Risk of Misclassification

I

The Risk of Misclassification,

when the Objects classified vary continuously, and the Examiners are fallible.

by Francis Galton

The particular Class to which an individual or object is assigned, may be a matter of serious importance.

A candidate for the Army, and certain other appointments, is required to pass a physical examination by a fallible examiner who decides whether he is fit to be accepted or not. But his decisions however conscientiously made, cannot be strictly trust-worthy, for the true class places of many candidates ~~must~~ will lie, so ~~sufficiently~~ near the ~~[illegible]~~ ideal line that separates the unfit from the fit, that such errors as examiners are apt to make will suffice to misclass them. Similarly in respect to the first, second and third classes in literary examinations, and their places of the candidates in those classes. How, it may be asked, is it possible to arrive at a strict numerical estimate of the risk of misclassification? What data do we require for the purpose; how can we hope to get those data, and when they are obtained, how are we to utilise them?

The ~~number of~~ interesting problems that fall under the same category ~~as the above, is exceedingly~~ are many ~~large~~ and various. Chief among them is that of Natural Selection, as to the degree in which it preserves the fittest. The children of each generation vary greatly in constitutional strength and in their aptitude for self-preservation, but the trials to which they are severally exposed, of infection, cold, hunger, &c, are so unequally distributed that those who survive and leave issue, are not necessarily the strongest. Many youths who were gifted above their fellows in body and mind, perish prematurely, owing to mishaps of exceptional severity, while many weakly children live and leave descendants, solely through their luck in never having been confronted with serious peril. Natural Selection is a highly fallible examiner; what, we may ask, is the measure of its success in preserving those who are best fitted to propagate the race? -

Other examples of the scope of the general problem might be taken from the verdicts "yes" or "no" of fallible juries; from the ~~graded~~ or [illegible] sentences passed on convicts by fallible judges; from the assortment of raw material into graded degrees of fineness by fallible experts.

Another class of problems includes these and goes a short step further; it relating to the chance that two independent examiners will both classify the same object correctly, (i.e. recovering) as in the anthropometric method of identification.

The example about to be given is taken from this latter class, & in the special case in which the objects ~~to be classified~~ are to be assorted into three equal classes. It is an important problem in the anthropometric system of ~~identification~~ discovering ~~by~~ whether the measurements ~~&whether~~ of an unknown person are already entered in a classified index of the measurements of convicts; the classification of each measurement ~~is here~~ being into one or other of the ~~three~~ approximately equal divisions of short, medium, or long.

(Space) → The first part of the inquiry is directed to finding the average chance ~~that~~ that a ~~any~~ single measure is classified correctly, the second part is directed to finding the average chance that two measurers will agree in any one case. ~~it and will afterwards~~ The problem in its practical applications ~~bearings~~, will be discussed more fully, afterwards, including the methods of obtaining the necessary data.

The applicability of the ordinary law of facility of error, will be assumed throughout this discussion, such as is expressed by the familiar formula that the chance of an error regardless of its sign, lying between $\underline{0}$ and $\underline{hx}$, will be

$$\frac{2}{\sqrt{\pi}}\int_0^{hx} e^{-h^2x^2}\,dx \text{ ,or, more briefly } \frac{2}{\sqrt{\pi}}\,\mathrm{Erf}\,hx$$

Here $\underline{h}$ is the measure of precision; its reciprocal $\frac{1}{h}$ being the measure ~~of~~ or modulus of fallibility, or, more, briefly the modulus. It is with this modulus that we shall chiefly be concerned

It will soon be seen that the only datum required for solving the first problem or any other of its class, is the ratio between (1) the modulus f7v

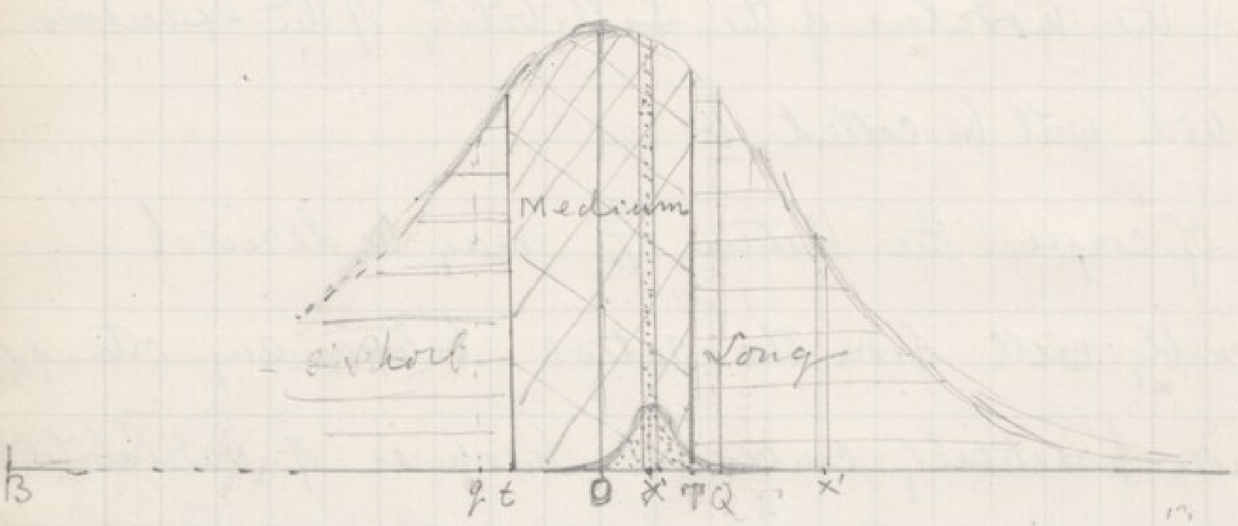
The words variability fallibility &c may be compared to those of price, value, &c, the ratios between the latter being quite independent of the coin in which they are appraised, whether of the shillings, francs or dollars.

of the variability of the objects that are to be classified, & which will be called $\underline{a}$ and (2) the modulus of the fallibility of the examiner, which will be called $\underline{b}$.

Of course, the datum $\frac{a}{b}$ may be derived equally well from the ratios between any other of the more practically convenient measures of variability, fallibility, &c, all of which are well known multiples of the modulus. Thus the probable error $\underline{r}$, is converted into the modulus $\frac{1}{h}$, by the well known equation of $rh = 0.4769$

The limits of the Medium division, when the three divisions of Short, Medium, and Long, are equally numerous, are easily found from f 8v

Fig 1 Scheme of Frequency of true values.



the Probability Integral Table. It there appears that out of 10,000 cases, 3286, or 47 less than a third, fall within the range of $hx = \pm 0.30$, while 3389, or 56 more than a third, fall within that of $hx = \pm 0.31$. It being ~~more~~ convenient to use round numbers, ~~and~~ while strict accuracy in the equality of the three classes ~~being~~ is unimportant, the value of $hx = \pm 0.30$ will be taken as the limits of the medium division.

Fig. 1 is ~~an~~ the greater part of an ordinary Scheme of Frequency, in which regard is paid to the signs of the errors, ~~are~~ being ~~regarded~~. ~~It is~~ bounded above by a curve of frequency, & below by its base. ~~The measures~~ It is supposed here to refer to ~~10,000~~ the true ~~dimensions~~ values of the same dimension, (say of head-length) in 10,000 different male adults

These values are all the ordinate standing at laid off horizontally from B, which is situated to the left of the curve. their termini are alone entered & form the curve. BO is the average length of all the values; consequently the ordinate at O divides the Scheme in two equal parts.

Let Q to the right of O, and q to its left be the positions of the base of the ordinates that respectively each divide the two halves of the scheme into equal areas. Consequently either OQ or Oq is the geometrical representation of the modulus of the curve; whose the numerical measure of the modulus (in some definite unit) being is $\underline{a}$. Take T, $\underline{t}$, so ~~that~~ $\frac{OT}{OQ}$, $\frac{Ot}{Oq}$, are respectively equal to ~~[illegible]~~ 0.30.

The ordinates at T & t thus form the limits of the medium division of the scheme, the areas to the left of $\underline{t}$ being the Shorts division of it and that, to the right of T being that of the Longs ~~[illegible]~~, the areas of all three divisions ~~being~~ are practically equal ~~to one another~~, and severally referring approximately to 3333½ ~~cases~~ different ~~dimensions~~ true values ~~along~~ of the lengths of head.

Let X be the centre of the base of a narrow column E, so narrow that the measures it contains are practically identical, and let ~~E~~ n be the number of ~~them~~ those measures, then ~~so~~ the area of the column is to ~~10,000~~ that of the whole scheme as ~~E~~ n to 10,000.

We will now consider the ~~chance~~ number out of the ~~E~~ n true values that an examiner, whose modulus of fallibility is b, will misclassify.

(1) When OX is less than OT. — The erroneous measures of the identical values in the column E will be distributed about X ~~with a~~ according to the law of facility, with a modulus $\underline{b}$, forming a little ~~heap or~~ scheme or heap of their own, as shown in Fig I. The ~~area~~ contents of this heap will of course be the

same in number as ~~that~~ of the column from which it was derived, namely c.

It will be understood that if the whole of the original scheme be divided into narrow columns whose contents are similarly distributed into heaps, each having a modulus ~~head~~ b, the superposition of the contents of these heaps will form a new scheme, having the modulus of $\sqrt{a^2+b^2}$, which will refer to the 10,000 <u>observed</u> values, or measures, just as the original scheme Fig 1 having the modulus <u>a</u> referred to the 10,000 <u>true</u> values.

~~Now~~ Let us trace the fortunes of the particular heap that we were considering. As OX is less than OT, the true values from which it was derived, are necessarily medium, but some of the observed values, in other words, a certain portion of the heap, will extend to the right, beyond T, and will be erroneously classified as Long. Another very small portion will extend to the left, beyond t, and will be erroneously classified as Short. It is easy to find the number of these transgressors ~~from~~ by means of the Probability Integral Table; they are respectively

$$e \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf}\left(\frac{OT - OX}{OQ} \right) \right\}$$

and

$$e \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf}\left(\frac{OX + Ot}{OQ} \right) \right\}$$

(2) OX greater than OT.— Here the true values contained in the column will be Long, but those of their erroneous measures which lie to the left of T are classed otherwise.

The number of these is

$$e \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \mathrm{Erf}\left(\frac{OX - OT}{OQ} \right) \cancel{\times \frac{1}{OQ}} \right.$$

of which those that lie to the left of t and are reckoned as Short is

$$e \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \mathrm{Erf} \left\{ \frac{Ot}{OQ} \cancel{\times \frac{1}{OQ}} \right\} \right.$$

and the remainder are reckoned as Medium.

The negative half of the Scheme being symmetrically opposite to the positive half, whatever results are obtained from the latter will, after interchanging the words Long & Short, be true of the former.

If Fig 1 and all its contents, be stretched or shortened either laterally or vertically or both, as it might be if it had been drawn on an elastic sheet of rubber, its internal relations will be obviously unaltered, the proportions of the transgressors from the several columns remaining the same. Consequently the ratio of $\underline{a}$ to $\underline{b}$ is the only datum with which we are concerned, their absolute values of a being unimportant. We can therefore select any unit of measurement we please for expressing the value of OQ ~~or the~~ ~~$\underline{a}$~~, and for all the other values, such as $\underline{b}$, ~~and~~ OT, which enter into the discussion. The unit selected will be $OQ = a = 1$; so that $a \times \frac{OT}{OQ}$, becomes 0.30. ~~instead of $a \times 0.30$~~ and ~~the modulus of a heap becomes $\frac{b}{a} = b$,~~ the modulus of a heap, ~~which~~ $= \frac{b}{a}$, becomes b; in other words when the general formula (in which k stands for the measure of precision, $\frac{a}{b} =$) is applied to a heap, $\frac{1}{b}$ must be substituted for k in it.

Consequently its measure of precision $k = \frac{a}{b}$

The foregoing formulae are now simplified as follows

For the Positive Half of the Scheme of True Values

The true values in any column, $\underline{e}$ in number, being as below	The number of them that will be misclassed
Medium	as Long $= e\{0.5 - \frac{1}{\sqrt{\pi}}\,\mathrm{Erf}\,\frac{1}{b}(0.3 - x)$
Medium	as Short $= e\{0.5 - \frac{1}{\sqrt{\pi}}\,\mathrm{Erf}\,\frac{1}{b}(0.3 + x)$
Long	as Short $= e\{0.5 - \frac{1}{\sqrt{\pi}}\,\mathrm{Erf}\,\frac{1}{b}(0.3 + x\} = S$
Long	as Medium $= e\{0.5 - \frac{1}{\sqrt{\pi}}\,\mathrm{Erf}\,\frac{1}{b}(x - 0.3)\} - S$

The total number of misclassifications of the 10,000 true values could be found by integration guided by the above formulae; practically it ~~is far~~ may be simpler to cut up the Scheme into narrow

columns, to work out their fortunes severally and to sum the results. It is easy to determine by trial the requisite narrowness. Thus if the columns are 0.06 in width and $\frac{a}{b}$ is as ~~small~~ large as 10, the total number of misclassments is 568, whereas if the columns are only 0.03 in width, the number is 576 (a slight change might have been made in these figures if the work had been carried out to an additional place of decimals). The difference between 568 & 576 is only 1 per cent, and therefore too small to affect the results as given in Table. Consequently, ~~so~~ columns of the breadth of 0.06 have been used for all values of $\frac{a}{b}$ that do not exceed 10. For higher values the breadth of 0.03 is used.

The actual work for the case of $\frac{a}{b} = 6$ is shown as an example, in the columns of Table , further on. In the mean time, the results are given in Table I

Table I

a = modulus of variability of the true values
b = " fallibility of the examiner

values of $\frac{a}{b}$	Percentage of the True Values that are classed by a fallible examiner into one or other of 3 equally numerous divisions	
	rightly	wrongly
2	73.3	26.7
4	86.1	13.9
6	90.5	9.5
8	92.9	7.1
10	94.3	5.7
12	95.2	4.8
14	95.9	4.1
16	96.5	3.5
18	96.9	3.1
20	97.2	2.8

The second part of the inquiry discusses the chance that two successive examiners will classify the same true dimension correctly.

The problem might be altered to determining the chance that they will classify it alike, both correctly or both erroneously. It will be found easy as we proceed, to dispose of this case, but it will not be considered in detail. Again, Space will be saved in the example, if the fallibility of the two examiners be considered the same. ~~When~~ The altered conditions when it is ~~they are~~ not the same, and further when the true dimensions measured may have changed during the interval between the two measurements, will be discussed afterwards. The example taken, is that in which $\frac{a}{b} = 6$.

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18–19

This is the back of
Table II

Table II. Example — $\frac{a}{b} = 6$.

N° of the Column	x	e	$\xi = \pm \frac{x\tau}{\sigma a}$ ξ	$(k = \frac{a}{b})$ $k\xi$	$\frac{1}{\sqrt{\pi}} \operatorname{Erf} k\xi$	0.500, minus $\frac{1}{\sqrt{\pi}} \operatorname{Erf} k\xi$ (F)	Long L (F×e)
			$\xi = 0.30 - x$				
+ 1	0.03	338	0.27	1.62	0.489	0.011	4
2	.09	336	.21	1.26	.463	.037	12
3	.15	330	.15	0.90	.398	.102	34
4	.21	324	.09	0.54	.277	.223	72
5	.27	315	.03	0.18	.100	.400	126
			$\xi = x - 0.30$			0.500, plus $\frac{1}{\sqrt{\pi}} \operatorname{Erf} k\xi$	248
6	.33	303	.03	0.18	.100	.600	182
7	.39	291	.09	0.54	.277	.777	226
8	.45	276	.15	0.90	.398	.898	248
9	.51	261	.21	1.26	.463	.963	251
10	.57	245	.27	1.62	.489	.989	242
11	.63	228	.33	1.98	.497	.997	227
12	.69	210	.39	2.34	.499	.999	210
Residue always classed as long		1543					1543
Totals for positive half		5000					3377
" " negative half		5000					1
Totals		10000					3378

Grand total of successes on one occasion 9048

No failures 959

3378
3378
3244
10,000

f.20br

Failures

248 true Medium called L
228 true Longs called M

476 in one half of scheme
952 in 10.000

9,048 successes

Successes on one occasion
This is the original failures problem – that in general
both occasions in special

Shorts called Short 1
~~Longs~~ called Short 1
2

954
9046

f.206v

A

Long

width of head

Long	Medium	Short

	Long	Medium	Short
Long			
M		x	
S			

Medium

L

Long

Medium

Sh

25

parts of 31.

x by Coridice 5

$\chi = \pm\frac{Xt}{\sigma Q}$ χ	$k = \frac{a}{\sigma}$ $k\chi$	$\frac{1}{\sqrt{\pi}}$ Erf $k\chi$	0.500 minus $\frac{1}{\sqrt{\pi}}$ Erf $k\chi$ (G)	Short S (G×e)	Medium M e−(S+L)	No. of successes $\frac{S^2}{e}$	$\frac{M^2}{e}$	$\frac{L^2}{e}$
0.33	1.98	0.497	0.003	1	333	–	328	–
0.39	2.34	0.499	0.001	–	324	–	312	–
				–	296	–	265	4
				–	252	–	196	16
				–	189	–	113	50
				–	121	–	48	109
				–	65	–	15	176
				–	28	–	3	223
				–	10	–	–	241
				–	3	–	–	239
				–	1	–	–	226
				–	–	–	–	210
					228			1543
				1	1622	0	1280	3037
				3377	1622	3037	1280	0
				3378	3244	3037	2560	3037

Grand total of successes on both occasions — 8634
failures 1366

Table II gives an outline of the calculation for the case when $\frac{a}{b} = 6$. All of it except the last 3 columns, are common to both the first and the second part of the inquiry; those 3 columns referring to the second part alone.

The Scheme fig 1 is supposed to have been divided into columns of the uniform breadth of $hx = 0.06$, which, as $h = \frac{1}{a}$ ~~[illegible]~~ and $\underline{a}$ is taken as the unit of measurement, becomes $x = 0.06$. Consequently 5 of these columns lie between O and $T = +0.30$, and a similar number between O and $t = -0.30$. In other words, the medium true values fall into 10 columns. The short & long divisions of the scheme are cut up into columns of the same breadth. The total number of true values to which the scheme refers is taken, as already mentioned, as 10,000, and in the calculations all values less than 0.5 have been ignored

Under the above conditions the number of ~~measures~~ values that will fall within the successive columns, reckoning from 0 outwards on either side is easily found from the Probability Integral Table. The columns are numbered for distinction as +1, +2, &c and −1, −2, &c, and the number of ~~measures~~ values contained in them severally, on the positive side, is entered in Table II under the heading e. Thus column +2 contains 336 values

Risk of
misclassification

II

Let us follow the fortunes of the 336 elements in that column as recorded in the second line of Table II which shews the number of the fallible measures of them that transgress beyond 'T' into the ~~the~~ long division. (The sign + need not now be prefixed to the column 2, as what is true of it is equally true mutatis mutandis of ~~column~~ −2). The base of column 2 ranges as we have seen from 0.06 to 0.12; Consequently X the centre of its base is situated at 0.09. In other words the value of $\frac{OX}{OQ}$, which we will call x is 0.09. ~~This~~ X= is the position of the ~~base~~ foot of the middle ~~line~~ ordinate of the heap, whence its elements are distributed on either side with a modulus of $\frac{b}{a}$; that is, with a measure of precision = $\frac{a}{b} = b$, ~~which~~ This corresponds to $\underline{h}$ in the general formula ~~and which~~ but, for distinction, will be here called k. Let the distance $\frac{XT}{OQ}$ be called ξ; ~~then we have to~~ we now find from the Probability Integral Table, the proportion of the elements in a ~~half~~ heap whose contents are reckoned as 1, that stray beyond the distance $k\xi$ on ~~one~~ the positive side only of $\underline{X}$. The ordinary Tables disregard the sign of the error so the entries in them are the double of what we want. They are those of $\frac{2}{\sqrt{\pi}}$ Erf $k\xi$ and not $\frac{1}{\sqrt{\pi}}$ Erf $k\xi$, thus for $k\xi = 1.26$ the tabular value is 0.9252, but we take the half of this, viz 0.463, the work ~~to~~ being here carried on to 3 decimal places only.

In short 136 × 0.463 out of the 136 cases are classified correctly as '<u>medium</u>' and the remainder of the <u>half-heap</u>, that is 136×0.5 are classified wrongly as 'long'. These are 12 in number.

Those that stray to the left beyond $\underline{t}$ are calculated in precisely the same way except that the distance $\frac{Ot}{OQ} = \chi$ has now to be substituted for ξ. No noticeable number of the 136 elements do this (as already mentioned, values below 0.5 are ignored).

Every one of the columns is treated in a similar way to find out the number

f.4

that each of them contribute to the 'long' division. When OX is greater than OT and they true values become 'long', the formula changes, and when column 12 has been reached all the elements are correctly classed as long, so it becomes unnecessary to employ the method of columns any farther. The whole of the residue of the half peak is clubbed together as shown in table II in the line headed 'Residue'.

The general result is as follows

~~2 the positive half of the scheme~~

248 true mediums are ~~wrongly~~ classed as long
<u>228</u> true longs " [?] " " medium
476 Total misclassed ~~failures~~ ~~in the positive half~~
<u>476</u> " in the negative half
952 misclassed ~~failures~~ out of 10,000 cases
or 9.5 per cent, as entered in Table II.

Positive half of the Scheme

248 true Mediums are misclassed Long

228 true Longs " " Medium

476

Negative half of the Scheme

248 true Mediums are misclassed Short

228 true Shorts " " Mediums

476

Total, – 952 misclassed out of 10,000 cases,

or 9.5 per cent, as entered in Table I.

The other entries in Table I are calculated by the same method. Thus it will be seen that the percentage of misclassments can always f6

be calculated when the relative sizes and the number of the classes is defined and when the rates of the variability of the objects to the fallibility of the examiner is known.

Next as regards the number of cases in which two examiners would concur in classifying alike. ~~We will first suppose their fallibility to be the same~~ Let their moduli of fallibility be respectively b and b_1; for an example we will suppose $b_1 = b$ ~~#~~ and $\frac{a}{b} = 6$ which is the case worked out in Table II. Let us take column No 5 which contains 315 elements, and of which 126 are misclassed. The position of ~~any~~ the measure of any one of

f.7

there in the heap is quite indeterminate, The fact of its being found in a particular part of the heap after one classification affording no clue whatever to its position in it after a second one. though the contour of the heap is constant; its constituents may interchange positions in any way, and any one given set of positions is just as likely to occur as any other. Hence the chance of any single element in column 5 being classified as Long in each of two trials is $\left(\frac{12h}{315}\right)^2$ and the number of the 315 elements that will be so classified is that fraction multiplied into 315 or $\frac{12h^2}{315}$. The working out of the calculation is seen in Table II, except that the number of successes and not of failures is given there.

The calculation when b is not equal to b' is an just as easily performed as when it is, after the number of correct (or wrong) classifications ~~successes (or failures)~~ have been worked out for the two cases separately, for instead of the headings as in Table II of $\frac{S^2}{e}$ $\frac{M^2}{e}$ $\frac{L^2}{e}$ we have to employ $\frac{SS'}{e}$ $\frac{MM'}{e}$ $\frac{LL'}{e}$.

In this way the first half of Table III has been calculated

TABLE. III.

The Chance of classifying alike, on two occasions

Value of k	one dimension only, Values of ~~m~~ k'				all the five dimensions Values of ~~m~~ k'			
	4	6	8	10	4	6	8	10
4	.80	.83	.84	.85	.29	.39	.42	.43
6	.83	.86	.88	.89	.39	.48	.53	.55
8	.84	.88	.90	.91	.42	.53	.59	.62
10	.85	.89	.91	.92	.43	.55	.62	.65

The entries in the second half are ~~merely~~ the 5th powers of those in the first half, or rather the 5th power of the figures that were calculated to an additional ~~another~~ place of decimals.

Before proceeding further ~~drawing~~ it will be well to show generally how the values of $\frac{a}{b}$ may be obtained and afterwards to speak of those in this particular case.

As a rule a & b have to be determined separately as follows

...and Holt F. Galton (annexe) No 222.

2 note bks "Risk of misclassification".

EXERCISE BOOK.

Name ____________________

Subject ____________________

School ____________________

f.1r

		E	δ	C	B	A
		pc	pc	pc	pc	pc
10°	I	1 3 1	21 57 21	5 13 5	4 11 4	9 24 9
30°	II	2 8 3	10 84 31	10 41 15	9 35 13	15 65 24
50°	III	7 27 10	3 93 34	8 62 23	8 57 21	8 86 32
70°	IV	8 49 18	2 97 36	7 81 30	8 78 29	3 95 35
90°	V	19 100 37	1 100 37	7 100 37	8 100 37	2 100 37
Median 18.5		IV+0.02	0+0.75	II+0.4	II+0.7	I+0.6
within 100 the places of the		8	14	48 30+9	54 30+	32 10+13
		70°	9°	39°	44°	23°

IV + 0.5/19, IV+0.02   0.5 : 19 :: x : 1

I − (9.5 × 1), 0.75   2.5 : 10 :: x : 1   x 0.25   1 − x = 0.75

II + 3.5/8 = 0.4   3.5 : 8 :: x : 1   x = 0.4

II + 5.5/8 = 0.7   50−26 = 26 : 41 :: x : 20

I = 9.5/15 = 0.6   15/9.5(.6

37 : 100 :: a : x

36 : 37 :: x : 100   37/3600(97.3

37/3200(86.5

37/3500(94.5

37/1500(4

37/1800(49

7 : 27 :: x : 10   27/70(3

10 : 57 :: x : 50   57/500(9

50−35

15 : 20 :: x : 22

15 : 22 :: x : 20   22/300(14

14+30

F.2

Perhaps the surest method of estimating the median rank places [each] of the five photographs is to find the exact rank ~~that~~ value ~~that it~~ for each that one half of the judgements fall short of & the other half exceed. As there are 37 judgements what is the rank value that ~~half is divided then~~ corresponds to 18½? We see that in E the rank IV corresponds to 18, which is close enough but none of the others fall so clearly on an even rank place. We therefore have recourse to percentiles ~~and~~ by supposing 100 class places to be occupied by 5 equally numerous classes, the 1st [grade] ~~from 0 to 20~~ ~~the graduation~~ standing between graduations 0° & 20°, the 2nd between 20° & 40° & so on. The middle position of these classes is that to which our ~~5 ran~~ places 1 to 5 severally belong that is to 10° 30° 50° 70° & 90°. To which points on these graduations do~~es~~ the [values of] above 18.5, which are now translated into 50°, severally belong.?

They are D at 9°; A at 23°; C at 39°; B at 46°; E at 70° as [judged by the] [illegible]

and not D at 10° A at 30° C at 50° B at 70° E at 90° as by [illegible]

H preferred to K $\overset{10}{x}$ times & K preferred to H $\overset{27}{37-x}$ times

their values are in that proportion $\frac{x}{37-x}$ & so for the rest

(work them all out) there are 10 equations $(4\times5)\times\frac{1}{2}$ & 5 unknown qual

(1) $\frac{e}{d} = \frac{2.5}{31.5}$

(2) $\frac{e}{c} = \frac{11.5}{25.5}$

(3) $\frac{e}{b} = \frac{12.5}{24.5}$

(4) $\frac{e}{a} = \frac{5}{32}$

(5) $\frac{d}{c} = \frac{34}{3}$

(6) $\frac{d}{b} = \frac{30}{7}$

(7) $\frac{d}{a} = \frac{24}{13}$

(8) $\frac{c}{b} = \frac{7}{30}$

(9) $\frac{c}{a} = \frac{6}{31}$

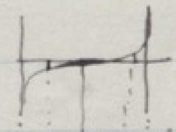
(10) $\frac{b}{a} = \frac{11.5}{25.5}$

From the first four of these if e be taken $= 1$ we have
$d = 12.6$; $c = 2.2$; $b = 2.0$; $a = 6.4$

f. 4r

Suppose ~~A~~ a rod ~~of~~ whose length ~~s~~ is $\underline{a}$, ~~A, BC~~ ~~saw~~ to be viewed by an observer, who afterwards guided ~~only~~ by his ~~then~~ ~~from~~ recollection ~~only~~, ~~marks off~~ registers what he judges to have been its length, in some suitable permanent manner. His recollection is fallible, so the marked length, which is measured and may be called α, usually differs more or less from $\underline{a}$. A second, third, and many other persons do the same, ~~and~~ producing ~~in this way~~ ~~an series of equal~~ ~~a large~~ the same number of independent estimates $\alpha_1, \alpha_2, \alpha_3$, &c of $\underline{a}$

(may fairly be assumed to be ~~are~~ subject to the normal law of frequency of error, ~~which~~ therefore) which, when arranged in order of magnitude at equal distances apart, will become ~~are~~ ordinates to the curve of Distribution, as ~~the~~ ~~form the ogival shape~~ shown in the figure.



The median ~~value of the~~ of the curve, or the ordinate at 50° will be ~~=~~ equal to $\underline{a}$, and ~~the~~ the ~~[illegible]~~ quartiles ~~=~~ deviations which, if the differences between ~~the~~ the median & the ordinates either at 25° or at 75°, may be called q, ~~If q~~ q which ~~and the median~~, be taken ~~equal to~~ as the unit of measurement, the length of ~~the~~ the deviations at ~~[illegible]~~ ~~5° or 15 20 25 30~~ every other grade is known in terms of that unit ~~the quartile~~. It will be convenient for the purposes of this paper to reprint here a brief abstract of ~~their values~~ these lengths taken from ~~my~~ Nat. Inh. p. .

~~The~~ actual ~~length of the quartile has~~

Let a second rod of length $\underline{b}$ be supposed to ~~be~~ treated in the same way, but we will further suppose that owing to difference of ~~precision~~ illumination, of in its terminals of the rod, ~~and~~ ~~of~~ ~~[illegible]~~ ~~of colour~~ or from other causes, the accuracy of the β estimates ~~β, of its length~~ is not necessarily ~~the~~ the same as ~~differs from~~ that of the α estimates. In other words the quartile of the β system is independent of that of the α system. Similarly as regards a third rod $\underline{c}$ and the quartile of the γ system ~~deduced from it~~.

~~So that the α system has an unknown quartile which shall be nevertheless~~ ~~to be taken as equal to and shall~~ serve

$$a = b + H \qquad 0 = b - c + H - K$$

$$a = c + K \qquad b = c + H - K$$

$$b = c + L \qquad \therefore H - K = L$$

F.5

as the unit of measurement for both the β & the γ systems the values of whose unknown quartiles shall then be taken as $\underline{u}$ & $\underline{v}$

Let the quartiles of the 3 systems be u, v, & w respectively. The values ~~have to be determined in the following indirect way~~ cannot be determined directly, in the problem we have really in view, but indirectly for the following ~~principle~~ data. What can really be obtained (I) 3 are systems of the respective forms $(\alpha_1-\beta_1), (\alpha_2-\beta_2)$, ... $(\alpha_1-\gamma_1), (\alpha_2-\gamma_2)$ &c, $(\beta_1-\gamma_1), (\beta_2-\gamma_2)$ &c., the quartiles to which are, by the well known theory, $\sqrt{u^2+v^2}$, $\sqrt{u^2+w^2}$, & $\sqrt{v^2+w^2}$.

II. the observed facts that in ~~so many~~ r per cent of the cases $\alpha-\beta$ is $+^{ve}$; in s per cent, $\alpha-\gamma$ is $+^{ve}$; and in t per cent $\beta-\gamma$ is $+^{ve}$.

~~Let us consider the significance of~~ Taking the first of these for an example the ~~whole~~ array of differences of the form $\alpha-\beta$ will produce ~~will form~~ an ogival curve of distribution as before whose quartile $= \sqrt{u^2+v^2}$. The median ~~difference~~ of value of the array ~~will be equal~~ will ~~be~~ equal ~~to~~ $\underline{a}-\underline{b}$ and supposing $\underline{a}$ greater than $\underline{b}$ (otherwise conversely) the $\alpha-\beta$ values will diminish as we proceed along the array to the left hand of the median & increase as we proceed to the right. There is one point in the array of difference going from left to right ~~at~~ which α ceases to be larger than β, thenceforward β is larger than α. The abscissa of this point is s per cent as given by observation. Let the value of the deviation at s per cent as ~~given~~ found by the Table above mentioned where the quartile $=1$ be called (Int. s)

then ~~we have the equation~~ the real deviation $= (\text{Int.}\ \gamma) \times \sqrt{u^2+v^2}$

in other words $a = b + (\text{Int.}\ \gamma) \times \sqrt{u^2+v^2}$

$$a = b + (\text{Int}-\underline{r}) \times \sqrt{u^2+v^2}$$
$$a = c + (\text{Int}-\underline{s}) \times \sqrt{u^2+w^2}$$
$$b = c + (\text{Int}-\underline{t}) \times \sqrt{v^2+w^2}$$

These are not independent equations as any two of them afford means for calculating the third, but they can be verified by independent observations.

The measure, was ~~has been~~ lost ~~of length~~ of a certain ~~distance~~ ~~interest~~ object which was as familiar to ~~a~~ the members of a large club ~~large number of persons~~ as is the distance between wickets to cricketers. Three ~~measures~~ ~~estimates~~ Estimates of the length of these objects which we will call A ~~(based on fallible recollection)~~ α, β, γ, ~~were offered~~ ~~will~~ are submitted to

~~The familiar object say the wickets of~~

~~An object that was familiar to all the members of a~~

~~It was~~ It is desired to reconstruct a yard measure from recollections only. Three ~~attempts~~ models were made, which will be called a, b, c, respectively. As they differed in length, ~~it was desirable to take~~ the independent opinions of many persons were sought ~~asking~~ as to their relative merits ~~as likenesses of the original~~. Each person was separately asked to range them in order of their merits beginning with the best. The first ranked them say as b, a, c; the second say as a, c, b; the third in some other arrangement, and so on. It was easy to see which was most frequently placed first

f.7

medians ~~which~~ whose values will be $(A-B)$, $(A-C)$, & $(B-C)$

In the $(a-b)$ system it ~~is~~ may be found that $a=b$, at that ~~point~~ centile in the curve of distribution of which the abscissa = . . Now Reference to the Integral Table of which a brief abstract is reprinted here from my Natural Inh. p. . shows that the value of the centile at that point $= \sqrt{r^2+s^2} \times$ . . Hence $a-b+(\sqrt{r^2+s^2}\times 1) = 0$

~~similarly $a-c+($ $)=0$~~ or $a - b = \sqrt{r^2+s^2} \times (+.74)$ +

similarly $a - c = \sqrt{r^2+t^2} \times (+1.47)$

a $b - c = \sqrt{s^2+t^2} \times (+1.30)$

Finding the supposition that $r = s = t = 1$, & $a = 1$

$a-b = \cancel{1}\times 0.74$ $\quad 0.26 = b$

$a-c = 1.47$ $\quad -0.47 = c$

$b-c = 1.30$ $\quad +0.73 = b-c$

a . b c

0.74 1.30

(2.04
1.44)

f. 8

~~Ed~~ (Three ~~artists painted a~~ portraits ~~of a public character~~ a, b & c, exist. How can the popular opinion of their relative merits ~~as likenesses, if true~~ be most fully appraised?

~~Ed~~ A ~~large~~ considerable number of persons ~~had been~~ were independently asked to rank the three portraits, of a ~~well known~~ person known to them, a b & c in what they each considered to be the order of their merits as likenesses.

It ~~resulted~~ proved that a was ranked first in .. % of the cases ~~second~~ in % & third in %

b was ~~ranked~~ ranked first, second & third in .., .., & .. % cases respectively

and c in .., .., .., & ... It was also ~~resulted~~ found that a was preferred to b in .. % of the cases, a was preferred to c in .. % and b to c in ... %

~~What are the relative merits of a b c, or~~ Required to find ~~the relation between~~ the estimated merits of a, b & c, relatively to one another.

It is reasonable to ~~make an~~ attempt to reply on the working hypothesis that likeness, like money value, may be treated as a linear variable, so that if the likeness between a & b be quantitatively appraised at 7, and that between a & c as 4, then the value of the likeness between b & c would be 7 − 4, or 3. It will be seen hereafter that the truth of the ~~hy~~ the results obtained through the hypothesis, admits of being tested. Let the true but unknown values of a, b & c, be A B & C

We can now fairly assume that the values of the estimated ~~the~~ merits of a, b & c, ~~respectively~~ are distributed according to the normal law of frequency of error about their several means with the probable errors of r, s, t respectively

then the values of the differences (a−b), (a−c), (b−c), will be distributed with the prob: errors of $\sqrt{r^2+s^2}$, $\sqrt{r^2+t^2}$ & $\sqrt{s^2+t^2}$ about their respective

ARITHMETICAL TABLES.

Numeration Table.

Units	
Tens	12
Hundreds	123
Thousands	1,234
Tens of Thousands	12,345
Hundreds of Thousands	123,456
Millions	1,234,567
Tens of Millions	12,345,678
Hundreds of Millions	123,456,789

Sterling Money Table.

4 Farthings	1 Penny	d.
12 Pence	1 Shilling	s.
2 Shillings	1 Florin	
2 Sh. & Sixpence	1 Half Crown	
5 Shillings	1 Crown	cr.
10 Shillings	1 Half Sov.	
20 Shillings, 1 Sov.	1 Pound	£
21 Shillings	1 Guinea	

Arithmetical Signs.

+ Plus; Sign of Addition
— Minus; Sign of Subtraction
× Sign of Multiplication
÷ Sign of Division
= Sign of Equality
: : : ; Sign of Proportion
√ Sign of the Square Root
∛ Sign of the Cube Root
° Degree, ′ Minute, ″ Second
∴ Therefore, ∵ Because

Troy Weight.

For Gold, Silver and Jewels.

24 Grains	1 Pennywgt.	dwt.
20 Pennywgts.	1 Ounce	oz.
12 Ounces	1 Pound	lb.

Apothecaries' Weight.

For Mixing Medicines.

20 Grains	1 Scruple	scr.
3 Scruples	1 Drachm	dr.
8 Drachms	1 Ounce	oz
12 Ounces	1 Pound	lb

Avoirdupois Weight.

For all Goods except Gold, Silver and Jewels.

16 Drachms	1 Ounce	oz.
16 Ounces	1 Pound	lb.
14 Pounds	1 Stone	st.
28 Pounds	1 Quarter	qr.
4 Quarters	1 Hundredwgt.	cwt.
20 Cwt.	1 Ton	tn.

Hay and Straw Weight.

36 lb. Straw	1 Truss
56 lb. Old Hay	1 Truss
60 lb. New Hay	1 Truss
36 Trusses	1 Load

Long or Lineal Measure.

12 Lines	1 Inch	in.
12 Inches	1 Foot	ft.
3 Feet	1 Yard	yd.
2 Yards	1 Fathom	f.
5½ Yards	1 Pole or Perch	
40 Poles	1 Furlong	fur.
8 Furlongs or 1760 Yds.	1 Mile	

Cloth Measure.

2¼ Inches	1 Nail
4 Nails	1 Quarter of a Yard
4 Quarters	1 Yard

Solid or Cubic Measure.

1728 Cubic Inches	1 Cubic Foot
27 Cubic Feet	1 Cubic Yard
24¾ Cubic Feet	1 Solid Perch mason's work
12 Cubic Feet	1 Solid Perch brickwork

Imperial Heaped Measure.

		Avoird. of Water lb.
3 Gallons	1 Bushel	80
3 Bushels	1 Sack	240
12 Sacks	1 Chaldron	2880

Imperial Dry Measure.

		Avoird. of Water. lb.	oz.
2 Glasses	1 Noggin	0	5
3 Noggins	1 Pint	1	4
2 Pints	1 Quart	2	8
4 Quarts	1 Gallon	10	0
2 Gallons	1 Peck	20	0
4 Pecks	1 Bushel	80	0
8 Bushels	1 Quarter	640	0

Square Measure.

144 Square Inches	1 Square Foot
9 Square Feet	1 Square Yard
30¼ Square Yards	1 Square Pole
40 Square Poles	1 Rood
4 Roods	1 Acre

Table of Motion.

60 Seconds (″)	1 Minute
60 Minutes (′)	1 Degree
30 Degrees (°)	1 Sign
12 Signs or 360°	the circle of the earth

Table of Time.

60 Seconds	1 Minute
60 Minutes	1 Hour
24 Hours	1 Day
7 Days	1 Week
4 Weeks	1 Month
365 Days	1 Year
366 Days	1 Leap Year
52 Weeks	1 Year
12 Calendar or 13 Lunar Months	1 Year

Days in the Month.

Thirty days hath September,
April, June, and November,
All the rest have thirty-one.
Excepting February alone, [clear,
Which has but twenty-eight days
And twenty-nine in each leap year.

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 ,, 4	2 ,, 6	2 ,, 8	2 ,, 10	2 ,, 12	2 ,, 14	2 ,, 16	2 ,, 18	2 ,, 20	2 ,, 22	2 ,, 24
3 ,, 6	3 ,, 9	3 ,, 12	3 ,, 15	3 ,, 18	3 ,, 21	3 ,, 24	3 ,, 27	3 ,, 30	3 ,, 33	3 ,, 36
4 ,, 8	4 ,, 12	4 ,, 16	4 ,, 20	4 ,, 24	4 ,, 28	4 ,, 32	4 ,, 36	4 ,, 40	4 ,, 44	4 ,, 48
5 ,, 10	5 ,, 15	5 ,, 20	5 ,, 25	5 ,, 30	5 ,, 35	5 ,, 40	5 ,, 45	5 ,, 50	5 ,, 55	5 ,, 60
6 ,, 12	6 ,, 18	6 ,, 24	6 ,, 30	6 ,, 36	6 ,, 42	6 ,, 48	6 ,, 54	6 ,, 60	6 ,, 66	6 ,, 72
7 ,, 14	7 ,, 21	7 ,, 28	7 ,, 35	7 ,, 42	7 ,, 49	7 ,, 56	7 ,, 63	7 ,, 70	7 ,, 77	7 ,, 84
8 ,, 16	8 ,, 24	8 ,, 32	8 ,, 40	8 ,, 48	8 ,, 56	8 ,, 64	8 ,, 72	8 ,, 80	8 ,, 88	8 ,, 96
9 ,, 18	9 ,, 27	9 ,, 36	9 ,, 45	9 ,, 54	9 ,, 63	9 ,, 72	9 ,, 81	9 ,, 90	9 ,, 99	9 ,, 108
10 ,, 20	10 ,, 30	10 ,, 40	10 ,, 50	10 ,, 60	10 ,, 70	10 ,, 80	10 ,, 90	10 ,, 100	10 ,, 110	10 ,, 120
11 ,, 22	11 ,, 33	11 ,, 44	11 ,, 55	11 ,, 66	11 ,, 77	11 ,, 88	11 ,, 99	11 ,, 110	11 ,, 121	11 ,, 132
12 ,, 24	12 ,, 36	12 ,, 48	12 ,, 60	12 ,, 72	12 ,, 84	12 ,, 96	12 ,, 108	12 ,, 120	12 ,, 132	12 ,, 144

frequency /
classification
Kall. Uhi Gry

p.10 xble of modulus
p.28 Experience at the laboratory
& 31 especially

p. 3, 11 + others now here.

f.1

On the probable frequency of mis-classifications of continuous variables by fallible examiners

(The worth of Classification when &c.

~~value~~
worth of ~~[illegible]~~

On the ~~reliance due to~~ trustworthiness of Classifications,
when the objects classified vary continuously
and the classifiers are fallible.

[Discussed with regard to the precision of measurement that is required for the efficient working of an Anthropometric system of Identification]

by

~~Francis Galton~~

On the worth of Classifications by Fallible Examiners

with especial reference to Anthropometric Discoveries

On the probable frequency of misclassification, as depending on the ratio between ~~when the measures of~~ the fallibility of the measurer ~~examiner~~ and of the variability of the objects measured, ~~is known~~

The particular class to which an object or an individual is assigned, may be a matter of serious importance. A candidate for the Army and certain other appointments ~~elsewhere~~ has to pass a physical examination by a fallible examiner, who determines whether he shall be accepted or not. Every degree of physical fitness exists among the body of the candidates, but the hard and fast classification of fit or unfit, has to be adhered to. So In respect to literary capacity, there is no natural frontier between first and second class ability (or acquirement), ~~and~~ while the examiners of literary work, ~~as~~ like all examiners, ~~elsewhere~~, are fallible. There ~~must~~ will always be candidates whose real rank lies so near the ideal frontier which separates ~~between~~ two classes, that such errors as examiners are apt to make, may suffice to misplace them. How, it may be asked, can we arrive at a strict numerical estimate of the worth of such classifications as these? What data do we require for the purpose; how can we get them, and, when they are obtained, how are we to utilise them?

The theory of natural selection contributes another problem of the same kind, in respect ~~as~~ to the degree in which it really preserves the fittest. A certain proportion of every species ~~of the yo those who are born~~, die in early life young and leave no issue. But they are not [illegible] ~~The~~ young ~~children~~ ~~in each generation~~ The children of each generation generally vary greatly in constitutional strength and in aptitudes for self-preservation but it is not necessarily the strongest who survive & leave issue, because

Another problem of the kind, but somewhat more complicated in its details, is raised by the requirements of the successful working of the anthropometric system of identification. What is the chance that the dimensions of the same criminal will be assigned to the same classes of short medium & long by different measurers?

but on the other hand ~~while~~, the risks of disease and danger to which children severally ~~they~~ are exposed, are ~~very~~ unequally distributed. Many youths who were gifted in body and mind above their fellows, have perished prematurely, owing to mishaps of exceptional severity. Many weakly children have lived ~~[illegible]~~ and left descendents, solely through their luck in ~~not~~ having been confronted by no ~~met with any~~ serious perils. Natural selection is a highly fallible examiner; what, we may ask, is the measure of its success in preserving those who are best fitted to propagate the race?

These few instances suffice to give a general some idea of the wide variety of problems f.5r 4

that fall within the scope of the title of this memoir. They include the verdicts, yes or no, of fallible juries, the graded sentences passed on convicts by fallible judges, the

It would not be ~~possible~~ feasible now to discuss ~~more than one~~ f4b

~~Many~~ problems of this kind ~~could not be adequately discussed in a single memoir~~ in an[illegible] manner so the plan will be adopted ~~is of~~ examining a single ~~problem~~ ~~case~~ case ~~of them~~ in detail, to serve as an example. The one ~~that was~~ selected for the purpose, is more complicated than the generality of ~~the~~ its class, thereby showing ~~not out how to discuss that~~ not only the point which is common to all of them, but also ~~how to~~ the method of dealing with ~~the~~ additional details of importance that occasionally present themselves ~~in some of them~~.

The selected problem (this at the bar

that fall within the scope of the title of this memoir. They include the verdicts, yes or no, of fallible juries, the graded sentences passed on convicts by fallible judges, the assortments of raw material into definite degrees of fineness for purposes of manufacture, and the retail charges in integral coins for numerous commodities that vary continously in their true values, ~~like oranges or eggs~~.

~~in full detail no more than~~

90 or p9 It is ~~the object of this memoir to discuss only one problem~~

~~A particular problem will here be~~

~~which includes all the side issues~~

~~discussed in detail, to serve as a~~

that are ~~likely to be needed~~

~~representative of the class.~~ It ~~is the one~~

~~that~~ lies at the base of the anthropometric system of identification invented by A. Bertillon, & It ~~and~~ has a double claim to present consideration;

f.5v

next page

* There is no need to ~~deb~~ describe the means by which the effects of correlation of dimensions are guarded which otherwise would cause the two frequency of all long or nearly all, or all short or nearly all to be much in excess of those in which long & short well intermixed

In the first place, the data required for its solution ~~are~~ can easily ~~to~~ be obtained; ~~In the second place,~~ the Bertillon system ~~has~~ is about to be ~~very recently been~~ introduced into the criminal administration of this country, and it is well to understand at its commencement, the degree of ~~with how much~~ precision with which the measurements ~~will have~~ ought to be made; in order that ~~the system~~ it may prove an eventual success. × in the second place

The principle of the anthropometric system consists in classifying each of 5 selected dimensions of the person who is measured, as short, medium, or long, the limits of the medium class being so laid down*, that the number of entries f.7 6

under each of these heads, in any large collection, shall always be approximately [illegible]* equal. The five selected measures are — (1) head-length; (2) head-breadth; (3) length of left middle finger; (4) of left cubit; & (5) of left foot. The set of 5 measures of each prisoner is written on a separate card, together with much other matter that does not concern us now, and it bears what may be called a ~~an Index-title~~ Title founded upon the 5 measures. The ~~index~~-title consists of 5 words or equivalent to those words symbols, their order (as — long, medium, medium, short, short), indicating the dimension to which they severally refer. The number of different titles is consequently 3 multiplied into itself 5 times

~~over~~ successively; that is, ~~to~~ 243. A corresponding number of compartments are provided and labelled, and the card of each prisoner is sorted into the appropriate compartment. The titles borne by these cards will be distinguished ~~by the name of the~~ as "Registered", or [R] titles. ~~Now~~, When it is desired to ascertain whether a person who has been apprehended for some crime, has ever been imprisoned before, he is measured, and his title, ~~as~~ determined ~~as derived from~~ by that measurement, ~~we~~ will be distinguished ~~this~~ as the Search, or [S] title. If the [S] always agreed with the [R] title, it would never be necessary to ransack more than one of the 243 compartments, ~~and~~ so the labour of search would be reduced 243 times. But, measurers being fallible and the dimensions of the same adult person differing somewhat from time to time, the (R) & (S) ~~two~~ titles do not always agree. In the first part ~~object~~ of this memoir the ~~is to~~ ~~determine~~ the chance of their agreement, ~~from the data~~ is shown to depend upon ~~when~~ two constants ~~are given~~, which may easily be determined from statistical observation.

In the sec part of the memoir ~~Its~~ second ~~object~~ is ~~to~~ determined, the average number of compartments that have to ~~must~~ be searched, before ~~we can be sure~~ the chance of failure ~~in~~ finding the (R) card, that corresponds to an (S) card which really exists in the Register ~~that exists really~~ ~~there~~ ~~contained in the collection~~, is reduced to some small specified value, say to 1 per thousand.

The applicability of the law of facility of error will be assumed throughout this discussion, such as is expressed by the familiar formula that the chance of an Error, regardless of its sign, lying between 0 and hx, will be

$$\frac{2}{\sqrt{\pi}} \int_0^{hx} e^{-h^2x^2} dx \quad \text{or, more briefly, } \frac{2}{\sqrt{\pi}} \operatorname{Erf} hx$$

Here $\underline{h}$ is the measure of precision, but the following discussion will be made in terms of its reciprocal, namely the modulus of fallibility or, more briefly, the Modulus. Consequently $\underline{h}$ will be replaced by values of the form $\frac{1}{a}$ and $\frac{1}{\cancel{x}\underline{c}}$, $\underline{a}$ and $\cancel{x}\,\underline{c}$ being moduli. The unit ~~measures~~ of fallibility that is ~~are~~ most conveniently ~~easily~~ obtained by statistical observation is ~~are~~ the probable error~~s~~ $\underline{r}$, which is ~~are~~ converted will be $\sqrt{(a^2+r^2)}$

Let $n =$ the ratio of a to r; ~~be called n~~ that is, let ~~or~~ $a = nr$.

The true head-length (or other dimension) is not strictly constant during adult life, but may varies slightly, ~~owing~~ chiefly on account of ~~to~~ changes in the thickness of ~~its covering~~ the softer tissues. ~~that cover it.~~ The dimension may therefore be said to have a normal value, about which the true lengths at different dates oscillate with a modulus $= \sigma$. Then the modulus of the difference between the true lengths at two different dates, taken at haphazard, would be $\sqrt{2} \times \sigma$, according to a well known rule. In other words, the fallibility of a true length at one time

of adult life in showing what the true length had been at another time, would have the modulus of $\sqrt{2} \times \sigma$

Now let the fallibility of the measurer [S], after being determined in the same way, as that of [R], be called s'. It follows that the (observed) measurement* of the length at the second date by [S] would determine its true length at the first date, with a fallibility whose modulus is $\sqrt{2\sigma^2 + s'^2} = b$ (i); ~~and again, that it~~ It also follows that the observed measure by [S] at the second date would determine its observed length as measured by {R} at the first date, with a fallibility whose modulus is ~~... called~~

$$\sqrt{(2\sigma^2 + s'^2 + r^2)}$$, ~~which we will~~ where it be called ~~c~~ c (this formula until we come to ~~in~~ the second part of this memoir ~~which~~ we shall have to use)

Let the ratio of a to s be called m; ~~or~~ that is let $a = m s$.

~~n and m are the two data that are needed for the solution of Problem I.~~

f. 13v

I propose to show ~~in this method~~ the way in which the case any one ~~special~~ case may be worked out, (consequently the way of making tables ~~that will~~ may be made to suit ~~each~~ any given ~~of~~ cases of the same kind. The particular kind of cases that will be dealt with is that ~~of a threefold division~~ where ~~of~~ all the cases have to be divided into three classes of approximately equal numbers, such as the short, medium & long, which are positively measures ~~employed~~ ~~classes of the~~ in the anthropometric system of identification.

Go back to fig 9

¶ It will be of course understood that n & m ~~are found~~ directly ~~independent of the unit by which variability or fallibility~~ actually ~~is measured~~, ~~if by the probable error~~ represent not only the ratio between ~~the~~ moduli but that between the prob errors, or that between their mean errors, ~~be whatever the unit may be by which the variability is~~ or between any other ~~multiples or~~ submultiples or multiples of the modulus]

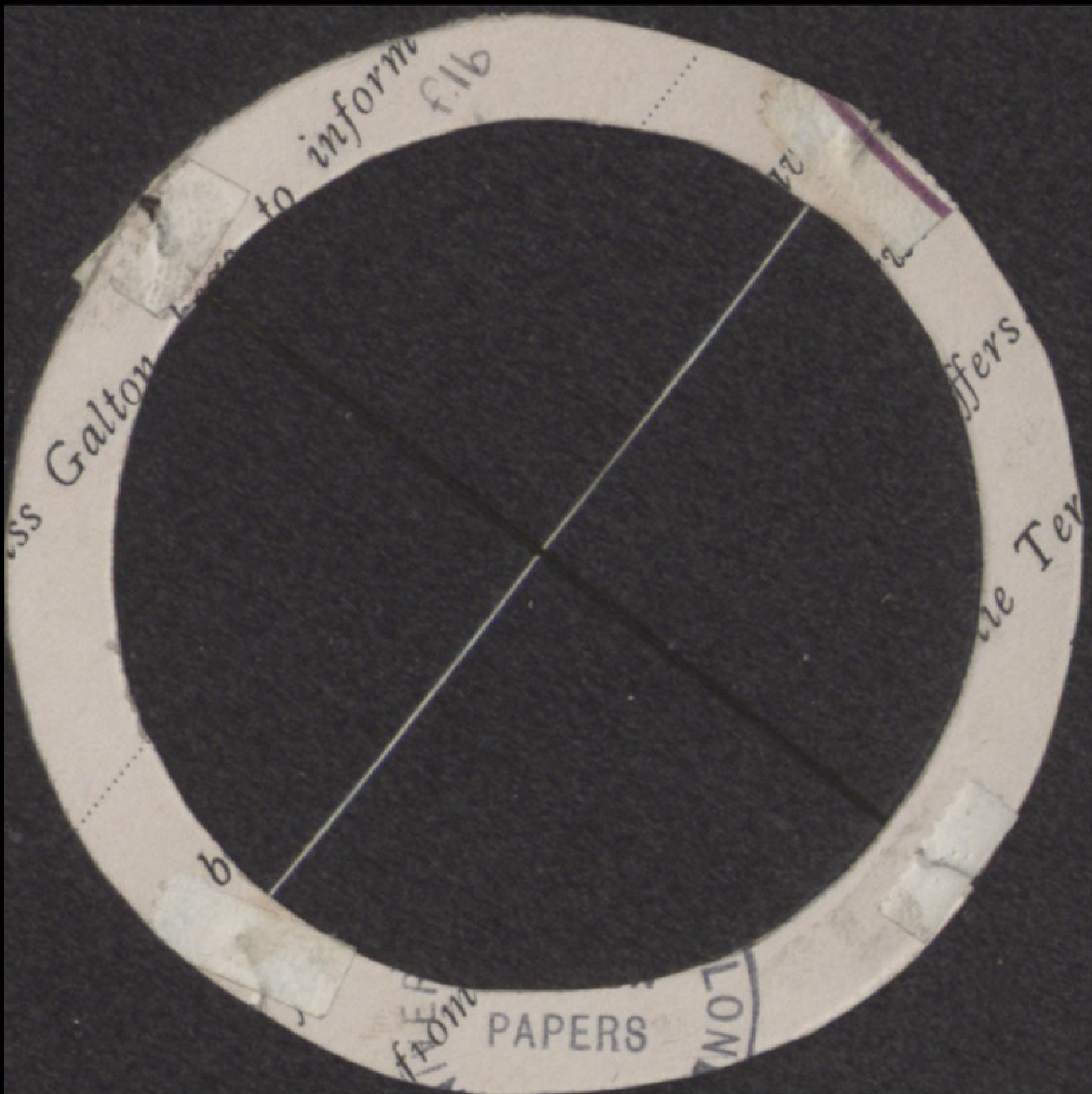
The only data that are needed for the solution of ~~the first~~ our principal problem[s] ~~are~~ is $\underline{n}$ and $\underline{m}$; the absolute value of $\underline{a}$ is of no importance whatever, that ~~as~~ will become apparent as we proceed, when the nature of the problem is more clearly grasped. So we will take th~~at~~e value for $\underline{a}$ that makes $h = 1$; in other words $\underline{a}$ will be adopted as the unit of all the measures employed in this discussion, whether they be those of the dimensions of the persons measured, or those of the fallibility of the measurers.

>

In assigning such limits to the medium class as shall fairly divide the <u>true</u> values of ~~each~~ any dimension into the three approximately equal divisions of short, medium & long, it is ~~sufficient~~ convenient to take round numbers that the~~se~~ ~~numbers~~ three divisions should be approximately equal.

measurements of horses ~~[illegible]~~


f.1a
SIT
ALTON
portun
to
er,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Ter
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PAPERS

which are more fully described in the Blue book ~~mentioned~~ mentioned above,

very probable from many considerations that the

photographic ~~is~~ are more to be depended on than ~~the~~

direct (measurements of horses ~~height~~). As regards the

measures of length there can be no doubt that

this is the case, it being a most simple and

straightforward process photographically but

almost impracticable on the living animal.

f.lcr

Before ~~In~~ concluding this report, also a brief notice

the valuable photographs of

~~only~~ can be made of the 35 triads (Subject, Sire and Dam)

~~and sire)~~ of purely bred shorthorn cattle. They

were taken by Mr. John Patten, Junior,

mostly from the herd at Alnwick under

which was all that was then asked for.

quasi standard conditions. They ~~have been were~~ photographs have been

received too recently ~~to have been~~ for adequate discussion

~~discussed as yet~~ at present.

1 between 0 and $\frac{1}{2} \times \frac{37}{5}$ or 3.7

2 3.7 and 3.7 + 7.4 = 11.1

3 + 7.4 18.5

4 + 7.4 25.9

5 7.4 33.3

3.7

37.0

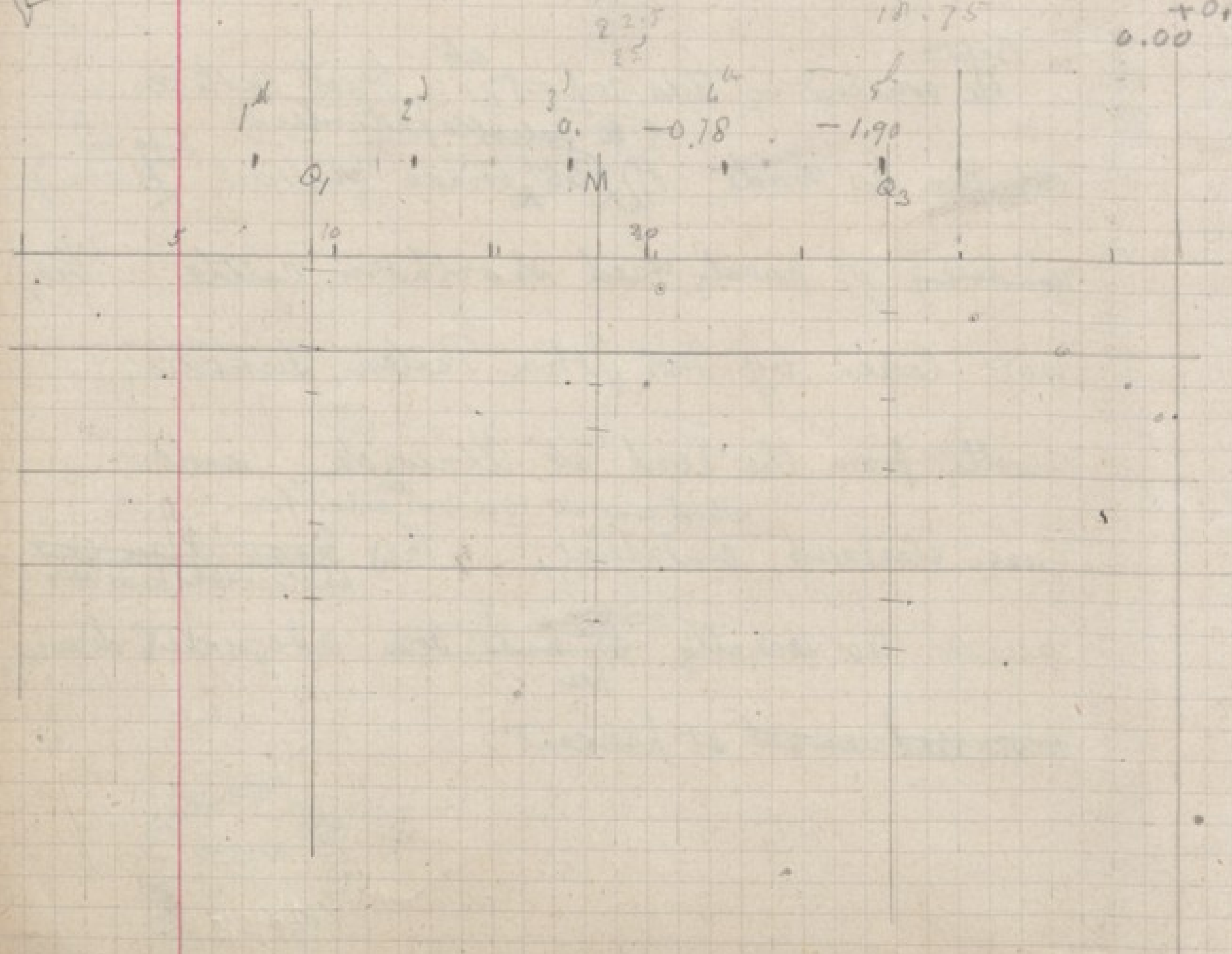
	f. lcv	per cent	devn
	1	10.0	1.90
	2	30.00	0.78
	3	50.00	0.00
	4	70.00	0.78
	5	90.00	1.90
		10	

		5 classes = PE in 10 classes		PE in 5 classes	M in 5 classes	in 10 classes
E	2 PE =	1.55	cl. 2.1	0.8	4.0	8.0
D	=	1.55	cl. 3.1	0.8	0.7	1.4
C		2.3	4.7	1.2	2.4	4.8
B		2.4	4.8	1.2	2.7	5.4
A		1.0	2.0	0.5	1.6	3.2

7.5
12.5
17.5
22.5
25

25
12.5
6.25
18.75

1.90
+0.12
1.78
+0.78
0.00

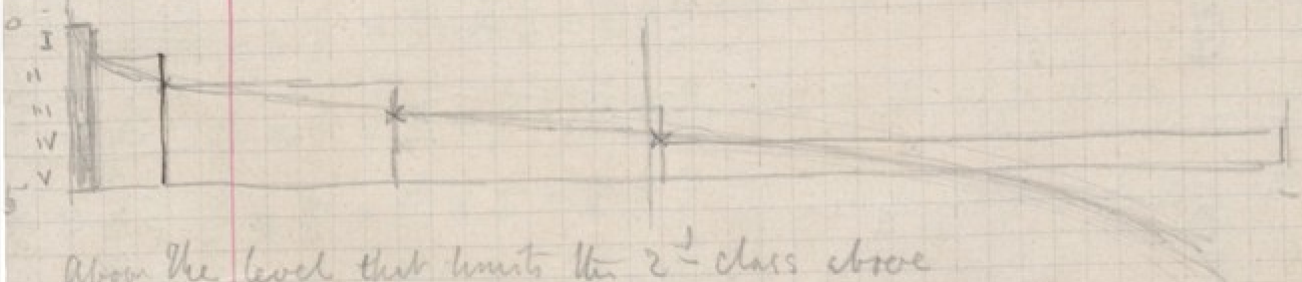


Estimated order of the 5 photos of 79 by Hollier 1899 (18)

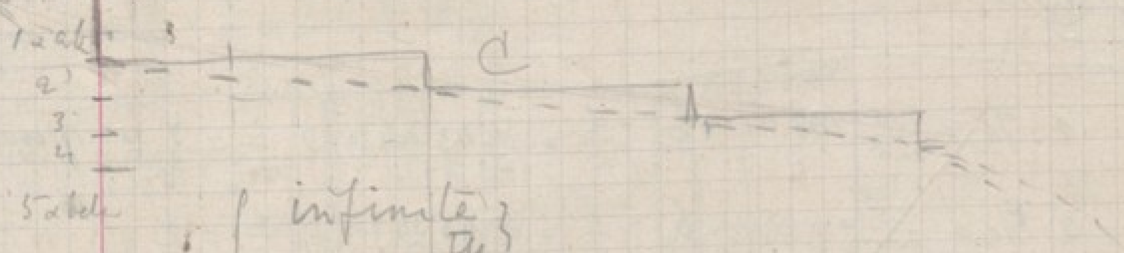
f.2r

		E	Sum	D	Sum	C	Sum	B	Sum	A	Sum
10	I	1	1	21	21	5	5	4	4	9	9
20	II	2	3	10	31	10	15	9	13	15	24
30	III	7	10	3	34	8	23	8	21	8	32
40	IV	8	18	2	36	7	30	8	29	3	35
50	V	19	37	1	37	7	37	8	37	2	37

	E	D	C	B	A	
Median class in 5 classes	IV + 0.2	0 + 0.75	II + 0.4	II + 0.7	I + 0.6	(right)
in 50 classes	48	7	24	27	16	
in 100 classes	80	14	48	54	32	
P.E. in unit distance between successive classes, 5 classes	0.8	0.8	1.2	1.2	0.5	
100 classes	15.5	15.5	23.0	24.0	10.0	



above the level that limits the 2nd class above
and below that which limits the 4th class below



(infinite ty }
(indefinite ty }

δ preferred to C | 5 | 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 &c — 34 d
C " to δ | 2, 22, 24, — 3 c

δ pref^d to B | 6 | — 30 d
B to δ | 3, 12, 14, 22, 24, 25, 37 — 7 b

C. pref to B | 8 | 2, 5, 16, 19, 22, 26, 31, 32, 35, (equal 6 7) — 7 c
B pref to C | — 30 b

E to A | 2, 8, 12, 18, 26 — 5 e
A to E | 4 | — 32 a

E to δ | 1 | 2 (one equality 22) 26 — 2½ e
δ to E | — d 34½

E to C | 2 | 4, 14, 18 (equality 19) 26, 27, 28, 29, 30, 31, 33 35 — 11½ e
C to E | — 25½ c

E to B | 3 | 1, 2 4, 5 (equality 6) 7, 18, 26, 27, 28, 29, 31, 35 — 12½ e
B to E | — 24½ b

δ to A | 7 | — 24 d
A to δ | 2, 3, 4, 13, 14, 16, 17, 22, 24, 31, 32, 33, 37 — 13 a

C to A | 9 | 2, 8, 10, 12, 22, 34 — 6 c
A to C | — 31 a

B to A | 10 | 3, 8, 9, 10, 11, 12 (equality 19), 21, 24, 25, 34, 36 — 11½ b
A to B | — 25½ a

$e = 1.0$
$d = 12.6$
$c = 2.2$
$b = 2.0$
$a = 6.4$
$5)\,24.2$
mean 4.8

			mean 3/9	3/14											
1	e/d =	$\frac{2.5}{31.5}$	0.08	0.1	0.1	5	$\frac{d}{c}$ =	$\frac{34}{3}$	11.3	6.4	7.4	8	$\frac{c}{b}$ =	$\frac{7}{30}$ $\frac{10}{7}$	17/100 (0.60
2	e/c =	$\frac{11.5}{25.5}$	0.45	0.37	0.39	6	$\frac{d}{b}$ =	$\frac{30}{7}$	4.3	34	3.6	9	$\frac{c}{a}$ =	$\frac{6}{31}$	
3	e/b	$\frac{12.5}{24.5}$	0.50	0.62	0.60	7	$\frac{d}{a}$ =	$\frac{24}{13}$				10	$\frac{b}{a}$ =	$\frac{11.5}{25.5}$	
4	e/a	$\frac{5}{32}$	0.16	0.15	0.15										

f3r 19

$\frac{c}{d} = \frac{3}{34}$, $\frac{b}{d} = \frac{7}{30}$, $\frac{c}{b} = \frac{7}{30} = \frac{1}{4}$

$\frac{c}{b} = \frac{3}{34} \times \frac{30}{7} = \frac{90}{238} = \frac{1}{2.4}$

(1) (5) $\frac{e}{d} \times \frac{d}{c} = \frac{e}{c} = \frac{2.5}{31.5} \times \frac{34}{3} = \frac{85}{94.5} = \frac{1}{1.1}$

(1) (6) $\frac{e}{d} \times \frac{d}{b} = \frac{2.5}{31.5} \times \frac{30}{7} = \frac{75}{220.5} = \frac{1}{3}$

(5) (8) $\frac{d}{c} \times \frac{c}{b} = \frac{34}{3} \times \frac{7}{30} = \frac{238}{90} = \frac{2.6}{1}$

(2) $\frac{e}{c} = \frac{11.5}{25.5} = \frac{1}{2.2}$

(3) $\frac{e}{b} = \frac{12.5}{24.5} = \frac{1}{2}$

(6) $\frac{d}{b} = \frac{30}{7} = \frac{4.3}{1}$

reciprocals 30 seconds ii 2nd case 3 C control 1

$\frac{e}{d} = \frac{e}{c} * \frac{c}{d} = \frac{e}{b} * \frac{b}{d} = \frac{e}{a} * \frac{a}{d}$ $= 0.07$

$\frac{e}{c} = \frac{e}{d} * \frac{d}{c} = \frac{e}{b} * \frac{b}{c} = \frac{e}{a} * \frac{a}{c}$ $= 0.45$

$\frac{e}{b} = \frac{e}{d} * \frac{d}{b} = \frac{e}{c} * \frac{c}{b} = \frac{e}{a} * \frac{a}{b}$ $= 0.51$

$\frac{e}{a} = \frac{e}{d} * \frac{d}{a} = \frac{e}{c} * \frac{c}{a} = \frac{e}{b} * \frac{b}{a}$ $= 0.16$

$\frac{d}{c} = \frac{d}{e} * \frac{e}{c} = \frac{d}{b} * \frac{b}{c} = \frac{d}{a} * \frac{a}{c}$

$\frac{d}{b} = \frac{d}{e} * \frac{e}{b} = \frac{d}{c} * \frac{c}{b} = \frac{d}{a} * \frac{a}{b}$

$\frac{d}{a} = \frac{d}{e} \div \frac{e}{a} = \frac{d}{c} \div \frac{c}{a} = \frac{d}{b} \div \frac{b}{a}$

$\frac{c}{b} = \frac{c}{e} * \frac{e}{b} = \frac{c}{d} * \frac{d}{b} = \frac{c}{a} * \frac{a}{b}$

$\frac{c}{a} = \frac{c}{e} * \frac{e}{a} = \frac{c}{d} * \frac{d}{a} = \frac{c}{b} * \frac{b}{a}$

$\frac{b}{a} = \frac{b}{e} * \frac{e}{a} = \frac{b}{d} * \frac{d}{a} = \frac{b}{c} * \frac{c}{a}$

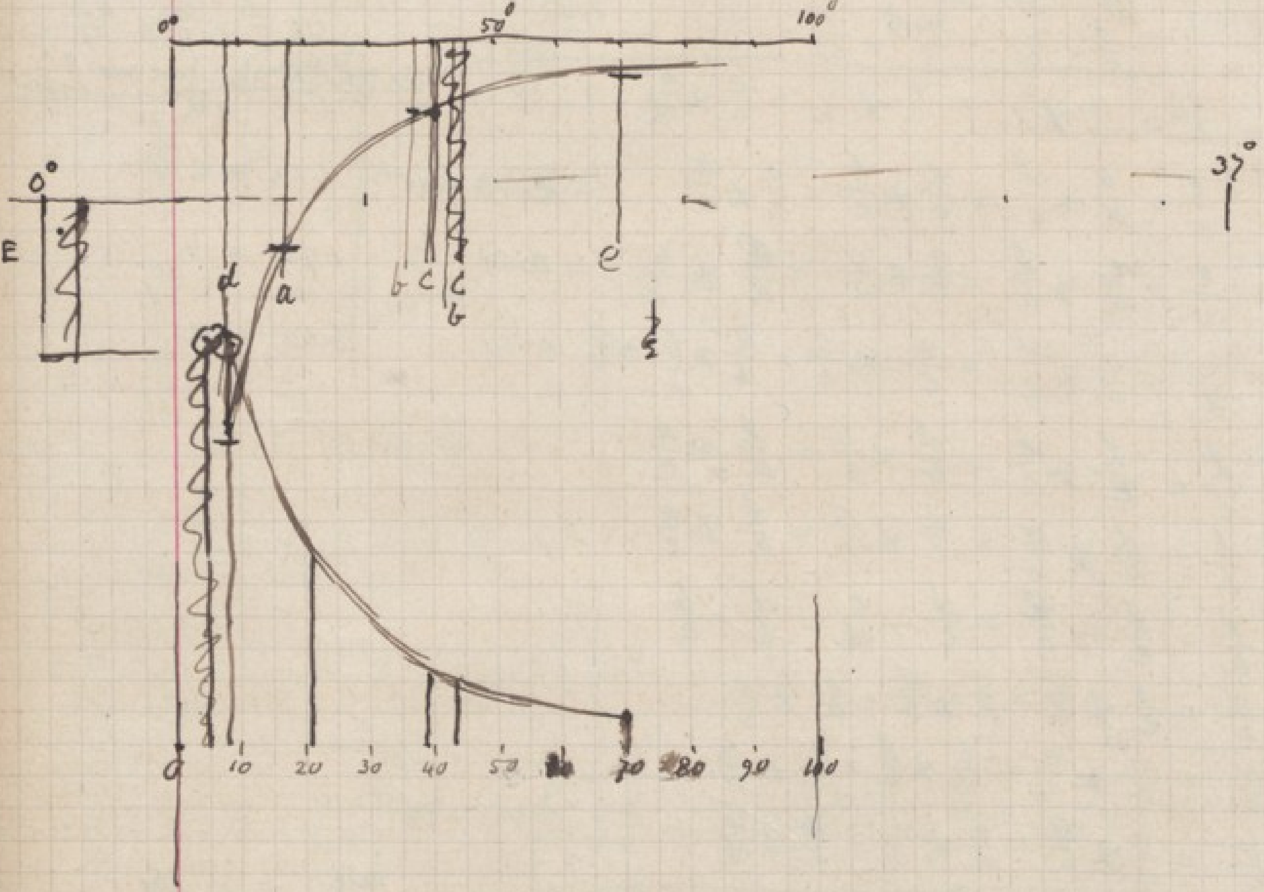
13.80	=	d
2.22	=	c
1.90	=	b
6.40	=	a

345/2500 (.08   255/1150 (0.45   reciprocal   245/1250 (0.51   115/255 (2.22   345/2400 (0.07   6.4/13.8 (2.1   222/1380 (6.2   190/138 (7   32/50   345/13.8   125/24   19/27.2

	calc	obs
$\frac{d}{c} = \frac{13.80}{2.22}$	= 6.2	11.3
$\frac{d}{b} = \frac{13.80}{1.90}$	= 7.0	4.3
$\frac{d}{a} = \frac{13.80}{6.40}$	= 2.1	1.9
$\frac{c}{b} \quad \frac{2.22}{1.90}$	1.2	0.6

f.3v

	relative magn?	medians at class place 0 to 100	class places	Sums 10° I	30° II	50° III	70° IV	90° V		class place 1 median	% left to right
e	= 1.0	~~90~~		1	3	10	18	37		70°	30
d	= 12.6	9°		21	31	34	36	37	30 +	9°	91
c	= 2.2	50°		5	15	23	30	37	20 × 3½/8	39°	51
b	= 2.0	70°		4	13	21	29	37		44°	56
a	= 6.4	23°		9	24	32	35	37		22°	78



$$70^\circ \quad E$$
$$9^\circ \quad D$$
$$30^\circ + \frac{3\frac{1}{2}}{8}\times 20 = 30 + \frac{70}{8} = 39^\circ \quad C$$
$$30^\circ + \frac{5\frac{1}{2}}{8}\times 20 = 30 + \frac{110}{8} = 30 + 14 = 44^\circ \quad B$$
$$10^\circ + \frac{9}{15}\times 20 = 10 + \frac{180}{15} = 10 + 12 = 22^\circ \quad A$$

~~Distribution~~ of Collective Judgement

A photographer sent (in the usual way) 5 portraits of myself for approval ~~that~~ ~~the one that sh^d be selected~~ ~~and~~ one to be chosen from which its negative to be preserved ~~& printed from~~ the others to be destroyed. ~~They all seemed~~ were all good ~~& I~~ I asked the opinions of friends ~~what~~ as to the order of merit in which they seemed to them to stand, & finding much difference of opinion began to note the results. The five photos making with a similar pattern shuffled were placed in any ~~the~~ friends ~~photos~~ hand asking him to arrange them in what appeared to him their order of merit the best uppermost. In this way I obtained 37 independent opinions ~~which to classification rubrication are often~~

The problem of the best way of analysing a collective judgement is of somewhat wide application but I do not know that it has hitherto been attempted. So I take this ~~simple~~ particular case as a subject to experiment on. to learn ~~to ascertain~~ much more than merely which photograph got the majority of suffrages. (1) ~~was~~ placed first by . .
(2) by . & so on. This answered the immediate question, but much more might be extracted bearing on the

Statistics by Intercomparison

Marks ~~[illegible]~~ [illegible] order of merit

~~[illegible]~~ as variously assigned by ~~several~~ many persons

In what cases do many different persons exercise independent judgement on the same series of articles or class of article?

Prices. ? out of a fixed income, too much to this or NO, one only wants a little Salt but must have that

? out of different classes of same article which have to be bought the price given for each sort? this has nothing to do with price of production it is the question of differential approval. — Not quite The purchasers should be persons of equal means People who are not wealthy would in no case be able to afford the costliest things.

? articles of a not costly kind are sold ~~as~~ as alternatives in shops for the same sort of customers — as

Free Choice among alternative pleasures — foods at a party walks, (colour of clothes?)

~~The~~ Collective Judgement, ~~as~~ Humanity personified (Hobbes Leviathan) is it not the Mean Man, but one who has the largeness of discourse = to that of all the individuals of his race & with tastes not mutually exclusive but proportionate to the sum of tastes in the Race at large. A racial representative, Not of the majority only. The idea of a Sovereign, or Father of his people — may of a Divinity.

Given the ~~[illegible]~~ of the individual people [illegible] to deduce [illegible] from the marks they would give [illegible]

It is one thing to say, I arrange these things in such and such an order of merit and another to arrange

Unconscious marks

exact relative merits of each of the five photographs If they had to be marked for their merits, how should they be separately marked.

A diagram in which the points on the abscissa represent the class places I to V and the ordinates ~~represent are proportional~~ at those places show to the nº of times in which each photo: was assigned to the place gives a ~~good~~ general idea of the peculiar opinions about each.

It also shows that the order I–V do not represent the way in which their merits are spaced out. This seems most satisfactorily indicated by the classplace 0°–100° ~~at~~ to which the median suffrage belongs. In the centenary Centile Scale the Class place I is at 10°, II at 30°, III at 50 IV at 70 & V at 90°, each in the middle of a space of 20°. Now, to take an instance, .. suffrages place .. in ~~order~~ II or lower .. suffrages place it in order III or lower, therefore the class place that corresponds to 18½ suffrage lies between II & III & is found by simple interpolating to corresponding with per-cent class place of . . Similarly with the rest ( ) at ( ) . . .

Next what is the relative merit attached by the collective judgement to the several photographs? Taken 2 & 2 together there are 10 different combinations. The records show the number of times that each member of the pair was preferred to the other. The ~~first~~ five ~~suffice~~ but the other five can be pressed into service to corroborate

f.5v

K L is less than p | if $r = \frac{t}{2}$, KL = i or less
if $r = \frac{t}{t}$, KL = p or more
if r equal a trifle less than $\frac{t}{t}$, KL = p

K|- - - p - - - - 14

p ranges between 1 per cent. a respectable approximation.

Imperfectly perceived difference = ~~δ~~ x
ranges between i ~~...~~ the largest size that is ~~strictly~~ ~~...~~ ~~perceptible~~ wholly imperceptible
and p ~~...~~ the smallest size ~~smallest~~ that is ~~clearly~~ unmistakeably perceptible

In dealing with the portraits in order of merit (as in order of price which depends upon many considerations)

let K be one of the portraits whose merit is k
and L be the next, whose merit = $k + x$

Required value of x when opinions are divided as to the relative merits of H and L

If the votes were quite almost unanimous ~~...~~ in favor of H having more merit than L then x would be greater than p
if almost unanimous (allows for ~~...~~ a few cantankerous persons who exaggerate trifles out of their due share) x might be taken as $= p$
If equally divided x .. .. $= \delta$.
the judgements being balanced on either side of a short line $= \delta$ at the utmost
Let t = total votes, r the number of the majority ~~in favor of H being greater~~

(A) then $x = \frac{r}{t-r}\, p$? as to a $\pm \delta$.

(1) Find $x_1\ x_2 - x_n$ by method of interpolated class places of votes corresponding to locus of median ~~...~~

(2) Find then ~~the~~ values of $p_1\ p_2$ — & equations (A).

contested opinions cover the range of difference of merit between i or say 0, and p
and a vote in favor of portrait K of $\frac{r}{t}$ means that
its locus is $K + \frac{r}{t}\, p$ (there being always a $\pm\delta$ uncertainty)
p is found by interpolated class place
& is checked by the other of the 10 equations that are available

f 6r 22

They are $\frac{e}{d}$ = ...

whence taking $e = 1$ we obtain the following values in order of merit according to the collective judgement

D - . . . .

The last point is to test the congruity of (1) & (2) by drawing a curve, which proves to be a flowing one, otherwise grave doubt would arise. Ordinates of the length determined by (2) are erected at the class places determined by (1) & ~~then~~ a line is drawn through them as in Fig: .

f.6v

	ED	E +	E −	EC	E +	E −	EB	E +	E −	EA	E +	E −	DC	D +	D −
1	4 1	3		4 3	1		4 ⊕		1	4 2	2		1 3		2
2	2 5		3	2 1	1		2 4		2	2 3		1	5 1	4	
3	5 3	2		5 4	1		5 1	4		5 2	3		3 4		1
4	3 2	1		3 4		1	3 5		2	3 1	2		2 4		2
5	4 1	3		4 3	1		4 ⊕		1	4 2	2		1 3		2
6	● 1	4		⊕ ⊕	½	½	⊗ ⊗	½	½	⊗ 2	3		1 ⊗		4
7	3 1		2	3 ⊗		2	3 ⊗		2	3 2	1		1 ⊕		4
8	4 1	3		4 3	1		4 2	2		4 5		1	1 3		2
9	5 1	4		5 4	1		5 2	3		5 3	2		1 4		3
10	5 1	4		5 3	2		5 2	3		5 4	1		1 3		2
11	5 1	4		5 4	1		5 2	3		5 3	2		1 4		3
12	4 2	2		4 3	1		4 1	3		4 5		1	2 3		1
13	5 2	3		5 3	2		5 4	1		5 1	4		2 3		1
14	4 3	1		4 5		1	4 2	2		4 1	3		3 5		2
15	5 1	4		5 4	1		5 3	2		5 2	3		1 4		3
16	5 2	3		5 3	2		5 4	1		5 1	4		2 3		1
17	5 2	3		5 4	1		5 3	2		5 1	4		2 4		2
18	2 1	1		2 5		3	2 3		1	2 4		2	1 5		4
19	5 1	4		5 3	2		5 4	1		5 2	3		1 3		2
20	⊗ 1	4		⊕ ⊗	½	½	⊗ 3	2		⊗ 2	3		1 ⊗		4
21	5 1	4		5 4	1		5 2	3		5 3	2		1 4		3
22	5 4	1		5 1	4		5 3	2		5 2	3		5 1	4	
23	5 1	4		5 4	1		5 3	2		5 2	3		1 4		3
24	5 4	1		5 3	2		5 1	4		5 2	3		4 3	1	
25	5 2	3		5 4	1		5 1	4		5 3	2		2 4		2
26	1 2		1	1 4		3	1 5		4	1 3		2	2 4		2
27	3 1	2		3 5		2	3 4		1	3 2	1		1 5		4
28	3 1	2		3 5		2	3 4		1	3 2	1		1 5		4
29	3 1	2		3 5		2	3 4		1	3 2	1		1 5		4
30	4 1	3		4 5		1	4 3	1		4 2	2		1 5		4
31	3 2	1		3 4		1	3 5		2	3 1	2		2 4		2
32	5 2	3		5 3	2		5 4	1		5 1	4		2 3		1
33	4 2	2		4 5		1	4 3	1		4 1	3		2 5		3
34	5 1	4		5 3	2		5 2	3		5 4	1		1 3		2
35	3 1	2		3 4		1	3 5		2	3 2	1		1 4		3
36	5 1	4		5 4	1		5 2	3		5 3	2		1 4		3
37	5 3	2		5 4	1		5 2	3		5 1	4		3 4		1
		35	2		24	13		24½	12½		32	5		3	34
		d	e		c	e		b	e		a	e		c	d

The larger the excess, the worse.

		D			D			C			C			B	
	DB	+	−	DA	+	−	CB	+	−	CA	+	−	BA	+	−
1	1 ⊗		4	1 2		1	3 ⊕		2	3 2	1		⊕ 2	3	
2	5 4	1		5 3	2		1 4		3	1 3		2	4 3	1	
3	3 1	2		3 2	1		4 1	3		4 2	2		1 2		1
4	2 5		3	2 1	1		4 5		1	4 1	3		5 1	4	
5	1 ⊗		4	1 2		1	3 ⊕		2	3 2	1		⊕ 2	3	
6	1 ⊗		4	1 2		1	⊕ ⊕	½	½	⊕ 2	3		⊕ 2	3	
7	1 ⊗		4	1 2		1	⊕ ⊕	½	½	⊕ 2	3		⊕ 2	3	
8	1 2		1	1 5		4	3 2	1		3 5		2	2 5		3
9	1 2		1	1 3		2	4 2	2		4 3	1		2 3		1
10	1 2		1	1 4		3	3 2	1		3 4		1	2 4		2
11	1 2		1	1 3		2	4 2	2		4 3	1		2 3		1
12	2 1	1		2 5		3	3 1	1		3 5		2	1 5		4
13	2 4		2	2 1	1		3 4		1	3 1	2		4 1	3	
14	3 2	1		3 1	2		5 2	3		5 1	4		2 1	1	
15	1 3		2	1 2		1	4 3	1		4 2	2		3 2	1	
16	2 4		2	2 1	1		3 4		1	3 1	2		4 1	3	
17	2 3		1	2 1	1		4 3	1		4 1	3		3 1	2	
18	1 3		2	1 4		3	5 3	2		5 4	1		3 4		1
19	1 4		3	1 2		1	3 4		1	3 2	1		4 2	2	
20	1 3		2	1 3		2	⊕ 3	2		⊕ 2	3		3 2	1	
21	1 2		1	1 3		2	4 2	2		4 3	2		2 3		1
22	4 3	1		4 2	2		1 3		2	1 2		1	3 2	1	
23	1 3		2	1 2		1	4 3	1		4 2	2		3 2	1	
24	4 1	3		4 2	2		3 1	2		3 2	1		1 2		1
25	2 1	1		2 3		1	4 1	3		4 3	1		1 3		2
26	2 5		3	2 3		1	4 5		1	4 3	1		5 3	2	
27	1 4		3	1 2		1	5 4	1		5 2	3		4 2	2	
28	1 4		3	1 2		1	5 4	1		5 2	3		4 2	2	
29	1 4		3	1 2		1	5 4	1		5 2	3		4 2	2	
30	1 3		2	1 2		1	5 3	2		5 2	3		3 2	1	
31	2 5		3	2 1	1		4 5		1	4 1	3		5 1	4	
32	2 4		2	2 1	1		3 4		1	3 1	2		4 1	3	
33	2 3		1	2 1	1		5 3	2		5 1	4		3 1	2	
34	1 2		1	1 4		3	3 2	1		3 4		1	2 4		2
35	1 5		4	1 2		1	4 5		1	4 2	2		5 2	3	
36	1 2		1	1 3		2	4 2	2		4 3	1		2 3		1
37	3 2	1		3 1	2		4 2	2		4 1	3		2 1	1	
		8	29		13	24		24	13		31	6		25	12

From page 19a

		Percents
A =	25.5	69
B =	11.5	31
A =	31	84
C =	6	16
B	30	81
C	7	19

TABLE DE MULTIPLICATION

2 fois 1 font 2	5 fois 1 font 5	8 fois 1 font 8
2 fois 2 » 4	5 fois 2 » 10	8 fois 2 » 16
2 fois 3 » 6	5 fois 3 » 15	8 fois 3 » 24
2 fois 4 » 8	5 fois 4 » 20	8 fois 4 » 32
2 fois 5 » 10	5 fois 5 » 25	8 fois 5 » 40
2 fois 6 » 12	5 fois 6 » 30	8 fois 6 » 48
2 fois 7 » 14	5 fois 7 » 35	8 fois 7 » 56
2 fois 8 » 16	5 fois 8 » 40	8 fois 8 » 64
2 fois 9 » 18	5 fois 9 » 45	8 fois 9 » 72
2 fois 10 » 20	5 fois 10 » 50	8 fois 10 » 80
2 fois 11 » 22	5 fois 11 » 55	8 fois 11 » 88
2 fois 12 » 24	5 fois 12 » 60	8 fois 12 » 96

3 fois 1 font 3	6 fois 1 font 6	9 fois 1 font 9
3 fois 2 » 6	6 fois 2 » 12	9 fois 2 » 18
3 fois 3 » 9	6 fois 3 » 18	9 fois 3 » 27
3 fois 4 » 12	6 fois 4 » 24	9 fois 4 » 36
3 fois 5 » 15	6 fois 5 » 30	9 fois 5 » 45
3 fois 6 » 18	6 fois 6 » 36	9 fois 6 » 54
3 fois 7 » 21	6 fois 7 » 42	9 fois 7 » 63
3 fois 8 » 24	6 fois 8 » 48	9 fois 8 » 72
3 fois 9 » 27	6 fois 9 » 54	9 fois 9 » 81
3 fois 10 » 30	6 fois 10 » 60	9 fois 10 » 90
3 fois 11 » 33	6 fois 11 » 66	9 fois 11 » 99
3 fois 12 » 36	6 fois 12 » 72	9 fois 12 » 108

4 fois 1 font 4	7 fois 1 font 7	10 fois 1 font 10
4 fois 2 » 8	7 fois 2 » 14	10 fois 2 » 20
4 fois 3 » 12	7 fois 3 » 21	10 fois 3 » 30
4 fois 4 » 16	7 fois 4 » 28	10 fois 4 » 40
4 fois 5 » 20	7 fois 5 » 35	10 fois 5 » 50
4 fois 6 » 24	7 fois 6 » 42	10 fois 6 » 60
4 fois 7 » 28	7 fois 7 » 49	10 fois 7 » 70
4 fois 8 » 32	7 fois 8 » 56	10 fois 8 » 80
4 fois 9 » 36	7 fois 9 » 63	10 fois 9 » 90
4 fois 10 » 40	7 fois 10 » 70	10 fois 10 » 100
4 fois 11 » 44	7 fois 11 » 77	10 fois 11 » 110
4 fois 12 » 48	7 fois 12 » 84	10 fois 12 » 120

The worth (59)
Classification

59.

Amended order

General scope of the problem

divides into two classes (1) ~~the one~~ simple the other includes (1) & goes further

Special case to illustrate both ~~Bertil.~~ directory

(1)
(2) How to obtain $\frac{a}{b}$ in this case & in others.
(at my laboratory &c, other values

Points omitted
Practical results

General ~~Statement~~ of scope

Divide into 2 classes of problems (1) simple (2) complex

(2) wh. includes I

Special case ~~left~~ directory

(1)
(2) How to obtain a & b for special cases & others.

obt. values of $\frac{a}{b}$ (at my laboratory
Bertillon

(2)

(Practical results of the working.

(points omitted in the theory.

~~The worth of Classification~~ (59) (The Risk of Misclassification) f.1 1

when the objects classified vary continuously and the classifiers are fallible

by ~~Francis~~ Galton

The particular class to which an object [2] or individual [1] is assigned, may be a matter of serious importance. A candidate for the Army, and certain other appointments, is required to pass a physical examination by a fallible examiner, who ~~determines~~ decides whether he is fit to be accepted or not.

and in many cases must decide erroneously. A hard & fast line idea of ~~ideal~~ fitness or unfitness is kept in view, but a trifling ~~difference~~ error in estimate [illegible] or candidate whose real place is near ~~the~~ line between fit & unfit is near that line, to the wrong side of it

~~Every degree of physical fitness exists among the body of the candidates, but the~~ hard and fast classification of either fit ~~or~~ unfit, has to be adhered to.

Similarly ~~So~~ in respect to literary examinations, ~~there is no~~ But their decisions however conscientiously made, cannot be wholly trustworthy, ~~natural frontier between first and second-class ability~~ for the true places of many candidates

~~Every degree of physical fitness exists among the candidates~~ must lie so near the ~~same~~ line that separates the fit from the unfit that such errors as examiners are ~~liable~~ apt to make will suffice to misclass them

~~or~~ acquirement artistic, ~~while the examiners of literary work are fallible, like all examiners~~. In any large number of candidates some ~~There~~ will always be found ~~some candidates~~ some whose true ~~real~~ rank lies so near the ideal frontier between a first & second ~~which separates two~~ classes, that such errors as examiners are apt to make, may misplace them. How, it may be asked, can we arrive at a strict numerical estimate of the risk ~~worth~~ of these mis- ~~such~~ classifications, ~~as these~~? What data do we require for the purpose; how can we hope to get them, and when they are obtained, how are we to utilise them?

The theory of Natural Selection contributes a very important problem of the same ~~this~~ kind, in respect to the degree in which it succeeds in ~~really~~ preserving the fittest. The children of each generation vary greatly in constitutional strength and in their

Other examples
juries
sentences
apportionment of [illegible]

These are the simple form of problems so far as theory is concerned

Another class of problems is compound, it includes the former as ex., & more complete so far as theory goes – it concerns

A consideration of the methods of dealing with
~~those calculations~~ problems of this kind
forms the first part of this memoir
The second part deals with the problems of the same general nature but of a somewhat

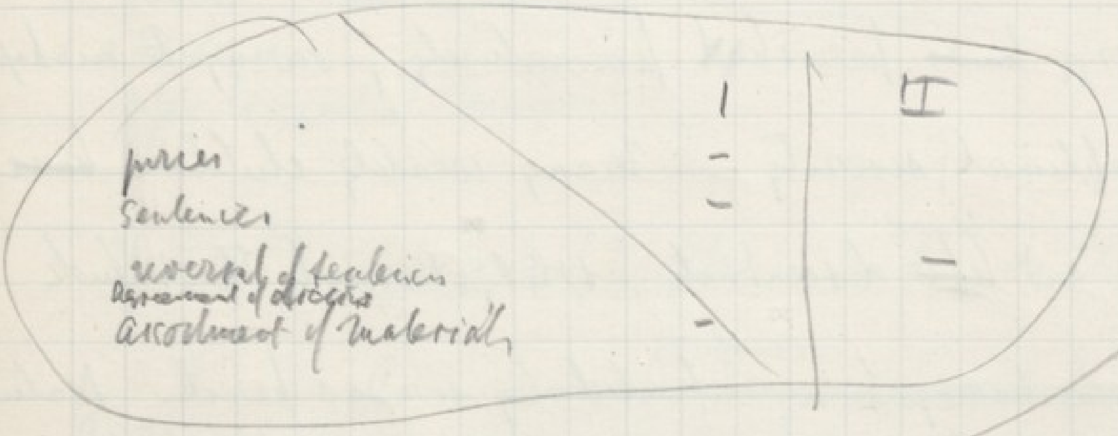
aptitude for self-preservation, but it is not necessarily the strongest that those who survive and leave issue, are ~~because~~ the trials of ~~disease~~ infection and cold and hunger to which they are severally exposed, are so unequally distributed, ~~that~~ Many youths who were gifted in body & mind above their fellows, ~~have~~ perish~~ed~~ prematurely, owing to mishaps of exceptional severity. Many weakly children ~~have~~ live~~d~~ and ~~left~~ leave descendants, solely through their luck in ~~never having been confronted by~~ escaping serious peril. Natural Selection is a highly fallible examiner; what, then, we may ask, is the measure of its success in preserving those who are best fitted to propagate the race?

[illegible]

Another problem ~~which is however~~ of a somewhat more complicated kind, ~~in its application~~, They concerns the ~~successful working~~ ~~probability of the~~ chance the chance that independent examiners ~~be~~ will ~~both successful~~ in classifying the same subjects correctly. Ellen(?) in respect to [illegible]

[illegible]

reversal of sentences



Each of these forms a separate study, to be approached in an III appropriate ~~special~~ way, ~~and there will be~~ so no attempt will be made ~~here~~ to deal fully with more than one class of them in this memoir, namely such as are similar in principle to that ~~in which~~ 1 the author: by it - of that where of the anthropometric system of identification, of identically ~~What~~

to know is the chance that ~~any~~ two same dimension of the same criminal will ~~be~~ class ~~assigned~~, by two different measurers, ~~for the purpose~~ alike, ~~as~~ ~~to the same~~ ~~class, short, medium or long, as the case may be?~~

These ~~few~~ examples of instances give some idea of the wide variety of problems that fall within the scope of the title to this memoir. examples might be taken from the Others refer to the verdicts, yes or no, of fallible juries, from the graded sentences passed on convicts by fallible judges, (and from the confirmation or reversal of a judgement in one court by a second tribunal. the assortment of raw materials into graded degrees of fineness, for purposes of manufacture, from

I propose in this memoir to show how any special case may be worked out, and to give ~~&~~ tables of values ~~formed~~, that are will be applicable a particular group; namely when ... classification is

to any ~~special cases of a specified order.~~ The

example ~~taken will be that~~ ~~in which~~ form of classification all ~~the~~ the

cases ~~have to be~~ are divided (1) into three classes, that

are (2) approximately equal in number., such as the

short, medium, and long classes of measures used in the

anthropometric system of identification

The applicability of the law of facility of error will be assumed throughout this discussion, such as is expressed by the familiar formula that the chance of an error regardless of its sign, lying between 0 and hx, will be

$$\frac{2}{\sqrt{\pi}}\int_0^{hx} e^{-h^2x^2}\,dx \qquad \text{or, more briefly, } \frac{2}{\sqrt{\pi}}\,\mathrm{Erf}\,hx$$

Here h is the measure of precision, ~~but~~ the following discussion will however be chiefly carried on in terms

Percentage of Accords

Register Series	Search Series								
Values of n	Values of m								
	2	3	4	5	6	7	8	9	10
2	59	63	66	67	69	70	70	70	70
3	63	73	74	74	79	81	81	82	82
4	66	74	79	82	83	83	84	84	84
5	67	79	82	82	83	84	84	84	84
6	69	79	83	84	84	85	85	85	85
7	69	81	83	84	84	84	85	85	86
8	69	81	84	84	85	85	86	87	88
9	69	82	84	84	85	85	86	89	90
10	69	82	84	84	85	86	88	90	92

These values are based on 16 calculated cases of which one was clearly erroneous & disregarded The remaining are interpolations f5c

$\frac{a}{\sigma} = 5$

risk to be neglected 1 in 10 $\frac{1}{k}x = \frac{a}{\sigma}\frac{1}{h}x$ $h = \frac{1}{5}k = 1$ $OI = OT = 0.300$

$kx = 0.370$ $hx = 0.074 = x$ as $h = 1$

$TJ = 0.074$ ~~.0908 .25 = .0834~~

$OI = .226$ $.2443 + .63 = .2506$

$OJ = .374$ $.3992 + .40 = .3032$

$.0526$

---

negligeable risk 1 in 100 $\frac{1}{k}x = \frac{a}{\sigma}\times\frac{1}{h}x$ $k = 0.01 \times h'$

$kx = 0.467$ $hx = 0.007 = x$

$OI = .293$ $.3183 + 31 = .3214$

$OJ = .307$ $.3286 + 70 = .3356$

$.0142$

---

negligeable risk 1 in 1000 $k = 0.001 \times h$

$kx = 0.476$ $hx = 0.005 = x$

$OI = .295$ $.3183 + 53 = .3236$

$OJ = .305$ $.3286 + 50 = .3336$

$.0100$

(variability, &c,)

of its reciprocal $\frac{1}{h}$, which is the modulus of fallibility, or more briefly, the "Modulus." [The unit of variability, &c, fallibility, that is most conveniently obtainable ~~through~~ from statistical observation, is the probable error $\underline{r}$, which is converted into the modulus $\frac{1}{h}$ by the well known equation of $rh = 0.4769$.]

Of course the ratio between the 2 moduli = that between the 2 prob. errors in the same

~~two data that are~~ ~~the~~ ratio between 1/ only datum required in the

The modulus of variability of the objects that to solve this or any other of the many problems I have in view are

~~[illegible]~~ to be classified (which will be called $\underline{a}$), and (2) the

of ~~course~~ ~~is supposed to be known~~, ~~and~~ ~~a~~ ~~$\underline{a}$~~ (~~to be equal to~~); ~~that of~~ the modulus of the fallibility of the examiner) ~~is also supposed to~~ (which will be called)

be ~~known~~, ~~and to be equal to~~ $\underline{b}$

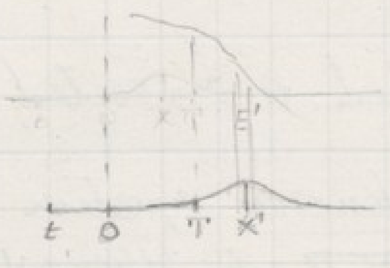
In ~~dividing a~~ selecting round numbers that shall divide a normal series into 3 ~~approximately~~ equal classes e.g. short-medium-long

~~It is convenient to take round numbers in~~ for

~~assigning such~~ will be taken as (limits to the medium division), ~~as~~

$$\begin{array}{r} 3333 \\ 3286 \\ \hline 47 \end{array} \qquad \begin{array}{r} 3333 \\ 3389 \\ \hline 56 \end{array}$$

Fig 1



(x)

Σ(x)

500 − ½ 9f. ½ xT

xT = (x − 0.30)

~~shall divide the whole number of cases into three equally numerous classes. As it appears from~~ we must refer to the Probability Integral table. It there appears that 3286 out of 10,000 cases, which is 47 too few, fall within the range of $hx = \pm 0.30$, and 3389 within that of $hx = \pm 0.31$. In the convenience of using round numbers, or 56 too many, the former of these two values which is both the lower and the plainer of the two, will be accepted as the limits of the medium class.

Fig 1 is an ordinary "Scheme", or figure bounded above by a curve of probability, and below by its base. ~~The curve~~ has a modulus (which OQ = $\underline{a}$), ~~and~~ it is supposed to refer to 10,000 true measures, or values ~~which are supposed to have been laid down~~ These are measured from the starting point B situated to the left of the curve, ~~and measured~~ horizontally, ~~from~~ along the base. BO ~~being~~ is the average length of all of these measures or values; and the points $\underline{t}$ and $\underline{T}$ are respectively situated at the distances, from O, of $\underline{a} \times (-0.30)$ and $\underline{a} \times (+0.30)$, consequently the three

we will now
~~Now let us~~ consider the ~~first of the two classes of~~ instances,
where fallibility is measured by a
viz ~~that of the~~ chance that ~~of~~ an ~~fallible~~ examiner
will misclassifying one of a series of variable
objects whose variability is measured by a

viz: (1) that

~~in~~ the Scheme lying the ordinate at

areas to the left of t, (2) that contained between the ordinates at t and T respectively, and (3) that lying to the right of the ordinate at T, are ~~each~~ approximately equal, ~~to one another~~, and severally refer to (about) 3333 short, medium, and long, true values ~~measures~~,

Let X fig I be the centre of the base of narrow a column, having ~~the finite~~ breadth = Δ, ~~which is~~ so narrow that the measures which it contains are practically identical, ~~is supposed to contain~~ & let E be the number of true measures, that it contains. ~~out of the 10,000. The centre of the base of the column being at X~~; let OX = x, and ~~we~~ we

~~will~~ First take the chance of misclassification ~~case~~ when OX is less than OT.

// Whenever each of the true values in the column E, which are all practically equal to BX, are measured by a fallible operator, he will under-foot note

Table

$\frac{a}{b}$	values of $(a^2+b^2)^{\frac{1}{2}}$ when ($a=1$)
2	2.24
4	4.12
6	6.08
8	8.06
10	10.05

* This differs little from a when $\frac{a}{b}$ is large

estimate the length of some and overestimate that of others in accordance with the law of facility, the ~~The modulus of his fallibility~~ being ~~is supposed to be known, and called b~~. Then consequently his erroneous ~~observed values~~ measures of the identical true values contained in the column, will form a little Scheme or heap of their own, as shown in Fig 1; its area will = E, and its modulus will = b. It will be understood If the whole of the original scheme be divided into narrow columns, whose contents are similarly distributed into heaps each with a modulus = b about the centres of their respective bases, the superposition of all these heaps will form a new Scheme, having ~~the~~ a modulus = $\sqrt{(a^2+b^2)}$, & which will refer to 10,000 more or less erroneous (or estimates, measures), not as the original scheme having the modulus a referred to the same number of true values

Reverting to the particular ~~column~~ heap that we were ~~considering~~ in fig 1, the true E values that it contains are necessarily ~~[illegible]~~ medium, because where OX is less than OT, but some ~~those~~ of their erroneous measures ~~which exceed~~ will extend to the right beyond BT, & will be falsely classed as long, while ~~and those~~ a very few ~~which~~ will extend to the left beyond ~~fall short of~~ Bt & will be falsely classed as ~~short~~

It is easy to find from the ~~Pro~~ Probability Integral table the number of these trangressors. They are respectively $E \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf}(OT - OX) \right\}$ and $E \times \left\{ 0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf}(OX + Ot) \right\}$

If x be greater than OT, as ~~[illegible]~~ in fig (1 a) ~~where X' is written in the place of X and the number of values in the column is E', then those~~ ~~then~~ the ~~[illegible]~~ true values contained in the column E,

~~and that~~ Of these, the number ~~those~~ that falls to the left of t are connected short

are $E \times \left(0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \theta t\right)$

$= E \times \left(0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \theta T\right)$

~~values~~ will be ~~are~~ long, but those erroneous measures of them that ~~transgr~~ transgress to the left beyond ~~fall short~~ of T, are classed otherwise; such of them as ~~if they~~ fall between T and t ~~they~~ being ~~are~~ falsely reckoned as medium, while those that ~~if they~~ fall to the left ~~short~~ of t being ~~they are~~ still more erroneously reckoned as short.

The total number ~~medium~~ of these fallacious measures (, the mediums and the shorts together, or all those that fall to the left of T being contained) is

$$E \times \left\{ \cdot 5 - \frac{1}{\sqrt{\pi}}\,\mathrm{Erf}(OX - OT) \right\}$$

It is unnecessary to consider the negative half of the scheme in the same detailed way, because it is ~~symmetrically~~ equal and opposite to the positive half. Whatever results are obtained from the positive half ~~latter~~ will, after interchanging the words long and short, hold true of the ~~former~~ negative half f.11v.

that if the whole of it [illegible] & all be simultaneously stretched or shortened as if it were printed on an elastic sheet of rubber, or if it be similarly heightened or depressed, the proportion of the transgressors will be unaltered. The ~~only constant needed is~~ being the ratio of a & b remaining the same, their absolute values are unimportant; so ~~Consequently~~ the units in which they are both alike measured, may ~~be~~ have any ~~the~~ value ~~that~~ we please to ~~assign~~ select and the most convenient by far in the following [illegible] ~~is that of~~ is to take a = 1 then the values of

~~$a = \frac{OX'}{a}$~~

the mod: of variability of the objects $= a = 1$

that of the fallibility of the exam[illegible] in a units $= \frac{b}{a}$

the length of OX ~~expressed in the a units~~ " $= \frac{OX}{a} = z$

OT and of Ot " " $= \frac{OT}{a}, \frac{Ot}{a} = 0.30$

~~the area of the~~

(amplify)

After a very little consideration of Fig 1, it becomes obvious that the unit in which the horizontal distances ~~in it~~ are measured throughout, can have no influence on the above results. It is well then, by taking a for the unit, ~~in~~ order to simplify the formulae, then ~~it a will~~

In other words h = the variability of the objects measured, divided by the fallibility of the examiner. It does not in the least matter in what unit ~~how~~ the variability and the fallibility are appraised so long as both are appraised by the same unit; it may be by their probable errors, ~~or else~~ by their mean errors, or otherwise.

~~a will~~ $\underline{a}$ henceforth be replaced by 1, and $\underline{b}$ by $\frac{b}{a}$. x will ~~express~~ be used to express $\frac{OX}{a}$ which is the number of the new units contained in OX ; $\underline{a} \times OT$ will become 0.30, instead of $\underline{a} \times 0.30$; and ~~when applying the general formula to a~~ heap, $\underline{h}$ becomes $\frac{a}{b}$.

The foregoing formulae thereby become simplified thus:-

For the Positive half of the scheme of true values

The true values in any column (E in number) being as below	The number of them that will be misclassed
Medium	as Long $= E\left\{0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \frac{a}{b}(0.3 - x)\right\}$
Medium	as Short $= E\left\{0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \frac{a}{b}(0.3 + x)\right\}$
Long	as Short $= E\left\{0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \frac{a}{b}(0.3 + x)\right\} \cancel{= V}$ $= S$, say
Long	as Medium $= E\left\{0.5 - \frac{1}{\sqrt{\pi}} \operatorname{Erf} \frac{a}{b}(x - 0.3)\right\} - \cancel{V}S$

f13v

(Foot note)*

When $\frac{a}{b} = 10$, and the columns are 0.06 in width, the total number of misclassments among the 10,000 true values, is (approximately) ~~appear to be~~ 568; when they are 0.03, or twice as narrow, ~~the number of misclassments appear~~ ~~to be~~ it is 576. {by working to an additional ~~another~~ place of decimals ~~might~~ possibly slightly change these figures} The difference between 568 & 576 is too small to affect the last place of the figures given in ~~the~~ Table.

The narrower columns, 0.03 in breadth, have been ~~were~~ used, (only) for the values of $\frac{a}{b}$ that exceed 10.

By integrating these values for the portions of the Scheme to which they severally apply, the number of false classifications in the 10,000 measures could be found. No attempt is made here to do this, but ~~To avoid the difficulties of such partial integrations~~ the columns of finite but of narrow limits, namely ~~will be~~ are taken of the uniform + convenient breadth of 0.06; therefore the medium area of the scheme ~~division~~ will be cut up into 10 ~~5 such~~ columns, 5 of them being situated in its positive half and 5 in negative halves ~~respectively~~; that is, ~~into 10 columns altogether; which is sufficient~~ These are quite narrow enough, so long as ~~when~~ $\frac{a}{b}$ is not greater than 10.* At that value

The columns are ~~will then be~~ worked out one by one, and their several results have been ~~are~~ ~~will then~~ summed giving the entries ~~results~~ in Table I.

(See III p.22 "Sebum follows the fortunes...")

f.14v

the true values of the

a = Variability of objects measured

b = Fallibility of examiner

Table I

Values of $\frac{a}{b}$ or	Percentage of True Values that are classed in one or other of 3 equally numerous classes	
	rightly	wrongly
2	73.3	26.7
4	86.1	13.9
6	90.5	9.5
8	92.9	7.1
10	94.3	5.7
12	95.2	4.8
14	95.9	4.1
16	96.5	3.5
18	96.9	3.1
20	97.2	2.8

This [illegible] a 2nd copy book

2nd part

On the chance ~~of~~ that two successive examiners ~~agreeing in~~ will classify~~ing~~ ~~an~~ the same object correctly

The fallibility of the two examiners a and a′ may not be the same, but, in order to save space, in the example about to be given of the work a & a′ will be alike $\frac{a}{b}$ and $\frac{a'}{b}$ being both taken $= 6$

over

17 f.166

(3) Value of $\frac{a}{b}$ by method of intercomparison

When we are satisfied that the law of facility is applicable, ~~[illegible]~~ it ~~is~~ becomes possible to find the value of $\frac{a}{b}$ by means of two classifications made by the same examiner(s), conducted solely by ~~according~~ to the method of inter-comparison, that is, without any individual measures or marks being ~~wanted~~ used. There are many cases in which ~~no measurements are satisfactory other~~ method of intercomparison is the only feasible ~~one~~ of classifying, ~~as appears to be the case~~ Thus in respect to general physical efficiency, ~~where~~ no ~~system~~ of marking has been yet devised which is generally satisfactory or even ~~that~~ relied on by individual examiners, although ~~Here~~ an examiner ~~has~~ who has two candidates before him ~~may~~ will trust his judgement to some extent, ~~to decide~~ as which of the two is the most effective physically for the particular purpose in view, ~~and~~ consequently, by sorting and changing the class places of a body of candidates he ~~can~~ is able to classify them by intercomparison into a series, beginning with the ~~one~~ candidate that seems the least efficient, and ending with the one that seems most efficient. Suppose an examiner to do this to the same body of candidates on two occasions ~~independently~~, under conditions that ensure the independence of the two attempts, no reference being permitted ~~to the note~~ on the second occasion to the record of the first ~~attempt~~ and the two classifications being separated by a sufficient time and ~~number~~ amount of ~~analogous~~ work to prevent the memory of the examiner helping him to classify alike on the two occasions. There will of course be many cases, in which the class place assigned to the same individual differs in the two classifications. It will now be shown how those differences may be used to determine ~~not range~~ the ratio between the variability of the candidates and the fallibility of the examiner, that is to afford the value of $\frac{a}{b}$ ~~can be determined by the differences in class places.~~

A difference of a class place has a different significance in different parts of the class. ~~The mediocrities about the middle~~ ~~requiring~~ it being far slighter ~~though~~ more ~~greater~~ ~~error~~ inexactness on the part of the examiner to produce that ~~when it occurs~~ mistake near either end of the class than it does near the middle, there being ~~is~~ much greater difference between the lowest & the next above him, or ~~lowest but one,~~ between the highest & the next below him ~~highest one but one~~ than there is between two adjacent mediocrities. The following table shews the value of a difference at each successive ~~for~~ class place in a class of 100, the probable error of the series, which is the ordinate at $50° \pm 25°$, being taken $a = 1$. ~~Thus take the be the~~ the proof ~~and~~ let Y be the ordinate at $50° + 38°$, then the table shows that $\frac{Y}{a} = 1.74$, and that the error in miscalculating one place at that point, or $\frac{E}{a}$, is equal to 0.08. This is twice as great as at the class place of $50° + 8°$

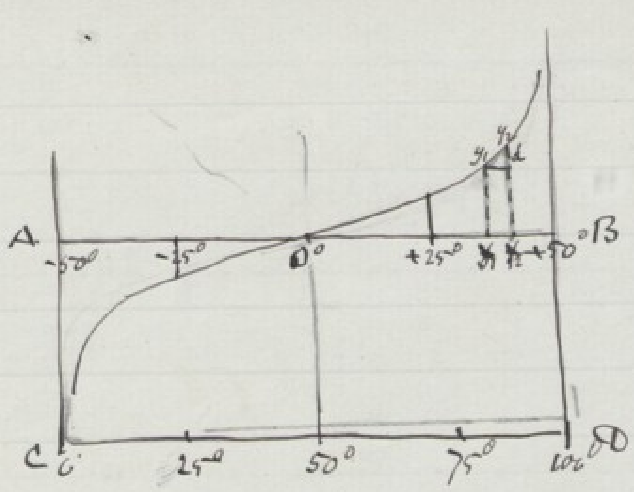
The misclassifications are practically independent if only a ~~few~~ fraction of the whole number are taken into account. If all were taken into account the case would be very different and unsuitable for the present purpose. Let us suppose then a series to be made of the differences in class place on the second occasion of those who ranked 1^{st} 5^{th} 10^{th} - - - - 100^{th} on the first occasion. [I will ignore the error ~~of~~ due to disregard of a half place throughout, as it is small and too troublesome to take account of ~~and it~~ here. In reality the 1^{st} person stands at class place 0.5 the second at 1.05, and the 100^{th} at 99.5.] This series of differences is then to be ~~replaced by~~ transformed into another consisting ~~one~~ series of the equivalent values of $\frac{E}{Q}$ ~~and~~ from which the probable error of that series $= \frac{e}{Q}$ is to be found. Now $\frac{e}{Q} = \frac{\sqrt{2}\,b}{Q}$

~~Hence~~ $\frac{b}{a}$, & its reciprocal $\frac{a}{b}$, are ~~known~~ determined.

0	0.00	0.04	0.07	0.11	0.15	0.19	0.22	0.26	0.30	0.34
	0.04	0.03	0.04	0.04	0.04	0.03	0.04	0.04	0.04	0
10	0.38	0.41	0.45	0.49	0.53	0.57	0.61	0.65	0.69	0.74
	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.	0.	0

f.162v

f.166 (2) 134



A diagram may express the theorem more clearly.

The entries in the Table under '~~Deviations~~' express the ratio between the lengths of the ordinates — to the curve of normal distribution, at the several grades in AB, regardless of ~~their~~ signs, (to the ordinate ~~at~~ $\pm 25°$), ~~which~~ the ordinate at $\pm 25°$ is the probable error of the System; namely that which any member of it is as likely as not to exceed or to fall short of. ~~In the~~ According to the above nomenclature the probable error of the system $= a = 1$

Let $x_r\, x_s$ be the class places in which the same candidate is placed in the 1st & 2nd exams respectively & $\frac{x_r y_r}{a}$, $\frac{x_s y_s}{a}$ be the true deviations at those class places, (in terms of a, the probable error of the System) and let $\frac{{}_r d_s}{a}$ = the difference between them. The Table gives ~~the~~ the values of $\frac{x_r y_r}{a}$ & of $\frac{x_s y_s}{a}$, consequently $\frac{{}_r d_s}{a}$ is known, which is the Error made by the examiner measured in terms of a

~~Let $d_1\, d_2 \ldots$ be the value difference between the ch~~

A series of values of ${}_r d_s$ is prepared & the probable error of this series is found; this, for reasons already given, $= \sqrt{2} \times$ probable Error of the examiner measured in terms of a $= \sqrt{2} \times \frac{b}{a}$, whence $\frac{a}{b}$ is determined.

Deviations and Differences at the Successive Percentile Grades, Counting outwards on either hand from the median; that is, from the 50th Percentile.

the deviation at the 25th percentiles is the probable Error, here equal to 1.00.

Units	Tens 0 Devn	0 Differces	10 Devn	10 Differces	20 Devn	20 Differces	30 Devn	30 Differces	40 Devn	40 Differces
0	0.00		0.38		0.78		1.25		1.90	
		0.04		0.03		0.04		0.05		0.09
1	0.04		0.41		0.82		1.30		1.99	
		0.03		0.04		0.04		0.06		0.09
2	0.07		0.45		0.86		1.36		2.08	
		0.04		0.04		0.05		0.06		0.11
3	0.11		0.49		0.91		1.42		2.19	
		0.04		0.04		0.04		0.05		0.12
4	0.15		0.53		0.95		1.47		2.31	
		0.04		0.04		0.05		0.06		0.13
5	0.19		0.57		1.00		1.53		2.44	
		0.03		0.04		0.05		0.06		0.16
6	0.22		0.61		1.05		1.59		2.60	
		0.04		0.04		0.05		0.07		0.19
7	0.26		0.65		1.10		1.66		2.79	
		0.04		0.04		0.05		0.08		0.26
8	0.30		0.69		1.15		1.74		3.05	
		0.04		0.05		0.05		0.08		0.40
9	0.34		0.74		1.20		1.82		3.45	
		0.04		0.04		0.05		0.08		infinite

The figures in the first columns are taken, with a little revision, from the author's "Natural Inheritance" Table 7, page 202.

This shd come at the end) f.17

Let us now consider how to obtain the values of a & b in any particular case. It is convenient to take b, or the fallibility of the examiner, the first.

(1) The direct method, when it is feasible to use it, is to take a large number of independent measures of the same object and thence to determine the probable error b′ (using b′ as a special case of the general term b)

(2) The best of the indirect methods, and that which is most generally feasible, is to take the differences between 2 independent measures of each of a multitude of objects, & to find the probable error of the system of these differences, which is equal to $\sqrt{2} \times b'$, whence b′ is found.

other indirect methods suitable to particular cases, form a study for the ingenuity of the inquirer who will probably have to content himself with approximate results, to be depended upon if they confirm one another. This is especially the case in respect to natural selection, where the problem is too ~~intricate~~ many sided to be discussed briefly here.

The value of $\underline{a}$ the variability of the true values to be measured cannot be obtained directly, but that of the variability of the observed values which we may call α is obtained easily.

Now $\alpha^2 = a^2 + b^2$, which ~~is to~~ can justly be treated as a simple equation, whence ~~the~~ $\underline{a}$ is to be obtained. [It must be recollected that we are speaking of schemes or systems, and not of particular values, ~~which~~ which alone the laws of 'inverse probability' is concerned. The our problem is simply this. A system ~~is known to~~ having the probable error α' is known to be formed of a system whose probable error is a' every one of whose constituents ~~is liable to a~~ has been subjected to a variation with a probable error $= b'$. Then α' and b' being known, there is only one possible value of $\underline{a}'$ that will satisfy the condition, namely that given in the above equation. This problem must not for a moment be confounded with the different problem of inverse probabilities.] ~~in which~~

f. 19v

	Direct	by diff.	Intercomparison $\frac{a}{b}$	by graded edge
Phys. exam			–	
Literary exam marks		–		
Nat. Sel				–
Anthropometry	–	–		
Assortment of material			–	
Jury verdict yes or no				
appraisement of damage				
Graded sentences				
Confirmation or reversal of judgment				

? doctors fallibility

Values of a and b, whether in the form of $a' \propto b'$ or otherwise can thus be found wherever classification is by measures or by marks as for multiform & for literary exams. ~~for most of the instances given at the beginning of this memoir.~~ Viz

[illegible] III p 26

	E		D	C	B	A
1			14	5	4	5
2		2	6	10	6	10
				1	1	1
3		2	2	5	5	5
4		5	1	2	4	2
		1	1			
5		13	1	2	1	2
⊕		2	–		4	
		25	25	25	25	25
		12	12	12	12	12
		37	37	37	37	37

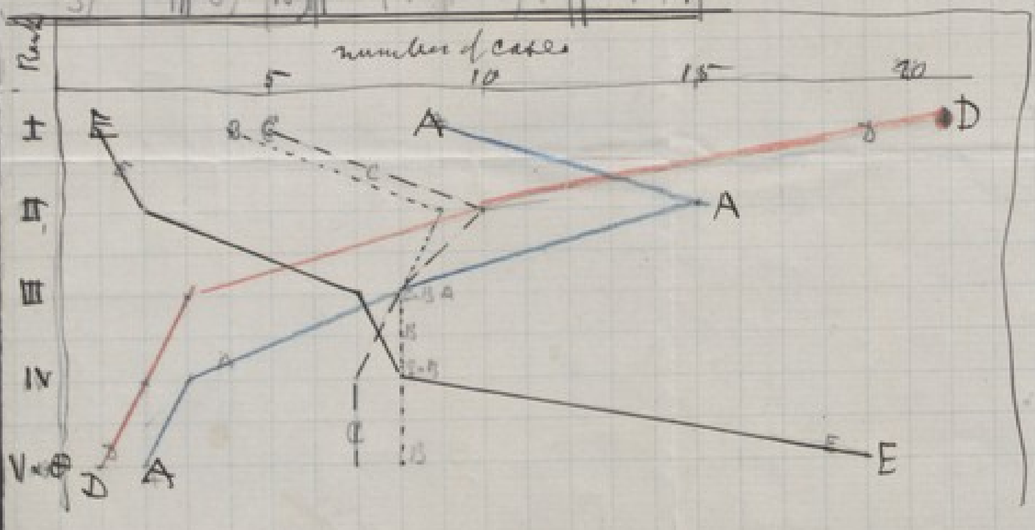
	E	D	C	B	A
I		14	5	4	5
II	2	6	10	6	10
			1	1	1
III	2	2	5	5	5
IV	5	1	2	4	2
V	13	1	2	1	2
⊗	2	–	–	4	–

Order of merit
D 5532
(A) equal 5533
(C) 5534
B 5535
E 5536

All 37 cases

Rank	E	Marks	D	Marks	C	Marks	B	Marks	A	Marks
I	1×5	5	21×5	105	5×5	25	4×5	20	9×5	45
II	2×4	8	10×4	40	10×4	40	9×4	36	15×4	60
III	7×3	21	3×3	9	8×3	24	8×3	24	8×3	24
IV	8×2	16	2×2	4	7×2	14	8×2	16	3×2	6
V	17 }19	19	1×1	1	7×1	7	4 }8	8	2×1	2
⊕	2		–		–		4		–	
	37	69	37	159	37	110	37	114	37	137

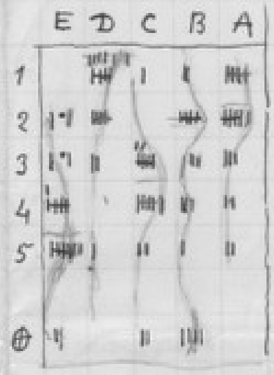
Order of merit
D 159 M+45
A 137 M+23
B 114 M
C 110 M − 4
E 69 M − 45



5 Photographs of F.G. by F. Hollier May/99

Order of merit as adjudged by different persons.
⊕ means bad, not worth classifying. ? count it as 7.

		E	D	C	B	A
1	Emma Galton	4	1	3	⊕	2
2	Temple	2	5	1	4	3
3	Bessy Wheler	5,	3	4	1	2
4	Augusta Galton	3	2	4	5	1
	Dora Galton omit by second [?] to error	4	1	4	2^b	2^a
5	Hilda Galton	4	1	3	⊕	2
6	Sophy Bree	⊕	1	⊕	⊕	2
7	Eva Biggs	3	1	⊕	⊕	2
8	Hesketh Biggs	4	1	3	2	5
9	Harrie Butler	5	1	4	2	3
	Emily Butler	5	1	3	2	4
10— 11	Olive Butler	5	1	4	2	3
12	Beth Butler	4	2	3	1	5
13	Arthur Butler	5	2	3	4	1
14	Harold Butler	4	3	5	2	1
15	Mr Henry	5	1	4	3	2
16	Karl Pearson	5	2	3	4	1
17	Gifi	5	2	4	3	1
18	Chumley	2	1	5	3	4
19	Mrs W. Leaf	5	1	3	4	2
	Revd Dr Butler	⊕	1	⊕	2^b	2^a
20— 21	Prof Jack	5	1	4	2	3
22	Maud Butler	4^b	4^a	1	3	2
23	Lady Strachey	5	1	4	3	2
24	Sir Richard Strachey	5	4	3	1	2
25—	Miss Strachey	5	2	4	1	3
26	Grace Moilliet	1	2	4	5	3
27	Ethel Galton	3	1	5	4	2
28	Frank Butler	3	1	5	4	2
29	Millicent Lethbridge	3	1	5	4	2
30	Wilson	4	1	5	3	2
31	Miss M. Coleridge	3	2	4	5	1
32	Mia Butler	5	2	3	4	1
33	Hugo de Vries	4	2	5	3	1
34	George G. Butler	5	1	3	2	4
35	Mr Sargeant	3	1	4	5	2
36	Gertrude Butler	5	1	4	2	3
37	Mrs Spencer Butler	5	3	4	2	1

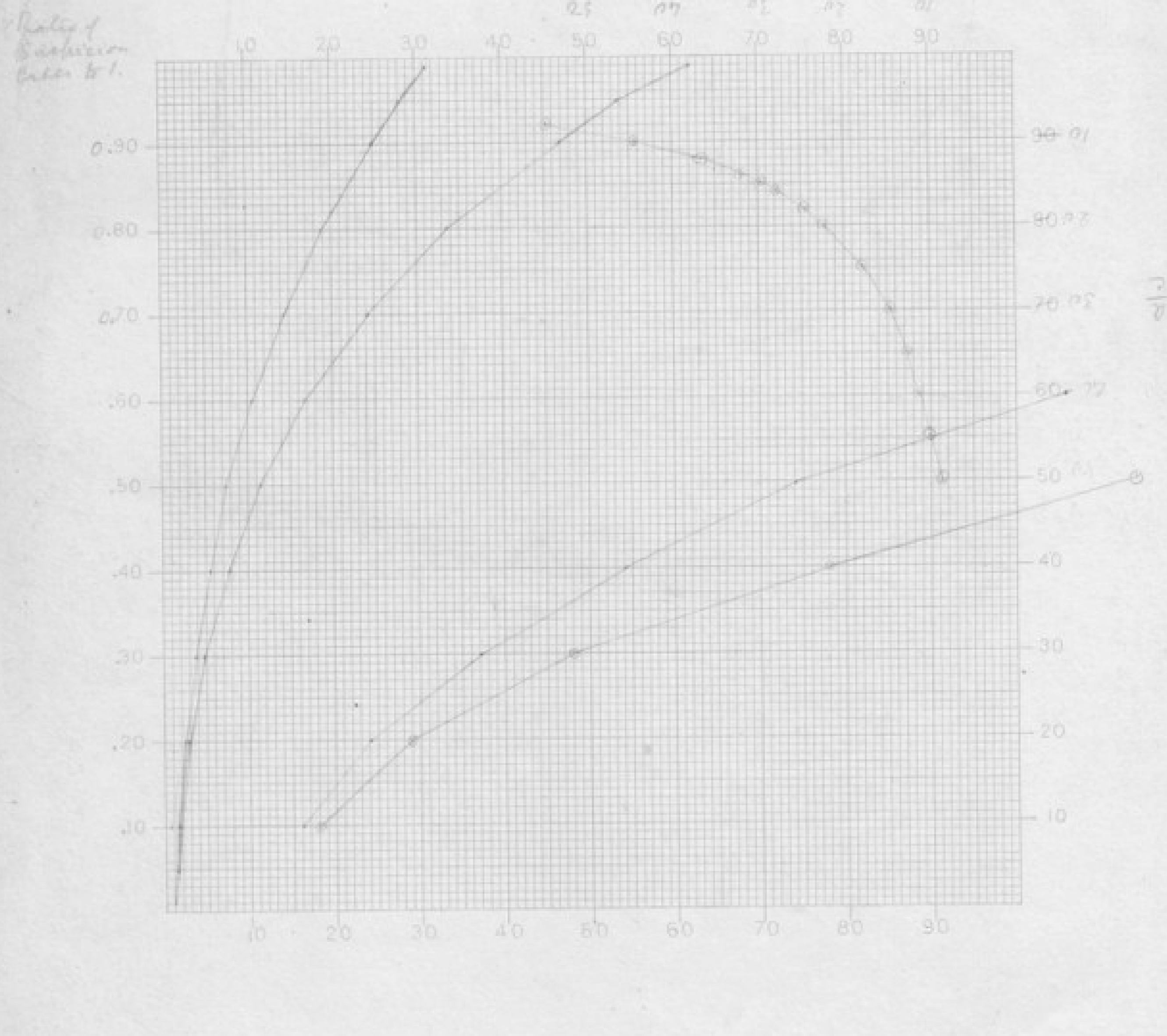


E is writing, looking up
D is the spread Eagle looking forward
C writing looking to camera
B elbow on knee fist under cheek
A. Bending over book, with eyeglass on

D is peculiarly characteristic, C & A distinctly good but might be other persons.

		Nos. Number of those who concluded this [?]	No. of cases 37
E	D		
5	1	9, 10, 11, 15, 19, 20, 21, 23, 24, 36	10 (about 1/4)
E	A		
5	1	13, 16, 17, 32, 37	5
E	A		
5	2	3, 15, 19, 23, 24	5
B	C		
3	2	13, 16, 17, 19, 23, 32	6

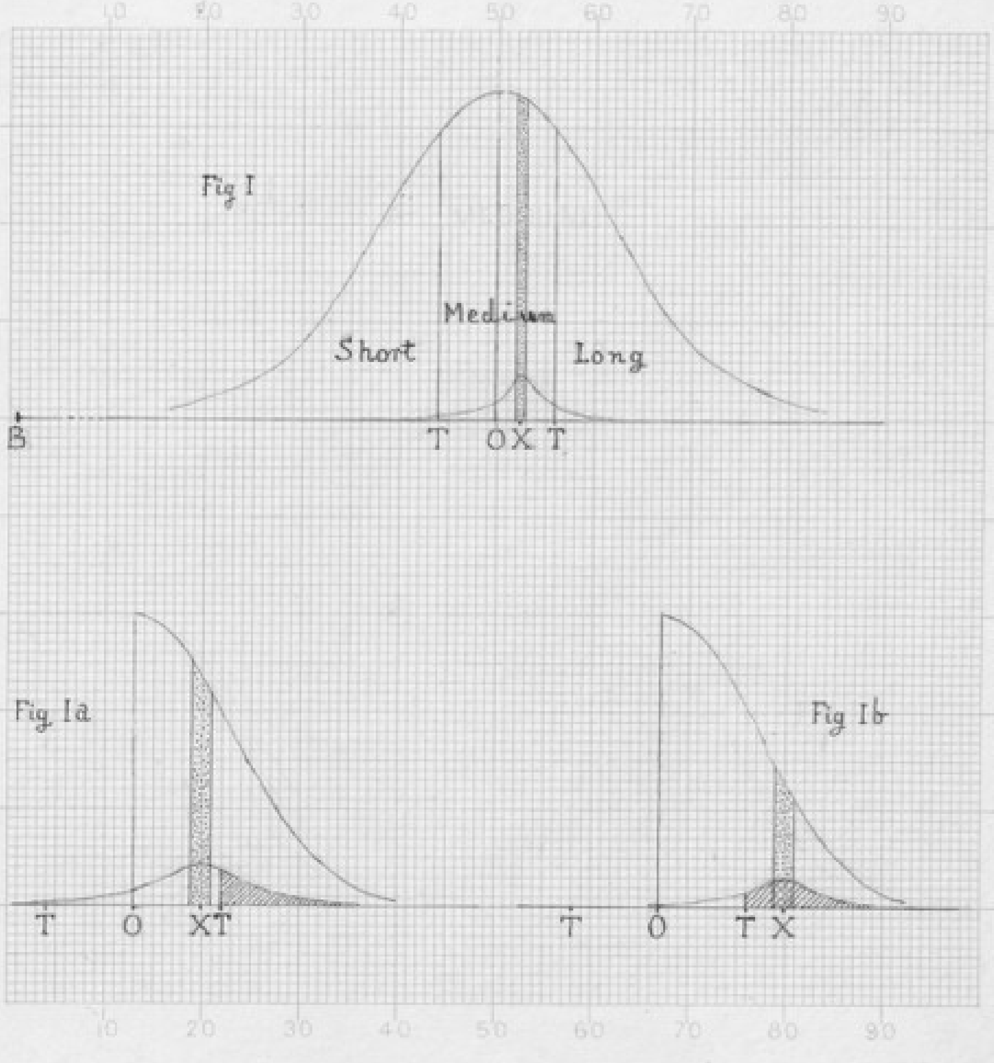
No. of compartments to be searched
according as 5 or 6 elements are used.
the chance of doubt between all 3 dimensions being equal



when ratio of ambiguous cases = 0, only 1 compartment has to be searched
= 1, all the compartments viz 2^5 in the one case $= 32$, 2^6 in the other $= 64$

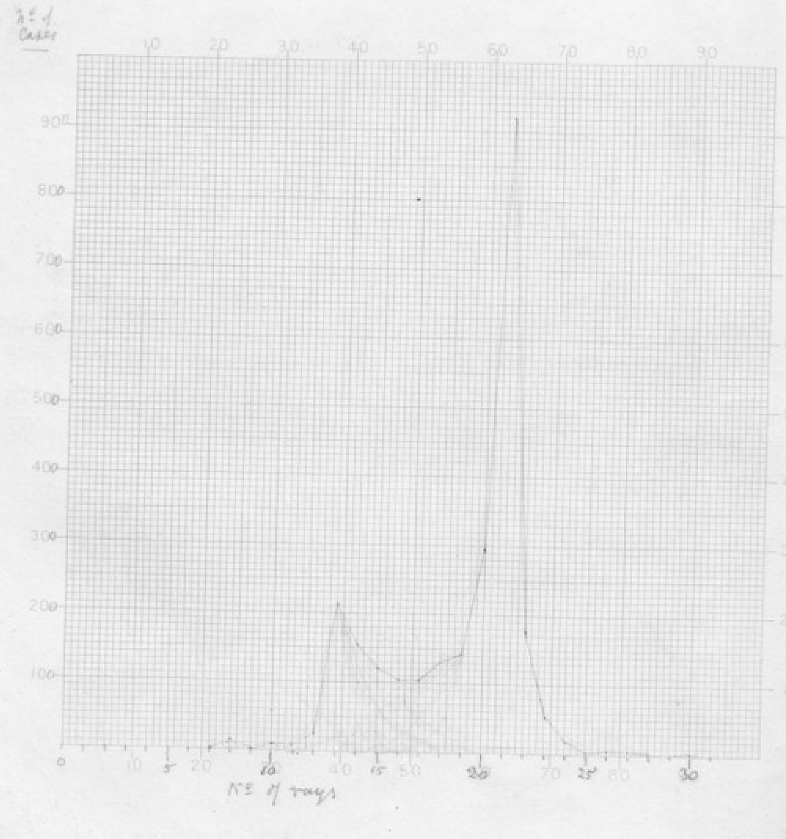
F. 4

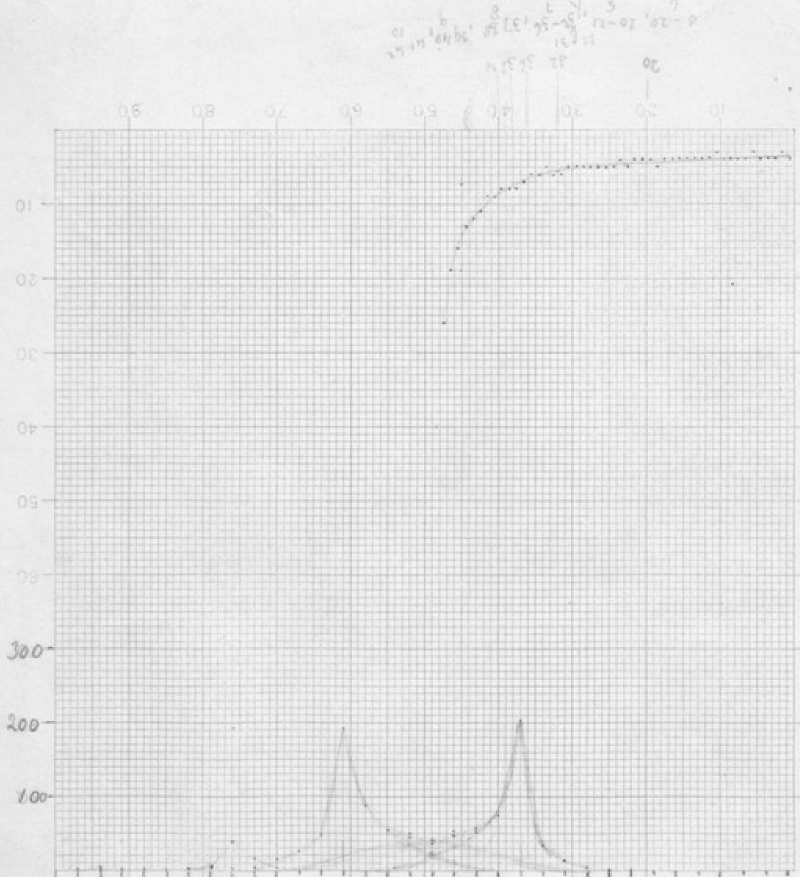
To Engraver
disregard the printed lines



f. 5

Chrys Myconis




300
200
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30