ship, valued at 5000 dollars, what is your share worth? Ans. 3750 dolls.—How do you find it?

20. A lot of ground is valued at 1000 dollars: what is $\frac{1}{2}$ of it worth? What is $\frac{2}{3}$ of it worth? What is $\frac{4}{5}$ of it worth? What is $\frac{7}{8}$ of it worth? What is $\frac{1}{10}$ of it worth? What is $\frac{1}{15}$ of it worth? What is $\frac{1}{20}$ of it worth?—What are these several answers?—How do you find them?

21. If you own a house worth 1200 dollars: for what sum would you sell $\frac{1}{4}$ of it? For what sum would you sell $\frac{3}{4}$ of it? For what sum would you sell $\frac{5}{6}$ of it? For what sum would you sell $\frac{9}{10}$ of it? For what sum would you sell $\frac{7}{8}$ of it?—What are these several answers?—How do you find them?

22. A cargo of flour is valued at 5500 dollars; how much must you pay for $\frac{3}{4}$ of it? How much for $\frac{7}{8}$ of it? How much for $\frac{11}{12}$ of it? How much for $\frac{7}{16}$ of it?—What are these several answers?—How do you find them?

23. A gentleman's income arising from public stock is 2000 dollars a year; how much is it for $\frac{1}{2}$ a year? How much for a $\frac{1}{4}$ of a year? How much for $\frac{3}{4}$ of a year? How much for $\frac{4}{5}$ of a year?—What are these several answers?—How do you find them?

REDUCTION OF VULGAR FRACTIONS.

A COMMON measure of a fraction is a number, that will divide the numerator and denominator without a remainder.

The greatest common measure is the greatest number, that will divide the numerator and denominator without a remainder; as, 3 is the greatest common measure of the fraction $\frac{9}{12}$, because it is the greatest number that will divide them without a remainder. It will then become $\frac{3}{4}$, which is of the same value of $\frac{9}{12}$.

A fraction is said to be in its lowest terms, when no number above a unit, will divide both parts of it without leaving a remainder ; thus, $\frac{5}{6}$ is in its lowest terms, because no number above 1 will divide the parts without a remainder.

CASE 1.

To reduce a fraction to its lowest terms.

RULE.—Divide both parts of the fraction by the greatest number that will leave no remainder; and, if no number above a unit will so divide them, the fraction is already in its lowest terms.

1. Reduce $\frac{12}{24}$ to its lowest terms. Ans. $\frac{1}{2}$.

Here by inspection, you see that 12 will divide both parts of the fraction without leaving a remainder, and it is the greatest number that will so divide it. The fraction then becomes $\frac{1}{2}$; which is of the same value with $\frac{12}{24}$.

2. Reduce $\frac{8}{12}$ to its lowest terms. Ans. $\frac{2}{3}$.

Here by inspection you find that 4 is the greatest number, which will divide both parts of the fraction without a remainder. Let it be divided, and it will become $\frac{2}{3}$, which is of the same value with $\frac{8}{12}$.

3. Reduce $\frac{16}{24}$ to its lowest terms. Ans. $\frac{2}{3}$.
4. Reduce $\frac{27}{36}$ to its lowest terms. Ans. $\frac{3}{4}$.
5. Reduce $\frac{25}{100}$ to its lowest terms. Ans. $\frac{1}{4}$.
6. Reduce $\frac{5}{15}$ to its lowest terms. Ans. $\frac{1}{3}$.
7. Reduce $\frac{6}{9}$ to its lowest terms. Ans. $\frac{2}{3}$.
8. Reduce $\frac{24}{48}$ to its lowest terms. Ans. $\frac{1}{2}$.

If the greatest common measure cannot readily be discovered by inspection, the division may be repeated, until you reduce the fraction so that no number above unit will divide it without a remainder. The fraction then will be in its lowest terms. Thus: let $\frac{72}{144}$ be reduced to its lowest terms. Divide, in the first place, by 2, which will reduce the fraction to $\frac{36}{72}$; which will divide by 4. It will then be reduced to $\frac{9}{18}$. Here I find by inspection that 9 will divide both parts of the fraction without a remainder. The division being made, the fraction becomes $\frac{1}{2}$, which is the same value of $\frac{72}{144}$. In like manner,

9. Reduce $\frac{48}{56}$ to its lowest terms. Ans. $\frac{6}{7}$.
10. Reduce $\frac{36}{60}$ to its lowest terms. Ans. $\frac{3}{5}$.
11. Reduce $\frac{90}{630}$ to its lowest terms. Ans. $\frac{1}{7}$.
12. Reduce $\frac{432}{576}$ to its lowest terms. Ans. $\frac{3}{4}$.
13. Reduce $\frac{164}{492}$ to its lowest terms. Ans. $\frac{1}{3}$.
14. Reduce $\frac{182}{196}$ to its lowest terms. Ans. $\frac{13}{14}$.

Or, divide the greater part of the fraction by the less, and this divisor by the number remaining; and so on till there is no remainder: the last divisor will be the greatest common measure; by which divide both parts of the fraction, and it will be reduced to its lowest terms.

15. Reduce $\frac{60}{125}$ to its lowest terms. Ans. $\frac{12}{25}$.

60)125(2
120
5)60(12
60
..

5)$\frac{60}{125}$=$\frac{12}{25}$ the lowest terms.

16. Reduce $\frac{288}{480}$ to its lowest terms. Ans. $\frac{3}{5}$.

288)480(1
288
192)288(1
192
96)192(2
192
...

96)$\frac{288}{480}$=$\frac{3}{5}$ the lowest terms.

In the first of these examples, I divide 125 by 60, and I have 5 remaining; by which I divide 60, the previous divisor, and nothing remains; 5 therefore is the greatest common measure. Divide both parts of the fraction by 5, and you have $\frac{12}{25}$, the answer, which are the lowest terms of the fraction. In the same manner proceed with the following fractions.

17. Reduce $\frac{84}{170}$ to its lowest terms. Ans. $\frac{42}{85}$.
18. Reduce $\frac{324}{648}$ to its lowest terms. Ans. $\frac{1}{2}$.
19. Reduce $\frac{9876}{88884}$ to its lowest terms. Ans. $\frac{1}{9}$.
20. Reduce $\frac{7654}{9876}$ to its lowest terms. Ans. $\frac{3827}{4938}$.

CASE 2.

To reduce several single fractions to others retaining the same value, but having a common denominator.

RULE.—Multiply each numerator into all the denominators except its own, for its respective numerator; then multiply all the denominators together for a common denominator.

NOTE.—Each fraction, in the first place, should be reduced to its lowest terms, if it is not so already.

EXAMPLES.

1. Reduce $\frac{3}{6}$, $\frac{8}{10}$, and $\frac{2}{4}$ to a common denominator.

Ans. $\frac{10}{20}$, $\frac{16}{20}$, $\frac{10}{20}$.

Before you begin the operation, reduce the fractions to their lowest terms; thus, $\frac{3}{6}$ divided by 3 becomes $\frac{1}{2}$: $\frac{8}{10}$ divided by 2 becomes $\frac{4}{5}$: and $\frac{2}{4}$ divided by 2 becomes $\frac{1}{2}$. These are of the same value with the original fractions $\frac{3}{6}$, $\frac{8}{10}$, and $\frac{2}{4}$. The fractions to be reduced to a common denominator are, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{1}{2}$.

$$\left.\begin{array}{l} 1\times5\times2=10 \\ 4\times2\times2=16 \\ 1\times2\times5=10 \end{array}\right\}\text{numerators.}$$

$2\times5\times2=20$ the common denominator.

The fractions become, $\frac{10}{20}$, $\frac{16}{20}$, $\frac{10}{20}$; which express the values of $\frac{3}{6}$, $\frac{8}{10}$, and $\frac{2}{4}$.

2. Reduce $\frac{2}{4}$, $\frac{4}{8}$, $\frac{11}{12}$, to a common denominator.

Ans. $\frac{24}{48}$, $\frac{24}{48}$, $\frac{44}{48}$.

Here you observe the two first fractions are not in their lowest terms. Let them be reduced, and they become $\frac{1}{2}$, and $\frac{1}{2}$; the last fraction is already in its lowest terms. You then have these fractions: $\frac{1}{2}$, $\frac{1}{2}$, $\frac{11}{12}$.

$$\left.\begin{array}{l} 1\times2\times12=24 \\ 1\times2\times12=24 \\ 11\times2\times2=44 \end{array}\right\}\text{the numerators.}$$

$2\times2\times12=48$ the common denominator.

The fractions become $\frac{44}{48}$, $\frac{24}{48}$, $\frac{24}{48}$. These fractions are of the same value with $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{11}{12}$, the given fractions. In like manner, let the following fractions be reduced to a common denominator.

3. Reduce $\frac{6}{9}$, $\frac{12}{24}$, and $\frac{7}{8}$, to a common denominator.

Ans. $\frac{32}{48}$, $\frac{24}{48}$, $\frac{42}{48}$.

4. Reduce $\frac{2}{3}$, $\frac{7}{21}$, and $\frac{24}{36}$, to a common denominator.

Ans. $\frac{18}{27}$, $\frac{9}{27}$, $\frac{18}{27}$.

5. Reduce $\frac{2}{4}$, $\frac{3}{5}$, and $\frac{2}{3}$, to a common denominator.

Ans. $\frac{15}{30}$, $\frac{18}{30}$, $\frac{20}{30}$.

6. Reduce $\frac{6}{10}$, $\frac{4}{8}$, $\frac{1}{9}$, and $\frac{6}{7}$, to a common denominator.

Ans. $\frac{378}{630}$, $\frac{315}{630}$, $\frac{70}{630}$, $\frac{540}{630}$.

7. Reduce $\frac{4}{10}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{6}{8}$, to a common denominator.

Ans. $\frac{120}{300}$, $\frac{100}{300}$, $\frac{120}{300}$, $\frac{225}{300}$.

8. Reduce $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$, to a common denominator.

Ans. $\frac{840}{960}$ $\frac{860}{960}$ $\frac{880}{960}$.

9. Reduce $\frac{4}{5}$, $\frac{1}{2}$, $\frac{2}{8}$, and $\frac{3}{9}$, to a common denominator.

Ans. $\frac{96}{120}$, $\frac{60}{120}$, $\frac{30}{120}$, $\frac{40}{120}$.

10. Reduce $\frac{6}{9}$, $\frac{7}{10}$, $\frac{2}{6}$, and $\frac{1}{4}$, to a common denominator.

Ans. $\frac{240}{360}$, $\frac{252}{360}$, $\frac{120}{360}$, $\frac{90}{360}$.

CASE 3.

To reduce a mixed number to an improper fraction:

RULE.—Multiply the whole number by the denominator of the fractional part; to the product add the numerator of the fractional part; and under the sum, place the denominator.

EXAMPLES.

1. Reduce $19\frac{7}{10}$ to an improper fraction. Ans. $\frac{197}{10}$.

Here you multiply 19 by 10, and add to the product 7, the numerator of the fractional part, which will make 197; under this, place 10, the denominator of the fractional part, thus, $\frac{197}{10}$; which is the answer. In like manner reduce the following mixed numbers.

2. Reduce $27\frac{2}{3}$ to an improper fraction. Ans. $\frac{83}{3}$.
3. Reduce $5\frac{7}{12}$ to an improper fraction. Ans. $\frac{67}{12}$.
4. Reduce $9\frac{3}{4}$ to an improper fraction. Ans. $\frac{39}{4}$.
5. Reduce $46\frac{7}{9}$ to an improper fraction. Ans. $\frac{421}{9}$.
6. Reduce $54\frac{16}{23}$ to an improper fraction. Ans. $\frac{1258}{23}$.
7. Reduce $125\frac{45}{100}$ to an improper fraction. Ans. $\frac{12545}{100}$.

CASE 4.

To reduce an improper fraction to a mixed number:

RULE.—Divide the numerator by the denominator.

NOTE.—This case is the reverse of the preceding one.

EXAMPLES.

1. Reduce $\frac{219}{17}$ to its proper terms. Ans. $12\frac{15}{17}$.
2. Reduce $\frac{354}{18}$ to its proper terms. Ans. $19\frac{12}{18}$.
3. Reduce $\frac{67}{12}$ to its proper terms. Ans. $5\frac{7}{12}$.
4. Reduce $\frac{421}{9}$ to its proper terms. Ans. $46\frac{7}{9}$.
5. Reduce $\frac{961}{17}$ to its proper terms. Ans. $56\frac{9}{17}$
6. Reduce $\frac{12588}{23}$ to its proper terms. Ans. $54\frac{16}{23}$
7. Reduce $\frac{13}{7}$ to its proper terms. Ans. $1\frac{6}{7}$
8. Reduce $\frac{144}{16}$ to its proper terms. Ans. 9.
9. Reduce $\frac{256}{4}$ to its proper terms. Ans. 64.
10. Reduce $\frac{1764}{14}$ to its proper terms. Ans. 126.

CASE 5.

To reduce a compound fraction to a single one:

RULE.—Multiply all the numerators together for a new numerator; and all the denominators together for a new denominator.

But, in the first place, reduce the fractions to their lowest terms, if they are not so already.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{4}{6}$ of $\frac{8}{10}$ to a single fraction. Ans. $\frac{16}{45}$.

Here you perceive $\frac{4}{6}$ and $\frac{8}{10}$ are not in their lowest terms. By dividing them by 2, they become $\frac{2}{3}$ and $\frac{4}{5}$, which are their lowest terms. The compound fraction to be reduced then will be, $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{4}{5}$.

$2 \times 2 \times 4 = 16$ product of the numerators.

$3 \times 3 \times 5 = 45$ product of the denominators. The single fraction therefore is $\frac{16}{45}$.

2. Reduce $\frac{12}{14}$ of $\frac{6}{10}$ of $\frac{2}{5}$ of $\frac{3}{9}$ to a single fraction. Ans. $\frac{36}{525}$.

Here the first, second, and fourth fractions are not in their lowest terms. Let them be reduced, and the fraction becomes $\frac{6}{7}$ of $\frac{3}{5}$ of $\frac{2}{5}$ of $\frac{1}{3}$.

$6 \times 3 \times 2 \times 1 = 36$ the product of the numerator.

$7 \times 5 \times 5 \times 3 = 525$ the product of the denominators. The single fraction is, therefore, $\frac{36}{525}$.

3. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a single fraction. Ans. $\frac{6}{24} = \frac{1}{4}$.

4. Reduce $\frac{1}{2}$ of $\frac{8}{9}$ of $\frac{6}{7}$ to a single fraction. Ans. $\frac{48}{126} = \frac{8}{21}$.

5. Reduce $\frac{1}{3}$ of $\frac{4}{7}$ of $\frac{11}{13}$ to a single fraction. Ans. $\frac{44}{272} = \frac{11}{68}$.

6. Reduce $\frac{5}{7}$ of $\frac{1}{3}$ of $\frac{8}{9}$ of $\frac{10}{17}$ to a single fraction. Ans. $\frac{400}{3213}$.

When the same figure or figures occur both in the numerator and denominator, they may be cancelled without affecting the result of the operation. This depends upon the well known axiom, that if equals be multiplied or divided by the same number, the product, or quotient will be equal accordingly.

7. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction. Ans. $\frac{2}{5}$.

Here you find 3 in the denominator of the first, and in the numerator of the second, fraction. Also 4 in the denominator of the second, and in the numerator of the third: all of which may be cancelled, leaving $\frac{2}{5}$ for the answer, which are the only remaining terms, or parts of the given fractions.

8. Reduce $\frac{7}{8}$ of $\frac{8}{16}$ of $\frac{3}{7}$ to a single fraction. Ans. $\frac{3}{16}$.

9. Reduce $\frac{5}{9}$ of $\frac{9}{12}$ of $\frac{8}{15}$ to a single fraction. Ans. $\frac{40}{360} = \frac{2}{9}$.

10. Reduce $\frac{3}{4}$ of $\frac{5}{5}$ of $\frac{7}{8}$ of $\frac{9}{10}$ to a single fraction. Ans. $\frac{189}{320}$.

CASE 6.

To reduce a fraction of a less denomination, to the fraction of a greater denomination, retaining the same value:

RULE.—Compound the given fraction with the several denominations, expressed in the form of a fraction, between its own, and the one, to which you would reduce it: this compound fraction reduce to a single one, as directed in the preceding case.

NOTE.—Observe to place the several denominations, with which the given fraction is to be compounded, as denominators, and a unit or 1, as the several numerators.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a cent to the fraction of a dollar. Ans. $\frac{1}{120}$.

Here between $\frac{5}{6}$ of a cent, and a dollar are two denominations, *cents* and *dimes*. You therefore compound it in the following manner, $\frac{5}{6}$ of $\frac{1}{10}$ of $\frac{1}{10}$; because 10 cents make a dime, and 10 dimes make one dollar.

Therefore $\frac{5}{6} \times \frac{1}{10} \times \frac{1}{10} = \frac{5}{600} \div 5 = \frac{1}{120}$.

The product of the numerators is 5, and that of the denominators 600. The fraction $\frac{5}{600}$, divided by 5, the greatest common measure, gives $\frac{1}{120}$ of a dollar, for the value of $\frac{5}{6}$ of a cent.

2. Reduce $\frac{7}{8}$ of an *oz.* troy to the fraction of a pound.

Ans. $\frac{7}{96}$. lb.

Here there is only one denomination between the ounce and the pound, namely, 12. Let $\frac{7}{8}$ be compounded with it, and you have $\frac{7}{8}$ of $\frac{1}{12}$, or $\frac{7}{8} \times \frac{1}{12} = \frac{7}{96}$. In like manner proceed in the following examples.

3. Reduce $\frac{5}{6}$ of a penny to the fraction of a £.

Ans. $\frac{5}{1440} = \frac{1}{288}$ £.

4. Reduce $\frac{4}{5}$ of a gallon to the fraction of a hhd. Ans. $\frac{4}{315}$.

5. Reduce $\frac{5}{9}$ of a pound avoirdupois to the fraction of a cwt.

Ans. $\frac{5}{1008}$.

6. Reduce $\frac{3}{9}$ of a cent to the fraction of a dollar.

Ans. $\frac{1}{300}$. dol.

7. Reduce $\frac{9}{10}$ of a minute to the fraction of a day.

Ans. $\frac{9}{14400} = \frac{1}{1600}$.

8. Reduce $\frac{2}{3}$ of an hour to the fraction of a year of 365 days.

Ans. $\frac{2}{2628} = \frac{1}{1314}$.

9. Reduce $\frac{3}{4}$ of a pint to the fraction of a barrel of 32 gallons. Ans. $\frac{3}{1024}$.

10. Reduce $\frac{7}{9}$ of a perch to the fraction of an acre.

Ans. $\frac{7}{1440}$

11. Reduce $\frac{5}{6}$ of a yard to the fraction of a mile.

Ans. $\frac{5}{10560} = \frac{1}{2112}$

CASE 7.

To reduce a fraction of a greater denomination to the fraction of a less denomination, retaining the same value:

RULE.—Compound the given fraction with the several denominations between it and the one, to which you would reduce it: this compound fraction reduce to a single one, by case 5th, and this again reduce to its lowest terms, by case 1.

NOTE.—Observe to place the several denominations, with which you compound the given fraction, as numerators, and a unit or 1, as the several denominators, contrary to the order of the preceding case.

EXAMPLES.

1. Reduce $\frac{5}{1008}$ of a cwt. to the fraction of a pound.

Ans. $\frac{5}{9}$ lb.

Here there is one denomination, namely, 112: which place as a numerator with a unit for its denominator, thus, $\frac{112}{1}$. Multiply this by $\frac{5}{1008}$, in the following manner:

$\frac{5}{1008} \times \frac{112}{1} = \frac{560}{1008}$; which reduced to its lowest terms, by dividing by 112, the greatest common measure, will give $\frac{5}{9}$.

2. Reduce $\frac{2}{27}$ of a pound troy to the fraction of an ounce.

Ans. $\frac{8}{9}$ oz.

Here there is only one denomination, namely, 12. Place this in the form of a fraction thus, $\frac{12}{1}$, and multiply it by $\frac{2}{27}$, and you have $\frac{2}{27} \times \frac{12}{1} = \frac{24}{27} = \frac{8}{9}$, by dividing by 3, the greatest common measure.

3. Reduce $\frac{1}{120}$ of a dollar to the fraction of a cent. Ans. $\frac{5}{6}$.

$$\frac{1}{120} \times \frac{100}{1} \times \frac{10}{1} = \frac{100}{120} = \frac{5}{6} \text{ of a cent.}$$

Here you divide by 20, the greatest common measure.

4. Reduce $\frac{5}{1440}$ of a £. to the fraction of a penny. Ans. $\frac{5}{6}$.

Here by compounding $\frac{5}{1440}$ with the denominations between a penny and a pound, you have $\frac{5}{1440} \times \frac{12}{1} \times \frac{20}{1}$; which being reduced, becomes $\frac{1200}{1440}$. This divided by the greatest common measure, gives $\frac{5}{6}$ of a penny.

5. Reduce $\frac{4}{2530}$ of a hhd. to the fraction of a pint. Ans. $\frac{4}{5}$.
6. Reduce $\frac{1}{1381}$ of a day to the fraction of a minute. Ans. $\frac{10}{11}$.
7. Reduce $\frac{7}{1440}$ of an acre to the fraction of a perch. Ans. $\frac{7}{9}$.
8. Reduce $\frac{5}{10560}$ of a mile to the fraction of a yard. Ans. $\frac{5}{6}$.
9. Reduce $\frac{1}{60}$ of a £. to the fraction of a shilling.

Ans. $\frac{3}{9} = \frac{1}{3}$s.

10. Reduce $\frac{3}{392}$ of a cwt. to the fraction of a lb. Ans. $\frac{6}{7}$ lb.

CASE 8.

To find the value of a fraction in the known parts of an integer, or whole number:

RULE.—Multiply the numerator of the fraction by the parts of the integer, and divide by the denominator.

NOTE.—The questions under this case are similar to the practical exercises, page 125 *et seq.*, and may be solved by the directions there given, which are in substance the same with the above rule.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of a dollar, or 100 cents, to its proper quantity.
Ans. 75 cents.

$$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75 \text{ cents.}$$

2. Reduce $\frac{12}{16}$ of a pound troy to its proper quantity.
Ans. 9 oz.

3. Reduce $\frac{4}{7}$ of a mile to its proper quantity.
Ans. 4 fur. 125 yds. 2 ft. 1 in. $2\frac{1}{7}$, b.c.

4. Reduce $\frac{6}{7}$ of a yard to its proper quantity.
Ans. 3 qr. $1\frac{5}{7}$ na.

5. Reduce $\frac{3}{10}$ of a day to its proper value. Ans. 7 h. 12 m.

6. Reduce $\frac{1}{8}$ of a dollar, or 100 cents to its proper value.
Ans. $12\frac{1}{2}$ cts.

7. Reduce $\frac{500}{630}$ of a hhd. to its proper quantity. Ans. 50 gal.

8. Reduce $\frac{18}{20}$ of a day to its proper quantity.
Ans. 16 h. 36 m. $55\frac{5}{13}$. sec.

9. Reduce $\frac{48}{89}$ of a hhd. to its proper quantity.
Ans. 33 gal. 3 qt. 1 pt. $\frac{75}{89}$.

10. Reduce $\frac{11}{12}$ of a £. to its proper quantity. Ans. 18*s*. 4*d*.

CASE 9.

To reduce any given sum, or quantity to a fraction of any greater denomination of the same kind:

RULE.—Reduce the given sum or quantity, to the lowest terms mentioned; and the integer to the same denomination, for a denominator to the fraction. Lastly, reduce the fraction to its lowest terms.

EXAMPLES.

1. Reduce 9 *oz.* troy, to the fraction of a pound. Ans. $\frac{3}{4}$*lb.*

Here $\frac{9}{12}$ is the fraction, which divided by 3, the greatest common measure, will give $\frac{3}{4}$.

2. Reduce 6*s*. and 8*d*. to the fraction of a £. Ans. $\frac{1}{3}$£.

Here you must reduce the shillings and pence to pence, which make 80, for the numerator of the fraction. Then re-

duce 20*s*. to pence, making 240, for a denominator, and the fraction is $\frac{80}{240}$, which divided by 80, the greatest common measure, gives $\frac{1}{3}$.

3. Reduce 1 *R*. and 30 *P*. to the fraction of an acre. Ans. $\frac{7}{16}$.

Here reduce 1 *R*. and 30 *P*. to perches, which make 70 perches: and 160 perches are an acre; therefore, $\frac{70}{160}$ will be the fraction, which reduced to its lowest terms, gives $\frac{7}{16}$.

4. Reduce 13 hours and 30 minutes to the fraction of a day. Ans. $\frac{9}{16}$.

5. Reduce 3 *qrs*. and 2 *na*. to the fraction of a yard. Ans. $\frac{7}{8}$ *yd*.

6. Reduce 1 month and 5 days to the fraction of a year, the month being 30 days, and the year 365 days. Ans. $\frac{7}{73}$.

7. Reduce 3 dolls. and 2 dimes to the fraction of an eagle. Ans. $\frac{8}{25}$.

8. Reduce 3 *fur*. and 25 *rods* to the fraction of a mile. Ans. $\frac{29}{64}$.

9. Reduce 125 yards and 2 feet to the fraction of a mile. Ans. $\frac{377}{5280}$.

ADDITION OF VULGAR FRACTIONS.

Rule.—Reduce the given fractions to a common denominator: add the several numerators together, under which place the common denominator: then reduce the fraction to its lowest terms.

Note.—If there are compound fractions, reduce them to single ones by case 5.

If there are whole numbers given, they may be omitted in the operation, and afterward added to the value of the fractional part.

EXAMPLES.

1. Add $\frac{1}{2}$ and $\frac{7}{8}$ together. Ans. $\frac{22}{16}=1\frac{6}{16}=1\frac{3}{8}$

$1 \times 8 = 8$
$7 \times 2 = 14$ } the numerators.
$\overline{22}$ = sum of the numerators.

$2\times8=16$ the common denominator. $\frac{22}{16}$ is the fraction, which, reduced to its lowest terms by dividing by 2, will be $\frac{11}{8}$. This, divided by the denominator, gives $1\frac{3}{8}$. In like manner add the following fractions.

2. Add $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$ together. Ans. $2\frac{11}{180}$.
3. Add $\frac{3}{7}$, $\frac{2}{5}$, and $\frac{7}{8}$ together. Ans. $1\frac{197}{280}$.
4. Add $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{10}$ together. Ans. $1\frac{1}{4}$.

5. Add $17\frac{1}{2}$ and $\frac{2}{3}$ together. Ans. $18\frac{1}{6}$.

Here you may omit the whole number, and find the sum of $\frac{1}{2}$ and $\frac{2}{3}$, to which afterward add the 17.

$1 \times 3 = 3$
$2 \times 2 = 4$ } numerators.

$\overline{7}$ the sum of the numerators.

$2 \times 3 = 6$, the denominator. The fraction is $\frac{7}{6}$, which divided by 6, the denominator, gives $1\frac{1}{6}$. To which add the integer, and you have $18\frac{1}{6}$.

6. Add 7, and $\frac{1}{2}$ of $\frac{2}{3}$ together. Ans. $7\frac{1}{3}$.

Here you reduce the compound fraction to a single one; which gives $\frac{2}{6}$. This, in its lowest terms, is $\frac{1}{3}$. To this add the integer 7, and the whole is $7\frac{1}{3}$.

7. Add $\frac{2}{4}$ of $\frac{7}{8}$, and $\frac{4}{6}$ of $\frac{19}{6}$ together. Ans. $1\frac{17}{48}$.

Here you see that $\frac{2}{4}$ and $\frac{4}{6}$ are to be reduced to their lowest terms, in the first place. The fraction then becomes, $\frac{1}{2}$ of $\frac{7}{8}$, and $\frac{2}{3}$ of $\frac{19}{6}$. Reduced to single fractions, we have $\frac{7}{16}$ and $\frac{38}{36}$. These reduced to a common denominator, and to their lowest terms, give $1\frac{17}{48}$.

8. Add $12\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{3}{4}$ together. Ans. $20\frac{11}{12}$.

Here omit the integers, and the fractions to be added will be $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; which reduced as above directed, will be $1\frac{11}{12}$: to which add the several integers, and you have $20\frac{11}{12}$

9. Add $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and $14\frac{20}{11}$ together. Ans. $14\frac{11}{12}\frac{1}{4}$.

10. Add $\frac{1}{3}$ of 95, and $\frac{7}{8}$ of 14 together. Ans. $43\frac{11}{12}$

Here you will make the integers improper fractions, by placing them above a unit, thus, $\frac{95}{1}$ and $\frac{14}{1}$. The fraction will then be $\frac{1}{3}$ of $\frac{95}{1}$, and $\frac{7}{8}$ of $\frac{14}{1}$. Reduced, to single fractions you have $\frac{95}{3}$ and $\frac{98}{8}$. These reduced, as in the preceding examples, you will have $43\frac{11}{12}$.

11. Add 19, 7, and $\frac{1}{2}$ of $\frac{2}{3}$ together. Ans $26\frac{1}{3}$.

To ADD fractions of money, weights, measures, &c. together:

RULE.—Find the value of each fraction separately, as in case 8, and add these values together as in compound addition.

12. Add $\frac{1}{2}$ an ounce troy, to $\frac{2}{3}$ of a pound. Ans. 8 *oz.* 10 *dwt.*

$\frac{1}{2} \times \frac{20}{1} = \frac{20}{2} = 10$ *dwt.* the value of half an oz.

$\frac{2}{3} \times \frac{12}{1} = \frac{24}{3} = 8$ *oz*, the value of two thirds of a pound.

These added together make 8 *oz.* 10 *dwt.*

13. Add $\frac{3}{4}$ of a mile, to $\frac{7}{10}$ of a furlong. Ans. 6 *fur.* 28 *poles.*

Here multiply $\frac{3}{4}$ by $\frac{8}{1}$, because 8 furlongs make a mile; and you have $\frac{24}{4} = 6$ furlongs, for the value. Proceed now to find the value of $\frac{7}{10}$ of a furlong. Multiply it by 40, because

40 poles make a furlong, and you will have $\frac{280}{10} = 28$ poles for the value of $\frac{7}{8}$ of a furlong. These sums added together make 6 *fur.* 28 *poles.*

14. Add $\frac{1}{2}$ a yard to $\frac{2}{3}$ of a foot. Ans. 2 *ft.* 2 *in.*

15. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ an hour together. Ans. 2 *d.* 14 *h.* 30 *m.*

16. Add $\frac{2}{3}$ of a dollar to $\frac{1}{2}$ a cent. Ans. 67 *c.* 1,6℘ *m.*

17. Add $\frac{1}{4}$ of an eagle to $\frac{7}{8}$ of a dollar. Ans. $3,37\frac{1}{2}$.

18. Add $\frac{1}{8}$ of a £. to $\frac{3}{4}$ of a shilling. Ans. 3*s.* 3*d.*

19. Add $\frac{2}{3}$ of a league, $\frac{3}{4}$ of a mile, and $\frac{3}{8}$ of a furlong together. Ans. 2 m. 270 rods.

20. Add $\frac{5}{6}$ of a hhd. to $\frac{3}{4}$ of a gal. Ans. 53 gal. 1 qt.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.—Reduce the fractions to a common denominator, and subtract the less numerator from the greater. Place the difference above the common denominator. Which fraction reduce to its lowest terms, if it is necessary.

EXAMPLES.

1. From $\frac{111}{112}$, take $\frac{3}{4}$. Ans. $\frac{27}{112}$.

$$\left.\begin{array}{r} 111\times\ 4=444 \\ 3\times 112=336 \end{array}\right\} \text{numerators.}$$

$$108=\text{difference of numerators.}$$

$$112\times 4=448 \text{ the common denominator.}$$

The fraction becomes $\frac{108}{448}$. Which reduced to its lowest terms, by dividing by 4, the greatest common measure, you have $\frac{27}{112}$.

2. From $\frac{97}{100}$, take $\frac{3}{7}$. Ans. $\frac{379}{700}$.

3. From $\frac{40}{41}$, take $\frac{5}{6}$. Ans. $\frac{35}{240}$.

4. From $\frac{7}{8}$, take $\frac{2}{5}$. Ans. $\frac{19}{40}$.

5. From $\frac{11}{15}$, take $\frac{4}{11}$. Ans. $\frac{61}{165}$.

6. From $96\frac{1}{3}$, take $14\frac{3}{7}$. Ans. $81\frac{19}{21}$.

Here you reduce the mixed numbers to improper fractions, as in case 3d. The fraction then will be $\frac{289}{3}$, and $\frac{101}{7}$: which reduced to a common denominator, will be

$$\left.\begin{array}{r} 289\times 7=2023 \\ 101\times 3=\ 303 \end{array}\right\} \text{the numerators.}$$

$$1720=\text{difference of the numerators.}$$

$$3\times 7=21 \text{ the common denominator.}$$

The fraction then becomes $\frac{1720}{21}=81\frac{19}{21}$.

7. From $71\frac{1}{2}$, take $\frac{17}{19}$. Ans. $70\frac{34}{38}$.
8. From 96, take $\frac{3}{5}$. Ans. $95\frac{2}{5}$.

Here you make 96 an improper fraction, thus, $\frac{96}{1}$; and reduce the fractions to a common denominator;

$$\left.\begin{array}{l} 96\times5=480 \\ 3\times1=\ \ 3 \end{array}\right\}\text{the numerators.}$$

$$477=\text{difference of the numerators.}$$

$$1\times5=5\text{ the common denominator.}$$

The fraction then will be $\frac{477}{5}=95\frac{2}{5}$.

Or: subtract the numerator of the fraction from the denominator for a new numerator, under which write the denominator, and add 1 or unit to the place of integers, thus:

From 96
Take $0\ \frac{3}{5}$
$95\ \frac{2}{5}$ the Ans. as before. This is a very short method, but is of very limited use. In like manner may any other examples of the kind be solved.

8. From 121, take $\frac{11}{12}$. Ans. $120\frac{1}{12}$.
9. From 79, take $\frac{7}{8}$. Ans. $78\frac{1}{8}$.
10. From $\frac{3}{4}$, take $\frac{1}{2}$ of $\frac{2}{3}$. Ans. $\frac{5}{12}$.

Here $\frac{1}{2}$ of $\frac{2}{3}$ is a compound fraction. It must be reduced to a single one, thus, $\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$ or $\frac{1}{3}$ by dividing by 2.

$$\left.\begin{array}{l} 3\times3=9 \\ 4\times4=4 \end{array}\right\}\text{the numerators.}$$

$$5=\text{the diff. of numerators.}$$

$4\times3=12$, the denominator. The fraction then becomes $\frac{5}{12}$.

11. From $\frac{3}{4}$ of $\frac{4}{8}$, take $\frac{1}{9}$. Ans. $\frac{19}{72}$.
12. From $14\frac{1}{4}$, take $\frac{2}{3}$ of 19. Ans. $1\frac{7}{12}$.
13. From $19\frac{2}{3}$, take $\frac{2}{3}$ of 19. Ans. 7.

To SUBTRACT fractions of money, weights, measures, &c.

RULE.—Find the proper value of the fractions separately, and subtract the less from the greater, as in compound subtraction.

14. From $\frac{1}{3}$ of a £. take $\frac{3}{4}$ of a shilling. Ans. 5*s*. 11*d*.
15. From $\frac{6}{10}$ of an oz. take $\frac{7}{8}$ of a dwt. Ans. 11 *dwt*. 3 *gr*.
16. From $\frac{2}{3}$ of a league, take $\frac{7}{10}$ of a mile.
Ans. 1 *m*. 2 *fur*. 16 *rods*.
17. From $\frac{5}{8}$ of a dollar, take $\frac{3}{4}$ of a dime. Ans. 55 *cts*.
18. From $\frac{2}{3}$ of an acre, take $\frac{1}{2}$ of a rod.
Ans. 2 *rods* $6\frac{2}{3}$ *perches*.
19. From $\frac{6}{7}$ of a day take $\frac{2}{3}$ of an hour.
Ans. 19 *h*. 54 *m*. $17\frac{1}{7}$ *sec*.

20. From $\frac{3}{8}$ of a great circle, take $\frac{1}{2}$ of a degree. Ans. 134° 30′.
21. From $\frac{5}{6}$ of an hhd. take $\frac{1}{2}$ of a gallon. Ans. 52 *gal.*
22. From $\frac{1}{5}$ of a ton wt. take $\frac{3}{8}$ of a lb. Ans. 2239 *lb.* 10 *oz.*

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.—Multiply the numerators of the given fractions for a new numerator; and the denominators together for a new denominator.

NOTE.—Mixed numbers are to be reduced to improper fractions, and compound fractions to single ones, if there are any such given; then proceed as the rule directs.

EXAMPLES.

1. Multiply $\frac{3}{4}$, by $\frac{2}{5}$. Ans. $\frac{6}{20} = \frac{3}{10}$.
2. Multiply $\frac{3}{7}$, by $\frac{4}{11}$. Ans. $\frac{12}{77}$.
3. Multiply $\frac{1}{3}$, by $\frac{5}{49}$. Ans. $\frac{5}{147}$.
4. Multiply $7\frac{1}{4}$, by $8\frac{1}{2}$. Ans. $61\frac{5}{8}$.

Here the fractions must be reduced to improper ones, as in case 3d. They then become $\frac{29}{4}$ and $\frac{17}{2}$, which, multiplied by the rule, are $\frac{493}{8} = 61\frac{5}{8}$.

5. Multiply $\frac{3}{5}$ of 8, by $\frac{7}{8}$ of 5. Ans. 21.

Here make the integers 8 and 5 improper fractions, thus, $\frac{8}{1}$, and $\frac{5}{1}$. The fractions will be compounded, thus, $\frac{3}{5}$ of $\frac{8}{1}$, and $\frac{7}{8}$ of $\frac{5}{1}$, or $\frac{24}{5}$ and $\frac{35}{8}$; which, multiplied by the rule, will be $\frac{840}{40} = 21$; by dividing by 40, the greatest common measure.

6. Multiply $\frac{7}{8}$, by $13\frac{9}{10}$. Ans. $12\frac{13}{80}$.

Here you must reduce $13\frac{9}{10}$ to an improper fraction, thus $\frac{139}{10}$. Then multiply $\frac{7}{8}$, and $\frac{139}{10}$, you will have $\frac{973}{80} = 12\frac{13}{80}$.

7. Multiply $\frac{2}{9}$ of $\frac{3}{5}$, by $\frac{5}{8}$ of $3\frac{2}{7}$. Ans. $\frac{23}{84}$.

Here you must reduce $3\frac{2}{7}$ to an improper fraction, thus, $\frac{23}{7}$. The fractions reduced to single ones are $\frac{6}{45}$, and $\frac{115}{56}$; which multiplied become $\frac{6900}{25200}$. This divided by 30, the greatest common measure, gives $\frac{23}{840}$.

8. Multiply $\frac{1}{3}$ of $\frac{4}{5}$, by $\frac{7}{10}$ of $\frac{11}{12}$. Ans. $\frac{77}{450}$.
9. Multiply $\frac{4}{5}$ of 91, by $71\frac{1}{2}$. Ans. $5205\frac{1}{5}$
10. Multiply $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$, by $\frac{6}{7}$ of $\frac{1}{2}$. Ans. $\frac{10}{112}$.
11. Multiply $\frac{2}{3}$ of $\frac{9}{10}$ of $\frac{1}{3}$ of $\frac{6}{9}$, by $\frac{1}{4}$ of 7. Ans. $\frac{7}{30}$.
12. Multiply $\frac{1}{4}$ of 112, by $\frac{3}{4}$ of 48. Ans. 1008.

DIVISION OF VULGAR FRACTIONS.

RULE.—Multiply the numerator of the dividend into the denominator of the divisor, for a new numerator, and the numerator of the divisor, into the denominator of the dividend, for a

new denominator. Then reduce the fraction to its lowest terms, if necessary.

NOTE.—Mixed numbers must be reduced to improper fractions, and compound fractions to single ones.

EXAMPLES.

1. Divide $\frac{17}{21}$, by $\frac{3}{5}$. Ans. $1\frac{23}{63}$.

$17\times5=85$ new numerator.

$21\times3=63$ new denominator.

The fraction is therefore $\frac{85}{63}=1\frac{23}{63}$.

2. Divide $\frac{2}{9}$, by $\frac{4}{7}$. Ans. $\frac{14}{36}=\frac{7}{18}$.

3. Divide $\frac{3}{7}$, by $\frac{3}{14}$. Ans. $\frac{42}{21}=2$.

4. Divide $\frac{7}{9}$, by $\frac{7}{10}$. Ans. $\frac{70}{63}=1\frac{7}{63}=1\frac{1}{9}$.

5. Divide $1\frac{1}{2}$, by $4\frac{4}{5}$. Ans. $\frac{15}{48}=\frac{5}{16}$.

Here you must reduce the mixed numbers to improper fractions, thus, $\frac{3}{2}$ and $\frac{24}{5}$, and proceed as in the preceding examples.

6. Divide $\frac{7}{8}$, by 4. Ans. $\frac{7}{32}$.

Here reduce 4 to an improper fraction, and then proceed as in the above.

7. Divide 4, by $\frac{7}{8}$. Ans. $\frac{32}{7}=4\frac{4}{7}$.

Here again you make 4 an improper fraction, and multiply the fractions as the rule directs.

8. Divide $\frac{1}{5}$ of 19, by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{38}{5}=7\frac{3}{5}$.

Here reduce the compound fraction to a single one, and the 19 to an improper fraction; and you will have $\frac{1}{5}$ of $\frac{19}{1}$ and $\frac{6}{12}$. Reduce now $\frac{1}{5}$ of $\frac{19}{1}$ to a single fraction, and you will have $\frac{19}{5}$. The fractions are $\frac{19}{5}$, and $\frac{6}{12}$, or $\frac{1}{2}$, dividing by 6 the common measure.

$19 \times 2 = 38$ the new numerator.

$1 \times 5 = \overline{5}$ the new denominator.

which divided gives as above.

9. Divide $\frac{1}{2}$ of $\frac{2}{3}$, by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{4}{6}=\frac{2}{3}$.

These reduced to single fractions become $\frac{2}{6}$, and $\frac{6}{12}$, or $\frac{1}{2}$.

$2 \times 2 = 4$ the new numerator.

$1 \times 6 = \overline{6}$ the new denominator,

which reduced gives as above.

10. Divide $\frac{5}{9}$ of 4, by $4\frac{5}{9}$. Ans. $\frac{180}{369}=\frac{20}{41}$.

Here $\frac{5}{9} \times \frac{4}{1} = \frac{20}{9}$, and $4\frac{5}{9}=\frac{41}{9}$. The fractions, thus reduced and multiplied, become $\frac{180}{369}$, which divided by 9, the greatest common measure, reduces it to $\frac{20}{41}$.

11. Divide $7\frac{1}{8}$ by $9\frac{5}{9}$. Ans. $\frac{33}{43}$.

12. Divide $\frac{7}{8}$ of 6 by $\frac{3}{4}$ of $\frac{3}{7}$ of $\frac{11}{12}$. Ans. $8\frac{10}{11}$.

13. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91. Ans. $71\frac{1}{2}$

QUESTIONS.—1. What is a vulgar fraction?—2. How do you represent it?—3. What is the upper number called?—4. What does it express.—5. What is the number below the line called?—6. What does it express?—7. How are vulgar fractions commonly divided?—8. What is meant by a proper fraction?—9. What is meant by an improper fraction?—10. What is meant by a compound fraction?—11. What by a mixed number?—12. How would you represent, or express a whole number in the form of a fraction?

13. What do you mean by a common measure of a fraction?

14. What do you mean by the greatest common measure of a fraction?—15. Will you give an example?

16. When is a fraction said to be in its lowest terms?—17. Will you give an example to show this?

18. How do you reduce fractions to their lowest terms?—19. But, if you cannot readily discover their greatest common measure, what do you do?—20. There is another method; and what is that?

21. How do you reduce fractions to a common denominator?

22. What should always be done previously?

23. How do you reduce mixed numbers to improper fractions?

24. How do you reduce compound fractions to single ones?

25. What should always be done previously?—26. How do you reduce a fraction to its proper value?—27. How do you add fractions, which have the same denominator?—28. If the given fractions have different denominators, what do you do?—29. After you have reduced them to a common denominator, what do you do?—30. If the fractions are mixed numbers or compound fractions, what must you previously do with them?

31. How do you add fractions of money, weights, and measures?

32. How do you subtract vulgar fractions, when the denominators are the same?—33. When they have different denominators, how do you proceed?—34. How do you subtract fractions of money, weights, and measures?

35. How do you multiply vulgar fractions?—36. When the fractions are mixed numbers, or compound fractions, how do you proceed?

37. How do you divide vulgar fractions?—38. If the fractions are mixt numbers, or compound fractions, how do you proceed?

DECIMAL FRACTIONS.

A DECIMAL fraction is a part or parts of a unit, denoted by a point or dot, prefixed to one or more figures; thus, .5, .45, .255. The first figure, after the point or dot, denotes so many tenths of a unit; the second figure denotes so many hundred parts of a unit; the third figure denotes so many thousand parts of a unit, &c.

The value of any decimal fraction may be expressed in the form of a vulgar fraction, thus, $\frac{5}{10}$, $\frac{45}{100}$, $\frac{225}{1000}$, &c.

A mixed number consists of a whole number and a decimal, thus, 125.7,—74.125,—65.25.

The value of a decimal fraction decreases from the decimal point in a tenfold proportion, thus .5, .05, .005. The .5 in the first place is ten times the value of it in the second, and a hundred times its value in the third place; and so on to any number of places.

Cyphers *added* to decimals neither increase nor diminish their value. But, if they are *prefixed* to them, they lessen their value, in a tenfold proportion. Thus, by adding a cypher to .5 we do not affect its value. It then becomes fifty hundredths of a unit, which is the same with 5 tenths; and so if two, three, or four cyphers were added to it. But placed before the decimal, they diminish its value by removing it farther from the decimal point, thus, .005, ,05, .5, all express different values. The first is one tenth part of the second, and the second one tenth part of the third; and so for any greater number of places.

TABLE OF DECIMAL PARTS.

.1	Tenth part of a unit.
2	Hundredth part of a unit.
3	Thousandth part of a unit.
4	X. Thousandth part of a unit.
5	C. Thousandth part of a unit.
6	Millionth part of a unit.
7	X. millionth part of a unit.
8	C. millionth part of a unit.
9	Thousand millionth part of a unit.

The denominations of Federal currency are arranged in a decimal or tenfold ratio. An eagle is ten times the value of a dollar; a dollar is ten times the value of a dime; a dime is ten times the value of a cent, and a cent ten times the value of a mill, thus:

1 Eagle = 10 dollars.	100 *c.*=1 *dol.*
1 dollar = 10 dimes.	75 *c.*= .75 " =7 *d.* 5 *c.*
1 dime = 10 cents.	50 *c.*= .5 " =5 *d.*
1 cent = 10 mills.	25 *c.*= .25 " =2 *d.* 5 *c.*
$\frac{1}{3}$ of a dollar = 33,3,3$\mathscr{3}$ cents.	$12\frac{1}{2}$ = .125 " =1 *d.* 2 *c.* 5*m.*
$\frac{2}{3}$ of a dollar = 66,6,6$\mathscr{6}$ cents.	or 12 *c.* 5 *m.*

Let the pupil be required to express the following sums in their proper decimals:—Three, tenths; twenty-six, hundredths; one hundred and fifteen, thousandths; five hundred and forty, thousandths; one hundred and twenty-five, thousandths; five, hundredths; fifteen, thousandths; one hundred and twelve, ten thousandths; twenty-four, millionths; nine, ten thousandths; three, thousandths; sixty-five, one hundred thousandths; eighty-four millionths.

Let him write, in figures, ten eagles, five dollars, six dimes, four cents, and seven mills; sixty-four cents and seven mills; five dollars, and twenty-five cents; three dollars, and two cents; six dollars, and one cent; four cents; two mills; one dime; one dollar, and twenty-five hundredths; two dollars, eight hundred and seventy five thousandths; ten dollars, and fifty five hundredths; nine dollars, one hundred and thirty-seven thousandths; six dollars, one hundred and twenty-five thousandths; one dollar, three hundred and seventy-five thousandths: and any other sums, the teacher may think proper.

ADDITION OF DECIMALS.

Addition of Decimals is performed like addition of whole numbers. Tenths must be placed under tenths, hundredths under hundredths, thousandths under thousandths, &c. In the sum, observe to make as many decimal places as are in any one of the given fractions.

If mixed numbers are given to be added together, place the integers, units under units, tens under tens, &c. beginning at the decimal point, from the right to the left. The fractional parts are to be placed on the right of the decimal point, as above directed, tenths under tenths, &c.

EXAMPLES.

Fractions.	*Mixed Numbers.*	*Mixed Numbers.*
.16457	146.02145	345.62171
.84321	364.14215	38.21054
.92456	562.52068	6.70204
.00256	142.35421	35.00164
.74021	813.21543	376.543
.54216	321.54216	75.04754
3.21727	2349.79608	877.12647

Let the pupil be required to add the following fractions:

1. .42105, .002541, .016421, .7042, .54, .25241.

2. .9874602, .745210, .000014, .34210, .2467, .789.

3. Let him add the following mixed numbers: 74.62741, 5.00421, 4261.3407, 174.32164, 9876.2542.

4. 10.6215, 107.35421, 84.542106, 42.81, 1.7542.

5. Also the following mixed numbers: 54.004, 12.54764, 1.2554, 746.2104, 25.46201, 246.00012, 4621.3456.

6. 333.3333, 64.6666, 246.7407, 246.22102.

7. William purchased the following articles, viz.; a copy of Virgil for $2.12,5, an exercise for .37,5 cts., a quire of paper for .18,7,5 cts., quills for .12,5 cts., ink for ,5 mills; what was the amount? Ans. $2.81,7,5.

8. Jane purchased an English grammar for .50 cts., a geography for $1.37,5 cts., a slate for .37,5 cts., paper and quills for .56,2,5 cts., ink for ,6 mills; how much money did she lay out? Ans. $2.81,8,5.

9. A farmer carried to market the following articles, which he sold for these several sums: viz. turkeys for $5.37,5, fowls for $3.75,4,5, butter for $1.25, potatoes for $3.87,5, turnips .93,7,5, Indian meal for $2.12,5; how much did he receive in all? Ans. $17.31,7.

10. A gentleman bought a chaise for $210.56,2,5, a horse for $116.75, a saddle for $18.75, a bridle for $4.87,5, a harness for $31.37,5; how much was the amount paid?

Ans. $382.31,2,5.

11. A lady laid out the following sums of money in a tour of shopping; and, on her return home, desired her daughter to tell her the amount: $1.37,5 for a pair of gloves, $2.87,5 for muslin, $5.12,5 for crape, $7.50 for silk, $3.93,7,5 for linen, .56,2,5 cts. for silk, and cotton thread; what was the amount?

Ans. $21.37,5.

12. A servant purchased in market a turkey for .87,5 cts., a pair of chickens for .56,2,5 cts., potatoes for .37,5 cts., other vegetables for $1.25, beef for $1.87,5; how much did he lay out for the whole? Ans. $4.93,7,5.

13. A gentleman purchased a suit of clothes: a coat for $18.75, a vest for $3.25, pantaloons for $7.87,5, linen for $5.56,2,5, a pair of boots for $5.75, stockings for $2.75, and a hat for $6.37,5; what was the amount? Ans. $50.31,2,5.

14. A merchant built a house, and when finished, the bills were as follow: The carpenter's bill $475.56,2,5, the bricklayer's bill $384.50, the lumber merchant's bill $450.37,5, the brickmaker's bill $275.12,5, the plasterer's bill $185.93,7,5, the painter's bill $94.17,9, the ironmonger's bill $49.85; how much did the house cost, including $1165.75, which he gave for the lot of ground? Ans. $3081.27,9.

SUBTRACTION OF DECIMALS.

Subtraction of decimals is performed like subtraction of whole numbers; but observe to make as many decimal places in the difference, as there are in either of the given numbers.

EXAMPLES.

Fractions.	*Mixed Numbers.*	*Mixed Numbers.*
.874698	4765.008476	47.62845
.23987426	2104.7628	31.046289
.63482374	2660.245676	16.582161

1. Take 47.504, from 1746.80456, what will be left? Ans. 1699,30056.
2. Take 1.755, from 5.8755, what will remain? Ans. 4.1205.
3. Take 74.8064 from 975.006, what will remain? Ans. 900.1996.
4. Take 25.0074 from 97.8462, what will be left? Ans. 72.8388.
5. A man borrowed $674.87,5, and paid $346.50; how much remains due? Ans. $328.37,5.
6. A servant went to market with $5.37,5, and purchased to the amount of $2.43,7,5; how much must he have remaining? Ans. $2.93,7,5.
7. Jane took with her, for the purchase of books and stationary, $4.87,5, and her bill was $3.62,5; how much change must she receive? Ans. $1.25.

8. A gentleman's income is $1145.87,5; and his family's expenses amount to $824.25; how much does he save yearly?
Ans. $321.62,5.

9. What is the difference between one dollar, and one mill?
Ans. 99 *c.* 9 *m.*

10. What is the difference between $99.99, and 4 dollars and 2 mills? Ans. $95.98,8.

11. A man commenced a journey with $174.85,5, and soon after he set out, he found that he had spent $47.95,5; what sum had he left? Ans. $126.90.

12. A merchant purchased a quantity of goods at a public sale for $1784.12,5, and sold $847.62,5 before he removed them; what amount had he left? Ans. $936.50.

MULTIPLICATION OF DECIMALS.

MULTIPLICATION of decimals is performed like multiplication of whole numbers; but observe to make as many decimal places in the product, as there are in the multiplicand and multiplier taken together.

EXAMPLES.

Fractions.	*Mixed Numbers.*	*Mixed Numbers.*
.4675	24.6021	742.0245
.27	24.02	1.54
32725	592042	29680980
9350	984084	37101225
.126225	492042	7420245
	591.042442	1142.717730

1. Multiply 743.56815, by 52.647. Ans. 39146.63239305.
2. Multiply 79.0342, by 102.51. Ans. 8101.795842.
3. Multiply 125.74, by 56.84. Ans. 7147.0616.
4. Multiply 274.174, by 125.57. Ans. 34428.02918.
5. Multiply 125.0478, by 12.042. Ans. 1505.8256076.
6. Multiply 70.3046, by 54.321. Ans. 3819.0161766.
7. Multiply 217.1462, by 740.025. Ans. 160693.616655.
8. Multiply .04621, by .00241. Ans. .0001117661.
9. Multiply .00027, by .00742. Ans. .0000019234.
10. Multiply .074, by .00025. Ans. .00001850.

1. A farmer sold 47.5 bushels of wheat at 75 cents a bushel; required the amount. Ans. $35.62,5.

2. A gentleman sold a farm containing 127.875 acres of land, at $11.37,5 an acre; required the amount. Ans. $1454.57,8,125.

3. A gentleman bought 4.25 yards of broad cloth, at $5.56,25 a yard; required the amount. Ans. $23.64.

4. A merchant bought a piece of cloth containing 31.6 yards, at $3.12,5 a yard; required the amount. Ans. $98,75.

5. A lady bought a piece of silk containing 7.55 yards, at $1.12.5 a yard; required the amount. Ans. $8.49,3,75.

6. Required the amount of 37.5 yards of muslin, at 16,5 cents a yard. Ans. $6.18,7,5.

7. Required the amount of 40.25 bushels of wheat, at 93,7,5 cents, a bushel. Ans. $37.73,44.

8. Required the amount of 5.625 cords of wood, at $3.75 a cord. Ans. $21.09,3,75.

9. A housekeeper purchased a bag of coffee containing 112.25 pounds, at 12.5 cts. a lb; what was the amount? Ans. $14.03,1,25.

10. A grocer bought a box of tea containing 41.657 lb. at $1.12,5 a lb.; what was the amount? Ans. $46.86,4,125.

11. A lady purchased the following articles; 4.25 yards of muslin at 18,7,5 cents a yard; 6.75 yards of linen at 56,2,5 cents a yard; 12,5 yards of tape at 5 mills a yard; and 5 skeins of silk at 3,5 cents each; what was the amount of her bill? Ans. $4.83,1,25.

12. The expenses of a family is $1,75,5 a day; how much will the amount of expenses be in a year, estimating 365.25 days? Ans. $641.01,3,75.

DIVISION OF DECIMALS.

Division of decimals is performed like division of whole numbers; but, observe that there must be as many decimal places in the divisor and quotient, taken together, as there are in the dividend.

Note.—If there are not a sufficient number of decimal places in the dividend, cyphers must be added.

EXAMPLES.

Divide 234.70525, by 64.25. Ans. 3.653.

64.25)234.70525(3.653
19275 · · ·
41955
38550
34052
32125
19275
19275

Divide .746894756, by .0216. Ans. 34.57846+

.0216).746894756(34.57846+

648·····

988
864

1249
1080

1694
1512

1827
1728

995
864

1316
1296

20

1. Divide 2508.92806, by 92.41035. Ans. 27.1498+.

Note.—Here there is not a sufficient number of decimal places in the dividend; cyphers must be added to it, so that the quotient may have three or more decimals, as in the answer.

2. Divide 1836.88305, by 23.15. Ans. 79.347.
3. Divide 3673.7661, by 158.694. Ans. 23.15.
4. Divide .00178600398, by .00463. Ans. .385746.
5. Divide 3632.46, by 78.00. Ans 46.57.
6. Divide 9, by .9. Ans. 10.
7. Divide .9, by 9. Ans. .1.
8. Divide .3, by 3. Ans. .1.
9. Divide 47.0546, by .0164. Ans. 2869,1829.+
10. Divide 174.62154, by 1.674. Ans. 104.31334.+
11. Divide 984.20462, by 4.026. Ans. 244.46214.+

PRACTICAL EXERCISES.

1. A miller bought 210.5 bushels of wheat for $195.75, how much was it a bushel? Ans. 92 cts. 9,9.+
2. A merchant bought 547.5 lbs. of tea for $497.87.5, how much was it a pound? Ans. 90 cts. 9,36.+
3. A piece of broad-cloth containing 25.335 yards, cost $116.76.5, how much was it a yard? Ans. $4.60,8,8.+
4. A gentleman boards at 3 dolls. 87,5 cts. a week; how many weeks can he board for 54 dolls. 75 cts.? Ans. 14.129 weeks.

5. A farmer raised upon one acre of land, 50.25 bushels of corn; how many acres will yield, at the same rate, 526.875 bushels? Ans. 10.485+acres.

6. A gentleman left 4567 dolls. 93,75 cts. to be divided equally among his 5 sons; what was the share of each?

Ans. $913.58,7,5.+

7. A farmer bought a plantation containing 425.5 acres for 2116 dolls. 87,5 cts.; how much was the land an acre?

Ans. $4.97,5.+

8. A merchant bought 575.25 bushels of salt for 174 dolls. 62,5 cts.; how much was it a bushel? Ans. 30 cts. 3,56.+

9. If 21.5 yards of cloth cost 46 dolls. 75 cts., how much was the cloth a yard? Ans. $2.17,67.+

10. A farmer sold a load of hay, weighing 23.25 cwt. for 15 dolls. 93,75 cts; how much was it a cwt?

Ans. 68 cts. 5,48 mills.

11. The sum of 846 dolls. 47,6 cts. was divided equally among 7 persons, how much was each one's share?

Ans. $120.92,5.

12. A labourer has 87,5 cts. a day; in how many days will he earn 48 dolls. 50 cts.? Ans. 55.428+ days.

13. The tax of a certain town amounts to 647 dolls. 75,5 cts. The population is 1000 taxable persons. How much must each one pay? Ans. 64 cts. 7,75+mills.

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal fraction:

Rule.—Add a sufficient number of cyphers, or decimal places to the numerator, and divide by the denominator.

Note.—There must be as many places of decimals in the quotient, as in the dividend.

EXAMPLES.

1. Reduce $\frac{1}{4}$ to a decimal fraction. Ans. .25

4)1.00
.25

2. Reduce $\frac{1}{2}$ to a decimal fraction. Ans. .5

1)1.0
.5

3. Reduce $\frac{3}{4}$ to a decimal fraction. Ans. .75

4)3.00
.75

3. Reduce $\frac{1}{3}$ to a decimal fraction. Ans. .33$\dot{3}$

3)1.000

.33$\dot{3}$

When the same figure recurs in the quotient, as in this example, it is called a repeater; and the true value of the fraction cannot be ascertained. It will continue to repeat to any indefinite number of places. It is usual to extend it to three or four places, after which it becomes so small in value, that, in ordinary cases, it may be rejected. The same is the case in the following example. A line is commonly drawn across the last figure, which implies that the figure is a repeater, and may be added indefinitely without continuing the operation.

When the same figure recurs at intervals of one, two, three, or more places, as it sometimes happens, it is called a circulating decimal.

4. Reduce $\frac{2}{3}$ to a decimal fraction. Ans. .66$\dot{6}$

3)2.0000

.66$\dot{6}$

5. Reduce $\frac{7}{8}$ to a decimal fraction. Ans. .875
6. Reduce $\frac{5}{6}$ to a decimal fraction. Ans. .83$\dot{3}$
7. Reduce $\frac{4}{5}$ to a decimal fraction. Ans. .8
8. Reduce $\frac{8}{9}$ to a decimal fraction. Ans. .8$\dot{8}$
9. Reduce $\frac{9}{10}$ to a decimal fraction. Ans. .9
10. Reduce $\frac{11}{12}$ to a decimal fraction. Ans. .91$\dot{6}$
11. Reduce $\frac{7}{16}$ to a decimal fraction. Ans. .4375
12. Reduce $\frac{26}{57}$ to a decimal fraction. Ans. .45614+
13. Reduce $\frac{3}{8}$ to a decimal fraction. Ans. .375
14. Reduce $\frac{12}{17}$ to a decimal fraction. Ans. .7058+
15. Reduce $\frac{14}{126}$ to a decimal fraction. Ans. .01111+
16. Reduce $\frac{216}{596}$ to a decimal fraction. Ans. .3624+
17. Reduce $\frac{2}{112}$ to a decimal fraction. Ans. .01785+
18. Reduce $\frac{1}{89}$ to a decimal fraction. Ans. .011236+
19. Reduce $2\frac{16}{19}$ to a mixed number. Ans. 11.3683+

Case. 2.

To reduce any sum or quantity to the decimal of a given denomination of the same kind:

Rule.—Reduce the given sum or quantity to its lowest terms for a numerator, and the proposed integer to the same denomination for a denominator. To the numerator affix a sufficient number of cyphers or decimal places, and divide by the denominator, as in the preceding case.

EXAMPLES.

1. Reduce 12*s*. and 6*d*. to the decimal of a £. Ans. .625.

s.	*d*.
12	6
12	

150, the numerator = the pence in 12*s*. 6*d*.

20 × 12 = 240, the denominator = the pence in 20*s*., or 1 £.

240)150.000(.625
1440 · ·
600
480
1200
1200

2. Reduce 7*s*. and 6*d*. to the decimal of a £. Ans. .375.

s.	*d*.
7	6
12	

20 × 12 = 240 the den. = the *d*. in 1 £.

90. the num. = the pence in 7*s*. 6*d*.

240).90.000(.375
720 · ·
1800
1680
1200
1200

3. Reduce 16*s*. to the decimal of a £. Ans. .8.
4. Reduce 76 yards, to the decimal of a mile. Ans. .04318+
5. Reduce 3 furlongs to the decimal of a mile. Ans. .375.
6. Reduce 112 perches to the decimal of an acre. Ans. .7.
7. Reduce 15 dwts. to the decimal of a pound troy. Ans. .0625.

dwt. in an oz. = 240)15.0000(.0625
1440 · ·
600
480
1200
1200

8. Reduce 7 minutes to the decimal of a day. Ans. .0048+
9. Reduce 14 gallons to the decimal of a hhd. Ans. .222*ż*.
10. Reduce 3 cwt. 3 qrs. to the decimal of a ton. Ans. .1875.

11. Reduce 2 qrs. and 1 nail to the decimal of a yard.
Ans. .5625.
12. Reduce 23 days to a decimal of a year. Ans. .006301+
13. Reduce 3 qrs. 16 lb. to the decimal of a cwt. Ans. .8928+
14. Reduce 12 drams to the decimal of a pound avoirdupois.
Ans. .046875.
15. Reduce 1 pint to the decimal of a gallon. Ans. .125.
16. Reduce 52 days to the decimal of a year of 365 days 6 hours. Ans. .142368.
17. Reduce 1 rood, 16 perches to the decimal of an acre.
Ans. .35.

CASE 3.

To reduce a decimal fraction to its proper value:

RULE.—Multiply it by the several denominations of the integer, and make as many decimal places in the product, as there are in the given decimal.

EXAMPLES.

1. Reduce .785416 of a £ to its value. Ans. 15*s*. 8½*d*.

.785416
20=1 £.
s. 15.708820
12=1 shilling.
d. 8.499840
4=1 penny.
qr. 1.999360

2. Reduce .76 of a £ to its value. Ans. 15*s*. 2¼*d*.
3. Reduce .861 of a cwt. to its value.
Ans. 3 qr. 12 lb. 6 oz. 14 dr.
4. Reduce .7 of a lb. troy to its value. Ans. 8 oz. 8 dwt.
5. Reduce .761 of a day to its value. Ans. 18 h. 15m. 50 sec.+
6. Reduce .67 of a league to its value.
Ans. 2 m. 3 rods, 1 yd. 1 in.+
7. Reduce .128 of an acre to its value.
Ans. 20 per. 14 yd. 4 ft.+
8. Reduce .216 of a year of 365 days to its value.
Ans. 78 d. 20 h. 9 m. 36 sec.
9. Reduce .5625 of a yard to its value. Ans. 2 qrs. 1 na.
10. Reduce .375 of a mile to its value. Ans. 3 fur.
11. Reduce .4875 of a dollar to its value. Ans. 48 cts. 7,5. m.
12. Reduce .6875 of a yard to its value. Ans. 2 qr. 3 na.
13. Reduce .07 of a hhd. to its value. Ans. 4 gal. 1 qt. 1.28 pt.

14. Reduce .078 of an acre to its value.
Ans. 12 per. 14 yd. 4.68 ft.
15. Reduce .46 of a great circle of 360 degrees to its value.
Ans. 165 deg. 36 m.
16. Reduce .146 of a sign of the zodiac to its value.
Ans. 4 deg. 22 m. 48 sec.
17. Reduce .875 of a quadrant or 90 degrees to its value.
Ans. 78 deg. 45 m.
18. Reduce .0425 of a century, or 100 years to its value.
Ans. 4 yrs. 3 mo.
19. Reduce .027 of a yard to its value. Ans. 2,916 b.c.
20. Reduce .00897 of a mile to its value.
Ans. 15 yds. 28 in. 1 b.c.
21. Reduce .79 lb. troy to its value.
Ans. 9 oz. 9 dwt. 14+ gr.
22. Reduce .56 of a yard to its value. Ans. 2.24 qr.
23. Reduce .04 of a year to its value.
Ans. 14 d. 14 h. 38 m. 24 sec.
24. Reduce .65 of a gallon to its value. Ans. 2 qt. 1.04 pt.
25. Reduce .7546 of a cwt. to its value.
Ans. 3 qr. 0 lb. 8 oz. 3.89+ dr.
26. Reduce .3467 of an acre to its value.
Ans. 1 rood, 15.47+ per.
27. Reduce .42891 of a mile to its value.
Ans. 3 fur. 17 roods, 1 yd. 1.1448 ft.
28. Reduce .186405 of a £. to its value.
Ans. 3*s*. 8*d*. 2.9488 *qr*.

QUESTIONS. 1. What is a decimal fraction?—2. How do you distinguish a decimal from a whole number or integer?—3. Will you give an example?—4. How much does the first figure to the right hand of the decimal point, signify?—5. How much does the second figure signify?—How much does the third signify?—How much does the fourth signify?—6. In what proportion does the value of a decimal decrease, from the decimal point?—7. Will you give an example?—8. Do cyphers, *added* to decimals in any way, affect their value?—9. If they are placed before them, what effect have they?—10. Will you explain this by an example, and assign the reason of this?

11. Will you repeat the table of decimal parts?—12. Are the denominations of Federal currency arranged in decimal proportion?—13. Will you explain this more fully?

14. What do you mean by a mixed number?—15. How are decimal fractions added together?—16. What do you observe in placing them for addition?—17. When they are added together, what rule do you observe in placing the *separatrix*, or decimal point?

18. When mixed numbers are given to be added together, how do you place the integers?—How do you place the decimals?—At what place do you begin?

19. How are decimals subtracted?—20. What rule do you observe in placing the decimal point?—21. How do you multiply decimals?—22. What rule do you observe in pointing off the product?—23. How do you divide decimals?—24. What rule do you observe in placing the decimal point?—25. When there are not sufficient decimal places in the dividend, what do you do?

26. How do you reduce a vulgar fraction, to a decimal?—27. What do you observe in placing the decimal point?—28. When is a decimal said to repeat?—When is it said to circulate?—29. Will you explain this by an example?—30. How do you reduce a decimal fraction to its proper value?—31. Will you explain this by an example?

REDUCTION OF INTEGERS.

Reduction is the operation of changing any sum or quantity of a given denomination to another denomination, retaining the same value.

Reduction is two-fold; descending and ascending.

Reduction descending brings higher denominations to lower, and is performed by multiplication.

When any sum or quantity is to be reduced to a lower denomination; multiply the highest denomination by the number, required of the next lower denomination to make 1 or unit of the higher; multiply this product by the number, required of the next lower denomination to make 1 or unit of the higher; and so continue to do, till you reduce it to the denomination required. But, if the given sum consists of several denominations, reduce the highest denomination to the next lower one, and to the product add the parts of the same denomination in the given sum; and so continue to do, till you come to the last or lowest denomination in the given sum.

Reduction descending is proved, by Reduction ascending.

EXAMPLES.

1. Reduce 26 Eagles to cents. Ans. 26000 cents.
2. Reduce 126 dollars to mills. Ans. 126000 mills.
3. Reduce 187 dimes to mills. Ans. 18700 mills.
4. Reduce 746 dollars to dimes, cents and mills.
Ans. 7460 dimes, 74600 cts., 746000 mills.
5. Reduce £25, 12 shillings, to shillings and pence.
Ans. 512 shillings, 6144 pence.

Here you multiply the pounds by 20, because 20 of the lower denomination or shillings, make 1 or unit of the higher; and to the product add the 12 shillings in the given sum; which make 512 shillings. This multiply by 12, because 12 pence make one shilling, and the product will be 6144. If there had been pence in the given sum, these must have been added; and the sum would have been the pence required. In like manner proceed to reduce any other sum or quantity to a lower denomination, whatever the denomination may be.

6. Reduce 15 lbs. troy weight to ounces. Ans. 180 oz.
7. Reduce 7 lbs. troy to dwt. and grains.
Ans. 1680 dwt. 40320 gr.
8. Reduce 12 lbs. 2 oz. 16 dwt. to grains. Ans. 70464 gr.
9. Reduce 25 tons to qrs. and pounds.
Ans. 2000 qrs. 56000 lbs.
10. Reduce 7 cwt. 3 qrs. 12 lbs. to pounds. Ans. 870 lbs.
11. Reduce 45 lbs. 11 oz. to drams. Ans. 11696 dr.
12. Reduce 5 lbs. 6 oz. to drams and scruples.
Ans. 528 dr. 1584 scru.
13. Reduce 6 lbs. 3 oz. to grains. Ans. 36000 gr.
14. Reduce 2 lbs. 5 oz. 4 drams 1 scruple to grains.
Ans. 14180 gr.
15. Reduce 6 miles to furlongs and rods.
Ans. 48 fur. 1920 rods.
16. Reduce 5 furlongs to rods and yards.
Ans. 200 rods, 1100 yds.
17. Reduce 24 degrees to geographical miles.
Ans. 1440 miles.
18. Reduce 15 degrees to statute miles of $69\frac{1}{2}$ to a degree.
Ans. 1042.5 miles.

When it is required to multiply by $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$, the operation may frequently be rendered easier, by taking the decimal values of the fraction, and annexing them to the integer; and multiply

by the sum. In the preceding example, reduce the ½ mile to its decimal value, or .5; which annexed to 69 becomes 69.5 for the statute miles in a degree. By this multiply the given degrees, and the product will be found 1042.5, or 1042 miles, and 5 tenths, or half a mile.

19. Reduce 120 yards to quarters and nails.
Ans. 480 qrs.. 1920 na.
20. Reduce 12 yards to nails. Ans. 192 na.
21. Reduce 127 ells English to quarters. Ans. 635 qrs.
22. Reduce 175 ells Flemish to quarters and nails.
Ans. 525 qrs. 2100 na.
23. Reduce 15 acres to roods and perches.
Ans. 60 R. 2400 P.
24. Reduce 24 acres, 3 roods to perches. Ans. 3960 P.
25. Reduce 46 acres, 2 roods, 19 perches, to perches.
Ans. 7459 P.
26. Reduce 4 acres, 1 rood, 39 per. to perches. Ans. 719 P.
27. Reduce 15 square feet to square inches.
Ans. 2160 sq. inches.
28. Reduce 4 hhds. 16 gals. to gallons and quarts.
Ans. 268 gals. 1072 qts.
29. Reduce 15 gals. to quarts and gills. Ans. 60 qt. 480 gills.
30. Reduce 54 gallons, 3 quarts, to quarts, pints, and gills.
Ans. 219 qt. 438 pt. 1752 gills.
31. Reduce 25 bushels, 2 pecks, to pecks and quarts.
Ans. 102 p. 816 qt.
32. Reduce 24 pecks, 1 pint, to pints. Ans. 385 pt.
33. Reduce 3 years, 7 months, to months. Ans. 43 mo.
34. Reduce 7 years, 4 mo. 22 days, to days. Ans. 2662 days.
35. Reduce 16 days, 22 hours, to hours and minutes.
Ans. 406 h. 24360 min.
36. Reduce 27 days, 10 hours, 16 seconds, to seconds.
Ans. 2368816 sec.
37. Reduce 16 degrees, 48 minutes, to minutes and seconds.
Ans. 1008 min. 60480 sec.
38. Reduce 4 signs, 13 degrees, to minutes. Ans. 7980′.
39. Reduce 15 lb. 7 oz. 10 dwt. 4 gr. troy weight, to grains.
Ans. 90004 gr.
40. In an ingot of silver, weighing 6 lb. 3 oz. 10 gr. how many grains? Ans. 36010 gr.
41. In 5 cwt. 2 qr. 16 lb. how many pounds? Ans. 632 lb.
42. In 4 miles, 6 furlongs, 22 rods, how many rods?
Ans. 1542 rods.

43. In 360 deg. how many geographic miles? Ans. 21600.
44. In 360 deg. how many statute miles? Ans. 25020.
45. In 456 yds. 3 qrs. of cloth, how many quarters and nails?
Ans. 1827 qr. 7308 na.
46. In 64 acres, 1 rood, 39 perches, how many perches?
Ans. 10297 per.
47. In 12 acres, 3 roods, 39 perches, how many perches?
Ans. 2079 per.
48. In 5 hhd. 12 gallons, 3 quarts, how many quarts?
Ans. 1311 qt.
49. In 46 bush. 3 pecks, how many quarts? Ans. 1496 qt.
50. In 5 years, 174 days, how many hours and minutes?
Ans. 48006 h. 2880360 min.
51. In 49 weeks, 4 days, 12 hours, how many hours and minutes? Ans. 7904 h. 474240 min.
52. In 9 signs, 12 degrees, 50 minutes, how many minutes?
Ans. 16970′.
53. A grocer purchased 16 cwt. 3 qrs. 12 lb. of sugar; how many pounds are there in the whole? Ans. 1888 lb.
54. A tobacconist bought 116 cwt. 2 qr. of tobacco; how many pounds were in the whole? Ans. 13048 lb.
55. A silversmith has 7 lb. 10 oz. 11 dwt. of silver; how many grains are in the whole? Ans. 45384 gr.
56. An apothecary has several sorts of drugs, weighing together, 47 lb. 6 oz. 4 dr.; how many scruples and grains are in the whole? Ans. 13692 scru. 273840 gr.
57. The sum of the distances of 4 places from Philadelphia is 156 miles, 5 fur. 6 poles; how many poles and yards are in the whole? Ans. 50126 poles, 275693 yds.
58. There are two places, one situated 40 degrees east of the meridian of Washington, the other 25 degrees west of it; what is the distance in geographic miles between them, they being situated on the equator? Ans. 3900 m.
59. A merchant bought 174 yards, 3 qr. 2 na. of broad-cloth; how many quarters and nails are in the whole?
Ans. 699 qr. 2798 na.
60. A farmer has several fields, containing in the whole 54 acres, 2 roods, 24 perches; how many perches in the whole?
Ans. 8744 per.
61. A gentleman has 4 lots of ground, containing as follow: No. 1. 2 acres, 1 rood, 30 perches—No. 2. 1 acre, 3 roods, 36 perches—No. 3. 2 roods, 39 perches—and No. 4. 5 acres, 0 roods, 10 perches; how many perches are in the whole?
Ans. 1635 per.

62. A merchant has several casks of wine, containing in the whole 156 gallons, 3 quarts; how many pints are in the whole?

Ans. 1254 pt.

63. There are 4 sisters; the age of the eldest is 13 years, 2 months, 16 days—the age of the next is 11 years, 1 month, 5 days—the age of the third is 8 years, 6 months, 11 days—and that of the youngest is 6 years, and 5 months; what is the sum of all their ages in days, hours, and minutes?

Ans. 14132 d. 339168 h. 20350080 min.

64. A gentleman has a field, in which is a large pond: the content of the field, including the pond is 47 acres, 2 roods, 36 perches; and surveying the ground, exclusive of the pond, he found it to be 26 acres, 3 roods, 39 perches; what is the content of the pond in perches? Ans. 3477 per.

65. A lady purchased for the use of her family the following articles: 18 yds. 3 qr. 2 na. of muslin—26 yds. 2 qr. 1 na. of linen—14 yds. 3 qr. 2 na. of crape—12 yds. 1 qr. of silk; and having returned home, desired her daughter to inform her, how many quarters, and nails were in the whole?

Ans. 290 qr. 1161 na.

66. A miller has in several bins the following quantities of wheat: in No. 1, he has 175 bu. 3 p. 6 qt.—in No. 2, 254 bu. 1 p. 7 qt.—in No. 3, 58 bu. 3 p. 5 qt.—and in No. 4, 546 bu. 2 p. 4 qt.; how many pecks, and quarts are in the whole?

Ans. 4143 p. 33150 qt.

REDUCTION ASCENDING.

Reduction ascending brings lower denominations to higher, and is performed by division. When any given sum or quantity of a lower denomination is to be reduced or brought to a higher denomination; you must divide by the number, required of the lower denomination to make 1 or unit of the higher; and so you must continue to do, till you bring the given sum, to the denomination required.

Reduction ascending is proved, by Reduction descending.

EXAMPLES.

1. In 467890 mills, how many dollars?

Ans. 467 dol. and 89 cts. over.

2. In 74654 dimes, how many dollars?

Ans. 7465 dol. and 4 dimes over.

3. In 246700 mills, how many dollars?

Ans. 246 dol. and 70 cts. over.

4. In 17460 pence, how many pounds?
Ans. £72, and 15*s*. over.

Here you divide, in the first place, by 12; because 12 pence make one shilling, or 1 of the next higher denomination, and the quotient will be the number of shillings: these you must divide by 20, because 20 shillings make £1, or 1 of the next higher denomination; the quotient thence arising will be pounds.

On the same principle, you proceed to bring any given sum or quantity of a lower denomination, to one that is higher, retaining the same value.

5. In 74616 farthings, how many shillings and pounds?
Ans. 1554*s*. £77, and 14*s*. over.
6. In 167400 grains troy, how many pounds?
Ans. 29 lb. and 15 dwt. over.
7. In 4765 dwt., how many ounces and pounds?
Ans. 238 oz. 19 lb. and 10 oz. over.
8. In 114 qrs. how many cwt.? Ans. 28 cwt. 2 qr. over.
9. In 17640 pounds, how many quarters and cwts.?
Ans. 630 qrs. 157 cwt. and 2 qrs. over.
10. In 467456 drams, how many pounds and quarters?
Ans. 1826 lb. 65 qr. and 6 lb. over.
11. In 1746880 ounces, how many tons?
Ans. 48 tons, and 14 cwt. 3 qrs. 14 lbs. over.
12. In 17645 grains apothecaries' weight, how many pounds?
Ans. 3 lbs. and 6 dr. 0 scru. 5 gr. over.
13. In 168474 feet, how many miles?
Ans. 31 miles, and 7 fur. 58 yds. over.
14. In 274755 feet, how many miles?
Ans. 52 miles, and 65 yds. over.
15. In 7468 rods, how many miles?
Ans. 23 miles, and 108 rods over.
16. In 11462 rods, how many furlongs?
Ans. 286 fur. and 22 rods over.
17. In 316 geographic miles, how many degrees?
Ans. 5° and 16′ over.
18. In 11456 geographic miles, how many degrees?
Ans. 190° and 56′ over.
19. In 164736 inches, how many miles?
Ans. 2 miles, and 4 fur. 176 yds. over.
20. In 14764 nails of cloth, how many yards?
Ans. 922 yds. and 3 qrs. over.
21 In 2746 quarters, how many ells English?
Ans. 549 ells, and 1 qr. over.

22. In 16217 quarters, how many yards?
Ans. 4054 yds. and 1 qr. over.

23. In 74678 perches of land, how many acres?
Ans. 466 acres, and 118 per. over.

24. In 174 roods, how many acres?
Ans. 43 acres, and 2 roods over.

25. In 4761 square feet, how many square yards?
Ans. 529 sq. yds.

26. In 17647 square yards, how many square perches?
Ans. 583,371+ sq. per.

27. In 1476 pints, how many gallons?
Ans. 184 gals. and 2 qts. over.

28. In 167840 gills, how many barrels of 32 gallons each?
Ans. 163 bbl. and 29 gals. over.

29. In 1789500 minutes, how many years of 365 days, 6 hours?
Ans. 3 yrs. and 3527 h. over.

30. In 7854 hours, how many days?
Ans. 327 d. and 6 h. over.

31. In 47820 seconds of a degree, how many degrees?
Ans. 13° and 17′ over.

32. In 7647 pounds, how many quarters?
Ans. 273 qr. and 3 lb. over.

33. In 10752 pounds, how many tons?
Ans. 4 tons, and 16 cwt. over.

34. In 16840 pints of wine, how many hogsheads?
Ans. 33 hhds. and 26 gal. over.

35. In 4746 grains of silver, how many ounces?
Ans. 9 oz. and 17 dwt. 18 gr. over.

QUESTIONS. 1. What is Reduction?—2. How is it divided?—3. How do you perform Reduction descending?—4. When any sum of a given denomination is to be reduced to a lower one, how do you proceed?—5. By what number do you multiply?—6. When the given sum consists of several denominations, with which do you begin?—7. What do you do with the other denominations?—8. How is Reduction descending proved?

9. How do you perform Reduction ascending?—10. By what numbers do you divide?—11. How is it proved?—12. When you have to multiply or divide by the fractions $\frac{1}{2}$ or $\frac{1}{4}$, how would you manage them?—13. Will you explain this by an example?

THE FOLLOWING TABLE shows the Currencies of the principal countries with which the United States have intercourse, reduced to Federal Currency, or Dollars, Cents, and Mills.

Great Britain and Ireland.

20*s.* sterling=$4.44,4,44=£1.
1*s.* " " .22,2,22 " 12*d.*
1*d.* " " 1,8,51 " 4 *qrs.*
1 *Guinea* " 4.66,6,66 " 21*s.*
4*s.*6*d.* or 54*d.*=1.00
20*s.* Irish=$4.10,25=£1.
1*s.* " " .20,5,2=12*d.*

France.

1 livre =$0.18,5,04= 20 *sols.*
1 sol " .00,9,25+=12 *den.*
1 denier " .00,0,77+
1 franc " .18,3,33=100 *cent.*
1 centime " .00,1,83.
1 crown=$1.10.

Holland.

1 florin or gilder=$0.40=20 *stiv.*
1 stiver =$0.02.
1 shilling " 12=6 *stivers.*
2½ florins " 1.00 " 1 *rix-dollar.*
20*s.* Flemish " 2.40 " 1£.
1 ducat " 2.00 " 5 *florins.*
1 groat " .01 " 8 *pennings.*

Spain.

1 dollar plate=$1.00 or 100 *cents.*
1 real plate " .10.
1 real vellon " .05.
1 marvedies " .00,1,47+

Russia.

1 ruble =$0.75* =100 *copecs.*
1 copec " .00,7,5 " 2 *denuscas.*
1 denusca " .00,3,75.

Leghorn—Italy.

1 piaster or pezzo=$0.86,5,55.
1 livre =$0.14,44=3⅓ *soldis.*
1 soldi " .04,3,33=12 *deniers.*
1 denier " .00,3,61+
1 ducat " 1.08,3=7½ *livres.*

Turkey.

1 piaster=$0.38,8 = 4 *solotas.*
1 asper " .04,8,5 " 4 *mangars.*
1 xeriff " 1.43,7 " 15 *solotas.*

Calcutta.

1 rupee=$0.50 =16 *anas.*
1 ana " .03,12 " 12 *pices.*

China.

1 tale =$1.48 =10 *maces*
1 rupee " .50
1 mace, " .14,8 " 10 *cand.*
1 candareen " .01,4,8 " 10 *caxas*

Jamaica—West Indies.

20 shillings=$3.00 =1£.
12 pence " .15 " 1 *shilling.*
1 penny " .01,2,5 " 4 *far.*
6*s.* 8*d.* or 80*d.*, 1 dollar.

Portugal.

1 millrea=$1.24=1000 *reas.*
1 rea " .00,1,24.
8 reas " .01.
800 reas " 1.00.

Trieste, Austria.

1 florin =$0.48,4,85=5¼ *livres.*
1 livre " .09,2,35 " 20 *soldi.*
1 soldi " .00,4,617.
1 cruitzer " .00,0,808.

East Indies.

1 rupee =$0.55,5,5=10 *fanams*
1 fanam " .05,5,5 " 8 *pices*
1 pice " .00,6,9+
1 pagoda " 1.94.

Cuba.

1 rial =$0.12,5.
8 rials " 1.00.

Hamburg.

1 mark-banco=$0.33,3,33=16 *sti.*
3 marks =$1.00=1 *rix-dollar*
2½ rix-dollars " 2.50 " 1 £. *Flem.*
1 stiver " .02,0,84.

British Possessions, N. A.

20 shillings=$4.00=1£.
1 shilling " .20 " 12*d.*
1 penny " .01,6,66=4 *far.*
5*s.* or 60*d.*=$1.00.

* This is the value of a ruble, by an essay, lately made in the mint of the U. States.

NOTE.—By the help of the preceding table, any sum of money, in the currency of any one of the above-mentioned places, may be easily reduced to dollars and cents.

If the given sum consists of one denomination only, multiply it by the value of a unit or 1 of that denomination in dollars, cents and mills, as given in the table, and the product will be the sum required. But, if there are several denominations in the given sum, then multiply each one by its respective value in dollars, cents, and mills; and the sum of these products will be the amount required.

THE FOLLOWING TABLE shows the currency of the several states, reduced to Dollars, Cents, and Mills.

N. E. States, Virginia & Kentucky.

20 shillings = $3.33,3,33 = 1£.
1 shilling ' .16,6,66 ' 12*d.*
1 penny ' .01,3,88 ' 4 *far.*
1 farthing ' .00,3,47.
6*s.* or 72*d.* ' 1.00.

New York and N. Carolina.

20 shillings = $2.50 = 1£.
1 shilling ' .12,5 ' 12*d.*
1 penny ' .01,04 ' 4 *far.*
1 farthing ' .00,2,6.
8*s.* or 96*d.* ' 1.00.

New Jersey, Pennsylvania, Delaware, and Maryland.

20 shillings = $2.66,6,66 = 1£.
1 shilling ' .13,3,33 ' 12*d.*
1 penny ' .01,1,11+ = 4 *far.*
1 farthing ' .00,2,78.
7*s.* 6*d.* or 90*d.* = $1.00.

South Carolina and Georgia.

20 shillings = $4.28,5,7 = 1£.
1 shilling ' .21,4,28 ' 12*d.*
1 penny ' .01,7,85 ' 4 *far.*
1 farthing ' .00,4,44.
4*s.* 8*d.* or 56*d.* = $1.00.

NOTE.—By the help of this table, any sum of money in the currency of any of the above mentioned states, may easily be reduced to dollars, cents, and mills.

If there are pounds, shillings, and pence in the given sum, multiply each denomination by the value of a pound, shilling, and penny, of that particular currency, given in the above table; and the sum of the products will be the sum required in dollars and cents. If pounds only are given; then multiply by the value of a pound in that particular currency as given in the table, and the product will be the answer in dollars, cents and mills.

TABLE OF FOREIGN AND DOMESTIC COINS.

This table exhibits their names, standard weight, and value in Dollars, Cents, and Mills.

Gold Coin.	St. Wt. dwt	gr.	Value. D.	C.	Silver Coin.	St. Wt. dwt	gr.	Val. D.	C.	U. S. Coin.	St. Wt. dwt.	gr.	Value. D.	C.
Johannes	18	0	16.00,0		Fr. Crown	19	6	1.10		Eagle	11	4½	10.00	
Doubloon	16	21	14.93,0		Spa. Dollar	17	0	1.00		Half Eagle	5	14½	5.00	
Moidore	6	18	6.		A rix-dol. of									
E. Guinea	5	6	4.66,6		Denmark,					Dollar	17	1¾	1.00	
F. Guinea	5	5	4.60		Sweden,					Half dollar	8	12⅞	.50	
Sp. Pistole	4	6	3.77,3		and Hamburg.									
Fr. Pistole	4	4	3.66,7			17	6	1.00		Dime	1	16.9	.10	

Other gold coin passes in the United States by its weight, at 89 cents per dwt., if it has sufficient fineness. Other silver coin is valued at 111 cents per ounce, by an act of congress.

To REDUCE sterling to dollars and cents, the dollar being valued at 54 pence.

If the given sum is pounds only: multiply by the value of a pound sterling, in dollars, cents, and mills, and the product will be the answer in hundredths of a mill; which reduce to dollars and cents, for the sum required. But, if there are also shillings and pence given, multiply the shillings by 22,2,22, and the pence by 1,8,51. The sum of these products will be the dollars, cents and mills required. Or,

Reduce the given sum to pence, which multiply by 100, and divide the product by 54; the quotient will be the answer in cents. See the table of foreign currency.

EXAMPLES.

1. Reduce £165, 12*s.* and 6*d*, sterling, to dollars and cents.

Ans. $736.10,3,7.

1st method. 165×4.44,44=733.32,60=dol. and cts. in £165.
12×.22,22= 2.66,66=dol. and cts. in 12*s.*
6×1,851= .11,11=cents in 6*d.*
736.10,37=the sum required.

2d method.

£	*s.*	*d.*
165	12	6
20		
3312		
12		
39750=the pence of the given sum.		
100		
3975000÷54=736,11,1+		

This gives a few mills more than the former method, and is the most correct. The difference is owing to the omission of the very small fractions of the pound, shilling and penny, in the table.

2. Reduce £455, 16*s.* and 9*d.* to dollars and cents.

Ans. $2025.94,4+

3. Reduce 154 guineas, to dollars and cents. Ans. $718.66,6.

Here again the latter method gives a few mills more than the former; for $4.66,6,6¢ are not exactly the value of a guinea.

Gold is the only legal tender in England. The dollar is merely an article of commerce, and consequently varies in value with the quantity in market. Its true value, as determined by the the British mint, is something less than 50 pence sterling. The Congress of the United States fixed its value at 54 pence sterling; which is too high.

4. A lady, having a legacy of £476, 10*s*. left to her by a relative in London, what sum in dollars and cents should she receive in New York for the same, the dollar being in London 51 pence sterling? Ans. $2242. 35+

£	*s.*
476	10
	20
	9530
	12
	114360
	100
	11486000÷51=2242,35+

When the value of the dollar is not 54 pence, you must always use the second method, and observe to divide by the current value of the dollar, for the true sum in dollars and cents.

5. A merchant in London draws upon his agent in Philadelphia, for the sum of £716, 8*s*. and 9*d*.; how much in dollars and cents will be the sum, the value of the dollar being 53 pence sterling? Ans. $3244.15,1.

To REDUCE dollars and cents to sterling, the dollar being valued at 54 pence.

Reduce the given sum to cents; which multiply by 54, and divide by 100, the quotient will be pence: these reduce to shillings, and pounds for the true sum. Or,

Reduce the given sum to hundredths of a mill, and divide by 4.44,4,44 for pounds; the remainder (if more than 22,2,22 the value of a shilling) divide by 22,2,22 for shillings; and the remainder (if more than 1,8,51 the value of a penny) divide by 1,8,51 for pence: but, if the remainder after the first division is less than 2,22,22, then divide by 1,8,51 for pence, and the remainder by 0,4,6+ the value of a farthing, for farthings.

EXAMPLES.

6. Reduce 17465.45 cents, to pounds sterling.

Ans. £3929 14*s*. $6\frac{3}{10}d$.

1st method.

17465.45
54
1.00)943134.30
12)943134.
2.0) 7859.4 6.
Ans. £3929, 14*s*. 6*d*.

2d method. 4.44,4,44)17465.45,0,00(3929£.
1333332 . . .
4132130
3999996
1321340
888888
4324520
3999996
22.2,22)3.24,524(14*s*.
2 2222
102304
88888
1.851)13.416(7*d*.
12957
,4.60)4.59(1*qr*.
460

Here after the first division $3.24,524 remain, which divided by the value of 1 shilling, or 22.2,22 cents, gives 14 shillings. The remainder is 13.416 cents, which divided by ,1.851, the value of a penny, gives 7 pence; the remaider 4.59 mills, divided by ,4,60 the value of a farthing, gives 1 farthing nearly.

The first method gives the true sum. The latter gives a few mills too much, because we do not divide by the exact value of a pound, shilling, &c. When the value of the dollar is not 54 pence, you must use the first method, but observe to multiply by the current value, and divide by 100.

7. A gentleman with his wife being about to make the tour of Europe, would invest his funds amounting to 2764 dolls. 37½ cts. in English funds; how much sterling will be the sum, the dollar being worth only 51 pence? Ans. £587. 8*s*. 7⅛*d*.

8. How much sterling will 3740 dolls. 25 cts. amount to, the dollar being 52 pence? Ans. £810. 7*s*. 9*d*.

9. How much sterling will 1574 dolls. 62.5 cts. amount to, the current value of the dollar being 53 pence?
Ans. £347. 14*s*. 7⅛*d*.

10. How many guineas will 846 dolls. 12.5 cts. purchase, the current value of the dollar being 52 pence?
Ans. 174 guineas, and 12*s*. 6½*d*. over.

Having reduced the given sum to shillings, you must divide by 21, the number of shillings in a guinea, and the quotient will be the answer.

11. How many guineas may be purchased for 1167 dolls. 87.5 cts. the dollar being valued at 51 pence?

Ans. 240 guineas, and 6*s.* 9$\frac{3}{8}$*d.* over.

To REDUCE the currency of France to dollars and cents:

If the given sum is livres, or livres, sols, and deniers; multiply the livres by the value of a livre as given in the table of foreign currency, and the sols and deniers by their respective values, as given in the said table. These several products, added together, will be the amount required. But,

If the given sum is francs and centimes, then multiply by the value of a franc, as given in the table, and the centimes by the value of a centime; the sum of these products will be the amount required. In all cases reduce the sum to dollars and cents.

EXAMPLES.

12. What will 4764 livres, 12 sols, 6 deniers, amount to in dollars and cents? Ans. $881.64,2.

13. A lady in Philadelphia has the sum of 14764 francs, 16 centimes, left to her by her father in Paris: what is the value in dollars and cents? Ans. $2706.75,9,28.

14. A gentleman in France, about to remove to the United States, invested his funds amounting to 11764 livres in public stock at 18.5 cents per livre; how much did he purchase?

Ans. $2176.34.

15. A merchant imported from France a case of silk, invoiced at 1087 francs, 75 centimes: what is the amount in dollars and cents? Ans. $199.42,0,5.

16. What is the value of 1476 livres, 10 sols, in dollars and cents, the current value of the livre being 17 cents 8 mills?

Ans. $262.81,7.

17. In 12784 francs, how many dolls. and cts.; the current value of the franc being 17 cts. 9 mills? Ans. $2288.33,6.

When the current value of the livre and franc, as in these last examples, is different from that given in the table, you must multiply by the current value, for the true sum. The same thing is to be observed in the reduction of any other currency.

To REDUCE dollars and cents to French currency:

Reduce the given sum to hundredths of a mill, so that there may be as many decimal places in the dividend, as there are in the divisor. Then divide by 18.5,04 cents for livres, and by 18.3,33 cents for francs, as given in the table of foreign cur-

rency; and the remainder, if there is any, by the value of a sol, denier, or centime as the case may be.

But, if the value of a livre and franc is different from that given in the table, observe always to divide by that value, for the true sum.

EXAMPLES.

18. Reduce 1074.27 cents, to livres.

Ans. 5805 livres, 12 sols, 2 deniers.

18.5,04)1074.27,000(5805 livres.
925 20
149070
148032
103800
92520
9.25)11,280(12 sols.
9 25
2030
1850
.77)1,80(2 deniers.
1 54
26

Here you add three cyphers or decimals to the cents, which reduce the sum to hundredths of a mill; these divide by 18.5,04, the quotient will be livres; the remainder divide by 9 mills 25 hundredths, the value of a sol, the quotient will be sols; the remainder, if any, divided by ,77 hundredths of a mill, the value of a denier, will give the deniers. But,

If the given sum is to be reduced to francs, you must divide by 18.3,33, and the quotient will be the francs; the remainder divided by 1,83, the value of a centime, will give the centimes.

19. Reduce 1764 dollars to francs.

Ans. 9621 francs, 99 centimes.

20. General Lafayette received of the congress of the United States the sum of 200,000 dollars; what is the amount in francs?

Ans. 1,090,928 francs, 93 centimes.

21. A lady has the sum of 1576 dollars, 15 cents, and desires to know how many livres and sols they would amount to: what is the number?

Ans. 8528 livres, 13 sols+.

22. The expense of a young lady at a boarding school in France is $285.50 cents; what is the amount in francs and centimes?

Ans. 1557 francs, 30 centimes.

23. Reduce $4789.17, to francs. Ans. 26,123 fr. 22 cen.

24. Reduce $13546.37,5, to livres, and also to francs.
Ans. 73207 liv. 16 sols, 4+den. 73890 fr. 66+cen.

To REDUCE the currency of Holland to dollars and cents:

Multiply the given florins by 40, the value of a florin in the table of foreign currency, the product will be the amount in cents: If shillings and stivers are also given, or either of them, multiply them by their respective values: the sum of all the products will be the amount required. If ducats, rix dollars, or pounds are given, multiply them by their respective values, as given in the above table, for their value in dollars and cents.

EXAMPLES.

25. A merchant in Amsterdam shipped to Philadelphia, a quantity of goods, valued at 17645 florins, 16 stivers; what is the value in dollars and cents? Ans. $7058.32.

26. A lady leaving Amsterdam sold her effects, which amounted to 1246 florins, 12 shillings: on her arrival at New York, she desired to know the amount of her funds in dollars and cents: what was the amount? Ans. $498.52.

27. A gentleman has in Dutch funds the sum of 1225 pounds, 16 shillings Flemish, which he is desirous to transfer to the currency of the United States; what is the sum in dollars and cents? Ans. $2941.92.

23. A gentleman, with his family, arrived from Holland to make a tour in the United States, and brought with him 5165 florins, 15 stivers; how much is the sum in dollars and cents? Ans. $2066.30.

29. A young lady has a fortune of 37491 guilders, 3 stivers, in the funds of the Dutch government, which she wishes to transfer to the Bank of the United States, and receive stock in exchange; how much stock must she receive, if it be transferred to her at par? Ans. $14996.46.

To REDUCE dollars and cents to the currency of Holland:

Reduce the given sum to cents or mills, and divide by the value of that particular denomination, mentioned in the question, as shown in the table of foreign currency.

EXAMPLES.

30. Reduce $2450.25, to florins. Ans. 6125 flor. 12+ stiv.

31. A merchant of New York remitted to Holland the sum of 2468 dollars, 50 cents; what will be the amount in pounds Flemish? Ans. £1028, 10s. 5 stiv.

32. A farmer of Virginia sent as an adventure to Amsterdam, 50 hhds. of tobacco, valued here at 4785.47 cents, which was sold so as to cover cost and charges; what was the amount in florins and stivers, when the freight and other charges amounted to 1264 dols. 12½ cts.? Ans. 15123 flor. 19¾ stiv.

33. Reduce $5467.15 cents, to florins and stivers.
Ans. 13667 flor. 17½ stiv.

PROMISCUOUS EXAMPLES.

See the note at the foot of the table of foreign currency.

34. Reduce 1624 piasters, 25 aspers of Turkey, to dollars and cents. Ans. $630.23,3,25.

35. Reduce 625 rubles, 26 copecs of Russia, to dollars and cents. Ans. $468.94,5.

36. Reduce 542 piasters, 5 livres of Leghorn, to dollars and cents. Ans. $469.85.

37. Reduce 1156 marks-banco, 12 stivers of Hamburg, to dollars and cents. Ans. $385.58⅓.

38. Reduce 256 tales, 5 mace, 4 candereens of China, to dollars and cents. Ans. $379.67,92.

39. Reduce 175 millreas, 645 reas of Portugal, to dollars and cents. Ans. $217.77,9,8.

40. Reduce 564 florins, 5 cruitzers of Austria, to dollars and cents. Ans. $273.49,6.

41. Reduce 316 pagodas, 25 fanams of India, to dollars and cents. Ans. $614.42,7,5.

42. A merchant of Jamaica, (West Indies,) shipped to Philadelphia goods amounting to 567 pounds, 10 shillings; how much is the sum in dollars and cents? Ans. $1702.50.

43. A merchant in Montreal, (Canada,) purchased a draft upon the Mechanics' Bank of the city of New York, for 476 pounds, 16 shillings; what is the sum in dollars and cents?
Ans. $1907.20.

To REDUCE New England and Virginia currency to dollars and cents:

6 shillings or 72 pence = 100 cents: or
18 pence = 25 cents, by dividing by 4, the greatest common measure: therefore

Multiply the Virginia or New England currency in pence, by 25, and divide by 18, for dollars and cents: Or,

Multiply the given sum, if in pounds only, or in pounds, shillings and pence, by the respective values of these denominations, as given in the table of domestic currency. The sum of

the products will be the amount required. See the table of domestic currency.

44. Reduce 146 pounds, 16 shillings, and 4 pence, to dollars and cents. Ans. $489.38,8,8.

First method. £146, 16*s*. 4*d*.= 35236 pence.

25

880900÷18=489.38,88.

Second method.

146×3,33,3,33=486.66,63
16× ,16,6,6¢= 2.66,66
4× 1,3,9 = 5,56

$489.38,85 nearly as before.

45. A lady in Virginia has an income of 450 pounds, 12 shillings a year; how much is it in dollars and cents?
Ans. $1501.99,9,9+

46. A gentleman in Richmond received an order for the collection of 1764 pounds, 5 shillings; what was the amount in dollars and cents? Ans. $5880.83,2,96.

47. A gentleman has an income of 746 pounds New England currency, and is desirous of knowing the amount in dollars and cents; what is it? Ans. $2486.66,6+

48. A gentleman placed his daughter at a boarding school in New Haven, at an expense of 117 pounds, 15 shillings per annum; what is the sum in dollars and cents? Ans. $392.50.

49. A lady of Boston purchased goods to the amount of 46 pounds, 17 shillings, and desired her daughter, who had just returned from school, to inform her of the amount in dollars and cents; what was the amount? Ans. $156.49,9+

To REDUCE dollars and cents to New England, or Virginia currency:

Reduce the given sum to cents, which multiply by 18 and divide the product by 25, the quotient will be pence; which reduce to pounds.

50. Reduce $1746.16, to Virginia currency.
Ans. £523, 16*s*. 11½*d*.

51. A merchant shipped an invoice of goods to Boston, amounting to $2416.75; what was the amount in New England currency? Ans. £725, 0*s*. 6*d*.

52. A gentleman has an income of 1746 dollars a year, but residing in New England, desires to know the amount in that currency; what is it? Ans. £524, 4*s*. 4*d*.

53. A gentleman has a son at a boarding school in New England, at an expense of $275,25 cts. per annum; what is the sum in that currency? Ans. £86, 0s. 3¾d.

54. A lady has in public stocks the sum of $10475,75 cts. but residing in Virginia, is desirous to know the amount in the currency of that state: what is the sum? Ans. £3273, 13s. 5¼d.

To REDUCE sterling to New England currency; and the contrary:

54 pence sterling=72 pence New England: or

3 pence do. = 4 pence do. by dividing both by 18, the greatest common measure: therefore,

Multiply sterling by 4, and divide by 3, the quotient will be New England, or Virginia currency: And

Multiply Virginia currency by 3, and divide by 4, the quotient will be sterling.

55. Reduce £476, 16s. sterling to Virginia currency.

Ans. £635, 14s. 8d.

£	s	
476	16	
	4	
3)1907	4	
635	14	8

56. A merchant in New England received from London an invoice of goods, amounting to £784, 12s. sterling: what is the sum in that currency? Ans. £1046, 2s. 8d.

57. A farmer in Virginia shipped to Liverpool a quantity of tobacco, amounting to £1564, 10s. Virginia currency; what is the amount in sterling? Ans. £1173, 7s. 6d.

58. A lady has a fortune of £1416, 5s. in the hands of a banker in London, which she is desirous to transfer to Boston; how much in New England currency will it amount to?

Ans. £1888, 6s. 8d.

59. A merchant of Richmond shipped to Liverpool a quantity of tobacco, amounting to £1746, 15s. Virginia currency; how much would it amount to in England, provided he neither gained nor lost by the shipment? Ans. £1310, 1s. 3d.

To REDUCE New York and North Carolina currency to dollars and cents; and the contrary:

96 pence = 100 cents; or

24 pence = 25 cents, by dividing by 4, the greatest common measure: therefore,

Reduce the given sum to pence, which multiply by 25, and divide the product by 24, for cents. And

Multiply the given sum in cents by 24, and divide the product by 25, the quotient will be pence, which reduce to pounds.

60. Reduce 125 pounds, 6 shillings and 8 pence, to dollars and cents. Ans. $313.33\frac{1}{3}$.

71. A lady has in cash 2174 pounds, 10 shillings, which she desires to invest in stock of the bank of the United States; how much can she purchase, the stock being at par? Ans. $5436.25.

62. A lady has a school worth to her the sum of 1756 dollars 45 cents per annum; how much is the income in New York currency? Ans. £702, 11*s*. $7\frac{1}{5}d$.

63. A lady made a purchase of goods in New York, and on her return home found her bill amounted to the sum of 56 pounds, 16 shillings, and 4 pence. Not understanding the currency very well, she gave it to her daughter, desiring her to tell the amount in dollars and cents: what was that amount?

Ans. $142.04.

64. A gentleman finding the expenses of his family, in the city of New York, to be annually $2162.68 cents, desired his son to tell him the amount in the currency of the state; what was the amount? Ans. £865, 1*s*. 5*d*.+

65. What will 4769 dollars, 14 cents, 6 mills, amount to in New York currency? Ans. £1907, 13*s*. 2*d*.

To REDUCE sterling to the currency of New York or North Carolina; and the contrary:

54 pence sterling = 96 pence New York: or

9 pence do. = 16 pence do. by dividing by 6, the greatest common measure: therefore,

Multiply the sterling by 16, and divide the product by 9, for New York currency. And

Multiply the New York currency by 9, and divide by 16, for sterling.

66. Reduce 416 pounds, 10 shillings sterling, to New York currency. Ans. £740, 8*s*. $10\frac{1}{2}+d$.

67. A merchant in Liverpool shipped to New York goods to the amount of 1764 pounds sterling; how much is the sum in New York currency? Ans. £3136.

68. A merchant wishes to purchase a draft on London for 785 pounds, 16 shillings. How much must he give in New York currency, the exchange being at par? Ans. £1396, 19*s*. $6\frac{1}{2}+d$.

69. Reduce 2457 pounds, 16 shillings to sterling.

Ans. £1382, 16*s*. 3*d*.

70. A gentleman remits to London 8740 pounds, 10 shillings: how much sterling is the sum, the exchange being 51 pence, per dollar? Ans. £4643, 7*s*. 9¾*d*.

When the dollar is not worth 54 pence, as in the above example, you must multiply by the current value, and divide by 96 for sterling; and the contrary, to reduce sterling to New York currency.

To REDUCE New Jersey, Pennsylvania and Maryland currency to dollars and cents; and the contrary:

90 pence = 100 cents; or,

9 pence = 10 cents, by dividing by 10, the greatest common measure: therefore,

Multiply the given sum, reduced to pence, by 10, and divide by 9 for cents. And

Multiply the sum in cents by 9, and divide by 10 for pence: which reduce to shillings and pounds.

71. Reduce 564 pounds, 10 shillings, Pennsylvania currency, to dollars and cents. Ans. $1505.33,33.

72. Reduce $10746.75 cents, to Pennsylvania currency. Ans. £4030, 0*s*. 7½*d*.

73. A merchant received an invoice of goods amounting to 6470 dollars: how much is the sum in Pennsylvania currency? Ans. £2426, 5*s*.

74. A gentleman in Maryland has an estate, which he values at 3650 pounds; how much is it worth in dollars and cents? Ans. $9733.33,33.

75. A gentleman in Philadelphia has in ground rents and real estate, the sum of 12640 pounds, 15 shillings; how much is the sum in dollars and cents? $33706.66,6¢.

76. A lady has, left by her father, an annual income of 1246 dollars, 49 cents; what is the sum in New Jersey currency? Ans. £467, 8*s*. 8*d*.

To REDUCE sterling to Pennsylvania currency; and the contrary:

54 pence sterling = 90 pence Pennsylvania; or,

3 pence do. = 5 pence do. by dividing by 18, the greatest common measure: therefore,

Multiply the sterling by 5, and divide the product by 3, for Pennsylvania currency. And,

Multiply Pennsylvania currency by 3, and divide by 5, for sterling.

77. Reduce 178 pounds, 18 shillings, and 6 pence sterling, to Pennsylvania currency. Ans. £319, 11*s*. 2+*d*.

78. Reduce 416 pounds, 10 shillings, Pennsylvania currency, to sterling. Ans. £249, 18*s*.

79. A merchant of Baltimore received from Liverpool an invoice of goods, amounting to 789 pounds, 10 shillings sterling; how much is the amount in Maryland currency?

Ans. £1315, 16*s*. 8*d*.

80. A gentleman having 7641 pounds, 15 shillings Pennsylvania currency, removes to London; how much sterling is the sum? Ans. £4585, 1*s*.

81. A lady placed her daughter at a boarding school in London, at an annual expense of 110 pounds, 5 shillings sterling; what is her bill in Pennsylvania currency? Ans. £183, 15*s*.

82. A gentleman commenced travelling, and took with him the sum of 1260 pounds, 15 shillings Maryland currency, which he placed in the hands of a banker in London, subject to his draft; what was the amount in sterling? Ans. £756, 9*s*.

To REDUCE South Carolina and Georgia currency to dollars and cents; and the contrary:

56 pence=100 cents: or
14 pence= 25 cents, by dividing by 4, the greatest common measure: therefore

Multiply the sum in South Carolina currency, reduced to pence, by 25, and divide the product by 14, for cents. And

Multiply the sum in cents by 14, and divide by 25, the quotient will be pence; which reduce to shillings and pounds.

83. Reduce 746 pounds, 10 shillings, Georgia currency to dollars and cents. Ans. $3199.28,5+.

84. Reduce 1074.25 cents, to South Carolina currency.

Ans. £250, 13*s*. 2*d*.

85. A merchant in Charleston sends to New York a quantity of cotton, amounting to 579 pounds, 10 shillings, Georgia currency; how much will it amount to in dollars and cents?

Ans. $2483.57,14+.

86. A gentleman has a plantation in South Carolina, valued at 7896 pounds, which he exchanges for property in Philadelphia; what must be the value of the property received, in dollars and cents? Ans. $33840.

87. A lady has a cotton plantation in Georgia, and is desirous of disposing of it, to remove to New York; what must be its value in the currency of South Carolina, when she receives for it 10140 dollars? Ans. £2266.

88. A father left his daughter the sum of 1265 pounds, 10 shillings, Georgia currency, to be paid to her without interest upon her marriage day; what is its value in dollars and cents?

Ans. $5423.57,14+.

To REDUCE South Carolina and Georgia currency to sterling; and the contrary:

54 pence sterling=56 pence South Carolina; or,

27 pence do. =28 pence do. by dividing by 2, the greatest common measure: therefore

Multiply the sterling by 28, and divide by 27 for South Carolina or Georgia currency. And

Multiply South Carolina currency by 27, and divide by 28 for sterling.

89. Reduce 312 pounds, South Carolina, to sterling.

Ans. £300, 17*s*. $1\frac{1}{2}+d$.

90. Reduce 516 pounds, 10 shillings sterling, to South Carolina.

Ans. £535, 12*s*. 7+*d*.

91. A merchant in Liverpool ships to his correspondent in Charleston, goods amounting to 1764 pounds, 10*s*. sterling; what is the sum in Georgia currency? Ans. £1829, 17*s*. $0\frac{1}{2}d$.

92. A merchant of Savannah remitted to London, the sum of 6174*l*. 5*s*. Georgia currency; for what sum must he have credit, the exchange being at par? Ans. £5953, 14*s*. $9\frac{3}{4}d$.

93. A planter of Georgia shipped to Liverpool, a quantity of cotton, amounting to the sum of 2160 pounds, 15 shillings, South Carolina currency; what sum in sterling must he receive for it, if he neither gains nor loses by the adventure?

Ans. £2083, 11*s*. 7+*d*.

To REDUCE New England and Virginia currency to New York currency; and the contrary:

72 pence Virginia = 96 pence New York; or,

3 pence do. = 4 pence do. by dividing by 24, the greatest common measure: therefore,

Multiply the New England currency by 4, and divide by 3, for New York currency. And

Multiply the New York currency by 3, and divide by 4, for New England.

94. Reduce 475 pounds 10 shillings, Virginia currency, to New York currency. Ans. £634.

95. Reduce 4791 pounds, New York, to New England currency. Ans. £3593, 5*s*.

96. A merchant of Virginia sent to New York a quantity of tobacco, amounting to 1140 pounds, 12 shillings; what will be the amount in the currency of New York? Ans. £1520, 16*s*.

97. A gentleman from Virginia, visiting with his family at Balston Springs, on his way purchased for his wife and daughters sundry articles in the city of New York, amounting to 55 pounds, 16 shillings, and 6 pence, North Carolina currency: how much is the sum in that of Virginia? Ans. £41, 17*s*. 4½*d*.

98. A young lady, at a boarding school in New Haven, finds her annual expense to be 95 pounds, 10 shillings, New England currency: what is it in that of New York? Ans. £127, 6*s*. 8*d*.

99. A gentleman from Virginia placed his eldest son at school in New York; and, for boarding, tuition, clothing, &c. his bill amounted to 135 pounds, 10 shillings: what is the sum in Virginia currency? Ans. £101, 12*s*. 6*d*.

To REDUCE Virginia currency to that of South Carolina and Georgia; and the contrary:

72 pence Virginia = 56 pence Georgia; or,

9 pence do. = 7 pence do. dividing by 8, the greatest common measure: therefore,

Multiply Virginia currency by 7, and divide by 9, for Georgia currency. And

Multiply Georgia currency by 9, and divide by 7, for Virginia.

100. Reduce 475 pounds, 10 shillings, Virginia currency, to that of South Carolina. Ans. £369, 16*s*. 8*d*.

101. Reduce 764 pounds, 15 shillings, Georgia currency, to that of New England or Virginia. Ans. £983, 5*s*.

102. A merchant of Charleston remitted his correspondent in Richmond the sum of 1216 pounds, 12 shillings, South Carolina currency: what is the sum in that of Virginia?

Ans. £1564, 4*s*.

103. The income of a cotton plantation was found to be 1024 pounds, 14 shillings, Georgia currency; what is the amount in that of New England? Ans. £1317, 9*s*. 5+*d*.

104. A cargo of provisions and lumber was shipped from Boston, invoiced at 3465 pounds, 5 shillings, and consigned to a merchant in Charleston: what is the amount in South Carolina currency? Ans. £2695, 3*s*. 10½+*d*.

105. A lady had an estate in Georgia, valued at 4768 pounds. But, for the purpose of educating her daughters, removed to Connecticut, and exchanged her property for lands and tenements there: what was the value of the property received in New England currency; the exchange being made even?

Ans. £6130, 5*s*. 8½+*d*.

To REDUCE New England and Virginia currency, to Pennsylvania, New Jersey and Maryland currency; and the contrary:

72 pence Virginia = 90 pence Maryland; or,

36 pence do. = 45 pence do. by dividing by 2, the greatest common measure: therefore

Multiply the pence New England currency by 45, and divide by 36, for pence Pennsylvania. And

Multiply the pence Pennsylvania by 36, and divide by 45, for Virginia currency.

106. Reduce 546 pounds, 10 shillings, New England currency, to Pennsylvania. Ans. £683, 2*s*. 6*d*.

107. Reduce 746 pounds, 15 shillings, Maryland currency, to Virginia. Ans. £597, 8*s*.

108. A merchant of Boston remitted to his correspondent in Philadelphia the sum of 1016 pounds, 5 shillings, New England currency; what is the amount in Pennsylvania currency? Ans. £1270, 6*s*. 3*d*.

109. A commission merchant of Baltimore sold goods for a manufacturer in Massachusetts, amounting to 1260 pounds, 16 shillings, Maryland currency; what is the sum in New England currency? Ans. £1008, 12*s*. $9\frac{1}{2}+d$.

110. A gentleman in New England possessed property which he valued at 4176 pounds, 17 shillings, but removing to Philadelphia, he exchanged the same for houses and lots there; what is their value in Pennsylvania currency, the exchange being made even? Ans. £5221, 1*s*. 3*d*.

111. A lady of Maryland has a fortune of 7467 pounds, 10 shillings, but marrying a gentleman of Richmond, is desirous of knowing the value of her estate in Virginia currency; what is the amount? Ans. £5974.

112. A young lady from Virginia, at a boarding school in Philadelphia, finds her expenses to be 145 pounds, 12 shillings per annum, in Pennsylvania currency; what is the amount in the currency of her native state? Ans. £116, 9*s*. 7+*d*.

113. If a young gentleman at Princeton college costs his father 175 pounds, 12 shillings, per annum, New Jersey currency; what will it be in Virginia or New England currency? Ans. £140, 9*s*. 7+*d*.

To REDUCE New York and North Carolina currency to New Jersey, Pennsylvania and Maryland currency; and the contrary:

96 pence New York = 90 pence Pennsylvania; or,

48 pence do. = 45 pence do. by dividing by 2. the greatest common measure: therefore

Multiply New York currency in pence by 45, and divide by 48, for Pennsylvania currency. And

Multiply Pennsylvania currency in pence, by 48, and divide by 45, for New York currency.

114. Reduce 125 pounds, 10 shillings, New York, to New Jersey currency. Ans. £117, 13*s*. 1½*d*.

115. Reduce £167, 12*s*. 6*d*. Maryland currency, to New York or North Carolina currency. Ans. £178, 16*s*.

116. A merchant of Philadelphia purchased goods in New York, to the amount of 1746 pounds, 17 shillings, New York currency; but desired his bill to be made out in Pennsylvania currency; what was the amount? Ans. £1637, 13*s*. 5¼*d*.

117. A gentleman of New York purchased an estate in New Jersey, valued at 1576 pounds, 10 shillings, in the currency of the state: what sum must he pay in New York currency?

Ans. £1681, 12*s*.

118. A manufacturer of Maryland consigned to a merchant in New York, goods amounting to 1846 pounds, 11 shillings; what is the sum in New York currency? Ans. £1969, 13+*s*.

119. A lady of Maryland has an income of 516 pounds, 10 shillings, but removing to New York, receives her money in the currency of that state; how much is the amount?

Ans. £550, 18*s*. 8*d*.

120. A young lady of New York had a fortune left to her by a rich relation, amounting to 5640 pounds, 17 shillings, which was to be paid on her marriage day, but removing with her husband to Pennsylvania, would know the amount in the currency of that state; what was it? Ans. £5288, 5*s*. 11¼*d*.

To REDUCE New York or North Carolina currency, to South Carolina or Georgia currency; and the contrary:

96 pence New York = 56 pence South Carolina; or,
12 pence do. = 7 pence do. by dividing by 8 the greatest common measure: therefore

Multiply New York currency by 7, and divide by 12, for South Carolina. And

Multiply Georgia currency by 12, and divide by 7, for New York currency.

121. Reduce 546 pounds, 10 shillings New York, to South Carolina currency. Ans. £318, 15*s*. 10*d*.

	546	10	
		7	
12)	3825	10	
	£318	15	10

122. Reduce 847 pounds, 12 shillings, and 6 pence, Georgia, to North Carolina currency. Ans. £1453, 1*s*. 5+*d*.

123. A merchant of Savannah shipped to New York a quantity of cotton amounting to 1042 pounds, 10 shillings; what is the amount in New York currency? Ans. £1787, 2*s*. 10¼*d*.

124. A merchant of Charleston purchased goods of a commission merchant in New York, amounting to the sum of 1574 pounds, 16 shillings of that currency; how much was the sum in the currency of his own state? Ans. £918, 12*s*. 8*d*.

125. A gentleman of South Carolina left home on an excursion of pleasure, with the sum of 416 pounds, 11 shillings; on his arrival in the city of New York he found that he had spent 27 pounds, 5 shillings; what sum had he remaining, in New York currency? Ans. £667, 7*s*. 5⅐*d*.

126. What sum in the currency of Georgia is equal to 2546 pounds, 5 shillings, and 9 pence, New York currency? Ans. £1485, 6*s*. 8*d*.

127. What sum of money in North Carolina currency is equal to 1850 pounds, 17 shillings, and 6 pence, in that of South Carolina? Ans. £1079, 13*s*. 6*d*.

128. A young lady is to receive the sum of 21 pounds, 14 shillings, and 6 pence, in money and presents, reckoning in Georgia currency, as soon as she shall be able to tell how much that sum will be in the currency of New York or North Carolina; what will be the sum? Ans. £37, 4*s*. 10¼*d*.

To REDUCE Pennsylvania currency to that of South Carolina, and the contrary:

90 pence Pennsylvania=56 pence South Carolina; or,
45 pence do. =28 pence do. by dividing by 2, the greatest common measure: therefore

Multiply Pennsylvania currency in pence by 28, and divide by 45, for South Carolina. And

Multiply South Carolina in pence by 45, and divide by 28, for Pennsylvania currency.

129. Reduce 749 pounds, 10 shillings Pennsylvania, to Georgia currency. Ans. £466, 7*s*. 1*d*.

130. Reduce 742 pounds, 12 shillings, and 6 pence, South Carolina to Maryland currency. Ans. £1193, 10*s*. 1+*d*.

131. A young lady at school in Philadelphia, is informed by her mother in Charleston that she shall send her in presents and

in money, the amount of 36 pounds, 7 shillings, and 5 pence, reckoning in Pennsylvania currency, as soon as she shall inform her, what that sum would be in the currency of Georgia; what is the sum? Ans. £22, 12*s*. 7+*d*.

132. A gentleman has a cotton plantation in Georgia, valued at 7416 pounds, which he will dispose of to a gentleman in New Jersey; what sum, reckoning in the latter currency, must he receive for it? Ans. £11918, 11*s*. 5*d*.

133. A merchant shipped to Charleston property, valued at 5476 pounds, 10 shillings, Pennsylvania currency; what sum will he receive, reckoning South Carolina currency?

Ans. £3407, 7*s*. $6\frac{1}{2}$+*d*.

SINGLE PROPORTION.

This is commonly called the Rule of Three, because three terms are always given to find a fourth. The operation is founded upon the principle that, if four numbers are proportional, the product of the means (or middle terms) is equal to the product of the extremes, (or the first and last terms.) And the product of the means, divided by either extreme, will give the other extreme.*

* Four numbers are proportional, when the first has the same ratio to the second, as the third has to the fourth, as; 2 : 6 :: 4 : 12; that is, 2 has the same ratio (or bears the same relation) to 6, that 4 has to 12.

The ratio, which one number has to another, is found by dividing the latter by the former. Thus, the ratio of 2 to 6 is 3, because 6 contains 2 three times: the ratio of 8 to 12 is 1.5, because 12 contains 8, one and a half times: and the ratio of 12 to 6 is represented by $\frac{6}{12}$. or 6 divided by 12: and so of other numbers. When there are four numbers such, that the ratio of the first to the second is the same with the ratio of the third to the fourth; these four numbers are proportional, as 4 : 8 :: 3 : 6, or as 12 : 4 :: 9 : 3. This is called direct proportion. When four numbers are proportional directly, they will be proportional in the following ways; for, in each case the product of the middle terms is equal to that of the extremes, as will appear by inspection:

1st. directly,	4 : 8 :: 3 : 6
2d. by transposition,	4 : 3 :: 8 : 6
3d. by inversion,	3 : 4 :: 6 : 8

This property of proportional numbers has been made the founda-

In all questions of the RULE OF THREE, the answer is to be considered the last term of the proportion. Of the three terms that are given, two are always of the same kind; and the third, or remaining term, must be always of the same kind with the answer, thus: If 2 bushels of wheat cost $2.50, what will 4 bushels cost? Here there are two terms alike, namely, 2 bushels and 4 bushels, which must stand in the first and second places in the statement; and from the nature of the question, the answer must be of the same kind with the remaining, or third term, which is money.

In stating the question, consider whether the answer must be greater than the third or remaining term; for then the first term in the proportion must be less than the second; but, if the answer must be less than the third term, then the first term must be greater than the second, for otherwise the product of the extremes could not be equal to that of the means, or middle terms.

Having stated the question, as above directed, reduce the first and second terms to the same denomination, if they are not so already. Reduce the third term to the lowest denomination mentioned, if it is a compound quantity. Then multiply the second and third terms together, and divide the product by the first term: the quotient will be the answer in the same kind, and of the same denomination, with the third term.

If any term has the fraction $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, added to it, the operation commonly may be rendered easier by reducing the fraction to a decimal, and adding it to the whole number or integer.

If the first and second terms will divide by a common measure, the operation may be shortened by dividing those terms by that number, and using the quotient in the operation.

Proportion is expressed by placing between the first and second terms, two dots or points, thus, : —between the second and third, four, thus, : : —and between the third and fourth, two, thus, : —as in this example, 4 : 8 : : 3 : 6, which means, that 4 is to 8, as 3 is to 6; or that 4 has the same ratio to 8, that 3 has to 6.

tion of the Rule of Three Inverse, as it is usually called. But it is a useless and unnecessary distinction. For any question in inverse proportion may, by changing the terms, be made direct, and *vice versa*.

Three numbers are proportional, when the first has the same ratio to the second as the second has to the third, as 4 : 8 : : 8 : 16, and the product of the extremes is equal to the square of the mean, or middle terms, as will appear from inspection.

EXAMPLES.

1. If 1 yard of broad-cloth costs $4.50 cents; what will 4 yards cost? Ans. $18.

yd. yds. cts.
1 : 4 : : 4.50 : $\frac{1}{4}$ proportion.
4
$18.00 Answer $18.00.

Here, from the nature of the question, the answer, or fourth term in the proportion, must be greater than the third term; therefore the first term must be less than the second; 1 must then be the first term, 4 the second; and 4.50, the third, or remaining term must be in the last place. The two first terms being of the same denomination, they require no reduction. You do not divide by the first term, because it is a unit, and would not reduce the dividend. Therefore the product of the means is the answer to the question, or the fourth term of the proportion; for multiplied by 1, it will equal the product of the middle terms.

2. If 6 bushels of wheat cost $7.50; what will 24 bushels cost? Ans. $30.

bush. bush. cts.
6 : 24 : : 7.50 : $\frac{1}{4}$ proportion; or, by dividing by 6, the common measure of 6 and 24, you have

bush. bush. cts.
1 : 4 : : 7.50 : $\frac{1}{4}$ proportion.
4
$30.00

Here again, the answer must be greater than the third term; therefore the first term must be less than the second. The first term is 6, and the second is 24; both of which will divide by 6 without a remainder; 6 is therefore their common measure. The quotients are 1 and 6, which you use in the operation, as will be seen above. In like manner, other questions may be reduced, whose first and second terms will divide by a common measure.

This operation is founded upon the principle, that if equal or proportional numbers are divided by the same number, the quotients will be equal, or proportional accordingly.

3. If 3 pounds of butter cost 50 cents; what will 25 pounds cost? Ans. $4.16,6¢.

lb. lb. cts.
3 : 25 : : 50 : $\frac{1}{4}$ prop.
50
3)12.50
4.16,6¢

Here the answer terminates in a repeating decimal of a cent, which is equal to $\frac{2}{3}$ of a unit. The answer, or fourth term in the proportion, is 4 dollars, $16\frac{2}{3}$ cents.

4. If 1 pair of stockings cost 75 cents, what will 14 pair cost?

Ans. $10.50.

pr. *pr.* *cts.*
1 : 14 : : 75 : $\frac{1}{4}$ prop.

14
300
75
10.50 the answer.

5. If 10 pounds of beef cost $62\frac{1}{2}$ cents; what will 116 pounds cost?

Ans. $7.25.

lb. *lb.* *cts.*
10 : 116 : : 62.5 : $\frac{1}{4}$ prop.

116
3750
625
625
10)7250.0
7.25 cents.

In the third term, the half cent, I make 5 tenths of a cent, or 5 mills, and annex it to the integer; the price becomes then 62 cents, 5 mills, for the third term. The first and second terms may be reduced by dividing by 2, their common measure, as directed, and shown in a preceding example.

6. If 12 oranges cost 75 cents; what will 125 cost?

Ans. $7.81,2,5.

or. *or.* *cts.*
12 : 125 : : 75 : $\frac{1}{4}$ prop.

7. If 24 yards of linen cost $14.50; what was the price of 1 yard?

Ans. 60 cts. 4,1+min.

yd. *yd.* *D.*
24 : 1 : : 14.50 : $\frac{1}{4}$ prop.

Here it is plain that the fourth term must be less than the third term; therefore the first term must be greater than the second; otherwise the product of the extremes would not equal that of the means.

8. If 1 bushel of corn costs 42 cents; what is the value of 52 bushels?

Ans. $21.84.

bu. *bu.* *cts.*
1 : 52 : : 42 : $\frac{1}{4}$ prop.

9. If 1 ton of hay sells for $8.50; for how much will $8\frac{1}{2}$ tons sell? Ans. $72.25.

ton. tons. D. cts.
1 : 8.5 :: 8.50 : $\frac{1}{4}$ prop.

Here the $\frac{1}{2}$ in the 2d term I change into 5 tenths of a ton, and annex it to the 8. It becomes then, 8.5 tons. The answer, or fourth term, must be greater than the third term, therefore, the first term must be greater than the second; for otherwise the product of the means would not equal that of the extremes.

10. A grocer bought a chest of tea containing 14 lbs. for $12.75 cents; how much was it a pound? Ans. $91.07+cts.

lb. lb. D. cts.
14 : 1 :: 12.75 : $\frac{1}{4}$ prop.

11. If you give 5 cents a loaf for bread; what will 48 loaves come to, at the same rate? Ans. $2.40.

lo. lo. cts.
1 : 48 :: 5 : $\frac{1}{4}$ prop.

12. If a man earns 75 cents a day; how much will he earn in $25\frac{3}{4}$ days? Ans. $19.31,2,5.

d. d. cts.
1 : 25.75 :: 75 : $\frac{1}{4}$ prop.

Here the $\frac{3}{4}$, in the second term, I reduce to a decimal, and annex it to the integer; and it becomes 25,75. The fourth term must be greater than the third, therefore, the first term must be less than the second; for otherwise the product of the means would not equal that of the extremes.

13. A gentleman bought a bag of coffee for the use of his family, weighing 127 lbs., for which he gave $15.25 cts.; what was it a pound? Ans. 12+cts.

lb. lb. ct.
127 : 1 :: 15.25 : $\frac{1}{4}$ prop.

14. A lady has an income of $675.75 cts.; how much may she spend every day, and have nothing left at the end of the year? Ans. $1.85+

d. d. ct.
365 : 1 :: 675.75 : $\frac{1}{4}$ prop.

15. A gentleman has an income of 1246 dol. 12 ct. 5 m.; how much may his family spend daily, and have 250 dollars left at the end of the year? Ans. $2.72,9,1+

Here you must subtract 250 dollars from the given sum; the difference will be what he spends in the support of his family. The question is performed exactly, after the deduction is made, as the preceding sum.

16. If a gentleman spends 4 dols. 62½ cts. every day, how much will that amount to in a year? Ans. $1688.12½.

d. day. ct.
1 : 365 : : 4.62,5 : ¼ prop.

17. A lady bought for the use of her family, a piece of cloth, containing 16 yds. 3 qrs. 2 na., at 1.25 cents a yard; what was the amount of the whole? Ans. $21.09+

yd. yd. qr. na. ct.
1 : 16 3 2 : : 1.25 : ¼ prop.

Here the second term consists of several denominations: it must be reduced to the lowest one mentioned, which is nails. The first term must also be reduced to nails. When this is done, then multiply the second and third terms together, and divide the product by the first term thus reduced; the quotient will be the fourth term of the proportion, or the answer to the question.

18. If 48 men can perform a certain piece of work in 24 days, how many men can do the same in 192 days? Ans. 6 men.

d. d. m.
192 : 24 : : 48 : ¼ prop. or, by dividing by 24 the greatest
d. d. m.
common measure, we have 8 : 1 : : 48 : ¼ prop.

Here it is evident from the nature of the question, that the last term of the proportion must be less than the third; therefore, the first term must be greater than the second; for otherwise, the product of the means could not equal that of the extremes.

19. A footman can perform a journey in 7 days, when the days are 12 hours long; how many days will he be, in performing the same, when the days are 10 hours long? Ans. 8⅖ days.

h. h. d.
10 : 12 : : 7 : ¼ prop.

Here the answer or fourth term in the proportion must be greater than the third term; therefore, the first term must be less than the second.

20. If 6 men can perform a piece of work in 12 days, how many men can do it in 24 days? Ans. 3 men.

24 : 12 : : 6 : ¼ prop.

21. A piece of silk containing 14 yds. 3 qrs., cost $17.62,5; what is that a yard? Ans. $1.19,4,9.

yd. qr. yd. D. ct. m.
14 3 : 1 : : 17.62,5 : ¼ prop.

Here the first term must be reduced to quarters, and the second term must be brought to the same denomination; after which, multiply the means, and divide by the first term, for the answer, or fourth term of the proportion.

22. The quarter bill of a young lady at school is $18.62½; what will her bill be for 2 quarters and a half, at the same rate? Ans. $46.56¼.

qr. qr. D. ct. m.
1 : 2.5 : : 18.62,5 : ¼ prop.

Here the half quarter, I make a decimal, and annex it to the whole number. The same I do with the ½ cent, making it 5 mills. The answer must be greater than the third term; therefore, the first term must be less than the second.

23. If 1 cwt. of sugar costs $12.50 cents, what will 6 cwt. 2 qr. 14 lb. come to, at the same rate? Ans. $82.81,2,5.

cwt. cwt. qr. lb. ct.
1 : 6 2 14 : : 12.50 : ¼ prop.

Here the second term consists of several denominations. It must be reduced to the lowest denomination mentioned. The first term must be reduced to the same denomination. After which, multiply the means, and divide the product by the first term; the quotient will be the answer, or fourth term in the proportion.

Observe always that the first and second terms must be of the same denomination; for, as often as either term is taken, so often must the other be taken also; otherwise, the proportion, and consequently the equality, of the terms will be destroyed.

24. A farmer of Virginia sold 22 hhds. of tobacco, weighing nett, in the whole, 156 cwt. 8 qrs. 16 lb. at $5.68 cents a cwt.; what is the amount of the whole? Ans. $891.15+

cwt. cwt. qr. lb.
1 : 156 3 16 : : 5.68 : ¼ prop.

25. A flour merchant sold 1574 bbls. of flour, at $5.12½ cents a barrel; what was the amount? Ans. $8066.75.

26. A gentleman bought a piece of ground, and agreed to give $26.37½ cts. an acre, and being surveyed, it was found to contain 5 acres, 2 roods, and 32 per.; what was the cost of the whole? Ans. $150.33,7,5.

acre. acre. r. per. ct.
1 : 5 2 32 : : 26.37,5 : ¼ prop.

27. A farmer made from an orchard of apples, 146 barrels of cider, which he afterwards sold at $3.12½ cents a barrel; what was the amount of the whole? Ans. $456.25.

28. A farmer raised from a field 1267 bushels of corn, which he sold at 47½ cents a bushel; what did the whole amount to? Ans. $601.82,5.

29. A merchant failing, was able to pay his creditors only 62½ cents in a dollar; how much will a person receive, whose claim is 746.25 cents? Ans. $466.40,6,25.

ct. ct. dol. cts.
100 : 62,5 : : 746.25 : ¼ prop.

30. A person purchased a piece of cloth for $54.75 cents, valuing it at $2.16¾ cents a yard; required the number of yards? Ans. 25 yds. $1\frac{8}{2}\frac{2}{1}\frac{5}{6}\frac{}{75}$. qr.

D. ct. m. D. ct. yd.
2.16,7,5 : 54,75 : : 1 : ¼ prop.

Here you must reduce the second term to tenths of a mill, that the first and second term may be of the same denomination. Then proceed as the rule directs.

31. A farmer rents a plantation, containing 316 acres, for 1.03 cts. an acre; what is the rent of the whole? Ans. $325.48.

32. A merchant sold 2 pieces of muslin, containing 74 yds. 2 qrs. 1 na. at 11½ cents a yard; what was the amount of the whole? Ans. $8.57,4+

yd. yds. qr. na. cts.
1 : 74 2 1 : : 11.5 : ¼ prop.

33. A gentleman pays 3.50 cents a week for board; what will 47½ weeks boarding cost him? Ans. $166.25.

34. If a pound of tea cost 93¾ cents, what will a box containing 14½ lbs. amount to? Ans. $13.59,3,75.

lb. lb. ct.
1 : 14.5 : : 93.7,5 : ¼ prop.

35. If 100 dollars in 12 months, gain 5 dollars; what sum will produce it in 8 months? Ans. $150.

m. m. D.
8 : 12 : : 100 : ¼ prop.

It is plain here, that the fourth term of the proportion, or answer to the question, must be greater than the third term; therefore, the first term must be less than the second. In this case, and in all others of the same kind, the interest or produce of the capital has no place in the statement. In the following

question, the sum at interest has no place in the statement. So in all other questions of the same kind.

36. If 600 dollars, in 18 months, produce 54 dollars interest, what will it produce in 10 months? Ans. 30 dollars.

m. *m.* *D.*
18 : 10 : : 54 : $\frac{1}{4}$ prop.

37. If a sum of money at interest yields in 12 months 126 dollars, what will the amount of interest be in 17 months?
Ans. $178.50.

m. *m.* *D.*
12 : 17 : : 126 : $\frac{1}{4}$ prop.

38. A gentleman lent his friend 1000 dollars for 10 months; how long must his friend in turn lend him 600 dollars to repay the favour? Ans. 16.66¢ months.

39. A lady has an income of $965.75 cents, and is desirous of saving at the end of the year, the sum of $342.87,5; how much may she spend weekly in her family? Ans. $11.97,8,3+

$965.75 = whole income.
342.87,5 ' sum saved.
$622.87,5 ' sum to be spent in 52 weeks.

40. If a staff 4 feet long casts a shadow 3 feet upon the level ground; and the shadow of a tree, standing perpendicularly to the ground, was measured, and found to be 154 feet; what was the height of the tree? Ans. 205$\frac{1}{3}$ feet.

ft. *ft.* *ft.*
3 : 154 : : 4 : $\frac{1}{4}$ prop.

41. If 4 cords of wood sell for $21.50 cents, what will 10$\frac{1}{2}$ cords come to at the same rate? Ans. $56.43,75.

32. A gentleman purchased his fuel for the winter for 72 dols. 25 cts.; how many cords of wood were there, reckoning it at 4.87\frac{1}{2}$ cents a cord? Ans. 14.825+cords, or 14$\frac{7}{8}$ cords.

43. The stage-fare being 5 cents a mile, how far can a person travel for $22.15 cents? Ans. 443 miles.

44. A lady purchased a set of silver, weighing in the whole 5 lb. 6 oz. 5 dwt. at 1.50 cents an ounce; what was the cost of the whole? Ans. $99.37,5.

oz. *lb.* *oz.* *dwt.* *cts.*
1 : 5 6 5 : : 1.50 : $\frac{1}{4}$ prop.

45. The floor of a parlour contains 32 square yards, how much carpet $\frac{3}{4}$ of a yard wide must be purchased to cover it?
Ans. 42.6ø yards.

yd. *yd.* *yds.*
.75 : 1 : : 32 : $\frac{1}{4}$ prop.

It is manifest, that it will require more yards of carpet than there are in the floor, because it is less than a yard in width; therefore, the fourth term, or answer must be greater than the third term; the first term will necessarily be less than the second. The $\frac{3}{4}$ I reduce to a decimal, which is .75 and use it in the operation.

46. A lady, intending to make a bed quilt, containing $8\frac{1}{2}$ square yards, desired her daughter to inform her how much muslin $1\frac{1}{4}$ yard wide would be required to line the same; how many did it take? Ans. 6.8 yds. or $6\frac{4}{5}$. yds.

yd. yd. yd.
1.25 : 1 :: 8.5 : $\frac{1}{4}$ prop.

Here reduce the fractions to decimals, and annex them to their respective integers, and multiply as the rule directs.

47. A miller purchased 576 bushels of wheat at \$1.12$\frac{1}{2}$ cents a bushel; what was the amount? Ans. \$648.

48. The rent of a house is 526 dollars a year; what will it amount to in 4 years and a half? Ans. \$2367.

49. The rent of a certain house is 647 dollars a year; what will it be a quarter? Ans. \$161.75.

50. If 12 yards of muslin cost \$2.25 cents; what is it a yard? Ans. 18 cts. 7,5 m.

51. If 10 persons in a family consume 4 bushels of corn in a month; how much will 25 persons use in the same time?

Ans. 10 bushels.

per. per. bu.
10 : 25 :: 4 : $\frac{1}{4}$ prop.

52. The expense of a young lady at a boarding school is \$275.50 a year; how much is it a week? Ans. \$5.29,8+.

53. If 75 men consume 40 bushels of corn in 5 months; how much will 216 men consume in the same time?

Ans. 114 $\frac{20}{75}$. bushels.

m. m. bu.
75 : 216 :: 40 : $\frac{1}{4}$ prop.

54. A certain field will afford pasture for 10 oxen, for 60 days; how long will the same pasture suffice for 24 oxen?

Ans. 25 days.

ox. ox. d.
24 : 10 :: 60 : $\frac{1}{4}$ prop.

55. A pipe will drain off a cistern of water in 12 hours; how many pipes of the same size will empty it in 30 minutes?

Ans. 24 pipes.

m. h. pipe.
30 : 12 :: 1 : $\frac{1}{4}$ prop.

56. The walls of a room measure 125 square yards; how many yards of paper, $\frac{2}{3}$ of a yard wide, will cover them?

Ans. 187.5+ yds.

.666$\not{6}$: 1 : : 125 : $\frac{1}{4}$ prop.

Here $\frac{2}{3}$ of a yard, reduced to a decimal, is .666$\not{6}$ of a yard, which will be the first term of the proportion.

57. A merchant purchased 476 barrels of flour, at \$4.87$\frac{1}{2}$ cts. a barrel; what did they amount to? Ans. \$2320.50.

58. If 4 thousand Spanish cigars cost \$45.50 cents, what will 16 thousand amount to? Ans. \$182.

4 : 16 : : 45.50 : $\frac{1}{4}$ prop.

59. A piece of work can be done by 16 men in 12 days; how many men can perform the same in 4 days? Ans. 48 men.

60. If a pair of shoes cost 87$\frac{1}{2}$ cents, what will 10 dozen pair amount to? Ans. \$105.

61. A lady purchased a yard of silk for \$1.21 cents; how much money must she have had to purchase 5$\frac{1}{2}$ yards?

Ans. \$6.65,5.

62. A lady purchased 6$\frac{1}{2}$ yards of merino stuff at \$3.50 cents a yard; what did the whole amount to? Ans. \$22.75.

63. A lady purchased 6$\frac{1}{4}$ yards of silk, and for the whole, gave \$10.25; what did it cost a yard? Ans. \$1.64.

64. A lady purchased 2$\frac{1}{4}$ yards of lace, at \$12.37$\frac{1}{2}$ cts. a yard; what did the whole come to? Ans. \$27.84+.

65. A servant purchased 24 lbs. of beef, at 5$\frac{1}{2}$ cts. a pound; what did the whole cost? Ans. \$1.32.

66. A gentleman purchased a dinner set of silver, weighing in the whole, 12 lbs. 6 oz. 10 dwt., at \$1.50 cents an ounce; what did the whole cost? Ans. \$225.75.

oz. lb. oz. dwt.

1 : 12 6 10 : : 1.50 : $\frac{1}{4}$ prop.

67. A merchant bought a bale of cloth, containing 375 yd$^{s.}$ at \$3.12$\frac{1}{2}$ a yard; what did the whole amount to?

Ans. \$1171.87,5.

68. How long must a board be, that is 9 inches wide, to make 12 square feet? Ans. 16 feet.

in. in. ft.

9 : 12 : : 12 : $\frac{1}{4}$ prop.

69. The earth revolves on its axis in 24 hours; how far will an inhabitant on the equator be carried by its motion in 1 hour?

Ans. 15 deg. or 1042$\frac{1}{2}$ statute miles.

Every great circle of the earth contains 360 degrees, which, multiplied by $69\frac{1}{2}$, will give the circumference of the equator: and 15 degrees of the equator equal $1042\frac{1}{2}$ statute miles, or 900 geographic miles.

70. Two persons travel on the same road, and in the same direction. One sets out 5 days before the other, and travels 20 miles a day; the other travels 25 miles a day; how long before the latter will overtake the former? Ans. 20 days.

Here the latter gains 5 miles a day; and, as the former was 100 miles a-head of him, it is plain that in 20 days, he would overtake him. The distance travelled will be 500 miles.

71. A gentleman, furnishing his house, purchased 24 chairs, at $3.50 each; what did they amount to? Ans. $84.

72. What will 5 dozen knives and forks amount to, at $1.75 a dozen? Ans. $8.75.

73. If 1 bushel of apples costs $37\frac{1}{2}$ cents; what will 25 barrels come to, each $2\frac{1}{2}$ bushels? Ans. $23.43,7,5.

74. A widowed lady hired a house for 350 dollars a year; but, by reason of sickness in her family, was enabled to pay no more than 150 dollars, which her landlord accepted for the whole amount; what was that in the dollar? Ans. 42 cts. 8,57+

75. A mother allows her daughter, at a boarding school, 3 cents a day for spending money; how much will that amount to in a year? Ans. $10.95.

76. If you buy 3 oranges for $10\frac{1}{2}$ cents; how many can you buy for 55 cents? Ans. 15.71+

77. If a horse will trot in a gig 8 miles in an hour; how far will he trot, at the same rate, in $3\frac{1}{2}$ hours. Ans. 28 miles.

78. If 25 quills cost $12\frac{1}{2}$ cents; what will 475 come to, at the same rate? Ans. $2.37,5.

79. If you buy 3 lb. of tea for, $4.16 cents what can you buy $10\frac{1}{2}$ for, at the same rate? Ans. $14.56.

80. If you pay $1.56 cents for 18 lb. of sugar; how much must you pay for 115 lb., at the same rate? Ans. $9.96,6$\not{\text{c}}$.

81. A merchant bought 5 pieces of muslin, each containing 26 yards, at 11 cents a yard; what did they amount to? Ans. $14.30.

82. If a gallon of wine cost $\$1.16\frac{1}{2}$; what will 28 gallons cost, at the same value? Ans. $32.62.

83. What will 2 hhds. of wine come to, at 1 doll. $87\frac{1}{2}$ cents a gallon? Ans. $236.25.

84. The interest of 100 dollars for a year, is 6 dollars; what will it be for $3\frac{1}{4}$ years? Ans. $19.50.

85. If the interest of a certain sum for 1 year, is $16.50; what will it be for $4\frac{3}{4}$ years? Ans. $78.37\frac{1}{2}$.

86. The income of a young lady is $765.50 cents a year; her expenses are $5.25 a week; what sum does she save at the end of the year? Ans. $492.50.

87. If 1 pair of gloves cost $87\frac{1}{2}$ cents; what will 15 pair come to, at the same rate? Ans. $13.12\frac{1}{2}$.

88. If a person earns $10.75 cents a month; how much will he earn in $6\frac{1}{2}$ months? Ans. $69.87\frac{1}{2}$.

89. If you purchase 55 yards of cloth, and pay at the rate of $1.75 cents a yard; what will the whole come to? Ans. $96.25.

90. A farmer raised 544 bushels of wheat, which he sold at $1.12\frac{1}{2}$ cents a bushel; 674 bushels of corn, which he sold at 45 cents a bushel; and 7 fat oxen, at $25.50 each; what was the profit of his farm? Ans. $1093.80.

91. If 1 lb. of raisins costs 20 cents; what will 3 boxes, each weighing 24 lb., come to? Ans. $14.40.

92. If 1 ream of paper costs $2.40 cents; what will 54 reams come to? Ans. $129.60.

93. What weight, at the end of a steelyard 39 inches from the center of motion, will equipoise 208 lbs., suspended $\frac{3}{4}$ of an inch from the center of motion, at the other end? Ans. 4 lb.

39 : .75 : : 208 : $\frac{1}{4}$ prop.

94. A garrison of 800 men have provisions for 8 months; how many must depart, that the provisions may last the remainder for 10 months? Ans. 160 men.

m. m. men.
10 : 8 : : 800 : $\frac{1}{4}$ prop.

The provisions will be found sufficient for 640 men; consequently 160 must depart.

95. How long must a board be, that is 9 inches wide, to make 16 square feet? Ans. 21 ft. 4 in.

96. The orbit of the earth is about 596,900,000 miles, in circumference; how many miles does the earth move in 1 day, to perform the whole in 365 days, and 6 hours? Ans. 1634223.

97. Sound moves at the rate of 1142 feet in a second of time; how far distant was a cannon fired, whose flash was observed 56 seconds before the report was heard?

Ans. 63,952 ft. or 12.112+ miles.

98. How far distant was a flash of lightning, which was observed 71 seconds before the thunder was heard?

Ans. 81,082 feet, or 15.356+ miles.

99. If 160 pounds be placed on the end of a lever 10 feet long, what weight will it raise on the other end, supposing the prop, or fulcrum, to be placed 1 foot from the weight? Ans. 1440 lbs.

ft. ft. lbs.
10—1=9 : 1 : : 160 : $\frac{1}{4}$ prop.

In questions of this kind, the proportion will be, as the length of the lever from the fulcrum, to the weight applied, is to its length from the fulcrum to the weight to be raised, so the weight applied is to the weight to be raised.

100. Venus performs her revolution around the sun in 224 of our days; how many degrees will she pass through in 75 days? Ans. 120 deg. 32 m. 8 sec.

224 : 75 : : 360 : $\frac{1}{4}$ prop.

Every circle, whether great or small, is supposed to be divided into 360 equal parts, called degrees; and each of these degrees to be divided into 60 equal parts, called minutes, and these again into 60 equal parts, called seconds.

QUESTIONS. 1. When are four numbers said to be proportional?—2. Will you explain this by an example?—3. How do you find the ratio which one number has to another?—4. Will you explain this by an example?—5. How will you find the ratio which 12 has to 8?—Which 5 has to 15?—Which 16 has to 4?—Which 12 has to 8?

6. When are four numbers said to be in *direct* proportion? 7. If four numbers are proportional directly, will they be proportional in any other way?—8. Will you take the proportion 4 : 8 : : 3 : 6, and show in what other way, the numbers will be proportional?—9. How can you prove that they are proportional?—10. In all cases, will the product of the means be equal to that of the extremes?—11. What other property have proportional numbers?—12. Is this the case, when there are only three numbers in the proportion, as well as, when there are four?—13. Will you show this by an example?

14. Is this property of proportional numbers, the principle on which the rule of three, is founded?—15. Why is it called the *rule of three?*—Is it correctly so called?—What then should it be called?—16. What is the fourth term in the proportion?—17. How do you proceed to find it?—18. On what principle is this process founded?

19. What do you observe of the three terms, that are given?—20. Which term in the statement of the question, must be of the same kind with the answer?—21. The two terms, that are

alike, must stand in the first and second places in the proportion; but how do you know the one to take the *first* place?—22. Will you explain this part, by an example?—23. Having stated your question, if the first and second terms are of different denominations, what do you do with them?—24. If the third term consists of different denominations, how do you proceed?—25. Of what denomination, and kind, will the answer or quotient be?—26. How do you find it?

27. If any term has the fraction $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, &c., added to it, what would you do with it?—20. If the first and second terms are divisible by a common measure, may the operation be shortened by dividing them by it, and using their quotients?—29. How do you express proportional numbers?—30. Will you show, by an example?

COMPOUND PROPORTION.

This is commonly called the Double Rule of Three. There are always five terms given to find a sixth, which is the answer to the question, and may be the fourth, fifth, or sixth term, in the proportion. The three first terms are a supposition, the other two are a demand.

In stating the question: put the leading term or the one, on which the proposition depends, in the first place; the term implying length of time, distance of place, profit, or increase, in the second place; and the remaining term of the supposition, in the third place.

Then place the terms of the demand under their correspondent terms, in the supposition. The answer will be of the same kind with the term in the supposition, under which the blank is.

If the blank is under the third term: multiply the first and second terms for a divisor, and the other three for a dividend; the quotient will be the answer. But, if the blank is under the first or second term: multiply the first, second, and sixth terms for a dividend, and the other two for a divisor; the quotient will be the answer.

EXAMPLES.

1. If 6 men eat 10 pounds of bread in 8 days; how much bread will 12 men eat in 24 days? Ans. 60 lb.

m.	*d.*	*lb.*	
6 ..	8 ..	10,	terms of the supposition.
12 ..	24		terms of the demand.

Here the blank is under the third term. The leading term in the question is men; therefore, 6 must be put in the first place; length of time must be in the second place; therefore 8 is the second term; and 10 must consequently, be in the third place. The product of the first and second terms will be the divisor; and the product of the remaining terms will be the dividend; the quotient will be the answer, and the sixth term in the proportion.

2. If 6 men eat 10 lbs. of bread in 8 days; in how many days will 12 men eat 30 lbs.? Ans. 12 days.

m.	*d.*	*lb.*	
6	8	10,	terms of the supposition.
12		30,	terms of the demand.

Here the blank is under the second term of the supposition. The product, therefore, of the third and fourth terms will be the divisor, and the product of the other three will be the dividend; the quotient will be the answer, and the fifth term in the proportion.

3. If 7 men can reap 84 acres of wheat in 12 days; how many men can reap 100 acres in 5 days? Ans. 20 men.

m.	*d.*	*a.*	
7	12	84,	the terms of the supposition.
	5	100,	the terms of the demand.

Here 7 men is the foundation of the supposition; it must therefore stand in the first place; length of time is the second term; therefore, 12 days must take the second place; and 84 acres must be in the third place. The blank is under the first term, because the answer is men. The product of the third and fifth terms, that is 5×84 will be the divisor; and the product of the remaining terms will be the dividend; the quotient will be the answer, and the fourth term in the proportion.

4. If 12 oxen eat 20 acres of grass in 16 days; how many acres will serve 24 oxen for 48 days? Ans. 120.

ox.	*d.*	*a.*	
12	16	20,	terms of the supposition.
24	48		terms of the demand.

The blank is under the third term; therefore the question is solved as the first example.

5. If 8 men in 4 days receive for their wages 7 dollars, 68 cts.; how much should 48 men receive for 16 days' labour?

Ans. $184.32 cts.

m.	*d.*	*cts.*	
8	4	7.68,	terms of the supposition.
48	16		terms of the demand.

6. If 500 dollars in 6 months, produce 12 dollars, 50 cts. interest; how much will 600 dollars produce in 18 months?

Ans. $45.

dol.	*m.*	*cts.*	
500 ..	6 ..	12.50,	terms of the sup.
600 ..	18		terms of the demand.

7. If 112 acres of meadow be mowed over by 16 men, in 7 days; how many acres can 24 men mow over, in 19 days?

Ans. 456 acres.

m.	*d.*	*a.*	
16 ..	7 ..	112,	the terms of the sup.
24 ..	19 ..		terms of the demand.

8. If 180 dollars in 9 months gain 8 dollars, 10 cents interest; how much will 100 dollars gain in 12 months? Ans. $6.

dol.	*m.*	*cts.*	
180 ..	9 ..	810,	terms of the sup.
100 ..	12		terms of the demand.

9. If 4 men earn 3 dollars, 20 cents, in 3 days; how many will earn 12 dollars, 60 cents, in 16 days? Ans. 3 men.

10. If a man travels 240 miles in 12 days, when the days are 12 hours long; how many days will he be in travelling 720 miles, when the days are 16 hours long? Ans. 27 days.

d.	*h.*	*m.*	
12 ..	12 ..	240,	terms of the sup.
	16 ..	720,	terms of the demand.

11. If 6 men can build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days; how long will it require 24 men, to build one 200 feet long, 8 feet high, and 6 feet thick? Ans. 80 men.

m.	*d.*	*ft.*	
6 ..	16 ..	$\overline{20\times6\times4}$,	the terms of the sup.
24		$\overline{200\times8\times6}$,	the terms of the demand.

Here the blank falls under the second term. Solved as the second example. The continued multiplication of the length, height and thickness of each wall, will give the cubic feet in it, or its contents.

12. If 8 cwt. of iron can be carried 128 miles for $12.80.; what will be the expense of carrying 4 cwt. 32 miles?

Ans. $1.60.

cwt.	*m.*	*D.*	
8 ..	128 ..	12.80,	terms of the sup.
4 ..	32		terms of the demand.

13. If 5 men make 150 pair of shoes in 20 days; how many men can make 1350 pair in 60 days? Ans. 15 men.

14. If baggage weighing 200 pounds may be carried 40 miles for 40 cents; how far may 2020 pounds be carried for $6.06 cts., at the same rate? Ans. 60 miles.

lb.	*m.*	*ct.*	
200	40	.40,	terms of the sup.
2020		6.06,	terms of the demand.

15. A certain edifice was erected by 20 men in 12 months; but, being demolished, it is required to build a similar one in 5 months; what number of men must be employed in the work? Ans. 48 men.

m.	*mo.*	
20	12	1 the terms of the sup.
	5	1 the terms of the demand.

Here the blank is under the first term. The question is solved, as the third example.

The effect produced must be the third term; therefore, that must be represented by a unit, or 1.

16. If the freight of a quantity of goods, weighing in the whole 12 cwt., is $38.25. for 60 miles; what will be the freight of 42 cwt. for 100 miles? Ans. $176.45,8+.

cwt.	*m.*	*D.*	
12	60	30.25,	terms of the sup.
42	100		terms of the demand.

17. If the freight of 22 cwt. cost $25.50, for 116 miles; how far may 65 cwt. be carried for 74 dollars? Ans. 113.9+miles.

cwt.	*m.*	*D.*	
22	116	25.50,	terms of the sup.
65		74,	terms of the demand.

18. If 240 dollars will pay the expenses of 6 scholars at a boarding school for 6 months; how long will 500 dollars pay the expenses of 10 scholars, at the same school? Ans. $7\frac{1}{2}$ months.

19. If 100 dollars in 12 months gain 5 dollars interest; how much will gain 25 dollars in 20 months? Ans. 250 dolls.

20. A wall to be built 27 feet high has been raised 9 feet by 12 workmen in 6 days; how many men must be employed to finish the remaining 18 feet in 4 days? Ans. 36 men.

m.	*d.*	*ft.*	
12	6	9,	terms of the sup.
	4	18,	terms of the demand.

21. If 15 men eat 36 cents worth of bread in 6 days, when

wheat is worth 72 cents a bushel; how many days will 30 men require to eat 1.60 cents worth, when wheat is 1.08 cts. a bushel?

Ans. 20 days.

m.	*d.*	*cts.*
15	6	.36× .72, terms of the sup.
30		1.60×1.08, terms of the demand.

22. A cellar 22.5 feet long, 17.3 feet wide, and 10.25 feet deep, may be dug by 6 men in 2.5 days of 12.3 hours long; in how many days of 8.2 hours long, can 9 men dig one, which measures 45 feet long, 34.6 feet wide, and 12.3 feet deep?

Ans. 12 days.

m.	*d.*	*ft.*
6	2.5×12.3	22.5×17.3×10.25, terms of the sup.
9	8.2	45×34.6× 12.3, terms of the demand.

Here the blank falls under the second term. This question is solved, as the second example; but observe that the 8.2, standing under the second term, is to be multiplied into the product of the third and fourth terms, for a divisor. See question 11.

Questions. 1. What is compound proportion commonly called?—2. How many terms are always given?—3. Is the answer to the question always the sixth term, or the last, in the proportion?—4. What are the first three terms in the proportion, called?—5. What are the remaining terms, called?—6. In stating the question, how do you proceed?—7. What term must stand in the first place?—What in the second place?—8. How do you place the terms of the demand?

9. When your question is stated, which terms do you multiply for a divisor?—Which for a dividend?—10. Do you always multiply the same terms for a dividend, as you do in single proportion?—11. What rule then do you observe?—12. Will you explain this matter by an example?

PRACTICE.

Many questions in single proportion may be solved in a short way or method, which has been denominated Practice. But this is properly no rule in arithmetic.

When the price of one article or thing is given, by making use of the parts of the integers, as given in the table below, the value of any number of articles of the same kind, and at the same rate, may be found.

TABLE OF PARTS OF THE INTEGER.

Pts.	Cts.
$\frac{7}{8}$	$87\frac{1}{2}$
$\frac{3}{4}$	75
$\frac{5}{6}$	$62\frac{1}{2}$
$\frac{3}{8}$	$37\frac{1}{2}$
$\frac{1}{8}$	$12\frac{1}{2}$
$\frac{1}{16}$	$6\frac{1}{4}$
$\frac{1}{3}$	$33\frac{1}{3}$
$\frac{2}{3}$	$66\frac{2}{3}$

of a dollar.

Cts.	Pts.
50	$\frac{1}{2}$
$33\frac{1}{3}$	$\frac{1}{3}$
25	$\frac{1}{4}$
20	$\frac{1}{5}$
$12\frac{1}{2}$	$\frac{1}{8}$
10	$\frac{1}{10}$
$6\frac{1}{4}$	$\frac{1}{16}$
5	$\frac{1}{20}$

of a dollar.

Mills.	Pts.
5	$\frac{1}{2}$
$2\frac{1}{2}$	$\frac{1}{4}$
2	$\frac{1}{5}$
1	$\frac{1}{10}$

of a cent.

Oz.	pts.
6	$\frac{1}{2}$
4	$\frac{1}{3}$
2	$\frac{1}{6}$
10 dwt.	$\frac{1}{24}$
or $\frac{1}{2}$ oz.	

of a pound troy.

Qrs.	lbs.	pts.
2, or	56	$\frac{1}{2}$
1, or	28	$\frac{1}{4}$
	14	$\frac{1}{8}$
	16	$\frac{1}{7}$
	7	$\frac{1}{16}$
	8	$\frac{1}{14}$
	4	$\frac{1}{28}$
	2	$\frac{1}{56}$

of a cwt. or 112 lbs.

Qrs.	pts.
2	$\frac{1}{2}$
1	$\frac{1}{4}$
2 na.	$\frac{1}{8}$
1 ‘	$\frac{1}{16}$

of a yard.

Qts.	pts.
2	$\frac{1}{2}$
1	$\frac{1}{4}$
1 pt.	$\frac{1}{8}$
$\frac{1}{2}$ pt.	$\frac{1}{16}$

of a gal.

Roods.	pts.
2	$\frac{1}{2}$
1	$\frac{1}{4}$
Perches.	
20	$\frac{1}{8}$
10	$\frac{1}{16}$
16	$\frac{1}{10}$
8	$\frac{1}{20}$
4	$\frac{1}{40}$

of an acre.

EXAMPLES.

1. What will 326 oranges come to, at $6\frac{1}{4}$ cents? Ans. $\$20.37\frac{1}{2}$.

$6\frac{1}{4}=\frac{1}{16})326$

$20\frac{6}{16}=20\frac{3}{8}=20.37\frac{1}{2}$.

2. What will 126 yards of muslin come to, at $12\frac{1}{2}$ cents a yard? Ans. \$15.75.

$12\frac{1}{2}=\frac{1}{8})126$

$15\frac{6}{8}=15\frac{3}{4}=15.75$.

3. What will 75 bushels of apples come to, at 25 cents a bushel? Ans. \$18.75.

$25=\frac{1}{4})75$

$18\frac{3}{4}=18.75$.

4. What will 174 gallons of molasses come to, at $33\frac{1}{3}$ cents a gallon? Ans. \$58.

$33\frac{1}{3}=\frac{1}{3})174$

58

5. What will 56 yards of muslin come to, at 50 cents a yard? Ans. \$28.

$50=\frac{1}{2})56$

28

6. What will 345 bushels of wheat amount to, at 75 cents a bushel? Ans. $258.75.

50=½)345
25=¼)172½
75 86¼
258¾=258.75.

7. What will 648 quills come to, at 5 mills, or ½ a cent each? Ans. $3.24.

8. What will 58 yards of tape come to, at ¼ of a cent, or 2.5 mills a yard? Ans. 14½ cents.

9. What will 29 peaches come to, at 5 mills, or ½ a cent each? Ans. 14½ cents.

10. What will 174 apples come to, at 2.5 mills, or ¼ of a cent each? Ans. 43¾ cents.

11. What will 17 lbs. of beef amount to, at 10½ cents a pound? Ans. $1.78½.

½)17
10½
170
8½
$1.78½

12. What will 146 yards of muslin come to, at 14¼ cents a yard? Ans. $20.80½.

¼)146
14¼
584
146
2044
36½
$20.80½

13. What will 12 cords of wood come to, at 4.75 cents a cord? Ans. $57.

50=½)12
25 ¼) 4¾
75 48=at 4 dollars.
6=at 50 cents.
3=at 25 do.
57=at 4.75.

14. What will 14 lbs. of tea come to, at $2.33\frac{1}{3}$ cents?
Ans. $32.66\frac{2}{3}$.

$33\frac{1}{3}=\frac{1}{3})14$
$2.33\frac{1}{3}$
28
$4\frac{2}{3}$
$32\frac{2}{3}=32.66\frac{2}{3}$

15. What will 5 cwt. 2 qrs. of sugar, come to, at 12 dollars, 50 cents a cwt.? Ans. $68.75.

2 qrs.$=\frac{1}{2})12.50$
5
62.50=amount for 5 cwt.
6.25= do. for 2 qrs.
68.75= do. for 5 cwt. 2 qrs.

In this, and in the following examples, multiply the price by the highest denomination in the given quantity, and take the parts for the other denominations; the sum will be the amount required. For the parts, see the preceding table.

16. What will 12 cwt. 2 qrs. 14 lb. of tobacco, come to, at 4 dollars, 75 cents, a cwt.? Ans. $60.56\frac{1}{4}$.

2 qrs.$=\frac{1}{2})$ 4.75
14 lb.$=\frac{1}{4})$ 12
57.00 =the amount of 12 cwt.
$2.37\frac{1}{2}$= do. 2 qrs.
$1.18\frac{3}{4}$= do. 14 lbs.
$\$60.56\frac{1}{4}$

Here I multiply $4.75 by 12, and take the parts for the 2 quarters, the half of a cwt. and place it under the product. I then take the parts for 14 lbs. which are a fourth part of two quarters, or 56 lbs. and place the result under the product also; the sum of the whole is the amount required.

17. A farmer bought a piece of land, containing 12 acres, 1 rood, 20 perches, at the rate of 16 dollars, 50 cents, an acre; what was the amount of the whole? Ans. $204.18\frac{3}{4}$

18. What is the amount of 46 gals. 2 qts. of wine, at 1 dol. 40 cents a gallon? Ans. $65.10.

19. What is the value of a tract of land, containing 25 acres, 2 roods, 20 perches, at the rate of 14 dols. 45 cents an acre?
Ans. $370.28+

20. A lady purchased for the use of her family, 42 yds. 2 qrs. 2 nails, of linen, at the rate of 44 cents, 4 mills a yard; what did the whole come to? Ans. $18.89,7+

21. A barrel of rice, weighing 2 cwt. 1 qr. 16 lbs. was sold, at 4 dols. 16 cents a cwt.; what did it come to? Ans. $9.95\frac{3}{7}$.

1 qr. $=\frac{1}{4}$) 4.16
16 lb. $=\frac{1}{7}$) 2

8.32 = the price of 2 cwt.
1.04 = do. of 1 qr.
.59$\frac{3}{7}$ = do. of 16 lb.

$9.95\frac{3}{7}$

22. A piece of broad cloth, containing $2\frac{3}{4}$ yards, cost 5 dols. 25 cents a yard; what did the whole amount to? Ans. $14.43\frac{3}{4}$.

2 qrs. $=\frac{1}{2}$) 5.25
1 do. $=\frac{1}{4}$) 2

10.50 = price of 2 yds.
2.62$\frac{1}{2}$ = do. of 2 qrs.
1.31$\frac{1}{4}$ = do. of 1 qr.

$14.43\frac{3}{4}$

23. A farmer sold a load of hay, weighing 16 cwt. 2 qrs. at 75 cents a cwt.; what did it come to? Ans. $12.37\frac{1}{2}$.

24. A gentleman bought a set of silver spoons, weighing 2 lb. 8 oz. 10 dwt. at 1 dol. 75 cents an ounce; what did the whole amount to? $56.87\frac{1}{2}$.

Here you reduce the pounds to ounces, to which add the 8 oz. and multiply the sum by the price; and take parts for the 10 dwt. which make $\frac{1}{2}$ an ounce.

25. A merchant bought several casks of wine, containing in the whole, 147 gals. 2 qts. 1 pt. at 1 dol. 17 cents a gallon; what was the amount? Ans. $172.72.

26. What will 416 yards of calico amount to, at 25$\frac{1}{2}$ cents a yard? Ans. $106.08.

27. What will 45$\frac{1}{2}$ barrels of flour come to, at 6 dols. 25 cts. a barrel? Ans. $284.37\frac{1}{2}$.

28. What will 18$\frac{1}{2}$ cords of wood cost, at 4 dols. 75 cents a cord? Ans. $87.87\frac{1}{2}$.

29. What will $2\frac{3}{4}$ tons of coal cost, at 7 dols. 50 cts. a ton? Ans. $20 62$\frac{1}{2}$.

30. What will 35$\frac{1}{2}$ lbs. of beef cost, at 8 cents a pound? Ans. $2.84.

31. A gentleman bought a barrel of sugar, weighing 2 cwt. 1 qr. 16 lb., at 11 dols. 25 cts. a cwt.; what did the whole come to? Ans. $26.91+.

32. The expenses of a young lady at a boarding school, is 250 dols. a year; how much will it be for two years and a quarter? Ans. $562.50.

33. At 1.25 cents a yard, what will 724½ yards of cloth come to? Ans. $905.62½.

34. If a man can travel 35 miles in a day, how far can he travel in 12¼ days? Ans. 428¾ miles.

35. At $3.50 cts. a week, how much will 15½ weeks' boarding amount to? Ans. $54.25.

36. What will 125 lbs. of flour come to, at 6¼ cts. a pound? Ans. $7.81,25.

37. What will 467 lbs. of pork come to, at 6¼ cts. a pound? Ans. $29.18,7,5.

38. What will 216 lbs. of bacon come to, at 6½ cents a a pound? Ans. $14.04.

39. What will 256 lbs. of cheese come to, at 12½ cents a pound? Ans. $32.

40. What will 24½ bushels of potatoes come to, at 24 cents a bushel? Ans. $7.88.

41. What will 59 lbs. of raw cotton come to, at 13½ cents a pound? Ans. $7.96,5.

42. If a gallon of vinegar cost 25 cents, what will a barrel come to, containing 32½ gallons? Ans. $8.12,5.

43. What will 18½ bushels of salt come to, at 27 cents a bushel? Ans. $4.99,5.

44. If a acre of land is worth 19 dollars, what is the value of 3 acres, 2 roods, 20 per., at the same rate? Ans. $68.87,5.

TARE AND TRETT.

TARE and TRETT are allowances, made upon some articles of merchandise by the seller.

Tare is an allowance, made for the weight of the barrel, box, bag, or whatever contains the commodity.

Trett is an allowance for waste, dust, &c., on some light articles, and is 4 pounds in every 104. It is commonly allowed with tare.

Gross weight is the weight of the goods, together with that which contains the articles.

Nett is the weight after all allowances are made.

When tare only is allowed, subtract the tare from the gross; the remainder is the nett.

EXAMPLES.

What is the nett weight of 10 barrels of sugar, weighing in the whole, 24 cwt. 2 qrs. 17 lbs. gross; tare in the whole, 2 qrs. 16 lbs.? Ans. 24 cwt. 0 qrs. 1 lb.

cwt.	*qr.*	*lb.*	
24	2	17	
00	2	16	=tare.
24	0	1	=nett weight.

2. What is the nett weight of 12 hhds. of tobacco, weighing 8 cwt. 1 qr. 6 lbs. each, gross; tare in the whole, 14 cwt. 2 qrs. 14 lbs.? Ans. 85 cwt. 0 qrs. 2 lbs.

cwt.	*qr.*	*lb.*	
8	1	6	
		12	
99	2	16	=the gross.
14	2	14	=the tare.
85	0	2	=the nett weight.

3. A gentleman in Virginia sold 25 hhds. of tobacco, weighing in the whole, 215 cwt. 3 qrs. gross; tare, 14 lbs. per cwt.; what is the nett weight of the whole? Ans. 188 cwt. 3 qrs. $3\frac{1}{2}$ lb.

	cwt.	*qr.*	*lb.*	
$14=\frac{1}{8}$)	215	3	00	
	26	3	$24\frac{1}{2}$	=the tare.
	188	3	$3\frac{1}{2}$	=the whole nett.

Here 14 pounds being the 8th part of 112 lbs., you may divide the gross by 8, and subtract the quotient from the gross; the remainder will be the nett. In like manner, proceed in any other example, where the tare is an aliquot part of 112 lbs.

4. A merchant bought 16 barrels of rice; weight in the whole, 42 cwt. 3 qrs. 16 lbs. gross; tare per cwt., 8 pounds; what is the nett weight? Ans. 39 cwt. 3 qrs. 8 lbs. 14 oz.

lb.	*cwt.*	*qr.*	*lb.*	*oz.*	
$8=\frac{1}{14}$)	42	3	16	0	
	3	0	7	2	=tare in the whole.
	39	3	8	14	=nett weight.

5. A gentleman in Kentucky sold 50 hogsheads of tobacco, weighing in the whole, 711 cwt. 1 qr. 10 lbs. gross; tare allowed for the hhd., 150 lbs. each; what is the nett weight? Ans. 644 cwt. 1 qr. 14 lbs.

Here multiply the tare by the number of hogsheads, the product will be the whole tare in pounds; which, bring to hun-

dred weight, quarters, and pounds, and subtract it from the gross; the remainder will be the nett.

5. A planter of Georgia sold 100 bales of cotton, weighing in the whole, gross 510 cwt. 2 qrs.; tare per bale, 7 lbs.; what is the nett weight? Ans. 504 cwt. 1 qr.

6. A merchant bought 4 hhds. of sugar, weighing in the whole, 56 cwt. 3 qrs. 21 lbs. gross; tare allowed, 16 lbs. per cwt.; what is the nett weight? Ans. 48 cwt. 3 qrs. 6 lbs.

7. A planter of Maryland sold 21 hhds. of tobacco, weighing in the whole, 160 cwt. 2 qrs. gross, but being a little damaged, he consented to allow 28 lbs. tare per cwt.; what was the nett weight of the whole? Ans. 120 cwt. 1 qr. 14 lbs.

8. A merchant bought 120 boxes of tea, weighing in the whole 5457 lbs.; tare allowed per box, 6 lbs.; what is the nett weight? Ans. 4737 lbs.

9. A merchant bought 16 boxes of raisins, each weighing 1 cwt. 1 qr. 17 lbs. gross; tare allowed per box, 21 lbs.; what was the nett weight? Ans. 19 cwt. 1 qr. 20 lbs.

10. A merchant sold 45 boxes of figs, weighing in the whole, 84 cwt. 2 qrs. gross; tare allowed per box, 22 lbs.; what is the nett weight? Ans. 75 cwt. 2 qrs. 18 lbs.

11. A planter in Louisiana, sold 16 hhds. of sugar, weighing on an average, 12 cwt. 2 qrs. 12 lbs. gross; tare allowed per cwt., 14 pounds; what is the nett weight of the whole? Ans. 176 cwt. 2 qrs.

12. A planter in Alabama sold 74 bales of cotton, weighing on an average 746 pounds gross; tare allowed per bale, 15 lbs.; what is the nett weight? Ans. 54094 lbs.

13. A merchant bought at a public sale, 15 boxes of dried currants, weighing in the whole, 17 cwt. 3 qrs. 11 lbs. gross; tare per cwt. 15 lbs.; what is the nett weight? Ans. 1731.28 lbs.

Here reduce the whole to pounds, which multiply by the tare per cwt.; divide the product by 112, the quotient will be the whole tare; which, subtract from the gross: the remainder will be the nett weight.

When trett is allowed with tare, find the tare as in the preceding examples, and subtract it from the gross weight; the remainder will be the *suttle pounds*, which divide by 26, the fourth part of 104 lbs., a cwt. suttle, the quotient will be the trett; subtract this from the suttle, the remainder will be the nett pounds, as in the following examples.

14. How much is the nett weight of 48 cwt. 2 qrs. 12 lbs. gross; tare per cwt. 11 lbs., and trett 4 lbs. for every 104 lbs.?

Ans. 4710.1 lbs.

cwt.	*qr.*
48	2
4	
194	
28	
1552	
388	
5432	=the gross.
533.5	=tare.
4898.5	=suttle.
188.4	=trett.
4710.1	=nett weight.

5432
11=tare per cwt.
112)59752(533½=tare.
560
375
336
392
336
56

suttle.
26)4898.5(188.4 tr.
26
229
208
218
208
105
104
1

In cases, where trett is allowed with tare, it is better to reduce the gross weight to pounds; from which, subtract the tare. Divide this remainder by 26 for the trett, which last, subtract from the suttle, and the remainder will be the nett weight in pounds. If the answer is required in hundred weight, quarters, and pounds, divide the nett by 28 for quarters, and these again by 4 for cwt., as in the following examples.

15. What is the nett weight of 177 cwt. 0 qrs. 22 lbs. gross; tare per cwt. 9 lbs., and trett 4 lbs. for every 104 pounds?

Ans. 156 cwt. 2 qrs. 22 lbs.

16. Required the nett weight of 17 boxes of raisins, weighing in the whole, 26 cwt. 1 qr. 18 lbs. gross; tare allowed, 10 lbs. per cwt.; trett 4 lbs. for every 104? Ans. 2590.3 lbs.

SIMPLE INTEREST.

Simple Interest is a sum allowed for the use of money, goods, or property.

The sum, for which interest is allowed, is called the *principal.*

The rate of allowance for 100 dollars, for 1 year, is called the *rate per cent.*

The *amount* is the principal and interest, added together.*

Every question of simple interest, is properly an example of the *Rule of Three.* Three terms are given to find a fourth proportion.

General Rule.—If the time given is one year, the fourth proportion is the answer to the question. But,

If the time given is less or more than one year; First, find the interest of the given sum for one year, by the rule of three, or otherwise: Then say, as 1 year in months, weeks, or days, as the case may be, is to the given time in years, months, weeks, or days; so will the interest of the given sum for one year, be to the interest required.

If the amount is sought, add the principal and interest together; their sum will be the amount.

EXAMPLES.

1. What is the interest of 125 dolls. for 1 year at 6 per cent.?

Ans. $7.50

D. *D.* *D.*
100 : 125 :: 6 : $\frac{1}{4}$ prop.

Here 100 dollars must be the first term; the given sum, which is 125 dollars, takes the second place; because the fourth proportion or answer must be greater than the third term or 6 dollars, the allowance made for the use of 100 dollars for one year.

2. What is the interest of 350 dolls. for 1 year, at 6 per cent.?

Ans. 21 dolls.

D. *D.* *D.*
100 : 350 :: 6 : $\frac{1}{4}$ prop.

3. What is the interest of 625 dollars for 1 year, at $5\frac{1}{2}$ per cent.?

Ans. $34.37,5.

D. *D.* *Ct.*
100 : 625 :: 5.5 : $\frac{1}{4}$ prop.

4. What is the interest of 722 dollars for 1 year, at $5\frac{3}{4}$ per cent.?

Ans. $41.51,5.

D. *D.* *Cts.*
100 : 722 :: 5.75 : $\frac{1}{4}$ prop.

* The rate of interest established by law, is 6 per cent.; but in the state of New York, the rate of interest is fixed at 7 per cent. The interest exacted by the banks, is 6 per cent., allowing only 360 days for the year. But money is frequently obtained at 5 per cent., and even at a lower rate. The rate of interest is generally agreed upon between the debtor and creditor; but, if nothing is said on that subject in the obligation, the rate is 6 per cent., except in the state of New York, where it is 7 per cent.

5. A lady has her fortune in money, at interest, at $5\frac{1}{4}$ per cent., amounting to 11642 dollars; what is her yearly income?

Ans. $611.20,5.

D. *D.* *Ct.*
100 : 11642 :: 5.25 : $\frac{1}{4}$ prop.

6. A gentleman has in bank stock 7446 dollars; what will he receive annually at $5\frac{1}{2}$ per cent.? Ans. $409.53.

D. *D.* *Ct.*
100 : 7446 :: 5.5 : $\frac{1}{4}$ prop.

7. A young lady has her fortune in money, amounting to $12140.50; what is her income, reckoning $6\frac{1}{4}$ per cent.?

Ans. $758.78,1,25.

D. *D.* *Ct.*
100 : 12140.50 :: 6.25 : $\frac{1}{4}$ prop.

When the rate of interest has $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ more than the integer, the fraction may be changed into a decimal, and added to the whole number, as in the above examples; or, $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of the principal may be taken, and added to the product of the principal and integer; the sum will be the whole interest.

8. What is the amount of $254 for one year, at 6 per cent?

Ans. $269.24

Here add the interest, found as above, to the principal, the sum will be the amount.

9. What is the amount of 7420 dollars, for one year, at $5\frac{3}{4}$ per cent? Ans. $7846.65.

10. What is the amount of 275 dols. 50 cts. for one year, at $5\frac{1}{4}$ per cent? Ans, $289.96,3,75.

11. What is the amount of 624 dols. at $5\frac{1}{2}$ per cent., for one year? Ans. $658.32.

12. What will be the interest of 240 dollars for 3 years, at 6 per cent? Ans. $43.20.

Here find the interest of the given sum for one year, as in the above examples. Then say, as 1 year is to the given time, so will the interest thus found, be to a fourth proportion, which will be the interest required. In like manner, proceed in the following examples.

13. What will be the interest of 525 dols. for 21 months, at $5\frac{1}{2}$ per cent.? Ans. $60.53,1,25.

Find the interest for one year as above directed; then say:

12 : 21 :: 28.87,5, the interest for 1 year : $\frac{1}{4}$ prop.

14. A gentleman has 478 dols. 50 cts. at interest at $5\frac{3}{4}$ per cent.; what will it be in $4\frac{1}{2}$ years? Ans. $123.81,1,8+.

Here again find the interest as above; then say:

1 : 4.5 :: 27.51,3,75 the interest for 1 year : $\frac{1}{4}$ prop.

15. What will be the interest of 256 dols. for 18 months, at $5\frac{3}{4}$ per cent.? Ans. $21.93.

100 : 256 :: 5.75 : $\frac{1}{4}$ prop.

Then say: 12 : 18 :: 14.62 the interest for 1 year : $\frac{1}{4}$ prop.

16. What will be the amount of a bond for 150 dols. 25 cts. at 6 per cent., for $3\frac{3}{4}$ years? Ans. $183.80,6,2.

Here find the interest of the given sum for one year; then find it for the given time as above directed. Add the interest thus found, to the principal, the sum will be the amount required.

17. A gentleman has a note of 520 dols., at $5\frac{1}{2}$ per cent., on which no interest has been paid for $2\frac{1}{2}$ years; what will be the amount? Ans. $591.50.

18. What will be the interest on a note of 1234 dols. 60 cts. for 27 months, at 7 per cent.? Ans. $194.44,9,5.

19. A lady has an obligation for 1576 dols. at $5\frac{1}{2}$ per cent.; what will be the interest of the same for 7 months? Ans. $50.56,33.

20. What will be the amount of a note for 1460 dols. for 10 months, at $6\frac{1}{4}$ per cent.? Ans. $1536.04+.

21. What will be the amount of 273 dols. 45 cts. at $5\frac{3}{4}$ per cent. for 17 months? Ans. $295.72,4,7.

25. What will 97 dols. 50 cts. amount to, at $6\frac{1}{2}$ per cent. for 33 months? Ans. $114.92,8.

When the time is the aliquot part of a year, or years, the operation may sometimes be shortened, by taking the parts as shown in the examples under *Practice*, which see.

23. What is the interest of 7342 dols. for $16\frac{1}{2}$ months, at 6 per cent.? Ans. $605.71,5.

Here, having found the interest for one year, say:

m. *m.*

12 : 16.5 :: so the interest thus found will be to the answer.

Here I change the $\frac{1}{2}$ month to a decimal, and add it to the integral part. It then becomes 16.5 months.

24. What will be the interest of 160 dols. at 6 per cent., for 15 months? Ans. 12 dols.

Having found the interest for one year, which is 9 dols. 60 cts., I take the fourth part of it, and add it to the interest of the

given sum for one year. The sum thus added is the interest required; thus:

m. yr.
3=¼)9.60=interest for one year.
2.40=interest for three months.
$12.00=interest for fifteen months.

25. What is the interest of 174 dols. 64 cts. for 3 years and 7 months, at 6 per cent.? Ans. $37.54,7+.

26. What is the interest of 370 dols. for 19½ months, at 6 per cent.? Ans. $36.07,5.

27. What is the interest of 576 dols. for 21 months, at 6 per cent.? Ans. $61.20.

28. What is the interest of 174 dols. for 5¾ months, at 6 per cent.? Ans. $5.00,2,5.

Find the interest of the given sum for a year; then say:

12 : 5.75 :: 10.44 : the interest for 1 year : ¼ proportion; or, take parts for the ¾ of a year, as shown under practice.

29. What is the interest of 3472 dols. 56 cts. for 9 months, at 6 per cent.? Ans. $156.26,5+.

30. A merchant in Philadelphia owes his correspondent in New York the following sums upon note; to wit:

120 dol. payable in 9 months,
210 do. do. 15 do.
390 do. do. 23 do.
874 do. do. 27 do.

What will be the interest, reckoning 6 per cent., and the principal added together on these several obligations?

Ans. $1777.99.

31. What will be the interest of 339 dols. 60 cts. for 16 weeks, at 5½ per cent.? Ans. $5.74,7.

Having found the interest for one year; namely, 18 dols. 67 cts. 3 mills, say

52 : 16 :: 18.67,8 : ¼ prop.

32. What is the interest of 650 dols. 27 cts. for 146 days, at 5¾ per cent.? Ans. $14.95.

Having found the interest of the given sum for one year, as above directed, say:

365 : 146 :: so is the interest for 1 year, to a ¼ prop.; or the answer.

33. What is the interest of 2780 dols. 75 cts. for 20 weeks, at 7 per cent.? Ans. $74.86,6+.

Having found the interest for one year, say:

w. *w.*
52 : 20 :: so is the interest for 1 year, to a $\frac{1}{4}$ proportion; or the answer.

34. What is the amount of 746 dols. for 245 days, at the rate of 6 per cent.? Ans. $776.04,4.

35. What is the interest of 152 dols. for 5 months, 21 days, (reckoning the month at 30 days) at 6 per cent.? Ans. $4.33,6+

Here reduce the time given to days, and work as directed in the preceding examples.

36. What will be the interest of a bond for 375 dols. for 2 years, 7 months, and 11 days, at 6 per cent.? Ans. $58.81,2,5.

Here reduce the given time to days, and having found the interest of the given sum for a year, say; as 365 days will be to the given time (in days), so will the interest found, be to the interest required.

37. What will be the amount of a note for 875 dols. 49 cts. for 5 years, 8 months, and 20 days, at 6 per cent.?

Ans. $1176.07,4,9+.

Proceed as in the above example, to find the interest for the given time; to which, add the principal, the sum will be the amount.

38. A rich uncle left a legacy of 1560 dols., to be paid to his niece on her arriving at the age of 18 years; at the time of the grant, she was 13 years, 3 months, and 12 days old; what will the legacy amount to for the time, at 6 per cent.? Ans. $2001.48.

39. A father, at his decease left three daughters: the eldest, 12 yrs. 5 mo. 10 days old—the second, 10 yrs. 1 mo. 5 days—and the youngest, 7 yrs. 10 mo. 22 days old. To each of them, he left the sum of 1860 dols., with directions to his executors, to put it at interest, at 6 per cent., and to pay the amount of their respective legacies into their hands, as they should arrive at the age of 18 years: what did each one receive?

Ans. { Eldest received, $2480.
The second, $2741.95.
The youngest, $2987.78.

Bank Interest. It is a custom with the banks, to allow 360 days to the year; which they divide into 6 equal parts of 60 days each. A note is discounted, commonly for 60 days only. If it is not paid at the end of that time, it is usually taken up, and another given for the same length of time, and so on.

The interest upon a note of 100 dollars, for 60 days, at 6 per cent., is 1 dollar, or 100 cents; that is, it is 1 cent upon a dollar. Therefore,

The interest of any sum of money in dollars, for 60 days, at 6 per cent., is exactly the same number of cents, or 1 cent in a dollar: for 90 days, it is $1\frac{1}{2}$ cents in 1 dollar; for 30 days, it is $\frac{1}{2}$ a cent in 1 dollar; and so for any greater or less number of days.

EXAMPLES.

1. What is the interest of 1542 dollars, for 60 days, at 6 per per cent.? Ans. $15.42.

2. What discount must be allowed at the bank, on a note for 574 dollars, at 6 per cent., payable in 60 days, after date?

Ans. $5.74.

3. What discount must be allowed on a note, for 1746 dollars, payable in 90 days after date, at 6 per cent.? Ans. $26.19.

Interest for 60 days=17.46, the face of the note.

Do. 30 days= 8.73

$26.19=interest for 90 days.

4. What will be the interest allowed at the banks, for discounting a note, for 750 dollars, payable in 30 days, after date?

Ans. $3.75.

30 days=$\frac{1}{2}$)750

3.75 the interest for 30 days.

5. What interest will be taken at the bank of the United States, on a note for 1576 dollars, offered for discount, for 60 days, at 6 per cent.? Ans. $15.76.

To find the PRINCIPAL, when the amount, time, and rate per cent., are given:

Find the amount of 100 dollars, for the time given, at the rate per cent.; then, as this amount is to 100 dollars, so is the amount given, to the principal required.

EXAMPLES.

1. What principal in 9 years, will amount to 725 dollars, at 5 per cent.? Ans. $500.

100 doll.

45=interest of 100 dolls. for 9 years at 5 per cent.

145=the amount of 100 dollars.

dol. *dol.* *dol.*

145 : 100 :: 725 : $\frac{1}{4}$ prop.

2. What principal will amount to 965 dols. 75½ cents, in 5 years, at 6 per cent.? Ans. $742.81,1,5+

100
30=interest for 5 years, at 6 per cent.
130=the amount of 100 dollars.

130 : 100 : : 965.75,5 : ¼ prop.

3. What sum, at 5½ per cent., will amount to 1565 dollars, in 4½ years? Ans. $1254.5+

4. What sum will amount to 571 dollars, 20 cents, in 4 years, at 5 per cent.? Ans. $476.

5. What sum will amount to 695 dollars, in 10 years, at 5¾ per cent.? Ans. $441.27.

6. What sum will amount to 425 dollars, 50 cents, in 2½ years, at 6 per cent.? Ans. $370.

To find the RATE PER CENT, when the amount, time, and principal, are given:

Subtract the principal from the amount, and the remainder will be the interest for the whole time: multiply the principal by the number of years: then say; as this product is to the whole interest, so will 100 dollars, be to the rate per cent.

EXAMPLES.

1. At what rate per cent., will 500 dollars amount to 725 dollars in 9 years? Ans. 5 per cent.

725=amount.
500=principal.
225=the whole interest.

500
9=the time.
4500 : 225 : : 100 : ¼ proportion.

2. At what rate of interest, will 600 dollars amount to 856 dols. 50 cents, in 9 years and 6 months? Ans. 4½ per cent.

600
time= 9.5
3000
5400
5700.0

856.50=amount.
600.00=principal.
256.50=whole interest.

5700 : 256.50 : : 100 : ¼ proportion.

Here I reduce the 6 months to its equivalent decimal of a year, and add it to the 9 years, making 9.5 years.

3. At what rate will 820 dols. amount to 1078 dols. 30 cts. in $4\frac{1}{2}$ years? Ans. 7 per cent.

820 = principal. 1078.30 = amount.
4.5 = time. 820.00 = principal.
4100 258.30 = whole interest.
3280
3690.0

3690 : 258.30 :: 100 : $\frac{1}{4}$ prop.

4. At what rate per cent. will 837 dols. amount to 1029 dols. 51 cts. in 4 years? Ans. $5\frac{3}{4}$ per cent.

5. At what rate per cent. will 624 dols. amount to 658 dols. 32 cts. in one year? Ans. $5\frac{1}{2}$ per cent.

6. At what rate per cent. will a bond for 150 dollars, 25 cents, amount to \$183.95,6,2 in $3\frac{3}{4}$ years? Ans. 6 per cent. nearly.

To find the TIME, when the principal, amount, and rate per cent. are given:

Subtract the principal from the amount, which will give the interest for the whole time. Then find the interest of the principal for one year. Then say: as this interest is to the interest for the whole time, so is 1 year to the time required.

EXAMPLES.

1. In what time will 500 dols. amount 725 dols. at 5 per cent.? Ans. 9 years.

500 = principal. 725 = amount.
5 = rate per cent. 500
25.00 = interest for 1 year. 225 = whole interest.

25 : 225 :: 1 : $\frac{1}{4}$ prop.

2. In what time will 820 dols. amount to 1078 dols. 30 cts. at 7 per cent.? Ans. $4\frac{1}{2}$ years.

820 = principal. 1078.30 = amount.
7 = rate per cent. 820.00 = principal.
57.40 = inter. for 1 year. 258.30 = whole interest.

57.40 : 258.30 :: 1 : $\frac{1}{4}$ prop.

3. In what time will 312 dols. amount to 377 dols. 52 cts. at 6 per cent.? Ans. $3\frac{1}{2}$ years.

4. In what time 746 dols. amount to \$776.04,4, at 6 per cent.? Ans. 245 days.

746 = principal. 776.04,4 = amount.
6 = rate per cent. 746.00 = principal.
44.76 = int. for 1 year. 30.04,4 = whole interest.

44.76 : 30.04,4 :: 365 : $\frac{1}{4}$ prop.

5. In what time will a legacy of 1560 dols. amount to 2001 dols. 94 cts. at 6 per cent.? Ans. 4 yrs. 8 mo. 19 days.

6. In what time will a bond for 150 dollars, 25 cents, amount to $183.95,6,2, at 6 per cent.? Ans. 3 yrs. 8 mo. 26 days, nearly.

7. A father left his son, at his decease, the sum of 1500 dollars, to be put to interest at the rate of 6 per cent. On his arriving at the age of 21 years, he received, principal and interest, 2332 dols.; what was his age at the decease of his father?

Ans. 11 yrs. 9 mo. 3 days.

COMPOUND INTEREST.

COMPOUND INTEREST is that, which arises from the principal and interest added together. When the interest becomes due, and is not paid, it is added to the principal, and interest is calculated upon the amount, as if it was the original principal, or sum.

Find the amount of the principal for the first year, at the given rate of interest. This amount becomes the principal for the second year. Find the amount of this sum or principal as before; which becomes the principal for the third year; and so on, for any number of years.

From the last amount, subtract the original sum; the remainder will be the compound interest.

EXAMPLES.

1. What is the compound interest of 350 dols. for 3 years, at 6 per cent.? Ans. $66.85,5,6.

D. *D.* *D.*
100 : 6 :: 350 : $\frac{1}{4}$ prop.
6

21.00, interest for the first year.
350.00, the principal sum.
371.00=amount; which is the principal for the 2d year.

100 : 6 :: 371 : $\frac{1}{4}$ prop.
6

22.26=the interest for the second year.
371.00=the principal for the second year.
393.26=amount; which is the principal for the 3d year.

100 : 6 : : 393.26 : $\frac{1}{4}$ prop.
6

23.59,5,6=interest for the 3d year.
393.26,0,0=the principal for the 3d year.
416.85,5,6=the amount for the 3d year.
350.00,0,0=the original sum.
$66.85,5,6=the whole compound interest.

2. What will 500 dollars amount to in 3 years at 6 per cent. compound interest? Ans. $595.50,7,5.

3. What will be the compound interest for 1200 dollars, for 4 years, at $5\frac{1}{2}$ per cent.? Ans. $286.58,9,52.

But the operation may be considerably shortened by reducing the rate per cent. to the decimal of 100, and adding it to 1 or unit; which will be the amount of 1 dollar for one year: Multiply the principal by this amount, continually, to the number of years given.

The following table exhibits the amount of 1 dollar for one year, at the several rates per cent.

Rates per cent.	Amounts.	Rates per cent.	Amounts.
3	1.03	$3\frac{1}{4}$	1.0325
$3\frac{1}{2}$	1.035	$3\frac{3}{4}$	1.0375
4	1.04	$4\frac{1}{4}$	1.0425
$4\frac{1}{2}$	1.045	$4\frac{3}{4}$	1.0475
5	1.05	$5\frac{1}{4}$	1.0525
$5\frac{1}{2}$	1.055	$5\frac{3}{4}$	1.0575
6	1.06	$6\frac{1}{4}$	1.0625
$6\frac{1}{2}$	1.065	$6\frac{3}{4}$	1.0675
7	1.07		

NOTE.—The above table is formed by reducing the rate per cent. to the decimal of 100 in the following manner.

Let it be required to reduce the rate of 6 per cent. to a decimal; or expressed in the form of a vulgar fraction it is $\frac{6}{100}$.

100)6.00(.06, the decimal value. Again,
6.00

Let it be required to reduce $6\frac{1}{2}$, and $5\frac{3}{4}$ per cent. to decimals.

100)6.500(.065
600
500
500

100)5.7500(.0575
500
750
700
500
500

These decimal values being added to 1, or unit, become 1.06—1.065—1.0575, which are the amounts of one dollar for one year, at their respective rates. In like manner, the other rates may be reduced to their value in decimals.

4. What is the compound interest of 250 dollars for 3 years, at 5 per cent.? Ans. $39.40,6,25.

principal=250 dollars.

1.05
1250
250
262.50=the amount of the 1st year.
1.05
131250
26250
275.62,50=the amount of the 2d year.
1.05
13781250
2756250
289.40,62,50=the amount of the 3d year.
250.00 deduct the original sum.
39.40,6,25 =compound interest for 3 years.

In like manner, let the pupil perform the following examples.

5. What will be the compound interest upon a note of 1152 dollars, at 5 per cent., for 6 years? Ans. $391.79.17+

6. What will be the amount at compound interest, of a note for 961 dollars, 20 cents, for 3 years, at $3\frac{1}{2}$ per cent.? Ans. $1065.69,9.

7. What will be the compound interest on a note of 750 dollars, at 5 per cent., for $5\frac{3}{4}$ years? Ans. $242.88,6,383625.

8. What will be the amount of a note for 1000 dollars, at $6\frac{1}{2}$ per cent., for 4 years? Ans. $1286.46,6,35.

9. A young lady, just 12 years old, had a legacy left her, of 1200 dollars, to be paid when she should arrive at the age of 18 years, with compound interest at 6 per cent.; what was the amount she received? Ans. $1787.33,4,066

10. What is the compound interest upon an obligation for 1796 dollars, 25 cents, at 6 per cent., for 7 years? Ans. $904.64,5,74672.

But compound interest, may be still more expeditiously calculated, by the help of the following tables, which give the amount of one dollar, from one quarter of a year, to forty years.

TABLE I.

Showing the Amount of one dollar from one Year to forty.

Yr.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1.0400000	1.0450000	1.0500000	1.0550000	1.0600000
2	1.0816000	1.0920250	1.1025000	1.1130250	1.1360000
3	1.1248640	1.1411661	1.1576250	1.1742413	1.1910160
4	1.1698585	1.1925186	1.2155062	1.2388246	1.2624769
5	1.2166529	1.2461819	1.2762815	1.3069598	1.3382256
6	1.2653190	1.3022601	1.3400956	1.3788426	1.4185191
7	1.3159317	1.3608618	1.4071004	1.4546789	1.5036302
8	1.3685690	1.4221006	1.4774554	1.5346862	1.5938480
9	1.4233118	1.4860951	1.5513282	1.6190939	1.6894789
10	1.4802442	1.5529694	1.6238946	1.7081440	1.7908476
11	1.5394540	1.6228530	1.7103393	1.8020919	1.8982985
12	1.6010322	1.6958814	1.7958563	1.9012069	2.0121964
13	1.6650735	1.7721961	1.8856491	2.0057732	2.1329282
14	1.7316764	1.8519449	1.9799316	2.1160907	2.2609039
15	1.8009435	1.9352824	2.0789281	2.2324756	2.3965581
16	1.8729812	2.0223701	2.1828745	2.3552617	2.5403517
17	1.9479005	2.1133768	2.2920183	2.4848011	2.6927727
18	2.0258161	2.2084787	2.4066192	2.6214652	2.8543391
19	2.1068491	2.3078603	2.5269502	2.7656458	3.0255995
20	2.1911231	2.4117140	2.6532977	2.9177563	3.2071355
21	2.2787680	2.5202411	2.7859625	3.0782329	3.3995636
22	2.3699187	2.6336520	2.9252607	3.2475357	3.6035374
23	2.4647155	2.7521663	3.0715237	3.4261502	3.8097496
24	2.5633041	2·8760138	3.2250999	3.6145885	4.0489346
25	2.6658363	3.0054344	3.3863549	3.8133910	4.2918707
26	2.7724697	3.1406790	3.5556726	4.0231279	4.5493829
27	2.8833685	3.2820095	3.7334563	4.2443999	4.8223459
28	2.9987033	3.4296999	3.9201291	4.4778419	5.1116867
29	3.1186514	3.5840364	4.1161356	4.7241232	5.4183870
30	3.2433975	3.7453181	4.3219423	4.9839469	5.7434912
31	3.3731334	3.9138574	4.5380394	5.2580671	6.0881007
32	3.5080587	4.0899810	4.7649414	5.5472608	6.4533867
33	3.6481831	4.2740301	5.0031885	5.8523600	6.8405899
34	3.7943163	4.4663015	5.2533479	6.1742398	7.2510253
35	3.9460889	4.6673478	5.5160152	6.5138230	7.6860868
36	4.1030325	4.8773784	5.7918101	6.8720832	8.1472520
37	4.2680898	5.0968604	6.0814069	7.2500478	8.6360871
38	4.4388134	5.3262192	6.3854772	7.6488004	9.1542523
39	4.6163659	5.5658990	6.7047511	8.0694844	9.7035074
40	4.8010206	5.8163645	7.0399887	8.5133060	10.2857178

TABLE II.

*Amount of One Dollar for a year, and for quarters, at Compound Interest.**

Rate per ct.	1 year.	For 3 qrs.	For 2 qrs.	For 1 qr.	Simple Int. of 1 dol. for 1 month.
3	1.03	1.022416	1.014889	1.007417	.002500
3½	1.035	1.026173	1.017349	1.008637	.002917
4	1.04	1.029852	1.019804	1.009853	.003333
4½	1.045	1.033563	1.022252	1.011065	.003750
5	1.05	1.037270	1.024695	1.012272	.004167
5½	1.055	1.040973	1.027132	1.013475	.004583
6	1.06	1.044671	1.029536	1.014674	.005000
6½	1.065	1.048364	1.031988	1.015868	.005417
7	1.07	1.052053	1.034408	1.017058	.005833

To calculate compound interest by the tables: multiply the amount of 1 dollar for the given years, and at the given rate, as found in the table for years, by the principal; the product will be the amount for that time, and at the given rate. Subtract the principal from the amount thus found; the remainder will be the compound interest required. But,

If the time is years, and quarters of a year: multiply the amount for the given term of years, by the proper tabular value for the respective part, or parts of a year; the product will be the amount of 1 dollar for the whole time. Multiply this by the given sum; the product will be the amount required.

* The foregoing tables are constructed in the following manner: The amount of one dollar, for instance, at 5 per cent., is 1.05, as has already been shown. This squared, will be the amount of 1 dollar for two years; cubed, will be the amount of it for three years; as may be seen by inspecting the tables; and so on, for any number of years, and for any rate per cent.

The table for quarters is formed upon the same principles. But because the parts of a year, are fractions of a year: for a quarter of a year, extract the biquadrate root of the amount of 1 dollar at the given rate, for the amount of 1 dollar for that time, which is 1.014764, the amount for ¼ of a year, at 6 per cent: for half a year, take the square root of the amount, thus: $\sqrt{1.06}=1.029563$, the amount for half a year: for three quarters of a year, the product of a half, and a quarter, thus: $1.029563\times1.014764=1.01467078+$, the amount for ¾ of a year; and so for any other rate.

11. What will be the compound interest of 450 dollars, for 3 years, at 6 per cent? Ans. $85.95,7,2.

1.191016 =tabular amount of 1 dol. for 3 years at 6 per ct.
450=the principal.

59550800
4764064

535.95,72=the amount.
450.00

$985.95,72=the compound interest required.

12. What will a bond for 750 dollars amount to, in 5 years, at $5\frac{1}{2}$ per cent., at compound interest? Ans. 980.21,9+

1.306959=tabular amt. of 1 dol. for 5 years at the given rate.
750

980.21,9,250=the amount for 5 years.

13. What will 786 dollars amount to, in 3 years and a quarter at 6 per cent.? Ans. $949.87,5,47+

1.191016=amount of 1 dol. for 3 years.
1.014674=amount of 1 dol. for 3 months, or 1 qr.

4764064
8337112
7146096
4764064
1191016
1191016

1,208492968784=the amount of 1 doll. for the given time.
786=the given sum.

7250957812704
9667943750272
8459450781488

949.87,5,473464224=the amount for the time required.

14. What is the compound interest of 629 dollars, for 7 years, at 6 per cent.? Ans. $316.78,3,3958.

15. What will be the compound interest on a bond, for 750 dollars, for $4\frac{3}{4}$ years, at $5\frac{1}{2}$ per cent.? Ans. $217.18,675+

16. A gentleman at the birth of his son, put out on compound interest, at 6 per cent. 250 dollars, to be paid to him with interest, when he arrived at the age of 21 years; what sum did he receive? Ans. $849.89,09+

17. A gentleman had an only daughter, and wishing to make provision for her, when she should arrive at the age of 18 years, on the day that she was 5 years old, put out at compound

interest at 6 per cent., the sum of 1500 dollars: what was the fortune of the young lady? Ans. $3199.39,23.

When the amount, time, and rate per cent. are given to find the principal: Divide the amount given, by the amount of 1 dollar at the rate, and for the given time; the quotient will be the principal.

18. What sum at compound interest, in 3 years will amount to $535.95,7,2, at 6 per cent.? Ans. 450 dols.

1.191016)535.957200(450
4764064 ··
5955080
5955080

19. What principal at compound interest, in $3\frac{1}{4}$ years will amount to $949.87,547346, at 6 per cent.? Ans. 786 dols.

20. What principal at compound interest, in 13 years will amount to $3199.39,23, at 6 per cent.? Ans. 1500 dols.

21. What sum at compound interest, in $4\frac{3}{4}$ years will amount to $967.18,6751814, at $5\frac{1}{2}$ per cent.? Ans. 750 dols.

22. What sum at compound interest, in 21 years will amount to $849.89,09, at 6 per cent.? Ans. 250 dols.

When the principal, rate per cent., and amount are given to find the time: Divide the amount by the principal, the quotient will be the power of the ratio, denoted by the number of years: involve the ratio till it equals this quotient, the power of the ratio will be the time: Or by the tables; find the quotient under the rate per cent., and in a line with it, in the first column, you will have the number of years.

23. In what time will 450 dols. amount to $520.93125, at 5 per cent.? Ans. 3 years.

450)520.93125(1.157625=the 3d power of the
450 · · · · · ratio, or 1.05.
709
450
2593
2250
3431
3150
2812
2700
1125
900

With this quotient enter the table, and you find it under the 5 per cent.; and in a line with it, in the first column, you will find 3 years for the time. Or, involve the ratio or amount of 1 dollar, at 5 per cent. to the third power, it will equal the quotient; which will, therefore, be the time as above. In like manner, proceed with the following examples.

24. In what time will a bond for $750 amount to $980.219250, at $5\frac{1}{2}$ per cent. compound interest? Ans. 5 years.

25. In what time will 629 dollars amount to $945.78,33958, at 6 per cent. compound interest? Ans. 7 years.

26. In what time will 250 dollars amount to $849.89090, at 6 per cent. compound interest? Ans. 21 years.

27. In what time will 1500 dollars amount to $3199.39,23, at 6 per cent. compound interest? Ans. 13 years.

When the principal, amount, and time are given to find the rate per cent.: Divide the amount by the principal, the quotient will be that power of the ratio, whose index is the same with the number of years. Extract such root of the quotient, as is indicated by the time: Or, with the quotient enter the table, and in a line with the number of years, find it; and in the top, you will have the rate per cent.

28. At what rate per cent. will $450 amount to $520.93125 in three years? Ans. 5 per cent.

450)520.93125(1.157625=quotient or amount of 1 dollar for the time, at the given rate.

450 ····

709

450

2593

2250

3431

3150

2812

2700

1125

900

2250

2250

$\sqrt[3]{1.157625}$=1.05=ratio, or rate per cent.

In the table, and opposite 3 years in the column of years, you will find the quotient under the rate per cent.; or extract the cube root, and you will find the root to be 1.05, the rate per cent., or amount of 1 dollar for one year. In like manner, proceed in the following examples.

29. At what rate per cent will 750 dolls. amount to 980 dolls. 21925 cents, in 5 years ? Ans. $5\frac{1}{2}$ per cent.

30. At what rate per cent. will 1500 dolls. amount to 3199 dolls. 39,23 cents, in 13 years ? Ans. 6 per cent.

QUESTIONS. 1. What do you understand by the term *Interest?* —2. How is it divided?—3. What is the principal?—4. What do you understand by the *rate per cent.?*—5. What do you understand by the term *amount?*—6. On what principle is the operation founded?—7. When the time is two, three, or more years, how do you perform the operation?—8. When the given time is less than a year, how do you find the interest?—9. Will you explain this by examples?—10. When the time is years, and parts of a year, how do you proceed to find the interest?—11. Will you explain this by examples?

12. In what does compound interest differ from simple interest? —13. How do you proceed to find the compound interest arising from a given sum of money, for two or more years?—14. Can you explain the process by an example?—14. May the operation be considerably shortened by decimals?—15. By what would you multiply the principal, or given sum?—16. How often is the multiplication to be repeated?—17. What would be the last product? —18. How then would you find the interest?—19. Can you explain this by an example?

20. What other method is there, still more expeditious, to calculate compound interest?—21. What do the tables give you?—22. By what do you multiply the tabular amount?—23. What will the product be?—24. Will you explain this by an example?—25. How are the tables constructed?—26. What then do the several columns contain?

27. How is the table for parts of a year constructed?—28. How would you find the amount of 1 dollar, for 1 quarter of a year, at any given rate of interest?—29. How would you find it for half a year?—30. How would you find it for three quarters of a year?

31. How would you proceed to find the amount of a given sum of money, at a given rate per cent., for any number of years, by the help of the tables?—32. How would you find the compound interest of any sum?—33. How would you proceed to find the amount, for any number of years, and a quarter, half, or three quarters of a year?—34. How would you obtain the interest only?

COMMISSION.

COMMISSION is an allowance of a certain PER CENT., to a person for buying or selling goods, or for transacting any business.

The operation is the same as the Rule of Three.

EXAMPLES.

1. What will be the commission for the purchase of goods, amounting to 1560 dols. at 2 per cent.? Ans. $31.20.

100 : 1560 :: 2 : $\frac{1}{4}$ prop.

2. What will be the commission on the purchase of 5675 dols. bank stock, at $1\frac{1}{2}$ per cent.? Ans. $85.12,5.

100 : 5675 :: 1.5 : $\frac{1}{4}$ prop.

3. What will be the commission on the sale of goods amounting to $1260.50, at $2\frac{1}{2}$ per cent.? Ans. $31.59,2,5.

100 : 1260.50 :: 2.5 : $\frac{1}{4}$ prop.

4. What will be the commission on the sale of goods, amounting to $746.25, at $3\frac{3}{4}$ per cent.? Ans. $27.98,4+.

100 :: 746.25 :: 3.75 : $\frac{1}{4}$ prop.

When the rate per cent. has a fractional part connected with it, reduce it to a decimal, and annex it to the integer, as in the above examples. When it is a fraction, reduce it to a decimal, and work with it according to the rules of decimals; as in the following example.

5. What will be the commission on the purchase of 1140 dols. of bank stock, at $\frac{3}{4}$ per cent.? Ans. $8.55.

100 : 1140 :: .75 : $\frac{1}{4}$ prop.

6. A commission merchant sold goods amounting to 1675 dols.; what is his commission at $3\frac{1}{2}$ per cent.? Ans. $58.62,5.

7. A planter consigned his crop of tobacco, amounting, when sold, to 2174 dols. to a merchant in New York; what must he allow for the sale, commission and other charges amounting in the whole to $5\frac{1}{4}$ per cent.? Ans. $114.13,5.

8. A manufacturer consigned goods amounting to 1825 dols. to a merchant in Philadelphia; what must he allow for the sale, at $2\frac{3}{4}$ per cent.? Ans. $50.18,75.

9. A merchant in the country directed his correspondent in New York to purchase for him sundry articles of merchandise, amounting in the whole to 1165 dollars; what will his commission be, at $2\frac{1}{4}$ per cent.? Ans. $26.21,25.

10. What will be the commission on the purchase of 50 shares of bank stock, at 112 dols. 50 cts. per share, at $\frac{1}{2}$ per cent.? Ans. $28.12,5.

Here find the amount of the stock, and calculate the commission on that sum.

11. What will be the commission on the sale of 125 shares of stock of the United States Bank, at $116\frac{1}{2}$ dollars a share, at $\frac{3}{4}$ per cent.? Ans. $109.21,8,75.

INSURANCE.

INSURANCE is the security against loss or damage in houses, vessels, merchandise, &c., procured by paying a certain sum of money to another person, for taking the risk or hazard upon himself.

In case of any loss or damage, in the property so secured, the insurer is bound to make it good, by paying the sum mentioned in the policy, or instrument of writing, in which the contract is made between the parties.

The operation is the same, as in commission.

EXAMPLES.

1. What will be the insurance on the sum of 575 dols. at $3\frac{1}{2}$ per cent.? Ans. $\$20.12\frac{1}{2}$.

100 : 575 :: 3.5 : $\frac{1}{4}$ prop.

2. What sum must be paid for the insurance of property against fire, valued at 2150 dols., at $2\frac{1}{2}$ per cent.? Ans. $\$53.75$.

3. A gentleman has a house in Philadelphia, valued at 7890 dols.; what sum must he pay, to be insured against loss or damage by fire, at $1\frac{1}{4}$ per cent.? Ans. $\$98.62\frac{1}{2}$.

4. A merchant has goods in his store, valued at 25675 dols.; what sum must he pay, to be secure against loss or damage by fire, the premium asked being $1\frac{3}{4}$ per cent.? Ans. $\$449.31\frac{1}{4}$.

5. A merchant having a ship at sea, valued at 12650 dols.; what must he pay, to be insured against the dangers of the sea, the premium demanded being $2\frac{3}{4}$ per cent.? Ans. $\$347.87\frac{1}{2}$.

6. A merchant having a ship on a voyage to Canton, valued at 15764 dols., and desiring to be secure against the dangers of the sea, both outward and homeward bound; what must he pay, the premium demanded being $5\frac{1}{2}$ per cent.? Ans. $\$867.02$.

7. A merchant shipped on board a brig bound to New Orleans, goods amounting to 2464 dols.; what must he pay, to be insured against the dangers of the sea, the premium being $2\frac{1}{4}$ per cent.? Ans. $\$55.46,25$.

8. A merchant having a ship, valued at 11750 dollars, put on board her on a voyage to Liverpool, goods amounting to 14754 dols.; what must he pay, to be insured against the dangers of the sea, on both vessel and cargo, the premium being $2\frac{1}{2}$ per cent.? Ans. $\$662.60$.

9. A gentleman having several houses in the city of New York, valued in the whole at 25760 dols.; what must he pay to be insured against fire, the premium demanded being $1\frac{3}{4}$ per cent.? Ans. $\$450.80$

10. A merchant having a house, valued at 10650 dols., and goods in his store amounting to 6740 dols.; what must he pay to be insured against the danger of fire, the premium being $1\frac{1}{2}$ per cent.?
Ans. $260.85.

11. What must be paid to insure a steam boat, running from New Orleans to Louisville, valued at 25675 dols.; the premium being $4\frac{1}{4}$ per cent.? Ans. $1091.18\frac{3}{4}$.

12. A merchant of Louisville shipped a quantity of tobacco for New York, valued at 11790 dols.; what must he pay for the insurance of the same to the place of destination; the premium to New Orleans being $\frac{3}{4}$ per cent., and from New Orleans to New York $2\frac{1}{2}$ per cent.? Ans. $383.17\frac{1}{2}$.

13. The fortune of a young lady consists in houses in the city of Philadelphia, valued at 14600 dols.; what must be paid by her guardian, to be secure against the risk of fire, the premium being $\frac{3}{4}$ per cent.? And how much must be paid in the whole for insurance, until she shall arrive at the age of 18, she being at the decease of her father just 8 years of age?
Ans. { $109.50 annual payment. / $1095 paid in the whole.

To FIND the sum, for which a policy is to be taken, in order to secure a given amount: take the premium from 100: then say, as the remainder is to 100, so will the sum adventured be to the policy to be taken out.

14. A gentleman wishes to secure 750 dols. on an adventure to London; what sum must be mentioned in the policy to cover that amount; the premium being 4 per cent.? Ans. $781.25.

96 : 100 :: 750 : $\frac{1}{4}$ prop.

15. A merchant shipped goods to Havanna, amounting to a certain sum; for his better security against the dangers of the sea, he would insure 1560 dols.: what must be the policy, the premium being $2\frac{1}{2}$ per cent.? Ans. $1600 dols.

16. A merchant shipped goods to South America, amounting to 10750 dols., and would insure to the amount of 5680 dols.; how much must be the policy to cover that sum, the premium being $3\frac{1}{2}$ per cent.? Ans. $5886+ dols.

17. A gentleman has perishable property, amounting to 12790 dols.; and wishes to secure a part of this, amounting to 8064 dols.; what must be the policy, the premium being $5\frac{1}{2}$ per cent.?
Ans. $8533.33.

18. A lady has property, on which she wishes to secure the sum of 7500 dols.; what must be the policy to cover that sum, the premium being $6\frac{1}{2}$ per cent.? Ans. $8021 39+.

19. A planter in Virginia effected insurance on his crop of

tobacco, to the amount of 1250 dols.; what must the policy be, the premium being $3\frac{3}{4}$ per cent.? Ans. $1298.90+.

20. A gentleman having houses, valued at 25000 dols., would secure against the risk of fire, the sum of 16000 dols.; what sum must be mentioned in the policy, to cover that amount; the premium being $3\frac{1}{4}$ per cent.? Ans. $16537.46,6+.

REBATE OR DISCOUNT.

Discount is an allowance made for the payment of money before it is due; and is generally at a given rate per cent.

The operation is the same, as the Rule of Three.

To find the present worth or sum to be received in hand: As the amount of 100 dols. at the rate, and for the time given, is to the given sum, so will 100 dols. be to the present worth, or payment.

Subtract this from the given sum, and the remainder will be the discount.

EXAMPLES.

1. What is the present value of 750 dols. due 12 months hence, at 7 per cent.? Ans. $700.93.45+.

107 : 750 :: 100 : $\frac{1}{4}$ prop.

Here 107 dols. is the amount of 100 dols. for 1 year at 7 per cent.; for if it were put at interest for 1 year, at that rate, it would produce 7 dollars. Therefore 100 dollars in hand is worth 107 at the end of 12 months; and so for a longer or shorter period of time. The rebate or discount is considered in business transactions, equal to the interest that would arise from the present worth, if put to interest at the given rate, and for the given time.

2. What discount must be made for the present payment of 1500 dols. due 12 months hence, at 6 per cent.?

Ans. $84.90,6+.

106 : 1500 :: 100 : $\frac{1}{4}$ prop.

Here, subtract the present worth, as found by the operation, from the given sum; the remainder will be the discount or sum allowed for the present payment.

3. A merchant sold goods to the amount of 1260 dols. on a credit of 9 months; but the purchaser proposed to pay him in hand, if he would make a discount of 8 per cent.; what sum did he pay in hand?

Ans. $1188.67,9+.

106 : 1260 :: 100 : $\frac{1}{4}$ prop.

Here the interest of 100 dols. for 9 months at 8 per cent., is 6 dollars, which add to 100, making 106 for the amount. Then proceed as directed above.

4. What sum must be discounted for the present payment of 760 dollars, due 18 months hence, allowing $5\frac{1}{2}$ per cent.?

Ans. $702.07,8,5.

m. *m.* *D.* *D.*
12 : 18 :: 5.5 : 8.25. Then say,
108.25 : 760 :: 100 : $\frac{1}{4}$ prop.

Here the interest of 100 dollars, for the time given, is 8 dollars, 25 cents, which add to 100, making 108.25, for the amount of 100 dollars for 18 months. Then proceed as above, to find the present worth: which subtract from the given sum, the remainder will be the discount.

5. A gentleman has due to him, on bond, the sum of 796 dollars, payable in 10 months; but being in want of money, will make a discount for present payment of 8 per cent; what sum will he receive? Ans. $746.25+

6. A merchant bought goods amounting to 3500 dollars, on a credit of 12 months; but proposed to the seller to pay him in hand, if he would make a discount of $8\frac{1}{2}$ per cent.; how much did he pay; and what was the discount? Ans. $3225.80+sum paid.
$274.20+the discount.

7. A gentleman bought a plantation, for 5760 dollars, payable one third in 12 months, after the signature of the deed; one third in 24 months, and the remainder in 36 months, without interest: but proposed to the seller to pay him all in hand, if he would make a discount of $7\frac{1}{2}$ per cent.—what was the sum actually paid, and what was the amount of the discount?

Ans. $5022.95,7, the sum paid in hand.
$737.04,3, the whole discount.

8. What is the difference between the interest of 560 dollars, for 5 years, at 6 per cent., and the discount for the same time, and at the same rate? Ans. $38.77.

Here you calculate the discount, on the given sum, for the rate and time, as directed above; also the interest upon the same sum, for the time and rate. The difference of these sums will be the amount required. The interest is the greater of the two sums, which shows, that the discount of any sum, and the interest of the same sum at the rate, and for the time given, are considerably different. But in ordinary transactions, the interest of the given sum for the time, is taken as the discount; and in small sums, for short periods, the difference is not very great.

9. A person has a note for 750 dollars, due 15 months hence;

but being in want of money, is willing to give $7\frac{1}{2}$ per cent. discount for ready money; what will be the sum received?

Ans. $685.71+

10. A merchant bought a quantity of cotton, amounting to 1475 dollars, on a credit of 9 and 15 months; that is, one half payable in 9 months, the other in 15 months: but proposes to pay the whole down, if the planter will make a discount of $8\frac{1}{2}$ per cent., on the whole; what will be the sum paid—and what will be the amount of the discount?

Ans. $1359.97+sum paid.
$115.02,4, the discount.

BARTER.

BARTER is the exchange of one commodity for another, by duly proportioning their quantities and values.

The operation is the same as the rule of Three; and, generally, two statements are required to determine the question.

EXAMPLES.

1. How much sugar, at 9 cents a pound, must be given in exchange for 6 cwt. 2 qrs. of tobacco, at 14 cents a pound?

Ans. 10 cwt. 0 qr. $12\frac{4}{9}$ lb.

1st statement. *lb.* 1 : *cwt.* 6 *qr.* 2 :: *cts.* 14 : $\frac{1}{4}$ prop.

4
26
28
208
52
728×14=10192=value of tobac. in cents.

2d statement. *cts.* 9 : *cts.* 10192 :: *lb.* 1 : $\frac{1}{4}$ prop.

1
9)10192=1132$\frac{4}{9}$ lb.=10 cwt. 0 qr. 12$\frac{4}{9}$ lb.

Here, in the first operation, find the value of the tobacco, at 14 cents a pound, which is 10192 cents. The next operation is to find the quantity of sugar, at 9 cents a pound, that would equal, in value, the tobacco as found above. The tobacco must be reduced to pounds, because the first term is 1 pound.

2. How much tea, at 1 dollar, 25 cents a pound, must be given for 112 lbs. of chocolate, at 50 cents a pound? Ans. 44.8 lb.

lb. *lbs.* *cts.*
1st statement. 1 : 112 :: 50 : $\frac{1}{4}$ prop.

50

56.00=the value of the chocolate.

dol. *dol.* *lb.*
2d statement. 1.25 : 56.00 :: 1 : $\frac{1}{4}$ prop.

1

1.25)56.00(44.8=the quantity of tea.
50 0
600
500
1000
1000

3. How many yards of muslin, at 16 cents a yard, must be given in exchange for 125 yards of linen, at 37$\frac{1}{2}$ cents a yard?

Ans. 292.968 yds.

yd. *yds.* *cts.*
1st statement. 1 : 125 :: 37.5 : $\frac{1}{4}$ prop.

37.5

$46.87,5=the value of the linen.

cts. *D. cts.* *yd.*
2d statement. 16 : 46.87,5 :: 1 : $\frac{1}{4}$ prop.

4. Two merchants barter. One has tea, which he will sell at $1.20 cash, but in exchange, he will have $1.25: the other has sugar, which he will sell at 10 cents a lb. cash; how must he rate his sugar, to lose nothing by the exchange? Ans. 10.416$\frac{6}{}$ cents.

1.20 : 1.25 :: 10 : $\frac{1}{4}$ prop.

5. How many yards of silk, at 87$\frac{1}{2}$ cents a yard, must be exchanged for 25 yards of broadcloth, at 4 dollars, 62$\frac{1}{2}$ cents a yard?

Ans. 132.14+yds.

yd. *yd.* *D.*
1st statement. 1 : 25 :: 4.62,5 : $\frac{1}{4}$ prop.

cts. *cts.* *yd.*
2d statement. 87.5 : 115.62,5 :: 1 : $\frac{1}{4}$ prop.

6. A merchant in Philadelphia sold goods to the amount of 450 dollars; for which he received $187.50, in cash, and the rest in flour, at 5.12\frac{1}{2}$ cents, a barrel; how much flour did it take to pay the balance? Ans. 51.22 barrels nearly.

$450=the whole debt.
187.50=cash paid.
262.50=the remaining sum.

5.12,5 : 262.50 :: 1 : $\frac{1}{4}$ prop.

7. A merchant has cotton, at 11 cents a pound, which he would exchange for 560 pounds of candles, at 8 cents a pound; how much cotton must he give? Ans. $407\frac{3}{11}$ lbs.

8. Two merchants barter. One has wine at $2.25 a gall. cash; but in exchange, he will have $2.37\frac{1}{2}$: the other has coffee at 16 cents cash; how must he rate the coffee to be no loser in the exchange? And how much must he give for a hhd. of wine?

Ans. 16.888 cents, the coffee in barter.

886+pounds must be given for 63 galls. of wine.

	cts.	*dol. cts.*	*cts.*
1st statement.	2.25 : :	2.37,5 : :	16 : $\frac{1}{4}$ prop.
2d statement.	16.888 :	2.37,5 : :	63 : $\frac{1}{4}$ prop.

9. Two merchants barter: One has 556 lbs. of sugar, valued at 10 cts. a pound; for which the other gave him 860 pounds of cheese; at what was the cheese rated? Ans. 6.4,65 cts. a lb.

	lb.	*lbs.*	*cts.*
1st statement.	1 :	556 : :	10 : $\frac{1}{4}$ prop.
	lbs.	*lb.*	*dols.*
2d statement.	860 :	1 : :	55.60 : $\frac{1}{4}$ prop.

10. Two merchants barter: The one has 40 yds. of cloth, which he rates at 98 cts. a yard; and the other 30 lbs. of tea, at 1 dol. 50 cents a pound; which party pays a balance; and what is that balance? Ans. $5.80, the bal. to be paid by the first party.

11. Two persons barter: One has 250 bushels of rye, which cost him 75 cts. a bushel; but he rates it at 85 cts.: the other has wheat, which cost him 95 cents a bushel; at what price must the wheat be rated, to be equal to the profit upon the rye? And how many bushels of wheat must be given for the 250 bushels of rye?

Ans. { $107.6,6¢, the price of the wheat.
197.36+, bush. given for the rye.

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the method of reducing several stated times, at which money is payable, to one mean or equated time, when the whole is to be paid at once; so that neither party may sustain loss. RULE:

Multiply each payment by the time when it is to be paid, and divide the sum of the products by the whole amount; the quotient will be the equated time, or the time when the whole sum is to be paid.

EXAMPLES.

1. A merchant has due to him 1500 dols., payable 500 dols. in 3 months; 500 dols. in 6 months; and the remainder in 9 months; but he agrees to receive the whole in one payment; at what time must it be made? Ans. 6 months.

$$500\times3=1500$$
$$500\times6=3000$$
$$500\times9=4500$$
$$\overline{9000}=\text{the sum of the products.}$$
$$9000\div1500=6 \text{ the equated time;}$$

Or time when the whole sum is to be paid. In like manner proceed with the following examples to find the equated time.

2. A planter sold a tract of land for 2500 dollars, payable in five equal payments of 500 dols. each; the first to be made in 6 months, the second, in 12 months, and so on; but afterward agreed to reduce them to one payment: what must be the time of payment? Ans. 18 months.

3. A gentleman sold a house for the sum of 2000 dols., payable 600 dols. in hand, 500 dols. in 6 months, 400 dols. in 12 months, and the remainder in 20 months; but he afterward agreed to reduce them all to one payment: when must it be made? Ans. 8.9 months.

4. A merchant sold goods amounting to 1800 dols.; payable, 600 dols. in 6 months, 600 dols. in 12 months, and the balance in 18 months; but the purchaser agreed to pay him 500 dols. in hand, provided he would extend the payment of the remainder, and reduce them to one: required the time of payment? Ans. $16\frac{8}{13}$ months.

$$600\times\ 6=\ 3600$$
$$600\times12=\ 7200$$
$$600\times18=10800$$
$$\overline{21600}=\text{the sum of the products.}$$

Which divide by $1800-500=1300$ dols., the sum remaining to be paid at the equated time.

Therefore, when a sum of money is to be paid at a given time, and a part is paid before the time: to find the time for the payment of the remainder; multiply the whole sum by the time when it is to be paid; divide the product by the difference of the whole sum and the payment made in hand, or at the particular time agreed upon; the quotient will be the equated time for the payment of the remainder.

5. A lady sold a property amounting to 1200 dollars, payable at the end of 12 months; but the purchaser agreed to pay her

400 dols. in hand, if she would consent to wait a longer time for the balance; when must the remainder be paid? Ans. 18 months.

6. A young lady had a legacy left by a relation, amounting to 1850 dols., payable in 18 months, but wishing to have a part of it in hand, agreed with the executor, if he would advance her 500 dols. to defer the payment of the remainder proportionally: when must she receive the balance? Ans. $24\frac{2}{3}$ months.

7. A gentleman has owing to him 4500 dollars, payable in 8 months, but being in want of money, proposes to his debtor to pay at the end of three months, the sum of 1600 dols.; and he will defer the payment of the balance proportionally; when must the remainder be paid? Ans. 12.413+ months.

8. A young gentleman has a sum of money due on an obligation, amounting, principal and interest, to 1500 dols., but is not to be paid for 16 months; being in want of money to spend in his amusements, offers to defer the balance, provided he can have 500 dols. in hand: how long must the balance be deferred?

Ans. 24 months.

LOSS AND GAIN.

Loss and Gain is the method of ascertaining the profit or loss upon the purchase and sale of goods, merchandise, or property of any kind.

The operation is commonly by two statements of the Rule of Three.

EXAMPLES.

1. A merchant bought 116 yards of cloth at $2.50 a yard, and sold the same at $2.87\frac{1}{2}$; what did he gain on the whole? Ans. $43.50.

1st statement. *yd.* 1 : *yds.* 116 :: *cts.* 2.50 : $\frac{1}{4}$ prop.

2.50

5800

232

$290.00 = the whole cost of the cloth.

2d statement. 1 : 116 :: 2.87,5 : $\frac{1}{4}$ prop.

This will give the amount for which the cloth was sold. From which subtract the cost; the remainder will be the sum gained. Or, take the difference of the cost per yard, and the sale per yard, which, in this case, is $37\frac{1}{2}$ cents, the gain per yard, and multiply it by the number of yards, the product will be the whole gain. This is an expeditious method, and may be practised in questions similar to the one above.

2. A merchant bought 560 yds. of calico, at 17½ cents a yard; and retailed the same at 21 cts. a yard; what did he gain in the whole? Ans. $19.60.

	yd.	*yds.*	*cts.*	
1st statement.	1	: 560 : :	17.5	: ¼ prop.
2d statement.	1	: 560 : :	21	: ¼ prop.

3. A grocer bought 5 bbls. of sugar, weighing in the whole, 950 pounds at 8½ cts., and retailed the same for 11 cts. a pound; what did he gain in the whole? Ans. $23.75.

	lb.	*lbs.*	*cts.*	
1st statement.	1	: 950 : :	8.5	: ¼ prop.
2d statement.	1	: 950 : :	11	: ¼ prop.

4. A merchant bought 2 hhds. of wine, containing 126 gals., at 1 dol. 75 cts. a gal, and retailed the same, at 2 dols. 12½ cents. a gal.; what did he gain in the whole? Ans. $47.12½.

	gal.	*gals.*	*cts.*	
1st statement.	1	: 126 : :	1.75	: ¼ prop.
2d statement.	1	: 126 : :	2.12,5	: ¼ prop.

5. A merchant bought 2 pieces of broad-cloth, containing 56 yds., at 4 dols. 75 cts. a yd.; but upon examination, found them damaged. He was, therefore, obliged to sell them for 4 dols. 12½ cts. a yard; how much did he lose by the bargain? Ans. $35.

	yd.	*yds.*	*cts.*	
1st statement.	1	: 56 : :	4.75	: ¼ prop.
2d statement.	1	: 56 : :	4.12,5	: ¼ prop.

6. A merchant bought 1000 yds. of muslin for 116 dols.; how must he retail the same to gain 25 dols. by the bargain?

Ans. 14$\frac{1}{10}$ cts. or 14 cts. 1 mill.

Here you add the gain 25 dols. to the cost, 116 dols., and divide the sum by 1000, the number of yards; the quotient will be the answer in cents. In like manner all similar questions may be readily solved.

7. A gentleman purchased 1500 lbs. of coffee for 172 dols. 50 cts. how must he sell the same to gain 32 dols. by his bargain?

Ans. 13 cts. 6 m. 33.

8. A merchant bought 250 bbls. of flour at 4 dols. 50 cts. a bbl; how must he sell the same to gain 55 dols. by the bargain?

Ans. $4.72.

9. A merchant bought 12 boxes of tea, in the whole 165 pounds, at 1 dollar, 45 cents a pound; but upon examination, it was found to be of an inferior quality, and he sold the same for 217 dollars, 80 cents; what did he lose on a pound? Ans. 13 cents.

Here find the cost of the tea by the rule of proportion; and from it take the sum for which he sold it: the remainder will be the whole loss: which divide by the number of pounds, the quotient will give the loss on a pound.

10. A lady purchased a quantity of millinery, for which she gave 184 dollars; and sold the same for 210 dollars; how much did she gain per cent.? Ans. 11.41+per cent.

D. *D.* *D.*
184 : 210 :: 100 : $\frac{1}{4}$ prop.

A merchant bought 100 yards of broad-cloth, for 425 dollars; how must he retail it a yard, to gain 10 per cent.? Ans. $4.67,5.

Here ten per cent. on 425 dollars, is 42 dollars, 50 cents; which added to 425 dollars, the cost, will be 467 dollars, 50 cents. This sum divided by the number of yards, will give the answer. In like manner proceed with all questions of the same kind.

12. A merchant bought 4 chests of tea, weighing nett 216 pounds, at 1 dollar, 16 cents a pound, and retailed the same at 1 dollar, 31 cents; and upon examining his account sales, found that he had lost 8 pounds in the weight; what was the gain on the whole—and what was the gain per cent.?

Ans. $21.92, the whole gain.
8.748, the gain per cent. nearly.

lb. *lb.* *cts.*
1st statement. 1 : 216 :: 1.16 : $\frac{1}{4}$ prop.
2d statement. 1 : 208 :: 131 : $\frac{1}{4}$ prop.
cost. *whole gain.* *dol.*
3d statement. 250.56 : 21.92 :: 100 : $\frac{1}{4}$ prop.

13. A merchant sells silk at 1 dollar, 12$\frac{1}{2}$ cents a yard, which cost him only 95 cents; how much does he gain per cent.?

Ans. 18.42+per cent.

14. A merchant bought 125 yards of broad-cloth, at 2 dollars, 12$\frac{1}{2}$ cents a yard ready money, and sold the same immediately, for 2 dollars, 35 cents, on a credit of 8 months; how much did he gain in the whole—and how much per cent., allowing him interest upon his money, at 6 per cent.? Ans. $17.50, whole gain.
6.588 per cent. nearly.

In this question, find the amount for which the cloth was sold, and from it subtract its cost; the remainder would be the gain, provided the money was paid in hand. Then find the interest on the cost of the cloth for 8 months, the credit given on the sale; which take from the above remainder, the balance will be the true gain, arising from the bargain. Having found the true gain, find the gain per cent., as in the 3d statement, in the 12th question.

15. A merchant purchased a quantity of coffee, and found upon

examination, that it was damaged; which obliged him to sell it, at 10 cents a pound, and by so doing, he lost just 15 per cent.; what was the cost? Ans. 11.764+cents.

16. A merchant bought a quantity of goods on a credit of 6 months, but being pressed for money, sold them at auction for 1650 dollars, by which he sustained a loss of 15 per cent. on the original cost; what did the goods cost him—and what was his actual loss? Ans. $1941.17,647, the cost nearly.
$241.67,647, true loss.

Here the loss would have been $291.37,647, if he had purchased for cash. But as he purchased on a credit of 6 months, he had the sum of 1650 dollars for that time; the interest of which at 6 per cent., is 49 dollars, 50 cents, which must be deducted from the above sum; the remainder will be the actual loss on the bargain.

27. A grocer purchased a hogshead of molasses, containing by the guage 116 gals., at 40 cts. a gal., and retailed it at 46 cents a gallon; but examining his account sales, found that it fell short 10 gallons: what was his gain in the whole; and the gain per cent.?

Ans. { $2.36 whole gain.
{ $5.08+ gain per cent.

18. A merchant is offered 500 yds. of cloth at 55 cts. cash, or 60 cts. on a credit of 6 months. Now he wishes to know, which will be the better bargain, to purchase for cash, provided he pays at the rate of 7 per cent. for money, or at a credit of 6 months?

Ans. $15.37½ saved by paying cash.

Here find the amount of the cloth at the cash, and credit prices, and take their difference. Then find the interest of the sum at cash price, for 6 months at the given rate. Take this from the above difference; the remainder will be the sum saved by paying cash, and borrowing the money.

FELLOWSHIP.

Fellowship is the rule or method of dividing the gain or loss of a joint adventure, or partnership; so that each one concerned may have, in proportion to his stock or capital invested: also, of dividing the estate of a bankrupt among his creditors, so that each one may share according to the sum due to him.

The operation is the same as the Rule of Three: as, the whole sum or stock is to the stock of each partner, so is the whole gain or loss to the share of each.

EXAMPLES.

1. Three persons entered into partnership in trade. The first put in 250 dollars; the second put in 350 dollars; and the third put in 500 dollars; and in 12 months, they found, by examining their books, that they had gained 460 dollars; how must the gain be divided between them, so that each may have his due proportion?

Ans. { $104.54,545+ share of the 1st.
146.36,363+ do. 2d.
209.09,09 + do. 3d.
$460.00,00 proof.

250=stock of 1st.
350=stock of 2d.
500=stock of 3d.
1100=whole stock.

1100 : 250 :: 460 : ¼ prop.
1100 : 350 :: 460 : ¼ prop.
1100 : 500 :: 460 : ¼ prop.

2. Two merchants entered into partnership for 12 months. The first put in 1500 dols., and the other, the sum of 2000 dols.; and by misfortune, they lost 750 dols.; how must it be divided between them?

Ans. { $321.42,85+ loss of the 1st.
$428.57,15 loss of the 2d.

1500=stock of the 1st.
2000=stock of the 2d.
3500=whole stock.

3500 : 1500 :: 750 : ¼ prop.

Here, having found the portion of loss, that is to be sustained by one of the partners, subtract it from the whole loss; the remainder will be the loss to be sustained by the other.

3. Three persons unite in an adventure. The first put in 500 dols.—the second, 400 dols.—and the third, 350 dols.; and they gained 850 dols.; how must the gain be divided between them?

Ans. { $340 the share of the 1st.
272 the share of the 2d.
238 the share of the 3d.

4. A merchant failing in business, was indebted to A. the sum of $1728.24; to B. $1540.76; and to C. the sum of $2371.17; and his estate was valued at $3361.74. How must this sum be divided among his creditors?

Ans. { $1030.09,21+ A's share.
918.34,71+ B's share.
1413.30,00+ C's share.
$3361.73,91 proof.

$1728.24=A's debt.
1540.76=B's debt.
2371.17=C's debt.
$5640.17=sum due.

5640.17 : 1728.24 :: 3361.74 : ¼ prop.
5640.17 : 1540.76 :: 3361.74 : ¼ prop.
5640.17 : 2371.17 :: 3361.74 : ¼ prop.

In questions of this kind, the whole sum or amount of debt is to

any particular debt, as the whole estate is to its particular share or part; as appears from the above question.

5. The estate of a deceased person amounted to the sum of 15500 dols.; but A. had a claim for 2500 dols.; B. had a claim for 5000 dols.; C. had a claim for 10000 dols.; and D. had a claim for 6550 dols.; how much will be each one's share?

Ans. { $1611.22,6 share of A.
3222.45,3 share of B.
6444.90,6 share of C.
4221.41,3 share of D.
15500.00,0 proof.

When the stocks are continued in company for different periods of time: Multiply each partner's stock by the time of its continuance in the company; and add all the products together. Then it will be: As the sum of the several products is to each particular product; so is the whole gain or loss, to its share of it.

6. Three persons engaged in commerce: The first advanced 600 dols. for 6 months; the second advanced 1000 dols. for 12 months, and the third advanced 1200 dols. for 15 months; when they dissolved partnership, and found that they had gained the sum of 500 dols.; how must it be divided?

Ans. { $76.27,11 share of the 1st.
169.49,18 share of the 2d.
254.23,71 share of the 3d.

1st stock 600× 9= 5400
2d stock 1000×12=12000
3d stock 1200×15=18000
Prod. of stock & time=35400

35400 : 5400 : : 500 : ¼ prop.
35400 : 12000 : : 500 : ¼ prop.
35400 : 18000 : : 500 : ¼ prop.

7. Two persons commenced a partnership to continue for 12 months: One advanced 1500 at the beginning; but it not being convenient for the other to advance at that time, 4 months afterward he put into the common stock, 2500 dollars. At the end of the 12 months, on settling their books, they found a loss of 400 dols.; what part must each one sustain of it?

Ans. { $189.47,36+ the part of the 1st.
210.52,64 the part of the 2d.

8. Three persons entered into company for 12 months. The first put in, at the beginning, 800 dols., and 5 months afterward, put in 100 dols. more; the second put in, at first, 1000 dols., but at the end of 6 months, took 200 dols. for his private concerns: The third put in, at first, 500 dols., and in three months having money on hand, he put in 250 dols. more: Being successful in

business, on settling their books, they found they had gained 1200 dols.; what must each one's share of the profit be?

Ans. { $421.12,4,3+ the part of the 1st.
441.56,7,3+ the part of the 2d.
337.30,8,3+ the part of the 3d.
1200.00,0,0 proof.

800×12=9600 } = stock put in by the 1st partner multiplied by the time.
100× 7= 700 }
1000× 6=6000 } = stock of the 2d partner multiplied by the time.
800× 6=4800 }
500×12=6000 } = stock of the 3d partner multiplied by the time.
250× 9=2250 }
29350=sum of the products. Hence the respective answers may readily be found by the rule.

THE FOLLOWING is a short and expeditious method of solving cases of partnership.

Divide the whole gain or loss by the whole stock in trade.

Multiply the quotient by each partner's stock, and the several products will be the respective gain or loss of each one.

10. Three persons traded in company. A. put into the company 500 dols.; B. put in 800 dols., and C. put in 1000 dols.; and they gained 500 dols.; what is the share of each?

Ans. { $108.69,5+.
173.91,2+.
217.39,2+.
500.00,0 proof.

11. Two persons entered into partnership for 12 months. The first put in 1200 dols., but it not being convenient for the other at the commencement; 4 months afterward, he put into the stock 2000 dols, and, on closing their books, they found they had gained 500 dols.; how must it be divided between them?

Ans. { $236.84,20+ share of the 1st.
264,05,78+ share of the 2d.

1200×12=14400 } =the product of stock and time.
2000× 8=16000 }
30400=the sum of the products.
500÷30400=16.4473+cents.
16.4473×14400=236.84,11+
16.4473×16000=263.15,68+
$499.99,80 proof.

12. A legacy of 1200 dollars was left to three sisters, Ann, Mary, and Susan, to be divided in the following manner: As often as Ann had five dollars and five-tenths, Mary had four, and three-tenths, and Susan had three, and two-tenths; what was the share of each?

Ans. $507.69,22+ the share of Ann.
396.92,31+ do. of Mary.
295.38,46+ do. of Susan.
$1199.99,99 proof.

Here, add 5.5 dollars, 4.3, and 3.2 together. By which divide the given legacy. Multiply the quotient thence arising by these sums respectively, the product will be the share of each. In like manner, proceed with the following question.

13. Three persons enter trade, and advance different sums of money; and at the end of 12 months, find they have gained 600 dollars; which they divide in the following manner, according to the stock advanced: as often as the first has 6 dollars, the second has 10 dollars, and the third has 12 dollars. What is the share of each?

Ans. $128.57,07+ share of the 1st.
214.28,44+ do. 2d.
257.14,13+ do. 3d.

ALLIGATION.

Alligation is the rule, by which we determine what quantity of any number of simples, whose prices are given, will be required to form a compound of any proposed value. And also, having the several quantities and rates given, we determine the value or mean rate of the mixture, composed of those several quantities.

CASE 1.

When several quantities and their rates are given, to find the mean rate of the mixture, or compound:

As the sum of the several quantities is to any part of the mixture, so is the total value of the compound to its value.

1. A grocer mixed 6 gallons of wine, at 67 cents a gallon; 7 gallons, at 80 cents; and 5 gallons, at 1 dollar, 20 cents; what is the value of one gallon of the mixture? Ans. 86.77+cents.

gall.		*cts.*
6	× 67 =	402
7	× 80 =	560
5	× 120 =	600
18		1562

gall. gall. cts.
18 : 1 :: 1562 : $\frac{1}{4}$ prop.

2. A goldsmith has 5 lbs. of silver bullion, of 8 oz. fine; 10 lbs. of 7 oz. fine; and 15 lbs. of 6 oz. fine; what is the value of 1 lb. of this mixture? Ans. $6\frac{2}{3}$ oz. fine.

3. A grocer would mix 12 pounds of tea, at 1 dollar, 75 cents a pound; 10 pounds, at 1 dollar, 50 cents; 8 pounds, at 1 dollar, 25 cents; and 5 pounds, at 95 cents a pound; what will be the value of 2 pounds of this mixture? Ans. $2.90

4. A farmer mixed 20 bushels of wheat, at 90 cents a bushel; 36 bushels of rye, at 75 cents; and 30 bushels of barley, at 50 cents a bushel; what is the value of 1 bushel of this mixture?

Ans. 69.7662+cents.

5. A goldsmith melted 6 oz. of silver, worth 90 cents an ounce, with 9 oz. worth 86 cents an ounce; what is the value of the mixture? Ans. 87.6 cents.

CASE 2.

When the prices of several simples are given, to find how much of each, at its respective price, must be taken to make a compound of any given rate, or price:

Put the prices under each other; and connect one, which is greater than the proposed price or rate with another, that is less. Take the difference of each price, and the mean rate, and place it opposite the price with which it is connected: these differences will be the quantities required.

EXAMPLES.

6. A farmer would mix rye worth 75 cents a bushel, with barley worth 90 cents, and oats worth 48 cents, so as to have a mixture worth 65 cents a bushel; what quantity must he take of each?

Ans. 35 bushels of oats, 17 bushels of rye, and 17 of barley.

mean rate, 65 { 90, 75, 48 } 17, 17, 25+10=35

7. A grocer would mix wine, at 80 cents a gallon, at 90 cents, and at 120 cents; so that the mixture could be sold, at 95 cents a gallon; how much of each sort must he take?

Ans. { 25 gals. at 80 cents. / 25 do. 90 do. / 20 do. 120 do. }

8. A grocer would mix sugar at 12 cents a pound, at 10 cents a pound, and at 8 cents a pound; so that the mixture may be worth 9 cents; what quantity must he take of each?

Ans. { 1 pound at 12 cents. / 1 do. 10 do. / 4 do. 8 do. }

When there are two or more rates, both greater, and both less than the mean rate, the question is susceptible of several answers, all equally true; as in the following examples. This arises from the various ways, in which the rates may be linked or connected together.

9. A person has four sorts of sugar: one sort, at 15 cents; another, at 12 cents; another, at 10 cents; and a fourth at 8 cents a pound: he would mix them in such proportion, that the mixture might be worth 11 cents; what quantities of each must he take?

Answer.

$$11\left\{\begin{array}{l}15\\12\\10\\8\end{array}\right\}\begin{array}{l}=3 \text{ at } 15 \text{ cents.}\\=1 \quad 12 \text{ do.}\\=1 \quad 10 \text{ do.}\\=4 \quad 8 \text{ do.}\end{array}$$

Answer.

$$11\left\{\begin{array}{l}15\\12\\10\\8\end{array}\right\}\begin{array}{l}=1 \text{ at } 15 \text{ cents.}\\=3 \quad 12 \text{ do.}\\=4 \quad 10 \text{ do.}\\=1 \quad 8 \text{ do.}\end{array}$$

$$11\left\{\begin{array}{l}15\\12\\10\\8\end{array}\right\}\begin{array}{l}=1+3=4 \text{ at } 15 \text{ cents.}\\= \quad 3 \quad 12 \text{ do.}\\= \quad 4 \quad 10 \text{ do.}\\=4+1=5 \quad 8 \text{ do.}\end{array}$$

$$11\left\{\begin{array}{l}15\\12\\10\\8\end{array}\right\}\begin{array}{l}= \quad 1 \text{ at } 15 \text{ cts.}\\=1+3=4 \quad 12 \text{ ‘}\\=4+1=5 \quad 10 \text{ ‘}\\= \quad 1 \quad 8 \text{ ‘}\end{array}$$

10. A grocer has several sorts of teas: one at 1 dollar, 50 cents a pound; another, at 1 dollar, 25 cents; a third, at 1 dollar, 15 cents; and a fourth, at 100 cents; and he would make a mixture worth 1 dollar 20 cents a pound; what quantity of each must he take?

Answer.

$$1.20\left\{\begin{array}{l}1.50\\1.25\\1.15\\1.00\end{array}\right\}\begin{array}{l}= 5 \text{ at } 1.50 \text{ cents.}\\=20 \quad 1.25 \text{ do.}\\=30 \quad 1.15 \text{ do.}\\= 5 \quad 1.00 \text{ do.}\end{array}$$

Answer.

$$1.20\left\{\begin{array}{l}1.50\\1.25\\1.15\\1.00\end{array}\right\}\begin{array}{l}=20 \text{ at } 1.50 \text{ cents.}\\= 5 \quad 1.25 \text{ do.}\\= 5 \quad 1.15 \text{ do.}\\=30 \quad 1.00 \text{ do.}\end{array}$$

Other answers may be obtained, by connecting the rates in different ways, as in the preceding examples; but these are sufficient for all practical purposes.

CASE 3.

When the price of several simples, the quantity of one of them, and the mean rate of the mixture, are given, to find the several quantities of the rest: proceed as in the preceding case to find the differences of the several simples, and the mean rate; then, by the rule of three: As the difference of the same name with the given quantity is to the remaining differences, so is the given quantity to the quantities required.

EXAMPLES.

11. A grocer would mix 30 pounds of sugar, at 14 cents a pound, with some at 10 cents, at 9 cents, and at 8 cents a pound;

how much of each sort must be mixed with the 30 pounds, that he may afford the mixture at 11 cents? Ans. 15 pounds of each sort.

$$11\left\{\begin{array}{rl} 14 & \\ 10 & \\ 9 & \\ 8 & \end{array}\right\}\begin{array}{l} =1+2+3=6 \\ =\quad 3 \quad 6:3::30:\frac{1}{4}\text{ prop.} \\ =\quad 3 \quad 6:3::30:\frac{1}{4}\text{ prop.} \\ =\quad 3 \quad 6:3::30:\frac{1}{4}\text{ prop.} \end{array}$$

Here 6 is the difference standing against the price of the given quantity; which is therefore the first term of the proportion, by the rule. The difference standing against each of the other quantities is the same, which shows that equal quantities are required. In like manner proceed with the following examples.

12. A merchant has several kinds of tea, and he would mix 25 pounds at 1 dollar, 75 cents, a pound, with some at 1 dollar, 60 cents, at 1 dollar, 40 cents, and at 1 dollar, 30 cents a pound; so that he may afford the mixture at 1 dollar, 50 cents; how much of each sort is required?

$$\text{Ans.}\left\{\begin{array}{ll} 31\frac{1}{4} \text{ lbs at } \$1.30 \text{ cents.} \\ 12\frac{1}{2} \text{ do.} \quad 1.40 \text{ do.} \\ 12\frac{1}{2} \text{ do.} \quad 1.60 \text{ do.} \end{array}\right.$$

13. A goldsmith would melt 10 oz. of silver of 18 carats fine, with some at 16 carats, 20 carats, and 24 carats fine, so that the mixture may be 22 carats fine; required the quantity of each?

$$\text{Ans.}\left\{\begin{array}{l} 10 \text{ oz. of 16 carats fine.} \\ 10 \text{ oz. of 20 carats fine.} \\ 10 \text{ oz. of alloy.} \\ 170 \text{ oz. of 24 carats fine.} \end{array}\right.$$

$$22\left\{\begin{array}{rl} 0 & \\ 18 & \\ 16 & \\ 20 & \\ 24 & \end{array}\right\}\begin{array}{l} = \quad - \quad - \quad 2 \\ = \quad - \quad - \quad 2 \\ = \quad - \quad - \quad 2 \\ = \quad - \quad - \quad 2 \\ =22+4+6+2=34 \end{array}$$

Alloy is considered of no value. It is therefore represented by a cypher. This question is susceptible of one answer only, there being but one rate greater than the mean rate.

CASE 4.

When the prices of the several simples, the quantity to be compounded, and the mean rate, are given, to find the quantity of each; proceed, as in the preceding cases to find the differences of the several simples and the mean rate; then, by the rule of three: As the sum of the differences is to the difference opposite each rate or price, so is the quantity to be compounded, to the quantity required.

EXAMPLES.

14. How much sugar at 10 cts. a pound, at 12 cts., and at 15

cts. a pound, will make a mixture of 20 pounds, worth 13 cents a pound?

Ans. { 5 lbs. at 10 cts.
5 lbs. at 12 cts.
10 lbs. at 15 cts.

13 { 10 = - 2
12 = - 2
15 =3+1=4
8

lb. lb. lbs.
8 : 2 :: 20 : $\frac{1}{4}$ prop.

15. A merchant has several sorts of coffee; namely, at 10 cts., at 12 cts. at 14 cts., and at 20 cts. a pound; and he would make a mixture of 40 pounds, which he could afford at 15 cts. a pound; what quantity of each must he take?

Ans. { 15 lbs. at 20 cts.
$8\frac{1}{3}$ lbs. at 14 cts.
$8\frac{1}{3}$ lbs. at 12 cts.
$8\frac{1}{3}$ lbs. at 10 cts.

16. A merchant has three sorts of wine, namely at 75 cts., at 90 cts., and at 1 dol. 20 cts. a gal.; and he would fill a cask containing 70 gals., so that he may afford to sell the mixture at 100 cents a gal.; what quantity of each sort must he take?

Ans. { $18\frac{2}{3}$ gals. at 75 cts.
$18\frac{2}{3}$ gals. at 90 cts.
$32\frac{2}{3}$ gals. at 120 cts.

SINGLE POSITION.

Single Position is the Rule for solving questions by making use of a supposed number in the room of the true one; and working with it, as if it was the true number.

Having proceeded with the supposed number, according to the conditions of the question; then, by the Rule of Three: as the result of the operation is to the supposed number, so is the given number to the one sought.

EXAMPLES.

1. A certain sum of money is to be divided among three persons in the following manner: The first is to have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, and the third, the remainder, which is 250 dols.; what is the sum?

Ans. 600 dols.

$\frac{1}{3}$ | 120=supposed number.
$\frac{1}{4}$ | 40
30
70
50=result of the operation. Now to find the true number, say as; 50 : 120 :: 250 : $\frac{1}{4}$ prop.

Here you suppose 120 to be the number. Then by the conditions of the question, take $\frac{1}{3}$ and $\frac{1}{4}$ of it, and add them together; and take their sum from the supposed number. This will be the result of the operation, and the first term of the proportion, by the rule. The fourth term will be the answer.

2. A person having a sum of money, spent $\frac{1}{3}$ and $\frac{1}{4}$ of it; and, upon examination, found that he had only 60 dols. left; what sum had he at first? Ans. 144 dols.

3. A person lent a sum of money at 6 per cent., and in 10 years it amounted to 1600 dollars; what was the original sum?

Ans. 1000 dols.

5. A lady being asked her age, replied; if $\frac{6}{10}$ of my age is multiplied by 7, and $\frac{2}{3}$ of my age is added to the product, the sum will be 292; what was her age? Ans. 60 years.

Suppose her age=30 yrs.; then by the conditions of the question $\frac{6}{10}$=18, which multiplied by 7=126; to which add $\frac{2}{3}$ of 30, and the sum will be the result of the operation. For the true number proceed according to the rule.

5. A lady having a certain sum of money, told her daughter, just going to school, that $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of it made 74 dols., and desired her to tell on returning home, what sum she had; what was the sum? Ans. 120 dols.

6. There is a number, which multiplied by 7, and the product divided by 6, the quotient will be 14; what is the number?

Ans. 12.

7. A teacher being asked how many scholars he had; to avoid a direct answer, and to show his knowledge of arithmetic, replied; if I had as many, half as many, and one quarter as many more, than I now have, I should have 88; how many scholars had he?

Ans. 32.

8. Two persons have the same annual salary or income: One saves $\frac{1}{3}$ part of his, and puts it out to interest; the other by dissipation, spends twice as much as the former, and thus runs in debt 120 dols. a year; what is their income? Ans. 360 dols.

Suppose 240 dollars the annual income. Then the first saves 80 and spends 160 dollars. The second will spend 320 dollars, by the question, or twice as much as the other. The difference between this sum, and the supposed number, will be the result of the operation. Whence the true sum may be found by the rule.

9. A young lady has a fortune in money, the interest of which, at 6 per cent. exceeds $\frac{1}{20}$ part of the principal by 100 dols.; what is her fortune? Ans. 10,000 dols.

10. Three persons live in the same family, whom I shall call A. B. and C. Speaking of their ages, B. said, my age is 1.5 tenths of

A's age; C. said my age is twice that of B.; and all together will make 132 years; what is the age af each one?

Ans. { 24 years=A's age. 36 years=B's age. 72 years=C's age.

11. A gentleman purchased a chaise, horse, and harness for 240 dols. The harness was half the price of the horse, and the chaise was double the price of both horse and harness; what was the price of each?

Ans. { $26.66,6¢ harness. 53.33,33 horse. 160. chaise.

12. Three merchants purchased 1200 dollars worth of sugar in company, and paid in the following manner: The first paid 3 times as much, as the second, and the second paid 4 times as much as the third; how much did each one pay?

Ans. { $847.05,8,82+=share of the 1st. 282.35,2,94+=share of the 2d. 70.58,8,23+=share of the 3d.

DOUBLE POSITION.

Double position is a rule, by which we determine the true number or numbers sought, by the use of two supposed numbers.

Rule: Suppose two numbers, and work with them according to the conditions of the question. Take the difference between the results of the operation, and the given number; and call these the errors.

Multiply the first supposed number by the second error, and the second supposed number by the first error, and note the product. If the results are both greater, or both less than the given number, take the difference of the errors for a divisor, and the difference of the products for a dividend; the quotient will be the answer: but if one is greater, and the other less than the given number, take the sum of the errors for a divisor, and the sum of the products for a dividend; the quotient will be the answer.

EXAMPLES.

1. It is required to divide 100 dollars between three persons in such manner, that the first may have twice as much, as the second, wanting 8 dollars; and the third, three times as much as the second, wanting 15 dollars: what is the share of each?

Ans. { $20.50=share of the 2d. 33.00=do. of the 1st. 46.50=do of the 3d.

1. Suppose the 2d person to have 12 dollars. Then by the question, the first will have 16 do.
and the third will have 21 do.
49 = the result of the oper.
100 = the given sum.
51 = 1st error: less.

2. Suppose the 2d person to have 16 dolls. Then by the question, the first will have 24 do.
and the third will have 33 do.
73 = the result of the oper.
100 = given sum.
27 = 2d error; less.

sup. num. err.
12 × 27 = 324.
16 × 51 = 816.
diff. of err. = 24 492 diff. of prod.

$\frac{492}{24}$ = $20.50, the share of the 2d.
33.00, the share of the 1st.
46.50, the share of the 3d.
100.00 proof.

2. A grazier had in his pasture 1000 head of oxen, sheep, and horses. There were 80 oxen more than horses, and three times as many sheep, as horses and oxen taken together; how many had he of each sort?

Ans. 165 oxen.
85 horses.
750 sheep.
1000, proof.

1st. Sup. he had 240 oxen. Then by the question there must be 160 horses. Three times as many sheep as horses and oxen = 1200
1600 = result of the operation.
1000 = the given number.
600 = 1st. error : greater.

2d. Sup. he had 200 oxen. Then by the question there would be 120 horses. And there would be 960 sheep.
1280 = the result of the operation.
1000 = the given number.
280 = 2d, error : greater.

Hence the answer may be readily found, as in the preceding example.

3. A labourer hired for forty days upon this condition, that he should receive 20 cents for every day he worked, and forfeit 10 cents for every day he was idle: At the expiration of the 40 days, he received 5 dollars; how many days did he work, and how many days was he idle?

Ans. { 30 = days he worked.
10 = days he was idle.

1st. Suppose he worked 25 days. Then by the question he would be idle 15 days; for both together, must equal 40, the term of his engagement.

$25 \times 20 = 500$
$15 \times 10 = 150$
3.50 = the result of the operation.
5.00 = given sum.
1.50 = 1st error: less.

2d. Suppose he worked 20 days. Then he would be idle, by the question, 20 days.

$20 \times 20 = 400$
$20 \times 10 = 200$
2.00 = the result of the operation.
5.00 = the given sum.
3.00 = 2d. error: less.

Having found the errors, the answer is easily obtained.

4. Three ladies contributed 240 dollars for charitable purposes. The first gave 24 dollars more than the second, and the third gave as much as both the others; how much did each one contribute?

Ans. { $72 was given by the first. 48 do. by the 2d. 120 do. by the 3d.

5. Three persons reside in the same family. The age of the youngest is 20 years; the age of the second is equal to that of the first, and half the age of the third; and the age of the third is equal to the sum of the ages of the other two; what are their several ages?

Ans. { 60 years the age of the second. 80 do. the age of the third.

1st. Suppose the age of the third person to be 60 years. Then by the question, the age of the 2d will be 50 years, being half the age of the third, and the whole of the first. Thus:

60 = age of 3d.
50 = age of 2d.
20 = age of the first.
70 = result of the operation, or sum of the ages of 1st and 2d persons.
age of 3d $= 60$
10 = 1st error: greater.

2d. Suppose 50 = the age of the third.
45 = the age of the 2d.
20 = the age of the first.
65 = result of the operation, or sum of the ages of 1st and 2d persons.
50
15 = 2d error: greater. Having the errors their ages are easily found.

6. A lady bought 15 yards of silk and lining, for a cloak, for which she gave $8.40 cents. The price of the silk was 96 cents a yard, and that of the lining 36 cents a yard; what was the number of yards of each?

Ans. { 5 yds. of silk. / 10 do. of lining.

7. Two young gentlemen began business with equal sums of money. The former gained in 12 months, a sum equal to $\frac{1}{4}$ of his capital stock; the other, by bad management, lost 540 dollars; when his stock was just half that of the former; what was the original stock of each? Ans. 1440 dollars.

1st. Suppose 600 dols. the original stock. Then by the question

150 dols.=the gain of the former;

750 =the amount of his capital.

600 dollars the capital of the latter.

540=his loss in trade.

60=the remaining stock, which must=$\frac{1}{2}$ the cap. of the former, 375

315=1st error: less.

2d. Suppose 900 dols. the original capital. Then by the question

225 =the gain in trade of the former.

1125=the amount of capital of the former.

900 dollars the capital of the latter.

540=his loss in trade.

360=the remaining stock; which equals $\frac{1}{2}$ of the other's capital, 562.5

202.5=2d error: less. Having the errors, the answer is readily found.

8. A lady married a gentleman, whose age on the day of their marriage, was to hers, as 9 is to 3. After they had lived together 15 years, his age was to hers, as 2 to 1; what was their respective ages?

Ans. { 45 the age of the man. / 15 the age of the lady.

9. A gentleman left his three daughters, at his decease, 10,000 dollars, to be divided in the following manner: the second was to have 1000 dollars more than the first; and the third as much, as both the others; how much had each one?

Ans. { 2000 dollars the share of the first. / 3000 do. do. of the 2d. / 5000 do. do. of the 3d.

PERMUTATION.

PERMUTATION is a rule, by which we find the number of changes, that may take place in any given number of things.

Rule. Multiply all the terms of the natural series continually, from 1 to the given number, inclusive; the last product will be the number of changes required.

EXAMPLES.

1. In how many different ways, can five persons place themselves at table, to dine? Ans. 120.

2. There is a chime of 12 bells; how many changes may be rung with them? Ans. 4790016000.

It would require 45 years, 195 days, and 18 hours, to ring them, allowing 3 seconds for every round or change.

3. In how many different positions may 8 persons seat themselves at dinner? Ans. 40320.

It would require 110 years, and 142 days to do it. If there had been one more, it would require the amazing period of 993.51129+ years, to accomplish the changes.

COMBINATION.

COMBINATION is a rule, by which we find how many different ways a less number may be taken out of a greater number of the same kind.

Rule. Multiply the natural numbers, from 1 up to the number to be combined, or taken at a time, and note the product: Multiply an equal number of terms beginning from the greatest number, out of which the election is to be made, decreasing by unit or 1, and note the product. The latter product, divided by the former, will give the answer.

EXAMPLES.

1. How many different ways may 6 letters be taken at a time out of 11? Ans. 462.

$1\times2\times3\times4\times5\times6 = 720$ the product of the first series.

$11\times10\times9\times8\times7\times6 = 332640$, the product of the 2d series.

$332640 \div 720 = 462$.

2. How many combinations of 4 letters may be made out of 24 letters of the alphabet? Ans. 10626.

$1\times2\times3\times4 = 24$ product of the first series.

$24\times23\times22\times21 = 255024$, product of the 2d series; which divided by 24 gives the answer.

3. A general, being asked what reward he should have for his services, replied, that he would be content to receive one cent, for every file of ten men, which he could make from a company of 90 men; how much did he receive? Ans. $57206454819.03.

DUODECIMALS, OR CROSS MULTIPLCATION.

This is a rule commonly used for finding the area of superficies, the content of solids, and the tonnage of ships; the dimensions being given in feet, inches and parts of an inch.

The denominations are:

12 fourths (marked thus, ′′′′) make 1 third.
12 thirds (marked thus, ′′′) make 1 second.
12 seconds (marked thus, ′′) make 1 inch.
12 inches make 1 foot.

ADDITION.

This is performed like Compound Addition, only observe to carry or add 1 for every 12, to the next higher denomination.

EXAMPLES.

	Ft.	*In.*	′′	′′′	′′′′
	22	8	6	10	11
	16	4	7	3	6
	25	3	8	9	3
Sum,	64	4	10	11	8

Ft.	*In.*	′′	′′′	′′′′
45	7	8	9	10
21	6	5	2	7
18	5	2	1	0

SUBTRACTION.

The operation is performed like Compound Subtraction, only observe to add 12 to the upper number, when it is less than the lower one; and add 1 to the lower number of the next higher denomination.

EXAMPLES.

	Ft.	*In.*	′′	′′′	′′′′
From	48	4	6	8	10
Take	27	8	9	4	7
Rem.	20	7	9	4	3

Ft.	*In.*	′′	′′′	′′′′
114	7	6	4	3
71	8	10	7	4
42	10	8	8	11

MULTIPLICATION.

Place the multiplier under the multiplicand, so that feet may stand under feet, inches under inches, &c. Multiply the highest

denomination of the multiplier into the lowest of the multiplicand; observing to place feet under the feet, inches under the inches, seconds under the seconds, &c. of the factors, in the several products; and for every 12 of a lower denomination, add 1 to the next higher denomination.

Observe that: Feet, multiplied by feet give feet.
Do. Feet, multiplied by inches give inches.
Do. Feet, multiplied by seconds give seconds.
Do. Inches, multiplied by inches give seconds.
Do. Inches, multiplied by seconds give thirds.
Do. Seconds, multiplied by seconds give fourths.

EXAMPLES.

1. Multiply 4 feet 8 inches, by 3 feet 6 inches.

Prod. 16 ft. 4 in.

Ft.	*In.*	
4	8	
3	6	
14	0	=the product of the multiplicand by 3 feet.
2	4	0=product of the multiplicand by 6 inches.
Prod.=16	4	0=Ans.

2. Multiply 9 feet 7 inches, by 6 feet 4 inches.

Prod. 60 ft. 8 in. 4″.

3. Multiply 5 feet 2 inches 4 seconds, by 7 feet 5 inches and 8 seconds.

Prod. 38 ft. 9 in. 9″ 2‴ 8⁗.

Ft.	*In.*	″		
5	2	4		
7	5	8		
36	4	4		=product of multiplicand by 7 ft.
2	1	11	8‴	=do of mult. by 5 inches.
	3	5	6	8⁗=prod. of mult. by 8 sec.
Product=38	9	9	2	8

4. There is a marble slab, whose length is 7 feet 8 inches, and breadth 2 ft. 4 in.; what is its superficial content?

Ans. 17 ft. 10 in. 8″.

5. The floor of a parlour is 17 ft. 9 in. long, and 16 ft. 10 in. broad; what is its superficial content? Ans. 298 ft. 9 in. 6″.

6. The wall on one side of a room is 18 ft. 5 in. long, and 10 ft. 9 in. broad; what is its superficial content?

Ans. 197 ft. 11 in. 9″.

7. A board is 16 ft. 8 in. long, and 1 ft. 7 in. wide; what is its superficial content? Ans. 26 ft. 4 in. 8″.

7. There is a yard, whose length is 47 ft. 10 in., and breadth 35 ft. 8 in.; what is its area? Ans. 1706 ft. 0 in. 8″

9. A partition wall is 104 ft. 8 in. long; and 6 ft. 8 in. high; what is its superficial content? Ans. 697 ft. 9 in. 4″.

10. A carpet is 18 ft. 4 in. long, and 15 ft. 9 in. broad; what is its content in square feet? Ans. 288 ft. 9 in.

12. An oil-cloth is 12 ft. 10 in. long, and 10 ft. 9 in. broad; what is its content in square feet? Ans. 138 ft. 11 in. 6″.

When the solid contents are required, there must be three dimensions given, namely, length, breadth and thickness: which must be multiplied continually into each other; the last product will be the answer: as in the following examples.

13. A load of wood measures 8 ft. 9 in. long, 3 ft. 5 in. high, and 3 ft. 10 in. wide; what are its solid contents?

Ans. 114 ft. 7 in. 2″ 6‴.

ft.	*in.*			
8	9			=length.
3	5			=height.
26	3	″		
3	7	9		
29	10	9		=the superficial content of one side.
3	10			=width.
89	8	3	‴	
24	10	11	6	
114	7	2	6	=solid content.

14. A piece of timber is 12 ft. 10 in. long, 1 ft. 7 in. wide, and 1 ft. 9 in. thick; what are the solid contents?

Ans. 35 ft. 6 in. 8″ 6‴.

15. A bale of goods measures 7 ft. 6 in. long, 4 ft. 3 in. wide, and 1 ft. 10 in. thick; what are its contents? Ans. 58 ft. 5 in. 3″.

16. A block of marble is 11 ft. 8 in. long, 4 ft. 5 in. broad, and 1 ft. 4 in. thick; what are its contents? Ans. 68 ft. 8 in. 5″ 4‴.

17. A gentleman purchased a pile of wood, the length of which was 32 ft. 6 in., its height 4 ft. 10 in., and the length of the wood was estimated at 3 ft. 10 in. only; what were the solid contents?

Ans. 602 ft. 1 in. 10″.

18. The floor of a parlour is 20 ft. 8 in. long, and 18 ft. 6 inches wide; what are its superficial contents? And how many yards of carpet just 3 ft. wide will be required to cover it?

Ans. { 382 ft. 4 in. =area of the floor.
{ 42 yds. 1 ft. 5½ in. of carpet cover it.

To find the number of yards of carpet to cover it; divide the area of the floor by the square feet in a yard, because the carpet is

supposed to be a yard or 3 feet wide. Or, you may reduce the 4 inches, which is $\frac{1}{3}$ of a foot, to a decimal, and annex it to the integer. The area, or content of the floor, will then be expressed thus: 382.3333 sq. feet; which divide by 9, and you have 42.48148+ yds; or 42 yds. 1 f. $5\frac{1}{2}$ in. nearly.

19. The floor of a room is 16 feet 7 inches long; and 15 feet 8 inches wide. A lady wishing to purchase carpet of 3 feet wide to cover it, desired her daughter to tell her, how many yards she must purchase; what number of yards were required?

Ans. 28 yds. 2 ft. 7+in.

To FIND the tonnage of merchant vessels: Multiply the length, breadth, and depth of the hold continually together, and divide the product by 95, the quotient will be the tonnage.

20. What is the tonnage of a vessel, whose length is 60 feet, breadth 20 feet, and depth of hold 8 feet? Ans. $101\frac{1}{19}$ tons.

21. What is the tonnage of a ship 74 feet keel, 26 feet beam, and depth of hold, 11 feet? Ans. $222\frac{74}{95}$ tons.

But in most cases, when, with the feet, there are parts of a foot given, it will be easier to change those parts to a decimal, and annex them to the integer, and proceed, as in multiplication of decimals.

22. What is the tonnage of a vessel, whose length is 69 feet 6 inches, breadth 22 feet 6 inches, and depth of the hold is 8 feet 6 inches? Ans. 139.914+tons.

69.5=length of the vessel.
22.5=breadth of the beam.
1563.75
8.5=depth of the hold.
13291.875÷95=139.914+tons.

INVOLUTION, OR THE RAISING OF POWERS.

A POWER is the product arising from multiplying any given number into itself, continually a certain number of times.

If the number is multiplied by itself once, it is called the *second power*, or *square* of that number, as; 4×4=16, the second power, or square of 4.

If it is multiplied into itself twice, it is called the *third power*, or cube of that number; as: 4×4×4=64, the third power, or cube of 4. If it is multiplied three times into itself, it is called the *fourth power*, or biquadrate of that number; as: 4×4×4×4=256, the fourth power, or biquadrate of 4.

If it is multiplied four times into itself, it is called the *fifth*

power, or *sursolid* of that number;; as: $4\times4\times4\times4\times4=1024$, the fifth power, or sursolid of 4; and so of all other numbers.

The number denoting the power, is called the index or exponent, of that power.

The index or exponent of the square, is 2: that of the cube, is 3; that of the biquadrate, is 4; and so of any other powers. It is usually written thus: 4^2, or $\overline{4}|^2$, which expresses the square of 4, or 4×4: 4^3, or $\overline{4}|^3$, which expresses the cube of 4, or $4\times4\times4$: 4^6, or $\overline{4}|^6$, which expresses the sixth power of 4, or $4\times4\times4\times4\times4\times4$; and so of any other powers.

When two or more powers of the same number are multiplied together, their product is that power of the number, whose index or exponent, is the sum of the exponents of the factors; thus, 16, the square of 4, multiplied by 64, the cube of 4, is 1024, the 5th power of 4; or $4^2\times4^3=4^5$, and so of all other powers.

EXAMPLES.

1. What is the square, or second power of 6?	Ans. 36.
2. What is the square, or second power of 464?	Ans. 215296.
3. What is the cube, or third power of 6?	Ans. 216.
4. What is the cube, or third power of 25?	Ans. 15625.
5. What is the biquadrate, or 4th power of 5?	Ans. 625.
6. What is the sursolid, or 5th power of 4?	Ans. 7776.
7. What is the square, or second power of .75?	Ans. .5625.
8. What is the square, or second power of .5?	Ans. .25.
9. What is the square, or second power of .25?	Ans. .0625
10. What is the square, or second power of $\frac{1}{4}$?	Ans. $\frac{1}{16}$.
11. What is the square, or second power of $\frac{1}{2}$?	Ans. $\frac{1}{4}$.
12. What is the square, or second power of $\frac{3}{4}$?	Ans. $\frac{9}{16}$.
13. What is the cube of $\frac{1}{4}$?	Ans. $\frac{1}{64}$.
14. What is the cube of $\frac{1}{2}$?	Ans. $\frac{1}{8}$.
15. What is the cube of .25?	Ans. .015625

It may be proper here to observe, that the power of any fraction, whether decimal or vulgar, is always less than its root; thus, the square of $\frac{1}{2}$ is only $\frac{1}{4}$, and the square of .25 is only .0625; and so of any other fractions.

EVOLUTION, OR EXTRACTION OF ROOTS.

THAT number, which being multiplied continually into itself, produces the given power, is called *the Root*.

Evolution is the finding of that number, which multiplied into itself continually, will make the given sum, or power. It is, therefore, the opposite of Involution.

Those roots, that cannot be exactly found, are called Irrational, or Surds.

Those that can be exactly found, are called Rational. The root of any power may be denoted or expressed, by placing this $\surd$ character before the power, with the exponent of that power written over it: thus, $\sqrt[3]{27}$ expresses the cube root of 27; and $\sqrt[4]{81}$ expresses the biquadrate root of 81; and so of any other roots. The square root is expressed simply by placing the character without the index, thus: $\surd 16$ denotes the square root of 16.

EXTRACTION OF THE SQUARE ROOT.

Any number multiplied into itself will produce a square. The extraction of the square root, therefore, is the finding of the number, which multiplied into itself, will make the given number.

Rule: Point off, or separate the given sum into periods of two figures each, if there are more than two figures, beginning at the unit's place. Place the greatest square number in the left hand period under the said period, and the root in the quotient: subtract the square number from the said period, and bring down the next period to the right hand of the remainder, if there is any. Double the quotient figure, and place it on the left hand of the dividend for a divisor; find by inspection, or trial, how many times this divisor will go in the dividend, reserving the right hand figure; place this in the quotient, and also on the right hand of the divisor. Multiply this divisor by the last quotient figure, and subtract the product from the dividend; and proceed, as before, until all the periods are brought down; and the whole quotient will be the root required.

EXAMPLES.

1. What is the square root of 625? Ans. 25.

$$\begin{array}{r} \dot{6}\dot{2}5(25 \\ 4 \\ \hline 45)225 \\ 225 \\ \hline \end{array}$$

$25 \times 25 = 625$ proof.

Here are three figures; which must constitute two periods. The two first on the right hand form one period, and the remaining figure must form another. The greatest root in 6 is 2, which I place in the quotient, and its square under the 6, and subtract it from 6. To the remainder 2, I bring down the left hand period 25. I next double the root, and place it on the left hand of the dividend, and find how often it will be contained in 22, reserving the 5. I find it will be contained 5 times, which I place in the quo-

tient to the right of the root, already found; and also to the right of the 4, already in the divisor. The divisor being thus completed, I multiply by the figure last placed in the quotient, and the product, I place under the dividend; when, on trial, I find nothing remains. The root is therefore, exactly found, and is a rational root.

In like manner proceed in the following examples.

2. What is the square root of 65536? Ans. 256.

```
  65536(256
  4 · ·
 45)255
    225
506)3036
    3036
```

3. What is the square root of 5499025? Ans. 2345.
4. What is the square root of 470596? Ans. 686.
5. What is the square root of 2125764? Ans. 1458.
6. What is the square root of 23059204? Ans. 4802.

If the given sum consists of a whole number, and a decimal, point it off into periods, both ways from the decimal point: and, if the decimal does not form an even number of periods of two places each, add a cypher to the right hand. If, after all the periods are brought down, there is a remainder, decimals may be added to it; and the operation be continued, at pleasure, or till nothing remains.

7. What is the square root of 30138.696025? Ans. 173.605.

```
  30138.696025(173.605
  1     · · · ·
 27)201
    189
343)1238
    1029
3466)20969
    20796
347205)1736025
    1736025
```

8. What is the square root of 14876.2357? Ans. 121.9681+.

In this example, the integer will form three periods, there being five figures. In the decimal part, there will be two periods, there being four figures. But there will be a remainder, after all the

periods are brought down. To which decimals may be added, and the operation continued at pleasure, or till nothing remains.

9. What is the square root of 36884? Ans. 192.052+.

```
  . . .
 36884(192.052+
 1  · ·
 ---
29)268
   261
   ---
382)784
    764
    ---
38405)20.00,00
      192025
      ------
384102)797500
       768204
       ------
        29296
```

10. What is the square root of 20? Ans. 4.472+.
11. What is the square root of 30? Ans. 5.4772+.
12. What is the square root of 40? Ans. 6.3245+.
13. What is the square root of .25? Ans. .5.
14. What is the square root of .75? Ans. .866+.

To EXTRACT the square root of a vulgar fraction: If the fraction is not in its lowest terms, reduce it to its lowest terms; then extract the root of the numerator, and the root of the denominator; which will be the parts of the new fraction required.

If the fraction is a surd, reduce it to a decimal, and extract the root, as in the foregoing examples. And, if the given sum is a mixed number, reduce it to an improper fraction, and proceed as above; or reduce the fractional part to a decimal, and annex it to the whole number, and extract the root of the whole, as in the foregoing examples. This latter method will be found, in general, the easiest and most expeditious.

15. What is the square root of $\frac{1}{4}$? Ans. $\frac{1}{2}$.
16. What is the square root of $\frac{49}{81}$? Ans. $\frac{7}{9}$.
17. What is the square root of $\frac{36}{81}$? Ans. $\frac{2}{3}$.
18. What is the square root of $\frac{224}{350}$? Ans. $\frac{4}{5}$.
19. What is the square root of $\frac{16}{25}$? Ans. $\frac{4}{5}$.
20. What is the square root of $\frac{7056}{92160}$? Ans. $\frac{7}{8}$.
21. What is the square root of $\frac{70}{84}$? Ans. .9128+.
22. What is the square root of $\frac{11}{12}$? Ans. .9574+.
23. What is the square root of $\frac{9}{13}$? Ans. .832+.
24. What is the square root of the mixed number $30\frac{1}{4}$? Ans. $5\frac{1}{2}$.

$30\frac{1}{4}=\frac{121}{4}$. $\sqrt{\frac{121}{4}}=\frac{11}{2}=5\frac{1}{2}$, by dividing by the denominator.

121

Here, I reduce the given sum to an improper fraction, which becomes $^{1}\frac{2}{4}^{1}$. The square root of which is $\frac{11}{2}$, or $5\frac{1}{2}$, the answer. It might have been very readily found by adding the decimal value of $\frac{1}{4}$ to the integer, making 30.25, thus:

30.25(5.5, the answer, as before.
25
105)525
525

25. What is the square root of $37\frac{36}{49}$? Ans. $6\frac{1}{7}$.
26. What is the square root of $272\frac{1}{4}$? Ans. $16\frac{1}{2}$.
27. What is the square root of $17\frac{16}{25}$? Ans. $4\frac{1}{5}$.
28. What is the square root of $76\frac{14}{17}$? Ans. 8.7649+.

APPLICATION.

1. A farmer planted 484 trees in a square; how many were there on each side? Ans. 22 trees.

2. A gentleman formed 400 perches of ground into a lot or garden, exactly in the form of a square; how many rods were on each side? Ans. 20 rods

3. An officer has a company of 400 men; if he formed them into a solid square, how many would there be on each side? Ans. 20 men.

4. A square pavement contains 1600 square stones of the same size; how many are there in one side? Ans. 40 stones.

5. A farmer would lay out a tract of land containing 140 acres, 2 roods, and 20 perches, into a square; how many rods will there be in one side of it? Ans. 150 rods.

5. A number of persons gave $12.25 for a charitable purpose; and each one gave as many cents, as there were persons; how many persons were there? Ans. 35 persons.

7. The area or content of a circle is 4276.5 feet; how many feet must the side of a square be, to contain the same quantity? Ans. 65.4 ft. nearly.

8. The area of a triangle is 144 yds.; how many yards long must the side of a square be to contain the same quantity? Ans. 12 yds.

9. A gentleman has an irregular fish pond, containing just 100 square rods. He is desirous to have another exactly in the form of a square; how long must each side be? Ans. 10 rods.

10. A gentleman is desirious of forming a court yard, containing 160 rods or 1 acre of ground, and to have it exactly in the form of a square; how long must one side be? Ans. 12.649+ rods.

To find a mean proportion between 2 given numbers; multiply the numbers together, and extract the square root of the product.

12. What is the mean proportion between 6 and 24? Ans. 12.

$24 \times 6 = 144$. The root of which will be the answer.

13. What is the mean proportion between 64 and 100? Ans. 80.

14. What is the mean proportion between 24 and 96? Ans. 48.

Hence it is manifest, when three numbers are proportional, the product of the extremes is equal to the square of the mean, or middle term.

To FIND any side of a right angle triangle, the other two being given: The sum of the squares of the two shortest sides will equal the square of the longest side, or hypothenuse; and the difference of the squares of the longest side and either of the others, will be the square of the third side. Prop. 47, 1 Book of Euclid.

15. There is a right angled triangle, whose base is 8 feet, and perpendicular 6 feet; what is the length of the hypothenuse?

Ans. 10 feet.

$6 \times 6 = 36$
$8 \times 8 = 64$
100 the square root of which is 10.

16. There is a right angled triangle, whose longest side is 10, and base 8; what is the perpendicular? Ans. 6.

$10 \times 10 = 100$ = the square of the hypothenuse.
$8 \times 8 = 64$ = the square of the base.
36 = the square of the perpendicular. The square root of which will be the perpendicular sought.

In like manner may the base be found, the perpendicular and hypothenuse being given.

17. The wall of a fortress is 17 feet high, which is surrounded by a ditch 20 feet from the wall; what must be the length of a ladder, that will reach from the outside of the ditch to the top of the wall? Ans. 26.2 ft.

Here the height of the wall may be considered the perpendicular of the right angled triangle, and the breadth of the ditch, as the base; while the ladder will be the hypothenuse.

18. On the margin of a river 24 yds. wide, stands a tree; from the top of which a line 36 yards long, will just reach to the other side of the stream; what is the height of the tree?

Ans. 26.83+yards.

Here the hypothenuse and base are given, to find the perpendicular; which may be done, as example 16.

19. Two ships sail from the same port; one sails due east 50 miles, and the other due south 84 miles; how far are they from each other? Ans. 97.75 miles.

This is solved as example 15, the hypothenuse of the triangle being required.

20. A ladder or pole, 40 feet long, placed in the middle of a street, will reach a window of a house on each side of the street 24 feet from the pavement; what is the width of the street?

Ans. 64 feet wide.

In this example the perpendicular and hypothenuse are given to find the base; which is found as example 16, and doubled will give the width of the street.

EXTRACTION OF THE CUBE ROOT.

The cube of any number is the product of that number, multiplied by its square, as: $3 \times 9 = 27$ the cube of 3.

The extraction of the cube root, is the operation of finding what number multiplied into itself twice, or multiplied by its square, will produce the given number.

Rule. Separate the given number into periods of three figures each, beginning at the unit's place. Find the greatest cube in the left hand period, and subtract it from the said period, placing the root in the quotient. Bring down the next period to the right hand of the remainder, if there is any.

Square the root, and multiply it by 3, the index or exponent of the cube, with the addition of two cyphers or decimals, for a divisor.

Find how many times this divisor is contained in the dividend, and place it in the quotient: Multiply the divisor by this figure, and note the product; multiply all the figures of the root, except the last one, by 3, the index of the given power, with the addition of a cypher or decimal; and multiply that product by the square of the last figure, and place the product under the preceding one; then cube the last figure in the root, and place it under the last ascertained product. Add these three sums or products together, and subtract it from the dividend.

Bring down the next period to the remainder, and proceed as above directed, till all the periods are brought down.

EXAMPLES.

1. What is the cube root of 17576? Ans. 26.

17576(26
$2^3 = 8$
1200)9576=dividend.
9576 subtrahend.

$2 \times 2 \times 3.00 = 1200$ divisor.
$1200 \times 6 = 7200$
$2 \times 3.0 \times 6 \times 6 = 2160$
$6 \times 6 \times 6 = 216$
9576=subt'nd.

2. What is the cube root of 12812904? Ans. 234.

12812904(234

$2^3 = 8$

1200)4812, dividend.
4167, subtrahend

156700)645904, dividend.
645904, subtrahend.

$2\times2\times3.00=1200$, 1st divisor.
$1200\times3=3600$
$2\times3.0\times9=540$
$3\times3\times3=27$
4167=1st sub.

$23\times23\times3.00=158700$, 2d divisor.
$158700\times4=634800$
$23\times3.0\times16=11040$
$4\times4\times4=64$
645904=2d subtrahend.

3. What is the cube root of 3796416?* Ans. 156.

* It may be proper, here, to explain more particularly this method of extracting the cube root. It is deduced from the binomial theorem, with which, it is not expected the pupil should, in this stage of his progress, be acquainted.

Let $a+b$ be any given root; its cube will be expressed thus: $a^3+3a^2b+3ab^2+b^3$. Now on this principle, let it be proposed to extract the root of $\dot{3}\dot{7}\dot{9}\dot{6}\dot{4}\dot{1}\dot{6}$. Here there are three periods, and consequently there will be three places in the root. The greatest cube in the first period is 1, which must be cubed and subtracted from 3. The 1^3 represents the first place, or a^3 in the theorem or formula. The remainder is 2; to which, bring down the next period, and we have 2796 for a dividend. Now to form a divisor for the next figure in the root, square the root already found, that is, $1\times1=1$, or in the formula a^2, in the second place. Multiply this by 3, the coefficient of a^2, with the addition of two cyphers, which answer no other purpose than to remove the product two places farther to the left hand; and, we have the divisor $1\times1\times3.00=300$. This will be contained in the dividend 5 times, which will be the second figure in the root.

Now to obtain a subtrahend, or sum to be subtracted from the dividend, multiply the divisor 300, by 5, the last ascertained root, and we have 1500, represented in the formula by $3a^2b$. Again multiply 1, the first figure in the root by 3, the coefficient of a in the third place of the formula, with the addition of one cypher, which only serves to remove the product one place farther toward the left hand. This product is multiplied by 5×5, or the square of the second figure in the root, represented in the formula by b^2 in the third place. To these two products, add the cube of 5; or $5\times5\times5$, which in the formula is represented by b^3, in the fourth place. The sum of these three products is 2375, the first subtrahend.

The remainder is 421; to which, bring down the remaining period, and we have 421416, for the 2d dividend.

Now, for a second divisor, square the root already found, that is, 15×15, expressed in the second place of the formula by a^2, and multiply by 300, for the reason already given, and the second divisor will be

In like manner may the cube root of any other number be found. If the given power consists of a whole number, and a decimal, you must point off both ways from the decimal point; and if there are not decimal places sufficient to constitute one or more periods of three places each, as the case may be; then add so many decimal places to the right hand, as will make the periods of three figures each; and proceed, as in the above examples, to find the root.

4. What is the cube root of 84604519? Ans. 439.
5. What is the cube root of 94818816? Ans. 456.
6. What is the cube root of 970299? Ans. 99.
7. What is the cube root of 2000376? Ans. 126.
8. What is the cube root of 15926.9725? Ans. 25.16+.

If, after all the periods are brought down, there should be a remainder, the operation may be continued by adding decimals, constituting one or more periods of three places each.

9. What is the cube root of 96? Ans. 4.56+.

$$
\begin{array}{l}
\dot{9}6(4.56+ \\
4^3=64 \\
4800)\overline{32}.000 \\
\quad 27\ 125 \\
\overline{607500)3875.000} \\
\quad 3693\ 816 \\
\overline{\quad 181184} \text{ Rem.}
\end{array}
$$

$4\times4\times3.00=\ 4800$, 1st divisor.

$$
\left.\begin{array}{r}
4800\times5=24000 \\
4\times3.0\times25=\ 3000 \\
5\times5\times5=\ \ 125
\end{array}\right\}
$$

27125, 1st subtrahend.

$45\times45\times3.00=\ 607500$, 2d div.

$$
\left.\begin{array}{r}
607500\times6=3645000 \\
45\times3.0\times36=\ \ 48600 \\
6\times6\times6=\ \ \ \ 216
\end{array}\right\}
$$

3693816, 2d sub.

10. What will be the cube root of 160, the decimal being continued to three places? Ans. 5.428+.

67500. The quotient figure will be 6, which will complete the root. For the subtrahend, multiply this divisor (that is, the product of 15×15, or a^2 in the second place of the formula by 3, the coefficient of a^2 with the addition of two cyphers) by the root last found, that is, by 6, which will make 405000. Multiply 15, the root previously found, by 3, with the addition of one cypher; and this product again, by the square of 6, represented by b^2 in the third place of the formula. Lastly, cube the 6, the last figure of the root, represented by b^3 in the last place of the formula. These three products added together, will be the second subtrahend, 421416, which completes the operation, for nothing remains.

If there are more than three periods, the same operation is to be repeated, until all are brought down. And if, after all the periods are brought down, there is a remainder, then decimals are to be added in periods of three, and the operation may be continued at pleasure, or till nothing remains.

To EXTRACT the cube root of a vulgar fraction:

If the fraction is not in its lowest terms, reduce it to its lowest terms, and extract the root of the numerator and of the denominator, for the parts of the new fraction. If the fraction is a surd, reduce it to a decimal, and extract the root.

A mixed number may be reduced to an improper fraction, and the root extracted as above; or the fractional part may be reduced to a decimal, and connected with the integer, and the root extracted.

11. What is the cube root of $\frac{8}{27}$? Ans. $\frac{2}{3}$.

$\sqrt[3]{8}=2$

$\sqrt[3]{27}=3$

12. What is the cube root of $\frac{64}{125}$? Ans. $\frac{4}{5}$.
13. What is the cube root of $\frac{352}{1188}$? Ans. $\frac{2}{3}$.
14. What is the cube root of $\frac{648}{3000}$? Ans. $\frac{3}{5}$.
15. What is the cube root of $\frac{4}{9}$? Ans. .763.
16. What is the cube root of $13\frac{2}{3}$? Ans. 2.3908+
17. What is the cube root of $7\frac{3}{5}$? Ans. 1.966+
18. What is the cube root of $9\frac{1}{6}$? Ans. 2.092+.

APPLICATION.

1. What is the length of one side of a cubical vessel, which contains 46656 solid inches? Ans. 36 inches.

2. A building in a cubical form contains 21952 feet; what is the length of one of its sides? Ans. 28 ft.

RULE.—The solid contents of similar figures are to each other, as the cubes of their similar sides or diameters.

3. If a globe of silver, 4 inches in diameter is worth 200 dollars, what will be the value of one, 10 inches in diameter? Ans. $3125.

4. If a globe of water 6 inches in diameter will weigh 1.5 pound; what will be the weight of one 24 inches? Ans. 96 lbs.

5. A ball of lead 2 inches in diameter weighs 1.5 lb.; what will be the weight of another 10 in. in diameter? Ans. $187\frac{1}{2}$ lbs.

6. A gentleman wishing to build a cubical cistern containing 729 cubic feet of water; how long must he make one of its sides? Ans. 9 feet.

7. A gentleman has an ingot of silver in the form of a cube, one side of which is 4 inches, which he values at 250 dols.; what must be the length of a cube of the like metal, which shall be worth 4 times as much? Ans. 6.349 in.

Here you cube the side of the given cube, which is 4, and multiply that by 4, the proportion between the given value of the

given cube, and that of the one sought. The cube root of the product will be the side sought. See the rule under the second example of this.

8. A gentleman has a cubical vessel, whose side is 2 feet, and he is desirous to make another of the same form, which shall contain 3 times as much; what must be the length of one side?

Ans. 2 ft. $10\frac{2}{3}$ in.

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is a series of numbers increasing or decreasing by a common difference. Five particulars are to be observed, viz.

The first term of the series } called extremes.
The last term of the series }
The common difference.
The number of terms; and
The sum of all the terms, or sum of the series.

Any three of them being given, the other two may be found.

When the first term, the common difference, and the number of terms are given, to find the last term, and the sum of all the terms:

RULE 1. Multiply the number of terms, less 1, by the common difference, and to the product add the first term; the sum will be the last term.

RULE 2. Add the first term, and last term together, and multiply this sum by the number of terms; half the product will be the sum of all the terms.

When the two extremes and the number of terms are given, to find the common difference:

RULE 3. Divide the difference of the extremes by the number of terms, less 1; the quotient will be the common difference.

When the first term, sum of the series, and number of terms are given to find the last term, and common difference; or when the last term, sum of the series, and number of terms are given to find the first term, and common difference:

RULE 4. From twice the sum of the series, subtract the product of the first term, or last term, (as the case may be,) and number of terms; and divide the difference by the number of terms, the quotient will be the last term, or first term, (as the case may be.) Having the two extremes, the common difference may be found by rule 3d.

When the first term, last term, and common difference are given, to find the number of terms:

RULE 5. Divide the difference of the extremes, by the common difference, the quotient+1, will be the number of terms.

EXAMPLES.

1. A lady purchased 20 yards of muslin, and agreed to give 1 cent, for the first yard, 3 cents for the second yard, 5 cents for the third yard, increasing two cents for every yard; what did she pay for the last yard, and what did the whole amount to?

Ans. { 39 cents for the last yd.
{ $4.00 the amount.

Here 1st. term= 1
Num. of terms=20
Com. diff. = 2

By rule 1st. 20—1=19
2 com. dif.
38
1=1st term add.
39=last term.

Having found the last term, the sum of all the terms is readily found by rule 2d.

2. How many strokes, does the hammer of a clock strike in 12 hours, beginning at 1 o'clock? Ans. 78.

Here are given, the 1st term= 1
the last term=12
the number of terms=12

The answer is found by rule 2d.

3. A labourer was hired to load a wagon with cabbages, placed in a row, at the distance of a yard from each other; how far must he travel to bring 100, one at a time, to the wagon, which is placed a yard from the first one? Ans. 10100 yds.

Here we have 1st term= 1
the last term=100
number of terms=100

By rule 2d.
100+1=101 sum of extremes.
50
5050
2
10100=distance.

Here you multiply the sum of the extremes by 50, half the number of terms; or multiply by the whole number of terms, and divide by 2. This product multiply by 2, because he travelled the distance twice over, once in going to the wagon, and once in returning after the next cabbage. This you may reduce to miles by dividing by the yards in a mile, viz. 1760, and you will find the distance to be 5 miles, 5 furlongs, and 200 yards.

If the cabbage had been placed at 2 yards distance, and the

wagon at 2 yards from the first one, you must have multiplied by 2, because he travelled the distance twice; and that product again by 2, because they were placed at the distance of 2 yards from each other, to obtain the answer in yards. In that case the distance would have been 11 miles, 3 furlongs, and 180 yards.

4. A merchant sold 100 yards of cloth, at 1 cent for the first yard, 10 for the second, 20 for the third, and so on increasing 10 cents on every yard; what did he receive for the last yard—and what did the whole amount to? Ans. { $9.91, price of the last yd. / $496, whole amount.

Here the 1st term=1
the num. of terms=100
com. diff. =10
} By rule 1st. 100—1=99
10
990
1 1st term.
991=last do.

Having found the last term, the answer will be easily found by rule 2d.

5. A merchant bought 100 yards of cloth, and, by agreement, was to give 10 cents for the first yard, 20 cents for the second, and so on, increasing 10 cents on every yard; what was the price of the last yard—and what did the whole amount to?

Ans. { Last yard=10 dol. / Amount=505 dol.

Here the 1st term=1
num. of terms=100
com. diff. =10
} By rule 1. 100—1=99
10 com. diff.
990
10
1000 last term.

Now, having found the last term, the sum of the series is found by rule 2.

First term= 10
Last term=1000
} The sum=1010
50=½ num. of terms.
505.00 cents.

It may here be proper to observe, that the result in this example is different from that in the preceding, owing to the difference in the price of the first yard of cloth. To obtain a true answer, you must add the price of the first yard, to wit, 10 cents in the room of a unit, or 1 for the last term. And, in the latter part of this example, you must consider the *price* of the first term, and not a unit or 1, as the first term.

The same observation will apply to other examples of a similar kind.

6. A merchant purchased 54 yards of cloth, and gave 20 cents for the first yard, 50 cents for the second yard, and so on, increasing 30 cents on every yard; what was the price of the last yard, and what was the amount of the whole?

Ans. { The last yard cost $16.10.
Whole amount $440.10.

7. A lady hired a servant for 12 years, and by agreement was to allow her 12 dols. for the first year, 14 dols. for the second year, 16 dols. for the third year, and so on, increasing 2 dols. in every year; what were her wages for the last year—what were her wages on an average, and what was the whole amount?

Ans. { She received the last year, 34 dols.
Whole sum received, 276 dols.
Annual average, 23 dols.

8. A person has 10 children, whose several ages differ alike from each other. The eldest is 48 years, and the youngest is 3 years old; what is the common difference of their ages?

Ans. 5 years.

48—3=45, which divide by 10—1=9: the answer is found by Rule 3.

9. A person proposes to travel from the city of New York to a certain place in 19 days; to travel 6 miles the first day, increasing by an equal excess until his last day's journey shall be 60 miles; what is the increase—and what is the distance, which he will travel?

Ans. { daily increase 3 miles.
whole distance 627 do.

Extremes, 60—6=54
No. of terms, 19—1=18=3 the common difference. By rule 3.

60+6=66×19=1254÷2=627. By rule 2d.

10. A lady being asked how many children she had; to avoid a direct answer replied, that her youngest child was 4 years old, and her eldest was just 32 years old, and that the difference of their ages was just 4 years; how many children had she? Ans. 8.

11. When the extremes are 3 and 45, and the common difference 6; what is the number of terms? Ans. 8.

Here 45—3=42÷6=7+1=8. By rule 5th.

12. A gentleman on a tour went three miles the first day, and increased every day by 5 miles, till on the last day, he travelled 58 miles; how many days did he travel? Ans. 12 days.

13. A gentleman, the first year he kept house, expended $500,

but afterward increased his expenses by an equal sum; and at the end of 12 years, found that he had expended altogether the sum of 12600 dollars; what was the annual increase of his expenses, and how much did he spend the last year?

Ans. { He spent the last year $1600.
Annual increase $100.

1st. term = $500
sum of series = 12600 } given.
num. terms = 12

12600 × 2 = 25200
500 × 12 = 6000
19200 ÷ 12 = 1600

last term, by Rule 4th. Having the two extremes, the common difference is found by rule 3d.

14. A gentleman found his family expenses increase in arithmetical progression, and, at the end of 15 years, his whole expenses amounted to 18,300 dollars; and his expenses the last year were 1500 dols.; what was the annual increase, and what did he spend the first year?

Ans. { $940, sum spent 1st year.
$40, common difference.

18300 × 2 = 36600, the product of the series by 2, by Rule 4.
1500 × 15 = 22500, prod. of the last term and number of years.
14100 ÷ 15 = 940, the sum spent the first year.

Having the two extremes, the common difference is found by Rule 3d.

15. A merchant began business with 3500 dols. and added 600 dols. annually to his capital; in what time would it amount to 12500 dols.?

Ans. 16 years.

12500 — 3500 = 9000 ÷ 600 = 15 + 1 = 16, by Rule 5th.

16. A young gentleman of fortune, on his arrival at the age 21 years, commenced by spending a certain sum; and by increasing his expenses annually in arithmetical progression for 13 years, found that he spent 2500 dollars the last year; and, in the whole, that he had diminished his estate 22750 dollars; what did he spend the first year, and what was the common difference?

Ans. { $1000.
$125 com. diff.

22750 × 2 = 45500 = prod. of sum of the series by 2. Rule 4th.
2500 × 13 = 32500 = prod. of last term, and number of years.
13000 ÷ 13 = 1000 = sum spent the first year.

Having the extremes, the common difference is found by Rule 3.

17. A young gentleman began his career by spending the first year after he came of age, 500 dols., and increased every year by an equal sum; and in 12 years found that he had spent, in all, the

sum of 9000 dols.; what did he spend the last year, and what was the common difference? Ans. { $1000, the sum spent the last year. / $45.45,45+ the com. difference.

9000 × 2 = 18000
500 × 12 = 6000
1200 ÷ 12 = 1000 sum spent the last year.

Having the extremes, the common difference is found by Rule 3.

GEOMETRICAL PROGRESSION.

Geometrical Progression is a series of numbers, increasing by one continual multiplier, as: 2, 4, 8, 16, &c. which increases by the continual multiplication of 2: or decreases by one continual divisor, as: 32, 16, 8, 4, 2, which decreases by the continual division of 2.

In every geometrical series, five particulars are to be observed:

1. The least term.
2. The greatest term.
3. The common ratio, or number, by which the series increases or diminishes.
4. The number of terms.
5. The sum of all the terms, or series.

Any three of these five being given, the other two may be found.

When the first term, the ratio, and number of terms are given to find the last term:

Rule 1. When the first term and ratio are alike; raise the ratio to that power indicated by the number of terms; the last product is the term required.

Rule 2. When the first term and ratio are different; raise the ratio to that power denoted by the number of terms less 1, and multiply this power by the first term; the last product is the term required.

When the first term, last term, and ratio are given, to find the sum of the series:

Rule 3. Multiply the last term by the ratio; from the product subtract the first term; and divide by the ratio, less 1 or unit; the quotient will be the sum of the series.

EXAMPLES.

1. The first term being 2, the ratio 2, and the number of terms 16; what is the last term of the series? Ans. 65536.

2 4 8 16 32 64=6 power of the ratio.

Num. terms=16
First term = 2
Ratio = 2

64
256
384
4096=12 power of the ratio.
16= 4 power of the ratio.
24576
4096
65536=16 power of the ratio.

Therefore, use rule 1.

2. What will be the last term of the series of 18 terms, the first term being 3, and the ratio three? Ans. 387420489.

3 9 27 81 243= 5 power of the ratio.

Num. terms=18
First term = 3
Ratio = 3

243
59049=10 power of the ratio.
343
14348907=15 power of the ratio.
27= 3 power of the ratio.
387420489=18 power of the ratio.

Therefore, use rule 1.

Here, it is obvious that all the powers of the ratio are not produced. That would have been quite unnecessary. Having produced any convenient power of the ratio, multiply it by itself, and you will have the power of the ratio denoted by double the former power; and so on, until you find the power denoted by the number of terms. Thus, in the first example, by multiplying the 6 power of the ratio by itself, you have the 12 power; and this again by the fourth power of the ratio, and you have the 16th power of the ratio, the power sought. In the same manner proceed with the following example, and all others of the like kind.

3. The first term being 4, the ratio 3, and the number of terms 10; what is the last term? Ans. 78732.

3 9 27 81=4 power of the ratio.

Num. terms=10
First term = 4
Ratio = 3

81
6561=8 power of the ratio.
3=the ratio.
19683=9 power of the ratio.
4 the first term.
73732=the last term.

Therefore, use rule 2d.

4. A farmer sold 24 sheep, and, by agreement, was to receive for the whole the sum that the last one came to, reckoning 2 cents for the first sheep, 4 cents for the second, and so on, in geometrical progression; what sum did he receive? Ans. $167772.16.

5. A wealthy gentleman having his family at dinner, consisting

of 10 children, proposed to make each of them a present of a sum of money; the youngest to have 10 cents, the next 30, and so on in triple proportion; how much did the eldest receive?

Ans. $1968.30.

Num. terms=10
First term =10
Ratio = 3

3 9 27 81=4 term of the ratio.
81
81
648
6561=8 term of the ratio.
3 the ratio.
19683=9 power of the ratio.
10
1968,30=the last term.

Therefore, use rule 2d.

6. A merchant sold 24 yards of cloth, at 2 cents for the first yard, 4 cents for the second, 8 for the third, and so on in duplicate proportion; how much did the cloth amount to? Ans. $335544.30.

Having found the last term by rule 1, find the sum of all the terms by rule 3d., which will be the answer in cents.

7. A lad, ignorant of progression, agreed to buy 18 oranges, and to pay, for the whole, the price of the last one, reckoning 1 cent for the first, 4 cents for the second, and so on in quadruple proportion; what sum would he have to pay for the oranges—and how much would be the average price? Ans. $171798691.84. $9544371.77.

Here the ratio and first term are different; therefore, use rule 2d.

Ratio= $\overline{4}|^{17}$=17179869184=the last term of the series; which, divided by 18, will give the average price=954437177 cents, nearly. In this example, the first term being unit or 1, it is not necessary to multiply by it.

8. A lady agreed to purchase 15 yards of silk, for a dress for her two daughters, and imprudently bargained upon these terms; to pay for the 1st yard 4 cents, for the 2d, 16 cents, for the 3d, 64 cents; and so on, in quadruple proportion; what sum did the whole amount to? Ans. $14316557.64.

Here the ratio and first term are different; the last term is found by rule 2d. And the sum of the series is found by rule 3d.

9. A wealthy gentleman married his daughter on a new year's day, and presented her with 5 dollars, or half an eagle, and promised to double the same on the first day of each month during that year, as her marriage portion, which the young bride readily accepted, being acquainted with the principles of geometrical progression; what sum did she receive? Ans. $20475.

Here the first term is 5, the ratio 2, and the number of terms 12. The last term is found by rule 2d, and the sum of the series by rule 3d.

10. A gentleman having a valuable horse, proposed to sell him upon the following terms, (to wit) 1 mill for the first nail, 2 mills for the second nail, 4 mills for the third nail, and so on through the 32 nails, which he had in his 4 shoes; a jockey considering him a great bargain, readily accepted the terms; what sum did the horse cost him? Ans. $4294967.29,5.

Here the first term and ratio are different, the last term is found by rule 2d; and the sum of the series by rule 3d.

11. If the posterity of Noah, who were 6 persons, at the flood, doubled in 20 years; how many inhabitants would there be in the world 2 years before the death of Shem, who lived 502 years after that event? Ans. 201336586 persons.

Here the first term is 6, the ratio 2, and the number of terms 25, or $\frac{500}{20}$. The last term is found by rule 2d.

12. A gambler lost 5 dollars at the first game of cards, and doubled the stake each game in the hope of recovering what he had lost; but the game was always against him, and the last one he played, he lost 5120 dols.; how much did he lose in all, and how many games did he play? Ans. { He lost $10235. / Number of games 11.

Here the ratio is 2, the first term 5, and the last term 5120. The sum of the series is found by rule 3d.

13. A manufacturer introduced a new branch of business into the country, and took 5 apprentices for 7 years; after which he took 5 more, and his former apprentices took each five, for the same length of time, and so on. Now, supposing all, who learned the business, were to be alive at the end of 42 years, what would be the number? Ans. 9331.

Here the first term=1
Number of terms=6
And ratio=6; for 42 years divided by 7 gives 6. And the 5 apprentices, with their masters, also make 6. The last term may be found by rule 2; and the sum of the series by rule 3.

14. A labourer engaged to work for a farmer for 12 days, at 1 cent for the first day, 3 for the second, 9 for the third, and so on, in geometrical proportion; what was the amount of his wages for the time? Ans. $2657.19 nearly.

Here the first term and ratio are different: the last term is found by rule 2d, and the sum of the series by rule 3d.

In a decreasing geometrical series, when the first term, ratio, and number of terms are given, to find the last term :

Rule 1. Divide the first time by the ratio, raised to the power denoted by the number of terms, less 1 ; the quotient will be the last term.

When the first term, last term, and ratio are given, to find the sum of the series :

Rule. 2. Multiply the first term of the decreasing series by the ratio: from the product subtract the last term, and divide the remainder by the ratio, less 1; the quotient will be the sum of all the terms.

15. What is the last term of a decreasing geometrical series, whose first term is 128, number of terms 4, and ratio 2 ? Ans. 16.

16. What is the last term of a decreasing geometrical series, whose first term is 729, ratio 3, and number of terms 5 ? Ans. 9.

17. What is the last term of a decreasing geometrical series, whose first term is 4096, ratio 4, and number of terms 6 ? Ans. 4.

These examples are performed by rule 1, and all others of the same kind.

18. In a decreasing geometrical series, whose first term is 64, ratio 2, and last term 4; what will be the sum of all the terms ? Ans. 124.

19. In a decreasing geometrical series, whose first term is 729, ratio 3, and last term 9 ; what will be the sum of all the terms ? Ans. 1089.

20. In a decreasing geometrical series, whose first term is 4096, ratio 4, and last term 16 ; what will be the sum of all the terms ? Ans. 5456.

These examples are performed by rule 2 of this : and all others of the like nature.

21. A certain number of persons settled in a country, and after a period 220 years, their number were 206848. Now, supposing they doubled their number every 20 years ; how many were there at first ? Ans. 101.

Here the first term of the series is 206848, the ratio 2, and the number of terms 11, or $\frac{220}{20}$. Solved by rule 1st of this.

ANNUITIES AT COMPOUND INTEREST.

An annuity is any sum of money payable yearly, half yearly, or quarterly, for a number of years, for life, or for ever. It draws interest, if it is not paid, when due. If it remains unpaid for any length of time, compound interest is computed.

When the annuity, time, and rate per cent. are given to find the amount :

RULE.—Involve the ratio to the power denoted by the time; from which take unit or 1; the remainder, divide by the ratio, less 1 or unit; the quotient multiplied by the annuity will give the amount, when the payments are made yearly.

EXAMPLES.

1. What will an annuity of 30 dollars, amount to in 4 years, at 5 per cent.? Ans. $129.30,375.

Here the ratio is 1.05, which, involved to the fourth power, is 1.21550625. From this take unit, or 1; the remainder is .21550625. This divide by the ratio, less 1 or .05; and multiply the quotient by 30, you will have the amount of the annuity for 4 years, thus:

.05).21550625

4,310125

30

129.30,3,75=the amount.

2. What will a pension of 96 dollars a year amount to, in 7 years, at 6 per cent.? Ans. $805.80,8352.

Here the ratio involved to the power denoted by the time, is 1.5036202; with which proceed, as in the above example, or as the rule directs.

3. A gentleman owns a ground rent of 84 dollars a year; what will it amount to, at 5½ per cent., if it remains unpaid for 6 years? Ans. $578.59,6284.

Here the ratio is, 1.055, which involved to the 6 power, is 1.3788426, with which proceed, by the rule, to find the amount.

4. A lady has an annual income of 500 dollars, arising from ground rent; what sum would it amount to, at 6 per cent., if not paid in 4 years? Ans. $2187.30,8.

When the amount, rate per cent., and time are given, to find the annuity:

RULE. Involve the ratio to the power denoted by the time, from which take unit or 1, for a divisor: Multiply the amount by the ratio, less unit or 1, for a dividend; the quotient arising from the division will be the annuity.

EXAMPLES.

1. What annuity being unpaid for 4 years, will amount to $129.3036, at 5 per cent.? Ans. $30.

129.3036=amount of the annuity.

.05=the ratio less 1, or unit.

6.465160

Which divide by .21550625, the ratio involved to the time, less unit or 1. The quotient will be the answer, by the rule.

2. What annuity remaining unpaid for 9 years, will amount to $1984.78,152, at 5 per cent.? Ans. 180.

3. What annuity will amount to $578.59,6284, in 6 years, at 5½ per cent.? Ans. $84.

When the annuity, amount, and rate per cent., are given, to find the time:

RULE. Multiply the amount by the ratio, less unit or 1. Divide the product by the annuity; and add unit or 1 to the quotient; which will be that power of the ratio, denoted by the time. Divide this continually by the ratio, until the quotient is unit or 1; the number of divisions will show the time.

EXAMPLES.

1. In what time will 30 dollars amount to, $129.30,75, at 5 per cent.? Ans. 4 years.

129.30375=the amount.
.05=ratio less unit or 1.

6.4651875÷30=.21550625: to which adding unit, or 1, you have 1.21550625. This sum divided by 1.05 continually, till the quotient is 1 or unit, the number of divisions will be the time, by the rule.

2. In what time will a pension of 96 dollars amount to $805.80,8,35, at 6 per cent.? Ans. 7 years.

When the annuity, time and rate per cent. are given, to find the present worth:

RULE. Involve the ratio to the time, and by it divide the annuity. Subtract the quotient from the annuity, and divide the remainder by the ratio less 1. The quotient will be the present worth.

EXAMPLES.

1. What is the present worth of an annuity of 30 dollars a year, to continue 5 years, at 4 per cent.? Ans. $133.55,4675.

Ratio=1.04 involved to the time=1.2166529: by this divide 30 the annuity, and you have 24.657813+, which take from the annuity, and there will remain 5.342187. This divide by .04, and the quotient will be the answer.

2. What is the present worth of a ground rent of $588.86,5, for 4 years, at 5½ per cent.? Ans. $2064 05,426+

3. What is the present value of a pension of 64 dollars, to continue 5 years, at 6 per cent.? Ans. $269.59,104.

When the present worth, time and rate per cent., are given, to find the annuity:

RULE 5th. Multiply the present worth by the ratio involved to the time; and this product by the ratio, less 1 or unit, for a dividend: divide by the ratio involved to the time, less unit or 1; and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to continue 5 years, may be purchased for $133.55,4,675, allowing 4 per cent. interest? Ans. $30.

$\overline{1.04}|^5 = 1.2166529$, the 5th power of the ratio: which multiply by the present worth; and this again by .04, the ratio less unit or 1. Then proceed as the rule directs, to find the annuity.

2. A gentleman, having in ready money $2064.05,426+, wishes to purchase an annuity for 4 years, allowing $5\frac{1}{4}$ per cent.; how much can he purchase? Ans. $588.86,5.

3. A lady, having the sum of 5000 dollars, is desirous of purchasing an annuity to continue 6 years, allowing 5 per cent. interest; what sum can she purchase? Ans. $985.08,756+

When the annuity, present worth, and ratio are given, to find the time:

RULE 6th. Multiply the present worth by the ratio, less unit or 1: subtract the product from the annuity, and divide the annuity by the remainder. The quotient will be the power of the ratio involved to the time: which divide by the ratio continually, till the quotient figure is unit or 1. And the number of divisions will show the time.

EXAMPLES.

1. How long may an annuity of 30 dollars a year, be purchased for $133.55,4678, allowing 4 per cent.? Ans. 5 years.

133.554675 = present value.
.05 = the ratio less 1.

$30 - \overline{6.677773375} = 23.32226625$: by this divide 30, the annuity, which will give the power of the ratio, indicated by the time. Proceed then by the rule to find the time.

2. A house is let for $588.8,65 a year; but the person having money in hand to the amount of $2064.05,4266, proposes to pay this sum, being allowed $5\frac{1}{2}$ per cent. for his money; how long may he occupy the premises? Ans. 4 years.

But every case of annuities may be much easier solved by the help of the following tables.

TABLE I. *Showing the Amount of One Dollar Annuity, from one Year to forty.*

yr.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1.	1.	1.	1.	1.
2	2.04	2.045	2.05	2.055	2.06
3	3.1216	3.137025	3.1525	3.168225	3.1836
4	4.246464	4.278191	4.310125	4.342266	4.374602
5	5.416322	5.470710	5.525631	5.581091	5.637093
6	6.632975	6.716892	6.801913	6.888051	6.975318
7	7.898294	8.019152	8.142008	8.266894	8.393837
8	9.214226	9.380014	9.549109	9.721573	9.897468
9	10.582795	10.802114	11.026564	11.256259	11.491316
10	12.006107	12.288210	12.577892	12.875354	13.180795
11	13.486351	13.841179	14.206787	14.583498	14.971643
12	15.025805	15.464032	15.917126	16.385590	16.869942
13	16.626838	17.159913	17.712983	18.286798	18.882138
14	18.291911	18.932109	19.598632	20.292572	21.015066
15	20.023588	20.784054	21.578563	22.408663	23.275971
16	21.824531	22.719337	23.657492	24.641140	25.672528
17	23.697512	24.741707	25.840366	26.996402	28.212881
18	25.645413	26.858084	28.132385	29.481205	30.905653
19	27.671229	29.063562	30.539004	32.102671	33.759993
20	29.778078	31.371423	33.065954	34.868318	36.785592
21	31.969202	33.783137	35.719252	37.786075	39.992728
22	34.247970	36.833378	38.505214	40.864309	43.392291
23	36.617888	38.937030	41.430475	44.111846	46.995828
24	39.082604	41.689196	44.501999	47.537998	50.815578
25	41.645908	44.565210	47.727099	51.152588	54.864513
26	44.311745	47.570645	51.113454	54.965979	59.156383
27	47.084214	50.711324	54.669126	58.989109	63.705766
28	49.967582	53.993333	58.402583	63.233510	68.528117
29	52.966286	57.423033	62.322712	67.711353	73.639798
30	56.084938	61.007069	66.438847	72.435478	79.058186
31	59.328335	64.752388	70.760790	77.419429	84.801677
32	62.701469	68.666245	75.298829	82.667498	90.889778
33	66.209527	72.756226	80.063771	88.224760	97.343165
34	69.857904	77.030256	85.066959	94.077122	104.183754
35	73.652225	81.496618	90.320307	100.251363	111.434780
36	77.598314	86.163966	95.836323	106.765188	119.120867
37	81.702246	91.041344	101.628139	113.637274	127.268118
38	85.970336	96.138205	107.709546	120.887324	135.904206
39	90.409150	101.464424	114.095023	128.536127	145.058458
40	95.025516	107.030329	120.799774	136.605146	154.761966

TABLE II. *Showing the present worth of One Dollar Annuity, from one Year to forty.*

Yr.	*4 per cent.*	*4½ per cent.*	*5 per cent.*	*5½ per cent.*	*6 per cent.*
1	0.96154	0.95694	0.95231	0.94786	0.94339
2	1.88609	1.87267	1.85941	1.81632	1.83339
3	2.77509	2.74876	2.72325	2.69798	2.67301
4	3.62989	3.58752	3.54295	3.50514	3.4651
5	4.45182	4.38997	4.32988	4.27028	4.21236
6	5.24214	5.15787	5.07569	4.99553	4.91732
7	6.40205	5.8927	5.78637	5.68297	5.58238
8	6.73274	6.59579	6.46321	6.33457	6.20979
9	7.43533	7.26879	7.10782	6.95220	6.80169
10	8.11089	7.91272	7.72173	7.53762	7.36008
11	8.76048	8.52892	8.3064	8.09254	7.88687
12	9.38500	9.11858	8.86325	8.61852	8.38384
13	9.98565	9.68285	9.39357	9.11708	8.85268
14	10.56312	10.22282	9.89864	9.58965	9.29498
15	11.41839	10.73954	10.37965	10.03759	9.71225
16	11.65229	11.23401	10.83777	10.46216	9.10589
17	12.16567	11.70719	11.27407	10.86461	10.47726
18	12.65929	12.15999	11.68958	11.24607	10.8276
19	13.13394	12.59329	12.08532	11.60765	11.15811
20	13.59032	13.00793	12.46221	11.95034	11.46992
21	14.02916	13.40472	12.82115	12.27524	11.76407
22	14.45111	13.78442	13.163	12.58317	12.04158
23	14.85684	14.14777	13.48857	12.87504	12.30338
24	15.24696	14.49548	13.79864	13.15170	12.55035
25	15.62208	14.82821	14.09394	13.41391	12.78335
26	15.98277	15.14661	14.37518	13.66250	13.00316
27	16.32959	15.45130	14.64303	13.89810	13.21052
28	16.66306	15.74287	14.89813	14.12142	13.40616
29	16.98371	16.02189	15.14107	14.33310	13.59072
30	17.29203	16.28889	15.37245	14.53375	13.76483
31	17.58849	16.54439	15.59281	14.72393	13.92908
32	17.87355	16.78889	15.80268	14.90420	14.08404
33	18.14764	17.02286	16.00255	15.07507	14.23023
34	18.41126	17.24676	16.1929	15.23703	14.36814
35	18.66461	17.46101	16.37419	15.39055	14.49825
36	18.90828	17.66604	16.54685	15.53607	14.62098
37	19.14258	17.86224	16.71129	15.67400	14.73678
38	19.36786	18.04999	16.86789	15.80474	14.84602
39	19.58448	18.22965	17.01704	15.92866	14.94907
40	19.79277	18.40158	17.15909	16.04612	14.92640

TABLE III.*

Rate per cent.	Half yearly payments.	Quarterly payments.
4	1.009902	1.014877
4½	1.011126	1.016720
5	1.012348	1.018559
5½	1.013567	1.020395
6	1.014781	1.022257

WHEN THE ANNUITY, time, and rate per cent. are given to find the amount.

RULE.—Take the tabular number in the column, under the rate per cent. and opposite to the time from table 1, and multiply it by the annuity; the product will be the amount. But,

If the payments are half yearly, or quarterly, multiply the amount found as above directed, for years, by the proper tabular amounts in table 3d; the product will be the true amount.

* Table first is constructed in the following manner: The interest, at the given rate per cent. is added to the annuity for the first year; which, with the annuity for the second year, becomes the principal for the second year. The interest arising on this amount, is added to the principal, together with the annuity for the third year, for the principal for the third year; and so on, for any number of years at pleasure. The table might, in this way, have been extended to any greater number of years, but these are sufficient for all practical purposes. The second table is constructed in the following manner: Divide the annuity by the ratio involved to the time of its continuance, and subtract the quotient from the annuity: Divide the remainder by the ratio less unit or 1, and the quotient will be the present worth, thus: Divide 1 by 1.04, the quotient is, .9615381; which subtract from 1, and the remainder will be .0384619, which divide by .04, and the quotient will be, 0.96154, the present worth of an annuity of 1 dollar for one year, as in the table opposite 1 year, and in the column under 4 per cent.; and so for any number of years, and at any rate per cent.

Table third is constructed in the following manner: Extract the square root of the ratio; and from the root, take unit or 1; by the remainder, divide the ratio less unit or 1; half the quotient will be the tabular value for half yearly payments, thus: $\sqrt{1.04}=1.019803902$; from which, take 1, and by the remainder .019801902, divide .04, the quotient will be 2.019804 nearly; half of which, will be the tabular value: to wit, 1.009902, as may be seen in the column of half yearly payments, and opposite 4 per cent. The tabular value of quarterly payments is found in a similar manner, only take the biquadrate root of the ratio, and proceed, as for half yearly payments, but observe to divide the quotient by 4.

EXAMPLES.

1. A young lady has an annual income of 500 dols., arising from ground rent; what will it amount to in 7 years, allowing 6 per cent. interest? Ans. $4196.93,85.

8.393837=amount of 1 dol. for 7 years at 6 per cent.
500

4196.91,35,00=the amount required.

2. A gentleman, at the age of 24 years, settled upon a salary of 800 dols. and by economy saved annually 250 dols., which he regularly invested in public stock, as well as the interest arising therefrom; what sum would the whole amount to in 26 years, or when he would be 50 years of age, allowing 6 cents interest?
Ans. $14789.09,575.

59.156383=tabular amount of 1 dollar.
250

$14789.095750=the amount required.

3. A gentleman was so well pleased at the birth of a daughter, that he determined annually, on her birth day, to invest the sum of 300 dols. at 5 per cent. as her portion of his estate, and also the interest arising thereform, until she should arrive at the age of 18 years; what did her portion amount to? Ans. $8439.71,55.

4. An officer, retired from service, has a pension of 450 dollars a year, which he invests in productive stock, as also the interest arising from it; to what sum will it amount in 15 years, allowing 5½ per cent. interest. Ans. $10083.89,83+.

5. What will an annuity of 30 dols., payable half yearly, amount to in 4 years, at 5 per cent.? Ans. $130.90,04.

Here find, in the first place, the amount for yearly payment as in the above examples, and multiply that amount by 1.012348, taken from the table for half yearly and quarterly payments; the product will be the true amount.

If the payments had been quarterly, you must have multiplied by, 1.018559; and in that case, the amount would have been $131.7035: hence it appears, that half yearly are more profitable than yearly, and quarterly more profitable than half yearly, payments.

This arises from the interest, which becoming due at short intervals, is then made a part of the principal, and draws interest also.

6. A gentleman, beside his family expenses, reserves the sum of 400 dollars semi-annually, and invests the same, as well as the interest arising therefrom, in public stocks; what sum would it amount to in 12 years, allowing the funds produced 6 per cent.?
Ans. $13695.45,733.

When the amount, rate, and time are given to find the annuity:

Rule.—Divide the amount by the number in the column under the ratio and opposite to the time: the quotient will be the annuity.

When the payments are half yearly or quarterly; divide the annuity, found as above, by the proper tabular amount for half yearly or quarterly payments.

EXAMPLES.

1. What annuity, being unpaid for seven years, will amount $4196.93,85, at 6 per cent.? Ans. 500.

4196.93,85÷8.393837=500.

2. What annuity will amount to $8439.71,55 in 18 years, at 5 per cent.? Ans. 300 dols.

3. What annuity being unpaid 4 years, will amount to $130.9004 at 5 per cent. payable half yearly? Ans. 30 dols.

Here divide the amount by 4.310125 the tabular amount of 1 dol. for 4 years at 5 per cent.; and the quotient thence arising by 1.012348, found in the table for quarterly payment, opposite the rate per cent.; this last quotient will be the true sum.

4. The rent of a house has remained unpaid for 6 years, and has amounted to the sum of $578.59,6,284, interest being 5½ per cent.; how much is the rent per annum? Ans. 84 dols.

When the annuity, time, and rate per cent. are given to find the present worth:

Rule.—Multiply the number in the column, table 2d, under the rate, and opposite the time, by the annuity; and the product will be the present worth.

When the payments are half yearly or quarterly, multiply the present worth, thus found, by the proper tabular amount for ½ yearly, or ¼ payments, table 3d, and the product will be the present worth.

EXAMPLES.

1. How much ready money will purchase an annuity of $415.81,25 to continue 4 years, allowing 6 per cent.? Ans. $1440.83,189375.

3.4651=present worth of 1 dol. for 4 years at 6 per cent.
415.8125=annuity.
1440.83,189375=the sum required.

2. How much ready money will purchase an annuity of 500 dols. to continue 7 years at 6 per cent.? Ans. $2791.19.

3. How much money will purchase an annuity of 450 dols. to continue 15 years, allowing 5½ per cent.? Ans. $4516.91,55.

4. What is the present value of an annuity of 20 dols., to continue 6 years, payable half yearly, at 5 per cent.?

Ans. $102.76,73.

5.07569=the present worth of 1 dol. for the time, and at the given rate.

20

101.51380=the present worth for yearly payments; which multiplied by 1.012348 taken from table 3d, the value of half yearly payments, will give $102.76,73 the present worth of an annuity payable half yearly for the time, and at the given rate. If the annuity had been payable quarterly, then the present worth for yearly payments must have been multiplied by 1.018559, taken from the same table in the column under quarterly payments. The product would be the present value of the annuity for quarterly payments. In that case the purchaser must pay $103.39,78. In like manner, the present value of any annuity, payable half yearly or quarterly, may be found.

When the present worth, time, and ratio are given to find the annuity.:

Rule.—Divide the present worth by the tabular value of 1 dollar, table 2d, taken from the column under the rate, and opposite to the time; and the quotient will be the annuity.

When the payments are half yearly or quarterly, divide the annuity as found for yearly payments, by the proper tabular value for half yearly or quarterly payments; table 3d.

EXAMPLES.

1. What annuity to continue for 4 years will purchase, allowing $5\frac{1}{2}$ per cent.? $2064.05,42661

Ans. $588.86,5.

2064.0542661÷3.50514=588.865 the annuity required.

2. A person has the sum of $2791.19, and wishes to purchase an annuity to continue 9 years, bearing 6 per cent. interest; how much can he purchase?

Ans. 500 dols.

2791.190÷5.58238=500, the annuity required.

3. A lady, having the sum of 5500 dollars in hand, wishes to purchase an annuity for her daughter, who is just 10 years of age, to continue till she shall arrive at the age of 25 years, bearing interest at the rate of 6 per cent.; how much can she purchase?

Ans. $566.29,3+.

5500.000÷9.71225=566.293+.

4. What annuity to continue 6 years, payable half yearly, may be purchased for $102.76,73, at 5 per cent.?

Ans. 20 dols.

102.7673000÷5.07569=20.246960, the annuity in yearly payments: which divided by 1.012348, taken from table 3d, in

the column under half yearly payments, and opposite to 5 per cent. gives 20 dollars for the annuity. In the same manner, proceed for quarterly payments.

ANNUITIES IN REVERSION.

An annuity is said to be in Reversion, when it is not to commence, till after a certain number of years.

When the annuity, time of reversion, time of continuance and ratio are given, to find the present worth :

Rule.—Divide the annuity by the ratio involved to the time of continuance, and subtract the quotient from the annuity, for a dividend : Multiply the ratio involved to the time of reversion, by the ratio, less 1, for a divisor ; the quotient thence arising will be the present worth.

Or, by the tables :

From table 2d, take the value of 1 dollar, in the column under the rate, and opposite the sum of the time of reversion and the continuance of the annuity. Also, take the value of 1 dollar from the same table, and opposite the time of reversion. Take the difference of these values, and multiply it by the annuity ; the product will be the present worth.

EXAMPLES.

1. What is the present worth of a pension of 96 dols. a year, to continue 7 years, at 6 per cent. ; but not to commence till after the expiration of 3 years ? Ans. $449.95,872

7.36008 = tabular value of 7+3=10 years.
2.67301 = tabular value of 3 years.
4.68707 = difference of tabular values.
96
449.95872 = the product, or present value.

2. A young man is to come to the possession of an annuity of 250 dols. a year, and to continue for 10 years, but not to commence till the expiration of 4 years, when he will be just 25 years of age ; but wanting ready money, he proposes to sell it, and allow 5½ per cent. to the purchaser ; what sum will he receive ?

Ans. $1521.12,75.

9.58965 = tabular value of 10+4=14 years.
3.50514 = tabular value of 4 years.
6.08451 = difference of tabular values.
250
1521.12750 = present value required.

3. What is the present value of an estate, worth 100 dols. a year, to continue 25 years, but not to commence till the expiration of 15 years, allowing 5 per cent. interest? Ans. $677.94,4.

4. A company of gentlemen propose to erect a splendid building for a public house, which will be in 3 years completed. They offer it on a lease for 10 years, at 1000 dols. a year; what is the present value of the lease at 6 per cent.? Ans. $6179.67.

5. A lady has a ground rent of 100 dollars a year, but which is so situated that she cannot come into possession till the expiration of 2 years: being in want of ready money, she proposes to sell it for 4 years, and allow at the rate of 5 per cent.; what sum must she receive for it? Ans. $321.77,723.

$\overline{1.05}|^4 = 1.215506$, the fourth power of the ratio, by the rule.

$\overline{1.05}|^2 = 1.102500$, the 2d power of the ratio, by the rule.

$100.000000 \div 1.215506 = 82.26203+$: which subtract from 100, the remainder is 17.73797. Divide this by the square of the ratio, multiplied by the ratio, less unit or 1; and the quotient will be the present worth, by the rule; thus: $1.102500 \times .05. = .055125$. By this divide 17.73797, and you will have $321.77,2+ the present value.

When the present worth, time, and ratio are given to find the annuity:

Rule.—Involve the ratio to a power, equal to the number of years from the time of purchase to the expiration of the annuity, for a dividend; and involve the ratio to the time of continuance, and take from the product unit or 1, for a divisor. Multiply the quotient thence arising by the present worth; and this product by the ratio, less unit or 1, and you have the annuity.

Or, by the tables: From table 2d, take the value of 1 dollar in the column under the rate, and opposite the sum of the time of reversion and the continuance of the annuity: also take the value of 1 dollar from the same table, and opposite the time of reversion; take the difference of these values, and divide the present worth by it; the quotient will be the annuity sought.

EXAMPLES.

1. What annuity to continue 7 years, but not to commence till after the expiration of 3 years, can be purchased for the sum of $449.95,872, allowing 6 per cent. interest? Ans. 96 dols.

7.36008 = value of 1 dol. for 7+3=10 years, table 2d.
2.67301 = value of 1 dol. for 3 years.

4.68707)449.95872(96.
4218363

2. What annuity to continue 25 years, but not to commence till the expiration of 15 years, may be purchased for the sum of $677.94,4, allowing 5 per cent. interest? Ans. 100 dols.

3. A gentleman purchased a lease of a house for ten years, but not to commence till the expiration of 3 years; for which he gave the sum of $6179.67, allowing 6 per cent. interest; what was the rent per annum? Ans. 1000 dols.

4. A farmer purchased a lease of some land, to continue 7 years, but not to commence till the expiration of 4 years; for which, he paid in hand $365.60,5; what was the annual rent, allowing 5 per cent. for his money? Ans. $76.8.

PERPETUITIES.

An annuity to continue for ever, is called a Perpetuity.

When the annuity and ratio are given, to find the present worth.

Rule. Divide the annuity by the ratio, less unit or 1. The quotient will be the present worth.

If the payments are quarterly, or half yearly, multiply the present worth for yearly payments, by the tabular value in table 3d; the product will be the present worth.

EXAMPLES.

1. What is the present worth of a perpetual annuity of 100 dollars, allowing 5 per cent. interest? Ans. 2000.

$100 \div 0.5 = 2000$ the present value.

2. What is the present value of a perpetual annuity of 250 dollars, allowing 6 per cent. interest? Ans. $4166.66,6.

3. What is the value of a perpetual annuity of 550 dollars, payable half yearly, at $5\frac{1}{2}$ per cent.? Ans. $10135.67.

$550.000 \div .055 = 1000$ present worth for yearly payments; which multiplied by 1.013567, the tabular value in table 3d, for half yearly payments, gives 10135.67 dollars, the present value required.

4. What sum may be given for the purchase of a freehold estate of the annual income of 300 dollars, payable quarterly, allowing 6 per cent. interest? Ans. $5111.28,5.

$300.00 \div .06 = 5000 \times 1.022257 = 5111.28,5,000.$

When the present worth and ratio are given to find the annuity.

Rule. Multiply the present worth by the ratio, less 1; the product will be the annuity.

EXAMPLES.

1. A gentleman purchased a freehold estate for the sum of 5000 dollars ready money, allowing 5 per cent. interest; what is the annual income? Ans. $250.

2. A lady of fortune, having 10000 dollars ready money, wishes to purchase a perpetual annuity; what annuity can she purchase with the above sum, allowing 6 per cent. interest? Ans. $600.

3. A young gentleman has a ground rent of a certain annual value; but wishing to go into business, offers to sell it for the sum of 6540 dollars, and allow $5\frac{1}{2}$ per cent. interest; what is the annuity? Ans. $359.70.

When the present worth and annuity are given to find the ratio:

RULE. Add the present worth and annuity together, and divide their sum by the present worth; the quotient will be the ratio.

EXAMPLES.

1. A freehold estate of the annual value of 250 dollars, can be purchased for 5000 dollars; what is the rate per cent. allowed?

Ans. 1.05 the ratio.

5000=the present worth.
250=the annual value on annuity.
$\overline{5250} \div 5000 = 1.05$.

2. A gentleman bought a ground rent of 550 dollars, for the sum of 16450 dollars; what is the rate per cent.?

Ans. 1.03,343 nearly.

3. A person, for the sum of 6540 dollars, purchased an annuity of $359.70; what is the rate per cent.? Ans. $5\frac{1}{2}$.

PERPETUITIES IN REVERSION.

When the perpetuity, time of reversion, and rate per cent., are given to find the present worth.

RULE. Multiply the ratio, involved to the time of reversion by the ratio, less unit or 1, for a divisor: by this divide the annuity; the quotient will be the present worth.

EXAMPLES.

1. What is the value of a perpetuity of 250 dollars, to commence 4 years hence, allowing 6 per cent. for present payment?

Ans. $3300.39,042+

$\overline{1.06}|^4 = 1.2624769 \times .06 = .075748614$. By which divide $250, the annuity, and the quotient will be, 3300.39,042+

2. A gentleman has a perpetuity of 696 dollars; but not to commence till the expiration of 4 years; which he wishes to sell,

allowing 4 per cent. to the purchaser; what is it worth on those terms? Ans. $14873.59,5.

3. A young lady received at the decease of her father, a ground rent of 200 dollars annual income, as her part of his estate; but it was so situated, that she could not come into possession of it, till after the expiration of 2 years; what is it worth in ready money, allowing $5\frac{1}{2}$ per cent.? Ans. $3267.09,9+

$\overline{1.055}|^2 = 1.113025 \times .055 = .0612163750$; by which divide the annuity, 200 dollars, and you have 3267.099+the present value of the perpetuity required.

When the present worth of the perpetuity, the time of reversion, and the rate per cent., are given to find the annuity:

Rule. Multiply the present worth, the ratio, less 1, and the ratio involved to the time of reversion together; the product will be the annuity.

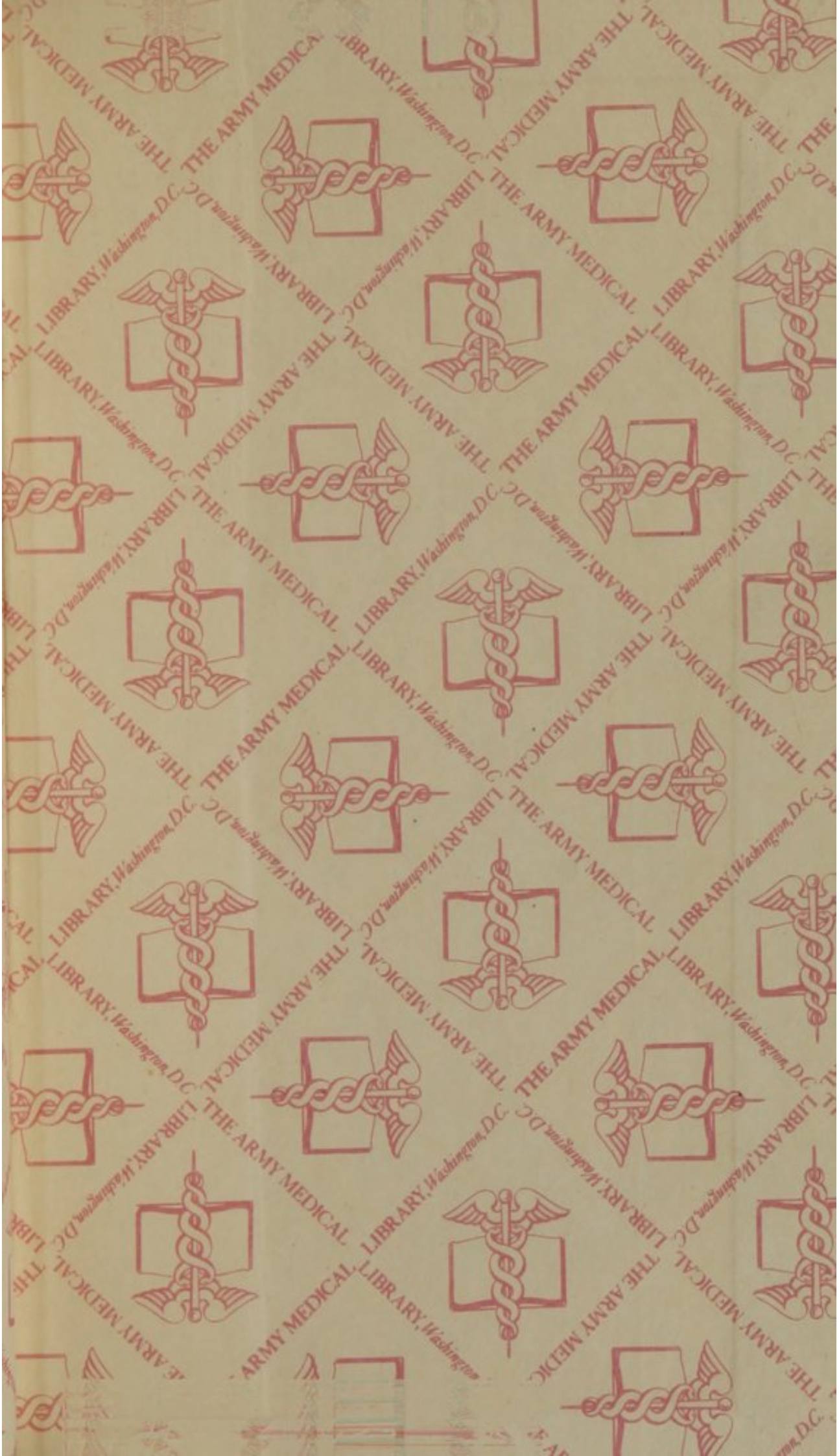
EXAMPLES.

1. The present worth of a perpetuity is $889.99,65, which does not commence until the expiration of 2 years, allowing 6 per cent.; what is the annuity? Ans. $60.

$\overline{1.06}|^2 = 1.1236 \times .06 = 067416$; which multiplied by the present worth, will give the answer.

2. A gentleman paid the sum of $3300.39,042+ for the purchase of a perpetuity, not to commence until the expiration of 4 years; what is the annuity, allowing 6 per cent.? Ans. $250.

THE END.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