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A SYNOPSIS

OF

Practical Philosophy,

ALPHABETICALLY ARRANGED,

CONTAINING

A GREAT VARIETY

OF

THEOREMS, FORMULÆ, AND TABLES,

FROM THE MOST ACCURATE AND RECENT AUTHORITIES,

IN VARIOUS BRANCHES OF

MATHEMATICS AND NATURAL PHILOSOPHY;

TO WHICH ARE SUBJOINED SMALL

TABLES OF LOGARITHMS.

DESIGNED AS A

MANUAL FOR ARCHITECTS, SURVEYORS, ENGINEERS, STUDENTS, NAVAL OFFICERS, AND OTHER SCIENTIFIC MEN.

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1843.

TO

JOHN MACNEILL, ESQ.,

CIVIL ENGINEER,

F. R. S., &c.

THE RE-PUBLICATION OF

THIS VERY USEFUL LITTLE WORK

IS INSCRIBED

BY HIS VERY HUMBLE SERVANT,

JOHN WEALE.

Dec. 31, 1842.

INTRODUCTION.

This small volume is intended, as its title page imports, partly as a Manual for the scientific man, to aid him in his researches, when, from his distance from home, or other circumstances, he is precluded from having access to more extended and elaborate works; and partly as a convenient appendage to the table of the general reader, for purposes of occasional reference; while to the Student it will supply the place of a syllabus, and furnish him with formulæ for the solution of problems in many useful branches of mixed Mathematics.

With respect to its plan, the reader, on turning to any article, will usually find entered first the Propositions or Formulæ applicable to it, illustrated, if necessary, by examples; to which are appended, such practical results and tables as the subject appeared to require, or the limits of the book to admit of.

The Propositions are very rarely accompanied by proofs; nor is any explanation given of the various terms employed, further than what is necessary to a due understanding of the several symbols introduced. The book professing merely to supply a combination of *facts*, calculated to aid the memory, or exercise the ingenuity, of the reader, any attempt at elementary instruction would have been altogether inconsistent with its scope and principle.

Most of the articles have been compiled and abridged from original sources, as will appear from referring to their several heads, where the names of the respective writers, from whom the extracts have been made, are usually inserted; and particular care has been taken throughout to admit nothing of a practical nature which has not been sanctioned by unexceptionable authorities: at all events, since, in every case which admits not of rigid demonstration, the authority has been most scrupulously quoted, the intelligent reader will at once be able to judge what degree of confidence it is entitled to.

The small Tables of Logarithms will probably be considered a valuable addition: by the help of these, any one, having the proper data, may exhibit arithmetically such formulæ as require a logarithmic computation with sufficient accuracy for all *temporary* purposes.

Some subjects, which, from their practical utility, might seem to claim a place in this Synopsis, have, in cases where long verbal descriptions or an expensive apparatus of plates were necessary for their illustration, been purposely omitted; it having been a leading object in the compilation to confine the volume within such limits as might render it conveniently portable. Other omissions doubtless there are, which may have proceeded from inadvertence, or a want of judgment in the selection; but these last will not, it is hoped, be found very considerable, either in point of number or importance.

To typographical accuracy every possible attention has been paid; without that, a book of this kind would be worse than useless.

A SYNOPSIS

OF

PRACTICAL PHILOSOPHY.

ABERRATION of *Light.*—(*Woodhouse, Vince.*)

1. If two lines be drawn from the earth, one in the direction of its motion, which will be a tangent to its orbit, and the other through the star, the angle they form is called the ∠ of the *earth's way;* and the aberration will wholly take place in the plane passing through these two lines; which is ∴ called the *plane of aberration.*

2. The greatest effect of aberration $= 20''.232$, or in round numbers $20''$; and generally the aberration in its own plane $= 20'' \times$ sin. of the ∠ of the earth's way.

The velocity of the earth : the velocity of light :: sin. $20''$: rad. :: 1 : 10324.

3. This aberration will affect the apparent position of the stars both in latitude, and longitude; declination, and right ascension. Hence the following Formulæ:—

Aberration in Latitude.

Aberrat. in lat. $= o$, when the earth is in syzygy with the star.

In any other position of the earth, aberration in lat. is

$20'' \times$ sin. of earth's distance from syzygy $\times$ sin. star's lat.

Hence the aberration in lat. is a max. when the earth is in quadratures with the star, and then $= 20'' \times$ sin. star's lat.

Aberration in Longitude.

Aberrat. in long. $= o$, when the earth is in quadratures with the star.

In any other position of the earth, aberration in longitude is

$$\frac{20'' \times \text{cos. earth's distance from syzygy}}{\text{cos. star's lat.}}$$

Hence aberrat. in long. is a max. when the earth is in syzygy with the star, and $= \frac{20''}{\text{cos. star's lat.}}$.

Aberration in Declination.

Aberration in declination $= o$, when tang. earth's dist. from syzygy $= \frac{\text{tan. position}}{\text{sin. star's lat.}}$.

In any other position of the earth, let $d =$ dist. of the earth, at the time of observation, from the position it had when aberrat. in declin. $= o$, D $=$ earth's dist. from syzygy at the same time, found by the last Article, then aberrat. in declination is

$$\frac{20'' \times \text{sin. } d \times \text{sin. position}}{\text{sin. D}}$$

Hence aberration in declination is a max^{m}. when $d = 90^{0}$. and then $= \frac{20'' \times \text{sin. position}}{\text{sin. D}}$.

Aberration in Right Ascension.

Aberration in right ascension $= o$, when the tang. earth's distance from syzygy $= \frac{\text{cotan. position}}{\text{sin. star's lat.}}$.

In any other position of the earth, let $d =$ dist. of the earth, at the time of observation, from the position it had when aberration in right ascension $= o$, D $=$ earth's distance from syzygy at the same time, found by last Art.; then aberrat. in right ascension is

$$20''. \frac{\text{sin. } d \times \text{cos. position}}{\text{cos. decl.} \times \text{sin. D}}.$$

Hence aberrat. in right ascension is a max. when $d = 90^{0}$, and $= 20''. \frac{\text{cos. posit.}}{\text{cos. dec.} \times \text{sin. D}}$.

4. The following are the Formulæ given by M. Cagnoli, in his Trigonometry, as being the most convenient for practice, and from which M. de Lambre has computed his Tables on Aberration.—*(See Vince & Playfair.)*

If L be the longitude of the sun at any time, and L′ the longitude of a star, the aberration of the star in lat. is

2

$$20''. 232 \times \text{sin. } (L' - L) \times \text{sin. lat.}$$

And the aberration in longitude is

$$\frac{-20''.232 \times \cos.(L'-L)}{\cos.\ lat.}.$$

If A be the right ascension, and D the declination of a star, L being the sun's longitude as before, the aberration in declination is

$$\sin.\ D\left(19''.17 \sin.(A-L) - o''.83 \sin.(A+L)\right) - 8'' \cos.\ L \times \cos.\ D.$$

And the aberration in right ascension is

$$-\frac{19''17 \times \cos.(A-L) - o''.83 \times \cos.(A+L)}{\cos.\ D}.$$

From these four last Formulæ all the effects of aberration may be computed.

5. In consequence of the aberration of light, the apparent place of a star will trace out upon a plane parallel to the ecliptic a circle, in which the true place of the star is similar to that of the sun in the circle described on the axis major of the earth's orbit as a diameter.

This circle, projected upon the plane of vision, is an ellipse, the $\frac{1}{2}$ ax. maj. $= 20''232$, and $\frac{1}{2}$ ax. min. $= 20''232 \times$ sin. star's lat. Hence a star in the pole of the ecliptic describes a circle, and a star in the ecliptic a straight line.

6. To make allowance for the aberration of a planet, let T be the instant for which the geocentric place is to be computed, $t =$ time light takes to move from the planet to the earth. Compute its geocentric place by the common rules for the time $T-t$, and it will be its geocentric place at the time T, corrected for aberration.

The aberration of the sun in longitude always $= 20''$, that being the space moved through by the sun or earth in $8'.\ 7\frac{1}{2}''$, which is the time in which light passes from the sun to the earth.

7. *Aberratic Curve.—(Wright's sol. Camb. Prob.)*

Let y and p denote the rad. vect. and perpendicular upon the tangent of the given orbit; y' and p' the corresponding ones to the aberratic curve. Also let $c =$ twice area described dat. tem.; then

$$p' = \frac{c}{y};$$

$$\&\ y' = \frac{c}{p}.$$

These two equations will give the equation to the aberratic curve.

Ex. 1. Let the given orbit be a parabola; then $p^2 = \frac{Ly}{4}$ (L = lat. rect.) $\therefore y' = \frac{c}{p} = \frac{2c}{\sqrt{Ly}} = 2\sqrt{\frac{cp'}{L}}$, $\therefore p' = \frac{L}{4c}\ y'^2$, but $p =$

$\frac{y^2}{2r}$ is an equation to the circle when the centre of the polar coordinates is in the circumference *(see Circle Equation to)*, ∴ the aberratic curve is a circle, whose rad. is $\frac{2c}{L}$.

2. Let the orbit be an ellipse, whose equation is $p^2 = \frac{b^2 y}{2a - y}$; Then

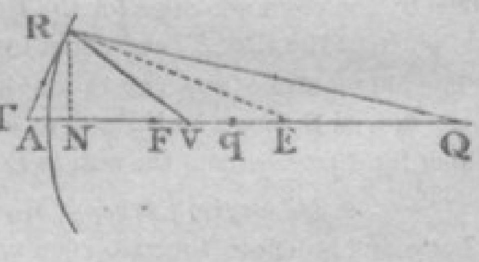
$$y' = \frac{c}{p} = \frac{c}{b}\sqrt{\frac{2a-y}{y}} = \frac{c}{b}\sqrt{\frac{2a}{c}p' - 1}, \therefore p' = \frac{y'^2 + \frac{c^2}{b^2}}{\frac{2ac}{b^2}}; \text{ so}$$

that the aberratic curve *(see Circle Equation to)* is a circle, whose radius is $\frac{ac}{b^2}$, and the distance of whose centre from the centre of coordinates is $\frac{c}{b}\sqrt{a^2 - b^2}$.

3. Let the orbit be the log. spiral, whose equation is $p = my$, then $y' = \frac{c}{p} = \frac{cm}{y} = \frac{cm}{\frac{c}{p'}} = mp'$, $\therefore p' = \frac{y'}{m}$; ∴ the aberratic curve is also a log. spiral.

ABERRATION in *Optics*.—*(Coddington.)*

I. *Aberration in reflection at spherical surfaces.*



Let $EQ = q$, $Eq = q'$, $EF = f$, $AN = v$ and the point v in the figure being the actual intersection of the reflected ray and axis, let $Ev = q'$, then $q' = \frac{qf}{q+f} + \frac{q^2 fv}{(q+f)^2} + \frac{q^3 fv^2}{(q+f)^3} + \&c.$

This in geometrical terms is equivalent to

$$Eq + \frac{QE^2}{QF^2} \cdot \frac{AN}{2} + \frac{QE^3}{QF^3} \cdot \frac{AN^2}{4EF} + \&c.$$

& ∴ $q' - q'$, or aberration in longitude, is $\frac{QE^2}{QF^2} \cdot \frac{AN}{2} + \frac{QE^3}{QF^3} \cdot \frac{AN^2}{4EF}$ &c. or, because v is small, is

$$\frac{QE^2}{QF^2} \cdot \frac{AN}{2}; \text{ or } \frac{QE^2}{QF^2} \cdot \frac{AT}{2} \text{ nearly.}$$

Cor. When Q E and Q F are given, aberration varies as A N varies as R N^2 nearly.

The following series for aberration is a little different from the preceding; but amounts nearly to the same thing; putting $\theta = \angle$ R E A.

$$\text{Aberrat.} = \frac{q^2}{(q+f)^2} f\ (\sec. \theta - 1) - \frac{q^2}{(q+f)^2} f^2\ (\sec. \theta - 1)^2 + \&c.$$

Cor. When the incident rays are parallel, aberration $= \frac{1}{2}$ A T

II. *Aberration in refraction at spherical surfaces.*

Let Δ & Δ' be the perpendicular distances of Q and q from the refracting surface, m the ratio of the sine of incidence : sine of refraction, $v =$ ver. sin. A N (see preceding figure ;) then

$$\text{Aberrat.} = (\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) v$$

$$\text{or} = (\Delta' - r)^2 \times \frac{m-1}{m} \left(\frac{m+1}{\Delta} - \frac{1}{r}\right) v,$$

& is $\therefore$ positive, if Δ be less than $(m+1)\,r$, & negative when Δ is above that value. When $\Delta = (m+1)\,r$, there is no aberration.

When the incident rays are parallel, or $\frac{1}{\Delta} = o$, this reduces to

$-\frac{(\Delta' - r)^2}{\Delta'}\, v$; or if F be the principal focal distance, it is

$$-\frac{(F - r)^2}{F}\, v,$$

III. *Aberration in a lens.*

We may consider this as consisting of two parts :—

(1.) The variation in the second focal distance arising from the aberration in the first (α.)

(2.) The additional aberration in the refraction at the second surface (β).

Let Δ'' be the distance of the focus after the 2d refraction, the rest as before ; then

$$\alpha = \frac{m\,\Delta''^2}{\Delta'^2}\ (\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) v.$$

For the 2d part we must alter our formula, by putting $\frac{1}{m}$ for m, v' for v, Δ' for Δ, Δ'' for Δ', r' for r ;

$$\beta = (\Delta'' - r')^2 \left(\frac{1}{m\,\Delta'} - \frac{1}{\Delta''}\right) v'$$

The whole aberration is therefore

$$\frac{m\,\Delta''^2}{\Delta'^2}(\Delta-r)^2\left(\frac{m}{\Delta}-\frac{1}{\Delta'}\right)v+(\Delta''-r')^2\left(\frac{1}{m\,\Delta'}-\frac{1}{\Delta''}\right)v'.$$

The aberration for a particular value of Δ varies as v, or as the *square of the radius of the aperture* nearly.

Let us examine what kinds of value the aberratiou in a lens assumes in different cases.

(1.) For the meniscus or concavo-convex lens, (r & r' being both positive.)

The aberration

$$A=\left\{m\,.\,\frac{\Delta''^2}{\Delta'^2}\,.\,(\Delta'-r)^2\left(\frac{m}{\Delta}-\frac{1}{\Delta'}\right)+(\Delta''-r')^2\left(\frac{1}{m\Delta'}-\frac{1}{\Delta''}\right)\right\}v.$$

(2.) For the double concave lens r' is negative,

$$A=\left\{m\,.\,\frac{\Delta''^2}{\Delta'^2}\,.\,(\Delta'-r)^2\left(\frac{m}{\Delta}-\frac{1}{\Delta'}\right)+(\Delta''+r')^2\left(\frac{1}{m\Delta'}-\frac{1}{\Delta''}\right)\right\}v.$$

(3.) For the double convex lens r is negative,

$$A=\left\{m\,.\,\frac{\Delta''^2}{\Delta'^2}\quad(\Delta'+r)^2\left(\frac{m}{\Delta}-\frac{1}{\Delta'}\right)+(\Delta''-r')^2\left(\frac{1}{m\Delta'}-\frac{1}{\Delta''}\right)\right\}v$$

To find the least circle of aberration into which all the homogeneal rays of the same pencil, refracted by a lens, are collected.

Let $a = \frac{1}{2}$ aperture of the lens, $b =$ distance of the point where the extreme ray cuts the axis from the focus of refracted rays, $c =$ distance of the same point from the lens, $x =$ rad. of the least circle of aberration, then

$$x=\frac{a\times b}{4\,c}$$

Cor. If the focal length of the refractor, and the focus of incidence be given, c is given and x varies as $a\,b$ varies as a^3; and on the same supposition the area of the least circle of aberration varies as a^6.

ACCELERATION of *Falling Bodies.—See Motion.*

ACCELERATION of *Stars on Mean Solar Time.—See Time.*

ACCELERATION of the *Moon.—See Moon.*

ADHESION, a term chiefly used to denote the force, with which the surface of a solid remains attached to the surface of a liquid, after they have been brought into contact.

In the year 1773, Guyton-Morveau ascertained experimentally the force of adhesion of eleven different metals to mercury. The surface of each metal was an inch (French) in diameter and polished. The following Table exhibits the weight in French grains necessary to separate each metal from the mercury.

Gold	446	Zinc	204
Silver	429	Copper	142
Tin	418	Antimony	126
Lead	397	Iron	115
Bismuth	372	Cobalt	8
Platinum	282		

ÆRAS, list of the most remarkable:—

	Julian Period.	*B.C.*
Creation of the world	706	4007
Deluge	2362	2351
Olympiads of the Greeks	3937	776
Rome built, or Roman æra	3961	752
Æra of Nabonassar of Chaldæns and Egyptians	3967	746
Death of Alexander	4390	323
Æra of the Seleucidæ	4401	312
First of Julius Cæsar	4669	44
Vulgar æra of Christ's birth	4713	*A.C.*
Hegira, Mahometan æra	5335	622
Yesdegird, Persian æra	5344	631

ÆRONAUTICS.

To calculate the height to which a balloon will ascend, under given circumstances.—*(Wright's solut. Camb. Prob.)*

Let W = weight of the balloon, and all its appendages in ounces, D = density of mercury at the time, δ the spec. grav. of the atmosphere at the surface of the earth, when the barometer stands at b feet, and $\frac{\delta}{n}$ that of the gas; c^3 the capacity of the balloon in cubic feet, x = height to which it will ascend in feet; then

$$x = \frac{b\,D}{\delta} \times \log. \frac{n\,\delta\,c^3}{n\,W + \delta\,c^3}.$$

Cor. If the gas be hydrogen or $n = 13$, $b = 30$ inches $= \frac{5}{2}$ feet, D = 14019 (density of water being 1000), and $\delta = \frac{6}{5}$, then

$$x = 42057 \times \log. \frac{78\,c^3}{65\,W + 6\,c^3}.$$

Ex. Given W = 20 stone, and the other elements as in the Cor. to determine the magnitude of the balloon necessary just to lift that weight from the ground.

$$\text{Here } x = o, \therefore \frac{78\,c^3}{65 \times 4480 + 6\,c^3} = 1$$

$\therefore c^3 = 4014$ cubic feet.

Short historical notice :—

October 15, 1783. M. Pilatre de Rozier was the first person who ever ascended in a balloon; it was inflated with heated air. He perished in a subsequent ascent, being the first who did so.

December 1st, 1783. M. M. Roberts and Charles first ascended with an hydrogen gas balloon.

September 15th, 1784. The first aerial voyage in England performed by Lunardi.

Jan. 7, 1785. M. Blanchard and Dr Jeffries passed from Dover to Calais.

August, 1785. Blanchard in one of his excursions from Lisle, traversed a distance of more than 300 miles without halting.

Sept. 21, 1802. Garnerin first descended in a parachute from London.

September 15, 1804. Gay Lussac ascended from Paris for scientific purposes, and rose to the enormous height of 22,912 feet; or 23,040, *i. e.* more than 4¼ miles above the level of the sea; being 1600 feet above the summit of the Andes; the barometer sunk to 12,95 inches. From this last ascent two results were obtained; (1) that the intensity of the magnetic power continues the same at all accessible distances from the earth's surface: and (2) that the proportions of oxygen and nitrogen, which constitute the atmosphere, do not vary sensibly in the most extended limits.

AIR *Atmospheric.—See Atmosphere.*

AIR *Pump.—See Pump.*

ANGULAR *Velocity.—See Central Forces.*

ANIMAL *Strength.—(Playfair.)*

1. The strength of men, and of all animals, is most powerful when directed against a resistance that is at rest: when the resistance is overcome, and when the animal is in motion, its force is diminished; lastly, with a certain velocity the animal can do no work, and can only keep up the motion of its own body.

2. A formula, having the three properties just mentioned, will afford an approximation to the law of animal force. Let P be the weight which the animal exerting itself to the utmost, or at a *dead pull*, is just able to overcome, W any other weight with which it is actually loaded, and v the velocity with which it moves when so loaded; c the velocity at which the power of drawing or carrying a load entirely ceases; then, till experience has led to a more accurate result, we may suppose the strength of animals to follow the law expressed by the formula

$$W = P\left(1 - \frac{v}{c}\right)^2.$$

This is Euler's Formula.

Cor. Hence the effect of animal force, or the quantity of work done in a given time, will be proportional to W v, or to P $v\left(1-\frac{v}{c}\right)^2$, and will be a maximum when $v=\frac{c}{3}$, and $W=\frac{4P}{9}$, *i. e.* when the animal moves with one-third of the speed with which it is able only to move itself, and is loaded with $\frac{4}{9}$ of the greatest load it is able to put in motion.

3. The quantities P and c can only be determined by experience. Euler supposes that for the work of men, P may upon an average be taken $=60$lb, and $c=6$ feet per second, or a little more than four miles an hour.

4. A man, according to this estimate, when working to the greatest advantage, should carry a load of 27lb, and walk at the rate of two feet in a second, or a mile and one-third an hour.

5. A horse, according to Desaguliers, drawing a weight out of a well, over a pulley, can raise 200lb. for eight hours together, at the rate of $2\frac{1}{2}$ miles an hour. Supposing in this case the horse to work to the greatest advantage, $P=450$, and $c=6\frac{3}{4}$ miles per hour. This estimate, however, seems to give too high a value to P. It will suit better with general experience to make $P=420$ and $c=7$.

6. It appears from Cavallo, that a horse can draw 25 cwt. on a level road in a cart weighing 10 cwt., with wheels six feet high. In a common cart, two horses easily draw 30 cwt. In a common waggon, six horses draw 80 cwt.: in three carts they might draw 90; in six, 150 cwt.: and three carts cost less than a waggon. A horse drew three tons up a railway rising 7 inches in 144: the draught was 327 pounds besides friction.—*(Young's Nat. Phil.)*

7. According to Coulomb's experiments on this subject, if w be the weight of the man's body, l an additional load, which he is made to carry, H the height to which he ascends in a given time, when walking freely, and h the height to which he ascends in the same time with the load l; then

$$h=\frac{wH\left(1-\frac{l}{2w}\right)}{w+l}.$$

Cor. 1. When $l=2w$, $L=o$.

Cor. 2. The greatest effect of a man's strength in raising a weight will be, when the weight of the man is to that of his load as $1:-1+\sqrt{3}$, or nearly as $4:3$.

8. The following experiments by Peron, with Regnier's dynamometer, shew that the strength of men depends considerably on the climate.

English	71,4	Van Diemen's Land	51,8
French	69,2	New Holland	50,6
Timor	58,7		

(Ency. Brit. Supplt.)

ANNUAL *Equation.—See Moon.*

ANNUITIES.

1. *Annuities at Simple Interest.*

1. Let A be the annuity, r the interest of £1. for one year, M the amount of the annuity for n years, then

$$M = n\,A + n.\ \frac{n-1}{2}.\ r\,A.$$

2. Let P be the present value of an annuity to continue for n years; the rest as before, then

$$P = \frac{n\,A + n.\ \frac{n-1}{2}.\ r\,A.}{1 + n\,r}$$

In these Equations any one of the quantities may be found, the rest being given.

Annuities at Compound Interest.

3. Let R = £1. and its interest for one year, the rest as before, then

$$M = \frac{R^n - 1}{R - 1} \times A.$$

$$\&\ P = \frac{1 - \frac{1}{R^n}}{R - 1} \times A$$

If the No. of years be infinite,

$$P = \frac{A}{R - 1},$$

which gives the value of a freehold estate, A being the annual rent.

4. The present value of an annuity to commence at the expiration of p years, and to continue q years, is the difference between its present value for $p + q$ years, and its present value for p years,

$$\text{or } P = \left(\frac{1}{R^p} - \frac{1}{R^{p+q}}\right) \times \frac{A}{R - 1}.$$

If the annuity, instead of being payable annually, be made payable half-yearly, quarterly, or at any other given interval, the above formulæ are still applicable, by calling R £1. and its interest for the given interval, and n the number of those intervals.

TABLE I.

Showing the amount of an Annuity of £1. for any number of years, not exceeding fifty; and for the different rates of interest from 3 to 6 per Cent.—(Encyc. Metrop.)

No. of Years.	3 perCent.	3½ perCent.	4 perCent.	4½ perCent.	5 perCent.	6 perCent.
1	1·00000	1.00000	1·00000	1·00000	1·00000	1·00000
2	2·03000	2·03500	2·04000	2·04500	2·05000	2·06000
3	3·09090	3·10622	3·12160	3·13702	3·15250	3·18360
4	4·18362	4·21494	4·24646	4·27819	4·31012	4·37461
5	5·30913	5·36246	5·41632	5·47070	5·52563	5·63709
6	6·46840	6·55015	6·63297	6·71689	6·80191	6·97531
7	7·66246	7·77940	7·89829	8·01915	8·14200	8·39383
8	8·89233	9·05168	9·21422	9·38001	9·54910	9·89746
9	10·15910	10·36849	10·58279	10·80211	11·02656	11·49131
10	11·46387	11·73139	12·00610	12·28820	12·57789	13·18079
11	12·80779	13·14199	13·48635	13·84117	14·20678	14·97164
12	14·19202	14·60196	15·02580	15.46403	15·91712	16·86994
13	15·61779	16·11303	16·62683	17·15991	17·71298	18·88213
14	17·08632	17.67698	18·29191	18·93210	19·59863	21·01506
15	18·59891	19·29568	20·02358	20·78405	21·57856	23·27596
16	20·15688	20·97102	21·82453	22·71933	23·65749	25·67252
17	21·76158	22·70501	23·69751	24·74170	25·84036	28·21287
18	23·41443	24·49969	25·64541	26·85508	28·13238	30·90565
19	25·11686	26·35718	27·67122	29·06356	30·53900	33·75999
20	26·87037	28·27968	29·77807	31·37142	33·06595	36·78559
21	28·67648	30·26947	31·96920	33·78313	35·71925	39·99272
22	30·53678	32·32890	34·24796	36·30337	38·50521	43·39229
23	32·45288	34·46041	36·61788	38·93702	41·43047	46·99582
24	34·42647	36·66652	39·08260	41·68919	44·50199	50·81557
25	36·45926	38·94985	41·64590	44·56521	47·72709	54·86451
26	38·55304	41·31310	44·31174	47·57065	51·11345	59·15638
27	40·70963	43·75906	47·08421	50·71132	54·66912	63·70576
28	42·93092	46·29062	49·96758	53·99333	58·40258	68·52811
29	45·21885	48·91079	52·96628	57·42303	62·32271	73·63979
30	47·57541	51·62267	56·08493	61·00706	66·43884	79·05818
31	50·00267	54·42947	59·32833	64·75238	70·76078	84·80167
32	52·50275	57·33450	62·70146	68·66624	75·29882	90·88977
33	55·07784	60·34121	66·20952	72·75620	80.06377	97·34316
34	57·73017	63·45315	69·85790	77·03025	85·06695	104·18375
35	60·46201	66·67401	73·65222	81·49661	90·32030	111·43477
36	63·27594	70·00760	77·59831	86·16396	95·83632	119·12086
37	66·17422	73·45786	81·70224	91·04134	101·62813	127·26811
38	69·15944	77.02889	85·97033	96·13820	107·70954	135·90420
39	72.23423	80·72490	90·40914	101·46442	114·09502	145·05845
40	75.40125	84·55027	95·02551	107·03032	120·79977	154·76196
41	78·66329	88·50953	99·82653	112·84668	127·83976	165·04768
42	82·02319	92·60737	104·81959	118·92478	135·23175	175·95054
43	85·48389	96.84862	110·01238	125·27640	142·99333	187·50757
44	89·04840	101·23833	115·41287	131·91384	151·14300	199·75803
45	92·71986	105·78167	121·02939	138·84996	159·70015	212·74351
46	96·50145	110·48403	126·87056	146·09821	168·68516	226·50812
47	100·39650	115·35097	132·94539	153·67263	178·11942	241·09861
48	104·40839	120·38825	139·26320	161·58790	188·02539	256·56452
49	108·54064	125·60184	145·83373	169·85935	198·42666	272·95840
50	112·79686	130·99791	152·60708	178·50302	209·34799	290·33590

TABLE II.

Showing the present value of an Annuity of £1. per annum, for any number of years, not exceeding fifty, and at different rates of interest, from 3 to 6 per cent.—(Encyc. Metrop.)

No. of Years.	3 perCent.	3½ perCent.	4 perCent.	4½ perCent.	5 perCent.	6 perCent.
1	·97887	·96618	·96153	·95693	·95238	·94339
2	1·91346	1·89969	1·88609	1·87266	1·85941	1·83339
3	2·82861	2·80163	2·77509	2·74896	2·72324	2·67301
4	3·71709	3·67307	3·62989	3·58752	3·54595	3·46510
5	4·57970	4·51505	4·45182	4·38997	4·32947	4·21236
6	5·41719	5·32855	5·24213	5·15787	5·07569	4·91732
7	6·23028	6·11454	6·00205	5·89270	5·78637	5·58238
8	7·01969	6·87395	6·73274	6·59588	6·46321	6·20979
9	7·78610	7·60768	7·43533	7·26879	7·10782	6·80169
10	8·53020	8·31660	8·11089	7·91271	7·72173	7·36008
11	9·25262	9·00155	8·76047	8·52891	8·30641	7·88687
12	9·95400	9·66333	9·38507	9·11858	8·86325	8·38384
13	10·63495	10·30273	9·98564	9·68285	9·39357	8·85268
14	11·29607	10·92052	10·56312	10·22282	9·89864	9·29498
15	11·93793	11·51741	11·11838	10·73954	10·37965	9·71224
16	12·56110	12·09416	11·65229	11·23401	10·83776	10·10589
17	13·16611	12·65132	12·16566	11·70719	11·27406	10·47725
18	13·75351	13·18968	12·65929	12·15999	11·68958	10·82760
19	14·32379	13·70983	13·13393	13·59329	12·08532	11·15811
20	14·87747	14·21240	13·59032	13·00793	12·46221	11·46992
21	15·41502	14·69794	14·02915	13·40472	12·82115	11·76407
22	15·93691	15·16712	14·45111	13·78442	13·16300	12·04158
23	16·44360	15·62041	14·85684	14·14777	13·48857	12·30337
24	16·93554	16·05836	15·24696	14·49547	13·79864	12·55035
25	17·41314	16·48151	15·62207	14·82820	14·09394	12·78335
26	17·87684	16·89035	15·98276	15·14661	14·37518	13·00316
27	18·32703	17·28536	16·32958	15·45130	14·64303	13·21053
28	18·76410	17·66701	16·66306	15·74287	14·89812	13·40616
29	19·18845	18·03576	16·98371	16·02188	15·14107	13·59072
30	19·60044	18·39204	17·29203	16·28888	15·37245	13·76483
31	20·00042	18·73627	17·58849	16·54439	15·59281	13·92908
32	20·38876	19·06886	17·87355	16·78889	15·80267	14·08404
33	20·76579	19·39020	18·14764	17·02286	16·00254	14·23022
34	21·13183	19·70068	18·41119	17·24675	16·19290	14·36814
35	21·48722	20·00066	18·66461	17·46101	16·37419	14·49824
36	21·83225	20·29049	18·90828	17·66604	16·54685	14·62098
37	22·16723	20·57052	19·14257	17·86223	16·71128	14·73678
38	22·49246	20·84108	19·36786	18·04999	16·86789	14·84601
39	22·80821	21·10249	19·58448	18·22965	17·01704	14·94907
40	23·11477	21·35507	19·79277	18·40158	17·15908	15·04629
41	23·41239	21·59910	19·99305	18·56610	17·29436	15·13801
42	23·70135	21·83488	20·18562	18·72354	17·42320	15·22454
43	23·98190	22·06268	20·37079	18·87421	17·54591	15·30617
44	24·25437	22·28279	20·54884	19·01838	17·66277	15·38318
45	24·51871	22·49545	20·72003	19·15634	17·77406	15·45583
46	24·77544	22·70091	20·88465	19·28837	17·88006	15·52436
47	25·02470	22·89943	21·04293	19·41470	17·98101	15·58902
48	25·26670	23·09124	21·19513	19·53560	18·07715	15·65002
49	25·50165	23·27656	21·34147	19·65129	18·16872	15·70757
50	25·72976	23·45561	21·48218	19·76200	18·25592	15·76186

TABLE III.

Showing the Annuity that £1. will purchase for any number of years, not exceeding fifty; at different rates of interest from 3 to 6 per cent.—(Encyc. Metrop.)

No. of Years.	3 perCent.	3½ perCent.	4 perCent.	4½ perCent.	5 perCent.	6 perCent.
1	1·03000	1·03500	1·04000	1·04500	1·05000	1·06000
2	·52261	·52640	·53019	·53399	·53780	·54543
3	·35353	·35693	·36034	·36377	·36720	·37410
4	·26902	·27225	·27549	·27874	·28201	·28859
5	·21835	·22148	·22462	·22779	·23097	·23739
6	·18459	·18766	·19076	·19387	·19701	·20336
7	·16050	·16354	·16660	·16970	·17281	·17913
8	·14245	·14547	·14852	·15160	·15472	·16103
9	·12843	·13144	·13449	·13757	·14069	·14702
10	·11723	·12024	·12329	·12637	·12950	·13586
11	·10807	·11109	·11414	·11724	·12038	·12679
12	·10046	·10348	·10655	·10966	·11282	·11927
13	·09402	·09706	·10014	·10327	·10645	·11296
14	·08852	·09157	·09466	·09782	.10102	·10758
15	·08376	·08682	·08994	·09311	·09634	·10296
16	·07961	·08268	·08582	.08901	·09226	·09895
17	·07595	·07904	·08219	·08541	·08869	·09544
18	·07270	·07581	·07899	·08223	·08554	·09235
19	·06981	·07294	·07613	·07940	·08274	·08962
20	·06721	·07036	·07258	·07687	·08024	·08718
21	·06487	·06803	·07128	·07460	·07799	·08500
22	·06274	.06593	·06919	·07254	·07597	·08304
23	·06081	·06401	·06730	·07068	.07413	·08127
24	·05904	·06227	·06558	·06898	·07247	·07967
25	·05742	·06067	·06401	·06743	·07095	·07822
26	·05593	·05920	·06256	·06602	·06956	·07690
27	·05456	·05785	·06123	·06471	·06829	·07569
28	·05329	·05660	·06001	·06352	·06712	·07459
29	·05211	·05544	·05887	·06242	·06604	·07357
30	·05101	·05437	·05783	·06139	·06505	·07264
31	·04999	·05337	·05685	·06044	·06413	·07179
32	·04904	·05244	·05594	·05956	·06328	·07100
33	·04815	·05157	·05510	·05874	·06249	·07027
34	·04732	·05075	·05431	·05798	·06175	·06959
35	·04653	·04999	·05357	·05727	·06107	·06897
36	·04580	·04928	·05288	·05660	·06043	·06839
37	·04511	·04861	·05223	·05598	·05983	·06785
38	·04445	·04798	·05163	·05540	·05928	·06735
39	·04384	·04738	·05106	·05485	·05876	·06689
40	·04326	·04682	·05052	·05434	·05827	·06646
41	·04271	·04629	·05001	·05386	·05782	·06605
42	·04219	·04579	·04954	·05340	·05739	·06568
43	·04169	·04532	·04908	·05298	·05699	·06533
44	·04122	·04487	·04866	·05258	·05661	·06500
45	·04078	·04445	·04826	·05220	·05626	·06470
46	·04036	·04405	·04788	·05184	·05592	·06441
47	·03996	·04366	·04752	·05150	·05561	·06414
48	·03957	·04330	·04718	·05118	·05531	·06389
49	·03921	·04296	·04685	·05088	·05503	·06366
50	·03886	·04263	·04655	·05060	·05477	·06344

2. *Of Life Annuities.—(Wood.)*

1. To find the probability that an individual of a given age will live any number of years.

Let A be the number in the tables of the given age, B, C, D .... X the number left at 1, 2, 3 .... t years; then $\frac{B}{A}$ is the probability that the individual will live one year; $\frac{C}{A}$ the probability that he will live two years, $\frac{X}{A}$ that he will live t years. Also $\frac{A-B}{A}$, $\frac{A-C}{A}$, $\frac{A-X}{A}$ are the probabilities that he will die in 1, 2, t years.

2. To find the probability that two individuals P and Q, whose ages are known, will live a year.

Let the probability that P will live a year, determined by the last Art. be $\frac{1}{m}$, and the probability that Q will live a year $\frac{1}{n}$; then the probability that they will both be alive at the end of that time is $\frac{1}{mn}$.

3. To find the probability that one of them at least will be alive at the end of any number of years.

Let $\frac{1}{p}$ be the probability that P will live t years, and $\frac{1}{q}$ the probability that Q will live the same time; then the prob. that one of them at least will be alive at the end of the time is $1 - \frac{\overline{p-1}.\ \overline{q-1}}{p\,q}$, or $\frac{p+q-1}{p\,q}$.

4. To find the present value of an annuity of £1. to be continued during the life of an individual of a given age, allowing compound interest for the money.

Let r be the amount of £1. for one year; A, B, C, &c. as in Art. 1, then the value required is $\frac{1}{A} \times \left(\frac{B}{r} + \frac{C}{r^2} + \frac{D}{r^3} + \&c.\right)$ to the end of the tables.

De Moivre supposes that out of 86 persons born, one dies every year, till they are extinct. On this supposition, the sum of the above series may be found thus. Let n be the number of years which any individual wants of 86; then will n be the number of persons living of that age, out of which one dies every year; then the sum of the above series or the present value of the annuity is $\frac{\overline{n-1}.\ r - n + \frac{1}{r\,n-1}}{n.\ (r-1)^2}$ = (if P be the present value of an annuity of £1. to continue certain for n years)

$$\frac{1 - \frac{r}{n} P}{r - 1}$$

5. The present value of the annuity to continue for ever from the death of the proposed individual is $\frac{r\,P}{n.\,\overline{r-1}}$.

6. To find the present value of an annuity of £1. to be paid as long as two specified individuals are both living.

Find by Art. 2. the probability that they will both be alive at the end 1, 2, 3, &c. years to the end of the Tables, call these probabilities a, b, c, &c. and r the amount of £1. in one year, then $\frac{a}{r} + \frac{b}{r^2} + \frac{c}{r^3} +$ &c. is the present value of the annuity required.

7. To find the present value of an annuity of £1. to be paid as long as either of two specified individuals is living.

Find by Art. 3. the probability that they will not both be extinct in 1, 2, 3, &c. years to the end of the tables, and call these probabilities A, B, C, &c. then the present value of the annuity is $\frac{A}{r} + \frac{B}{r^2} + \frac{C}{r^3} +$ &c.

TABLE I.

Mean Standard Table of the Decrements of Life in Great Britain, 1824. —(*Dr Young's Phil. Trans.* 1826.)

Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.
0	20531	100003	30	705	46527	60	938	21810	90	164	589
1	9106	79472	31	712	45822	61	942	20872	91	130	425
2	4780	70366	32	719	45110	62	943	19930	92	87	295
3	2854	65586	33	726	44391	63	944	18987	93	60	208
4	1880	62732	34	734	43665	64	943	18043	94	44	148
5	1341	60852	35	742	42931	65	942	17100	95	31	104
6	979	59511	36	751	42189	66	939	16158	96	19	73
7	752	58532	37	759	41438	67	933	15219	97	14	54
8	603	57780	38	768	40679	68	926	14286	98	9	40
9	494	57177	39	776	39911	69	915	13360	99	6	31
10	423	56683	40	785	39135	70	903	12445	100	6	25
11	377	56260	41	795	38350	71	888	11542	101	5	19
12	349	55883	42	804	37555	72	871	10654	102	5	14
13	337	55534	43	813	36751	73	850	9783	103	4	9
14	337	55197	44	821	35938	74	826	8933	104	2	5
15	347	54860	45	831	35117	75	801	8107	105	1	3
16	381	54513	46	839	34286	76	768	7306	106	.25	2
17	393	54132	47	848	33447	77	733	6538	107	.25	1.75
18	422	53739	48	857	32599	78	697	5805	108	.25	1.50
19	458	53317	49	866	31742	79	654	5108	109	.25	1.25
20	497	52859	50	874	30876	80	610	4454	110	.25	1.0
21	540	52362	51	882	30002	81	559	3844	111	.25	.75
22	581	51822	52	890	29120	82	513	3285	112	.25	.50
23	621	51241	53	898	28230	83	460	2772	113	.25	.25
24	656	50620	54	906	27332	84	408	2312	114	0	0
25	678	49964	55	913	26426	85	357	1904			
26	682	49286	56	917	25513	86	307	1547			
27	687	48604	57	923	24596	87	258	1240			
28	692	47917	58	929	23673	88	215	982			
29	698	47225	59	934	22744	89	178	767			

Dr Young's formula expressing the decrement of human life is

$$y = 368 + 10\,x - 11\,(156 + 2\,o\,x - x^2)^{\frac{3}{4}} + \frac{1}{285 + 2.05\,x^2 + 2\left(\frac{x}{10}\right)^6}$$

$$-5.5\left(\frac{x}{50}\right)^{10} + \frac{5.5_2}{4000}\left(\frac{x}{50}\right)^{20} - 5500\left(\frac{x}{100}\right)^{40}.$$; y being the number of deaths among 100000 persons, in the year that completes the age x. ; $y = 368 + 10\,x$ may be employed as sufficiently correct for the middle portion of life, being certainly much nearer to the truth than Demoivre's hypothesis, who makes $y = \frac{100000}{86} = 1163$ throughout life.

TABLE II.

Showing the value of an Annuity on a single life at every age, deduced from the observations made at Northampton.—(Encyc. Metrop.)

Age.	4 per Cent.	5 per Cent.	6 per Cent.	Age.	4 per Cent.	5 per Cent.	6 per Cent.
1	13·465	11·563	10·107	49	11·475	10·443	9·563
2	15·633	13·420	11·724	50	11·234	10·269	9·417
3	16·462	14·135	12·348	51	11·057	10·097	9·273
4	17·010	14·613	12·769	52	10·849	9·925	9·129
5	17·248	14·826	12·962	53	10·637	9·748	8·980
6	17·482	15·041	13·156	54	10·421	9·567	8·827
7	17·611	15·166	13·275	55	10·201	4·382	8·670
8	17·662	15·226	13·337	56	9·977	9·193	8·509
9	17·625	15·210	13·335	57	9·749	8·999	8·343
10	17·523	15·139	13·285	58	9·516	8·801	8·173
11	17·393	15·043	13·212	59	9·280	8·599	7·999
12	17·251	14·937	13·130	60	9·039	8·392	7·820
13	17·103	14·826	13·044	61	8·795	8·181	7·637
14	16·950	14·710	12·953	62	8·547	7·966	7·449
15	16·791	14·588	12·857	63	8·291	7·742	7·253
16	16·625	14·460	12·755	64	8·030	7·514	6·052
17	16·462	14·334	12·655	65	7·761	7.276	6·841
18	16·309	14·217	12·562	66	7·488	7·034	6·625
19	16·167	14·108	12·477	67	7·211	6·787	6·405
20	16·033	14·007	12·398	68	6·930	6·536	6·179
21	15·912	13·917	12·329	69	6·647	6·281	5·949
22	15·797	13·833	12·265	70	6·361	6·023	5·716
23	15·680	13·746	12·200	71	6·075	5·764	5·479
24	15·560	13·658	12·132	72	5·790	5·504	5·241
25	15·438	13·567	12·063	73	5·507	5·245	5·004
26	15·312	13·473	11·992	74	5·230	4·990	4·796
27	15·184	13·377	11·917	75	4·962	4·744	4·542
28	15·053	13·278	11·841	76	4·710	4·511	4·326
29	14·918	13·177	11·763	77	4·457	4·277	4·109
30	14·781	13·072	11·682	78	4·197	4·035	3·884
31	14·639	12·965	11·598	79	3·921	3·776	3·641
32	14·495	12·854	11·512	80	3·643	3·515	3·394
33	14·347	12·740	11·423	81	3.377	3·263	3·156
34	14·195	12·623	11·331	82	3·122	3·026	2·926
35	14·039	12·502	11·236	83	2·887	2·797	2·713
36	13·889	12·377	11·137	84	2·708	2·627	2·551
37	13·716	12·249	11·035	85	2·543	2·471	2·402
38	13·548	12·116	10·929	86	2·393	2·328	2·266
39	13·375	11·979	10·819	87	2·251	2·193	2·138
40	13·197	11·837	10·705	88	2·131	2·080	2·031
41	13·018	11·695	10·589	89	1.967	1·924	1·882
42	12·838	11·551	10·473	90	1·758	1·723	1·689
43	12·657	11·407	10·356	91	1·474	1·447	1·422
44	12·472	11·258	10·235	92	1·171	1·153	1·136
45	12·283	11·105	10.110	93	·827	·816	·806
46	12·089	10·947	9·980	94	·530	·524	·518
47	11·890	10·784	9·846	95	·240	·238	·236
48	11·685	10·616	9·707	96	·000	·000	·000

TABLE III.

Showing the value of an Annuity on two joint lives, deduced from observations made at Northampton, the difference of ages being 5 years.—(Encyc. Metrop.)

Ages.	4 per Cent.	5 per Cent.	Ages.	4 per Cent.	5 per Cent.
1- 6	10·741	9·479	47-52	8·147	7·582
2- 7	12·581	11·100	48-53	7·965	7·424
3- 8	13·319	11·755	49-54	7·780	7·262
4- 9	13·775	12·165	50-55	7·593	7·098
5-10	13·933	12·315	51-56	7·409	6·936
6-11	14·068	12·447	52-57	7·225	6·774
7-12	14·111	12·498	53-58	7·039	6·609
8-13	14·089	12·492	54-59	6·850	6·442
9-14	13·992	12·421	55-60	6·659	6·272
10-15	13·841	12·302	56-61	6·465	6·100
11-16	13·664	12·158	57-62	6·270	5·925
12-17	13·480	12·009	58-63	6·070	5·744
13-18	13·303	11·864	59-64	5·867	5·561
14-19	13·130	11·723	60-65	5·658	5·372
15-20	12·961	11·585	61-66	5·447	5·180
16-21	12·799	11·452	62-67	5·285	4·986
17-22	12·646	11·327	63-68	5·017	4·786
18-23	12·500	11·209	64-69	4·798	4·585
19-24	12·361	11·096	65-70	4·573	4·378
20-25	12·229	10·989	66-71	4·349	4·169
21-26	12·105	10·890	67-72	4·124	3·960
22-27	11·987	10·796	68-73	3·901	3·752
23-28	11·866	10·699	69-74	3·683	3·547
24-29	11·743	10·600	70-75	3·471	3·347
25-30	11·618	10·499	71-76	3·270	3·159
26-31	11·489	10·396	72-77	3·070	2·971
27-32	11·359	10·289	73-78	2·869	2·780
28-33	11·225	10·181	74-79	2·659	2·580
29-34	11·088	10·069	75-80	2·448	2·381
30-35	10·948	9·954	76-81	2·258	2·195
31-36	10·805	9·837	77-82	2·077	2·013
32-37	10·659	9·716	78-83	1·899	1·838
33-38	10·508	9·591	79-84	1·751	1·750
34-39	10·354	9·463	80-85	1·608	1·573
35-40	10·196	9·331	81-86	1·478	1·447
36-41	10·037	9·198	82-87	1·356	1·329
37-42	9·877	9·062	83-88	1·259	1·235
38-43	9·716	8·927	84-89	1·164	1·145
39-44	9·550	8·787	85-90	1·054	1·038
40-45	9·381	8·643	86-91	·902	·892
41-46	9·210	8·497	87-92	·738	·734
42-47	9·037	8·350	88-93	.554	·547
43-48	8·862	8·200	89-94	·373	·369
44-49	8·683	8·046	90-95	·177	·175
45-50	8·503	7·891	91-96	·000	·000
46-51	8·326	7·737			

TABLE IV.

Showing the value of an Annuity on two joint lives, deduced from observations made at Northampton, the difference of ages being 10 years.—(Encyc. Metrop.)

Ages.	4 per Cent.	5 per Cent.	Ages.	4 per Cent.	5 per Cent.
1-11	10·782	9·544	44-54	8·130	7·569
2-12	12·438	11·010	45-55	7·948	7·411
3-13	13·019	11·528	46-56	7·763	7·249
4-14	13·374	11·850	47-57	7·574	7·084
5-15	13·479	11·954	48-58	7·382	6·915
6-16	13·578	12·052	49-59	7·186	6·742
7-17	13·599	12·083	50-60	6·989	6·568
8-18	13·569	12·070	51-61	6·795	6·395
9-19	13·482	12·006	52-62	6·600	6·222
10-20	13·355	11·906	53-63	6·399	6·042
11-21	13·217	11·797	54-64	6·196	5·860
12-22	13·078	11·686	55-65	5·986	5·671
13-23	12·934	11·570	56-66	5·774	5·479
14-24	12·784	11·450	57-67	5·559	5·283
15-25	12·630	11·324	58-68	5·341	5·084
16-26	12·470	11·193	59-69	5·121	4·883
17-27	12·311	11·063	60-70	4·900	4·680
18-28	12·158	10·939	61-71	4·679	4·476
19-29	12·013	10·820	62-72	4·458	4·272
20-30	11·873	10·707	63-73	4·236	4·066
21-31	11·742	10·600	64-74	4·019	3·864
22-32	11·615	10·498	65-75	3·806	3·665
23-33	11·485	10·393	66-76	3·606	3·477
24-34	11·352	10·285	67-77	3·405	3·289
25-35	11·217	10·175	68-78	3·199	3·095
26-36	11·078	10·062	69-79	2·979	2·887
27-37	10·936	9·946	70-80	2·757	2·675
28-38	10·791	9·826	71-81	2·542	2·470
29-39	10·642	9·703	72-82	2·334	2·271
30-40	10·490	9·576	73-83	2·141	2·085
31-41	10·336	9·448	74-84	1·991	1·941
32-42	10·182	9·320	75-85	1·856	1·811
33-43	10·027	9·190	76-86	1·739	1·699
34-44	9·869	9·058	77-87	1·633	1·597
35-45	9·706	8·921	78-88	1·546	1·514
36-46	9·540	8·781	79-89	1·427	1·400
37-47	9·370	8·636	80-90	1·278	1·255
38-48	9·195	8·487	81-91	1·078	1·061
39-49	9·015	8·333	82-92	·864	·852
40-50	8·834	8·177	83-93	·614	·606
41-51	8·658	8·025	84-94	·403	·398
42-52	8·483	7·875	85-95	·187	·185
43-53	8·308	7·724	86-96	·000	·000

3. *Of Assurances on Lives.*

This article has been already extended beyond its due limits; the following Table is therefore all that can be inserted on this subject.

Terms of Assurance proposed by the Amicable Society, for assuring the sum of £100. upon the life of any healthy person from the age of 8 to 72.

Age.	For one year.	For 7 years.	For the whole life.	Age.	For one year.	For 7 years.	For the whole life.
	£. s. d.	£. s. d.	£. s. d.		£. s. d.	£. s. d.	£. s. d.
8 to 14	0 14 6	0 18 0	1 14 6	44	1 17 0	2 1 0	3 13 0
15	0 15 0	0 19 0	1 15 6	45	1 18 0	2 2 0	3 15 0
16	0 15 6	1 0 0	1 16 6	46	1 19 0	2 3 0	3 17 6
17	0 17 0	1 1 0	1 17 6	47	2 0 0	2 4 6	4 0 0
18	0 18 6	1 2 0	1 18 6	48	2 1 6	2 6 0	4 2 6
19	1 0 0	1 3 6	1 19 6	49	2 3 0	2 8 0	4 5 0
20	1 1 6	1 4 6	2 0 6	50	2 4 6	2 10 0	4 8 0
21	1 2 6	1 5 0	2 1 6	51	2 6 0	2 12 0	4 11 0
22	1 3 6	1 5 6	2 2 6	52	2 7 6	2 14 0	4 14 0
23	1 4 0	1 6 0	2 3 6	53	2 9 6	2 16 0	4 17 0
24	1 4 6	1 6 6	2 4 6	54	2 11 6	2 18 0	5 0 0
25	1 5 0	1 7 0	2 5 6	55	2 13 6	3 0 0	5 3 6
26	1 5 6	1 7 6	2 6 6	56	2 15 6	3 2 0	5 7 6
27	1 6 0	1 8 0	2 7 6	57	2 17 6	3 4 0	5 11 6
28	1 6 6	1 8 6	2 8 6	58	2 19 6	3 6 6	5 15 6
29	1 7 0	1 9 0	2 9 6	59	3 1 6	3 9 6	6 0 0
30	1 7 6	1 9 6	2 10 6	60	3 4 0	3 12 6	6 5 0
31	1 8 0	1 10 0	2 11 6	61	3 6 6	3 15 6	6 10 0
32	1 8 6	1 10 6	2 12 6	62	3 9 0	3 19 0	6 15 6
33	1 9 0	1 11 0	2 14 0	63	3 11 6	4 2 6	7 1 0
34	1 9 6	1 11 6	2 15 6	64	3 14 6	4 7 0	7 7 6
35	1 10 0	1 12 0	2 17 0	65	3 18 0	4 12 0	7 14 6
36	1 10 6	1 13 0	2 18 6	66	4 2 0	4 17 6	8 2 0
37	1 11 0	1 14 0	3 0 0	67	4 6 0	5 4 0	8 10 0
38	1 11 6	1 15 0	3 1 6	68	4 13 6	5 13 6	8 19 6
39	1 12 0	1 16 0	3 3 0	69	5 1 6	6 4 0	9 9 0
40	1 13 0	1 17 0	3 5 0	70	5 10 6	6 17 6	9 19 6
41	1 14 0	1 18 0	3 7 0	71	6 1 0	7 14 6	10 10 0
42	1 15 0	1 19 0	3 9 0	72	6 13 0	8 16 0	11 2 0
43	1 16 0	2 0 0	3 11 0				

Ex. Let it be proposed to determine the annual payment to be made by a person aged 42, to insure £1000. payable at his decease.

	£.	s.
By Table, annual payment per cent.	3	9
Multiply by		10
	£34	10

ANOMALISTIC *Year.*—*See Earth* Elements of.

ANOMALY, in *Astronomy.*—*(Maddy, Playfair.)*

Given the mean anomaly (m), to find the true (v), (usually called Kepler's Problem.)

1*st Method.*—If the eccentricity (e) be *very* small,

$$\tan. \tfrac{1}{2}\, v = \frac{1+e}{1-e}.\ \tan. \tfrac{1}{2}\, m.$$

2*d Method.*—Let u be the eccentric anomaly, and let the true and mean anomalies be measured from *aphelion ;* then we have the following equations :—

$$m = u + e \sin. u.$$

$$\& \tan. \tfrac{1}{2}\, v = \sqrt{\frac{1-e}{1+e}}.\ \tan. \tfrac{1}{2}\, u.$$

Therefore, eliminating u between these two equations, the relation between m and v may be found.

If the anomalies are measured from *perihelion,*

$$m = u - e \sin. u.$$

$$\& \tan. \tfrac{1}{2}\, v = \sqrt{\frac{1+e}{1-e}}.\ \tan. \tfrac{1}{2}\, u.$$

The following is the series for v in terms of m.

$$v = m - \left(2\,e - \frac{1}{4}\,e^3 + \frac{5}{96}\,e^5\right) \sin. m + \left(\frac{5}{4}\,e^2 - \frac{11}{24}\,e^4 + \frac{17}{192}\,e^6\right)$$

$$\sin. 2\,m + \left(\frac{13}{12}\,e^3 - \frac{43}{64}\,e^5\right) \sin. 3\,m + \left(\frac{103}{96}\,e^4 - \frac{451}{480}\,e^6\right) \sin 4\ m$$

$$+\ \frac{1097}{960}\,e^5 \sin. 5\,m + \frac{1223}{960}\,e^6 \sin. 6\,m.$$

Note.—The constant coefficients must be reduced into degrees and minutes, by multiplying each of them by $57^0.29578$, the number of degrees in an arc equal to the radius.

3*d Method.*—To find the true anomaly in terms of the mean, in a series ascending by powers of e.

$$v = m + 2 \sin m.\ e + \frac{5}{4} \sin. 2\,m.\ e^2 + \&c.$$

And to find m in terms of v,

$$m = v - (2\,c + e.\ \overline{1-c^2}) \sin. v + (c^2 + e\,c.\ \overline{1-c^2}) \sin. 2\,v - \&c.$$

$$\text{where } c = \frac{1-\sqrt{1-e^2}}{e}.$$

The radius vector r may also be expressed in terms of the mean anomaly, supposing the mean distance 1.

$$r = 1 + \frac{1}{2}e^2 - \left(e - \frac{3}{8}e^3 + \frac{5}{192}e^5\right)\cos. m + \left(-\frac{1}{2}e^2 + \frac{1}{3}e^4 - \frac{1}{16}e^6\right)$$

$$\cos. 2m - \left(\frac{3}{8}e^3 - \frac{45}{128}e^5\right)\cos. 3m + \left(-\frac{1}{3}e^4 + \frac{2}{5}e^6\right)\cos. 4m$$

$$-\left(\frac{125}{384}e^5\right)\cos. 5m + \left(-\frac{27}{80}e^6\right)\cos. 6m.$$

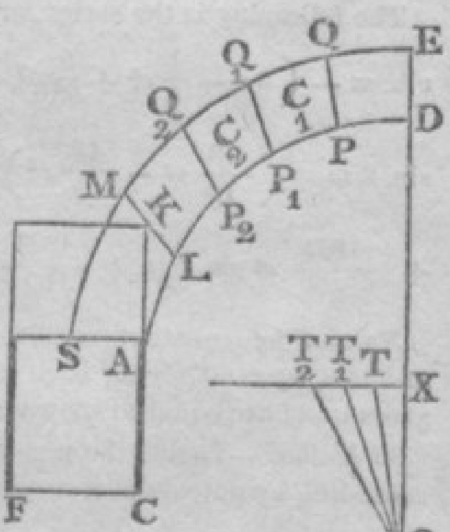
In the case of the sun, e being small (viz. ·016814) its powers above the 3d. may be neglected, and in this case $y = (1^{o}.\ 55'.\ 26'', 35)\sin. m + (1'.\ 12'', 68)\sin. 2m + (1''.\ 05)\sin. 3m$.

And $r = 1 + \frac{1}{2}e^2 - e\cos. m - \frac{1}{2}e^2\cos. 2m$.

When m is computed from the apogee instead of the perigee, the signs of the terms involving the *odd* multiples of m must be changed.

ARCHES, *Equilibrium* of.—(*Whewell, Playfair.*)

1. In an arch which is in equilibrium, the weights of the voussoirs are as the differences of the tangents of the angles which their joints make with the vertical.



Hence if O T be in the line of the joint P Q or parallel to it, OT_1, parallel to P_1Q_1, &c., and TT_1, be horizontal, the weights of the voussoirs C, C_1 &c. will be as the portions TT_1, T_1T_2 &c.

Cor. 1. If the arch is a circle, the weights of the voussoirs are as the differences of the tangents of the arches, reckoned from the crown. This is nothing more than the general proposition above, applied to a particular case.

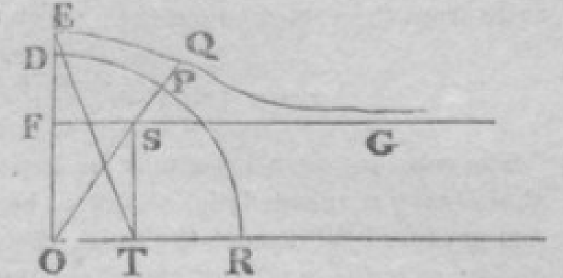
Note.—As the stones themselves cannot always be made in the proportion thus required; the wedges, of which they make parts, are supposed to be extended upward by courses of masonry. The whole mass included between the planes of the joints produced, as far as that masonry extends, is understood to make up the weight of the voussoirs.

Cor. 2. The *horizontal* pressure is represented by O X, and is the same at each joint.

Cor. 3. The pressures at the joints are represented by O T, $O\,T_1$ &c. and are therefore as the secants of the $\angle^s$ which the joints make with the vertical. If θ be the $\angle$ of any joint with the vertical, and H the horizontal pressure, H sec. θ is the pressure at that joint.

Cor. 4. The line $X\,T_2$ will represent the whole weight of the mass between D E and $P_2\,Q_2$, and similarly for any other joint; hence H tan. θ is the weight of any portion.

2. The intrados being a circle, with the joints in the direction of the radii, to find the extrados, so that the voussoirs may be in equilibrium.



Let P be any point of the intrados, O its centre, put D O P $= \theta$, O D $=$ O P $= l$, O Q $= r$, O E $= k$, then

$$r^2 = l^2 + (k^2 - l^2) \text{ sec.}^2\, \theta.$$

Hence we have the following construction. Make O R horizontal, R F $=$ O E, F G horizontal. Let O P meet F G in S, draw S T vertical, and take O Q $=$ E T; the *locus* of Q will be the extrados.

Cor. 1. The extrados has F G for an asymptote.

Cor. 2. To find the equation to the extrados. Let O be the origin of the coordinates, x and y corresponding coordinates to the point Q, O D $= l$, O F $= a$; then the Equation to the curve is

$$x = \frac{y\,\sqrt{(l^2 + a^2 - y^2)}}{\sqrt{(y^2 - a^2)}}.$$

The extrados, in the case of a circular arch, is therefore a curve of the 4th order, very much resembling the conchoid of Nicomedes. It has an asymptote F G and also a point of contrary flexure, so that it coincides very nearly with the curve in which a road is usually carried over a bridge.

3. In an elliptic arch, or one of which the intrados is a semi-ellipse, if 2 a be the span of the arch or the major axis of the ellipse, and b the height of the arch or the semi-conjugate axis; then if from any point in

the curve a perpendicular x be let fall on the longer axis, and W be the weight of the key-stone, the weight V of any voussoir is

$$\frac{a^2 \mathrm{W}}{x^2 \sqrt{\left(a^2 + \frac{a^2 - b^2}{a^2} x^2\right)}}.$$

4. If the weights of the voussoirs are all equal, the arch of equilibrium is a catenarian curve, the same that a chain of uniform thickness would assume, if hanging freely; the horizontal distance of the points of suspension being equal to the span of the arch, and the depth of the lowest points of the chain being equal to the greatest height of the arch.

The equation to the catenary, if x and y be the corresponding coordinates from the vertex along the axis or vertical line, is

$$y = a + \text{h, l.} \frac{a + x + \sqrt{2ax + x^2}}{a}.$$

The constant quantity a may be determined by experiment; for the chain being suspended; let a tangent be drawn to any point of the curve, and produced till it meet the axis; then as the subtangent is to the ordinate, so is the length of the chain, between the given point and the vertex, to the quantity a. When a is found, the curve can be constructed.

5. The pressure of an arch on the piers or abutments which support it, may be estimated by considering the parts of the arch, which rest immediately on the abutments to a certain height, as parts of the abutments themselves; and the remainder of the arch as a wedge; tending to separate the abutments from one another.

Thus the part A L M S (see above Fig.) which would remain in its place though there were no pressure from above, may be regarded as a part of the pier, and L M E D &c., the remainder of the arch, as a wedge tending to overthrow the pier by its pressure on the plane M L. On these suppositions the thickness of the piers, so that their weight shall enable them to resist this pressure, may be determined.

Let the $\angle$ which M L makes with the vertical $= \theta$, twice the area M L D E $= a^2$, C K $= h$, and F C $= x$, then

$$x = -\frac{a^2}{2h \cos.^2 \theta} + a \sqrt{\left(\frac{2}{\sin. 2\theta} + \frac{a^2}{4h^2 \cos.^4 \theta}\right)}.$$

In the above demonstration, the hypothesis is that the pier A F, if the weight of the arch were too great to be sustained, would fall, by turning round F as a fulcrum. Now this is not what would happen; the part of the abutment behind S M would be thrust out in the horizontal direction, till the arch had room to fall; it is therefore against the masonry immediately behind the part A M, and chiefly in a horizontal direction, that the force is exerted.

ARCHIMEDES' *Spiral.—See Spiral.*

ARCS *Circular to find length of, in terms of the radius.—(Vince.)*

TABLE,
For finding the length of Circular Arcs to Radius Unity.

Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	0,0174533	60	1,0471976	1	0,0002909	1	0,0000048
2	0,0349066	70	1,2217305	2	0,0005818	2	0,0000097
3	0,0523599	80	1,3962634	3	0,0008727	3	0,0000145
4	0,0698132	90	1,5707963	4	0,0011636	4	0,0000194
5	0,0872665	100	1,7453293	5	0,0014544	5	0,0000242
6	0,1047198	120	2,0943951	6	0,0017453	6	0,0000291
7	0,1221730	150	2,6179939	7	0,0020362	7	0,0000339
8	0,1396263	160	3,1415927	8	0,0023271	8	0,0000388
9	0,1570796	210	3,6651914	9	0,0026180	9	0,0000436
10	0,1745329	240	4,1887902	10	0,0029089	10	0,0000485
20	0,3490659	270	4,7123890	20	0,0058178	20	0,0000970
30	0,5235988	300	5,2359878	30	0,0087266	30	0,0001454
40	0,6981317	330	5,7595865	40	0,0116355	40	0,0001939
50	0,8726646	360	6,2831853	50	0,0145444	50	0,0002424

Circular arc = radius = $57^0. 2957795 = 57^0. 17'. 44'', 8.$

Ex. What is the length of a circular arc of $37^0. 42'. 58''$?

	30^0 ——	0·5235988
	7^0 ——	0·1221730
	$40'$ ——	0·0116355
	$2'$ ——	0·0005818
	$50''$ ——	0·0002424
	$8''$ ——	0·0000388
Length required		0·6582703

ARCS *Semi-diurnal.—(Vince.)*

TABLES *of Semi-diurnal Arcs.*

Latitude and Declination of the *same* kind.

Declination.	LATITUDE.						
	50°	51°	52°	53°	54°	55°	56°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 8	6. 8	6. 9	6. 9	6. 9	6. 9	6. 10
2	6. 13	6. 13	6. 14	6. 14	6. 15	6. 15	6. 16
3	6. 18	6. 18	6. 19	6. 19	6. 20	6. 21	6. 22
4	6. 22	6. 22	6. 24	6. 25	6. 26	6. 27	6. 28
5	6. 27	6. 27	6. 29	6. 30	6. 31	6. 32	6. 34
6	6. 32	6. 33	6. 34	6. 36	6. 37	6. 38	6. 40
7	6. 37	6. 38	6. 40	6. 41	6. 43	6. 44	6. 46
8	6. 42	6. 43	6. 45	6. 47	6. 48	6. 50	6. 52
9	6. 47	6. 48	6. 50	6. 52	6. 54	6. 56	6. 58
10	6. 52	6. 54	6. 56	6. 58	7. 0	7. 2	7. 5
11	6. 57	6. 59	7. 1	7. 3	7. 6	7. 8	7. 11
12	7. 2	7. 4	7. 7	7. 9	7. 12	7. 15	7. 18
13	7. 7	7. 10	7. 12	7. 15	7. 18	7. 21	7. 24
14	7. 13	7. 15	7. 18	7. 21	7. 24	7. 28	7. 31
15	7. 18	7. 21	7. 24	7. 27	7. 31	7. 34	7. 39
16	7. 24	7. 27	7. 30	7. 33	7. 37	7. 41	7. 45
17	7. 29	7. 33	7. 36	7. 40	7. 44	7. 48	7. 52
18	7. 35	7. 38	7. 42	7. 46	7. 51	7. 55	8. 0
19	7. 41	7. 45	7. 49	7. 53	7. 58	8. 2	8. 7
20	7. 47	7. 51	7. 55	8. 0	8. 5	8. 10	8. 15
21	7. 53	7. 57	8. 2	8. 7	8. 12	8. 18	8. 24
22	7. 59	8. 4	8. 9	8. 14	8. 20	8. 26	8. 32
23	8. 6	8. 11	8. 16	8. 22	8. 28	8. 34	8. 41
24	8. 12	8. 18	8. 24	8. 30	8. 36	8. 43	8. 51
25	8. 19	8. 25	8. 31	8. 38	8. 45	8. 53	9. 1
26	8. 27	8. 33	8. 39	8. 47	8. 54	9. 2	9. 11
27	8. 34	8. 41	8. 48	8. 56	9. 4	9. 13	9. 23
28	8. 42	8. 49	8. 57	9. 5	9. 14	9. 24	9. 35
29	8. 50	8. 58	9. 6	9. 14	9. 25	9. 36	9. 49
30	8. 59	9. 8	9. 17	2. 26	9. 38	9. 50	10. 4
31	9. 9	9. 18	9. 28	9. 38	9. 51	10. 5	10. 22
32	9. 19	9. 28	9. 39	9. 52	10. 6	10. 23	10. 44

Latitude and Declination of *different* kinds.

Declination.	LATITUDE.						
	50°	51°	52°	53°	54°	55°	56°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5. 59	5. 58	5. 58	5. 58	5. 58	5. 58	5. 58
2	5. 54	5. 53	5. 53	5. 53	5. 53	5. 52	5, 52
3	5. 49	5. 49	5. 48	5. 48	5. 47	5. 47	5. 46
4	5. 44	5. 44	5. 43	5. 42	5. 42	5. 41	5. 40
5	5. 39	5. 39	5. 38	5. 37	5. 36	5. 35	5. 34
6	5. 35	5. 34	5. 33	5. 31	5. 30	5. 29	5. 28
7	5. 30	5. 29	5. 27	5. 26	5. 25	5. 23	5. 22
8	5. 25	5. 23	5. 22	5. 21	5. 19	5. 17	5. 16
9	5. 20	5. 18	5. 17	5. 16	5. 13	5. 12	5. 10
10	5. 15	5. 13	5. 11	5. 10	5. 8	5. 5	5. 3
11	5. 10	5. 8	5. 6	5. 4	5. 2	4. 59	4. 57
12	5. 5	5. 3	5. 0	4. 58	4. 56	4. 53	4. 51
13	5. 0	4. 57	4. 55	4. 52	4. 50	4. 47	4. 44
14	4. 54	4. 52	4. 49	4. 47	4. 44	4. 41	4. 37
15	4. 49	4. 46	4. 44	4. 41	4. 37	4. 34	4. 31
16	4. 45	4. 41	4. 38	4. 34	4. 31	4. 27	4. 24
17	4. 38	4. 35	4. 32	4. 28	4. 23	4. 21	4. 17
18	4. 33	4. 29	4. 26	4. 22	4. 18	4. 14	4. 9
19	4. 27	4. 23	4. 19	4. 15	4. 11	4. 7	4. 2
20	4. 21	4. 17	4. 13	4. 9	4. 4	3. 59	3. 54
21	4. 15	4. 11	4. 6	4. 2	3. 57	3. 52	3. 46
22	4. 9	4. 4	4. 0	3. 55	3. 50	3. 44	3. 38
23	4. 3	3. 58	3. 53	3. 47	3. 42	3. 36	3. 29
24	3. 56	3. 51	3. 46	3. 40	3. 34	3. 27	3. 20
25	3. 49	3. 44	3. 38	3. 32	3. 25	3. 18	3. 11
26	3. 42	3. 37	3. 30	3. 24	3. 17	3. 9	3. 1
27	3. 35	3. 29	3. 22	3. 15	3. 8	2. 59	2. 50
28	3. 28	3. 21	3. 14	3. 6	2. 58	2. 49	2. 38
29	3. 20	3. 12	3. 5	2. 56	2. 47	2. 37	2. 26
30	3. 11	3. 4	2. 55	2. 46	2. 36	2. 25	2. 13
31	3. 3	2. 54	2. 45	2. 35	2. 24	2. 12	1. 57
32	2. 53	2. 24	2. 44	2. 23	2. 11	1. 57	1. 40

Explanation of the Tables.

The first is a Table of semi-diurnal arcs, when the latitude of the place and the declination of the body are of the same kind; the 2d. when the latitude and declination are of different kinds. The first column of each

Table contains the declination of the body from 1° to 32°, and at the top of each succeeding column is set down the latitude of the place from 50° to 56° both inclusive.

For the *sun*, the arc gives the time of its setting, and if it be subtracted from twelve o'clock, you get the time of its rising.

For a *star*, add and subtract the equation to and from the time at which the star passes the meridian, and you have the time of its setting and rising.

The time so given is the hour when the centre of the sun appears in the horizon, the eye being at the surface of the earth; thereby taking into consideration the effect of refraction.

Example.—In latitude 52°. 12'., and declination of the sun 23°. 28'., what is the time of its rising and setting?

		h.	m.
Latitude 52°. declination 23°.	arc	8.	16
——— 53. ———	arc	8.	22
1			6

Hence 1° : 12' :: 6*m* : 1*m*, to be added to 8*h*. 16*m*.

		h.	m.
Latitude 52°, declination 23°	arc	8.	16
——— 24	arc	8.	24
1			8

Hence 1° : 28' :: 8*m* : 4*m*, to be added also to 8*h*. 16*m*.

Therefore the semi-diurnal arc = 8*h*. 16*m*. + 1*m*. + 4*m*. = 8*h*. 21*m*. the time of setting; and 3*h*. 39*m*. = time of rising.

AREAS *of Curves*, whose Equations are given.

Let x and y be the abscissa and ordinate of the curve, then

$$\text{Area} = \text{fl. } y\,dx.$$

Ex. 1.—Area of a triangle = base × ½ perpendicular.

2. Area of the common parabola = $\frac{2}{3}\,xy$ = $\frac{2}{3}$ of the circumscribing rectangle. Or if the general equation is $a^{n-1}x = y^n$, area = $\frac{n}{n+1} \times xy$.

3. Area of circle whose radius = 1 is 3. 14159 &c. or if rad. = r, and π = 3. 14159 &c. area = $\pi\, r^2$; or in terms of circumference C = C. $\frac{r}{2}$.

4. Area of ellipse, if a and b = ½ ax. maj. and min., = $\pi.\,a\,b$.

5. Area of cycloid = 3 times area of the generating circle.

6. In the hyperbola the area between the asymptotes $= \text{fl.} \frac{m^2\,dx}{x}$, (assuming $yx = m^2$), the hyperbola being equi-lateral; $\therefore$ area $= m^2$ log. $x + C$; and assuming it $= o$ when $x = m$, we shall have m^2 log. $\frac{x}{m}$ as the general expression for the area.

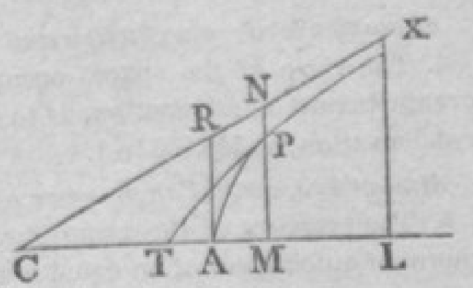
If $m = 1$, the areas are the hyperbolic logarithms of the corresponding abscissæ; and hence the origin of the term *hyperbolic* as applied to logarithms.

For Areas of Spirals.—See Spiral.

ARITHMETICAL *Progression.—See Progression.*

ASSURANCE *on Lives.—See Annuities.*

ASYMPTOTES, *to draw.*



Find the value of $\frac{y\,dx}{dy} =$ subtangent MT; $\therefore AT = \frac{y\,dx}{dy} - x$ is known. Now suppose x to become infinite, and T to move on to C; then if AC be finite the curve admits an asymptote. Next find the ratio of TM : MP, which, if we again suppose x infinite, gives us the ratio of CL : Lx; then by similar Δs CL : Lx :: CA : AR, of which proportion the three first terms are known, and $\therefore$ AR can be determined. Join CR, and produce it indefinitely, and CR is the asymptote required.

Ex. 1.—To draw an asymptote to the common hyperbola.

Here $AT = \frac{2ax + x^2}{a + x} - x$ (when x is infinite) $= a = AC$. Again TM : MP :: $\frac{2ax + x^2}{a + x} : \frac{b}{a}\sqrt{2ax + x^2}$ :: (when x is infinite) $x : \frac{bx}{a}$ :: CL : Lx :: CA (a) : AR, $\therefore AR = b$; Hence from A draw $AR = b$; take C the centre, and join CR, and produce it indefinitely, and CRx is the asymptote.

Ex. 2.—Let the equation be $y^3 = ax^2 + x^3$.

Proceed just as before, and we get $CL = x$, $Lx = x$; $AC = \frac{a}{3}$, $\therefore$

$x : x :: \frac{a}{3} : AR = \frac{a}{3}$ $\therefore$ &c.

ATMOSPHERE.

Atmospheric air, properties of.

1. Fluidity, elasticity, expansibility, and gravity.

Atmospheric air, composition of.

2. Nitrogen 79 parts, oxygen 21, and about 1 part in 1000 of carbonic acid gas. It also contains about 1 per cent. of water in the state of elastic vapour. If the calculation be made by weight, there will be, in every 100 measures of atmospheric air, $23\frac{1}{4}$ of oxygen, and $76\frac{3}{4}$ of nitrogen.

Atmospheric air, specific gravity of.

3. Specific gravity of air : that of water :: 1 : 832 or 833, when reduced to the pressure of 30 inches of the barometer, and the mean temperature of 55°. of the thermometer. 100 cubic inches of air at the surface of the sea, when the thermometer is at 60°, weigh $30\frac{1}{2}$ grains.

Atmospheric air, rarefaction and condensation of.

4. The ratio of the spaces occupied by a given quantity of air in its greatest state of rarefaction, is to the same under the highest degree of condensation, as 550,000 to 1.

Atmosphere, weight or pressure of.

5. The pressure of the atmosphere in its mean state is equal to a column of quicksilver of an equal base and 30 inches high, or to a column of water of 34 feet in height. Hence its weight on every square inch is nearly equal to 15lbs. Mr Cotes computed that the pressure of this ambient fluid on the whole surface of the earth is equivalent to that of a globe of lead of 60 miles in diameter; and admitting the surface of a man's body to be about 15 square feet, he must sustain 32,400 lbs, or nearly $14\frac{1}{2}$ tons weight. But since the variation in the height of the mercurial column may occupy a range of 3 inches, every square inch base on any body may at one time be pressed more than it is at others by a weight equal to three cubic inches of mercury. Hence it may be easily shewn that the difference in the weight of air, sustained by our bodies, in different states of the asmosphere, is often near a ton and a half.

Atmosphere homogeneous, height of.

6. Let H = height of homogeneous atmosphere, δ its uniform density, b the height of the barometer in feet, and D the density of the mercury, then

$$H = \frac{b\,D}{\delta}.$$

At a medium $\delta : D :: 1_9^2 : 13600$; and b at a mean = 30 inches = $2\frac{1}{2}$ feet,

$$\therefore H = \frac{2\frac{1}{2} \times 13600}{1^2} = 27818 \text{ feet}, = \text{rather more than } 5\frac{1}{4} \text{ miles.}$$

Atmosphere, density of.

7. The density of the air is in proportion to the force which compresses it, or to its elasticity, or inversely as the spaces within which the same quantity of it is contained.

8. If altitudes be taken from the earth's surface in arithmetical progression, the density of the air decreases in geometrical progression.

9. Given the altitude above the earth's surface, to find the density of the air; and conversely.

Let $y =$ density at the distance x from the earth's surface, δ the density at the surface, and h the height of the homogeneous atmosphere, then

$$y = \delta \times e^{\frac{-x}{h}}, \text{ or by Art. 6, } y = \delta \times e^{-\frac{\delta x}{b D}}.$$

Or conversely, having given the density to find the altitude, we have

$$x = h \times \text{hyp. log. } \frac{\delta}{y}; \text{ or in common logs. nearly } x = 1000 \times \log. \frac{\delta}{y}.$$

In the above formulæ δ and y denote the atmospherical pressures at the surface and altitude x, for which we may substitute M and m, the altitudes of the mercury in the barometer at those distances; we shall then have

$$x = 1000 \times \log. \frac{M}{m}.$$

This gives only the approximate height; for the correct formula—*see Barometer.*

10. If, instead of supposing gravity constant, we assume it to vary inversely as the n^{th} power of the distance, we shall have, putting the earth's radius $= r$,

$$y = \delta. \frac{1}{e^{\frac{r}{(n-1)h} \cdot \frac{(r+x)^{n-1} - r^{n-1}}{(r+x)^{n-1}}}},$$

which is a general Equation, expressing the relation between the altitude and density.

Cor. If F varies as $\frac{1}{D^2}$, $y = \delta. \frac{1}{e^{\frac{r}{h} \cdot \left(\frac{1}{r} - \frac{1}{r+x}\right)}}$;

hence if $r + x$ increase in harmonical progression, $\frac{1}{r+x}$ is in arithmetic, and ∴ the densities themselves will decrease in geometric.

11. TABLE *exhibiting the comparative density of the air at the several corresponding heights.*

Height in miles.	Rarity.	Height.	Rarity.
0	1	35	1024
$3\frac{1}{2}$	2	42	4096
7	4	49	16384
14	16	56	65536
21	64	63	262144
28	256	70	1048576

And by pursuing the calculation, it might easily be shown that a cubic inch of the air we breathe would be so much rarified at the height of 500 miles, that it would fill a sphere equal in diameter to the orbit of Saturn.

Atmosphere, refractive and reflective powers of.

12. The altitude above the earth's surface at which the atmosphere begins to have any sensible effect on the rays of light to refract them = 77.25 miles; and the altitude at which reflection begins = 39.64 miles, = about half the altitude at which refraction begins.—(*Vince.*)

How much farther than this the atmosphere may extend, it is impossible to ascertain; it must, however, at all events, be limited in its extent by the centrifugal force of the earth, and the attraction of the moon.

For terrestrial refraction, and the refraction of the heavenly bodies—*see Refraction.*

Atmosphere, motion of.

13. To determine the velocity with which atmospheric air will rush into a vacuum, let h = height of homogeneous atmosphere, and v the required velocity, $g = 32\frac{1}{6}$ feet, then

$$v = \sqrt{2gh} = 8\sqrt{h} \text{ nearly, = at a medium 1339 feet.}$$

14. To find the velocity with which air rushes into a medium rarer than itself, put V = velocity with which it rushes into a vacuum, D the natural density of the air, and δ the density of the air contained in the vessel into which it is supposed to run; then

$$v = V\sqrt{\frac{D-\delta}{D}}.$$

15. To find the time in which air will fill a vacuum of given dimensions, put C = capacity of the vessel in cubic feet, A the area of the section of the orifice, h = height of homogeneous atmosphere; then

$$t = \frac{C}{4A\sqrt{h}}.$$

Atmosphere, law of repulsion in the particles of.

16. In general if the particles of a fluid repel each other with forces varying inversely as the n^{th} power of their distances or as $\frac{1}{\delta n}$, & d represent the density of any part, and c the compressive force upon it; then

$$c \text{ varies as } d^{\frac{n+2}{3}} \text{ or varies as } \frac{1}{\delta^{n+2}}.$$

It appears by experiment, that the compressive force of atmospheric air varies as the density, $\therefore \frac{n+2}{3} = 1$ or $n = 1$; consequently the particles of air repel each other with forces which vary inversely as their distances.

Cor. This fluid will be elastic, if $n + 2$ be positive.

Atmosphere, temperature of.

17. Various formulæ for the mean temperature of any place at the level of the sea.

Playfair's formula.

$t = 58^0 + 27^0 \times \cos 2$ latitude.—Fahrenheit.

When 2 latitude is greater than 90^o, cos 2 latitude is negative.

Leslie's formula.

$t = \cos^2$ lat. $\times 29^0$.—Centigrade.

Daubisson's formula.

$t = 27^0 \times \cos^2$ lat.—Centigrade.

Brewster's formula.

For the old world, $t = 81\frac{1}{2}^o \times \cos$ lat.—Fahrenheit.
For the new, $t = 81\frac{1}{2}^0 \times \cos^2$ lat. $\times$ 1.13.

Atkinson's formula.

Deduced from Humboldt's observations in the new world.—(*See Mem. Astron. Soc.)*

$t = 97^0, 08 \times \cos^{\frac{3}{2}}$ lat.—10^0, 53.—Fahrenheit.

TABLE *of mean temperature at the level of the sea in different latitudes, calculated from Leslie's formula.*

Lat.	Cent.	Fahr.	Lat.	Cent.	Fahr.
0°	29°	84.2	54	10.02	50.0
5	28.78	83.8	55	9.54	49.2
10	28.13	82.6	56	9.07	48.3
15	27.06	80.7	57	8.60	47.5
20	25.61	78.1	58	8.14	46.6
25	23.82	74.9	59	7.69	45.8
30	21.75	71.1	60	7.25	45.0
35	19.46	67.0	65	5.18	41.3
40	17.01	62.6	70	3.39	38.1
45	14.50	58.1	75	1.94	35.5
50	11.98	53.6	80	0.86	33.6
51	11.49	52.7	85	0.22	32.4
52	10.99	51.8	90	0.0	32.0
53	10.50	50.9			

Mean temperature of London, as observed at the apartments of the Royal Society for 20 years, from 1790 to 1809, = 50° 94. The greatest annual temperature during that time was 53°. 2, the least 48°. 5.

18. In ascending from the level of the sea, this mean temperature decreases nearly uniformly, though accurately the decrease seems somewhat slower as we ascend. Playfair calculates the diminution of heat at the rate of 1° for 270 feet nearly, when not far from the surface of the earth. Leslie allows 300 feet at the earth's surface; and at 1, 2, 3, 4, and 5 miles altitude, 295, 277, 252, 223, and 192 feet respectively, for every degree of Fahrenheit.

Hence to find the mean temperature at any height h above the level of the sea, we must subtract from the formulæ in the last Art. $\frac{h}{270}$ according to Playfair, $\frac{h}{300}$ according to Leslie, and $\frac{h}{251 + \frac{h}{200}}$, according to Atkinson.

19. The temperature of profuse fountains gives very accurately the mean temperature of any place; and by this method the altitude of any place above the level of the sea may be nearly ascertained. Thus suppose t = temperature of the spring (Fahrenheit), T = mean temperature due to that parallel, found by the above Table or formulæ, then

$$(T - t) \times 300 = \text{height above the level of the sea in feet.}$$

If the altitude be very considerable, 300 is too large a multiplier, and a correction must be applied thus: Let h = height found by the above rule, then

$$\frac{h \times (T - t)^2}{48600} = \text{correction to be subtracted from } h.$$

According to Atkinson *(see Mem. Astron. Soc.)* the height in feet due to any given depression of the thermometer n, is

$$h = \left\{251.3 + \tfrac{3}{2}(n-1)\right\} n.$$

$$\text{and } n = \frac{h}{251 + \frac{h}{200}} \text{ nearly.}$$

which two formulæ apply to both hemispheres.

20. To find the mean temperature of any day, under any parallel, and with any elevation.

Let λ be the mean longitude of the sun, computed from the 1st of aries for any day of the year, the mean temperature of which is y; then in these latitudes.

$$y = 58^0 + 37^0 \cos 2 \text{ lat.} - \frac{h}{270} + 15^0 \times \sin(\lambda - 30^0)$$

21. On ascending into the atmosphere, there is a certain height in every latitude, where the mean temperature is below 32^0; the curve joining all these points, is called the line of perpetual congelation; to find its height in any latitude.

$$H = 7642 + 7933. \cos. 2 \text{ L.} \quad \textit{(Playfair.)}$$

TABLE *of the height of the curve of congelation in different latitudes, as computed by Leslie.*

Lat.	Ht. of curve in feet.	Lat.	Ht. of curve in feet.
0^0	15207	54	5290
5	15095	55	5034
10	14764	56	4782
15	14220	57	4534
20	13478	58	4291
25	12557	59	4052
30	11484	60	3818
35	10287	65	2722
40	9001	70	1778
45	7671	75	1016
50	6334	80	457
51	6070	85	117
52	5808	90	00
53	5548		

In our latitudes, the altitude of the point of congelation may be found with sufficient precision by multiplying the mean temperature—32° by 300, and correcting as in Art. 19.

We will conclude this Article with the following short Tables and observations :—

TABLE *exhibiting the different gradations of the mean annual temperature in Western Europe and North America, continuing the scale to the Equator.—(Humboldt.)*

Lat.	Old World.	New World.	Difference.
0°	81°. 5	81°. 5	0
20	77. 9	77. 9	0
30	70. 7	67. 1	3.6
40	63. 5	54. 5	9
50	50. 9	38. 3	12.6
60	41. 0	25. 0	16
70	33. 0	0. 0	33

The difference of mean temperature between summer and winter (reckoning each to consist of three months), is nothing at the equator, and constantly increases as we approach the pole, as shown in the following Table :—

	Lat.	Mean temperat. of winter.	Mean temperat. of summer.	Differ.
Algiers	37°	61°. 5	80°. 2	18°.7
Buda	47½	34. 0	70. 5	26.5
Upsal	60	25. 0	60. 2	35.2

The following Table of mean annual temperature, drawn up principally by M. dè. Humboldt, is worth the attention of meteorologists. Those cities, to which an asterisk is attached, are singularly situated with respect to climate, either by their elevation above the level of the ocean, or by circumstances independent of the latitude :—

	Lat.	Temp.
Melville Island	74°. 47′	1. 33°
Umeo	63. 50	33. 25
Petersburgh	59. 56	38. 84
Upsala	59. 51	41. 90
Stockholm	59. 20	42. 26
Copenhagen	55. 41	45. 68
Berlin	52. 31	46. 58
London	51. 31	50. 36
Paris	48. 50	51. 26
Vienna	48. 13	50. 54

	Lat.	Temp.
Geneva *	46. 12	50. 18
Milan	45. 28	55. 76
Marseilles	43. 17	57. 74
Toulon*	43. 3	63. 50
Rome	41. 53	60. 26
Naples	40. 50	64. 40
Madrid*	40. 25	59. 00
Havannah	23. 10	78. 08
Mexico*	19. 25	62. 60
Vera Cruz*	19. 11	77. 72
La Plata*	2. 24	74. 66
Equator at the level of the sea	0. 0	80. 60
Quito*	0. 14	59. 00

From a general and extensive review of the various experimental data respecting the temperatures observed at different places on the earth's surface, the Editor of the Annales de Chimie deduces the following consequences.—(*Ann. de Chimie*, xxvii. 432.)

In no place on the earth's surface, nor at any season, will a thermometer raised 2 or 3 metres above the soil, and sheltered from all reverberation, attain the 37° of Reaumur, or 46° centigrade, or 114°. 8 Fahrenheit.

On the open sea, it will never attain 25° Reaumur, or 31° centigrade, or 87°. 8 Fahrenheit.

The greatest degree of cold ever observed on our globe in the air, is 40° Reaumur, or 50° centigrade below Zero, (58° Fahrenheit.)

The temperature of the water of the ocean in any latitude, or at any season, never rises above 24° Reaumur, or 30° centigrade, (86° Fahrenheit.)

AXIS, *to find the angle at which a curve cuts.—(Higman.)*

Find the value of $\frac{dy}{dx}$ in the given curve, take $y = o$, and we shall get the tangent of the angle required.

Ex. Let the Equation be $y = \frac{x}{a} \cdot \sqrt{a^2 - x^2}$.

Here $\frac{dy}{dx} = \frac{\sqrt{a^2 - x^2}}{a} - \frac{x^2}{a\sqrt{a^2 - x^2}}$; now $y = o$, when $x = o$, and when $x = a$, and the values of $\frac{dy}{dx}$ are 1 and infinite respectively; $\therefore$ the curve cuts the axis at an angle of 45° at the origin, and at right angles when $x = a$.

AXIS, *rotation of bodies about.*—*See Rotation.*

B

BALANCE. *(Playfair.)*

The balance, when well constructed, must have the following properties. (1.) It should rest in a horizontal position, when loaded with equal weights. (2.) It should have great sensibility, i. e. the addition of a small weight in either scale should disturb the equilibrium, and make the beam incline sensibly from the horizontal position. (3.) It should have great stability, i. e. when disturbed, it should quickly return to a state of rest.

That the first requisite may be obtained, the beam must have equal arms; and the centre of suspension must be higher than the centre of gravity. Were these centres to coincide, the sensibility would be the greatest possible, but the other two requisites of level and stability would be entirely lost.

The 2d requisite is the sensibility of the balance. If a be the length of the arm of the balance, and b the distance between the centre of suspension and the centre of gravity, P the load in either scale, and W the weight of the beam, the sensibility of the balance is as $\frac{a}{b\,(2\,P + W)}$; it is $\therefore$ greater, the greater the length of the arm, the less the distance between the two centres, and the less the weight with which the balance is loaded.

Lastly, the stability is proportional to $(2\,P + W)\,b$. The diminution of b $\therefore$, while it increases the sensibility, lessens the stability of the balance. The lengthening of a will, however, increase the former of these quantities, without diminishing the latter.

Hence the merit of balances depends upon the quantities a, b, and W.

BALLOON.—*See Æronautics.*

BALLS *iron and leaden, weight of.*—*See Shot.*

BAROMETER.

1. *Barometer, scale of.*

The usual scale of the Barometer is 31 very dry, or hard frost; 30. 5. settled fair or frost; 30 fair or frost; 29. 5. changeable; 29 rain or snow; 28. 5. much rain or snow; 28 stormy.—*(Young's Nat. Phil.)*

2. *Barometer, measurement of heights by.*

Professor Robison's formula in feet, without logarithms.

Let f = mean temperature of air at the two stations; d = difference of Barometric heights in *tenths* of an inch; m = mean Barometric heights; δ = difference of mercurial temperatures; then;

$$\text{Height} = \frac{30 \times \left(87 + 0.21\ (f - 32^0)\right) \times d}{m} \mp \delta \times 2.83,$$

— when the attached thermometer is highest at the lower station, and *v, v.*

Sir G. Shuckburgh's formula in fathoms.

Let l = difference of logarithms of the heights of Barometer in inches; d = difference of mercurial temperatures; f as before; then

$$\text{Height} = (10000l \mp 0.440d) \times (1 + \overline{f - 32^0} \times .00244),$$

— when the attached thermometer is highest at the lower station, and *v, v.*

Playfair's formula in fathoms, which does not differ much from La Place's.

Let b and β be the height of the Barometer at the *lowest* and *highest* stations, t and t' the temperatures of the air (Fahr.) at those stations, q and q' the temperatures of the mercury in the two stations; then

$$\text{Height} = 10000 \left\{1 + .00244.\ \left(\frac{t + t'}{2} - 32^0\right)\right\} \log. \frac{b}{\beta\left(1 + \frac{q - q'}{10000}\right)}$$

Formula Encyc. Metrop.

The height in *feet* is

$$60347\ \left(1 + \frac{t + t'}{900}\right) \log. \frac{b}{\beta\left(1 + \frac{q - q'}{9742}\right)}$$

where t and t' denote the number of degrees *above* the freezing point of Fahrenheit. This formula differs very little from the last.

3. *Barometer, correction of observed heights in.*

When the mercury in the tube of a Barometer sinks, and the surface of that in the basin rises; to determine the correction.

Let a = the section of the tube, and b = that of the basin, supposed cylindrical; then apparent diminution of height : the real diminution :: $b - a$: b. In the best Barometers there is a contrivance for bringing the mercury in the basin always to the same level, which obviates the necessity of this correction.

Barometer, correction of observed heights in, as far as regards a change of temperature.

Given the temperature of the mercury in a Barometer, measured by the attached thermometer; to reduce the observed height to what it would have been at any other temperature, as for instance 32°.

Let b = observed height of Barometer, f = temperature; then true height at temperature 32° = (see Art. 2) $b \times \left(1 - \frac{f - 32^\circ}{10000}\right)$.

4. *Barometer, range of.*

Annual range of Barometer does not exceed from ¼ to ½ an inch in the torrid zone; about two inches at Liverpool, the same at St Petersburg; at Melville Island, as observed by Capt. Parry, $1\frac{8}{10}$. The extreme variation scarcely any where exceeds 3 inches, viz. from 28 to 31 inches. In the apartments of the Royal Society (the barometer being 81 feet above low water), during a period of 22 years, viz. from 1800 to 1821, both inclusive, the mean height was 29.86; the greatest height 30.77; the least height 28.18; and consequently the greatest range 2.59; the mean annual range during the same period was 1.92. The barometer was once observed at Middlewick, as high as 31.00. Greatest height ever observed by Sir G. Shuckburgh, in London, was 30.957. In these climates, the barometer is generally lowest at noon and at midnight. The mean height is greatest at the Equinoxes, but greater in summer than in winter.

5. *Barometer, mean height of.*

Mean height of the Barometer in various places, from Erxleben, and others.—(Young's Nat. Phil.)

Upsal	30. 15
S. Carolina	30. 09
Mean level of the sea. Fleuriau	30. 095
Atlantic. Burckhardt.	30. 09
Mediterranean. Do.	30. 04
Mean in England and Italy. Shuckburgh	30. 04
Mean level of the sea as usually estimated	30. 00
Fort St George	30. 00
Columbo	29. 98
Dover	29. 90
London R. S. 81 feet above the level of low water. The mean of any year scarcely differing 0.5.	29. 89
Leyden	29. 84
Kendal	29. 80
Padua	29. 80

Panama	29. 80
Porto Bello	29. 80
Liverpool	29. 74
Turin	29. 62
Petersburg	29. 57
Gottingen	29. 37
Paris	29. 31
Basle	28. 82
Nuremberg	28. 69
Zurich	28. 29
Clausthal	27. 89
Chur	27. 71
M. St Gothard	23. 05
Quito	21. 37

We shall close this article with the following Proposition :—

If a Barometer tube be in part only filled with mercury, and then its open end be immersed in a basin of the same fluid, the mercury will sink below the point called the standard altitude, or the point at which it would have stood if no air had been left in; and the standard altitude will be to the depression below that altitude, as the space occupied by the air after the immersion, to the space occupied before.

This Proposition may be applied to the solution of two problems; for we may either give the quantity of air left in before immersion, to find the altitude of the mercury after immersion; or we may give the altitude of the mercury after immersion, to find the quantity of air left in before.

Ex. Let 5 inches of air be left in a tube of 35 inches before inversion, to find the altitude of the mercury after.

Let x = depression below the standard altitude — then $30 : x :: x + 5 : 5$, $\therefore x = 10$.

BARS *Iron, to find the weight of.—(Gregory.)*

The following is an approximate rule for finding the weight of cast iron bars :—

Take $\frac{7}{144}$ of the product of the breadth and thickness, each in eighths of an inch; the result is the weight of one foot in length, in avoirdupois pounds.

Hence an inch square cast iron bar would require 9 feet, or 108 inches in length for $\frac{1}{4}$ cwt. For *wrought iron* square bars, allow 100 inches in length of an inch square bar to $\frac{1}{4}$ cwt.

BELLOWS *Hydrostatical.*—*See Fluids pressure of.*

BINOMIAL THEOREM.

This series, in its most simple form, is as follows :—

$$(a+b)^n = a^n + n\,a^{n-1}b + n.\frac{n-1}{2}a^{n-2}b^2 + n.\frac{n-1}{2}.\frac{n-2}{3}$$

$$a^{n-3}b^3 + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}a^{n-4}b^4 + \&c.$$ where n is a whole number or fraction, positive or negative.

If b be negative, the odd powers of b will be also negative.

Cor. 1. The n^{th} term of the series is $n.\frac{n-1}{2}.\frac{n-2}{3}\ldots\frac{n-\overline{n-2}}{n-1}.$ $a^{n-\overline{n-1}}\,b^{n-1}$.

Cor. 2. If n be a positive whole number, the series will consist of $n+1$ terms, but in every other case, the number of terms will be unlimited.

Cor. 3. If n be a whole positive number, the whole sum of the indices $= .n\,\overline{n+1}$.

Cor. 4. If n be a whole positive number, and b also positive, the sum of the coefficients of $(a+b)^n = 2^n$; but if b be negative, the sum of the coefficients $= o$; this appears by expanding the series, and making $a = b$.

Cor. 5. If we call the index $\frac{m}{n}$, and put $\frac{b}{a} = Q$, and let A, B, C, D, &c. represent the 1st, 2d, 3d, &c. terms of the series, with their proper signs, we shall have $(a+b)^{\frac{m}{n}} =$

$$a^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}.BQ + \frac{m-2n}{3n}.CQ + \frac{m-3n}{4n},DQ + \&c.$$

This is the most convenient practical form in the case of fractional or negative indices.

Ex. 1. $(a^2+z^2)^{\frac{1}{2}} = a + \frac{z^2}{2a} - \frac{z^4}{8a^3} + \frac{z^6}{16a^5} - \&c.$

2. $(1-x)^{\frac{1}{4}} = 1 - \frac{x}{4} - \frac{3x^2}{4.8} - \frac{3.7.x^3}{4.8.12} - \&c.$

3. $(a-z)^{\frac{1}{3}} = a^{\frac{1}{3}} - \frac{z}{3a^{\frac{2}{3}}} - \frac{z^2}{9a^{\frac{5}{3}}} - \frac{5z^3}{81a^{\frac{8}{3}}} - \&c.$

4. $(a + x)^{-2} = \frac{1}{a^2} - \frac{2x}{a^3} + \frac{3x^2}{a^4} - \frac{4x^3}{a^5} +$ &c.

5. $\frac{1}{2az + z^2} = \frac{1}{2az} - \frac{1}{4a^2} + \frac{z}{8a^3} -$ &c.

In expanding a trinomial, quadrinomial, mutinomial, consider every term, except the 1st., as the 2d term of a binomial, and then proceed according to the rule.

Ex. 1. $(a + b + c)^2 = (a + \overline{b + c})^2 = a^2 + 2a.\,\overline{b + c} + (b + c)^2$ $= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc.$

2. $(a + b + c)^3 = (a + \overline{b + c})^3 = a^3 + 3a^2.\,\overline{b + c} + 3a.\,(b + c)^2$ $+ (b + c)^3.$

3. $(a + b + c + d + \&c.)^n = a^n + n\,a^{n-1}.\,(b + c + d + \&c.)$

$+ n.\,\frac{n-1}{2}\,a^{n-2}\,(b + c + d + \&c.)^2 + n.\,\frac{n-1}{2}.\,\frac{n-2}{3}.\,a^{n-3}.$

$(b + c + d + \&c.)^3 +$ &c. but *see Demoivre's Analyt. p.* 87.

BISSEXTILE.—*See Calendar.*

BOILING *point of various liquids.—See Heat.*

BRIDGE.—*See Arches equilibrium of.*

BRIDGES.

List of a few of the most remarkable modern Bridges, with the date of their erection, the lengths of the chord and versed sine of the centre arch in feet, &c. &c.

STONE BRIDGES.

Place.	*Date.*	*No. Arches.*	*Chord.*	*Ver. sin.*	*Curve.*
Avignon, Rhone	1188	18	110¾	45¾	circular.
Brioude, Allier	1454	—	183	70¼	circular.
This is the largest stone arch in existence.					
Florence, Arno	1569	1	95¼	14¾	elliptical.
The Rialto, Venice	1591	1	96¾	20½	
Grenoble, Drac	1611	—	150	62¼	circular.
Orleans, Loire	——	9	106½	29¾	false ellipse.
Pont Royal, Seine	1685	5	82	——	
Neuilly	1774	5	128	32	false ellipse.
Nantes, Seine	1765	3	128	38¼	elliptical.

Place.	Date.	No. Arches.	Chord.	Ver. sin.	Curve.
Maxence, Oise	1785	—	76¾	6¼	
Pont de la Concorde, Paris	1791	—	93¾	9¾	circular.
Saumur, Loire	1770	12	60	21	elliptical.
Bridge of Jena, Paris	1815	—	91½	10¾	circular.
Ulm, Danube	1806	1	181¼	22¼	circular.
Burton, Trent	1200	34	1545 long, longest in Britain.		
London Bridge	1176	20	70½	22¾	circular.
Llanwrst, Conway	1600	3	58	17	circular.
Pont y Pryd, Taaf	1752	1	140	35	circular.
Blackfriars	1771	—	100	41½	elliptical.
Waterloo	1818	9	120	32	elliptical.
Westminster	1750	15	76	—	circular.

IRON BRIDGES.

	Chord.	Ver. sin.
Colbrook Dale	100	45
Sunderland	240	30
Buildwas	130	27
Bristol	100	15
Boston	85	5½
Southwark	240	24
Bonar	150	20

SUSPENSION BRIDGES.

	Chord.
Menai Bridge	560
Its suspended weight 490 tons.	
Berwick	432
Dryburgh	261
Middleton, Tees	70
Proposed Bridge at Runcorn	1000

C

CABLES *strength of.*—*See Cords.*

CALENDAR.

The civil year consists of 365 days; the real tropical year of 365*d.* 5*h.* 48*m.* 51,6*s.* The excess therefore of the tropical year amounts to nearly 24 hours, or one day, in four years. Hence the necessity of intercalating a day every fourth year, effected by making February contain 29 days. This correction was first applied by Julius Cæsar, and the year on which it fell was called by him Bissextile, by us Leap year. As it occurs every 4th year, and every 100th year was a leap year in the Julian account, it follows that every year divisible by four is a leap year. This correction is evidently too great by nearly twelve minutes, which would amount to one day in about 129 years. By the omission of this second correction, an error crept into the calendar, which was first amended by Pope Gregory, in 1582, who wishing to bring the vernal equinox to the 21st of March, the day on which it happened in the year 325, when the council of Nice was held, suppressed 10 days. The correction of the stile did not take place in England till 1752, at which time a suppression of 11 days became necessary. This is called by us the new stile. To correct the error in future, three intercalary days are omitted every 400 years. Thus the centenary years 1700, 1800, 1900, which ought to have been leap years, were ordered not to be so; and the same in 2100, 2200, 2300, and so on for succeeding centuries. The error of the calendar, as at present constituted, will not amount to one day in less than 4237 years.

CAPILLARY *Tubes.—(Playfair.)*

Glass tubes so called of which the diameter is less than $\frac{1}{10}$ of an inch.

1. The height to which water rises, and mercury sinks, in capillary tubes, varies inversely as the diameter of the tubes.

If the bore is $\frac{1}{100}$th of an inch, the rise is 5,3 inches.

2. If a capillary tube, composed of two cylinders of different bores, be immersed in water, first with the widest part downward, and afterwards with the narrowest, the water will rise in both cases to the same height.

3. If two plates of glass be kept parallel and near to one another, and if their ends be immersed in water, the water will ascend between them to half the height it would rise to in a tube having its diameter equal to the distance of the plates.

When the plates make an angle with one another, if they be immersed with the line of their intersection vertical, the water will ascend between them and form an hyperbola.

4. To find the diameter of a capillary tube, put into the tube some mercury, whose weight in grains $= w$, and let it occupy a length of the tube $= l$, then

$$\text{Diameter} = \sqrt{\frac{w}{l}} \times ,09123 \text{ in inches.}$$

CATENARY *Equations, &c. to.*

Let x, y, and z be the abscissa, ordinate, and curve, then the Equations to the curve are,

$$dy = \frac{a\,dx}{z}.$$

$$dx = \frac{z\,dz}{\sqrt{a^2+z^2}}.$$

$$z = \sqrt{2\,a\,x + x^2}.$$

$$y = a \times h.\,l.\,\frac{z+x}{z-x}.$$

$$z = \frac{a}{2}\left(\varepsilon^{\frac{y}{a}} - \varepsilon^{-\frac{y}{a}}.\right)$$

$$y = a \times h.\,l.\,\frac{x + a + \sqrt{x^2 + 2\,a\,x}}{a}.$$

$$\text{Subtangent} = \frac{z\,y}{a}.$$

$$\text{Area} = y\,\sqrt{a^2 + z^2} - a\,z.$$

$$\text{Surface} = 2\,\pi\left(y\,z + a^2 - a\,\sqrt{a^2+z^2}\right)$$

$$\text{Content of solid} = \pi\left(\overline{2\,a^2 + y^2} \times \overline{a + x} - 2\,a \times \overline{y\,z + a^2}\right).$$

CAUSTICS.—*(Coddington.)*

I. *Caustics produced by reflection.*

1. Given a point, from which a thin pencil of rays proceeding fall on a curved reflector, to determine their intersections after reflection.

Let the incident ray .... $= u$
the reflected ray .... $= v$
$\angle$ of incidence .... $= \phi$
Perpendicular on the tangent .... $= p$
Principal focal distance of reflector ... $= f$

then we have the following equations,

$$d u + d v = o.$$

$$v = \frac{u f . \cos \phi}{u - f \cos \phi}; \text{ or } \frac{1}{u} + \frac{1}{v} = \frac{1}{f \cos \phi}.$$

$$\text{also } v = \frac{d u}{\frac{2 d p}{p} - \frac{d u}{u}} = \frac{d u}{d \log. \frac{p^2}{u}}.$$

2. Given the radiant point and the reflecting surface, to find the caustic.

Let p and u be the perpendicular and radius vector of the reflecting curve, p' and $u' =$ do. of the caustic, the rest as before, then

$$u'^2 = (u + v)^2 - \frac{4 p^2 v}{u}.$$

$$\text{and } p' = 2 p \sqrt{1 - \frac{p^2}{u^2}}.$$

For v put its value $\frac{u f \cos. \phi}{u - f \cos \phi}$ or $\frac{d u}{d \log \frac{p^2}{u}}$ and for p the proper function of u given by the equation to the original curve, let u be then eliminated, and we shall have an equation in u' and p', which will be that of the caustic.

Ex. Let the reflecting curve be the log. spiral.

$$\text{Here } p = m u, \; v = \frac{d u}{\frac{2 d p}{p} - \frac{d u}{u}} = u;$$

$p' = 2 m u' \sqrt{1 - m^2}$; $u'^2 = 4 u^2 - 4 \frac{m^2 u^2 u}{u} = 4 u^2 (1 - m^2)$; hence $p' = m u'$; the caustic is therefore another log. spiral.

An equation in rectangular coordinates may also be obtained, but the method is too long for insertion here.

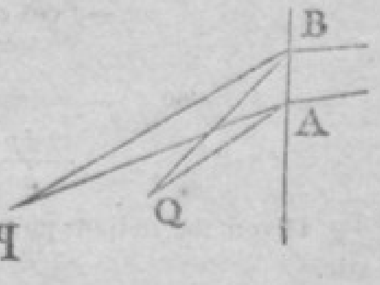
There are some simple cases in which it is easy to determine the nature of the caustic by geometrical investigation; for instance, when the reflecting curve is a circular arc, and parallel rays are incident in the plane of the circle; or when the focus of incident rays is in the circumference of the circle, the caustic in either case may be proved geometrically to be an epicycloid. When the reflecting curve is a common cycloid, and the rays are incident parallel to its axis, the caustic is also a common cycloid.

II. *Caustics produced by refraction.*

1. Required the focus of a thin pencil of rays, after being refracted obliquely at a plane surface.

Let Q and q be the foci of incident and refracted rays, θ and θ' the $\angle$s. of incidence and refraction, then

$$q\,A : Q\,A :: \frac{\sin\theta}{\cos^2\theta} : \frac{\sin\theta'}{\cos^2\theta'}$$



2. Required the same at a curved surface.

Let the incident ray .... $= u$
the refracted ray .... $= v$
$\angle$ of incidence .... $= \phi$
$\angle$ of refraction .... $= \phi'$
Radius of curvature of the surface at the point of incidence .... $= r$

then

$$v = \frac{u\,r\cos\phi' . \tan\phi}{u\tan\phi - (u + r\cos\phi)\tan\phi'}$$

$$\text{and } \frac{1}{v} = \frac{1}{r\cos\phi'} - \frac{(u + r\cos\phi)\sin\phi'}{u\,r\cos\phi'^2 . \tan\phi}.$$

When u is infinite, or the incident rays are parallel,

$$v = \frac{r\cos\phi' . \tan.\phi}{\tan\phi - \tan\phi'} = \frac{r\cos\phi'^2 . \sin\phi}{\sin(\phi - \phi')}.$$

When ϕ is a right angle, or u a tangent to the surface,

$$v = r\cos\phi'.$$

When v is infinite, or the refracted rays parallel,

$$u = \frac{r\cos\phi\tan\phi'}{\tan\phi' - \tan\phi} = r\cos\phi^2 . \frac{\sin\phi'}{\sin(\phi' - \phi)}.$$

For further information on this subject, *see Coddington's Optics.*

CENTRAL FORCES.

1. *Of the motion of bodies in circular orbits.*

Let V = velocity of a body in a circle, R = radius, P = periodic time, F = accelerating force, $\pi = 3.14159$ &c. then

$$F = \frac{V^2}{R}, \text{ or } = \frac{4\pi^2 R}{P^2}.$$

Cor. 1. Hence $V = \sqrt{F \times R}$, or $= \frac{2\pi R}{P}$;

$$\text{and } P = 2\pi\sqrt{\frac{R}{F}}, \text{ or } = \frac{2\pi R}{V}.$$

Ex. 1. If F varies as $\frac{1}{R^2}$ or $= \frac{\phi}{R^2}$ ($\phi =$ absolute force),

$V = \sqrt{\frac{\phi}{R}}$, and $P = \frac{2\pi R^{\frac{3}{2}}}{\phi^{\frac{1}{2}}}$. And in general if F varies as $\frac{1}{R^{2n-1}} = \frac{\phi}{R^{2n-1}}$; $V = \frac{\phi^{\frac{1}{2}}}{R^{n-1}}$, and $P = \frac{2\pi R^n}{\phi^{\frac{1}{2}}}$.

Ex. 2. If the body revolve at the earth's surface $F = g = 32\frac{1}{6}$ feet, and R = the earth's radius, $\therefore V = \sqrt{gr}$, and $P = 2\pi\sqrt{\frac{R}{g}}$. If the body revolve at any other distance x from the earth's centre, $V = R\sqrt{\frac{g}{x}}$, and $P = \frac{2\pi}{R}\sqrt{\frac{x^3}{g}}$.

Cor. 2. The same formulæ are applicable to the centrifugal force; Hence if $v =$ velocity of the earth round its axis, and $p =$ time of its revolving round its axis, centrifugal force at the equator $= \frac{v^2}{R}$, or $= \frac{4\pi^2 R}{p^2}$.

Ex. Centrifugal force at the Equator : centripetal :: $\frac{4\pi^2 R}{p^2} : g$ :: $\frac{R}{p^2} : \frac{g}{4\pi^2}$.

2. *Of the centripetal force of bodies revolving in any trajectories.*

Let P V = chord of curvature passing through the centre of force, $y =$ radius vector, $p =$ perpendicular on the tangent, $a =$ area described dat. temp. then

$$F = \frac{V^2}{\frac{1}{2} PV}, \text{ or } = \frac{4 a^2 dp}{p^3 dy}.$$

Ex. 1. If a body revolve in an ellipse, (force tending to the centre)

$$F = \frac{4 a^2 \times CP}{AC^2 \times CB^2}.$$

Ex. 2. If bodies revolve in the conic sections the force tending to the focus, $F = \frac{8a^2}{Ly^2}$, where L = lat. rect.

Cor. The space through which a body P must fall, the force at P continuing uniform, to acquire the velocity in the curve $= \frac{PV}{4}$. If the curve be a circle, space $= \frac{R}{2}$.

3. *Of the linear velocity of bodies revolving in trajectories round a centre of force.*

$$\text{Here } V = \sqrt{F \times \tfrac{1}{4} PV}, \text{ or } = \frac{2a}{p}.$$

And velocity (V) in any point of a curve : velocity (v) of a body revolving in a circle at the same distance :: $\sqrt{PV} : \sqrt{pv} :: \sqrt{\frac{dy}{y}} : \sqrt{\frac{dp}{p}}$.

Ex. 1. In an ellipse (the centre of force being in the centre), $V = \phi^{\frac{1}{2}} \times CD$. Also $V : v :: CD : CP$.

Ex. 2. In conic sections, having the centre of force in the focus, $V = \sqrt{\frac{\phi L}{2}} \times \frac{1}{SY}$; or, by substitution, we have in the parabola $V = \sqrt{\frac{2\phi}{SP}}$. In ellipse and hyperbola, $V = \sqrt{\frac{\phi \times PH}{AC.SP}}$.

Ex. 3. In the conic sections (force in the focus) $V : v :: \sqrt{HP} : \sqrt{AC}$. In the parabola, this ratio becomes that of $\sqrt{2} : 1$; in ellipse, that of $\sqrt{2} - : 1$; in the hyperbola, that of $\sqrt{2} + : 1$.

Ex. 4. In the ellipse velocity at any distance SP : velocity at the mean distance :: $\sqrt{HP} : \sqrt{SP}$.

4. *Of the angular velocities of bodies revolving in trajectories.*

Let a = area described dat. temp. y = distance, then

$$\text{Angular velocity varies as } \frac{a}{y^2}.$$

Ex. 1. In the conic sections (force in the focus) angular velocity varies as $\frac{L^{\frac{1}{2}} \times \phi^{\frac{1}{2}}}{y^2}$.

Ex. 2. Angular velocity in a conic section : do. in a circle at the same distance :: $(\frac{1}{2} L)^{\frac{1}{2}}$: $(SP)^{\frac{1}{2}}$.

Ex. 3. Angular velocity in ellipse : mean angular velocity :: $\frac{1}{SP^2}$: $\frac{1}{AC.CB}$.

Ex. 4. Angular velocity at mean distance : mean angular velocity :: CB : CA.

Let α be the angular velocity in any curve, then the rate at which it decreases, or

$$d\,\alpha \text{ varies as } \frac{\sqrt{y^2 - p^2}}{p\,y^4}.$$

Ex. 1. In a parabola, the decrement of the angular velocity is a maximum, when $y = \frac{8a}{7}$ ($a = \frac{1}{4}$ L. R.)

Ex. 2. In the ellipse, the decrement is a max. when $3y^2 - 7ay + 4b^2 = o$.

5. *Of the paracentric velocity in any curve.*

$$\text{Par. velocity varies as } \frac{\sqrt{y^2 - p^2}}{p\,y}.$$

Ex. 1. In parabola, par. velocity is a maximum when $y = 2a$.

Ex. 2. In ellipse, $y = \frac{b^2}{a} = \frac{1}{2}$ L.

6. *Of the centrifugal force of bodies revolving in trajectories.*

Let a = twice area described in 1″, then

$$\text{Centrifugal force} = \frac{a^2}{y^3}.$$

And centripetal force : centrifugal :: $2\,SP^3$: $SY^2 \times PV$, or :: $\frac{y^3}{p^3}$: $\frac{dy}{dp}$.

Ex. 1. In an ellipse (force in the centre), centripetal : centrifugal force :: CP^4 : $AC^2 \times CB^2$.

Ex. 2. In conic sections (force in the focus) centrifugal force $= \frac{L \times \phi}{2\,SP^3}$; and centripetal : centrifugal force :: SP : $\frac{1}{2}$ L.

7. *Of the periodic times of bodies revolving in trajectories.*

Let A = whole area, a = area dat. temp., then

$$\text{P. T.} = \frac{\text{A}}{a}.$$

Ex. 1. In ellipses (force in the centre), the periodic times $= \frac{2\pi}{\phi^{\frac{1}{2}}}$, and are therefore equal in all ellipses.

Ex. 2. In an ellipse (force in the focus). $\text{PT} = \frac{2\pi\,\text{AC}^{\frac{3}{2}}}{\sqrt{\phi}}$.

CENTRE *of Gravity.—(Vince, Playfair.)*

1. To find the centre of gravity of two given bodies, divide the distance between them in the inverse ratio of their quantities of matter, and the point so determined is the centre of gravity.

2. To find the centre of gravity of any number of bodies placed in the same straight line.

Let the bodies be A, B, C, D, &c. and their distances from a given point in the straight line be a, b, c, d, &c. then the distance of their centre of gravity from this point is

$$\frac{\text{A}\,a + \text{B}\,b + \text{C}\,c + \text{D}\,d, \text{ \&c.}}{\text{A} + \text{B} + \text{C} + \text{D}, \text{ \&c.}}.$$

3. In general, the distance of the centre of gravity of any system of bodies from a given plane, is equal to the sum of the products of all the masses, into their distances from the plane, divided by the sum of the masses.

Cor. If any of the bodies in this and the last Art. lie on the other side of the point or plane, their distances must be reckoned negative.

4. Any number of bodies being given in position, to find their centre of gravity.

The bodies must be referred to three planes given in position, cutting one another at right angles, one of them horizontal, and of course the other two vertical. Let the bodies be A, B, C, D, their distances from the given horizontal plane, a, b, c, d; their distances from one of the vertical planes a', b', c', d', and from the other a'', b'', c'', d''; then if we take $x = \frac{\text{A}\,a + \text{B}\,b + \text{C}\,c + \text{D}\,d}{\text{A} + \text{B} + \text{C} + \text{D}}$, the centre of gravity of the system is in a horizontal plane at the distance x from the given horizontal plane. Again take $x' = \frac{\text{A}\,a' + \text{B}\,b' + \text{C}\,c' + \text{D}\,d'}{\text{A} + \text{B} + \text{C} + \text{D}}$ and the centre of gravity is in a plane parallel to the first of the two vertical planes, and distant from it by the line x'. Lastly, take in the intersection of these planes a point

distant from the second vertical plane by a quantity $x'' =$
$\frac{A a'' + B b'' + C c'' + D d''}{A + B + C + D}$; and this point will be the centre of gravity of the given bodies, as is evident from the last Art.

5. If a body be placed upon a horizontal plane, and a line drawn from its centre of gravity perpendicular to that plane, the body will be sustained or not, according as the perpendicular falls within or without the base.

6. If a body be suspended by a point, it will not remain at rest till the centre of gravity is in the line which is drawn through that point perpendicular to the horizon.

Cor. Hence to find the centre of gravity of any plane mechanically, suspend it by a given point in or near its perimeter, and when it is at rest, draw across it a vertical line passing through that point. Suspend it in like manner by another point, and draw a vertical line as before. The intersection of these lines is the centre of gravity of the plane.

7. If any momenta be communicated to the parts of a system, its centre of gravity will move in the same manner that a body equal to the sum of the bodies in the system would move, were it placed in that centre, and the same momenta communicated to it in the same directions.

8. In any machine kept in equilibrium by the action of two weights, if an indefinitely small motion be given to it, the centre of gravity of the weights will neither ascend nor descend.

9. Formulæ for finding the centre of gravity of a body considered as an area, solid, surface, or curve.

Let x, y, and z, represent the abscissa, ordinate, and curve, D = distance of vertex from the centre of gravity; then

For an area, $D = \frac{\text{fl. } y x d x}{\text{fl. } y d x}$.

For a solid, $D = \frac{\text{fl. } y^2 x d x}{\text{fl. } y^2 d x}$.

For a surface, $D = \frac{\text{fl. } y x d z}{\text{fl. } y d z}$.

For a curve line, $D = \frac{\text{fl. } x d z}{z}$.

Ex. 1. In a triangle and conical surface, let a be the line from the vertex bisecting the base, then $D = \frac{2a}{3}$.

2. In parabola, $D = \frac{3}{5}$ altitude.

3. In a ½ circle distance from centre $= \frac{4r}{3\pi}$.

4. In a cycloid, $D = \frac{7a}{12}$.

5. In a sector of a circle, distance from centre $= \frac{2}{3}\ \frac{\text{rad. chord}}{\text{arc}}$.

6. In a circular arc, distance from centre $= \frac{\text{rad.} \times \text{chord}}{\text{arc}}$.

7. In cones and all regular pyramids, $D = \frac{3}{4}$ altitude.

8. In a paraboloid, $D = \frac{2}{3}$ altitude.

9. In ½ sphere and ½ spheroid $D = \frac{5r}{8}$.

10. In the surface of a ½ sphere, $D = \frac{r}{2}$.

CENTRE *of Gyration*.

Let A, B, C, &c. be the bodies, or the particles of which the body is composed, S the point round which the particles revolve, D = distance of the centre of gyration from the axis, then

$$D = \sqrt{\left(\frac{A \times SA^2 + B \times SB^2 + C \times SC^2 + \&c.}{A + B + C \times \&c.}\right)}$$

Or if ds be the differential of the body at the distance x from the axis,

$$D = \sqrt{\left(\frac{\text{fl. } x^2\, ds}{s}\right)}.$$

Ex. 1. In a straight line, $D = \frac{\text{Length}}{\sqrt{3}}$.

2. In a circle revolving in its own plane, round its centre, or in a cylinder, $D = \frac{r}{\sqrt{2}}$.

3. In the periphery of a circle revolving about its diameter, $D = \frac{r}{\sqrt{2}}$

4. In the plane of a circle revolving round its diameter, $D = \frac{r}{2}$.

5. In a sphere revolving round its diameter $D = r\sqrt{\frac{2}{5}}$.

6. In the surface of a sphere, $D = r\sqrt{\frac{2}{3}}$.

7. In a cone about its axis, $D = r\sqrt{\frac{3}{10}}$.

CENTRE *of Oscillation.*

1. Let D = distance of the point of suspension from the centre of oscillation, δ = distance from the centre of gravity, then

$$D = \frac{A + S A^2 + B + S B^2 + C + S C^2 + \&c.}{(A + B + C + \&c.) \times \delta}.$$

Or if $d\,s$ be the differential of the body at the distance x from the axis,

$$D = \frac{\text{fl. } x^2\,d\,s}{s \times \delta}.$$

2. If S be the point of suspension, G the centre of gravity, O the centre of oscillation,

$$G\,O \text{ varies as } \frac{1}{S\,G}.$$

Cor. If O be made the point of suspension, S will be the centre of oscillation; or the centre of oscillation and the point of suspension are convertible.

3. If R be the centre of gyration,

$$S\,G : S\,R :: S\,R : S\,O.$$

Ex. 1. In a straight line, $D = \frac{2\,L}{3}$.

2. In an isosceles triangle vibrating flat ways, $D = \frac{3}{4}$ alt.

3. In a circle flat ways, $D = \frac{5}{4} r$.

4. In a parabola flat ways, $D = \frac{5}{7}$ alt.

5. In a sphere, $D = a + \frac{2\,r^2}{5\,a}$ (a = distance of the point of suspension from the centre of the sphere.)

6. In a cone, $D = \frac{4}{5}\text{ axis} + \frac{(\text{rad. of base})^2}{5\text{ axis}}$.

7. In a circle vibrating edgeways, $D = \frac{3}{2} r$.

8. In a sector of a circle edgeways, $D = \frac{3\text{ arc} \times \text{rad.}}{4\text{ chord}}$.

9. In a rectangle edgeways, suspended by one angle, $D = \frac{2}{3}$ diagonal

10. In a parabola edgeways, suspended by the vertex, $D = \frac{5}{7}$ axis $+$ $\frac{1}{3}$ parameter.

To find the centre of oscillation practically, suspend the body freely by the point of suspension, and make it vibrate in small arcs, counting the vibrations it makes in any given time, as one minute. Call the number in a minute n, then will the distance of the centre of oscillation be $\frac{140850}{n^2}$ inches. For a still more accurate method—*see Captain Kater's Paper in the Phil. Trans. for* 1818.

CENTRE *of Percussion.*

When the percutient body revolves about a fixed point, the centre of percussion is the same as the centre of oscillation. But when the body moves with a parallel motion, the centre of percussion is the same as the centre of gravity.

CENTRE *of Pressure.*

Centre of pressure of a fluid against a plane, is that point against which a force being applied equal and contrary to the whole pressure, it will sustain it, so as that the body pressed on will not incline to either side. This, according to some writers, is the same as the centre of percussion, supposing the axis of suspension to be at the intersection of this plane with the surface of the fluid; while others assert, that though the *distance* of this intersection from the centre of pressure is the same as that of the centre of percussion, yet that they do not in general lie in the same line, and consequently are not the same point. The centre of pressure upon any plane *parallel* to the horizon, or upon any plane where the pressure is uniform, is the same as the centre of gravity of that plane.

CENTRIFUGAL *Force.—See Central Forces.*

CENTRIPETAL *Force.—See Central Forces.*

CERES.

This planet was discovered by M. Piazzi, of Palermo, Jan. 1, 1801. For its elements, &c.—*see Planets, elements of.*

CHANCES, *doctrine of.—(Wood.)*

1. If an event may take place in n different ways, and each of these be equally likely to happen, the *probability* that it will take place in a specified way is $\frac{1}{n}$, certainty being represented by unity.

See Permutations

CHAPMAN, wife of Mr. G., F.S.A., Marlborough-hill, N.W., 23rd inst.
COLLYER, Mrs. T., Loraine-place, Holloway, 19th inst.
ELTON, Mrs. W. W., Heathfield Lodge, near Taunton, 21st inst.—stillborn.
IRVINE, Mrs. J., Claughton, Cheshire, 20th inst.
MARKS, Mrs. B., Greville House, Maida-hill, 17th inst.
MARSHALL, Mrs. W. J., Enholmes, Patrington, 18th inst.
MELLOR, Mrs. J. J., The Ferns, Bury, Lancashire, 18th inst.
MORGAN, Mrs. J., Jun., Henrietta-street, Brunswick-square, 22nd inst.
PALMER, wife of Captain H., Adjutant R. Glam. L. I. Militia, Llandaff, 21st inst.
PARHAM, Mrs. H. M., Norrington, Wilts, 23rd inst.
PICARD, Mrs. C. F., Bedford-square, 21st inst.
PULFORD, Mrs. G. C., Montague Villa, Lancaster-road, Kensington Park, 22nd inst.

DAUGHTERS.

BALFOUR, Mrs. A., St. Petersburg, 17th inst.
CROXTON, Mrs. G., Richmond-road, Dalston, 19th inst.
GILPIN, Mrs. E. O., Russell-place, Nottingham, 21st inst.
HAMMOND, wife of Dr., Bentley, Hants, 22nd inst.
HARRISON, Mrs. T. E., Whitburn, 22nd inst.
HESSEY, wife of Major, Madras Staff Corps, Ootacamund, Neilgherry-hills, India, 17th ult.
HOGARTH, wife of Rev. G., The Vicarage, Barton-on-Humber, Lincolnshire, 22nd inst.
HOLDEN, wife of Mr. G. C., Military Store Staff, Upnor Castle, near Rochester, 22nd inst.
MAY, Mrs. J. G., Northam Devon, 19th inst.—prematurely, stillborn.
PALMER, Mrs. J. C., Eastbourne, 21st inst.
SIMMONS, Mrs. L., South Hayes, Bath, 17th inst.
URWICK, Mrs. S. J., Falcon-road, Battersea, 23rd inst.

MARRIAGES.

CREAGH—CROZIER—At Lymington, Hants, Lieut.-Col. C. O. Creagh, 86th Regt., eldest son of the late Gen. Sir M. Creagh, K.H., to Harriet F., eldest daughter of Mr. F. H. Crozier, of The Elms, Lymington, late Madras C.S., 22nd inst.

MONTIER—HEPBURN—At St. Mark's, Myddelton-square, Mr. J. Montier, of Tunbridge Wells, to Mary C., younger daughter of the late Mr. G. Hepburn, of Chancery-lane and Brixton-rise, 21st inst.

PARR—GRIFFITH—At St. James's, London, Mr.

THE SCIENCE OF BETTING.

To the EDITOR *of the* PALL MALL GAZETTE.

SIR,—It may be interesting to your readers to know that this subject was brought forward and the principle published in the Senate House Problem proposed at Cambridge January 8, 1839. I remember when I was a freshman that it was considered by some to be likely to stimulate undergraduates to book-making ; by others as a caution to them that those who professed book-making would probably only utilize their bets to their own ends. I append the problem.—Yours,

A.

Nov. 21, 1866.

The odds against m horses are n_1 to 1, n_2 to 1,...n_m to 1 : show that, except in the particular case in which the sum of the reciprocals of $n_1 + 1$, $n_2 + 1 \ldots n_m + 1$ is unity, a person may so arrange his bets as to win a given sum whichever horse be successful ; and that he must bet against or back every horse according as the above sum is greater or less than unity. Taking the odds in the St. Leger (1838) against the horses annexed to them as follows :—7 to 4 Don John, 9 to 4 Ion, 9 to 2 Lanercost, 9 to 1 Saintfoin, 15 to 1 Cobham, 20 to 1 Alzira, 33 to 1 Hydra ; arrange the bets, 1st—so as to win £378 10*s.* in any case ; 2nd—so as to win £378 10*s.* if either Don John or Lanercost be first, otherwise to be even.

2. If an event may happen in a ways, and fail in b ways, any of these being equally probable, the chance of its happening is $\frac{a}{a+b}$; and the chance of its failing is $\frac{b}{a+b}$.

Ex. 1. The probability of throwing an ace with a single die in one trial is $\frac{1}{6}$; the probability of not throwing an ace is $\frac{5}{6}$; and the probability of throwing an ace or a deuce is $\frac{2}{6}$.

Ex. 2. If n balls a, b, c, d, &c. be thrown promiscuously into a bag, and if two balls be drawn out, the probability that these will be a and b is $\frac{2}{n(n-1)}$.

3. If two events be independent of each other, and the probability that one will happen be $\frac{1}{m}$, and the probability that the other will happen $\frac{1}{n}$; the probability that they will both happen is $\frac{1}{mn}$.

Cor. 1. The probability that both do not happen is $\frac{mn-1}{mn}$.

Cor. 2. The probability that they will both fail is $\frac{(m-1).(n-1)}{mn}$.

Cor. 3. The probability that one will happen, and the other fail, is $\frac{m+n-2}{mn}$.

Cor. 4. If there be any number of independent events, and the probabilities of their happening be $\frac{1}{m}, \frac{1}{n}, \frac{1}{r}$ &c. respectively, the probability that they will all happen is $\frac{1}{mnr}$ &c. When $m = n = r$ &c. the probability is $\frac{1}{m^v}$, v being the number of events.

Ex. 1. The probability of throwing an ace and then a deuce with one die is $\frac{1}{36}$.

Ex. 2. If 6 white and 5 black balls be thrown promiscuously into a bag, the probability that a person will draw out first a white, and then a black ball, is $\frac{3}{11}$. And the probability of drawing a white ball, and then two black balls is $\frac{4}{33}$.

Ex. 3. The probability of throwing an ace with a single die in two trials is $\frac{11}{36}$.

4. If the probability of an event's happening in one trial be $\frac{a}{a+b}$, the probability of its happening t times exactly in n trials is

$$\frac{n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdots\cdots \frac{n-t+1}{t} a^{t} b^{n-t}}{(a+b)^n}.$$

Cor. 1. The probability of the event's failing exactly t times in n trials is

$$\frac{n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdots\cdots \frac{n-t+1}{t} a^{n-t} b^{t}}{(a+b)^n}.$$

Cor. 2. The probability of the event's happening *at least* t times in n trials, is

$$\frac{a^n + n a^{n-1} b + n \cdot \frac{n-1}{2} a^{n-2} b^2 \cdots\cdots \text{to } n-t+1 \text{ terms.}}{(a+b)^n}$$

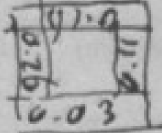
5. In astronomical or other observations, let a, b, c, d, &c. be the differences between the mean of the observations, and the observations themselves; n the number of observations; $\pi = 3.14159$ &c.; then the mean error, or the greatest *probable* error is

$$\frac{\sqrt{(a^2 + b^2 + c^2 + \&c.)}}{n \sqrt{\pi}}.—(\textit{La Place.})$$

6. Let n be the number of times an event has happened, where n is very large, then the chance that the same event will occur again is $\frac{n+1}{n+2}$. Thus supposing 5000 years the greatest antiquity to which history goes back; then the probability that the sun will rise to-morrow is 1826214 to 1.—*(La Place.)*

4 for each of the 6
24 equally probable errors, ~~for the~~ faces of a die
(the last decimal may not be quite correct)
Probable error = 1

~~0.03~~	~~0.76~~	~~1.63~~
~~.11~~	~~0.85~~	~~1.78~~
~~[illegible]~~	~~0.94~~	~~1.94~~
~~[illegible]~~	~~1.04~~	~~2.14~~
~~.35~~	~~1.14~~	~~2.40~~
~~.43~~	~~1.26~~	~~2.74~~
~~.50~~	~~1.37~~	~~3.40~~ better say 3.60
~~.59~~	~~1.56~~	
~~.68~~		



The signs of the errors are tossed for separately

This is closely correct:

0.03	.67	1.50
.11	.76	1.63
.19	.85	1.78
.27	.94	1.95
.35	1.04	2.15
.43	1.14	2.40
.51	1.25	2.75
.59	1.37	3.40 better say 3.60

As the 4 highest values 2.14 2.40, 2.75, 3.40 differ much inter se, mark them with red. Then whenever a red is thrown, disregard its value and throw again with another die marked as below:—

2.29	2.59	3.06
2.32	2.64	3.15
2.35	2.68	3.25
2.39	2.72	3.36
2.43	2.77	3.49
2.47	2.83	3.65
2.51	2.90	4.00
2.55	2.98	4.55

CHRONOLOGY.

A short Chronological TABLE *of remarkable discoveries and inventions, and of the most eminent Mathematicians and Philosophers.*

	B. C.
First eclipse of the moon on record, observed at Babylon	720
Thales predicts an eclipse	600
Anaximander, globes and maps	600
Anaxagoras, eclipse—Pythagoras, astron.	530
Plato, geom.—Meton, Metonic cycle	430
Aristotle, Eudoxus	360
Obliquity of ecliptic first observed	359
A transit of the Moon over Mars observed	357
Euclid, geom.	300
Papirius Cursor, first sun dial at Rome	293
Dionysius, Astron. Æra	285
Apollonius, Archimedes, Aristarchus, Eratosthenes, about	270
Hipparchus, the father of Astronomy	162
	A. D.
Ptolemy, *Almagest*, born	69
Diophantus, analysis	280
Pappus and Theon	380
Proclus, Diocles, about	500
Figures employed by the Arabs	813
A conjunction of all the planets observed, Sept. 16	1186
Alphonso, Astron. tables—Bacon R.	1250
Figures employed in England	1253
Mariner's compass said to be used at Venice	1260
A clock at Westminster Hall	1288
Spina invented spectacles at Pisa	1299
Windmills invented	1299
Gunpowder invented	1330
Decimal arithmetic introduced	1402
Printing invented by Faust	1441
Made public by Gutenberg	1458
Regiomontanus or Muller, astron.	1460
Watches made at Nuremberg	1477
First voyage round the world by Magellan	1522
Variation of the compass by Cabot	1540
Copernicus, Cardan, Vieta, about	1550
Dip of the magnetic needle observed	1576
Telescopes discovered by Jansen	1590

Tycho Brahe, Bacon, Galileo, Kepler, Des Cartes	1600
Thermometers invented by Drebel	1610
Napier, logarithms	1614
Vernier's index made known	1631
Cassini observes a transit of Mercury	1636
A transit of Venus first observed by Horrox	1639
Barometers by Torricelli	1643
Pendulum applied to clocks by Huygens	1649
Cavalerius, Fermat, Pascal, Wallis, Hevelius	1650
Air pump by Otto Guericke	1653
Royal Society established in London	1662
Foundation of the Royal Observatory at Greenwich	1675
Micrometer of Kircher	1677
Newtonian Philosophy published	1686
Savery had invented steam engines	1696
Bernouilli J., Barrow, Hooke, Leibnitz, Reaumur, Flamstead, Picard, Cotes, Taylor, Halley,	1650 to 1700
Aberration of light by Bradley	1727
Achromatic glasses invented	1729
Franklin, identity of lightning and electricity	1747
Harrison, time pieces	1750
Clairaut, Maclaurin, De Moivre, Simpson, Bouguer, Bernouillis, Dollond, Maupertuis	1750
New stile introduced into Britain	1752
Galvanism	1791
Telegraph invented by the French	1794
D'Alembert, Euler, Landen, Lalande, Maskelyne, Waring, &c.	from 1750 to 1800

For a List of the most remarkable Æras—see Æra.

CIRCLE *Equations to.*

1. Let x and y be rectangular coordinates; then if the origin be at the centre,

$$y = \sqrt{r^2 - x^2}.$$

If at the extremity of the diameter,

$$y = \sqrt{2\,r\,x - x^2}.$$

And in general if x', y' be the coordinates to the centre, the equation is, when the axes are rectangular,

$$(x - x')^2 + (y - y')^2 = r^2.$$

Hence every equation of two dimensions of the form $A\,x^2 + A\,y^2 +$

$Bx + Cy + D = o$, where the coefficients of x^2 and y^2 are the same, and the term involving xy is wanting, is an equation to the circle; as for example $2y^2 + 2x^2 - 4y - 4x + 1 = o$.

2. When the circle is considered as a spiral, let a = distance of the centre of the polar coordinates from the centre of the circle, y = rad. vect. p = perpendicular on the tangent; then

$$p = \frac{r^2 - a^2 + y^2}{2r}.$$

When the pole is in the circumference,

$$p = \frac{y^2}{2r}.$$

CISSOID *of Diocles, Equations, &c. to.*

$$y^2 = \frac{x^3}{2a - x}.$$

Or, when considered as a spiral,

$$\varrho = \frac{2a \sin.^2 \theta}{\cos \theta}.$$

$$\text{Subtangent} = \frac{2x.(2a - x)}{6a - 2x}.$$

$$\text{Area} = \frac{3\pi a^2}{2} = 3 \text{ area of the generating semicircle.}$$

$$\text{Content of solid} = -\pi\left(\frac{x^3}{3} + ax^2 + 4a^2x - 8a^3 \log. \frac{2a}{2a - x}\right)$$

which is infinite when $x = 2a$.

CLEPSYDRA.—*See Fluids, discharge of.*

CLIMATE.—*See Atmosphere.*

CLOCK, *to correct going of.*—*See Pendulum.*

CLUSTERS *of Stars.*—*See Nebulæ.*

COHESION, *or Attraction of Cohesion.*—*See Elastic Bodies, equilibrium of.*

COINAGE.—*See Money.*

COLD *Artificial.*—*See Frigorific Mixtures.*

COLLISION *of Bodies.*—*(Wood, Whewell.)*

I. *Of the impact of perfectly hard bodies.*

1. Let A and B be the quantities of matter contained in two perfectly

hard bodies, a and b their velocities before impact, v the common velocity after impact, then

$$v = \frac{A\,a \pm B\,b}{A + B}.$$

$+$ or $-$, according as they move in the same or opposite directions before impact.

Cor. 1. When the bodies are equal, $v = \frac{a \pm b}{2}$.

Cor. 2. When B is at rest or $b = o$, $v = \frac{A\,a}{A + B}$.

2. In the direct impact of two perfectly hard bodies A and B, if $g =$ velocity gained by B, and $l =$ velocity lost by A after impact;

$$g = \frac{A\,.\,\overline{a \mp b}}{A + B}.$$

$$\text{and } l = \frac{B\,.\,\overline{a \mp b}}{A + B}.$$

$-$ or $+$, according as they move in the same or opposite directions.

Cor. $g + l = a \mp b$.

II. *Of the impact of perfectly elastic bodies.*

1. Let $r =$ relative velocity of two bodies $= a \mp b$, according as they move in the same or opposite directions, $g =$ velocity gained by B in the direction of A's motion, $l =$ velocity lost by A in that direction; then in the direct impact of two perfectly elastic bodies,

$$g = \frac{2\,A\,r}{A + B}.$$

$$\text{and } l = \frac{2\,B\,r}{A + B}.$$

Cor. 1. If $A = B$, B's velocity after impact $= a$, and A's velocity $= \pm\, b$, i. e. the bodies interchange velocities.

Cor. 2. In the congress of perfectly elastic bodies, the relative velocity after impact = the relative velocity before.

Cor. 3. If a' and b' be the velocities after impact,

$$A\,a^2 + B\,b^2 = A\,a'^2 + B\,b'^2.$$

Cor. 4. If there be a row of elastic bodies A, B, C, D, &c. at rest, and a motion communicated to A, and thence to B, C, D, &c.; then, if the bodies are equal, they will all remain at rest after impact, except the last, which will move off with a velocity equal to that with which the first moved. If they decrease in magnitude, they will all move in the direction of the first motion, the velocity of each succeeding body being greater than that communicated to the preceding; and the contrary, if the bodies increase in magnitude.

Cor. 5. The velocity thus communicated from A through B to C, when B is greater than one of the two A & C, and less than the other, exceeds the velocity which would be communicated immediately from A and C; and is a maximum when B is a mean proportional between A and C.

Cor. 6. If there be n bodies in geometrical progression, whose common ratio is r,

$$\text{Velocity of first} : \text{velocity of last} :: (1 + r)^{n-1} : 2^{n-1}$$

Cor. If the number of mean proportionals interposed between two given bodies A and X be increased without limit, the ratio of A's velocity to that communicated to X approximates to the ratio of $\sqrt{X} : \sqrt{A}$.

III. *Of the impact of imperfectly elastic bodies.*

1. In the direct impact of two imperfectly elastic bodies A and B, if the compressing force be to the force of elasticity $:: 1 : e$,

$$g = \frac{\overline{1 + e} \cdot A r}{A + B}.$$

$$\text{and } l = \frac{\overline{1 + e} \cdot B r}{A + B}.$$

Cor. 1. The relative velocity before impact : do. after $:: 1 : e$.

Cor. 2. In imperfectly elastic bodies $A a_2 + B b_2$ is greater than $A a'^2 + B b'^2$— (*See last Art. Cor.* 3.)

Cor. 3. If there be n imperfectly elastic bodies in geometrical progression, whose common ratio is r,

$$\text{Velocity of first} : \text{velocity of last} :: (1 + r)^{n-1} : (1 + e)^{n-1}.$$

IV. *Of the impact of bodies against immoveable planes.*

1. When a perfectly hard body impinges obliquely on a perfectly hard immoveable plane, after impact it will move along the plane, and velocity before impact : do. after :: radius : sin. of incidence.

2. If the body be perfectly elastic, the $\angle$ of incidence $=$ $\angle$ of reflection; and velocity before incidence $=$ velocity after.

3. If the body be imperfectly elastic, the velocity before incidence : velocity after reflection :: sin. of $\angle$ of reflection : sin. $\angle$ of incidence;

and compressing force : force of elasticity :: tan. ∠ reflection : tan. ∠ of incidence.

4. If an imperfectly elastic body fall from a given distance a upon an immoveable plane, the whole space described, from the beginning to the end of the motion, is

$$\frac{1+e^2}{1-e^2} \times a.$$

And the whole time of the body's motion is

$$\frac{1+e}{1-e} \times \sqrt{\frac{2a}{g}} \quad (g = 32\tfrac{1}{6} \text{ feet.})$$

COMBINATIONS.—*See Permutations.*

COMPASS, *points of.*

To reduce points of the compass to degrees reckoned from the meridian, and conversely.

N. E. Quadrant.	S. E. Quadrant.	Points.	D. M.	S. W. Quadrant.	N. W. Quadrant.
N.	S.	0	0. 0.	S.	N.
N. by E.	S. by E.	1	11. 15.	S. by W.	N. by W.
N. N. E.	S. S. E.	2	22. 30.	S. S. W.	N. N. W.
N. E. by N.	S. E. by S.	3	33. 45.	S. W. by S.	N. W. by N.
N. E.	S. E.	4	45. 0.	S. W.	N. W.
N. E. by E.	S. E. by E.	5	56. 15.	S. W. by W.	N. W. by W.
E. N. E.	E. S. E.	6	67. 30.	W. S. W.	W. N. W.
E. by N.	E. by S.	7	78. 45.	W. by S.	W. by N.
E.	E.	8	90. 0.	W.	W.

COMPASS, *variation and dip of.—See Variation.*

CONCHOID *of Nicomedes, Equations to, &c.*

$$(a+y)^2 \times (b^2 - y^2) = x^2 y^2.$$

Or, referred to the centre of revolution of its generating line ϱ, the equation is

$$\varrho = \frac{a}{\cos.\theta} + b.$$

$$\text{Area} = \tfrac{1}{2} b \times \left(\text{arc of quadrant} - \text{arc rad. } b \text{ \& sin. } y + \frac{y\sqrt{b^2-y^2}}{2}\right)$$

$$- \frac{ab}{2} \times \text{h. l. } \frac{b - \sqrt{b^2-y^2}}{b+\sqrt{b^2-y^2}}.$$

Content of the whole solid, formed by a revolution round the asymptote, $= \pi b^2 \times \left(\frac{1}{2} \pi a \times \frac{2b}{3} \right)$.

CONDENSER.—*See Pump condensing.*

CONDITION, *Equations of.—See Equation.*

CONE.—*Equation to the section of a right Cone.—(Francœur.)*

Let the vertical angle of the cone $= \beta$, the angle which the cutting plane makes with the side $= \alpha$, and the distance of this plane from the vertex $= c$, then the equation to the section is

$$y^2 = \frac{\sin. \alpha}{\cos.^2 \frac{\beta}{2}} \left(c x \sin. \beta - x^2 \sin. (\alpha + \beta) \right).$$

Cor. 1. If $\alpha + \beta$ be less than 180°, or the plane cut both sides of the cone, the section is an ellipse.

Cor. 2. If $\alpha + \beta = 180°$, or the plane be parallel to the side, the section is a parabola.

Cor. 3. If $\alpha + \beta$ be greater than 180°, or the plane cut the opposite cones, the section is an hyperbola.

Cor. 4. The $\frac{1}{2}$ major and $\frac{1}{2}$ minor axes of the ellipse and hyperbola are $\frac{c \sin. \beta}{2 \sin. (\alpha + \beta)}$, and $\frac{c \sin. \beta}{2 \cos. \frac{\beta}{2} \sin. (a + \beta)} \cdot \sqrt{\sin. \alpha. \sin. (\alpha + \beta)}$.

Cor. 5. The lat. rect. of the parabola $= 4 c \sin.^2 \frac{\beta}{2}$.

Cor. 6. The parallel and subcontrary sections of an oblique cone are circles.

CONGELATION.—*See Heat.*

CONGELATION, *point of perpetual.—See Atmosphere.*

CONIC SECTIONS, *properties of.*

PARABOLA.

Latus rectum or $L = 4 S A$.

$T N = 2 A N$.

$S Y^2 = S P. S A$; i. e. $p = \sqrt{a r}$.

$Q v^2 = 4 S P. P v$.

$SP = \frac{2\,AS}{1 + \cos.\,\theta}$ or $= \frac{2\,AS}{2\cos.^2 \frac{\theta}{2}}$, where $\theta = \angle$ traced out by rad. vect. SP.

Ch. curv. $= 4\,SP$.

Diam. curv. $= \frac{4\,SP^{\frac{3}{2}}}{\sqrt{SA}}$.

Equation to the curve $y^2 = a\,x$ $(a = L)$.

Note.—The general equation to a parabolic curve is $a^{n-1}\,y = x^n$.

If $n = 3$, it is called the cubical parabola.

If $n = \frac{3}{2}$, it is called the semi-cubical parabola.

Ellipse.

$SP + PH = 2\,AC$.

$AS.\,SM = BC^2$.

$L = \frac{2\,BC^2}{AC}$.

$SY^2 = BC^2.\frac{SP}{PH}$; i. e. $p = b\sqrt{\frac{r}{2\,a - r}}$.

$SP.\,PH = CD^2$.

$AC^2 + CB^2 = CP^2 + CD^2$.

$AC.\,CB = CD.\,PF$, or if the perpendicular PF be called P,

$P = \frac{a\,b}{\sqrt{a^2 + b^2 - \varrho^2}}$.

$Q\,v^2 = \frac{Pv.\,vG \times CD^2}{CP^2}$.

$BC = a\sqrt{1 - e^2}$, where e = eccentricity $= \frac{SC}{AC}$.

$CP = \frac{b}{\sqrt{(1 - e^2\cos.^2\theta)}} = \sqrt{\frac{1 - e^2}{1 - e^2\cos.^2\theta}}$.

$SP = \frac{b^2}{a}.\frac{1}{1 + e\cos.\,\theta} = \frac{a\,(1 - e^2)}{1 + e\cos.\,\theta}$.

Ch. curv. through centre $= \frac{2\,CD^2}{CP}$.

Ch. curv. through focus $= \frac{2\,C\,D^2}{A\,C}$.

Diameter curv. $= \frac{2\,C\,D^2}{P\,F}$, or $= L \times \frac{S\,P^3}{S\,Y^3}$.

Equation to the curve, when referred to its principal diameters.

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} = 1.$$

And when the coordinates originate at the vertex,

$y^2 = \frac{b^2}{a^2}\ (2\ a\,x - x^2)$.

Or $y^2 = \frac{b^2}{a^2}\ (a^2 - x^2)$ **when the origin is at the centre.**

HYPERBOLA.

$H\,P - S\,P = 2\ A\,C.$

$A\,S.\ S\,M = B\,C^2.$

$L = \frac{2\,B\,C^2}{A\,C}.$

$S\,Y^2 = B\,C^2.\ \frac{S\,P}{P\,H}$, i. e. $p = b\sqrt{\frac{r}{2\,a + r}}$.

$S\,P.\ P\,H = C\,D^2.$

$A\,C^2 - C\,B^2 = C\,P^2 - C\,D^2.$

$A\,C.\ C\,B = C\,D.\ P\,F$, or $P = \frac{a\,b}{\sqrt{\varrho^2 - (a^2 - b^2)}}$.

$Q\,v^2 = \frac{P\,v.\ v\,G \times C\,D^2}{C\,P^2}.$

$B\,C = a\ \sqrt{(e^2 - 1.)}$

$C\,P = \frac{b}{\sqrt{(e^2 \cos.^2 \theta - 1)}} = a\sqrt{\frac{e^2 - 1}{e^2 \cos.^2 \theta - 1}}.$

$S\,P = \frac{b^2}{a} \cdot \frac{1}{1 + e \cos.\ \theta} = \frac{a\ (e^2 - 1)}{1 + e \cos.\ \theta}.$

Ch. curv. through centre $= \frac{2\,C\,D^2}{C\,P}$.

Ch. curv. through focus $= \frac{2\,C\,D^2}{A\,C}$.

Diam. curv. $= \frac{2\,C\,D^2}{P\,F}$ or $= L.\ \frac{S\,P^3}{S\,Y^3}$.

Equation to the curve, when referred to its principal diameters,

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = -1.$$

$$\text{or } \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1.$$

And when the coordinates originate at the vertex, $y^2 = \frac{b^2}{a^2}$ $(2\,a\,x + x^2)$.

Or $y^2 = \frac{b^2}{a^2}(x^2 - a^2)$, when the origin is at the centre.

Equation to the hyperbola, when referred to its asymptotes, is $x\,y = \frac{a^2 + b^2}{4}$, where y is parallel to the other asymptote.

If the hyperbola is equilateral, $x\,y = \frac{a^2}{2}$.

The general equation to an hyperbolic curve is $y\,x^n = a^{n+1}$

Note.—The general Equation to the Conic Sections, referred to their axes is

$y^2 = m\,x + n\,x^2$, where m is the latus rectum, and the conic section is a parabola, ellipse, or hyperbola, according as $n = o$, or is negative, or positive.

CONTACT *of Curves.*—(*Higman.*)

Let there be two curves, whose equations are $y = f(x)$, and $y' = \phi(x')$, and suppose them (1) to have a point in common, so that when $x = x'$, $y = y'$: (2) that $x = x'$, $y = y'$, and $\frac{d\,y}{d\,x} = \frac{d\,y'}{d\,x'}$: (3) that besides the preceding conditions $\frac{d^2\,y}{d\,x^2} = \frac{d^2\,y'}{d\,x'^2}$ and so on; then will the distance between the curves be infinitely greater in the first case, than it is in the second; infinitely greater in the second than it is in the third; and so on continually.

CONTINUED *Fractions.*—*See Fractions.*

COORDINATES *Polar, to find the relation between.*—(*Higman.*)

If the relation between the rectangular coordinates x and y in any curve be given, that between the polar ones ϱ and θ may be determined; and conversely.

For $x = \varrho$ cos. θ, and $y = \varrho$ sin. θ; substitute these values in the given equation, and the polar one will be found.

Ex. Let $y^2 = \frac{x^3}{a - x}$ By substitution

$$\varrho^2 \sin.^2 \theta = \frac{\varrho^3 \cos.^3 \theta}{a - \varrho \cos. \theta};$$

$$\therefore a \sin.^2 \theta = \varrho \cos. \theta \times (\sin.^2 \theta + \cos.^2 \theta) = \varrho \cos. \theta \times 1$$

$$\therefore \varrho = \frac{a \times \sin.^2 \theta}{\cos. \theta}.$$

CORDS, *strength of.—(Gregory.)*

The best mode of estimating the strength of a cord of hemp is to multiply by 200 the square of its number of inches in girth, and the product will express in pounds the practical strain it may be safely loaded with. For cables, multiply by 120, instead of 200. The ultimate strain is probably double this.

For the *utmost* strength that a cord will bear before it breaks, a good estimate will be found by taking $\frac{1}{5}$ of the square of the girth of the cord, to express the tons it will carry. This is about double the rule for practice just given; and is, even for an ulterior measure, too great for tarred cordage, which is always weaker than white.

In cables, the strength when twisted, is to the strength when the fibres are parallel, as about 3 to 4.

The following is the breaking strain, by experiment, in the best bower cables at present employed in the British navy.—(Encyc. Metrop.)

Sizes, circum. in inches.	No. of threads in each.	Breaking strain.		
		TONS.		
23	2736	114	0	0
21	2268	89	0	0
18	1656	63	0	0
$14\frac{1}{2}$	1080	40	0	0

From the experiments of Mr. Labillardiere, it appears, that if we call the strength of flax 1000; that of the American aloe will be 596; of hemp 1390; of New Zealand flax 1996; and of silk 2894.—*(Young's Nat. Phil.)*

COSINES, *figure of.—See Figure.*

CUBATURES *of Solids.—See Solids.*

CUBE *Roots of Numbers.—See Involution.*

CURVATURE *radius of, in any curve, whose equation is given.*

Let x, y, and z represent the abscissa, ordinate, and curve, then

$$\text{Rad.} = \frac{d\,z^3}{-d\,x\,d^2 y} \text{ (}d\,x\text{ being constant)} = \frac{d\,x^2}{-d^2 y}\left(1 + \frac{d\,y^2}{d\,x^2}\right)^{\frac{3}{2}}$$

$$\text{Or Rad.} = \frac{d\,z^3}{d\,y\,d^2 x} \text{ (}d\,y\text{ being constant)} = \frac{d\,y^2}{d^2 x}\left(1 + \frac{d\,x^2}{d\,y^2}\right)^{\frac{3}{2}}$$

For the Curvature of Spirals—see Spiral.

CYCLE.

A circulation of time between the returns of the same event.

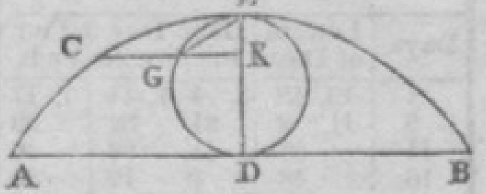
Cycle of the sun, a space of 28 years, in which time the days of the month return again to the same days of the week, and the sun's place to the same degrees of the Ecliptic on the same days, so as not to differ 1° in 100 years; and the leap years return again in respect to the days of the week on which the days of the months fall. To find it, add 9 to the given year of Christ, and divide the sum by 28, and the quotient is the number of cycles elapsed since his birth, and the remainder is the cycle for the given year; if nothing remain the cycle is 28.

Cycle of the moon, or golden number, a revolution of 19 years, in which time the conjunctions, oppositions, and all other aspects of the moon, return on the same days of the months as they did 19 years before, but about 1½ hours sooner. To find it, add 1 to the given year of Christ, and divide the sum by 19, and the quotient is the number of cycles elapsed from the birth of Christ, and the remainder is the cycle for the given year, or the golden number; and if nothing remain, 19 is the cycle.

Cycle of Indiction, a revolution of 15 years, but has no dependance on the motions of the heavenly bodies. It was used by the Romans for indicating the times of certain payments made by the subjects to the republic, established by Constantine, A. D. 312. To find it, subtract 312 from the given year, and divide by 15.

Julian period. From the multiplication of the Solar cycle of 28 years, into the Lunar of 19, and Indiction of 15, arises the Julian period of 7980 years, in which time they all return again in the same order. The Julian period, commencing before all the known epochs, is, as it were, a common receptacle of them all, and to which they may all be reduced *(see Æra.)* To find it, add to any year of Christ, 4713, and it gives the year of the Julian period; or subtract for any time *before* Christ.

CYCLOID, *principal properties of.*



1. Circ. arc E G = G C.

2. Tangent at C is parallel to the chord E G.

3. Cycloidal arc E C = 2 chord E G.

4. Area of cycloid = 3 times area of the generating circle.

5. Solid generated by the revolution of the cycloid about its base A B : its circumscribing cylinder :: 5 : 8.

6. Centre of gravity of the whole cycloid = $\frac{5}{8}$ of the axis from the vertex.

7. Rad. curv. at E = 2 D E.

8. Equations to the cycloid; put a = diameter E D; x and y the co-ordinates E K, K C; z = arc E C; z' = arc E G; then

$$d z = a^{\frac{1}{2}} x^{-\frac{1}{2}} d x.$$

$$\text{and } y = z' + \sqrt{a x - x^2}.$$

For the oscillation of a body in a cycloid, *see Pendulum.*

D

DAMS.—*See Fluids.*

DATES.—*See Chronology.*

DAY *of the week to find.—See Dominical Letter.*

DAYS, *length and increase of, &c.*

TABLE,

Shewing, with sufficient accuracy for common purposes, the length and increase of the days in this country, at different seasons of the year, together with the beginning and end of twilight.

JANUARY.					FEBRUARY.				
Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.	Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.
1	7. 50	0. 6	6. 0	6. 0	1	9. 4	1. 20	5. 31	6. 29
6	56	12	5. 58	2	6	20	36	24	37
11	8. 6	22	54	6	11	40	56	16	45
16	16	32	49	11	16	58	2. 14	7	54
21	30	46	44	16	21	10. 16	32	4. 58	7. 3
26	44	1. 0	38	22	26	38	52	49	12

MARCH.

Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.
1	10. 48	3. 4	4. 44	7. 17
6	11. 8	24	32	29
11	28	44	22	40
16	48	4. 4	12	50
21	12. 8	24	2	8. 1
26	26	42	3. 50	13

APRIL.

Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.
1	12. 50	5. 6	3. 33	8. 28
6	13. 10	26	21	40
11	30	46	8	53
16	50	6. 6	2. 54	9. 7
21	14. 8	24	40	21
26	26	42	26	35

MAY.

Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.
1	14. 44	7. 0	2. 7	9. 55
6	15. 2	18	1. 52	10. 10
11	18	34	30	33
16	34	50	7	56
21	48	8. 4	0. 32	11. 48
26	16. 0	16	No real night.	

JUNE.

Days.	Length of Day.	Day inc.	Day breaks	Twi. ends.
1	16. 12	8. 28		
6	22	38	No real night	
11	28	44	but constant	
16	32	48	day or twi-	
21	34	50	light.	
26	34	50		

JULY.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	16. 30	0. 4		
6	24	10		
11	16	18	No real night	
16	6	28		
21	15. 56	38		
26	42	52	0. 44	11. 14

AUGUST.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	15. 24	1. 10	1. 22	10. 35
6	8	26	42	15
11	14. 50	44	2. 0	9. 57
16	34	2. 0	18	40
21	16	18	33	25
26	13. 56	38	48	10

SEPTEMBER.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	13. 34	3. 0	3. 5	8. 54
6	16	18	19	40
11	12. 56	38	32	27
16	36	58	43	16
21	16	4. 18	54	5
26	11. 58	36	4. 5	7. 54

OCTOBER.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	11. 38	4. 56	4. 17	7. 41
6	18	5. 16	23	31
11	10. 58	36	38	21
16	38	56	48	11
21	20	6. 14	57	2
26	0	34	5. 6	6. 53

NOVEMBER.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	9. 38	6. 56	5. 15	6. 44
6	20	7. 14	22	37
11	4	30	29	30
16	8. 48	46	35	24
21	32	8. 2	42	18
26	20	14	48	12

DECEMBER.

Days.	Length of Day.	Day dec.	Day breaks	Twi. ends.
1	8. 8	8. 26	5. 54	6. 6
6	7. 58	36	57	3
11	52	42	58	2
16	46	48	59	1
21	44	50	6. 0	0
26	46	0in 2	5. 59	1

DEGREE, *decimal parts of.—See Time.*

DEGREES, *&c. converted into Time.—See Time.*

DEGREES *of Latitude and Longitude.*

TABLE *of the lengths of different degrees in fathoms, computed by Col. Lambton, for every three degrees from the Equator to the Pole.—(Phil. Trans.* 1818.*)*

Lat.	Degrees on the Meridian.	Degrees on the Perpendicular.	Degrees of Longitude.
0	60459,2	60848,0	60848,0
3	60460,8	60848,4	60765,0
6	60465,6	60850,1	60516,8
9	60473,5	60852,8	60103,6
12	60484,5	60856,5	59526,7
15	60498,4	60861,1	58787,3
18	60515,1	60866,7	57887,7
21	60534,3	60873,2	56830,0
24	60556,0	60880,5	55628,1
27	60579,8	60888,5	54252,0
30	60605,5	60897,1	52738,4
33	60632,7	60906,2	51080,2
36	60661,3	60915,8	49281,9
39	60690,8	60925,7	47348,2
42	60721,3	60935,7	45284,0
45	60751,8	60946,1	43095,4
48	60782,3	60956,4	40787,8
51	60812,5	60966,5	38367,5
54	60842,1	60976,5	35841,1
57	60870,7	60986,1	33215,4
60	60898,0	60995,2	30497,6
63	60923,7	61003,8	27695,2
66	60947,5	61011,8	24815,7
69	60969,1	61018,9	21867,2
72	60988,3	61025,6	18857,9
75	61005,1	61031,0	15796,0
78	61018,9	61035,8	12690,1
81	61029,9	61039,5	9548,7
84	61037,8	61042,1	6380,6
87	61042,6	61043,7	3194,8
90	61044,3	61044,3	

DEGREE *French.*

The French usually divide the circumference of the circle into 400°, each degree into 100′, and each minute into 100″. Hence if $n =$ number of French degrees, &c. the corresponding number of English $= n - \frac{n}{10}$; i. e. from the number we must subtract the same, after the decimal point has been removed one place to the left.

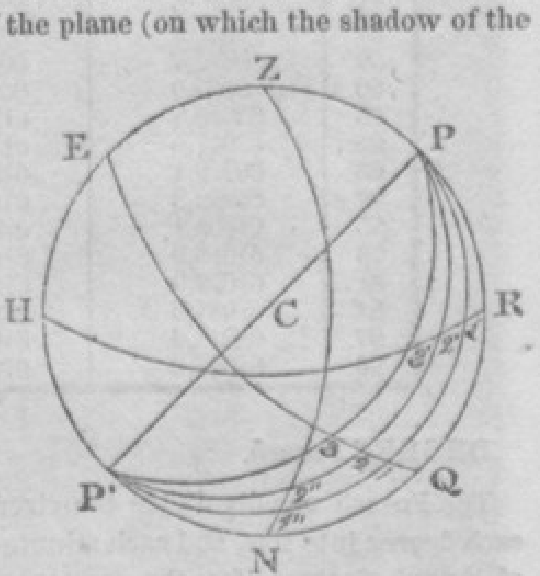
Exs. What number of degrees, minutes, &c. in the English scale correspond to 71°. 15′., and to 26°. 0735, in the French scale.

71.15	26.0735
7.115	2.60735
64.035	23.46615
60	60
2.100	27.96900
60	60
6.000	58.14000 Answer 23°. 27′. 58″.
64°. 2′. 6″.	

DEW.—*See Rain.*

DIALLING.

In all Dials universally, the style or gnomon is parallel to the earth's axis, and, on account of the great distance of the sun, may be imagined actually to coincide with it. In like manner the dial plate is parallel to, and supposed actually to coincide with, some great circle of the earth; and the hours may be conceived to be traced out by the shadow of the axis of the earth (here supposed hollow) upon one of these great circles. Hence there may be an infinite number of different kinds of dials, as they depend upon the position of the plane (on which the shadow of the earth's axis falls) with respect to the meridian and horizon. Thus if the shadow be received upon the Equator E Q, the dial is called an Equatorial Dial; if upon H R (a great circle of the earth in the plane of the horizon), a Horizontal one; if upon Z N, which is in the plane of the prime vertical, a North or South Dial, &c. &c. And in these three last cases, it is obvious that the shadow of the earth's axis, when the sun is on the meridian, or at 12 o'clock, will cut these several circles in Q, R, and N. At 1 o'clock, or when the ∠ Q P 1 is 15°, it will cut them at 1, 1′, 1″; at 2 o'clock, or when the ∠ at P is 30°, in 2, 2′, 2″, &c.; which are ∴ the 12 o'clock, 1 o'clock, 2 o'clock, &c. marks.



Equatorial Dial.

In this Dial, since the sun moves uniformly 15° per hour, the ∠s. at P, and consequently the arcs of the circle Q E, which measure them, will increase uniformly. Hence we have only to take from Q the arcs 15°, 30°, 45°, &c., and they will be 1 o'clock, 2 o'clock, 3 o'clock, &c., marks. This Dial, unless graduated on both sides, will only shew the hours for the six summer months, viz. from the vernal to the autumnal equinoxes.

Horizontal Dial.

Here the arcs R 1', 1' 2' &c. are not equal, but must be calculated by the resolution of the right ∠d. Δs. P R 1', P R 2', &c., where R P 1' = 15°, R P 2' = 30°, &c., then we shall have

$$\tan. R\,1' = \sin. lat. \times \tan. 15^{\circ}.$$

$$\tan. R\,2' = \sin. lat. \times \tan. 2 \times 15^{\circ}.$$

&c. &c.

This Dial shews the hour throughout the year, whenever the sun is above the horizon. In order to fix a horizontal dial, find the time by the sun's alt. when it is at or near the solstices, and set a well regulated watch to that time; then when the watch shews 12 o'clock, at that instant set the dial to 12 o'clock, and it stands right.

Vertical North and South Dials.

Here to find the arcs N 1'', N 2'', &c., we have in the right ∠d. Δ P' N 1'',

$$\tan. N\,1'' = \text{cosin. lat.} \times \tan. 15^{\circ}.$$

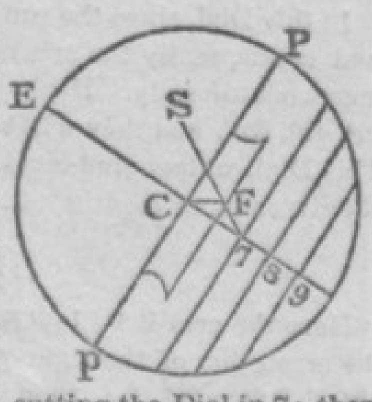
$$\tan. N\,2'' = \cos. lat. \times \tan. 2 \times 15^{\circ}.$$

&c. &c.

If P be the North Pole, this represents a South Dial. The construction for the Vertical North Dial is nearly the same. In this Dial the number of hours shewn in a day can never exceed twelve, which is the case at both the equinoxes; at any other season of the year, the number of hours shewn is less.

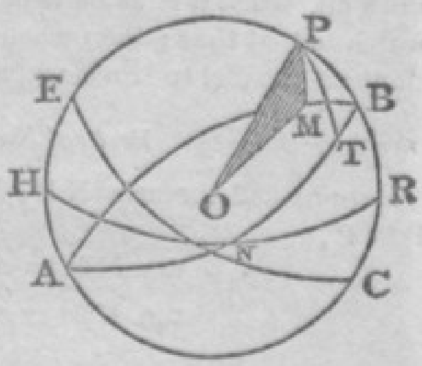
To find whether a wall be full south for a vertical south Dial, erect a gnomon perpendicular to it, and hang a plumb line from it; then when the watch shews 12, if the shadow of the gnomon coincide with the plumb line, the wall is full south.

Vertical East Dial.



Here the plane of the Dial is in the meridian, and the gnomon a parallelogram perpendicular to it (as represented in the Fig.) and the shadows upon the plane will evidently be all parallel to the gnomon, and to one another. Moreover, at 6 o'clock, the sun, being due east, will be in the plane of the gnomon, and ∴ cast the shadow perpendicularly upon the Dial or on P*p*. To find the 7 o'clock mark, let S be the sun at that hour, and S F a ray proceeding from it cutting the Dial in 7; then in the plane right ∠d. Δ C F 7, C 7, = C F × tan. ∠ C F 7 = height of style × tan. 15°. C 8 = height of style × tan. 2 × 15°. &c. Similarly may be constructed a vertical West Dial. The East Dial will not shew the hour after 12 o'clock at noon, nor the West Dial before.

General Problems.



1. Given the latitude of the place, and the position of the plane of the Dial, both with respect to the meridian and horizon; it is required to find the elevation of the style, the distance of the sub-style from the meridian, and the arc intercepted between the meridian and any other given hour line.

Let B O A be the plane of the dial, given in position both with respect to the horizon H R, and the meridian P E A C; then in the right angled Δ B N R, the ∠s. B N R, N B R are given, ∴ B R may be found; but P R = latitude, ∴ P B is known. Now let a plane pass through O P, and let it be turned about till it becomes perpendicular to B O A, and let it cut the circumference of B A in M, then P M is that meridian which is perpendicular to B O A, ∴ in the right angled Δ P M B, P B and ∠ P B M are known, ∴ P M = elevation of the style, and M B, the distance of the substyle from the meridian, may be found. Draw P T, making an ∠ of 15° with P B; then will T be the 1 o'clock mark, and to find it we have P B, and B P T = 15°, and ∠ P B T = supplement of N B R, ∴ B T may be found, and so on for the other hours.

2. To determine the curve, traced out by the extremity of the shadow of a vertical gnomon on a horizontal plane.—*(Maddy.)*

Conceive a line A B to be the gnomon, A P the shadow, A N the direction of the meridian shadow. Draw P N perpendicular to A N, and let A N $= x$, P N $= y$, A B $= a$, $l =$ latitude of the place, $\delta =$ sun's declination; then

$$y^2 = \frac{(\cos.^2 l - \sin.^2 \delta).\ x^2 + 2a \sin. l \cos. l\, x + (\sin.^2 l - \sin.^2 \delta).\ a^2.}{\sin.^2 \delta}$$

Cor. If $\cos. l = \sin. \delta$, or $l = 90^0 - \delta$, the curve is a parabola, if $\cos. l$ is greater than $\sin. \delta$, or l less than $90^0 - \delta$, an hyperbola, if $\cos l$ is less than $\sin. \delta$, or l greater than $90^0 - \delta$, an ellipse.

DIFFERENTIALS.

TABLE I.

Differentiation of Algebraic and Transcendental Functions; and of the higher orders of Differentials.

Quantity.	Differential.
$a x$	$a\, d x$.
$a x + b y - \frac{z}{c} + e$ ...	$a\, d x + b\, d y - \frac{d z}{c}$.
x^n	$n x^{n-1} d x$.
$x^{\frac{n}{m}}$	$\frac{n}{m} x^{\frac{n}{m} - 1} d x$.
$\left(a^m + x^m\right)^{\frac{1}{n}}$	$\frac{1}{n}\left(a^m + x^m\right)^{\frac{1}{n} - 1} \times m x^{m-1} d x$.
$x y$	$x\, d y + y\, d x$.
$x^m y^n$	$m y^n x^{m-1} d x + n x^m y^{n-1} d y$.
$\frac{x}{y}$	$\frac{y\, d x - x\, d y}{y^2}$.
Hyp. log. x	$\frac{d x}{x}$.
Hyp. log. $1 + x$	$\frac{d x}{1 + x}$.
e^x	$e^x\, d x \times$ h. l. e.
y^x	$y^x\, d x \times$ h. l. $y + x y^{x-1} d y$.

Sin. z	dz cos. z.
Cos. z	$-dz$ sin. z.
Sin. mz	$m\,dz$ cos. mz.
Cos. mz	$-m\,dz$ sin. mz.
Ver. sin. z	dz sin. z.
Tan. z	$\frac{dz}{\cos.^2 z}$.
Cot. z	$-\frac{dz}{\sin.^2 z}$.
Sec. z	$\frac{dz \sin. z}{\cos.^2 z} = dz \tan. z \times \sec. z$.
Cosec. z	$-\frac{dz \cos. z}{\sin.^2 z}$.
$(\text{Sin.})^m z$	$m\,(\sin.)^{m-1} z\,dz \cos. z$.
$(\text{Cos.})^m z$	$-m\,(\cos.)^{m-1} z\,dz \sin. z$.
2d. differential x^2 (dx constant)	$2\,dx_2$.
2d. diff. x^2 (dx variable)	$2\,dx^2 + 2\,x\,d^2 x$.
2d. diff. $y_2 = 2\,ax - x^2$ (dx constant)	$y\,d^2 y + dy^2 = -dx^2$.
2d. diff. $2\,xy + ay^2 - bx^2 = o$	$dx\,dy + x\,d^2 y + dy\,dx + a\,dy^2 + ay\,d^2 y - b\,dx^2 = o$.

TABLE II.

Of Integrals, containing a few of the most usual forms that occur in the practical solution of Problems.

DIFFERENTIAL.	INTEGRAL.
$z^n\,dz$	$\frac{z^{n+1}}{n+1}$.
$\frac{dz}{z^n}$	$-\frac{1}{(n-1)}\frac{1}{z^{n-1}}$.
$(a \pm z^n)^{m-1} z^{n-1}\,dz$ ...	$\pm \frac{1}{mn}(a \pm z^n)^m$.
Ex. 1. $\frac{z\,dz}{\sqrt{a^2+z^2}}$ ...	$\sqrt{a^2+z^2}$.

Ex. 2. $\frac{x\,dx}{(a^2-x^2)^{\frac{3}{2}}}$ ... $\frac{1}{\sqrt{a^2-x^2}}$.

$x\,dy$ ... ... $xy -$ fl. $y\,dx$.

$x^m dx\,(a+bx^n)^p$ m, n, and p any numbers whatever; integral may be found if $\frac{m+1}{n}$ or $\frac{m+1}{n}+p$ be a whole number.

Ex. 1. $\frac{z^m dz}{(a+bz)^p}$ ... $\frac{1}{b^{m+1}}\left(\frac{z^{m-p+1}}{m-p+1} - \frac{maz^{m-p}}{m-p} + \frac{m(m-1)\,a^2 z^{m-p-1}}{2\,(m-p-1)}\right.$ — &c. where m is an integer, and $z = a + bx$.

Ex. 2. $\frac{a\,dx}{(a^2+x^2)^{\frac{3}{2}}}$ ... $\frac{x}{a\sqrt{a^2+x^2}}$.

$\frac{dx}{x}$ ... ... h. l. x.

$\frac{dx}{x \pm a}$ ... ... h. l. $(x \pm a)$.

$\frac{n x^{n-1} dx}{a^n + x^n}$ ... ... h. l. $(a^n + x^n)$.

$\frac{dx}{\sqrt{x^2 \pm a^2}}$ ... ... h. l. $\left(x + \sqrt{x^2 \pm a^2}\right)$

$\frac{dx}{\sqrt{x^2 \pm 2ax}}$ ... ... h. l. $\left(x \pm a + \sqrt{x^2 \pm 2ax}\right)$.

$\frac{2a\,dx}{a^2 - x^2}$ ... ... h. l. $\frac{a+x}{a-x}$.

$\frac{2a\,dx}{x^2 - a^2}$ ... ... h. l. $\frac{x-a}{x+a}$.

$\frac{2a\,dx}{x\sqrt{a^2+x^2}}$ ... ... h. l. $\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}+a}$.

$\frac{2a\,dx}{x\sqrt{a^2-x^2}}$ ... ... h. l. $\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$.

The last 9 forms of fluents may be found by a table of hyperbolic logarithms, or by a table of common logarithms, by multiplying the logarithm by 2.30258509, which will give the corresponding hyp. log.

$\frac{a\,dy}{\sqrt{a^2-y^2}}$ ... ... circ. arc rad. a and sin. y.

$\frac{a\,dx}{\sqrt{2ax-x^2}}$ ... ... circ. arc rad. a and ver. sin. x.

$\frac{a^2\,dt}{a^2+t^2}$ ... ... circ. arc rad. a and tan. t.

$\frac{a^2\,ds}{s\sqrt{s^2-a^2}}$ ... ... circ. arc rad. a and sec. s.

$\frac{-a\,dx}{\sqrt{a^2-x^2}}$ ... ... circ. arc rad. a and cos. x.

The five last forms may be found by a table exhibiting the length of circ. arcs for all degrees, &c. of the quadrant to rad. 1 *(see Arc)*; for by multiplying these arcs by a, we shall have their lengths to radius a. Thus, if the integral of $\frac{a\,dy}{\sqrt{a^2-y^2}}$ were required, when y is the sin. 30°, we have the length of an arc of 30° to rad. 1 = 0.5235987; hence length to rad. a is $a \times 0.5235987$ = integral required; and so for the rest.

$\frac{x^{mn-1}\,dx}{(a \pm x^n)^{m+1}}$ ... ... $\frac{1}{mna} \times \frac{x^{mn}}{(a \pm x^n)^m}$.

$\frac{(a \pm xn)^{m-1}\,dx}{x^{mn+1}}$ ... $\frac{-1}{mna} \times \frac{(a \pm x^n)^m}{x^{mn}}$.

$\left(\frac{m\,dx}{x} + \frac{n\,dy}{y} + \frac{r\,dz}{z}\right) x^m y^n z^r$ ... ... $x^m y^n z^r$.

$\frac{x^{-1}\,dx}{a \pm x^n}$ ... ... $\frac{1}{na}$ h. l. $\frac{x^n}{a \pm x^n}$.

$\frac{x^{\frac{1}{2}n-1}\,dx}{a - x^n}$ ... ... $\frac{1}{n\sqrt{a}}$ h. l. $\frac{\sqrt{a}+\sqrt{x^n}}{\sqrt{a}-\sqrt{x^n}}$.

$\frac{x^{\frac{1}{2}n-1}\,dx}{\sqrt{\pm a + x^n}}$ ... ... $\frac{2}{n}$ h. l. $\sqrt{x^n} + \sqrt{\pm a + x^n}$.

$\frac{x^{\frac{1}{2}n-1}dx}{\sqrt{a-x^n}}$ ... ... $\frac{2}{n} \times$ arc sin. $\sqrt{\frac{x^n}{a}}$.

$\frac{x^{-1}dx}{\sqrt{a \pm x^n}}$ ... ... $\frac{1}{n\sqrt{a}} +$ h. l. $\frac{\pm\sqrt{a \pm x^n} \pm \sqrt{a}}{\sqrt{a \pm x^n} + \sqrt{a}}$.

$\frac{x^{-1}dx}{\sqrt{-a+x^n}}$ ... ... $\frac{2}{n\sqrt{a}} \times$ arc. sect. $\sqrt{\frac{x^n}{a}}$.

$dx\sqrt{ax-x^2}$ ... ... ½ circ. seg. to diam. a and ver. sin. x.

$a^x x^n dx$ ... ... $\frac{a^x}{k}\left(x^n - \frac{nx^{n-1}}{k} + \frac{n(n-1)x^{n-2}}{k^2}\right.$ ......

$\left.\pm \frac{1.2.3.\ldots\ldots n}{k^n}\right)$ if n be an integer and positive and $k =$ h. l. x.

$\frac{dx}{\sin. x}$ ... ... log. tan. $\frac{x}{2}$.

$\frac{dx}{\cos. x}$ ... ... log. tan. $\left(\frac{\pi}{4} + \frac{x}{2}\right)$

sin. mx. cos. nx. dx ... $-\frac{1}{2}\left(\frac{\cos.(m+n)x}{m+n} + \frac{\cos.(m-n)x}{m-n}\right)$.

DIP *of the Horizon.—See Horizon.*

DIP *of the Magnetic Needle.—See Variation.*

DISCHARGE *of Fluids.—See Fluids discharge of.*

DISCOUNT.—*See Interest.*

DISCOVERIES, *dates of.—See Chronology.*

DIVING BELL.—*(Bland.)*

Having given the form of a Diving Bell, and the depth it has descended, to determine how high the water will have risen.

Let x = abscissa, measured from the vertex, occupied by the air, h = depth of the vertex below the surface of the water; M and m the capacities of the whole bell and of that part occupied by the air, a the altitude of the barometer; then

$$14a : 14a + h + x :: m : M.$$

from which proportion x may be found.

Ex. Let the bell be a paraboloid, whose equation is $y^2 = 4\,c\,x$.

$$\text{Here } M = 2\,c\,\pi\,b^2 \;\&\; m = 2\,c\,\pi\,x^2.$$

$$\therefore\; x^3 + (14\,a + h)\,x^2 = 14\,a\,b^2.$$

from which equation x may be found.

DOMINICAL *Letter to find.*

Rule.—Divide the centuries by 4, and take twice what remains from 6, then add the remainder to the odd years above the even centuries, and their fourth. Divide their sum by 7, and the remainder taken from 7 will leave the number answering to the letter required.

Thus to find the letter for 1826.

The centuries 18 divided by 4 leave 2, the double of which taken from 6 leaves 2, to which add the odd years 26 and their fourth part 6, their sum 34 divided by 7 leaves 6, which taken from 7 leaves 1, answering to A the first letter of the alphabet.

If it be leap year, the above rule gives the Dominical letter from the end of February to the end of the year; the letter immediately succeeding will be the Dominical letter for the beginning of the year, from January to the end of February.

Knowing the Dominical letter, we may, by the help of the following Table, find the day of the week answering to any given day of the month, and conversely.

TABLE.

Months.	*Dominical Letters.*						
January, October	A	B	C	D	E	F	G
February, March, November	D	E	F	G	A	B	C
April, July	G	A	B	C	D	E	F
May	B	C	D	E	F	G	A
June	E	F	G	A	B	C	D
August	C	D	E	F	G	A	B
September, December	F	G	A	B	C	D	E
	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31				

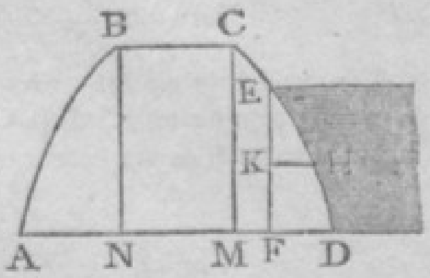
Here the first vertical column contains the several months in the year, and that part of the other columns immediately opposite contains the dominical letters: the under part contains the days of the month on which the Sundays happen; and hence the other days of the week are easily found.

DOUBLE *Stars.*—*See Stars.*

DYKE.—*(Bland.)*

A mound or obstacle opposed to the effort made by a fluid to spread itself.

1. Let A B C D be a vertical section of a dyke opposed to the stagnant fluid. Its parts are supposed to be so connected, as to yield to the pressure of the fluid, either by turning altogether round the point A, or by sliding along the horizontal base D A.



2. Supposing the dyke to yield by turning round A, to determine when there will be an equilibrium.

Let H K $= x$, E K $= y$, E F $= a$, A D $= b$, F D $= c$, $s =$ specific gravity of fluid, $s' =$ do. of dyke, Q $=$ the product of the area A B C D multiplied by the distance of A from the vertical passing through the centre of gravity of the area; then in the case of equilibrium,

$$\tfrac{1}{6}\, s\, a^3 = s'\, Q + s.\ \text{fl.}\ (b - c + x).\ y\, dx.$$

3. Supposing the dyke to yield by sliding along its horizontal base: to determine when there will be an equilibrium; neglecting the vertical pressure of the fluid.

The base being horizontal, the mass which it sustains is supported against the horizontal force of the fluid only by its adhesion to the base, and the resistance arising from friction. Supposing these resistances $=$ n times the weight of the dyke (n being determined by experiment); and let P $=$ the area of the section A B C D; then

$$P = \frac{s}{s'} \times \frac{a^2}{2\, n}.$$

4. If A B and C D be straight lines, i. e. if the sides of the dyke be rectilinear, and A D, B C horizontal; to determine the equation of equilibrium of Art. 2.

Let C M $= h$, M D $= e$, A N $= e'$, then $b^2 + \left(\frac{s\, c}{s'\, h}.\ a - e\right) b -$

$\frac{s(a^3 + c^2 a)}{3 s' h} + \frac{1}{3}(e^2 - e'^2) = o$ an equation which includes all the cases of rectilinear sloping banks.

Cor. If the slopes be $= o$, or the dykes vertical, $e = o$, $e' = o$, and $c = o$,

$$\therefore b = \sqrt{\frac{s a^3}{3 s' h}}.$$

5. If the sides be rectilinear, and A D, B C horizontal, to determine the equation of equilibrium of Art. 3.

$$\text{Here } b = \frac{s}{s'} \times \frac{a^3}{2 h n} + \tfrac{1}{2}(e + e').$$

The preceding equations have been investigated on the supposition of a perfect connexion of all the parts of the dyke; they are ∴ only applicable to such as are constructed of masonry.

E

EARTH, *Elements of.*

The principal elements of the earth, according to the determination of La Place, in the last edition of his Systeme du Monde, are as follows:—

Equatorial Diameter	7924	English miles.
Polar do.	7908	....
Mean do.	7916	....
Mean circumference	24869	....
Mean length of a degree	69.08	....
Surface	196862256	square miles.
Solidity	259726936416	cubic miles.

Density of earth about 5 times that of common water.

Mass of earth is $\frac{1}{337086}$ of the mass of the sun.

Weight of a body at Equator : do. at the Poles :: 1 : 1.00569.

Length of seconds Pendulum at Equator = 39.027 in.; do. at Poles = 39.197 in.

Centrifugal force at Equator is about $\frac{1}{289}$ of gravity, and ∴ if the rotatory motion of the earth were 17 times greater than it is, bodies at the Equator would have no weight.

Mean distance of earth from sun = 93,321,724 English miles.

Aphelion distance = 94,889,528 miles.

Perihelion do. = 91,753,920 miles.

Eccentricity = .0168, ½ ax. maj. being 1.

Time of sidereal revolution 365*d.* 6*h.* 9*m.* 11,5*s.*
Do. tropical .... 365*d.* 5*h.* 48*m.* 51,6*s*; being less than in the time of Hipparchus by 11,2.″

Mean velocity of earth in its orbit 59′. 10,7″ each day.

Velocity in its perihelion .... 1°. 1′. 9,9.″

Do. in aphelion .... 57.′ 10,7″

Revolution about the line of the apsides, or anomalistic year, 365*d.* 6*h.* 14*m.* 2*s.*

Tropical revolution of apsides performed in 20,931 years.

Inclination of axis to Ecliptic 23°. 27′. 57″., which decreases at the rate of 52,1″. in a century, but this decrease can never exceed 2°. 42′.

Nutation of axis = 19,3″.

Precession of the equinoxes 50,1″ annually, or 1°. 23′. 30″. in a century. A complete revolution performed in 25868 years.

Length of sidereal day 23*h.* 56*m.* 4,1*s.*; and has not varied the hundredth part of a second since the time of Hipparchus.

The interval between the vernal and autumnal equinoxes is (on account of the excentricity of the earth's orbit and its unequal velocity therein) nearly eight days longer than the interval between the autumnal and vernal equinoxes. These intervals are at present nearly as follows:—

	d. h. m.		d. h. m.
From spring equinox to summer solstice	92. 21. 45.		
From summer solstice to autumnal equinox	93. 13. 35.	=	185. 35. 20.
From autumn equinox to winter solstice	89. 16. 47.		
From winter solstice to spring equinox	89. 1. 42.	=	178. 18. 29.
	Difference		7. 16. 51.

EARTH, *figure of.*—(*Playfair, Maddy.*)

1. To find the radius of curvature at any point of the terrestrial meridian, supposing the earth to be an oblate spheriod.

Let a and b be the Equatorial and Polar ½ axes, r the rad. of curv. to the latitude λ, $c = a - b =$ compression, $m = 57^{o}.2957795$ the number of degrees in an arc = radius; then

$$r = a - 2c + 3\,c\ \sin.\ 2\ \lambda.$$

$$\text{or} = a - \frac{c}{2} - \frac{3\,c}{2}\ \cos.\ 2\ \lambda.$$

and if $D =$ length of a degree in lat. λ, $r = m\,D$

$$\therefore D = \frac{a}{m}\left(1 - \frac{c}{2\,a} - \frac{3\,c}{2\,a}\ \cos.\ 2\ \lambda.\right)$$

Cor. 1. At the Equator $m\,D = a - 2\,c$; at the Pole $m\,D = a + c$; and in lat. $45^{o}. = a - \frac{1}{2}\,c$. Hence if E, P, and M = the degree at the Equator, Pole, and lat. $45^{o}.$; $M = \frac{1}{3}\,(P + E)$.

Cor. 2. The excess of a degree in any lat. above that at the Equator, or $D - E$, varies as $\sin.^{2}\,\lambda$.

2. The lengths of two degrees of latitude, of which the middle points are in given latitudes, being known by admeasurement, the Equatorial and Polar diameters of the earth may be calculated from the following formulæ.

Let D and D′ be the given degrees (the least, or that nearest the Equator being D) λ and λ' the latitudes of their middle points, then

$$c = \frac{m.\ (D' - D.)}{3\sin.\ (\lambda' + \lambda) \times \sin.\ (\lambda' - \lambda)}.$$

and the compression, or ellipticity of the earth

$$= \frac{c}{a} = \frac{D' - D}{3\,D.\ \sin.\ (\lambda' + \lambda) \times \sin.\ (\lambda' - \lambda)}.$$

from which two equations a and c, and consequently a and b, may be found.

The following are the five arcs, which have been measured with the greatest care :—

Latitude.	*Degrees in Fathoms.*	*Country.*	*By whom.*
0°. 0′. 0″.	60480.2 ...	Peru	Condamine, &c.
11. 0. 0.	60486.6 ...	India	Major Lambton.
45. 0. 0.	60759.4 ...	France ...	Cassini, &c.
52. 2. 2.	60826.6 ...	England...	Colonel Mudge.
66. 20. 10.	60952.4 ...	Lapland...	Swanberg, &c.

By combining these in pairs, and taking the mean, we get the following results.

$$a : b :: 312 : 311.$$

$$D = 69.044 - .3299 \times \cos. 2\lambda \text{ in miles,}$$

or $D = 60759.472 - 290.576 \times \cos. 2\lambda$ in fathoms, which expresses the degrees of the meridian in any latitude.

$$\frac{c}{a} = .0032 = \frac{1}{312.5}.$$

$$c = 12.680 \text{ miles.}$$

$$a = 3962.349 \text{ miles.}$$

$$b = 3949.669 \text{ miles.}$$

Hence circumference of elliptic meridian = 24855.84 miles ; do. of equator = 24896.16 miles ; ∴ difference = 40 miles nearly.

3. The figure of the earth may also be determined, by comparing a degree of the meridian with the degree of a great circle perpendicular to the meridian in the same latitude, by the following formulæ.

Let Δ be the degree of the curve perpendicular to the meridian, the rest as before, then

$$c = \frac{m}{2}(\Delta - D) \times \frac{1}{\cos.^2 \lambda}.$$

$$\text{and } \frac{c}{a} = \frac{\Delta - D}{2\,\Delta \cos.^2 \lambda} \text{ nearly.}$$

4. To find the compression by means of a second's pendulum, considering the earth as a spheroid of equilibrium.

Let p and p' be the lengths of two pendulums oscillating seconds in latitudes λ and λ', c the compression, the equatorial radius being unity; then

$$c = \frac{p - p'}{p \sin.^2 \lambda - p' \sin.^2 \lambda'}.$$

5. Comparison of the figure of the earth, deduced from actual admeasurement of a degree in different latitudes, with that deduced from the theory of gravity.

If a homogeneous fluid revolve on an axis, it will form itself into an oblate spheroid, of which the Polar ½ axis : radius of Equator :: attraction at Equator — centrifugal force at Equator : attraction at the Pole.

In the case of the earth, this ratio will be :: 229 : 230.

If the earth be not homogeneous, but composed of strata that increase in density towards the centre, the spheroid will have less oblateness than if it were homogeneous, and it is demonstrable that if the density increase so that it be infinite at the centre, the ellipticity $= \frac{1}{578}$, which is the case of the least ellipticity; $\frac{1}{230}$ is the case with the greatest.

Hence as the ellipticity of the earth has been shewn to be less than $\frac{1}{230}$ (viz. $\frac{1}{312}$), it is evident that if the earth is a spheroid of equilibrium, it is denser towards the interior. This has been indisputably proved to be the case by actual experiment.—*See Mountain, attraction of.*

But after all, whether the earth be a spheroid of equilibrium, whether the N. and S. ½ spheres be equal and similar to each other, and what is the ratio of an arc of the meridian, measured in a given latitude, to the whole meridian, are questions to which complete solutions have not yet been given.

6. TABLE of the ellipticities of the earth.

Authors.	Ellipticities.	Principles.
Newton	$\frac{1}{230}$	Theory of Gravity.
Playfair	$\frac{1}{312}$	Mensuration of Arcs.
Lambton	$\frac{1}{310}$	Do.
Sabine	$\frac{1}{312.6}$ to $\frac{1}{314.3}$ ...	Vibration of Pendulum.
Treisnecker......	$\frac{1}{329}$	Occultation of Stars.
La Place	$\frac{1}{334}$	Precession and Nutation.
	$\frac{1}{305}$	Theory of the Moon.

Upon the whole, the ellipticity probably lies between $\frac{1}{307}$ and $\frac{1}{314}$. But Captain Sabine, from some very recent experiments on the length of the Pendulum *(see Pendulum)*, states the ellipticity at $\frac{1}{288.4}$. For Tables of Degrees of Latitude and Longitude, *see Degree.*

EARTH's *Surface, extent of.—(Encyc. Britt. Supplt.)*

The extent of the four great divisions of the world is as follows :—

		Sq. Eng. Miles.
Europe, with its Isles		3,432,000
Africa, with Madagascar		11,420,000
Continental Asia	16,890,000	21,090,000
Isles, including New Holland and Polynesia	4,200,000	
South America	6,420,000	15,300,000
North do.	8,100,000	
Islands	160,000	
Greenland (supposed)	620,000	
Total		51,242,000

The ocean, with all its inland bays and seas, covers an area of 145,600,000 square miles, or nearly $\frac{3}{4}$ of the surface of the globe. About $\frac{7}{12}$ of the great body of waters lie in the southern hemisphere; and $\frac{5}{12}$

in the northern. In the one the ocean : the land :: 7 : 5; and in the other :: 13 : 12.

If we suppose the mean depth of the ocean to be two miles, the cubic content will be 290,000,000 of cubic miles.

Comparative superficial extent of the frigid, temperate, and torrid zones, taking the whole area of the globe as unity.—*(Lacroix.)*

The frigid zones occupy	$\frac{83}{1000}$.
The temperate zones	$\frac{519}{1000}$.
The torrid zone	$\frac{398}{1000}$.

EARTH, *density of.*—*See Mountain, attraction of.*

EARTH, *internal temperature of.*—*(Encyc. Britt. Supp.)*

In descending below the surface of the earth, a considerable increase of temperature is observed, as the following examples prove.

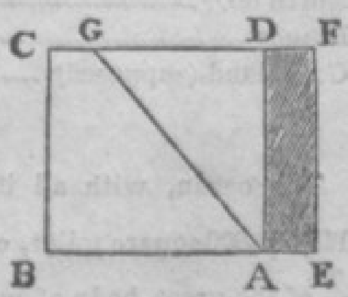
At Giromagny, in the Vosges, annual temperature at surface is 49°; at 110 yards depth, 53°. 6; at 336 yards, 65°. 8; at 472 yards, 74°. 6.

In Saxony, in four of the deepest mines, annual temperature at surface is 46°. 4; at 170 to 200 yards depth, 54°. 5; at 280 yards, 58°; at 360 yards, 62°. 6.

In the coal mine of Killingworth, the deepest in Britain, annual temperature at surface is 48°; at 300 yards, 70°; at 400 yards, 77°. In seven others of the deepest coal mines in Britain, a corresponding gradation was observed.

In these British mines, the increment of temperature is about 1° for 15 yards of descent. In the Vosges it is about 1° for 20 yards, and in Saxony 1° for 22 yards. Taking 20 yards as a mean, if the increase follows the same arithmetical ratio to a considerable depth, we should find the temperature of the Bath waters (116°) at 1320 yards below the surface; and that of boiling water at 3300 yards, or nearly two miles.

EARTH, *pressure of against walls.*—*(Gregory.)*



Let D A E F be the vertical section of a wall, behind which is placed a bank or terrace of earth, of which a prism, whose section is represented by D A G, would detach itself and fall down, were it not prevented by the wall. Then A G is called the *line of rupture*, or the natural slope, or natural declivity. In sandy or loose earth, the ∠ B A G seldom exceeds 30°; in stronger earth it becomes 37°; and in some favourable cases more than 45°.

1. If h = A D, x = A E, $\theta = \angle$ D A G, and S and s represent the specific gravities of the wall and earth, the state of equilibrium is expressed by this equation,

$$\tfrac{1}{2}\, x^2.\, S = \tfrac{1}{6}\, h^2.\, s.\, \tan.^2 \tfrac{1}{2}\, \theta.$$

Ex. Suppose the wall to be 39.37 feet high, of brick, specific gravity 2000, and the bank of earth specific gravity 1428, and the natural slope 53°; then

$$\tfrac{1}{2}\, x^2.\, 2000 = \tfrac{1}{6} \times 39.37^2 \times 1428 \times \tan._2 26\tfrac{1}{2}^{\circ},$$

$$\therefore x = 9.6 \text{ feet} = \text{thickness of wall.}$$

The following practical results may be found useful.

Values of D G *for different materials.*

Bank of vegetable earth	D G = .618 h.
Do. of sand	D G = .677 h.
Do. of vegetable earth mixed with small gravel	D G = .646 h.
Do. of rubbles	D G = ·414 h.
Do. of vegetable earth mixed with large gravel	D G = .618 h.

Thickness of walls, both faces vertical.

1. Wall brick, 109 lbs. per cubic foot, bank vegetable earth carefully laid course by course	D F = .16 h.
2. Wall unhewn stones, 135 lbs. per cubic foot, earth as before	D F = .15 h.
3. Wall brick, earth clay well rammed	D F = .17 h.
4. Wall unhewn stones, earth as before	D F = .16 h.
5. Wall of hewn freestone, 170 lbs. per cubic foot, bank vegetable earth	D F = .13 h.
6. Do. bank clay	D F = .14 h.
7. Bank of earth mixed with large gravel, wall of bricks	D F = .19 h.
Do. of unhewn stone	D F = .17 h.
Do. of hewn freestone	D F = .16 h.
8. Bank of sand.	
Wall of bricks	D F = .33 h.
Do. of unhewn stones	D F = .30 h.
Do. of hewn freestone	D F = .26 h.

When the earth of the bank is liable to be much saturated with water the proportional thicknesses of the walls must at least be doubled.

2. For walls with an interior slope, or a slope towards the bank, let the base of the slope be $\frac{1}{n}$ of the height, then

$$DF = h\sqrt{\left(\frac{1}{3n^2} + m.\frac{s}{S}\right)} - \frac{h}{n}$$

where $m = .0424$ for vegetable or clayey earth, mixed with large gravel; $m = .0464$ if the earth be mixed with small gravel; $m = .1528$ for sand; and $m = .166$ for semifluid earths.

Ex. Let the height of a wall be 20 feet, and $\frac{1}{20}$ of the height for the base of the slope, suppose also the specific gravity of the wall and bank to be 2600 and 1400, and the earth semifluid; then

$$DF = 20\sqrt{\left(\frac{1}{1200} + .166 + \frac{14}{26}\right)} - \frac{20}{20}$$

= 5 feet, while the thickness of the wall at the bottom will be 6 feet.

EASTER, *to find it on any year.—(Delambre.)*

1. Divide the year proposed by 19 .... Call remainder a.
2. Divide the same number by 4 .... Call remainder b.
3. Divide it also by 7 .... Call remainder c.
4. Divide $(19\,a + M)$ by 30 .... Call remainder d.
5. Divide $(2\,b + 4\,c + 6\,d + N)$ by 7 .... Call remainder e.
6. Then Easter day will fall either on $(22 + d + e)$ of March; or on $(d + e - 9)$ of April.

Values of M and N in the above calculation.

	M.	N.
From 1700 to 1799	23	3
1800 to 1899	23	4
1900 to 1999	24	5

Exceptions to this rule:

1. If the computation give April 26, substitute the 19th.
2. If it give .... April 25, substitute the 18th.

ECCENTRICITY *of a Planet's orbit.—(Woodhouse, Playfair.)*

Let e be the eccentricity of the orbit, g the greatest equation of the centre, found by observation, and put $\frac{g}{57^0.\,29578} = h$, then

$$e = \tfrac{1}{2}\,h - \frac{11}{768}\,h^3 - \frac{587}{983040}\,h^5 - \&c.$$

In the earth's orbit h is very small, $\therefore e = \frac{1}{2} h$ nearly.

The secular diminution $= 18''.79$, and $\therefore$ if this diminution continued uniform (which, however, we have not a right to suppose) the earth's orbit would become a circle in about 36300 years.

ECHO.

That an echo may return one syllable as soon as it is pronounced, the reflecting surface should be 80 or 90 feet distant; for a dissyllablic echo 170 feet, &c. This is upon the supposition that sound proceeds at the rate of 1142 feet per second, and that the ear can distinguish the succession of two sounds or syllables, when the interval between them is $\frac{1}{7}$th of a second.—*(Playfair.)*

An echo in Woodstock Park repeats 17 syllables by day, and 20 by night. An echo on the north side of Shipley church in Sussex, repeats 21 syllables.—*(Young's Nat. Phil.)*

ECLIPSES.—*(Woodhouse, Playfair.)*

1. *Eclipses of the Moon.*

1. The length of the earth's shadow varies, according to the distance of the sun and earth, between the limits of 212,896, and 220,238 semidiameters of the earth; its mean length being 216,531. And in general if r be the earth's radius, $\frac{D}{2}$ the apparent semidiameter, and p the horizontal parallax of the sun, the length of the shadow, reckoned from the earth's centre,

$$= \frac{r}{\sin.\left(\frac{D}{2} - p\right)} \text{ or } = \frac{r}{\sin.\ \frac{109\ D}{220}}.$$

2. Hence half the angle subtended at the earth's centre by the section of the shadow, at the distance of the moon, (if P be the horizontal parallax of the moon) is

$$P + p - \frac{D}{2}.$$

From this formula the apparent diameters of the earth's shadow may be computed for various distances of the sun and moon, as in the following Table.

		Apparent diam. of earth's shadow.
Sun in perigee	Moon in apogee	1°. 15′. 24″.3036
	at mean distance	1. 23. 2·31
	in perigee	1. 30. 40.3164

		Apparent diam. of earth's shadow.		
Sun at mean distance	Moon in apogee	1.	15.	56.8656
	at mean distance	1.	23.	34.872
	in perigee	1.	31.	12.8784
Sun in apogee	Moon in apogee	1.	16.	28.2936
	at mean distance	1.	24.	6.3
	in perigee	1.	31.	44.3064

3. The distance of the centres of the moon and of the earth's shadow, when the moon's disk just touches the shadow (if d = moon's diameter) is

$$P + p - \frac{D}{2} + \frac{d}{2}.$$

Cor. If $P = 57'.\ 1''$, $p = 8'',8$, and $\frac{D}{2} = 16'.\ 1''.3$, we have the mean apparent $\frac{1}{2}$ diameter of the earth's shadow $= 41'.\ 8''.5$, which is nearly three apparent $\frac{1}{2}$ diameters of the moon. Hence since the moon in the space of an hour moves over a space nearly equal to its diameter, the moon may be entirely within the shadow, or a total eclipse may endure, about two hours.

4. The apparent $\frac{1}{2}$ diameter of a section of the penumbra at the moon's orbit =

$$P + p + \frac{D}{2}.$$

And the distance of the moon's centre and of the centre of the shadow, when the moon first enters the penumbra, is

$$P + p + \frac{D}{2} + \frac{d}{2}.$$

5. To find the time, duration, and magnitude of a lunar eclipse.

Let m = moon's motion in longitude,
n = moon's motion in latitude,
s = sun's (or the shadow's centre's) motion in longitude,
λ = moon's latitude when in opposition,
t = time from opposition,
c = distance of moon and earth's shadow,

and let $\frac{n}{m - s} = \tan.\ \theta$.

$$\text{then } t = \frac{1}{n}\left\{-\lambda \sin.^2 \theta \pm \sin.\ \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}\right\}$$

from which expression may be deduced values of the time, corresponding to any assigned values of c, as in the following instances.

(j) To determine the time at which the moon first enters the penumbra, for c put $P + p + \frac{D}{2} + \frac{d}{2}$; t has two values, and the second value will denote the time at which the moon quits the penumbra.

(ij) To determine the time at which the moon enters the umbra, put $c = P + p + \frac{d}{2} - \frac{D}{2}$.

(iij) To determine the time when the whole disk has just entered the shadow, we must deduct d from the preceding value, and make $c = P + p - \frac{d}{2} - \frac{D}{2}$; and similarly for other phases.

(iiij) To find the middle of the eclipse, we have $t = -\lambda \frac{\sin.^2 \theta}{n}$, and in that case the distance of the centres (c) is $= \lambda \cos. \theta$.

(v) The nearest approach of the centres being known, the magnitude of the eclipse is easily ascertained. Thus on the supposition that $\lambda \cos. \theta$ is less than the distance $\left(P + p + \frac{d}{2} - \frac{D}{2}\right)$ at which the moon's limb just touches the shadow, some part of the moon's disk is eclipsed; and the portion of the diameter of the eclipsed part is

$$P + p + \frac{d}{2} - \frac{D}{2} - \lambda \cos. \theta.$$

The portion of the diameter of the non-eclipsed part is the moon's apparent diameter d, minus the preceding expression, and therefore is

$$\lambda \cos. \theta + \frac{d}{2} + \frac{D}{2} - P - p.$$

If this expression should be equal nothing, the eclipse would be *just* a total one. If the expression should be negative, the eclipse may be said to be *more than* a total one, since the upper boundary of the moon's disk would be below the upper boundary of the section of the shadow.

(vj) If in the expression

$$\frac{2}{n} \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}.$$

we substitute for c, $P + p + \frac{d}{2} - \frac{D}{2}$ we have the time from the moon's first entering to her finally quitting the shadow or *umbra*. And if in the same expression we substitute for c, $P + p + \frac{d}{2} + \frac{D}{2}$, we have the whole time of an eclipse, from the moon's first entering, till her finally quitting the penumbra.

6. Ecliptic limits. When the mean opposition is 12°. 36′ distant from the node, there can be no eclipse; and when it is less than 9°. distant from it, there must be an eclipse. Between these limits 12°. 36′ and 9°. the matter is uncertain, and must be decided by the calculation of the true place of the moon.

II. *Eclipses of the Sun.*

1. Let r, R be the radii of the moon and earth, the rest as before; then the length of the moon's shadow

$$= \frac{r}{\sin. \left\{ \left(\frac{D}{2} - p \frac{r}{R} \right) \frac{P}{P - p} \right\}}$$

By means of this formula, we have

	Length of shadow.	*Moon's distance.*
Sun in apogee, moon in perigee	59.730	55.902
Sun in perigee, moon in apogee	57.760	63.862

Hence in the latter case, the moon's shadow never reaches the earth, and the eclipse cannot any where be total.

The moon's mean motion about the centre of the earth is 33′ in an hour; and the shadow of the moon ∴ traverses the surface of the earth, when it falls on the surface perpendicularly, with a velocity of about 380 miles in a minute. When the shadow falls obliquely, its velocity appears greater in the inverse ratio of the sine of the obliquity.

The duration of a total eclipse in any given place cannot exceed 7*m*. 58*s*.

An annular eclipse may last 12*m*. 24*s*.

2. The apparent ½ diameter of the moon's shadow $= \frac{d - D}{2} + \frac{P}{P - p}$.
Hence when $d = D$ apparent ½ diameter $= o$, or the vertex of the conical shadow just reaches the earth. When d is less than D, the expression is negative, in other words the shadow never reaches the earth.

In a similar manner may the formulæ for the penumbra of the earth be transformed and adapted to the case of the moon.

(iii) The solar ecliptic limits = 17°. 21′. 27″. If the conjunction happens nearer to the node than this, there may be an eclipse. If it be more distant, there can be none.

Solar eclipses are more difficult of computation than lunar ones; nor is it possible to enter here upon the methods that have been employed. We shall ∴ conclude this article with an account of the number of eclipses that may take place in a year.

III. *Eclipses, number of.*

In the space of 18 years, there are usually about 70 eclipses, 29 of the moon, and 41 of the sun.

Seven is the greatest number of eclipses that can happen in a year, and two the least.

If there are seven, five must be of the sun, and two of the moon. If there are only two, they must be both of the sun; for in every year there are at least two eclipses of the sun.

There can never be more than three eclipses of the moon in a year; and in some years there are none at all.

Though the number of solar eclipses is greater than of lunar in the ratio of 3 to 2, yet more lunar than solar eclipses are visible in any particular place, because a lunar eclipse is visible to an entire hemisphere, and a solar is only visible to a part.

ECLIPTIC, *obliquity of.—(Woodhouse, Vince.)*

The mean obliquity of the Ecliptic in January 1, 1827 = 23°. 27′. 43″.7. For the variations in the obliquity, *see Precession.* But besides these variations in the obliquity, arising from solar inequality and nutation, the former of which passes through all its changes in the period of half a year, and the latter in 9 years and 3½ months, the obliquity of the Ecliptic has, as far back as observation goes, been diminishing from the action of the planets, particularly Venus and Jupiter. This diminution, called the secular diminution, is *at present* 52″ in a century. There is, however, a mean to the obliquity which it cannot pass, and round which it oscillates backwards and forwards. According to La Grange, the inclination will never vary more than 5°. 23′ from the year 1700.

Hence if we have given the mean obliquity for any time, and wish to find the true obliquity, we must correct the given mean obliquity by the secular diminution, the solar inequality, and the nutation. The analytical expression for the obliquity, including these corrections, is

$$E - \frac{0''.52 \times n}{365} + 0''.4345 \times \cos.\ 2\ \text{sun's longitude} + 9''.63 \times \cos.\ N$$

E being the mean obliquity at the beginning of the year, N the supplement of the node, and n the number of days from the beginning of the year.

ELASTIC *bodies, equilibrium of.—(Whewell.)*

This subject may be comprised under three heads. (1.) Elasticity of Extension and Compression, as in the case of a string stretched by a force. (2.) Elasticity of Flexure, as when wires and laminæ of different metals and other substances exert a force to unbend themselves when forcibly bent. (3.) The Elasticity of Torsion, as when twisted threads of metal exert a force to untwist themselves. Our view of these several subjects must necessarily be very limited and imperfect.

1. *Elasticity of Extension.*

1. When an elastic string of given length is stretched by a given force, to find its length.

The increase of length is proportional to the tension. Let ι be the measure of the *extensibility* of the string, whose length at first is a; t the force or weight with which the string is stretched, which of course measures the tension; then the increase of length $= a \iota t$, and the length l when stretched will ∴ be

$$a + a \iota t, \text{ or } a\ (1 + \iota t)$$

We may determine ι, if we know the original length of the string, and its length for any given value of t. It may be convenient to know it in terms of the force which will draw out the string to *double* its length. Let E be this force; hence

$$a\ (1 + \iota\ \mathrm{E}) = 2\ a, \text{ and } \iota = \frac{1}{\mathrm{E}}.$$

Hence the length of the string under a tension t becomes

$$= a\left(1 + \frac{t}{\mathrm{E}}\right).$$

E may be expressed by a length of the given string, whose weight would draw the string a to double its length. E is then called the *modulus of elasticity.*

2. A uniform elastic string hangs vertically, stretched by its own weight: to find its length.

The same notation being retained,

$$l = a + \frac{\iota\ a^2}{2} \text{ or } = a + \frac{a^2}{2\ \mathrm{E}}.$$

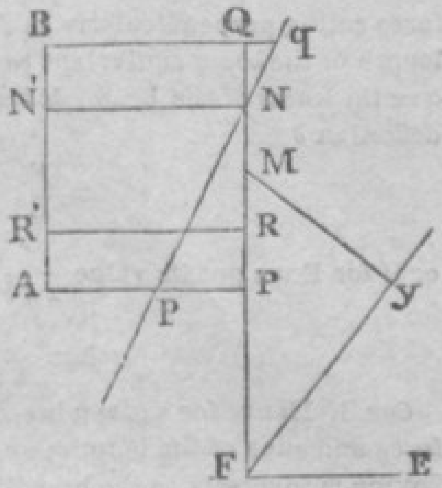
Cor. 1. If $a = \mathrm{E}$, $l = \frac{3\ \mathrm{E}}{2}$.

Cor. 2. Since $l = a\left(1 + \frac{\iota\ a}{2}\right)$, it appears that the weight of the string stretches it half as much, as if it were all collected at the lowest point.

2. *Elasticity and resistance of solid materials.*

Here we suppose that all solid bodies may be considered as made up of elastic fibres capable of extension and compression; and that the resistance to extension is proportional to the extension in each fibre.

When a solid body is acted on by any force, it may be partly extended and partly compressed. Thus let a mass A B Q P be acted upon by a force F compressing it in the direction E F. The surface P N Q may be brought into the direction p N q; in this case all the fibres R R′ which are on one side of N are shortened; all those on the other side of N are lengthened. N N′ remains the same as in the natural state. N is called the *neutral point;* and the line which separates the parts of the body which are compressed from those which are elongated is called the *neutral line.*



1. When a rectangular prismatic mass is compressed by a force parallel to the direction of the axis: to find the neutral line.

Let P M = M Q = a, M F = h, M N = n, then

$$n = \frac{(2\,a)^2}{12\,h} = \frac{\text{P Q}^2}{12\,\text{M F}}$$

Cor. 1. If $h = \frac{1}{3}\,a$, $n = a$, or the neutral point is in the surface, and the whole beam is compressed.

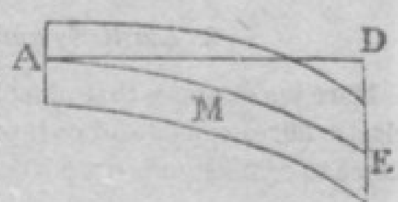
2. When a rectangular prism is acted upon by any force in any direction; to find the neutral point at any part.

Let a force f act in the line y F on a prism A B P Q, then the same notation being retained, we have as before

$$n = \frac{a^2}{3\,h}.$$

Cor. If the force act perpendicularly to the axis, h is infinite, $n = o$, and the neutral point is in the axis.

3. When a rectangular prismatic beam is made to deviate a little from a straight line by the action of a given force perpendicular to it, to find the deflexion.



Since the force is perpendicular to the beam, and the beam is nearly a straight line, we may *(by Cor. last Art.)* suppose the neutral point coincident with the axis. Let A M E represent the axis bent by a force acting perpendicularly to A D its original position; and let F be a length of the beam equivalent to the force f, $l =$ length $b =$ breadth, and $a =$ thickness of the beam, E the modulus of elasticity, then the whole deflexion δ

$$= \frac{F}{E} \cdot \frac{l^3}{a^2}$$

or if for F we put its value $\frac{f}{2ab}$,

$$\delta = \frac{f l^3}{2 E a^3 b}.$$

Cor. 1. Hence for a given breadth and thickness, the deflexion is as the force and cube of the length; and for a given weight and length, the deflexion is inversely as the breadth and cube of the thickness.

Cor. 2. Let the direction of the tangent at E make an $\angle \theta$ with the tangent at A; then θ may be called the *angular deflexion*, and we have

$$\tan \theta = \frac{F}{E} \cdot \frac{3 l^2}{2 a^2} = \frac{3 f l^2}{4 a^3 b}.$$

The angular deflexion is as the force and square of the length.

4. When a rectangular prismatic beam in a horizontal position is bent by its own weight; (its thickness being vertical) to find the deflexion.

The same notation being retained, the whole deflexion

$$\frac{3 l^4}{8 E a^2}.$$

Cor. In this and the last Art. δ being observed, E may be found.

5. A rectangular prismatic beam is compressed by a given force acting in a direction parallel to the axis; to find the deflexion.

Let a be $\frac{1}{2}$ the thickness of the beam, $l = \frac{1}{2}$ the length, $h =$ distance of the force from the axis; then if E be very large compared with F, we have the deflexion

$$= h \left(\sec. \frac{l \sqrt{3 F}}{a \sqrt{E}} - 1\right).$$

Cor. If the force act at the extremities of the axis, $h = o$, and there will be no deviation except

$$\frac{l^2}{a^2} = \frac{\pi^2 E}{12 F} = .8225 \frac{E}{F}.$$

Hence we may find the weights which columns of given materials will support. Thus, if in fir-wood the modulus E be 10,000,000 feet, a bar, an inch square, and 10 feet long, may begin to bend, when

$$F = .8225 \times \frac{1}{(120)^2} \times 10{,}000{,}000 = 571 \text{ feet.}$$

3. *Elasticity of Torsion.*

1. Let f and f' be the forces necessary to twist a metallic thread, from the position in which it would naturally hang, through the ∠s. θ and θ'; then if θ and θ' be very small,

$$\frac{f}{f'} = \frac{\theta^3}{\theta'^3}.$$

On this principle depends the *Torsion Balance* of Coulomb, which has been employed for the purpose of measuring very small repulsive and attractive forces. In some cases the instrument was constructed with so much delicacy, that each degree of torsion required a force of only $\frac{1}{122400}$ of a grain.

Height of the Modulus of Elasticity in thousands of feet.—(Encyclop. Brit. Supplem.)

Iron and steel	10,000	Fir wood	10,000
Copper	5,700	Elm	8,000
Brass	5,000	Beech	8,000
Silver	3,240	Oak	5,060
Tin	2,250	Box	5,050
Crown glass	9,800	Ice	,850

The following Table is the result of experiments by Mr. Rennie, published in the first part of the Phil. Trans. for 1818.

Mr. Rennie found a cubical inch of the following bodies crushed by the following weights :—

	lbs. av.
Elm	1284
American Pine	1606
White Deal	1928
English Oak	3860

Cubes of 1½ inch.

	Sp. gr.	
Chalk		1127
Red Brick	2168	1817
Derby Grit	2316	7070
Portland	2428	10284

	Sp. gr.	
Craigleith White Freestone	2452	12346
Yorkshire Paving	2507	12856
White Statuary Marble	2760	13632
Bromley Fell Sandstone, near Leeds	2506	13632
Cornish Granite	2662	14302
Dundee Sandstone	2530	14918
Compact Limestone	2584	17354
Purbeck	2599	20610
Black Brabant Marble	2697	20742
Very Hard Freestone	2528	21254

Cubes of different metals of ¼ inch were crushed by the following weights :—

	lbs. av.		lbs. av.
Cast Iron	9773	Wrought Copper	6440
Cast Copper	7318	Cast Tin	966
Fine Yellow Brass	10304	Cast Lead	483

Bars of different metals six inches long, and ¼ inch square, were suspended by nippers, and broken by the following weights :—

	lbs. av.		lbs. av.
Cast Iron, horizontal	1166	Gun Metal	2273
Ditto, vertical	1218	Copper hammered	2112
Cast Steel	8391	Cast Copper	1192
Blistered Steel hammered	8322	Fine Yellow Brass	1123
Shear Steel do.	7977	Cast Tin	296
Swedish Iron do.	4504	Cast Lead	114
English Iron do.	3492		

ELASTIC *bodies, theory of.—See Collision.*

ELLIPSE, *principal properties of.—See Conic Sections.*

ELLIPTICITY *of the Earth.—See Earth, figure of.*

EMBANKMENT.—*See Dyke, and Earth pressure of.*

EPOCH.—*See Æra.*

EQUATIONS *of condition.—(Playfair, Maddy.)*

Any equation expressing the relation that obtains among the coefficients of another equation, is called an *Equation of condition.* These equations are used in determining by observation the constant coefficients in an assumed or given function of a variable quantity. Thus let us suppose that the form of the function is known from theory, but that the constant quantities that enter into it, are to be determined by observation; required, considering that every observation is liable to error, in what way these quantities may be most accurately determined.

RULE.—Substitute the quantities known by observation for y and x, in the given formula (each observation being supposed to afford a value both of x and y), and thus, as many equations of condition will be obtained, as there are observations. If these exceed the number of quantities to be found, or of the equations wanted, let there be composed from the addition of them into separate sums, as many equations as are necessary, each consisting of as many of the given equations as the question admits of. From the equations thus obtained, the quantities sought may be determined with the least probability of error.

Suppose the general formula to be

$$y = \text{A sin.}\, x + \text{B sin. } 2\,x,$$

and that from observation we have eight values of x and y, viz.

Values of x.	Values of y.
140°	73′.5
135	80.2
130	87.0
125	94.1
120	99.5
115	104.5
110	107.5
105	110.2

Hence,

$$.6428\text{ A} - .9848\text{ B} = 73.5$$
$$.7071\text{ A} - 1.0000\text{ B} = 80.2$$
$$.7660\text{ A} - .9848\text{ B} = 87.0$$
$$.8191\text{ A} - .9337\text{ B} = 94.1$$
$$.8660\text{ A} - .8660\text{ B} = 99.5$$
$$.9063\text{ A} - .7660\text{ B} = 104.5$$
$$.9397\text{ A} - .6428\text{ B} = 107.5$$
$$.9660\text{ A} - .5000\text{ B} = 110.2$$

By adding the first four into one, and also the second four, we get

$$2.9350\text{ A} - 3.9033\text{ B} = 334.8, \text{ and}$$
$$3.6780\text{ A} - 2.7748\text{ B} = 421.7;$$

and therefore,

$$\text{A} = \frac{3.9033 \times 421.7 - 2.7748 \times 334.8}{3.678 \times 3.9033 - 2.935 \times 2.7748},$$

$$\text{or A} = 1^{0}.55^{\prime}.00.$$

In like manner, $B = 1'.2$; so that the equation becomes,

$$y = (1^{\circ}.\ 51'.2)\ \sin.\ x + (1'.2)\ \sin.\ 2\ x.$$

This is nearly the equation of the centre in the earth's orbit.

In this way all the elements of any of the planetary orbits may be determined *simultaneously*, or corrected if they are already nearly known. In the construction of Astronomical Tables, the number of equations combined has amounted to many hundreds.

In the example above, no method was to be followed, but that of dividing the original equations into two parcels or groups, from the sums of which the new equations were to be deduced. But when it happens in the given equations, that the terms involving the same unknown quantity have different signs, the best way is to order all the equations so that one of the unknown quantities, as A, shall have the same sign throughout; and then to add them together, for the first of the derivative equations. Let the same be done with B, C, &c. whatever be the number of the quantities sought. Thus, each of the unknown quantities will occur in one of the equations, with the greatest possible coefficient; and the coefficients of the same unknown quantity, in the different equations, will become by that means as unequal as they can be rendered, which contributes to make the divisor by which that quantity is to be found, as large, and itself of course, as accurate as the case will admit of.

Ex. Let the equations be

$$3 - x + y - 2\,z = o$$
$$5 - 3\,x - 2\,y + 5\,z = o$$
$$21 - 4\,x - y - 4\,z = o$$
$$14 + x - 3\,y - 3\,z = o$$

changing the signs of the last equation, and adding,

$$15 - 9\,x + y + 2\,z = o$$

similarly for y, $37 - 5\,x - 7\,y \quad = o$

for z, $\quad 33 - \ x - y - 14\,z = o$

From these equations $x = 2.486$
$y = 3.517$
$z = 1.928$

Second Method.

Let $m + a\,x + b\,y + c\,z + \&c. = o$,
$m' + a'\,x + b'\,y + c'\,z + \&c. = o$,
$m'' + a''\,x + b''\,y + c''\,z + \&c. = o$,
&c. .... $= o$,

be the equations; multiply the first by a, the second by a', and so on; then by addition,

$$m a + m' a' + \&c. + (a^2 + a'^2 + \&c.)\, x + (a b + a' b' + \&c.)\, y + (a c + a' c' + \&c.)\, z = o,$$

Similarly

$$(m b + m' b' + \&c.) + (a b + a' b' + \&c.)\, x + (b^2 + b'^2 + \&c.)\, y + (b c + b' c' + \&c.)\, z + \&c. = o,$$

$$(m c + m' c' + \&c.) + (a c + a' c' + \&c.)\, x + (b c + b' c' + \&c.)\, y + (c^2 + c'^2 + \&c.)\, z + \&c. = o,$$

$$\&c. + \&c. \ldots\ldots\ldots\ldots\ldots\ldots = o.$$

By this means as many equations are formed as there are unknown quantities, and from them x, y, z, &c. may be determined.

The method applied to the example in the preceding article gives the reduced equations

$$-\ 88 + 27\, x + 6\, y = o,$$
$$-\ 70 + 6\, x + 15\, z = o,$$
$$-\ 107 + y + 54\, z = o.$$

From whence $x = 2{,}470$, $y = 3{,}551$, $z = 1{,}916$.

The above mode of reducing the linear equations, which is called the *Method of Least Squares*, was invented by Gauss.

EQUATION *of Payments.*

Common rule.

Let p and p' be the sums due at the end of the times n and n'; x = equated time

$$\text{then } x = \frac{p n + p' n'}{p + p'}.$$

i.e. equated time is found by multiplying each sum by the time at which it is due, and dividing by the sum of the payments.

This rule is erroneous in principle, being founded upon the supposition that the receiver gains *interest* upon the latter sum by receiving it before it is due; whereas in fact he ought only to gain the *discount*. In most questions, however, that occur in business, the error is so trifling, that the above rule will always be made use of as the most eligible method.

Correct rule.

Let r = interest of £1. for one year, the rest as before, put $\frac{p r (n + n') + p' + p}{p r} = a$, and $\frac{p r n n' + p' n' + p n}{p r} = b$, then

$$x = \frac{a \pm \sqrt{a^2 - 4 b}}{2}.$$

EQUATION *of Time.*

The equation of time, relatively to its causes, depends on two circumstances; (1) the obliquity of the ecliptic; and (2) the unequal angular motion of the sun in its orbit. The equation of time, as arising from the first cause, would be the difference of the sun's longitude and its right ascension converted into time. In the first and third quadrants, apparent time would precede true; in the second and fourth quadrants, true time would precede apparent; and at the Tropics and Equinoxes, true and apparent time would coincide. Also upon this supposition, the equation would be a maximum at 4 points, viz. when the cosine of the sun's declination is a mean proportional between radius, and the cosine of the obliquity of the ecliptic.

The equation of time, as arising from the second cause, would be the difference between the true and mean anomaly. Hence true and apparent time would coincide at the higher and lower apsides. From the higher to the lower apside, apparent time would precede true; from the lower to the higher apside, true time would precede apparent. The equation, in this case, would be greater at two points than at any other, viz. when the earth's distance from the sun is a mean proportional between the $\frac{1}{2}$ axes of its orbit. To find it, when both causes are considered together, let A be the sun's *true* right ascension, M his *mean* longitude, ν the equation of the Equinoxes in longitude; then $\nu \times$ cos. obliquity = the equation of the Equinoxes in right ascension, and

$$\text{Equation of time} = \frac{A - M - \nu \times \text{cos. obliquity}}{15}.$$

which is to be added to apparent time if positive, and subtracted if negative.

As the sun's true right ascension is deduced from the true longitude and the apparent obliquity of the ecliptic, both of which vary from one age to another; hence tables of the equation of time, constructed for any one time, are not true for another. The following Table, therefore, taken from the Nautical Almanack for 1828, or leap year, though inapplicable when any very nice determinations of the time are required, may yet be useful for regulating common clocks or watches, as the error for the next half century will only amount to a few seconds.

TABLE.

Equation of Time for every Day in the Year 1828.

Days.	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	Add	Add	Add	Add	Sub.	Sub.	Add	Add	Sub.	Sub.	Sub.	Sub.
	m s	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*
1	3 35	13 52	12 35	3 54	3 5	2 33	3 25	5 57	0 15	10 25	16 17	10 38
2	4 4	14 0	12 23	3 36	3 13	2 24	3 37	5 53	0 34	10 43	16 17	10 14
3	4 32	14 7	12 10	3 18	3 19	2 14	3 48	5 49	0 53	11 2	16 17	9 50
4	4 59	14 13	11 56	3 0	3 26	2 4	3 58	5 44	1 12	11 20	16 16	9 26
5	5 27	14 18	11 42	2 42	3 31	1 54	4 9	5 38	1 32	11 38	16 14	9 1
6	5 54	14 23	11 28	2 25	3 37	1 43	4 19	5 32	1 51	11 55	16 11	8 35
7	6 20	14 27	11 17	2 7	3 41	1 33	4 29	5 25	2 11	12 12	16 8	8 9
8	6 46	14 30	10 59	1 50	3 45	1 21	4 39	5 18	2 32	12 29	16 3	7 43
9	7 11	14 32	10 43	1 33	3 48	1 10	4 48	5 10	2 52	12 45	15 58	7 15
10	7 36	14 34	10 28	1 17	3 51	0 58	4 57	5 1	3 12	13 0	15 52	6 48
11	8 1	14 35	10 12	1 1	3 53	0 46	5 5	4 52	3 33	13 15	15 45	6 20
12	8 25	14 35	9 55	0 45	3 55	0 34	5 13	4 43	3 54	13 30	15 37	5 52
13	8 48	14 34	9 39	0 29	3 55	0 1	5 20	4 32	4 15	13 44	15 29	5 23
14	9 10	14 33	9 22	0 11	3 56	0 9	5 27	4 22	4 36	13 58	15 19	4 55
				Sub.		Add						
15	9 32	14 30	9 5	0 1	3 56	0 4	5 34	4 10	4 57	14 11	15 9	4 25
16	9 54	14 28	8 47	0 16	3 55	0 17	5 40	3 58	5 18	14 24	14 58	3 56
17	10 15	14 24	8 30	0 30	3 54	0 30	5 45	3 46	5 39	14 36	14 46	3 27
18	10 35	14 19	8 12	0 44	3 52	0 43	5 50	3 33	6 0	14 47	14 34	2 57
19	10 54	14 14	7 54	0 58	3 49	0 56	5 54	3 20	6 21	14 58	14 20	2 27
20	11 12	14 9	7 36	1 11	3 46	1 9	5 58	3 6	6 42	15 8	14 6	1 58
21	11 30	14 2	7 18	1 24	3 43	1 22	6 1	2 51	7 3	15 18	13 51	1 28
22	11 47	13 55	7 0	1 36	3 39	1 35	6 4	2 37	7 24	15 27	13 35	0 58
23	12 3	13 47	6 41	1 48	3 34	1 47	6 6	2 21	7 45	15 35	13 18	0 28
												Add
24	12 18	13 39	6 23	1 59	3 29	2 0	6 7	2 5	8 5	15 43	13 1	0 2
25	12 33	13 29	6 4	2 10	3 24	2 13	6 8	1 49	8 26	15 50	12 42	0 32
26	12 47	13 20	5 46	2 20	3 18	2 25	6 8	1 33	8 46	15 56	12 23	1 2
27	13 0	13 9	5 27	2 30	3 12	2 38	6 8	1 16	9 7	16 1	12 4	1 32
28	13 12	12 59	5 8	2 40	3 5	2 50	6 7	0 58	9 26	16 6	11 43	2 0
29	13 23	12 47	4 50	2 49	2 57	3 2	6 5	0 41	9 46	16 10	11 22	2 30
30	13 33		4 31	2 57	2 50	3 14	6 3	0 23	10 6	16 13	11 0	3 0
31	13 43		4 13		2 41		6 0	0 4		16 15		3 29

The above Table contains the equation of time for leap year; but the equation may be found for other years as follows. For the first year after leap year take one-fourth of the difference between the equations for the given and preceding days, which is to be added to the equation for the given day, if at that time the equation is decreasing; but subtracted if it is increasing. In the second after leap year, take half the difference between the equations; and in the third, take three-fourths of the difference, and apply this correction in the same manner as before.

Note.—The word *add* in the Table denotes that the equation of time, as there expressed, must be *added* to the apparent time, shewn by a Dial or other instrument, in order to give the mean or equated time. In those columns to which the word *sub* is prefixed, it implies that the equation of time must be *subtracted* from the apparent time, in order to give the true or correct time.

If it be proposed to convert mean time into apparent, this is done by a contrary process, by applying the equation of time to the mean time given, with its title or sign changed, viz. subtracting instead of adding, and adding instead of subtracting.

EQUILIBRIUM of *Floating Bodies.*—(*Playfair, Bland.*)

1. When the centre of gravity of a floating body is in the same vertical line with the centre of gravity of the fluid displaced, the body remains in equilibrium.

2. If in a floating body, of which the transverse section is the same from one end of the body to the other, a be the length of the water line, c^2 the area of the section of the immersed part, d the distance between the centre of gravity of the whole and the centre of gravity of the immersed part, and i an indefinitely small inclination from the position of equilibrium, the momentum of the force tending to restore the equilibrium is

$$\left(\frac{a^3}{12\,c^2} - d\right) \text{W sin. } i.$$

If $\frac{a^3}{12\,c^2}$ is greater than d, the force tends to restore the body to its state of equilibrium, or the equilibrium is that of *stability*.

If $\frac{a^3}{12\,c^2} = d$, there is no force tending either to restore or destroy the equilibrium; or the equilibrium is that of *indifference*.

If $\frac{a^3}{12\,c^2}$ be less than d, the force becomes negative, and tends to overset the body; or the equilibrium is that of *instability*.

When W remains the same, the stability is proportional to $\left(\frac{a^3}{12\,c^2} - d\right)$ sin. i.

When the centre of gravity of the body is lower than the centre of gravity of the immersed part, d is negative, and the quantity $\frac{a^3}{12\,c^2} - d$ is affirmative, whatever be the magnitude of $\frac{a^3}{12\,c^2}$.

If in the axis of the solid, or in the line passing through the two centres, there be taken a point distant from the centre of the immersed part by $= \frac{a^3}{12\,c^2}$, this point is called the *metacentre*; and the stability will be positive or negative or nothing, according as the metacentre is above, below, or coincident with the centre of gravity of the floating body.

3. If a rectangular parallelopiped float in a fluid, with its altitude a perpendicular to the surface; if its breadth be b, and its specific gravity n, that of the fluid being 1, its stability will be as $\frac{b^2}{6} - n\,(1-n)\,a^2$.

When it has no stability, $\frac{b^2}{6} - n\,(1-n)\,a^2 = o$, and $a = \frac{b}{\sqrt{6\,n\,(1-n)}}$, and $n = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{b^2}{6\,a^2}}$.

Cor. 1. When $\frac{b^2}{6\,a^2}$ is less than $\frac{1}{4}$, or when the height of the solid has a greater proportion to the base of the section than $\sqrt{2} : \sqrt{3}$, two values may assigned to the specific gravity of the body, which will cause it to float in the equilibrium of indifference.

Cor. 2. If $n = \frac{1}{2}$, as is nearly the case with fir, $a = b\sqrt{\frac{2}{3}} = \frac{5\,b}{6}$ nearly. The truth of this conclusion may be shewn by experiment.

EQUILIBRIUM *of an Elastic Body.—See Elastic Bodies equilibrium of.*

EQUILIBRIUM *of a Point.—See Forces composition of.*

EQUINOXES, *precession of.—See Precession.*

ERRORS *in Time, in Astronomy.—See Time.*

EVAPORATION.

Mean monthly evaporation from the surface of water, from the experiments of Dr Dobson, of Liverpool, in the years 1772, 1773, 1774, and 1775.—*(Phil. Trans., and Manchester Memoirs.)*

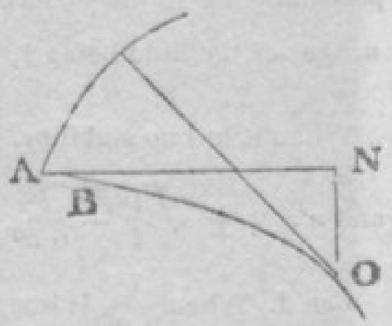
	Inches.		*Inches.*
January	1.50	July	5.11
February	1.77	August	5.01
March	2.64	September	3.18
April	3.30	October	2.51
May	4.34	November	1.51
June	4.41	December	1.49

From some very accurate experiments made by Mr Dalton, the mean annual evaporation, over the whole surface of the globe, has been estimated at 35 inches; this gives 94,450 cubic miles for the water annually evaporated over the whole globe.—*see Rain.*

EVECTION.—*See Moon.*

EVOLUTES *of Curves.—(Higman.)*

To find the equation to the evolute.



Let A N $= \alpha$, and N O $= -\beta$, then may the relation between α and β be found by eliminating x and y from the equations

$$y = f(x),$$

$$y - \beta = -\frac{1 + \frac{dy^2}{dx^2}}{\frac{d^2 y}{dx^2}},$$

$$x - \alpha = -\frac{dy}{dx}(y - \beta).$$

Ex. Required the evolute of the parabola.

$$\text{Here } y^2 = mx$$

$$y - \beta = \frac{4y^3}{m^2} + y.$$

$$x - \alpha = -\frac{2y^2}{m} - \frac{m}{2}.$$

Find values of y and x from the two last equations, substitute them in the first, and we shall have

$$\beta^2 = \frac{16}{27m}\left(\alpha - \frac{m}{2}\right)^3 = \frac{16}{27m}\alpha'^3, \text{ if } \alpha' = \alpha - \frac{m}{2}.$$

∴ the evolute is the semicubical parabola.

EVOLUTION.—*See Involution.*

EXPANSION *of liquids and solids by Heat.—See Heat.*

EXPANSION *of Water.—See Heat.*

EYE, *dimensions of, &c.—(Coddington.)*

The proportions of the spaces occupied by the three humours of the eye vary in different animals, as may be seen from the following Table, taken from *M. Cuvier's Anatomie Comparee,* which shews the parts of the *axis* lying in the several humours.

	Aqueous Humour.	*Chrystal-line.*	*Vitreous Humour.*
Man	$\frac{3}{22}$	$\frac{4}{22}$	$\frac{15}{22}$
Dog	$\frac{5}{21}$	$\frac{8}{21}$	$\frac{8}{21}$
Ox	$\frac{5}{37}$	$\frac{14}{37}$	$\frac{18}{37}$
Sheep	$\frac{4}{17}$	$\frac{11}{17}$	$\frac{12}{17}$
Horse	$\frac{9}{43}$	$\frac{16}{43}$	$\frac{18}{43}$
Owl	$\frac{8}{27}$	$\frac{11}{27}$	$\frac{8}{27}$
Herring	$\frac{1}{7}$	$\frac{5}{7}$	$\frac{1}{7}$

The radii of the surfaces of the chrystalline are in

Man as	12 to 16
Dog	12 to 14
Ox	6 to 21
Rabbit	14 to 14
Owl	16 to 14

The specific gravities of the different parts are as follows, that of distilled water being 1.

	In the Ox.	*In the Cod Fish.*
Aqueous humour	1	1
Vitreous humour	1.016	1.013
Chrystalline lens (mean)	1.114	1.165
Outer part of ditto	1.070	1.140
Inner	1.160	1.200

As to their refractive powers, they must be more considerable than their density indicates, on account of the inflammable particles which enter into their composition.

Dr. Wollaston makes the refracting power of the vitreous humour equal to that of water, and that of the chrystalline lens of the ox greater in the ratio of from 1.38 to 1.447 to 1. Dr. Brewster gives the following Table, deduced from experiments made on a recent human eye :—

Refracting power of	Water	1.3358
	The Aqueous humour	1.3366
	— Vitreous humour	1.3394
	— outer coat of chrystalline	1.3767
	— middle	1.3786
	— central parts	1.3990
	— whole-chrystalline	1.3839

Dr. Brewster also gives the following dimensions :—

	Inch.
Diameter of the chrystalline	0.378
——— cornea	0.400
Thickness of the chrystalline	0.172
——— cornea	0.042

If the humours of the eye be too convex or too flat, an imperfection in vision is in either case the consequence : a concave lens will remedy the former defect, and a convex one the latter. The following problems embrace nearly every thing connected with the theory of spectacles.

1. Given the distance at which a short-sighted person can see distinctly, to find the focal length of a concave glass which will enable him to see distinctly at any other given distance.

Let $\Delta'' =$ distance at which he can see distinctly, Δ a greater distance at which he wishes to view objects, $F =$ focal length of the required lens, then *(see Refraction* iii, *Art.* 2.)

$$\frac{1}{\Delta''} = \frac{1}{F} + \frac{1}{\Delta}; \text{ and } F = \frac{\Delta\,\Delta''}{\Delta - \Delta''}.$$

Cor. If Δ be indefinitely great, $F = \Delta''$.

2. Given the distance at which a long-sighted person can see distinctly, to find the focal length of a convex glass which will enable him to see distinctly at any other given distance.

Let $\Delta'' =$ distance at which he can see distinctly, Δ a shorter distance at which he wishes to view objects, $F =$ focal length of the lens, then

$$\frac{1}{\Delta''} = \frac{1}{\Delta} - \frac{1}{F}; \text{ and } F = \frac{\Delta\,\Delta''}{\Delta'' - \Delta}.$$

Cor. If Δ'' be indefinitely great, or the eye require parallel rays, $F = \Delta$.

TABLE,

Of the focal length of the convex or magnifying glasses, commonly required at various ages.—(Kitchiner.)

Years of age.	*Inches. Focus.*	*Remarks.*
40 ...	36	
45 ...	30	Convex Spectacles are seldom wanted except to read by candle light, till 45 or 50.
50 ...	24	
55 ...	20	
58 ...	18	
60 ...	16	Concave glasses called No. 1, are equivalent to a convex of 24 inches focus; No. 2 to a 21 inch convex; No. 3 to an 18 inch.
65 ...	14	
70 ...	12	
75 ...	10	
80 ...	9	

The following is an easy method of finding which of two concave or convex glasses magnifies most. Hold one in each hand about one foot from your eye, and about five feet from a window frame, and the lens, through which the panes of glass appear *least*, magnifies *most*. This is the readiest way of ascertaining their comparative power.

F

FIGURE *of the sines, &c.*

Figure of the sines, cosines, tangents, secants; to find the area of.

1. Figure of the sines.
   Let $\theta =$ arc or abscissa, then area $= r \times$ ver. sin. θ.
   When θ is a quadrant, area $= r^2$.

2. Figure of the cosines.
   Area $= r \times$ sin. θ.
   When θ is a quadrant, area $= r^2$.

3. Figure of the tangents.
   Area $= r^2 \times$ h. l. $\frac{\text{sec. } \theta}{r}$.
   When θ is a quadrant, area is infinite.

Cor. The solid, generated by the revolution of the figure of the tangents about its base, is equal to a cylinder, the base of which is the circle, and height = excess of tang. θ above θ.

4. Figure of the secants.

$$\text{Area} = r^2 \times \text{h. l.} \frac{\text{sec. } \theta + \text{tan. } \theta}{r}.$$

When θ is a quadrant, area is infinite.

Cor. The solid, generated by the revolution of the figure of the secants about its base, is equal to a cylinder, the base of which is the circle, and the altitude = tan. θ.

FLEXURE *point of contrary in curves.—See Inflexion.*

FLOATING *bodies.—See Specific Gravity, and Equilibrium.*

FLUENTS.—*See Differentials.*

FLUIDS, *pressure of.—(Vince, Bland.)*

1. The pressure of a fluid against any surface, in a direction perpendicular to it, is as the area of the surface, multiplied into the depth of its centre of gravity below the surface of the fluid, multiplied into the specific gravity of the fluid; and is ∴ equal to the weight of a cylinder of the same fluid, the area of whose bottom = the given surface, and altitude the depth of the centre of gravity.

Hence the pressure is entirely independent of the weight of the fluid.

Ex. Compare the pressure on the area of a parabola with that on its circumscribing rectangle, both being immersed perpendicularly to the vertex.

The areas are as 2 : 3, and the depths of their centres of gravity as $\frac{3}{5} : \frac{1}{2}$; ∴ the pressures are as 4 : 5.

2. Hence if a vessel be filled with a fluid, the pressure on any part : the whole weight of the fluid :: the area of that part × the depth of its centre of gravity : the solid content of the fluid.

Ex. 1. In a cone, pressure on base = 3 weight of fluid.
2. In a cube, pressure on any side = $\frac{1}{2}$ weight of fluid.
3. In a sphere, pressure on surface = 3 weight.
4. In a paraboloid, pressure on base = 2 weight.
5. In a cylinder, pressure on bottom : pressure on the side :: diamater of base : 2 altitude.

3. If a solid of revolution be filled with fluid, to find the pressure perpendicular to the surface.

Let the height of the solid = h, x and y the coordinates, then the pressure on the curve surface

$$= 2\pi \text{ fl. } y\, d\, \Sigma.\, (h - x) + \text{C}.$$

Ex. Let the surface be a segment of a sphere, with its vertex downwards.

Here $y\, d\, \Sigma = r\, d\, x$, $\therefore$ pressure $= 2\, \pi\, r$ fl. $(h - x) .\ d\, x = 2\, \pi\, r\, (h\, x - \frac{1}{2}\, x^2)$; and for the whole segment $x = h$, $\therefore$ pressure $= \pi\, r\, h^2$.

4. Upon the principle that fluids press equally in all directions, and in proportion to their perpendicular depths, depends the principle of the hydrostatical paradox or hydrostatic bellows.

In the hydrostatic bellows, as the area of the orifice of the pipe : area of the bellows board :: weight of the water in the pipe above the bellows board : the weight sustained on the board.

Cor. Supposing a given quantity of fluid to be poured into the tube, to determine how much the weight will rise.

Let z = required height, x and y = the area of the sections of the tube and bellows, and let the quantity poured into the tube $= l\, x$,

$$\text{then } z = \frac{l\, x}{x + y}.$$

5. If two fluids communicate in a bent tube, their perpendicular altitudes, above the plane where they meet, are inversely as their specific gravities.

Hence the *same* fluid will stand at the same altitude on each side. Thus water may be conveyed by pipes from a spring on the side of a hill to a reservoir of equal height on another hill.

For Centre of Pressure, *see Centre.*

A few practical inferences from the foregoing propositions.

1. In a vertical gate, dam, or sluice exposed to the pressure of water, the pressure on a square foot at the depth of d feet $= 1000\, d$ in ounces. And if it be rectangular and b its breadth, and D its depth in feet, the pressure by *Art.* 2. *Ex.* 2. $= 1000 \times \frac{1}{2}\, b\, D^2 = 500 \times b\, D^2$ in ounces.

2. If the transverse section of a canal be in the form of a trapezium, widest at the top, then if B and b be the breadth at the top and bottom respectively, and d the depth in feet, and it be required to find the pressure on a gate, which, standing across the canal, would dam the water up, we have area of trapez. $= \frac{1}{2}\, \overline{B + b}.\ d$; and depth of centre of gravity $= \frac{\overline{2\, b + B}\, .\ d}{3.\ \overline{B + b}}$; $\therefore$ the whole pressure in ounces $= 500.\ \frac{2\, b + B}{3}.\ d^2$.

3. The strongest angle of position for a pair of gates for the lock of a canal or river $= 109^{\circ}.\ 28'$.

4. The thickness of pipes to convey water is as $\frac{h\,d}{c}$; where h is the height of the head of water, d the diameter of the pipe, and c the cohesion of a bar of the same material as the pipe, and an inch square. In the same metal, thickness varies as $h\,d$. This result obviously only gives the *proportional* thickness : to determine the actual thickness, we must have a series of experiments on which to found our computation. But these do not appear to have been carried on upon a sufficiently large scale to inspire us with any confidence in the results. In fact, the thickness of pipes is generally determined in practice by experiment, or rather by imitating, as near as circumstances will allow, some other work of a similar kind.

Should we, however, suppose, with Dr Gregory, that a pipe of cast iron 15 inches diameter, and $\frac{3}{4}$ of an inch thick, will be strong enough for a head of water of 600 feet; and a pipe of oak of the same diameter, and two inches thick, would sustain a head of 180 feet, we should have for any other head h and diameter d, thickness of cast iron pipes $= \frac{h\,d}{12000}$, and thickness of oak pipes $= \frac{h\,d}{1350}$.

For the pressure of fluids against dykes—*see Dyke.*

FLUIDS *discharge of, through very small apertures in the bottom or sides of vessels.—(Vince, Bland, Playfair.)*

1. The velocity at the aperture is equal to that acquired in falling freely through $\frac{1}{2}$ the altitude of the fluid above the orifice, and the velocity at the *vena contracta* equal to that acquired in falling through the whole height.

Cor. 1. Hence if $h =$ height of the fluid above the orifice, $g = 32\frac{1}{6}$ feet, the velocity at the orifice $= \sqrt{g\,h}$, and velocity at the vena contracta $= \sqrt{2\,g\,h}$.

Cor. 2. If any pressure be exerted on the surface of the fluid, the velocity of the issuing fluid will be increased. Thus when water is projected into a vacuum, as the pressure of the atmosphere is equal to that of a column of water of 34 feet, $v = \sqrt{2g.\,(h + 34)}$. And in general, if h' be the height of the column of fluid, which would exert the same pressure as is applied at the upper surface,

$$v = \sqrt{2g\,(h + h')}.$$

Cor. 3. It is found by experiment that the section of the vena contracta is distant from the orifice a little less than the radius of the orifice, and its magnitude is about $\frac{5}{8}$ of the magnitude of the orifice.

2. If a cylindrical or prismatic vessel, whose altitude is h, and the area of whose section is A, empty itself through a very small orifice a at the bottom, the time t of emptying itself

$$= \frac{2}{\sqrt{g}} \times \frac{A}{a} \sqrt{h} = ,3526 \times \frac{A}{a} \sqrt{h}.$$

and the time that the surface takes to sink from the depth h to any other depth h'

$$= ,3526 \times \frac{A}{a} (\sqrt{h} - \sqrt{h'}.)$$

Cor. The construction of the clepsydra depends upon this Proposition. If the whole depth through which the water sinks in 12 hours be divided into 144 parts, it will sink through 23 of these in the first hour, 21 in the second, 19 in the third, and so on according to the series of the odd numbers.

Any vessel may serve for a clepsydra, but in order that the fluid may descend (which is most commodious) through equal portions of the vertical axis in equal portions of time, the vessel must be a paraboloid of the fourth order.

3. M. Prony deduces from actual experiment, the following formula for computing the discharge due to any altitude, and with any given orifice. Let Q = quantity of water discharged in cubic feet, d = diameter of orifice in inches, H = height of the head of the water in feet, T = time in seconds. then

$$Q = 3.9103\, d^2\, T \sqrt{H}\cdot$$

If instead of the aperture a pipe of one or two inches in length be inserted, the discharge is increased in the ratio of 13 to 10 nearly; in that case

$$Q = 5.1086\, d^2\, T \sqrt{H}\cdot$$

4. Bossut has found that the discharges due to equal intervals of time, through horizontal tubes of the same diameter, and under the same height of water, but of different lengths, not differing greatly from each other, will be very nearly in the inverse ratio of the square roots of these lengths.

5. To find the time of emptying vessels in general; let $g = 32\frac{1}{6}$ feet, x = depth of fluid at any point of time, z = area of surface at the depth x, a = area of orifice; then the velocity with which the surface descends

$$= \frac{a \sqrt{gx}}{z}.$$

$$\& \ t = \text{fl.}\ \frac{z\,dx}{a\sqrt{gx}}.$$

Cor. If any pressure be exerted on the surface of the fluid, and $h' =$ the height of a column of the fluid which would exert the same pressure,

$$t = \text{fl.}\ \frac{z\,dx}{a\sqrt{g \times (x + h')}}.$$

Ex. 1. If equal hemispheres are emptied by orifices in the vertex and base, time in the first case : time in last :: 7 : 12; the actual time in the first case being $\frac{14\pi r^{\frac{5}{2}}}{15\,a\sqrt{g}}$, and in the latter $\frac{8\pi r^{\frac{5}{2}}}{5\,a\sqrt{g}}$.

2. In paraboloids, the times are as 1 : 2.
3. In cones, the times are as 3 : 8.
4. In a sphere, time of emptying upper half : time of emptying lower :: $8\sqrt{2} - 7 : 7$.
5. To determine the time in which a cylinder will empty itself into a vacuum, its upper surface being exposed to the pressure of the atmosphere.

Let $h =$ height of the vessel, and $h' =$ the height of a column of fluid, equal to the weight of the atmosphere. Then by Cor. Art. 5.

$$t = \frac{2z}{a\sqrt{g}} \times \left\{ (h + h')^{\frac{1}{2}} - h'^{\frac{1}{2}} \right\}.$$

6. If upon the altitude of a fluid in a vessel as diameter we describe a ½ circle, the horizontal space described by the fluid from a perpendicular orifice at any point in the diameter equals twice the ordinate of the ½ circle drawn from that point, and ∴ varies as sin. θ, where $\theta =$ the arc of a circle, whose diameter is the depth of the fluid, and versed sine the depth of the orifice.

7. In jets d'eau, the differences between the heights of the jets and of the reservoirs, are as the squares of the heights of the jets themselves. i.e. if H and H′ be the heights of two reservoirs, h and h' the heights of the actual jets,

$$H - h : H' - h' :: h^2 : h'^2.$$

FLUIDS, *resistance of.—(Vince, Bland.)*

The resistance to a body moving in a fluid arises from the inertia, the tenacity, and the friction of the fluid. But the resistance here considered is that arising solely from the inertia of the fluid. The following articles are also deduced upon an hypothesis which cannot obtain in real practice; because it supposes first, that the medium in which the body moves falls in behind the body in motion, as fast as this moves forward, which is not the case, except the velocity is very small; and secondly, that the particles are so constituted, that after the body strikes them their action entirely ceases; whereas the particles, after they are struck. must necessarily diverge, and act upon other particles behind them. Hence will arise some difference between theory and experiment.

1. Required the resistance to a plane, moving in a fluid, in a direction perpendicular to its surface.

Let a = area of the plane; v its velocity, w its weight, δ the density of the fluid; $g = 32\frac{1}{6}$ feet. R the resistance, R′ the retarding force, then

$$R = \frac{a \delta v^2}{2g}.$$

$$\& \; R' = \frac{R}{w} = \frac{a \delta v^2}{2 g w}.$$

Cor. If the body be a cylinder (rad. $= r$) moving in the direction of its axis,

$$R = \frac{\pi r^2 \delta v^2}{2g}.$$

2. If the direction of motion be not perpendicular to the face of the plane, but inclined to it at any angle θ, the resistance *perpendicular* to the plane, is

$$\frac{a \delta v^2 \sin.^2 \theta}{2g}.$$

And the resistance *in the direction* of its motion, is

$$\frac{a \delta v^2 \sin.^3 \theta}{2g}.$$

And in a direction perpendicular to that of its motion, is

$$\frac{a \delta v^2 \sin.^2 \theta \times \cos. \theta}{2g}.$$

Ex. At what $\angle$ must the rudder of a vessel be inclined to the stream, that the effect produced may be a maximum?

The effect varies (by the 3d Formula) as $\sin.^2 \theta \times \cos. \theta = \max.$, $\therefore$ $\sin. \theta = \sqrt{\frac{2}{3}}$.

3. If a plane figure, or a solid generated by the revolution of a plane figure round its axis, move in a fluid in the direction of its axis; to determine the ratio of the resistances on the curve or surface, and on the base.

In a plane figure,

Res. on base : that on the curve :: y : fl. $\frac{dy}{1+\frac{dx^2}{dy^2}}$

And in a solid,

Res. on base : that on the surface :: $\frac{1}{2}y^2$: fl. $\frac{y\,dy}{1+\frac{dx^2}{dy^2}}$

Ex. 1. Let the curve be a semicircle.

Res. on base : res. on curve :: $y : y - \frac{y^3}{3r^2}$, which, when $y = r$, becomes as 3 : 2.

Ex. 2. Let the solid be a sphere,

Res. on base : res. on surface :: $\frac{1}{2}y^2 : \frac{1}{2}y^2 - \frac{y^4}{4r^2}$ :: 2 : 1 when $y = r$.

Hence resistance to a cylinder is double that of the inscribed sphere.

Cor. Hence if n = density of a globe, whose radius is r, and the specific gravity of the fluid be 1,

$$R = \frac{\pi r^2 v^2}{4g}.$$

and $R' = \frac{R}{w} = \frac{\pi r^2 v^2}{4g} \div \frac{4\pi n r^3}{3} = \frac{3v^2}{16gnr}$; or if z = space fallen through to acquire the velocity v

$$R' = \frac{3z}{8nr}.$$

4. Let a sphere of given diameter be projected in a resisting medium, whose specific gravity is to that of the sphere as 1 : n. Having given the velocity of projection, to find the velocity of the sphere at any distance x, and the time of description.

Let e = No. whose hyp. log. = 1, and suppose when $x = o$, $z = a$, then

$$V = \frac{\sqrt{2ga}}{e^{\frac{3x}{16nr}}}.$$

$$\text{and } T = \frac{16nr}{3\sqrt{2ga}} \times \left(e^{\frac{3x}{16nr}} - 1\right)$$

5. Let a spherical body descend in a fluid from rest by the action of

gravity (the rest as before), to find the velocity at any point of the descent, and the time of description.

$$\text{Here } V = \sqrt{\frac{16\,g\,r.\,\overline{n-1}}{3}} \times \sqrt{1 - e^{\frac{-3x}{8nr}}}.$$

$$\text{and } T = \sqrt{\frac{4n^2 r}{3g.\,\overline{n-1}}} \times \text{hyp. log.} \frac{1 + \sqrt{1 - e^{\frac{-3x}{8nr}}}}{1 - \sqrt{1 - e^{\frac{-3x}{8nr}}}}.$$

Cor. 1. If x be increased sine limite, $e^{\frac{-3x}{8nr}}$ vanishes, and $V = \sqrt{\frac{16\,g\,r.\,\overline{n-1}}{3}}$ = the greatest velocity that can be acquired by a spherical body descending in a fluid.

FLUID *elastic.—See Atmosphere.*

FLUXIONS.—*See Differentials.*

FORCES, *the composition and resolution of.—(Whewel'.)*

1. If any two forces act at the same point, the force, which is equivalent to the two, is represented in *direction and magnitude* by the diagonal of the parallelogram, of which the sides represent the magnitude and direction of the component forces.

Cor. If p and q be the component forces, which contain an angle θ, the resultant will be $\sqrt{p^2 + 2\,p\,q \cos.\theta + q^2}$.

2. Forces may be represented by lines parallel to their direction, and proportional to them in magnitude.

Cor. 1. If two sides of a Δ taken in order represent the magnitude and direction of two forces, the third side will represent a force equivalent to them both.

Cor. 2. If three forces, represented in magnitude and direction by the three sides of a Δ taken in order, act on a point, they will keep it at rest; and conversely.

Cor. 3. If three forces keep a body in equilibrium, and three lines be drawn making with the directions of the forces three equal angles towards the same parts, these three lines will form a Δ, whose sides will represent the three forces respectively.

Cor. 4. If three forces keep a point at rest, they are each inversely as the sine of the $\angle$ contained by the other two.

Cor. 5. If the ∠ between two given forces be diminished, the resultant is increased.

Cor. 6. If any number of forces be represented by sides of a polygon taken in order, their resultant will be represented by the line which completes the polygon.

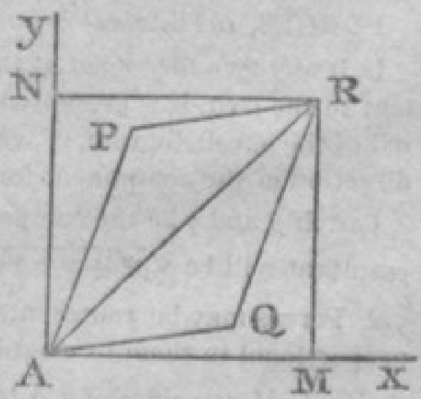
Cor. 7. A number of forces which are represented by all the sides of a polygon taken in order, acting upon a point, will keep it at rest.

3. If the edges of a parallelopiped drawn from the same point, represent three component forces, the diagonal will represent the resultant.

Cor. 1. If any number of forces be represented by sides, taken in order, of a polygon, which is not in the same plane, their resultant will be represented by the line which completes the polygon.

Cor. 2. If any number of forces be represented by all the sides, taken in order, of a polygon, they will keep a point at rest.

4. To find, by means of equations among the symbols, which the forces and their positions introduce, the resultant of two forces acting at a point.



If we suppose a line, as A x, to pass through A, we may determine the positions, both of the components and resultant, by the ∠s. which they make with this line.

Let p and q be the forces in A P, A Q; α, β the ∠s. which they make with A x. Resolve p into two forces in the directions A x, and A y perpendicular to A x, then the resolved parts will be p cos. α, p sin. α. In like manner q is equivalent to q cos. β in the direction A x, and q sin. β in the direction A y. Hence the forces are equivalent to

$$p \cos. \alpha,\ q \cos. \beta \text{ in A } x.$$
$$p \sin. \alpha,\ q \sin. \beta \text{ in A } y.$$

And the resultant of p and q will be the resultant of these four forces. If we put

$$p \cos. \alpha + q \cos. \beta = X.$$
$$p \sin. \alpha + q \sin. \beta = Y.$$

and take in A x, A y, A M = X, A N = Y, and complete the rectangle A M R N, A R will be the resultant of p and q, and if r be this resultant, and θ the ∠ which it makes with A x, we have

$$r = \sqrt{X^2 + Y^2},\ \tan.\ \theta = \frac{Y}{X}.$$

whence the magnitude and position of the resultant are known.

Cor. 1. By putting the values of X and Y in the expression for r, we shall get

$$r = \sqrt{\left\{p^2 + 2\,p\,q \cos.\ (\alpha - \beta) + q^2\right\}}$$

which agrees with the result obtained in Cor. Art. 1.

Cor. 2. If we call ϕ and ψ the ∠s. P A R and Q A R, we shall have

$$\sin.\ \phi = \frac{q \sin.\ (\alpha - \beta)}{\sqrt{\left\{p^2 + 2p\,q \cos.\ (\alpha - \beta) + q^2\right\}}}$$

$$\&\ \sin.\ \psi = \frac{p \sin.\ (\alpha - \beta)}{\sqrt{\left\{p^2 + 2\,p\,q \cos.\ (\alpha - \beta) + q^2\right\}}}$$

5. To find the resultant of any number of forces, $p_1, p_2, p_3 \ldots\ldots\ldots p_n$ in the same plane; their directions making with the line A x angles $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots\ldots \alpha_n$ respectively.

By proceeding precisely as before, we shall have, by putting

$$p_{,} \cos.\ \alpha_{,} + p_2 \cos.\ \alpha_2 + p_3 \cos.\ \alpha_3 \ldots\ldots + p_n \cos.\ \alpha_n = X$$

$$p_{,} \sin.\ \alpha_{,} + p_2 \sin.\ \alpha_2 + p_3 \sin.\ \alpha_3 \ldots\ldots + p_n \sin.\ \alpha_n = Y$$

$$r = \sqrt{(X^2 + Y^2)};\ \tan.\ \theta = \frac{Y}{X}.$$

6. To find the resultant of forces, whose directions are not all in the same plane.

In the preceding case, the forces were resolved in the directions of two lines at right ∠s. to each other. In this case we must resolve them in the directions of three lines each at right ∠s. to the other two, and meeting together in a point. Let us suppose these three lines to be A x, A y, A z, and let p be a force, and α, β, γ the ∠s. which it makes with A x, A y, A z; the force will then be equivalent to three forces

$$p \cos.\ \alpha \text{ in A } x,\ p \cos.\ \beta \text{ in A } y,\ p \cos.\ \gamma \text{ in A } z.$$

Hence if we have forces $p_1, p_2, p_3 \ldots\ldots p_n$

making with A x angles $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_n$

with A y angles $\beta_1, \beta_2, \beta_3 \ldots\ldots \beta_n$

with A z angles $\gamma_1, \gamma_2, \gamma_3 \ldots\ldots \gamma_n$

and make

$$p_1 \cos. \alpha_1 + p_2 \cos. \alpha_2 + p_3 \cos. \alpha_3 \ldots\ldots + p_n \cos. \alpha_n = X$$

$$p_1 \cos. \beta_1 + p_2 \cos. \beta_2 + p_3 \cos. \beta_3 \ldots\ldots + p_n \cos. \beta_n = Y.$$

$$p_1 \cos. \gamma_1 + p_2 \cos. \gamma_2 + p_3 \cos. \gamma_3 \ldots\ldots + p_n \cos. \gamma_n = Z$$

the forces will be equivalent to X in A x, Y in A y, and Z in A z.

If R be the resultant, and θ, η, ζ the $\angle$s. which it makes with A x, A y, A z respectively, we shall have

$$R = \sqrt{(X^2 + Y^2 + Z^2)}$$

$$\cos. \theta = \frac{X}{R}, \cos. \eta = \frac{Y}{R}, \cos. \zeta = \frac{Z}{R}$$

One of the three last Equations is superfluous.

7. When a point is acted upon by any forces, to find the conditions of equilibrium.

In order that there may be an equilibrium, the resultant of all the forces must be o. And in order that this may be the case, it is evident we must have in Art. 5, $X = o$, $Y = o$; and in Art. 6, $X = o$, $Y = o$, $Z = o$. Hence we have for the conditions of equilibrium in the former case

$$p_1 \cos. \alpha_1 + p_2 \cos. \alpha_2 + p_3 \cos. \alpha_3 + \ldots\ldots = o$$

$$p_1 \sin \alpha_1 + p_2 \sin. \alpha_2 + p_3 \sin. \alpha_3 + \ldots\ldots = o$$

And in the latter case

$$p_1 \cos. \alpha_1 + p_2 \cos. \alpha_2 + p_3 \cos. \alpha_3 + \ldots\ldots = o$$

$$p_1 \cos. \beta_1 + p_2 \cos. \beta_2 + p_3 \cos. \beta_3 + \ldots\ldots = o$$

$$p_1 \cos. \gamma_1 + p_2 \cos. \gamma_2 + p_3 \cos. \gamma_3 + \ldots\ldots = o$$

FORCE.—*See Motion.*

FORCE *moving, or motive.—See Momentum.*

FORCES, *centripetal and centrifugal.—See Central Forces.*

FRACTIONS *continued.*

Continued fractions are very useful when we have a fraction or ratio in very large numbers which are prime to one another, as by their means we may find an approximate value in less terms.

To represent $\frac{a}{b}$ in a continued fraction.

Divide as in the rule for finding the greatest common measure, thus

$$\begin{array}{l} b)\,a\,(p \\ \quad c)\,b\,(q \\ \quad\quad d)\,c\,(r \\ \quad\quad\quad e)\,d\,(s \\ \quad\quad\quad\quad \&c.\ \&c. \end{array} \qquad \text{Then } \frac{a}{b} = p + \cfrac{1}{q + \cfrac{1}{r + \cfrac{1}{s}}}\ \&c.$$

The first approximation is p, which is too small, the next $p + \frac{1}{q}$, which is too large, the next $p + \cfrac{1}{q + \frac{1}{r}}$, which is too small; and thus we may form a series of fractions, each succeeding one being nearer the true value of the proposed fraction than the one which preceded it.

This series of fractions requires some trouble in their formation after the first two or three; but the 3d, 4th, &c. may be expeditiously found thus. Arrange the figures of the quotients in a line, as p, q, r, s, t, &c. let the successive fractions be $\frac{c}{d}, \frac{e}{f}, \frac{g}{h}, \frac{k}{l}, \frac{m}{n}$, &c. then to find any of them after the 2d, as $\frac{g}{h}$, we have $\frac{g}{h} = \frac{re + c}{rf + d}$; $\frac{k}{l} = \frac{sg + e}{sh + f}$; $\frac{m}{n} = \frac{tk + g}{tl + h}$, &c. where the law of formation is evident.

Ex. To approximate to $\frac{277288}{87968}$, proceeding as if finding the greatest common measure we have for the quotients

3, 6, 1, 1, 2, 1, &c.

Now first approximation $= p = 3$; 2d. $= p + \frac{1}{q} = 3 + \frac{1}{6} = \frac{19}{6}$, $\therefore$ we have by the rule the following series of fractions

$3, \frac{19}{6}, \frac{22}{7}, \frac{41}{13}, \frac{104}{33}, \frac{145}{46}$ where 3 is too small, $\frac{19}{6}$ too large, &c.

FRACTIONS *vanishing*.

If $u = \frac{P}{Q}$, where P and Q are functions of x, which are both $= o$, when $x = a$, then the value of u, in this case, is the same as the values in this case of $\frac{dP}{dQ}, \frac{d^2P}{d^2Q}, \frac{d^3P}{d^3Q}$, &c.

Hence the value of a vanishing fraction may be found by differentiation, as in the following examples :—

Ex. 1. Required the value of $\frac{x^2 - a^2}{x - a}$ when $x = a$.

Here $\frac{d\,P}{d\,Q} = \frac{2\,x\,d\,x}{d\,x} = 2\,x = 2\,a$.

Ex. 2. Required the value of $\frac{x^{n+1} - x}{x - 1}$, when $x = 1$.

Here $\frac{d\,P}{d\,Q} = (n + 1)\,x^n - 1 = n$.

But if it so happen that on substituting a instead of x in $\frac{d\,P}{d\,Q}$, this fraction also becomes $\frac{o}{o}$, we must treat it in the same manner as the first, and so on, till we arrive at a value of which one term at least is finite.

Ex. Let $\frac{P}{Q} = \frac{a\,x^2 + a\,c^2 - 2\,a\,c\,x}{b\,x^2 - 2\,b\,c\,x + b\,c^2}$, which $= \frac{o}{o}$ when $x = c$.

Here $\frac{d\,P}{d\,Q} = \frac{2\,a\,x - 2\,a\,c}{2\,b\,x - 2\,b\,c}$ which also $= \frac{o}{o}$ when $x = c$,

But $\frac{d^2\,P}{d^2\,Q} = \frac{a}{b}$ which is the value of $\frac{P}{Q}$ in this case.

FREEZING.—*See Congelation.*

FRICTION.—*(Playfair.)*

The following must only be considered as a short abstract of the most interesting general results on the subject of Friction, as deduced from experiments made by Coulomb and others.

1. The retardation which friction opposes to motion is nearly uniform, or the same for all velocities.

2. The force of friction is the greater, the greater the force with which the surfaces, moving on one another, are pressed together, and is commonly equal to between ⅓ and ¼ of that force ; but it is very little affected by the extent of the surfaces.

3. Friction may be distinguished into two kinds, that of sliding, and that of rolling bodies. The force of the latter is very small compared with that of the former.

4. The distance to which a given body will be moved by percussion in

opposition to friction, is as the square of the velocity communicated to it. Thus a nail is driven by a blow of no great force, into a piece of wood where the mere friction is sufficient to retain it against a great force applied to draw it out.

5. When motion begins, the intensity of friction diminishes; it does not, however, change afterwards as the velocity changes, but continues, as already said, to retard with a uniform force. Coulomb found the friction of wood sliding on wood to become less when the body began to move, than it had been the instant before in the ratio nearly of 2 to 9.

6. Friction may be measured by placing the body on a plane of variable inclination, and increasing that inclination till the body begin to slide. If the weight of the body = W, and the inclination of the plane when the body begins to slide $= \theta$, the friction $= W \times \tan. \theta$.

7. Time is often required for friction to attain its maximum, and in this respect different substances differ much from one another.

8. Friction is diminished by unctuous substances; those that are thinnest and least tenacious are the best; plumbago reduced to powder, and rubbed on the surface of wood, metal, stone, &c. serves greatly to diminish friction.

9. The effect of friction may be diminished by drawing a body in a line inclined at a certain angle to the plane on which it rests. Thus if the weight of a body be to its friction on a horizontal plane as n to 1, it will be drawn with the greatest ease in the direction which makes with that plane an angle, having for its tangent $\frac{1}{n}$.

10. The friction of cylinders rolling upon an horizontal plane is in a direct ratio of their weights, and in the inverse ratio of their diameters.

11. The momentum of friction is diminished by friction wheels in the ratio of the radius of the axis of any one of the wheels (they are supposed equal) to the perpendicular height of the axis that rests upon them, above the line joining their centres.

12. In wheel carriages, the plane on which they move, and the line of draught, being both horizontal, the advantage for surmounting an immoveable obstacle, of a given height, is as the square root of the radius of the wheel.

Let the whole weight to be moved be W, the radius of the wheel r, f the force which drawing horizontally will raise the carriage over an immoveable obstacle of the height h; then $f = W \times \sqrt{\frac{2h}{r}}$.

13. The stiffness of ropes, or the force requisite to bend them has a great analogy to friction. In different ropes, the forces requisite to bend them are in the direct ratio of their diameters and their tensions jointly, and in the inverse ratio of the radii of the cylinders round which they are bent.

14. The friction of a rope that is wound round a cylinder increases in geometrical progression, while the number of turns increases in arithmetical progressio ..

If the turns be represented by the numbers 0, 1, 2, 3, 4, &c. the resistance made by the rope may be represented by the numbers 1, 2. 4, 8, 16, &c.

15. Though friction destroys motion, and generates none, it is of essential use in mechanics. It is the cause of stability in the structure of machines; and is necessary to the exertion of the force of animals.

FRIGORIFIC *Mixtures.*—(*Ure.*)

Tables of Frigorific Mixtures, sufficient for all useful philosophical purposes.

FRIGORIFIC MIXTURES WITHOUT ICE.

Mixtures.		*Thermometer sinks from* + 50°.	*Degree of cold produced.*
Muriate of Ammonia ...... Nitrate of Potash .... Water	5 parts. 5 16	To + 10°.	40°.
Nitrate of Ammonia .... Water	1 part. 1	To + 4°.	46
Nitrate of Ammonia ...... Carbonate of Soda .... Water	1 part. 1 1	To — 7°.	57
Sulphate of Soda .... Diluted nitric acid	3 parts. 2	To — 3°.	53
Sulphate of Soda .... Nitrate of Ammonia ...... Diluted nitric acid	6 parts. 5 4	To — 14°.	64
Phosphate of Soda .... Diluted nitric acid	9 parts. 4	To — 12°.	62
Sulphate of Soda .... Muriatic acid	8 parts. 5	To 0°.	50
Sulphate of Soda .... Diluted sulphuric acid.....	5 parts. 4	To + 3°.	47

N.B. If the materials are mixed at a warmer temperature than 50°, the effect will be proportionably greater.

FRIGORIFIC MIXTURES WITH ICE.

Mixtures.	
Snow or pounded ice ...... 2 parts. Muriate of Soda 1	To — 5° from any temperature.
Snow or ice .... 5 parts. Muriate of Soda .... 2 Muriate of Ammonia 1	To — 12° from any temperature.
Snow .... 3 parts. Diluted sulphuric acid ... 2	From + 32° to — 23°.
Snow .... 8 parts. Muriatic acid 5	From + 32° to — 27°.
Snow .... 7 parts. Diluted nitric acid 4	From + 32° to — 30°.
Snow .... 4 parts. Muriate of lime 5	From + 32° to — 40°.
Snow .... 3 parts. Potash 4	From + 32 to — 51°.

Greatest artificial cold yet measured — 91°.

G

GAUGING.—*(Hutton.)*

Rule for finding the dimensions of a cask, in wine, ale, or imperial gallons.

Let B = bung diameter, H = head diameter, L = length of cask, all in inches; then

$$(39\,B^2 + 25\,H^2 + 26\,B\,H) \times \frac{L}{114}$$

is the content in inches, which being divided by 231 for wine gallons; or by 282 for ale gallons; or by 277.274 for imperial gallons, will be the content required.

GEOMETRICAL *Progression.—See Progression.*

GEORGIUM *Sidus.*

This planet was discovered by Dr Herschel, March 13, 1781. For its elements, &c.—*see Planets, elements of;* and for its satellites, *see Satellites.*

GOLDEN *Number.—See Cycle.*

GRAVITY, *Centre of.—See Centre of Gravity.*

See also MOTION, accelerated.

GRAVITY *specific.*—(*Vince, Bland.*)

1. Of the specific gravities of a body and fluid, having given the one, to find the other.

Case 1. When the body is heavier than the fluid.

Let $w =$ weight lost by the body when immersed in the fluid, W its whole weight in vacuo, $s =$ spec. grav. of the fluid, S = that of the body; then

$$w : W :: s : S.$$

whence s or S may be found.

Cor. 1. If different bodies be weighed in the same fluid, S is as $\frac{W}{w}$, from whence we may compare the spec. grav. of two bodies.

Cor. 2. If the same body is weighed in different fluids, s is as w; from whence we can compare the spec. grav. of two fluids.

Case 2. When the body Q is lighter than the fluid in which it is weighed.

Connect it with a heavier body P, so that together they may sink. Find the weight lost by P + Q, and the weight lost by P, when immersed; then the difference = the weight lost by Q; and ∴ its specific gravity may be found by the last case.

2. If the specific gravity of air be called m, that of water being 1, and W the weight of any body in air, and W′ its weight in water; then its weight *in vacuo* is nearly

$$W + m\,(W - W').$$

3. If σ be the specific gravity of a body ascertained by weighing it in air and water, and m the specific gravity of the air at the time when the experiment was made; the correct specific gravity, or that which would have been found, if the body had been weighed in vacuo, instead of air, is

$$\sigma + m\,(1 - \sigma).$$

4. If a body float on a fluid, the part immersed (Q) : the whole body (P + Q) :: sp. grav. (s) of the body : sp. grav. (S) of the fluid.

Cor. Hence if the same body float on different fluids, Q is as $\frac{1}{S}$; on which principle the *Hydrometer* is constructed. For let the instrument be successively immersed in two fluids, and the magnitudes of the parts immersed be observed. Then the magnitude of the part immersed in the first : that immersed in the second :: spec. grav. of the second fluid : to that of the first.

A considerable improvement has been made in the hydrometer, by placing a small brass cup on the top of the stem, into which small weights may be put, so as to sink it in different fluids to the same point of the stem. Let W = weight necessary to make it sink in one fluid, and $W + w$ the weight necessary to make it sink to the same point in another; then if one of them is water, the spec. grav. of the other $= 1.000 \times \left(1 \pm \frac{w}{W}\right)$.

5. If a lighter fluid rest upon a heavier, and their spec. grav. be as $a : b$, and a body, whose spec. grav. is c, rest with one part P in the upper fluid, and the other part Q in the lower, then

$$P : Q :: b - c : c - a.$$

6. If a and b be the spec. grav. of two fluids or solids to be mixed together, P and Q their magnitudes, and c the spec. grav. of the compound,

$$P : Q :: b - c : c - a, \text{ and}$$

$$\text{weight of } P : \text{weight of } Q :: a.\ (b - c) : b\ (c - a).$$

Cor. Hence from the first proportion,

$$c = \frac{Pa + Qb}{P + Q}.$$

And from the second, if W and w = weights of P and Q,

$$c = \frac{(W + w)\ ab}{Wb + wa}.$$

7. Of the magnitude and weight of a body, having given the one to find the other.

Let M = magnitude in cubic feet, S = its spec. grav. that of water being 1000, W = weight in avoirdupois ounces, then

$$W = M \times S.$$

for 1 cubic foot rain water weighs 1000 oz. very nearly.

Or let W = its weight in grains, and S its spec. grav. that of water being 1, B its bulk in cubic inches, then

$$B = \frac{W}{252{,}576\ S}.$$

Cor. If the weight is expressed in pounds Troy, it must be multiplied (to reduce it to grains) by 5760; if in pounds avoirdupois, by 7002.

We may thus find the magnitude of bodies which are too irregular to admit of the application of the common rules of mensuration; or we may, by knowing the spec. grav. and magnitude, find the weight of bodies which are too ponderous to be submitted to the action of the balance or steel yard.

8. To determine the magnitude of an irregular solid, and the capacity of an irregular vessel.

(j) Weigh the solid in air, and water; then since a cubic foot of rain water weighs 1000 ounces,

1000 oz. : weight lost :: 1 cubic foot : magnitude required.

(jj) Weigh the vessel when empty, and full of water, and you have the weight of water it contains, then

1000 oz. : weight of water :: 1 cub. foot : capacity required.

(jjj) To determine the diameter of any small sphere, whose spec. grav. is s, its weight in grains (w) being known

$$\delta = 1.9612 \sqrt[3]{\frac{w}{s}}.$$

TABLE OF SPECIFIC GRAVITY.

Extracted from Davies, Lavoisier, Young, and other authentic sources.

Note.—Water at 60° is assumed 1000 specific gravity.

MINERAL PRODUCTIONS.

Platina, purified	19500
hammered	20336
Pure gold, cast	19258
hammered	19361
Mercury	13568
Lead, cast	11352
Silver, pure, cast	10474
hammered	10510
Bismuth, cast	9822
Copper, cast	8788
wire	8878
Brass, cast	8395
wire	8544
Cobalt, cast	7812
Nickel, cast	7807
Iron, cast	7207
bar	7788
Steel, hard, not screwed	7816
soft, not screwed	7833
Loadstone	4800
Tin, cast	7291
Zinc, cast	7190
Antimony, cast	6702

Tungstein	6066
Arsenic, cast	5763
Molybdena	4738
Spar. ponderous	4430
Ruby, oriental	4283
Garnet, Bohemian	4188
Sapphire of Puy	4076
Topas, oriental	4010
Beryl, or oriental aquamarine	3548
Diamond, rose coloured	3531
white	3521
lightest	3501
Glass, flint	3329
white	2892
bottle	2732
green	2642
Fluor	3191
Serpentine, green	2988
Mica, black	2900
Basaltes, from the Giants' Causeway	2864
Marble, white, Parian	2837
green	2741
red	2725
white, of Carrara	2716
Emerald, Peruvian	2775
Porphyry, red	2765
Jasper	2764
Alabaster, white, antique	2730
Calcarious spar, rhombic	2715
pyramidal	2714
Slate	2671
Pitch stone	2669
Onyx, pebble	2664
Chalcedony, transparent	2664
Granite, Egyptian, red	2654
Rock crystal, pure	2653
Amorphous quartz	2647
Agate, onyx	2637
Carnelian	2613
Sardonyx	2602
Purbeck stone	2601
Flint, white	2594

Flint, blackish	2581
Agate, oriental	2590
Prase	2580
Portland stone	2570
Mill-stone	2483
Paving-stone	2415
Touchstone	2415
Porcelain, Chinese	2384
Lapis obsidianus	2348
Selenite	2322
Grindstone	2142
Salt	2130
Sulphur, native	2033
Nitre	2000
Brick	2000
Plumbago	1860
Alum	1720
Asphaltum	1400
Coal, Scots	1300
Newcastle	1270
Staffordshire	1240
Jet	1238
Ice, *probably*	930
Pumice-stone	914

LIQUIDS.

Sulphuric acid	1840
Ph. London	1850
Nitrous acid, Ph. London	1550
Nitric acid	1217
Water of the Dead Sea	1240
Sea Water	1026
Muriatic acid	1194
Water of the Seine, filtered	1001
Naphtha	708

ELASTIC FLUIDS.

	Kirwan. Barometer, 30.	*Lavoisier.* Thermometer 52°.
Sulphureous acid gas	2·265	——
Carbonic acid gas	1·500	·00176
Nitrous gas	1·194	——

The weight of a cubic foot dry air at
32° Fahr & 30 inch Bar = 566.5691 grains

	Kirwan. Barometer, 30.	*Lavoisier.* Thermometer 52°.
Hepatic gas	1·106	——
Oxygen gas	1·103	·00137
Atmospheric air	1·000	·00128
Nitrogen gas	·985	·00120
Ammoniacal gas	·600	——
Hydrogen gas	·084	·000096

VEGETABLE PRODUCTIONS.

Sugar, white	1606
Gum Arabic	1452
Honey	1450
Catechu	1398
Pitch	1150
Copal, opaque	1140
Yellow amber	1078
Malmsey, Madeira	1038
Cider	1018
Vinegar, distilled	1009
Water at 60°	1000
Bourdeaux wine	994
Burgundy wine	991
Turpentine liquid	991
Camphor	988
Linseed oil	940
Elastic gum	933

ANIMAL SUBSTANCES.

Pearl	2750
Coral	2680
Sheep's bone, recent	2222
Oyster shell	2092
Ivory	1917
Stag's horn	1875
Ox's horn	1840
Isinglass	1111
Egg of a hen	1090
Human blood	1053
Milk cow's	1032
Wax, white	968
yellow	965

Spermaceti	943
Butter	942
Tallow	942
Fat of hogs	937
—— veal	934
—— mutton	923
—— beef	923
Ambergrease	926
Lamp oil	923

WOODS.

Pomegranate tree	1354
Lignum vitæ	1333
Box, Dutch	1328
Ebony	1177
Heart of oak, 60 years felled	1170
Oak, English, just felled	1113
the same, seasoned	743
usually stated at	925
Bog oak, of Ireland	1046
Teak, of the East Indies	from 745 to 657
Mahogany	from 1063 to 637
Pear tree trunk	646
Medlar tree	944
Olive wood	927
Logwood	931
Beech	852
Ash	from 845 to 600
Yew, Spanish	807
—— Dutch	788
Alder	800
Elm	from 800 to 600
Apple tree	793
Plum tree	755
Maple	755
Cherry tree	715
Quince tree	705
Orange tree	705
Walnut	671
Pitch pine	660
Red pine	657
Yellow pine	529

White pine	420
Fir, of New England	553
of Riga	753
of Mar Forest, Scotland	696
Cypress	644
Lime tree	604
Filbert wood	600
Willow	585
Cedar	560
Juniper	556
Poplar, white Spanish	529
common	383
Sassafras wood	482
Larch, of Scotland	530
Cork	240

GREGORIAN *Calendar.—See Calendar.*

GULDINUS' *Property.—See Solids and Surfaces.*

GUNNERY, *leading principles of.—(Hutton.)*

1. To find the initial velocity of a shot.

Let P = weight of powder, B of the ball, v the initial velocity, then

$$v = 2000 \sqrt{\frac{P}{B}}.$$

Cor. 1. The initial velocity of a shot varies from 1600 to 2000 feet per second.

Cor. 2. $B v^2 = (2000)^2 . P$, i.e. the *effect* of a shot is nearly as the quantity of gunpowder.

2. If w = weight of any ball, d its diameter.

$$w = .5236 \, d^3 \text{ in pounds.}$$

3. To find the resistance of the air to any ball or projectile.

Let d = diameter of ball, v its velocity, r = resistance in avoirdupois pounds, then

$$r = \frac{d^2}{1000} \left(\frac{2 v^2}{3000} - v. \right)$$

Ex. Resistance to a ball, whose diameter = 2.78 inches (or weight 3 lbs.), when thrown with a velocity of 1800 feet per second, = 176 lbs., more than 58 times its own weight.

4. Supposing the air to resist according to the law just assigned, required the height to which a ball will ascend perpendicularly.

Let d = diameter of ball, c the velocity of projection, h = height ascended, then

$$h = 760\, d \times \log. \left(\frac{c^2 - 150\, c}{21090\, d} + 1. \right)$$

Ex. A ball of 1.05 lbs., discharged with a velocity of 2000 feet, will ascend to the height of 2920 feet; *in vacuo* it would have ascended to the height of $11\frac{3}{4}$ miles.

5. If a body descending in the atmosphere has acquired such a velocity that the resistance is equal to its weight, the accelerating and retarding forces being equal, its motion will become uniform; to find this terminal velocity,

$$\frac{2\, v^2}{3000} - v = 523.6\, d.$$

a quadratic equation, from whence v may be found.

Ex. For an iron ball of 1 lb. the terminal velocity = 244 feet; for one of 42 lbs. it is 456.

6. The best charge of powder is about $\frac{1}{5}$ or $\frac{1}{6}$ of the weight of the ball; for battering $\frac{1}{3}$: a 24-pounder with 16 pounds of gunpowder at an elevation of 45° ranges 20,250 feet, about $\frac{1}{5}$ of the range that would take place in a vacuum. The resistance is at first 400 pounds or more, and reduces the velocity in a second from 2000 to 1200 feet in the first 1500 feet.—*(Young's Nat. Phil.)*

GUNPOWDER.—*See Gunnery and Steam.*

GYRATION, *Centre of.*—*See Centre of Gyration.*

H

HARMONICAL *Progression.*—*See Progression.*

HARVEST *Moon.*—*(Maddy.)*

To find the retardation of the Moon's rising on successive nights.

Let the moon's daily motion = m, the inclination of the moon's orbit to the horizon = n, latitude of the place = l, moon's declination = δ, then the difference of the times of rising on succeeding days (D) is

$$D = \frac{m. \sin. n}{\sqrt{\cos.^2 \delta - \sin.^2 l}}.$$

add table of capacity of heat

Mechanical value of heat

The heat required to raise 1 lb of water 1° Fahr would lift 1 lb weight 772 feet. or 772 lbs 1 foot.

1 lb 1° Centigrade = force to 1 lb 1390 feet.

A body falling through 772 feet has acquired a velocity of about 223 feet in a second

Hence may be explained the phænomenon of the Harvest Moon, premising that when the 1st point of Aries rises, the ecliptic makes the least angle with the horizon. For if the moon's orbit be supposed to coincide with the ecliptic (which it does nearly) sin. n is least when the moon rises in Aries; therefore the numerator of the above expression is then least; and because $\cos.^2 \delta = 1$, the denominator is then greatest; ∴ on both accounts D is least, and if the sun be at the same time in Libra, the moon is then at the full; therefore the full moon, which takes place near the autumnal equinox rises nearly at the same time for several nights, and as this is near the time of harvest in north latitudes, it is called the Harvest Moon.

HEAT, *various Tables relating to.*

TABLE I.

Table of the effects of heat on different substances according to Fahrenheit's thermometer and Wedgwood's.—(Wedgwood.)

	Fahr.	*Wedg.*
Extremity of the scale of Wedgwood ...	32277°	240°
Greatest heat of his small air furnace ...	21877	160
Chinese porcelain softened	——	156
Cast iron melts	17977	130
Greatest heat of a common smith's forge ...	17327	125
Derby porcelain vitrifies	——	112
Welding heat of iron greatest	13427	95
least	12777	90
Fine gold melts	5237	32
Fine silver melts	4717	28
Swedish copper melts	4587	27
Brass melts	3807	21
Enamel colours burnt on	1857	6
Red heat fully visible in day light ...	1077	0
in the dark ...	947	— 1
Mercury boils	600 ...	$-3\frac{673}{1000}$
Water boils	212 ...	$-6\frac{658}{1000}$
Vital heat	97 ...	$-7\frac{542}{1000}$
Water freezes	32 ...	$-8\frac{42}{1000}$
Proof spirit freezes	0 ...	$-8\frac{289}{1000}$

		Fahr.	*Wedg.*
Mercury freezes	---- ---- ----	— 40 ...	— $8\frac{596}{1000}$

TABLE II.

Table of the congealing or concreting temperatures of various liquids by Fahrenheit's scale.—(Ure.)

Sulphuric ether	—	46°
Liquid ammonia	—	46
Nitric acid sp. gr. 1.424	—	45.5
Sulphuric acid sp. gr. 1.6415	—	45
Mercury	—	39
Nitric acid sp. gr. 1.3290	—	2.4
Brandy	—	7.0
Alochol 1, water 1.	—	7
Alcohol 1, water 3	+	7
Oil of turpentine		14
Strong wines		20
Blood		25
Vinegar		28
Sea water		28
Milk		30
Water		32
Olive oil		36
Sulphuric acid, sp. gr. 1.741		42
Tallow		92
Spermaceti		112
Yellow wax		142
White do.		155
Tin		442
Lead		612
Zinc		680

The concreting temperature of the bodies above tallow in this Table, is usually called their freezing or congealing point, and of tallow and the bodies below it the fusing or melting point.

TABLE III.

Table of the boiling points by Fahrenheit's scale of a few of the most important liquids, under a mean barometrical pressure of 30 inches.—(Ure.)

Ether sp. gr. 0.7365 at 48°	Gay Lussac	100°
Alcohol sp. gr. 0.813	Ure	173.5

Nitric acid sp. gr. 1.500	Dalton	210
Water		212
Muriatic acid sp. gr. 1.094	Dalton	232
Do. 1.047	Do.	222
Nitric acid 1.16	Do.	220
Oil of turpentine	Ure	316
Sulphuric acid, sp. gr. 1.30	Dalton	240
Do. 1.848	Ure	600
Linseed oil		640
Mercury		656

TABLE IV.

Boiling temperature of water.

Height of the boiling point in Fahrenheit's Thermometer at different heights of the Barometer.

Barom.	Ht. of boiling point.
31°. 0	213°. 57
30. 5	212. 79
30. 0	212. 00
29. 5	211. 20
29. 0	210. 38
28. 5	209. 55
28. 0	208. 69
27. 5	207. 84
27. 0	206. 96

And in general Dr Horsley's rule deduced from De Luc is, height $= \frac{99}{8990000} \times$ log. z.—92.804, where z = height of Barometer in 10ths of an inch.

In an exhausted receiver water boils at 98°. or 100°. in Papin's digester at 412°.

From this variation in the height of the boiling point, arising from the variation of the pressure of the atmosphere, an ingenious instrument called the Thermometrical Barometer has been invented by Mr Wollaston, for ascertaining the heights of mountains; it appearing from General Roy's experiments, that a difference of 1°. in the boiling point corresponds to 535 feet in height. Let ∴ n = difference of boiling points at the bottom and top of a mountain, then $1° : n° :: 535$ feet $: n \times 535$ = approximate height. To correct it for the temperature of the air, let m = mean temperature of the top and bottom, ascertained by a common thermometer, then *(see Barometer)* $n. 535 \times (1 + \overline{m - 32°} \times .00244)$ = correct height.—*(Phil. Trans.)*

TABLE V.

Linear expansion of solids by heat.

Dimensions which a bar takes at 212° whose length at 32° is 1.000000.—(Ure.)

Glass tube	Smeaton	1.00083333
Do.	Roy	1.00077615
Deal	Roy, as glass	———

Platina	Troughton	1.00099180
Cast iron prism	Roy	1.00110940
Steel rod	Roy	1.00114470
Iron	Smeaton	1.00125800
Iron wire	Troughton	1.00144010
Gold	Ellicot	1.00150000
Copper	Troughton	1.00191880
Brass	Laplace	1.00186671
Brass wire	Smeaton	1.00193000
Silver	Troughton	1.0020826
Tin	Laplace	1.00217298
Lead	Smeaton	1.00286700

TABLE VI.

Expansion of liquids.

Dilatation of the volume of liquids by being heated from 32° to 212°.—(Ure.)

Mercury	Lord C. Cavendish	0.018870	$\frac{1}{53}$
Do.	Roy	0.017000	$\frac{1}{59}$
Do.	Shuckburgh	0.018510	$\frac{1}{54}$
Do.	Du Long and Petit	0.0180180	$\frac{1}{55.5}$
Do.	Do. from 212° to 392°	0.1301843	$\frac{1}{54.25}$
Do.	Do. from 392° to 572°	0.0188700	$\frac{1}{53}$
Water	Kirwan from 39°. its max. dens.	0.04332	$\frac{1}{23.08}$
Muriatic acid sp. gr. 1.137	Dalton	0.0600	$\frac{1}{17}$
Nitric acid, sp. gr. 1.40	Do.	0.1100	$\frac{1}{9}$
Sulphuric acid sp. gr. 1.85	Do.	0.0600	$\frac{1}{17}$
Alcohol	Do.	0.1100	$\frac{1}{9}$
Water saturated with salt	Do.	0.0500	$\frac{1}{20}$

VII. Latent heat of vapour = 940° + that of water
= + that of ice

Sulphuric ether	.... Do.	....	0.0700	$\frac{1}{14}$
Fixed oils	.... Do.	....	0.0800	$\frac{1}{12.5}$

Of gases $\frac{1}{480}$ of volume for each 1° Fahr Gay Lussac but by Regnault more accurate $\frac{1}{491}$

TABLE VII.

Expansion of water.—(Ure.)

The maximum density of water is at 39°., and it is a singular fact that the expansion of water is the same for any number of degrees above or below the maximum of density; thus the density of water at 32° and at 46° is precisely the same. The following Table, the result of experiments by Sir Charles Blagden and Mr Gilpin, shews this in a clear light.

Sp. Gr.	*Bulk of water.*	*Temperat.*		*Bulk of water.*	*Sp. Gr.*
	1.00000	39°		1.00000	
1.00000	1.00000	38	40	1.00000	1.00000
0.99999	1.00001	37	41	1.00001	0.99999
0.99998	1.00002	36	42	1.00002	0.99998
0.99996	1.00004	35	43	1.00004	0.99996
0.99994	1.00006	34	44	1.00006	0.99994
0.99991	1.00008	33	45	1.00008	0.99991
0.99988	1.00012	32	46	1.00012	0.99988

This law of maximum density does not prevail in the case of sea water; on the contrary, Dr Marcet found that sea water gradually increases in weight down to the freezing point.

HORIZON, *Dip or depression of.*

In observing an altitude at sea with the sextant or reflecting circle, the image of the object is made to coincide with the visible horizon, but as the eye is elevated above the surface of the sea by the height of the ship's deck, the visible horizon will be below the true horizontal plane.

The following Table gives the dip or apparent depression of the horizon for different elevations of the eye, allowing $\frac{1}{10}$ for terrestrial refraction. The dip must be always subtracted from the observed altitude when taken by the fore observation, but added to it in the back observation.

TABLE I.—*Of the dip of the horizon.*

H. of Eye.	Dip of Horiz.	H. of Eye.	Dip. of Horiz.	H. of Eye.	Dip of Horiz.	H. of Eye.	Dip of Horiz.
Feet.	′ ″	Feet.	′ ″	Feet.	′ ″	Feet.	′ ″
1	1 0	6	2 26	16	3 58	32	5 37
1¼	1 7	6½	2 32	16½	4 2	33	5 42
1½	1 13	7	2 38	17	4 5	34	5 47
1¾	1 19	7½	2 43	17½	4 9	35	5 53
2	1 24	8	2 48	18	4 12	36	5 58
2¼	1 29	8½	2 53	18½	4 16	37	6 2
2½	1 34	9	2 58	19	4 19	38	6 7
2¾	1 39	9½	3 3	19½	4 23	39	6 12
3	1 43	10	3 8	20	4 26	40	6 17
3¼	1 47	10½	3 12	21	4 33	42	6 26
3½	1 51	11	3 17	22	4 39	44	6 35
3¾	1 55	11½	3 21	23	4 46	46	6 44
4	1 59	12	3 26	24	4 52	48	6 52
4¼	2 3	12½	3 31	25	4 58	50	7 1
4½	2 6	13	3 35	26	5 4	55	7 21
4¾	2 10	13½	3 39	27	5 10	60	7 41
5	2 13	14	3 43	28	5 16	70	8 18
5¼	2 17	14½	3 47	29	5 22	80	8 53
5½	2 20	15	3 51	30	5 27	90	9 25
5¾	2 23	15½	3 55	31	5 32	100	9 56

If the land is not sufficiently distant to afford a free horizon, it may be sometimes necessary to obtain an altitude referred to the surface of the sea at some known or estimated distance. Under such circumstances, the dip may be taken from the following Table.

TABLE II.—*The dip at different distances from the Observer.*

Miles.	Height of the eye in feet.					
	5	10	15	20	25	30
¼	11′	23′	34′	45′	57′	68′
½	6	12	17	23	28	34
¾	4	8	12	15	19	23
1	3	6	9	12	15	17
1¼	3	5	7	10	12	14
1½	3	4	6	8	10	12
2	2	4	5	7	8	9
2½	2	3	4	6	7	8
3	2	3	4	5	6	7
3½	2	3	4	5	6	6
4	2	3	4	5	5	6
5	2	3	4	4	5	6
6	2	3	4	4	5	5

See Refraction terrestial p. 240
& Levelling

Height in feet of object ~~visible~~ required to ~~above~~ see on horizon distant D miles

$h^{ft} = \frac{2}{3} D^2$

this is not corrected for refraction

HOUR, *decimal parts of.—See Time.*

HYDROMETER.—*See Gravity specific.*

HYDROSTATICAL *paradox, or bellows.—See Fluids, pressure of.*

HYPERBOLA, *principal properties of.—See Conic Sections.*

I

ICEBERG.

According to the experiments of Boyle and Mairan, the volume of solid compact ice is to that of sea water as 10 to 9; therefore the volume of ice which rises above the surface of the water is to that which sinks below it as 1 to 9. Supposing ∴ a cylinder of ice to rise above the surface of the sea 200 feet, which does not exceed the height of some ice islands described by navigators, its depth under water would be 1800 feet, and its whole height 2000 feet. But it is probable that this considerably exceeds the actual height of the Polar Icebergs. For first, the shape of these floating bodies is probably somewhat pyramidal, the part immersed being the broader end. And in the next place, as Mr Wales observes, the ice, which composes these masses, is comparatively light and porous, being chiefly snow and salt water frozen together, and bearing not perhaps a greater proportion to the weight of salt water than that of 5 to 6, or 6 to 7 at the utmost.

Icebergs in both $\frac{1}{2}$ spheres are sometimes carried by currents as low as 40^0 latitude.

JETS *d'eau.—See Fluids, discharge of.*

IMPACT *of hard and elastic bodies.—See Collision.*

IMPERIAL *weights and measures.—See Weights.*

INCLINED *Plane.*

I. *Equilibrium of bodies upon inclined planes.*

Let P = power, W = weight, p = pressure, H = height of the plane, B = base, and L = length, $\alpha = \angle$ of inclination of the plane, $\beta = \angle$ which the direction of the power makes with a perpendicular to the plane, $\gamma = \angle$ which the direction of the power makes with a perpendicular to the horizon; then when a body is sustained upon the plane, we have the following proportions:—

$$P : W :: \sin. \alpha : \sin. \beta.$$

$$P : p :: \sin. \alpha : \sin. \gamma.$$

$$p : W :: \sin. \gamma : \sin. \beta.$$

Cor. 1. When the power acts parallel to the plane,

$$P : W :: H : L.$$
$$P : p :: H : B.$$
$$p : W :: B : L.$$

Cor. 2. When the power acts parallel to the base,

$$P : W :: H : B.$$
$$P : p :: H : L.$$
$$p : W :: L : B.$$

Cor. 3. If W and α be invariable, P varies as $\frac{1}{\sin. \beta}$, $\therefore$ P is least, when it acts in the direction of the plane; and is indefinitely great, when it acts perpendicular to the plane.

Cor. 4. If P and α be invariable, W varies as sin. β; $\therefore$ W is the greatest, when P acts in the direction of the plane, and the least when P acts perpendicular to the plane.

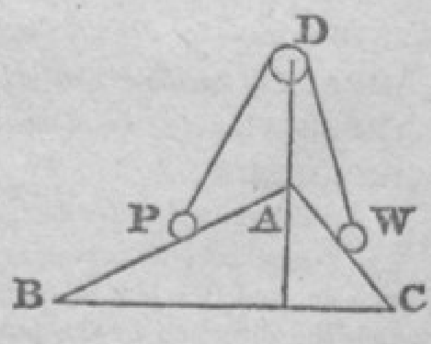
Cor. 5. If P and α be given, p varies as sin. γ, and $\therefore$ is the greatest when P acts parallel to the base.

Cor. 6. If two weights P and W sustain each other on two planes, whose lengths are L and l, and which have a common altitude by means of a string passing over a pulley fixed at the intersection of the planes,

$$P : W :: L : l.$$

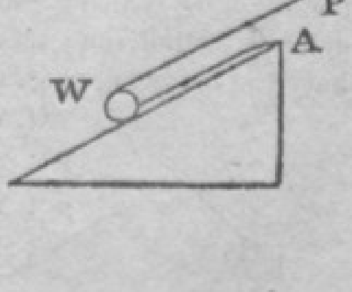
Cor. 7. If the pulley be above the intersection of the planes, as in the annexed figure,

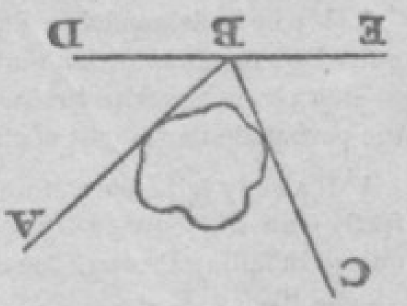
P : W :: sin. B C A × cos. D P A : sin. A B C × cos. D W A.



Cor. 8. If a string fixed at A pass round the weight W, and then be parallel to the plane,

$$P : W :: \tfrac{1}{2} H : L.$$





Cor. 9. If a body be supported on two inclined planes, the pressures on A B, B C and the weight of the body are represented by sin. C B E, sin. A B D, and sin. C B A respectively.

II. *Inclined planes, motion of bodies down.*

1. The force which accelerates or retards a body's motion upon an inclined plane, is to the force of gravity, as the height of the plane to its length.

Hence if $g = 32\frac{1}{6}$ feet, accelerating force $= \frac{g\,H}{L}$; or if $\alpha =$ plane's inclination, accelerating force $= g \times \sin.\ \alpha$.

Cor. 1. Hence if in the formulæ for the rectilinear descent of bodies (*see Motion*) we substitute $g \times \frac{H}{L}$, or $g \times \sin.\ \alpha$ for F, we shall have, if the body descends from rest,

$$v = \frac{H}{L} \times g\,t = \sin.\ \alpha \times g\,t.$$

$$s = \frac{H}{L} \times \frac{g\,t^2}{2} = \sin.\ \alpha \times \frac{g\,t^2}{2}.$$

$$s = \frac{L}{H} \times \frac{v^2}{2g} = \frac{v^2}{2\,g \times \sin.\ \alpha}.$$

Cor. 2. The velocity acquired in falling down the whole length of an inclined plane varies as $\sqrt{H}$.

Cor. 3. The time of descent down the whole length of an inclined plane varies as $\frac{L}{\sqrt{H}}$. Or if the inclination be given, i.e. if H varies as L, T varies as $\sqrt{L}$.

2. If chords be drawn in a circle from the extremity of that diameter which is perpendicular to the horizon, the velocities which bodies acquire by falling down them are proportional to their lengths; and the times of descent are equal.

Cor. The times of descent down chords in different circles are as the square roots of the diameters.

3. If a body descend down a system of inclined planes, the velocity acquired, on the supposition that no motion is lost in passing from one plane to another, is equal to that which would be acquired in falling through the perpendicular height of the system.

4. If a body fall from a state of rest down a curve surface which is perfectly smooth, the velocity acquired is equal to that which would be acquired in falling through the same perpendicular height.

5. The times of descent down similar systems of inclined planes, similarly situated, are as the square roots of their lengths, on the supposition that no velocity is lost in passing from one plane to another.

INFLEXION, *point of in curves.*

To ascertain the point of contrary flexure in any curve, find the 2d differential of the equation of the curve, supposing dx constant, and we shall have a finite value of $\frac{-d^2y}{dx^2}$, which must be put equal to either zero or infinity. By means of this equation, and that of the curve, we can determine those values of x and y, which belong to the point or points of contrary flexure.

Ex. 1. Let the equation be $y = 3x + 18x^2 - 2x^3$.

$$\text{Here } -\frac{d^2y}{dx^2} = 12x - 36 = o, \therefore x = 3.$$

2. Let the curve be the cubical parabola, whose equation is $y^3 = a^2x$.

Here $\frac{-d^2y}{dx^2} = \frac{2}{9}x - \frac{5}{3}a^{\frac{2}{3}} = o, \therefore x = o$, or the point of inflexion is at the vertex.

For the point of inflexion in spirals—*see Spirals.*

In general there cannot be a point of contrary flexure, unless the first differential coefficient, which does not vanish, for a particular value of the abscissa, be of an odd order.—*See Maxima and Minima.*

INTEGRAL.—*See Differential.*

INTEREST.

Interest simple.

Let P = principal, r = interest of £1. for one year, I the interest of P, and M its amount in the time n; then we have the following equations, from which any of the quantities may be found, the rest being given.

$$I = n\,r\,P.$$

$$M = P + n\,r\,P = (1 + n\,r.)\,P.$$

$$\text{Discount of M £} = M - \frac{M}{1 + nr}.$$

The following Tables will much facilitate the computation of simple Interest :—

TABLE I.

Of the Interest of £1. for any number of days at different rates of Interest.

No. of Days.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	·0000821	·0000958	·0001095	·0001232	·0001369
2	·0001641	·0001916	·0002191	·0002465	·0002739
3	·0002465	·0002876	·0003287	·0003698	·0004109
4	·0003287	·0003835	·0004383	·0004931	·0005479
5	·0004109	·0004794	·0005479	·0006164	·0006849
6	·0004931	·0005753	·0006575	·0007397	·0008219
7	·0005753	·0006712	·0007671	·0008630	·0009589
8	·0006575	·0007671	·0008767	·0009863	·0010958
9	·0007397	·0008630	·0009863	·0011095	·0012328
10	·0008219	·0009589	·0010958	·0012328	·0013698
20	·0016438	·0019178	·0021917	·0024657	·0027397
30	·0024657	·0028767	·0032876	·0036986	·0041095
40	·0032876	·0038356	·0043835	·0049315	·0054794
50	·0041095	·0047945	·0054794	·0061643	·0068493
60	·0049315	·0057534	·0065753	·0073972	·0082191
70	·0057534	·0067123	·0076712	·0086301	·0095890
80	·0065753	·0076712	·0087671	·0098630	·0109589
90	·0073972	·0086301	·0098630	·0110958	·0123287
100	·0082191	·0095890	·0109589	·0123287	·0136986
200	·0164382	·0191780	·0219178	·0246574	·0273972
300	·0246573	·0287670	·0328767	·0369861	·0410958

This Table it is obvious will furnish, by the addition of two or three of its numbers, the interest for any number of days, and the following will in the same way find it for any number of years.

TABLE II.

Of the Interest of £1. for any number of years not exceeding 25, at different rates of Interest.

No. of Years	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	·0300000	·0350000	·0400000	·0450000	·0500000
2	·0600000	·0700000	·0800000	·0900000	·1000000
3	·0900000	·1050000	·1200000	·1350000	·1500000
4	·1200000	·1400000	·1600000	·1800000	·2000000
5	·1500000	·1750000	·2000000	·2250000	·2500000
6	·1800000	·2100000	·2400000	·2700000	·3000000
7	·2100000	·2450000	·2800000	·3150000	·3500000
8	·2400000	·2800000	·3200000	·3600000	·4000000
9	·2700000	·3150000	·3600000	·4050000	·4500000
10	·3000000	·3500000	·4000000	·4500000	·5000000
11	·3300000	·3850000	·4400000	·4950000	·5500000
12	·3600000	·4200000	·4800000	·5400000	·6000000
13	·3900000	·4550000	·5200000	·5850000	·6500000
14	·4200000	·4900000	·5600000	·6300000	·7000000
15	·4500000	·5250000	·6000000	·6750000	·7500000
16	·4800000	·5600000	·6400000	·7200000	·8000000
17	·5100000	·5950000	·6800000	·7650000	·8500000
18	·5400000	·6300000	·7200000	·8100000	·9000000
19	·5700000	·6650000	·7600000	·8550000	·9500000
20	·6000000	·7000000	·8000000	·9000000	1·0000000
21	·6300000	·7350000	·8400000	·9450000	1·0500000
22	·6600000	·7700000	·8800000	·9900000	1·1000000
23	·6900000	·8050000	·9200000	1.0350000	1·1500000
24	·7200000	·8400000	·9600000	1·0800000	1·2000000
25	·7500000	·8750000	1·0000000	1·1250000	1.2500000

To find the Interest of any sum, for a given time, by the preceding Tables:

Add together the interests for the several periods corresponding with the proposed rate of per cent. and that sum multiplied by the principal will be the interest required.

Interest compound.

Let R = £1. and its interest for one year = $1 + r$, M the amount of P £ in n years, then

$$M = P R^n$$

Discount of M £ = $M - \frac{M}{R^n}$ where n must be greater than one year, otherwise only simple interest can be allowed.

If besides the interest being converted into principal at the end of every year, the sum P is at the same time annually invested in capital then at the end of n years,

$$M = \frac{P R (R^n - 1)}{R - 1}.$$

TABLE I.

Showing the sum to which one pound will increase when improved at Compound Interest during any number of years not exceeding 50.

Years.	2½ per Cent.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	1·02500	1·030000	1·040000	1·050000	1·060000
2	1·05063	1·060900	1·081600	1·102500	1·123600
3	1·07689	1·092727	1·124864	1·157625	1·191016
4	1·10381	1·125509	1·169859	1·215506	1·262477
5	1·13141	1·159274	1·216653	1·276282	1·338226
6	1·15969	1·194052	1·265319	1·340096	1·418519
7	1·18869	1·229874	1·315932	1·407100	1·503630
8	1·21840	1·266770	1·368569	1·477455	1·593848
9	1·24886	1·304773	1·423312	1·551328	1·689479
10	1·28008	1·343916	1·480244	1·628895	1·790848
11	1·31209	1·384234	1·539454	1·710339	1·898299
12	1·34489	1·425761	1·601032	1·795856	2·012196
13	1·37851	1·468534	1·665074	1·885649	2·132928
14	1·41297	1·512590	1·731676	1·979932	2·260904
15	1·44830	1·557967	1·800944	2·078928	2·396558
16	1·48451	1·604706	1·872981	2·182875	2·540352
17	1·52162	1·652848	1·947901	2·292018	2·692773
18	1·55966	1·702433	2·025817	2·406619	2·854339
19	1·59865	1·753506	2·106849	2·526950	3·025600
20	1·63862	1·806111	2·191123	2·653298	3·207135
21	1·67958	1·860295	2·278768	2·785963	3·399564
22	1·72157	1·916103	2·369919	2 925261	3·603537
23	1·76461	1·973587	2·464716	3 071524	3·819750
24	1·80873	2·032794	2·563304	3·225100	4·048935
25	1·85394	2·093778	2·665836	3·386355	4·291871
26	1·90029	2·156591	2·772470	3.555673	4·549383
27	1·94780	2·221289	2·883369	3·733456	4·822346
28	1·99650	2·287928	2·998703	3·920129	5·111687
29	2·04641	2·356566	3·118651	4·116136	5·418388
30	2 09757	2·427262	3·243398	4·321942	5·743491
31	2·15001	2·500080	3·373133	4·538039	6 088101
32	2·20376	2·575083	3·508059	4·764941	6·453387
33	2·25885	2·652335	3·648381	5·003189	6·840590
34	2·31532	2·731905	3·794316	5·253348	7·251025
35	2·37321	2·813862	3·946089	5·516015	7·686087
36	2·43254	2·898278	4·103933	5·791816	8·147252
37	2·49335	2·985227	4·268090	6·081407	8·636087
38	2·55568	3·074783	4·438813	6·385457	9·154252
39	2·61957	3·167027	4·616366	6·704751	9·703507
40	2·68506	3·262038	4·801021	7·039989	10·285718
41	2·75219	3·359899	4·993061	7·391988	10·902861
42	2·82100	3·460696	5·192784	7·761588	11·557033
43	2·89152	3·564517	5·400495	8·149667	12·250455
44	2·96381	3·671452	5·616515	8·557150	12·985482
45	3·03790	3·781596	5·841176	8·985008	13·764611
46	3·11385	3·895044	6·074823	9·434258	14·590487
47	3·19170	4·011895	6·317816	9·905971	15·465917
48	3·27149	4·132252	6·570528	10·401270	16·393872
49	3·35328	4·256219	6·833349	10·921333	17·377504
50	3·43711	4·383906	7·106683	11·467400	18·420154

TABLE II.

Shewing the present value of one pound to be received at the end of any number of years not exceeding 50.

Years.	2½ per Cent.	3 per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	·975610	·970874	·961538	·952381	·943396
2	·951814	·942596	·924556	·907029	·889996
3	·928599	·915142	·888996	·863838	·839619
4	·905951	·888487	·854804	·822702	·792094
5	·883854	·862609	·821927	·783526	·747258
6	·862297	·837484	·790315	·746215	·704961
7	·841265	·813092	·759918	·710681	·665057
8	·820747	·789409	·730690	·676839	·627412
9	·800728	·766417	·702587	·644609	·591898
10	·781198	·744094	·675564	·613913	·558395
11	·762145	·722421	·649581	·584679	·526788
12	·743556	·701380	·624597	·556837	·496969
13	·725420	·680951	·600574	·530321	·468839
14	·707727	·661118	·577475	·505068	·442301
15	·690466	·641862	·555265	·481017	·417265
16	·673625	·623167	·533908	·458112	·393646
17	·657195	·605016	·513373	·436297	·371364
18	·641166	·587395	·493628	·415521	·350344
19	·625528	·570286	·474642	·395734	·330513
20	·610271	·553676	·456387	·376889	·311805
21	·595386	·537549	·438834	·358942	·294155
22	·580865	·521893	·421955	·341850	·277505
23	·566697	·506692	·405726	·325571	·261797
24	·552875	·491934	·390121	·310068	·246979
25	·539391	·477606	·375117	·295303	·232999
26	·526235	·463695	·360689	·281241	·219810
27	·513400	·450189	·346817	·267848	·207368
28	·500878	·437077	·333477	·255094	·195630
29	·488661	·424346	·320651	·242946	·184557
30	·476743	·411987	·308319	·231377	·174110
31	·465115	·399987	·296460	·220359	·164255
32	·453771	·388337	·285058	·209866	·154957
33	·442703	·377026	·274094	·199873	·146186
34	·431905	·366045	·263552	·190355	·137912
35	·421371	·355383	·253415	·181290	·130105
36	·411094	·345032	·243669	·172657	·122741
37	·401067	·334983	·234297	·164436	·115793
38	·391285	·325226	·225285	·156605	·109239
39	·381741	·315754	·216621	·149148	·103056
40	·372431	·306557	·208289	·142046	·097222
41	·363347	·297628	·200278	·135282	·091719
42	·354485	·288959	·192575	·128840	·086527
43	·345839	·280543	·185168	·122704	·081630
44	·337404	·272372	·178046	·116861	·077009
45	·329174	·264439	·171198	·111297	·072650
46	·321146	·256737	·164614	·105997	·068538
47	·313313	·249259	·158283	·100949	·064658
48	·305671	·241999	·152195	·096142	·060998
49	·298216	·234950	·146341	·091564	·057546
50	·290942	·228107	·140713	·087204	·054288

INTERPOLATIONS.—(*Woodhouse, Vince.*)

If a, a', a'', &c. are successive values of a quantity a, differing by a constant interval 1, and if the 1st, 2d, 3d, &c. differences be d', d'', d''', &c.; then any intermediate value (y), distant from a by the interval x, is equal to
$$a + x d' + x. \frac{x-1}{2} d'' + x. \frac{x-1}{2} . \frac{x-2}{3} d''' \text{ \&c.}$$

Note.—In taking the differences, the *preceding* quantity must always be subtracted from the succeeding; they will ∴ be positive or negative according as the series of quantities is increasing or decreasing.

If the law of the quantities be such that their last differences always become $= o$, we shall get at any intermediate time the *accurate* value of that quantity; but if the differences do not at last become accurately $= o$, we shall then get only an approximate value.

In general the quantities d', d'', &c. diminish very fast, and it will not often be necessary to proceed farther than d'''.

Ex. 1. Given the squares of 2, 3, 4, and 5, to find the square of $2\frac{1}{2}$.

4, 9, 16, 25 ...... quantities
5, 7, 9 .... 1st order of differences.
2, 2 .... 2d do.
0 .... 3d do.

Here $a = 4$, $d' = 5$, $d'' = 2$, $d''' = 0$, x the required interval $= \frac{1}{2}$; ∴
$$y = 4 + \frac{1}{2} \times 5 - \frac{1}{8} \times 2 = 6,25.$$

Ex. 2. Given the log. of 110 = 2.04139, of 111 = 2.04532, of 112 = 2.04922, and of 113 = 2.05308; required the log. of 110.5.

2.04139, 2.04532, 2,04922, 2,05308
.00393, .00390, .00386
—. 00003, —. 00004

Here $a = 2.04139$. $d' = .00393$, $d'' = -.00003$, and $x = \frac{1}{2}$, ∴
$$y = 2.04139 + \frac{1}{2} \times .00393 - \frac{1}{8} \times -.00003 = 2.043359.$$

Ex. 3. Given five places of a comet as follows; on Nov. 5th at 8*h.* 17*m.* in Cancer 2°. 30′ = 150′; on the 6th at 8*h.* 17*m.* in 4°. 7′ = 247′; on the 7th at 8*h.* 17*m.* in 6°. 20′ = 380′; on the 8th at 8*h.* 17*m.* in 9°. 10′ = 550′; on the 9th at 8*h.* 17*m.* in 12°. 40′ = 760′. To find its place on the 7th at 14*h.* 17*m.*

First subtract 5*d.* 8*h.* 17*m.* from 7*d.* 14*h.* 17*m.*, and there remains 2*d.* 6*h.* = 2,25 for the interval of time between the first observation and the given time at which the place is required; this ∴ is the value of x, to which we want to find the corresponding value of y; hence

150, 247, 380, 550, 760
97, 133, 170, 210
36, 37, 40
1, 3
2

Here $a = 150$, $d' = 97$, $d'' = 36$, $d''' = 1$, $d'''' = 2$; hence $y = 150 + 97 \times 2{,}25 + \frac{36}{2} \times 2{,}25 \times 1{,}25 + \frac{1}{2.3} \times 2{,}25 \times 1{,}25 \times {,}25 + \frac{2}{2.3.4} + 2{,}25 \times 1{,}25 \times {,}25 \times -{,}75 = 418', 96 = 6^0.\ 58'.\ 57''$, the place required.

But besides the use of the above equation, to find the value of any term of a series from its position being given, the converse is often required, *i. e.* having given any term, to find its position or distance from the first term.

Ex. On March, 1783, the sun's declination at noon at Greenwich was as follows:—On the 19th, N. 28'. 41'' = 1721''; on the 20th, N. 5' = 300''; on the 21st, S. — 18'. 41'' = — 1121''; to find the time of the equinox.

$$1721,\ 300,\ -1121$$
$$-1421,\ -1421$$
$$0$$

Here $a = 1721$, $d' = -1421$, hence $y = 1721 - 1421 \times x$; now when the sun comes to the Equator, y the declination becomes $= o$; $\therefore 1721 - 1421\,x = o$, and $x = \frac{1721}{1421} = 1d.\ 5h.\ 3m.\ 53s.$, the time from the 19th; hence 20*d.* 5*h.* 3*m.* 53*s.* is the time required.

We have here supposed that the quantities to be interpolated were taken at equal intervals of time; for a formula when the intervals are unequal, *see Vince's Astronomy*, vol. 2.

INVOLUTION *and Evolution.*

TABLE *of the first nine powers of numbers.*

1st	2d	3d	4th	5th	6th	7th	8th	9th
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

TABLE *of squares, cubes, square roots, cube roots, and reciprocals, of of all numbers from* 1 *to* 100.—*(Barlow.)*

Num.	Squares.	Cubes.	Sq. Roots.	Cu. Roots.	Reciprocals.
1	1	1	1	1	1
2	4	8	1·4142136	1·2599210	·500000000
3	9	27	1·7320508	1·4422496	·333333333
4	16	64	2·0000000	1·5874011	·250000000
5	25	125	2·2360680	1·7099759	·200000000
6	36	216	2·4494897	1·8171206	·166666667
7	49	343	2·6457513	1·9129312	·142857143
8	64	512	2·8284271	2·0000000	·125000000
9	81	729	3·0000000	2·0800837	·111111111
10	100	1000	3·1622777	2·1544347	·100000000
11	121	1331	3·3166248	2·2239801	·090909091
12	144	1728	3·4641016	2·2894286	·083333333
13	169	2197	3·6055513	2·3513347	·076923077
14	196	2744	3·7416574	2·4101422	·071428571
15	225	3375	3·8729833	2·4662121	·066666667
16	256	4096	4·0000000	2·5198421	·062500000
17	289	4913	4·1231056	2·5712816	·058823529
18	324	5832	4·2426407	2·6207414	·055555556
19	361	6859	4·3588989	2·6684016	·052631579
20	400	8000	4·4721360	2·7144177	·050000000
21	441	9261	4·5825757	2·7589243	·047619048
22	484	10648	4·6904158	2·8020393	·045454545
23	529	12167	4·7958315	2·8438670	·043478261
24	576	13824	4·8989795	2·8844991	·041666667
25	625	15625	5·0000000	2·9240177	·040000000
26	676	17576	5·0990195	2·9624960	·038461538
27	729	19683	5·1961524	3·0000000	·037037037
38	784	21952	5·2915026	3·0365889	·035714286
29	841	24389	5·3851648	3·0723168	·034482759
30	900	27000	5·4772256	3·1072325	·033333333
31	961	29791	5·5677644	3·1413806	·032258065
32	1024	32768	5·6568542	3·1748021	·031250000
33	1089	35937	5·7445626	3·2075343	·030303030
34	1156	39304	5·8309519	3·2396118	·029411765
35	1225	42875	5·9160798	3·2710663	·028571429
36	1296	46656	6·0000000	3·3019272	·027777778
37	1369	50653	6·0827625	3·3322218	·027027027
38	1444	54872	6·1644140	3·3619754	·026315789
39	1521	59319	6·2449980	3·3912114	·025641026
40	1600	64000	6·3245553	3·4199519	·025000000
41	1681	68921	6·4031242	3·4482172	·024390244
42	1764	74088	6·4807407	3·4760266	·023809524
43	1849	79507	6·5574385	3·5033981	·023255814
44	1936	85184	6·6332496	3·5303483	·022727273
45	2025	91125	6·7082039	3·5568933	·022222222
46	2116	97336	6·7823300	3·5830479	·021739130
47	2209	103823	6·8556546	3·6088261	·021276600
48	2304	110592	6·9282032	3·6342411	·020833333
49	2401	117649	7·0000000	3·6593057	·020408163
50	2500	125000	7·0710678	3·6840314	·020000000

Num.	Squares.	Cubes.	Sq. Roots.	Cu. Roots.	Reciprocals.
51	2601	132651	7·1414284	3·7084298	·019607843
52	2704	140608	7·2111026	3·7325111	·019230769
53	2809	148877	7·2801099	3·7562858	·018867925
54	2916	157464	7·3484692	3·7797631	·018518519
55	3025	166375	7·4161985	3·8029525	·018181818
56	3136	175616	7·4833148	3·8258624	·017857143
57	3249	185193	7·5498344	3·8485011	·017543860
58	3364	195112	7·6157731	3·8708766	·017241379
59	3481	205379	7·6811457	3·8929965	·016949153
60	3600	216000	7·7459667	3·9148676	·016666667
61	3721	226981	7·8102497	3·9364972	·016393443
62	3844	238328	7·8740079	3·9578915	·016129032
63	3969	250047	7·9372539	3·9790571	·015873016
64	4096	262144	8·0000000	4·0000000	·015625000
65	4225	274625	8·0622577	4·0207256	·015384615
66	4356	287496	8·1240384	4·0412401	·015151515
67	4489	300763	8·1853528	4·0615480	·014925373
68	4624	314432	8·2462113	4·0816551	·014705882
69	4761	328509	8·3066239	4·1015661	·014492754
70	4900	343000	8·3666003	4·1212853	·014285714
71	5041	357911	8·4261498	4·1408178	·014084507
72	5184	373248	8·4852814	4·1601676	·013888889
73	5329	389017	8·5440037	4·1793390	·013698630
74	5476	405224	8·6023253	4·1983364	·013513514
75	5625	421875	8.6602540	4·2171633	·013333333
76	5776	438976	8·7177979	4·2358236	·013157895
77	5929	456533	8·7749644	4·2543210	·012987013
78	6084	474552	8·8317609	4·2726586	·012820513
79	6241	493039	8·8881944	4·2908404	·012658228
80	6400	512000	8·9442719	4·3088695	·012500000
81	6561	531441	9·0000000	4·3267487	·012345679
82	6724	551368	9·0553851	4·3444815	·012195122
83	6889	571787	9·1104336	4·3620707	·012048193
84	7056	592704	9·1651514	4·3795191	·011904762
85	7225	614125	9·2195445	4·3968296	·011764706
86	7396	636056	9·2736185	4·4140049	·011627907
87	7569	658503	9·3273791	4·4310476	·011494253
88	7744	681472	9·3808315	4·4479602	·011363636
89	7921	704969	9·4339811	4·4647451	·011235955
90	8100	729000	9·4868330	4·4814047	·011111111
91	8281	753571	9·5393920	4·4979414	·010989011
92	8464	778688	9·5916630	4·5143574	·010869565
93	8649	804357	9·6436508	4·5306549	·010752688
94	8836	830584	9·6953597	4·5468359	·010638298
95	9025	857375	9·7467943	4·5629026	·010526316
96	9216	884736	9·7979590	4·5788570	·010416667
97	9409	912673	9·8488578	4·5947009	·010309278
98	9604	941192	9·8994949	4·6104363	·010204082
99	9801	970299	9·9498744	4·6260650	·010101010

The use of the first five columns is obvious: the column of reciprocals is useful for converting a vulgar into a decimal fraction, as in the following example.

Express $\frac{3}{28}$ as a decimal.

By Table $\frac{1}{28}$ —— is —— .035714286

$\therefore \frac{3}{28}$ ——— is —— .107142858

JULIAN *Period.—See Cycle.*

JUNO.

This planet was discovered by Mr Harding, at Lilienthal, September 1st, 1804. For its elements, &c.—*see Planets, elements of.*

JUPITER.—*See Planets, elements of.*

JUPITER'S *Satellites.—See Satellites.*

L.

LAND *Surveying.—See Surveying.*

LATITUDE *Geographical.—(Woodhouse.)*

1*st Method, by the Altitudes of circumpolar stars.*

Co-latitude = half the sum of the greatest and least zenith distances corrected for refraction.

Or the latitude may be found by Captain Kater's method, from an observed altitude of the pole star when out of the meridian thus.—*(Galbraith.)*

To the constant log. 5.314425, add the log. tangent of the star's polar distance p, and the log. cos. of the meridian distance t in degrees, the sum of these will be log. of an arc u in seconds. Now to the log. secant p add the log. cosine u, and cosine of the zenith distance z; the sum will be the cosine of $(\psi \pm u)$ an arc which being increased or diminished by the arc u, will be the co-latitude ψ.

To find t, calculate the time of the star's meridian passage *(see Time)*, the difference between which and the time of observation gives t.

In the application of u attention must be paid to the sign of the arc t, according to its situation in the circle which the star describes round the pole in its diurnal revolution. If t is in the 1st or 4th quadrant it is additive; but if in the 2d or 3d, it is subtractive.

Ex. On the 22d of February, 1826, at 7*h.* 42*m.* 49*s.* mean time, the altitude of the Pole star was observed to be 51o. 58′. 11″.; required the latitude.

First to find the mean solar time when the star was upon the meridian.

	h.	m.	s.		
Star's app. R. A.	0	58	15,2	App. alt.	51o. 58′ 11″
Sun's R. A. at noon	22	21	18,3	Refract.	— 45,4
	2	36	56,9	True alt.	51 57 25,6
Diff. *(see Time Table 6)*		—	25,7	z =	38 2 34,4
	2	36	31,2		
Equat. of time for noon	+	13	50,7		
Star upon meridian	2	50	21,9		
Time of observation	7	42	49,0		
Distance of star from meridian in mean time	4	52	27,1	= 73o. 18′. 47″. *(see Time Table.)*	

Constant log.	5.314425		
p = 1°. 36′. 48″ tang. ...	8.449716	secant	0.000172
t = 73. 18. 47 cosine ...	9.458097		
u = 0. 27. 18,2 = 1668″,2	3.222238	cosine	9.999986
	z = 38o. 2′. 34,4″	cosine	9.896278
$(\psi - u)$ = 38°. 0′. 58,2″		cosine	9.896436
ψ =	38. 28. 46.4		
λ =	51. 31. 13,6		

2d Method, by the zenith distances of stars near the zenith.

This method determines merely the difference of latitude by means of the zenith sector, measuring small zenith distances with great exactness.

Ex. By observation at the College of Mazarin.

Z. D. of γ *Draconis* reduced to January, 1750	2°. 40.′ 15″
At Greenwich Z. D. reduced to same epoch	0. 3. 4,5
Difference of latitude	2. 37. 10,5
Hence if latitude of Greenwich be	51. 28. 39,5
Latitude of Observatory of College of Mazarin	48. 51. 29

This method, which is capable of great accuracy, was employed in the Trigonometrical Survey of England.

3d Method by observations of Altitudes made near the Meridian, and reduced to the Meridian.

Let z', z'', &c. $z_{\prime}$, $z_{\prime\prime}$, &c. be n zenith distances to the east and west of the meridian.

$h,'$ $h,''$ &c. $h_{\prime}$ $h_{\prime\prime}$ &c. the corresponding times determined by means of a time-keeper.

δ', δ'', &c. $\delta_{\prime}$, $\delta_{\prime\prime}$, &c., the calculated corrections, or the reductions to the meridian: then the true or corrected meridional zenith distances will be (if the passage of the star be above the pole.)

$$z' - \delta',\ z'' - \delta''\ \&c.\ z_{\prime} - \delta_{\prime},\ z_{\prime\prime} - \delta_{\prime\prime}\ \&c.$$

and the true mean meridional zenith distance will be

$$= \frac{z' - \delta' + z'' - \delta'' + \&c. + z_{\prime} - \delta_{\prime} + z_{\prime\prime} - \delta_{\prime\prime} + \&c.}{n} =$$

$$\frac{z' + z'' + \&c. + z_{\prime} + z_{\prime\prime} + \&c.}{n} - \frac{\delta' + \delta'' + \&c. + \delta_{\prime} + \delta_{\prime\prime} + \&c.}{n}$$

In the above formula $\delta = \frac{2}{\sin. 1''} \times \sin.^2 \frac{h'}{2}. \ \frac{\cos. d. \cos. L}{\sin. z}$, where d is known from the Tables; and for L the *approximate* value of the latitude may be taken, and for z the observed meridional zenith distance.

Having thus found the meridional Z. D. and knowing the N. P. D. the latitude may be ascertained.

This method of determining the latitude was used by the French Astronomers in measuring an arc of the meridian, and is capable of determining the latitude within the fraction of a second. It is peculiarly adapted to Borda's *Circle of Repetition.*

Latitude of a vessel at sea by the sextant.

Method by the Meridional Altitude of the Sun.

If the latitude and the declination be of the same denomination, then the latitude =

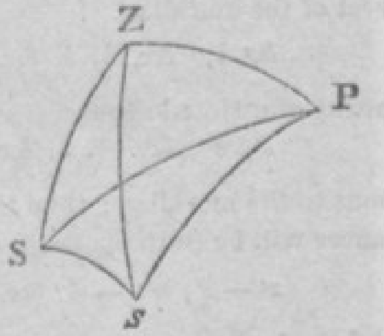
Z. D. of sun + declination of sun:

or if declination be greater than latitude, = declination sun — Z. D. of sun.

If the latitude and declination be of different denominations, then latitude =

Z. D. of sun — declination of sun.

This method is commonly used at sea, but as the sun must be on the meridian, clouds may prevent its being used. A subsidiary method therefore is provided, in which the latitude may be computed from two observed altitudes of the sun, and the interval of time between the observations.



Let Z be the zenith, P the pole, S, *s* two positions of the sun; then the following are the steps in this process.

(1.) Find S *s*; let t = interval of time, p = P S then

$$2 \sin.^2 \frac{S\,s}{2} = \sin.^2 p.\ 2.\ \sin.^2 \frac{t}{2};$$

$$\therefore \log.\sin.\frac{S\,s}{2} = \log.\sin.\,p + \log.\sin.\frac{t}{2} - 10.$$

(2.) Find $\angle$ S *s* P;

$$\tan.\ S\,s\,P = \frac{\cot.\frac{t}{2}}{\cos.\,p};$$

$$\therefore \log.\tan.\ S\,s\,P = 10 + \log.\cot.\frac{t}{2} - \log.\cos.\,p.$$

(3.) Find $\angle$ Z *s* S;

Let a and a' be the observed altitudes, then $\sin.^2 \frac{1}{2} Z\,s\,S$

$$= \frac{\cos.\frac{1}{2}(S\,s + a' + a).\ \sin.\frac{1}{2}(S\,s + a' - a)}{\sin.\ S\,s \times \cos.\,a};\ \therefore 2\log.\sin.\tfrac{1}{2} Z\,s\,S =$$

$$20 + \log.\cos.\tfrac{1}{2}(S\,s + a' + a) + \log.\sin.\tfrac{1}{2}(S\,s + a' - a) - \log.\sin.\ S\,s - \log.\cos.\,a.$$

Hence Z *s* P = S *s* P − Z *s* S is known.

(4.) Find Z P;

Assume θ such that

$$\tan.^2 \theta = \frac{\cos.\,a'.\ \sin.\,p.\ \text{ver.}\sin.\ Z\,s\,P}{\text{ver.}\sin.\,(90^0 - a' - p)};$$

$$\text{then}\ \sin.\frac{Z\,P}{2} = \sin.\frac{90^0 - a' - p}{2} \times \sec.\,\theta;$$

$$\therefore \log.\sin.\frac{Z\,P}{2} = 10 + \log.\sin.\tfrac{1}{2}(90^0 - a' - p) - \log.\cos.\,\theta.$$

LEAP *Year.—See Calendar.*

LEMNISCATA, *equation to.*

$$a y = x \sqrt{a^2 - x^2}$$

Lemniscata of James Bernouilli.

$$(x^2 + y^2)^2 = a^2 (x^2 - y^2)$$

or considered as a spiral

$$\varrho = a \sqrt{\cos. 2\, \theta}.$$

LENGTHS *of curves.—See Rectification.*

LENS.—*See Refraction.*

LEVELLING.

Two or more places are on a true level, when they are equally distant from the centre of the earth; and a line equally distant from that centre in all its points, is called the line of true level. This line is nearly an arc of a circle, and will evidently pass below the line of apparent level, which, as determined by the instrument, will be a tangent or a parallel to a tangent at the earth's surface at the point of observation. Hence the depression of the true below the apparent level is always equal to the excess of the secant of the arc of distance above the radius of the earth. To find this depression, let L be the arc of distance in English miles, D the depression in feet; then

$$D = \frac{2\, L^2}{3}.$$

TABLE *shewing the height of the apparent above the true level for every 100 yards of distance on the one hand, and for every mile on the other.*

Distance of base.	*Diff. of level.*	*Distance of base.*	*Difference of level.*
Yards.	Inches.	Miles.	Feet. In.
100	0.026	$\frac{1}{4}$	0. $0\frac{1}{2}$
200	0.103	$\frac{1}{2}$	0. 2
300	0.231	$\frac{3}{4}$	0. $4\frac{1}{2}$
400	0.411	1	0. 8
500	0.643	2	2. 8
600	0.925	3	6. 0
700	1.260	4	10. 7
800	1.645	5	16. 7
900	2.081	6	23. 11
1000	2.570	7	32. 6
1100	3.110	8	42. 6
1200	3.701	9	53. 9
1300	4.344	10	66. 4
1400	5.038	11	80. 3
1500	5.784	12	95. 7
1600	6.580	13	112. 2
1700	7.425	14	130. 1

Example. Suppose a spring to be on one side of a hill, and a house on an opposite hill, with a valley between them; and that the spring seen from the house appears by a levelling instrument to be on a level with the foundation of the house, which suppose is at a mile distance from it; then *(by Table)* the spring is eight inches above the true level of the house; and this difference would be barely sufficient for the water to be brought in pipes from the spring to the house, the pipes being laid all the way in the ground.

In the above Table, the effects of refraction have not been considered, which, however, should not be neglected, if the distances are considerable. In that case, the correct formula is

$$D = \frac{4\,L^2}{7};$$

which expression includes the effects both of curvature and refraction. *See Refraction terrestrial.*

LEVER.

Levers may be divided into three kinds. In levers of the first kind, the fulcrum is between the power and the weight, as in the balance, steelyard, scissors, poker, &c. In levers of the second kind, the weight is between the fulcrum and the power, as in oars, doors, cutting knives fixed at one end, &c. In levers of the third kind, the power acts between the fulcrum and the weight, as in tongs, sheers for sheep, muscles of animals, &c.

1. Two weights or forces, acting perpendicularly upon a straight lever, will balance each other, when they are reciprocally proportional to their distances from the fulcrum.

Cor. 1. When the power and weight act on the same side of the fulcrum, and keep each other in equilibrio, the weight sustained by the fulcrum is equal to the difference between the power and the weight.

Cor. 2. If the same body be weighed at the two ends of a false balance (one arm of which is longer than the other), its true weight is a mean proportional between the apparent weights,

Cor. 3. If a weight be placed upon a lever supported upon two props, the pressures upon the props are inversely proportional to their distances from the weight.

2. If two forces, acting upon the arms of *any* lever, keep it at rest, they are to each other inversely as the perpendiculars drawn from the centre of motion to the directions in which the forces act; or inversely as the arms, multiplied into the sines of the angles, which the direction of the forces make with them.

Cor. If a man, balanced in a common pair of scales, press upwards, by means of a rod, against any point of the beam, except that from which the scale is suspended, he will preponderate.

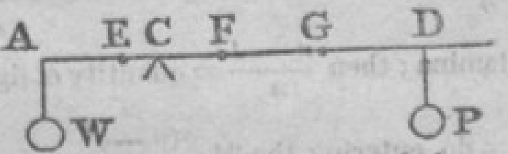
3. In a compound lever, where one is made to turn another, there is an equilibrium, when W : P :: the product of all the arms taken alternately, beginning with that to which the power is applied : the product of all the other arms.

4. Any weights will keep each other in equilibrio on the arms of a straight lever, when the products, which arise from multiplying each weight by its distance from the fulcrum, are equal on each side of the fulcrum.

Cor. 1. If in the above Propositions we would allow for the weight of the lever itself, we must suppose its weight to be united in the centre of gravity, and to act there as a third force added to the power or the weight, according to the side of the fulcrum on which it is placed.

Cor. 2. If the weights do not act perpendicularly to the arms of the lever, we must for the distances substitute the perpendiculars, (see Art. 2.)

Cor. 3. Let A D be the common steelyard, whose fulcrum is C, and let the moveable weight P, when placed at E, keep the lever at rest;



then when W and P are suspended upon the lever, and the whole remains at rest, $W \times AC = P \times DC + P \times EC = P \times DE$; $\therefore$ W varies as E D; the graduation must $\therefore$ begin from E, and if P when placed at F support a weight of one pound at A, take F G, G D, &c. equal to one another and to E F; and when P is placed at G it will support two pounds; and when at D it will support three pounds, &c.

LIFE *Annuities.—See Annuities Life.*

LIFE *Assurances.—See Annuities.*

LIGHT, *Phænomena of.*

Light, propagation of.

1. In a free medium the force and intensity of light, which propagates itself in rays emanating from the same point, are inversely as the squares of the distances from that point.

Prob. Having given the position of two lights of known intensities, to determine the nature and equation of the surface, of which every point shall be equally illuminated by the two lights.

Let A and B be the two points at which the lights are placed, *m* and *n* their intensities at any assumed unit of distance, and let $a = AB$; then it may be shewn that the required surface is a sphere of which the radius $= \frac{a}{n-m}\sqrt{mn}$, and whose centre has for an abscissa $\frac{ma}{m-n}$.

Cor. If $m = n$ the radius is infinite, as also the abscissa from the centre; in this case the surface is a plane perpendicular to the middle of the line A B.

Light, velocity of.

2. Light takes up about 16½ minutes in passing over a space = the diameter of the earth's orbit, which is nearly 190 millions of miles; ∴ it travels at the rate of almost 200,000 miles per secnod.

Light, diminution of, under various circumstances.

3. If the spaces through which light passes through a uniformly dense diaphonous medium increase in arithmetical progression, the quantity will decrease in geometrical progression.

Let the space be divided into equal portions or laminæ, and suppose $\frac{1}{n}$th part of the whole light to be lost or absorbed in its passage thro' the 1st lamina; then $\frac{n-1}{n}$ = quantity of light entering the 2d lamina; $\frac{(n-1)^2}{n^2}$ = do. entering the 3d; $\frac{(n-)^3}{n^3}$ = do. entering the 4th, &c.

TABLE *from Bouguer, shewing the intensity of the sun's light at different altitudes, and the thickness of air it has to penetrate at each angle.*

Sun's altitude.	*Thickness of air in toises.*	*Intensity of light the whole being* 10,000.
90°	3911	8123
80	3971	8098
70	4162	8016
60	4516	7866
50	5104	7624
40	6086	7237
30	7784	6613
20	11341	5474
15	14880	4535
10	21745	3149
5	39893	1201
3	58182	454
1	100930	47
0	138823	6

4. According to Leslie, in passing through sea water, light is diminished four times for every five fathoms of vertical descent; and Bouguer asserts, that the whole effect of the sun's light would be lost by passing through 679 feet of sea water, and that the same effect would take place by its passing through 3,110,310 feet of air.

5. Bouguer computes that of 300,000 rays which the moon receives; 172,000, or perhaps 204,100 are absorbed; and that the light of the sun : ditto of the full moon :: 300,000 : 1.

6. Euler makes the light of the sun equal to that of 6560 candles at one foot distance; that of the moon to a candle at 7½ feet; of Venus to a candle at 421 feet; and of Jupiter to a candle at 1620 feet, partly from Bouguer's experiments. Hence the sun would appear like Jupiter, if removed to 131,000 times his present distance.—*(Young's Nat. Phil.)*

Light, refrangibility of.

7. The sun's light consists of rays which differ in refrangibility and colour.

The 7 *primary* colours are red, orange, yellow, green, blue, indigo, and violet, of which the red rays are the least refrangible, and the violet ones the most; while green and blue are the colours which have a mean degree of refrangibility. Sir Isaac Newton found their degrees of refrangibility in passing out of glass into air to be as the numbers 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, and 78, those being the values of the sines of refraction to the common sine of incidence 50. Some substances, however, separate the different coloured rays more widely than others, and the *dispersive* power of media does not appear to depend at all upon their mean refracting power.

To find a measure of the *dispersing* power, take a constant small $\angle\ \theta$ for the $\angle$ of refraction, the $\angle$ of incidence will then be $m\,\theta$ and will differ according to the value of m. The difference between these two or $(m-1)\,\theta$ is the refraction; and if m_r and m_v be values of m for red and violet rays, the difference of refraction will be $(m_v-1)\,\theta-(m_r-1)\,\theta$ or $(m_v-m_r)\,\theta$. Its ratio to the refraction will consequently be $\frac{m_v-m_r}{m}$, taking the mean value of m : this is the usual measure of the dispersing power.

In flint glass its value is about 0.05; in crown glass 0,033.

8. Having given the refracting powers of two mediums, to find the

ratio of the focal lengths of two lenses formed of these substances, which, when united, will produce images nearly free from colour.

Let ϱ and ϱ' be the focal lengths of the lenses, $1 + r$ and $1 + v$ the *ratio of refraction* belonging to the red and violet rays respectively in the 1st lens, and $1 + r'$ and $1 + v'$ = ditto of the other; then

$$\frac{\varrho'}{\varrho} = -\frac{v' - r'}{v - r}.$$

Hence it appears that ϱ' and ϱ must be of different signs, or one lens concave and the other convex; and that they are as the respective dispersive powers of the substances of which the lenses are made.

The common practice of opticians, is to use flint glass and crown glass, the dispersive powers of which are in the ratio of 50 to 33; and ∴ a compound lens, in which the separate focal lengths for the same kind of homogeneous light, are as 50 : 33 will make the red and violet rays, converge accurately to one point.

9. Having given the aperture of any lens, and the foci to which rays of different colours, belonging to the same pencil, converge; to find the least circle of aberration through which these rays pass.

Let D = diameter of the least circle of aberration, α = aperture of the lens, the rest as before; then

$$D = \frac{v - r}{v + r}.\ \alpha.$$

Suppose, for instance, the lens be of crown glass, $v = .56$, $r = .54$; $\therefore \frac{v - r}{v + r} = \frac{1}{55}$; D ∴ is $\frac{1}{55}$ of the aperture.

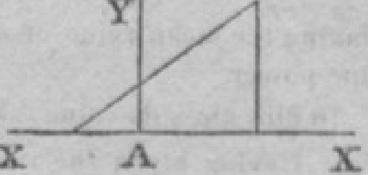
Light, aberration of—see Aberration.

For a concise account of other physical properties of light, such as the phænomena of coloured rings, double refraction, polarization of light, &c. *see Coddington's Optics;* these subjects, as requiring diffuse explanations, cannot here be entered upon.

LINE *right.*

Equations and Problems relating to, the co-ordinates being supposed *rectangular.—(Hamilton.)*

1. The equation to a straight line is $y = a x + b$, where a is the tangent of the angle which the line makes with the axis X A x, and b is the distance from A at which it intersects the axis A y.



2. Required the equation to a straight line passing through a given point, whose co-ordinates are x', y'.

Any point of which the co-ordinates are x, y being assumed in the line, we have $y = a x + b$; also $y' = a x' + b$; $\therefore$ equation required is

$$y - y' = a\ (x - x').$$

For the sake of brevity it is usual to designate the point, whose co-ordinates are x', y', as the point (x', y'); and the straight line, whose equation is $y = a x + b$, as the straight line $y = a x + b$.

2. Required the equation to the line which passes through *two* given points (x', y') and (x'', y'').

$$y - y' = \frac{y'' - y'}{x'' - x'}\ (x - x').$$

3. Required the angle formed by the intersection of two given lines.

Let $y = a x + b$ and $y = a'x + b'$ be the given lines, and θ the given angle; then

$$\tan.\ \theta = \frac{a - a'}{1 + a\,a'}$$

$$\sin.\ \theta = \frac{a - a'}{\sqrt{(1 + a^2)\ (1 + a'^2)}}$$

$$\cos.\ \theta = \pm \frac{1 + a\,a'}{\sqrt{(1 + a^2)\ (1 + a'^2)}}$$

the positive sign being used when the $\angle$ is acute, the negative when it is obtuse.

4. Required the equation to a straight line drawn through a given point (x', y'), and making an angle $\tan.^{-1} m$ with the line $y = a x + b$.

Here $y - y' = \dfrac{a - m}{1 + a m}\ (x - x')$.

Hence (1) when the lines are perpendicular,

$$y - y' = -\frac{1}{a}\ (x - x').$$

(2) When they are parallel,

$$y - y' = a\ (x - x').$$

5. Required the distance (r) between two points (x, y) and (x', y').

$$r = \sqrt{\left\{ (x' - x)^2 + (y' - y)^2 \right\}}$$

When x' and $y' = o$, $r = \sqrt{(x^2 + y^2)}$; which therefore expresses the distance of a point from the origin.

5. If (p) be the perpendicular dropped from a given point $(x', y',)$ on the straight line $y = a\,x + b$; then

$$p = \pm \frac{y' - a\,x' - b}{\sqrt{(1 + a^2)}}.$$

LITUUS.—*See Spiral.*

LOGARITHMS.

1. Properties of Logarithms.

$\text{Log.}\ a \times b = \text{Log.}\ a + \text{Log.}\ b.$

$\text{Log.}\ \frac{a}{b} = \text{Log.}\ a - \text{Log.}\ b.$

$\text{Log.}\ a^{m} = m\ \text{Log.}\ a.$

$\text{Log.}\ a^{\frac{1}{m}} = \frac{1}{m}\ \text{Log.}\ a.$

$-\ \text{Log.}\ a = \text{Log.}\ \frac{1}{a}.$

Ex. 1. $\text{Log.}\ \frac{2^{20} \times 3^{7} \times 2.031}{17 \times 9350} = 20\ \text{log.}\ 2 + 7\ \text{log.}\ 3 + \text{log.}\ 2.013 -$ $(\text{log.}\ 17 + \text{log.}\ 9350).$

Ex. 2. $\text{Log.}\ \sqrt[5]{\frac{317^2 \times \sqrt{3}\ \sqrt[3]{5}}{251}} = \frac{1}{5}\ (2\ \text{log.}\ 317 + \frac{1}{2}\ \text{log.}\ 3 + \frac{1}{3}$ $\text{log.}\ 5 - \text{log.}\ 251).$

2. Given a number, to find its Logarithm.

Let $1 + x$ be the number, m the modulus,

$$\text{then log.}\ 1 + x = m \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \&c. \right)$$

$$\text{and log.}\ 1 - x = m \times \left(- x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \&c. \right)$$

$$\text{Hence log.}\ \frac{1 + x}{1 - x} = 2\,m \times \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \&c. \right)$$

Or since $N = \dfrac{1 + \dfrac{N - 1}{N + 1}}{1 - \dfrac{N - 1}{N + 1}}$, we may for x substitute $\frac{N - 1}{N + 1}$, and we shall have

$$\text{Log. } N = 2m \times \left\{ \frac{N-1}{N+1} + \frac{1}{3}\left(\frac{N-1}{N+1}\right)^3 + \frac{1}{5}\left(\frac{N-1}{N+1}\right)^5 + \&c. \right\}$$

both of which last series converge very fast.

Ex. If $N = 2, \frac{N-1}{N+1} = \frac{1}{3}$; $\therefore$ log. 2 = .3010300.

In hyp. logarithms $m = 1$, in the common system $m = \frac{1}{2.30258509} =$ .43424948. And since different systems of logs. are as their moduli, if any common log. be divided by this modulus, it gives the corresponding hyp. log.; or if any hyp. log. be multiplied by it, it gives the corresponding common logarithm.

3. Given a logarithm, to find its number.

Let $1 + x =$ No., y its log. m the modulus.

$$\text{then } 1 + x = 1 + \frac{y}{m} + \frac{y^2}{2\,m^2} + \frac{y^3}{2.3.\,m^3} + \&c.$$

If $m = 1$, $1 + x = 1 + y + \frac{y^2}{2} + \frac{y^3}{2.\,3} + \&c. =$ No. whose hyp. log. is y.

4. *Modular ratio* is the ratio of which the modulus is the measure, or the number of which the modulus is the logarithm, and $= 1 + 1 + \frac{1}{2} + \frac{1}{2.\,3} + \&c. : 1$; or 2.7182818 : 1; which is therefore the same for every system, being independent of m and y.

Hence in Napier's or hyp. logs., where the modulus is 1, the log. of 2.7182818 is 1; in Brigg's or the common system, log. 2.7182818 is .43424948.

Hence also since in every system the log. of the base is 1; 2.7182818 is the base of Napier's logs.; in Brigg's the base is 10.

In general if $a =$ base of any system, whose modulus is m, $m = \frac{1}{\text{h. l. } a}$.

The following Table of Logarithmic series will be found useful on various occasions.

1. $\text{Log. } a = \frac{1}{M} \times \left\{ (a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c. \right\}$

2. $\text{Log. } a = \frac{1}{M} \times \left\{ \left(\frac{a-1}{a}\right) + \frac{1}{2}\left(\frac{a-1}{a}\right)^2 + \frac{1}{3}\left(\frac{a-1}{a}\right)^3 + \&c. \right\}$

3. $\text{Log. } a = \frac{2}{M} \times \left\{ \left(\frac{a-1}{a+1}\right) + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c. \right\}$

4. $\text{Log. } \frac{a}{b} = \frac{1}{M} \times \left\{ \left(\frac{a \backsim b}{b}\right) - \frac{1}{2}\left(\frac{a \backsim b}{b}\right)^2 + \frac{1}{3}\left(\frac{a \backsim b}{b}\right)^3 - \&c. \right\}$

5. $\text{Log. } \frac{a}{b} = \frac{1}{M} \times \left\{ \left(\frac{a \backsim b}{a}\right) + \frac{1}{2}\left(\frac{a \backsim b}{a}\right)^2 + \frac{1}{3}\left(\frac{a \backsim b}{a}\right)^2 + \&c. \right\}$

6. $\text{Log. } \frac{a}{b} = \frac{2}{M} \times \left\{ \left(\frac{a \backsim b}{a+b}\right) + \frac{1}{3}\left(\frac{a \backsim b}{a+b}\right)^3 + \frac{1}{5}\left(\frac{a \backsim b}{a+b}\right)^5 + \&c. \right\}$

7. $\text{Log. } a = \text{log. } (a-1) + \frac{1}{M} \times \left\{ \frac{1}{a} + \frac{1}{2a^2} + \frac{1}{3a^3} + \frac{1}{4a^4} + \&c. \right\}$

8. $\text{Log. } a = \text{log. } (a-1) + \frac{1}{M} \times \left\{ \frac{1}{a-1} - \frac{1}{2(a-1)^2} + \frac{1}{3(a-1)^3} - \&c. \right\}$

9. $\text{Log. } a = \text{log. } (a-1) + \frac{2}{M} \times \left\{ \frac{1}{a-1} + \frac{1}{3(a-1)^3} + \frac{1}{5(a-1)^5} + \&c. \right\}$

10. $\text{Log. } a = \frac{1}{M} \times \left\{ (a - a^{-1}) - \frac{1}{2}(a^2 - a^{-2}) + \frac{1}{3}(a^3 - a^{-3}) - \&c. \right\}$

11. $\text{Log. } (a+z) = \text{log. } a + \frac{1}{M} \times \left\{ \frac{z}{a} - \frac{1}{2}\frac{z^2}{a^2} + \frac{1}{3}\frac{z^3}{a^3} - \frac{1}{4}\frac{z^4}{a^4} + \&c. \right\}$

12. $\text{Log. } (a-z) = \text{log. } a - \frac{1}{M} \times \left\{ \frac{z}{a} + \frac{1}{2}\frac{z^2}{a^2} + \frac{1}{3}\frac{z^3}{a^3} + \frac{1}{4}\frac{z^4}{a^4} + \&c. \right\}$

13. $\text{Log. } (a \pm z) = \text{log. } a \pm \frac{2}{M} \times \left\{ \left(\frac{z}{a+z}\right) + \frac{1}{3}\left(\frac{z}{a+z}\right)^3 + \frac{1}{5}\left(\frac{z}{a+z}\right)^5 + \&c. \right\}$

14. $\text{Log. } a = \frac{m}{M} \times \left\{ (m\sqrt{a} - 1) - \frac{1}{2}(m\sqrt{a} - 1)^2 + \frac{1}{3}(m\sqrt{a} - 1)^3 - \&c. \right\}$

LOGARITHMIC *Curve, Equation to, &c.—(Higman.)*

$$y = a^x.$$

The curve consists of one branch infinite on each side of the origin, to which the axis of abscissas is an asymptote.

If $x = o$, $y = 1$; and if $x = 1$, $y = a$.

If the abscissas increase in arithmetic progression, the ordinates increase in geometric.

The subtangent is a constant quantity, and = modulus of the system of logarithms, whose base = a.

Area between any two ordinates y and $b = m\,(y - b)$, where m is the modulus or subtangent.

$$\text{Content} = \frac{\pi m}{2}\,(y^2 - b^2).$$

$$\text{Arc} = \sqrt{(m^2 + y^2)} - \sqrt{(m^2 + b^2)}$$

$$+ m \log. \frac{b\left(\sqrt{(m^2 + y^2)} - m\right)}{y\left(\sqrt{(m^2 + b^2)} - m\right)}.$$

$$\text{Surface} = \pi \left(y \sqrt{(m^2 + y^2)} - b \sqrt{(m^2 + b^2)} \right.$$

$$\left. + m^2 \log. \frac{y + \sqrt{(m^2 + y^2)}}{b + \sqrt{(m^2 + b^2)}} \right).$$

LOGARITHMIC *Spiral.—See Spiral.*

LONGITUDE *Geographical.—(Woodhouse, Vince.)*

1*st Method, by a chronometer.*

Suppose a chronometer to be adjusted to mean solar time at Greenwich, then if its motion were equable, and of the proper rate, we should always know, whatever the place, the time at Greenwich. Compute ∴ the apparent, and by means of the equation of time, the mean time, at the place of observation. The difference between this latter time, and that shewn by the chronometer, would be the longitude, east or west of Greenwich.

2*d Method, by an eclipse of the moon or of Jupiter's satellites.*

Having the times calculated when the eclipse begins and ends at Greenwich, observe the times when it begins and ends at any other place; the difference of these times, converted into degrees, gives the difference of longitudes.

3*d. Method, by the moon's distance from the sun or a fixed star.*

The steps by which we find the longitude by this method are these:

From the observed altitudes of the moon and the sun or a fixed star, and their observed distance, compute the moon's true distance from the sun or star.

From the Nautical Almanack find the time at Greenwich when the moon was at that distance.

From the altitude of the sun or star, find the time at the place of observation.

The difference of the times thus found, gives the difference of the longitudes.

Formula for deducing the true from the observed distance.

Conceive S, M to be the true places of the star and moon in two vertical circles S Z, M Z forming at the zenith Z the $\angle$ M Z S; then since both parallax and refraction take place entirely in the direction of vertical circles, some point *s* above S, in the circle Z S, will be the apparent place of the star, and *m* below M (since in the case of the moon the depression by parallax is greater than the elevation by refraction) will be the apparent place of the moon: let

D (S M) be the true, *d* (*s m*) the apparent distance,

A, *a* ($90^0 - Z M, 90^0 - Z S$) the true altitudes,

H, *h* ($90^0 - Z m, 90^0 - Z s$) the apparent altitudes,

$$\text{then if } F = \frac{\cos. A. \cos. a}{\cos. H. \cos. h},$$

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A+a)\left[1 - \frac{\cos. \frac{1}{2}(H+h+d) . \cos. \frac{1}{2}(H+h-d)}{\cos.^2 \frac{1}{2}(A+a)} . F\right]$$

or if we make the fraction, on the right hand side of the equation $= \sin.^2 \theta$, we shall have

$$\sin^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A+a) . \cos.^2 \theta.$$

$$\text{and } \sin. \frac{D}{2} = \cos. \tfrac{1}{2}(A+a) . \cos. \theta.$$

The true distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich corresponding to this true distance. To do this, we must observe that the true distance is computed in the Naut. Almanack for every three hours for the meridian of Greenwich. Hence considering that distance as varying uniformly, the time corresponding to any other distance may be thus computed. Look into the Naut. Almanack, and take out two distances, one next greater, and the other next less, than the true distance deduced from observation, and the difference D of these distances gives the access of the moon to, or recess from, the sun or star in three hours; then take the difference *d* between the moon's distance at the beginning of that interval, and the true distance deduced from observation, and then say, D : *d* :: 3 hours : the time the moon is acceding to or receding from the sun or star by the quantity *d*, which added to the time at the beginning of the interval, gives the apparent time at Greenwich corresponding to the given true distance of the moon from the sun or star.

Having thus found the time at Greenwich, compute the time at the place of observation from the corrected altitude of the sun or star, the sun's or star's north polar distance (furnished by Tables) and the latitude.

The difference between this latter time and the time at Greenwich, is the longitude.

The other methods of finding the longitude are, by an occultation of a fixed star by the moon; by a Solar eclipse; and by the passage of the moon over the meridian.

LOOKING *Glass, method of judging of.—(Coddington.)*

To find the thickness of a looking glass, bring a pin or other slender object into contact with the fore surface of the glass, and observe its image, as shown by reflection; then the thickness of the glass will be equal to $\frac{3}{4}$ths of the apparent distance between the objects and its image.

In a looking glass it is not only necessary that each plane should be perfect, but they must be also parallel to each other. If the images of a candle seen very obliquely, and under different degrees of obliquity, and from all parts of the glass, do not always keep pretty nearly at equal distances from one another, it is a proof that the sides of the glass are neither plane nor parallel.

Another method of trying the goodness of a glass is as follows:—Stick a pin or slender wire in the bar of a window sash, so that the pin may be nearly horizontal, and in the plane of the window. Then hold the looking-glass, and turn it about so as to see the image of the pin very obliquely and from all parts of the glass. In this case two images will be visible; and if these images keep always straight, parallel, and at regular distances one from another, the glass may be considered as being well figured. These phænomena will be more conspicuous if two pins be stuck parallel to one another, and at a small distance asunder.

With respect to the polish of a glass; we may observe, cæteris paribus, that the darker the colour of the glass of the speculum is, the better generally is the polish.

For the theory of plane mirrors—*see Reflection.*

LUNAR *inequalities.—See Moon.*

M

MACLAURIN'S *Theorem.—See Taylor's Theorem.*

MAGNETIC *Needle, variation and dip of.—See Variation.*

MARS.—*See Planets, elements of.*

MARS *phases of.—See Venus.*

MAXIMA *and Minima of quantities.*

1. To determine in what cases any quantity y, depending upon x, may become a maximum or minimum, we must find the differential of the equation which expresses the relation that they bear to each other, and make the quantity $\frac{dy}{dx} = o$. The resulting equation, combined with the original one, will give the values of x and y in which y is a maximum or minimum.

2. To determine when y is a maximum and when a minimum; find the value of $\frac{d^2y}{dx^2}$, and if it be negative, y is a maximum; if it be positive, a minimum.

3. If $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both vanish, but $\frac{d^3y}{dx^3}$ remain, then y will be neither a maximum or minimum at that place, but will pass through a point of contrary flexure parallel to the abscissa. In like manner, if dy, d^2y, d^3y vanish, but d^4y remain, the ordinate y will be a maximum or minimum; and if dy, d^2y, d^3y, and d^4y vanish, but d^5y remain, it will pass through a point of contrary flexure, and so on alternately. This follows immediately from Taylor's theorem.

4. If a quantity be a maximum or minimum, any power or root, multiple, or part, of the original quantity, will be a max. or min.

Ex. 1. To divide a right line a into two parts, such that their rectangle may be either a max. or min.

Here $ax - x^2$ = maximum or minimum. Suppose $y = ax - x^2$, then $\frac{dy}{dx} = a - 2x = o$; $\therefore x = \frac{a}{2}$. To find whether this solution gives a max. or min., take the differential of the equation $\frac{dy}{dx} = a - 2x$; $\therefore$

$\frac{d^2 y}{d x^2} = -2$ a negative quantity; $\therefore$ the value $x = \frac{a}{2}$ gives a max.: also $y = \frac{a^2}{4}$.

Ex. 2. To divide a given line into two parts x and y, so that $\frac{x}{y} + \frac{y}{x}$ = min.

Here $x = y$.

Ex. 3. To inscribe the greatest rectangle in a given triangle and parabola.

Let x = that part of the perpendicular measured from the vertex, which determines the required rectangle, a = perpendicular, then

(1) $x = \frac{a}{2}$. (2) $x = \frac{a}{3}$.

Ex. 4. To inscribe the greatest cylinder in a given cone.

Using the same notation $x = \frac{2\,a}{3}$.

Ex. 5. To inscribe the greatest rectangle in a given ellipse.

Let x = the part of the $\frac{1}{2}$ ax. maj. measured from the centre, which determines the rectangle; then $x = \frac{a}{\sqrt{2}}$.

Ex. 6. To find y a max. in the equation $(x^3 + y^3)^2 = a^4 x^2$.

Here $x = \frac{a}{\sqrt{3}}$, and $y = \sqrt[3]{\frac{2}{3\sqrt{3}}}$.

When a quantity is a max. or min. it frequently shortens the operation to assume its logarithm a max. or min.. Thus to find when $\sqrt{x^2 - a\,x + b} \times \sqrt[3]{m - x^3}$ is a max. or min. assume log. $\sqrt{x^2 - a\,x + b} \times \sqrt[3]{m - x^3}$ a max. or min. or log. $\sqrt{x^2 - a\,x + b}$ + log. $\sqrt[3]{m - x^3}$ max. or mix.; hence $\frac{1}{2} \times \frac{2x\,d\,x - a\,d\,x}{x^2 - a\,x + b} - \frac{1}{3}\,\frac{3\,x^2\,dx}{m - x^3} = o$.

6. If a, b, c, d, &c. be the real roots of the equation $\frac{d\,y}{d\,x} = o$, taken in the order of their magnitude, they will render y a minimum and maximum alternately.

Cor. If there be m roots equal to a, and n roots equal to b, then there will be one minimum value of y for the root a, and one maximum for b, if m and n be odd; and neither max. nor min. values when they are even.

MEASURES.—*See Weights and Measures.*

MECHANICAL *Powers.*

The simple mechanical powers into which more complex machines are resolved, are these :—1. The Lever; 2. The Wheel and Axle; 3. The Pulley; 4. The Inclined Plane; 5. The Wedge; 6. The Screw; for which see the respective heads.

The following property is common to all the mechanical powers, and indeed to all machines whatsoever; it is known by the name of the principle of *Virtual velocities.*

If a power and weight sustain each other on any machine, and the whole be put in motion, the velocity of the power : the velocity of the weight :: the weight : the power. In other words, if the equilibrium of a machine be disturbed by a quantity indefinitely small, and if the velocity of each force be multiplied into its quantity, the sum of these products, reckoning the forces which are in opposite directions positive and negative with respect to one another, will be equal to nothing.

MERCURY.—*See Planets, elements of.*

MERCURY, *Phases of.—See Venus.*

MERCURY, *Transit of.—See Transit.*

MERIDIAN, *Transit of a star or planet over.—See Time.*

MERIDIAN, *to place a Telescope in.—See Telescope.*

MERIDIONAL *parts.—See Projection.*

METEOROLOGY.—*See Atmosphere, Rain.*

MICROSCOPE.—*(Wood, Coddington.)*

1. The visual angle of an object, when seen through a single microscope, is to its visual angle, when seen with the naked eye at the least distance of distinct vision, as that least distance to the focal length of the glass.

Let c = least distance of distinct vision, which in common eyes is about 7 or 8 inches, F = the focal length of the lens; then

$$\text{Magn. power} = \frac{c}{F}.$$

Ex. Let F = .02 inch, and c = 7 inches, then the number of times that the length of the object is magnified $= \frac{7}{.02} = 350$; and the number of times the surface is magnified = 122500.

2. Required the same in the double microscope.

Let Δ = distance of the object from the object glass, F and f = focal lengths of the object and eye glasses, c as before, then

$$\text{Magn. power} = \frac{c\,F}{\Delta\, f}.$$

MILL, *Water.—See Wheel.*

MILL, *Wind.—See Windmill.*

MODULUS.—*See Logarithms.*

MODULUS *of Elasticity.—See Elastic bodies, equilibrium of.*

MOMENTUM, *and the moving or motive force which produces it.*

Let M be the momentum or the moving force which produces it; Q the quantity of matter moved; A the accelerating force; then

$$\text{M varies as } Q \times A, \text{ or A varies as } \frac{M}{Q}.$$

Cor. Since A varies as $\frac{V}{T}$, or varies as $\frac{V^2}{S}$ we have

$$\text{M varies as } \frac{Q \times V}{T} \text{ or varies as } \frac{Q \times V^2}{S}.$$

$\therefore$ when T is given, M varies as $Q \times V$, and when S is given, M varies as $Q \times V^2$ i.e. the moving force, estimated by its effect produced in a *given time*, is as $Q \times V$; but when estimated by its effect produced through a *given space*, is as $Q \times V^2$.

MONEY.—*(Enc. Brit. Supp.)*

Principal gold coins of different countries, with their value in sterling nearly.

			STERLING. s.	d.
AUSTRIA	Souverain		13	11
	Ducat, coins of this name are current in Bavaria, Bern, Brunswick, Cologne, Denmark, Francfort, Hamburgh, Hanover, Holland, Poland, Prussia, Russia, Saxony, Sweden, Treves, Wirtemberg, and Zurich; and vary in sterling value	from to	9 9	2 6
BAVARIA	Carolin		20	4
	Max. d'or, or Maximilian		13	7
BERN	Pistole, also used in Brunswick		18	8
DENMARK	Christian d'or		16	6
ENGLAND	Guinea		21	0
	Sovereign		20	0

		s.	d.
FRANCE	Louis, since 1786	18	10
	Napoleon, or piece of 20 francs	15	10
	New Louis	ditto.	
GENEVA	Pistole, old	16	4
	Pistole, new	14	2
GENOA	Sequin	9	5
HANOVER	George d'or	16	5
	Gold florin	6	11
HOLLAND	Ryder	24	10
MALTA	Louis	19	1
MILAN	Sequin	9	5
	Doppia or pistole	15	8
	40 lire piece of 1808	31	10
NAPLES	Two ducat piece or sequin	6	7
NETHERLANDS	Gold lion, or 14 florin piece	20	9
PARMA	Pistole, or doppia of 1796	17	0
	Maria Theresa of 1818	15	10
PERSIA	Tomaun	10	0
PIEDMONT	Pistole since 1785	22	3
	Sequin	9	4
	Marengo (20 francs)	14	8
PORTUGAL	Dobra of 12,800 rees	71	1
	Moidore	26	11
	Piece of 16 testoons, or 1600 rees	8	9
	New crusado of 480 rees	2	7
PRUSSIA	Frederick of 1800	16	3
ROME	Sequin since 1760	9	3
	Scudo of the Republic	64	11
RUSSIA	Ruble of 1799	3	0
	Gold poltin of 1777	1	5
	Imperial of 1801	32	2
SARDINIA	Carlino	30	8
SAXONY	Augustus of 1784	16	4
SICILY	Ounce of 1751	10	4
SPAIN	Doubloon of 1772	65	10
	Pistole of 1801	15	11
	Coronilla, gold dollar or vintem of 1801	4	0
SWITZERLAND	Pistole of 1800	18	9
TURKEY	Sequin fonducli of 1789	7	7
	Half misseir of 1818	2	2
	Yermeebeshlek	12	5

		s.	d.
Tuscany	Zecchino or sequin	9	6
	Ruspone	28	6
United States	Eagle	43	7
Venice	Zecchino or sequin	9	6
Wirtemberg	Carolin	20	1
East Indies ...	Rupee of Bombay and Madras	29	2
	Pagoda, Star	7	5

Principal silver coins of different countries, with their value in sterling nearly.

			s.	d.
Austria	Rix-dollar, coins of the same name are current in Baden, Bavaria, Brunswick, Denmark, Hamburgh, Hanover, Hesse Cassel, Holland, Lubec, Poland, Prussia, Saxony, Spain, Sweden, Switzerland, United States, Wirtemberg; and vary in sterling value	from	4	2
		to	4	8
	Copftsuck, or 20 creutzer piece		0	3
	17 creutzer piece		0	7
Bern	Patagon or crown		4	9
	Piece of 10 batzen		1	2
Bremen	Piece of 48 grotes		2	4
Brunswick ...	Guilder of 1795		2	4
Denmark	Ryksdaler of 1798		4	6
	Mark, or ⅙ ryksdaler		0	7
	Piece of 24 skillings		0	10
England	Crown (old)		5	0
	Shilling		1	0
	Crown (new)		4	8
	Shilling		0	11
France	Ecu of 6 livres		4	8
	Piece of 24 sous		1	0
	Piece of 30 sous		1	2
	Franc and Franc (Louis)		0	10
Geneva	Patagon		4	1
	Piece of 15 sous of 1794		0	5
Genoa	Scudo		5	4
Hamburgh	Piece of 8 schillings		0	7
Hanover	Florin		2	4
Hesse Cassel	Florin		2	1

		s.	d.
Hesse Cassel	Ecu (1815)	4	1
	Bon gros	0	1
Holland	Ducatoon	5	6
	Florin or guilder	1	8
	12 stiver piece	1	1
Lubeck	Mark	1	3
Lucca	Scudo	4	4
	Barbone	0	4
Malta	Ounce of 30 tari	3	11
Milan	Scudo	3	9
	Lira	0	7
	Piece of 30 soldi	0	11
Modena	Scudo of 1796	3	4
Naples	Ducat	3	5
	Piece of 12 Carlini	4	2
Netherlands	Ducatoon of Maria Theresa	5	2
	Crown	4	7
	5 stiver piece	0	4
	Florin of 1816	1	9
Parma	Ducat of 1796	4	2
	Piece of 3 lire	1	1
Persia	Real	1	3
Piedmont	Scudo (1770)	5	8
	5 franc piece	4	0
Poland	Florin or gulder	1	0
Portugal	New crusado (1809)	2	5
	Seis vintems, or 120 rees (1802)	0	6
	Testoon (1802)	0	6
Prussia	Florin	2	4
	Florin of Silesia	2	0
	Drittel, or 8 good groschen	1	0
Rome	Scudo (1799)	4	3
	Testone (1785)	1	3
	Paolo (1785)	0	5
Russia	Ruble (1805)	3	3
	10 copeck piece (1802)	0	4
Sardinia	Scudo or crown	3	9
Saxony	Piece of 16 groschen	2	0
	⅙ thaler (1809)	0	6
Sicily	Scudo	4	1
	Piece of 40 grains	1	4

		s.	d.
SPAIN	Peceta of 2 reals of new plate (1775)	0	10
	Real of new plate	0	5
SWITZERLAND	Ecu of 40 batzen (1796)	4	9
	Florin of 40 schillings (1793)	1	1
TURKEY	Piastre of Selim (1801)	1	1
	Piastre (1818)	0	9
TUSCANY	Piece of 10 Paoli (1801)	4	5
	Scudo Pisa (1803)	4	6
	Lira (1803)	0	7
UNITED STATES	Dime, or one-tenth of a dollar (1796)	0	6
VENICE	Piece of 2 lire	0	4
WIRTEMBERG	Copftsuck	0	8
EAST INDIA ...	Rupee of Sicca and Calcutta	2	0
	——— Bombay, new, or Surat	1	11
	Fanam, Cananore, Bombay	0	5
	——— Pondicherry	0	3
	Gulden of the Dutch East India Company	1	9

Weight of English gold and silver coin.

	DWTS.	GR.
Guinea	5	$9\frac{39}{89}$.
Sovereign	5	$3\frac{171}{623}$.
Half guinea	2	$16\frac{64}{89}$.
Angel or ½ Sovereign	2	$13\frac{397}{623}$.
Crown	18	$4\frac{4}{11}$.
Half-Crown	9	$2\frac{2}{11}$.
Shilling	3	$15\frac{3}{11}$.
Sixpence	1	$19\frac{7}{11}$.

MONSOON.—*See Wind.*

MOON, *elements and principal phænomena of.—(Vince, Playfair.)*

Secular motion for 100 years, of which 25 are bissextiles $10^s\ 7^{\circ}\ 53'\ 12''$

Secular motion of the apogee		3ˢ 19° 11′ 15″
Secular motion of the node		4 14 11 15
Epoch of the mean longitude for 1750		6 8 22 20
Epoch of the longitude of the apogee for 1750		5 20 54 56
Epoch of the longitude of the node for 1750		9 10 19 59
Mean equation of the orbit		6 18 32
	y. d. h. m. s.	
Tropical revolution	27 7 43 4,7	
Sidereal revolution	27 7 43 11,5	
Synodic revolution	29 12 44 2,8	
Anomalistic revolution	27 13 18 33,9	
Revolution in respect to the node	27 5 5 35,6	
Tropical revolution of the apogee	8 311 8 34 57,6	
Sidereal revolution of do.	8 312 11 11 39.4	
Tropical revolution of the node	18 228 4 52 52,0	
Sidereal revolution of do.	18 223 7 13 17,7	
Diurnal motion of the moon in respect to the Equinox		13° 10′ 35,0″
Diurnal motion of the apogee		0 6 41,1
Diurnal motion of the node		0 3 10,6
Inclination of orbit to Ecliptic		5 9 0
Inclination of axis to orbit		88 17 0
Mean apparent diameter as seen from earth		31 8
——— sun		4,6
Greatest parallax		1 1 22,99
Least ———		53 50,99
Greatest distance from earth in ½ diam. of earth	63,8419	
Least ———	55,9164	
Mean distance ———	59,8791	
Eccentricity, mean distance being 1	,05518	
Mean diameter in miles	2180	
Density (earth's density being 1)	0,6149	
Quantity of matter (earth's being 1)	0,01245	
Gravity at surface (earth's being 1)	0,1677	

The least difference between the times of the moon's rising on two successive nights in the latitude of London is 17*m*; and the greatest difference 1*h*. 17*m*.

MOON, *inequalities affecting the orbit of, usually called the Lunar inequalities.*

Of these the following are the most important:—

1. Evection, or a correction applied to the equation of the centre,

arising from an increase of the eccentricity of the moon's orbit at the quadratures, and a diminution of it at the syzygies. Let M and S be the mean longitudes of the moon and sun, x the mean anomaly of the moon, then the evection is

$$1^{\circ}.\ 21'.\ 5'',5 \times \sin.\ \left(2\,(M - S) - x\right).$$

The evection runs through all its changes in 31*d.* 19*h.* 28*m.* nearly. It is called the second lunar inequality, the equation of the centre being the first.

2. Variation.—i.e. a variation in the moon's velocity, which is nothing in syzygy and quadratures, and greatest at the octants. It =

$$35'.\ 42'' \times \sin.\ 2\,(M - S).$$

Its period is half a lunar month. This is the third lunar inequality.

3. The annual equation or fourth lunar inequality, is an irregularity in the moon's motion, arising from the variation of the sun's distance from the earth. It is

$$11'.\ 11'',9 \times \sin.\ \text{mean anomaly of sun.}$$

Its period is an anomalistic year.

These three inequalities were known before Newton's time: they are applied as corrections to the equation of the centre in determining the moon's longitude.

Other inequalities there are which have a much longer period; one for instance, discovered by Laplace, depending upon the position of three lines, the axis of the moon's orbit, the axis of the earth's orbit, and the line of the moon's nodes which takes up a period of 85 years, and amounts to

14'' × sin. (2 longitude node × longitude perigee of moon — 3 longitude perigee of sun).

Others again there are which do not run through the circle of their changes but in the course of several thousand years, and are usually expounded by their aggregate in 100 years. The moon's nodes, the apogee, the eccentricity, the inclination of the orbit, the moon's mean motion, are all subject to secular inequalities. Of these, the most remarkable is the acceleration of the moon's mean motion (depending on a change in the eccentricity of the earth's orbit), by which her velocity continually encreases, and periodic time decreases from age to age. For many ages to come, it may be nearly expressed by this formula, where n denotes the number of centuries from 1700.

$$10''.\ 181621268\ n^2 + 0''.\ 0185384408\ n^3.$$

The tables of the moon's motion contain at present 28 equations for the moon's longitude, 12 for her latitude, and 13 for the horizontal parallax; their greatest probable error does not exceed twelve seconds.

MOON, *libration of.*

The libration in longitude, or the alternate appearance and disappearance of small segments of the moon on the east and west limbs, arises from the uniform angular velocity round her axis, and the variable angular velocity in her orbit. It nearly = the equation of the orbit, or about $7\frac{1}{2}^{0}$ at its maximum.

The libration in latitude, or the alternate appearance and disappearance of the north and south poles, arises from the moon's axis not being perpendicular to the plane of her orbit.

Diurnal libration, or the appearance of a small segment of the western limb at the rising, and of the eastern limb at the setting of the moon, arises from the spectator being situated on the surface, instead of at the centre of the earth.

MOON, *augmentation of diameter of.*

When the moon is at different altitudes above the horizon, it is at different distances from the spectator, and therefore there is a change of the apparent diameter. Let the altitude of the moon reckoned from the earth's centre or true altitude $= a$; apparent altitude, or altitude reckoned from the earth's surface = A; then increase of $\frac{1}{2}$ diameter =

$$\text{Hor. } \tfrac{1}{2} \text{ diam.} \times \frac{\sin. (\frac{1}{2} a + \frac{1}{2} A) \times \sin. (\frac{1}{2} a - \frac{1}{2} A)}{\cos. a}.$$

Hence the following Table :—

Augmentation of moon's semi-diameter.

Altitude	*Augmentation.*	*Altitude*	*Augmentation.*
0° —	0″	40° —	10″
5 —	1	45 —	11
10 —	3	50 —	12
15 —	4	55 —	13
20 —	6	60 —	14
25 —	7	70 —	15
30 —	8	80 —	15
35 —	9	90 —	16

MOON, *Harvest.—See Harvest Moon.*

MOON'S *mean age to find.*

The moon's mean age at any time may be found for the next 50 years, by the following Tables :—

TABLE I.

Epacts of Years.

Years.	*Epacts.*	*Years.*	*Epacts.*	*Years.*	*Epacts.*
1827	2*d* 8*h* 24′	B 1844	10*d* 22*h* 14′	1861	18*d* 12*h* 4′
B 1828	13 23 35	1845	21 13 26	1862	29 3 16
1829	24 14 47	1846	2 15 53	1863	10 5 43
1830	5 17 14	1847	13 7 4	B 1864	21 20 55
1831	16 8 26	B 1848	24 22 16	1865	2 23 22
B 1832	27 23 37	1849	6 0 43	1866	13 14 33
1833	9 2 4	1850	16 15 55	1867	24 5 45
1834	19 17 16	1851	27 7 6	B 1868	6 8 12
1835	0 19 43	B 1852	9 9 34	1869	16 23 24
B 1836	12 10 55	1853	20 0 45	1870	27 14 35
1837	23 2 6	1854	1 3 12	1871	8 17 3
1838	4 4 34	1855	11 18 24	B 1872	20 8 14
1839	14 19 45	B 1856	23 9 35	1873	1 10 42
B 1840	26 10 56	1857	4 12 2	1874	12 1 53
1841	7 13 24	1858	15 3 14	1875	22 17 4
1842	18 4 35	1859	25 17 26	B 1876	4 20 32
1843	28 19 47	B 1860	7 20 53		

TABLE II.

Epacts of Months.

Months.	*Epacts.*	*Months.*	*Epacts.*
January	0*d* 0*h* 0′	July	3*d* 19*h* 36′
February	1 11 16	August	5 6 52
March	29 11 16	September	6 18 8
April	1 9 48	October	7 5 24
May	1 21 4	November	8 16 40
June	3 8 20	December	9 3 55

In Leap Years, a day is to be subtracted from the sum of the Epacts, in the months of January and February.

Rule.—Add the Epacts of the given year and month, and the proposed time reduced to the meridian of Greenwich. If this sum exceeds a mean lunation or 29*d.* 12*h.* 44*m.*, deduct it therefrom, and the remainder is the moon's mean age.

MOON's *time of passing Meridian.—See Time.*

MOON *Eclipses of.—See Eclipses.*

MOTION *accelerated.—(Whewell.)*

Formulæ for accelerated motion, whether the body is acted upon by constant or variable forces, gravity being represented by $g = 32\frac{1}{6}$ feet, its effect produced in 1 second.

Force constant.

$$v = f\,t \;\ldots\ldots\ldots\ldots\ldots\ldots\; s = \tfrac{1}{2} f\,t^2.$$

From these two equations we obtain, by simple eliminations, the following results:

$$s = \tfrac{1}{2} f\,t^2 = \tfrac{1}{2}\,t\,v = \frac{v^2}{2f};$$

$$v = f\,t = \frac{2\,s}{t} = \sqrt{(2\,f\,s)};$$

$$t = \frac{v}{f} = \frac{2\,s}{v} = \sqrt{\frac{2\,s}{f}};$$

$$f = \frac{v}{t} = \frac{v^2}{2\,s} = \frac{2\,s}{t^2}.$$

If gravity be the constant force, substitute g for f in the above formulæ.

Let a body be projected with a given velocity u, and acted on in the same direction by a constant force f; it is required to determine the relation of the space, time, and velocity.

Let s be the space described in the time t, then

$$s = t\,u + \tfrac{1}{2} f\,t^2;$$

$$v = u + f\,t.$$

If the body is projected in a direction opposite to that in which the force acts;

$$s = t\,u - \tfrac{1}{2} f\,t^2;$$

$$v = u - f\,t.$$

Ex. 1. Space described by gravity in $10'' = 1608\frac{1}{3}$ feet; and velocity acquired $= 321\frac{2}{3}$ feet.

Ex. 2. A body is projected upwards with a velocity of 100 feet; to find how high it will ascend in $2''$.

$$s = t\,u - \tfrac{1}{2} g\,t^2 = 2 \times 100 - \tfrac{1}{2} \times 32.2 \times 4 = 135.6 \text{ feet.}$$

Ex. 3. Spaces described by gravity in the 1st, 2d, 3d, &c. seconds are $\frac{g}{2}, \frac{3\,g}{2}, \frac{5\,g}{2}, \frac{7\,g}{2}$ &c.

After a body has fallen 2·9932 inches (which will occupy 0.1245 seconds) it will fall through the next half-inch in one hundredth of a second.

No. of ~~hundredths~~ of a second, from the beginning	distance reached in inches	successive intervals fallen through	diffces
0. 1245	2·9932		
0. 1265	3·0901	0.0969	
			.0017
0. 1285	3. 1887	0.0986	
			.0013
0. 1305	3. 2886	0.0999	
			.0015
0. 1325	3.3900	0.1014	
			.0018
0. 1345	3.4932	0.1032	

Hence the error is at the most 3.2 per cent if the half inch interval be divided into 10 equal parts and each of these is taken to represent the space fallen through in each successive thousandth of a second (i.e. 1/20 inch space = 1/1000 inch second) and the beginning of the graduation may be at 3 inches.

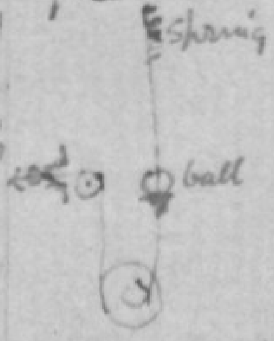
Recorded velocity = 3 times the real velocity feet per second	Height to which the ball is tossed in inches
5	0.52
6	0.75
7	1.01
8	1.33
9	1.68
10	2.07
11	2.51
12	2.98
13	3.49
14	4.08
15	4.66
16	5.30
17	5.98
18	6.71
19	7.47
20	8.28
21	9.13
22	10.02
23	10.95
24	11.93
25	12.94
26	14.00
27	15.09
28	16.23
29	17.41
30	18.03

$S = \frac{v^2}{2f}$ in feet, $f = 32.2$ feet

$= v^2 \times 0.0155$ feet

$= v^2 \times 0.1860$ inches

In the table v is only one third of the recorded velocity, to suit the requirements of the machine.



A body allowed to fall, reaches the distances as below, at the expiration of each successive hundredth of a second.

No of hundredths of a second	distance reached in inches	intervals fallen through	second difference is constant after the first term
0	0	00	
1	0.0193	0.0193	.0193
2	0.0772	0.0579	.0386
3	0.1737	0.0965	.0386
4	0.3088	0.1351	.0386
5	0.4825	0.1737	.0386
6	0.6948	0.2123	.0386
7	0.9457	0.2509 ✓	&c
8	1.2352	0.2895 ✓	
9	1.5633	0.3281 ✓	
10	1.9300	0.3667 ✓	
11	2.3353	0.4053 ✓	
12	2.7792	0.4439 ✓	
13	3.2617	0.4825 ✓	
14	3.7828	0.5211 ✓	
15	4.3425	0.5597 ✓	
16	4.9408	0.5983 ✓	
17	5.5777	0.6369 ✓	
18	6.2532	0.6755 ✓	
19	6.9673	0.7141 ✓	
20	7.7200	0.7527 ✓	
21	8.5113	0.7913 ✓	
22	9.3412	0.8299 ✓	
23	10.2097	0.8685 ✓	
24	11.1168	0.9071 ✓	
25	12.0625	0.9457 ✓	
26	13.0468	0.9843 ✓	
27	14.0697	1.0229 ✓	
28	15.1312	1.0615 ✓	
29	16.2313	1.1001 ✓	
30	17.3700	1.1387 ✓	.0386
31	18.5473	1.1773 ✓	
32	19.7632	1.2159 ✓	
33	21.0177	1.2545 ✓	
34	22.3108	1.2931 ✓	
35	23.6425	1.3317 ✓	.0386
36	25.09		
37	26.44		
38	27.79		
39	29.34		
40	30.8800		

A body allowed to fall, under the action of gravity below, of the effect of each succeeding hundredth of a second.

Hundredths of a second	distance fallen in inches	intervals fallen through	second difference constant after the first
0	0	0	
1	0.0193	0.0193	.0193
2	0.0772	0.0579	[illegible]
3	0.1737	0.0965	.0386
4	0.3088	0.1351	.0386
5	0.4825	0.1737	.0386
6	0.6948	0.2123	.0386
7	0.9457	0.2509	
8	1.2352	0.2895	
9	1.5633	0.3281	
10	1.9300	0.3667	
11	2.3353	0.4053	
12	2.7792	0.4439	
13	3.2617	0.4825	
14	3.7828	0.5211	
15	4.3425	0.5597	
16	4.9408	0.5983	
17	5.5777	0.6369	
18	6.2532	0.6755	
19	6.9673	0.7141	
20	7.7200	0.7527	
21	8.5113	0.7913	
22	9.3412	0.8299	
23	10.2097	0.8685	
24	11.1168	0.9071	
25	12.0625	0.9457	
26	13.0468	0.9843	
27	14.0697	1.0229	
28	15.1312	1.0615	
29	16.2313	1.1001	
30	17.3[illegible]	1.13[illegible]	
31	[illegible]	1.1773	
32	[illegible]	1.21[illegible]	
33	[illegible]	1.25[illegible]	
34	[illegible]	1.29[illegible]	
35	[illegible]	1.3317	
40	30.8800		

Ex. 4. If a body has fallen t'', the space described in the last $n'' = \frac{g}{2}(2nt - n^2)$.

Ex. 5. Space described in the second immediately previous to the last $n'' = \frac{g}{2}(2t - 2n - 1)$.

Force variable.

$$v = \frac{ds}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad f = \frac{dv}{dt}.$$

When the force is a function of the space, we may hence find the velocity and time. For multiplying the above equations crossways, to eliminate dt,

$$v\,dv = f\,ds.$$

If we put for f its value in terms of s, we can integrate the last equation, and thus obtain $\frac{1}{2}v^2 = \text{fl.}\, f\,ds$, whence v is known.

After this t is determined by the equation

$$dt = \frac{ds}{v}. \quad \therefore t = \text{fl.}\,\frac{ds}{v}.$$

Ex. 1. Let a body fall from rest from the distance a towards a centre of force varying directly as the distance; to determine the motion.

Let m be the absolute force tending to the centre, then

$$f = mx, \therefore v\,dv = mx\,ds = -mx\,dx \therefore v^2 = C - mx^2 = m(a^2 - x^2).$$

$$\text{And } v = m^{\frac{1}{2}}(a^2 - x^2)^{\frac{1}{2}}.$$

$$dt = \frac{ds}{v} = -\frac{dx}{m^{\frac{1}{2}}(a^2 - x^2)^{\frac{1}{2}}}$$

$$\therefore t = \frac{1}{m^{\frac{1}{2}}}.\ \text{arc}\left(\cos. = \frac{x}{a}\right)$$

Ex. 2. Required the same when f varies as $\frac{1}{D^2}$.

$$\text{Here } v\,dv = -\frac{m\,dx}{x^2};$$

$$\therefore v = \sqrt{\frac{2m}{a}} \times \sqrt{\frac{a-x}{x}}.$$

$$\text{Also } dt = -\frac{dx}{v} = \sqrt{\frac{a}{2m}} \times \frac{-x^{\frac{1}{2}}\,dx}{\sqrt{(a-x)}} \quad \therefore t = \sqrt{\frac{a}{2m}} \times$$

(arc + sin.) whose ver. sin. is $a - x$.

Ex. 3. Required the same when the force is as $\frac{1}{D^n}$.

$$v\,d\,v = f\,d\,s = -\frac{m\,d\,x}{x^n}.$$

$$v = \sqrt{\frac{2m}{n-1}} \cdot \sqrt{\frac{a^{n-1} - x^{n-1}}{a^{n-1}\,x^{n-1}}}$$

$$d\,t = \sqrt{\frac{(n-1) \cdot a^{n-}}{2m}} \times \frac{-x^{\frac{n-1}{2}}\,d\,x}{\sqrt{\left(a^{n-1} - x^{n-1}\right)}},$$ which

can be integrated only in particular cases.

MOVING *Force.—See Momentum.*

MOUNTAINS, *height of the principal, from the best authorities; together with the rocks of which they are composed.*

		Feet.
Dhawalageri (Napaul) slaty primitive rocks, as gneiss, mica slate, schorl rock; details unknown . .		27,677
36 Peaks of the Himalaya mountains, observed and calculated by Captain Hodgson . . .	Do. from to	25,749 17,017
Jamaturi (Napaul) do.	do. .	25,500
Chimborazo (highest of the Andes)	All these high summits of the Andes are of volcanic matter resting on primitive rocks, such as gneiss, mica slate, &c.	21,470
Cajambe (Quito Andes) . .		19,480
Antisana (highest volcano, Andes)		19,150
Cotopaxi (volcano, Andes) . .		18,875
Mount St Elie		18,090
Popocatepetl (volcano of Puebla Mexico)		17,720
Cotocatche (Andes)		16,450
Tonguragua (volcano, Quito)		16,270
Mouna Roa (Owhyhee), volcanic from top to bottom; all the rocks of the island are igneous		15,871
Mont Blanc (Alps, highest in Europe) granite, syenite, hornblende slate, in vertical layers		15,665

Pic Nethou (Maladetta)	11.060	start from Rencluse 6.832
Pic Maladetta	10.870	
Tusse de Maupas	10.210	... Cabane Lys 3613
Pic de la Pique	7.850	... Hospice 4462
Port Venasque	7.920	

Mont Rosa (Alps), talc slate and serpentine	15,527
Ortler Spiltze (Tyrol) alpine or Jura limestone, with organic remains	15,430
Mount Cervin (Switzerland) primitive slaty rocks	14,780
Mount Ophir (Sumatra)	13,842
Peak of Jungfraa (Switzerland) alpine limestone	13,735
Pambamarca (Andes)	13,500
Bræt-horn (Switzerland) granite and gneiss	12,815
Sochonda (China) primitive, probably granite	12,800
Finisteraaharn (Alps) granite and gneiss	12,210
Lake of Toluca (Mexico)	12,195
Peak of Teneriffe, volcanic from top to bottom	12,176
Town of Micupampa (Peru)	11,670
Mulahacen (Spain)	11,670
Peak of Venlatta (Spain)	11,390
Mont Perdu (Pyrenees) calcareous, with organic remains	11,265
Le Viguemal (do.) summit granite, flanks calcareous	11,010
Mount Ætna (Sicily) volcanic ashes, scoria, lava, &c.	10,955
Italitzkoi (Altaic chain, Asia)	10,735
Pic Blanc (Alps)	10,205
Quito	9,630
Awatsha (volcano, Kamtchatka)	9,600
Mount Libanus (Turkey)	9,585
Real del Monte (mine, N. Spain) poryhyry slate	9,125
Imbabura (Quito) a great volcanic dome resting on primitive rocks	8,960
Mont St Gothard (Alps) granite and gneiss	8,930
Peak of Lomnitz (Carpathian Mountains) primitive rocks, details unknown	8,610
Mount Valina (highest of the Apennines)	8,300
Sneebutten (Norway) gneiss, mica slate, and other primitive slates	8,295
Blue Mountains (Jamaica)	8,180
Volcano, Isle of Bourbon	7,680

Mexico	7,525
Mount Cenis (Alps) transition slate, &c.	6,780
Mount Olympus (Turkey) primitive limestone, with serpentine, syenite, porphyry . . . , ...	6,500
Stony Mountains (N. America)	6,250
Mont d'Or (France)	6,100
Roettruck (Sweden) gneiss and mica slate	6,000
Mount Reculet (Switzerland)	5,590
Puy de Dome (France) ancient volcanic rocks (trachyte) .	5,225
La Souffriere (Guadaloupe) volcanic	5,110
Hecla (Iceland) volcanic from top to bottom, scoria, lava, tuff, porphyry, slate, &c.	5,010
Mount Ida (Turkey)	4,960
Ben Nevis (highest in Britain) feldspathic slate, greenstone slate, &c.	4,370
Jorulla (volcano, Mexico) volcanic scoria	4,265
Ben Lawers (Scotland)	4,015
Mount Vesuvius (Italy) volcanic ashes, scoria, lava . .	3,985
Ben Wyvis (Scotland)	3,720
Snowden (highest in Wales) transition slate, with organic remains, greenstone slate, &c.	3,571
Town of Caracas	3,490
Ben Lomond (Scotland) feldspathic slate, greenstone slate, &c.	3,250
Sca Fell (Cumberland) chloritic slate, greenstone slate, compact felspar, &c.	3,166
Helvellyn do.	3,055
Skiddaw, clay slate, with crystals of chiastolite	3,022
Cadir Idris, compact felspar, greenstone slate, &c. . . .	2,914
Cross Fell, mountain limestone, and millstone grit. . . .	2,901
Cheviot, porphyry	2,658
Plynlimmon	2,463
Whernside (Ingleton Fells) mountain limestone, gritstone, &c.	2,384

Ingleborough, mountain limestone, gritstone, &c. . . .	2,361
Madrid	2,276
Pennigent do.	2,270
Whernside (Kettlewell) do.	2,263
Fountains Fell do.	2,180
Snea Fell (Isle of Man) clay slate	2,004
Pendle Hill, mountain limestone, millstone, grit. . . .	1,803
Rye Loaf Hill do.	1,753
Malvern Hills, Syenitic granite	1,444
Cataract of Tequendama (S. America)	600
St Peter's, summit of the Cross	535
Pyramids (Egypt)	500
Natural Bridge of Iconozo (S. America)	300
Caspian sea below the ocean	306
Gay Lussac—highest altitude ever attained by a balloon, Sept. 16, 1804	22,900
Highest flight of the Condor	21,000
Height attained by Humboldt up the Andes, June 23, 1802	19,400
Highest limit of lichen plants	18,225
Lower limit of perpetual snow on the Equator . . .	15,739
Farm House of Antisana	13,435
Highest limit of trees	11,125
Superior limit of oaks in the torrid zone	10,500
Convent on Mont St Bernard (Switzerland) . . .	8,040
Do. of St Gothard (Alps)	6,810

MOUNTAINS, *visibility of.—See Refraction.*

MOUNTAINS, *attraction of.*

Dr Maskelyne was the first who satisfactorily proved the attraction of mountains by their effect in drawing the plumb line from its vertical direction. The mountain selected was Schehallien in Scotland, the mean height of which above the surrounding valley is 2000 feet, and above the level of the sea 3550. The attraction of this mountain was found = 5″,8 : from which Dr Hutton calculated the mean density of the earth to be near 5 times that of water, or as 99 to 20, and almost double the density of rocks near the earth's surface. Mr Cavendish, upon totally different principles, found the density of the earth to be to that of water as 5.48 : 1. The internal parts of the earth are ∴ much denser than those at the surface; though in what manner the dense parts are disposed of must be uncertain.

MOUNTAIN, *correction for height of.—See Refraction.*

MOUNTAINS, *visibility of.—See Refraction.*

N

NEBULÆ, *and Clusters of Stars.—(Herschel.)*

On Herschel's Catalogue of new Nebulæ, and clusters of stars.

The telescope used was a Newtonian reflector of 20 feet focal length, and $18\frac{7}{10}$ inches aperture. The sweeping power was 157. The field of view 15′. 4″.

The Nebulæ are divided into classes like the double stars. *(See Stars double.)* Thus in the 1st class, the degree of brightness of the Nebulæ has been the leading feature, as most likely to point out those which ordinary instruments may be expected to reach. The 1st class ∴ contains the brightest of them; the 2d those which shine but with a feeble light; and in the 3d are placed all the very faint ones. It should be observed, that what Herschel calls bright, or very bright among those of the first class, are commonly less distinguishable than what Messier, in his *Catalogue des Nebules* (given in Wollaston) calls faint; on account of the superiority in the instruments of the former observer.

Besides this general division, there are added a 4th and 5th class, which contain Nebulæ deserving a more particular description. The 4th class contains Planetary Nebulæ, i.e. stars with burs, with milky chevelure, with short rays, remarkable shapes, &c. The 5th class very large Nebulæ.

The 6th, 7th, and 8th classes contain clusters of stars sorted according to their apparent compression, like the Catalogue of double stars, so that the closest and richest clusters take up the first or 6th class; the brightest, largest, and pretty much compressed ones, the second or 7th class; and those which consist only of scattered and less collected large stars, are put into the last.

Note.—When a superior power and telescope increase the brightness of a nebula, but at the same time only make the tinge of it more uniformly united, and of a milky appearance, it may be concluded to be purely nebulous; but when by using a superior instrument, its appearance is a mixture of nebulosity and extremely fine points, so that we can almost see stars, the nebula is said to be *easily resolvable*, and may be concluded to be a cluster of stars.

Conjecture on the nature of nebulæ, not resolvable.

In the Philosophical Transactions for 1811, Herschel has started a new conjecture respecting the nature of nebulæ. He no longer considers them as clusters of stars, which assume a nebulous appearance by reason of their immense distance, but that they consist of a luminous and extremely rare substance. That this substance, at its first formation, is pretty equally diffused through the nebula; but that in the course of ages, this matter, by the preponderance of some part of it, forms one or more centres, to which all the other matter gravitates; that in consequence of this, the nebula gradually decreases in size, and increases in density, till at last a nucleus is formed; and the nebula becomes planetary surrounded by nebulous matter; which last again is finally absorbed by the central body; and the whole then is, or has all the appearance of, a fixed star. This connexion between nebulous matter and a fixed star, and the conversion of the one into the other, he endeavours to establish, by arranging the nebulæ into classes, according to their supposed age and degree of condensation, beginning with extensive and uniformly diffused nebulosity, and establishing the connexion between this and a fixed star by such nearly allied intermediate steps, as makes it not improbable that every succeeding state of the nebulous matter is the result of the action of gravitation upon it while in a foregoing one, and by such steps the successive condensation of it has been brought up to the planetary condition. From this the transit to the stellar form requires but a very small additional compression of the nebulous matter; and in Herschel's observations of many of these it became doubtful whether they were not stars already.

The steps by which he arrives at this conclusion are nearly as follows:—

1. Extensively diffused nebulosity.
2. Nebulosities joined to nebulæ.
3. Nebulæ of various shapes, but nearly uniform brightness.
4. Nebulæ that are gradually a *little* brighter in the middle.
5. Nebulæ which are gradually brighter in the middle.
6. Nebulæ which are gradually *much* brighter in the middle.

7. Nebulæ that have a cometic appearance.

8. Nebulæ that are suddenly much brighter in the middle.

9. Round nebulæ increasing gradually in brightness up to a nucleus in the middle.

10. Nebulæ that have a nucleus.

11. Round nebulæ that are of an almost uniform light.

12. Nebulæ that draw progressively towards a period of final condensation.

13. Planetary nebulæ.

14. Stellar nebulæ.

15. Stellar nebulæ nearly approaching to the appearance of stars.

Clusters of stars.

We have seen according to Herschel's doctrine, that extensive nebulosities are in process of time broken up into separate and distinct nebulæ; and that these last again, after becoming gradually more and more condensed, form stars. Upon the same principle he accounts for the formation of clusters of stars. He conceives that in rich portions of the heavens, as for instance the milky way, various centres of attraction are formed, to which the neighbouring stars gravitate; that thus the whole is broken up into separate systems or clusters of stars. That these clusters at first are of various irregular figures, and consist of stars coarsely and unequally scattered over the mass; that by the progress of condensation they become more insulated and detached from the neighbouring stars, their figures are more regular and spherical, and the stars more rich and closely connected; till they at length form those minute and beautiful phænomena which are undoubtedly the most interesting objects for our finest telescopes. He arranges them as follows, according to their degree of condensation.

1. Aggregation of stars, or patches of stars, which seem beginning to form clusters.

2. Irregular clusters of various unascertained sizes.

3. Clusters variously extended and compressed.

4. Considerably compressed clusters of stars.

5. Gradual concentration and insulation of clusters of stars.

6. Globular clusters of stars requiring a very fine telescope.

7. More distant globular clusters of stars.

8. Still more distant globular clusters.

NEUTRAL *Point.—See Elastic bodies, equilibrium of.*

NIGHT-GLASS, *or Sweeper.*

These are Telescopes of two, or two and a half feet in length, with large apertures, the object glass either a single lens of 3 or 3½ inches diameter, or an achromatic of 2½; their magnifying power, 6, 8, or 10 times; field of view 5 or 6 degrees: they are occasionally furnished with a system of cross wires, and a diagonal eye piece. Their use is for a rapid survey of any part of the heavens, and for fixing upon such objects as may be proper for examination with finer telescopes. They are also useful, provided the observations are recorded, in detecting minute changes in the heavens upon a subsequent review; or in searching for any object supposed to be moveable, as an asteroid. For this purpose delineations of the telescopic constellations, near the place where it is suspected to be, should be drawn upon paper; and after some days interval, the moving star will be discovered. This can only be done with a night glass of very low magnifying power. Herschel's small sweeper was a Newtonian reflector of 2 feet focal length, aperture 4,2 inches, magnifying power 24, and field of view 2° 12′.—*(Phil. Trans.)*

NONIUS.—*See Vernier.*

NORMAL.—*See Subnormal.*

NORMAL, *equation to.—See Tangent.*

NUTATION *of the Earth's axis.—See Precession.*

O

OBSERVATORY.

TABLE,

Of the Latitudes and Longitudes of the principal Observatories of Europe, from the most recent and accurate determinations.—(Lax.)

	Latitude N.	Longitude.
Amsterdam	52° 22′ 17″	0h 19m 33s E
Armagh	54 21 15	0 26 30 W
Berlin	52 31 45	0 53 29 E
Berne	46 56 55	0 29 45
Bologna	44 30 12	0 45 26
Bremen	53 4 38	0 35 12

	Latitude N.	Longitude.
Brunswick	52° 16′ 29″	0h 42m 8s
Buda	47 29 44	1 16 10
Cadiz	36 32 0	0 25 9 W
Cambridge	52 12 36	0 0 17 E
Cassel	51 19 20	0 38 21
Coimbra	40 12 30	0 33 39 W
Copenhagen	55 41 4	0 31 38 E
Cracow	50 3 38	1 19 49
Dantzic	54 20 48	1 14 32
Dorpat	58 22 47	1 46 48
Dresden	51 2 50	0 54 52
Dublin	53 23 13	0 25 22 W
Edinburgh	55 57 21	0 12 41
Florence	43 46 41	0 45 3 E
Geneva	46 12 0	0 24 38
Genoa	44 25 0	0 35 52
Glasgow	55 51 32	0 17 4 W
Gotha	50 56 8	0 42 56 E
Gottingen	51 31 50	0 39 46
Greenwich	51 28 39	0 0 0
Koenigsburg	54 42 12	1 21 57
Leipsic	51 20 16	0 49 27
Lilienthal	53 8 30	0 35 37
Lisbon	38 42 24	0 36 34 W
London	51 30 49	0 0 23
Madrid	40 24 57	0 14 49
Marseilles	43 17 49	0 21 29 E
Milan	45 28 2	0 36 46
Moscow	55 45 45	2 30 12
Munich	48 8 20	0 46 18
Naples	40 50 15	0 57 3
Nuremberg	49 26 55	0 44 17

	Latitude N.	Longitude.
Oxford	51 45 39	0h 5m 1s W
Padua	45 24 2	0 47 26 E
Palermo	38 6 44	0 53 28
Paris	48 50 14	0 9 21
Pavia	45 10 47	0 36 39
Petersburg	59 56 23	2 1 15
Pisa	43 43 11	0 41 36
Portsmouth	50 48 3	0 4 24 W
Rome	41 53 54	0 49 59 E
Slough	51 30 20	0 2 24 W
Stockholm	59 20 31	1 12 14 E
Strasburgh	48 34 56	0 30 59
Toulouse	43 35 46	0 5 46
Turin	45 4 0	0 30 41
Upsal	59 51 50	1 10 36
Utrecht	52 5 31	0 20 29
Venice	45 25 32	0 49 24
Verona	45 26 7	0 44 5
Vienna	48 12 40	1 5 31
Wilna	54 41 2	1 41 12

OPERA-GLASS.—*Kitchiner.*

An Opera-glass should not magnify more than three, or at the most four times; this also makes a pleasant prospect glass. If it have besides a power magnifying twice, it will be an excellent assistant in giving a general view of the constellations, and will be a good finder for sweeping the sky for a comet. The best Opera-glasses at present made by opticians, have an achromatic object glass of one and a half inch in diameter, magnifying four times; the price in a plain mounting, about two guineas and a half.

OPTICS, *laws of.*

The theory of Optics reposes on three *laws*, which depend for their proof upon observation and induction.

1. The rays of light are straight lines.

2. The angles of incidence and reflexion are in the same plane and equal.

The angles of incidence and refraction are in the same plane, and their sines bear an invariable ratio to one another for the same medium.

For the various subjects connected with this branch of science, see the respective heads.

OSCILLATION, *Centre of.—See Centre.*

P

PALLAS.

This planet was discovered by Dr Olbers, of Bremen, March 28, 1802. For its elements—*see Planets, elements of.*

PARABOLA, *principal properties of.—See Conic Sections.*

PARACENTRIC *velocity.—See Central Forces.*

PARALLAX.—*(Woodhouse, Playfair.)*

1. If P be the horizontal parallax of a heavenly body, p the parallax at a zenith distance z.

$$p = P \times \sin. z.$$

Cor. If R be the radius of the earth; r the tabular radius, d the distance of the body, then

$$d = \frac{r}{P} \times R.$$

To adapt this to computation, r must be expressed in degrees, minutes, &c. then

$$d = \frac{57^{o}.\ 2957795}{P} \times R.$$

2. If two observers under the same meridian, but at a great distance from one another, observe the zenith distances of the same planet, when it passes the meridian on the same day, they can from thence determine the horizontal parallax.

Let L and L′ be the two latitudes, z and z' the observed zenith distances, then

$$P = \frac{z + z' - (L \pm L')}{\sin. z + \sin. z'}$$

This formula was employed by Lacaille, at the Cape of Good Hope, and Wargentin, at Stockholm, for finding the parallax of Mars. It cannot be successfully applied to the Sun, or to Jupiter, Saturn, or the Georgian; for where the parallax does not exceed 10 or 12 seconds, the probable errors of observation will bear so large a proportion to it, as materially to affect the certainty of the result.

The moon, however, whose parallaxes are considerable, is a proper instance for the method, though in that case it will require some modification; as we must take into consideration the spheroidical figure of the earth; thus

Let R be the radius of the equator, r and r' the radii of the earth at the two places of observation, z and z' the zenith distances found as before, but corrected for the ∠'s between the vertical and the radius, then the horizontal parallax at the equator is

$$\frac{z + z' - (L \pm L')}{r \sin. z + r \sin. z'} \times R.$$

3. The parallax of a planet in R. A. being found by observation, to find its horizontal parallax.

Let s be the R. A. in time, taken out of the meridian, then

$$P = \frac{15\, s \times \cos. \text{dec.}}{\cos. \text{lat.} \times \sin. \text{hour angle}}.$$

If the R. A. be taken both before and after the meridian, and h and h' be the two hour angles, and s the sum of the parallaxes in R. A. on the east and west of the meridian,

$$P = \frac{15\, s \times \cos. \text{dec.}}{\cos. \text{lat.} \times (\sin. h + \sin. h')} \text{ or } =$$

$$\frac{15\, s \times \cos. \text{dec.}}{2 \cos. \text{lat.} \times \sin. \frac{h + h'}{2} \times \cos. \frac{h - h'}{2}}$$

4. The greatest horizontal parallax of the sun and planets.

Sun .... 8″,75	Venus ...... 29″,16	Jupiter ...... 2″,08
Mercury 14″,58	Mars .... 17″,50	Saturn ...... 1″,027
		Georgian ... 0,″415.

For Sun's parallax in altitude—*see Sun.*

5. Parallax of the fixed stars.

If the *annual* parallax does not exceed 1″, the distances of the fixed stars cannot be less than 206265 times the radius of the earth's orbit. It is probable, however, that the parallax of a star of the second magnitude is not more than $\frac{1}{5}$ of a second; and of a star of the sixth magnitude, not more than $\frac{1}{20}$ or $\frac{1}{30}$ of that quantity.

PENDULUMS, *oscillation of, &c.—(Wood, Whewell, &c.)*

1. Let T = time of vibration of a simple pendulum in a cycloidal arc, L = length, F = accelerating force, g = force of gravity = 32⅙ feet, $\pi = 3.14159$, &c., n = number of vibrations in a given time T′, then

$$T = \sqrt{\frac{\pi^2 L}{F}}, \text{ or in case of gravity } T = \sqrt{\frac{\pi^2 L}{g}}.$$

$$\text{and } n = \sqrt{\frac{F T'^2}{\pi^2 L}}, \text{ or in case of gravity } n = \sqrt{\frac{g T'^2}{\pi^2 L}}.$$

Cor. Hence if x = space fallen through by gravity in 1″ in any latitude, and L = length of a seconds pendulum, then if x be given, $L = \frac{2x}{\pi^2}$; and if L be given, $x = \frac{\pi^2 L}{2}$.

By help of this last formula x is found more exactly than can be done by direct experiment. In the latitude of London L = 39.126 inches, hence $x = 16.09$ feet.

2. To find the vibration of a pendulum in a circular arc, let a = ver. sin. of ½ arc of vibration, r radius of the circle; then

$$T = \sqrt{\frac{\pi^2 r}{g}} \times \left(1 + \frac{a}{8r} + \frac{9a^2}{256r^2} + \&c.\right) =, \text{ when } a \text{ or}$$

the arc is very small, $\sqrt{\frac{\pi^2 r}{g}}$ = time of vibration in a cycloid. Hence the formulæ above given are applicable to bodies vibrating in very small circular arcs.

3. If a pendulum vibrating in a circular arc keeps true time whilst oscillating through δ degrees on each side of the vertical; then when it oscillates through D degrees, the seconds lost in 24 hours, if D is greater than δ,

$$= \frac{5}{3}\left(D^2 - \delta^2\right)$$

or if D is less than δ, time gained

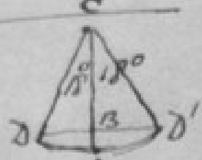
$$= \frac{5}{3}\left(\delta^2 - D^2\right)$$

$g = 386$ inches

Length of a $\frac{1}{2}$ second pendulum
9.777 inches.

" a $\frac{2}{3}$ sec = 17.40 — i.e. 3 beats (double) in 2 seconds, or 90 in 60 sec

" a $\frac{1}{4}$ sec = 2.44



A pendulum swinging from D' to D through 18° on each side of the vertical CA traverses the undermentioned angles when passing from A to D in equal intervals of time

2°	48'	20"
5.	32.	30
8.	8	40
10	33	0
12	42.	0
14.	32.	40
16.	1	30
17.	6.	40
17.	46.	30
18.	0	0

If the above be a ½ pendulum, swinging through the single oscillation D'AD in .5 sec and if BD BD' be each equal to 260 mm: which is the case when CB = 800 mm then the readings on the chord DD' for each hundredth of a second are as below. The graduations run symmetrically outwards from B both to D & to D'

$\frac{1}{50}$ths of a second		millims
25	25	0
26	24	16
27	23	32
28	22	47
29	21	62
30	20	77
31	19	92
32	18	107
33	17	121
34	16	135
35	15	149
36	14	162
37	13	174
38	12	186
39	11	197
40	10	207
41	9	216
42	8	225
43	7	234
44	6	241
45	5	247
46	4	251
47	3	254
48	2	257
49	1	259
50	0	260

4. To correct the going of a clock.

Let L = present length of the pendulum, t'' = No. of seconds gained or lost in the time T'', x = quantity by which the pendulum must be altered; then

$$x = \pm \frac{2 L t''}{T''} \text{ nearly.}$$

5. Let x = height of a mountain upon which a seconds pendulum loses n'' per hour; then

$$x = n + \frac{n}{9} \text{ miles nearly.}$$

6. If the force of gravity be slightly altered, to find the number of seconds gained or lost in a day by a seconds pendulnm.

Let g = force of gravity when pendulum vibrates seconds,

N = No. of seconds in a day,

$g(1 + h)$ = force of gravity when slightly increased,

t = seconds gained in consequence, then

$$t = \frac{N h}{2}.$$

7. Given the number of vibrations (v) of a pendulum in air, to find the number V in a vacuum.—*(Galbraith.)*

Let m be the spec. grav. of the pendulum, that of air being 1; then

$$V = v\left(1 + \frac{1}{2m} + \frac{1}{8m^2} + \&c.\right)$$

8. If n' be the number of oscillations performed in 24 hours by the experimental pendulum, n the true number, e the expansion for a change of 1° Fahrenheit, t the standard temperature, and t' the observed; then

$$n = n' + \tfrac{1}{2} n' e (t' - t)$$

9. To reduce the length of the pendulum from any height to the level of the sea, the true length being denoted by l, the observed by l', the height above the sea by a, and the radius of the earth by r; then

$$l = l' + \frac{2 a l'}{r}.$$

Some allow one-third for the effect of the dense strata immediately under the pendulum, in which case

$$l = l' + \frac{4 a l'}{3 r}.$$

In a similar manner $v = v' + \dfrac{2 v' a}{3r}$.

10. To find the length of the seconds pendulum at the level of the sea, in any latitude λ.

$$\text{Length} = 39.0117150 + 0.2102732 \sin.^2 \lambda.$$

TABLE,

Of the length of the seconds Pendulum in vacuo, at the level of the sea, by Sir G. Shuckburgh's standard (see Weights and Measures), observed at the following places, by Capt. Kater's method, and with his apparatus.

		Latitude.			*Length of Pendulum.*
Captain Sabine	Melville Island	74	47	13,4	39,207
Do.	Hare Isl. Baffin's Bay	70	26	17	39,1984
Captain Kater	Unst	60	45	28,01	39,17146
Captain Sabine	Brassa, Shetland	60	10	0	39,16929
Captain Kater	Portsay	57	40	58,65	39,16159
Do.	Leith Fort	55	58	40,8	39,15554
Do.	Clifton	53	27	43,12	39,14600
Do.	Arbury Hill	52	12	55,32	39,14250
Do.	London	51	31	8,40	39,13929
Do.	Shanklin Farm	50	37	23,94	39,13614
Captain Basil Hall	San Blas	21	30	24	39,03776
Mr Goldingham	Madras	13	4	9	39,026302
Captain Basil Hall	Galapagos	0	32	19	39,01717
Do.	Rio de Janeiro	22	55	22 S	39,04381
Sir T. Brisbane	Paramatta	33	48	43 S	39,07696
Captain Sabine	St Thomas	0	24	41 N	39,02074
Do.	Maranham	2	31	43 S	39,01214
Do.	Ascension	7	55	48 S	39,02410
Do.	Sierra Leone	8	29	28 N	39,01997
Do.	Trinidad	10	38	56 N	39,01884
Do.	Bahia	12	59	21 S	39,02425
Do.	Jamaica	17	56	7 N	39,03510
Do.	New York	40	42	43 N	39,10168
Do.	Drontheim	63	25	54 N	39,17456
Do.	Hammerfest	70	40	5 N	39,19519
Do.	Greenland	74	32	19 N	39,20335
Do.	Spitzbergen	79	49	58 N	39,21469

For the ellipticity of the earth as deduced from these experiments—*see Earth, figure of.*

The following Table shews the seconds gained in one day by a pendulum vibrating seconds at the Equator, in different latitudes, when it remains of the same length.—(Vince.)

Lat.	Seconds gained.	Lat.	Seconds gained.
5^0	1″,7	50^0	134″,0
10	6,9	55	153,2
15	15,3	60	171,2
20	26,7	65	187,5
25	40,8	70	201,6
30	57,1	75	213,0
35	75,1	80	221,4
40	94,3	85	226,5
45	114,1	90	228,3

PERCUSSION, *Centre of.—See Centre.*

PERMUTATIONS *and Combinations.*

Permutations.

1. The number of permutations that can be formed out of n quantities,

Taken 2 and 2 together .... $= n. (n-1.)$

Taken 3 and 3 together .... $= n. (n-1.) (n-2.)$

&c. &c.

Taken r and r together .... $= n. (n-1.) (n-2) \ldots (n-r+1.)$

Thus the number of changes that can be made with 3 bells out of 8 = 8. 7. 6. = 336.

If $n = r$ i.e. if the permutations respect all the quantities at once, then (since $n - r = o$) the number will be $n. (n-1.) (n-2) \ldots\ldots 3. 2. 1.$ Thus the number of permutations which can be formed ont of the letters composing the word *virtue*, are 6. 5. 4. 3. 2. 1 = 720.

2. The number of permutations that can be made out of n things where there are p of one kind, r of another, q of another, &c.

$$= \frac{n. (n-1.) (n-2.) (n-3) \ldots\ldots\ldots 2. 1}{1. 2. 3 \ldots p \times 1. 2. 3 \ldots\ldots r \times 1. 2. 3 \ldots q}.$$

Thus the number of permutations that can be made of the letters in the word *Bacchanalia* (since a occurs four times, c twice)

$$= \frac{11. 10. 9. 8. 7. 6. 4. 3. 2. 1}{1. 2. 3. 4 \times 1. 2} = 831600.$$

Combinations.

1. The number of combinations that can be formed out of n things,

Taken 2 and 2 together $= n. \frac{n-1}{2}$

—— 3 and 3 —— $= n. \frac{n-1}{2}. \frac{n-2}{3}$

&c. &c.

—— r and r —— $= n. \frac{n-1}{2}. \frac{n-2}{3} \ldots\ldots \frac{n-r+1}{r}$

Thus how many combinations can be made of two letters out of the 26 letters of the alphabet,

$$\text{No.} = 26. \frac{25}{2} = 325.$$

2. The total number of combinations of n things taken 1 and 1; 2 and 2; 3 and 3, &c. together,

$$= 2^n - 1.$$

Thus all the possible combinations that can be made of a common suit of 13 cards taking them by one's, two's, three's, &c. at a time $= 2^{13} - 1 = 8191$.

3. All the possible permutations and combinations that any number of things can be made to undergo when taken by two's, three's, &c., up to the whole number of things given, is expressed by the sum of the geometrical series $n + n^2 + n^3 + n^4 \ldots\ldots\ldots\ldots + n^n$; and ∴

$$= \frac{n^n - 1}{n - 1} \times n.$$

Thus the whole number of permutations and combinations that can be made of the 4 letters a, b, c, d, when they are taken by two's, three's, and four's

$$= \frac{4^4 - 1}{4 - 1} \times 4 = 340.$$

4. Supposing there are m sets of things, one set containing n things, another p, another q, &c., then the total number of combinations that can be formed by taking one from each set

$$= n \times p \times q \ldots\ldots\ldots \text{ to } m \text{ factors.}$$

Thus suppose there are 4 companies, in one of which there are 6 men, in another 8, in each of the other two 9, then the number of changes that can be made by taking one out of each company $= 6\ 8.\ 9.\ 9 = 3888.$

PIPES, *leaden and iron, weight of.—(Gregory.)*

Let l be the length in feet, d the interior diameter, and t the thickness both in inches and parts of an inch, W the weight in hundred weights; then,

In a leaden pipe, $W = ,1382\ l\,t\ (d + t.)$

In a cast iron pipe, $W = ,0876\ l\,t\ (d + t.)$

PIPES *for conveying Water.—See Fluids, pressure of.*

PLANE *inclined.—See Inclined Plane.*

PLANETS, *Elements, &c. of.* *see also Satellites – Moon*

What are usually called the elements of a planet's orbit are in number seven.

1. The longitude of the ascending node of the orbit.
2. The inclination of the orbit to the plane of the Ecliptic.
3. The mean motion of the planet round the sun.
4. The mean distance of the planet from the sun.
5. The eccentricity of the orbit.
6. The longitude of the aphelion.
7. The epoch at which the planet is in the aphelion.

Elements and general view of the Planetary System.—(Laplace, Maskelyne, Vince, Playfair.)

Names of the Planets.	*Sidereal Periods of the Planets.*	*Mean distances or ½ axes of the orbits.*	*Ratio of the eccentricities to the ½ axes at the beginning of 1801.*	*Mean longitudes reckoned from the mean Equinox at the epoch of the mean noon of Jan. 1, 1801, Greenwich.*
	d.			°. ′ ″
Mercury	87.969258	0.387098	0.205514	166. 0. 48.2
Venus ...	224.700824	0.723332	0,006853	11. 33. 16,1
Earth	365.256384	1,000000	0,016853	100. 39. 10
Mars.....	686,979619	1.523694	0.093134	64. 22. 57.5
Vesta	1335.205 ...	2.373000	0.093220	267. 31. 49
Juno	1590.998 ...	2·667163	0.254944	290. 37. 16
Ceres	1681.539 ...	2,767406	0.078349	264. 51. 34
Pallas	1681.709 ...	2,767592	0,245384	252. 43. 32
Jupiter ..	4332,596308	5.202791	0,048178	112. 15. 7
Saturn ...	10758,969840	9.538770	0,056168	135. 21. 32
Georgian	30688.712687	19,183305	0,016670	177. 47. 38

Names of the Planets.	Mean longitudes of the Perihelion for the same epoch as before.	Inclinations of orbits to the Ecliptic, for Jan. 1, 1801.	Longitudes of the ascending nodes on the Ecliptic, Jan. 1, 1801.
	°. ′ ″	° ′ ″	° ′ ″
Mercury	74. 21. 46	7. 0. 1	45. 57. 31
Venus ...	128. 37. 0,8	3. 23. 32	74. 52. 38.6
Earth	99. 30. 5	0. 0. 0	0. .0 0
Mars	332. 24. 24	1. 51. 3.6	48. 14. 38
Vesta	249. 43. 0	7. 8. 46	171. 6. 37
Juno	53. 18. 41	13. 3. 28	103. 0. 6
Ceres	146. 39. 39	10. 37. 34	80. 55. 2
Pallas ...,	121. 14. 1	34. 37. 7.6	172. 32. 35
Jupiter ..	11. 8. 35	1. 18. 51	98. 25. 34
Saturn ...	89. 8. 58	2. 29. 34.8	111. 55. 46
Georgian	167. 21. 42	0. 46. 26	72. 51. 14

Names of the Planets.	Mean diameter in English miles.	Mean dist. from the sun in mil. of miles.	Mean appar. diam. as seen from the earth.	Mean diameter as seen from the sun.	Apparent diam. of sun as seen from each.	Diurnal rotation on their own axes.	Inclination of axes to orbits.
The Sun	883246	......	32′ 1″.5	......	......	25d 14h 8m 0s	82° 44′0″
Mercury	3224	37	10	16″	80′	1 0 5 28	....
Venus ...	7687	68	58	30	45,7	0 23 20 54	....
Earth ...	7912	95	......	17.2	32	1 0 0 0	66 32
Mars	4189	144	27	10	21,33	0 24 39 22	59 22
Ceres	163	263	1	......	......	....	....
Pallas	80	265	0.5	......	......	....	....
Juno	1425	252	3	......	......	....	....
Vesta	238	225	0.5	......	......	....	....
Jupiter...	89170	490	39	37	6,15	0 9 55 37	90 near.
Saturn ...	79042	900	18	16	3,37	0 10 16 2	60 prob.
Georgian	35112	1800	3.5	4	1,64	....	....

Names of the Planets.	*Place of Aphelia in Jan. 1800.*	*Secular motions of the aphelia: progressive.*	*Secular motions of nodes.*	*Greatest Equation of the Centre.*	*Elongation when stationary.*	*Arc of retrogradation.*	*Time of retrogradation.*
Sun	....	....	....	....	......	......	...
Mercury	8s 14° 20′ 50″	1° 33′ 45″	1° 12′ 10″	23° 40′ 0″	18° 00′	13° 30′	23 *d.*
Venus ...	10 7 59 1	1 21 0	0 51 40	0 47 20	28 48	16 12	42
Earth	9 8 40 12	0 19 35	....	1 55 30,9	......	......	...
Mars	5 2 24 4	1 51 40	0 46 40	10 40 40	136 48	16 12	73
Ceres	4 25 57 15	....	....	9 20 8	......	......	...
Pallas	10 1 7 0	....	....	28 25 0	......	......	...
Juno	7 29 49 33	....	....	....	......	......	...
Vesta?....	2 9 42 53	....	....	....	......	......	...
Jupiter ..	6 11 8 20	1 34 33	0 59 30	5 30 38	115 12	9 54	121
Saturn ...	8 29 4 11	1 50 7	0 55 30	6 26 42	108 54	6 18	139
Georgian	11 16 30 31	1 29 2	1 44 35	5 27 16	103 30	3 36	151

Names of the Planets.	*Synodic revolution.*	*Densities.*	*Quantities of matter.*	*Gravity on surface.*	*Intensities of light and heat.*
Sun	......	0,25226	333928	27,7	......
Mercury	118 *d.*	2,5833	0,16536	1,0333	6,25
Venus ...	584	1,024	0,88993	0,9771	2,04
Earth	......	1	1	1	1
Mars	780	0,6563	0,08752	0,3355	0,44375
Jupiter ..	399	0,20093	312,101	2,3287	0,036875
Saturn ...	378	0,10349	97,762	1,0154	0,01106
Georgian	370	0,21805	16,837	0,9285	0,00276

For the telescopic appearance of the Planets—*see Telescope.*

PLANETS, *disturbances occasioned by their mutual action upon each other.—(Playfair.)*

The orbit of every planet, by the action of the other planets, is changed in all its elements but two; the mean motion, and the mean distance from the sun. Thus in Mercury and Venus the line of the nodes, the inclination to the Ecliptic, the line of the apsides, the eccentricity, and consequently the greatest equation of the centre all vary. In Mars the eccentricity, the lines of the apsides and nodes, vary by the action of Venus, the Earth, and Jupiter; as also his *place* in his orbit, which is not the case with Mercury and Venus, in consequence of their nearness to the Sun. In Jupiter and Saturn, the place in their orbit, the motion of the apsides, and the change of eccentricity, are chiefly produced by their action on each other; but in the disturbance of the inclination the other planets have a sensible effect. Uranus, from his great distance, suffers no disturbance in his motion but from Jupiter and Saturn.

Two interesting results are obtained from the investigation of the planetary disturbances. 1. That both in the system of primary and secondary planets, two elements of every orbit remain secure against all disturbances, the mean distance, and the mean motion, *i. e.* the transverse axis of the orbit, and the periodic time. 2. That all the inequalities in the planetary motions are periodical, and after a certain time run through the same series of changes. This accurate compensation of the planetary inequalities arises from three conditions; 1st. that the eccentricities of the orbits are small; 2d. that the planets all move in the same direction, *i. e.* from west to east; 3d. that the planes of their orbits are but little inclined to one another.

PLANET, *time of its passing over the meridian.—See Time.*

POLAR *Seas.—See Seas, Polar.*

POLYGONS *regular, to find the area of.*

Let s represent the length of one of the equal sides, n the number of them; then

$$\text{Area} = s^2 \times \frac{n}{4} \tan.\left(\frac{90n - 180^\circ}{n}\right).$$

Hence the following Table :—

Trigon	$= s^2 \times 0.4330127$
Tetragon	$= s^2 \times 1.0000000$
Pentagon	$= s^2 \times 1.7204774$
Hexagon	$= s^2 \times 2.5980762$
Heptagon	$= s^2 \times 3.6339124$
Octagon	$= s^2 \times 4.8284271$
Nonagon	$= s^2 \times 6.1818242$
Decagon	$= s^2 \times 7.6942088$
Undecagon	$= s^2 \times 9.3656399$
Dodecagon	$= s^2 \times 11.1961524$

 24 H/y S²×

POPULATION, *increase of.—(Bridge.)*

Of the method of finding the increase of population in any country, under given circumstances of births and mortality.

Let P represent the population of a country at any given period; $\frac{1}{m}$ the fractional part of the population which die in a year (or ratio of mortality;) $\frac{1}{b}$ the proportion of births in a year; then, if A represent the state of the population at the end of n years,

$$A = P\left(1 + \frac{m-b}{mb}\right)^n.$$

$$\text{Or Log. } A = \text{Log. } P + n \times \log.\left(1 + \frac{m-b}{mb}\right).$$

Of the quantities A, P, m, b, n, any four being given, the fifth may be found.

Ex. 1. Suppose the population of Great Britain, in the year 1800, to have been ten millions; that $\frac{1}{40}$th part die annually; and the number of births are $\frac{1}{30}$th, and that no emigration takes place during the present century; What will be the state of its population in the year 1900?

Here A = 22930000.

Ex. 2. Suppose the population of France in the year 1792 to have been 27000000; the *ratio of mortality*, during the 18th century, to have been $\frac{1}{30}$th, and the number of births $\frac{1}{26}$th; What was the state of its population in the year 1700?

Here P = 16864396.

Ex. 3. Suppose the population of North America to have been five millions in the year 1800; in how many years will it amount to 16 millions, taking the ratio of mortality at $\frac{1}{45}$th, and the annual proportion of births at $\frac{1}{24}$th?

Here n = 60.3 years.

Ex. 4. The population of a province, in the year 1760, was estimated at 500000 persons; in the year 1800 it amounted to 720000 persons; from the bills of mortality it appeared, that, upon an average, $\frac{1}{50}$th part of the population had died annually; no register was kept of the births; What was the annual proportion of *them* during that period?

Here $b = 34.4$.

The annual proportion of births was about $\frac{1}{34}$th.

Supposed Population of the World.—(Enc. Brit. Suppl.)

Europe	185,000,000
Asia (with Australia and Polynesia)	270,000,000
Africa	55,000,000
America	40,000,000
	550,000,000

POWERS *of numbers.—See Involution.*

POWERS *Mechanical.—See Mechanical Powers.*

PRECESSION *of the Equinoxes.—(Woodhouse, Vince, Playfair.)*

I. The mean annual precession $= 50'',34$, which gives nearly 1^0 for the precession in $71\frac{1}{2}$ years, or about 25745 years for the entire revolution of the pole of the Equator round that of the Ecliptic. The part of the precession arising from the action of the sun $= 15'',3$, that from the moon $= 35''$. If the effect of the sun be reduced to $12'',5$; that of the moon will be triple of it, which is agreeable to the latest results deduced from the theory of the tides.

The precession affects the situation of stars in Declination or North Polar distance, and in right ascension; hence the following Formulæ.

Annual Precession in Declination.

This $= 50'',34 \times$ sin. obliquity $\times$ cos. star's R. A.

Cor. When the right ascension (R. A.) is between 90^0 and 270^0, the declination is diminished by the effect of precession. And when the R. A. is between 0^0 and 90^0, or between 270^0 and 360^0, the declination is increased.

Annual Precession in R. A.

This $= 50'',34 \times$ (cos. I $+$ sin. I $\times$ sin. star's R. A. $\times$ tan. star's declination) where I $=$ obliquity of ecliptic. In this expression the first part, $50'',34$ cos. I is common to all stars.

Cor. The precession in R. A. is nothing when the angle of position is a right $\angle$; it is also positive when that $\angle$ is acute, and negative when obtuse.

II. *Precession Solar, inequality of.*

The mean annual precession has been stated at $50'',34$; but it cannot have been equably produced. For the sun is sometimes in the Equator, when its force causing precession is nothing; at other times more than 23° distant, when its force is greatest. Hence the sun's action in producing precession must continually vary from the Equinox in March to the solstice in June. The correction due to this solar inequality is called the semi-annual Solar Equation. In consequence of this solar inequality, the pole of the earth describes, half yearly according to the order of the signs, round the place of the mean pole a circle whose radius = 0″,4345.

The solar inequality affects the precession of the stars in longitude, declination, and right ascension, also the obliquity of the ecliptic; hence the following formulæ.

Equation of precession in longitude.

This = 1″, 1 × sin. 2 sun's longitude.

Substitute this expression for 50″,34 in the above formulæ for precession, and we shall have the equations of precession in declination and R. A.

Correction of the obliquity.

This = 0″,4345 × cos. 2 sun's longitude.

The variation in the obliquity of the ecliptic arising from the sun, is called the *correction* of the obliquity; that from the moon is called the *equation* of the obliquity.

III. *Precession, lunar inequality of.*

The lunar inequality of precession is called *Nutation*, to distinguish it from the solar inequality. In consequence of the lunar action, the true pole of the earth describes about the place of the mean pole, in 18 years 7 months, contrary to the order of the signs, an ellipse of which the major axis = 19″,2, and minor axis = 15″.

The nutation affects the precession of the stars in longitude, declination, and R. A. and also the obliquity of the ecliptic; hence the following formulæ.

Equation of the Equinoxes in R. A.

The variation in the precession, or in the equinoctial points, usually called the *Equation of the Equinoxes in R. A.* is

17″,2 sin. longitude moon's node.

This affects the longitude of all the stars equally.

Nutation in declination.

Let A be the R. A. of a star, D its declination, N the longitude of the moon's node; then nutation in declination =

1″,1 sin. (A + N) + 8″,5 sin. (A — N).

Nutation in Right Ascension.

This = 8″,5 tan. dec. cos. (N − A) + 1″,1 tan. dec. cos. (N + A) and if to this be added the equation of the equinoxes, the whole effect of nutation will =

$$\pm 8,''5 \text{ tan. dec. cos. } (N - A) + 1'',1 \text{ tan. dec. cos. } (N + A) + 17'', 2 \text{ sin. } N.$$

Equation of the obliquity.

This = 9″,63 cos. N.

PRESSURE *of Earth against walls.—See Earth, pressure of.*

PRESSURE *of Fluids.—See Fluids.*

PRESSURE, *centre of.—See Centre.*

PRISM.—*See Refraction.*

PROGRESSION, *Arithmetical, Geometrical, and Harmonical.*

I. *Arithmetical Progression.*

All the cases of Arithmetical progression may be solved by the following formulæ :—

1. Let a = first term, l = last, b = common difference, n = number of terms, s = sum of the series ; then

Of the quantities a, l, b, n, any three being given, the other may be found by the equation

$$l = a + \overline{n - 1}.\ b.$$

2. Of the quantities a, b, n, s, any three being given, the other may be found by the equation

$$s = (2\,a + \overline{n - 1}.\ b).\ \frac{n}{2}.$$

3. Of the quantities a, l, n, s, any three being given, the other may be found by the equation

$$s = (a + l).\ \frac{n}{2}.$$

II. *Geometrical Progression.*

1. All the cases of Geometrical Progression may be solved by the following formulæ :—

Let a = first term, l = last, r = common ratio, n = number of terms, s = sum of the series ; then

Of the quantities a, l, r, n, any three being given, the other may be found by the equation

$$l = a r^{n - 1}$$

2. Of the quantities a, r, n, s, any three being given, the other may be found by the equation

$$s = \frac{ar^n - a}{r - 1}.$$

3. Of the quantities a, l, r, s, any three being given, the other may be found by the equation

$$s = \frac{lr - a}{r - 1}.$$

4. When n, or the number of terms is infinite, then of the quantities a, r, s, any two being given, the other may be found by the equation

$$s = \frac{a}{1 - r}.$$

III. *Harmonical Progression.*

1. Let a, b, c be in Harmonical Progression; then $a : c :: a - b : b - c$.

2. Let a, b, c, &c. be as before, then

$\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$, &c. are in arithmetical progression.

3. Let a and b be the two first terms of an Harmonical Progression, to continue the series.

$$a,\ b,\ \frac{ab}{2a - b},\ \frac{ab}{3a - 2b},\ \frac{ab}{4a - 3b},\ \&c.$$

4. To find an harmonic mean (x) between two quantities a and b.

$$x = \frac{2ab}{a + b}.$$

5. If between two quantities a and b, an harmonic mean x, and an arithmetical mean y, be inserted,

$$a : x :: y : b.$$

6. If between two quantities a and b an arithmetic mean x, a geometric mean y, and an harmonical z, be inserted

$$x : y :: y : z.$$

7. If a fourth proportional be found to three quantities in Arithmetical progression, the three last terms are in Harmonical progression.

PROJECTILES *in a vacuum.—(Whewell.)*

Formulæ for finding the range, altitude, and time of flight, of bodies projected along planes inclined to the horizon.

1. Let r = range, A = greatest altitude, t = time of flight, v = velo-

city of projection, $h =$ height due to this velocity, $\alpha = \angle$ of projection above the horizontal plane, $\iota =$ elevation of the plane above the horizon, $g = 32\frac{1}{6}$ feet; then we have the following equations.

$$r = \frac{2v^2}{g} \cdot \frac{\sin.(\alpha - \iota)\cos.\alpha}{\cos.^2\iota} = 4h\frac{\sin.(\alpha - \iota)\cos.\alpha}{\cos.^2\iota}.$$

$$A = \frac{v^2}{2g} \cdot \frac{\sin.^2(\alpha - \iota)}{\cos.^2\iota} = h\frac{\sin.^2(\alpha - \iota)}{\cos.^2\iota}.$$

$$t = \frac{2v}{g} \cdot \frac{\sin.(\alpha - \iota)}{\cos.\iota} = \sqrt{\frac{2h}{g}} \cdot \frac{2\sin.(\alpha - \iota)}{\cos.\iota}.$$

$$\text{Greatest range} = \frac{2h}{1 + \sin.\iota}.$$

2. When $\iota = o$; r will be the *horizontal* range, and the above equations will become

$$r = \frac{v^2}{g} \cdot \sin.2\alpha = 2h\sin.2\alpha.$$

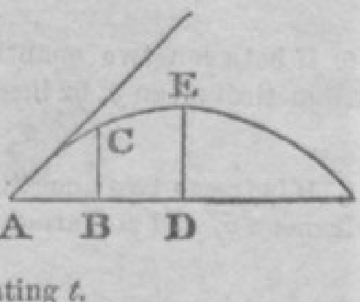
$$A = \frac{v^2}{2g} \cdot \sin.^2\alpha = h\sin.^2\alpha.$$

$$t = \frac{2v}{g} \cdot \sin.\alpha = \sqrt{\frac{2h}{g}} \cdot 2\sin.\alpha.$$

Greatest range $= 2h$.

3. The curve described by a projectile is a parabola, the principal parameter of which $= 4h\cos.^2\alpha$, and the velocity at any point is that acquired by falling from the directrix.

4. To find an equation to the curve, referred to horizontal and vertical co-ordinates.



Let $AB = x$, $BC = y$, $t =$ any time; then

$$x = vt\cos.\alpha.$$

$$y = vt\sin.\alpha - \frac{gt^2}{2} \text{ and eliminating } t.$$

$$y = x\tan.\alpha - \frac{g}{2v^2}\frac{x^2}{\cos.^2\alpha},$$

the equation to the curve.

Cor. 1. If, as before, $h = \frac{v^2}{2g}$,

$$y = x \tan. \alpha - \frac{x^2}{4\,h \cos.^2 \alpha}.$$

Cor. 2. To find where the curve meets the horizontal plane, we must put $y = o$, $\therefore x \tan. \alpha - \frac{x^2}{4\,h \cos.^2 \alpha} = o$, $\therefore x = 4\,h \tan. \alpha \cos.^2 \alpha = 4\,h \sin. \alpha \cos. \alpha = 2\,h \sin. 2\,\alpha$, which agrees with Art. 2.

Cor. 3. If v does not enter the conditions of the problem, we have, by eliminating v,

$$y = x \tan. \alpha - \frac{g\,t^2}{2}.$$

Cor. 4. To find the ∠ which the curve makes with the horizon at any point.

Let ϕ be this angle, $\tan. \phi = \frac{d\,y}{d\,x}$, and differentiating the value of y,

$$\tan. \phi = \tan. \alpha - \frac{x}{2\,h \cos.^2 \alpha}.$$

Ex. 1. Let a body be projected from the top of a tower horizontally with a velocity acquired in falling down its height; at what distance from the base will it strike the horizon?

$$y = x \tan. \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here if a = altitude of tower, $y = -a$, $\alpha = o$, and $v^2 = 2\,g\,a$, $\therefore$

$-a = -\frac{x^2}{4\,a}$, and $x = 2\,a$.

Ex. 2. A body is projected at an ∠ of 45°, with a velocity of 50 feet per second; find its horizontal range.

$$y = x \tan. \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here $\alpha = 45°$, $v = 50$, $\therefore$ when $y = o$

$$x = \frac{2500}{g}.$$

Ex. 3. A projectile is thrown across a plain 120 feet wide, to strike a mark 30 feet high, the velocity of projection being that acquired down 80 feet; required the ∠ of projection.

$$y = x \tan. \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here $y = 30$, $x = 120$, $v^2 = 160\,g$, $\therefore$

$$1 = 4 \tan.\ \alpha - \frac{3}{2\,(\cos.)^2\,\alpha}$$

$$\therefore (\tan.)^2\,\alpha - \frac{8}{3}\tan.\ \alpha = -\frac{5}{3},$$

$$\text{and } \tan.\ \alpha = 1 \text{ or } \frac{5}{3}, \text{ and } \alpha = 45^{0}.$$

Ex. 4. A body projected from the top of a tower at an $\angle$ of 45_0 above the horizontal direction, fell in $5''$ at a distance from the bottom equal to its altitude; required the altitude.

$$y = x \tan.\ \alpha - \frac{g\,t^2}{2}.$$

Let a = height, then $\alpha = 45^0$, $t = 5$, and $y = -a$,

$$\therefore -a = a \tan.\ 45^0 - \frac{g}{2}.\ 25,$$

$$\therefore a = 200.$$

PROJECTILES, *resistance of air to.—See Gunnery.*

PROJECTION, *principles of.—(Vince.)*

I. *Orthographic Projection.*

1. The figure of a straight line is a straight line in the projection.

2. The figure of the projection of a circle is an ellipse, of which the minor axis is the cosine of inclination of the circle to the plane of projection. Hence if the circle be parallel to the plane of projection, the projection will be a circle equal to it. If the circle be perpendicular to the plane of projection, the circle is projected into its diameter; any arc, reckoned from its intersection with the plane, into its versed sine; and the remainder of the quadrant into the sine of that remainder, or into the cosine of the first mentioned arc.

3. In this projection the area of the circle : the area of the ellipse into which it is projected :: radius : cosine of inclination of the plane of the body to the plane of projection; hence the area of the circle will be diminished in the ratio of radius : the cos. of this inclination. And this is true whatever be the form of the projected body. Also the projection is not similar to the body. Hence equal parts upon the surface of a sphere will not be projected into parts either equal or similar.

This projection is not convenient for maps, but is used in the construction of solar eclipses.

II. *Stereographic Projection.*

1. The projection of an arc, measured from the pole, is equal to the tangent of half that arc.

2. The projection of every circle is a circle.

3. The projection of all circles parallel to the plane of projection will be concentric circles, the radii of which are the tangents of ½ the distances of the circles from the pole.

4. The projection of every great circle passing through the pole is a straight line.

5. The radius of projection of any other great circle is the secant of the angle between the plane of the circle and the plane of projection.

From these Arts. it appears, that the projection of the parts of the sphere will not properly represent, in magnitude and situation, the parts themselves.

6. If the place of the eye be the pole of the earth, the meridians will be projected into straight lines (Art. 4); and the parallels to the equator will be projected into circles (Art. 3). This is called the *Polar Projection.*

7. If the eye be placed in the equator 90^0 distant from the point from which the longitude is reckoned, the projection of the radius of any meridian will be the secant of its longitude (Art. 5). And the radius of projection of the parallels of latitude is the cotangent of their latitude. This is called an *Equatorial Projection.*

The stereographic projection is chiefly used in delineating maps of the world.

III. *Mercator's Projection.*

1. In this projection the meridians are parallel lines, the degrees of longitude are all equal; the parallels of latitude are also parallel lines, but unequal, a degree of latitude being to a degree of longitude :: rad. : cos. latitude, and ∴ the length of a degree of longitude being constant, the length of a degree of latitude will be inversely as the cosine of latitude, and will ∴ increase in going towards the pole.

2. To find the length of the meridian on this projection for any number of degrees of latitude.

Let x = required length, r = earth's radius then

$$x = r \times \text{h. l.}\ \frac{\text{cot. } \frac{1}{2} \text{ comp. latitude}}{r}.$$

If ∴ we take the latitude $= 1^0, 2^0, 3^0$ .... 90^0 we can construct a

Table shewing the length of the meridian on the projection for every degree of latitude; in like manner it may be constructed for every minute. Such a table is called a table of *Meridional Parts.*

This projection is of great use in navigation, on account of its being constructed by right lines only; the rhumb lines or lines of azimuth being also straight lines.

Suppose for example a ship wants to go from any place A to B laid down upon Mercator's map, and it is required to find the rhumb or point of the compass it must sail upon; we have only to join A B, and that is the rhumb. Now to determine what rhumb this is, there is always in these maps one or more points, from which are drawn 32 straight lines, representing the 32 points of the compass. Apply ∴ one edge of a parallel ruler to the line A B, and bring the other edge over the point from which the lines of the compass are drawn, and it immediately gives the direction in which the ship must sail.

PULLEY.

1. In the single fixed pulley, there is an equilibrium when the power and weight are equal.

2. In the single moveable pulley whose strings are parallel, P : W :: 1 : 2.

3. In a system where the same string passes round any number of pullies, and the parts of it between the pullies are parallel, P : W :: 1 : n, n being the number of strings at the lower block.

Cor. If we consider the weight of the pullies, it is only requisite to add the weight of the lower block; hence if a be this block,

$$W = n P - a.$$

4. In general in the single moveable pulley, P : W :: rad. : 2 cosine of the angle which either string makes with the direction in which the weight acts; or :: sin. $\frac{1}{2}$ angle which the two strings make with each other : sin. of the whole angle.

5. In a system where each pulley hangs by a separate string, and the strings are parallel, P : W :: 1 : 2^n where n is the number of moveable pullies.

Cor. 1. Hence $W = 2^n P$. If the weight of the pullies be taken into the account, and a = weight of each, $W = 2^n P - a(2^n - 1)$; hence the weight W is less as a is greater.

Cor. 2. When the strings are not parallel, P : W :: $(\text{rad.})^n$: $2^n \times$

cos. $\alpha \times$ cos. $\beta \times$ cos. γ &c. where α, β, γ, &c. are the angles which the strings make with the direction in which the weight acts in each case.

6. In a system of n pullies each hanging by a separate string, where the strings are attached to the weight, $P : W :: 1 : 2^n - 1$.

Cor. Supposing the weight of each pulley $= a$, then the part of the weight sustained by the pullies $= a \times (2^n - n - 1)$; and $\therefore$ $W = (2^n - 1) P + (2^n - n - 1) a$.

PULLEY, *on the ascent or descent of bodies over.*

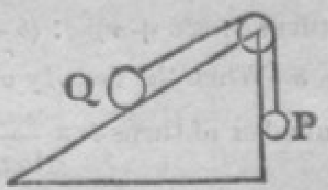
1. If two bodies P and Q are connected by a string and hung over a fixed pulley, the accelerating force, supposing P the heaviest, is $g \times \frac{P - Q}{P + Q}$. Substitute this for F in the formulæ for the rectilinear descent of bodies *(see Motion)* and we get

$$v = \frac{P - Q}{P + Q} \times g\,t.$$

$$s = \frac{P - Q}{P + Q} \times \frac{g\,t^2}{2}.$$

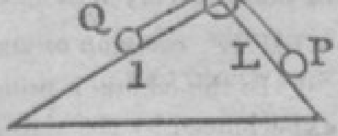
$$s = \frac{P + Q}{P - Q} + \frac{v^2}{2\,g}.$$

2. If two bodies P and Q are connected together by a cord going over a fixed pulley, and one of them Q descends down an inclined plane, we have the moving force of $Q = Q \times \frac{H}{L}$; hence the moving force of Q when connected with $P = \frac{Q\,H}{L} - P = \frac{Q\,H - P\,L}{L}$ and accelerating force $= g \times \frac{Q\,H - P\,L}{L \times P + Q}$.



If P draws up Q, accelerating force $= g \times \frac{P\,L - Q\,H}{L \times P + Q}$, which may be substituted for F as in the last Art.

Let both bodies P and Q move upon inclined planes, whose lengths are L and l respectively, and having a common altitude H, and let Q be the descending body; then moving force of Q



$= \frac{QH}{l}$; do. $P = \frac{PH}{L}$; $\therefore$ the moving force of the system $=$ $\frac{LQ - lP}{Ll} \times H$, and accelerating force of $Q = g \times \frac{\overline{LQ - lP} \times H}{Ll \times \overline{P + Q}}$, which may be substituted as before.

PUMP. *Air Pump.*

1. If b represent the capacity of one of the barrels, and r that of the receiver together with the pipes and gages connected with it; then the quantity of air extracted after every turn : the quantity before that turn :: $b : 2b + r$. And the quantity left in : the quantity before :: $b + r : 2b + r$.

Cor. Hence if P represent the quantity of air in the machine before the first turn, the quantity left in after n turns is

$$P. \left(\frac{b+r}{2b+r}\right)^n.$$

And the quantity exhausted is $P - P. \left(\frac{b+r}{2b+r}\right)^n =$

$$P. \frac{(2b+r)^n - (b+r)^n}{(2b+r)^n}$$

2. The density of the air in the receiver at first : the density after t turns :: $(2b+r)^t : (b+r)^t$.

3. When the density of the air is diminished in the ratio of $n : 1$, the number of turns $t = \frac{\log. n}{\log. \overline{2b+r} - \log. \overline{b+r}}$.

4. As the air is exhausted, the mercury will rise in the gage, and the defects of the mercury in the gage from the standard altitude, after each successive turn, form a geometric series, the ratio of whose terms is $2b + r : b + r$. And the ascents of the mercury at each successive turn form a geometric series, the ratio of whose terms is $2b + r : b + r$.

PUMP *condensing, or condenser.*

If b represent the capacity of the barrel of the syringe, and r that of the receiver, then after t descents of the sucker, the density of the air in the receiver, will be to the density at first in the ratio of $r + tb : r$.

PUMP, *common or sucking.*

1. In the common pump the force necessary to overcome the resistance experienced by the piston, in ascending, is equal to the weight of a co-

lumn of water, having the same base as the piston, and an altitude equal to that of the surface of the water in the body of the pump above that in the reservoir.

2. In a sucking pump, if the height of the lower or fixed valve above the surface of the water $= a$, the length of the stroke of the piston $= b$, and the height of a column of water in equilibrium with the pressure of the atmosphere $= h$, the height to which the water is raised by the first stroke is

$$\frac{a+b+h-\sqrt{(a+b+h)^2-4bh}}{2}.$$

3. The same notation being retained, and c being put for $a+b$ or the greatest height to which the piston ascends, b must be greater than $\frac{c^2}{4h}$ otherwise the water will not rise above the piston.

4. Height to which water will rise in a vacuum in different states of the barometer.

Barom. in inches.	*Height of water in feet.*
28	31.66
28½	32.23
29	32.79
29½	33.36
30	33.92
30½	34.49
31	35.05

Hence the valve of the piston in the common pump must be nearer to the surface of the water in the reservoir than 33 feet, otherwise the water can never rise above it.

PYROMETER, *Wedgwood's, for measuring very high temperatures.*

The scale of this Pyrometer, or the point marked 0 commences at red heat fully visible in day light, and is equivalent to 1077½° of Fahrenheit's scale, and one degree of the former is $= 130°$ of the latter. The extremity of Wedgwood's scale is 240°, but the highest heat he measured with it is 160°. It appears ∴ that this pyrometer includes an extent of about 32000 of Fahrenheit's degrees, or about 54 times as much as that between the boiling and freezing points of mercury, by which mercurial ones are naturally limited; that if the scale be produced downward in the same manner as Fahrenheit's has been supposed to be produced upward for an ideal standard, the freezing point of water would fall nearly 8° below 0 of Wedgwood's and the freezing point of mercury a little below $8\frac{1}{2}$, and that there are 8o from the freezing of water to full ignition.

Q

QUADRATRIX *of Dinostrates, Equation to.*

$y\sqrt{r^2 - s^2} = s(r - x)$, where r = radius, and s the sine of the circ. arc, by the help of which the curve is generated.

The radius of the generating circle is a mean proportional between the quadrantal arc and the base of the quadratrix.

If with the base of the quadratrix as radius, there be described a quadrantal arc, this will be equal in length to the radius of the generating circle.

QUADRATURE *of Curves.—See Area.*

R

RADIUS *vector of a planet's orbit.—See Anomaly.*

RAIN, *quantity of at different places.*

Mean annual quantity of rain for 30 years, as observed at the apartments of the Royal Society. The gage is placed 75 feet 6 inches above the ground.

Years.	Inches.	Years.	Inches.
1790	16.052	1800	18.925
1791	15.310	1801	19.197
1792	19.489	1802	13.946
1793	17.128	1803	17.922
1794	18.466	1804	20.973
1795	16.864	1805	20.396
1796	14.779	1806	20.427
1797	22.697	1807	14.206
1798	19.411	1808	18.475
1799	19.662	1809	20.711

Years.	Inches.	Years.	Inches.
1810 and 1811	——	1817	15.299
1812	18.348	1818	11.636
1813	15.953	1819	13.727
1814	16.367	1820	18.381
1815	12.968	1821	23.567
1816	15.174		

Average of the 30 years	17.548
Greatest mean quantity during this period	23.567
Least do.	11.636

Mean quantity of rain for each month during the above period of 30 *years.*

	Inches.		Inches.
January	1.253	July	1.979
February	1.004	August	1.489
March	0.884	September	1.564
April	1.269	October	1.712
May	1.476	November	1.985
June	1.411	December	1.520

It appears from observation, that the quantity of rain, as shewn by two gages, is not materially influenced by the height of the places above the level of the sea, provided the heights of the gages above the ground are equal; but it is a singular fact, which has not been satisfactorily accounted for, that it is very considerably affected by the height of the gages above the surface of the earth, though all other circumstances are the same. This will appear by a comparison of the following results, given in the Philosophical Transactions.

Quantity of rain observed by Mr Daines Barrington, for upwards of four months in 1770, as shewn by two gages, the one placed upon Mount Rennig, in Wales, the other on the plain below at about half a mile distant; the perpendicular height of the mountain being 1350 feet, and each gage being at the same height above the surface of the ground.

	Inches.
Bottom of mountain	8.766
Top of mountain	8.165

Quantity of rain observed by Dr Heberden, from July 7, 1766, to July 7, 1767, as shewn by three gages, one placed below the top of a house, a second upon the top of a house, and the third upon Westminster Abbey.

	Inches.
Lowest gage	22.608
Middle gage	18.139
Highest	12.099

The same result was obtained from the two gages belonging to the Royal Society, the one placed 75 feet 6 inches above the ground, the other a few feet distant from the other and 11 feet 6 inches lower.

Mean annual quantity of rain, as shewn by the two gages.

Year.	Lower gage in inches.	Higher.
1812	22.03	18.348
1813	18.296	15.953
1814	20.723	16.367

These facts should be attended to, in order to prevent any inaccurate conclusions from a comparison of different gages.

Estimate by Humboldt of the quantity of rain in different latitudes.

Latitude.	*Eng. inches.*	*Latitude.*	*Eng. inches.*
0°	96	45°	29
19	80	60	17

Professor Leslie has given the following empirical rule for the annual deposit of rain and dew in any latitude.

Quantity = 75 (1 — sin. lat.) + 8 = depth in inches.

Annual fall of rain at different places, according to Dalton and others. —*(Young's Nat. Phil.)*

	Inches.
Granada, Antilles	126
Cape François, St Domingo	120
Calcutta	81
Bombay	64
Charlestown	50.9
Pisa	43.2
Rome	39.0
Venice	36.1
Padua	34.5
Zurich	33.1
Madeira	31.0
Leyden	30.2

	Inches.
Hague	28.4
Algiers	27.0
Utrecht	24.7
Lisle	24.0
Dublin	22.2
Edinburgh	22
Berlin	20.6
Petersburgh	17.2
Upsal	16.7
Keswick, Cumberland, 7 years	67.0
Kendal, Westmorland, 25 years	53.9
Garsdale, Westmorland, 3 years	52.3
Lancaster, 20 years	39.7
Townley, Lancashire, 15 years	41.5
Dover, 5 years	37.5
Liverpool, 18 years	34.4
Manchester, 33 years	36.1
Bristol, 3 years	29.2
Chatsworth, Derbyshire, 15 years	27.8
Barrowby, near Leeds, 6 years	27.5
Fyfield, Hampshire, 7 years	25.9
Norwich, 13 years	25.5
Lyndon, Rutlandshire, 21 years	24.3
Near Oundle, Northamptonshire, 14 years	23.0
South Lambeth, 9 years	22.7
Dalton's mean for all England	31.3
Dalton's mean, for rain and dew together, for all England	36.0
M. Cotte's mean annual quantity of rain falling at 147 places from N. lat. 11°. to 60°.	34.7

The superficies of the globe consists of 170,981,012 square miles; supposing therefore that the mean annual quantity of rain for the whole globe to be 34 inches, the quantity of rain falling annually will amount to somewhat more than 91,751 cubic miles of water, which must be supplied by evaporation from the surface of the earth and sea.—*See Evaporation.* The dry land amounting to 52,745,253 square miles, the quantity of rain falling on it will amount to 30,960 cubic miles. The quantity of water running annually into the sea is estimated at 13,140 cubic miles, a quantity of water equal to which must be supplied by evaporation from the *sea*, otherwise the land would be soon completely drained of its moisture.

The area of England and Wales = 46,450 square miles, taking therefore Dalton's mean at 36 inches, we shall have the annual quantity of rain and dew falling in England and Wales = 28 cubic miles of water.

RAINBOW.—(*Wood.*)

1. If a ray of light refracted into a sphere, emerge from it after any given number of reflections, to find the angle contained between the directions in which it is incident and emergent.

Let ϕ and ϕ' = angles of incidence and refraction, p = number of reflections, then the deviation, or inclination of the emergent to the incident ray is

$$2\phi - 2(p+1)\,\phi' \text{ or } 2(p+1)\,\phi' - 2\phi.$$

Cor. In the primary rainbow $p = 1$, $\therefore$ deviation $= 4\phi' - 2\phi$; in the secondary bow $p = 2$; $\therefore$ deviation $= 6\phi' - 2\phi$, or its supplement.

2. The rays of any colour will fall most copiously on the eye, when the rays emerge parallel, in which case only they are efficient.

3. When rays emerge parallel, the increment of the angle included between the incident and emergent rays = 0, and tangent of incidence : tangent of refraction :: $p + 1 : 1$.

Cor. Hence it is easy to shew that if the ratio of refraction $= m$, $\cos.\phi = \sqrt{\dfrac{m^2-1}{p^2+2p}}$.

Ex. 1. If the pencil be parallel red rays incident upon a sphere of water, and suffer two refractions and one reflection, as in the primary bow, $m = \frac{4}{3}$, and $p = 1$ $\therefore \cos.\phi = \sqrt{\frac{7}{27}}$ when the rays emerge parallel, or $\phi = 59^\circ.\ 23'$; also $\phi' = 40^\circ.\ 12'$; $\therefore$ deviation, or angle between the incident and emergent rays, $= 4\phi' - 2\phi = 42^\circ.\ 2'$.

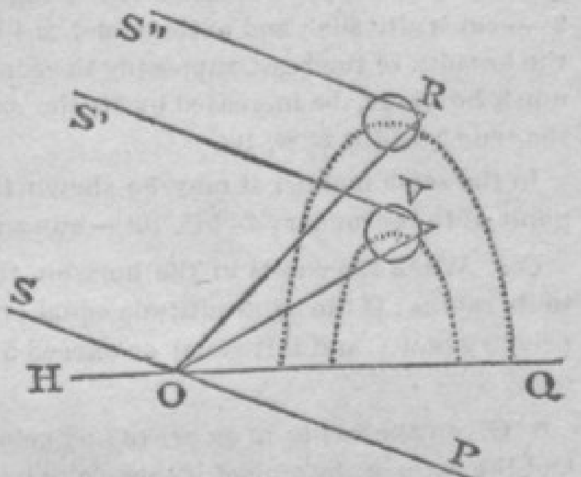
When the violet rays are thus incident and emergent, $\phi = 58^\circ.\ 40'$, $\phi' = 39^\circ.\ 24'$, and deviation $= 40^\circ.\ 16'$.

Ex. 2. If the pencil be parallel red rays incident upon a sphere of water, and suffer two refractions and two reflections, as in the secondary bow, $p = 2$, and $\therefore$ when the rays emerge parallel, $\phi = 71^\circ.\ 50'$; $\phi' =$

$45^\circ. 27'$, $\therefore 6\phi' - 2\phi = 129^\circ. 2'$, the supplement of which, or the deviation, $= 50^\circ. 58'$.

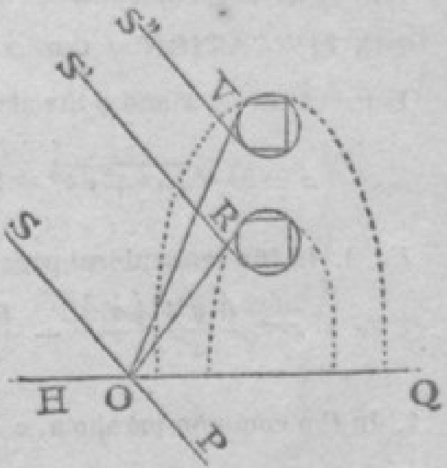
When violet rays are thus incident, $\phi = 71^\circ. 26'$, $\phi' = 44^\circ. 47'$, and the deviation $= 54^\circ. 10'$.

4. Construction of the primary and secondary Rainbow.



The red rays we have seen are efficacious when the $\angle$ between the incident and emergent rays $= 42^\circ. 2'$, and the violet rays when the same $\angle = 40^\circ. 16'$; hence if H Q be the horizon, S, S′, S″ rays proceeding from the sun, O the eye of the spectator, and the $\angle$ P O R ($= \angle$ S″ R O) be taken $= 42^\circ. 2'$ the drop R will transmit the red rays to the eye; and if P O V ($=$ S′ V O) be taken $= 40^\circ. 16'$ the drop V will transmit the violet rays. The drops betwixt R and V will transmit to the eye the other colours in their proper order.

If O R and O V revolve about the axis O P, every drop of water in the surface of the cones thus described will respectively transmit to the eye a small parallel pencil of red and violet rays; and thus a red and violet arc, whose radii (measured by the angles which they subtend at the eye) are $42^\circ. 2'$, and $40^\circ. 16'$ respectively, will appear in the falling rain opposite to the sun; and the same may be said of the other colours.



The parallel pencils of red &c. rays which emerge from other drops fall above or below the eye.

The secondary rainbow is formed by two refractions and two reflections. In this case, as we have seen, the violet rays are efficacious when the $\angle$ contained by the incident and emergent rays $= 54^\circ. 10'$, and the red rays when the same $\angle = 50^\circ. 58'$. Hence as in the primary bow, if $\angle$ P O V $= 54^\circ. 10'$, the drop V will transmit the violet rays to the eye; and if P O R $= 50^\circ. 58'$ the drop R will transmit the red rays.

Hence the colours in the two bows lie in a contrary order, the red forming the exterior ring of the primary, and the interior ring of the secondary bow.

5. To find the altitude and breadth of the rainbow.

In the primary, the altitude of the highest point of the red arc $= 42^{0}. 2' -$ sun's altitude ; and of the violet $= 40^{0}. 16' -$ sun's altitude. Hence the breadth of the bow, supposing the sun a point $= 1^{0}. 46'$; this breadth must, however, be increased by $30'$ the sun's apparent diameter, and $\therefore$ the true breadth $= 2^{0}. 16'$.

In the same manner it may be shewn that the altitude of the highest point of the secondary $= 54^{0}. 10' -$ sun's altitude; and breadth $= 3^{0}. 42'$.

Cor. When the sun is in the horizon, the altitude of the bow is equal to its radius ; if the sun's altitude equal or exceed $42^{0}. 2'$, there can be no primary bow ; and if it equal or exceed $54^{0}. 10'$, there can be no secondary.

6. Given the radius of an arc of any colour in the primary rainbow, to find the ratio of the sine of incidence to the sine of refraction, when rays of that colour pass out of air into water.

The radius of the arc $= 4\,\phi' - 2\,\phi$; let the tangent of $2\,\phi' - \phi$, half this angle, be a, z the tangent of ϕ'; then

$$2\,z^3 - 3\,a\,z^2 - a = o.$$

The value of z being thus obtained, the angles ϕ' and ϕ and consequently their sines may be found from the tables.

RECIPROCALS *of numbers.—See Involution.*

RECIPROCAL *Spiral.—See Spiral.*

RECTIFICATION *of Curves.*

Let $z =$ curve, x and y the abscissa and ordinate ; then

$$z = \text{fl.}\ \sqrt{d\,x^2 + d\,y^2} = \text{fl.}\ d\,x\sqrt{1 + \frac{d\,y^2}{d\,x^2}}.$$

Ex. 1. In the semicubical parabola, where $a\,x^2 = y^3$,

$$z = \frac{(9\,y + 4\,a)^{\frac{3}{2}}}{27\,a^{\frac{1}{2}}} - \frac{8\,a}{27}.$$

2. In the common parabola, $z = \frac{1}{2\,b} \times (y^4 + b^2\,y^2)^{\frac{1}{2}} + \frac{1}{2}\,b$

$$\times\ \text{h. l.}\ \frac{y + \sqrt{y^2 + b^2}}{b}.$$

Rate of travel, to find.

Suppose a wheel 1 mile in circumference; it would make 1 revolution in 1 hour (60×60 secs) when going at the rate of 1 mile an hour, 2 revolns when going 2 miles an hour, & so on.

Suppose a wheel of 1 inch in circumference, it would make 1 revolution in $\frac{60 \times 60}{1760 \times 36}$ secs

$= \frac{10}{176}$ secs when going at 1 mile an hour and r revolutions, in same period, when going r miles an hour.

Generally, a wheel n inches in circumference will make r revolutions in $\frac{10.n}{176}$ secs when going r miles an hour.

Carriage wheel in circumference	in diameter	appropriate period
8 feet 9½ inches	2 feet 9½ inch	6 seconds
10 " 3¼ "	3 " 3¼ "	7 "
11 " 8¾	3 " 8¾ "	8 "
13 " 2½	4 " 2½ "	9 "
14 " 8	4 " 8 "	10 "
16 " 1½	5 " 1½	11 "

Therefore, suppose the circumference of the carriage wheel to be 14 ft 8 inch, it is only necessary to note the number of revolutions it makes in 10 secs and that will be the number of miles per hour at which it is travelling.

3. In a circular arc, whose tangent is t,

$$z = t - \frac{t^3}{3a^2} + \frac{t^5}{5a^4} - \frac{t^7}{7a^6} + \&c.$$

For rectification of Spirals—*see Spiral.*

REFLEXION *in Optics.*—(*Coddington.*)

1. *Reflexion at plane surfaces.*

1. To find the direction in which a ray of light, emanating from a given point, takes after reflexion at a plane mirror.

Let the ray proceed from a point Q, and a perpendicular Q C be drawn to the surface of the reflector, and let the ray after reflexion cut Q C produced in q; then will Q and q be on opposite sides of C, and Q C will $=$ C q.

2. To find the same when the ray is reflected alternately by two plane mirrors inclined to each other at any given angle.

Let $\phi_{,}$ be the $\angle$ of incidence at the first reflexion

ϕ_2 ——————— second

$\vdots$ $\qquad\qquad\qquad\qquad\qquad\qquad \vdots$

ϕ_n ——————— n^{th}

ι the inclination of the two mirrors; then we shall have this series of equations,

$$\phi_1 - \phi_2 = \iota$$
$$\phi_2 - \phi_3 = \iota$$
$$\phi_3 - \phi_4 = \iota$$
$$\vdots \quad \vdots$$
$$\phi_1 - \phi_n = (n-1)\,\iota$$
$$\text{or} \quad \phi_n = \phi_1 - (n-1)\,\iota$$

If now $\phi_{,}$ be any multiple of ι, as $(n-1)\,\iota$, we shall have somewhere $\phi_n = 0$, i.e. some reflected ray will be perpendicular to one of the mirrors, and these of course will end the series of reflexions. If $\phi_{,}$ be not a multiple of ι, some value of n will make $(n-1)\,\iota$ greater than $\phi_{,}$, and then ϕ_n will be negative. This shews that the ray will at length be turned back upon itself in a direction contrary to what it at first proceeded in.

To find the angle between the 1st incident and last reflected ray, let

q_1 represent the first incident ray and q_2, q_3 ...... the rays after the 1st, 2d, 3d, &c. reflexions, and let the $\angle$'s. between them be denoted by $(q_1\ q_2)$ $(q_1\ q_3)$ &c. and we shall have

$$(q_1\ q_3) = (q_3\ q_5) = \text{.........} (q_{n-1}\ q_{n+1}) = 2\ \iota$$

and $(q_1\ q_{2n+m}) = 2\ n\ \iota$, provided m be an odd number.

$$\text{Also } (q_1\ q_4) = 2\ \phi_{\prime} - 2\ \iota$$

$$(q_1\ q_6) = 2\ \phi_{\prime} - 4\ \iota$$

$$\vdots \qquad \vdots$$

$$(q_1\ q_{2m}) = 2\ \phi_{\prime} - (2\ m - 2)\ \iota$$

II. *Reflexion at spherical surfaces.*

1. Rays meeting in a point being incident on a spherical reflecting surface; to determine the directions of the reflected rays.

Let r = radius of the surface, q and q' = distance of the foci of incident and reflected rays from the *centre* of the reflector, then when the incident rays are nearly coincident with the axis,

$$\frac{1}{q'} = \frac{1}{q} + \frac{2}{r}.$$

If q is infinite, or the rays parallel,

$$\frac{1}{q} = \frac{2}{r} \text{ or } q' = \frac{r}{2}.$$

This is technically called the *principal focal distance* of the reflector, and if we call it f, we have $f = \frac{r}{2}$, and $\therefore$ by substituting in the first equation,

$$\frac{1}{q'} = \frac{1}{q} + \frac{1}{f}.$$

These formulæ may be obtained in another form, which is often more convenient, thus:—

Let Δ and Δ' = distance of the foci of incident and reflected rays from the *surface* of the mirror, then $\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{2}{r}$; r being negative if the mirror be convex.

If Δ is infinite, or the rays parallel,

$$\frac{1}{\Delta'} = \frac{2}{r}, \text{ or } \Delta' = \frac{r}{2} = f$$

and $\therefore$ as before $\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{1}{f}$.

Cor. If E be the centre of the mirror, Q and q the foci of incident and reflected rays, F the principal focus of parallel rays,

$$Fq : FE :: FE : FQ$$

where Q and q lie on the same side of F, move in opposite directions, and meet at the centre and surface of the reflector.

III. *Reflexion, images produced by.*

1. When an object is placed between two parallel *plane* mirrors, a row of images is formed, which are gradually fainter as they are more remote, and at length become invisible.

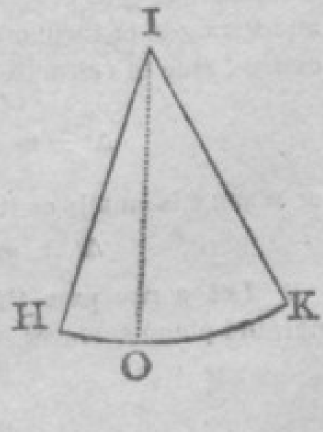
Now let A and B represent the two mirrors, O the object between them, and let O' be the image of O at the mirror A, O'' the image of O' at the second mirror B, O''' the image of O'' at the mirror A &c.

Also let $O_{\prime}$ be the image of O at the second mirror B, $O_{\prime\prime}$ the image of $O_{\prime}$ at A, $O_{\prime\prime\prime}$ of $O_{\prime\prime}$ at B &c. and let it be required to find the distances $O\,O'$, $O\,O''$ .... $O\,O_{\prime}$, $O\,O_{\prime\prime}$ &c.

Put $OA = a$, $OB = b$, $AB = c$ or $a + b$. Then

$$\left.\begin{array}{l} O\,O' = 2a \\ O\,O'' = 2c \\ O\,O''' = 2c + 2a \\ \vdots \quad\quad \vdots \end{array}\right\} \text{Again} \left\{\begin{array}{l} O\,O_{\prime} = 2b \\ O\,O_{\prime\prime} = 2c \\ O\,O_{\prime\prime\prime} = 2c + 2b \\ \vdots \quad\quad \vdots \end{array}\right.$$

2. Suppose now that the mirrors, instead of being parallel, are inclined to each other, in this case the number of images will be limited, and will evidently lie in the circumference of a circle, whose centre is the intersection of the two planes, and radius the distance of the object from that intersection.



Now let H I and K I be the mirrors, O the object, then as before there will be two series of images

O', O'', O''' &c. and $O_{\prime}$, $O_{\prime\prime}$, $O_{\prime\prime\prime}$ &c.

to determine the distances $O\,O'$, $O\,O''$ &c. measured along the circumference of the circle, put HIO or $HO = \theta$, OIK or $OK = \theta'$, HIK or $HK = \iota$, then

$$\left.\begin{array}{l} O\,O' = 2\,\theta \\ O\,O'' = 2\,\iota \\ O\,O''' = 2\,\iota + 2\,\theta \\ O\,O'''' = 2\,\iota + 2\,\theta + 2\,\theta' = 4\,\iota \\ \vdots \qquad \vdots \end{array}\right\} \text{Again} \left\{\begin{array}{l} O\,O_{,} = 2\,\theta' \\ O\,O_{,,} = 2\,\iota \\ O\,O_{,,,} = 2\,\iota + 2\,\theta' \\ \vdots \qquad \vdots \end{array}\right.$$

and the number of images in the first series is the least whole number greater than $\frac{\pi - \theta}{\iota}$; and the number of images in the other series is the least whole number greater than $\frac{\pi - \theta'}{\iota}$.

If ι be a measure of π, the whole number of images is $\frac{2\,\pi}{\iota}$; and in this case two images of the different series coincide.

3. If the object placed before a *spherical* reflector be a circular arc concentric with it, the image will also be a circular arc concentric with and similar to the object, and its position and magnitude may be determined by the proportion

$$F\,q : FE :: FE : FQ.$$

4. If the object placed before a spherical reflector be a straight line, the image is a conic section; and is a parabola, ellipse, or hyperbola, according as the distance of the object from the centre of the mirror is equal to, greater, or less than, half its radius.

REFRACTION *in Optics.—(Coddington.)*

I. *Refraction at plane surfaces.*

1. Given the direction in which a ray falls on a plane surface bounding a refracting medium; to find the direction of the refracted ray.

Let the ray proceed from the point Q, and a perpendicular Q C be drawn to the surface of the refracting medium, and let the ray after refraction cut Q C or Q C produced in q; put Δ and $\Delta' =$ C Q and Cq; θ and $\theta' =$ ∠'s. of incidence and refraction; m the ratio of the sine of incidence : sine of refraction, usually called the *ratio of refraction;* then

$$\Delta' = m\,\Delta\left(1 + \frac{m^2 - 1}{2\,m^2}\tan.\,\theta^2\right)$$

or when θ is small, as it is usually supposed to be,

$$\Delta' = m\,\Delta \text{ nearly.}$$

2. Let a ray pass through a refracting substance bounded by two parallel plane surfaces; to determine its direction after emergence.

Let D = distance of the foci of incident and emergent rays, T the thickness of the medium, then

$$D = \frac{m-1}{m} T.$$

3. To determine the refraction which a ray experiences in passing through a medium bounded by planes not parallel; for example a triangular prism of glass,

Let ι = vertical angle of the prism.

$\phi = \angle$ of incidence.

$\psi = \angle$ of emergence.

$\delta = \angle$ of deviation of the incident and emergent rays; then

$$\delta = \phi + \psi - \iota,$$

or if the $\angle$ of the prism and the $\angle$'s. of incidence and emergence be exceedingly small,

$$\delta = (m-1)\,\iota.$$

Cor. The $\angle$ of deviation is a minimum when the incident and emergent rays make equal $\angle$'s. with the sides of the prism.

II. *Refraction at spherical surfaces.*

1. A ray of light is refracted at a spherical surface bounding two different media; given the point where it meets the axis, required the point where the refracted ray meets the axis.

Let r = radius of the spherical surface,

Δ and Δ' = distances of Q and q from the refracting surface; then we may tabulate the results as follows:—

Case.	*Refracting Medium.*	*Surface.*	*Equation.*
1	Denser	Concave	$\frac{1}{\Delta'} = \frac{m-1}{m\,r} + \frac{1}{m\,\Delta}$
2	Denser	Convex	$\frac{1}{\Delta'} = -\frac{m-1}{m\,r} + \frac{1}{m\,\Delta}$
3	Rarer	Concave	$\frac{1}{\Delta'} = -\frac{m-1}{r} + \frac{m}{\Delta}$
4	Rarer	Convex	$\frac{1}{\Delta'} = \frac{m-1}{r} + \frac{m}{\Delta}$

In order to find the *principal focal distance*, which we call f (*see Reflexion* II) we have of course only to make Δ infinite in the equations just given; we have then

Case 1. $\frac{1}{f} = \frac{m-1}{m\,r}$, or $f = \frac{m}{m-1}\,r$.

Case 2. $\frac{1}{f} = -\frac{m-1}{m\,r}$, or $f = -\frac{m}{m-1}\,r$.

Case 3. $\frac{1}{f} = -\frac{m-1}{r}$, or $f = -\frac{1}{m-1}\,r$.

Case 4. $\frac{1}{f} = \frac{m-1}{r}$ or $f = \frac{1}{m-1}\,r$.

It is important to observe that in all cases the distance of the principal focus from the surface, is to its distance from the centre, as the sine of incidence to the sine of refraction.

If we introduce the distance f into the formulæ, we shall have in

Cases 1 & 2, $\frac{1}{\Delta'} = \frac{1}{f} + \frac{1}{m\,\Delta}$,

3 & 4, $\frac{1}{\Delta'} = \frac{1}{f} + \frac{m}{\Delta}$.

The following are corresponding values of Δ and Δ', in different positions of the conjugate foci Q and q.

Case 1. $\begin{cases} \Delta = \infty,\ r,\ 0,\ -\frac{r}{m-1},\ \infty \\ \Delta' = \frac{m\,r}{m-1},\ r,\ 0,\ \infty,\ \frac{m\,r}{m-1} \end{cases}$

Case 2. $\begin{cases} \Delta = \infty,\ \frac{r}{m-1},\ 0,\ -r.\ \infty \\ \Delta' = -\frac{m\,r}{m-1},\ \infty,\ 0,\ -r,\ \frac{m\,r}{m-1} \end{cases}$

Case 3. $\begin{cases} \Delta = \infty,\ \frac{m\,r}{m-1},\ r,\ 0,\ \infty \\ \Delta' = -\frac{r}{m-1},\ -\infty,\ r,\ 0,\ -\frac{r}{m-1} \end{cases}$

$$\text{Case 4.}\begin{cases}\Delta = \infty\,,\ 0, -r, -\dfrac{mr}{m-1}, \infty \\ \Delta' = \dfrac{r}{m-1}, 0, -r, \infty\,, \dfrac{r}{m-1}\end{cases}$$

2. To find the direction of a ray after refraction through a lens.

The method here is to consider a ray refracted at the first surface, as incident on the second, and there again refracted.

Let Δ'' be the distance of the focus after the second refraction, r' the radius of the second surface, t the thickness of the lens, the other symbols as above; then

$$\frac{1}{\Delta''} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) + \frac{1}{\Delta}.$$

To find the *principal* focal distance F, put Δ infinite in the above expression, i.e. suppose the rays parallel, and we have

$$\frac{1}{F} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$\text{and then } \frac{1}{\Delta''} = \frac{1}{F} + \frac{1}{\Delta}.$$

If we put $\frac{1}{\varrho}$ for $\frac{1}{r} - \frac{1}{r'}$ in the above equation,

$$\frac{1}{F} = \frac{m-1}{\varrho} \text{ or } F = \frac{\varrho}{m-1}.$$

Hence arise different values of $\frac{1}{F}$ according as $\frac{1}{r} - \frac{1}{r'}$ is positive or negative.

In the *concavo-convex* lens, either r is less than r' and F positive; or when the lens is turned the contrary way and r greater than r' they are both negative, we have then

$$\frac{1}{F} = (m-1)\left(\frac{1}{r'} - \frac{1}{r}\right)$$

In the *meniscus*, either r is greater than r', both being positive, and then

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r'} - \frac{1}{r}\right)$$

or r is less than r' and both are negative, so that

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

In the *double concave* lens, r' is negative,

$$\frac{1}{F} = (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

In the *double convex*, r is negative,

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

In the *plano-concave*, either r' is infinite, or r is infinite and r' negative; therefore putting r for the single radius

$$\frac{1}{F} = \frac{m-1}{r},\ F = \frac{r}{m-1}.$$

In the *plano-convex*

$$\frac{1}{F} = -\frac{m-1}{r},\ F = -\frac{r}{m-1}.$$

When in the *double concave* or *double convex* lens the radii are equal,

$$\frac{1}{F} = \pm (m-1)\ \frac{2}{r},\ \text{or } F = \pm \frac{r}{2.(m-1)}.$$

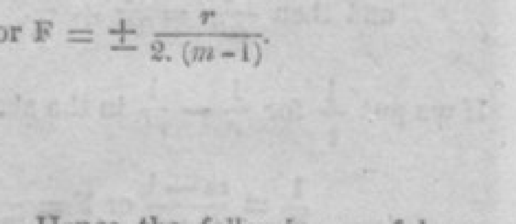
Since $\frac{1}{\Delta''} = \frac{1}{F} + \frac{1}{\Delta}$

$\Delta'' = \frac{\Delta F}{\Delta + F}$. Hence the following useful proportions, it being understood that F and f are the principal foci of rays coming in a contrary direction.

A q : A Q :: A F : A Q + A F

or Q f : f A :: Q A : A q

& ∴ Q f : Q A :: Q A : Q q



The following are corresponding values of Δ and Δ'' for a *concave lens* :—

$$\infty \ldots 2F \ldots F \ldots \frac{F}{2} \ldots 0 \ldots -\frac{F}{2} \ldots -F \ldots -2F \ldots -3F \ldots -\infty$$

$$F \ldots \frac{2}{3}F \ldots \frac{F}{2} \ldots \frac{F}{3} \ldots 0 \ldots -F \ldots \infty \ldots 2F \ldots \frac{3}{2}F \ldots F$$

From Penny Cycl: "Lens"

F = focal distance with its sign; f = its numerical value
R & R' – radii with their signs; r & r' their numer: values
let r be the radius of the side at wh: the light enters,

Then $\frac{1}{F} = (\mu - 1) \cdot \left(\frac{1}{R} + \frac{1}{R'}\right) + \frac{(\mu-1)^2}{\mu} \cdot \frac{t}{r^2}$

or correct F as found from the approx: formula $\frac{1}{F} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R'}\right)$ by subtracting from its algebraical value

$\frac{(\mu-1)^2}{\mu} \frac{F^2 t}{R^2}$ F being found from the preceding.

Thus application, *mean approx

1. Plano convex. $\frac{1}{F} = \frac{\mu - 1}{R'}$ or $f = \frac{r'}{\mu - 1}$
2. Convexo plane. $\frac{1}{F} = \frac{\mu-1}{R} + \frac{(\mu-1)^2 t}{\mu R^2}$; $f = \frac{r}{\mu-1} - \frac{t}{\mu}$*
3. Double convex $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} + \frac{1}{s}\right) + \frac{(\mu-1)^2}{\mu} \frac{t}{r^2}$
4. Plano concave $\frac{1}{F} = -\frac{\mu-1}{r'}$ or $f = \frac{r'}{\mu-1}$

Concavo-plane $\frac{1}{F} = -\frac{\mu-1}{r} + \frac{(\mu-1)^2}{\mu} \cdot \frac{t}{r^2}$; $f = \frac{r}{\mu-1} + \frac{t}{\mu}$*

The following are for a *convex* one :—

$$\infty, 2F, F, \frac{9}{10}F, \frac{F}{2}, \left(\frac{F}{n}\right), 0, -\frac{F}{2}, \left(-\frac{F}{n}\right), -F, -3F. -\infty$$

$$-F, -2F, \infty, 9F, F, \left(\frac{F}{n-1}\right), 0, -\frac{F}{3}, \left(-\frac{F}{n+1}\right), -\frac{F}{2}, -\frac{3}{4}F, -F.$$

If it be not thought proper to neglect t the thickness of the lens, then in the case of parallel incident rays the equation is

$$\frac{1}{F} = -(m-1)\left\{\frac{1}{r'} - \frac{1}{r}\left(1 + \frac{m-1}{m}\frac{t}{r}\right)^{-1}\right\}$$

3. If the lens be a sphere.

Put q = distance of the focus of incident rays from the centre ; q' = do. of focus of rays after the first refraction ; q'' = do. after two refractions ; then

$$\frac{1}{q''} = -2\,\frac{m-1}{mr} + \frac{1}{q}.$$

Cor. 1. To find the *principal* focus, suppose q infinite, or $\frac{1}{q} = o$;

$\therefore F = -\frac{mr}{2(m-1)}$. The negative sign meaning that the focus is on the opposite side from whence the light proceeds.

Cor. 2. If the sphere be glass, $m = \frac{3}{2}$, and $F = \frac{3}{2}r$. If water, $m = \frac{4}{3}$, and $F = 2r$.

4. If there be a compound lens, or a system of lenses placed close to each other, whose principal focal distances are $F, F', F'' \ldots\ldots F^{(n)}$, and F_n is the distance of the focus after all the refractions ; then

$$\frac{1}{F_n} = \frac{m-1}{\varrho} + \frac{m'-1}{\varrho'} + \frac{m''-1}{\varrho''} \ldots\ldots + \frac{m^{(n)}-1}{\varrho^{(n)}}$$

$$\text{or} = \frac{1}{F} + \frac{1}{F'} + \frac{1}{F''} \ldots\ldots + \frac{1}{F^{(n)}}.$$

Hence if, with Mr Herschel, we call the reciprocal quantity $\frac{1}{F}$ the *power* of a lens, we have the following enunciation,

" *The power of any system of lenses is the sum of the powers of the component lenses.*"

III. *Refraction, images produced by.*

1. The image of a straight line, formed by a plane refracting surface, is a straight line : and if a be the perpendicular distance of any point of the object from the surface, and m the ratio of refraction, the distance of its image $= m\,a$.

Cor. In the case of water, $m = \frac{3}{4}$. Thus the image of the bed of a river is nearer to the surface than the bed itself by $\frac{1}{4}$ of the whole depth.

2. If the object placed before a lens or a sphere be a circular arc concentric with it, the image will also be a circular arc concentric with and similar to the object.

3. The image of a straight line formed by a lens or sphere is the arc of a conic section.

4. The sun's image formed by a lens is a circle, and nearly in the principal focus : and the density of his rays when viewed with a reflector or refractor varies as $\frac{\text{area of aperture} \times \text{power}}{(\text{Focal length})^2}$.

TABLE,

Of the refractive and dispersive power of different substances, with their densities compared with that of water, which is taken as the unit.

The substances marked (*) are *combustible.*

The refraction is supposed to take place between the given substance and a vacuum.

Substance.	*Ratio of refraction*	*Dispersive power.*	*Density.*
Chromate of lead (strongest)	2.974	0.4	5.8
Realgar	2.549	0.267	3.4
Chromate of lead (weakest)	2.503	0.262	5.8
* Diamond	2.45	0.038	3.521
* Sulphur (native)	2.115		2.033
Carbonate of lead (strongest)	2.084	} 0.091	6.071
——— weakest	1.813		4.000
Garnet	1.815	0.033	3.213
Axinite	1.735	0.030	

If m be the index of refraction from vacuum into a medium A & m' that from vacuum into a medium B then $\frac{m}{m'}$, is the index when the light passes from the A to B.

Substance.	*Ratio of refraction*	*Dispersive power.*	*Density.*
Calcareous Spar (strongest)	1.665	0.04	2.715
——— weakest	1.519		
* Oil of Cassia	1.641	0.139	
Flint glass	1.616	0.048	3.329
——— another kind	1.590		
Rock crystal	1.562	0.026	2.653
Rock salt	1.557	0.053	2.130
Canada balsam	1.549	0.045	
Crown glass	1.544	0.036	2.612
Selenite	1.536	0.037	2.322
Plate glass	1.527	0.032	2.488
Gum arabic	1.512	0.036	1.452
* Oil of almonds	1.483		0.917
* Oil of turpentine	1.475	0.042	0.869
Borax	1.475	0.030	1.718
Sulphuric acid	1.440	0.031	1.850
Fluor spar	1.436	0.022	3.168
Nitric acid	1.406	0.045	1.217
Muriatic acid	1.374	0.043	1.194
* Alcohol	1.374	0.029	0.825
White of egg	1.361	0.037	1.090
Salt water	1.343		1.026
Water	1.336	0.035	1.000
Ice	1.307		0.930
Air	1.00029		0.0013
Oxygen	1.00028		0.0014
* Hydrogen	1.00014		0.0001
Nitrogen	1.00029		0.0012
Carbonic acid gas	1.00045		0.0018

REFRACTION, *terrestrial.—(Vince, Playfair, &c.)*

1. To determine it, let E = apparent elevation of a mountain from a point in the plain below; D = apparent depression of that point from the top of the mountain observed at the same moment; A = ∠ subtended at the earth's centre by the distance between them; then

$$\text{Refraction} = \frac{A + E - D}{2}.$$

The terrestrial refraction found by this theorem, when the elevation is not very great, varies from $\frac{1}{4}$ to $\frac{1}{24}$ of the ∠ A, but in the mean state of the atmosphere $= \frac{1}{14}$ of A, which, in taking the elevation of any object, must be subtracted from the observed ∠ E to give the correct elevation. Also the radius of curvature of the ray varies from twice to 12 times the earth's radius, but in the mean state of the atmosphere = 7 times earth's radius. When the ray is not horizontal it = $\frac{\text{7 times earth's radius}}{\text{sin. appar. zen. dist.}}$.

2. But in determining the height of a mountain, a correction may be made at once both for the curvature of the earth and for refraction thus. Let L = horizontal distance of the object in English miles, then the correction for curvature in feet is $\frac{2 L^2}{3}$, *(see Levelling)* and for refraction is $\frac{2 L^2}{21}$; ∴

$$\frac{2 L^2}{3} - \frac{2 L^2}{21} = \frac{4 L^2}{7}$$ = feet which must be added to computed height, and it will give correct height both for curvature and refraction.

3. To determine the most distant point on the earth's surface that can be seen from the top of a given height with and without refraction.

Let h = given height in miles, r = earth's radius, then in the mean state of the atmosphere, the distance of the farthest visible point = $\sqrt{\frac{7 r h}{3}}$; and distance, if there was no refraction, $= \sqrt{2 r h}$; ∴ distance which the eye can reach with refraction : do. without :: $\sqrt{7}$: $\sqrt{6}$ :: 14 : 13 nearly.

Cor. $\sqrt{\frac{7 r}{2}} = 96.1$ miles, ∴ the distance of the farthest visible point in miles, allowing for refraction, $= 96.1 \sqrt{h}$. Or by the last Art. if h' = height in *feet*, $\frac{4 L^2}{7} = h'$, ∴ $L = \frac{\sqrt{7 h'}}{2}$.

$= 1.323 \times \sqrt{h'}$

By the last formula the following Table was computed:—

TABLE,

Shewing the distance of the farthest visible point in miles that can be seen from the top of a given height, taking into account the effect of refraction.

Height in feet.	Dist. in miles.	Height in feet.	Dist. in miles.
5	2.96	500	29.5
10	4.18	700	35.0
15	5.12	1000	41.8
20	5.91	1500	51
25	6.61	2000	59
30	7.25	2500	66
35	7.82	3000	72
40	8.37	3500	78
50	9.35	4000	83
60	10.25	5000	94
70	11.1	6000	102
100	13.2	7000	110
150	16.2	8000	118
200	18.7	9000	125
250	20.9	10000	132
300	22.9	15000	162
400	26.4	20000	187

Ex. 1. The topmast of a ship 50 feet high was just visible to a spectator situated 20 feet above the level of the sea; required the distance of the ship.

	Feet.		*Miles.*
By table	50	give	9.35
	20	do.	5.91
Required distance			15.26

Ex. 2. The summit of Mouna Roa (whose height is supposed 15,000 feet) was observed at 180 miles distance; required the height of the observer.

	Miles.
Observed distance	180
Distance due to 15000 feet	162
Difference	18

which answers to a little less than 200 feet altitude.

TABLE *of distances at which mountains are said to have been observed.*

	Authorities.	Miles.
Himalaya mountains	Sir W. Jones	244
Mount Ararat	Bruce	240
Mouna Roa, Sandwich Isles (53 leagues)		180
Chimborazo (47 leagues)		160
Peak of Teneriffe from Cape of Lanzerota		135
Do. from ship's deck		115
Peak of Azores	Humboldt	126
Temaheud	Morier	100
Mount Athos	Dr Clarke	100
Adam's Peak		95
Ghaut at the back of Tellichery		94
Golden Mount from ship's deck		93
Pulo Pera from the top of Penang		75
Ghaut at Cape Comorin		73
Pulo Penang from ship's deck		53

The last six observations, and that of the Peak of Teneriffe, were made by a writer in the Calcutta Monthly Journal.

REFRACTION *of the heavenly bodies.—(Vince, Maddy.)*

1. The refraction of a star in the zenith is nothing, is greatest in the horizon, and at considerable altitudes is nearly as the tangent of the zenith distance. Or more nearly as tan. $(z - 3r)$, if z = zenith distance, and r the refraction found by the common rule.

Cor. Refraction $= 57'' \times \tan. (z - 3r.)$

2. To determine the refraction of a star by observation.

Observe the altitude and azimuth of a star of a known declination at the same moment: from the azimuth, the polar distance, and the complement of latitude, compute the altitude; the difference between this and the observed altitude is the refraction.

3. To determine how much the apparent time of rising and setting of a star is affected by refraction,

$$\text{Time} = \frac{\text{hor. refract.}}{15^0 \times \text{cos. lat.} \times \text{sin. star's azim.}}$$

Hence the time is least, when the star is in the equator. Or if l = latitude, δ = star's declination, r = hor. refraction.

$$\text{Time} = \frac{r}{15^{\circ} \sqrt{\left(\cos.\ (l + \delta).\ \cos.\ (l - \delta)\right)}}$$

4. Twilight is occasioned by the refraction and reflexion of the sun's rays passing through the atmosphere, and continues till the sun descends about 18° degrees below the horizon.

To find the duration of twilight.

Let h and h' be the hour angles corresponding to the beginning and end of twilight, l the latitude, and δ the sun's declination; then

$$\cos.\ h = -\tan.\ l.\ \tan.\ \delta$$

$$\cos.\ h' = -\sin.\ 18^{\circ}.\ \sec.\ l.\ \sec.\ \delta - \tan.\ l.\ \tan.\ \delta$$

hence $h' - h$ may be deduced.

Cor. Twilight will continue all night, if $l + \delta$ be greater than 72°.

To find the time of year when twilight is shortest.

$$\sin.\ \delta = -\tan.\ 9^{\circ}.\ \sin.\ l$$

$$\text{and } \sin.\ h = \sin.\ 9^{\circ}.\ \sec.\ l$$

The first equation gives the sun's declination, or the time when the twilight is shortest; and the second gives the duration of it.

Ex. In latitude 52°, the time of shortest twilight will fall about March 2, and October 11; and the duration will be about 1*h*. 58*m*.

5. The refraction varies with the state of the barometer and thermometer.

Dr Maskelyne's Formula.

Let a = height of barometer in inches, h = height of Fahrenheit's thermometer, z = zenith distance, $r = 57''$ tan. z; then

$$\text{Refraction} = \frac{a}{29.6} \times \tan.\ (z - 3\,r) \times 57'' \times \frac{400}{350 + h}.$$

Dr Young's Formula.

$$.0002825 = v.\ \frac{r}{s} + (2.47 + .5\,v^2)\,\frac{r^2}{s^2} + 3600\,v\,\frac{r^3}{s^3} + 3600\,(1.235 + .25\,v^2)\,\frac{r^4}{s^4}\ \ldots\ldots$$

r being the refraction, v the sine of altitude, and s the cosine.

From this last formula, the following Table, taken from the Nautical Almanack for 1827, is computed.

TABLE OF REFRACTIONS.

App. Altitu.	Refr. B. 30 Th.50°	Diff. for 1′ Alt.	Diff. for +1 B.	Diff. for -1° Fa	App. Altitu.	Refr. B 30 Th.50°	Diff. for 1′ Alt.	Diff. for +1 B.	Diff. for -1° Fa
D.M.	M.S.	S.	S.	S.	D.M.	M.S.	S.	S.	S.
0. 0	33.51	11,7	74	8,1	4. 0	11.52	2,2	24,1	1,70
5	32.53	11,3	71	7,6	10	11.30	2,1	23,4	1,64
10	31.58	10,9	69	7,3	20	11.10	2,0	22,7	1,58
15	31. 5	10,5	67	7,0	30	10.50	1,9	22,0	1,53
20	30.13	10,1	65	6,7	40	10.32	1,8	21,3	1,48
25	29.24	9,7	63	6,4	50	10.15	1,7	20,7	1,43
30	28.37	9,4	61	6,1	5. 0	9.58	1,6	20,1	1,38
35	27.51	9,0	59	5,9	10	9.42	1,5	19,6	1,34
40	27. 6	8,7	58	5,6	20	9.27	1,5	19,1	1,30
45	26.24	8,4	56	5,4	30	9.11	1,4	18,6	1,26
50	25.43	8,0	55	5,1	40	8.58	1,3	18,1	1,22
55	25. 3	7,7	53	4,9	50	8.45	1,3	17,6	1,19
1. 0	24.25	7,4	52	4,7	6. 0	8.32	1,2	17,2	1,15
5	23.48	7,1	50	4,6	10	8.20	1,2	16,8	1,11
10	23.13	6,9	49	4,5	20	8. 9	1,1	16,4	1,09
15	22.40	6,6	48	4,4	30	7.58	1,1	16,0	1,06
20	22. 8	6,3	46	4,2	40	7.47	1,0	15,7	1,03
25	21.37	6,1	45	4,0	50	7.37	1,0	15,3	1,00
30	21. 7	5,9	44	3,9	7. 0	7.27	1,0	15,0	,98
35	20.38	5,7	43	3,8	10	7.17	,9	14,6	,95
40	20.10	5,5	42	3,6	20	7. 8	,9	14,3	,93
45	19.43	5,3	40	3,5	30	6.59	,8	14,1	,91
50	19.17	5,1	39	3,4	40	6.51	,8	13,8	,89
55	18.52	4,9	39	3,3	50	6.43	,8	13,5	,87
2. 0	18.29	4,8	38	3,2	8. 0	6.35	,7	13,3	,85
5	18. 5	4,6	37	3,1	10	6.28	,7	13,1	,83
10	17.43	4,4	36	3,0	20	6.21	,7	12,9	,82
15	17.21	4,3	36	2,9	30	6.14	,7	12,6	,80
20	17. 0	4,1	35	2,8	40	6. 7	,7	12,3	,79
25	16.40	4,0	34	2,8	50	6. 0	,6	12,1	,77
30	16.21	3,9	33	2,7	9. 0	5.54	,6	11,9	,76
35	16. 2	3,7	33	2,7	10	5.47	,6	11,7	,74
40	15.43	3,6	32	2,6	20	5.41	,6	11,5	,73
45	15.25	3,5	32	2,5	30	5.36	,6	11,3	,71
50	15. 8	3,4	31	2,4	40	5.30	,5	11,1	,71
55	14.51	3,3	30	2,3	50	5.25	,5	11,0	,70
3. 0	14.35	3,2	30	2,3	10. 0	5,20	,5	10,8	,69
5	14.19	3,1	29	2,2	10	5.15	,5	10,6	,67
10	14. 4	3,0	29	2,2	20	5.10	,5	10,4	,65
15	13.50	2,9	28	2,1	30	5. 5	,5	10,2	,64
20	13.35	2,8	28	2,1	40	5. 0	,5	10,1	,63
25	13.21	2,7	27	2,0	50	4.56	,4	9,9	,62
30	13. 7	2,7	27	2,0	11. 0	4.51	,4	9,8	,60
35	12.53	2,6	26	2,0	10	4.47	,4	9,6	,59
40	12.41	2,5	26	1,9	20	4.43	,4	9,5	,58
45	12.28	2,4	25	1,9	30	4.39	,4	9,4	,57
50	12.16	2,4	25	1,9	40	4.35	,4	9,2	,56
55	12. 3	2,3	25	1,8	50	4.31	,4	9,1	,55

TABLE OF REFRACTIONS.

App. Altitu.	*Refr. B. 30 Th.50°*	*Diff. for 1′ Alt.*	*Diff. for +1 B.*	*Diff. for -1° Fa*	*App. Altitu.*	*Refr. B 30 Th.50°*	*Diff. for 1′ Alt.*	*Diff. for +1 B.*	*Diff. for -1° Fa*
D.M	M.S.	S.	S.	S.	D.	M.S.	S.	S.	S.
12. 0	4.28,1	,38	9,00	,556	42	1. 4,6	,038	2,16	,130
10	4.24,4	,37	8,86	,548	43	1. 2,4	,039	2,09	,125
20	4.20,8	,36	8,74	,541	44	1. 0,3	,034	2,02	,120
30	4.17,3	,35	8,63	,533	45	58,1	,034	1,94	,117
40	4.13,9	,33	8,51	,524	46	56,1	,033	1,88	,112
50	4.10,7	,32	8,41	,517	47	54,2	,032	1,81	,108
13. 0	4. 7,5	,31	8,30	,509	48	52,3	,031	1,75	,104
10	4. 4,4	,31	8,20	,503	49	50,5	,030	1,69	,101
20	4. 1,4	,30	8,10	,496	50	48,8	,029	1,63	,097
30	3.58,4	,30	8,00	,490	51	47,1	,028	1,58	,094
40	3.55,5	,29	7,89	,482	52	45,4	,027	1,52	,090
50	3.52,6	,29	7,79	,476	53	43,8	,026	1,47	,088
14. 0	3.49,9	,28	7,70	,469	54	42,2	,026	1,41	,085
10	3.47,1	,28	7,61	,464	55	40,8	,025	1,36	,082
20	3.44,4	,27	7,52	,458	56	39,3	,025	1,31	,079
30	3.41,8	,26	7,43	,453	57	37,8	,025	1,26	,076
40	3.39,2	,26	7,34	,448	58	36,4	,024	1,22	,073
50	3.36,7	,25	7,26	,444	59	35,0	,024	1,17	,070
15. 0	3.34,3	,24	7,18	,439	60	33,6	,023	1,12	,067
30	3.27,3	,22	6,95	,424	61	32,3	,022	1,08	,065
16. 0	3.20,6	,21	6,73	,411	62	31,0	,022	1,04	,062
30	3.14,4	,20	6,51	,399	63	29,7	,021	,99	,060
17. 0	3. 8,5	,19	6,31	,386	64	28,4	,021	,95	,057
30	3. 2,9	,18	6,12	,374	65	27,2	,020	,91	,055
18. 0	2.57,6	,17	5,98	,362	66	25,9	,020	,87	,052
19. 0	2.47,7	,16	5,61	,340	67	24,7	,020	,83	,050
20	2.38,7	,15	5,31	,322	68	23,5	,020	,79	,047
21	2.30,5	,13	5,04	,305	69	22,4	,020	,75	,045
22	2.23,2	,12	4,79	,290	70	21,2	,020	,71	,043
23	2.16,5	,11	4,57	,276	71	19,9	,020	,67	,010
24	2.10,1	,10	4,35	,264	72	18,8	,019	,63	,038
25	2. 4,2	,09	4,16	,252	73	17,7	,018	,59	,036
26	1.58,8	,09	3,97	,241	74	16,6	,018	,56	,033
27	1.53,8	,08	3,81	,230	75	15,5	,018	,52	,031
28	1.49,1	,08	3,65	,219	76	14,4	,018	,48	,029
29	1.44,7	,07	3,50	,209	77	13,4	,017	,45	,027
30	1.40,5	,07	3,36	,201	78	12,3	,017	,41	,025
31	1.36,6	,06	3,23	,193	79	11,2	,017	,38	,023
32	1.33,0	,06	3,11	,186	80	10,2	,017	,34	,021
33	1.29,5	,06	2,99	,179	81	9,2	,017	,31	,018
34	1.26,1	,05	2,88	,173	82	8,2	,017	,27	,016
35	1.23,0	,05	2,78	,167	83	7,1	,017	,24	,014
36	1.20,0	,05	2,68	,161	84	6,1	,017	,20	,012
37	1.17,1	,05	2,58	,155	85	5,1	,017	,17	,010
38	1.14,4	,05	2,49	,149	86	4,1	,017	,14	,008
39	1.11,8	,04	2,40	,144	87	3,1	,017	,10	,006
40	1. 9,3	,04	2,32	,139	88	2,0	,017	,07	,004
41	1. 6,9	,04	2,24	,134	89	1,0	,017	,03	,002

Explanation of the Table of Refractions.

The apparent altitude being found in the first column, the second shows the refraction when the barometer stands at 30 inches, which is its mean height on the level of the sea, and the thermometer at 50° of Fahrenheit. The third column contains the difference to be subtracted or added for every minute of altitude, reckoned from the nearest number in the first column. The fourth shows the number of seconds to be added for every inch that the height of the barometer exceeds 30, or to be subtracted for each inch that it wants of 30; and the last contains the number of seconds to be subtracted for each degree that the thermometer stands above 50°, or to be added for each degree that its height wants of 50°.

Ex. At 7°. 18'. 13". Bar. 29.87. Ther. 66°. required refraction.

Alt. 7°. 20'. R. 7'. 8"	Diff. Alt. ",9	B. 14", 3	Th. ",93
+ 1.62	1'. 47" = 1'. 8	− .13	− 16
7. 9,62	+ 1,62	1,86	14,88
16,74			1,86
Ref. = 6. 52,83			16,74

REFRANGIBILITY *of light.—See Light.*

RESISTANCE *of air to Projectiles.—See Gunnery.*

RESISTANCE *of Fluids.—See Fluids.*

RIVER.—*(Du Buat, Robison.)*

1. Let V = velocity of the stream per second in inches, R the quotient arising from the division of the section of the stream, expressed in square inches, by its perimeter *minus* the superficial breadth of the stream in linear inches, S the slope the numerator being unity, i.e. the quotient arising from dividing the length of the stream, supposing it extended in a straight line, by the difference of level of its two extremities, or let it be the cotangent of the inclination or slope;—then the section and velocity being both supposed uniform,

$$V = \sqrt{R - \frac{1}{10}} \left(\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h. l.}\left(S + \frac{16}{10}\right)} - \frac{3}{10} \right)$$

When R and S are very great

$$V = R^{\frac{1}{2}} \left(\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h. l. } S} - \frac{3}{10} \right) \text{ nearly.}$$

The slope remaining the same, the velocities are as $\sqrt{R - \frac{1}{10}}$ or as $\sqrt{R}$, when R is very great.

The velocity will become nothing by making the declivity so small that $\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h.l.}\left(S + \frac{16}{10}\right)} - \frac{3}{10} = 0$; but if $\frac{1}{S}$ is less than $\frac{1}{500000}$ or than $\frac{1}{10}$th of an inch to an English mile, the water will have sensible motion.

In the above formula R is called the *radius of the section.*

2. In a river the greatest velocity is at the surface and in the middle of the stream, from which it diminishes towards the bottom and sides, where it is least; and it has been found by experiment, that if v = velocity of the stream in the middle in inches, then the velocity at the bottom is

$$v - 2\sqrt{v} + 1.$$

3. The mean velocity, or that with which (were the whole stream to move) the discharge would be the same with the real discharge, is equal to half the sum of the greatest and least velocities, as computed in the last Prop. Hence the mean velocity $= v - \sqrt{v} + \frac{1}{2}$.

4. Suppose that a river having a rectangular bed is increased by the junction of another river equal to itself, the declivity remaining the same; required the increase of depth.

Let the breadth of the river $= b$, the depth before the junction $= d$, and after it $= x$; then

$$x^3 - \frac{3\,d^3}{b + 2\,d}\,x = \frac{4\,b\,d^3}{b + 2\,d},$$ a cubic equation which can always be resolved by Cardan's rule.

5. To find the fall of water under bridges, let the breadth of the river in feet $= b$; the breadth between the piers $= c$; the velocity in a second $= v$; $g = 32\frac{1}{6}$ feet; then the fall of the river will be

$$\left\{\left(\frac{25\,b}{21\,c}\right)^2 - 1\right\}\frac{v^2}{2g}.$$

Thus at London bridge $b = 926$, $c = 236$, reduced by the piles to $196\frac{3}{4}$, $v = 3\frac{1}{6}$, hence the fall is 4.739; by observation 4.75.—*(Young's Nat. Phil.)*

6. When the sections of a river vary, the quantity of water remaining the same, the mean velocities are inversely as the areas of the sections.

7. The following Table abridged from Dr Robison serves at once to compare the surface, bottom, and mean velocities in rivers according to the principles of Arts. 2, 3.—*(Gregory.)*

Velocity in Inches.			*Velocity in Inches.*		
Surface.	*Bottom.*	*Mean.*	*Surface.*	*Bottom.*	*Mean.*
4	1	2.5	56	42.016	49.008
8	3.342	5.67	60	45.509	52.754
12	6.071	9.036	64	49.0	56.5
16	9.0	12.5	68	52.505	60.252
20	12.055	16.027	72	56.025	64.012
24	15.194	19.597	76	59.568	67.784
28	18.421	23.210	80	63.107	71.553
32	21.678	26.839	84	66.651	75.325
36	25.0	30.5	88	70.224	79.112
40	28.345	34.172	92	73.788	82.894
44	31.742	37.871	96	77.370	86.685
48	35.151	41.570	100	81.0	90.5
52	38.564	45.282			

8. Eytelwein, a German mathematician, gives the following formula for the mean velocity of the stream of a canal. Let v be the mean velocity of the current in English feet, a the area of the vertical section of the stream, p the perimeter of the section, or sum of the bottom and two sides, l the length of the bed of the canal corresponding to the fall h, all in feet; then

$$v = -0.109 + \sqrt{9582 \frac{a\,h}{p\,l} + 0.0111}$$

9. To find experimentally the velocity of the water in a river, and the quantity which flows down in a given time, observe a place where the banks of the river are steep and nearly parallel, and by taking the depth at various places in crossing make a true section of the river. Stretch a string at right ∠'s. over it, and at a small distance another parallel to the first. Then take an apple, orange, or a pint or quart bottle partly filled with water so as just to swim in it, and throw it into the water above the strings. Observe when it comes under the first string by means of a quarter second pendulum or a stop watch, and observe also when it arrives at the second string. By this means the velocity of the upper surface, which in practice may *frequently* be taken for that of the whole, will be obtained. The section of the river at the second string must then be ascertained by taking various depths as before, and the mean of the two will be obtained by adding both together and taking half the sum for the mean section. Then the area of the mean section in square feet being multiplied by the distance between the strings in feet will give the contents of the water in solid feet which passed from

one string to the other during the time of observation; and this by the rule of three may be adapted to any other portion of time. This operation may often be greatly abridged by noticing the arrival of the floating body opposite to two stations on the shore, especially when it is not convenient to stretch a string across. Where a time piece is not at hand the observer may easily construct a quarter second or other pendulum.

RIVERS, *proportional lengths of, and supposed quantity of water discharged per annum.—(Ency. Brit. Suppl.)*

	Rivers.	Length.	Qy. of water.
Europe	Thames	1	1
	Rhine	$4\frac{1}{2}$	13
	Loire	4	10
	Po	$2\frac{1}{4}$	6
	Elbe	$4\frac{1}{2}$	8
	Vistula	$4\frac{1}{4}$	12
	Danube	$9\frac{3}{4}$	65
	Dneiper	$7\frac{3}{4}$	36
	Don	$7\frac{1}{2}$	38
Asia ...	Wolga	14	80
	Euphrates	$9\frac{3}{4}$	60
	Indus	$11\frac{1}{2}$	133
	Ganges	10	148
	Kang-tse or Great river of China	$21\frac{1}{2}$	258
	Amour, Chinese Tartary	16	166
	Lena, Asiatic Russia	$13\frac{1}{2}$	125
	Oby do.	15	179
Africa	Nile	$18\frac{1}{2}$	250
America	St Lawrence including Lakes	$22\frac{1}{2}$	112
	Mississippi	19	338
	Plata	$13\frac{1}{2}$	490
	Amazon, not including Araguay	$22\frac{3}{4}$	1280

To deduce the approximate lengths of the rivers in miles from the proportional lengths we may multiply the latter by 180. To convert the proportional discharge into known measures we may multiply by 1800 to obtain the number of *cubic feet per second*, or by .4 or $\frac{4}{10}$ to find the annual discharge in *cubic miles.*

Proportional lengths according to Major Rennell.

EUROPE	Thames	1
	Rhine	5¼
	Danube	7
	Volga	9½
ASIA ...	Indus	6¾
	Euphrates	8½
	Ganges	9½
	Burrampooter	9½
	Ava River	9½
	Jenicei	10
	Obi	10½
	Amour	11
	Lena	11½
	Hoang-Ho	13½
	Kian Ku	15½
AFRICA	Nile	12½
AMERICA	Mississippi	8
	Amazon	15¾

ROOFS, *equilibrium of.—(Whewell.)*

1. A roof A C A′, consisting of beams forming an isosceles triangle with its base horizontal, supports a given weight at its vertex C: the weights of the beams being also given; it is required to find the horizontal force at A and A′.

Let B be the weight of the beam A C, C the weight at C, α the angle which A C makes with the horizon, H the horizontal pressure at A; then

$$H = \frac{B + C}{2 \tan. \alpha}.$$

If there is no beam joining A A′, this horizontal pressure H must be counteracted by the supports on which the ends A, A′ are placed.

If the roof A C A′ support a covering of uniform thickness, the formula will still be true including in the weight B, the weight of that portion of the covering which rests upon the beam.

The weight C at the point C may arise from a longitudinal beam perpendicular to the plane A A′ C.

2. Any number of given beams, arranged as sides of a polygon, in a vertical plane, support each other, and support also given weights at the ∠s; it is required to find the horizontal pressure at the points of support.

Let B and $B_{,}$ be the weights of two contiguous beams, α and $\alpha_{,}$ the angles they make with the horizon, and C the given weight at the ∠, or point of junction; then

$$H = \frac{\frac{1}{2}(B + B_{,}) + C}{\tan.\alpha - \tan.\alpha_{,}}.$$

This horizontal pressure is the same at all the angles.

Cor. If we suppose the weights of the beams $= o$, $H = \frac{C}{\tan.\alpha - \tan.\alpha_{,}}$; If we suppose no weights, except the beams,

$$H = \frac{\frac{1}{2}(B + B)}{\tan\alpha - \tan\alpha_{,}}.$$

3. To find the position of the beams, having given their weights B_1, B_2, B_3 &c. the weights C_1, C_2, C_3 &c. and the position of two of them.

By the last Prop. we have the following equations, α_1, α_2, α_3 being the ∠s which the beams make with the horizon.

$$H(\tan.\alpha_{,} - \tan.\alpha_2) = \tfrac{1}{2}(B_1 + B_2) + C_1$$

$$H(\tan.\alpha_2 - \tan.\alpha_3) = \tfrac{1}{2}(B_2 + B_3) + C_2$$

&c. &c.

If there be n beams there will be $n - 1$ weights C_1, C_2 &c. and $n - 1$ equations. The number of unknown quantities is $n + 1$, viz. the n tangents, $\tan.\alpha_1$, $\tan.\alpha_2$ &c. and the pressure H. Hence if we know two of the ∠s α_1, α_2, we can find the rest.

ROOTS *of numbers.—See Involution.*

ROPES, *rigidity of.—See Friction.*

ROTATION *of bodies about a fixed or moveable axis.*

The following Proposition is of the greatest use in Mechanics, and is general under the circumstances there mentioned, whether bodies move in right lines or have a rotatory motion. It applies with peculiar facility to the investigation of the motion of revolving bodies, and by the help of it the most difficult problems admit of a simple and easy solution.

Prop. If a system of bodies be connected together and supported at any point which is not the centre of gravity, and then left to descend by that part of their weight which is not supported; $2g$ multiplied into the sum of all the products of each body into the space it has perpendicularly descended will be equal to the sum of all the products of each body into the square of its velocity, g being $= 32\frac{1}{6}$ feet.—(*Mr Dawson, Sedbergh.*)

A demonstration of this Prop. may be seen in Leybourn's Mathematical Repository.

Ex. 1. Let a cylinder whose weight $=$ W, moveable about a horizontal axis passing through the centre, be put in motion by a weight P attached to a string wound round it; required the force accelerating the body P, and the space descended in t seconds.

Let $s =$ space perpendicularly descended by P, $v =$ velocity acquired in the time t, $r =$ radius of the cylinder, $x =$ distance of the centre of gyration from the centre of the cylinder; then by the Prop.

$$2g \times Ps = Pv^2 + W \times v^2 \frac{x^2}{r^2} = Pv^2 + W \times \frac{v^2}{2}$$

$$\text{but } s = \frac{tv}{2}, \therefore v = \frac{2gPt}{2P + W}, \text{ or if } t = 1$$

$$v = \frac{2gP}{2P + W} = \text{accelerating force.}$$

To find s, put $v^2 = \frac{4s^2}{t^2}$ in the leading equation, and we shall have

$$s = \frac{gPt^2}{2P + W}.$$

Ex. 2. A given cylinder with a thread wound round it is suffered to unwrap itself and descend; required the time of its descent through a given space.

The same notation being retained

$$2g \times Ws - Wv^2 + Wv^2 \times \frac{x^2}{r^2} = W \times \left(v^2 + \frac{v^2}{2}\right)$$

$$= \frac{3Wv^2}{2} \text{ but } v = \frac{2s}{t},$$

$$\therefore t = \sqrt{\frac{3s}{g}}.$$

Ex. 3. P and W are hung over a fixed pulley, to find how far P will descend in t''.

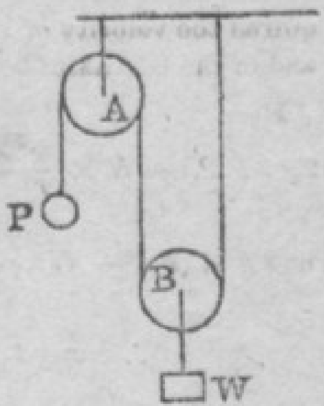
Let r = radius of pulley, w = its weight, x = distance of the centre of gyration from its centre; then

$$2g \times (P - W)\,s = (P + W)\,v^2 + w\,r^2 \times \frac{x^2}{r^2}$$

$$= (P + W)\,v^2 + w.\,\frac{v^2}{2},\ \text{but } v = \frac{2s}{t},$$

$$\therefore s = \frac{g\,t^2\,(P - W)}{2P + 2W + w}.$$

Ex. 4. Let A and B represent a single fixed and moveable pulley as represented in the annexed figure; required the space which the descending weight P describes in a given time.



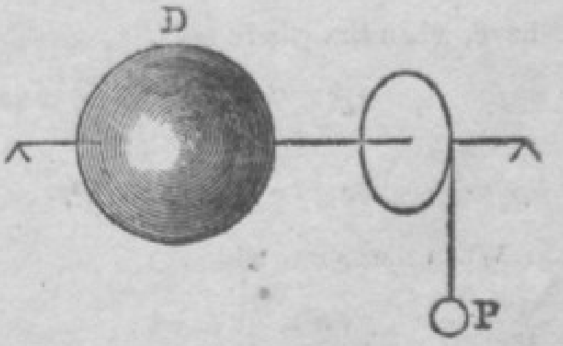
Let w = weight of each pulley, v = velocity of P, then $\frac{v}{2}$ = velocity of W; also $\frac{v}{\sqrt{2}}$ = velocity of the centre of gyration of A, and $\frac{v}{2\sqrt{2}}$ = velocity of the same centre in B; then

$$2g \times \left(P s - W.\,\frac{s}{2}\right) = P\,v^2 + W \times \frac{v^2}{4} + w \times \frac{v^2}{2} + w \times \frac{v^2}{8}$$

$$= P\,v^2 + W \times \frac{v^2}{4} + w \times \frac{5\,v^2}{8},\ \text{but } v = \frac{2s}{t},\ \therefore$$

$$s = \frac{2g\,t^2 \times (2P - W)}{8P + 2W + 5w}$$

Ex. 5. A sphere D, whose radius is ϱ and weight W, is put in motion by a weight P acting by means of a string going over a wheel whose radius is r: required the velocity acquired in the time t.



Let v = velocity of P, s the space descended by P in t'', x = distance of the centre of gyration of the sphere from its centre; then

$$2g \times P s = P v^2 + W\,v^2 \times \frac{x^2}{r^2};\ \text{but } x = \varrho \times \sqrt{\frac{2}{5}},$$

$$\therefore 2g \times Ps = Pv^2 + Wv^2 \times \frac{2\rho^2}{5r^2}$$

$$\text{but } s = \frac{tv}{2}, \therefore v = \frac{5gPtr^2}{5r^2P + 2W\rho^2}.$$

Or by substituting $\frac{2s}{t}$ for v; s or t may be found.

Ex. 6. Let a weight P, fastened to a string going over a wheel, by its descent cause two weights W, W′ to be wound up on two axles. Required the velocity of P after it has descended t''; the radii of the wheel and of the two axles being r, ρ, ρ'.

Here

$$2g \times \left(Ps - W \times \frac{s\rho}{r} - W' \times \frac{s\rho'}{r} \right) = Pv^2 + Wv^2 \times \frac{\rho^2}{r^2} + W'v^2 \times \frac{\rho'^2}{r^2}$$

$$\text{or } 2g \times (Pr^2 - Wr\rho - W'r\rho') \times \frac{tv}{2} = (Pr^2 + W\rho^2 + W'\rho'^2)v^2$$

$$\therefore v = gt \times \frac{Pr^2 - Wr\rho - W'r\rho'}{Pr^2 + W\rho^2 + W'\rho'^2}.$$

Here the weight of the wheel and axles are not taken into the account.

Ex. 7. The force which accelerates the centre of gravity of a sphere, while it rolls down an inclined plane, is to the force by which it would be accelerated, were the sphere to slide, in the ratio of 5 to 7.

Let W = weight of the sphere, s = space descended along the plane, v = velocity generated in time t when the sphere rolls, v' = do. when it slides, then since the distance of the centre of gyration $= r\sqrt{\frac{2}{5}}$, we have, when the sphere rolls,

$$2g \times W \times s \times \frac{H}{L} = Wv^2 + W \times v^2 \times \frac{2}{5}.$$

$$\text{or } gtv \times \frac{H}{L} = \frac{7}{5}v^2.$$

When the sphere slides,

$$gtv \times \frac{H}{L} = v'^2;$$

$$\therefore v : v' :: 5 : 7.$$

S

SATELLITES.—*(Vince, Playfair.)*

1. *Of Jupiter.*

Jupiter's satellites were discovered by Galileo in 1610. The times of their rotation are the same with the periodic times round the primary. Occultations happen to the first and second satellites at every revolution; the third very rarely escapes an occultation, but the fourth more frequently, by reason of its distance. The three first are eclipsed in every revolution, the fourth not always. In the first satellite we never can see both the immersion and emersion; the other three satellites *may* have both visible, but it depends on the position of the earth. The first satellite is the most proper for finding the longitude, its tables being the most correct. The observer should be settled at his telescope three minutes before the expected time of an immersion of the first satellite, six or eight minutes before that of the second or third; and at least a quarter of an hour before that of the fourth. If the longitude be different from that of Greenwich, allowance must be made for it. The telescopes proper for observing these eclipses are reflecting ones of 18 inches or 2 feet, or the 46 inch achromatic with three object glasses.

There is a singular analogy between the three first satellites, discovered by Laplace, viz. that if m', m'', m''', are the mean motions of the 1st, 2d, and 3d, satellites of Jupiter,

$$m' + 2\,m''' = 3\,m''.$$

Also if L′, L″ L‴, are the mean longitudes of these satellites,

$$L' - 3\,L'' + 2\,L''' = 180^{0}.$$

The last equation shews that the three satellites can never be eclipsed at the same time.

Table of the Satellites.

	I.	II.	III.	IV.
Sidereal Revolution.	*d. h. m. s.* 1 18 27 33	*d. h. m. s.* 3 13 13 42	*d. h. m. s.* 7 3 42 33	*d. h. m. s.* 16 16 32 8
Synodic Revolution.	1 18 28 36	3 13 17 54	7 3 59 36	16 18 5 7
Mean distance, the radius of the Planet being 1.	5.812964	9.248679	14.752401	25.946860
Mass.	.0000173281	.0000232355	.0000884972	.0000426591
Greatest duration of an eclipse.	*h. m.* 2 16	*h. m.* 2 54	*h. m.* 3 34	*h. m.* 4 48

TABLE

Of the apparent distances of Jupiter's satellites from its limb at the time of an eclipse, in ½ diameters of Jupiter and decimal parts, for every tenth day of Jupiter's distance from opposition or conjunction. *Note.*—Before the oppositions of Jupiter the immersions and emersions happen to the west of Jupiter; after opposition they happen to the east; in an astronomical telescope the appearance will be contrary.

Distance of Jupiter from opposition to the Sun.	*Distance of the Satellites from Jupiter's limb at the eclipses, in semidiameters of Jupiter.*				*Distance of Jupiter from conjunction with the Sun.*	*Distance of the Satellites from Jupiter's limb at the eclipses, in semidiameters of Jupiter.*			
Days.	I.	II.	III.	IV.	*Days.*	I.	II.	III.	IV.
10	0.20	0.33	0.50	0.85	10	0.15	0.25	0.35	0.55
20	0.40	0.66	1.05	1.66	20	0.30	0.45	0.70	1.25
30	0.60	0.95	1.50	2.65	30	0.40	0.67	1.05	1.70
40	0.75	1.20	1.90	3.35	40	0.55	0.90	1.40	2.50
50	0.90	1.40	2.25	3.95	50	0.70	1.00	1.80	3.20
60	1.00	1.60	2.50	4.40	60	0.80	1.25	2.00	3.50
70	1.05	1.70	2.66	4.70	70	0.90	1.40	2.25	3.95
80	1.10	1.75	2.75	4.85	80	1.00	1.55	2.45	4.33
90	1.10	1.75	2.75	4.85	90	1.05	1.66	2.60	4.60
100	1.10	1.70	2.70	4.80	100	1.10	1.75	2.70	4.80

The difficulty of observing the immersions, and particularly the emersions of Jupiter's satellites, may be attributed to the observer not having his eye well directed to the spot at which the satellite first issues from the shadow. The discordancies will be materially diminished by the above Table, particularly if a diagram be formed from it, representing the disk of Jupiter at the several times mentioned in the Table, and the proportional distances of the several satellites as there expressed.

2. *Saturn's satellites.*

The 4th satellite of Saturn was discovered by Huygens, in 1655; and the 1st, 2d, 3d, and 5th by Cassini, within the years 1671 and 1684. Herschel discovered two others in 1789 interior to the other five, but which, to prevent confusion, are called the 6th and 7th, the 7th being the innermost. They revolve nearly all in the same plane, inclined to Saturn's orbit at an ∠ of about 30°; hence they are eclipsed seldomer than Jupiter's. The 5th satellite (like those of Jupiter) revolves round its axis in the same time as round Saturn; a remarkable instance of analogy among the secondary planets.

Table of the Satellites.

	VII.	VI.	I.	II.	III.	IV.	V.
Sidereal Revolution.	*h. m. s.* 22.37.30	*d.h.m.s* 1.8.53.9	1.21.18.26	2.17.44.51	4.12.25.11	15.22.41.14	79.7.54.37
Synodic Revolution.			1.21.18.55	2.17.45.51	4.12.27.55	15.23.15.23	79.22.3.13
Mean dist. rad. of planet being 1.	3.080	3.952	4.893	6.268	8.754	20.295	59.154

3. *Satellites of the Georgian Planet.*

These six satellites were discovered by Dr Herschel, in 1787 and 1788. They all move in a plane which is nearly *perpendicular* to the plane of the planet's orbit, and *contrary to the order of the signs.*

Table of the Satellites.

	I.	II.	III.	IV.	V.	VI.
Sidereal Revolution.	*d. h. m. s.* 5.21.25.20	8.16.57.47	10.23.3.59	13.10.56.30	38.1.48.0	107.16.39.56
Mean dist. rad. of planet being 1.	13.120	17.022	19.845	22.752	45.507	91.008

SATURN. *For its elements, &c.—See Planets elements of. And for its satellites—See Satellites.*

Saturn's ring.—(Vince.)

Galileo announced his discovery of Saturn's ring in 1610. Dr Herschel and others have since ascertained that it consists of two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet. The dimensions of the rings are as follows:—

	Miles.
Inside diameter smaller ring	146345
Outside do.	184393
Inside diameter larger ring	190248
Outside do.	204883
Breadth of inner ring	20000
——— of outer ring	7200
Space between rings	2839
——— between planet and ring	70277
Mean thickness of ring	4500

Time of rotation .... 10*h*. 32*m*. 15,4*s*.

When Saturn's geocentric longitude is 5*s*. 20°. or 11*s*. 20°, his ring is invisible to us. When he is in 2*s*. 20°. or 8*s*. 20°. we may see it to most advantage; its minor axis is then nearly half its major. But the following Table will shew both the apparent figure of the ring, and of the orbits of the six first satellites, at all times, and as seen either from the sun or the earth.

N.B. When the geocentric latitude and longitude are taken, we get the appearance as seen from the earth; the heliocentric latitude and longitude being assumed, gives the appearance as seen from the sun.

For the ring and six first Satellites.

Arg. Long. Sat. + 13°. 43′. 30″

Deg.	O. VI. − +	I. VII. − +	II. VIII. − +	Deg.
0	0,000	0,260	0,451	30
3	0,027	0,284	0,464	27
6	0,054	0,306	0,476	24
9	0,081	0,328	0,486	21
12	0,108	0,348	0,495	18
15	0,135	0,368	0,503	15
18	0,161	0,384	0,509	12
21	0,187	0,405	0,514	9
24	0,212	0,421	0,518	6
27	0,236	0,437	0,520	3
30	0,260	0,451	0,521	0
	+ − XI. V.	+ − X. IV.	+ − IX. III.	

To the quantity taken from the Tables, apply the latitude of Saturn expressed in minutes divided by 4000, with the sign —, when the latitude is north, and +, when it is south; and the result gives the minor axis of the ring or of the orbits, the major axis being unity.

Ex. On April 22, 1767, the geocentric latitude of Saturn was 1°. 10′ south, and longitude 2*s*. 16°. 55′; hence for the ring and six first satellites

2s. 16°. 55′.		
13 43		
3 0 38	....	− 0.521
70 / 4000	....	+ 0.017
Minor axis	....	− 0,504

The sign + shews that that half of the ring, or of the orbits, which is most distant, is more *north* than the centre of Saturn, and the sign − shews it to be more south.

SCREW.

When there is an equilibrium upon the screw, P : W :: the distance between two contiguous threads, measured in the direction of the axis : the circumference of the circle which the power describes.

Hence if $d =$ distance between the threads, $a =$ radius of the circle described by the power, $P : W :: d : 2 \pi a$; $\therefore P = \frac{W d}{2 \pi a}$, from which equation any three of the four quantities P, W, a, d being given, the fourth may be found.

In the endless screw, which works in, and turns a dented wheel, let $a =$ length of the lever, $R =$ radius of the wheel, $r =$ do. of the axle, the rest as before; then

$$P : W :: d r : 2 \pi a R;$$

$$\therefore P = \frac{W d r}{2 \pi a R}.$$

SEA WATER, *specific gravity of.*

Table of the specific gravity of sea water in various parts of the globe, as ascertained by Dr Marcet.

Arctic Ocean	1.02664	Sea of Marmora	1.01915
Northern hemisphere	1.02829	Black Sea	1.01418
Equator	1.02777	White Sea	1.01901
Southern hemisphere	1.02882	Baltic	1.01523
Yellow Sea	1.02291	Ice—sea waters	1.00057
Mediterranean	1.02930	Lake of Ourmia	1.16507
Dead Sea	1.21100		

From the preceding facts, Dr Marcet concludes,

1. That the Southern Ocean contains more salt than the Northern in the ratio of 1,02919 to 1.02757,

2. That the mean spec. grav. of sea water near the equator is 1.02777; intermediate between that of the N. and S. hemispheres.

3. That there is no notable difference in sea waters under different meridians.

4. That there is no satisfactory evidence that the sea at great depths is more salt than at the surface.

5. That the sea in general contains more salt where it is deepest, and most remote from land, and that its saltness is always diminished in the vicinity of large masses of ice.

6. That small inland seas, though communicating with the ocean, are much less salt than the open ocean.

7. But that the Mediterranean contains rather larger proportions of salt than the ocean.

SEA WATER, *saline contents of.*

Sea water contains in solution, muriate of soda, sulphate of soda, muriate of lime, and muriate of magnesia; Dr Wollaston has also ascertained that it contains potash, though in a proportion less than $\frac{1}{2000}$th part of sea water at its average density. The following analysis of sea water, brought from the middle of the North Atlantic, as given by Dr Marcet, may serve as a specimen. The quantity operated upon was 500 grains:—

Muriate of soda	13,3	grs.
Sulphate of soda	2,33	
Muriate of lime	0,975	
Muriate of magnesia	4,955	
	21,460	

Analysis of the water of the Dead Sea, by Dr Marcet; the quantity operated upon being 100 grains.

Muriate of lime	3,920	grs.
Muriate of magnesia	10,246	
Muriate of soda	10,360	
Sulphate of lime	0,054	
	24,580	

SEA WATER, *temperature of.*

In Baffin's Bay, the Mediterranean Sea, and the Tropical Seas, the temperature of the sea diminishes with the depth, according to the observations of Phipps, Ross, Parry, Sabine, Saussure, Ellis, and Peron, but this diminution is not subject to any regular law. At the depth of 100 fathoms the difference is sometimes no more than 1°, and sometimes as great as 20°. Sometimes the coldness attains its maximum at 100 fathoms, and sometimes it increases to 400 and 500. Humboldt thinks that, on a mean, the change is about six times more rapid than in the atmosphere, or about 1° in 50 feet; but the facts are too anomalous to be easily brought under any general rule. It is a remarkable fact that in the Arctic or Greenland seas the temperature of the sea increases with the depth. This singular result was first obtained by Mr Scoresby, and has been confirmed by the later observations of Franklin, Beechy, and Fisher.

The following are some of Mr Fisher's results obtained on board the Dorothea:—

Latitude.	*Longitude*	*Depth in Fath.*	*Temp. at Bottom.*	*Temp. of Surface.*
Between 79° 50′ & 80. 14.	11°. 30′. E	40 ...	35°.5 ..	31°.8
		60 ...	36.0 ...	32.0
		100 ...	36.3 ...	32.0
		124 ...	36.7 ...	33.5
		140 ...	36.5 ...	32.0
		188 ...	42.5 ...	33.0
		304 ...	39.0 ...	31.0

And similar results were obtained by Lieut. Beechy and Mr Scoresby. The greatest difference found by Lieut. Parry was 6° at a depth of 246 fathoms; and the greatest obtained by Capt. Sabine was 7½° at a depth of 680 fathoms.

SEAS POLAR.—*(Enc. Brit. Supp.)*

Short chronological notice of the principal navigators, who have explored the Polar seas, from the voyages of Davis to the present time, with the highest latitude reached by each.

Year.	NORTH.	*Highest Lat.*
1585	Davis, three voyages	72°. 12′ Davis Strait
1594	Barentz, three voyages	80. 11. Spitzbergen.
1602	Weymouth	—— Resolution Island

Year.	North.	Highest Lat.	
1605	Hall and Knight	66. 55.	W. coast of Greenland.
1606	Knight	56. 48.	Labrador.
1607	Hudson, four voyages	78. 56.	E. coast of Greenland.
1611	Button	65. 0.	Southampton Island.
1612	Hall	67. 0.	W. coast of Greenland.
1614	Gibbon	——	Labrador.
1615	Baffin, two voyages	78. 0.	Baffin's Bay.
1631	Fox	——	Fox's Farthest.
1741	Middleton	66. 14.	Cape Hope.
1746	Moor and Smith	——	Repulse Bay.
1773	Phipps and Lutwidge	80. 48.	W. coast of Spitzbergen.
1779	Cook and Clarke	70. 41.	Behring's Strait.
1787	Lowenorn	66. 30.	E. coast of Greenland.
1791	Duncan	——	Chesterfield Inlet.
1806	Scoresby	81. 30.	Longitude 19° E.
1819	Parry's second voyage	75. 35.	Melville Island.
	Farthest point westward	74. 26. 25.	Long. 113. 46. 43. W.
Do.	Capt. Franklin, land expedition	67. 48.	Coppermine River, and 5 or 600 miles to the eastward.
1826	Franklin & Richardson, do.	——	Mackenzie's River, and from 113 to 149. 38. W. Long.
1827	Parry's 4th	81. 5½. and on the ice to 82. 45½.	20°. E. Long.
	South.		
1774	Cook	71. 10.	Long. 101 to 110 W.
1822	Weddell	74. 15.	Long. 34. 16. 45. W.

No human beings are found in the Southern Ocean below the 55th parallel of latitude, and none beyond the 50th, except on Patagonia and Terra del Fuego.

It is impossible to enter here into any of those points of scientific research which these expeditions have been the means of communicating. It may not, however, be uninteresting to subjoin the result of Captain Parry's observations on the temperature of Melville Island, in 1819 and 1820, as indicating a very extraordinary degree of cold.

		Greatest Temperat.	Least.	Mean.
1819	September	+ 37°	— 1°	+ 22°.54
	October	+ 17.5	— 28	— 3.46
	November	+ 6	— 47	— 20.60
	December	+ 6	— 43	— 21.79
1820	January	— 2	— 47	— 30.09
	February	— 17	— 50	— 32.19
	March	+ 6	— 40	— 18.10
	April	+ 32	— 32	— 8.37
	May	+ 47	— 4	+ 16.66
	June	+ 51	+ 28	+ 36.24
	July	+ 60	+ 32	+ 42.41
	August	+ 45	+ 22	+ 32.68
			Annual temperature	+ 1.33

According to Leslie's Table *(see Atmosphere)* the temperature of Melville Island should have been nearly 36°, whereas it is only $1°\frac{1}{5}$.

	Inches.
Greatest height of barometer was	30.86
Least do.	29.00

SEA, *extent of.—See Earth.*

SEASONS, *length of.—See Earth, elements of.*

SECANTS, *figure of.—See Figure.*

SEMIDIURNAL *arcs.—See Arcs Semidiurnal.*

Series – see Progression

SHIPS, *tonnage of.*

To find the tonnage of Ships.

Rule 1.—Multiply the length of the keel, taken within the vessel, or as much as the ship treads upon the ground, by the length of the midship beam, taken also within, from plank to plank, and that product by half the breadth, taken as the depth; then divide the last product by 94, and the quotient will give the tonnage.

If the length of a ship's keel be 80 feet, and the midship-beam 30; required the tonnage. *Ans.* 382.9787 + tons.

Rule 2.—Shipwrights take the dimensions on the outside of the light mark, as the ship swims, being unladen, to find the content of the empty ship. But if the measure of the ship be taken from the light mark to her full draught of water, when laden, it will give the burden of the ship; and then the length, breadth, and depth multiplied together, and the product divided by 100 for men of war (which gives an allowance for guns, anchors, &c. that are all burden but no tonnage) and by 95 for merchant ships, will give the tonnage.

N.B. A hundred solid feet make a ton.

Required the tonnage of a ship, whose length is 300 feet, breadth 50, and depth 30.

Ans. $4736\frac{16}{19}$ tons.

Rule 3.—At London, shipwrights multiply the length of the keel by the extreme breadth of the ship, taken from outside to outside, and that product by half the breadth; and this they divide by 94 for merchant ships, and by 100 for men of war; the quotients are the tonnage of the vessels of their respective classes.

Required the tonnage of an eighty gun ship, the length of whose keel is 149 feet 4 inches, and her extreme breadth 49 feet 8 inches.

Ans. 1841·86 + tons.

The following method is used in the Royal Navy:—

Rule 4.—Let fall a perpendicular from the foreside of the stern at the height of the hawse holes, and another from the back of the main port at the height of the wing transom; from the distance between these perpendiculars deduct $\frac{3}{5}$ of the extreme breadth, and as many times 2½ inches as there are feet in the height of the wing transom above the upper edge of the keel, the remainder is the length of the keel for tonnage. Then multiply the length of the keel by the extreme breadth, and that product by half the breadth; divide this product by 94 for the tonnage.

Given the length of the keel 68 feet, and the extreme breadth 22; required the tonnage.

Ans. $175\frac{6}{94}$ tons.

Ship-building.

A man-of-war of 74 guns requires about 3000 loads of timber, of 50 cubic feet each; worth, at £5. a load, £15,000. A tree contains about two loads, and 3000 loads would cover fourteen acres. The value of shipping in general is estimated at £8. or £10. a ton.

It is said that 180,000 pounds of hemp are required for the rigging of a first-rate man-of-war.—(*Young's Nat. Phil.*)

Note.—The above calculation of fourteen acres to a 74 gun ship is probably much too low. It will be nearer the truth to suppose each tree to contain only a load and a half of timber, and that every acre contains 35 trees fit for naval purposes; this gives 57 acres of land for a 74 gun ship. *See Report of the Board of Commissioners of Woods and Forests*, 1812.

SHOT, *pile of.*

Shot or shells are usually piled up in a pyramidal form, the base being an equilateral triangle, square, or rectangle.

The following formulæ give the total number of balls in any of these piles:—

$$\text{Triangular pile} = \frac{n.\ (n+1.)\ (n+2.)}{6}.$$

$$\text{Square pile} = \frac{n.\ (n+1.)\ (2n+1.)}{6}.$$

$$\text{Rectangular pile} = \frac{m.\ (m+1.)\ (3n-m+1)}{6}.$$

Where n in the two first formulæ denotes the number of balls in the side of the base; and in the last n is the number of balls in the length of the base, and m the number of those in the breadth.

SHOT, *weight of.—(Hutton.)*

Let W be the weight in pounds, δ = diameter in inches, then

$$\text{In iron balls, } W = \frac{9}{64} \times \delta^3.$$

$$\text{In leaden, } \quad W = \frac{3}{14} \times \delta^3.$$

In iron shells, if D and δ be the external and internal diameters, $W = \frac{9}{64} \times (D^3 - \delta^3.)$

SHOT.—*See Gunnery.*

SIDEREAL *time.—See Time.*

SINES, *figure of.—See Figure.*

SINES, *arithmetic of.—See Trigonometry.*

SIPHON, *oscillatory motion of water in.—(Playfair.)*

1. Let an inverted siphon, partly filled with water, be composed of three rectilinear tubes of equal diameters, of which the intermediate one is horizontal, and the two others inclined to the horizon at any angles , θ'; and let an oscillatory motion be communicated to the water; re-

quired the time of the water's oscillating in either of the legs from the lowest to the highest points.

Let L = length of the whole canal, $g = 32\frac{1}{6}$ feet; then

$$T = \pi \sqrt{\frac{L}{g \times (\sin. \theta + \sin. \theta')}}.$$

When the two ascending tubes are vertical,

$$T = \pi \sqrt{\frac{L}{2g}}.$$

Cor. Hence if the legs are vertical, the time of one oscillation = the time in which a pendulum would vibrate, whose length is $\frac{1}{2}$ L.

2. The vibratory motion of water in the form of waves may be compared to the above reciprocation in a siphon or bent tube. And hence if a be the altitude of a wave, and b half the breadth, the time of one undulation, i.e. the time, from the wave being highest at any point, to its being highest at that point again, is

$$\frac{\pi}{\sqrt{2g}} \sqrt{a + b}.$$

and the space which the wave appears to pass over in a second is

$$\frac{b \sqrt{2g}}{\pi \sqrt{a + b}}.$$

Cor. 1. If a be neglected, the velocity of the wave becomes $\frac{\sqrt{2gb}}{\pi}$, which is the velocity as determined by Newton, Princip. lib. 2. Prop. 46.

Cor. 2. Hence a pendulum whose length = $\frac{1}{2}$ its distance between any two consecutive highest and lowest points will make two vibrations during the time of one complete undulation; or if the pendulum is four times the preceding, i.e. equal to the distance of any two consecutive waves, the time of one undulation equals the time in which this latter pendulum would perform one vibration.

SLUICES.—*See Fluids.*

SOLAR *inequality.—See Precession.*

SOLAR *mean time.—See Time.*

SOLIDS *the five regular, surface and solidity of.*

Names.	*Surface.*	*Solidity.*
Tetraedron	$s^2 \times 1.7320508$	$s^3 \times 0.1178513$
Hexaedron	$s^2 \times 6.0000000$	$s^3 \times 1.0000000$
Octaedron	$s^2 \times 3.4641016$	$s^3 \times 0.4714045$
Dodecaedron	$s^2 \times 20.6457288$	$s^3 \times 7.6631189$
Icosaedron	$s^2 \times 8.6602540$	$s^3 \times 2.1816950$

SOLIDS, *contents of.*

Let x and y be the abscissa and ordinate of any curve; then if $\pi = 3.14159$ &c.

$$\text{Solid content} = \pi y^2 dx.$$

Ex. 1. Content of cylinder $= \pi y^2 x$.

2. Content of cone $= \frac{1}{3} \pi y^2 x = \frac{1}{3}$ of circumscribing cylinder.

3. Content of paraboloid $= \frac{1}{2} \pi y^2 x = \frac{1}{2}$ circumscribing cylinder.

4. Content of sphere $= \frac{2}{3}$ of circumscribing cylinder.

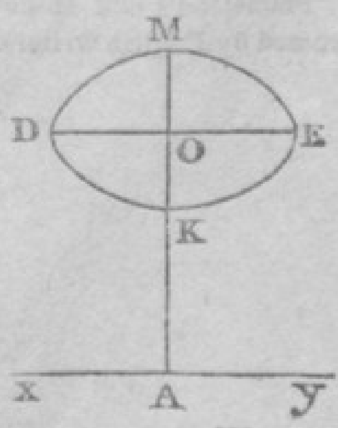
5. Content of spheroid round ax. maj.

$$= \frac{4 \pi b^2 a}{3}. \quad \text{Do. round ax. min.} = \frac{4 \pi a^2 b}{3}.$$

6. Content of pyramid $= \frac{1}{3}$ content of prism of the same base and altitude.

Guldinus' property.

Let M D E K be any plane figure revolving about an axis xy in its own plane, then the solid generated is equal to the circumference described by the centre of gravity multiplied into the area of the figure.



Ex. Let D M E K be a circle, then the solid will represent the ring of an anchor; in this case if r = radius of circle, and a = A O, the solid $= 2 \pi a \times \pi r^2 = 2 \pi^2 a r^2$.

SOUND, *velocity of.*—(*Phil. Trans.* 1823.)

The velocity with which vibrations are propagated through the air, is the same that a heavy body would acquire by falling through half the height of the *homogeneous* atmosphere, or that which the atmosphere would be reduced to, if it were everywhere of the same density, and the same temperature with the air at the surface of the earth.

The height of this homogeneous atmosphere has been computed at 4343 fathoms, when the temperature is that of freezing. If this height be called H, then v, the velocity of the aerial vibrations, $= \sqrt{2gH}$. Hence $v = 1057$, which is too small, see infra.

The velocity of sound has been variously given by different philosophers, as appears from the following Table:—

	Feet.	
Newton	968	per second.
Roberts	1300	
Boyle	1200	
Walker	1338	
Flamstead, Halley, and Derham	1142	
Florentine Academy	1148	
French Academy	1172	

More modern determinations.

Millington	1130	Chili.
Bengenberg	1095	Dusseldorf.
La Caille	1106½	Montmartre.
La Place	1133	
Lacaille	1130	

Flamstead's and Halley's measure, or 1142, is the one generally assumed by English writers.

Result of Mr Goldingham's elaborate series of experiments at Madras.

Months.	*Barometer in Inches.*	*Thermometer, Fah.*	*Hygrometer, dry.*	*Velocity of Sound in a Second in Feet.*
January,	30.124	79o.05	6o.2	1101
February,	30.126	78 .84	14 .70	1117
March,	30.072	82 .30	15 .22	1134
April,	30,031	85 .79	17 .23	1145
May,	29,892	88 .11	19 .92	1151
June,	29.907	87 .10	24 .77	1157
July,	29,914	86 .65	27 .85	1164
August,	29,931	85 .02	21 .54	1163
September,	29,963	84 .49	18 .97	1152
October,	30.058	84 .33	18 .23	1128
November,	30.125	81 .35	8 .18	1101
December,	30.087	79 .37	1 .43	1099

Mr Goldingham concludes, that for each degree of the thermometer 1.2 feet may be allowed in the velocity of sound for a second; for each degree of the hygrometer 1.4 feet; and for $\frac{1}{10}$th of an inch of the barometer 9.2 feet. He concludes that 10 feet per second is the difference of the velocity of sound between a calm and in a moderate breeze, and 21$\frac{1}{4}$ feet in a second, or 1275 in a minute, is the difference, when the wind is in the direction of the motion of sound, or opposed to it.—*See Phil. Trans.* 1823.

SPECIFIC *Gravity.—See Gravity specific.*

SPECTACLES.—*See Eye.*

SPHERE, *doctrine of.*

In what is usually called the doctrine of the sphere is merely included the solution of the following problem :—

In a spherical triangle, whose sides are the co-declination D, the co-latitude of the place L, the zenith distance Z, and two of whose angles are the hour angle from noon H, and azimuth α; if any three of these quantities be given, the other two may be found by the rules and formulæ of Trigonometry.

For the solution of the several cases—*see Trigonometry spherical.*

SPHERE, *Equations to, when the axes are rectangular.*—(*Hamilton.*)

Let r = radius, and suppose x', y', z' to be the coordinates of the centre, and x, y, z those of any point on the surface; then the general equation is

$$(x - x')^2 + (y - y')^2 + (z - z')^2 = r^2.$$

If the origin be at the centre, x', y', and z' each $= o$, and the equation becomes

$$x^2 + y^2 + z^2 = r^2.$$

SPHERICAL *excess.*

Spherical excess in Trigonometry is the excess of the sum of the three angles of any spherical Δ above two right angles. Now in surveying a country where the sides of the Δ's are usually 14 or 15 miles each, the spherical excess, with a fine instrument, is plainly discernable; and in strict accuracy the sides of the Δ's ought to be calculated by the rules of spherical Trigonometry, which would be a most tedious process, where many hundreds of such operations are to be performed. Legendre has therefore furnished us with the following rule, which combines sufficient exactness, with all the conciseness that can be expected, viz.:—

A spherical Δ being proposed, of which the sides are very small with regard to the radius of the sphere, if from each of its angles one-third of the excess of the sum of its three ∠'s above two right ∠'s be subtracted, the angles so diminished may be taken for the ∠'s of a rectilineal Δ, the sides of which are equal in length to those of the proposed spherical triangle.

SPIRALS.—*(Higman, Vince.)*

1. *Spirals, Equations to.*

In the spiral of Archimedes, let r = rad. vect. θ = ∠ traced out by r; then

$$r = \frac{b}{2\pi} . \theta, \text{ or } r = a\theta; \text{ if } a = \frac{b}{2\pi}.$$

In the reciprocal or hyperbolic spiral,

$$r = \frac{a}{\theta}.$$

In the logarithmic spiral,

$$r = a^{\theta}.$$

In the lituus,

$$r = \frac{a}{\theta^{\frac{1}{2}}}.$$

The spiral of Archimedes, the reciprocal spiral, and the lituus are particular cases of the equation $r = a\,\theta^n$.

If n be $+$, the spirals begin at the pole, and recede to an infinite distance; but if n be $-$, the spirals begin at an infinite distance, and reach the pole after an infinite number of revolutions.

2. *Spirals to draw tangents to.*

$$\text{Subtangent} = \frac{r^2\, d\,\theta}{d\,r}.$$

Ex. 1. In the spiral of Archimedes $r = a\,\theta$,

$$\therefore \text{Subtangent} = \frac{r^2}{a}, \text{ and hence } p = \frac{r^2}{\sqrt{a^2 + r^2}}.$$

Ex. 2. In the reciprocal spiral,

$$\text{Subtangent} = a, \text{ and } p = \frac{a\,r}{\sqrt{a^2 + r^2}}.$$

Ex. 3. In the logarithmic spiral,

$$\text{Subtangent} = \frac{a\,r}{b} \text{ and } p = a\,r.$$

3. *Spirals to find the areas of.*

$$\text{Area} = \text{fl.}\, \frac{r^2\, d\,\theta}{2}.$$

Ex. 1. In the spiral of Archimedes,

$$\text{fl.}\, \frac{r^2\, d\,\theta}{2} = \text{fl.}\, \frac{\pi\, r^2\, d\,r}{b}, \text{ since } \theta = \frac{2\,\pi\,r}{b} \text{ (Art. 1)};$$

$$\therefore \text{Area} = \frac{\pi\, r^3}{3\,b}.$$

Ex. 2. In the reciprocal spiral,

$$\text{fl.}\, \frac{r^2\, d\,\theta}{2} = -\,\text{fl.}\, \frac{a\,d\,r}{2}, \text{ since } \theta = \frac{a}{r};$$

$$\therefore \text{Area} = -\frac{a\,r}{2} + \text{C}.$$

Suppose the area to vanish when $r = b$, then will the area, intercepted between two radii b and r, $= \frac{a}{2}\,(b - r)$.

Ex. 3. In the logarithmic spiral,

Area between two radii b and $r = \frac{m}{4}\,(r^2 - b^2)$, m being the modulus.

Ex. 4. In the lituus,

$$\text{Area} = a^2 \log. \frac{b}{r}.$$

4. *Spirals to find the lengths of.*

$$d z^2 = d r^2 + r^2 d \theta^2.$$

$$\text{or } d z = \frac{r\, d r}{\sqrt{r^2 - p^2}}. \ (p = \text{perpendicular on the tangent}).$$

Ex. 1. In the spiral of Archimedes,

$\text{Arc} = \frac{1}{a} \text{fl. } d r \sqrt{a^2 + r^2}$, and $\therefore$ = a parabolic arc, whose latus rectum is $2\,a$, and whose ordinate is r *(see Rectification.)*

Ex. 2. In the reciprocal spiral,

Arc = arc of a logarithmic curve contained between the ordinates b and r; the subtangent of the curve being equal to the subtangent of the spiral.

Ex. 3. In the logarithmic spiral,

$$\text{Arc} = \sqrt{(1 + m^2)}\ (r - b)$$

Ex. 4. In the involute of a circle,

$$\text{Arc} = \frac{p^2}{2\,a}\ (a = \text{radius of the circle}).$$

5. *Spirals, curvature of.*

$$\text{Rad. of curv.} = \frac{r\, d r}{d p}.$$

$$\text{Ch. curv.} = \frac{2\, p\, d r}{d p}.$$

Ex. 1. In the logarithmic spiral,

$$\text{Rad. curv.} = \frac{r}{m}, \text{ and ch. curv.} = 2\,r.$$

Ex. 2. In the spiral of Archimedes,

$$\text{Rad. curv.} = \frac{(r^2 + a^2)^{\frac{3}{2}}}{r^2 + 2\,a^2}$$

Ex. 3. In the reciprocal spiral,

$$\text{Ch. curv.} = \frac{2\,r\,(a^2 + r^2)}{a^2}.$$

6. *Spirals, point of contrary flexure in.*

Here the rad. of curvature is either infinite or nothing; $\therefore \frac{r\,d\,r}{d\,p} = 0$ or infinity, and $d\,p$ is infinite or nothing.

Ex. Let $r = a\,\theta^n$, then when $d\,p = o$, $r = a\,(-n.\overline{n+1})^{\frac{n}{2}}$. Hence in the case of the lituus, where $n = -\frac{1}{2}$, $r = a\,\sqrt{2}$.

SPRINGS *hot, temperature of a few of the principal.—(Ure.)*

Matlock	66°	Borset	132°
Bristol	74	Aix	143
Buxton	82	Carlsbad	165
Bath	114	The Geyzers (Iceland)	212
Berege	120		

SPRINGS, *temperature of.—See Atmosphere.*

SQUARES *minimum, method of.—See Equations of Condition.*

SQUARE *roots of numbers.—See Involution.*

STANDARD *measures.—See Weights and Measures.*

STARS, *Catalogue of.—(Naut. Alm.)*

A Catalogue of 60 principal Fixed Stars for Jan. 1, 1823.

No.	Names of Stars.	A.R.	An. Var.	N.P.D.	An. Var.
		H. M. S.	S.	° ′ ″	″
1	γ Pegasi	0. 4. 8,1	+ 3,08	75.48. 2	—19,9
2	α Cassiopeiæ	0.30.31,3	3,31	34.26. 6	—19,7
3	Polaris	0.57.46,5	15,01	1.38. 8	—19,4
4	α Arietis	1.57.13,1	3,36	67.22.44	—17,2
5	α Ceti	2.53. 2,3	3,12	86.36.37	—14,4
6	α Persei	3.11.44,3	4,20	40.46.39	—13,3
7	Aldebaran	4.25.46,6	3,43	73.51.18	— 7,7
8	Capella	5. 3.37,8	4,41	44.11.37	— 4,3
9	Rigel	5. 6. 2,2	2,88	98.24.48	— 4,5
10	β Tauri	5.15. 6,8	3,78	61.33. 7	— 3,7
11	γ Orionis	5.15.38,7	3,20	83.49. 8	— 4,0
12	δ	5.22.58,3	3,06	90.26.18	— 3,1
13	ι	5.27.14.3	3,03	91.19.23	— 2,6
14	ζ	5.31.50,1	3,01	92. 2.38	— 2,4
15	α	5.45.35,6	3,25	82.38. 4	— 1,1
16	β Aurigæ	5.46.32,9	4,39	45. 4.56	— 1,2
17	Sirius	6.37.20,9	2,64	106.28.49	+ 4,8
18	Castor	7.23.17,6	3,85	57.43.59	+ 7,2

No.	Names of Stars.	A.R.	An. Var.	N.P.D.	An. Var.
		H. M. S.	s.	° ′ ″	″
19	Procyon	7.30. 2,2	3,17	84.19.43	+ 8,9
20	Pollux	7.34.28,5	3,69	61.33.17	+ 8,1
21	α Hydræ	9.18.53,5	2,95	97.53.44	+ 15,3
22	Regulus	9,58.56,3	3,21	77.10.16	+ 17,3
23	α Ursæ Majoris ...	10.52.43,5	3,83	27.17.44	+ 19,2
24	β Leonis	11.40. 1,7	3,07	74.26.18	+ 20,1
25	γ Ursæ Majoris ...	11.44.28,6	3,20	35.19.15	+ 19,9
26	δ	12. 6.37,2	3,00	31,59. 0	+ 20,2
27	Spica Virginis ...	13.15.52,9	3,14	100.14. 0	+ 18,9
28	ζ Ursæ Majoris ...	13.16.46,8	2,41	34. 8.51	+ 18,9
29	η	13.40.33,5	2,38	39.48. 0	+ 18,2
30	α Draconis	13.59.35,9	1,62	24.46.31	+ 17,3
31	Arcturus	14. 7,35,6	2,73	69.58.29	+ 19,0
32	ε Bootis	14.37.15,6	2,61	62.10.28	+ 15,5
33	2 α Libræ	14.41. 6,4	+ 3,30	105.17.56	+ 15,4
34	β Ursæ Minoris ...	14.51.19,6	— 0,32	15. 7.16	+ 14,7
35	α Cor. Bor.	15.27.12,0	+ 2,54	62.41. 1	+ 12,5
36	α Se-pentis	15.35.33,5	2,95	83. 0.37	+ 11,7
37	δ Ophiuchi	16. 5. 5,0	3,13	93.13.49	+ 9,9
38	Antares	16.18.34,2	3,66	116. 1.43	+ 8,7
39	α Herculis	17. 6.35,0	2,73	75.24. 0	+ 4,6
40	β Draconis	17.26.25,9	1,34	37.33.49	+ 2,9
41	α Ophiuchi	17.26.43,5	2,78	77.18.11	+ 3,2
42	γ Draconis	17.52.30,1	+ 1,38	38.29.11	+ 0,7
43	δ Ursæ Minoris	18.29.22,3	- 19,12	3.25.11	— 2,4
44	α Lyræ	18.30.57,0	+ 2,03	51.22.31	— 3,0
45	β	18.43.33,0	2,20	56.50.12	— 3,8
46	ζ Aquilæ	18.57.16,8	2,75	76.23.31	— 4,8
47	δ Draconis	19.12.29,7	0,02	22.38.59	— 6,2
48	δ Aquilæ	19.16.34,6	3,00	87.13.47	— 6,6
49	γ	19.37.50,8	2,85	79.48.39	— 8,5
50	α	19.42. 8,9	2,93	81.35.30	— 8,9
51	β	19.46.37,3	2,95	84. 1.40	— 8,6
52	2 α Capricorni	20. 8.13,7	3,34	103. 5. 7	— 10,8
53	α Cygni	20.35.24,2	2,05	45.20.52	— 12,5
54	1st 61 Cygni	20.58.58,6	2,77	52. 6.55	— 17,6
55	α Cephei	21.14.21,0	1,42	28. 9.43	— 15,0
56	β Aquarii	21.22.14,2	3,15	96.20.38	— 15,3
57	β Cephei	21.26.20,4	0,81	20.12.54	— 15,7
58	α Aquarii	21.56.41,5	3,09	91,10.31	— 17,0
59	α Pegasi	22.55.57,2	2,93	75.44.42	— 19,0
60	α Andromedæ	23.59.15,6	3,08	61.53.12	— 19,8

STARS *double.*

On Herschel's Catalogue of double Stars.

The first Catalogue of double stars was made with a Newtonian telescope of not quite seven feet focus, and with only 4½ inches aperture, charged with a power of 222. The second Catalogue with an aperture of six inches and a quarter, with a power of 227, and 460; when the stars were detected, he used a gradual variety of powers from 460 to 6000.

These double stars are divided into several different classes. In the first are placed all those which require a very superior telescope, the utmost clearness of air, and every other favourable circumstance to be seen at all, or well enough to judge of them. Their distance is so extremely small (seldom exceeding two diameters of the largest) that it cannot be accurately measured by the micrometer, but may be more correctly estimated by the eye in measures of their own apparent diameters. It should be observed, that since it will require no common stretch of power and distinctness to see these double stars, it will ∴ not be amiss to go gradually through a few preparatory steps of vision, such as the following:—for instance, when η Coron. Borealis (one of the most minute double stars) is proposed to be viewed, let the telescope be some time before directed to α Geminorum, or if not in view to either of the following stars, ζ Aquarii, μ Draconis, ρ Herculis, α Piscium, or the curious double-double star ε Lyræ. These should be kept in view for a considerable time, that the eye may acquire the habit of seeing such objects well and distinctly. The observer may next proceed to the ξ Ursæ Majoris, and the beautiful treble star in Monoceros' right foot; after these to *i* Bootis, which is a fine miniature of α Geminorum, to the star preceding α Orionis, and to *n* Orionis. By this time both the eye and the telescope will be prepared for a still finer picture, which is η Coronæ Borealis. It will be in vain to attempt this latter, if all the former, at least *i* Bootis, cannot be distinctly perceived to be fairly separated; because it is almost as fine a miniature of *i* Bootis as that is of α Geminorum. To try stars of unequal magnitude, it will be expedient to take them in some such order as the following: α Herculis, ω Aurigæ, δ Geminorum, *k* Cygni, ε Persei, and *b* Draconis; from these the observer may proceed to a most beautiful object ε Bootis. As the foregoing remarks have suggested the method of seeing how far the power and distinctness of our instruments will reach, we may next add the way of finding how much light we have. The observer may begin with the pole star, and α Lyræ, then go to the star south of ε Aquilæ, the treble star near *k* Aquilæ, and last of all to the star following *o* Aquilæ. Now if his telescope has not a great deal of good light, he will not be able to see some of the small stars that accompany them.

In the second class of double stars are put all those that are proper for estimations by the eye, or very delicate measures of the micrometer. To compare the distances with the apparent diameters, the power of the telescope should not be much less than 200, as they will otherwise be too close for the purpose. It will be necessary here to notice that the estimation made with one telescope cannot be applied to those made with another, nor can the estimations made with different powers, though with the same telescope, be applied to each other; therefore if we would wish to compare any such observations together with a view to see whether a change in the distance has taken place, it should be done with the very same telescope and power, even with the very same eye-glass or glasses.

In the third class are placed all those double stars that are more than 5 but less than 15″ asunder. In the same manner that the stars in the 1st and 2d classes will serve to try the goodness of the most capital instruments, these will afford objects for telescopes of inferior power, such as magnify from 40 to 100 times. The observer may take them in this or the like order; ζ Ursæ Maj., γ Delphini, γ Arietis, π Bootis, γ Virginis, ι Cassiopeæ, μ Cygni. And if he can see all these he may pass over into the second class, and direct his instrument to some of those that are pointed out as objects for the very best telescopes, where he will soon find the want of superior power.

The 4th, 5th, and 6th classes contain double stars that are from 15 to 30″; from 30″ to 1′, and from 1′ to 2′ or more asunder.—*Phil. Trans. vol.* 72, 75.

For a list of a few of the most remarkable double stars—*see Telescope.*

STARS *changeable.*

Catalogue of twenty-eight changeable stars.—(Baron de Zach.)

Names of Stars.	Right Ascension in 1800.	Declination in 1800.
46 Andromeda,	1h 11′	44° 29′ N.
o Balænæ *Mira*,	2 09	3 54 S.
β Persei *Algol*,	2 55	40 11 N.
Unicorn,	6 13	8 51
5 23 γ Canis Major,	6 55	15 21
16 ψ Leonis,	9 33	14 56
Leo 420 Mayer	9 37	12 21
16 c Virgo,	12 10	4 26 N.
10 Virgo,	12 28	8 06 N.
Virgo,	13 04	15 28 S.
ϖ Hydra	13 19	22 15 S.
97 Virgo,	14 02	8 57 S.
Bootes,	14 04	12 57 N.

Names of stars.	Right Ascension. in 1800.	Declination in 1800.
1 Libra A,	14 05	26 19 S.
15 Virgo,	14 37	2 53 N.
50 Northern Crown,	15 40	28 47 N.
81 Hercules,	16 24	33 57 N.
α Hercules,	17 06	14 38 N.
50 Sobieski's Shield.	18 37	5 54 S.
20 β Lyra,	18 43	33 08 N.
34 σ Sagittarius,	18 43	26 32 S.
χ Swan,	19 39	33 16 N.
Swan No. 295. P.	19 41	32 57 N.
η Antinous,	19 42	0 30 N.
25 Southern Fish,	19 43	33 08 S.
34 Swan, near γ	20 10	37 25 N.
δ Cepheus,	22 22	57 23
Aquarius,	23 24	16 23

Of this catalogue of Variable Stars, Nos. 2, 3, 7, 11, 18, 19, 20, 22, 24, and 26, belong to the list of fifteen as given by Mr Pigott. Some of the others belong to the list of those which he suspected to be variable.

STARS, *clusters of.—See Nebulæ.*

STEAM, *elasticity and density of.—(Encyc. Brit. Sup.)*

Let E be the No. of atmospheres expressing the elasticity, f the temperature reckoned from 212°; then

$$E = (1 + .004 f)^5$$

From hence is obtained the following Table of the elasticities and densities :—

Atmospheres.	*Temperature.*	*Compar. Density.*	*Atmospheres.*	*Temperature.*	*Compar. Density.*
1	212°	1.000	30	456	21.334
2	249	1.806	40	485	28.210
3	273	2.742	50	509	34.383
4	292	3.565	60	529	40.404
5	307	4.366	70	547	46.285
6	320	5.150	80	562	52.093
7	331	5.917	90	577	57.766
8	341	6.678	100	590	63.371
9	350	7.433	1000	957	465.33
10	358	8.170	2000	1105	815.31
15	388	11.820	3000	1202	1193.00
20	417	15.232			

Note.—Bernouilli makes the expansive force of gunpowder equal to 10,000 atmospheres; Rumford, from the bursting of a barrel of iron, 50,000, from some more direct experiments from 20,000 to 40,000. The utmost that can be justly inferred from the bursting of the barrel is in reality about 30,000, since the tension could by no means be equal through every part of its substance.—*(Young's Nat. Phil.)*

STEELYARD.—*See Lever.*

STILE *new.*—*See Calendar.*

STRENGTH *animal.*—*See Animal strength.*

SUBNORMAL, *formula for.*

Let x and y = abscissa and ordinate of any curve; then

$$\text{Subnormal} = \frac{y\,dy}{dx}.$$

$$\text{and normal} = y \times \frac{\sqrt{dx^2 + dy^2}}{dx} = y\sqrt{1 + \frac{dy^2}{dx^2}}.$$

Ex. Let the curve be the common parabola, then subnormal $= \frac{L}{2}$, and normal $= \sqrt{y^2 + \frac{L^2}{4}}$, where L = lat. rect.

SUBTANGENT.—*See Tangent.*

Sum of terms – see Progression

SUN *eclipses of.*—*See Eclipse.*

SUN *elements of.*—*See Planets elements of.*

SUN, *table of mean right ascension of.*—*See Time.*

SUN, *time of passing Meridian.*—*See Time.*

SUN, *Right Ascension and Declination of.*

TABLE I.

Sun's Right Ascension for every Day in the Year 1828.

Days.	*January.*	*February.*	*March.*	*April.*	*May.*	*June.*
	h m s	h m s	h m s	h m s	h m s	h m s
1	18 44 5	20 56 36	22 49 40	0 43 11	2 34 27	4 37 12
2	18 48 30	21 0 41	22 53 24	0 46 49	2 38 16	4 41 18
3	18 52 55	21 4 45	22 57 7	0 50 27	2 42 6	4 45 24
4	18 57 19	21 8 47	23 0 51	0 54 6	2 45 56	4 49 31
5	19 1 43	21 12 50	23 4 33	0 57 45	2 49 47	4 53 38
6	19 6 7	21 16 51	23 8 16	1 1 24	2 53 38	4 57 45
7	19 10 30	21 20 51	23 11 57	1 5 3	2 57 30	5 1 52
8	19 14 52	21 24 51	23 15 39	1 8 42	3 1 23	5 6 0
9	19 19 14	21 28 50	23 19 20	1 12 22	3 5 16	5 10 8
10	19 23 36	21 32 48	23 23 1	1 16 2	3 9 10	5 14 17
11	19 27 57	21 36 45	23 26 41	1 19 42	3 13 4	5 18 25
12	19 32 17	21 40 42	23 30 22	1 23 23	3 16 59	5 22 34
13	19 36 37	21 44 38	23 34 1	1 27 3	3 20 55	5 26 43
14	19 40 57	21 48 33	23 37 41	1 30 45	3 24 51	5 30 52
15	19 45 15	21 52 27	23 41 21	1 34 26	3 28 48	5 35 2
16	19 49 33	21 56 21	23 45 0	1 38 8	3 32 45	5 39 11
17	19 53 50	22 0 14	23 48 39	1 41 50	3 36 43	5 43 20
18	19 58 7	22 4 6	23 52 18	1 45 33	3 40 42	5 47 30
19	20 2 23	22 7 57	23 55 56	1 49 16	3 44 40	5 51 39
20	20 6 38	22 11 48	23 59 35	1 52 59	3 48 40	5 55 49
21	20 10 52	22 15 38	0 3 13	1 56 43	3 52 40	5 59 59
22	20 15 6	22 19 27	0 6 51	2 0 27	3 56 41	6 4 8
23	20 19 19	22 23 16	0 10 29	2 4 12	4 0 42	6 8 18
24	20 23 31	22 27 4	0 14 7	2 7 57	4 4 43	6 12 27
25	20 27 42	22 30 52	0 17 45	2 11 43	4 8 45	6 16 36
26	20 31 52	22 34 38	0 21 23	2 15 29	4 12 48	6 20 45
27	20 36 1	22 38 25	0 25 1	2 19 15	4 16 51	6 24 54
28	20 40 10	22 42 10	0 28 39	2 23 2	4 20 54	6 29 3
29	20 44 18	22 45 55	0 32 17	2 26 50	4 24 58	6 33 11
30	20 48 25		0 35 55	2 30 38	4 29 2	6 37 20
31	20 52 31		0 39 33		4 33 7	

Days.	July.	August.	September.	October.	November.	December.
	h m s	h m s	h m s	h m s	h m s	h m s
1	6 41 28	8 46 14	10 42 14	12 30 19	14 26 39	16 30 36
2	6 45 36	8 50 6	10 45 52	12 33 57	14 30 35	16 34 56
3	6 49 44	8 53 58	10 49 29	12 37 35	14 34 32	16 39 16
4	6 53 51	8 57 50	10 53 6	12 41 13	14 38 30	16 43 37
5	6 57 58	9 1 41	10 56 43	12 44 52	14 42 28	16 47 59
6	7 2 5	9 5 31	11 0 20	12 48 31	14 46 27	16 52 21
7	7 6 11	9 9 21	11 3 56	12 52 11	14 50 27	16 56 44
8	7 10 18	9 13 10	11 7 33	12 55 51	14 54 28	17 1 7
9	7 14 23	9 16 59	11 11 9	12 59 31	14 58 30	17 5 31
10	7 18 29	9 20 47	11 14 45	13 3 12	15 2 33	17 9 55
11	7 22 34	9 24 34	11 18 21	13 6 53	15 6 36	17 14 20
12	7 26 38	9 28 21	11 21 56	13 10 35	15 10 41	17 18 44
13	7 30 42	9 32 7	11 25 32	13 14 17	15 14 46	17 23 10
14	7 34 46	9 35 53	11 29 8	13 18 0	15 18 52	17 27 35
15	7 38 49	9 39 38	11 32 43	13 21 44	15 22 59	17 32 1
16	7 42 51	9 43 23	11 36 19	13 25 28	15 27 6	17 36 27
17	7 46 53	9 47 7	11 39 54	13 29 12	15 31 15	17 40 53
18	7 50 55	9 50 51	11 43 29	13 32 57	15 35 24	17 45 19
19	7 54 55	9 54 34	11 47 5	13 36 43	15 39 34	17 49 45
20	7 58 56	9 58 16	11 50 40	13 40 29	15 43 45	17 54 12
21	8 2 56	10 1 58	11 54 16	13 44 16	15 47 57	17 58 38
22	8 6 55	10 5 40	11 57 51	13 48 3	15 52 9	18 3 5
23	8 10 53	10 9 21	12 1 27	13 51 52	15 56 22	18 7 31
24	8 14 51	10 13 2	12 5 3	13 55 41	16 0 36	18 11 58
25	8 18 49	10 16 42	12 8 39	13 59 30	16 4 51	18 16 25
26	8 22 45	10 20 22	12 12 15	14 3 21	16 9 7	18 20 51
27	8 26 42	10 24 2	12 15 51	14 7 12	16 13 23	18 25 17
28	8 30 37	10 27 41	12 19 28	14 11 4	16 17 40	18 29 44
29	8 34 32	10 31 20	12 23 4	14 14 56	16 21 58	18 34 10
30	8 38 26	10 34 58	12 26 42	14 18 50	16 26 17	18 38 35
31	8 42 20	10 38 36		14 22 44		18 43 1

This Table is adapted to Leap Year, particularly the year 1828, and is only intended to answer the purposes of information when no great degree of accuracy is required, and the Nautical Almanack not at hand.

In order to adapt it to common years, *one-fourth* of the difference between the given and preceding days is to be subtracted from the right ascension in the table for the first after Leap Year, *one-half* for the second after Leap Year, and *three-fourths* for the third; and in the months of January and February, the right ascension is to be taken for the day following that given.

This Table may be employed in finding the apparent time by the altitude of a star, for finding the time of a star's transit when that is required, for obtaining the latitude by a meridian altitude, &c.

TABLE II.

Sun's Declination for every Day in the Year 1828.

Days.	*January.*	*February.*	*March.*	*April.*	*May.*	*June.*
	South.	South.	South.	North.	North.	North.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	23 4 22	17 17 44	7 28 10	4 38 49	15 9 11	22 5 43
2	22 59 29	17 0 43	7 5 18	5 1 52	15 27 8	22 13 37
3	22 54 8	16 43 23	6 42 21	5 24 50	15 44 50	22 21 7
4	22 48 20	16 25 46	6 19 18	5 47 43	16 2 17	22 28 14
5	22 42 5	16 7 53	5 56 9	6 10 29	16 19 27	22 34 57
6	22 35 23	15 49 42	5 32 56	6 33 9	16 36 22	22 41 17
7	22 28 14	15 31 15	5 9 38	6 55 43	16 53 0	22 47 12
8	22 20 58	15 12 32	4 46 15	7 18 10	17 9 22	22 52 44
9	22 12 36	14 53 34	4 22 50	7 40 30	17 25 26	22 57 52
10	22 4 8	14 34 21	3 59 21	8 2 42	17 41 13	23 2 36
11	21 55 14	14 14 53	3 35 48	8 24 46	17 56 43	23 6 55
12	21 45 54	13 55 10	3 12 13	8 46 42	18 11 54	23 10 50
13	21 36 9	13 35 14	2 48 36	9 8 29	18 26 48	23 14 20
14	21 25 59	13 15 5	2 24 57	9 30 6	18 41 22	23 17 26
15	21 15 24	12 54 43	2 1 16	9 51 35	18 55 38	23 20 7
16	21 4 24	12 34 8	1 37 35	10 12 54	19 9 35	23 22 24
17	20 53 0	12 13 21	1 13 52	10 34 2	19 23 12	23 24 16
18	20 41 13	11 52 22	0 50 10	10 55 1	19 36 29	23 25 43
19	20 29 2	11 31 13	0 26 27	11 15 48	19 49 27	23 26 45
20	20 16 27	11 9 52	0 2 45 S	11 36 24	20 2 4	23 27 22
21	20 3 30	10 48 22	0 20 56 N	11 56 49	20 14 20	23 27 35
22	19 50 11	10 26 41	0 44 36	12 17 1	20 26 16	23 27 22
23	19 36 29	10 4 52	1 8 14	12 37 2	20 37 51	23 26 45
24	19 22 26	9 42 52	1 31 50	12 56 50	20 49 5	23 25 44
25	19 8 1	9 20 44	1 55 24	13 16 25	20 59 57	23 24 17
26	18 53 16	8 58 28	2 18 56	13 35 48	21 10 28	23 22 26
27	18 38 10	8 36 5	2 42 24	13 54 56	21 20 36	23 20 10
28	18 22 43	8 13 33	3 5 49	14 13 51	21 30 23	23 17 30
29	18 6 57	7 50 55	3 29 10	14 32 32	21 39 47	23 14 25
30	17 50 52		3 52 27	14 50 59	21 48 49	23 10 56
31	17 34 27		4 15 40		21 57 28	

Days.	*July.* North.	*August.* North.	*September* North.	*October.* South.	*November* South.	*December.* South.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	23 7 2	17 59 28	8 13 9	3 16 33	14 31 39	21 52 8
2	23 2 44	17 44 9	7 51 16	3 39 51	14 50 44	22 1 9
3	22 58 2	17 28 33	7 29 15	4 3 7	15 9 35	22 9 44
4	22 52 56	17 12 39	7 7 6	4 26 20	15 28 11	22 17 53
5	22 47 26	16 56 29	6 44 51	4 49 30	15 46 32	22 25 37
6	22 41 32	16 40 2	6 22 28	5 12 36	16 4 37	22 32 54
7	22 35 14	16 23 19	5 59 59	5 35 39	16 22 26	22 39 45
8	22 28 33	16 6 21	5 37 25	5 58 37	16 39 59	22 46 9
9	22 21 29	15 49 6	5 14 44	6 21 31	16 57 14	22 52 6
10	22 14 1	15 31 36	4 51 59	6 44 19	17 14 12	22 57 36
11	22 6 11	15 13 52	4 29 8	7 7 2	17 30 52	23 2 38
12	21 57 58	14 55 53	4 6 12	7 29 40	17 47 14	23 7 13
13	21 49 22	14 37 39	3 43 13	7 52 11	18 3 18	23 11 21
14	21 40 23	14 19 11	3 20 9	8 14 35	18 19 2	23 15 0
15	21 31 3	14 0 30	2 57 2	8 36 52	18 34 27	23 18 12
16	21 21 20	13 41 35	2 33 51	8 59 2	18 49 32	23 20 56
17	21 11 16	13 22 27	2 10 38	9 21 4	19 4 17	23 23 12
18	21 0 50	13 3 7	1 47 22	9 42 58	19 18 41	23 24 59
19	20 50 3	12 43 35	1 24 4	10 4 43	19 32 44	23 26 19
20	20 38 55	12 23 50	1 0 43	10 26 19	19 46 27	23 27 10
21	20 27 26	12 3 54	0 37 22	10 47 46	19 59 47	23 27 33
22	20 15 36	11 43 46	0 13 59N	11 9 3	20 12 46	23 27 27
23	20 3 27	11 23 28	0 9 25 S	11 30 10	20 25 22	23 26 53
24	19 50 57	11 2 58	0 32 50	11 51 7	20 37 35	23 25 51
25	19 38 7	10 42 18	0 56 15	12 11 53	20 49 26	23 24 21
26	19 24 58	10 21 28	1 19 39	12 32 28	21 0 53	23 22 22
27	19 11 30	10 0 28	1 43 4	12 52 51	21 11 57	23 19 55
28	18 57 42	9 39 18	2 6 28	13 13 2	21 22 36	23 17 0
29	18 43 36	9 17 59	2 29 51	13 33 1	21 32 52	23 13 37
30	18 29 11	8 56 31	2 53 13	13 52 47	21 42 43	23 9 46
31	18 14 28	8 34 54		14 12 20		23 5 27

This Table, like the last, is for the year 1828, or Leap Year. The correction for any other year must be made as before.

SUN'S *Semidiameter, &c.—(Naut. Alm.)*

TABLE,

Of Sun's Semidiameter, and of the time of his semidiameter passing the meridian.

	Time of Sun's ½ diam passing Meridian.	*Semi-diameter.*		*Time of Sun's ½ diam passing Meridian.*	*Semi-diameter.*
Jan.	M. S.	M. S.	*July.*	M. S.	M. S.
1	1. 10,8	16. 17,8	1	1. 8;5	15. 45,5
7	1. 10,5	16. 17,7	7	1. 8,3	15. 45,5
13	1. 10.1	16. 17,4	13	1. 8,0	15. 45,8
19	1. 9,5	16. 16.9	19	1. 7,5	15. 46,1
25	1. 8,9	16. 16,3	25	1. 7,0	15. 46,6
Feb.			*Aug.*		
1	1. 8,1	16. 15,3	1	1. 6,5	15. 47,4
7	1. 7,4	16. 14,4	7	1. 6,0	15. 48,3
13	1. 6,7	16. 13,2	13	1. 5,5	15. 49,3
19	1. 6,1	16. 12,0	19	1. 5,0	15. 50,4
25	1. 5,5	16. 10,7	25	1. 4,6	15. 51,6
Mar.			*Sept.*		
1	1. 5,2	16. 9,7	1	1. 4,2	15. 53,1
7	1. 4,8	16 8,2	7	1. 3,9	15 51,6
13	1. 4,5	16. 6,6	13	1. 3,8	15. 56,1
19	1. 4,3	16. 4,9	19	1. 3,8	15. 57,7
25	1. 4,2	16. 3,3	25	1. 3,9	15. 59,3
April.			*Oct.*		
1	1. 4,2	16. 1,3	1	1. 4,1	16. 1,0
7	1. 4,4	15. 59,7	7	1. 4,4	16. 2,6
13	1. 4,6	15. 58,1	13	1. 4,8	16. 4,3
19	1. 4,9	15. 56,5	19	1. 5,3	16. 5,9
25	1. 5,4	15. 54,9	25	1. 5,9	16. 7,5
May.			*Nov.*		
1	1. 5,8	15. 53,5	1	1. 6,7	16. 9,3
7	1. 6,3	15. 52,1	7	1. 7,4	16. 10,8
13	1. 6,8	15. 50,9	13	1. 8,1	16. 12,1
19	1. 7,2	15. 49,7	19	1. 8,7	16. 13,3
25	1. 7,7	15. 48,7	25	1. 9,4	16. 14,5
June.			*Dec.*		
1	1. 8,1	15. 47,6	1	1. 10,0	16. 15,4
7	1. 8,3	15. 46,9	7	1. 10,5	16. 16,2
13	1. 8,5	15. 46,3	13	1. 10,8	16. 16,9
19	1. 8,6	15. 45,9	19	1. 10,9	16. 17,4
25	1. 8,6	15. 45,6	25	1. 11,0	16. 17,7

SUN'S *parallax in altitude.*

Altitude.	*Parallax.*	*Altitude.*	*Parallax.*
0°	9″	60°	4″
10	9	65	4
20	8	70	3
30	8	75	2
40	7	80	2
50	6	85	1
55	5	90	0

SURFACES *of Solids.*

Let $y =$ ordinate of any curve, $z =$ length; then

$$\text{Surface} = \text{fl. } 2\,\pi\,y\,d\,z.$$

Ex. 1. Surface of cone $= 2\,\pi\,b \times \frac{s}{2}$, where $b = \frac{1}{2}$ base, and $s =$ slant side, $=$ circumference of base $\times \frac{1}{2}$ slant side.

2. Surface of sphere $= 4\,\pi\,r^2 =$ four times the area of one of its great circles.

3. Surface of paraboloid $= \frac{\pi.\,(4y^2 + a^2)^{\frac{3}{2}}}{6\,a} - \frac{\pi\,a^2}{6}$.

4. Surface of cycloid $= \frac{8\,\pi\,a^2}{3}$ ($a =$ diameter of generating circle.)

Guldinus' property.

Let M D E K (*see Fig. Art. Solid*) be any plane figure, revolving about an axis $x\,y$ in its own plane; then the area of the surface generated by the perimeter of this figure, is equal to the circumference described by the centre of gravity of the perimeter multiplied into the perimeter.

Ex. Let D M E K be a circle, then the solid will represent the ring of an anchor, and if $r =$ radius of circle, and $a =$ A O, the surface $= 2\,\pi\,a \times 2\,\pi\,r = 4\,\pi^2\,a\,r$.

SURVEYING.

I. *Surveying Land.*

1. The area of a triangle $=$ base $\times \frac{1}{2}$ perpendicular altitude: or $=$ the product of any two sides $\times$ natural sine of their included $\angle$; or when three sides A B, A C, B C are given, their half sum being S, area $=$ $\sqrt{\left\{S \times (S - AB) \times (S - AC) \times (S - BC)\right\}}$

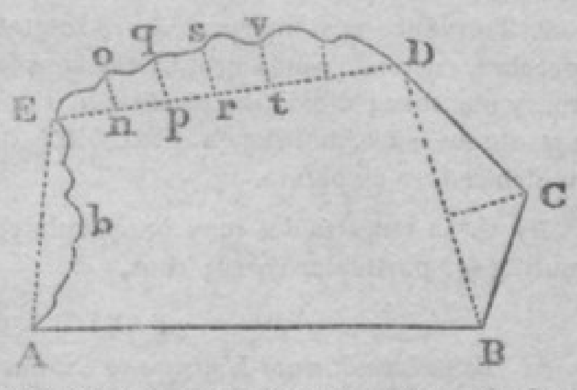
2. The area of a trapezium = base × ½ sum of the perpendiculars. And the area of a trapezoid = ½ sum of the parallel sides × perpendicular distance between them.

3. To find the area of any irregular polygon, divide it into trapeziums, or trapezoids, or triangles, and find their areas separately; and their sum is the area of the polygon.

4. To find the area of a long irregular figure E *q* D *p* bounded on one side by a curve.

Divide E D into any number of equal parts, and measure the perpendiculars, *n o*, *p q*, *r s*, *t v* &c. then the area is found nearly by adding together all the perpendiculars, dividing the sum by the number of perpendiculars increased by unity, and multiplying by the chord of the curve.

5. To find, by the foregoing rules, the content of the irregular field ABCDE, which will include most of the cases likely to occur in practice.



Find the area of the trapezium A B D E by Art. 3, the △ B D C by Art. 1; and the curvilinear areas E *q* D, E *b* A by Art. 4; add the three first areas together and subtract the last, for the content of the field.

Land is measured by a chain 22 yards long, and divided into 100 equal parts or links, each link being 7.92 inches: 10 square chains, or 100,000 square links, is one acre, viz.:—

625 square links is 1 perch.
25,000 square links or 40 perches, 1 rood.
100,000 square links or 4 roods, 1 acre.

The perch (which in statute measure is 16½ feet) varies by custom in different parts of England; and with it, consequently, varies the acre in proportion.

In Devonshire and part of Somersetshire, 15; in Cornwall, 18; in Lancashire and Yorkshire, 21; and in Cheshire and Staffordshire, 24 feet are accounted a perch.

Hence the following Table will give the number of square feet in a square perch, in the above-mentioned counties.

Statute perch	16,5 × 16.5	=	272.25 square feet.	
Devonshire perch	15 × 15	=	225	do.
Cornwall perch	18 × 18	=	324	do.
Lanc. and Yorks. perch	21 × 21	=	441	do.
Cheshire and Staff. perch	24 × 24	=	576	do.

Rules for reducing Statute Measure to Customary, and the contrary.

1. To reduce statute measure to customary, multiply the number of perches statute measure, by the square feet in a square perch statute measure; divide the product by the square feet in a square perch customary measure, and the quotient will be the answer in square perches; which reduce to roods and acres, by dividing by 40 and 4.

2. To reduce customary measure to statute, multiply the number of perches, customary measure, by the square feet in a square perch customary measure; divide the product by the square feet in a square perch statute measure, and the quotient will be the answer in square perches; which reduce as before.

By these rules tables may be calculated to save the trouble of computing for particular cases; thus,

TABLE I.

To reduce Statute Measure to Customary of 21 feet to a perch.

Stat. Acre.	Customary. A.	R.	P.	Stat. Rood.	Customary. R.	P.
1	0	2	18,7	1	0	24,7
2	1	0	37,5	2	1	9,4
3	1	3	16,3	3	1	34,1
4	2	1	35,0	*Stat. Perch*	*Customary.*	P.
5	3	0	13,8			
6	3	2	32,6			
7	4	1	11,4	1		0,6
8	4	3	30,1	5		3,0
9	5	2	8,9	10		6,1
10	6	0	27,7	15		9,2
20	12	1	15,4	20		12,3
30	18	2	3,1	25		15,4
40	24	2	30,8	30		18,5
50	30	3	18,5	35		21,6

TABLE II.

To reduce Customary Measure of 21 feet to a perch, to Statute.

Cust. Acre.	*Statute.* A.	R.	P.	*Cust. Rood.*	*Statute.* A.	R.	P.
1	1	2	19,17	1		1	24,79
2	3	0	38,24	2		3	9,58
3	4	3	17,44	3	1	0	34,39
4	6	1	36,64	*Cust. Perch*	*Statute.*	R.	P.
5	8	0	15,84				
6	9	2	34,88				
7	11	1	14,8	1			1,619
8	12	3	33,28	5			8,099
9	14	2	12,48	10			16,198
10	16	0	31,68	15			24,297
20	32	1	23,36	20			32,396
30	48	2	15,4	25		1	0,495
40	64	3	36,88	30		1	8,594
50	80	3	38,56	35		1	16,693

Ex. 1. In 36A. 1R. 10P. statute how many acres, &c. customary measure of 21 feet to a perch ?

Reduce to perches, which will be found 5810, ∴ 5810 × 272,25 = 1851772,50; this divided by 441 gives 3586,7 perches; divide by 40 and 4 and the result is 22A. 1R. 26,7P. The same answer may be had from Table I.

Ex. 2. Reduce 22A. 1R. 27P. customary, to statute measure.

Here the number of perches is 3587, which, multiplied by 441, and divided by 272,25, gives 36A. 1R. 10P. The same result may be obtained from Table II.

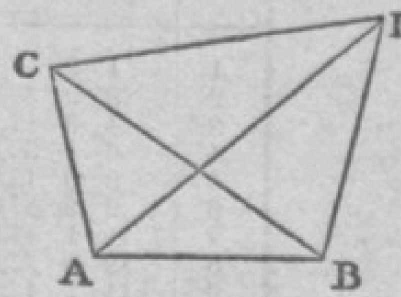
II. *Surveying Trigonometrically.*

1. These large surveys have been undertaken principally for the accomplishment of one or other of these three objects, viz. (1) For finding the difference of longitude between two moderately distant and noted meridians, as the meridians of the observatories at Greenwich and Paris. (2) For the exact determination of the principal places in a country, with a view to give greater accuracy to maps. (3) For the measurement of a degree in various situations, in order to determine from thence the figure and magnitude of the earth.

These important objects can only be attained, by the greatest possible degree of accuracy in the instruments employed, the operations performed, and the computations required.

The following must only be considered a mere outline of the method pursued in surveying a country: the niceties necessary to be attended to, in order to render such survey available for scientific purposes, cannot be here described.

2. To carry on a measurement by a series of triangles.



Let a base line A B be measured,* and having fixed upon two objects C and D, observe the ∠'s B A C, B A D, A B C, A B D; then in the △ A B C, the ∠'s B A C, A B C, being known, their supplement A C B is known, ∴ A C and C B may be found by Case 1. Plane Trigonometry. The relative bearings and distances therefore of A, B, C are thus determined. Again in the △ A B D, the ∠'s D A B, D B A being known, A D B is known, and ∴ D B may be found. Lastly, in the △ D B C, the sides B C, B D and the included ∠ C B D are known, ∴ the remaining ∠'s B C D, B D C may be found, and consequently also the side C D (see Case 2. Plane Trigonometry). The bearings ∴ and distances of B, C, D are also known. In the same way, by considering either A C, C D or D B as a new base, and fixing upon two other points; the measurement may be continued at pleasure.

In conducting geodetical operations, the following rules by Hutton should be observed, to diminish the probability of error.

(1) When one side only of a triangle is to be determined, the measured base should be nearly equal to the required side.

(2) When two sides of a triangle are to be determined, the triangle should, if possible, be equilateral.

(3) When the base cannot be equal to one or both the required sides, it should be as long as possible, and the two angles at the base equal, and not less than 20 or 30 degrees.

In the late survey of England the base first measured was upon Hounslow Heath. By continuing the measurement to Salisbury Plains, the

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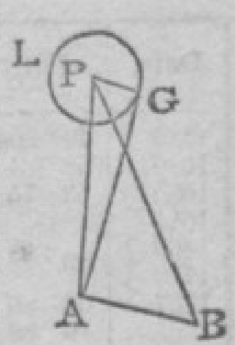
* To reduce a base on an elevated level to that at the surface of the sea, let r = rad. of earth at the surface of the sea, $r + h$ the rad. referred to the level of the base measured, the altitude h being determined by the barometer, B the length of the measured base at the altitude h, then the correction is $\frac{B\,h}{r}$ nearly, which must be subtracted from the measured base, to give the true base at the level of the sea.

distance of two objects was there found by calculation to be 36574,4 feet, and, by actual admeasurement, the distance was found to be 36574,3 feet, differing very little more than an inch from the computed distance.

We shall close this necessarily imperfect article with the methods of finding the difference of longitudes and difference of latitudes of places upon the earth's surface, as practiced in the late government survey of this country.

Difference of Longitude.

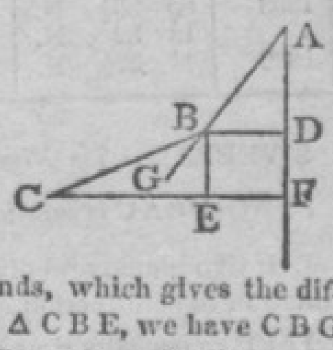


Let P be the pole, L G the circle described by the pole star, A and B the two places. Take by the instrument the ∠ G A B = ∠ contained by B and the pole star when at its greatest azimuth; then knowing P A, P G, we may find P A G the greatest azimuth, which added to G A B gives P A B; hence in the spherical Δ P A B, we have P A, P B and ∠ P A B to find ∠ A P B the difference of longitude. Hence if A be Greenwich, the longitude of B is known.

N.B. For four minutes either before or after the pole star's greatest elongation, it moves only about a second in azimuth; hence a good pocket watch gives the time of greatest azimuth with sufficient accuracy.

Difference of Latitude.

In finding the difference of longitude above, the latitude was supposed known; this was found by geometrical admeasurement thus,



Let A, B, and C be three places, A F the meridian of A. Find the ∠ B A D = supplement of ∠ P A B in the former figure (and which may be found, if A's latitude is known, as was shewn in the last Article); and find by observation A B, B C, and ∠ A B C; then in the Δ A B D we can find A D in feet, and ∴ (knowing the dimensions of the earth) in seconds, which gives the difference of latitudes of A and B. Again, in the Δ C B E, we have C B G the supplement of A B C and G B E (= B A D) and ∴ C B E; hence B E may be found, and ∴ A F, = difference of latitude of A and C; and thus we may proceed through any number of Δ's. If A be Greenwich, or any place whose latitude is accurately known, the latitude of the rest will be had.

In the same Δ's, we can also find B D, C F, or the perpendicular distance of each place from A's meridian.

The latitudes thus determined are more accurate than those deduced from astronomical observation; since the best instruments could not have given the zenith distances nearer than about one second, answering in these parts to 101 feet on the surface of the earth.

TABLE

Of the different lengths of a degree, as measured in various parts of the earth, the time of its measurement, the latitude of its middle point, &c. (Barlow.)

Date.	Latitude.	Extent in Eng. miles and dec.	Measurers.	Countries.
1525	49° 20½′ N.	68·763	M. Fernel	France.
1620	52 4 N.	66· 91	Snellius............	Holland.
1635	53 15 N.	69·545	Norwood	England.
1644		75·066	Riccioli	Italy.
1660 }	49 22 N.	{ 68·945	Picard............. }	France.
1718 }		{ 69·119	Cassini .... }	
1737	66 20 N.	69·403	Maupertuis, &c.	Lapland.
	49 22 N.	69·121 }	Cassini and La	
1740	45 00 N.	69·092 }	Caille	France.
		{ 68·751	Juan and Ulloa }	
1744	0 0	{ 68·732	Bouguer .... }	Peru.
		{ 68·713	Condamine ...... }	
1752	33 18½ S.	69·076	La Caille	C. of Good Hope
1755	43 0 N.	68·998	Boscovich .... }	
1764	44 44 N.	69·061	Beccaria .... }	Italy.
1766	47 40 N.	69·142	Leisganig	Germany.
1768	39 12 N.	68·893	Mason & Dixon	America.
1802	51 29 54½ N.	69.146	Lt.-Col. Mudge	England.
1803	66 20½ N.	69·292	Swanberg, &c. ..	Lapland.
	12 32 N.	68·743	Lambton	Misore.
1808	44 52½ N.	68·769	Biot, Arago, &c.	France.

SWEEPER.—*See Nightglass.*

SYNODICAL *revolution of the planets.*

1. If two planets revolve in circular orbits, to find the time from conjunction to conjunction.

Let P = periodic time of the earth, p = that of the planet, suppose an inferior, t = the time required; then

$$t = \frac{Pp}{P - p}.$$

For a superior planet,

$$t = \frac{Pp}{p - P}.$$

This will also give the time between two oppositions, or between any two similar situations.

Cor. Since $t = \frac{P\,p}{P - p}$, $p = \frac{P\,t}{t + P}$.

Therefore, from the earth's period (P) known, and the synodic (s) observed, we can determine the periodic time (p) of the planet.

For the synodical periods of the planets—*see Planets elements of.*

2. To find the same for three bodies.

Let $T =$ time between the conjunctions of the 1st and 2d found as above; $t =$ do. between the 2d and 3d.

Then if $m =$ least common multiple of T and t, $m =$ time between two conjunctions of the three bodies.

T

TANGENTS, *method of drawing.*

1. Method of drawing a tangent to any curve, whose equation is given.

Let x and $y =$ abscissa and ordinate; then

$$\text{Subtangent} = \frac{y\,d\,x}{d\,y}.$$

Ex. 1. In parabola, subtangent $= 2\,x$.

2. In circle and ellipse, subtan. $= \frac{2\,a\,x - x^2}{a - x}$.

3. In hyperbola, subtan. $= \frac{2\,a\,x + x^2}{a + x}$.

2. To find the equations to the tangent and normal.

Let $y' = a\,x' + b$ be the equation to the tangent; then $a = \frac{d\,y}{d\,x}$;

also since the curve and line have a point in common, $y = a\,x + b$,

and $y' - y = \frac{d\,y}{d\,x}\,(x' - x)$ which is the required equation.

Again let y'' and x'' be the coordinates to the normal, then since it passes through a point whose coordinates are x and y, and is perpendicular to a line whose equation is $y' - y = \frac{d\,y}{d\,x}\,(x' - x)$, the equation to the normal will be $y'' - y = -\frac{d\,x}{d\,y}\,(x'' - x)$. *(See Line, Art. 4.)*

Ex. In the parabola, equation to the tangent is $y' - y = \frac{2\,a}{y}\,(x' - x)$; and that to the normal $y'' - y = -\frac{y}{2\,a}\,(x'' - x)$.

For tangents to Spirals—*see Spiral.*

TAYLOR'S *Theorem.—(Higman.)*

If x and y be the coordinates to any point of a curve, and if, when x becomes $x + h$, y becomes y'; then will

$$y' = y + \frac{d\,y}{d\,x}\,h + \frac{d^2\,y}{d\,x^2}\,\frac{h^2}{1.2} + \frac{d^3\,y}{d\,x^3}\,\frac{h^3}{1.2.3} + \&c.$$

Cor. 1. If when x becomes $x - h$, y becomes $y_{,}$ then will

$$y_{,} = y - \frac{d\,y}{d\,x}\,h + \frac{d^2\,y}{d\,x^2}\,\frac{h^2}{1.2} - \frac{d^3\,y}{d\,x^3}\,\frac{h^3}{1.2.3} + \&c.$$

Cor. 2. If $h = d\,x$.

$$y' = y + d\,y + \frac{d^2\,y}{1.2} + \frac{d^3\,y}{1.2.3} + \&c.$$

Cor. 3. The above theorem may be expressed in general terms thus :— The variable of a function being supposed to consist of two parts x and h, to develope the function in a series of powers of one of the parts h.

Maclaurin's Theorem.

To expand a function in a series of ascending integral and positive powers of the variable.

Let u = any function of x, then $u = (u) + \left(\frac{d\,u}{d\,x}\right)x + \left(\frac{d^2\,u}{d\,x^2}\right)\frac{x^2}{1.2} + \left(\frac{d^3\,u}{d\,x^3}\right)\frac{x^3}{1.2.3} + \&c.$ where (u), $\left(\frac{d\,u}{d\,x}\right)$, $\left(\frac{d^2\,u}{d\,x^2}\right)$, &c. denote the values of u, $\frac{d\,u}{d\,x}$, $\frac{d^2\,u}{d\,x^2}$ &c. when $x = 0$.

This theorem is only a particular case of Taylor's, for take $x = 0$ in Taylor's series, and we have

$$f(h) = (u) + \left(\frac{d\,u}{d\,x}\right)h + \left(\frac{d^2\,u}{d\,x^2}\right)\frac{h^2}{1.2} + \&c.$$

which is the same as the theorem above, if for h we write x.

Ex. 1. To expand $(x+h)^m$.

Let $u = x^m$ and $w = (x+h)^m$. By differentiation we have $\frac{du}{dx} = m\,x^{m-1}$; $\frac{d^2u}{dx^2} = m.\,(m-1)\,x^{m-2}$; $\frac{d^3u}{dx^3} = m.\,(m-1).\,(m-2)\,x^{m-3}$ &c.

Hence, by Taylor's theorem, $w = x^m + m\,x^{m-1}\,h + m.\frac{m-1}{2}\,x^{m-2}\,h^2$ + &c.

Ex. 2. To expand $(a+x)^m$ by Maclaurin's theorem.

Let $u = (a+x)^m$; $\therefore \frac{du}{dx} = m.\,(a+x)^{m-1}$; $\frac{d^2u}{dx^2} = m.\,(m-1.)\,(a+x)^{m-2}$ &c. &c. Now let $x = 0$; then $(u) = a^m$, $\left(\frac{du}{dx}\right) = m\,a^{m-1}$; $\left(\frac{d^2u}{dx^2}\right) = m.\,(m-1.)\,a^{m-2}$ &c. $\therefore$

$$u = (a+x)^m = a^m + m\,a^{m-1}\,x + m.\,\frac{m-1}{2}\,a^{m-2}\,x^2 + \text{\&c.}$$

Ex. 3. To expand a^x in a series.

Let $u = a^x$, then $\frac{du}{dx} = k\,u\ (k = \text{h.\,l.}\,a)$, $\frac{d^2u}{dx^2} = k^2\,u$, &c. Now let $x = 0$, then $u = 1$; $\therefore$, by Maclaurin's theorem,

$$a^x = 1 + \frac{k\,x}{1} + \frac{k^2\,x^2}{1.\,2} + \frac{k^3\,x^3}{1.\,2.\,3.} + \text{\&c.}$$

Ex. 4. To expand log. $(x+h)$.

Let $u = l.\,x$, and $w = l\,(x+h)$; $\therefore$

$$\frac{du}{dx} = \frac{m}{x},\ \frac{d^2u}{dx^2} = -\frac{m}{x^2},\ \frac{d^3u}{dx^3} = \frac{2\,m}{x^3}\ \text{\&c.}$$

$\therefore$ by Taylor's theorem,

$$w = u + m\left(\frac{h}{x} - \frac{h^2}{2\,x^2} + \frac{h^3}{3\,x^3} - \frac{h^4}{4\,x^4} + \text{\&c.}\right)$$

Cor. If $x = 1$, we have

$$l\,(1+h) = m\left(\frac{h}{1} - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \text{\&c.}\right)$$

Ex. 5. Expand sin. x in a series.

Let $u = \sin.\,x$. Take the successive differentials of sin. x, and find their value when $x = 0$, and we shall have by Maclaurin's theorem

$$\sin. x = x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \&c.$$

In like manner,

$$\cos. x = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \&c.$$

TELESCOPE, *theory of.—(Coddington, Wood.)*

1. *Astronomical Telescope.*

Let F, F′ represent the focal lengths of the object and eye glass; then the magnifying power is $\frac{F}{F'}$.

Cor. The linear magnitude of the greatest visible area is measured by the ∠, which the diameter of the eye glass subtends at the centre of the object glass, increased by the difference between the ∠'s which the diameter of the object glass subtends at the image, and at the eye glass.

2. *Galileo's telescope.*

The magnifying power as before $= \frac{F}{F'}$.

Cor. The linear magnitude of the field of view, when the eye is placed close to the concave lens, is measured by the angle which the diameter of the pupil subtends at the centre of the eye glass, increased by the difference between the ∠'s which the diameter of the object glass subtends at the pupil, and at the image.

3. *Herschel's and Newton's telescope.*

Let f and F′ be the focal lengths of the speculum and eye glass; then the magnifying power $= \frac{f}{F'}$.

Cor. The field of view is nearly equal to the apparent magnitude of the eye glass seen from the speculum.

4. *The Gregorian and Cassegrain's telescope.*

Let f, f', F, be the focal lengths of the great and small mirror, and the lens respectively, l the distance of the mirrors; then the magnifying power of the Gregorian is nearly

$$\frac{(l - f')^2}{f' F},$$

and of Cassegrain's is

$$\frac{(l + f')^2}{f' F}.$$

5. In refracting telescopes, if A be the linear aperture of the object glass, the density of rays in the picture upon the retina varies as

$$\frac{A^2 F'^2}{F^2}.$$

And in the Newtonian telescope, as

$$\frac{A_2 F'^2}{f^2}.$$

6. *To place a telescope in the meridian by the pole star.—(Wollaston.)*

Calculate the time of the meridian passage of the star correctly, and apply that to your chronometer. Then having the star in the field of your telescope (the instrument being first truly adjusted, and the adjusting screw for azimuth between your finger and thumb) and keeping it bisected, or covered by your meridian wire till the exact instant calculated, clamp the instrument there in azimuth, and you will find it very nearly in the meridian indeed.

Having thus placed the telescope *very nearly* in the meridian; we may adjust it accurately so, by either of the following formulæ :—

Formula for correcting the error of a Meridian Telescope by the observation of any circumpolar star above and below the pole.

If the western interval be greater than the eastern one, the telescope points to the east of that end of the true meridian which lies under the the elevated pole (be that N. or S.) and v. v.

The angle of this deviation may be investigated thus:—

To the log. of half the difference between the intervals in seconds (or the difference between either interval and 12*h*. sid. time.)

Add the log. tangent of the star's P D.

And the log. secant of the lat. of the station.

The sum (abating 20 from the Index) will give the log. of a number of seconds of sid. time; which converted into degrees, &c. will express the angular deviation of the instrument from the true meridian, to be applied as above.

This method depends not at all upon knowing truly the R A of the star; nor its P D with any very great accuracy: the Z D or alt. read off with the instrument, as it passes the meridian, will give the latter with fully sufficient precision.

Formula from which the above rule is deduced.—*(Maddy.)*

Deviation $= \frac{1}{2}\,\frac{180^0 - (t - t')}{\cos.\,l.\,\tan.\,\delta}$; where t and t' are the two intervals, δ the star's declination, and l the latitude of the place.

Formula for the correction of a Meridian Telescope by the observation of two stars differing considerably in polar distance.

If the *southern* of two stars passes a meridian telescope *too soon* for the calculated difference of apparent R A between them (whether its passage be before or after the northern one, is immaterial) the telescope when turned down towards the *south* horizon will point to the *east* of the true meridian, and v. v. This holds universally, whether the latitude of the station be N. or S.

The angle of this deviation from the meridian may be found thus:—

The quantity of sidereal time, by which the observed difference of R A varies from the calculated difference between the stars, being reduced to seconds of time;

To the log. of that number of seconds; add
the log. cosines of the declination of each star;
the log. cosecant of the difference between them in declination;
and the log. secant of the lat. of the station:

The sum (abating 40 from the Index) will give the log. of a number of seconds of sidereal time; which reduced to degrees, &c. will express the angle made by the instrument and the true meridian.

Formula from which the above rule is deduced.—*(Maddy.)*

Let $T - T'$ be the difference of right ascensions of the two stars from the Tables.

$t - t'$ the difference of right ascension as observed by the telescope, δ and δ' the declinations, l the latitude, then

$$\text{Deviation} = \left\{ T - T' - (t - t') \right\} \cdot \frac{\cos. \delta. \cos. \delta'}{\cos. l. \sin. (\delta - \delta')}.$$

7. *To find the field of view of a telescope.*

Direct the telescope to a star in the equator, or very near it, which will answer quite well enough for all usual purposes, and observe the number of seconds occupied in its passage across the field of view, and multiply this number by 4, to obtain *in degrees* a measure of the field.

It would evidently be inconsistent with the limits of this small work to enter into any explanation of the nature, use, and adjustment, of mathematical instruments; nevertheless as a telescope is in the hands of almost every one at all conversant in scientific pursuits, the following practical observations on this instrument, selected from the works of eminent practical astronomers, may not be unacceptable to the inexperienced observer.

Proper size of telescopes.

The smallest achromatic that can be used with success for astronomical purposes is the 3½ feet, aperture 2¾ inches.—*(Kitchiner.)*

Magnifying powers of telescopes.

For day purposes, a power of 90 or 100 is the maximum that can be generally used in this country, except on very fine days, and on objects uncommonly well lighted up. In telescopes of different apertures, the maximum power for day purposes is had by multiplying the diameter of the object speculum or glass in inches by 30. For astronomical purposes the rotatory motion of the earth prevents the application of a much higher power than 300 being used with any advantage: when a higher power than 300 is used, it requires uncommon dexterity both to find the object and manage the instrument. The following powers are proper for a fine achromatic. (1) A comet eye piece, made with two plano convexes not magnifying more than 12 or 15 times, which is also a delightful eye piece for viewing nebulæ and the milky way. (2) For a series of powers for *planetary* observations, multiply the diameter of the object glass in inches by 20, 30, 40, 50 and 60; this last is the maximum that can be used for the planets, and requires a very perfect telescope, and every circumstance to be favourable, to admit of its application with good effect. (3) A positive eye piece magnifying 300 times for close double stars; yet unless the telescope be an uncommonly fine one, a higher power than 200 only renders the object less distinct. (4) A circle of six single double convex lenses magnifying 50, 100, 150, 200, 300, and 400 times, but when the highest power is used, the distinct field of view is reduced to a very small diameter.—*(Kitchiner.)*

Eye glasses for telescopes.

In very delicate observations Herschel observes, no double eye glass should be used, as that occasions a too great waste of light. With the double eye glass he could not see the belts of Saturn, which he very plainly saw with the single one. Of single glasses he decidedly prefers concave to convex glasses, as they give a much more distinct image. Their very small field of view is a considerable imperfection, but in objects such as double stars, or the satellites of Saturn, and the Georgian, this inconvenience is not so material.—*(Phil. Trans.)*

Best criterion of a good telescope.

The most difficult object to define in the day time, and the best test of the distinctness and correctness of our instruments, is the dial plate of a watch, when the sun shines upon it, placed about 100 feet from the glass. In the night time a fixed star of the first magnitude is the best test, as the least defect in the figure or adjustment of the object glass is immediately seen by the star not appearing round, but surrounded by false lights and luminous accompaniments. For a test of the perfection of a telescope as to its *light and distinctness*, the pole star is as proper as any, as the small accompanying star is not visible except in a very perfect instrument. The examination of a bright object on a dark ground, as a card by daylight or Jupiter by night, with high magnifying powers, affords the severest test of the *perfect achromaticity* of a telescope, by the production of green and purple borders about their edges in the contrary case.—*(Kitchiner.—Mem. Astr. Soc.)*

On the evenings and situations favourable or otherwise to astronomical observations.

The rule upon which almost all the rest are founded is *that an uniform temperature* is necessary for the proper performance of a telescope. Upon this principle the following facts, the results of long experience, may be satisfactorily explained.

(1) A frost after mild weather, and a thaw after frost, will derange the telescope, till either the frost or mild weather are sufficiently settled. (2) No telescope just brought out of a warm room can act properly. (3) No delicate observation with high powers can be made when looking through a door, window, or slit, in the roof of on observatory; even a confined place in the open air is detrimental. (4) Windy weather is unfavourable. (5) Stars seen over the roof of a house, when very near, are not distinct, being disturbed probably by warm exhalations from the roof. (6) Dry air is unfavourable; but those evenings wherein the air is saturated with moisture, so as to drop down the tube of the telescope, are particularly favourable to distinct vision.

Upon the whole Dr Herschel observes that to use the highest magnifying powers to the greatest advantage, the air must be very clear, the moon absent, no twilight, no haziness, no violent wind, no sudden change of temperature; under all these circumstances a year that will afford 100 hours must be called a very productive one.—*(Herschel, Phil. Trans.)*

Rules necessary to be observed for examining delicate objects with success.

(1) If the telescope has been kept in a warm room, the cap of the object end should be taken off, the eye piece taken out, and the air suffered to pass through the tube for ten minutes, that it may acquire the temperature of the open air.—*(Kitchiner.)*

(2) The observer should in like manner be exposed in the open air for 15 or 20 minutes, and the eye carefully kept from all stimulating and

bright objects, so that the pupil may be in its most expanded state; it requiring at least 20 minutes before the eye can admit a view of very delicate objects (such as faint nebulæ); and the observation of a star, though only of the 2d or 3d magnitude, disorders the eye again, so as to require nearly the same time for the re-establishment of its tranquillity. —*(Herschel, Phil. Tran.)*

(3) We should never use a greater magnifying power than we absolutely want; the lower the power, the more beautiful and brilliant the object appears. In objects however that require great nicety to discern, such as the spheroidical shape of the planets, &c. it is proper in the first instance to use a considerable power, till the eye is accustomed to the phænomenon, after which the power may be gradually lowered.—*(Herschel, Phil. Trans.)*

(4) It may be proper to observe, in order to prevent disappointment, that in the prints usually given of Jupiter, Saturn, &c., the outlines and all the other features of the engraving are far more distinct than we can ever see them in the telescope in one view, it being the very intention of a copper-plate to collect together in one view all that has been successfully discovered by repeated and occasional perfect glimpses, and to represent it united to our conceptions. And this is the case with all drawings in books of Astronomy.—*(Hersch. Phil. Trans.)*

(5) In attempting to determine the apparent shape or magnitude of any planetary body or satellite, it is useful to compare it with some other known object of a similar kind. Thus to form an idea of the peculiar shape of Saturn, compare it with Jupiter several times in succession. To form some notion of the apparent magnitudes of Juno, Pallas, Ceres, and Vesta, compare them with each other, or with Jupiter's satellites.—*(Hersch. Phil. Trans.)*

(6) When we wish to discover very delicate and minute objects, which, with the finest instruments, are only to be seen under the most favourable circumstances, it is indispensable that we should be in a position of the greatest ease; no cramped or painful posture must distort the body or irritate the mind, the whole powers of which must be concentrated in the eye.—*(Kitchiner.)*

(7) In adjusting the telescope to close double stars, Dr. Herschel advises the observer previously to adjust the focus of his glass with the utmost delicacy on a star known to be single, of as nearly as possible the same altitude, magnitude, and colour, as the star which is to be examined, carefully observing whether it be round and well defined, or surrounded by little flitting appendages, as is the case when the object glass is not quite perfect.—*(Phil. Trans.)*

(8) To those who have not been long in the habit of observing double stars, it is necessary to mention, that when first seen they will appear nearer together than after a certain time; nor is it so soon as might be expected that we see them at their greatest distance. I have known it take up two or three months before the eye was sufficiently acquainted with the object to judge with the requisite precision.—*(Hersch. Phil. Trans.)*

(9) It is a singular fact, that a double star, where one of them is of the last degree of faintness, may be best seen by directing the eye to another part of the field. In this way a faint star in the neighbourhood of a large one will often become very conspicuous, though it will totally disappear, as if suddenly blotted out, when the eye is turned full upon it. The small companion of 23 (*h*) Ursæ Maj., is a remarkable instance of this; also ζ Persei, 7 Tauri, 43 Persei, ι Leporis. The lateral portions of the retina, less fatigued by strong lights, and less exhausted by perpetual attention, are probably more sensible to faint impressions than the central ones, which may serve to account for this phænomenon.—*(Hersch. jun., Phil. Trans.)*

Of the powers necessary for observing various celestial objects.

Comets may be advantageously seen with a power of about 15.

The *sun*, *moon*, and *nebulæ*, with powers of from 45 to 60.

Jupiter and his moons from 80 to 130; but for estimating the brightness and apparent magnitude of the satellites, a lower power than 180 should not be attempted. *The belts of Jupiter* are scarcely discernable in a one foot achromatic, but may be seen with an 18 inch of $1\frac{3}{10}$ aperture, and power of 40; and are easily visible in a two feet, with an aperture of $1\frac{6}{10}$, with a power of from 30 to 60. *Note.* The 3d satellite is considerably larger than any of the rest; the 1st is a little larger than the 2d, and nearly of the size of the 4th.

Saturn. The best powers for general purposes are from 130 to 200. To view him with effect, he should not be more than two, or, in very fine nights, three hours from the meridian. The phænomena most worthy of observation in this planet, are the following: his belts; the singular compression at his poles; his double ring; the shadow of the ring upon the planet, and of the planet upon the ring; and his seven satellites. The ring *may* be seen in the 18 inch telescope with a power of 40; but for observing the division of the ring, its shadow upon the planet, his belts, and the compression at his poles, we should not have a less power than 200. As to his satellites, the visibility of these minute and extremely faint objects depends more upon the penetrating, than on the magnifying power, of our telescopes; and with a ten feet Newtonian, charged with a power of only 60, Sir W. Herschell saw all the five old satellites.

Georgium Sidus. The satellites of this planet were discovered by Herschel with a power of 157; but their faint scintillations were only perceived by interrupted glimpses, but magnifiers of 300, 460, 600, and 800, were most effective. It is in vain, however, to attempt any considerable power, unless in telescopes which have a prodigious quantity of light; and in Herschell's ten feet telescope, with none of its highest powers, could he possibly ascertain even the existence of the satellites.

Ceres, Pallas, Juno, and Vesta.—Herschel applied a distinct magnifying power of 500 or 600 and even higher to these asteroids, and yet no disc was discoverable, any more than in very small stars.

Double stars. In a fine telescope the powers employed should be from 200 to 400.

List of a few double stars, which are proper objects for common telescopes.

ζ Ursæ Majoris.
γ Delphini.
β. Cygni.
γ Arietis.
γ Andromedæ.
δ Orionis.
σ Orionis double.
11 Monocerotis appears double.
ε Lyræ appears double.
ϑ Serpentis.
β Lyræ.
ι Delphini.
π Bootis.
γ Virginis.
ι Cassiopeæ.
μ Cygni.

List of a few double stars, in which proximity or faintness renders one of them difficult to be seen, and which are proper objects for the finest telescopes.

ζ Aquarii.
μ Draconis.
β Orionis.
α Piscium.
ξ Ursæ Majoris.
ω Aurigæ.
δ Geminorum.
k Cygni.
ε Persei.
α Lyræ.
ς Herculis.
α Geminorum.
ξ Libræ Seq.
π Bootis.
ε Lyræ.
11 Monocerotis.
κ Bootis.
β Scorpii.
Polaris.
γ Herculis.

49 Serpentis.	44 Bootis.
γ Leonis.	δ Serpentis.
ξ Libræ Præ.	ε Bootis.
70 Ophiuchi.	95 Herculis.
α Herculis.	β Serpentis.
ζ Coronæ.	μ Herculis.
ξ Bootis.	δ Herculis.

Performance of different telescopes.

Dr Kitchiner has seen the small star accompanying Polaris with a $2\frac{3}{4}$ feet achromatic, aperture $1\frac{3}{4}$ inches; and the small star accompanying Rigel; but the telescope was exquisitely perfect.

ε Bootis, α Herculis, γ Andromedæ, β Cygni, ζ Aquarii, Pole Star, Castor, Rigel, may be seen with a fine 44 inch achromatic, of $2\frac{3}{4}$ aperture; but not one instrument in a hundred will shew them without a false light round the larger star.

With an exquisite achromatic of 46 inches focus and a treble object glass of $3\frac{3}{8}$ inches aperture, Dr Kitchiner has seen the Pole star with the following powers, 40, 80, 150, 250, 350, 420, 700, and even with 1123 times the small star was still visible. This shews only how far magnifying power could be carried with this instrument, as it was with evident detriment to vision when higher than 80.

With a most perfect achromatic of 44 inches focus, aperture $2\frac{3}{4}$ inches made by Dollond, Mr Walker made the following observations. With a negative power of 180, he saw ε Bootis double; ι Bootis; η Coronæ Borealis. Three satellites of Saturn; the shadow of his ring on the planet; and a belt; *d* Serpentis; δ Herculis; the Pole star; ι Bootis, and *h* Draconis; powers 423 single eye glass, and 180, and 133 negative powers,— Rigel with 133, and the star in Monoceros' right foot treble with powers 133, 180, and 423.

The ordinary powers used by Messrs South and Herschel, *(see Phil. Trans.)* in forming their catalogue of double stars, was 179; though occasionally a lower power of 105, and a higher one of 273 were also used.

TEMPERATURE *of Atmosphere.—See Atmosphere.*

THERMOMETER.

	Freezing point.	*Boiling point.*
Fahrenheit's Thermometer	32°	212
Reaumur's do.	0	80
Centrigrade do.	0	100

To convert the degrees of Reaumur into those of Fahrenheit, and the contrary.

$$F = \frac{R \times 9}{4} + 32^0 \text{ — and } R = \frac{(F - 32^0) \times 4}{9}.$$

To convert the centrigrade to Fahrenheit and the contrary.

$$F = \frac{C \times 9}{5} + 32^0 \text{ — and } C = \frac{(F - 32^0) \times 5}{9}.$$

To convert the Centrigrade to Reaumur and the contrary.

$$R = \frac{C \times 4}{5} \text{ —— and } C = \frac{R \times 5}{4}.$$

THERMOMETRICAL *Barometer.—See Heat.*

TIDES.—*(Vince and Robison from Bernouilli.)*

1. If a fluid sphere at rest be attracted by a distant body S also at rest, it will put on the form of a spheroid; and if P and Q represent respectively the attraction of the spheroid at the extremities of the minor and major axes, m be the addititious force of S upon P, and n that upon the point E

$$\text{Major axis} : \text{Minor} :: P + m : E - 2n.$$

Cor. If the sphere were the earth, and S the sun or moon; then, upon the above supposition, the difference of the diameters or height of the tide, as caused by the sun, would = 2,033 feet; and the height, as caused by the moon, = 5,412 feet; ∴ in syzygy the height would be 7,445 feet.

2. The altitude of the high tide above the level of the water, if there had been no tide, is double of the depression of the low tide below.

3. Find (1) The elevation of the water at any point above the natural level of the undisturbed ocean. (2) The depression below the natural level at any point. (3) The falling of the water from the highest point, and (4) The rising of the water from the lowest point.

Put θ = angular distance of the point from the place of high water, or the hour ∠ from the time of high tide; m = perpendicular height of high above low water; then the equations will stand thus:

$$(1)\ \text{Elevation} = \frac{3 \cos.^2 \theta - 1}{3} \times m.$$

$$(2)\ \text{Depression} = \frac{3 \sin.^2 \theta - 2}{3} \times m.$$

$$(3)\ \text{Fall} \ldots\ldots\ldots = m \times \sin.^2 \theta.$$

$$(4)\ \text{Rise} \ldots\ldots\ldots = m \times \cos.^2 \theta.$$

Cor. To find the distance of high tide from the point where the water is at the same height at which it would have been if there had been no tide, put $3\cos.^2\theta - 1 = 0$; $\therefore \cos.\theta = \frac{1}{\sqrt{3}} = \cos. 54^{\circ}. 44'$.

4. To find the elevation and depression as before, produced by the joint action of the sun and moon.

Let m = perpendicular height of high above low water, as caused by the sun, n = ditto arising from the moon, θ = hour angle from the time of high tide for the sun, θ' = ditto for the moon; then the elevation above the natural level is

$$\frac{3\cos.^2\theta - 1}{3} \times m + \frac{3\cos.^2\theta' - 1}{3} \times n;$$

and depression is $\frac{3\sin.^2\theta - 2}{3} \times m + \frac{3\sin.^2\theta' - 2}{3} \times n$.

Cor. 1. If the sun and moon be in syzygy, $\theta = \theta'$;

$\therefore$ elevation $= (m + n)\cos.^2\theta - \frac{m+n}{3}$;

and depression $= (m + n)\sin.^2\theta - \frac{2}{3}(m + n)$.

Hence at high water, elevation $= \frac{2}{3}(m + n)$, and at low water, depression $= \frac{1}{3}(m + n)$.

Cor. 2. If the moon is in quadrature, elevation at $S = \frac{2}{3}m - \frac{1}{3}n$, and depression at $M = \frac{1}{3}m - \frac{2}{3}n$; also the elevation at S above the *inscribed sphere* $= m - n$, and the elevation at M above the same $= n - m$. Hence since n is greater than m in the ratio (according to Bernouilli) of $2\frac{1}{2} : 1$, it is plain that when the moon is in quadrature, it is high water under the moon, and low water under the sun.

Cor. 3. Supposing the sun and moon to be in any other position, and it were required to find an intermediate point between them where there is high tide; in this case we must take the expression $\frac{3\cos.^2\theta - 1}{3} \times m + \frac{3\cos.^2\theta' - 1}{3} \times n$, and make the differential $= 0$, and we shall get $m : n :: \sin. 2\theta : \sin. 2\theta'$. Hence we have only to divide an arc $2(\theta + \theta')$ into two parts, so that the ratio of the sines may be given; and the half of each part will give θ and θ', and thus we get the point where the tide is highest.

Cor. 4. By computing by the last Cor. the $\angle$'s θ and θ' for every day from the new or full moon, we might get the time of the high tide when compared with the passage of the sun and moon over the meridian; and thus from these we might construct a table, shewing the theoretical times of high tide during the month.

Hitherto we have supposed the luminary to be in the equator: we come in the next place to consider the effect arising from the declination of the moon.

5. Let D = moon's declination, L = latitude of the place, θ = hour ∠ from high water; then the height of the water from the lowest point is

$$(\cos. D \times \cos. L \times \cos. \theta + \sin. D \times \sin. L)^2 \times m.$$

Hence we may consider the following cases:—

I. To find the interval from high to low tide, put cos. D × cos. L × cos. θ + sin. D × sin. L = 0; $\therefore \cos. \theta = -\frac{\sin. D \times \sin. L}{\cos. D \times \cos. L}$.

II. When the latitude of the place = comp. of moon's declination, cos. θ = − 1; $\therefore \theta = 180^\circ$, i.e. the interval between high and low tide = 12 hours, i.e there is only one high and one low tide in 24 hours.

III. When the distance of the place from the pole is less than the moon's declination, the expression in Art. 5 never can become = 0 within the limits of cos. θ; ∴ there is only one high and one low tide in 24 lunar hours. And if we make cos. θ = 1, and cos. θ = − 1, we have the difference of the altitudes of the two tides = 4 cos. D × cos. L × sin. D × sin. L × m.

IV. When D = L, make cos. θ = 1, and we have the greatest altitude = m; also $\cos. \theta = \frac{\sin 2 D}{\cos. 2 D}$ = interval from high to low water.

V. When the moon is in the equator, the altitude of the tide = $\cos.^2 L \times m$.

VI. The height of the tide, when the moon passes the meridian, = $(\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$; and when the moon is at the opposite meridian, the height is $(-\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$. Hence when the moon is in the equator, sin. D = o, and the height of both tides is equal. To a place on the north of the equator, when the moon has south declination, sin. D becomes negative, and the latter tides are the greatest; but when the moon has north declination, sin. D is positive, and the former is the greatest. Hence, to us in this case, the high tide is greater when the moon is above the horizon than when below. The difference of the two tides is always what is given in Case III.

VII. The height of the two tides, when the moon passes the meridian, being $(\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$, and $(-\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$, the mean height is $(\cos.^2 D \times \cos.^2 L + \sin.^2 D \times \sin.^2 L) \times m$. Hence the same north and south declination of the moon give the same mean altitude.

VIII. Under the equator the mean height $= \cos.^2 D \times m$.

The general phænomena of the tides agree very well with the conclusions deduced from the theory of gravity, indeed much more accurately than could have been expected, when we consider that the theory supposes the whole surface of the earth to be covered with deep waters; that there is no inertia of the waters; that the major axis of the spheroid is constantly directed to the moon; and that there is an equilibrium of all the parts; none of which suppositions are strictly founded in fact.

As a sequel to this Article we will subjoin a few of the principal phænomena of the tides, as deduced from actual observation.—*(Playfair.)*

The time from one high water to the next, is, at a mean, 12*h.* 25*m.* 24*s.* The instant of low water is not exactly in the middle of this interval; the tide in general taking 9 or 10 minutes more in ebbing than in flowing.

At new and full moon, or at the spring tides, the interval between the consecutive tides is the least, viz. 12*h.* 19*m.* 28*s.* At the quadratures, or neap tides, the interval is greatest, viz. 12*h.* 30*m.* 7*s.*

The gradual subsidence of the waters is such, that the diminution of heights are nearly as the squares of the times from high water.

The time of high water in the open sea is from 2 to 3 hours after the moon has been on the meridian, either above or under the horizon; but on the shores of large continents, and where there are shallows and obstructions, there are great irregularities in this respect; but for any given place the hour of high water is always nearly at the same distance from that of the moon's passage over the meridian.

The highest of the spring tides is not the tide that immediately follows the syzygy, but is in general the third, and in some cases the fourth.

At Brest, the spring tides rise to 19,317 feet; and those of the neap to 9,151. In the Pacific Ocean, the rise, in the first case, is 5 feet; in the second, 2 or 2,5. Indeed it may happen, that although the greatest elevation produced by the joint action of the sun and moon, in the open sea, does not exceed 8 or 9 feet, the tide in some singular situations may amount considerably higher. For instance, in the harbour of Annapolis-Royal, it sometimes rises 120 feet; the water accumulating to this astonishing height in consequence of its being stopped in the Bay of Fundy as in a hook.

The greater the rise of high water above the level of a fixed point, the greater is the depression of the corresponding low water relatively to the same point.

The height of the tide is affected by the vicinity of the moon to the earth, and increases, cæteris paribus, when the parallax and apparent diameter of the moon increase, but in a higher ratio.

The rise of the tide is affected by the declination of the luminaries; it is greatest, cæteris paribus, at the equinoxes, and least at the solstices.

When the moon is in the northern signs, the tide of the day, in all northern latitudes, is somewhat greater than the tide of the night: and the contrary when the moon is in the southern signs.

If the tides be considered relatively to the whole earth, and to the open sea, there is a meridian about 30° eastward of the moon, where it is always high water; on the west side of this circle, the tide is flowing; on the east, it is ebbing; and on the meridian, at right ∠'s to the same, it is every where low water.

In high latitudes, whether south or north, the rise and fall of the tide are inconsiderable. It is probable that at the poles there are no tides.

The tides, in narrow seas, and on shores far from the main body of the ocean, are not produced in those seas by the direct action of the luminaries, but are waves propagated from the great diurnal undulation, and moving with much less velocity. For instance, the high water transmitted from the tide in the Atlantic, reaches Ushant between three and four hours after the moon has passed the meridian. This wave then divides itself into three; one passing up the British Channel, another ranging along the west side of Ireland and Scotland, and the third entering the Irish Channel. The first of these flows through the channel at about 50 miles an hour, and reaches the Nore about 12 at night. The second moves more rapidly, so as to reach the North of Ireland by six, and the Orkneys by nine, and the Naze of Norway by 12; and in 12 hours more it reaches the Nore, where it meets the morning tide, that left the mouth of the channel only eight hours before. Thus these two tides travel round Britain in about 28 hours, in which time the primitive tide has gone round the whole circumference of the earth and nearly 45 degrees more.

TABLE

Of the time of High Water at the full and change of the Moon, at the principal ports and places on the coasts of Great Britain and Ireland.

Places.	Situation.	Time H.	Time M.
Aberdeen	Scotland	12	45
Aberdovy	Wales	7	30
Aberistwith	Wales	7	30
Achill Head	Ireland	6	0
Agnes (St)	Scilly Isles	4	40
Air Point	Isle of Man	10	30
Aldborough	England	10	45
Alne River	England	2	45
Amlwick Point	Anglesea	10	30
Arran Isle	Scotland	11	15
Arundel	England	9	20
Balta	Shetland	3	0
Baltimore	Ireland	3	45
Bamff	Scotland	11	30
Bantry Bay	Ireland	3	45
Barmouth	Wales	8	0
Barnstaple Bar	England	5	30
Beachy, on Shore	England	9	45
Beachy Offing ...	England	11	0
Beaumaris	Wales	10	15
Berwick	England	2	15
Blakeney	England	6	0
Blyth	England	2	45
Bolt Head	England	5	55
Boston	England	7	15
Brassa Sound ...	Shetland	10	0
Bree Bank	North Sea	3	30
Bridgewater	England	6	45
Bridlington	England	4	30
Bridport	England	6	45
Brighton	England	10	0
Bristol *	England	7	0
Burnt Island ...	Scotland	2	30
Caernarvon Bar	Wales	9	0
Cairston	Orkney	9	0
Calf of Man	St Geo. Cha	10	30
Cantire (Mull) ...	Scotland	9	0
Cardigan Bar ...	Wales	7	0
Carlingford	Ireland	9	0
Carmarthen	Wales	6	0
Chatham	England	1	0
Chester-Bar	England	10	30
Chichester Harb.	England	11	30
Clear Cape	Ireland	4	30
Cornwall Cape ...	England	4	25
Cork Harb. Ent.	Ireland	4	30
Cowes	I. of Wight	11	15
Cromartie	Scotland	11	45
Cuckold's Point	Ri. Thames	2	15
Dartmouth	England	6	10
Deal	England	11	15
Dee (River)	Scotland	12	45
Dingle Bay	Ireland	3	20
Dover Pier	England	11	16
Downs	England	11	15
Dublin	Ireland	9	30
Dudgeon Lights	North Sea	6	0
Dunbar	Scotland	2	15
Dundalk Bay ...	Ireland	10	45
Dundee	Scotland	2	15
Dungarvon	Ireland	4	30
Dungeness	England	11	15
Eddystone	Eng. Chan.	5	15
Exmouth Bar ...	England	6	25
Falmouth	England	5	30
Flamboro' Head	England	4	30
Flats (Kentish)	England	11	0
Foreland (N) ...	England	11	15
Foreland (S)	England	11	6
Fowey	England	5	30
Galloper	Ri, Thames	12	45
Galway Bay	Ireland	4	30
Galloway (Mull)	Scotland	11	15
Goodwyn	Downs	1	30
Gravesend	England	1	30
Gunfleet	Ri. Thames	12	6
Harwich	England	11	30
Hastings	England	10	36
Helen's (St)	England	11	45
Holyhead Bay ...	Wales	10	0
Hull	England	6	0
Humber R. Ent.	England	5	15
Ives (St)	England	4	30
Kenmare River	Ireland	3	30
Kentish Knock	Ri. Thames	11	30
Kinsale	Ireland	5	0

Places.	Situation.	Time H.	Time M.
Land's End	England	4	30
Leith Pier	Scotland	2	20
Lewis Islands	Scotland	6	0
Liverpool	England	11	8
London	England	2	45
Lyme Regis	England	6	45
Margate Roads	England	11	45
Milford Haven	England	6	0
Montrose	Scotland	1	30
Mount's Bay	England	4	55
Needles	England	9	45
Newcastle	England	4	0
Nore Light	Ri. Thames	12	30
Orfordness	England	11	0
Orkney Isles	Scotland	10	30
Pentland Frith	Scotland	10	30
Penzance	England	4	30
Plymouth Sound	England	5	15
Portland Race	England	9	15
Portland Road	England	6	15
Portsmouth Har.	England	11	30
Ramsgate	England	11	20
Rye Harbour	England	10	36
Saltees	Ireland	5	40
Seaford	England	10	16
Selsea Harbour	England	11	15
Shannon R. Ent.	Ireland	3	45
Sheerness	England	12	0
Shields	England	3	0
Skerries	Ireland	4	45
Sligo	Ireland	6	45
Solebay	England	10	30
Southampton	England	11	45
Spithead	England	9	30
Sunderland	England	3	0
Swansea	Wales	6	0
Swin	Ri. Thames	12	0
Tay Bar	Scotland	2	0
Tees River	England	3	30
Tinmouth	England	3	0
Torbay	England	6	10
Trallee Bay	Ireland	3	45
Waterford Harb.	Ireland	5	30
Wexford Harb.	Ireland	7	30
Weymouth	England	6	30
Whitby	England	3	45
Wicklow	Ireland	9	0
Wisbeach	England	7	30
Yarmouth Roads	England	8	45
Yarmouth Sands	England	10	30
Yorkshire Coast	England	6	0
Youghall	Ireland	5	0

To find the time of high water on a given day at any place where the time of high water at full and change is known.

Let the time of the moon's passing the meridian of the given place be found in the Nautical Almanack, and to this time apply the correction, from the following Table, corresponding to her meridian passage and semidiameter, and to the result add the time of high water at full and change at the given place, as given in the preceding Table, and the sum will be the time of high water on the given day. If this sum exceed 12*h* 24*m*, or 24*h* 49*m*, subtract those times from it, and the remainder will be nearly the time of high water on the afternoon of the given day.

Corrections to be applied to the time of the moon's meridian passage in finding the time of high water.

Moon's Mer. Pass.	Moon's Semidiameter.			Moon's Mer. Pass.	Moon's Mer. Pass.	Moon's Semidiameter.			Moon's Mer. Pass.
	14′ 30″	15′ 30″	16′ 30″			14′ 30″	15′ 30″	16′ 30″	
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 0	− 0 4	0 0	+ 0 5	12 0	6 0	− 0 55	− 1 2	− 1 12	18 0
0 30	− 0 10	− 0 8	− 0 5	12 30	6 30	− 0 46	− 0 51	− 0 58	18 30
1 0	− 0 17	− 0 16	− 0 15	13 0	7 0	− 0 32	− 0 31	− 0 37	19 0
1 30	− 0 24	− 0 25	− 0 25	13 30	7 30	− 0 17	− 0 16	− 0 14	19 30
2 0	− 0 31	− 0 34	− 0 36	14 0	8 0	− 0 1	+ 0 3	+ 0 9	20 0
2 30	− 0 38	− 0 41	− 0 46	14 30	8 30	+ 0 8	+ 0 15	+ 0 24	20 30
3 0	− 0 44	− 0 49	− 0 55	15 0	9 0	+ 0 14	+ 0 21	+ 0 32	21 0
3 30	− 0 50	− 0 56	− 1 4	15 30	9 30	+ 0 16	+ 0 24	+ 0 36	21 30
4 0	− 0 55	− 1 2	− 1 12	16 0	10 0	+ 0 15	+ 0 23	+ 0 34	22 0
4 30	− 0 58	− 1 6	− 1 16	16 30	10 30	+ 0 12	+ 0 19	+ 0 29	22 30
5 0	− 1 0	− 1 8	− 1 19	17 0	11 0	+ 0 7	+ 0 14	+ 0 23	23 0
5 30	− 0 59	− 1 7	− 1 18	17 30	11 30	+ 0 2	+ 0 7	+ 0 15	23 30
6 0	− 0 56	− 1 2	− 1 12	18 0	12 0	− 0 4	0 0	+ 0 5	24 0

Ex. Required the time of high water at London, Sept. 2, 1823, the time of the moon's transit being 22*h.* 39*m.*, and her ½ diameter 16′. 26″, by the Naut. Alm.

	h.	m.
Moon's transit	22	39
Correction from the above Table	+ 0	29
	23	8
High water at full and change by 1st Table	2	45
	25	53
Subtract	24	49
Time required	1	4

TIMBER *measuring.*

The customary rule for the measurement of timber is erroneous; for, according to the common rule, a tree frequently contains one-fourth more timber than it is estimated at. The following formulæ give both the customary and true content.

Let L = the length of the tree in feet and decimals, and G the mean girth taken in inches; then

$$\frac{L\,G^2}{2304} = \text{cubic feet customary.}$$

$$\frac{L\,G^2}{1807} = \text{cubic feet true content.}$$

If G as well as L be in feet,

$.08\ L\ G^2 =$ cubic feet true content.

Sometimes a certain allowance is made in girting a tree for the thickness of the bark, which is generally one inch to every foot in girt, or $\frac{1}{12}$ of the whole girt; in that case,

$$\frac{L\ G^2}{2742} = \text{cubic feet customary.}$$

$$\frac{L\ G^2}{2150} = \text{cubic feet true content.}$$

If the tree tapers regularly from one end to the other, take half the sum of the girts at the two ends for the mean girt. If the tree do not taper regularly, but is unequal, being thick in some places and small in others, it is usual to take several different dimensions, the sum of which divided by the number of them is accounted the mean girt. But when the tree is very irregular, it is best to divide it into several lengths, and to find the content of each separately. That part of a tree, or of the branches, whose ¼ girt is less than ½ a foot, is not accounted timber.

TIMBER, *on the strength and stress of.—See Elastic bodies, equilibrium of.*

TIME, *equation of.—See Equation of Time.*

TIME, *various tables relating to.—(Vince.)*

TABLE I.

For converting degrees, minutes, and seconds into sidereal time.

Deg. Min.	Hou. Min. Min. Sec.	Deg. Min.	Hou. Min. Min. Sec.	Sec.	Dec. of Sec.
1	0. 4	30	2. 0	1	,067
2	0. 8	40	2. 40	2	,133
3	0. 12	50	3. 20	3	,2
4	0. 16	60	4. 0	4	,266
5	0. 20	70	4. 40	5	,333
6	0. 24	80	5. 20	6	,4
7	0. 28	90	6. 0	7	,466
8	0. 32	100	6. 40	8	,533
9	0. 36	200	13. 20	9	,6
10	0. 40	300	20. 0	10	,666
20	1. 20				

Ex. Reduce 74° 39′ 57″ into time.

70°	4*h*	40*m*	0*s*
4°	0	16	0
30′	0	2	0
9′	0	0	36
50″	0	0	3,333
7″	0	0	0,466
Time required	4	58	39,799

TABLE II.

For converting sidereal time into degrees, minutes, and seconds.

Hou.	Deg.	Min. Sec.	Deg. Min. Min. Sec.	Dec. of Sec.	Sec.
1	15	1	0. 15	,1	1,5
2	30	2	0. 30	,2	3,0
3	45	3	0. 45	,3	4,5
4	60	4	1. 0	,4	6,0
5	75	5	1. 15	,5	7,5
6	90	6	1. 30	,6	9,0
7	105	7	1. 45	,7	10,5
8	120	8	2. 0	,8	12,0
9	135	9	2. 15	,9	13,5
10	150	10	2. 30		
11	165	20	5. 0		
12	180	30	7. 30		
16	240	40	10. 0		
20	300	50	12. 30		

The manner of applying this Table is evident from the last Example.

TABLE III.

Decimal parts of an Hour.

1′	,01666	1″	,00028
2	,03333	2	,00056
3	,05	3	,00083
4	,06666	4	,00111
5	,08333	5	,00139
6	,1	6	,00167
7	,11666	7	,00194
8	,13333	8	,00222
9	,15	9	,00250
10	,16666	10	,00277
20	,33333	20	,00556
30	,5	30	,00833
40	,66666	40	,01111
50	,83333	50	,01388

TABLE IV.

Decimal parts of a Degree.

Min.	Dec.	Min.	Dec.	Sec.	Dec.	Sec.	Dec.
1	,01667	31	,51667	1	,00028	31	,00861
2	,03333	32	,53333	2	,00056	32	,00889
3	,05000	33	,55000	3	,00083	33	,00917
4	,06667	34	,56667	4	,00111	34	,00914
5	,08333	35	,58333	5	,00138	35	,00972
6	,10000	36	,60000	6	,00167	36	,01000
7	,11667	37	,61667	7	,00194	37	,01028
8	,13333	38	,63333	8	,00222	38	,01056
9	,15000	39	,65000	9	,00250	39	,01083
10	,16667	40	,66667	10	,00278	40	,01111
11	,18333	41	,68333	11	,00306	41	,01139
12	,20000	42	,70000	12	,00333	42	,01167
13	,21667	43	,71667	13	,00361	43	,01194
14	,23333	44	,73333	14	,00389	44	,01222
15	,25000	45	,75000	15	,00417	45	,01250
16	,26667	46	,76667	16	,00414	46	,01278
17	,28333	47	,78333	17	,00472	47	,01306
18	,30000	48	,80000	18	,00500	48	,01333
19	,31667	49	,81667	19	,00528	49	,01361
20	,33333	50	,83333	20	,00556	50	,01389
21	,35000	51	,85000	21	,00583	51	,01417
22	,36667	52	,86667	22	,00611	52	,01444
23	,38333	53	,88333	23	,00639	53	,01472
24	,40000	54	,90000	24	,00667	54	,01500
25	,41667	55	,91667	25	,00694	55	,01528
26	,43333	56	,93333	26	,00722	56	,01556
27	,45000	57	,95000	27	,00750	57	,01583
28	,46667	58	,96667	28	,00778	58	,01611
29	,48333	59	,98333	29	,00806	59	,01639
30	,50000	60	1,00000	30	,00833	60	,01667

TABLE V.

Decimal Numbers for each Day in the Year.

D	MONTHS.											
D	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug	Sept.	Oct.	Nov.	Dec.
1	0.000	0.085	0.162	0.246	0.329	0.414	0.496	0.581	0.666	0.747	0.832	0.914
2	0.003	0.088	0.164	0.249	0.331	0.416	0.499	0.583	0.668	0.750	0.835	0.917
3	0.006	0.091	0.167	0.252	0.334	0.419	0.502	0.586	0.671	0.753	0.838	0.920
4	0.008	0.093	0.170	0.255	0.337	0.422	0.504	0.589	0.673	0.755	0.840	0.922
5	0.011	0.096	0.173	0.258	0.340	0.425	0.507	0.592	0.675	0.758	0.843	0.925
6	0.014	0.099	0.175	0.260	0.342	0.427	0.509	0.594	0.678	0.760	0.845	0.928
7	0.017	0.102	0.178	0.263	0.345	0.430	0.512	0.597	0.681	0.763	0.848	0.931
8	0.019	0.104	0.181	0.266	0.348	0.433	0.515	0.600	0.684	0.766	0.851	0.933
9	0.022	0.107	0.184	0.269	0.351	0.436	0.518	0.602	0.687	0.769	0.854	0.936
10	0.025	0.109	0.186	0.271	0.353	0.438	0.520	0.605	0.689	0.772	0.856	0.939
11	0.028	0.112	0.189	0.274	0.356	0.441	0.523	0.608	0.692	0.775	0.859	0.942
12	0.030	0.115	0.192	0.277	0.359	0.444	0.526	0.610	0.695	0.777	0.862	0.944
13	0.033	0.118	0.195	0.280	0.362	0.447	0.529	0.613	0.698	0.780	0.865	0.947
14	0.036	0.120	0.197	0.282	0.364	0.449	0.531	0.616	0.701	0.782	0.867	0.950
15	0.039	0.123	0.200	0.285	0.367	0.452	0.534	0.619	0.703	0.785	0.870	0.953
16	0.041	0.127	0.203	0.288	0.370	0.455	0.537	0.622	0.706	0.788	0.873	0.955
17	0.044	0.129	0.206	0.291	0.373	0.458	0.540	0.625	0.709	0.791	0.876	0.958
18	0.046	0.131	0.208	0.293	0.375	0.460	0.542	0.627	0.711	0.793	0.878	0.961
19	0.049	0.134	0.211	0.296	0.378	0.463	0.545	0.630	0.714	0.796	0.882	0.964
20	0.052	0.137	0.214	0.299	0.381	0.466	0.548	0.633	0.717	0.799	0.884	0.966
21	0.056	0.140	0.217	0.302	0.383	0.468	0.551	0.636	0.720	0.802	0.887	0.969
22	0.057	0.142	0.219	0.304	0.386	0.471	0.553	0.638	0.722	0.804	0.890	0.971
23	0.060	0.145	0.222	0.307	0.389	0.473	0.556	0.641	0.725	0.807	0.893	0.974
24	0.063	0.148	0.225	0.309	0.392	0.476	0.559	0.644	0.728	0.810	0.895	0.977
25	0.066	0.151	0.227	0.312	0.395	0.479	0.562	0.647	0.731	0.813	0.898	0.980
26	0.068	0.153	0.230	0.315	0.397	0.482	0.564	0.649	0.733	0.815	0.900	0.983
27	0.071	0.156	0.233	0.318	0.400	0.485	0.567	0.652	0.736	0.818	0.903	0.985
28	0.074	0.159	0.236	0.320	0.403	0.487	0.570	0.655	0.739	0.821	0.906	0.988
29	0.077	0.162	0.239	0.323	0.406	0.490	0.573	0.657	0.742	0.824	0.909	0.991
30	0.079		0.241	0.326	0.408	0.493	0.575	0.660	0.744	0.826	0.912	0.994
31	0.082		0.244		0.411		0.578	0.663		0.829		0.997

TABLE VI.

For reducing Sidereal to Mean Solar Time.

Hou.	Min. Sec.	Min.	Sec.	Sec.	Sec.
1	0. 9, 83	1	0,16	1	0,00
2	0. 19, 66	2	0,33	2	0,01
3	0. 29, 49	3	0,49	3	0,01
4	0. 39, 32	4	0,66	4	0,01
5	0. 49, 15	5	0,82	5	0,01
6	0. 58, 98	6	0,98	6	0,02
7	1. 8, 81	7	1,15	7	0,02
8	1. 18, 64	8	1,31	8	0,02
9	1. 28, 47	9	1,47	9	0,02
10	1. 38, 30	10	1,64	10	0,03
11	1. 48, 13	11	1,80	11	0,03
12	1. 57, 96	12	1,97	12	0,03
13	2. 7, 78	13	2,13	13	0,04
14	2. 17, 61	14	2,29	14	0,04
15	2. 27, 44	15	2,46	15	0,04
16	2. 37, 27	16	2,62	16	0,04
17	2. 47, 10	17	2,78	17	0,05
18	2. 56, 93	18	2,95	18	0,05
19	3. 6, 76	19	3,11	19	0,05
20	3. 16, 59	20	3,28	20	0,05
21	3. 26, 42	30	4,91	30	0,08
22	3. 36, 25	40	6,55	40	0,11
23	3. 46, 08	50	8,19	50	0,14
24	3. 55, 91	60	9,83	60	0,16

RULE. Subtract the numbers found in the table corresponding to the given sidereal time from that time, and it reduces it to mean solar time.

Ex. Reduce 17*h*. 19*m*. 23*s*. sidereal time into mean solar time.

17*h*		2*m*	47,10*s*	
19*m*		0	3,11	
20*s*		0	0,05	
3*s*		0	0,01	
		2	50,27	
	17*h*	19	23	
Mean solar time	17	16	32,73	

TABLE VII.—*For converting Mean Solar into Sidereal Time.*

Hou.	Min. Sec.	Min.	Sec.	Sec.	Sec.
1	0. 9, 86	1	0,16	1	0,00
2	0. 19, 71	2	0,33	2	0,01
3	0, 29, 57	3	0,49	3	0,01
4	0. 39, 43	4	0,66	4	0,01
5	0. 49, 28	5	0,82	5	0,01
6	0. 59, 14	6	0,99	6	0,02
7	1. 8, 99	7	1,15	7	0,02
8	1. 18, 85	8	1,31	8	0,02
9	1. 28, 71	9	1,48	9	0,02
10	1. 38, 56	10	1,64	10	0,03
11	1. 48, 42	11	1,82	11	0,03
12	1. 58, 28	12	1,97	12	0,03
13	2. 8, 13	13	2,14	13	0,04
14	2. 17, 99	14	2,30	14	0,04
15	2. 27, 85	15	2,46	15	0,04
16	2. 37, 70	16	2,63	16	0,04
17	2. 47, 56	17	2,79	17	0,05
18	2. 57, 42	18	2,96	18	0,05
19	3. 7, 27	19	3,12	19	0,05
20	3. 17, 13	20	3,28	20	0,05
21	3. 26, 98	30	4,93	30	0,08
22	3. 36, 84	40	6,57	40	0,11
23	3. 46, 70	50	8,21	50	0,14
24	3. 56, 55	60	9,86	60	0,16

RULE. Add the acceleration or the numbers found in the table corresponding to the given mean solar time, to that time, and it reduces it to sidereal time.

The application of this rule is evident, from the last example.

Time, sidereal and mean solar.

Given the hour in mean solar time, to find the sidereal time.

RULE. To the given mean solar time apply the equation of time at the preceding noon from the Naut. Alm., but with a contrary sign, which gives the time since the sun's centre was on the meridian; reduce this time so corrected to sidereal time, by adding the acceleration from Tab. VII.; to which add the sun's R. A. at preceding noon from the Naut. Alm.

Or thus at short—

Sid. time = mean solar time $\mp$ equation of time at prec. noon + acceleration for that hour + sun's R. A. at prec. noon.

Hence conversely,

Mean sol. time = sid. time — sun's R. A. at prec. noon — acceleration for the hour so deduced by Tab. VI. $\pm$ equation of time at prec. noon.

This last rule also gives the time of a star's passage over the meridian in mean solar time, the star's R. A. being substituted for sid. time.

Ex. Given mean solar time 5*h.* 19*m.* 17,4*s.*, Nov. 8, 1827; to find the corresponding sidereal time,

	d.	*m.*	*s.*
	5	19	17,4
Equation of time	+	16	6,9
	5	35	24,3
			55,1
	5	36	19,4

By Tab. VII.	*s.*
5*h*	49,28
30*m*	4,93
5*m*	0,82
20*s*	0,05
4*s*	0,01
	55,09

	5	36	19,4
R. A. Sun. N. Alm.	14	51	25,8
Sid. Time	20	27	45,2

When the longitude is different from that of Greenwich, a proportional correction must be made for the difference.

If the Naut. Alm. is not at hand, sidereal time may be found very nearly by the following Table, merely adding the sun's *mean* R. A. in the table to the time of day where you are.—*(Woodhouse.)*

Sun's mean R. A.

	Hours.	Days.	M.	S.
Jan. 6	19	1	3	56
21	20	2	7	53
Feb. 5	21	3	11	49
20	22	4	15	46
Mar. 7	23	5	19	43
22	0	6	23	39
Apr. 7	1	7	27	36
22	2	8	31	32
May 7	3	9	35	29
22	4	10	39	28
June 7	5	11	43	22
22	6	12	47	18
July 7	7	13	51	15
22	8	14	55	12
Aug. 7	9	15	59	8
22	10			
Sept. 6	11			
21	12			
Oct. 6	13			
21	14			
Nov. 6	15			
21	16			
D c. 6	17			
21	18			

Ex. Given as before; to find the sidereal time.

	h.	*m.*	*s.*
Nov. 6th	15	0	0
2d		7	53
	15	7	53
Given time	5	19	17,4
Sid. time	20	27	10,4

Or more accurately by adding the acceleration 55,1*s*, as found in the last example, to the given time, we should have sid. time = 20. 28. 5,5.

From this same Table and the Table of R. A. of the principal stars *(see Stars, catalogue of)*, may also be found the time of a star's transit over the meridian in mean solar time nearly without the aid of the Naut. Alm.; the rule being,

Star's R. A. — sun's mean R. A. — acceleration = mean solar time at the time of the star's transit.

To find the time of the moon or a planet's passing the meridian.—(Woodhouse.)

Let the increment of sun's R. A. in 24*h*. be *a*.; do. of a planet or the moon be A: let also the difference between the R. A. of the heavenly body and that of the sun at the *preceding noon*, expressed in sidereal time, be *t*; then time of a planet's transit =

$$t - \frac{a - A}{24} t + \left(\frac{a - A}{24}\right)^2 t.$$

Or when the planet is retrograde, time =

$$t - \frac{a + A}{24} t + \left(\frac{a + A}{24}\right)^2 t.$$

In the case of the moon, the time =

$$t + \frac{A - a}{24} t + \left(\frac{A - a}{24}\right)^2 t.$$

And in the case of a star, the time =

$$t - \frac{a\,t}{24} + \left(\frac{a}{24}\right)^2 t.$$

Time error in, corresponding to any small given error or variation in the declination, latitude, or altitude.—(Woodhouse.)

(1) *Declination.*

Let t be the exact time from noon, δ = change of declination, ι = variation in the time, then

$$\iota = \delta \text{ (tan. declination} \times \text{cot. } t - \text{tan. lat.} \times \text{cosec. } t)$$

This formula is used in finding the time from equal altitudes of the sun, when there is a change of declination, in the interval between the two observations, which there is always, except at the solstices.

(2) *Given the error in latitude to find the error in time.*

Let λ = error in latitude, ι = do. in time, then

$$\iota = \lambda \text{ (tan. dec.} \times \text{cosec. } t - \text{tan. lat.} \times \text{cot. } t.)$$

This formula is useful at sea; for between the observation which determines the latitude from the sun's meridian altitude, and the observation of the altitude, the observer, if on board a ship, may have changed his place, and if so may have probably changed his latitude.

(3) *Given the error in altitude to find the error in time.*

Let α be the error in altitude, then

$$\iota = \frac{\alpha}{\text{sin. azim.} \times \text{cos. lat.}}.$$

Hence for a given error in altitude ι is the least when the body is on the prime vertical, the altitude ∴ should be taken near the east or west points.

Time of sun's passing the meridian, or the horizontal or vertical wire of a telescope.—(Vince.)

(1) Let $d'' =$ diameter of the sun estimated in seconds of a great circle; then the time of passing the meridian is

$$\frac{d'' \times \text{sec. declin.}}{15''}.$$

The same will do for the moon if $d'' =$ its diameter.

(2) The time of passing an horizontal wire is

$$\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}.$$

The same expression must also give the time which the sun takes in rising.

If $d'' = 1980''$ the horizontal refraction, we have the time that refraction accelerates the rising of the sun =

$$132'' \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}.$$

(3) The time in which the sun would pass the vertical wire of a telescope is

$$\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. }\theta \times \text{cos. dec.}},$$ where $\theta = \angle$ formed by the circles of altitude and declination.

TORSION, *elasticity of.—See Elastic Bodies, equilibrium of.*

TRADE-WIND.—*See Wind.*

TRANSITS *of Mercury and Venus.—(Vince.)*

Let P = the periodic time of the earth, p that of Venus or Mercury. Now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position in respect to the node. Let the earth perform x revolutions whilst the planet performs y revolutions; then will $Px = py$, $\therefore \ \frac{x}{y} = \frac{p}{P}$. Now $P = 365{,}256$ and for *Mercury*, $p = 87{,}968$, therefore, $\frac{x}{y} = \frac{p}{P} = \frac{87{,}968}{365{,}256} =$ (by resolving it into its continued fractions) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a state of equality, the further you go. The first period, or that of one year, is not sufficiently exact; the period of six years will sometimes bring on a return of the transit at the *same* node; that of seven years more frequently; that of 13 years still more frequently, and so on. Now there was a transit of Mercury at its descending node, in May, 1786; hence by continually adding 6, 7, 13, 33, 46, &c. to it, you get all the years when the transit may be *expected* to happen at that node. In 1789 there was a transit at the ascending node, and therefore by adding the same numbers to that year you will get the years in which the transits may be expected to happen at that node. The next transits at the descending node will happen in 1799, 1832, 1845, 1878, 1891; and at the ascending node, in 1802, 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894. For *Venus*, $p = 224{,}7$; hence $\frac{x}{y} = \frac{p}{P} = \frac{224{,}7}{365{,}256} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The transits at the same node will therefore sometimes return at 8 years, but oftener in 235, and still oftener in 713, &c. Now in 1769 a transit happened at the descending node in June, and the next transits at the same node will be in 2004, 2012, 2247, 2255, 249 , 2498, 2733, 2741, and 2984. In 1639 a transit happened at the ascending node in November, and the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846, and 2854. These transits are found to happen, by continually adding the periods, and finding the years when they may be *expected*, and then computing, for each time, the shortest geocentric distance of Venus from the sun's centre at the time of conjunction; and if it be less than the semidiameter of the sun, there will be a transit.

TRANSIT *of a star and planet over the Meridian.—See Time.*

TRANSIT *instrument, to bring it into the Meridian.—See Telescope.*

TRAPEZIUM, *area of.—See Surveying.*

TRIANGLE, *plane and spherical area of.—See Surveying and Trigonometry.*

TRIGONOMETRY.—(*Woodhouse, Barlow.*)

I. Plane Trigonometry.

Solution of the cases of right angled triangles.

Let a be the base, b the perpendicular, c the hypothenuse, and A, B, C the angles opposite.

Given.	*Sought.*	*Solution.*	*Given.*	*Sought.*	*Solution.*
c, B	a	$a = c.\ \cos. B$	a, c	b	$b = \sqrt{c^2 - a^2}$
	b	$b = c.\ \sin. B$		B	$\cos. B = \frac{a}{c}$
	A	$A = \frac{\pi}{2} - B$			
a, B	b	$b = a.\ \tan. B$	a, b	c	$c = \sqrt{a^2 + b^2}$
	c	$c = a.\ \sec. B$		B	$\tan. B = \frac{b}{a}$
b, B	a	$a = b.\ \cot. B$			
	c	$c = b.\ \text{cosec}. B$			

Solution of the cases of oblique angled triangles.

Let a, b, c be the sides of the Δ; A, B, C the $\angle$'s opposite to them.

Case 1.

Given two sides and an $\angle$ opposite to one of them; or two $\angle$'s and a side; to find the rest.

Solution.—The sides are proportional to the sines of the opposite $\angle$'s.

Note. When two sides and an $\angle$ opposite to one of them are given, the case is sometimes ambiguous, viz. when the side adjacent is greater than the side opposite to the given $\angle$, and that $\angle$ is acute. But in practical cases there will be found some circumstance or other to remove the ambiguity.

Case 2.

Given two sides a and b, and included $\angle$ C.

Solution 1st. Tan. $\frac{A - B}{2} = \frac{a - b}{a + b}$. Tan. $\frac{A + B}{2}$. Hence $A + B$ and $A - B$ are known, and consequently A and B.

Solution 2d. Let a be greater than b. Find in the tables an $\angle\, \theta$ such that $\tan.\ \theta = r.\ \frac{a}{b}$; then $r.\ \tan.\ \frac{A-B}{2} = \tan.\ \frac{A+B}{2}.\ \tan.\ (\theta - 45^o)$.

The latter method is the most concise in those cases in which the logs. of a and b are given.

Case 3d.

Given a, b, c to find A, B, C.

Solution 1st. Let $S = \frac{a+b+c}{2}$; then $\left(\sin.\ \frac{A}{2}\right)^2 = r^2 \times \frac{(S-b).\ (S-c).}{c\, b}$

Solution 2d. $\left(\text{Cos.}\ \frac{A}{2}\right)^2 = r^2 \times \frac{S.\ (S-a)}{b\, c}$.

Solution 3d. $\left(\text{Tan}\ \frac{A}{2}\right)^2 = r^2 \times \frac{(S-b).\ (S-c)}{S.\ (S-a)}$.

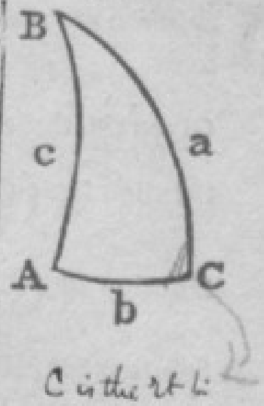
If the $\angle$ sought be less than 90° use 1st method.

If ———— greater than 90° use 2d method.

The third method may be used in all cases, except when the $\angle$ sought is nearly 180°. When the $\angle$ sought is very small, and great accuracy is required, a peculiar computation is necessary.

II. Spherical Trigonometry.

Solution of the six cases of right angled spherical triangles.



Given.	Soug.	Values of the terms required.	Cases where the terms required are less than 90°.
c, b	B	$\text{Sin. B} = \frac{\text{sin. } b}{\text{sin. } c}$	If b be less than 90°
	A	$\text{Cos. A} = \frac{\text{tan. } b}{\text{tan. } c}$	If c and b are of same affection
	a	$\text{Cos. } a = \frac{\text{cos. } c}{\text{cos. } b}$	Do.
b, B	c	$\text{Sin. } c = \frac{\text{sin. } b}{\text{sin. B}}$	Ambiguous
	a	$\text{Sin. } a = \frac{\text{tan. } b}{\text{tan. B}}$	Do.
	A	$\text{Sin. A} = \frac{\text{cos. B}}{\text{cos. } b}$	Do.
b, A	c	$\text{Tan. } c = \frac{\text{tan. } b}{\text{cos. A}}$	If b and A are of same affection
	B	$\text{Cos. B} = \text{cos. } b \times \text{sin. A}$	If b be less 90°.
	a	$\text{Tan. } a = \text{sin. } b \times \text{tan. A}$	If A be less 90°.
c, A	b	$\text{Tan. } b = \text{tan. } c \times \text{cos. A}$	If c and A be of same affection
	a	$\text{Sin. } a = \text{sin. } c \times \text{sin. A}$	If A be acute
	B	$\text{Tan. B} = \frac{\text{cot. A}}{\text{cos. } c}$	If c and A be of same affection
a, b	c	$\text{Cos. } c = \text{cos. } a \times \text{cos. } b$	If a and b are of same affection
	A	$\text{Tan. A} = \frac{\text{tan. } a}{\text{sin. } b}$	If a be less than 90°.
A, B	c	$\text{Cos. } c = \text{cot. A} \times \text{cot. B}$	If A and B are of same affection
	a	$\text{Cos. } a = \frac{\text{cos. A}}{\text{sin. B}}$	If A be acute

If the Δ, instead of being right angled, is a quadrantal Δ, the surest, and perhaps the most expeditious method is to take the supplemental or polar Δ, and solve it by the above table, taking the supplements of the given sides for the ∠'s of the polar Δ, and the supplements of the given ∠'s for the sides.

Solution of the six cases of oblique ∠d spherical Δs.

Case 1.

Given the three sides a, b, c to find A.

Solution 1st. $\left(\text{Cos.}\,\frac{A}{2}\right)^2 = r^2 \times \frac{\text{sin. S. sin. }(S-a)}{\text{sin. }b\text{. sin. }c}$.

Solution 2d. $\left(\text{Sin.}\,\frac{A}{2}\right)^2 = r^2 \times \frac{\text{sin. }(S-b)\text{. sin. }(S-c)}{\text{sin. }b\text{. sin. }c}$.

Solution 3d. $\left(\text{Tan.}\,\frac{A}{2}\right)^2 = r \quad \frac{\text{sin. }(S-b)\text{. sin. }(S-c)}{\text{sin. S. sin. }(S-a)}$.

Sometimes one of these methods may be more convenient than another, see corresponding case in Plane Trigonometry.

Case 2.

Given A, B, C to find a, &c.

Solution 1st. Let $S' = \frac{A+B+C}{2}$, then

$$\left(\text{Sin.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{-\text{cos. S'. cos. }(S'-A)}{\text{sin. B. sin. C}}.$$

Solution 2d. $\left(\text{Cos.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{\text{cos. }(S'-B.)\text{ cos. }(S'-C)}{\text{sin. B. sin. C}}$.

Solution 3d. $\left(\text{Tan.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{-\text{cos. S'. cos. }(S'-A)}{\text{cos. }(S'-B.)\text{ cos. }(S'-C)}$.

S′ is greater than 90° and less than 270°, ∴ — cos. S′ is positive, and whole quantity is positive.

Case 3.

Given a, b and included ∠ C. Required A and B.

$$\text{Solution 1st.}\quad \text{Tan.}\,\frac{A+B}{2} = \frac{\text{cos. }\frac{a-b}{2}}{\text{cos. }\frac{a+b}{2}}\text{. Cot. }\frac{C}{2}$$

$$\text{and Tan.}\,\frac{A-B}{2} = \frac{\text{sin. }\frac{a-b}{2}}{\text{sin. }\frac{a+b}{2}}\text{. Cot. }\frac{C}{2}$$

from whence A + B and A — B, and consequently A and B may be found, as also c.

Solution 2d. But if c be required alone, then it may be thus determined independently of A and B.

Assume $(\tan.\ \theta)^2 = \dfrac{\sin.\ a.\ \sin.\ b.\ \text{v. sin. C}}{\text{v. sin. }(a-b)}$; then $2\left(\sin.\ \frac{c}{2}\right)^2$

$\dfrac{\text{v. sin. }(a-b.)\ (\sec.\ \theta)^2}{r}$.

Case 4.

Given A, B, and included side c. Required a, b and C.

$$\text{Solution 1st.}\quad \text{Tan.}\ \frac{a+b}{2} = \frac{\cos.\ \frac{A-B}{2}}{\cos.\ \frac{A+B}{2}} \times \tan.\ \frac{c}{2}.$$

$$\text{and Tan.}\ \frac{a-b}{2} = \frac{\sin.\ \frac{A-B}{2}}{\sin.\ \frac{A+B}{2}} \times \tan.\ \frac{c}{2}.$$

From whence $a+b$ and $a-b$ and $\therefore a$ and b may be found.

Solution 2d. Or C may be determined independently of a and b thus

Assume $(\tan.\ \theta)^2 = \dfrac{\text{ver. sin. } c.\ \sin.\ A.\ \sin.\ B}{\left(\cos.\ \frac{A+B}{2}\right)^2}$, then $\left(\sin.\ \frac{C}{2}\right)^2 =$

$\dfrac{\left(\cos.\ \frac{A+B}{2}\right)^2.\ (\sec.\ \theta)^2}{r^2}$.

Case 5.

Given a, b and A opposite to a; to find the rest.

To find B, $\sin.\ B = \dfrac{\sin.\ A.\ \sin.\ b}{\sin.\ a}$.

To find C, $\cot.\ \dfrac{C}{2} = \tan.\ \tfrac{1}{2}(A+B).\ \dfrac{\cos.\ \frac{1}{2}(a+b)}{\cos.\ \frac{1}{2}(a-b)}$.

To find c, $\sin.\ c = \sin.\ a.\ \dfrac{\sin.\ C}{\sin.\ A}$.

Case 6.

Given A, B and a opposite to A.

$\sin.\ b = \sin.\ a.\ \dfrac{\sin.\ B}{\sin.\ A}$; then C and c as in the last case.

To find the area of a spherical Δ.

Let A, B, C be the three angles, then

Area = A + B + C − 180°. or, if r = radius of the sphere, area = $r \times (A + B + C - 180^0)$.

III. Trigonometrical Formulæ.

1. If s = sin. and c = cos. of an arc A ; the arcs, of which s is the sine, are comprehended within the two formulæ.

$$2 n \pi + A, \text{ and } (2 n + 1) \pi - A, \text{ where } \pi = 180^0.$$

Do., of which $-s$ is the sine, are

$$(2 n + 1) \pi + A, \text{ and } (2 n + 2) \pi - A.$$

Do., of which c is the cosine, are

$$2 n \pi + A \text{ and } (2 n + 2) \pi - A.$$

Do., of which $-c$ is cosine, are

$$(2 n + 1) \pi - A \text{ and } (2 n + 1) \pi + A$$

in all which cases n may be 0, 1, 2, 3, &c.

2. $\text{Sin.} \left(\frac{\pi}{2} + A\right) = \text{sin.} \left(\frac{\pi}{2} - A\right).$

3. $\text{Cos. A.} = \text{sin.} \left(\frac{\pi}{2} - A\right) = \text{sin.} \left(\frac{\pi}{2} + A\right).$

4. $\text{Sin. A.} = \text{cos.} \left(\frac{\pi}{2} - A\right) = - \text{cos.} \left(\frac{\pi}{2} + A\right).$

$$= \frac{\text{cos. A}}{\text{cot. A}} = \sqrt{1 - \text{cos.}^2 A} = \frac{1}{\text{cosec. A}}$$

$$= \text{cos. A. tan. A} = \frac{1}{\sqrt{1 + \text{cot.}^2 A}}$$

$$= \frac{\text{tan. A}}{\sqrt{1 + \text{tan.}^2 A}} = 2 \text{ sin. } \tfrac{1}{2} \text{ A. } 2 \text{ cos. } \tfrac{1}{2} \text{ A.}$$

$$= \sqrt{\frac{1 - \text{cos. } 2 A}{2}} = \frac{2 \text{ tan. } \frac{1}{2} A}{1 + \text{tan.}^2 \frac{1}{2} A}$$

$$= \frac{2}{\text{cot. } \frac{1}{2} A + \text{tan. } \frac{1}{2} A} = \frac{1}{\text{cot. A} + \text{tan. } \frac{1}{2} A}.$$

5. $\text{Cos. A} = \frac{\text{sin. A}}{\text{tan. A}} = \text{sin. A. cot. A.} = \frac{1}{\text{sec. A}}$

$$= \sqrt{1-\sin.^2 A} = \frac{1}{\sqrt{1+\tan.^2 A}}.$$

$$= \frac{\cot. A}{\sqrt{1+\cot.^2 A}} = \cos.^2 \tfrac{1}{2} A - \sin.^2 \tfrac{1}{2} A.$$

$$= 1 - 2\sin.^2 \tfrac{1}{2} A = 2\cos.^2 \tfrac{1}{2} A - 1$$

$$= \sqrt{\frac{1+\cos. 2A}{2}} = \frac{1-\tan.^2 \frac{1}{2} A}{1+\tan.^2 \frac{1}{2} A}$$

$$= \frac{\cot. \frac{1}{2} A - \tan. \frac{1}{2} A}{\cot. \frac{1}{2} A + \tan. \frac{1}{2} A} = \frac{1}{1+\tan. A \tan \frac{1}{2} A}.$$

6. $\text{Tan. } A = \frac{\sin. A}{\cos. A} = \frac{1}{\cot. A}$

$$= \sqrt{\frac{1}{\cos.^2 A} - 1} = \frac{\sin. A}{\sqrt{1-\sin.^2 A}}$$

$$= \frac{\sqrt{1-\cos.^2 A}}{\cos. A} = \frac{2\tan. \frac{1}{2} A}{1-\tan.^2 \frac{1}{2} A}.$$

$$= \frac{2\cot. \frac{1}{2} A}{\cot.^2 \frac{1}{2} A - 1} = \frac{2}{\cot. \frac{1}{2} A - \tan \frac{1}{2} A}$$

$$= \cot. A - 2\cot. 2A = \frac{1-\cos. 2A}{\sin. 2A}$$

$$= \frac{\sin. 2A}{1+\cos. 2A} = \sqrt{\frac{1-\cos. 2A}{1+\cos. 2A}}.$$

Formulæ relating to two arcs.

1. $\text{Sin. } (A+B) = \sin. A \,.\, \cos. B + \cos. A \,.\, \sin. B.$
2. $\text{Sin. } (A-B) = \sin. A \,.\, \cos. B - \cos. A \,.\, \sin. B.$
3. $\text{Cos. } (A+B) = \cos. A \,.\, \cos. B - \sin. A \,.\, \sin. B.$
4. $\text{Cos. } (A-B) = \cos. A \,.\, \cos. B + \sin. A \,.\, \sin. B.$
5. $\text{Tan. } (A+B) = \frac{\tan. A + \tan. B}{1-\tan. A \,.\, \tan. B}.$
6. $\text{Tan. } (A-B) = \frac{\tan. A - \tan. B}{1+\tan. A \,.\, \tan. B}.$
7. $\frac{\text{Sin. } (A+B)}{\text{Sin. } (A-B)} = \frac{\tan. A + \tan. B}{\tan. A - \tan. B} = \frac{\cot. B + \cot. A}{\cot. B - \cot. A}.$

8. $\dfrac{\text{Cos. }(A+B)}{\text{Cos. }(A-B)} = \dfrac{\text{cot. }B - \text{tan. }A}{\text{cot. }B + \text{tan. }A} = \dfrac{\text{cot. }A - \text{tan. }B}{\text{cot. }A + \text{tan. }B}$.

9. $\dfrac{\text{Sin. }A + \text{sin. }B}{\text{Sin. }A - \text{sin. }B} = \dfrac{\text{tan. }\frac{1}{2}(A+B)}{\text{tan. }\frac{1}{2}(A-B)}$.

10. $\dfrac{\text{Cos. }B + \text{cos. }A}{\text{Cos. }B - \text{cos. }A} = \dfrac{\text{cot. }\frac{1}{2}(A+B)}{\text{tan. }\frac{1}{2}(A-B)}$.

11. $\text{Sin. }A \,.\, \text{cos. }B = \frac{1}{2}\text{ sin. }(A+B) + \frac{1}{2}\text{ sin. }(A-B)$.

12. $\text{Cos. }A \,.\, \text{sin. }B = \frac{1}{2}\text{ sin. }(A+B) - \frac{1}{2}\text{ sin. }(A-B)$.

13. $\text{Sin. }A \,.\, \text{sin. }B = \frac{1}{2}\text{ cos. }(A-B) - \frac{1}{2}\text{ cos. }(A+B)$.

14. $\text{Cos. }A \,.\, \text{cos. }B = \frac{1}{2}\text{ cos. }(A+B) + \frac{1}{2}\text{ cos. }(A-B)$.

15. $\text{Sin. }A + \text{sin. }B = 2\text{ sin. }\frac{1}{2}(A+B)\text{ cos. }\frac{1}{2}(A-B)$.

16. $\text{Cos. }A + \text{cos. }B = 2\text{ cos. }\frac{1}{2}(A+B)\text{ cos. }\frac{1}{2}(A-B)$.

17. $\text{Tan. }A + \text{tan. }B = \dfrac{\text{sin. }(A+B)}{\text{cos. }A \,.\, \text{cos. }B}$.

18. $\text{Cot. }A + \text{cot. }B = \dfrac{\text{sin. }(A+B)}{\text{sin. }A \,.\, \text{sin. }B}$.

19. $\text{Sin. }A - \text{sin. }B = 2\text{ sin. }\frac{1}{2}(A-B) \,.\, \text{cos. }\frac{1}{2}(A+B)$.

20. $\text{Cos. }B - \text{cos. }A = 2\text{ sin. }\frac{1}{2}(A-B) \,.\, \text{sin. }\frac{1}{2}(A+B)$.

21. $\text{Tan. }A - \text{tan. }B = \dfrac{\text{sin. }(A-B)}{\text{cos. }A \,.\, \text{cos. }B}$.

22. $\text{Cot. }B - \text{cot. }A = \dfrac{\text{sin. }(A-B)}{\text{sin. }A \,.\, \text{sin. }B}$.

23. $\left.\begin{array}{l}\text{Sin.}^2 A - \text{sin.}^2 B = \\ \text{Cos.}^2 B - \text{cos.}^2 A = \end{array}\right\} \text{sin. }(A-B) \,.\, \text{sin. }(A+B)$.

24. $\text{Cos.}^2 A - \text{sin.}^2 B = \text{cos. }(A-B) \,.\, \text{cos. }(A+B)$.

25. $\text{Tan.}^2 A - \text{tan.}^2 B = \dfrac{\text{sin. }(A-B) \,.\, \text{sin. }(A+B)}{\text{cos.}^2 A \,.\, \text{cos.}^2 B}$.

26. $\text{Cot.}^2 B - \text{cot.}^2 A = \dfrac{\text{sin. }(A-B) \,.\, \text{sin. }(A+B)}{\text{sin.}^2 A \,.\, \text{sin.}^2 B}$.

27. $\text{Sin. }B = \text{sin. }(A+B) \,.\, \text{cos. }A - \text{sin. }A \,.\, \text{cos. }(A+B)$.

28. $\text{Cos. }B = \text{sin. }(A+B)\text{ sin. }A + \text{cos. }A \,.\, \text{sin. }(A+B)$.

Note.—To express the formulæ to rad. *r*, multiply each term by that power of *r* that will make each term of the same dimensions as that term which has the highest dimensions.

Expressions for the sines and cosines of multiple arcs.

1. $\text{Cos.}\ (n+1)\ A = 2\cos.\ n\ A.\ \cos.\ A - \cos.\ (n-1)\ A.$

2. $2\ \text{Cos.}\ m\ A = (2\cos.\ A)^m - m\ (2\cos.\ A)^{m-2} + m.\frac{m-3}{2}$

$(2\cos.\ A)^{m-4} - \frac{m.\ (m-4).\ (m-5)}{2.\ 3}\ (2\cos.\ A)^{m-6} + \&c.$

3. $\text{Sin.}\ (n+1)\ A = 2\sin.\ n\ A.\ \cos.\ A - \sin.\ (n-1)\ A.$

4. $\text{Sin.}\ m\ A = m\sin.\ A - \frac{m.\ (m^2-1)}{2.\ 3}\ (\sin.\ A)^3 + \frac{m.\ (m^2-1.)\ (m^2-9)}{2.\ 3.\ 4.\ 5}$

$(\sin.\ A)^5$ &c. (m, odd.)

5. $\text{Sin.}\ m\ A = \cos.\ A\ \Big(m.\ \sin.\ A - \frac{m.\ (m^2-4)}{2.\ 3}\ (\sin.\ A)^3 +$

$\frac{m.\ (m^2-4)\ (m^2-16)}{2.\ 3.\ 4.\ 5}\ (\sin.\ A)^5\ \&c.\Big)$ (m even.)

6. Let $2\cos.\ A = x + \frac{1}{x}$ then $2\cos.\ n\ A = x^n + \frac{1}{x^n}$ (n any No.)

7. $(\text{Cos.}\ A + \sqrt{-1}\ \sin.\ A)^m = \cos.\ m\ A + \sqrt{-1}\ \sin.\ m\ A.$

and $(\cos.\ A - \sqrt{-1}\ \sin.\ A)^m = \cos.\ m\ A - \sqrt{-1}\ \sin.\ m\ A.$

whence we have in another form

8. $\text{Cos.}\ m\ A = (\cos.\ A)^m - \frac{m.\ (m-1)}{2}\ (\cos.\ A)^{m-2.}\ (\sin.\ A)^2 +$

$\frac{m.\ (m-1)\ (m-2)\ (m-3)}{2.\ 3.\ 4}\ (\cos.\ A)^{m-4}\ (\sin.\ A)^4 - \&c.$

and $\sin.\ m\ A = m\ (\cos.\ A)^{m-1}\sin.\ A - \frac{m.\ (m-1)\ (m-2)}{2.\ 3}(\cos.\ A)^{m-3}$

$(\sin.\ A)^3$ &c.

9. Also if e = No. whose hyp. log. = 1 we have in terms of the impossible quantity $\sqrt{-1}$

$$\text{Cos.}\ n\ A = \frac{e^{n\ A\ \sqrt{-1}} + e^{-n\ A\ \sqrt{-1}}}{2},\ \&\ \sin.\ n\ A = \frac{e^{n\ A\ \sqrt{-1}} - e^{-n\ A\ \sqrt{-1}}}{2\sqrt{-1}}$$

Expressions for the powers of the sine and cosine of an arc.

1. $2^{n-1}\ (\cos.\ A)^n = \cos.\ n\ A + n.\ \cos.\ (n-2)\ A + n.\ \frac{n-1}{2}\ \cos.$ $(n-4)\ A + \&c.$

Note. If n be even the last term must always be $\frac{1.3.5.7 \ldots n-1.}{1.2.3 \ldots \frac{n}{2}} 2^{\frac{n}{2}-1}$

Exs. $2\ (\cos. A)^2 = \cos. 2A + 1.$

$2^2\ (\cos. A)^3 = \cos. 3A + 3 \cos. A.$

$2^3\ (\cos. A)^4 = \cos. 4A + 4 \cos. 2A + 3.$

$2^4\ (\cos. A)^5 = \cos. 5A + 5 \cos. 3A + 10 \cos. A.$

$2^5\ (\cos. A)^6 = \cos. 6A + 6 \cos. 4A + 15 \cos. 2A + 10.$

&c. &c.

2. $2^{n-1} (\sin. A)^n = \pm \cos. nA \mp \cos. (n-2). A \pm n. \frac{n-1}{2} \cos.$ $(n-4)$ A &c. where the upper sign must be used when n is 4, 8, 12, &c. and the lower when n is 2, 6, 10, &c., and in both cases the last term is as before.

3. $2^{n-1} (\sin. A)^n = \pm \sin. nA \mp \sin. (n-2) A \pm n. \frac{n-1}{2} \sin.$ $(n-4)$ A, &c., where the upper sign must be used when n is 1, 5, 9, &c., and the lower when n is 3, 7, 11, &c.

Exs. $2\ (\sin. A)^2 = -\cos. 2A + 1.$

$2^2\ (\sin. A)^3 = -\sin. 3A + 3 \sin. A.$

$2^3\ (\sin. A)^4 = \cos. 4A - 4 \cos. 2A + 3.$

$2^4\ (\sin. A)^5 = \sin. 5A - 5 \sin. 3A + 10 \sin. A.$

$2^5\ (\sin. A)^6 = -\cos. 6A + 6 \cos. 4A - 15 \cos. 2A + 10.$

Series for the sine and cosine in terms of the arc.

1. $\text{Sin.}\ x = x - \frac{x^3}{2.3} + \frac{x^5}{2.3.4.5} + \&c.$

2. $\text{Cos.}\ x = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \&c.$

Value of the sine in some of the most simple cases.

$\text{Sin.}\ 0^\circ = 0.$

$\text{Sin.}\ 9^\circ = \frac{1}{4}\sqrt{3+\sqrt{5}} - \frac{1}{4}\sqrt{5-\sqrt{5}}.$

$\text{Sin.}\ 15^\circ = \frac{1}{2}\sqrt{1+\tfrac{1}{2}} - \frac{1}{2}\sqrt{1-\tfrac{1}{2}}.$

$\text{Sin.}\ 18^\circ = \frac{1}{4}.(\sqrt{5}-1).$

$\text{Sin.}\ 27^\circ = \frac{1}{4}\sqrt{5+\sqrt{5}} - \frac{1}{4}\sqrt{3-\sqrt{5}}.$

$\text{Sin.}\ 30^\circ = \frac{1}{2}.$

Sin. 36° $= \frac{1}{4}\sqrt{10-2\sqrt{5}}$.

Sin. 45° $= \frac{1}{\sqrt{2}}$.

Sin. 54° $= \frac{1}{4}(\sqrt{5}+1)$.

Sin. 60° $= \frac{\sqrt{3}}{2}$.

Sin. 63° $= \frac{1}{4}\sqrt{5+\sqrt{5}} + \frac{1}{4}\sqrt{3-\sqrt{5}}$.

Sin. 72° $= \frac{1}{4}\sqrt{10+2\sqrt{5}}$.

Sin. 81° $= \frac{1}{4}\sqrt{3+\sqrt{5}} + \frac{1}{4}\sqrt{5-\sqrt{5}}$.

Sin. 90° $= 1$.

TWILIGHT.—*See Refraction.*

U, V.

VARIATION *and dip of the Magnetic Needle.*

TABLE I.

Shewing the variation of the Needle in various parts of the earth, from Professor Hansteen, of Christiania.

Authority.	*Date.*	*Variation.*	*Latitude.*	*Longitude*	*Place.*
Luchtemacher	1649	1° 30′ E	55 41′	12 35 E	Copenhagen.
Bartholin	1672	3 35 W			
Lous, sen.	1730	10 37			
Lous, jun.	1765	15 5			
Do.	1779	17 5			
Bugge	1784	18 0			
Wleugel	1817	18 5			
Elvius	1718	5 37 W	59 20	18 4 E	Stockholm.
Wilcke	1763	11 48			
Do.	1771	13 4			
Do.	1786	15 34			
Cronstrand	1817	15 36			
Holm	1761	13 50 W	63 26	10 22 E	Drontheim.
Berlin	1779	18 0			
Vibe	1786	19 0			

Authority.	*Date.*	*Variation.*	*Latitude.*	*Longitude*	*Place.*
		° ′	° ′		
Holm	1761	15 15 W	59 55	10 42 E	Christiania.
Hansteen	1817	20 3			
Billings	1735	1 5 W	52 17	104 11 E	Irkutsk.
Schubert	1805	0 32 E			
Mayer	1726	3 15 W	59 56	30 19 E	Petersburgh.
Krafft	1774	4 50			
Henry	1805	11 0			
Do.	1812	7 16			
Cook	1779	6 19 E	53 1	158 48	Kamtschatka.
Krusenstern ...	1805	5 20			
Kirch	1717	10 42 W	52 32	13 21 E	Berlin.
Do.	1751	14 16			
Bernouilli	1770	16 9			
Schulze	1785	18 3			
Bode	1805	18 2			
V. Swinden ...	1797	19 40 W	46 12	6 9 E	Geneva.
	1804	21 13			
Bellarmatus ...	1541	7 0 E	48 50	2 20 E	Paris.
Picard	1666	0 0			
Cassini	1687	5 12 W			
La Hire	1707	10 10			
Maraldi	1720	13 0			
Do.	1740	15 30			
Do.	1760	18 30			
Le Monnier ...	1780	20 35			
Cotte	1800	22 12			
Bouvard	1814	22 34			
Kendrick	1745	18 0 W	53 21	353 41 E	Dublin.
Harding	1791	27 23			
Burrows	1580	11 15 E	51 31	00 00	London.
Gunter	1622	5 56½			
Gellibrand	1634	4 6			
Bond	1657	0 0 W			
Gellibrand	1665	1 22½			
Halley	1672	2 30			
Do.	1692	6 0			
Graham	1723	14 17			
Do.	1745	17 0			
Do.	1745	17 0			
Do.	1746	17 10			
Do.	Ma. 21	17 10			
Do.	Ap. 22	17 15			
Do.	May 4	17 18			
Do.	14	17 20			
Do.	16	17 15			
Do.	De. 18	17 25			
Do.	1747	17 30			
Do.	1747	17 40			
Do.	1748	17 40			
Heberden	1773	21 9			
Cavendish	1774	21 16			

Authority.	Date.	Variation.	Latitude.	Longitude	Place.
		° ′	° ′		
Cavendish	1775	21 43	51 31	00 00	London.
Gilpin	1786	23 17			
Do.	1787	23 19			
Do.	1788	23 32			
Do.	1789	23 19			
Do.	1790	23 39			
Do.	1791	23 36			
Do.	1792	23 36			
Do.	1793	23 49			
Do.	1794	23 56			
Do.	1795	23 57			
Do.	1796	24 0			
Do.	1797	24 1			
Do.	1798	24 0·6			
Do.	1799	24 1·8			
Do.	1800	24 3·6			
Do.	1801	24 4·2			
Do.	1802	24 6·7			
Do.	1803	24 8·8			
Do.	1804	24 8·4			
Do.	1805	24 8·8			
Do.	1809	24 11·0			
Do.	1814	24 16·7			
Do.	Jul.	24 17·9			
Do.	Au.	24 21·2			
Do.	Sep.	24 20·5			
Martinius	1638	7 39 E	38 42	350 51	Lisbon.
Do.	1668	0 50 W			
Ross	1762	17 32			
Lowenorn	1782	19 51			
Auzout	1670	2 15 W	41 54	12 28	Rome.
	1788	17 12			
Mathews	1721	5 12 W	19 0	71 45	Bombay.
Yeates	1817	0 0			
Fontenay	1690	2 25 W	22 13	113 35	Canton.
Yeates	1817	0 0			
Mathews	1722	2 52 W	13 15	79 57	Madras.
Yeates	1817	1 0 E			
Wallis	1766	14 10 W	32 36	342 57	Madeira.
Mudge	1820	19 59			
Fleurieu	1769	15 43 W	28 27	343 45	Teneriffe.
Bligh	1788	20 1			
Krusenstern ...	1803	16 1			
Keeling	1609	21 0 W	20 10	57 28	Isle of France.
Yeates	1817	15 0			
Daunton	1614	1 45 W	33 55	18 24	Table Bay.
Caille	1752	19 0			
Bonsoe	1804	25 4			
Davis	1610	7 13 E	15 55	354 12	St Helena.
Halley	1677	0 40			
Wallis	1768	12 47 W			
Krusenstern ...	1806	17 18			

Authority.	*Date.*	*Variation.*	*Latitude.*	*Longitude*	*Place.*
Mathews	1723	12 20 W	12 47	56 0	Socotra Isl.
Yeates	1817	6 0			
Mathews	1726	4 31 E	17 6	283 14¼	P.R., Jamaica
Yeates	1817	6 0			
Vancouver	1795	14 49 E	33 0	287 46	Valparaiso.
Basil Hall	1821	14 43			
Cook	1774	4 30 E	27 6	250 14	Easter Island.
La Perouse	1786	3 10			
Cook	1779	8 6 E	19 28	204 0	Owhyee.
Broughton.......	1796	8 15			
Tasman	1643	7 15 E	21 9	184 55	Tongataboo.
Cook	1777	9 44			
Oxley	1817	7 47 E	33 40	148 21	New Holland.

TABLE II.

Shewing the dip of the Needle in various parts of the earth.—(Hansteen.)

Authority.	*Date.*	*Dip.*	*Latitude.*	*Longitude*	*Place.*
		° ′			
Lous	1773	71 45 N	———	———	Copenhagen.
Wleugal	1813	71 26			
Schubert	1805	67 0 N	———	———	Irkutsk.
Euler	1755	73 30 N	———	———	Petersburgh.
Kraft	1802	76 42			
	1755	71 45 N	———	———	Berlin.
Euler	1769	72 45			
Humboldt	1805	69 53			
Richer	1671	75 0 N	———	———	Paris.
La Caille	1754	72 15			
Cassini	1791	70 52			
Humboldt	1806	69 12			
Conn. de tems.	1814	68 36			
Norman	1576	71 50 N			
Gilbert	1600	72 0 N			
Ridley	1613	72 30 N			
Bond	1676	73 30 N			
Whiston	1720	73 45 75 10			
Graham	1723	74 42 74 42			
Nairne	1772	72 19	———	———	London.
Cavendish	1775	72 31			
Gilpin	1786	72 8·1			
Do.	1787	72 2·5			
Do.	1788	72 4·0			
Do.	1789	71 54·8			
Do.	1790	71 53·7			
Do.	1791	71 23·7			

Authority.	*Date.*	*Dip.*	*Latitude.*	*Longitude*	*Place.*
		° ′			
Gilpin	1795	71 11·4			
Do.	1797	70 59·4			
Do.	1798	70 55·4			
Do.	1799	70 52·2			
Do.	1801	70 35·6	———	———	London.
Do.	1803	70 32·0			
Do.	1805	70 21·0			
Sabine	1821	70 3·2			
Kircher	1640	65 40 N	———	———	Rome.
Humboldt	1806	63 48			
Abercrombie ..	1775	5 15 N	———	———	Madras.
Mudge	1820	63 47 N	———	———	Madeira.
La Caille	1751	43 0 S	———	———	Good Hope.
Bayley	1775	45 19			
La Caille	1754	9 0 S	———	———	St. Helena.
Cook	1775	11 25			
Panton	1776	4 37 N	———	———	Socotra.
Vancouver	1795	44 15 S	———	———	Valparaiso.
Basil Hall	1821	38 46			
Cook	1777	40 51 S	———	———	Owhyee.
Vancouver	1793	41 24			
Cook	1777	39 1 S	———	———	Tongataboo.
Cook	1776	61 52 N	———	———	Teneriffe.
Mudge	1820	58 22			
Basil Hall	1821	12 11½ N	2 13 S	280 15	Guayaquil.
Humboldt	1805	61 35 N	40 50 N	14 16	Naples.
Do.	1799	13 22 N	0 13 S	281 15	Quito.
Do.	1805	64 45 N	44 25 N	8 58	Genoa.
Do.	1805	67 10 N	———	———	Lucern.

The following recent observations on the dip and variation were selected by Mr. Barlow, as being entitled to the greatest credit :—

Place.	*Date.*	*Lat.*	*Long.*	*Dip.*	*Variation*	*Authority.*
Tristan da Acunha	1821	37 0	12 10	37 53 S	12 0 W	Marryat.
Trinidad	1821	20 30	29 0	10 27 S	5 0 W	Do.
St. Jago	1820	14 54	23 32	48 0 N	15 55 W	Mean of Do.
Teneriffe	Do.	28 28	16 16	58 22 N	20 47 W	and Mudge's
Madeira	Do.	32 38	17 54	63 47 N	23 7 W	observations
Madrid	1799	40 25	3 29	67 41 N	19 59 W	Humboldt.
Paris	1814	48 50	2 20	68 36 N	22 34 W	Bouvard.
London	1818	51 31	0 0	70 34 N	24 30 W	Kater, the dip.
Berlin	1805	52 32	13 21	69 53 N	18 2 W	Humboldt.
Copenhagen	1813	55 41	12 35	71 26 N	18 22 W	Wleugel.
Davis' Strait.........	1820	64 0	61 50	83 43 N	60 20 W	Parry.
Regent's Inlet	Do.	72 45	89 41	88 26 N	118 16 W	Do.
Baffin's Bay	Do.	73 0	61 30	84 30 N	82 2 W	Do.
Possession Bay	Do.	73 31	77 22	86 4 N	108 46 W	Do.
Melville Island	Do.	74 47	110 48	88 43 N	127 48 E	Do.

VARIATION *diurnal.*

The horizontal needle, besides its annual change in direction, is also subject to a daily change, amounting at certain seasons of the year to about 14′ or 15′. According to the most recent observations, it appears that the needle attains its maximum direction eastward about 7 o'clock, or ½ past 7 in the morning, that it continues moving westward till two o'clock in the afternoon; it then returns to the eastward till the evening; it has then again a slight westerly motion, and in the course of the night, or early in the morning, attains the bearing it had 24 hours before, or very nearly. It has also been admitted by all observers, that the daily motion during the summer months is the greatest, and during the winter months the least; but the particular month in the summer when the daily change is the greatest, is a little uncertain. Canton and Wargentin make it about July; but Col. Beaufoy found it greater in June and August than in July.

Table of the mean monthly diurnal variation of the compass from April 1817 *to March* 1819. *By Colonel Beaufoy, at Stanmore Heath.*

From April 1817 to March 1819.	Difference morning, noon, evening.	Mean difference. ′ ″	From April 1817 to March 1819.	Difference morning, noon, evening.	Mean difference. ′ ″
April	n. — m.	11 48	October	n. — m.	8 46
	n. — e.	8 30		n. — e.	
	e. — m.	3 18		e. — m.	
May	n. — m.	9 53	Novem.	n. — m.	7 10
	n. — e.	7 32			
	e. — m.	2 21			
June	n. — m.	11 15	December	n. — m.	4 7
	n. — e.	7 50			
	e. — m.	3 25			
July	n. — m.	10 43	January	n. — m.	5 3
	n. — e.	6 34			
	e. — m.	4 9			
August	n. — m.	11 26	February	n. — m.	6 3
	n. — e.	8 34			
	e. — m.	2 52			
Septem.	n. — m.	9 44	March	n. — m.	8 22
	n. — e.	7 26		n. — e.	7 7
	e. — m.	2 18		e. — m.	1 15

VELOCITY *angular.*—*See Central Forces.*

VELOCITY *paracentric.*—*See Central Forces.*

VENUS.—*See Planets, elements of.*

VENUS, *transit of.*—*See Transit.*

VENUS, *phases of.*—*(Vince.)*

In the case of Mercury, Venus, and Mars, if $\theta =$ exterior $\angle$ of elongation, i. e. $=$ supplement of the $\angle$, which the earth and sun subtend at the planet, the visible enlightened part : the whole disc :: ver. sin. θ : diameter.

Hence Mercury and Venus will have the same phases, from their inferior to their superior conjunction, as the moon has from the new to the full; and the same from the superior to the inferior conjunction, as the moon has from the full to the new. Mars will appear gibbous in quadratures, as the $\angle\ \theta$ will then differ considerably from two right $\angle$s; and consequently the versed sine from the diameter. For Jupiter, Saturn, and the Georgian, the $\angle\ \theta$ never differs enough from two right $\angle$s to make them appear gibbous, so that they always appear to shine with a full face. In the case of the moon, the $\angle\ \theta$ very nearly equals the $\angle$ of elongation; $\therefore$ the visible enlightened part of the moon varies very nearly as the ver. sin. of its elongation.

Venus is brightest between its inferior conjunction and its greatest elongation; and its elongation at that time from the sun $= 39^0.\ 44'$. Also at that time the visible enlightened part : whole disc :: 0,53 : 2. Venus therefore appears a little more than one-fourth illuminated, and answers to the appearance of the moon when five days old. This situation happens about 36 days before and after its inferior conjunction.

Mercury is brightest between its greatest elongation and superior conjunction; the elongation of Mercury at this time $= 22^0.\ 18\frac{3}{4}'$.

VERNIER.

As instruments are now usually constructed, the following is a general rule for finding the value of each division on any vernier.

Find the value of each of the divisions or sub-divisions of the limb to which the vernier is applied. Divide the number of minutes or seconds thus found by the number of divisions on the vernier, and the quotient will give the value of the vernier division. Thus suppose each sub-division of the limb to be $5'$ or $300''$, and that the vernier has 20 divisions, then $\frac{300}{20} = 15'' =$ value of the vernier.

VESTA.—This planet was discovered by Dr. Olbers, of Bremen, March 29, 1807. For its elements, &c., *see Planets, elements of.*

VOLCANOES.

The total number of Volcanoes known is about 205, of which Europe contains 13 or 14. Of the whole number, it is computed that 107 are in islands, and 98 on the great continents. The most remarkable are Ætna, Vesuvius, the Lipari islands, Iceland, Kamschatka, Japan, and so along the eastern coast of Asia and the Indian islands; Cape Verd, Canary, and other African islands; an immense range of them, at least 60 in number, running from north to south on the Continent of America, and occupying the summits of many of the Andes, as well as the Mexican and Californian ridges; for a few of the principal of which, see *Mountains, height of.*

URANUS, *or Georgium Sidus.—See Planets, elements of.*

W.

WATER *boiling, temperature of.—See Heat.*

WATER, *expansion of.—See Heat.*

WATER MILL.—*See Wheel.*

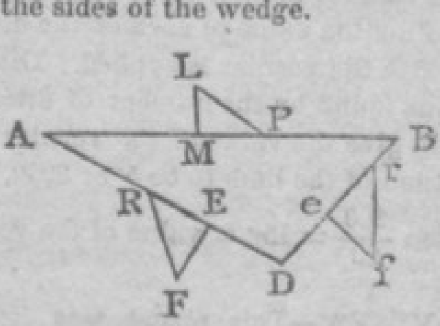
WAVES, *motion of.—See Siphon.*

WEDGE.

1. When three forces, acting perpendicularly upon the sides of a scalene wedge, keep each other in equilibrio, they are proportional to those sides.

Cor. When the directions of the forces are not perpendicular to the sides, the effective parts must be found, and there will be an equilibrium when those parts are to each other as the sides of the wedge.

2. In general let A B C represent a section of the wedge, and let a power P, represented in magnitude and direction by L P, act upon A B the back of the wedge, and let it be counteracted by two resistances R and R′, which are represented in quantity and direction by F R, *f r*; then when the wedge is at rest,

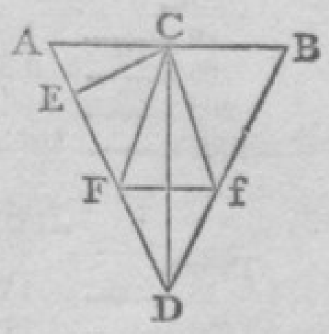


$$P : R + R' :: \frac{AB}{\sin. LPM} : \frac{AD}{\sin. FRE} + \frac{BD}{\sin. fre}$$

Cor. 1. If the wedge be isosceles, $AD = DB$, and if resistances act at equal angles,

$$P : R + R' :: \frac{AB}{\sin. LPM} : \frac{2AD}{\sin. FRE}.$$

Cor. 2. If power act at right $\angle$s to the back, $P : R + R' :: \frac{\frac{1}{2}AB}{r} : \frac{AD}{s.\,FRE} :: \frac{\sin. \frac{1}{2}\angle \text{ of wedge}}{r} : \frac{r}{s.\,FRE} ::$ sin. $\frac{1}{2}\angle$ of wedge × sin. FRE : (rad.)^2.



Cor. 3. If the resistances in the last Corollary act perpendicularly on the sides of the wedge, $P : R + R' :: AC : AD$. If the directions of the resistances be perpendicular to the back, $P : R + R' :: AC^2 : AD^2$. And lastly, if they act parallel to the back, $P : R + R' :: CE : AD$.

Cor. 4. In the demonstration of the proposition, it has been supposed that the sides of the wedge are perfectly smooth; if, on account of the friction, the resistances C F, C *f* are wholly effective, we have

$$P : R + R' :: \sin. CFf, \text{ or } FCA : \text{rad.}$$

The power applied to the wedge is usually percussion, and almost the only instance in which it is used for the purpose of equilibrium is in the construction of *arches*, built of truncated wedges.

WEIGHTS *and Measures, Tables of.*

Weights.

TROY WEIGHT.

Grains.			
24	1 Pennyweight.		
480	20	1 Ounce.	
5760	240	12	1 Pound.

By this weight, gold, silver, jewels, and precious stones are weighed. It is also used for ascertaining the strength of spirits, for experiments in Nat. Philosophy, and for comparing the different weights with each other.

Standard gold consists of 22 parts of fine gold, and 2 parts of alloy; and standard silver contains 37 parts of fine silver, and 3 of alloy.

The standard price of gold is £3. 17*s.* 10½*d.* per ounce, or £46. 14*s.* 6*d.* per pound, a pound being coined into 44½ guineas. A pound of standard silver is now coined into 66 shillings, instead of 62 shillings, as formerly.

By the Act of Parliament passed in June, 1824, all the weights remain as they were, the Act only declaring that the Imperial standard Pound Troy shall be the unit or only standard measure of weight from which all other weights shall be derived and computed; that this Troy pound is equal to the weight of 22.815 cubic inches of distilled water weighed in air at the temperature of 62° of Fahrenheit's thermometer, the barometer being at 30 inches; and that there being 5760 grains in a Troy pound, there will be 7000 such grains in a pound avoirdupoise.

APOTHECARIES WEIGHT.

Grains.				
20	1 Scruple.			
60	3	1 Dram.		
480	24	8	1 Ounce.	
5760	288	96	12	1 Pound.

AVOIRDUPOIS WEIGHT.

Drams.					
16	1 Ounce.				
256	16	1 Pound.			
7168	448	28	1 Quarter.		
28672	1792	112	4	1 Cwt.	
573440	35840	2240	80	20	1 Ton.

Avoirdupois weight is used for all coarse and heavy goods, such as butcher's meat, groceries, bread, cheese, butter, tea, &c., and all metals, except gold and silver.

The statute stone is 14lb., but it varies in different places; in London 8lb. make a stone of butcher's meat.

An avoirdupois pound : pound Troy :: 175 : 144 or :: 11 : 9 nearly; and an avoirdupois pound = 1lb. 2oz. 11dwts. 16 gr. Troy; and a Troy ounce = 1oz. 1,55dr. avoirdupois.

WOOL WEIGHT.

Pounds.						
7	1 Clove.					
14	2	1 Stone.				
28	4	2	1 Tod.			
182	26	13	6½	1 Wey.		
364	52	26	13	2	1 Sack.	
4368	624	312	156	24	12	1 Last.

In the northern counties woolstaplers allow 30lb. to the tod, and 8 tods to the pack.

HAY AND STRAW.

36lb. of straw		make 1 truss.
56lb. of old hay		make 1 truss.
60lb. of new hay		make 1 truss.
36 trusses		1 load.

BREAD.

	lbs.	*oz.*	*dr.*
A Peck Loaf weighs	17	6	0
A Half Peck	8	11	0
Quartern	4	5	8

Measures.

CLOTH MEASURE.

Inch.				
2¼	1 Nail.			
9	4	1 Quarter of a Yard.		
36	16	4	1 Yard.	
27	12	3	1 Flemish Ell.	
45	20	5	1 English Ell.	

LONG MEASURE.

Barley corns.						
3	1 Inch.					
36	12	1 Foot.				
108	36	3	1 Yard.			
594	198	16½	5½	1 Pole.		
23760	7920	660	220	40	1 Furlong.	
190080	63360	5280	1760	320	8	1 Mile.

Also,

4 Inches	1 Hand.
1½ Feet	1 Cubit.
6 Feet	1 Fathom.
3 Miles	1 League.
60 Geographical Miles	1 Degree.
69½ English Miles	1 Degree nearly.

By the late Act of Parliament it is declared, that the Imperial standard yard (which is the same as the old yard) shall be the unit or only standard measure of extension, wherefrom all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be derived and computed; and that the Imperial standard yard, when compared with a pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea, is in the proportion of 36 inches to 39.1393 inches.

Note.—The following standards, accurately measured, give these results:—

	Inches.
General Lambton's scale, used in India	35.99934
Sir G. Shuckburgh's scale	35.99998
General Roy's scale	36.00088
Royal Society's standard	36.00135
Ramsden's bar	36.00249

SQUARE OR LAND MEASURE.

Feet.				
9	1 Yard.			
272¼	30¼	1 Pole.		
10890	1210	40	1 Rood.	
43560	4840	160	4	1 Acre.

For further observations on this measure—*see Surveying*.

WINE MEASURE.

Pints.							
2	1 Quart.						
8	4	1 Gallon.					
336	168	42	1 Tierce.				
504	252	63	1½	1 Hogshead.			
672	336	84	2	1⅓	1 Punch.		
1008	504	126	3	2	1½	1 Pipe.	
2016	1008	252	6	4	3	2	1 Tun.

This measure is used for wines, brandies, rum, honey, oil, vinegar, &c.

A cask of rum, which contains from 95 to 110 gallons, is usually called a puncheon; a foreign pipe of wine varies from 110 to 140 gallons.

ALE AND BEER MEASURE.

Quarts.					
4	1 Gallon.				
36	9	1 Firkin.			
72	18	2	1 Kilderkin.		
144	36	4	2	1 Barrel.	
216	54	6	3	1½	1 Hogshead.
432	108	12	6	3	2 — 1 Butt.

By the late Act the old Wine and Ale Gallons are abolished, and the Imperial standard gallon substituted in their place. This is declared to contain ten pounds avoirdupoise weight of distilled water weighed in air at the temperature of 62° of Fahrenheit, the barometer being at 30 inches. From this standard gallon all other measures of capacity, as well for wine, ale, beer, spirits, &c., as for dry goods not measured by heap measure, shall be derived and computed. Two of these gallons make a peck, and 8 such gallons make a bushel, and 8 such bushels a quarter of corn, or other dry goods not measured by heaped measure.

The above bushel of 8 Imperial gallons is also to be used for coals, culm, fish, potatoes, fruit, and all other goods commonly sold by heaped measure, which goods are to be heaped up in the form of a cone of at least six inches in height, the base of the cone being 18½ inches diameter.

The Imperial gallon contains	277.274	cubic inches.
The old wine gallon	231	do.
The old corn	268.8	do.
The old ale	282	do.

TABLE OF FACTORS,

For converting old measures into new, and the contrary.

	By decimals.			By vulgar fractions nearly.		
	Corn Measure.	Wine Measure.	Ale Measure.	Corn. Measure.	Wine Measure.	Ale Measure.
To convert old measures to new.	.96943	.83311	1.01704	$\frac{31}{32}$	$\frac{5}{6}$	$\frac{60}{59}$
To convert new measures to old.	1.03153	1.20032	.98324	$\frac{32}{31}$	$\frac{6}{5}$	$\frac{59}{60}$

N.B. For reducing the prices, these numbers must all be reversed.

Ex. Reduce 63 gallons wine measure to the equivalent number in Imperial measure.

$$63 \times .83311 \text{ or } 63 \times \frac{5}{6} = 52\tfrac{1}{2} \text{ Imperial gallons nearly.}$$

DRY OR CORN MEASURE.

Pints.
8 — 1 Gallon.
16 — 2 — 1 Peck.
64 — 8 — 4 — 1 Bushel.
256 — 32 — 16 — 4 — 1 Coomb.
512 — 64 — 32 — 8 — 2 — 1 Quarter.
2560 — 320 — 160 — 40 — 10 — 5 — 1 Wey.
5120 — 640 — 320 — 80 — 20 — 10 — 2 — 1 Last.

Also,

2 bushels make 1 boll.

3 bushels —— 1 sack.

36 bushels —— 1 chaldron of coals at London.

68 bushels —— 1 do. Newcastle.

The London chaldron weighs 28½ cwt.; and the Newcastle chaldron 53 cwt.

A bushel, water measure, is 5 pecks. 8 chaldrons a keel.

MEASURE ITINERARY.

	Yds.		Yds.
Mile of Russia	1100	Mile of Poland	4400
of Italy	1467	of Spain	5028
of England	1760	of Germany	5867
of Scotland & Ireland	2200	of Sweden	7333
Old league of France	2200	of Denmark	7333
Small league do.	2933	of Hungary	8800
Mean league do.	3667		
Great league do.	4400		

TABLE OF MISCELLANEOUS ARTICLES.

24	Sheets of Paper make	1 quire.
20	Quires	1 ream.
21½	Quires	1 printer's ream.
2	Reams	1 bundle.
10	Reams	1 bale.
60	Skins	1 roll of parchment.
12	Dozen of any thing	1 gross.
12	Gross	1 great gross.
6	Score	1 great hundred.
500	Bricks	1 load.
384	Bricks	1 cubic yard.
40	Solid feet of hewn timber	1 load.
50	Solid feet of unhewn timber	1 load.
100	Acres	1 hide of land.
20	Stone of flour	1 sack.
56	Pounds of butter	1 firkin.
64	Pounds of soap	1 firkin.

A square cistern of only 5 feet in the side will where the annual rainfall is about 28 inches (28.12 inches) catch enough water to supply 1 gallon per diem.

1 hectare = 1140 sq yards less than 2½ acres

1 franc a square metre = £162 an acre
1 acre contains 4047 metres (very nearly)

19½	Cwt. of lead	1 fother.
84	Pounds of tea	1 chest.
168	Pounds of rice	1 bag.
112	Pounds of raisins	1 barrel.

FRENCH WEIGHTS & MEASURES.

A few of the principal old French Measures.

A point	,0148025 English inches.
A line	,088815
An inch	1,06578
A foot	12,78936
A toise	6,394665 English feet.

According to Gen. Roy, an English fathom : a French toise :: 1000 c 1065,75.

New, or Metre, System.

In the new system the metre is the ten millionth part of the quadrant of the meridian = 3.281 English feet. The Are is the square decametre, and the litre the cubic decimetre.

Lineal Measure.

Millimetre	0.03937 English inches.
Centimetre	0.39371
Decimetre	3.93710
Metre	39.37100
Decametre	393.71000
Hecatometre	3937.10000
Chillometre	39371.00000
Myriometre	393710.00000

Superficial Measure.

Are	119.6046 Eng. square yards.
Decare	1196.0460
Hecatare	11960.4600

Measure of capacity.

Millilitre	,06103 Eng. cubic inches.
Centilitre	,61028
Decilitre	6.10280
Litre	61.02800

Decalitre	610.28000
Hecatolitre	6102.80000
Chiliolitre	61028.00000
Myriolitre	610280.00000

New French Weights.

	English Troy grains.
Milligramme	.01544
Centigramme	.15445
Decigramme	1.54457
Gramme	15.44579
Decagramme	154.45793
Hecatogramme	1544.57938
Chiliogramme	15445.79386
Myriogramme	154457.93860

WHEEL *and Axle.*

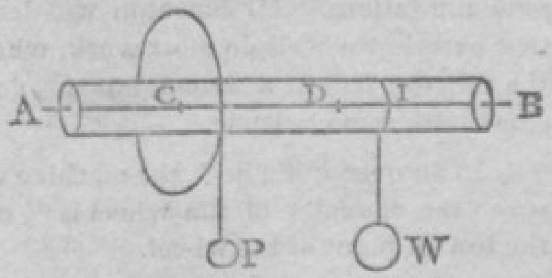
1. There is an equilibrium upon the wheel and axle, when the power is to the weight, as the radius of the axle to the radius of the wheel.

Cor. 1. Hence if the diameter of the wheel increases in the same proportion as the power decreases, the force with which the wheel is turned remains unaltered. This principle is introduced in the construction of the fuzee of a watch, and of the mainspring on the tumbler of gun locks.

Cor. 2. If 2 R be the thickness of the ropes by which the power and weight act, there will be an equilibrium when P : W :: rad. of axle + R : rad. of wheel + R. Hence the ratio of the power to the weight is greater in this case than the former.

Cor. 3. In a combination of wheels and axles, where the circumference of the first axle is applied to the circumference of the second wheel, by means of a string or by tooth and pinion, and the 2d axle to the 3d wheel, &c. there is an equilibrium, when P : W :: the product of the radii of all the axles : the product of the radii of all the wheels; or :: the product of all the teeth in the pinions : the product of all the teeth in the wheels.

If the wheels be all equal to each other, and the axles equal to each other; or if each wheel be to the axle on which it is fixed in a constant ratio, as for instance that of r to s, and if n be the number of wheels or axles, $P : W :: r^n : s^n$.



2. Required the pressure upon the pivots A and B of the wheel and axle when in equilibrio.

Let A = weight of the axle, w = weight of the wheel, and let D be the centre of gravity of P and W, then the pressure on A from the axle $= \frac{1}{2}$ A; do. from the wheel $= \frac{w \times BC}{AB}$; do. from P and W $= (P + W) \times \frac{BD}{AB}$; $\therefore$ the whole pressure on $A = \frac{1}{2} A + \frac{w \times BC + (P + W) \times BD}{AB}$. In like manner the pressure on $B = \frac{1}{2} A + \frac{w \times AC + (P + W) \times AD}{AB}$.

WHEEL *undershot and overshot.—(Playfair.)*

1. An undershot wheel (i. e. one carried round by the impulse of a stream flowing *under* it) produces the greatest effect, or does the most work, when the wheel moves with one-third of the velocity of the water.

For if V and v be the velocities of the water and floatboard, the effect in a given time will vary as $(V - v)^2 \times v$, which is a maximum when $v = \frac{1}{3}$ V.

If h = height due to velocity V, and a^2 = the section of the stream, the effect when a maximum $= \frac{8 a^2 h}{27}$, i. e. $\frac{8}{27}$ of the water expended.

Mr Smeaton found in *practice*, that when the effect is a maximum, v equalled from $\frac{1}{3}$ to $\frac{1}{2}$ V; and that its effect instead of $\frac{8}{27}$, was about $\frac{1}{3}$ of the water expended. He also found that the expence of water being the same, the effect is as the square of the velocity, and when the section of the water is the same, the effect is as the cube of the velocity.

2. In overshot wheels (i. e. when the wheel receives the water into buckets at or near the highest point) if A be the quantity of water issuing in a second, and h the height due to the velocity of the circumference of the wheel, and r the radius, the effect of the machine is proportional to A. $(2 r - h.)$

Cor. Hence the effect will be the greater the less h is, or the less the velocity of the wheel; this however in practice is found to be subject to some limitations. Mr Smeaton was led from experiment to conclude that overshot wheels do most work, when their circumferences move at the rate of 3 feet in a second, but this determination is also to be understood with some latitude.

3. In an overshot wheel, the machine will be in its greatest perfection; when the diameter of the wheel is ⅘ of the height of the water above the lowest point of the wheel.

4. The power of the overshot wheel is greater, cæteris paribus, than that of the undershot, nearly in the ratio of 13 to 5.

WIND.

Winds may be divided into *constant*, or those which always blow in the same direction; *periodical*, or those which blow half a year in one direction, and half a year in the contrary direction, which last are called *monsoons;* and *variable*, which are subject to no rules.

I. *Constant or Trade Winds.*

The trade wind at the Equator blows constantly from the east: from the Equator to the northern tropic, or even as far as the parallel 25° or 30°, it declines towards the N.E., and the more so the further you recede from the Equator: and from the Equator to the southern tropic, or to the parallel 25° or 30°, it has a S.E. direction. The line however that separates the opposite trade winds is not precisely the Equator, but the second or third parallel north. To a certain extent also they follow the course of the sun, reaching a little further into the southern ½ sphere, and contracting their limits in the north, when the sun is on the south side of the Equator; and making a reverse change when he declines to the north. In a zone of variable breadth in the middle of this tract, calms and rains prevail, caused probably by the mingling and ascending of the opposite aerial currents. The phænomenon of the trade winds may be thus explained. The air towards the poles being denser than that at the Equator, will continually rush towards the Equator; but as the velocity of the different parts of the earth's surface, from its rotation, increases as you approach the Equator; the air which is rushing from the north will not continue upon the same meridian, but it will be left behind; that is, in respect to the earth's surface, it will have a motion from the east; and these two motions combined produce a N.E. wind on the north side of the Equator. And in like manner there must be a S.E. wind on the south side. The air which is thus continually moving from the Poles to the Equator, being rarified when it comes there, ascends to the top of the atmosphere, and then returns back to the Poles.

II. *Periodical Winds, or Monsoons.*

Such would probably be the regular course of the trade winds supposing the parts between and near the tropics were open sea. But high lands change or interrupt their regular course. For instance, in the Indian Ocean the trade wind is curiously modified by the lands which surround it on the north, east, and west. There, the southern trade wind blows regularly as it ought to do from the E. and S.E., from 10° S. latitude to the tropic; but in the space from 10° S. latitude to the Equator, N.W. winds blow during our winter (from October to April); and S.E. in the other six months, while in the *whole* space north of the Equator S.W. winds blow during summer, and N.E. during winter. These winds are called monsoons. It was observed above, that the regular trade wind blows in the Indian Ocean from 10° S. latitude to the tropic, but there is an exception to this in all that part of the Indian Ocean which lies between Madagascar and Cape Comorin; for there, between the months of April and October, the wind blows from the S.W., and in the contrary direction from October to April. But of both the constant and periodical winds it may be observed, that they blow only at sea; at land the wind is always variable.

Particulars of the Trade Winds, from Robertson.—(Young's Natural Philosophy.)

1. For 30° on each side of the Equator, there is almost constantly an easterly wind in the Atlantic and Pacific Oceans: it is called the trade wind: near the Equator it is due east, further off it blows towards the Equator, and is N.E. or S.E.

2. Beyond 30° latitude, the wind is more uncertain.

3. The monsoons are, perhaps erroneously, deduced from a superior current in a contrary direction.

4. In the Atlantic, between 10° and 28° N. latitude, about 300 miles from the coast of Africa, there is a constant N.E. wind.

5. On the American side of the Caribbee Islands the N.E. wind becomes nearly E.

6. The trade winds extend 3° or 4° further N. and S. on the W. than on the E. side of the Atlantic.

7. Within 4° of the Equator, the wind is always S.E.: it is more E. towards America, and more S. towards Africa. On the coast of Brazil, when the sun is far northwards, the S.E. becomes more S., and the N.E. more E., and the reverse when the sun is far southwards.

8. On the coast of Guinea, for 1500 miles, from Sierra Leone to St.

Thomas, the wind is always S. or S.W. probably from an inclination of the trade wind towards the land.

9. Between lat. 4° and 10°, and between the longitudes of Cape Verd and the Cape Verd Islands, there is a track of sea very liable to storms of thunder and lightning. It is called the rains. Probably there are opposite winds that meet here.

10. In the Indian Ocean, between 10° and 20° S. latitude, the wind is regularly S.E. From June to November, these winds reach to within 2° of the Equator: but from December to May the wind is N.W. between lat. 3° and 10° near Madagascar, and from 2° to 12° near Sumatra.

11. Between Sumatra and Africa, from 3° S. latitude to the coasts on the N. the monsoons blow N.E. from September to April, and S.W. from March to October: the wind is steadier, and the weather fairer, in the former half year.

12. Between Madagascar and Africa, and thence northwards to the Equator, from April to October there is a S.S.W. wind, which further N. becomes W.S.W.

13. East of Sumatra, and as far as Japan, the monsoons are N. and S. but not quite so certain as in the Arabian gulf.

14. From New Guinea to Sumatra and Java, the monsoons are more N.W. and S.E. being on the south of the Equator; they begin a month or six weeks later than in the Chinese seas.

15. The changes of these winds are attended by calms and storms.

III. *Winds variable.*

In the temperate zones the direction of the winds is by no means so regular as between the tropics. In the north temperate zone, however, they blow most frequently from the S.W., in the south temperate zone, from the N.W.; but changing frequently to all points of the compass, and in the north temperate zones blowing, particularly during the spring, from the north-east.

From an average of 10 years of the register kept by order of the Royal Society, it appears that at London the winds blow in the following order:—

Winds.	*Days.*	*Winds.*	*Days.*
South-west	112	South-east	32
North-east	58	East	26
North-west	50	South	18
West	53	North	16

It appears from the same register, that the S.W. wind blows at an average more frequently than any other wind during every month of the year, and that it blows longest in July and August; that the N.E. blows most constantly during January, March, April, May, and June; and most seldom during February, July, September, and December; and that the N.W. wind blows oftener from November to March, and more seldom during September and October than any other months.—(*Phil. Trans.* 66, 2.)

The following Table of the winds at Lancaster, has been drawn up from a register kept for seven years at that place:—

	Days.		Days.
S.W.	92	S.E.	35
N.E.	67	N.	30
S.	51	N.W.	26
W.	47	E.	17

The following Table is an abstract of nine years observation made at Dumfries, by Mr Copland:—

	Days.		Days.
S.	28½	N.	36½
W.	69	N.W.	25½
E.	68	S.E.	18½
S.W.	50½	N.E.	14½

The following Table exhibits a view of the number of days during which the westerly and easterly winds blow in a year at different parts of the island. Under the term westerly are included the N.W., W., S.W., and S.; the term easterly is taken in the same latitude:—

Years of observation	Places.	Westerly.	Easterly.
10	London	233	132
7	Lancaster	216	149
51	Liverpool	190	175
9	Dumfries	227.5	137.5
10	Branxholm, 54 miles S.W. of Berwick	232	183
7	Cambuslany, near Glasgow	214	151
8	Hawkhill, near Edinburgh	229.5	135.5
	Mean	220.3	144.7

IV. *Wind, velocity of.*

The following Table, drawn up by Mr Smeaton, will give the reader a pretty precise idea of the velocity of the wind in different circumstances.—(*Phil. Trans.* 1757.)

Miles per Hour.	*Feet per Second.*	*Perpendicular force on one square foot in avoirdupois pounds and parts.*	
1	1.47	.005	Hardly perceptible.
2	2.93	.020	Just perceptible.
3	4.4	.044	
4	5.87	.079	Gently pleasant.
5	7.33	.123	
10	14.67	.492	Pleasant, brisk.
15	22.00	1.107	
20	29.34	1.968	Very brisk.
25	36.67	3.075	
30	44.01	4.429	High wind.
35	51.34	6.027	
40	58.68	7.873	Very high wind.
45	66.01	9.963	
50	73.35	12.300	Storm or tempest.
60	88.02	17.715	Great storm.
80	117.36	31.490	Hurricane.
100	146.7	49.200	Hurricane that tears up trees and carries buildings before it.

WINDMILL.—(*Playfair.*)

1. The impulse of a stream of air, striking with a velocity of v feet per second, on a plane whose area in feet $= a^2$, inclined at an angle θ to the direction of the stream, is in avoirdupois pounds,

$$\frac{v^2\, a^2 \sin.^2 \theta}{440}.$$

2. The sails of windmills are so constructed as to have different inclinations to the plane of their motions at different distances from the axis; greatest nearer the centre, and least at their extremities. This is done in order to make the momentum of the wind nearly the same as all different distances from the centre of motion.

3. Supposing the sail of a windmill to be a plane, inclined to the axis at an angle θ, the effect of the wind to turn the sail in a plane, at right angles to its axis, will be the greatest when $\cos.\ \theta \times \sin.^2 \theta$ is a maximum, or when $\cos.\ \theta = \frac{1}{3}$.

This gives $\theta = 54^\circ\ 44'$, and therefore the inclination of the sail to the plane of its motion, or what is called the angle of *weather*, is $35^\circ\ 16'$. This is true only when the sail is at rest or just beginning to move.

A Simple Rule for finding the Day of the Week corresponding to any given Day of the Month and Year.

MR. H. W. W., in NATURE, vol. xlvii. p. 509, gives a simple rule for finding the day of the week corresponding to any given date. It seems that this rule could be made still more simple. Thus, let

A = number of the given year.

B = number of the day in the year.

C = number of leap years from A.D. 1 to the beginning of the given year—viz. $(A-1) \div 4$, neglecting the remainder. Add these numbers together, and from the total subtract D = the number of secular years, which were ordinary years (100, 200, 300, 500, &c.). The sum is then divided by 7, and the remainder is the day of the week.

Example: June 18, 1815. $1815 + 169 + 453 - 14 = 2423 \div 7$. The remainder = 1. Therefore the day is Sunday.

This method holds good for any century according to the Gregorian Calendar. For the Julian reckoning, the rule is the same, only we must omit the number D, and write -2 in its place. The rule is then good without any change for any century.

Example: Oct. 14, 1066. $1066 + 287 + 266 - 2 = 1617 \div 7$. The remainder = 0 = 7th day, Saturday.

C. BRAUN.

Mariaschein, Bohemia, June 15.

? divisible by 4 without a remainder

When the sail is in motion, and of course near the extremities of the sail, when it moves faster, the angle of weather must be less.

Maclaurin makes the weather to vary from 26° 34′, at the point of the sail nearest the centre, to 9° at its extremity. Mr Smeaton, however, by experiment has found the following angles to answer as well as any. The radius is supposed to be divided into six parts, and ⅙th reckoning from the centre is called I, the extremity being denoted 6.

No.	*Angle with the axis.*	*Angle with the plane of motion.*
1	72°	18
2	71	19
3	72	18 middle.
4	74	16
5	77½	12½
6	83	7 extremity.

4. From Smeaton's experiments it appears, that a windmill works to the greatest advantage, when it is so constructed that the velocity of the sails is to their velocity when they go round without any load, as a number between 6 and 7 is to 10; and also that the load, when the mill works in this manner, is to the load that would just keep it from moving, nearly as 8,5 to 10.

5. With different velocities of wind the load that gives the maximum *effect* varies nearly as the square of the velocity, and the *effect* itself as the cube.

WIRE, *time of sun's passing.—See Time.*

Y

YEAR, *length of.—See Earth elements of, and Calendar.*

TABLE I.

Containing the Logarithms of all numbers from 1 to 1000; and of all the even numbers from 1000 to 10,000. The Logarithms of the odd numbers from 1000 to 10,000 may be had by simple subtraction.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

U 3

N.	0	2	4	6	8	N.	0	2	4	6	8
100	000000	000868	001734	002598	003461	130	113943	114611	115278	115943	116608
1	4321	5181	6038	6894	7748	1	7271	7934	8595	9256	9915
2	8600	9451	010300	011147	011993	2	120574	121231	121888	122544	123198
3	012837	013680	4521	5360	6197	3	3852	4504	5156	5806	6456
4	7033	7868	8700	9532	020361	4	7105	7753	8399	9045	9690
5	021189	022016	022841	023664	4486	5	130334	130977	131619	132260	132900
6	5306	6125	6942	7757	8571	6	3539	4177	4814	5451	6086
7	9384	030195	031004	031812	032619	7	6721	7354	7987	8618	9249
8	033424	4227	5029	5830	6629	8	9879	140508	141136	141763	142389
9	7426	8223	9017	9811	040602	9	143015	3639	4263	4885	5507
110	041393	042182	042969	043755	044540	140	146128	146748	147367	147985	148603
1	5323	6105	6885	7664	8442	1	9219	9835	150449	151063	151676
2	9218	9993	050766	051538	052309	2	152288	152900	3510	4120	4728
3	053078	053846	4613	5378	6142	3	5336	5943	6549	7154	7759
4	6905	7666	8426	9185	9942	4	8362	8965	9567	160168	160769
5	060698	061452	062206	062958	063709	5	161368	161967	162564	3161	3758
6	4456	5206	5953	6699	7443	6	4353	4947	5541	6134	6726
7	8186	8928	9668	070407	071145	7	7317	7908	8497	9086	9674
8	071882	072617	073352	4085	4816	8	170262	170848	171434	172019	172603
9	5547	6276	7004	7731	8457	9	3186	3769	4351	4932	5512
120	079181	079904	080626	081347	082067	150	176091	176670	177248	177825	178401
1	082785	083503	4219	4934	5647	1	8977	9552	180126	180699	181272
2	6360	7071	7781	8490	9198	2	181844	182415	2985	3555	4123
3	9905	090611	091315	092018	092721	3	4691	5259	5825	6391	6956
4	093422	4122	4820	5518	6215	4	7521	8084	8647	9209	9771
5	6910	7604	8298	8990	9681	5	190332	190892	191451	192010	192567
6	100371	101059	101747	102434	103119	6	3125	3681	4237	4792	5346
7	3804	4487	5169	5851	6531	7	5900	6453	7005	7556	8107
8	7210	7888	8565	9241	9916	8	8657	9206	9755	200303	200850
9	110590	111263	111934	112605	113275	9	201397	201943	202488	3033	3577

N.	0	2	4	6	8	N.	0	2	4	6	8
160	204120	204663	205204	205746	206286	190	278754	279211	279667	280123	280578
1	6826	7365	7904	8441	8979	1	281033	281488	281942	2396	2849
2	9515	210051	210586	211121	211654	2	3301	3753	4205	4656	5107
3	212188	2720	3252	3783	4314	3	5557	6007	6456	6905	7354
4	4844	5373	5902	6430	6957	4	7802	8249	8696	9143	9589
5	7484	8010	8536	9060	9585	5	290035	290480	290925	291369	291813
6	220108	220631	221153	221675	222196	6	2256	2699	3141	3584	4025
7	2716	3236	3755	4274	4792	7	4466	4907	5347	5787	6226
8	5309	5826	6342	6858	7372	8	6665	7104	7542	7979	8416
9	7887	8400	8913	9426	9938	9	8853	9289	9725	300161	300595
170	230449	230960	231470	231979	232488	200	301030	301464	301898	302331	302764
1	2996	3504	4011	4517	5023	1	3196	3628	4059	4491	4921
2	5528	6033	6537	7041	7544	2	5351	5781	6211	6639	7068
3	8046	8548	9049	9550	240050	3	7496	7924	8351	8778	9204
4	240549	241048	241546	242044	2541	4	9630	310056	310481	310906	311330
5	3038	3534	4030	4525	5019	5	311754	2177	2600	3023	3445
6	5513	6006	6499	6991	7482	6	3867	4289	4710	5130	5551
7	7973	8464	8954	9443	9932	7	5970	6390	6809	7227	7616
8	250420	250908	251395	251881	252368	8	8063	8481	8898	9314	9730
9	2853	3338	3822	4306	4790	9	320146	320562	320977	321391	321805
180	255273	255755	256237	256718	257198	210	322219	322633	323046	323458	322871
1	7679	8158	8637	9116	9594	1	4282	4694	5105	5516	5926
2	260071	260548	261025	261501	261976	2	6336	6745	7155	7563	7972
3	2451	2925	3399	3873	4346	3	8380	8787	9194	9601	330008
4	4818	5290	5761	6232	6702	4	330414	330819	331225	331630	2034
5	7172	7641	8110	8578	9046	5	2438	2842	3246	3649	4051
6	9513	9980	270446	270912	271377	6	4454	4856	5257	5658	6059
7	271842	272306	2770	3233	3696	7	6460	6860	7260	7659	8058
8	4158	4620	5081	5542	6002	8	8456	8855	9253	9650	340047
9	6462	6921	7380	7838	8296	9	340444	340841	341237	341630	2028

N.	0	2	4	6	8	N.	0	2	4	6	8
220	342423	342817	343212	343606	343999	250	397940	398287	398634	398981	399328
1	4392	4785	5178	5570	5962	1	9674	400020	400365	400711	401056
2	6353	6744	7135	7525	7915	2	401401	1745	2089	2433	2777
3	8305	8694	9083	9472	9860	3	3121	3464	3807	4149	4492
4	350248	350636	351023	351410	351796	4	4834	5176	5517	5858	6199
5	2183	2568	2954	3339	3724	5	6540	6881	7221	7561	7901
6	4108	4493	4876	5260	5643	6	8240	8579	8918	9257	9595
7	6026	6408	6790	7172	7554	7	9933	410271	410609	410946	411283
8	7935	8316	8696	9076	9456	8	411620	1956	2293	2629	2964
9	9835	360215	360593	360972	361350	9	3300	3635	3970	4305	4639
230	361728	362105	362482	362859	363236	260	414973	415307	415641	415974	416308
1	3612	3988	4363	4739	5113	1	6641	6973	7306	7638	7970
2	5488	5862	6236	6610	6983	2	8301	8633	8964	9295	9625
3	7356	7729	8101	8473	8845	3	9956	420286	420616	420945	421275
4	9216	9587	9958	370328	370698	4	421604	1933	2261	2590	2918
5	371068	371437	371806	2175	2544	5	3246	3574	3901	4228	4555
6	2912	3280	3647	4015	4382	6	4882	5208	5534	5860	6186
7	4748	5115	5481	5846	6212	7	6511	6836	7161	7486	7811
8	6577	6942	7306	7670	8034	8	8135	8459	8783	9106	9429
9	8398	8761	9124	9487	9819	9	9752	430075	430398	430720	431042
240	380211	380573	380934	381296	381656	270	431364	431685	432007	432328	432649
1	2017	2377	2737	3097	3456	1	2969	3290	3610	3930	4249
2	3815	4174	4533	4891	5249	2	4569	4888	5207	5526	5844
3	5606	5964	6321	6677	7034	3	6163	6481	6799	7116	7433
4	7390	7746	8101	8456	8811	4	7751	8067	8384	8701	9017
5	9166	9520	9875	390228	390582	5	9333	9648	9964	440279	440594
6	390935	391288	391641	1993	2345	6	440909	441224	441538	1852	2166
7	2697	3048	3400	3751	4101	7	2480	2793	3106	3419	3732
8	4452	4802	5152	5501	5850	8	4015	4357	4669	4981	5293
9	6199	6548	6896	7245	7592	9	5604	5915	6226	6537	6848

N.	0	2	4	6	8	N.	0	2	4	6	8
280	447158	447468	447778	448088	448397	310	491362	491642	491922	492201	492481
1	8706	9015	9324	9633	9941	1	2760	3040	3319	3597	3876
2	450249	450557	450865	451172	451479	2	4155	4433	4711	4989	5267
3	1786	2093	2400	2706	3012	3	5544	5822	6099	6376	6653
4	3318	3624	3930	4235	4540	4	6930	7206	7483	7759	8035
5	4845	5150	5454	5758	6062	5	8311	8586	8862	9137	9412
6	6366	6670	6973	7276	7579	6	9687	9962	500236	500511	500785
7	7882	8184	8487	8789	9091	7	501059	501333	1607	1880	2154
8	9392	9694	9995	460296	460597	8	2427	2700	2973	3246	3518
9	460898	461198	461499	1799	2098	9	3791	4063	4335	4607	4878
290	462398	462697	462997	463296	463594	320	505150	505421	505693	505964	506234
1	3893	4191	4490	4788	5085	1	6505	6776	7046	7316	7586
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720	857332	857453	857574	857694	857815	750	875061	875177	875293	875409	875524
1	7935	8056	8176	8297	8417	1	5640	5756	5871	5987	6102
2	8537	8657	8778	8898	9018	2	6218	6333	6449	6564	6680
3	9138	9258	9379	9499	9619	3	6795	6910	7026	7141	7256
4	9739	9859	9978	860098	860218	4	7371	7487	7602	7717	7832
5	860338	860458	860578	0697	0817	5	7947	8062	8177	8292	8407
6	0937	1056	1176	1295	1415	6	8522	8637	8752	8866	8981
7	1534	1654	1773	1893	2012	7	9096	9211	9325	9440	9555
8	2131	2251	2370	2489	2608	8	9669	9784	9898	880013	880127
9	2728	2847	2966	3085	3204	9	880242	880356	880471	0585	0699

N.	0	2	4	6	8	N.	0	2	4	6	8
760	880814	880928	881042	881156	881271	790	897627	897737	897847	897957	898067
1	1385	1499	1613	1727	1841	1	8176	8286	8396	8506	8615
2	1955	2069	2183	2297	2411	2	8725	8835	8944	9054	9164
3	2525	2638	2752	2866	2980	3	9273	9383	9492	9602	9711
4	3093	3207	3321	3434	3548	4	9821	9930	900039	900149	900258
5	3661	3775	3888	4002	4115	5	900367	900476	0586	0695	0804
6	4229	4342	4455	4569	4682	6	0913	1022	1131	1240	1349
7	4795	4909	5022	5135	5248	7	1458	1567	1676	1785	1894
8	5361	5474	5587	5700	5813	8	2003	2112	2221	2329	2438
9	5926	6039	6152	6265	6378	9	2547	2655	2764	2873	2981
770	886491	886604	886716	886829	886942	800	903090	903199	903307	903416	903524
1	7054	7167	7280	7392	7505	1	3633	3741	3849	3958	4066
2	7617	7730	7842	7955	8067	2	4174	4283	4391	4499	4607
3	8179	8292	8404	8516	8629	3	4716	4824	4932	5040	5148
4	8741	8853	8965	9077	9190	4	5256	5364	5472	5580	5688
5	9302	9414	9526	9638	9750	5	5796	5904	6012	6119	6227
6	9862	9974	890086	890197	890309	6	6335	6443	6551	6658	6766
7	890421	890533	0645	0756	0868	7	6874	6981	7089	7196	7304
8	0980	1091	1203	1314	1426	8	7411	7519	7626	7734	7841
9	1537	1649	1760	1872	1983	9	7949	8056	8163	8270	8378
780	892095	892206	892317	892429	892540	810	908485	908592	908699	908807	908914
1	2651	2762	2873	2985	3096	1	9021	9128	9235	9342	9449
2	3207	3318	3429	3540	3651	2	9556	9663	9770	9877	9984
3	3762	3873	3984	4094	4205	3	910091	910197	910304	910411	910518
4	4316	4427	4538	4648	4759	4	0624	0731	0838	0944	1051
5	4870	4980	5091	5201	5312	5	1158	1264	1371	1477	1584
6	5423	5533	5644	5754	5864	6	1690	1797	1903	2009	2116
7	5975	6085	6195	6306	6416	7	2222	2328	2435	2541	2647
8	6526	6636	6747	6857	6967	8	2753	2859	2966	3072	3178
9	7077	7187	7297	7407	7517	9	3284	3390	3496	3602	3708

N.	0	2	4	6	8	N.	0	2	4	6	8
820	913814	913920	914026	914132	914237	850	929419	929521	929623	929725	929827
1	4343	4449	4555	4660	4766	1	9930	930032	930134	930236	930338
2	4872	4977	5083	5189	5294	2	930440	0542	0643	0745	0847
3	5400	5505	5611	5716	5822	3	0949	1051	1153	1254	1356
4	5927	6033	6138	6243	6349	4	1458	1560	1661	1763	1865
5	6454	6559	6664	6770	6875	5	1966	2068	2169	2271	2372
6	6980	7085	7190	7295	7400	6	2474	2575	2677	2778	2879
7	7506	7611	7716	7820	7925	7	2981	3082	3183	3285	3386
8	8030	8135	8240	8345	8450	8	3487	3589	3690	3791	3892
9	8555	8659	8764	8869	8973	9	3993	4094	4195	4296	4397
830	919078	919183	919287	919392	919496	860	934498	934599	934700	934801	934902
1	9601	9706	9810	9914	920019	1	5003	5104	5205	5306	5406
2	920123	920228	920332	920436	0541	2	5507	5608	5709	5809	5910
3	0645	0749	0853	0958	1062	3	6011	6111	6212	6313	6413
4	1166	1270	1374	1478	1582	4	6514	6614	6715	6815	6916
5	1686	1790	1894	1998	2102	5	7016	7117	7217	7317	7418
6	2206	2310	2414	2518	2622	6	7518	7618	7718	7819	7919
7	2725	2829	2933	3037	3140	7	8019	8119	8219	8320	8420
8	3244	3348	3451	3555	3658	8	8520	8620	8720	8820	8920
9	3762	3865	3969	4072	4176	9	9020	9120	9220	9320	9419
840	924279	924383	924486	924589	924693	870	939519	939619	939719	939819	939918
1	4796	4899	5003	5106	5209	1	940018	940118	940218	940317	940417
2	5312	5415	5518	5621	5725	2	0516	0616	0716	0815	0915
3	5828	5931	6034	6137	6240	3	1014	1114	1213	1313	1412
4	6342	6445	6548	6651	6754	4	1511	1611	1710	1809	1909
5	6857	6959	7062	7165	7268	5	2008	2107	2207	2306	2405
6	7370	7473	7576	7678	7781	6	2504	2603	2702	2801	2901
7	7883	7986	8088	8191	8293	7	3000	3099	3198	3297	3396
8	8396	8498	8601	8703	8805	8	3495	3593	3692	3791	3890
9	8908	9010	9112	9215	9317	9	3989	4088	4186	4285	4384

N.	0	2	4	6	8	N.	0	2	4	6	8
880	944483	944581	944680	944779	944877	910	959041	959137	959232	959328	959423
1	4976	5074	5173	5272	5370	1	9518	9614	9709	9804	9900
2	5469	5567	5665	5764	5862	2	9995	960090	960185	960281	960376
3	5961	6059	6157	6256	6354	3	960471	0566	0661	0756	0851
4	6452	6551	6649	6747	6845	4	0946	1041	1136	1231	1326
5	6943	7041	7140	7238	7336	5	1421	1516	1611	1706	1801
6	7434	7532	7630	7728	7826	6	1895	1990	2085	2180	2275
7	7924	8022	8119	8217	8315	7	2369	2464	2559	2653	2748
8	8413	8511	8609	8706	8804	8	2843	2937	3032	3126	3221
9	8902	8999	9097	9195	9292	9	3316	3410	3504	3599	3693
890	949390	949488	949585	949683	949780	920	963788	963882	963977	964071	964165
1	9878	9975	950073	950170	950267	1	4260	4354	4448	4542	4637
2	950365	950462	0560	0657	0754	2	4731	4825	4919	5013	5108
3	0851	0949	1046	1143	1240	3	5202	5296	5390	5484	5578
4	1338	1435	1532	1629	1726	4	5672	5766	5860	5954	6048
5	1823	1920	2017	2114	2211	5	6142	6236	6329	6423	6517
6	2308	2405	2502	2599	2696	6	6611	6705	6799	6892	6986
7	2792	2889	2986	3083	3180	7	7080	7173	7267	7361	7454
8	3276	3373	3470	3566	3663	8	7548	7642	7735	7829	7922
9	3760	3856	3953	4049	4146	9	8016	8109	8203	8296	8390
900	954243	954339	954435	954532	954628	930	968483	968576	968670	968763	968856
1	4725	4821	4918	5014	5110	1	8950	9043	9136	9229	9323
2	5207	5303	5399	5495	5592	2	9416	9509	9602	9695	9789
3	5688	5784	5880	5976	6072	3	9882	9975	970068	970161	970254
4	6168	6265	6361	6457	6553	4	970347	970440	0533	0626	0719
5	6649	6745	6840	6936	7032	5	0812	0904	0997	1090	1183
6	7128	7224	7320	7416	7512	6	1276	1369	1461	1554	1647
7	7607	7703	7799	7894	7990	7	1740	1832	1925	2018	2110
8	8086	8181	8277	8373	8468	8	2203	2295	2388	2481	2573
9	8564	8659	8755	8850	8946	9	2666	2758	2851	2943	3035

Y

N.	0	2	4	6	8	N.	0	2	4	6	8
940	973128	973220	973313	973405	973497	970	986772	986861	986951	987040	987130
1	3590	3682	3774	3866	3959	1	7219	7309	7398	7488	7577
2	4051	4143	4235	4327	4420	2	7666	7756	7845	7934	8024
3	4512	4604	4696	4788	4880	3	8113	8202	8291	8381	8470
4	4972	5064	5156	5248	5340	4	8559	8648	8737	8826	8916
5	5432	5524	5616	5707	5799	5	9005	9094	9183	9272	9361
6	5891	5983	6075	6167	6258	6	9450	9539	9628	9717	9806
7	6350	6442	6533	6625	6717	7	9895	9983	990072	990161	990250
8	6808	6900	6992	7083	7175	8	990339	990428	0516	0605	0694
9	7266	7358	7449	7541	7632	9	0783	0871	0960	1049	1137
950	977724	977815	977906	977998	978089	980	991226	991315	991403	991492	991580
1	8181	8272	8363	8454	8546	1	1669	1758	1846	1935	2023
2	8637	8728	8819	8911	9002	2	2111	2200	2288	2377	2465
3	9093	9184	9275	9366	9457	3	2554	2642	2730	2819	2907
4	9548	9639	9730	9821	9912	4	2995	3083	3172	3260	3348
5	980003	980094	980185	980276	980367	5	3436	3524	3613	3701	3789
6	0458	0549	0640	0730	0821	6	3877	3965	4053	4141	4229
7	0912	1003	1093	1184	1275	7	4317	4405	4493	4581	4669
8	1366	1456	1547	1637	1728	8	4757	4845	4933	5021	5108
9	1819	1909	2000	2090	2181	9	5196	5284	5372	5460	5547
960	982271	982362	982452	982543	982633	990	995635	995723	995811	995898	995986
1	2723	2814	2904	2994	3085	1	6074	6161	6249	6337	6424
2	3175	3265	3356	3446	3536	2	6512	6599	6687	6774	6862
3	3626	3716	3807	3897	3987	3	6949	7037	7124	7212	7299
4	4077	4167	4257	4347	4437	4	7386	7474	7561	7648	7736
5	4527	4617	4707	4797	4887	5	7823	7910	7998	8085	8172
6	4977	5067	5157	5247	5337	6	8259	8347	8434	8521	8608
7	5426	5516	5606	5696	5786	7	8695	8782	8869	8956	9043
8	5875	5965	6055	6144	6234	8	9131	9218	9305	9392	9479
9	6324	6413	6503	6593	6682	9	9565	9652	9739	9826	9913

TABLE II.

LOGARITHMIC

SINES, COSINES, TANGENTS, AND COTANGENTS,

TO EVERY EVEN MINUTE OF THE QUADRANT.

Note.—From this Table may also be found the Logarithmic Secants and Cosecants; the logarithm of the secant of any arc being = 20 — log. cosine; and the logarithmic cosecant = 20 — log. sine.

	0 Deg.				1 Deg.				2 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	0.000000	10.000000	0.000000	Infinite.	8.241855	9.999934	8.241921	11.758079	8.542819	9.999735	8.543084	11.456916	60
2	6.764756	00	6.764756	13.235244	256094	29	256165	743835	549995	26	550268	449732	58
4	7.065786	00	7.065786	12.934214	269881	25	269956	730044	557054	17	557336	442664	56
6	241877	9.999999	241878	758122	283243	20	283323	716677	563999	08	564291	435709	54
8	366816	99	366817	633183	296207	15	296292	703708	570836	9.999699	571137	428863	52
10	463725	98	463727	536273	308794	10	308884	691116	577566	89	577877	422123	50
12	542906	97	542909	457091	321027	05	321122	678878	584193	80	584514	415486	48
14	609853	96	609857	390143	332924	9.999899	333025	666975	590721	70	591051	408949	46
16	667845	95	667849	332151	344504	94	344610	655390	597152	60	597492	402508	44
18	718997	94	719003	280997	355783	88	355895	644105	603489	50	603839	396161	42
20	764754	93	764761	235239	366777	82	366895	633105	609734	40	610094	389906	40
22	7.806146	9.999991	7.806155	12.193845	8.377499	9.999876	8.377622	11.622378	8.615891	9.999629	8.616262	11.383738	38
24	843934	89	843944	156056	387962	70	388092	611908	621962	19	622343	377657	36
26	878695	88	878708	121292	398179	64	398315	601685	627948	08	628340	371660	34
28	910879	86	910894	089106	408161	58	408304	591696	633854	9.999597	634256	365744	32
30	940842	83	940858	059142	417919	51	418068	581932	639680	86	640093	359907	30
32	968870	81	968889	031111	427462	44	427618	572382	645428	75	645853	354147	28
34	995198	79	995219	004781	436800	38	436962	563038	651102	64	651537	348463	26
36	8.020021	76	8.020045	11.979955	445941	31	446110	553890	656702	53	657149	342851	24
38	043501	73	043527	956473	454893	23	455070	544930	662230	41	662689	337311	22
40	065776	71	065806	934194	463665	16	463849	536151	667689	29	668160	331840	20
42	8.086965	9.999968	8.086997	11.913003	8.472263	9.999809	8.472454	11.527546	8.673080	9.999518	8.673563	11.326437	18
44	107167	64	107202	892797	480693	01	480892	519108	678405	06	678900	321100	16
46	126471	61	126510	873490	488963	9.999793	489170	510830	683665	9.999493	684172	315828	14
48	144953	58	144996	855004	497078	86	497293	502707	688863	81	689381	310619	12
50	162681	54	162727	837273	505045	78	505267	494733	693998	69	694529	305471	10
52	179713	50	179763	820237	512867	69	513098	486902	699073	56	699617	300383	8
54	196102	46	196156	803844	520551	61	520790	479210	704090	43	704646	295354	6
56	211895	42	211953	788047	528102	53	528349	471651	709049	31	709618	290382	4
58	227134	38	227195	772805	535523	44	535779	464221	713952	18	714534	285465	2
60	241855	34	241921	758079	542819	35	543084	456916	718800	04	719396	280604	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	89 Deg.				88 Deg.				87 Deg.				

	3 Deg.				4 Deg.				5 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	8.718800	9.999404	8.719396	11.280604	8.843585	9.998941	8.844644	11.155356	8.940296	9.998344	8.941952	11.058018	60
2	723595	9.999391	724204	275796	847183	23	848260	151740	943174	22	944852	055148	58
4	728337	78	728959	271041	850751	05	851846	148154	946034	00	947734	052266	56
6	733027	64	733663	266337	854291	9.998887	855403	144597	948874	9.998277	950597	049403	54
8	737667	50	738317	261683	857801	69	858932	141068	951696	55	953441	046559	52
10	742259	36	742922	257078	861283	51	862433	137567	954499	32	956267	043733	50
12	746802	22	747479	252521	864738	32	865906	134094	957284	09	959075	040925	48
14	751297	08	751969	248011	868165	13	869351	130649	960052	9.998186	961866	038134	46
16	755747	9.999294	756453	243547	871565	9.998795	872770	127230	962801	63	964639	035361	44
18	760151	79	760872	239128	874938	76	876162	123838	965534	39	967394	032606	42
20	764511	65	765246	234754	878285	57	879529	120471	968249	16	970133	029867	40
22	8.768828	9.999250	8.769578	11.230422	8.881607	9.998738	8.882869	11.117131	8.970947	9.998092	8.972855	11.027145	38
24	773101	35	773866	226134	884903	18	886185	113815	973628	68	975560	024440	36
26	777333	20	778114	221886	888174	9.998699	889476	110524	976293	44	978248	021752	34
28	781524	05	782320	217680	891421	79	892742	107258	978941	20	980921	019079	32
30	785675	9.999189	786486	213514	894643	59	895984	104016	981573	9.997996	983577	016423	30
32	789787	74	790613	209387	897842	39	899203	100797	984189	72	986217	013783	28
34	793859	58	794701	205299	901017	19	902398	097602	986789	47	988842	011158	26
36	797894	42	798752	201248	904169	9.998599	905570	094430	989374	22	991451	008549	24
38	801892	26	802765	197235	907297	78	908719	091281	991943	9.997897	994045	005955	22
40	805852	10	806742	193258	910404	58	911846	088154	994497	72	996624	003376	20
42	8.809777	9.999094	8.810683	11.189317	8.913488	9.998537	8.914951	11.085049	8.997036	9.997847	8.999188	11.000812	18
44	813667	77	814589	185411	916550	16	918034	081966	999560	22	9.001738	10.998262	16
46	817522	61	818461	181539	919591	9.998495	921096	078904	002069	9.997797	004272	995728	14
48	821343	44	822298	177702	922610	74	924136	075864	004563	71	006792	993208	12
50	825130	27	826103	173897	925609	53	927156	072844	007044	45	009298	990702	10
52	828884	10	829874	170126	928587	31	930155	069845	009510	19	011790	988210	8
54	832607	9.998993	833613	166387	931544	10	933134	066866	011962	9.997693	014268	985732	6
56	836297	76	837321	162679	934481	9.998388	936093	063907	014400	67	016732	983268	4
58	839956	58	840998	159002	937398	66	939032	060968	016824	41	019183	980817	2
60	843585	41	844644	155356	940296	44	941952	058048	019235	14	021620	978380	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine	Sine.	Cotang.	Tang.	′
	86 Deg.				85 Deg.				84 Deg.				

	6 Deg.				7 Deg.				8 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.019235	9.997614	9.021620	10.978380	9.085894	9.996751	9.089144	10.910856	9.143555	9.995753	9.147803	10.852197	60
2	9.021632	588	4044	5956	7947	720	9.091228	10.908772	5349	717	9632	0368	58
4	4016	561	6455	3545	9990	688	3302	6698	7136	681	9.151454	10.848546	56
6	6386	534	8852	1148	9.092624	657	5367	4633	8915	646	3269	6731	54
8	8744	507	9.031237	10.968763	4047	625	7422	2578	9.150686	610	5077	4923	52
10	9.031080	480	3609	6391	6062	594	9468	0532	2451	573	6877	3123	50
12	3421	452	5969	4031	8066	562	9.101504	10.898496	4208	537	8671	1329	48
14	5741	425	8316	1684	9.100062	530	3532	6468	5957	501	9.160457	10.839543	46
16	8018	397	9.040651	10.959349	2048	498	5550	4450	7700	464	2236	7764	44
18	9.040342	369	2973	7027	4025	465	7559	2441	9435	427	4008	5992	42
20	2625	341	5284	4716	5992	433	9559	0441	9.161164	390	5774	4226	40
22	9.044895	9.997313	9.047582	10.952418	9.107951	9.996400	9.111551	10.888449	9.162885	9.995353	9.167532	10.832468	38
24	7154	285	9869	0131	9901	368	3533	6467	4600	316	9284	0716	36
26	9400	257	9.052144	10.947856	9.111842	335	5507	4493	6307	278	9.171029	10.828971	34
28	9.051635	228	4407	5593	3774	302	7472	2528	8008	241	2767	7233	32
30	3859	199	6659	3341	5698	269	9429	0571	9702	203	4499	5501	30
32	6071	170	8900	1100	7613	235	9.121377	10.878623	9.171389	165	6224	3776	28
34	8271	141	9.061130	10.938870	9519	202	3317	6683	3070	127	7942	2058	26
36	9.060460	112	3348	6652	9.121417	168	5249	4751	4744	089	9655	0345	24
38	2639	083	5556	4444	3306	134	7172	2828	6411	051	9.181360	10.818640	22
40	4806	053	7752	2248	5187	100	9087	0913	8072	013	3059	6941	20
42	9.066962	9.997024	9.069938	10.930062	9.127060	9.996066	9.130994	10.869006	9.179726	9.994974	9.184752	10.815248	18
44	9107	9.996994	9.072113	10.927887	8925	032	2893	7107	9.181374	935	6439	3561	16
46	9.071242	964	4278	5722	9.130781	9.995998	4784	5216	3016	896	8120	1880	14
48	3366	934	6432	3568	2630	963	6667	3333	4651	857	9794	0206	12
50	5480	904	8576	1424	4470	928	8542	1458	6280	818	9.191462	10.808538	10
52	7583	874	9.080710	10.919290	6303	894	9.140409	10.859591	7903	779	3124	6876	8
54	9676	843	2833	7167	8128	859	2269	7731	9519	739	4780	5220	6
56	9.081759	812	4947	5053	9944	823	4121	5879	9.191130	700	6430	3570	4
58	3832	782	7050	2950	9.141754	788	5966	4034	2734	660	8074	1926	2
60	5894	751	9144	0856	3555	753	7803	2197	4332	620	9713	0287	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	83 Deg.				82 Deg.				81 Deg.				

′	9 Deg. Sine.	9 Deg. Cosine.	9 Deg. Tang.	9 Deg. Cotang.	10 Deg. Sine.	10 Deg. Cosine.	10 Deg. Tang.	10 Deg. Cotang.	11 Deg. Sine.	11 Deg. Cosine.	11 Deg. Tang.	11 Deg. Cotang.	′
0	9.194332	9.994620	9.199713	10.800287	9.239670	9.993351	9.246319	10.753681	9.280599	9.991947	9.288652	10.711348	60
2	5925	580	9.201345	10.798655	9.241101	307	7794	2206	1897	897	9999	0001	58
4	7511	540	2971	7029	2526	262	9264	0736	3190	848	9.291342	10.708658	56
6	9091	499	4592	5408	3947	217	9.250730	10.749270	4480	799	2682	7318	54
8	9.200666	459	6207	3793	5363	172	2191	7809	5766	749	4017	5983	52
10	2234	418	7817	2183	6775	127	3648	6352	7048	699	5349	4651	50
12	3797	377	9420	0580	8181	081	5100	4900	8326	649	6677	3323	48
14	5354	336	9.211018	10.788982	9583	036	6547	3453	9600	599	8001	1999	46
16	6906	295	2611	7389	9.250980	9.992990	7990	2010	9.290870	549	9322	0678	44
18	8452	254	4198	5802	2373	944	9429	0571	2137	498	9.300638	10.699362	42
20	9992	212	5780	4220	3761	898	9.260863	10.739137	3399	448	1951	8049	40
22	9.211526	9.994171	9.217356	10.782644	9.255144	9.992852	9.262292	10.737708	9.294658	9.991397	9.303261	10.696739	38
24	3055	129	8926	1074	6523	806	8717	6283	5913	346	4567	5433	36
26	4579	087	9.220492	10.779508	7898	759	5138	4862	7164	295	5869	4131	34
28	6097	045	2052	7948	9268	713	6555	3445	8412	244	7168	2832	32
30	7609	003	3607	6393	9.260633	666	7967	2033	9655	193	8463	1537	30
32	9116	9.993960	5156	4844	1994	619	9375	0625	9.300895	141	9754	0246	28
34	9.220618	918	6700	3300	3351	572	9.270779	10.729221	2132	090	9.311042	10.688958	26
36	2115	875	8239	1761	4703	525	2178	7822	3364	038	2327	7673	24
38	3606	832	9773	0227	6051	478	3573	6427	4593	9.990986	3608	6392	22
40	5092	789	9.231302	10.768698	7395	430	4964	5036	5819	934	4885	5115	20
42	9.226573	9.993746	9.232826	10.767174	9.268734	9.992382	9.276351	10.723649	9.307041	9.990882	9.316159	10.683841	18
44	8048	703	4345	5655	9.270069	335	7734	2266	8259	829	7430	2570	16
46	9518	660	5859	4141	1400	287	9113	0887	9474	777	8697	1303	14
48	9.230984	616	7368	2632	2726	239	9.280488	10.719512	9.310685	724	9961	0039	12
50	2444	572	8872	1128	4049	190	1858	8142	1893	671	9.321222	10.678778	10
52	3899	528	9.240371	10.759629	5367	142	3225	6775	3097	618	2479	7521	8
54	5349	484	1865	8135	6681	093	4588	5412	4297	565	3733	6267	6
56	6795	440	3354	6646	7991	044	5947	4053	5495	511	4983	5017	4
58	8235	396	4839	5161	9297	9.991996	7301	2699	6689	458	6231	3769	2
60	9670	351	6319	3681	9.280599	947	8652	1348	7879	404	7475	2525	0
′	80 Deg. Cosine.	80 Deg. Sine.	80 Deg. Cotang.	80 Deg. Tang.	79 Deg. Cosine.	79 Deg. Sine.	79 Deg. Cotang.	79 Deg. Tang.	78 Deg. Cosine.	78 Deg. Sine.	78 Deg. Cotang.	78 Deg. Tang.	′

′	12 Deg.				13 Deg.				14 Deg.				′
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.317879	9.990404	9.327474	10.672526	9.352088	9.988724	9.363364	10.636636	9.383675	9.986904	9.396771	10.603229	60
2	9066	351	8715	1285	3181	666	4515	5485	4687	841	7846	2154	58
4	9.320249	297	9953	0047	4271	607	5664	4336	5697	778	8919	1081	56
6	1430	243	9.331187	10.668813	5358	548	6810	3190	6704	714	9990	0010	54
8	2607	188	2418	7582	6443	489	7953	2047	7709	651	9.401058	10.598942	52
10	3780	134	3646	6354	7524	430	9094	0906	8711	587	2124	7876	50
12	4950	079	4871	5129	8603	371	9.370232	10.629768	9711	523	3187	6813	48
14	6117	025	6093	3907	9678	312	1367	8633	9.390708	459	4249	5751	46
16	7281	9.989970	7311	2689	9.360752	252	2499	7501	1703	395	5308	4692	44
18	8442	915	8527	1473	1822	193	3629	6371	2695	331	6364	3636	42
20	9599	860	9739	0261	2889	133	4756	5244	3685	266	7419	2581	40
22	9.330753	9.989804	9.340948	10.659052	9.363954	9.988073	9.375881	10.624119	9.394673	9.986202	9.408471	10.591529	38
24	1903	749	2155	7845	5016	013	7003	2997	5658	137	9521	0479	36
26	3051	693	3358	6642	6075	9.987953	8122	1878	6641	072	9.410569	10.589431	34
28	4195	637	4558	5442	7131	892	9239	0761	7621	007	1615	8385	32
30	5337	582	5755	4245	8185	832	9.380354	10.619646	8600	9.985942	2658	7342	30
32	6475	525	6949	3051	9236	771	1466	8534	9575	876	3699	6301	28
34	7610	469	8141	1859	9.370285	710	2575	7425	9.400549	811	4738	5262	26
36	8742	413	9329	0671	1330	649	3682	6318	1520	745	5775	4225	24
38	9871	356	9.350514	10.649486	2373	588	4786	5214	2489	679	6810	3190	22
40	9.340996	300	1697	8303	3414	526	5888	4112	3455	613	7842	2158	20
42	9.342119	9.989243	9.352876	10.647124	9.374452	9.987465	9.386987	10.613013	9.404420	9.985547	9.418873	10.581127	18
44	3239	186	4053	5947	5487	403	8084	1916	5382	480	9901	0099	16
46	4355	128	5227	4773	6519	341	9178	0822	6341	414	9.420927	10.579073	14
48	5469	071	6398	3602	7549	279	9.390270	10.609730	7299	347	1952	8048	12
50	6579	014	7566	2434	8577	217	1360	8640	8254	280	2974	7026	10
52	7687	9.988956	8731	1269	9601	155	2447	7553	9207	213	3993	6007	8
54	8792	898	9893	0107	9.380624	092	3531	6469	9.410157	146	5011	4989	6
56	9893	840	9.361053	10.638947	1643	030	4614	5386	1106	079	6027	3973	4
58	9.350992	782	2210	7790	2661	9.986967	5694	4306	2052	011	7041	2959	2
60	2088	724	3364	6636	3675	904	6771	3229	2996	9.984944	8052	1948	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	77 Deg.				76 Deg.				75 Deg.				

	15 Deg.				16 Deg.				17 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.412996	9.984944	9.428052	10.571948	9.440338	9.982842	9.457496	10.542504	9.465935	9.980596	9.485339	10.514661	60
2	3938	876	9062	0938	1218	769	8449	1551	6761	519	6242	3758	58
4	4878	808	9.430070	10.569930	2096	696	9400	0600	7585	442	7143	2857	56
6	5815	740	1075	8925	2973	624	9.460349	10.539651	8407	364	8043	1957	54
8	6751	672	2079	7921	3847	551	1297	8703	9227	286	8941	1059	52
10	7684	603	3080	6920	4720	477	2242	7758	9.470046	208	9838	0162	50
12	8615	535	4080	5920	5590	404	3186	6814	0863	130	9.490733	10.509267	48
14	9544	466	5078	4922	6459	331	4128	5872	1679	052	1627	8373	46
16	9.420470	397	6073	3927	7326	257	5069	4931	2492	9.979973	2519	7481	44
18	1395	328	7067	2933	8191	183	6008	3992	3304	895	3410	6590	42
20	2318	259	8059	1941	9054	109	6945	3055	4115	816	4299	5701	40
22	9.423238	9.984190	9.439048	10.560952	9.449915	9.982035	9.467880	10.532120	9.474923	9.979737	9.495186	10.504814	38
24	4156	120	9.440036	10.559964	9.450775	9.981961	8814	1186	5730	658	6073	3927	36
26	5073	050	1022	8978	1632	886	9746	0254	6536	579	6957	3043	34
28	5987	9.983981	2006	7994	2488	812	9.470676	10.529324	7340	499	7841	2159	32
30	6899	911	2988	7012	3342	737	1605	8395	8142	420	8722	1278	30
32	7809	840	3968	6032	4194	662	2532	7468	8942	340	9603	0397	28
34	8717	770	4947	5053	5044	587	3457	6543	9741	260	9.500481	10.499519	26
36	9623	700	5923	4077	5893	512	4381	5619	9.480539	180	1359	8641	24
38	9.430527	629	6898	3102	6739	436	5303	4697	1334	100	2235	7765	22
40	1429	558	7870	2130	7584	361	6223	3777	2128	019	3109	6891	20
42	9.432329	9.983487	9.448841	10.551159	9.458427	9.981285	9.477142	10.522858	9.482921	9.978939	9.503982	10.496018	18
44	3226	416	9810	0190	9268	209	8059	1941	3712	858	4854	5146	16
46	4122	345	9.450777	10.549223	9.460108	133	8975	1025	4501	777	5724	4276	14
48	5016	273	1743	8257	0946	057	9889	0111	5289	696	6593	3407	12
50	5908	202	2706	7294	1782	9.980981	9.480801	10.519199	6075	615	7460	2540	10
52	6798	130	3668	6332	2616	904	1712	8288	6860	533	8326	1674	8
54	7686	058	4628	5372	3448	827	2621	7379	7643	452	9191	0809	6
56	8572	9.982986	5586	4414	4279	750	3529	6471	8424	370	9.510054	10.489946	4
58	9456	914	6542	3458	5108	673	4435	5565	9204	288	0916	9084	2
60	9.440338	842	7496	2504	5935	596	5339	4661	9982	206	1776	8224	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	74 Deg.				73 Deg.				72 Deg.				

	18 Deg.				19 Deg.				20 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.489982	9.978206	9.511776	10.488224	9.512642	9.975670	9.536972	10.463028	9.534052	9.972986	9.561066	10.438934	60
2	0759	124	2635	7365	3375	583	7792	2208	4745	894	1851	8149	58
4	1535	042	3493	6507	4107	496	8611	1389	5438	802	2636	7364	56
6	2308	9.977959	4349	5651	4837	408	9429	0571	6129	709	3419	6581	54
8	3081	877	5204	4796	5566	321	9.540245	10.459755	6818	617	4202	5798	52
10	3851	794	6057	3943	6294	233	1061	8939	7507	524	4983	5017	50
12	4621	711	6910	3090	7020	145	1875	8125	8194	431	5763	4237	48
14	5388	628	7761	2239	7745	057	2688	7312	8880	338	6542	3458	46
16	6154	544	8610	1390	8468	9.974969	3499	6501	9565	245	7320	2680	44
18	6919	461	9458	0542	9190	880	4310	5690	9.540249	151	8098	1902	42
20	7682	377	9.520305	10.479695	9911	792	5119	4881	0931	058	8873	1127	40
22	9.498444	9.977293	9.521151	10.478849	9.520631	9.974703	9.545928	10.454072	9.541613	9.971964	9.569648	10.430352	38
24	9204	209	1995	8005	1349	614	6735	3265	2293	870	9.570422	10.429578	36
26	9963	125	2838	7162	2066	525	7540	2460	2971	776	1195	8805	34
28	9.500721	041	3680	6320	2781	436	8345	1655	3649	682	1967	8033	32
30	1476	9.976957	4520	5480	3495	347	9149	0851	4325	588	2738	7262	30
32	2231	872	5359	4641	4208	257	9951	0049	5000	493	3507	6493	28
34	2984	787	6197	3803	4920	167	9.550752	10.449248	5674	398	4276	5724	26
36	3735	702	7033	2967	5630	077	1552	8448	6347	303	5044	4956	24
38	4485	617	7868	2132	6339	9.973987	2351	7649	7019	208	5810	4190	22
40	5234	532	8702	1298	7046	897	3149	6851	7689	113	6576	3424	20
42	9.505981	9.976446	9.529535	10.470465	9.527753	9.973807	9.553946	10.446054	9.548359	9.971018	9.577341	10.422659	18
44	6727	361	9.530366	10.469634	8458	716	4741	5259	9027	9.970922	8104	1896	16
46	7471	275	1196	8804	9161	625	5536	4464	9693	827	8867	1133	14
48	8214	189	2025	7975	9864	535	6329	3671	9.550359	731	9629	0371	12
50	8956	103	2853	7147	9.530565	444	7121	2879	1024	635	9.580389	10.419611	10
52	9696	017	3679	6321	1265	352	7913	2087	1687	538	1149	8851	8
54	9.510434	9.975930	4504	5496	1963	261	8702	1298	2349	442	1907	8093	6
56	1172	844	5328	4672	2661	169	9491	0509	3010	345	2665	7335	4
58	1907	757	6150	3850	3357	078	9.560279	10.439721	3670	249	3422	6578	2
60	2642	670	6972	3028	4052	9.972986	1066	8934	4329	152	4177	5823	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	71 Deg.				70 Deg.				69 Deg.				

	21 Deg.				22 Deg.				23 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.554329	9.970152	9.584177	10.415823	9.573575	9.967166	9.606410	10.393590	9.591878	9.964026	9.627852	10.372148	60
2	4987	055	4932	5068	4200	064	7137	2863	2473	9.963919	8554	1446	58
4	5613	9.969957	5686	4314	4824	9.966961	7863	2137	3067	811	9255	0745	56
6	6299	860	6439	3561	5447	859	8588	1412	3659	704	9956	0044	54
8	6953	762	7190	2810	6069	756	9312	0688	4251	596	9.630656	10.369344	52
10	7606	665	7941	2059	6689	653	9.610036	10.389964	4842	488	1355	8645	50
12	8258	567	8691	1309	7309	550	0759	9241	5432	379	2053	7947	48
14	8909	469	9440	0560	7927	447	1480	8520	6021	271	2750	7250	46
16	9558	370	9.590188	10.409812	8545	344	2201	7799	6609	163	3447	6553	44
18	9.560207	272	0935	9065	9162	246	2921	7079	7196	054	4143	5857	42
20	0855	173	1681	8319	9777	136	3641	6359	7783	9.962945	4838	5162	40
22	9.561501	9.969075	9.592426	10.407574	9.580392	9.966033	9.614359	10.385641	9.598368	9.962836	9.635532	10.364468	38
24	2146	9.968976	3171	6829	1005	9.965929	5077	4923	8952	727	6226	3774	36
26	2790	877	3914	6086	1618	824	5793	4207	9536	617	6919	3081	34
28	3433	777	4656	5344	2229	720	6509	3491	9.600118	508	7611	2389	32
30	4075	678	5398	4602	2840	615	7224	2776	0700	398	8302	1698	30
32	4716	578	6138	3862	3449	511	7939	2061	1280	288	8992	1008	28
34	5356	479	6878	3122	4058	406	8652	1348	1860	178	9682	0318	26
36	5995	379	7616	2384	4665	301	9364	0636	2439	067	9.640371	10.359629	24
38	6632	278	8354	1646	5272	195	9.620076	10.379924	3017	9.961957	1060	8940	22
40	7269	178	9091	0909	5877	090	0787	9213	3594	846	1747	8253	20
42	9.567904	9.968078	9.599827	10.400173	9.586482	9.964984	9.621497	10.378503	9.604170	9.961735	9.642434	10.357566	18
44	8539	9.967977	9.600562	10.399438	7085	879	2207	7793	4745	624	3120	6880	16
46	9172	876	1296	8704	7688	773	2915	7085	5319	513	3806	6194	14
48	9804	775	2029	7971	8289	666	3623	6377	5892	402	4490	5510	12
50	9.570435	674	2761	7239	8890	560	4330	5670	6465	290	5174	4826	10
52	1066	573	3493	6507	9489	454	5036	4964	7036	179	5857	4143	8
54	1695	471	4223	5777	9.590088	347	5741	4259	7607	067	6540	3460	6
56	2323	370	4953	5047	0686	240	6445	3555	8177	9.960955	7222	2778	4
58	2950	268	5682	4318	1282	133	7149	2851	8745	843	7903	2097	2
60	3575	166	6410	3590	1878	026	7852	2148	9313	730	8583	1417	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	68 Deg.				67 Deg.				66 Deg.				

	24 Deg.				25 Deg.				26 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.609313	9.960730	9.648583	10.351417	9.625948	9.957276	9.668673	10.331327	9.641842	9.953660	9.688182	10.311818	60
2	9880	618	9263	0737	6490	158	9332	0668	2360	537	8823	1177	58
4	9.610447	505	9942	0058	7030	040	9991	0009	2877	413	9463	0537	56
6	1012	392	9.650620	10.349380	7570	9.956921	9.670649	10.329351	3393	290	9.690103	10.309897	54
8	1576	279	1297	8703	8109	803	1306	8694	3909	166	0742	9258	52
10	2140	165	1974	8026	8647	684	1963	8037	4423	042	1381	8619	50
12	2702	052	2650	7350	9185	566	2619	7381	4936	9.952918	2019	7981	48
14	3264	9.959938	3326	6674	9721	447	3274	6726	5450	793	2656	7344	46
16	3825	825	4000	6000	9.630257	327	3929	6071	5962	669	3293	6707	44
18	4385	711	4674	5326	0792	208	4584	5416	6474	544	3930	6070	42
20	4944	596	5348	4652	1326	089	5237	4763	6984	419	4566	5434	40
22	9.615502	9.959482	9.656020	10.343980	9.631859	9.955969	9.675890	10.324110	9.647494	9.952294	9.695201	10.304799	38
24	6060	368	6692	3308	2392	849	6543	3457	8004	168	5836	4164	36
26	6616	253	7364	2636	2923	729	7194	2806	8512	043	6470	3530	34
28	7172	138	8034	1966	3454	609	7846	2154	9020	9.951917	7103	2897	32
30	7727	023	8704	1296	3984	488	8496	1504	9527	791	7736	2264	30
32	8281	9.958908	9373	0627	4514	368	9146	0854	9.650034	665	8369	1631	28
34	8834	792	9.660042	10.339958	5042	247	9795	0205	0539	539	9001	0999	26
36	9386	677	0710	9290	5570	126	9.680444	10.319556	1044	412	9632	0368	24
38	9938	561	1377	8623	6097	005	1092	8908	1549	286	9.700263	10.299737	22
40	9.620488	445	2043	7957	6623	9.954883	1740	8260	2052	159	0893	9107	20
42	9.621038	9.958329	9.662709	10.337291	9.637148	9.954762	9.682387	10.317613	9.652555	9.951032	9.701523	10.298477	18
44	1587	213	3375	6625	7673	640	3033	6967	3057	9.950905	2152	7848	16
46	2135	096	4039	5961	8197	518	3679	6321	3558	778	2780	7220	14
48	2682	9.957979	4703	5297	8720	396	4324	5676	4059	650	3409	6591	12
50	3229	863	5366	4634	9242	274	4968	5032	4558	522	4036	5964	10
52	3774	746	6029	3971	9764	152	5612	4388	5058	394	4663	5337	8
54	4319	628	6691	3309	9.640284	029	6255	3745	5556	266	5290	4710	6
56	4863	511	7352	2648	0804	9.953906	6898	3102	6054	138	5916	4084	4
58	5406	393	8013	1987	1324	783	7540	2460	6551	010	6541	3459	2
60	5948	276	8672	1328	1842	660	8182	1818	7047	9.949881	7166	2834	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	65 Deg.				64 Deg.				63 Deg.				

′	27 Deg.				28 Deg.				29 Deg.				′
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.657047	9.949881	9.707166	10.292834	9.671609	9.945935	9.725674	10.274326	9.685571	9.941819	9.743752	10.256248	60
2	7542	752	7790	2210	2084	800	6284	3716	6027	679	4348	5652	58
4	8037	623	8414	1586	2558	666	6892	3108	6482	539	4943	5057	56
6	8531	494	9037	0963	3032	531	7501	2499	6936	398	5538	4462	54
8	9025	364	9660	0340	3505	396	8109	1891	7389	258	6132	3868	52
10	9517	235	9.710282	10.289718	3977	261	8716	1284	7843	117	6726	3274	50
12	9.660009	105	0904	9096	4448	125	9323	0677	8295	9.940975	7319	2681	48
14	0501	9.948975	1525	8475	4919	9.944990	9929	0071	8747	834	7913	2087	46
16	0991	845	2146	7854	5390	854	9.730535	10.269465	9198	693	8505	1495	44
18	1481	715	2766	7234	5859	718	1141	8859	9648	551	9097	0903	42
20	1970	584	3386	6614	6328	582	1746	8254	9.690098	409	9689	0311	40
22	9.662459	9.948454	9.714005	10.285995	9.676796	9.944446	9.732351	10.267649	9.690548	9.940267	9.750281	10.249719	38
24	2946	323	4624	5376	7264	309	2955	7045	0996	125	0872	9128	36
26	3433	192	5242	4758	7731	172	3558	6442	1444	9.939982	1462	8538	34
28	3920	060	5860	4140	8197	036	4162	5838	1892	840	2052	7948	32
30	4406	9.947929	6477	3523	8663	9.943899	4764	5236	2339	697	2642	7358	30
32	4891	797	7093	2907	9128	761	5367	4633	2785	554	3231	6769	28
34	5375	665	7709	2291	9592	624	5969	4031	3231	410	3820	6180	26
36	5859	533	8325	1675	9.680056	486	6570	3430	3676	267	4409	5591	24
38	6342	401	8940	1060	0519	348	7171	2829	4120	123	4997	5003	22
40	6824	269	9555	0445	0982	210	7771	2229	4564	9.938980	5585	4415	20
42	9.667305	9.947136	9.720169	10.279831	9.681443	9.943072	9.738371	10.261629	9.695007	9.938836	9.756172	10.243828	18
44	7786	004	0783	9217	1905	9.942934	8971	1029	5450	691	6759	3241	16
46	8267	9.946871	1396	8604	2365	795	9570	0430	5892	547	7345	2655	14
48	8746	738	2009	7991	2825	656	9.740169	10.259831	6334	402	7931	2069	12
50	9225	604	2621	7379	3284	517	0767	9233	6775	258	8517	1483	10
52	9703	471	3232	6768	3743	378	1365	8635	7215	113	9102	0898	8
54	9.670181	337	3844	6156	4201	239	1962	8038	7654	9.937967	9687	0313	6
56	0658	203	4454	5546	4658	099	2559	7441	8094	822	9.760272	10.239728	4
58	1134	069	5065	4935	5115	9.941959	3156	6844	8532	676	0856	9144	2
60	1609	9.945935	5674	4326	5571	819	3752	6248	8970	531	1439	8561	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	62 Deg.				61 Deg.				60 Deg.				

	30 Deg.				31 Deg.				32 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.698970	9.937531	9.761439	10.238561	9.711839	9.933066	9.778774	10.221226	9.724210	9.928420	9.795789	10.204211	60
2	9407	385	2023	7977	2260	9.932914	9346	0654	4614	263	6351	3649	58
4	9844	238	2606	7394	2679	762	9918	0082	5017	104	6913	3087	56
6	9.700280	092	3188	6812	3098	609	9.780489	10.219511	5420	9.927946	7475	2525	54
8	0716	9.936946	3770	6230	3517	457	1060	8940	5823	787	8036	1964	52
10	1151	799	4352	5648	3935	304	1631	8369	6225	629	8596	1404	50
12	1585	652	4933	5067	4352	151	2201	7799	6626	470	9157	0843	48
14	2019	505	5514	4486	4769	9.931998	2771	7229	7027	310	9717	0283	46
16	2452	357	6095	3905	5186	845	3341	6658	7428	151	9.800277	10.199723	44
18	2885	210	6675	3325	5602	691	3910	6090	7828	9.926991	0836	9164	42
20	3317	062	7255	2745	6017	537	4479	5521	8227	831	1396	8604	40
22	9.703749	9.935914	9.767834	10.232166	9.716432	9.931383	9.785048	10.214952	9.728626	9.926671	9.801955	10.198045	38
24	4179	766	8414	1586	6846	229	5616	4384	9024	511	2513	7487	36
26	4610	618	8992	1008	7259	075	6184	3816	9422	351	3072	6928	34
28	5040	469	9570	0430	7673	9.930921	6752	3248	9820	190	3630	6370	32
30	5469	320	9.770148	10.229852	8085	766	7319	2681	9.730217	029	4187	5813	30
32	5898	171	0726	9274	8497	611	7886	2114	0613	9.925868	4745	5255	28
34	6326	022	1303	8697	8909	456	8453	1547	1009	707	5302	4698	26
36	6753	9.934873	1880	8120	9320	300	9019	0981	1404	545	5859	4141	24
38	7180	723	2457	7543	9730	145	9585	0415	1799	384	6415	3585	22
40	7606	574	3033	6967	9.720140	9.929989	9.790151	10.209849	2193	222	6971	3029	20
42	9.708032	9.934424	9.773608	10.226392	9.720549	9.929833	9.790716	10.209284	9.732587	9.925060	9.807527	10.192473	18
44	8458	274	4184	5816	0958	677	1281	8719	2980	9.924897	8083	1917	16
46	8882	123	4759	5241	1366	521	1846	8154	3373	735	8638	1362	14
48	9306	9.933973	5333	4667	1774	364	2410	7590	3765	572	9193	0807	12
50	9730	822	5908	4092	2181	207	2974	7026	4157	409	9748	0252	10
52	9.710153	671	6482	3518	2588	050	3538	6462	4549	246	9.810302	10.189698	8
54	0575	520	7055	2945	2994	9.928893	4101	5899	4939	083	0857	9143	6
56	0997	369	7628	2372	3400	736	4664	5336	5330	9.923919	1410	8590	4
58	1419	217	8201	1799	3805	578	5227	4773	5719	755	1964	8036	2
60	1839	066	8774	1226	4210	420	5789	4211	6109	591	2517	7483	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang	Cosine.	Sine.	Cotang.	Tang.	′
	59 Deg.				58 Deg.				57 Deg.				

′	33 Deg.				34 Deg.				35 Deg.				′
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.736109	9.923591	9.812517	10.187483	9.747562	9.918574	9.828987	10.171013	9.758591	9.913365	9.845227	10.154773	60
2	6498	427	3070	6930	7936	404	9532	0468	8952	187	5764	4236	58
4	6886	263	3623	6377	8310	233	9.830077	10.169923	9312	010	6302	3698	56
6	7274	098	4175	5825	8683	062	0621	9379	9672	9.912833	6839	3161	54
8	7661	9.922933	4728	5272	9056	9.917891	1165	8835	9.760031	655	7376	2624	52
10	8048	768	5279	4721	9429	719	1709	8291	0390	477	7913	2087	50
12	8434	603	5831	4169	9801	548	2253	7747	0748	299	8449	1551	48
14	8820	438	6382	3618	9.750172	376	2796	7204	1106	121	8986	1014	46
16	9206	272	6933	3067	0543	204	3339	6661	1464	9.911942	9522	0478	44
18	9590	106	7484	2516	0914	032	3882	6118	1821	763	9.850058	10.149942	42
20	9975	9.921940	8035	1965	1284	9.916859	4425	5575	2177	584	0593	9407	40
22	9.740359	9.921774	9.818585	10.181415	9.751654	9.916687	9.834967	10.165033	9.762534	9.911405	9.851129	10.148871	38
24	0742	607	9135	0865	2023	514	5509	4491	2889	226	1664	8336	36
26	1125	441	9684	0316	2392	341	6051	3949	3245	046	2199	7801	34
28	1508	274	9.820234	10.179766	2760	167	6593	3407	3600	9.910866	2733	7267	32
30	1889	107	0783	9217	3128	9.915994	7134	2866	3954	686	3268	6732	30
32	2271	9.920939	1332	8668	3495	820	7675	2325	4308	506	3802	6198	28
34	2652	772	1880	8120	3862	646	8216	1784	4662	325	4336	5664	26
36	3033	604	2429	7571	4229	472	8757	1243	5015	144	4870	5130	24
38	3413	436	2977	7023	4595	297	9297	0703	5367	9.909963	5404	4596	22
40	3792	268	3524	6476	4960	123	9838	0162	5720	782	5938	4062	20
42	9.744171	9.920099	9.824072	10.175928	9.755326	9.914948	9.840378	10.159622	9.766072	9.909601	9.856471	10.143529	18
44	4550	9.919931	4619	5381	5690	773	0917	9083	6423	419	7004	2996	16
46	4928	762	5166	4834	6054	598	1457	8543	6774	237	7537	2463	14
48	5306	593	5713	4287	6418	422	1996	8004	7124	055	8069	1931	12
50	5683	424	6259	3741	6782	246	2535	7465	7475	9.908873	8602	1398	10
52	6060	254	6805	3195	7144	070	3074	6926	7824	690	9134	0866	8
54	6436	085	7351	2649	7507	9.913894	3612	6388	8173	507	9666	0334	6
56	6812	9.918915	7897	2103	7869	718	4151	5849	8522	324	9.860198	10.139802	4
58	7187	745	8442	1558	8230	541	4689	5311	8871	141	0730	9270	2
60	7562	574	8987	1013	8591	365	5227	4773	9219	9.907958	1261	8739	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	56 Deg.				55 Deg.				54 Deg.				

	36 Deg.				37 Deg.				38 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.769219	9.907958	9.861261	10.138739	9.779463	9.902349	9.877114	10.122886	9.789342	9.896532	9.892810	10.107190	60
2	9566	774	1792	8208	9798	158	7640	2360	9665	335	3331	6669	58
4	9913	590	2323	7677	9.780133	9.901967	8165	1835	9988	137	3851	6149	56
6	9.770260	406	2854	7146	0467	776	8691	1309	9.790310	9.895939	4371	5629	54
8	0606	222	3385	6615	0801	585	9216	0784	0632	741	4892	5108	52
10	0952	037	3915	6085	1134	394	9741	0259	0954	542	5412	4588	50
12	1298	9.906852	4445	5555	1468	202	9.880265	10.119735	1275	343	5932	4068	48
14	1643	667	4975	5025	1800	010	0790	9210	1596	145	6452	3548	46
16	1987	482	5505	4495	2132	9.900818	1314	8686	1917	9.894945	6971	3029	44
18	2331	296	6035	3965	2464	626	1839	8161	2237	746	7491	2509	42
20	2675	111	6564	3436	2796	433	2363	7637	2557	546	8010	1990	40
22	9.773018	9.905925	9.867094	10.132906	9.783127	9.900240	9.882887	10.117113	9.792876	9.894346	9.898530	10.101470	38
24	3361	739	7623	2377	3458	047	3410	6590	3195	146	9049	0951	36
26	3704	552	8152	1848	3788	9.899854	3934	6066	3514	9.893946	9568	0432	34
28	4046	366	8680	1320	4118	660	4457	5543	3832	745	9.900086	10.099914	32
30	4388	179	9209	0791	4447	467	4980	5020	4150	544	0605	9395	30
32	4729	9.904992	9737	0263	4776	273	5503	4497	4467	343	1124	8876	28
34	5070	804	9.870265	10.129735	5105	078	6026	3974	4784	142	1642	8358	26
36	5410	617	0793	9207	5433	9.898884	6549	3451	5101	9.892940	2160	7840	24
38	5750	429	1321	8679	5761	689	7072	2928	5417	739	2679	7321	22
40	6090	241	1849	8151	6089	494	7594	2406	5733	536	3197	6803	20
42	9.776429	9.904053	9.872376	10.127624	9.786416	9.898299	9.888116	10.111884	9.796049	9.892334	9.903714	10.096286	18
44	6768	9.903864	2903	7097	6742	104	8639	1361	6364	132	4232	5768	16
46	7106	676	3430	6570	7069	9.897908	9160	0840	6679	9.891929	4750	5250	14
48	7444	487	3957	6043	7395	712	9682	0318	6993	726	5267	4733	12
50	7781	298	4484	5516	7720	516	9.890204	10.109796	7307	523	5784	4216	10
52	8119	108	5010	4990	8045	320	0725	9275	7621	319	6302	3698	8
54	8455	9.902919	5536	4464	8370	123	1247	8753	7934	115	6819	3181	6
56	8792	729	6063	3937	8694	9.896926	1768	8232	8247	9.890911	7336	2664	4
58	9128	539	6589	3411	9018	729	2289	7711	8560	707	7852	2148	2
60	9463	349	7114	2886	9342	532	2810	7190	8872	503	8369	1631	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	53 Deg.				52 Deg.				51 Deg.				

	39 Deg.				40 Deg.				41 Deg.				
′	Sine.	Cosine.	Tang.	Cotang	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.798872	9.890503	9.908369	10.091631	9.808067	9.884254	9.923813	10.076187	9.816943	9.877780	9.939163	10.060837	60
2	9181	298	8886	1114	8368	042	4327	5673	7233	560	9673	0327	58
4	9495	093	9402	0598	8669	9.883829	4840	5160	7524	340	9.940193	10.059817	56
6	9806	9.889888	9918	0082	8969	617	5352	4648	7813	120	0694	9306	54
8	9.800117	682	9.910435	10.089565	9269	404	5865	4135	8103	9.876899	1204	8796	52
10	0427	477	0951	9049	9569	191	6378	3622	8392	678	1714	8286	50
12	0737	271	1467	8533	9868	9.882977	6890	3110	8681	457	2223	7777	48
14	1047	064	1982	8018	9.810167	764	7403	2597	8969	236	2733	7267	46
16	1356	9.888858	2498	7502	0465	550	7915	2085	9257	014	3243	6757	44
18	1665	651	3014	6986	0763	336	8427	1573	9545	9.875793	3752	6248	42
20	1973	441	3529	6471	1061	121	8940	1060	9832	571	4262	5738	40
22	9.802282	9.888237	9.914044	10.085956	9.811358	9.881907	9.929452	10.070548	9.820120	9.875348	9.944771	10.055229	38
24	2589	030	4560	5440	1655	692	9964	0036	0406	126	5281	4719	36
26	2897	9.887822	5075	4925	1952	477	9.930475	10.069525	0693	9.874903	5790	4210	34
28	3204	614	5590	4410	2248	*261	0987	9013	0979	680	6299	3701	32
30	3511	406	6104	3896	2544	046	1499	8501	1265	456	6808	3192	30
32	3817	198	6619	3381	2840	9.880830	2010	7990	1550	232	7318	2682	28
34	4123	9.886989	7134	2866	3135	613	2522	7478	1835	009	7826	2174	26
36	4428	780	7648	2352	3430	397	3033	6967	2120	9.873784	8336	1664	24
38	4734	571	8163	1837	3725	180	3545	6455	2404	560	8844	1156	22
40	5039	362	8677	1323	4019	9.879963	4056	5944	2688	335	9353	0647	20
42	9.805343	9.886152	9.919191	10.080809	9.814313	9.879746	9.934567	10.065433	9.822972	9.873110	9.949862	10.050138	18
44	5647	9.885942	9705	0295	4607	529	5078	4922	3255	9.872885	9.950370	10.049630	16
46	5951	732	9.920219	10.079781	4900	311	5589	4411	3539	659	0879	9121	14
48	6254	522	0733	9267	5193	093	6100	3900	3821	434	1388	8612	12
50	6557	311	1247	8753	5185	9.878875	6610	3390	4104	208	1896	8104	10
52	6860	100	1760	8240	5778	656	7121	2879	4386	9.871981	2405	7595	8
54	7163	9.884889	2274	7726	6069	438	7632	2368	4668	755	2913	7087	6
56	7465	677	2787	7213	6361	219	8142	1858	4949	528	3421	6579	4
58	7766	466	3300	6700	6652	9.877999	8653	1317	5230	301	3929	6071	2
60	8067	254	3813	6187	6943	780	9163	0837	5511	073	4437	5563	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	50 Deg.				49 Deg.				48 Deg.				

A a

	42 Deg.				43 Deg.				44 Deg.				
′	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	Sine.	Cosine.	Tang.	Cotang.	′
0	9.825511	9.871073	9.954437	10.045563	9.833783	9.864127	9.969656	10.030344	9.841771	9.856934	9.984837	10.015163	60
2	5791	9.870846	4945	5055	4054	9.863892	9.970162	10.029838	2033	690	5343	4657	58
4	6071	618	5454	4546	4325	656	0669	9331	2294	446	5848	4152	56
6	6351	390	5961	4039	4595	419	1175	8825	2555	201	6354	3646	54
8	6631	161	6469	3531	4865	183	1682	8318	2815	9.855956	6860	3140	52
10	6910	9.869933	6977	3023	5134	9.862946	2188	7812	3076	711	7365	2635	50
12	7189	704	7485	2515	5403	709	2694	7306	3336	465	7871	2129	48
14	7467	474	7993	2007	5672	471	3201	6799	3595	219	8376	1624	46
16	7745	245	8500	1500	5941	234	3707	6293	3855	9.854973	8882	1118	44
18	8023	015	9008	0992	6209	9.861996	4213	5787	4114	727	9387	0613	42
20	8301	9.868785	9516	0484	6477	758	4719	5281	4372	480	9893	0107	40
22	9.828578	9.868555	9.960023	10.039977	9.836745	9.861519	9.975226	10.024774	9.844631	9.854233	9 990398	10.009602	38
24	8855	324	0531	9469	7012	280	5732	4268	4889	9.853986	0903	9097	36
26	9131	093	1038	8962	7279	041	6238	3762	5147	738	1409	8591	34
28	9407	9.867862	1545	8455	7546	9.860802	6744	3256	5405	490	1914	8086	32
30	9683	631	2052	7948	7812	562	7250	2750	5662	242	2420	7580	30
32	9959	399	2560	7440	8078	322	7756	2244	5919	9.852994	2925	7075	28
34	9.830234	167	3067	6933	8344	082	8262	1738	6175	745	3430	6570	26
36	0509	9.866935	3574	6426	8610	9.859842	8768	1232	6432	496	3936	6064	24
38	0784	703	4081	5919	8875	601	9274	0726	6688	247	4441	5559	22
40	1058	470	4588	5412	9140	360	9780	0220	6944	9.851997	4947	5053	20
42	9.831332	9.866237	9.965095	10.034905	9.839404	9.859119	9.980286	10.019714	9.847199	9.851747	9.995452	10.004548	18
44	1606	004	5602	4398	9668	9.858877	0791	9209	7454	497	5957	4043	16
46	1879	9.865770	6109	3891	9932	635	1297	8703	7709	246	6463	3537	14
48	2152	536	6616	3384	9.840196	393	1803	8197	7964	9.850996	6968	3032	12
50	2425	302	7123	2877	0459	151	2309	7691	8218	745	7473	2527	10
52	2697	068	7629	2371	0722	9.857908	2814	7186	8472	493	7979	2021	8
54	2969	9.864833	8136	1864	0985	665	3320	6680	8726	242	8484	1516	6
56	3241	598	8643	1357	1247	422	3826	6174	8979	9.849990	8989	1011	4
58	3512	363	9149	0851	1509	178	4331	5669	9232	738	9495	0505	2
60	3783	127	9656	0344	1771	9.856934	4837	5163	9485	485	10.00000	0000	0
′	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	Cosine.	Sine.	Cotang.	Tang.	′
	47 Deg.				46 Deg.				45 Deg.				

TABLE III.

Table of useful numbers with their logarithms.—(Babbage.)

Constants.		Log.	Ar. comp Log.
Diameter = 1, circumference = π	3.1415927	0.4971499	9.5028501
——— area of circle = $\frac{\pi}{4}$	0.7853982	9.8950899	0.1049101
——— content of sphere = $\frac{\pi}{6}$	0.5235988	9.7189986	0.2810014
$\sqrt{\pi}$	1.7724539	0.2485750	9.7514250
π^2	9.8696044	0.9942997	9.0057003
Hyp. log. π	1.1447299	0.0587030	9.9412970
Length of arc 1″ = sin. 1″	.000004848	4.6855749	5.3144251
——— 2″ = sin. 2″	.000009696	4.9866049	5.0133951
——— 3″ = sin. 3″	.000014544	5.1626961	4.8373039
——— 1′ = sin. 1′	.000290888	6.4637261	3.5362739
——— 1° ———	.017453293	8.2418774	1.7581226
Sin. 1°	.017452406	8.2418553	1.7581447
360° expressed in seconds	1296000	6.1126050	3.8873950
Radius reduced to seconds	206264.8	5.3144251	4.6855749
Radius reduced to minutes	3437.74677	3.5362739	6.4637261
Radius reduced to degrees	57.295780	1.7581226	8.2418774
Number whose hyp. log. is unity	2.718281829	0.4342945	9.5657055
Modulus of common logs.	.434294482	9.6377843	0.3622156
French toise = in metres	1.949040	0.2898200	9.7101800
——— = in English yards	2.1315308	0.3286916	9.6713084
——— = in English feet	6.3945925	0.8058129	9.1941871
French metre = in English yards	1.0936331	0.0388716	9.9611284
——— = in English feet	3.2808992	0.5159929	9.4840071
——— = in English inches	39.37079	1.5951741	8.4048259
French foot = in English feet	1.0657654	0.0276616	9.9723384
French are = in English acres	.02471143	8.3928979	1.6071021
French gramme = imperial lbs. Troy	.00268098	7.4282928	2.5717072
French gramme = in imperial lbs. avoirdupois	.00220606	7.3436173	2.6563827
French litre in imperial gallons	.22009687	9.3426139	0.6573861

Constants.		*Log.*	*Ar. comp Log.*
Centes. degree = in sex. degrees	.9	9.9542425	0.0457575
—— minute = —— minutes	.54	9.7323938	0.2676062
—— second = —— seconds	.324	9.5105450	0.4894550
Mean circumference of the earth in miles	24856	4.3954319	5.6045688
Diameter	7912	3.8[illegible]	6.1017137
Radius of Equator	3962.349	3.5979528	6.4020472
Semipolar axis	3949.069	3.5965608	6.4034392
Difference	12.680	1.1031193	8.8968807
Circumference of the Equator	24896	4.3961296	5.6038704
Geographical mile in feet	6075.6	3.7835892	6.2164108
24 hours expressed in seconds	86400	4.9365137	5.0634863
Diurnal acceleration of stars in mean solar seconds	235.9093	2.3727451	7.6272549
Sidereal day (*23h. 56m. 4.09s.*) in mean solar days	.99726967	9.9988126	0.0011874
Solar mean day (*24h. 3m. 56.5551s*) in sidereal days	1.00273791	0.0011874	9.9988126
Sidereal revolution of earth in mean solar days	365.25636	2.5625978	7.4374022
Tropical revolution of earth in mean solar days	365.24224	2.5625810	7.4374190
Cubic inch of distilled water in grains (Bar. 30 in. Fah. Therm 62°)	252.458	2.4021891	7.5978109
An ounce of water in cubic inches	1.73298	0.2387924	9.7612076
Cubic inches in the Imperial gallon	277.276	2.4429124	7.5570876
Length of seconds pendulum at London	39.1393	1.5926130	8.4073870
Force of gravity at London in feet	32.19084	1.5077222	8.4922778

EXPLANATION AND USE OF THE TABLES.

TABLE I.

1. *To find the log. of any given number.*

If the given number be under 100, its log. is found in the first page of the Table, immediately opposite to it. Thus log. 66 is 1.819544.

If the No consist of three figures, find the given number in the column under N, and opposite to it in the next column, marked 0 at the top, is the decimal part of the logarithm required, before which put an index, which is always less by unity than the number of integral figures in the natural number. Thus log. 448 is 2.651278. If the number should consist wholly of decimals, the index of the log. is then *negative*, and it is indicated by the place occupied by the first figure in the decimal. Thus the index of the log of .04 is —2; of .006 is —3. But to avoid the confusion that might arise by the addition and subtraction of negative indices, it is customary to take the arithmetical complement of the negative indices, and to consider these complements as positive; thus 8 is put as the index of .04; 7 as the index of .006.

If the No. consist of four figures, the three first are to be found as before in the side column under N; and under the 4th at the top will be found the logarithm required, to which prefix the index as before. Thus log. 7218 is 3.858417. If the No. be odd, and ∴ not contained in the Table, take the difference of the logs. of the Nos. next greater and less than the given one; and add ½ this difference to the less log. Thus if log. 7217 were required, we have by Table

Log. 7218	3.858417
Log. 7216	3.858297
	120

the ½ of which, or 60, added to 3.858297 gives 3.858357, the log required.

If the No. consist of 5 figures or more, find the difference between the logs. answering to the first four figures of the given No., and the next immediately following; multiply this difference by the remaining figures in the given number, strike off as many figures from the right hand as there are in the multiplier; and the remainder added to the log., answering to the first 4 figures, will be the log. required nearly. Thus if log. 100176 were required, we have by last case,

Log. 1001	000434
1002	000868
	434

∴ 434 × 76 is 32984. From this cut off two figures, and it becomes 329.84 or 330 nearly. Whence to 000434 add 330 and supply the index, and we have the required log. = 5.000764.

2. *To find the natural No. corresponding to any given logarithm.*

Look in the different columns for the decimal part of the given log.; but if you cannot find it exactly, take the next less tabular log., and in a line with the log. found in the col. on the left marked N, you have three figures of the number sought, and at the top of the column in which the log. is, you have one figure more, which annex to the other three. As, however, the Table contains only the logs. of the even Nos, it should be observed that if the given log. falls between any two of the tabular logs, and differs considerably from both; in that case we must find the log. of the intermediate odd No. as directed above, and compare it with the given one; by which means the 4th figure of the No. sought (whether it be even or odd) may be correctly ascertained. The number of integers is always one more than the number expressed by the index. Thus the

No. answering to 2.993789 is 985.8. If the number be required to a greater No. of places than four, find the difference between the given and the next less log. To this annex on the right hand as many ciphers as there are figures required above four. Divide the whole by the difference between the next less and next greater log., and the quotient annexed to the four figures formerly found will be the natural number required. Thus required the No. to 6 places answering to the log. 4.687956. The nearest less log. than this is 687886 corresponding to which is the No. 4874. The difference between 687956 and 687886 is 70, to this annex 2 ciphers and it becomes 7000, which being divided by 89, the difference between the next less and next greater log. gives 79, ∴ the number required is 48747.9.

TABLE II.

1. *To find the logarithmic sine, cosine, &c. answering to any given degree or minute.*

Find the given degrees at the top of the page, if less than 45°, and the minutes in the left hand column; opposite to which, and under the word sine, cosine, &c. is the number required. But if the given degrees be greater than 45° and less than 90°, find them at the bottom, and the required sine, cosine, &c. will be found above the word sine, cosine, &c. opposite to the given number of minutes in the *right* hand column. If the given arc exceed 90°, find the sine, cosine, &c. of its supplement. Thus the log. sine of 23°. 28′ is 9.600118; and the cotangent of 55°. 57′ is 9.829805. If the No. of minutes be odd, and ∴ not contained in the Table, proceed as directed for the odd numbers, Table I.

To find the logarithmic sine, tangent, &c. of an arc expressed in degrees, minutes, and seconds.

Find the sine, tangent, &c. corresponding to the given degree and minute, and also that answering to the next greater minute; multiply the difference between them by the given number of seconds, and divide the product by 60; then the quotient added to the sine, tangent, &c. of the given degree and minute, or subtracted from the cosine, cotangent, &c. will give the quantity required nearly.

Ex. Required the log. sine of 23°. 27′ 40″.

Log. sin. 23° 27′	....	9.599827
23 28	....	9.600118
Difference	....	291

which multiplied by 40, and divided by 60, gives 194, and this added to 9.599827 gives the required logarithm 9.600021.

2. *To find the degrees and minutes answering to any given logarithmic sine, tangent, &c.*

Find the nearest log. to that given in the proper column: if the title be at the top of the column, you have the number of degrees at the top of the page, and the minutes in the column on the left hand; but should the title be at the bottom of the column, you have the degrees at the bottom of the page, and the minutes in the column on the right hand. If the given log. seems to belong to the odd minutes, proceed as directed Art. 2. Table I. Thus log. sin. 9.457584 answers to 16°. 40′. Log. tan. 10.535401 answers to 73°. 45′. But if the seconds in the arc are also required, we seek in the proper column for the logarithm which is next less than the given one, when the logs. in the column are increasing; but next greater, when they are decreasing, and take the degrees and minutes corresponding to that logarithm for the degrees and minutes in the required arc. Then to the difference between the logarithm so found and the given log. we annex two ciphers, and divide the result by $\frac{5}{3}$ of the difference between the next less and next greater log.; and the quotient is the seconds to be added to the degrees and minutes before taken out.

Ex. Required the degrees, minutes, and seconds corresponding to the log. sin. 9.641357.

The sin. 25°. 58′. is 9.641324 which is the log. *next less* than the given one. The difference of these two logs. is 33, which by adding two ciphers becomes 3300, and this divided by $\frac{5}{3}$ of 260, or by 433, gives 8 nearly for the number of seconds; ∴ required arc is 25°. 58′. 8″.

When the arc is small, a particular process is necessary as follows :—

To find the log. sine of a small arc less than 3°.

Add 4.685575 to the common log. of the arc reduced to seconds; from the sum subtract one-third of the log. secant less radius of the arc, and the remainder will be the required log. sine.

To find the log. tangent of a small arc.

Add together the common log. of the arc, reduced to seconds, ⅔ of the log. secant less radius of the arc, and 4.685575; and the sum will be the required tangent. We have hence the following rules for performing the reverse operations :—

To find a small arc whose log. sine is given.

To ⅓ of the log. secant of the arc in the Table, whose log. sine most nearly corresponds with the given log. sine, add the given log. sine, and 5.314425, and the sum will be the common log. of the seconds in the required arc.

To find a small arc when its log. tangent is given.

To the log. tangent add 5.314425, and from the sum subtract ⅔ of the log. secant of the arc in the Table, whose tangent most nearly agrees with the given tangent; and the remainder will be the log. of the seconds in the required arc.

Ex. 1. Required the log. sine of 1°. 28′. 13″. or the log. cosine of 88°. 31′. 47″.

1°. 28′. 13″ = 5293″ .... log.	3.723702
Constant No.	4.685575
	8.409277
⅓ log. secant 1°. 28′ .... sub.	.000047
1°. 28′. 13″. log. sine	8.409230

Ex. 2. Required the arc to the log. sine 7.963214.

⅓ log. sec. 0°. 32′	.000006
	7.963214
Constant No.	5.314425
1895″ .... log.	3.277645

Whence the required arc is 31′. 35″

Hence the arc to log. cosine 7.963214 is 89°. 28′. 25″.

FINIS.

ERRATA.

Page 16. line 3. for Young's *read* Young.
P. 21. l. 26. This series is the same as the last, the higher powers of e being neglected.
P. 22. l. 7. for y *read* y (= equation of the centre.)
P. 55. l. 5. for $A + S A^2 + B + S B^2 + C + S C^2 +$ &c. *read* $A \times S A^2 + B \times S B^2 + C \times S C^2 +$ &c.
P. 86. l. 3. for spheriod *read* spheroid.
P. 88. l. 16. for with *read* of.
P. 107. March 7, *read* 11. 14; April 14, *read* 0. 14; June 13, *read* 0. 21.
P. 147. In some copies the Figure has been inverted by mistake.
P. 169. l. 20. for .43424948 *read* .43429448.
P. 175. l. 21. for mix. *read* min.
P. 252. l. 26. for $2 g \times W s - W v^2$; *read* $2 g \times W s = W v^2$.
P. 273. l. 11. for Berege *read* Barege.
P. 302. Art. Thermometer, for Centrigrade *read* Centigrade.

Durham: Printed by Francis Humble.

When arc between two points in a staff or base line subtends 2°
then the error of 10" in measurement will cause an error
of .0014 in the calculated distance between observer
& staff At 1000 yards distance 35 ~~yards~~ subtends
2°. (nearly). ∴ the error of 10" would give 1.4 yards error
in estimated distance ∴ ultimately (**168** yards at 60 geog. miles.

Meteorological constants

__Expansion of air__ for 1° Fahr = 0.0020361 of its bulk
(Regnault. see B. 92. in Guyot's tables. (122 MS paging))
It used to be considered (Gay-Lussac) = 0.00208 or $\frac{1}{480}$th of its bulk.

__Weight of a cubic foot dry air__ at 32° & under 30 inches Bar. Press.
is 566.5654 grains Troy
(Regnault. B. 92. Guyot, as above)

__The ~~effect~~ heat of condensation__ of vapour into an inch of rain is sufficient to raise the temperature of entire atmosphere (assumed at 30 inches) exactly $1^{\circ}\frac{1}{3}$ Fahr.
(the specific heat of ~~water~~ air is taken as 1.8) F.G.

If the entire atmosphere (30 inches) be heated $1^{\circ}\frac{1}{3}$ it would occupy as much space as 30.81 not so heated.

__Difference velocity rotation__ of 2 points on earth's surface in our latitudes (52°) ~~of~~ which one is __a yard__ to the North of another. = .002105 __inch__ in __1 sec.__
or approximately = $\frac{1}{8}$th __inch__ in 1 __minute__

or 100 yd — 1 foot — of meridian

as an average for all latitudes
the difference in question =

$$\frac{1}{90 \times 60 \times 2000} \times \frac{360 \times 60 \times 2000 \times 36}{24 \times 60 \times 60}$$

(degrees; metres in 1°; yards in 1′ — decimal; metres in 1°; yards in 1′; inches in 1 yard — hours in 1 day; minutes in 1 h; seconds in 1 m)

$$= \frac{1}{90} \times \frac{\not{4} \times \not{60} \times 36}{\not{24} \times \not{60} \times 60}$$

$$= \frac{36}{90 \times 4 \times 60} = \frac{1}{600} \text{ inches in 1 yd in 1 second}$$

$$= \frac{1}{10} \text{ inch in 1 yd in 1 minute}$$

(turn over)

continued

Also th diff. of E or W movement of 2 points in our latitudes 1° lat apart = **12** miles an hour

The deflection of an air particle supposing it unchecked in its lateral tendency on being transferred to a point n° N or S & remaining there for h hours = **5** n h.

To measure the rate per hour, of walking – driving &c.

l = length of pace &c in inches. or
r = radius of wheel.
then number of paces taken in $l \times$ ~~0.05[illegible]9~~ 0.05682 seconds
or in $r \times$ 0.3593 seconds
is identical with the rate in miles per hour.

l	seconds.	radius	second
1	0.56	1	3.59
2	1.12	2	7.19
3	1.68	3	10.78
4	2.24	4	14.37
5	2.80	5	17.97
6	3.36	6	21.56
7	3.92	7	25.15
8	4.48	8	28.74
9	5.44	9	32.34

It is more convenient to ~~use~~ (which are the estimated economy) 10 times the above numbers, treating the result as miles and decimals to one place, of a mile

Example. $l = 25$
for 20 – – 11.2
for 5 – – – 2.8
for 25 – 14.0. seconds.

Tadini's formula for quantity of water flowing through a river, (in metres per)
Let l = width of stream in metres, h = depth
p = fall per metre

$$Q = 50\, l h \sqrt{ph}.$$

Rain guage ~~area~~ diameter of a circular rain guage such that one inch fall is one ounce (℥1) of water
= 2.8934 inches or 2 9/10th. very nearly.

$r \times 1.314159$ = 1.901 . . . . — {1lb Troy 12 oz water = 22.815 inches water}
area — Cubic inches in one oz water

$2r = d$ = 2.8934 inches.

Weight of a cubic foot of air at 30 inches of mercury (Report on ventilation warming & British Sp. p. 5.)

at 30° Fahr	569.2 grains
40	557.8
50	546.8
60	536.3
70	526.2

(or about 1 grain lighter for each 1° of temperature) between say 30° & 100°.

Weight of water in a saturated cubic foot of air

30° Fahr	2 grains
41°	3
49°	4
56°	5
61°	6
66°	7
70°	8

Table of Chords

0°30'	00873	21°0'	35837	41°0'	65606
1°0'	01745	21°30'	36650	41°30'	66262
1.30	02618	22	37461	42	66913
2	03490		38262		67559
2.30	04362	23	39073	43	68200
3	05234		39875		68835
	06105	24	40674	44	69466
4	06976		41469		70091
	07846	25	42262	45	70711
5	08716		43051		71325
	09585	26	43837	46	71934
6	10452		44620		72537
	11320	27	45399	47	73135
7	12186		46175		73728
	13053	28	46947	48	74314
8	13917		47716		74896
	14781	29	48481	49	75471
9	15643		49242		[illegible]
	[illegible]				

					77142
11	19081	31	51504	51	77715
	19937		52250		78261
12	20791	32	52992	52	78801
	21644		53730		79335
13	22495	33	54464	53	79864
	23345		55194		80386
14	24192	34	55919	54	80902
	25038		56641		81412
15	25882	35	57358	55	81915
	26724		58070		82413
16	27564	36	58779	56	82904
	28402		59447		83389
17	29237	37	60181	57	83867
	30071		60876		84339
18	30902	38	61566	58	84805
	31730		62251		85264
19	32557	39	62932	59	85717
	33381		63608		86163
20	34202	40	64279	60	86603

Table of Chords

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21°0'	35837
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	39875
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	41469
25	42262
	43051
26	43837
	44626
27	45399
	46175
28	46947
	47716
29	48481
	49242

41°0'	65606
41°30'	66262
42	66913
	67559
43	68200
	68835
44.	69466
	70091
45	70711
	71325
46	71934
	72537
47	73135
	73728
48	74314
	74896
49	75471
	[illegible]

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	60876
38	61566
	62251
39	62932
	63608
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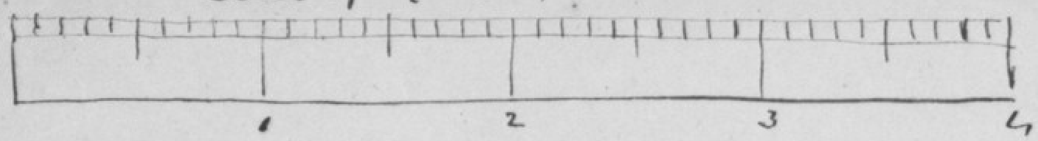
	77112
51	77715
	78261
52	78801
	79335
53	79864
	80386
54	80902
	81412
55	81915
	82413
56	82904
	83389
57	83867
	84339
58	84805
	85264
59	85717
	86163
60	86603

61°.0'	87462
30'	87882
62	88295
	88701
63	89101
	89493
64	89879
65	90631
	90996
66	91355
	91706
67	92050
	92388
68	92718
	93042
69	93359

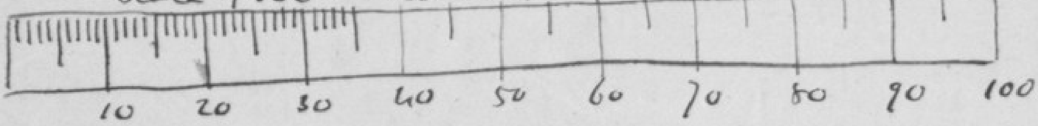
71	94264
	94552
	94832
72	95106
	95372
73	95630
	95[illegible]82 8
74	96126
	96363
75	96593
	96815
76	97030
	97237
77	97437
	97630
78	97815
	[illegible]
79	98163
	98325
80	98481
	98629

81°.0'	98769
30'	98902
82	99027
	991[illegible]4
83	[illegible]255
	99357
84	99452
	99540
85	99619
	99692
86	99756
	99813
87	99863
	99905
88	99939
	99966
89	99985
	99996
90	100000

Scale of 4 inches



scale of 100 millimetres



Aberration of Light
Aberration in Optics
Adhesion (of solids to Liquids)
Aeras, dates of
Aeronautics
Animal strength
Annuities
Anomaly (astronomy)
Arches
Arcs length of circular.
Arcs' semidiurnal.
Asymptotes to draw
Atmosphere
Axes to find the angle at which
" a curve cuts.

Balance
Barometer
Bars of Iron weight of.
Binomial Theorem
Bridges

Calendar
Capillary tubes
Catenary Equations
Caustics
Central forces (orbits)
Centre of gravity
" gyration
" oscillation
" percussion
" pressure
Chances, doctrine of
Chronology
Circle equations to
Cissoid.
Collision of bodies
Compass.
Conchoid
Cone
Conic Sections
Contact of curves.
Coordinates
Cords, strength of
Curvature, radius of
Cycle
Cycloid

Days length & increase of
Degrees of Lat & Long:
Dialling
Differentials

Diving Bell
Dominical Letter
Dyke
Earth elements of &c
Earth pressure agst walls
Easter to find
Eccentricity of orbits
Echo
Eclipses
Ecliptic obliquity of.
Elastic bodies
Equations of condition
Equation of payments
Equation of time
Equilibrium floating bodies.
Evolutes of curves
Eye dimensions of.

Figure of Sines Cos: &c.
Fluids, pressure of.
" discharge of
" resistance of
Forces composition & resolution of
Fractions, continued. vanishing
Friction
Frigorific mixtures.

Guaging
Gravity Specific
Gunnery
H
Harvest moon
Heat. tables
Horizon dip

Iceberg
Inclined plane
Inflexion, point of in curves.
Interest
Involution & Evolution

Latitude
Lemniscata
Levelling
Lever
Light
Logarithms
Logarithmic curve
Longitude
Looking glass, judging of.

P. I. 1

Entire & unimpaired heat of a vertical sun will commun[icate] 1° Cent per hour to a sheet of water 1 foot thick

Humboldt says (quoted Climate) the temp. at moment of Sunset is very nearly = the mean temp of day ☉

The rate of penetration of Solar heat into the ground is just about 1 inch a day see Forgaltain observations. Climate 76

θ = density of air (Trace 780)

[h]eat evolved by changing density

[of] air from 0 to θ is $25^{\circ}\left(\theta - \frac{1}{\theta}\right)$ Centigrade

[th]en if air be condensed 30 times

[th]e heat given out is $25^{\circ}\left(30 - \frac{1}{30}\right)$ Centigrade.

[o]r $45^{\circ}\left(\theta - \frac{1}{\theta}\right)$ in degrees Fah

[Wh]en bar at bottom 29.500
at top 26.444

[g]ives .896 the density at summit

[th]e reciprocal $\left(\frac{1}{\theta}\right)$ of which is 1.116

[th]at 1.116 − .896 = .22 & .22 × 25 = 5°.5

[w]hich corresponds to experiment.

F. I ✓

Joseph Murphy Edinburgh New Phil Journal
No 36 Oct 1843 p. 235

When 1 in weight of vapour is condensed amount of heat liberated wd raise temp of an equal weight of water by 1178° Fahr — the temp: at which condensation takes place.

To find to what temp it wd raise an equal weight of air divide by .2377 (= spec: heat air at constant pressure)

Again to find effect when 1 in volume of vapour is condensed we take $\frac{5}{8}$th of above because vapour has only $\frac{5}{8}$th spec: gravity of air.

Example. Suppose condensation to take place at 80°

the heating effect on same volume of air

$$= \frac{5}{8} \cdot \frac{1178-80}{.2377} = 2877$$

This will expand the air. Air at 32° expands $\frac{1}{492}$ for each 1° and air at 80° expands $\frac{1}{540}$. Consequently the Expanding effect at 80° will be (after subtracting the destroyed volume of the condensed vapour $\frac{2877}{540}$ = 4.35

In other words for Every cubic foot of vapour condensed 4.35 cubic feet will be added to the volume of the air. hence powerful ascentional current — & cumuli

See a late paper by Prof: Thomson of Glasgow (Manchester Phil Soc) says temp of saturated air diminishes 1° for each 294 of ascent. ~~Dry~~ air 1° for each 183 ft.