Calculations. Sep-Oct 1903

F.1

Photographic alteration of scale
of a picture in one dimension
and in perspective

THE

ALBION
Exercise Book

Text in a separate book

Name,

Diagrams in here

Subject,

Rough calculation by Crelle's Table

φ	$\cos\varphi$	$\sin\varphi$	$d\cos\varphi$	$d+\sin\varphi$	S: $\frac{d\cos\varphi}{d+\sin\varphi}$	$\tan\varphi = \sin\psi$	ψ	$\cos\psi$	$s.\cos\psi$	$ds\cos\psi$	$s.\sin\psi$	$d-s\sin\psi$	$\frac{ds\cos\psi}{d-s\sin\psi}$	t
										9.51	.170	9.830		
0°	1.000	.000	10.0	10.0	1.00	.000	0°	1.00	1.00	10.0	.00	10.00	.1[illegible]	1.00
5°	.996	.087	9.96	10.1	.987	.087	5°.0'	.996	.983	9.83	.086	9.914		0.99
10°	.985	.174	9.85	10.2	.9166	.176	10°.8'	.985	.951	9.51	.170	9.830		0.97
15°	.966	.259	9.66	10.3	.9138	.268	15°.33'	.964	.904	9.04	2.58	7.58		0.93
20°	.940	.342	9.40	10.3	.913	.364	21°.21'	.937	.855	8.55	.332	9.668	.89	0.88
25°	.906	.423	9.06	10.4	.871	.466	27°.47'	.885	.772	7.72	.406	9.594		0.80
30°.0'	.866	.500	8.66	10.5	.825	.577	34°.52'	.820	.676	6.76	.476	9.524	.71	0.71
35°.0'	.819	.574	8.19	10.6	.773	.700	35.15 44°.26'	.715	.553	5.53	.541	9.459		0.59
40°.0	.766	.643	7.66	10.6	.723	.839	57°.2'	.544	.393	3.93	.606	9.394	.0650	0.42
41°.0	.755	.656	7.55	10.7	.706	.869	60°.21'	.496	.350	3.50	.614	9.386		0.37
42°.0	.743	.669	7.43	10.7	.695	.900	64°.10	.438	.304	3.04	.626	9.374		0.33
43°.0	.731	.682	7.31	10.7	.684	.933	68°.50	.360	246	2.46	.638	9.362	.262	0.26
44°.0'	.719	.695	7.19	10.7	.672	.966	75°.0'	.259	.174	1.74	.649	9.351	.186	0.19
44°30'	.713	.701	7.13	10.7	.667	.983	79°.25'	184 .992	.662	1.23 6.62	.656	9.344	[illegible]	0.13
45°	.707	.707	7.07	10.7	.620	1.00	90°.0	.000	.000	.000	10.0	.000	.00	0.00

d=5	$\cos\varphi$	$\sin\varphi$	$5\cos\varphi$	$5+\sin\varphi$	S	$\tan\varphi = \sin\psi$		$\cos\psi$	$s.\cos\psi$	$5s.\cos\psi$		$d-s.\sin\psi$		d=5	d=10
10°	985	174	492	5.17	.952	176	10°8'	.985	938	419	168	4.832	.868	.97	97
20°	940	342	4.55	5.34	.853	364	21°.21'	.937	800	400	310	4.690	.853	.85	88
30°	866	500	4.33	5.50	.787	577	34°.52'	820	645	322	454	4.546	.713	.71	71
35°	819	574	4.09	5.57	.735	700	44°.26'	715	526	263	514	4.486	.586	.59	59
40°	766	643	3.83	5.64	.679	839	57°.2	544	369	184	570	4430	.415	.41	42
									$s\cos\psi$						
5°	996	087	498	5.09	978	087	5°0'	996	974	4.87	085	4915	992	99	99
10°	984	174	492	5.17	952	176	10°.8	984	492	4.68	167	4.83	969	97	97

f. 4v

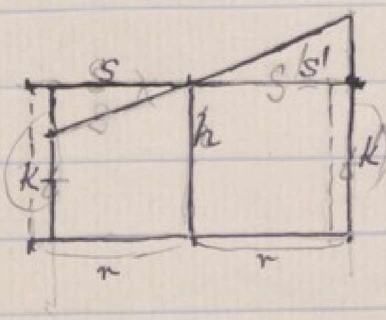
1)

$$k = \frac{d}{d+\sin\varphi} \; ; \qquad k' = \frac{d}{d-\sin\varphi} .$$

$$s = \frac{d\cos\varphi}{d+\sin\varphi} \; ; \qquad s' = \frac{d.\cos\varphi}{d-\sin\varphi}$$

$$\tan\varphi = \sin\psi$$

$$t = \frac{ds\cos\psi}{d-s\cdot\sin\psi} \; ; \qquad t' = \frac{d.s'\cos\psi}{d+s'\sin\psi}$$



f. 5r

$\tan\varphi = \sin\psi$ f. 5v

$s = \dfrac{d.\cos\varphi}{d+\sin\varphi}$, $s' = \dfrac{d.\cos\varphi}{d-\sin\varphi}$ | $k = \dfrac{d}{d+\sin\varphi}$ $k' = \dfrac{d}{d-\sin\varphi}$

~~d = k ... d+sin φ~~ ~~d sin ψ~~ e ~~d cos φ~~

logs

~~d, 4, sin φ, 0.5, cos φ~~

	logarithms		
$d = 3$	0.4771	0.4147	0.4147
$\varphi = 30^\circ$		0.5441	0.3979
$\sin\varphi = 5000$		$\bar{9}.8706$	0.0168
$\cos\varphi = 8660$		$s = .7423$	$s' = 1.040$
$d\cos\varphi = 2.5980$	0.4147	0.4771	0.4771
$d+\sin\varphi = 3.5000$	0.5441	0.5441	0.3979
$d-\sin\varphi = 2.5000$	0.3979	$\bar{9}.9330$	0.079
		$k = .8570$	$k' = 1.200$

	log		
$d = 4$	0.6021	0.5396	0.5396
$d.\cos\varphi = 3.4640$	0.5396	0.6532	0.5441
$d+\sin\varphi = 4.5000$	0.6532	$\bar{9}.8864$	$\bar{9}.9955$
$d-\sin\varphi = 3.5000$	0.5441	s. 7695	s' 9897
		~~0.5396~~ 0.6021	0.6021
		0.6532	0.5441
		$\bar{9}.9489$	0.0580
		k 8890	k' 1.143

	logs		
$d = 6$	0.7782	0.7157	0.7157
$d\cos\varphi = 5.1960$	0.7157	0.8129	0.7404
$d+\sin\varphi = 6.5000$	0.8129	$\bar{9}.9028$	$\bar{9}.9753$
$d-\sin\varphi = 5.5000$	0.7404	s .7994	s' = .9448
		0.7782	0.7782
		0.8129	0.7404
		$\bar{9}.9653$	0.0378
		k 9232	k' 1.091

d	s	s'	k	k'
3	7423	1.040	.857	1.200
4	7698	9897	8890	1.143
5	7872	9623	9090	1.112
6	7994	9448	9232	1.091

$d = 5$; $\varphi = 25°$; $\tan\varphi = \cdot 4663 = \sin 27°.48'$ $\psi = 27°.48'$ $\log\cos\psi$ 9.9467 $\log\sin\psi$ 9.6687 (2)

F.6

$\log 5 = 0.6990$ | $\sin\varphi = .4226$, $\cos\varphi = .9063$ | $d+\sin\varphi = 5.423$, $d.\cos\varphi = 4.5315$, $d-\sin\varphi = 4.5774$ | $\log d+\sin\varphi$ .7342 ✓, $\log d.\cos\varphi = .6562$ ✓, $\log d-\sin\varphi$ .6606 ✓

$\frac{d\cos\varphi}{d+\sin\varphi}$ { logs .6562, .7342; $\log s$ 9.9220; $s =$ ~~.~~.8356

$\frac{d\cos\varphi}{d-\sin\varphi}$ { logs 6562, 6606; $\log s'$ 9.9956; $s' =$ .9900

$ds\cos\psi$: $\log d$ .6990, $\log s$ 9.9220, $\log\cos\psi$ 9.9467; $ds\cos\psi =$ 0.5677

$\log d$, .6990; $\log s'$ 9.9956; $\log\cos\psi$ 9.9467; $\log ds'\cos\psi = 0.6413$

$s.\sin\psi$: $\log s$ 9.9220, $\log\sin\psi$ 9.6687; 9.5907; .3896; $d - s\sin\psi$ 4.6104; log .6637

$\log s'$ .9900 [sic]; $\log\sin\psi$ 9.6687; 9.6587; .4557; $d+s'\sin\psi$ ~~5.456~~ 5.456; log .7369

$\frac{ds\cos\psi}{d-s\sin\psi}$ 0.5677, 0.6637; 9.9040; $t =$ .8017

$\frac{ds'\cos\psi}{d+s'\sin\psi}$ 0.6413, 0.7369; 9.9044; $t' =$ .8024

very close
right to 3rd place of decimals

$\log d$ 6990
$\log(d+\sin\varphi)$ 7342
9.9658

$h_0 = \cdot 9243$
$1-h_0 = .0757$ $\log = 8.8791$
$\log s = 9.9220$
8.9571 $\log\tan 5°.11'$

$\log d$ 6990
$\log(d-\sin\varphi)$ 6606
.0384

$k_0 = 1.092$
$k_0 - 1 = .092$ $\log = 8.9638$
$\log s' = 9.9956$
8.9682 $\log\tan 5°.19'$

$d = 5$; $\phi = 30^\circ$; $\tan\phi = .5774 = \sin 35^\circ.16^\circ = \sin\psi$ $\log\sin\psi = 9.7614$ F.7

$\log\cos\psi = \cancel{9.8165}$ 9.9119 ×4

$\log 5 = 0.6990$ | $\sin\phi = .5000$
$d + \sin\phi = 5.500$ $\log = .7404$
$d - \sin\phi = 4.500$ $\log = .6532$ | $\cos\phi = .8660$
$d\cos\phi = 4.3300$
$\log d\cos\phi = .6365$

	logs
$d\cos\phi$	.6365
$d+\sin\phi$	.7404

$\log s = 9.8961$

$s = .7872$

	logs
$d\cos\phi$	.6365
$d-\sin\phi$	.6532

$\log s' = 9.9833$

$s' = .9623$

$\log d$	.6990
$\log s$	9.8961
$\log\cos\psi$	9.9119
$\log d.s\cos\psi$	0.5070

$\log d$,	.6990
$\log s'$	9.9833
$\log\cos\psi$	9.9119
$\log ds'\cos\psi$	0.5942

$\log s$	9.8961
$\log\sin\psi$	9.7614
$\log s.\sin\psi$	9.6575
$s.\sin\psi$	.4544
$d - s\sin\psi$	4.5456; log .6577

$\log s'$	9.9833
$\log\sin\psi$	9.7614
	9.7447
$s'.\sin\psi$	.5555
$d + s'\sin\psi$	5.5555; log .7447

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d $\dfrac{ds\cos\psi}{d - s\sin\psi}$ logs 0.5070, 0.6577

$\log t = \bar{9}.8493$

$\dfrac{ds'\cos\psi}{d + s'\sin\psi}$ 0.5942, 0.7447

$\log t'$ $\bar{9}.8495$

$t = .7069$

$\log d$	6990
$\log(d + \sin\phi)$	7404
	9.9586

$\log d$	.6990
$\log(d - \sin\phi)$	6532
	.0458

$h_0 = 9090$
$1 - h_0 = .0910$ log = 8.9590. log s = 9.8961; 9.0629 = log tan 6°.36'.

$k_0 = 1.112$
$k_0 - 1 = 0.112$ log = 9.0453; log s' = 9.9833; 9.0620

$d = 5^\circ.\ \varphi = 20^\circ \begin{cases} \tan 20^\circ = .3640 = \sin 21^\circ.21' \\ \sin 20 = .3420 \\ \cos 20^\circ = .9397 \end{cases}$ $\psi = 21^\circ.21'$ $\log \sin\psi\ 9.5612$, $\log\cos\psi\ 9.9692$

$\sin\varphi = 3420$
$d + \sin\varphi = 5.3420$ log $= .7277$
$d - \sin\varphi = 4.6580$ log $= 6682$

$\cos\varphi = .9397$
$d\cos\varphi = 4.6985$
$\log d.\cos\varphi = .6719$

$\log\frac{d\cos\varphi}{d+\sin\varphi} = \log s = \frac{.6719}{.7277}$
$\log s = 9.9432$
$s = .8776$

$\log\frac{d\cos\varphi}{d-\sin\varphi} = \log s' = \frac{6719}{6682}$
$\log s' = 0037$
$s' = 1.009$

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log d .6990
log s 9.9432
log $\cos\psi$ 9.9692
log $ds\cos\psi$ 0.6114x
~~$ds\cos\psi$ = .4087~~

log d, 0.6990
log s' 0.0037
log $\cos\psi$ 9.9692
log $ds'\cos\psi$ = 0.6719x
~~$ds'\cos\psi$ = 4698~~

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log s 9.9432
+ log $\sin\psi$ 9.5612
9.5044
$s.\sin\psi$ .3195
$d - s\sin\psi$ 4.6805x
$\log(d - s\sin\psi)$ .6702
log $ds\cos\psi$ .6114
9.9412. $\log\frac{ds\cos\psi}{d - s.\sin\psi}$

$t = .8734$

log s' 0.0037
+ log $\sin\psi$ 9.5612
9.5649
$s'\sin\psi$ .3672
$d + s'\sin\psi$ 5.3672x
$\log(d + s'\sin\psi)$ 7298 7298
log $ds'\cos\psi$ ~~6698~~ 6719 $ds'\cos\psi$
~~9.9400~~ 9 $d + s'\sin\psi$
9.9421

$t = 8752$

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log d .6990
$\log(d + \sin\varphi)$ 7277
9.9713

$h_1 = .9361$
$1 - h_1 = .0639$ log $= 8.8055$
log s = 9.9432
8.8623 = log tan $2^\circ.24'$

log d .6990
$\log(d - \sin\varphi)$ 6682
.0308

$k_1 = 1074$
$k_1 - 1 = 0.074$ log $= 8.8692$
log s' = 0.0037
8.8655 = log tan $2^\circ.25'$

59 8

$d = 5$; $\varphi = 10^\circ$; $\tan\varphi = .1763 = \sin\psi$ — $\log\sin\psi$ 9.2460

$\sin\varphi = .1736$ — log 9.2397 — $\log\cos\psi$ 9.9931

$\cos\varphi = .9848$ — log 9.9934 — $\psi = 10^\circ 9'$

~~d = 10~~	1.0000	$d\cdot\cos\varphi$ 9.8480	~~0.9980~~ 0.9934
~~$d + \sin\varphi = 10.500$~~	1.0212		
~~$d - \sin\varphi = 9.500$~~	0.9777		
$\log d\cos\varphi$	0.9934	$\log d\cos\varphi$	0.9934
$-\log(d+\sin\varphi)$	1.0212	$-\log(d-\sin\varphi)$	0.9777
s = 9380	9.9722	s' = 1.037	0.0157
$\log d$	1.0000	$\log d$	1.0000
$+\log s$	9.9722	$\log s'$	0.0157
$+\log\cos\psi$	9.9931	$\log\cos\psi$	9.9931
$(\log(d.s.\cos\psi))$	0.9653	$(\log(d.s'\cos\psi))$	1.0088
$\log s$	9.9722	$\log s'$	0.0157
$+\log\sin\psi$	9.2460	$+\log\sin\psi$	9.2460
s = 1653	9.2182	s' = 1829	9.2617
$d - s\sin\psi$ 9.8347	0.9928	$d + s'\sin\psi$ 10.1829	1.0076
	0.9653		1.0088
	0.9928	$\log(d+s'.\sin\psi)$	1.0076
t = 9891	9.9725	t' = ~~1.017~~ 1002	0.0012

$d = 5 \; ; \varphi = 35^\circ$ $\tan\varphi = .7002 = \sin\psi$ 44.27 $\sin\varphi = .5736$ $\cos\varphi = .8192$ $\psi = 44^\circ 27'$

$\log\sin\psi$ 9.8453
$\log\cos\psi$ 9.8536

10

	logs		logs
$d+\sin\varphi$ 5.5736	.7462	$\cos\varphi$ .8192	—
$d-\sin\varphi$ 4.4264	.6460	$d.\cos\varphi$ 4.0960	.6123
$\log d\cos\varphi$	6123	$\log d\cos\varphi$	6123
$-\log(d+\sin\varphi)$	7462	$-\log(d-\sin\varphi)$	6460
$\log s$	9.8661	$\log s'$	9.9663
$s = .7347$		$s' = .9253$	
$\log d$	0.6990	$\log d$	0.6990
$\log s$	9.8661	$\log s'$	9.9663
$\log\cos\psi$	9.8536	$\log\cos\psi$	9.8536
$\log ds\cos\psi$	0.4187	$\log ds'\cos\psi$	0.5189
$\log s$	9.8661	$\log s'$	9.9663
$+\log\sin\psi$	9.8453	$+\log\sin\psi$	9.8453
$s.\sin\psi$ .5145	9.7114	$s'.\sin\psi$ .6480	9.8116
$d-s.\sin\psi$ 4.4855	0.6518	$d+s'.\sin\psi$ 5.6480	0.7519
$\log d.s.\cos\psi$	0.4187	$\log ds'\cos\psi$	0.5189
$-\log(d-s\sin\psi)$	0.6518	$-\log(d+s'\sin\psi$	0.7519
$\log t$	9.7669	$\log t'$	9.7670
$t = .5847$		$t' = .5848$	

$\log d$ .6990
$\log(d+\sin\varphi)$ .7462
9.9528

$k_0 = .8970$
$1-k_0 = .1030$ $\log = 9.0128$
$\log s = 9.8661$
$9.1467 = \log\tan 8^\circ.0'$

$\lambda = 8^\circ.0'$

$\log d$ .6990
$\log(d-\sin\varphi$ .6460
.0530

$k_0 = 1.130$
$k_0 - 1 = .130$ $\log = 9.1139$
$\log s' = 9.9663$
9.1476

F. 10

$\tan\varphi = \sin\psi$ .8391
$\cos\varphi$ ~~.5456~~ .7660
$d = 5$; $\varphi = 40°$; $\sin\varphi$ .6428 ; $\psi = 57° \, 4'$

~~log sin~~ ~~[illegible]~~ 312
log cos ψ 9.7353
log sin ψ 9.9239

logs		logs
d + sin φ 5.6428 log .7515	cos φ .7660	
d − sin φ 4.3572 log .6392	d × cos φ 3.8300	log .5832

log d.cos φ 0.5832
− log(d + sin φ) 0.7515
s = .6787 log s → $\bar{9}$.8317

log d 0.6990
log s $\bar{9}$.8317
log cos ψ $\bar{9}$.7353
log d.s.cos ψ 0.2660

log s $\bar{9}$.8317
+ log sin ψ $\bar{9}$.9239
s.sin ψ .5697 log s.sin ψ 9.7556
d − s.sin ψ 4.4303 log .6464

log ds cos ψ 0.2660
− log(d − s.sin ψ) 0.6464
t = .4164 log t $\bar{9}$.6196

log d .6990
log (d + sin φ) .7515
h_0 = .8861 9.9475
$1 - h_0$ = 0.1142 log = 9.0577
log s = 9.8317
log tan 9°.32 = 9.2260

log d.cos φ .5832
− log(d − sin φ) .6392
s' = .8790 log s' $\bar{9}$.9440

log d, 0.6990
log s' $\bar{9}$.9440
log cos ψ $\bar{9}$.7353
log d.s'.cos ψ .3783

log s' 9.9440
+ log sin ψ 9.9239
s'.sin ψ .7377 log s'.sin ψ 9.8679
d + s' sin ψ 5.7377 log .7588

log d.s'.cos ψ 0.3783
− log(d + s' cos ψ) 0.7588
t = .4164 log t' 9.6195

log d .6990
log (d − sin φ) .6392
.0598
k_0 = 1.147
$k_0 - 1$ = 0.147 log = 9.1673
log s' = 9.9440
log tan 9°.30 0.2233

$\lambda = 9°.30'$

$d = 5 \quad \varphi = 45^\circ \quad \tan\varphi = 1.0000 = \sin 90^\circ \quad \cos\varphi = .7071 \quad \sin\varphi = .7071 \quad \psi = 90^\circ$

$\cos 90^\circ = 0 \quad \sin 90^\circ = 1 \quad \log\cos\psi = \text{neg inf} \quad \log\sin\psi = 0.0000$

logs

$d + \sin\varphi = 5.7071 \quad \log = 7564$

$d - \sin\varphi = 4.2919 \quad 6326$

$\cos\varphi = 7071$

$d.\cos\varphi = 3.5355 \quad 5485$

$\log d.\cos\varphi \quad 5485$

$-\log(d + \sin\varphi) \quad 7564$

$\log s = 9.7921$

$s = 6195$

$\log d\cos\varphi \quad 5485$

$-\log(d - \sin\varphi) \quad 6326$

$\log s' = 9.9159$

$s' = .8239$

$\log d \quad 6990$

$\log s \quad \bar{9}.7921$

$\log\cos\psi$ — infinity neg

$\log d s.\cos\psi$ — o. infinity

$\log d \quad .6990$

$\log s' \quad \bar{9}.9158$

$\log\cos\psi$ — infinity neg

$\log d s'\cos\psi$ — o. neg inf

$\log s \quad 9.7921$

$\log\sin\psi \quad 0.000$

$\log s.\sin\psi \quad 9.7921$

$\log s' \quad 9.9158$

$\log\sin\psi \quad 0.0000$

$\log s'.\sin\psi \quad 9.9158$

$s.\sin\psi = 6195$

$d - s.\sin\psi \; 4.3805 \quad \log = 6415$

$s'.\sin\psi \; 8237$

$d + s'\sin\psi \; 5.8237 \quad \log = 7652$

$\log d s\cos\psi$ — inf neg

$-\log(d - s.\sin\psi) \quad .6415$

$\log t$ — inf neg

$t = 0$

$\log d s'\cos\psi$ — inf neg

$-\log(d + s'\cos\psi) \quad .7652$

$\log t'$ — inf neg

$t' = 0$

$\log d \quad .6990$

$\log(d + \sin\varphi) \quad .7564$

9.9326

$h_0 = .8762$

$1 - h_0 = .1238 \quad \log = 9.0927$

$\log s = 9.7921$

$9.3006 = \log\tan 11^\circ 18'$

$\log d \quad .6990$

$\log(d - \sin\varphi) \quad .6326$

$.0664$

$k_0 = 1.165$

$k_0 - 1 = 0.165 \quad \log = 9.2175$

$\log s' = 9.9159$

$9.3016 = \log\tan 11^\circ 19'$

$d = 2$

$\log d$	0.3010
$\log d + \sin\varphi$	.3979
$k = .5000$	9.9031

$k' = 11.1334$

$\log d$	0.3010
$\log d - \sin\varphi$	0.1761
	0.1249

$d = 2$ $\tan\varphi = .5774 = \sin\psi$ $\log d = 0.3010$
$\varphi = 30^\circ$ $\cos\varphi = .8660$ $\psi = 35^\circ 16'$ $\log\cos\psi = 9.9119$
$r = 1$ $\sin\varphi = .5000$ $\log\sin\psi = 9.7614$

		logs			logs
$d+\sin\varphi$	2.5000	0.3979	$d = 2.$	log	0.3010
$d-\sin\varphi$	1.5000	0.1761	$\cos\varphi = .8660$	log	9.9375
			$d \times \cos\varphi$	log =	0.2385
	$\log d.\cos\varphi$	0.2385		$\log d\cos\varphi$	0.2385
	$-\log(d+\sin\varphi)$	0.3979		$-\log(d+\sin\varphi)$	0.1761
					9.9909
$s = 0.6928$	$\log s$	9.8406	$s' = 1.154$	$\log s'$	0.0624
	$\log d$	0.3010		$\log d$	0.3010
	$\log s$	9.8406		$\log s'$	0.0624
	$\log\cos\psi$	9.9119		$\log\cos\psi$	9.9119
	$\log ds\cos\psi$	0.0535 (F)		$\log d s'\cos\psi$	0.2753 (F')
$k = .8000$	$\log s$	9.8406	$k' = 1.1334$ see opposite page	$\log s'$	9.0624
	$+\log\sin\psi$	9.7614		$+\log\sin\psi$	9.7614
	$\log s.\sin\psi$	9.6020		$\log s'\sin\psi$	9.8238
$s.\sin\psi = .3999$			$s'\sin\psi = .6665$		
$d - s.\sin\psi = 1.6001$	$\log(d-s.\sin\psi)$	0.2041 G	$d+s'\sin\psi = 2.6665$	$\log(d+s'\sin\psi)$	0.4259 G'
$t = .7069$	$\log F - \log G$	9.8494	$t' = .7069$	$\log F' - \log G'$	9.8494
$d = 10$ rest as above			$d = 10$	log	1.0000
$d+\sin\varphi$	10.5000	1.0212	$\cos\varphi$ 0.8660	$\log\cos\varphi$	9.9375
$d-\sin\varphi$	9.5000	0.9777		$\log d.\cos\varphi$	0.9375
	$\log d\cos\varphi$	0.9375		$\log d\cos\varphi$	0.9375
	$-\log(d+\sin\varphi)$	1.0212		$-\log(d-\sin\varphi)$	0.9777
$s = .8247$	$\log s$	9.9163	$s' = .9116$	$\log s'$	9.9598
	$\log d$	1.0000		$\log d$	1.0000
	$\log s$	9.9163		$\log s'$	9.9598
	$\log\cos\psi$	9.9119		$\log\cos\psi$	9.9119
	$\log(d.s.\cos\psi)$	0.8282 F		(F) $\log(d.s'\cos\psi)$	0.8717
	$\log s$	9.9163		$\log s'$	9.9598
	$+\log\sin\psi$	9.7614		$+\log\sin\psi$	9.7614
.4761	$\log(s.\sin\psi)$	9.6777	.5262	$\log(s'\sin\psi)$	9.7212
$d - s.\sin\psi$ 9.5239	$\log(d-s.\sin\psi)$	9.9788 G	$d+s'.\sin\psi = 10.5262$ G	$\log(d+s'\sin\psi)$	1.0222
$t = .7068$	$\log F - G$	9.8494	$t =$	$\log F - G$	9.8493
	$\log d$	1.0000		$\log d$	1.0000
	$\log(d+\sin\varphi)$	1.0212		$\log(d-\sin\varphi)$	0.9777
$k = .9524$		0.9788	$k' = 1.053$		0.0223

values of t
for $\varphi = 30^0$

d	
infin	·71
10	·71
5	·71
1	·71

The effect of d is not felt in the first two decimal places

$d = 1$ $\varphi = 30^0$ $\tan\varphi = .5774 = \sin\psi$
$\cos\varphi = .8660$
$\sin\varphi = .5000$ $\psi = 35^\circ 16$

$\log d = 0$
$\log\cos\psi = 9.9119$
$\log\sin\psi = 9.7614$

	Logs			Logs
$d + \sin\varphi = 1.5000$	0.1761			
$d - \sin\varphi = 0.5000$	9.6990		$d.\cos\varphi$	9.9375
$\log d\cos\varphi$	9.9375		$\log d\cos\varphi$	9.9375
$-\log(d+\sin\varphi)$	0.1761		$-\log(d-\sin\varphi)$	9.6990
$s = .5773$ $\log s =$	9.7614	$s' = 1732$	$\log s' =$	0.2385
$\log d$	0.0000		$\log d =$	0.0000
$\log\cos\psi$	9.9119		$\log\cos\psi$	9.9119
$\log d s\cos\psi$	9.6733		$\log d s'\cos\psi$	0.1504
$\log s$	9.7614		$\log s'$	0.2385
$+\log\sin\psi$	9.7614		$+\log\sin\psi$	9.7614
$\log s.\sin\psi$	9.5228	$s'\sin\psi = .9999$	$\log s'\sin\psi$	9.9999
$s.\sin\psi = .3333$ $\log d.s.\cos\psi$	9.6733		$\log d s'\cos\psi$	0.1504
$d - s.\sin\psi = .6667$ — $\log =$	9.8174	$d + s'\sin\psi = 1.9999$ —	$\log =$	0.3010
$t = .7176$	9.8559	$t' = .7069$		9.8494

not quite right

$\psi^t = \cos\varphi_t \cdot \cos\psi$

φ	ψ		diff			
0°						
1°	1° 0	9997			1.000	1
2°	2° 0	9988	9	6	999	2
3°	3° 0	9973	15	7	997	2
4°	4° 1'	·9951	22	6	·995	3
5°	5° 1'	·9923	28	5	·992	3
6°	6° 2'	·9890	33	7	·989	4
7°	7° 3'	·9850	40	6	·985	5
8°	8°. 5'	·9804	46	6	·980	5
9°	9°. 7'	·9752	52	6	·975	6
10°	10°. 9'	·9694	58	7	·969	6
11°	11°. 13'	·9629	65	6	963	7
12°	12°. 16'	·9558	71	7	956	8
13°	13°. 21'	·9480	78	5	948	8
14°	14°. 26'	·9397	83	8	940	9
15°	15°. 33'	·9306	91	6	931	10
16°	16°. 40'	·9209	97	7	921	10
17°	17°. 48'	·9105	104	6	911	11
18°	18° 48'	·8995	110	8	900	12
19°	20°. 8'	·8877	118	7	888	13
20°	21. 21	·8752	125		875	

φ	ψ	t				
20°	21° 21'	.8752	125	6	875	
21°	22°. 34'	.8621	131	9	862	13
22°	23°. 50'	.8481	140	6	848	14
23°	25°. 7'	.8335	146	9	833	15
24°	26°. 26'	.8180	155	8	818	15
25°	27°. 48'	.8017	163	7	802	16
26°	29°. 12'	.7847	170	10	785	17
27°	30°. 37'	.7667	180	9	767	18
28°	32°. 7'	.7478	189	9	748	19
29°	33°. 40'	.7280	198	11	728	20
30°	35°. 16'	.7071	209	10	707	21
31°	36° 56'	.6852	219	12	685	22
32°	38°. 40	.6621	231	12	662	23
33°	40°. 30'	.6378	243	14	638	24
34°	42°. 25'	.6121	257	16	612	26
35°	44°. 27'	.5848	273	16	585	27
36°	46°. 36'	.5559	289		556	29
37°	48°. 54'	.5262	297	22	526	30
38°	51°. 23'	.4919	331	28	492	34
39°	54°. 4'	4566	359	34	456	36
40°	57° 3'	4167	393		417	39

5

f. 15v

φ	ψ	t					
			393	39		39	
40°	57° 3′	4167		43	·417		5
			436			44	
41°	60° 23′	3731		62	·373		6
			498			50	
42°	64° 13′	3233		94	·323		9
			592			59	
43°	68° 50′	2641		181	·264		18
			773			77	
44°	74° 57′	1868		1085	·187		110
			1868			187	
45	90°	— 0000			.000		

44°30′	79° 20′	1321

page	d	φ	bases of first projection s	s'	s+s'	2nd projection final width of each side t	sides of first projection h	k	cot. of first projection slope
8	5	10°	.9519	1.021	1.973	.970	.966	1.036	
6	5	20°	.8776	1.009	1.886	.874	.936	1.074	2°.24'
2	5	25°	.8356	.9900	1.8256	.802	.924	1.092	5° 15 ab
4	5	30°	.7872	.9623	1.7495	.707	.909	1.111	6°.36'
10	5	35°	.7347	.9253	1.6600	.585	.897	1.136	8°.0'
12	5	40°	.6787	.8790	1.51577	.416	.886	1.149	9°.30'
14	5	45°	.6195	.8239	1.4434	.000	.876	1.165	11.18

f. 16v

$OA_1 = s \qquad OB_1 = s'$

$s : d :: r\cos\varphi : d + r\sin\varphi \qquad s = \dfrac{dr\cos\varphi}{d + r\sin\varphi}$

Similarly, taking change of sign in account $\qquad s' = \dfrac{dr\cos\varphi}{d - r\sin\varphi}$

$OA_2 = t \qquad OB_2 = t'$

$t : s\cos\psi :: d : d - s\sin\psi \qquad t = \dfrac{d.s\cos\psi}{d - s.\sin\psi}$

Similarly, taking change of sign in account $\qquad t' = \dfrac{d.s'\cos\psi}{d + s'\sin\psi}$

h_0 at a_1 is projected as h_1 at A_1: [illegible] the projection of the vertical rod height at a_1 when it stands

$h_1 : h_0 :: d : d + r\sin\varphi \qquad h_1 = \dfrac{h_0 d}{d + r\sin\varphi}$

similarly $k_1 \qquad k_1 = \dfrac{k_0 d}{d - r\sin\varphi}$

h_1 at a_2 projected as h_2 at A_2 — let the height of [illegible] call its projection at A_2 be called h_2

$h_2 : h' :: d : d - s.\sin\psi \qquad h_2 = \dfrac{d h_1}{d - s\sin\psi}$

when we substitute for s & ψ $\qquad k_2 = \dfrac{d k_1}{d + s'\sin\psi}$

$$h_2 = \frac{d}{d - \frac{dr\cos\varphi\ \tan\varphi}{d + r\sin\varphi}} \times \frac{h_0 d}{d + r\sin\varphi}$$

$$= \frac{h_0 d^2(d + r\sin\varphi)}{d^2 + rd\sin\varphi - dr\sin\varphi} \times \frac{1}{d + r\sin\varphi}$$

$$= h_0\frac{d^2}{d^2} = h_0$$

$$k_2 = \frac{d}{d + \frac{dr\cos\varphi\ \tan\varphi}{d - r\sin\varphi}} \cdot \frac{d k_0}{d - r\sin\varphi}$$

$$= \frac{k_0 d^2(d - r\sin\varphi)}{d^2 - dr\sin\varphi + dr\sin\varphi} \cdot \frac{1}{d - r\sin\varphi}$$

$$= k_0$$

slope $= \lambda \qquad \tan\lambda = \dfrac{h - h_0}{s} = \dfrac{k_0 - k}{s'} = \dfrac{1 - h_0}{s} = \dfrac{k_0 - 1}{s'}$

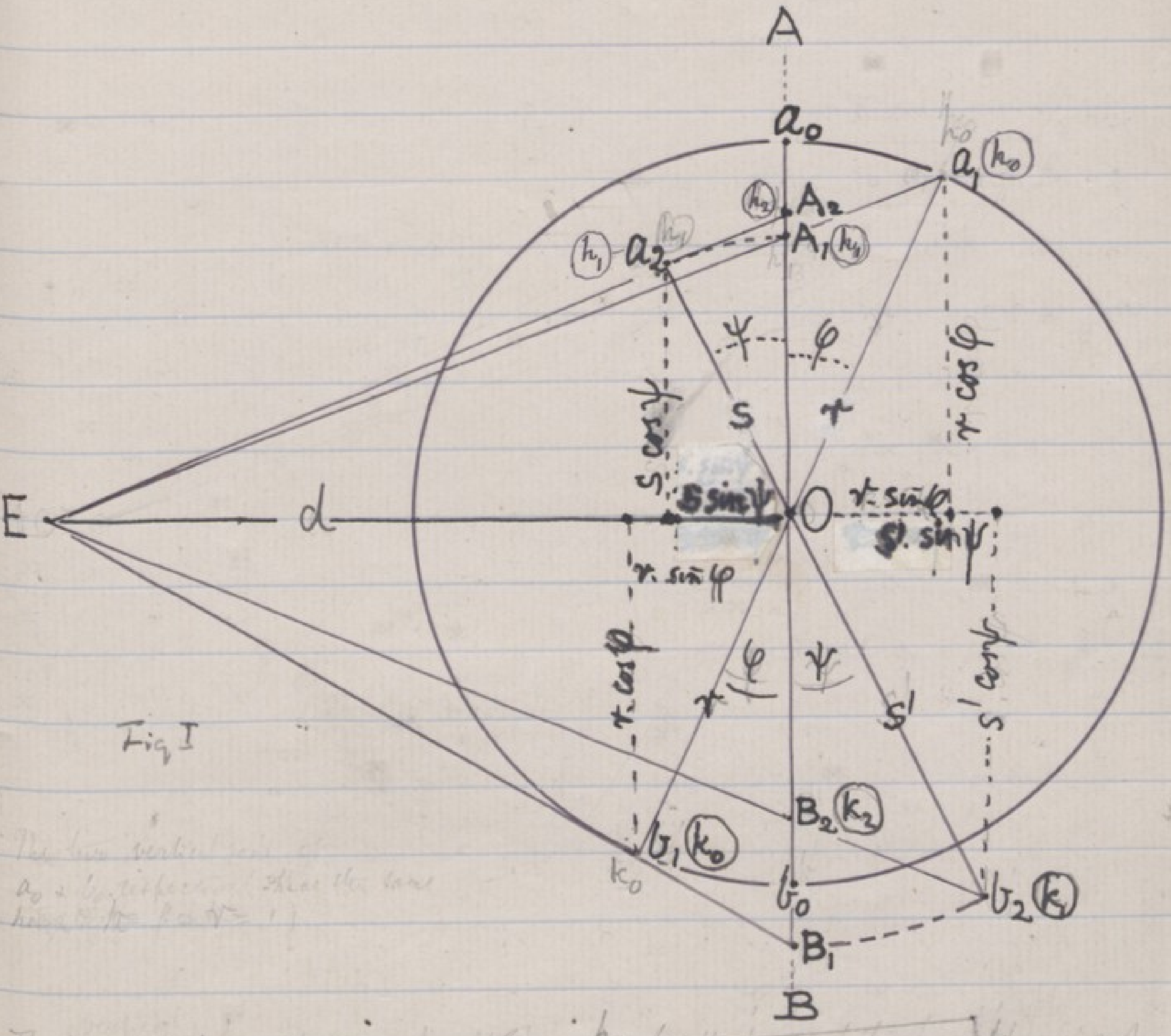

A
a_0
a_1 (k_0)
k_0
(k_2) A_2
(h_1) a_2
A_1 (h_3)
ψ
φ
$s.\cos\psi$
s
r
$r.\cos\varphi$
E
d
$s\sin\psi$
O
$r.\sin\varphi$
$s.\sin\psi$
$r.\sin\varphi$
$r.\cos\varphi$
φ
ψ
r
s'
$s'\cos\psi$
B_2 (k_2)
b_1 (k_0)
k_0
b_0
b_2 (k_1)
B_1
B

Fig I

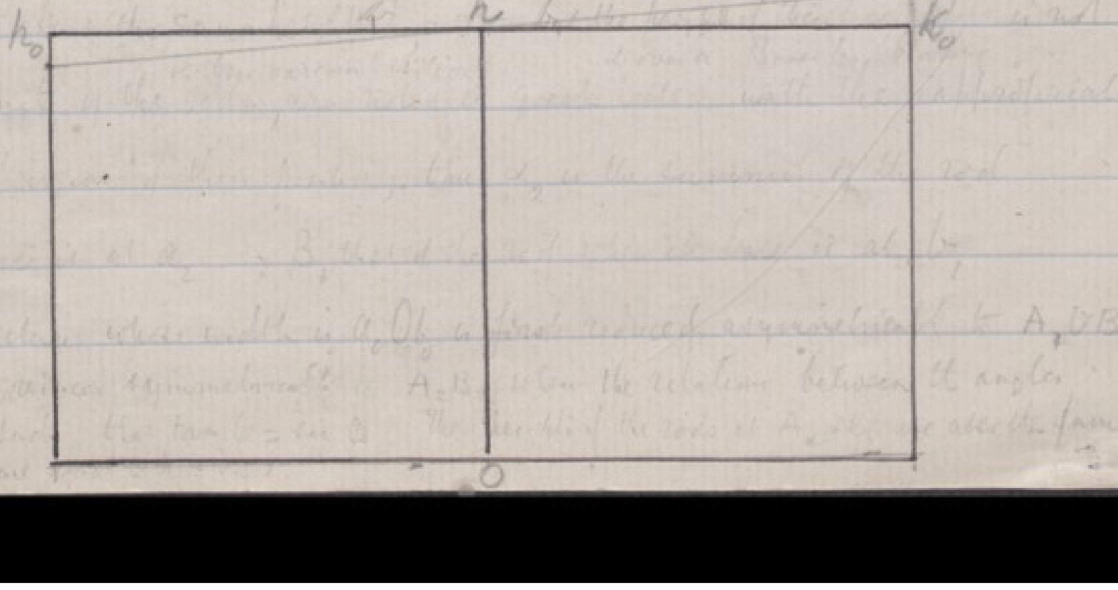

h_0
h
k_0
O

Fig II

f. 18v

ARITHMETICAL TABLES.

Numeration Table.

Units,	1
Tens,	1 2
Hundreds,	1 2 3
Thousands,	1 2 3 4
Tens of Thousands,	1 2, 3 4 5
Hundreds of Thousands,	1 2 3, 4 5 6
Millions,	1, 2 3 4, 5 6 7
Tens of Millions,	1 2, 3 4 5, 6 7 8
C. of Millions,	1 2 3, 4 5 6, 7 8 9

Sterling Money Table.

4 Farthings	1 Penny	*d.*
12 Pence	1 Shilling	*s.*
2 Shillings	1 Florin.	
2 Shillings & Sixpence	1 Half Crown	
5 Shillings	1 Crown	*cr.*
10 Shillings	1 Half Sov.	
20 Shillings, 1 Sov. or	1 Pound	£
21 Shillings	1 Guinea.	

Arithmetical Signs.

+ Plus; Sign of Addition.
− Minus; Sign of Subtraction.
× Sign of Multiplication.
÷ Sign of Division.
= Sign of Equality.
: :: : Sign of Proportion.
√ Sign of the Square Root.
∛ Sign of the Cube Root.
° Degree, ′ minute, ″ second.
∴ Therefore.

Troy Weight.

For Gold, Silver, and Jewels.

24 Grains	1 Pennyweight	*dwt.*
20 Pennyweights	1 Ounce	*oz.*
12 Ounces	1 Pound	*lb.*

Apothecaries' Weight.

For Mixing Medicines.

20 Grains	1 Scruple	*scr.*
3 Scruples	1 Dram	*dr.*
8 Drams	1 Ounce	*oz.*
12 Ounces	1 Pound	*lb.*

Avoirdupois Weight.

For all Goods except Gold, Silver, and Jewels.

16 Drams	1 Ounce	*oz.*
16 Ounces	1 Pound	*lb.*
14 Pounds	1 Stone	*st.*
28 Pounds	1 Quarter	*qr.*
4 Quarters	1 Hundredweight	*cwt.*
20 Cwt	1 Ton	*tn.*

Hay and Straw Weight.

36 lb. Straw	1 Truss.
56 lb. Old Hay	1 Truss.
60 lb. New Hay	1 Truss.
36 Trusses	1 Load.

Long or Lineal Measure.

12 Lines	1 Inch	*in.*
12 Inches	1 Foot	*ft.*
3 Feet	1 Yard	*yd.*
2 Yards	1 Fathom	*f.*
5½ Yards	1 Pole.	
40 Poles	1 Furlong	*fur.*
8 Furlongs or 1760 yards	1 Mile.	

Cloth Measure.

2¼ inches	= 1 nail.
4 nails	= 1 quarter of a yard.
4 quarters	= 1 yard.

Solid or Cubic Measure.

1728 cubic inches	= 1 cubic foot.
27 cubic feet	= 1 cubic yard.
24¾ cubic feet	= 1 solid perch mason's work.
12¾ cubic feet	= 1 solid perch brickwork.

Imperial Heaped Measure.

		Lbs. Avoird. of water.
8 gallons	= 1 bushel	= 80
3 bushels	= 1 sack	= 240
12 sacks	= 1 chaldron	= 2880

Imperial Dry Measure.

		Avoird. of water. lb.	oz.
2 glasses	= 1 naggin	= 0	5
4 naggins	= 1 pint	= 1	4
2 pints	= 1 quart	= 2	8
4 quarts	= 1 gallon	= 10	0
2 gallons	= 1 peck	= 20	0
4 pecks	= 1 bushel	= 80	0
8 bushels	= 1 quarter	= 640	0

Square Measure.

144 square inches	= 1 square foot.
9 square feet	= 1 square yard.
30¼ square yards	= 1 square pole.
40 square poles	= 1 rood.
4 roods	= 1 acre.

Table of Motion.

60″ seconds	= 1 minute.
60′ minutes	= 1 degree.
30° degrees	= 1 sign.
12s signs, or 360°	= the circle of the earth.

Table of Time.

60 Seconds	1 Minute.
60 Minutes	1 Hour.
24 Hours	1 Day.
7 Days	1 Week.
4 Weeks	1 Month.
365 Days	1 Year
366 Days	1 Leap Year.
52 Weeks	1 Year.
12 Calendar or 13 Lunar Months	1 Year.

Days in the Months.

Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Excepting February alone, [clear.
Which has but twenty-eight days
And twenty-nine in each leap year.

MULTIPLICATION TABLE.

2 TIMES	3 TIMES	4 TIMES	5 TIMES	6 TIMES	7 TIMES	8 TIMES	9 TIMES	10 TIMES	11 TIMES	12 TIMES
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 — 4	2 — 6	2 — 8	2 — 10	2 — 12	2 — 14	2 — 16	2 — 18	2 — 20	2 — 22	2 — 24
3 — 6	3 — 9	3 — 12	3 — 15	3 — 18	3 — 21	3 — 24	3 — 27	3 — 30	3 — 33	3 — 36
4 — 8	4 — 12	4 — 16	4 — 20	4 — 24	4 — 28	4 — 32	4 — 36	4 — 40	4 — 44	4 — 48
5 — 10	5 — 15	5 — 20	5 — 25	5 — 30	5 — 35	5 — 40	5 — 45	5 — 50	5 — 55	5 — 60
6 — 12	6 — 18	6 — 24	6 — 30	6 — 36	6 — 42	6 — 48	6 — 54	6 — 60	6 — 66	6 — 72
7 — 14	7 — 21	7 — 28	7 — 35	7 — 42	7 — 49	7 — 56	7 — 63	7 — 70	7 — 77	7 — 84
8 — 16	8 — 24	8 — 32	8 — 40	8 — 48	8 — 56	8 — 64	8 — 72	8 — 80	8 — 88	8 — 96
9 — 18	9 — 27	9 — 36	9 — 45	9 — 54	9 — 63	9 — 72	9 — 81	9 — 90	9 — 99	9 — 108
10 — 20	10 — 30	10 — 40	10 — 50	10 — 60	10 — 70	10 — 80	10 — 90	10 — 100	10 — 110	10 — 120
11 — 22	11 — 33	11 — 44	11 — 55	11 — 66	11 — 77	11 — 88	11 — 99	11 — 110	11 — 121	11 — 132
12 — 24	12 — 36	12 — 48	12 — 60	12 — 72	12 — 84	12 — 96	12 — 108	12 — 120	12 — 132	12 — 144

Photographic alterations of the scale of a picture in one dimension only.

EXERCISE BOOK.

Calculations in a separate book

Name..........

Subject..........

Standard..........

f. 2

Desiring for purposes in which I was interested to reproduce photographs that should be reduced in one direction only I thought of turning the picture through a small angle, of photographing it in that position and then photographing the negative after turning it through a small angle in the opposite direction. A square would be turned into a four sided figure with the nearer side longer than the further one by the first process, and the action would be reversed by the second and possibly as I hoped the result might be a figure narrower than the square and approaching to a rectangle with useful precision. On working out the problem I however found to my surprise that the result would be not an approximate but an exact rectangle if the second angle of rotation ψ, bore to the first angle of rotation φ, the simple relation of ~~tan~~ $\sin\psi = \tan\varphi$. This is true whatever be the value of φ and whatever be the distance of the camera, as I shall shortly show.

This fact seems worthy of record ~~and~~ to justify ~~to~~ the publication of the annexed tables as the power of accurate photographic retraction in one direction only may be useful in other ways than those which I had in view. ~~The process is simple~~ Of course the picture might be hung so far from the camera that ~~the a simple inclination would suffice for the required reduction~~ its perspective appearance when inclined would differ insensibly from a rectangle & the resultant photograph might be enlarged; but that is hardly practicable for the enlargement would ~~have to~~ be too great for good photographic work. On my method the process is very simple a symmetrical camera is used with ~~the~~ a lens of small aperture half way between the ~~vertical~~ axis of the picture & the focussing screen, It must be so small that a moderate difference in distance is attended by no considerable change of focus. The frame that holds the picture stands on a horizontal circle, graduated like a theodolite. The table gives the values of φ & ψ suitable for the required lateral reduction. The picture is inclined to $90^\circ + \varphi$ from the optical axis and is photographed in that position on sensitive paper in the camera. After development the negative is substituted for the picture, the frame is turned to $90° - \psi$ and a fresh photograph is made on paper in the camera, which ~~this~~ will be a positive, reduced as desired.

Show specimens

represents a horizontal section of the part concerned passing through the optical axis which

I will now explain the problem ~~which~~ is one of simple perspective. In fig I, E is the eye. EO the optical axis. ~~A~~ AB is the line where the vertical plane of projection cuts the plane of the paper. ~~A~~$a_0 O b_0$ is the lower edge of the support of the ~~frame~~ picture, which can be rotated in the first instance through an angle φ (measured from OA) into the position $a_1 O b_1$, and ~~afterward~~ then backwards past its original position through an angle ψ measured from OA ~~beyond it~~ beyond it. Let $Oa_0 = Ob_0 = r$ and $EO = d$. The projection of Oa_1 will be OA_1 & for the present the attention will be confined to what takes place in the plane of the paper. Then

$A_1O : r\cos\varphi$ as $d : d + r\sin\varphi$ $\quad A'O = \dfrac{d \cdot r\cos\varphi}{d + r\sin\varphi} = S_1$ say

f. 7v

[illegible] i, [illegible] t t, required value of $\frac{t}{r}$

The [illegible]

$$s = \frac{d r \cos\varphi}{d + r \sin\varphi} \qquad t = \frac{d s \cos\psi}{d - s \sin\psi}$$

$$t = \frac{\dfrac{d \cos\psi \times d r . \cos\varphi}{d + r \sin\varphi}}{d - \dfrac{\sin\psi \, d r \cos\varphi}{d + r \sin\varphi}} = \frac{d^2 r \cos\varphi . \cos\psi}{d^2 + d r \sin\psi - d r \sin\psi . \cos\varphi}$$

$$\left(\sin\psi . \cos\varphi = \frac{\sin\varphi}{\cos\varphi} . \cos\varphi = \sin\varphi\right)$$

$$= \frac{d^2 r . \cos\varphi . \cos\psi}{d^2 + d r \sin\varphi - d r \sin\varphi}$$

$$\frac{t}{r} = \ldots \cos\varphi . \cos\psi$$

Using analogous relation for Ob_0

$OB_1 : r\cos\varphi$ as $d : d - r\sin\varphi$, $\quad OB_1 = \dfrac{dr\cos\varphi}{d - r\sin\varphi} = s'$

Take a negative of the picture in this position

Restore the frame to its original position and ~~Now so The~~ negative ~~of the picture~~ ~~will now~~ replace the picture with its the point O being the same in both. Let A_1O be called s, & OB_1 be s'.

~~Then~~ The position of the negative in the frame will then be A_1OB_1.

Rotate the frame through ψ into the position a_2Ob_2.

The projection of Oa_2 will be OA_2 &

$$t : s\cos\psi \text{ as } d : d - s.\sin\psi \quad \text{or} \quad t = \frac{ds\cos\psi}{d - s\sin\psi}$$

Similarly $\quad t' = \dfrac{ds'\cos\psi}{d + s'.\sin\psi}$

Let OA_2 be called t and OB_2 be t'

In order that the original square of the picture may be reduced to a symmetrical rectangle t must be equal to t'

$$\text{or} \quad s(d + s'.\sin\psi) = d(d - s\sin\psi)$$

Substituting for s and s'

$$\frac{dr.\cos\varphi}{d + r\sin\varphi}\left\{d + \frac{dr\cos\varphi}{d - r\sin\varphi}.\sin\psi\right\} = \frac{dr\cos\varphi}{d - r\sin\varphi}\left\{d - \frac{dr\cos\varphi}{d + r\sin\varphi}.\sin\psi\right\}$$

which after effecting the multiplication reduces itself to

$$2\sin\varphi = 2\cos\varphi.\sin\psi$$

$$\text{or} \quad \tan\varphi = \sin\psi.$$

$s = \dfrac{dr\cos\varphi}{d + r\sin\varphi}$ $\quad s' = \dfrac{dr\cos\varphi}{d - r\sin\varphi}$ $\quad \tan\psi = \sin\varphi$

$t = \dfrac{ds\cos\psi}{d - s\sin\psi}$ $\quad t' = \dfrac{ds\cos\psi}{d - s'\sin\psi}$ $\quad \operatorname{cotan}\psi = \cos\varphi$

(1) $\underset{r=1}{d=0}$ $\quad s = s' = 0 \quad t \ \frac{0}{\alpha} = t'$

$\underset{r=1}{d=1}$ $\quad s = \dfrac{\cos\varphi}{1+\sin\varphi} \quad s' = \dfrac{\cos\varphi}{1-\sin\varphi}$, when $\sin\varphi = 1$ $s' = \frac{\cos\varphi}{0} = \alpha$

$\underset{r=1}{d = \text{very large}}$ $\quad s = \cos\varphi =$

$t = s\cos\psi.\cos\varphi.$ If $\varphi = 30$ $\ t = \cdot 7069$

the effect then, of changes in value of d for $\varphi = 30$ does not affect the first 2 decimals

The projections of the sides of the square will now be considered. ~~each~~ let that at a_0 be called h_0 that at b_0 be k_0 and the axis of the square at O which never changes its position will be called h. Then $h_0 = k_0 = h$.

The projection of h_0 when it has been moved to a_1 will stand above A_1 and will be of a height which will be called h_1

$h_1 : h_0$ (that is h) as $d : d + r \sin\varphi$ $\quad h_1 = \dfrac{dh}{d + r.\sin\varphi}$

similarly using the analogous notation of k with the b & B series

$$k_1 = \frac{dh}{d - r.\sin\varphi}$$

When h_1 stands at a_2 let its projection standing at A_2 be called h_2

then $h_2 : h_1$ as $d : d - s.\sin\psi$ $\quad h_2 = \dfrac{dh_1}{d - s.\sin\psi}$

$$\& \quad k_2 = \frac{dk_1}{d + s'.\sin\psi}$$

on reducing these equations after substituting for s, s', & ψ,

we get $h_2 = h_0 = k_0 = k_2$.

As $\underline{d}$ disappears, the reduction for any angle φ is independent of $\underline{d}$.

Tables are annexed that give the results for different values of φ which ~~cover~~ pass through every degree of reduction from 1 to 0. Conversely tables can hardly

f. 10r

be calculated directly, but to find the values of φ & ψ for any required degree of reduction, recourse must be had to interpolation.

It follows from the equation $\tan \varphi = \sin \psi$ that when $\varphi = 45^\circ$ $\psi = 90^\circ$. In this case the ~~frame~~ plane of the picture coincides with the vertical plane passing through the optical axis, and ~~the~~ its perspective is a straight line. Up even to $\varphi = 40^\circ$ the reduction proceeds by stages that do not ~~[illegible]~~ increase with excessive rapidity; thus at 10° the reduction is .97, at 20° .87; at 30° .71 at 40° it is .41 but between 40° & 45° it descends from ~~the latter figure~~ .41 to zero. at an ~~with~~ rapidly increasing rate. Probably ~~then~~ owing to the necessity of attending to the exact angle extreme reductions wd rarely be used ~~to the latter extent~~, but to reductions of at least one-half, ~~they~~ are ~~perfectly~~ easy.

$r = r' = 1 \quad \varphi = 30°$

d	s	s'	k	k'
2°	6928	1.154	8000	1.334
3°	7423	1.040	857	1.200
4°	7698	9897	8890	1.143
5°	7872	9623	9090	1.112
6°	7994	9448	9232	1.091
7°	8081	9327	9333	1.076
8°	8151	9215	9412	1.067
9°	8204	9168	9473	1.059
10°	8247	9116	9546	1.053
15°				
20°	8433	8882	9982	1.026
30°	8519	8810	9835	1.018
40°	8553	8770	9877	1.013
50°	8574	8748	9901	1.010

F. 11

(This Table sh'd be recalculated with at least 5-place logarithms; it also requires revision.)

For $r = 1.0$ and $d = 5.0$

Angles of rotation		Widths in 1st projn		Final widths	Sides in 1st projn		1st projn
φ	ψ	s	s'	t	h_1	k_1	slope of top
10°	10°.9'	·952	1.021	·970	·966	1.036	2°. 0
20°	21°.21'	·878	1.009	·874	·936	1.074	2°.24' ?
25°	27°.48'	·836	·990	·802	·924	1.092	5°.15
30°	35°.16'	·787	·962	·707	·909	1.111	6°.36
35°	44°.27'	·735	·925	·535	·897	1.130	8°.0'
40°	57°.4'	·679	·879	·416	·886	1.147	7°.30'
45°	90° -	·620	·824	0.000	·876	1.165	11.18

~~Similar tables to be calculated for values of d~~
~~= 20 = 10 & perhaps others~~

It will be observed that a photograph which has been so taken that lines intended to appear vertical have converged, admits of being adjusted by these ~~first of the two~~ processes. In the first instance the slope has to be eliminated by an appropriate angle φ of rotation, best perhaps effected by the eye but it could be done by measurement & the column "slope" in the Table. The picture will then be narrowed & wd be brought into just proportion by the combined processes so as to narrow it proportionately in the direction crossways. The result would be the negative of a smaller picture in correct proportion.

=

Over

f. 13

Resemblance

Resemblance between two things may be defined either as the tendency of the one to suggest the other, or as the tendency of the one to be mistaken under specified conditions for the other. It is really an ~~personal~~ individual judgment, one observer noting a strong likeness where another sees none, but the independent estimates of many persons give a statistical assurance that is of value in estimating absolute resemblance. For example, I happened to be photographed by a person who ~~was good~~ as a usual matter of business took my likeness in six different aspects & sent the proofs to me to choose from. I marked them on the back with the letters A to F, and asked friends as occasion offered, to arrange them in what they severally considered to be their order of merit as likenesses. Each ~~was~~ friend was taken apart from the rest to do this that his judgment might be quite independent. The noted results ~~were noted~~ were curious. No picture gained the first place in general estimation, ~~but~~ there were not a few gross incongruities between the judgements but it was ~~easily~~ easy to arrange them in ~~an~~ the order that most nearly expressed "average opinion". By some such method, grades of resemblance may be treated from the statistical view-point, but I do not propose now to deal with this part of the subject; ~~it is~~ intending to confine my remarks to the measurement of ~~individual~~ grades of resemblance as perceived by an individual. Here in its simplest form we stumble across the difficulty that individual judgment is not stable, resemblances are suggested under some surroundings and not in others but that difficulty can be ~~[illegible]~~ practically eliminated by making the comparisons under definite & stated conditions. The question must be approached in its simplest forms. ~~the one~~ one of the earlier comparisons that ~~we~~ I will consider is ~~whether~~ how far the ~~two~~ photographs, alike in aspect, shades & ~~tone~~ and background, ~~of two~~ of two different ~~persons~~ busts, appear to resemble one another when they are placed side by side. but this, simple as it may be is far too complex for a first step.

Let us begin with two plain rectangles, set with their corresponding sides parallel. Are they or are they not alike could they under any & what circumstances be mistaken

f. 15r

for one another in respect to shape as distinguished from size?

1°0'0" 8.241 8853
9215
1206 0362

f. 15v

		tan	cos
φ	1° - -	8.241 9215	φ 9.999 9338
ψ	~~1 30~~ 50 1° 0' 1/3		9338 ψ 9.999 ~~8484~~
t		·99995	9.999 ~~7822~~ .8676
φ	2 - -	8.543 0838	9.999 7354
ψ	2° 0' 4"		9.999 7350
t		·99878	9.999 4704
φ	3	8.719 3958	9.999 4044
ψ	3 0 15"		9.999 4027
t		·99726	9.998 8081
φ	4° - -	8.844 6437	9.998 9408
ψ	4 0' 35"		9.998 9357
		·99572	9.997 8765
φ	5°	8.941 9518	9.998 3442
ψ	5 1' 10"		9.998 3313
t		·99237	9.996 6755

f. 16r

				log tan	log Cos
φ	6°	–	–	9.021 6202	9.997 6143
ψ	6°	2'	–		9.997 5877
t				.989000	9.995 2020
φ	7°	–	–	9.089 1438	9.996 7507
ψ	7	3'	10"		9.996 7014
t				.98503	9.993 4521
φ	8°			9.147 8025	9.995 7528
ψ	8°	4'	50		9.995 6665
				.98043	9.991 4193
φ	9°	–	–	9.199 7125	9.994 6199
ψ	9	6'	50		9.994 4823
t				.97522	9.989 1022
φ	10			9.246 3188	9.993 3515
ψ	10	9	20		9.993 1419
t				.96938	9.986 4934

f. 16v

	°	′	″	log tan.	log cosine
φ	11°	–	–		9.991 9466
ψ	11	12	31.	9.288 6523	9.991 6366
t				·96290	$\bar{9}$.983 5832
φ	12°	–	–	9.327 4745	9.990 4044
ψ	12	16	20		9.989 9607
t				·95580	9.980 3651
φ	13°			9.363 3641	9.988 7239
ψ	13	21	–		9.988 1029
t				·94804	9.9768268
φ	14°	–	–	9.396 7711	9.986 9041
ψ	14	26′	10″		9.986 0665
t				·93966	9.9729706
φ	15°	–	–	9.4280525	9.984 9438
ψ	15	32	31″		9.983 8228
t				·93061	9.9687666

f. 17r

	°	′	″	log tan	log cosin
φ	16	–	–	9.457 4964	9.982 8416
ψ	16	39	50		9.981 3671
t				·92089	9.964 2087
φ	17°	–	–	9.485 3390	9.980 5963
ψ	17	48	10		9.978 6892
				·91051	9.959 2855
φ	18°	–	–	9.511 7760	9.978 2063
ψ	18	57	40		9.975 7715
t				·89945	9.953 9778
φ	19°	–	–	9.536 9719	9.975 6701
ψ	20	8	29		9.972 5935
t				88770	9.948 2636
φ	20°	–	–	9.561 0659	9.972 9858
ψ	21°	20	40		9.969 1405
t				·87520	9.942 1263

	°	'	''	log tan	log cos
φ	21°	–	–	9.584 1774	9.970 1517
ψ	22	34'	20		9.965 3882
t				·86206	9.935 5399
φ	22°	–	–	9.606 4096	9.967 1659
ψ	23°	49	50		9.961 2997
t				·84813	9.928 4656
φ	23°	–	–	9.627 8519	9.964 0261
ψ	25°	7'	10''		9.956 8524
t				·83345	9.920 8785
φ	24°	–	–	9.648 5831	9.960 7302
ψ	26	26'	20		9.952 0218
t				81800	9.912 7520
φ	25°	–	–	9.668 6725	9.957 2757
ψ	27°	47'	40''		9.946 7598
t				·80174	9.904 0355

f. 18r

	°	′	″	log tan	log cos
φ	26°	–	–	9.688 1818	·9.953 6602
ψ	29°	11′	30		9.941 0108
t			+0.9	78465	9.894 6710
φ	27°	–	–	9.707 1659	9.949 8809
ψ	30°	37′	50″		9.934 7360
t				·78669	9.884 6169
φ	28°	–	–	9.725 6744	9.945 9349
ψ	32	7′	20		9.927 8402
t				·74778	9.873 7751
φ	29	–	–	9.743 7520	9.941 8193
ψ	33°	39	40″		9.920 2958
t				·72797	9.862 1151
φ	30	–	–	9.761 4394	9.937 5306
ψ	35	15′	50″		9.911 9571
t				·70711	9.849 4877

f. 18v

	°	′	″	log tan	log cos
φ	31°	–	–	9.778 7737	9.933 0656
ψ	36	55	50		9.902 7448
t				·65549	9.8358104
φ	32°	–	–	9.795 7892	9.928 4205
ψ	38	40′	20″		9.892 5028
t				·66210	9.8209233
φ	33°	–	–	9.812 5174	9.923 5914
ψ	40°	29′	50″		9.881 0635
t				·63776	9.8046549
φ	34°	–	–	9.828 9874	9.918 5742
ψ	42°	25′	0″		9.868 2088
t				·61205	9.7867830
φ	35°	–	–	9.845 2268	9.913 3645
ψ	44°	26′	40″		9.853 6555
t				·58482	9.7670200

	°	′	″		
φ	36°	–	–	9.861 2610	9.907 9576
ψ	46	35	50		9.837 6343
t				·55590	9.744 9919
φ	37°	–	–	9.877 1144	9.902 3486
ψ	48°	54′	0″		9.817 8133
t				·52621	9.720 1619
φ	38°	–	–	9.892 8098	9.896 5321
ψ	51	22	40		9.795 3117
t				·49186	9.691 8438
φ	39°	–	–	9.908 3692	9.890 5026
ψ	54°	4′	29″		9.768 4351
t				·45597	9.658 9377
φ	40°	–	–	9.923 8135	9.884 2540
ψ	57	2	40′		9.735 5896
t				·41672	9.619 8436

f. 19v

	°	′	″	log tan	log cos.
φ	41°	–	–	9.939 1631	9.877 7799
ψ	60°	22	31		9.694 0092
t				.37307	9.571 7891
φ	42°	–	–	9.954 4374	9.871 0735
ψ	64°	12′	40″		9.638 5457
t				.32332	9.509 6292
φ	43	–	–	9.969 6559	9.864 1275
ψ	68	49′	50″		9.557 6603
t				.26411	9.421 7878
φ	44	–	–	9.984 8372	9.856 9341
ψ	74	56′	50″		9.414 4865
t				.18682	9.271 4206
φ	44°	30′	–″	9.992 4197	9.853 2421
ψ	79	19	31″		9.267 7297
t				.13212	9.120 9718

f. 20r.

45.			0.000 0000	9.849 4850 – ∞ inf.

φ	ψ	t	φ	ψ	t
1°	1°. 0'	1.000	21°	22°. 34'	.862
2°	2°. 0'	.999	22°	23°. 50'	.848
3°	3°. 0'	.997	23°	25°. 7'	.833
4°	4°. 1'	.995	24°	26°. 26'	.818
5°	5°. 1'	.992	25°	27°. 48'	.802
6°	6°. 2'	.989	26°	29°. 12'	.785
7°	7°. 3'	.985	27°	30°. 37'	.767
8°	8°. 5'	.980	28°	32°. 7'	.748
9°	9°. 7'	.975	29°	33°. 40'	.728
10°	10°. 9'	.969	30°	35°. 16'	.707
11°	11°. 13'	.963	31°	36°. 56'	.685
12°	12°. 16'	.956	32°	38°. 40'	.662
13°	13°. 21'	.948	33°	40°. 30'	.638
14°	14°. 26'	.940	34°	42°. 25'	.612
15°	15°. 33'	.931	35°	44°. 27'	.585
16°	16°. 40'	.921	36°	46°. 36'	.556
17°	17°. 48'	.911	37°	48°. 54'	.526
18°	18°. 48'	.910	38°	51°. 23'	.492
19°	20°. 8'	.888	39°	54°. 4'	.456
20°	21°. 21'	.875	40°	57. 3'	.417

φ	ψ	t
~~41°~~	~~57°.3~~	
41°	60°.23'	.373
42°	64°.13'	.323
43°	68°.56'	.264
44°	74°.57'	.187
45	90°.0'	.000

φ	ψ	log tan	log cos
φ. 18°.37′	41° 10	9.527 4508	9.976 6597
ψ			9.973 8444
t		89230	9.950 5041
		1770	
φ. 18.40′	19°.44′.40″	9.528 7021	9.976 5318
			9.973 6860
		89170	9.950 2178
		3′ 50	
18°.35′	19.38′ 50″	9.526 6150 =	9.976 7447 =
		=	9.973 9498
		89268	9.950 6945 — 19/13/0/15
	5′	98 2.268	80
			460
18°.20′		9.520 3052	9.977 3772
	19° 21′ 0″		9.974 7475
		89562	9.952 1247
		5′ 294	
φ 18.10′		9.516 0575	9.977 7938
ψ.	19.9′.20″		9.975 2623
t		99859	9.953 0561
		97	
		141	

f. 22v

φ	17°.40′	9.503 1092	9.979 0192
ψ	18°.34′.20″		9.976 7730
t		90323	9.955 7922
	1.8° =	8995	

10′ make 37 unit of diff

37 : 10 :: 32 : x   32   x = 37/320 (8′.40″

296

240

tan ψ = tan φ

~~t = cos ψ . cos φ = cotan φ . cos φ~~

~~log t = log cotan φ = cos φ~~

~~use a moveable card with log t on it & apply to the columns to log cotan & find value of φ that satisfies~~ the Equation

~~t = cos ψ × cos φ~~ $\frac{\sin \psi}{\cos \psi} = \tan \varphi$

$\varphi = 17^\circ$ $t = 0.9105$ 9105

$= 18^\circ$ $t = 0.8995$ 9000

110 105

$105 : 110 :: x : 60$

$11 \overline{) 630} \; 57' . 16''$

55

80

77

$\frac{3}{3 \times 60} = ''/ \frac{180}{16}$

$\varphi = 17^\circ\ 57'\ 16$	9.510 6285	9.978 3156
$\psi = 18\ 54\ 30$		9.975 9087
+	89996	9.954 2243
$\varphi = 17^\circ . 57' . 30$	9.510 7003	9.978 3088
$18^\circ . 54' . 40''$		9.975 9015
	8991	9.954 2103
$\varphi = 17^\circ . 57'$ –	9.510 4849	9.978 3295
$\psi = 18^\circ . 54' . 10''$		9.975 9231
$t =$	.90002	9.954 2526

f. 23v

		t	
$\varphi =$	$25° - -$	802	
"	$26° - -$	785	$17/\frac{120}{119}(7$
		17 : 60 :: 2 : t	
$\varphi =$	$25°.7' -$	9.670 9774	9.956 8623
ψ	$27°.57'.20$		9.946 1139
		79979	9.9029762
$\varphi =$	$25.5'$	9.670 3197	9.956 9806
	$27°\ 54'\ 35''$		9.946 3037
		80016	9.903 2843
φ	$25.6'0''$	9.670 6486	9.956 9215
ψ	$27.56.0''$		9.946 2030
t		80006	9.903 1245

f. 24ar

30° 7071

31° 6852

219 : 60′ : 71 : x′

219) 4260 (19′. 26″
219
2070
1971
99 × 60″
219) 5940 (26″
438
1560
1314

φ	30° 19′ 20″	9.767 0617	9.936 1113
ψ	35° 47′ 40″		9.909 0854
t =		·70016	9.845 1967

φ 34 — 612
35 585
27 : 60 = 12 : x

x = 27) 720 (26′ 40″
54
180
162
18 × 60
27) 1080″ (40″
108

φ =	34°. 26′. 40″	9.836 2318	9.916 2828
ψ =	43° 18′ 10″		9.861 9760
t =		60015	9.778 2588

f. 24av

34) 1560 (45'. 5"
136
200
170
30 × 60

34) 1800" (53
170
100

φ 37° - - 526
38° 492

34 : 60 :: 26 : x

φ	37° 45' 0	9.888 8996	9.898 0060
ψ	50° 44' 20"		9.801 3046
t		.500 40	9.699 3106

φ 17° 57'	9.978 3293
ψ 18° 54'	9.975 9303
	9.954 2596
t 9000.3	

φ 25° 6	9.956 9215
ψ 27.56	9.946 2032
t 8000 6	9.903 1247

φ 30° 19	9.936 1360
ψ 35 48	9.909 0550
t 70015	9.845 1910.

φ 34.27	9.916 2539
ψ 43.18	9.861 9958
t 60013	9.778 2497

φ 37.45	9.8980060
ψ 50.44	9.801 3561
t 50045	9.699 3621

t unequal 6' Chords = 2 sin ½ φ ½ ψ

grade t	½φ	½ψ	sines ½φ	½ψ	chords φ	ψ
9	9°	9° 30'	156	165	312	330
8	12° 30'	14° -	216	242	432	484
7	15° 6	17° 54'	261	307	522	614
6	17° 12	21° 36'	296	368	592	736
5	18° 54'	25° 24'	324	429	648	858

f. 24b

Photographic change of Scale in One Dimension only

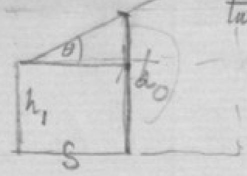
Made a pasteboard plate-holder & tried 
The plate & screen were not parallel however I got promising results

Improved the plate holder somewhat, using wood. put (H) in one of the corners
circles for distortion. Results better – sent them to Pearson to see

Made as good a plate-holder as I could. and rearranged
the camera with firm wooden cheeks. With care it ought to give
results nearly as good as a workmanlike article. The apparatus
was wholly readjusted, and a new disc for angle of rotation made & placed
Also I determined distance from plate to screen = 26" as near as I
could measure. Also determined not to use a portrait above 2" square.
This allows for the magnification of the advanced side in the φ process.

Oct 27. Did a few ~~[illegible]~~ of the diagram of the problem
Kopft – (glass) 1 neg. of φ for ·5 reduction; (paper) 1 ditto, also 1 full.

Oct 28 Neg: $\underline{b}$ in camera on paper ¼ plate of  φ of 0.5 reduction, fixed it, washed
for 10 minutes, laid it on a glass lantern plate, smoothly turning the
long ends round to back. Mounted it in camera & took the positive 0.5
from it C. Then restored b to the working trough & washed & fixed $\underline{b}$
and $\underline{C}$



$\tan \text{slope} = \dfrac{h_0 - h_1}{s} = \tan \theta$ say

$h_1 = \dfrac{h_0 d}{d + r.\sin\varphi}$

$S = \dfrac{d r \cos\varphi}{d + r.\sin\varphi}$

0. 8062

9.8938

$$\tan\theta = h_0 - \frac{h_0 d}{d + r\sin\varphi} \times \frac{d + r.\sin\varphi}{d r \cos\varphi}$$

$$= \frac{\cancel{h_0 d} + h_0 r \sin\varphi - \cancel{h_0 d}}{\cancel{d + r.\sin\varphi}} \times \frac{\cancel{d + r\sin\varphi}}{d r \cos\varphi} = h$$

$\frac{h_0}{d_0}$ $\frac{1}{6.4} = .156$ $\frac{}{2} = 0.0781$

$$= \frac{h_0 r \sin\varphi}{d r \cos\varphi} = \frac{h_0}{d}\tan\varphi = \tan\theta$$

$\varphi = 90^0 = 0$

$\varphi = 90^0 = 0$

when d_0 is large $\theta = 0$

φ	$\tan\varphi$	$\frac{h_0}{d}\tan\varphi$	θ
20^0	.3640	.2842	$15^0.51$

$h_0 = 1 \quad d = 2 \times 6.4 = 12.8$

$\frac{h_0}{d} = .0781$

F. 24cv

0692747/00683375 0820

0054599 9 .2×.2=.0402

00683375 .08×.8 =.064

.0781

38|.7813|.06099 61 13/.78

$s = \frac{d\cos\varphi}{d+\sin\varphi}$; $s' = \frac{d\cos\varphi}{d-\sin\varphi}$; $k = \frac{d}{d+\sin\varphi}$; $k' = \frac{d}{d-\sin\varphi}$

$\varphi = 30^\circ$ $\cos\varphi = 0.8660$ $\sin\varphi = 0.5000$

		logs								
$d = 7$		0.8451	0.7826	0.7826	$d=8$	0.9031	0.9031		0.8406	0.8406
$d\times\cos\varphi$	6.0620	0.7826	0.8129	0.8751	$d\times\cos\varphi$	6.9280	0.8406		0.9294	0.8751
$d+\sin\varphi$	7.5000	0.8751	~~0.0322~~ 9.9697	~~9.9700~~ 9.9075	$d+\sin\varphi$	8.5000	0.9294		9.9112	9.9645
$d-\sin\varphi$	6.5000	0.8129	s' .9326	s .8081	$d-\sin\varphi$	7.5000	0.8751		s .8151	.9215 s'
			0.8451	0.8451					0.9031	0.9031
			0.8751	0.8129					0.9294	0.8751
			9.9700	0.0322					9.9737	0.0280
			k .9333	1.076					k .9412	1.067 k

	vals	logs				logs		
$d = 9$		0.9542	0.8917	0.8917	$d = 20$	1.3010	1.2385	1.2385
$d\times\cos\varphi$ =	7.7940	0.8917	0.9777	0.9294	$d\cos\varphi$ 17.32	1.2385	1.3118	1.2900
$d+\sin\varphi$ =	9.5000	0.9777	9.9140	9.9623	$d+\sin\varphi$ 20.5	1.3118	9.9267	9.9485
$d-\sin\varphi$	8.5000	0.9294	s .8204	s' .9168	$d-\sin\varphi$ 19.5	1.2900	.8433	.8882
			0.9542	0.9542			1.3010	1.3010
			0.9777	0.9294			1.3118	1.2900
			9.9765	0.0248			9.9992	0.0110
			.9473	1.059			.9982	1.026

	logs		
$d = 30$	1.4771	1.4148	1.4148
$d\times\cos\varphi$ = 25.98	1.4148	1.4843	1.4698
$d+\sin\varphi$ 30.5	1.4843	9.9305	9.9450
$d-\sin\varphi$ 29.5	1.4698	.8519	.8810
		1.4771	1.4771
		1.4843	1.4698
		9.9928	0.0073
		.9835	1.016

	logs		
$d=7$	0.8451		
$d\cos\varphi$ 6.062	0.7826	.7826	.7826
$d+\sin\varphi$ 7.500	0.8751	.8751	.8129
$d-\sin\varphi$ 6.5000	0.8129	9.9075	9.9697
		.8081	.9327
		.8451	.8451
		.8751	.8129
		9.9700	0.0322
		.9333	1.076

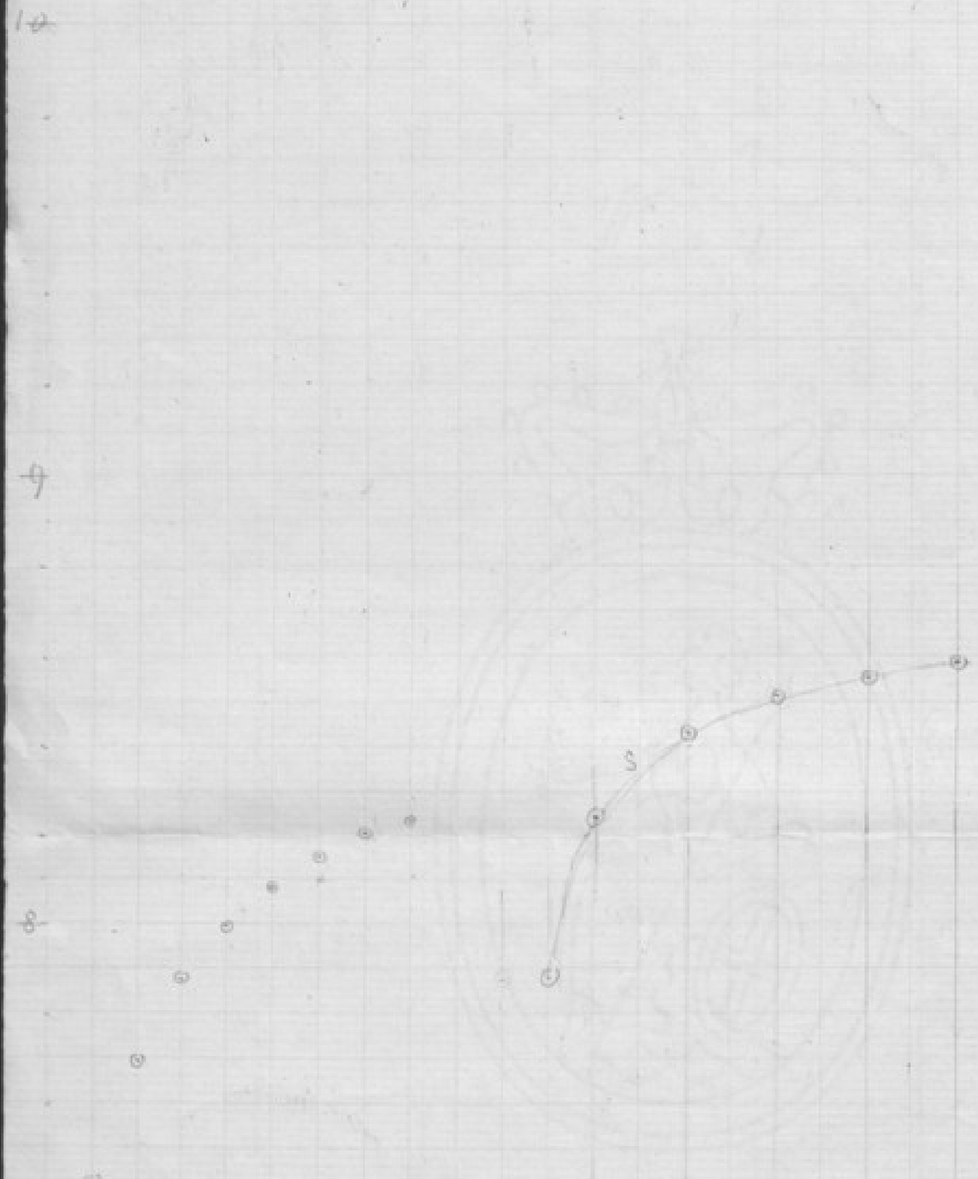
$d = 40$	1.6021	1.5396	1.5396	$d=50$	1.6990	~~1.6365~~	~~1.6365~~
$d\cos\varphi$ 34.64	1.5396	1.6075	1.5966	$d\cos\varphi$ 43.30	1.6365	~~1.6994~~	~~1.6906~~
$d+\sin\varphi$ 40.5	1.6075	9.9321	9.9430	$d+\sin\varphi$ 50.5	1.7033	~~9.9371~~	~~9.9459~~
$d-\sin\varphi$ 39.5	1.5966	s .8553	.8770	$d-\sin\varphi$ 49.5	1.6946	~~.8652~~	~~.8829~~
		1.6021	1.6021			~~1.6990~~	~~1.6990~~
		1.6075	1.5966			~~1.6994~~	~~1.6906~~
		9.9946	0.0055			~~9.9996~~	~~0.0084~~
		.9877	1.013			~~.9991~~	~~1.0080~~

1.6365	1.6365
1.7033	1.6946
9.9332	9.9419
.8574	.8748
1.6990	1.6990
1.7033	1.6946
9.9957	0.0044
.9901	1.010

log $d\cos\varphi$ log $d\cos\varphi$
log $d+\sin\varphi$ log $d-\sin\varphi$

d	$s-s'$ difference	
2	0.0595	114
3	0.0275	143
4	0.0174	264
5	0.0122	175
6	0.0087	121
7	0.0070	112
8	0.0053	47
9	0.0043	52
10		
20	0.0066	
30	0.0034	
40	0.0021	
50		

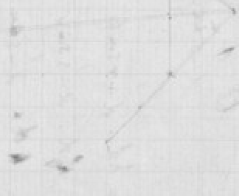
f. 24dv



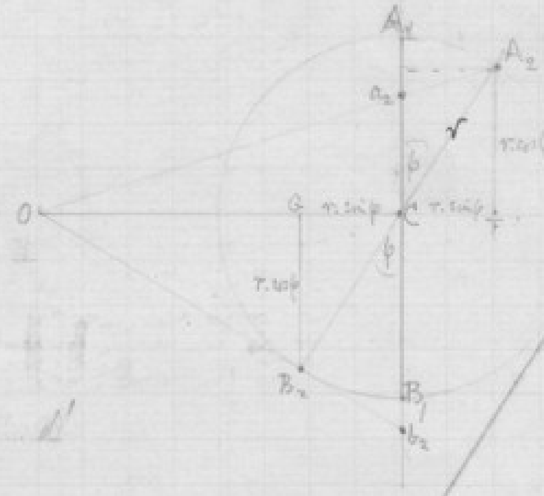
$d = 10$

φ	$\cos\varphi$	$\sin\varphi$	$d\cos\varphi$	$d-\sin\varphi$	$s' = \frac{d\cdot\cos\varphi}{d-\sin\varphi}$	$\sin\psi = \frac{\sin\varphi}{s'}$	ψ	$\cos\psi$	$s'\cos\psi$	$d s'\cos\psi$	$s'\sin\psi$	[illegible]	$d+s'\sin\psi$	$d\,s'\cos\psi$	$t' = \frac{d\,s'\cos\psi}{d+s'\sin\psi}$	t'
0°	1.000	0.000	10.000	10.000	1.00	0.000	0°	1.000	1.000	10.00	0.000	~~10.00~~	10.000	10.000	1.000	1.00
5°	.9962	.087	9.96	9.913	1.00	.087	5°.0'	.996	.996	9.96	.087	~~9.913~~	10.087	9.96	.987	99
10°	.985	.174	9.85	9.826	1.00	.176	10°.8'	.985	.985	9.85	.176	~~9.824~~	10.176	9.85	.966	97
15°	.966	.259	9.66	9.741	.992	.268	15°.33'	.963	.955	9.55	.266	~~9.734~~	10.266	9.55	.928	93
20°	.940	.342	9.40	9.658	.973	.364	21°.21'	.9310	.906	9.06	.354	~~9.646~~	10.354	9.06	.875	88
25°	.906	.423	9.06	9.577	.946	.466	27°.47'	.885	.837	8.37	.441	~~9.559~~	10.441	8.37	.805	80
30°	.866	.500	8.66	9.500	.912	.577	35°.14'	.817	.745	7.45	.526	~~9.474~~	10.526	7.45	.710	71
35°	.819	.574	8.19	9.426	.869	.700	44°.26	.714	.620	6.20	.608	~~9.392~~	10.608	6.20	.585	59
40°	.766	.643	7.66	9.357	.818	.839	57°.2'	.544	.445	4.45	.686	~~9.314~~	10.686	4.45	.416	42
41°	.755	.656	7.55	9.344	.808	.869	60°.21'	.495	.400	4.00	.702	~~9.298~~	10.702	4.00	.374	37
42°	.743	.669	7.43	9.331	.796	.900	64°.10	.436	.347	3.47	.716	~~9.284~~	10.716	3.47	.325	33
43°	.731	.682	7.31	9.318	.784	.932	68.45	.362	.284	2.84	.731	~~9.269~~	10.731	2.84	.266	26
44°	.719	.695	7.19	9.305	.772	.966	74.59	.259	.200	2.00	.746	~~9.254~~	10.746	2.00	.187	19
44½°	.713	.701	7.13	9.299	.767	.983	79.25	.184	.141	1.41	.754	~~9.246~~	10.754	1.41	.131	13
45°	.707	.707	7.07	9.293	.760	1.000	90°.0'	0.000	0.000	0.000	.760	~~9.240~~	10.760	0.00	0.000	0.00

F. 24er



f. 24ev



(1) $a_2C = r\cos\phi \cdot \dfrac{l}{l + r\sin\phi}$

$a_2O = \sqrt{\{(a_2C)^2 + l^2\}}$

(2) $b_2C = r'\cos\phi \cdot \dfrac{l}{l - r\sin\phi}$

$b_2O = \sqrt{\{((b_2C)^2 + l^2\}}$ $1/_{h}$

in vertical proportions

(3) $a_2\alpha_2 = h \cdot \dfrac{l}{l + r\sin\phi}$ call this

(4) $b_2\beta_2 = h' \cdot \dfrac{l}{l - r'\sin\phi}$ call this

write a similar formula to 3, in which $s = a_2C = \left\{r\cos\phi \cdot \dfrac{l}{l + r\sin\phi}\right\}$
take the place of r & equate it to h; & ψ take the place of ϕ

$$h\left\{\frac{l}{l + \left\{r\cos\phi \cdot \frac{l}{l + r\sin\phi}\right\}\sin\psi}\right\} = h$$

$$l = \frac{l + r\cos\phi \cdot \sin\psi}{l + r\sin\phi} =$$

$$1 = \frac{r\cos\phi \cdot \sin\psi}{l + r\sin\phi} \qquad \sin\psi = \frac{l + r\sin\phi}{r\cos\psi}$$

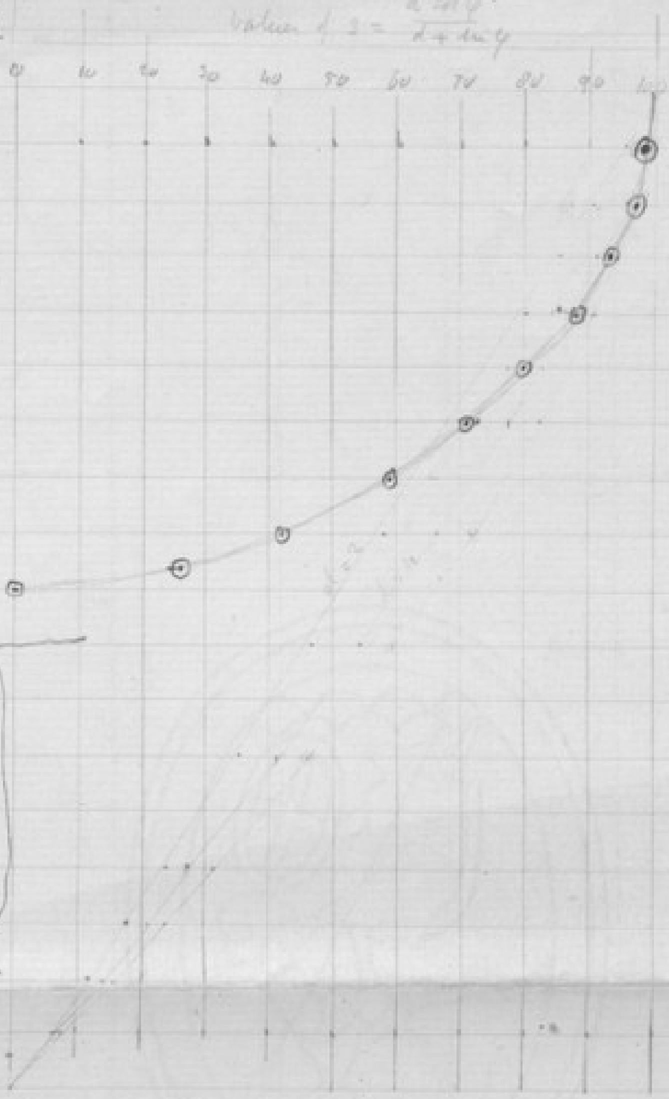
Similarly for 4 in which $s = b_2C = \left\{r'\cos\phi \cdot \dfrac{l'}{l - r\sin\phi}\right\}$

$$h'\frac{l}{l - \left\{r'\cdot\cos\phi \cdot \frac{l}{l - r\sin\phi}\right\}\sin\psi}$$

$$l = \frac{l - r'(\cos\phi \cdot \sin\psi}{l - r\sin\phi}$$

$$l - r\sin\phi = 1 - r'\cos\phi\ \sin\psi$$

φ	ψ	t
0°	0	1.00
5°	5°	0.99
10°	10°.8'	.97
15°	15°33	.93
20°	22°21	.88
25°	29°.47	.80
30°	34°.52	.71
35	44°.26	.59
40°	57°.2	.42
45°	90°.0	.00
41° 4Ƒ	60.21	.37
42° 4[illegible]	64.10	.33
43° 4[illegible]30	68.54	.26
44°	75.0	.19
44°.30'	79.28	.07



$s = \frac{d r \cos\varphi}{d + r \sin\varphi}$ $t = \frac{d s \cos\psi}{d - s \sin\psi}$

$\tan\varphi = \sin\psi$

φ (tangentes)	ψ (Sinus)	s	cos ψ	sin ψ	d.s cos ψ	s. sin ψ	d − s. sin ψ	t
5° 0.0875	5°. 1'	.989	.996	.087	~~.[illegible]~~ .985	.086	9.914	.994
10° 0.1763	10°. 9	.969	.985	.176	.954	.171	9.837	.970
15° 0.2679	15°. 32	.941	.964	.268	.907	.252	9.758	.930
20° 0.3640	21° 21'	.909	.931	.364	.846	.331	9.671	.875
25 0.4663	27°. 48	.870	.885	.466	.770	.405	9.595	.803
30 0.5774	35. 16	.825	.816	.577	.673	.476	9.524	.707
35 0.7002	44. 27	.778	.714	.700	.555	.545	9.465	.587
38 0.7813	51°. 23'	.742	.624	.781	.463	.578	9.422	.492
40 0.8391	57°. 3	.720	.544	.839	.392	.604	9.406	.417
41 0.8693	60° 23'	.708	.494	.864	.350	.612	9.390	.373
42 0.9004	64° 13'	.696	.435	.900	.303	.626	9.374	.324
43 0.9325	68. 50'	.685	.360	.932	.247	.638	9.362	.264
44		.672	.260	.966	.174	.649	9.35	.173

f. 248.v

Required $\frac{S}{r}$ when $d = 2r$ for values of $\varphi = 30^\circ$, $S = \frac{d\,r\cos\varphi}{d + r\sin\varphi}$

$d = 2,\ \varphi = 30^\circ \quad S = \frac{2\cos 30^\circ}{2 + \sin 30^\circ} = \frac{1.7320}{2.5000}$ log = 0.2385, log = 0.3979, 0.6928 log = 9.8406

$d = 2,\ \varphi = 45^\circ \quad S = \frac{1.4142}{2.7071}$ log = 0.1504, log = 0.4325, log 0.5223 = 9.7179

$d = 2,\ \varphi = 60^\circ \quad S = \frac{1.0000}{2.8660}$ log = 0.0000, 0.4573, log 0.3489 = 9.5427

$d = 2,\ \varphi = 80^\circ \quad S = \frac{0.3472}{2.9848}$ log = 9.5406, log = 0.4749, 0.1163 log = 9.0657

$d = 2,\ \varphi = 85^\circ \quad S = \frac{0.1744}{2.9962}$ log = 9.2415, log = 0.4766, 0.05820, 8.7649

$d = 2,\ \varphi = 90^\circ \quad S = \frac{0}{3} = 0$

$d = 2,\ \varphi = 20^\circ \quad S = \frac{1.8794}{2.3420}$ log = 0.2740, log = 0.3696, 9.9044 = $\log^{-1}$ 0.8024

$d = 2,\ \varphi = 10^\circ \quad S = \frac{1.9696}{2.1736}$ log = 0.2945, log = 0.3373, 9.9572 = $\log^{-1}$ 0.9061

$d = 2,\ \varphi = 40^\circ,\ S = \frac{1.5320}{2.6428}$ log = 0.1853, log = 0.4221, 9.7632 = $\log^{-1}$ 0.5797

$d = 2\ \varphi = 0^\circ \quad S = \frac{2}{2+0} = 1$

$d = 2,\ \varphi = 5^\circ \quad \frac{1.9924}{2.0871}$ log = 0.2993, log = 0.3196, 9.9797 = $\log^{-1}$ 0.9543 ✓

r = 1, d = 2

φ	S
0°	1.000
5	0.954
10	0.906
20°	0.802
30	
40°	0.580
45	0.522
50	
60	0.349
80	0.116
85	0.058
90	0.004

$S = \frac{d\cos\varphi}{d + \sin\varphi}$ given S and d find φ

S = 0.5, d = 2; $0.5 = \frac{2\cos\varphi}{2 + \sin\varphi}$; $1 + 0.5\sin\varphi = 2\cos\varphi$

$t = \cos\varphi . \cos\psi$

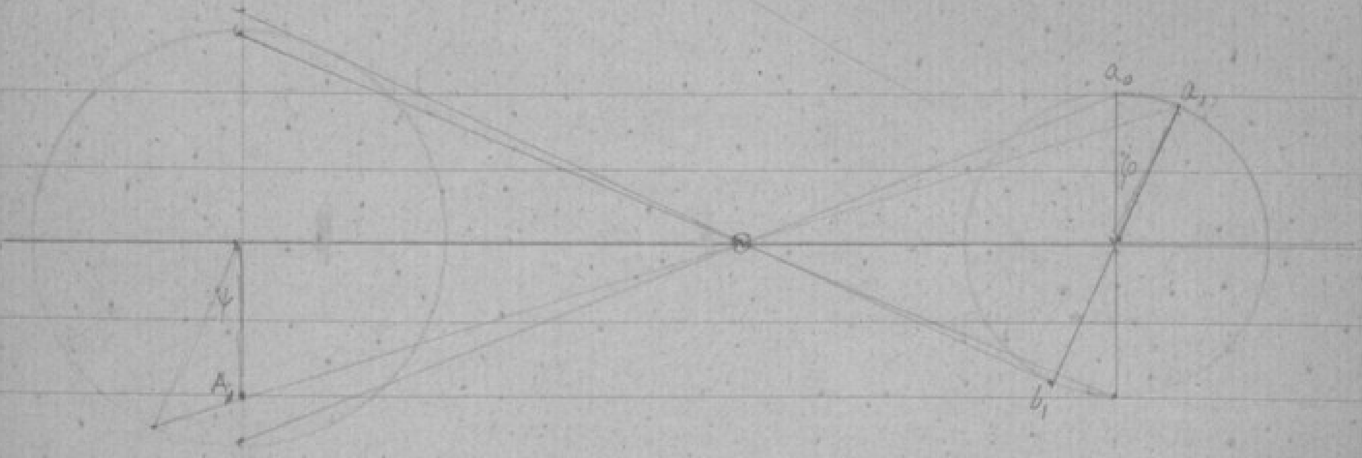
To nearest minute worked afresh

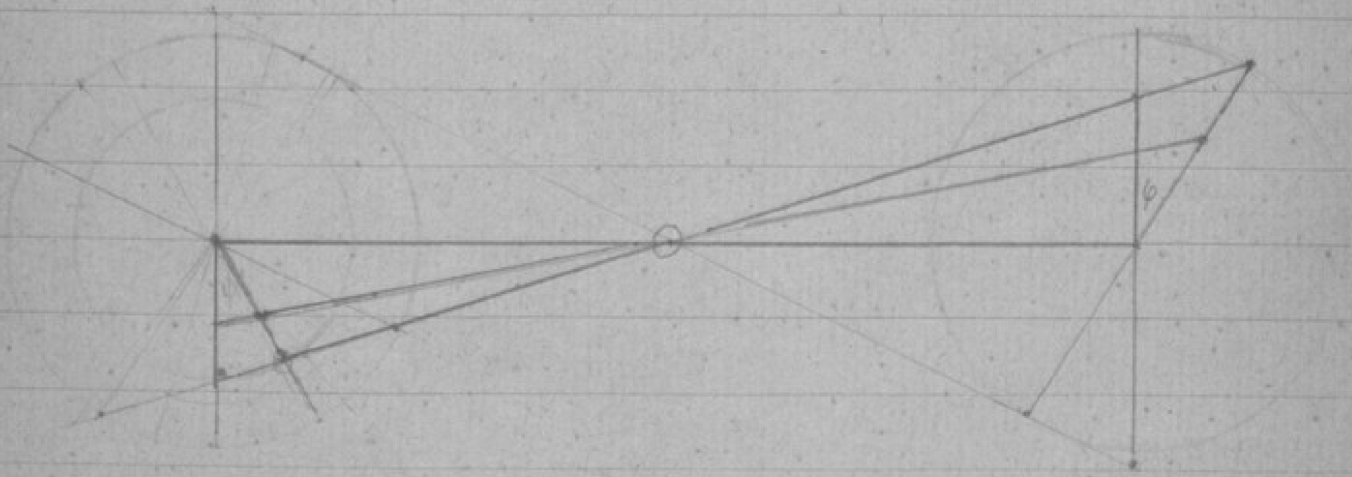
t	φ	ψ	φ	ψ	t
.9	17° 57′ 0″	18° 54′ 10	17° 57′	18° 54′	90003
.8	25° 6′ 0	27° 56′ 0″	25° 6′	27° 56′	80006
.7	30° 19′ 20″	35 47 40	30° 19′	35° 48	70015
.6	34° 26′ 40″	43 18 10	34° 27′	43° 18′	60013
.500	37° 45′	50° 44′ 20″	37° 45′	50° 44′	50045

$h_0 = 1$, $d = 2 \times 6''.4 = 12''.8$ $\frac{h_0}{d} = 0.0781$ $\tan\theta = \frac{1}{12.8}\tan\varphi = 0.0781 \tan\varphi$

φ	$\tan\varphi$	$\frac{h_0}{d}\tan\varphi$	θ	φ	$\tan\varphi$	$\frac{h_0}{d}\tan\varphi$	θ
1°	.0175	.0014	0°.5	21°	.3839	.0299	
2°	.0349	.0027	0°.9'	22°	.4040	.0315	
3°	.0524	.0041	0.14'	23°	.4245	.0330	
4°	.0699	.0055	0° 19	24°	.4452	.0347	
5°	.0875	.00693	0°. 24	25°	.4663	.0363	
6°	.1051	.00820	0°. 28	26°	.4877	.0380	
7°	.1228	.00960	0. 33	27°	.5095	.0397	
8°	.1405	.01161	0. 38	28°	.5317	.0414	
9°	.1584	.01233	0. 42	29°	.5543	.0432	
10°	.1763	.01374	0. 47	30°	.5774	.0450	
11°	.1944	.01515	0. 52	31°	.6009	.0469	
12°	.2126	.01655	0. 57	32°	.6249	.0487	
13°	.2309	.01804		33°	.6494		
14°	.2493	.01944		34°	.6745		
15°	.2679	.02093		35°	.7002		
16°	.2867	.02241		36°	.7265		
17°	.3057	.02389		37°	.7536		
18°	.3249	.02538		38°	.7813		
19°	.3453	.02694		39°	.8098		
20°	.3640	.02843	1°. 38'	40°	.8391		

f. 40





ARITHMETICAL TABLES.

Numeration Table.

Units	......
Tens	12
Hundreds	123
Thousands	1,234
Tens of Thousands	12,345
Hundreds of Thousands	123,456
Millions	1,234,567
Tens of Millions	12,345,678
Hundreds of Millions	123,456,789

Sterling Money Table.

4 Farthings ....1 Penny ...d.
12 Pence...............1 Shilling...s.
2 Shillings ....1 Florin
2 Sh. & Sixpence 1 Half Crown
5 Shillings ....1 Crown. .cr.
10 Shillings.......... 1 Half Sov.
20 Shillings, 1 Sov. 1 Pound ...£
21 Shillings...........1 Guinea

Arithmetical Signs.

+ Plus; Sign of Addition
— Minus; Sign of Subtraction
× Sign of Multiplication
÷ Sign of Division
= Sign of Equality
: : : : Sign of Proportion
√ Sign of the Square Root
∛ Sign of the Cube Root
° Degree, ' Minute " Second
∴ Therefore, ∵ Because

Troy Weight.

For Gold, Silver and Jewels.
24 Grains.......1 Pennywgt...dwt.
20 Pennywgts 1 ounce ....oz.
12 Ounces......1 Pound ....lb.

Apothecaries Weight.

For Mixing Medicines.
20 Grains......1 Scruple ....scr.
3 Scruples...1 Drachm ....dr.
8 Drachms...1 Ounce...........oz.
12 Ounces .....1 Pound ....lb.

Avoirdupois Weight.

For al. Goods except Gold, Silver, and Jewels.
16 Drachms 1 Ounce ....oz.
16 Ounces ...1 Pound ....lb.
14 Pounds ...1 Stone................st.
28 Pounds ...1 Quarter ....qr.
4 Quarters 1 Hundredwght.cwt
20 Cwt. ......1 Ton....................tn.

Hay and Straw Weight.

36 lb. Straw.........................1 Truss
56 lb. Old Hay.....................1 Truss
60 lb. New Hay ....1 Truss
36 Trusses ....1 Load

Long or Lineal Measure.

12 Lines.........1 Inch ....in.
12 Inches.......1 Foot ....ft.
3 Feet ....1 Yard ....yd.
2 Yards ....1 Fathom ....f.
5½ Yards ....1 Pole or Perch
40 Poles ....1 Furlong ...fur.
8 Furlongs or 1760 Yds. 1 Mile

Cloth Measure.

2¼ inches......1 Nail
4 Nails... ....1 Quarter of a Yard
4 Quarters...1 Yard

Solid or Cubic Measure.

1728 Cubic Inches 1 Cubic Foot
27 Cubic Feet ...1 Cubic Yard
24¾ Cubic Feet ...1 Solid Perch mason's work.
12 Cubic Feet ...1 Solid Perch brick work

Imperial Heaped Measure.

		Avoird. of Water. lb.
8 Gallons	1 Bushel	80
3 Bushels	1 Sack	240
12 Sacks	1 Chaldron	2880

Imperial Dry Measure.

		Avoird. of Water. lb.	oz.
2 Glasses	1 Noggin	0	5
8 Noggins	1 Pint	1	4
2 Pints	1 Quart	2	8
4 Quarts	1 Gallon	10	0
2 Gallons	1 Peck	20	0
4 Pecks	1 Bushel	80	0
8 Bushels	1 Quarter	640	0

Square Measure.

144 Square Inches 1 Square Foot
9 Square Feet...1 Square Yard
30¼ Square Yards 1 Square Pole
40 Square Poles 1 Rood
4 Roods............1 Acre

Table of Motion

60 Seconds (") ......1 Minute
60 Minutes (') ......1 Degree
30 Degrees (°) ......1 Sign
12 Signs or 360° ...the circle of the earth

Table of Time.

60 Seconds.....................1 Minute
60 Minutes.....................1 Hour
24 Hours ....1 Day
7 Days ....1 Week
4 Weeks...........................1 Month
365 Days..........................1 Year
366 Days ....1 Leap Year
52 Weeks ....1 Year
12 Calendar or
13 Lunar Months..........1 Year

Days in the Month.

Thirty days hath September,
April, June and November,
All the rest have thirty-one,
Excepting February alone, [clear
Which has but twenty-eight days
And twenty-nine in each leap year

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1 are 2	1 are 3	1 „ 4	1 „ 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 „ 11	1 are 12
2 „ 4	2 „ 6	2 „ 8	2 „ 10	2 „ 12	2 „ 14	2 „ 16	2 „ 18	2 „ 20	2 „ 22	2 „ 24
3 „ 6	3 „ 9	3 „ 12	3 „ 15	3 „ 18	3 „ 21	3 „ 24	3 „ 27	3 „ 30	3 „ 33	3 „ 36
4 „ 8	4 „ 12	4 „ 16	4 „ 20	4 „ 24	4 „ 28	4 „ 32	4 „ 36	4 „ 40	4 „ 44	4 „ 48
5 „ 10	5 „ 15	5 „ 20	5 „ 25	5 „ 30	5 „ 35	5 „ 40	5 „ 45	5 „ 50	5 „ 55	5 „ 60
6 „ 12	6 „ 18	6 „ 24	6 „ 30	6 „ 36	6 „ 42	6 „ 48	6 „ 54	6 „ 60	6 „ 66	6 „ 72
7 „ 14	7 „ 21	7 „ 28	7 „ 35	7 „ 42	7 „ 49	7 „ 56	7 „ 63	7 „ 70	7 „ 77	7 „ 84
8 „ 16	8 „ 24	8 „ 32	8 „ 40	8 „ 48	8 „ 56	8 „ 64	8 „ 72	8 „ 80	8 „ 88	8 „ 96
9 „ 18	9 „ 27	9 „ 36	9 „ 45	9 „ 54	9 „ 63	9 „ 72	9 „ 81	9 „ 90	9 „ 99	9 „ 108
10 „ 20	10 „ 30	10 „ 40	10 „ 50	10 „ 60	10 „ 70	10 „ 80	10 „ 90	10 „ 100	10 „ 110	10 „ 120
11 „ 22	11 „ 33	11 „ 44	11 „ 55	11 „ 66	11 „ 77	11 „ 88	11 „ 99	11 „ 110	11 „ 121	11 „ 132
12 „ 24	12 „ 36	12 „ 48	12 „ 60	12 „ 72	12 „ 84	12 „ 96	12 „ 108	12 „ 120	12 „ 132	12 „ 144


BOARD OF CONGREGATIONAL MINISTERS.

f. 25

height $h = 31.5$ mm ab

$\tan \frac{\theta}{2} = \frac{31}{214} = 0.144$

measured from above

$\theta =$

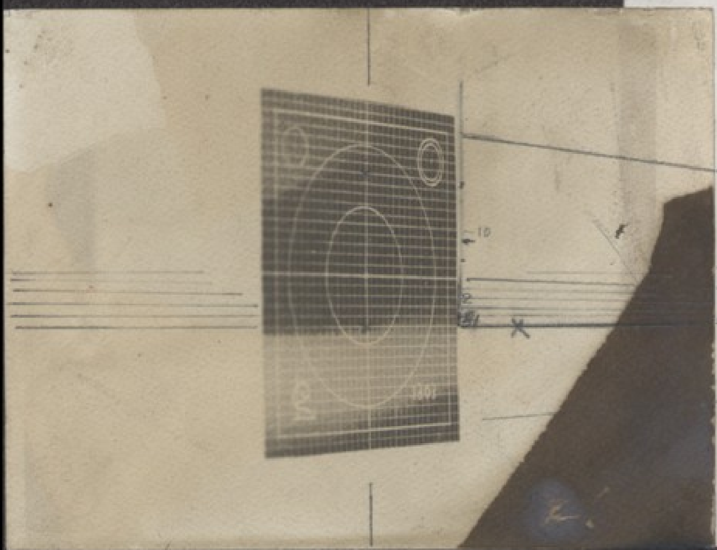
$\tan \theta = \frac{23.4}{214}$ measured $= 0.11$

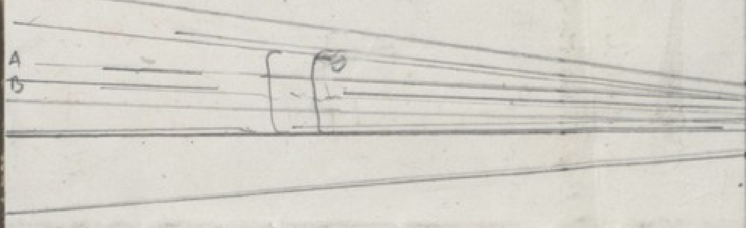
$214 \,/\, 31.0 \,(\, .144$
214
960
856
1040

$214 \,/\, 23.5 \,(\, 0.11$
214
210

$= \tan 6^\circ .17$

by protractor by measure $- 5^\circ$





Tape
String
India rubber bands
Box to put things in
Circulars & tins &c. — Instruction about

f. 2v

Freshly registered Sundry
Cabinet — for ·6 corrected

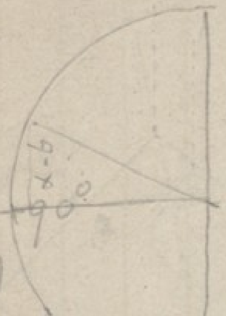

0
10
20
30
40
50
60
70
80
90

$180^\circ = EQ_1C + (90^\circ - \theta) + \alpha$

$180^\circ + \theta - \alpha = EQ_1C + 90^\circ$

$EQ_1C = 90 + (\theta - \alpha)$

$\sin EQ_1C = \cos(\theta - \alpha)$



$180^\circ = EP_1C + \theta + 90^\circ + \alpha$

$180 - (\theta + \alpha) = EP_1C + 90$

$EP_1C = 90^\circ - (\theta + \alpha)$

$\sin EP_1C = \cos(\theta + \alpha)$

$90^\circ - (\gamma - \phi)$

$90^\circ - \gamma + \phi$

$-90 + \phi$

$d_1 = d \frac{\sin\alpha}{\sin(Q_1)} = d \frac{\sin\alpha}{\cos(\theta+\alpha)}$

$\therefore d_1 : d :: \sin\alpha : \sin Q_1$

$S_1 = \frac{d_1 \sin\alpha}{\sin(\alpha-\theta)}$

$(Q_1) = 180 - \{ \ldots 90 - \theta \}$

$= 90 - (\alpha + \theta)$

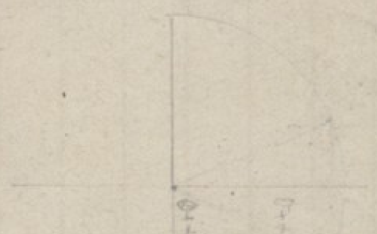
$\sin Q_1 = \cos(\alpha - \theta)$

$(P_1) = 180 - \{ \alpha + 90 + \theta \}$

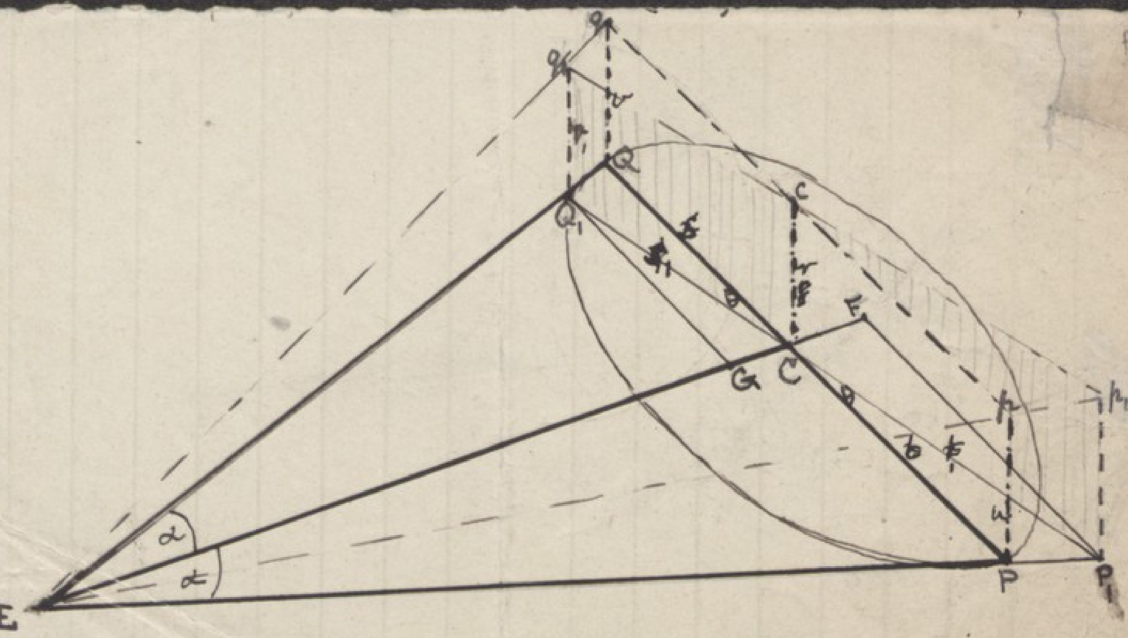
$= 90^\circ - (\alpha + \theta)$

$\therefore d :: \sin\alpha : \sin(P_1)$

$: \cos(\alpha + \theta)$



f. 4v



EC is the optical axis

The Ellipse and all continuous lines are in a horizontal plane passing through ECQ and P

· The dark dotted lines are all perpendicular to the horizontal plane, the other lines fall into neither of the foregoing groups

pPQq is the half of a square (of which two sides are horizontal) which the plane ECQP bisects ~~2 of the sides of the~~ It can rotate round Cc

$p_1P_1Q_1q_1$ is its projection as seen from E on a ~~plane~~ vertical plane passing through Cc ~~which has been rotated~~ as in the drawing, through an angle θ, Q_1 being nearer E than P_1

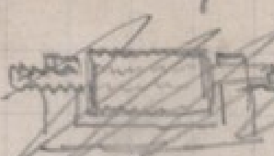
f.5

Under edge of clip to be roughened, to keep
parting from paper — Screw elevator

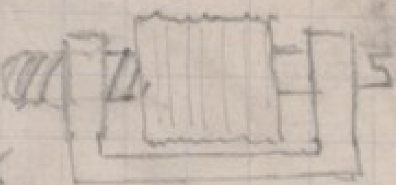
Jambs of wire holder not to shake 
being bevelled

Upper edge of wire holder to be smoothed to
admit of wire being pushed by finger

If screw adjustment is provided, it shd be of this
sort. The inner screw
passes tightly through
the jacket screw & they
are soldered together
So a screw projects from one
side of the jacket & a rod from the other



~~not for same thing~~



This wd be convenient with an elastic thread (Ind?, Hair)

The thread must not project beyond face of holder but
must be let in a little way, or it will be rubbed & injured

The vertical thread to be nearest the portrait — the
horizontal to be as near behind as may be, without
touching it.

$t_1 : t \cos\theta = d : d + t \sin\theta$

f. 6r

$t_2 = t = 1$

$$t_1 = \frac{d \cdot \cos\theta}{d + \sin\theta} \qquad t_2 = \frac{t_1 d \cos\varphi}{d - t_1 \sin\varphi} = 1$$

$$1 = \frac{d \cdot \cos\varphi \left(\frac{d \cdot \cos\theta}{d + \sin\theta} \right)}{d - \sin\varphi \left(\frac{d \cdot \cos\theta}{d + \sin\theta} \right)} = \frac{d^2 \cdot \cos\theta \cdot \cos\varphi}{d^2 + d \sin\theta - d \cos\theta \sin\varphi}$$

$$d + \sin\theta - \cos\theta \sin\varphi = d \cdot \cos\theta \cos\varphi$$

$s = t = 1$ $\tan\theta = \sin\varphi$

$$s_2 + t_2 = \frac{d s_1 \cos\varphi}{d + s_1 \sin\varphi} + \frac{d t_1 \cos\varphi}{d - t_1 \sin\varphi}$$

$$= \frac{d \cos\varphi . \frac{d \cos\theta}{d - \sin\theta}}{d + \sin\varphi \frac{d \cos\theta}{d - \sin\theta}} + \frac{d \cos\varphi \frac{d \cos\theta}{d + \sin\theta}}{d - \sin\varphi . \frac{d \cos\theta}{d + \sin\theta}}$$

$$= \frac{d^2 \cos\varphi . \cos\theta}{d^2 - d \sin\theta + d \sin\varphi . \cos\theta} + \frac{d^2 \cos\varphi . \cos\theta}{d^2 + d \sin\theta - d \sin\varphi . \cos\theta}$$

$$= d . \cos\varphi . \cos\theta \left\{ \frac{1}{d - (\sin\theta - \sin\varphi . \cos\theta)} + \frac{1}{d + (\sin\theta - \sin\varphi \cos\theta)} \right\}$$

$$= d . \cos\varphi . \cos\theta \left\{ \frac{d + (\sin\theta - \sin\varphi . \cos\theta) + d - (\sin\theta - \sin\varphi . \cos\theta)}{d^2 - \sin^2\theta - 2 \sin\theta \sin\varphi \cos\theta + \sin^2\varphi \cos^2\theta} \right.$$

$$= d^2 \cos\varphi . \cos\theta \left\{ \frac{1}{d^2 - \sin^2\theta - 2 \sin\theta . \cos\varphi . \cos\theta + \sin^2\varphi . \cos^2\theta} \right\}$$

$2 \sin\theta \cos\theta \{$ $+ \frac{\sin^2\theta}{\cos^2\theta} \cos^2\theta$

$$= \left\{ \frac{d^2 . \cos\varphi . \cos\theta}{d^2 - 2 \sin\theta . \cos\varphi . \cos\theta} \right)$$

f. 75

$$s_2 + t_2 = d.\cos\phi \left\{ \frac{s_1}{d + s_1 \sin\phi} + \frac{t_1}{d - t_1 \sin\phi} \right\}$$

$$\frac{s_1}{d + s_1 \sin\phi} = \frac{\dfrac{ds \cos\theta}{d - s.\sin\theta}}{\dfrac{d(d - s.\sin\theta) + ds.\cos\theta.\sin\phi}{d - s.\sin\theta}} = \frac{s.\cos\theta}{d - s\sin\theta + s\cos\theta.\sin\phi}$$

$$\frac{t_1}{d - t_1 \sin\phi} = \frac{\dfrac{dt \cos\theta}{d + t \sin\theta}}{d - \dfrac{dt.\cos\theta.\sin\phi}{d + t\sin\theta}} = \frac{t\cos\theta}{d + t.\sin\theta - t\cos\theta\sin\phi}$$

$$s_2 + t_2 = d.\cos\theta\cos\phi \left\{ \frac{s}{d - s.\sin\theta + s\cos\theta\sin\phi} + \frac{t}{d + t.\sin\theta - t\cos\theta.\sin\phi} \right\}$$

$$= \cancel{d\cos\theta\cos\phi} \left\{ \frac{\cancel{sd + st\sin\theta - st\cos\theta\sin\phi + td - ts\sin\theta + ts\cos\theta\sin\phi}}{d^2 + dt\sin\theta - dt\cos\theta\sin\phi - st\sin} \right\}$$

f. 7v

$$t_1 = \frac{d\cos\theta}{d+\sin\theta} \qquad t_2 = \frac{d\cos\phi \cdot t_1}{d - t_1\sin\phi}$$

$$t_2 = \frac{\dfrac{d^2\cos\theta\cos\phi}{d+\sin\theta}}{d + \dfrac{d\cos\theta}{d+\sin\theta}\sin\phi} = \frac{d^2\cos\theta\cos\phi}{d^2 + d\sin\theta - d\cos\theta\cdot\sin\phi} = 1$$

$$d^2\cos\theta\cos\phi = d^2 + d\sin\theta - d\cos\theta\sin\phi$$

$$t_2 = \frac{d.\cos\phi\,\dfrac{d\cos\theta}{d+\sin\theta}}{d - \dfrac{d.\cos\theta}{d+\sin\theta}.\sin\phi} = \frac{d^2\cos\theta\cos\phi}{d^2 + d\sin\theta - d\cos\theta\sin\phi} = 1$$

$$d^2 + d\sin\theta - d\cos\theta\sin\phi = d^2\cos\theta.\cos\phi$$

$$\sin\theta - \cos\theta.\sin\phi = \cos\theta.\cos\phi$$

$$\sin\theta = 2\cos\theta\cos\phi$$

$$\tan\theta = 2\cos\phi \qquad \text{wrong}$$

when $s = t = 1$

$$s_2 + t_2 = d \cos\theta \cos\phi \left\{ \frac{1}{d - \sin\theta + \cos\theta \sin\phi} + \frac{1}{d + \sin\theta - \cos\theta \sin\phi} \right\}$$

$$= d.\cos\theta \cos\phi \left\{ \frac{d + \sin\theta - \cos\theta.\sin\phi + d - \sin\theta + \cos\theta.\sin\phi}{(\quad\quad) \times (\quad\quad)} \right.$$

$$= \frac{d^2 \cos\theta.\cos\phi}{d^2 + \cancel{d \sin\theta} - \cancel{d \cos\theta \sin\phi} - (\cancel{d \sin\theta} - \sin^2\theta + \sin\theta.\cos\theta.\cos\phi + \cancel{d \cos\theta \cos\phi} + \sin\theta \cos\theta \sin\phi) - \cos^2\theta \sin^2\phi}$$

$$= \frac{d^2 \cos\theta \cos\phi}{d^2 - \sin^2\theta + \sin\theta \cos\theta \cos\phi + \sin\theta.\cos\theta.\sin\phi - \cos^2\theta \sin^2\phi}$$

which is too cumbrous for use

reduction in width

f.9 (6

$t_1 : t\cos\theta = d : d + t\sin\theta \qquad t_1 = \dfrac{td\cos\theta}{d + t_1\sin\theta}$

$v_1 : v = d : d + t\sin\theta \qquad v_1 = \dfrac{vd}{d + t\sin\theta}$

$t_2 : t_1\cos\varphi = d : d - t_1\sin\varphi \qquad t_2 = \dfrac{t_1 d\cos\varphi}{d - t_1\sin\varphi}$

$v_2 : v_1 = d : d - t_1\sin\varphi \qquad v_2 = \dfrac{v_1 d}{d - t_1\sin\varphi}$

$$v_2 = \frac{vd^2}{d + t\sin\theta} \times \frac{1}{d - \left\{\frac{td\cos\theta}{d + t\sin\theta}\right\}\sin\varphi} = \frac{vd^2(d + t\sin\theta)}{(d + t\sin\theta)\times(d^2 + dt\sin\theta) - td\cos\theta\sin\varphi}$$

$$= \frac{vd}{d + t\sin\theta - t\cos\theta\sin\varphi}$$

If $v = t = 1$, $d + \sin\theta - \cos\theta\sin\varphi = 0$

$\tan\theta = \sin\varphi$

reduction $= \dfrac{s_2 + t_2}{2s} = \dfrac{1}{2}\left\{\dfrac{ds_1\cos\varphi}{d + s_1\sin\varphi} + \dfrac{t_1 d\cos\varphi}{d - t_1\sin\varphi}\right\} = \dfrac{d\cos\varphi}{2}\left\{\dfrac{s_1}{d + s_1\sin\varphi} + \dfrac{t_1}{d - t_1\sin\varphi}\right\}$

$$= \frac{d\cos\varphi}{2}\left\{\frac{\frac{ds\cos\theta}{d - s\sin\theta}}{d + \frac{ds\cos\theta}{d - s\sin\theta}\sin\varphi} + \frac{\frac{dt\cos\theta}{d + t\sin\theta}}{d + \frac{dt\cos\theta}{d + t\sin\theta}\sin\varphi}\right\} = \frac{d^2\cos\theta\cos\varphi}{2}\left\{\frac{1}{d^2 - ds\sin\theta + ds\cos\theta\sin\varphi} + \frac{1}{d^2 + dt\sin\theta + dt\cos\theta\sin\varphi}\right\}$$

see over; clear

f. 10 (5

Reduction in width.

$$s_1 = \frac{d\,s\cos\theta}{d - s\sin\theta} \qquad t_1 = \frac{t\,d\cos\theta}{d + t\sin\theta}$$

$$s_2 = \frac{d\,s_1\cos\varphi}{d + s_1\sin\varphi} \qquad t_2 = \frac{t_1\,d\cos\varphi}{d - t_1\sin\varphi}$$

required $\dfrac{s_2 + t_2}{s + t}$

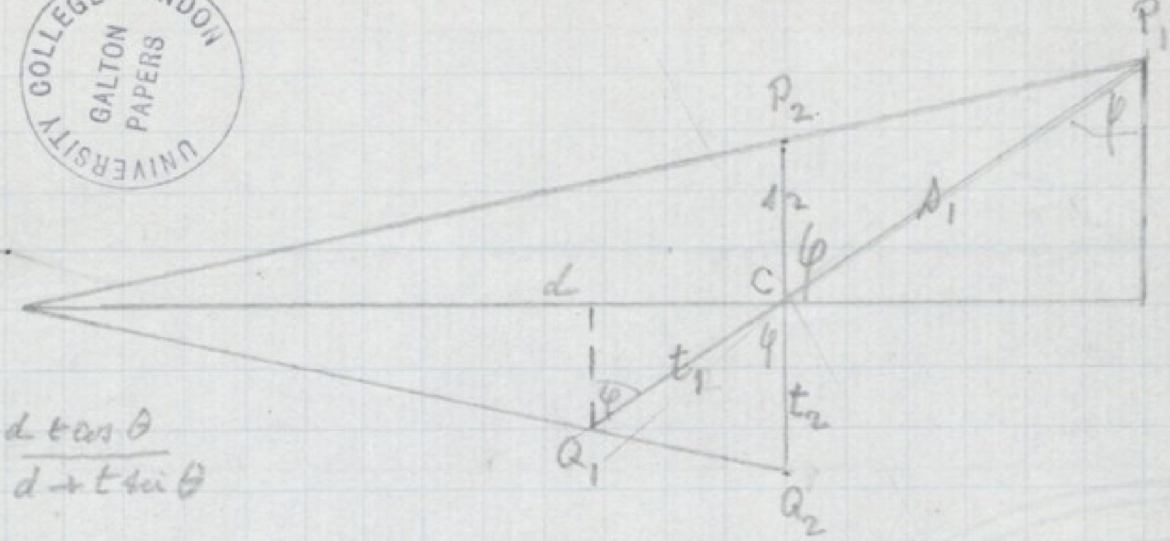
If $s = t = 1$ and $\tan\theta = \sin\varphi$

$$s_1 = \frac{d\cos\theta}{d - \sin\theta} \qquad t_1 = \frac{d\cos\theta}{d + \sin\theta}$$

$$s_2 = \frac{d\,s_1\cos\varphi}{d + s_1\sin\varphi} \qquad t_2 = \frac{d\,t_1\cos\varphi}{d - t_1\sin\varphi}$$

required $\dfrac{1}{2}\left(s_2 + t_2\right)$

~~These equations have to be calculated separately; they do not clear themselves~~



$$t_1 = \frac{d\,t\cos\theta}{d + t\sin\theta}$$

$$t_2 : t_1 \sin\varphi = d : d - t_1\cos\varphi \qquad t_2 = \frac{d \cdot t_1 \sin\varphi}{d - t_1\cos\varphi}$$

$$t_2 = d\left\{\frac{\sin\varphi \cdot \dfrac{d\,t\cos\theta}{d+t\sin\theta}}{d - \cos\varphi \cdot \dfrac{d\,t\cos\theta}{d+t\sin\theta}}\right\} = d\left\{\frac{d\,t\cos\theta\sin\varphi}{d^2 + dt\sin\theta - dt\sin\theta\cos\varphi}\right\}$$

$$1 = \frac{d\cdot\cos\theta\cdot\sin\varphi}{d + \sin\theta - \sin\theta\cdot\cos\varphi}$$

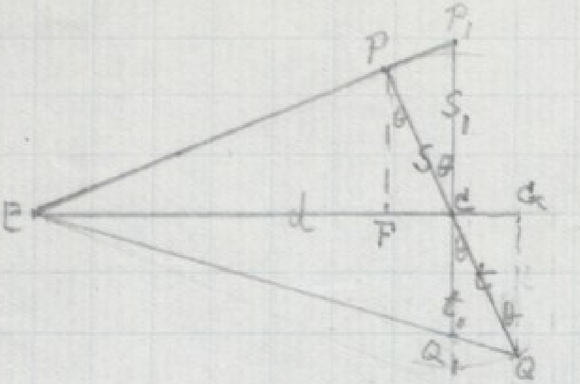
if $t_2 = t = 1$

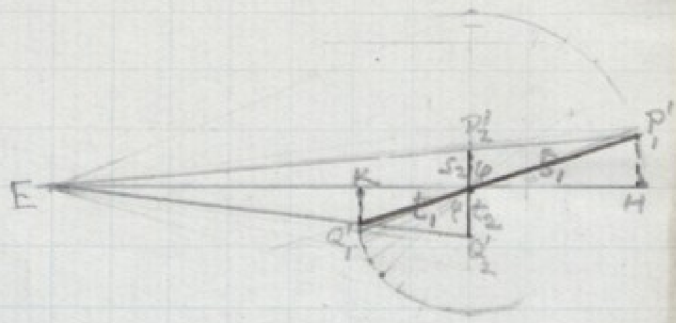
$$d + \sin\theta - \sin\theta\cdot\cos\varphi = d\cos\theta\cdot\sin\varphi$$

let $\tan\theta = \sin\varphi$

$d +$

(reduction in widths) f. 11v





(In calculation of t also) Let $t = 1 = t_2$

$$t_2 = \frac{\dfrac{d.\cos\theta\,\cos\varphi}{d + \sin\theta}}{d - \dfrac{d\cos\theta\,\sin\varphi}{d + \sin\theta}} = \frac{d.\cos\theta.\cos\varphi}{d^2 + d\sin\theta - d\cos\theta\sin\varphi} = 1$$

$$d.\cos\theta\cos\varphi = d^2 + d\sin\theta - d\cos\theta.\sin\varphi$$

$t = 1$

$$t.\cos\theta : t_1 = d + \sin\theta : d$$

$$t_1 = \frac{d\cos\theta}{d + \sin\theta} \qquad t_2 : t_1\cos\varphi = d : d - t_1\sin\varphi$$

$$t_2 = \frac{d.\cos\varphi.t_1}{d - t_1\sin\varphi}$$

right
repeated. see other p. 3

For reduction in width only

f. 12

3a
(see 3)

$$u_2 = d u_1 \frac{1}{d + s_1 \sin\varphi}$$

$$= d\left\{\left(\frac{du}{d - s.\sin\theta}\right) \times \frac{1}{d + \sin\varphi\left(\frac{d s\cos\theta}{d - 1.\sin\theta}\right)}\right.$$

$$= d\left\{\left(\frac{\cancel{d} u}{\cancel{d - s.\sin\theta}}\right) \cancel{\times} \frac{d - 1.\sin\theta}{\cancel{d}(d - 1.\sin\theta) + \cancel{d} s\cos\theta.\sin\varphi}\right\}$$

$$= \frac{du}{d - s.\sin\theta + s\cos\theta.\sin\varphi}$$

$u_2 = u = 1$
$s = 1$

$$d - \sin\theta + \cos\theta.\sin\varphi = d$$

$$\sin\theta = \cos\theta \sin\varphi \qquad \tan\theta = \sin\varphi$$

For reduction in width only — for proof of $\tan\theta = \sin\varphi$ f.13 3

right
best

$$u_2 = u_1 \frac{d}{d + s_1 \sin\varphi} = \left(\frac{du}{d - s.\sin\theta}\right) \times \left[\frac{d}{d + \frac{ds\cos\theta}{d - s.\sin\theta}.\sin\varphi}\right]$$

$$= \frac{d^2u(d - s.\sin\theta)}{(d - s.\sin\theta) \times (d^2 - ds.\sin\theta) + ds\cos\theta.\sin\varphi)}$$

$$= \frac{du}{d - s\sin\theta + s\cos\theta.\sin\varphi}$$

If $u_2 = u$ & taking $u = s = 1$

$$d - \sin\theta + \cos\theta.\sin\varphi = d$$

$$\sin\varphi = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

right

For reduction in width only — independent of $\tan\theta = \sin\varphi$ F. 14 (2

I.

$$s_1 : PF = EC : EF \;; \qquad s_1 = \frac{d\, s_1 \cos\theta}{d - s_1 \sin\theta}$$

$$u_1 : u = EC : EF \qquad u_1 = \frac{du}{d - s_1 \sin\theta}$$

Similarly $$s_2 : s_1 \cos\varphi = d : d + s_1 \sin\varphi \qquad s_2 = \frac{d s_1 \cos\varphi}{d + s_1 \sin\varphi}$$

$$u_2 : u_1 = d : d + s_1 \sin\varphi \qquad u_2 = \frac{d u_1}{d + s_1 \sin\varphi}$$

$$d + s_1 \sin\varphi = d + \frac{ds \cdot \cos\theta \sin\varphi}{d - s \sin\theta} = d \left\{ \frac{(d - s \sin\theta) \times (ds \cos\theta \cdot \sin\varphi)}{d - s \sin\theta} \right\}$$

$$u_2 = \frac{d u_1 (d - s \sin\theta)}{(d - s_1 \sin\theta) \times (d s_1 \cos\theta \cos\varphi)} = \frac{du\, du\, (d - s \sin\theta)}{ds (d - s_1 \sin\theta) \times (\cos\theta \cdot \cos\varphi)}$$

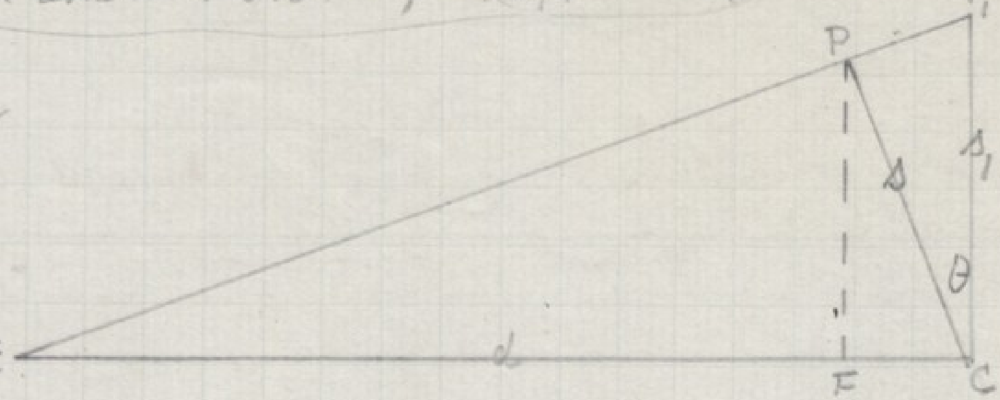
$u_2 = u \cdot s \times$

$\cos\theta \cdot \cos\varphi =$

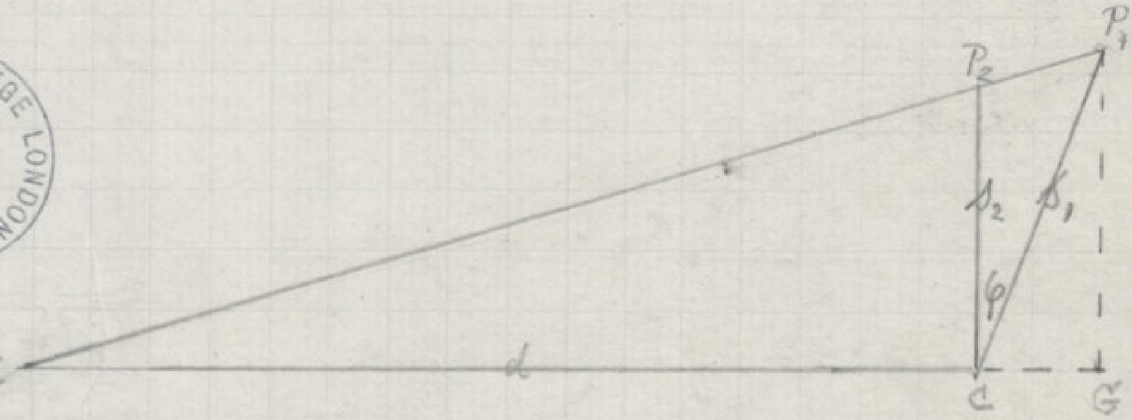
(1) (For reduction in width only (Proof of $\tan\theta = \sin\varphi$))

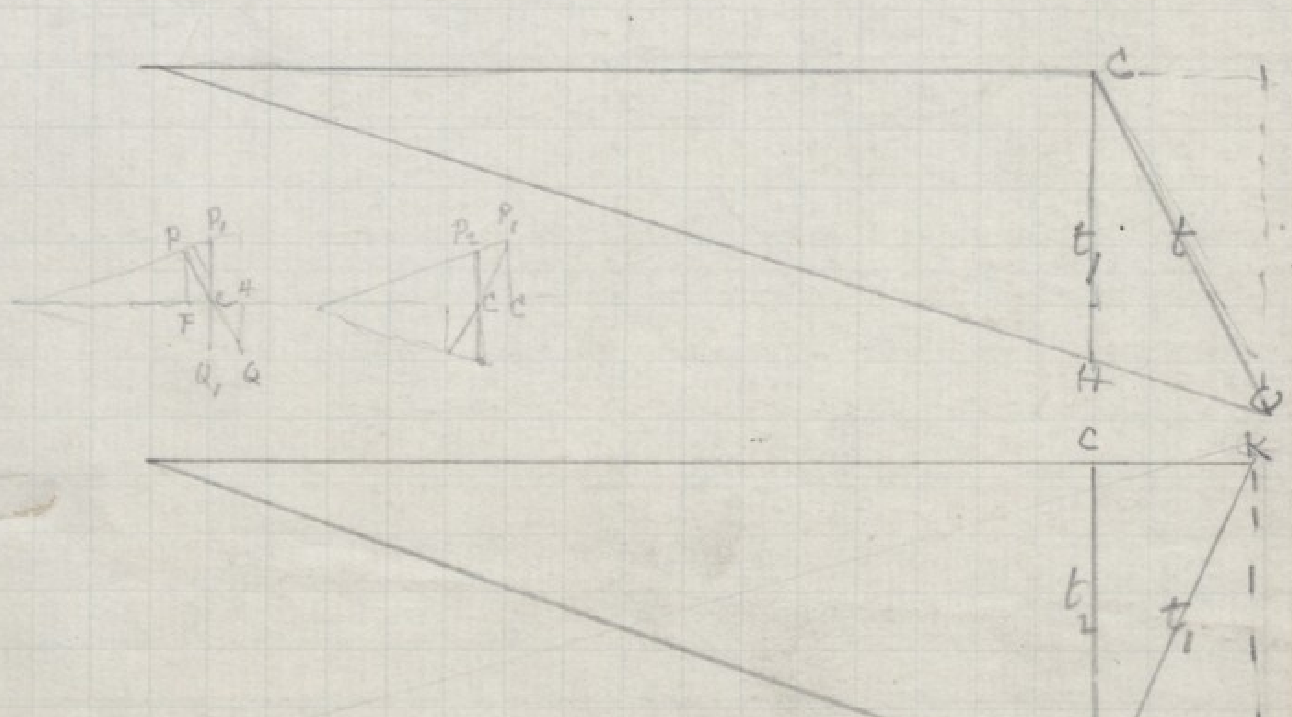
Right

I



II





A line S_n ~~of vari~~ is any one of a statical group of ~~[S] values~~ variables, $S_1 S_2 \ldots S_n \ldots$, S_n ~~and~~ is the sum of the lines A_n B_n C_n and Z_n, ~~the [A] [B] [C] & [Z] values~~ $A_1, A_2 \ldots B_1, B_2 \ldots C_1 C_2 \ldots$ and $Z_1 Z_2 \ldots$ being independent & normal variables. ~~[illegible] a Table~~ of S_n A_n B_n & C_n are known but Z_n is not known, required the mean & the probable error of each of the above series viz S, A, B, C, Z and s, a, b, c, z.

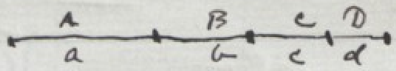
~~By well known theorem~~ ~~$Z = S - \{A+B+C\}$~~ ~~$z^2 = s^2$~~

$S = A+B+C+Z$ and by well known theorem $s^2 = a^2+b^2+c^2+z^2$ (Recollect that This latter is ~~a [illegible]~~ an ordinary equation, ~~and must not be confounded~~ does not refer to that has no connection with the ~~[illegible]~~ inverse probability of a single event)

~~In these cases~~ It is well to verify statistical theory by occasional experiment

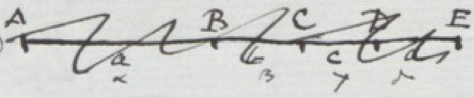
I made ~~Take~~ a roulette in wh ~~there one~~ an index was then over 10 compartments, & ~~each the~~ suitably shaded From a table of ordinates such as that I have given in Nat Inher p – (see also Roy. R. Soc. .) ten groups ~~classes~~ of ordinates (in which individual errors are are formed) having equally likely to fall, whose their general prob: error being = 1, is easily constructed.) ~~The ordinates~~ are . . . . They ~~probability of occurrence~~, (the means of these ten groups ~~are~~ inscribed on the ten compartments

By adding the successive indications of the roulette to a constant (say 10) that represents the mean, an artificial table of ("polygonal") variables is rapidly constructed.



(variable length whose unknown ~~mean~~ value is)

A line of ~~the total length~~ S, with an unknown prob. error q_s



~~The line AB~~ is the sum of

~~the~~ four, independently, ~~variable lines a, b, c, d AB, BC, CD, and DE~~ smaller lines that vary

whose ~~respective~~ ~~their~~ unknown mean lengths ~~are~~ are A, B, C, ... Z ~~known with~~ the corresponding unknown prob. errors ~~of~~ are a b c & z, and there

~~and~~ the normal law of frequency ~~being observed~~ is known to prevail throughout.

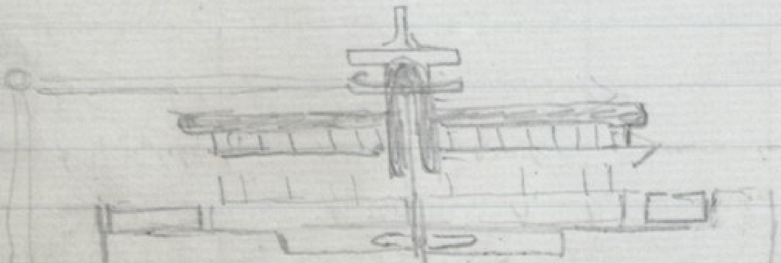
(call them $S_1\ S_2 - S_n$ .. &c) with their

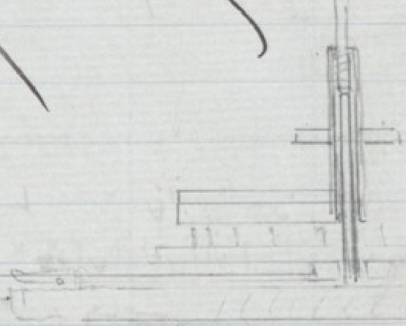
A ~~table~~ statistical ~~table~~ list has been compiled ~~giving the many~~ of various values of S & ~~their~~ the accompanying

values of A, B, & C, .. From this table it is required to deduce S, A B C & Z, and s, a b c z

By well known theorem (1) $Z_x = S_x - (A_x + B_x + C_x)$ & (2) $z^2 = s_x^2 - (a_x^2 + b_x^2 + c_x^2)$

S_n S





Adjustments – first set the centre of the screen equidistant from the axes & each seeing a facsimile of the position on the other

(I) ~~Suppose~~ that i.e. the vertical screen has been made truly ⊥^{r} to the horizontal turntable as it should be

i.e. suppose the face of the plate to be^{s} vertically above the axis as it should be

then a vertical line can be found & drawn on the plate which shall remain fixed however the screen be rotated

(II) This being done, a horizontal line can be ~~drawn~~ found & drawn on the plate, that will be always ⊥^{r} to the vertical line however the screen be rotated. It will be shortened & the ends of the image will be blurred, but it will always remain horizontal

Then mark the prolongation of these lines upon the vertical face of the screen & adjust the threads of the adjuster to match

For (1) & (2) it would be well ~~to make~~ to use a sliding T

The working parts sh^{d} be simple handy & very accurate – polish is of no importance – rather the contrary

First make the 2 kinds of slide

Then fix sizes & thickness of disc & how to clamp it

The line AB drawn on the front side of the plate is identical with the axis of revolution. This has to be carefully attended to.

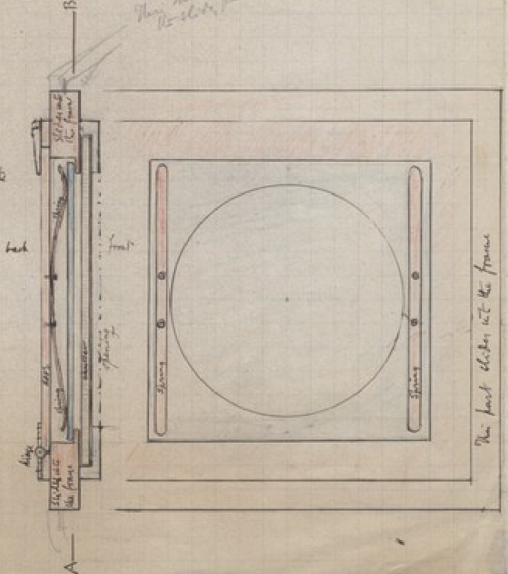
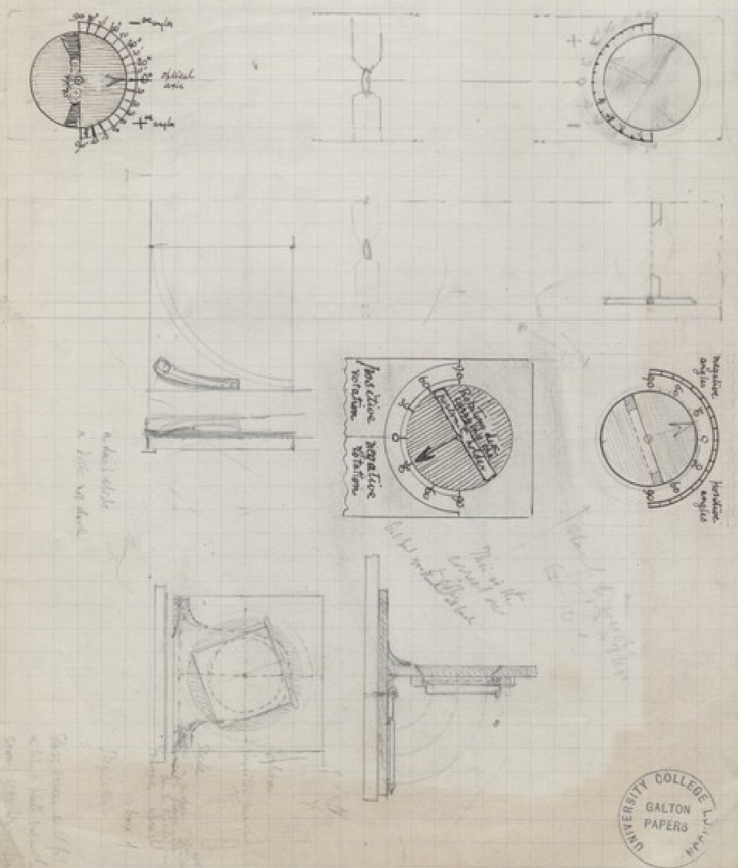
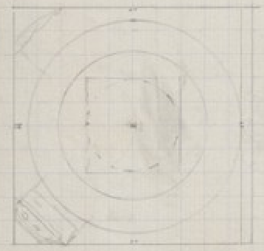
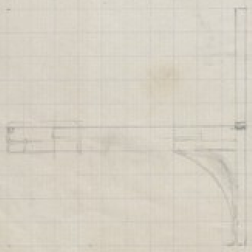


Fig 3 is a diagrammatic representation of the working part of the camera, used







2^{d} stage $\qquad$ s_1 recedes from E, t_1 approaches E

In Triangle EQ_2C

$$\text{angle } Q_2EC = \beta \qquad \tan\beta = \frac{s_1}{d} \qquad (6)$$

$$\text{angle } EQ_2C = 180^\circ - \beta - 90^\circ - \varphi = 90^\circ - (\beta+\varphi) \quad \text{its sine} = \cos(\beta+\varphi)$$

$$s_2 : d :: \sin\beta : \cos(\beta+\varphi) \qquad s_2 = d . \frac{\sin\beta}{\cos(\beta+\varphi)} \qquad (7)$$

u_2 is the perpendicular standing on Q_2

$$u_2 : u_1 :: d + s_2 \sin\varphi : d \qquad u_2 = \frac{u_1}{d} . \{d + s_2 \sin\varphi\} \qquad (8)$$

In triangle EP_2C

$$\text{angle } P_2EC = \gamma \qquad \tan\gamma = \frac{t_1}{d} \qquad (9)$$

$$\text{angle } EP_2C = 180^\circ - \gamma - (90^\circ - \varphi) = 90^\circ - (\gamma - \varphi) \; ; \text{ its Sine} = \cos(\gamma-\varphi)$$

$$t_2 : d :: \sin\gamma : \cos(\gamma-\varphi) \qquad t_2 = d \, \frac{\sin\gamma}{\cos(\gamma-\varphi)} \qquad (10)$$

~~In triangle CE~~

$$v_2 : v_1 :: d - t_2 \cos\varphi : d \qquad v_2 = \frac{v_1}{d} \{d - t_2 \cos\varphi\} \qquad (11)$$

$$u_2 = \frac{1}{d} \times u_1 \times \left\{ d + d \frac{\sin\beta}{\cos(\beta+\varphi)} . \sin\varphi \right\} = u_1 \left\{ 1 + \frac{\sin\beta . \sin\varphi}{\cos(\beta+\varphi)} \right\}$$

$$= v \left\{ 1 - \frac{\sin\alpha}{\cos(\alpha-\theta)} \right\} \left\{ 1 + \frac{\sin\beta . \sin\varphi}{\cos(\beta+\varphi)} \right. = v \text{ for requirement to be fulfilled}$$

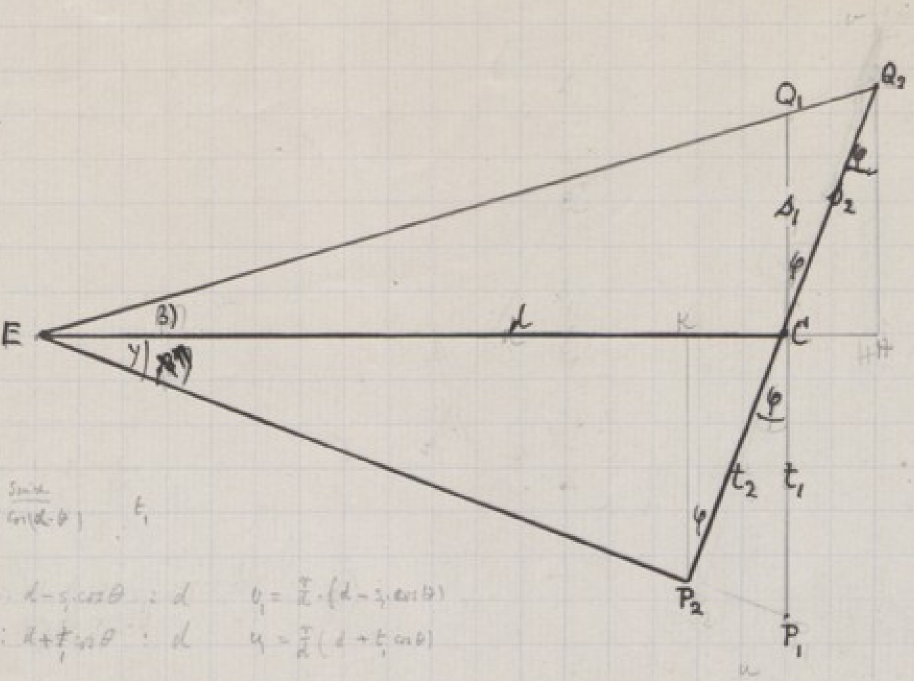
$$\frac{\cos(\alpha-\theta) - \sin\alpha}{\cos(\alpha-\theta)} \times \frac{\cos(\beta+\varphi) + \sin\beta . \sin\varphi}{\times \cos(\beta+\varphi)} = 1$$

$$\left(\cos(\alpha-\theta) - \sin\alpha\right) \times \left(\cos(\beta+\varphi) + \sin\beta . \sin\varphi\right) = \cos(\alpha-\theta) \times \{\cos(\beta+\varphi)\}$$

$$\left.\begin{array}{l} \cancel{\cos(\alpha-\theta) \times \cos(\beta+\varphi} \\ + \cos(\alpha-\theta) \times \sin\beta . \sin\varphi \\ - \sin\alpha \times \cos(\beta+\varphi) \\ - \sin\alpha \times \sin\beta . \sin\varphi \end{array}\right\} = \cancel{\cos(\alpha-\theta) \times \cos(\beta+\varphi)}$$

$$\cos(\alpha-\theta) \times \sin\beta . \sin\varphi = \cos(\beta+\varphi) . \sin\alpha \times \sin\alpha . \sin\beta . \sin\varphi$$

2^{nd} transformation



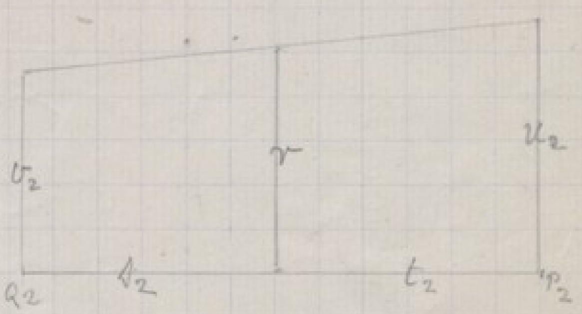
$s_1 = d \cdot \frac{\sin\alpha}{\cos(\alpha-\theta)}$ $\quad t_1$

$v_1 : r :: d - s_1\cos\theta : d \qquad v_1 = \frac{r}{d}\cdot(d - s_1\cos\theta)$

$u_1 : r :: d + t_1\cos\theta : d \qquad u_1 = \frac{r}{d}(d + t_1\cos\theta)$

$v_2 : r :: d + s_2\cos\phi : d$

$u_2 : r :: d - t_2\cos\phi : d$



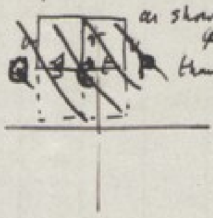
Required Q_1G, P_1F, Q_1q_1, P_1p_1

$\tan QEC = \frac{Qc}{EC} = \tan PEC$, say $\tan\alpha = \frac{r}{d}$, ~~so~~

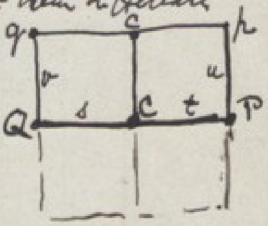
~~for convenience designate by so the length CP & by S the length CQ though they are equal.~~

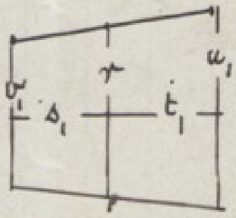
~~Though~~ and each Qq, Cc, and Pp, Qc & Pc are all equal, ~~but~~ it is convenient to designate them differently

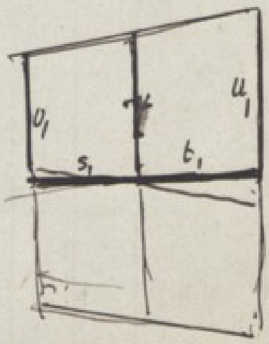
v, r, u, s, t



as shown more clearly ~~as~~ in the Elevation than in the perspective diagram







1st stage. s_1 approaches E; t_1 recedes from E

In the triangle EQ_1C

the angle at $E = \alpha$

that at $Q_1 = 90^\circ - \alpha + \theta = 90^\circ - (\alpha - \theta)$. its sine $= \cos(\alpha-\theta)$

$s_1 : \sin\alpha :: d : \cos(\alpha-\theta)$ $\qquad s_1 = \dfrac{d.\sin\alpha}{\cos(\alpha-\theta)}$

u_1 the perpendicular at Q_1 : u (or r) that at Q $:: d - s_1\sin\theta : d$ $\qquad u_1 = \dfrac{r.(d-s_1\sin\theta)}{d.\cos(\alpha-\theta)}$

$u_1 d = u \times (d - s_1 \sin\theta)$

2nd stage. s_2 recedes from E, t_2 approaches E.

In the triangle EQ_2C

the angle at $E = \alpha$

that at $Q_2 = 180^\circ - \{\beta + 90^\circ + \varphi\} = 90^\circ - (\beta+\varphi)$; its sine $= \cos(\beta+\varphi)$

$s_2 : \sin\beta :: d : \cos(\beta+\varphi)$ $\qquad s_2 = \dfrac{d.\sin\beta}{\cos(\beta+\varphi)}$

u_2 the perpr at Q_2 : $u_1 :: d + s_2\sin\varphi : d$ $\qquad \cancel{u_2 = \dfrac{u_1 \times (d+s_2\sin\varphi)}{d}}$

$u_2 d = u_1(d + s_2\sin\varphi)$ $\qquad = u_2 = \dfrac{u_1}{d}(d + s_2\sin\varphi)$

$u_2 = \dfrac{u_1}{d}(d+s_2\sin\varphi)$ $\qquad \cancel{= r \times (d + s_1\sin\theta)}$

(Both stages)

$$\cancel{u_1 = \frac{r}{d}\cdot\frac{d - \dfrac{d.\sin\alpha.\sin\theta}{\cos(\alpha-\theta)}}{d.\cos(\alpha-\theta)}}$$

$$\cancel{= \frac{r}{d}\cdot\frac{\dfrac{d.\cos(\alpha-\theta) - d\sin\alpha.\sin\theta}{\cos(\alpha-\theta)}}{d.\cos(\alpha-\theta)}} \qquad u_1 = \frac{r}{d}\cdot\frac{\cos(\alpha-\theta) - \sin\alpha.\sin\theta}{\cos^2(\alpha-\theta)}$$

$\cancel{u_2 = \frac{r}{d^2}(d+s_2\sin\varphi)}$

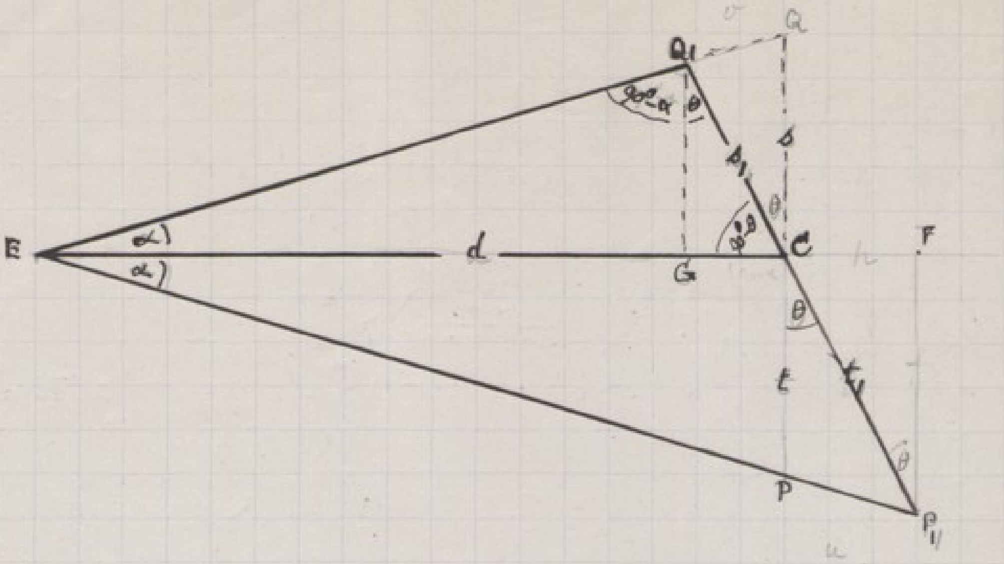
$$\cancel{u_2 = \frac{u_1}{d}\times\left\{d + \frac{d.\sin\beta.\sin\varphi}{\cos(\beta+\varphi)}\right\}}$$

$$\cancel{= \frac{r}{d^2}\cdot\frac{\cos(\alpha-\theta) - \sin\alpha.\sin\theta}{\cos^2(\alpha-\theta)} \times d\left\{1 + \frac{\sin\beta.\sin\varphi}{\cos(\beta+\varphi)}\right\}} = r \text{ by requirement to be fulfilled}$$

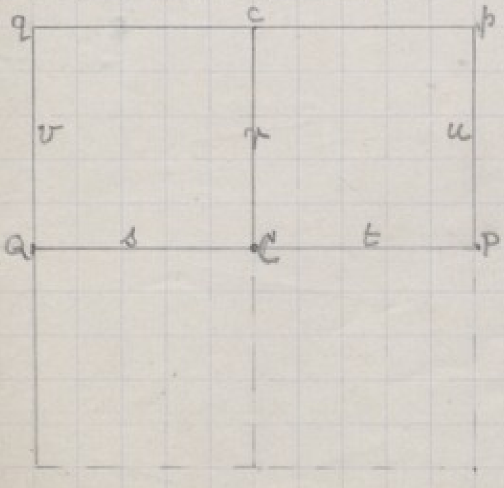
$$\cancel{\{\cos(\alpha-\theta) - \sin\alpha.\sin\theta\} \times \{\cos(\beta+\varphi) + \sin\beta.\sin\varphi\} = d.\cos^2(\alpha-\theta).\cos(\beta+\varphi)}$$

$$\cancel{\cos(\alpha-\theta).\cos(\beta+\varphi) + \cos(\alpha-\theta).\sin\beta.\sin\varphi - \sin\alpha.\sin\theta.\cos(\beta+\varphi) - \sin\alpha\sin\theta.\sin\beta.\sin\varphi = d\cos^2(\alpha-\theta).\cos(\beta+\varphi)}$$

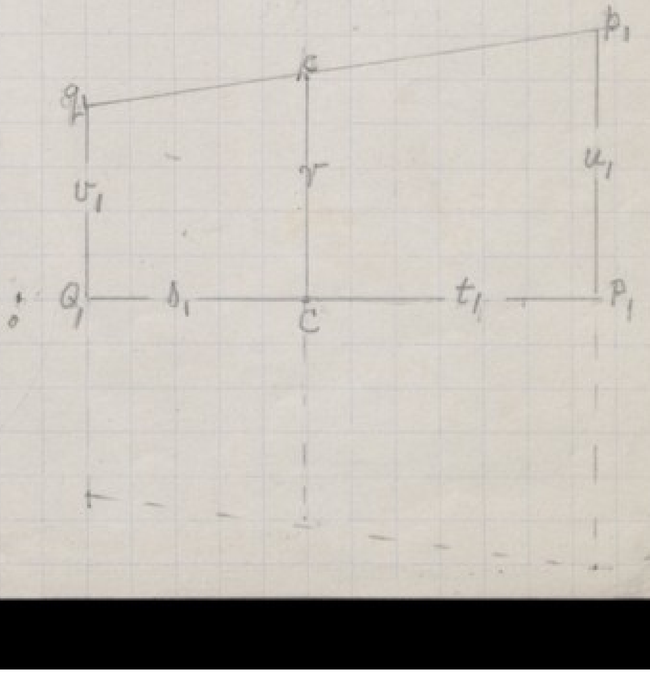
1st transformation



Front elevation of portrait
Here it is a square that can be rotated round Cc
the half that lies above the horizontal plane
which passes through the optical axis EC is in dark lines



Front Elevation after first transformation



The principal plane stands ⊥ to the optical axis EO and OP is its intersection with the plane of the paper.

E is the eye

The Plane of paper is the horizontal plane through optical axis

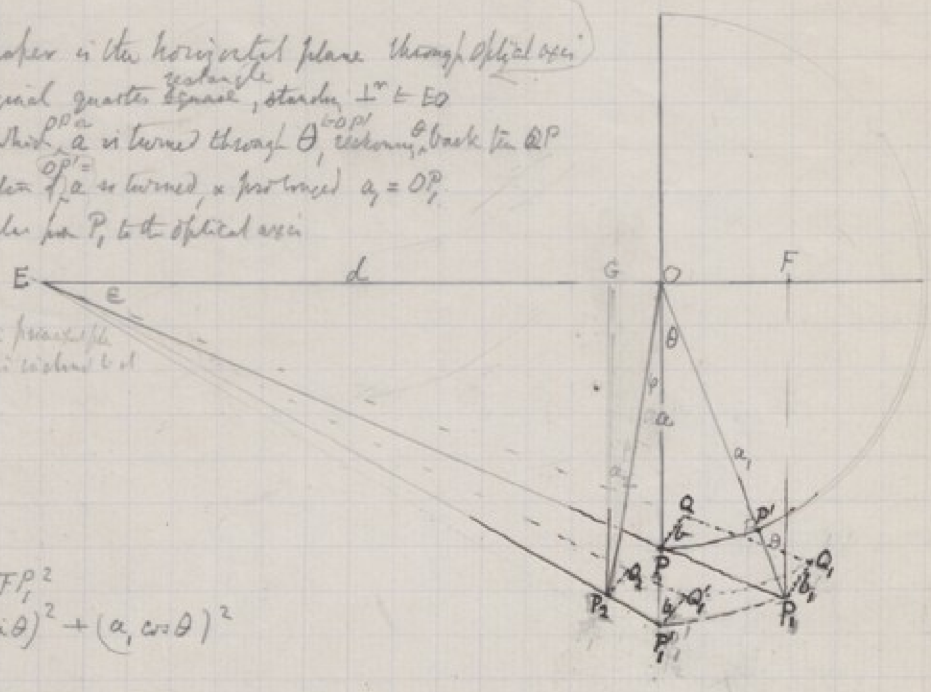
OP = a one side of original quarter ~~square~~ rectangle, standing $\perp^r$ to EO

O the axis round which OP a is turned through θ, reckoning θ back to OP (< OP′)

P_1 is the intersection of OP′ = a so turned, & prolonged $a_1 = OP_1$

P_1F is the perpendicular from P_1 to the optical axis

EO = d



Projection of portrait in principle is to a sensitive plate that is inclined to it

Find EP_1

$P_1F = a_1 \cos\theta$

$OF = a_1 \sin\theta$

$EP_1^2 = EF^2 + FP_1^2$

$= (d + a_1 \sin\theta)^2 + (a_1\cos\theta)^2$

$= d^2 + 2a_1 d \sin\theta + a_1^2\sin^2\theta + a_1^2\cos^2\theta \;=\; d^2 + 2a_1 d\sin\theta + 1$

Q is the top of the perpendicular at P; QP = b, the vertical side of the quarter rectangle; Q_1 that at P_1, EQ_1 being EQ prolonged. $PQ_1 = b_1$

$Q_1 b_1 : b$ (or a) $:: EQ_1 : EP :: EF : d :: d + a_1 \sin\theta : d$

$b_1 = \frac{b}{d}\{d + a_1 \sin\theta\}$

Place a_1 in the principle plane where P_1 takes the position of P_1' and $OP_1' = a_1$. Join E P′. then its projection upon sensitive plane inclined forwards through angle θ from the principal plane is $OP_2 = a_2$

$\tan\epsilon = \frac{a}{d}$

$a_1\cos\theta : a :: d + a_1\sin\theta : d$

$a_1 = \frac{a}{d}\cdot\frac{d + a_1\sin\theta}{\cos\theta}$

P_1Q_1 (or b_1) : PQ (or b) :: $d + a_1\sin\theta : d$

$b_1 = \frac{b}{d}(d + a_1\sin\theta)$

1st stage. s_1 approaches E, t_1 recedes from E,

$s = t = r$

in triangle EQ_1C, $\begin{cases} \text{the angle at } E = \alpha \\ \text{the angle at } Q_1 = 90^\circ - \alpha + \theta = 90^\circ - (\alpha - \theta) \\ \text{its sine} = \cos(\alpha - \theta) \\ \cancel{\text{the angle at } C = 90^\circ \theta} \\ \cancel{\text{its sine} = \cos\theta} \end{cases}$ $\tan\alpha = \frac{s}{d}$ (1)

$s_1 : \sin\alpha :: \cancel{\cos(\alpha-\theta) : \cos\theta}$
$:: d : \cancel{\cos\theta}\ \cos(\alpha - \theta)$ $\qquad s_1 = d . \frac{\sin\alpha}{\cos(\alpha-\theta)}$ (2)

u is the perpendicular ~~of length r~~ standing at Q; its length $= r$.
u_1 is the perpendicular ——— " " Q_1.

$u_1 : u :: d - s_1\cos\theta : d$ $\qquad u_1 = \frac{r}{d}(d - s_1\cos\theta)$ (3)

---

in triangle EP_1C $\begin{cases} \text{the angle at } E = \alpha \\ \text{the angle at } P_1 = 180^\circ - 90^\circ - \theta - \alpha = 90^\circ - (\theta + \alpha) \\ \text{its sine} = \cos(\theta + \alpha) \end{cases}$

$t_1 : d :: \sin\alpha : \cos(\theta + \alpha)$ $\qquad t_1 = d\ \frac{\sin\alpha}{\cos(\theta+\alpha)}$ (4)

v is the perpendicular standing at P; its length $= r$
v_1 - - - - - - - P_1.

$v_1 : \cancel{v\,(\text{or } r)} :: d + t_1\sin\theta : d$ $\qquad v_1 = \frac{r}{d}(d + t_1\sin\theta)$ (5)

$u_1 = \frac{r}{d}\left\{d - d\,\frac{\sin\alpha}{\cos(\alpha-\theta)}\cos\theta\right\} = r\left\{1 - \frac{\sin\alpha}{\cos(\alpha-\theta)}\right\}$

$v_1 = \frac{r}{d}\left\{d . \frac{\sin\alpha}{\cos(\alpha+\theta)} . \sin\theta\right\} = r\left\{\frac{\sin\alpha . \sin\theta}{\cos(\alpha+\theta)}\right\}$

$\mathcal{N}_1 \quad s_1 = \dfrac{d.\sin\alpha}{\cos(\alpha-\theta)}$ Also $\quad s_1 = s\,\dfrac{1}{\cos\theta}$

There are at last correct

$u_1 = \dfrac{u}{d}\cdot\{d - s_1\sin\theta\}$

$s_2 = \dfrac{d.\sin\beta}{\cos(\beta+\varphi)}$

$14 \quad u_2 \qquad \dfrac{u_1}{d}(d + s_2 \sin\varphi$

$$u_1 = \frac{u}{d}\left\{d - \frac{d.\sin\alpha.\sin\theta}{\cos(\alpha-\theta)}\right\}$$

$$= u\,\frac{(\cos(\alpha-\theta) - \sin\alpha.\sin\theta)}{\cos(\alpha-\theta)}$$

$$u_1 = \frac{u}{d}\left\{d - s.\frac{\sin\theta}{\cos\theta}\right\}$$

$$= u - \frac{us}{d}.\tan\theta. \quad \text{if } u \text{ be taken} = 1$$

$$u_1 = 1 - \frac{s}{d}.\tan\theta$$

$$u_2 = \frac{u_1}{d}\left\{d + \frac{d\sin\beta.\sin\varphi}{\cos(\beta+\varphi)}\right\}$$

$$= \frac{u_1(\cos(\beta+\varphi) + \sin\beta.\sin\varphi)}{\cos(\beta+\varphi)}$$

$$= u.\frac{(\cos(\alpha-\theta) - \sin\alpha.\sin\theta)\times(\cos(\beta+\varphi) + \sin\beta.\sin\varphi)}{\cos(\alpha-\theta)\times\cos(\beta+\varphi)}$$

if $u_2 = u = 1$

$$\cancel{\cos(\alpha-\theta)\times\cos(\beta+\varphi)} = \cancel{\cos(\alpha-\theta)\cos(\beta+\varphi)}$$

$$0 = \begin{cases} +\cos(\alpha-\theta)(\sin\beta.\sin\varphi) \\ -\sin\alpha.\sin\theta.\cos(\beta+\varphi) \\ -(\sin\beta.\sin\varphi).(\sin\alpha.\sin\theta) \end{cases}$$

$$0 = \sin\beta.\sin\varphi\{(\cos(\alpha-\theta) - (\sin\alpha.\sin\theta)\} -$$
$$-\sin\alpha.\sin\theta\{\cos(\beta+\varphi)\}$$

2^{nd} - trans formation

F. 25

[4]

$\tan\beta = \frac{s_1}{d}$, $\tan\gamma = \frac{t_1}{d}$

in triangle $\Sigma Q_2 C$ the angle at ~~Q~~ $C = 90^\circ + \varphi$ and that at $Q_2 = 180^\circ - \beta - 90^\circ - \varphi$

$= 90^\circ - (\beta + \varphi)$; its sine $= \cos(\beta + \varphi)$

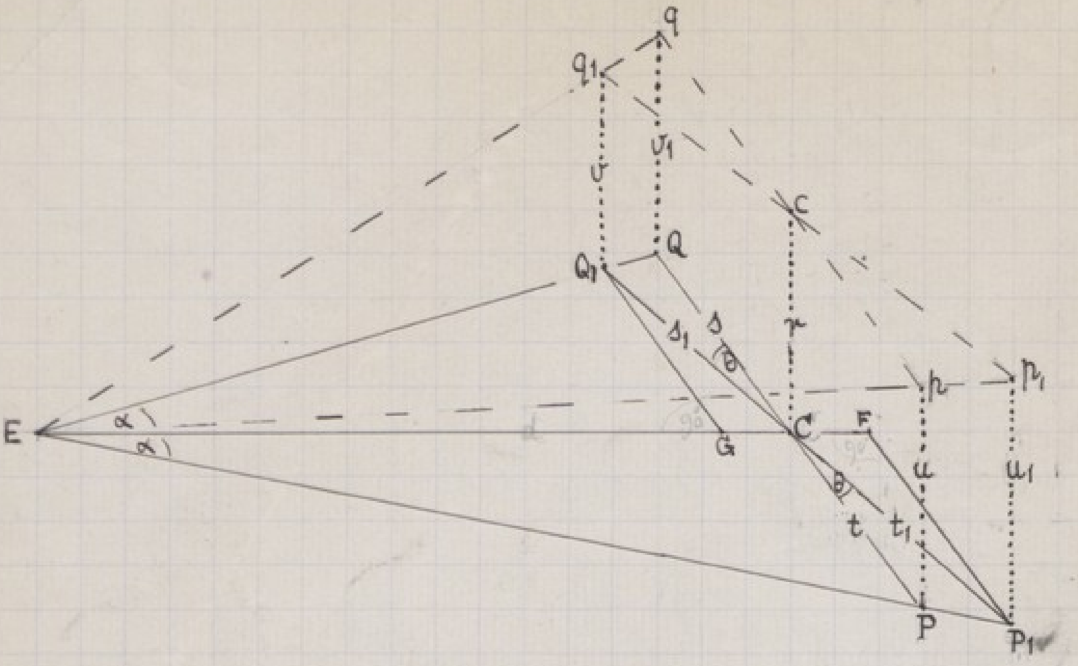
$s_2 : d :: \sin\beta : \cos(\beta+\varphi)$ $\qquad s_2 = d . \frac{\sin\beta}{\cos(\beta+\varphi)}$ (5)

$u_2 : u_1 :: d + s_2 . \sin\varphi : d$ $\quad (u_1 = r)$ $\qquad u_2 = \frac{r}{d}(d + s_2 \sin\varphi)$ (6)

in triangle $\Sigma P_2 C$ the angle at $P_2 = 90^\circ - \varphi$; its sine $= \cos\varphi$.

$t_2 : d :: \sin\gamma : \cos\varphi$ $\qquad t_2 = d \frac{\sin\gamma}{\cos\varphi}$

f. 26



(2 short sliding boards each bearing a lens ~~[illegible]~~,)

(say 14½ in. focus)

The apparatus consists of a base board B, a lens fixed in the middle, and 2 removable turntables ~~with appurtenances~~ The latter are exactly alike one another, are symmetrically placed, one at either end of the base board ~~at a total distance apart equal to four times the equivalent focal length of the lens.~~

~~at double the focal distance~~ The sliding boards are pushed to or fro until the position of the ~~central~~ pin round which ~~the~~ each turntable rotates, ~~is at twice~~ ~~is at a distance from the lens~~ is equal to twice ~~the~~ the length of the Equivalent focus of lens, measured from its ~~to the~~ side of the optical centre.

In the following description face means the side facing the lens; back means the ~~opposite~~ ~~other~~ side away from ~~the lens~~ it.

~~In~~ Each ~~the~~ turntable there are ~~consists~~ of ~~4~~ parts (1) a disc, with ~~hole~~ ~~a hole~~ a small hole in its centre that fits ~~the~~ a pin fixed in the baseboard ~~an arrow to~~ round which it turns, (2) A Cross, that is as follows a cross-shaped frame, carrying ~~with~~ threads ~~(2 [illegible] briefly called the Cross,)~~ that intersect at right angles. It which is hinged to the disc ~~on whose back~~ ~~[illegible] the threads are in vertical axial thread~~ & so arranged that when the cross is ~~at right angles~~ stands ~~upright~~ at right angles to ~~the disc~~ the disc, ~~lies exactly~~

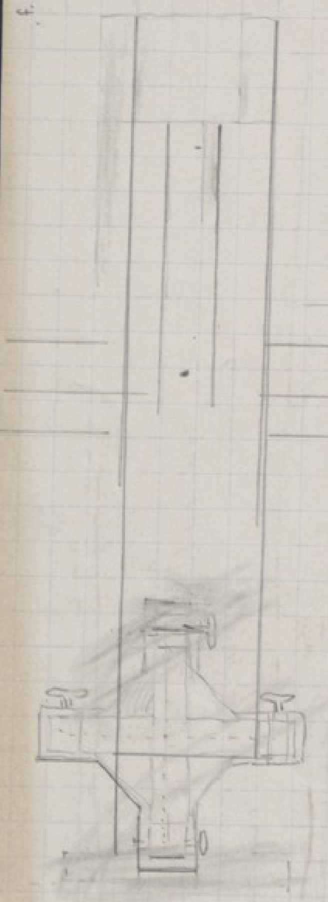
~~in the axial thread~~ its axial thread (which then becomes vertical) ~~is in the same~~ shall be in the ~~same~~ vertical line ~~as the axis~~ with the centre of the pin round which the disc rotates, ~~and shall~~

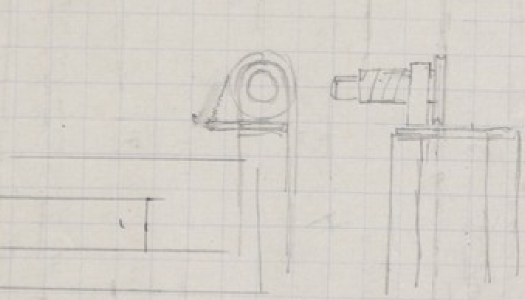
~~is at the same time just~~ The thread projects a very little beyond the back ~~plane of the~~ of the ~~central~~ framework of the cross. The horizontal thread,

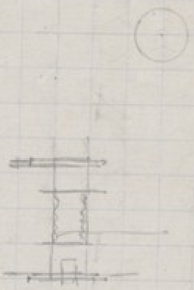
~~is as nearly as possible in the same plane as~~ lies ~~as~~ close behind the vertical thread, ~~as it can be~~ ~~without touching it~~ & is therefore practically in the same plane; ~~as~~

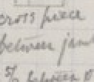
~~Hence~~ The point where the threads ~~inter~~ ~~appear to~~ intersect, when the Cross ~~is in the above position~~ stands vertically on the board, shortly lies in the ~~[illegible]~~ optical axis ~~plane~~ of the lens ~~[illegible]~~ the adjustments for effecting this will be described shortly

3 ~~the~~ a standard ~~appurtenances standard~~ frame, ~~[illegible]~~ fixed perpendicularly to the disc, it has ~~[illegible]~~ a circular aperture, in it of 3¾ inch diameter, and ~~with~~ many screw holes (suitably disposed ~~about~~ in its face) to receive clamps (like the holes ~~any~~ in the face plate of a lathe). by which the removable appurtenances about to be described can be securely ~~are~~ attached to it. The frame is erected ~~to the~~ somewhat behind the axis of the disc, ~~at a [illegible]~~ ~~to [illegible] the following~~ for reasons that ~~shortly to be~~ explained in the next paragraph but one.

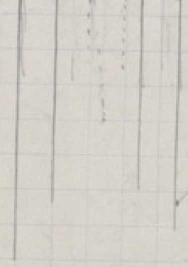


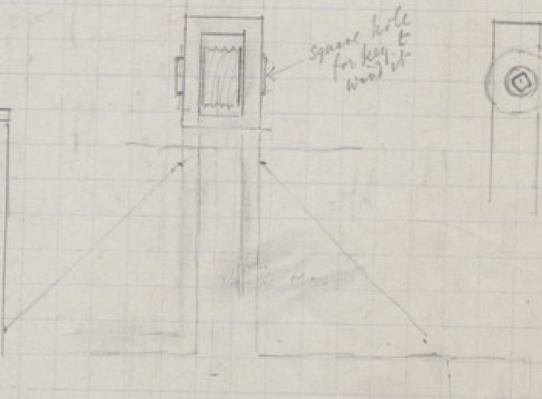






3/4 cross piece wide
1/4 between jambs
1/2 a 5/8 between outsides
3/8 width of [illegible]





Can allow 1 inch width

4. Three Removable appurtenances. (a) A flanged plate F, with slider attached, into which a ~~plate holder~~ either plate or a portrait holder can be inserted & fixed tightly. F ~~is secured to~~ is adjustable on the frame V & is finally secured to it by the clamps screwed into the holes just mentioned. The portrait holder P has tags for securing the portrait at its corners only. ~~being~~ The face of the portrait is flush with that of the flanged plate F. The exact distance of V behind the ~~position of the frame in connection with~~ depends on the sum of the thicknesses of the F & C, ~~has &~~ It under these conditions ~~must be such that~~ the axis of the turntable lies in the face of the portrait.

When about to adjust the portrait, the turntable is ~~lifted~~ lifted off the pin and laid on supports, so that the Disc D ~~shall be for the time~~ vertical ~~& the~~ (frame F horizontal &) shall be for the time. The flanged plate with the portrait holder slid into it and holding the portrait, all firmly fixed, is ~~then~~ laid upon F; then the hinged Cross is lowered down upon the portrait, the back of its axial wires will then lie ~~(especially the axial one)~~ in exactly the same plane as the face of the portrait, & that of the horizontal will be practically in the same plane. The tags that hold the portrait do not interfere with the position, because the arms of the cross pass between them. They leave room for an adjustment of F to at least 1/4 inch in any direction from its median position, as well as of 30° of rotation. ~~By help of fiducial points on the portrait &~~ F is ~~then~~ adjusted (while in a horizontal position, by sliding it about) until the fiducial points in the portrait are intersected by the cross wires. Then F is securely clamped to V. Finally, the turntable is replaced on its pin.
The object of the aperture in V is to admit of using a transparent portrait.

The plate holder C. requires no adjustment, but its ~~thickness must~~ must be made, like ~~correspond to that~~ of the portrait-holder b so far that the face of the sensitive plate shall be at ~~occupy~~ exactly the ~~position relation~~ same distance from the frame as that of the portrait would have been if it were inserted in its place. Sometimes the ~~one~~ has to be used in the ~~same~~ turntable & sometimes the other.

Graduations. The disc bears an arrow head, pointing backwards, and towards (from the axis at right angles to V) i.e. The base board carries a graduated (degrees) circle concentric with ~~graduation round~~ the disc & just outside it. The graduation that corresponds to the optical axis is marked 0°. The graduations are carried to 90° on either side & to −90° on the other. Consequently, when the arrow head points to 0° the portrait is ⊥r to the optical axis. If the turntable be ~~is~~ revolved until the arrow head points to, say, + 30°, it shows that the right hand side of the portrait, or of any curved [illegible] has advanced towards it & the left side has retreated (the amount of advance or retreat being in the case = ~~to a corresponding degree~~ (r. sin 30°)). Similarly for any other position of the turntable round its axis.

It is practically most convenient to ~~graduate the base board~~ graduate the base board ~~at intervals~~ at intervals of 10 only (numbered as they are), & to ~~add single degree~~ graduations, as the marked ones, on the disc, as far as ± 10° on either side of the arrow head.

It is ~~very~~ essential that the workmanship should be solid and exact. That is to say that the frame F shd be truly at right angles to the disc D; that the disc should revolve without shake, at right angles to ~~the~~ its apparent ~~[illegible]~~ axis; that the faces, whether of the portrait, or of the sensitive place, or of the axial [illegible] of the cross prolongation of that axis should coincide truly with the.

~~Means for adjusting the cross wires are~~.

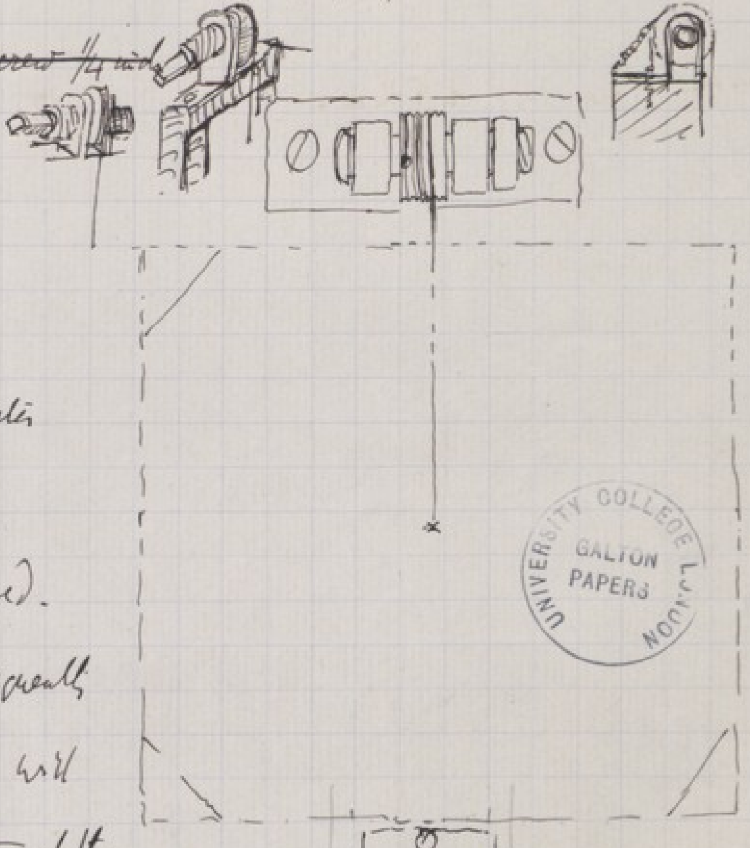
It is presumed that the focal distances are exact (insert description see 5) f. 30r

4

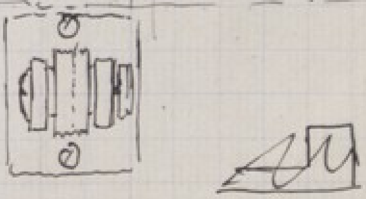
Adjustments. These ~~have to~~ are simple but exact. Everything (primarily) depends on the solidity of the cross & its hinges & the apparatus generally, which then ~~& security on the [illegible] & sufficient means~~ ~~for adjusting~~ the cross wires are ~~They are adjustable~~ independently adjusted ~~as will~~ ~~be shown.~~ ~~The [illegible] by using neither the~~ the empty horizontal holder C) ~~& placing a ground glass in front of~~ ~~When correctly in place~~ Then if ~~each of~~ the turntables A & B be placed with their ~~screw~~ heads at D, raise each of the two crosses ~~raised~~ & secure them with the face against their respective F. ~~resting against~~ laying the crosses, & setting them each against their corresponding F so ~~that they shall~~ both ~~be vertical &~~ Place a ground glass in front of one of them (say that on A).

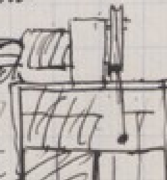
Then (1) the image of B on the ground glass of A of the cross wires of ~~the~~ B ought to ~~fall on the glass~~ (fixed a small aperture to the back) correspond with the cross wires themselves as seen against A. (2) Let B be rotated by an assistant first to one side, then to the other, whilst you yourself are stationed at A, and regard only the vertical (or axial) wire. Its position should be wholly unaffected by the rotation of B. If ~~such~~ not, the wire of B must be adjusted by turning the screws till it is so. (3) Keep B ~~square~~ with its pointer at 0° ~~Rotate B as before~~ and now regard the ~~horizontal wire~~ two wires. The point of intersection of the image of the two wires of B ~~with that of the vertical axis will be unchanged~~ should correspond with that of the ~~vertical~~ two wires of A. If not, the horizontal line is either above or below the optical axis ~~line joining the two intersections & the centre of the lens~~ is ~~vertically~~ tilted ~~vertically in the plane of the optical axis~~ & must in B first be adjusted ~~in the~~ till the image of the intersection of the B wires, seen at A stands half-way ~~between the~~ as far as it stood before, from the intersection of the actual A wires. When the ~~plane~~ adjustment is correct, the rotation of B will have no further effect on the image of its horizontal line beyond that of shortening it & blurring it through its ends being out of focus. The position of a line drawn through the middle of the blurring will always remain constant in position ~~horizontal~~ when the adjustment is correct. Then reverse the process stationing yourself at B and the other at A.

~~An exact adjustment is so important, the means of adjusting must~~ be ~~both effective & easy solid & easy to work~~. The threads of a well made apparatus is unlikely to be as much as 1/10 inch wrong in any ~~respect~~ direction; it therefore suffices to arrange to little ~~more than~~ that amount of play on either side of the median. A ~~portion of a fine screw 1/4 inch~~

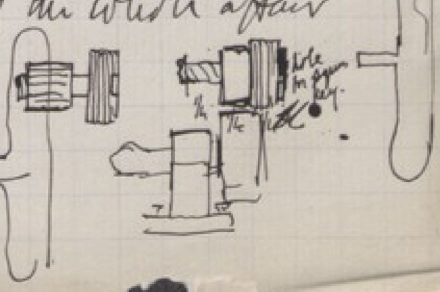


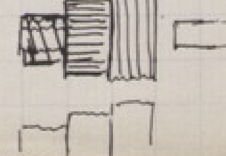
A cylinder 3/4" inch long, 1/2" diameter is turned down for 1/2 inch of its length to a diameter of 1/4 inch. A screw is turned on both portions. In the shorter & thicker portion it ~~should be~~ is of a width equal to the diameter of the thread (to be used). & less deep than its diameter. Consequently the edge of the thread wound round it will project slightly. The thinner portion of the screw passes rather stiffly through a block (which is fastened to the flanged frame F. and 1/4 inch thick,) projecting (above it) a little less than the bigger portion of the screw.





As the width of the whole affair has to be restricted, the screw may be turned by a key. inserted into a square hole made along ~~to~~ the axis of its thicker portion.





First transformation

$d \cdot \tan\alpha = t = r$

Call $EQ_1C = \zeta$; $180^\circ = (90-\theta) + \zeta + \alpha$ $\quad \zeta = 90^\circ + (\theta - \alpha)$ $\quad \sin EQ_1C = \cos(\theta-\alpha)$

$EP_1Q = 180^\circ - 2\alpha - \zeta = 90^\circ - 2\alpha - \theta + \alpha = 90^\circ - (\theta - \alpha)$ $\quad \sin EP_1C = \cos(\theta+\alpha)$

$s_1 : d :: \cancel{\sin(90^\circ-\theta) : \sin\alpha}$ $\quad \sin\alpha : \sin EQ_1C$ $\quad = \sin\alpha : \cos(\theta-\alpha)$ $\qquad s_1 = \frac{d \cdot \cos\theta}{\sin\alpha}$ $\qquad \frac{d \sin\alpha}{\cos(\theta-\alpha)}$ (1)

$t_1 : d :: \cancel{\sin(90^\circ+\theta) : \sin\alpha}$ $\quad \sin\alpha : \sin EP_1C$ $\quad = \sin\alpha : \cos(\theta+\alpha)$ $\qquad t_1 = \frac{d \sin\theta}{\sin\alpha}$ $\qquad \frac{d \sin\alpha}{\cos(\theta+\alpha)}$ (2)

$EP_1 : EF = EP : d$

$EP_1 : (d + t_1 \sin\theta) = \frac{d}{\cos\alpha} : d$
$= 1 : \cos\alpha$ $\qquad EP_1 = \frac{1}{\cos\alpha} \times \left(d + \frac{d \sin\theta}{\sin\alpha}\right) = d\left(\frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\alpha . \sin\alpha}\right)$ (3) ✓

$EQ_1 : EQ = (d - CG) : d = (d - s_1 \sin\theta) : d = \left(d - \frac{d . \cos\theta}{\sin\alpha}\right) : d = \left(1 - \frac{\cos\theta}{\sin\alpha}\right)$

$EQ_1 : \frac{d}{\cos\alpha} =$

$EQ = \frac{d}{\cos\alpha} - \frac{d \cos\theta}{\cos\alpha . \sin\alpha}$ $\qquad EQ_1 = d\left\{\frac{1}{\cos\alpha} - \frac{\cos\theta}{\cos\alpha \sin\alpha}\right\}$ (4)

$EP_1 : EP = EF : EC$ $\qquad EP = \frac{d}{\cos\alpha}$; $EF = d + t_1 \sin\theta$; $EC = d$

$EP_1 = \frac{1}{d}\left\{\frac{d}{\cos\alpha} \times (d + t_1 \sin\theta)\right\} = \frac{1}{\cos\alpha} \times \left(d + \frac{d . \sin\theta}{\sin\alpha}\right) = d\left(\frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\alpha . \sin\alpha}\right)$ (3) ✓

$\tan\beta = \frac{r}{d}$

$P_1p_1 : Pp = EP_1 : EP$ $\qquad Pp = r$; $EP_1 = d\left(\frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\alpha . \sin\alpha}\right)$; $EP = d$

$P_1p_1 = \frac{1}{d} \cdot \left\{ rd \left(\frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\alpha \sin\alpha}\right)\right\}$; $\quad P_1p_1 = r\left(\frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\alpha . \sin\alpha}\right)$ (5)

$Q_1q_1 : Qq = EQ_1 : EQ$ $\qquad Qq = r$; $EQ_1 = d\left\{\frac{1}{\cos\alpha} - \frac{\cos\theta}{\cos\alpha . \sin\alpha}\right\}$; $EQ = d$

$Q_1q_1 = \frac{1}{d} \times \left\{ rd \left(\frac{1}{\cos\alpha} - \frac{\cos\theta}{\cos\alpha . \sin\alpha}\right)\right\}$; $\quad Q_1q_1 = r\left(\frac{1}{\cos\alpha} - \frac{\cos\theta}{\cos\alpha \sin\alpha}\right)$ (6)

Appears made very gimcracky – there is board on it &c other & it had got made

Lens – convenient good [illegible] & its depth – but [illegible] it need [illegible] any lens w^d do – 4½ f.c ?

Turntable or a board that can be slid a little to & fro on base board to ensure the exact distance from lens

Ground glass in front of cross – all one to portrait holder

Covering it all in

f. 33

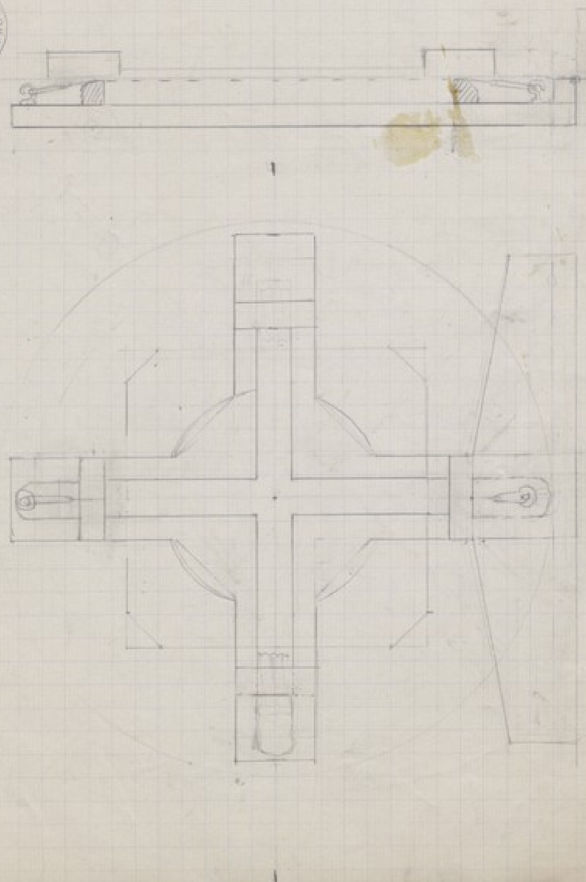
(1

The apparatus consists of B, a base board; L, a lens fixed above its middle; D_1, D_2, two sliding boards each with a short pin projecting from the middle; T_1, T_2, two removable turntables that ~~fit on to~~ ~~the~~ revolve round the pins. The latter are exactly alike one another and are symmetrically placed at either end of the base board, each at a distance from the optical centre of the lens equal to twice the length of its equivalent focus. The object of the sliding boards is to enable the adjustment to be made exactly. If however the data have been determined with precision the sliding boards might be dispensed with and the pins fixed directly into the base board.

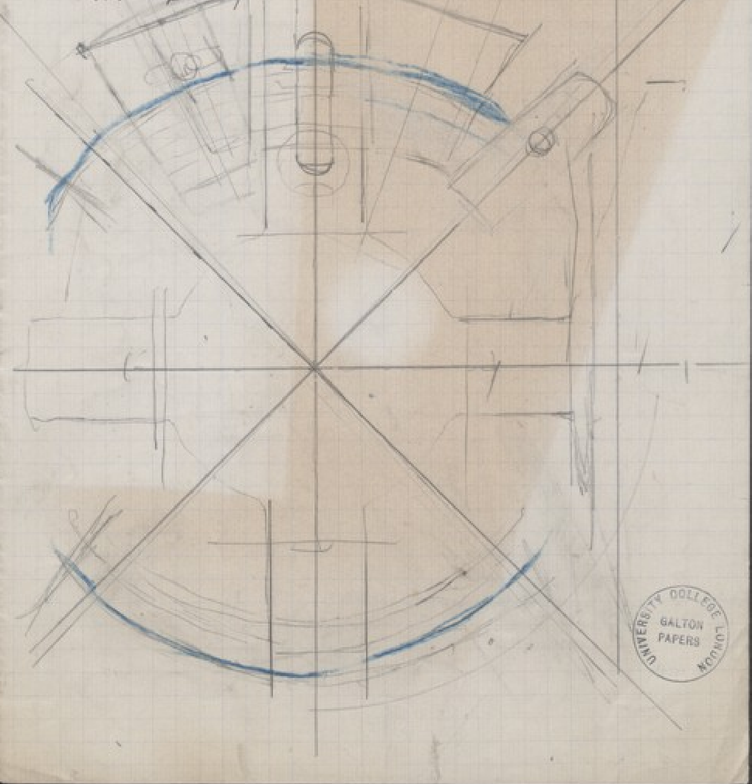
The lens, L, whose focal length determines the size of the apparatus has been taken at inches. So far as the reductions are concerned ~~the~~ it is used with so small an aperture, that a plain lens would suffice, but this gives too little light by far for making the adjustments. It is also very convenient to have ready means of varying the size of the aperture I have therefore employed a good lens with an iris diaphragm

The turntables T consist of parts

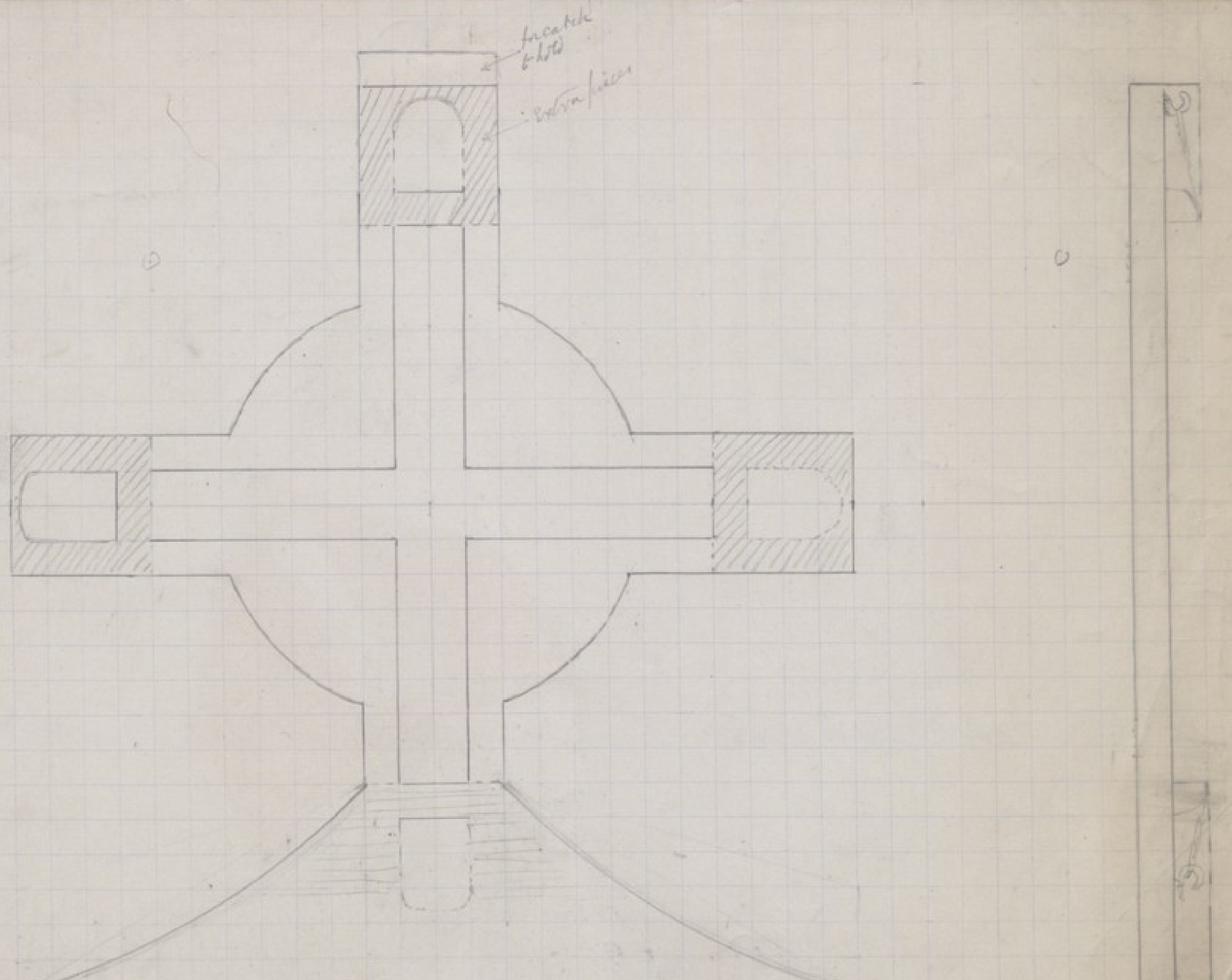
1. A disc D with a hole in its centre to fit the pin, round which it turns.



2. A frame, R, securely fixed at right angles to the disc D; its position on it ~~will be~~ is determined by the conditions explained in   It has a circular aperture of ...

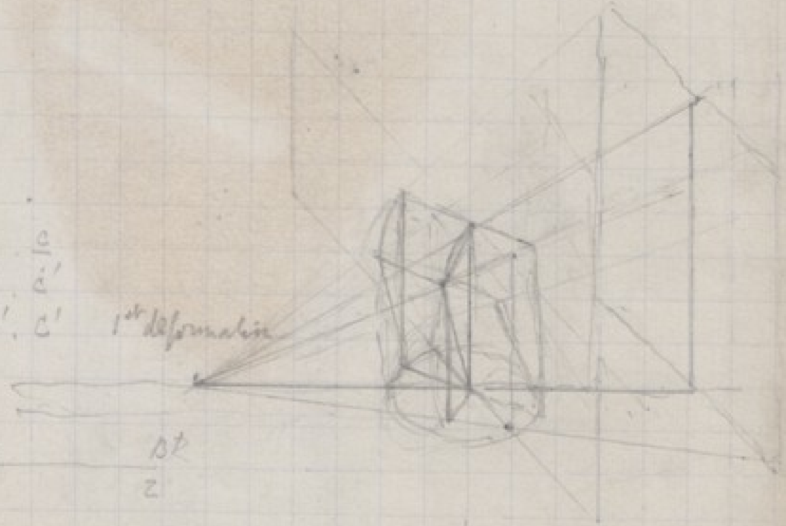


f. 36r

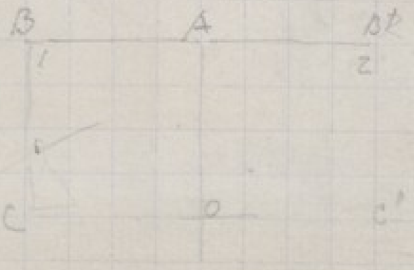


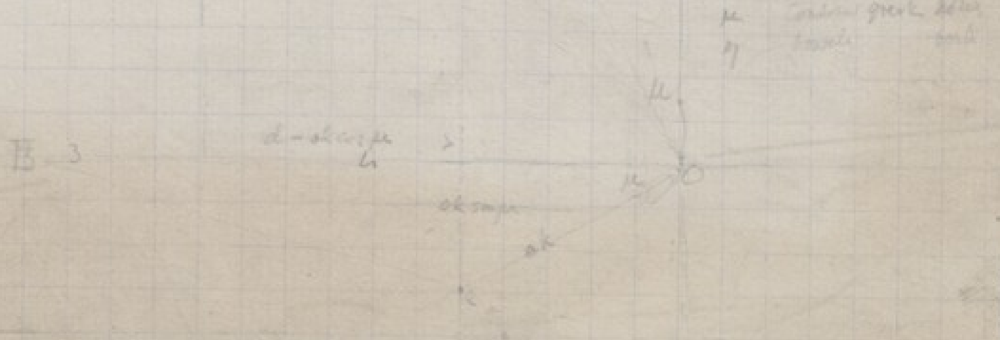
Relative proportions in planes
XY from transverse μ
XZ from vertical η

Projection in plane of
XY of A, B, C   a b c $\underline{b}$ $\underline{c}$
XZ of " " "   a' b' c' $\underline{b}'$ $\underline{c}'$
✳ YZ of " " "   A B C   B', C'   1st deformation



Repeat with ✳ for 2d deformat.
with other letters
Equate the result with k' and
$k' = ok$





$ok =$
Length of hor projection of OK = $r \cos\eta$
angle of Ok with = μ
dist of K from E = $\sqrt{(d - ok\cos\mu)^2 + (ok\sin\mu)^2}$
= $\sqrt{(d - r\cos\eta\cdot\cos\mu)^2 + (r\cos\eta\sin\mu)^2}$

Reduction in breadth only
by photography

(increase of breadth does not yet come out rightly)

f. 38 (1)

Photographic ~~enlarge~~ reduction or enlargement in one direction only

The process about to be described was intended for use in composite portraiture to which it is well adapted. In this, the portraits all having the same aspect (say full face) ~~are full face~~ all taken under light falling in the same direction, are reduced to a common standard height, (say that the distance between the line passing through the centres of the pupils of the eyes & that passing between the middle of the lips is in all cases made equal to 0.4 inch). The portraits are then adjusted by suitable register marks and their images successively thrown in superposition on the same sensitive plate.

The results of good care as is well known an idealised and unexpectedly good average portrait of them all, but I was never content with the results because of a certain want of fit horizontally. A broad & a narrow face, that share the same family likeness cannot be exactly fitted & there is a consequent blur. By this process a fit can be ensured not only in the dimension specified above but in the distance between the centres of the pupils also, the residual blurring becoming greatly reduced. In the course of the process, opportunity is given of correcting another ~~occasional~~ drawback sometimes very noticeable, ~~in lunatics~~ of a want of parallelism between the eye brows & mouth, the face being shorter on one side than on the other.

I give ~~here some diagrams showing both~~ Fig 1 examples of the broadening & the narrowing, ~~reduction & enlargement~~ ~~of the same figure~~ of the same square in which is enclosed a circle, ~~some letters & other marks~~ with letters at the corners ~~convenient for explanation are inserted~~ which have been affected by this process.

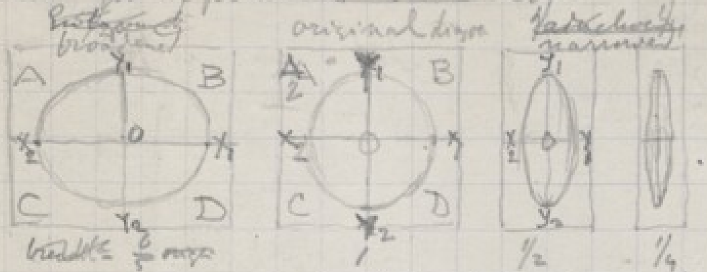
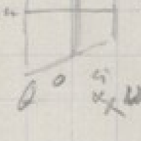


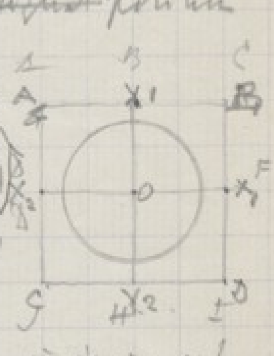
Fig 1

3.5
1.75

~~It is easily understood that the~~ A square rectangle turned round ~~on~~ a vertical axis ~~would~~ become distorted perspectively  In Fig 2 ~~is an example of~~ where the original diagram of Fig I, has been, when turned round XX through 60° is viewed from a point in the line of intersection of the two optical planes XX & YY at a distance of 2 times its height from ~~the~~ an median line. The greater the distance ~~from~~ of the view point from O the more nearly does the figure become a rectangle & if it were convenient to photograph ~~the~~ square in the above position from a considerable distance and to magnify the result to the required size, the problem of reduction in one dimension only would be practically solved. But it is not ~~feasible~~ or convenient to do so & an other method must be employed. When photography is employed as it was in produced Fig 2 the aperture of the lens must approach the "pin-hole" dimension, or the ~~advanced~~ sides of the rectangle would be greatly blurred through ~~from~~ being out of focus with its axis, the one side much nearer to the eye & the other side much further.

The size of the aperture suitable for the purpose depends on the character of the portrait. In the present instance it was f/. In composite portraiture less precision is needed. Of course this means long exposures, but the work is so mechanical, especially when artificial light is used, that it is not a serious drawback. Otherwise, When a piece of exposed test paper has assumed a given tint, the exposure is stopped; a minute or two error the time being of little consequence.

Principle of the process. Regard at first, only the upper half of the diagram in Fig. I. The eye is supposed to be situated at a point E, in the optical axis passing through O, and at a distance from O equal to d. ~~However the square and~~ Let the image of the square however it be turned from its ~~original~~ primary position which is ⊥ to the optical axis, to be always projected on the plane passing through its ~~original~~ primary position.



Then the vertical axis of the half square OX_1 will always remain unchanged in position and length, (but the advanced side say BX_1 will be lengthened & the retreating side will be shortened), the horizontal axis X_1OX_2 (which is made up of the original portions OX_1 & OX_2) will always remain horizontal, but its length will ~~vary~~ becoming narrower ~~the more~~ as the angle θ of rotation is increased, until it ~~becomes 0~~ is reduced to zero when $\theta = 90°$. It will be remembered that every projection of a straight line in the diagram on to the ~~original~~ primary plane will remain straight. Consequently in all circumstances the perspective figure, Fig 3 made by rotating the square, ~~as in Fig~~ will be bounded by straight lines.

It then occurred to me that if the perspective representing image of the upper half of the original square which may be called $X'_2 A', Y'_1 B' X'_1$

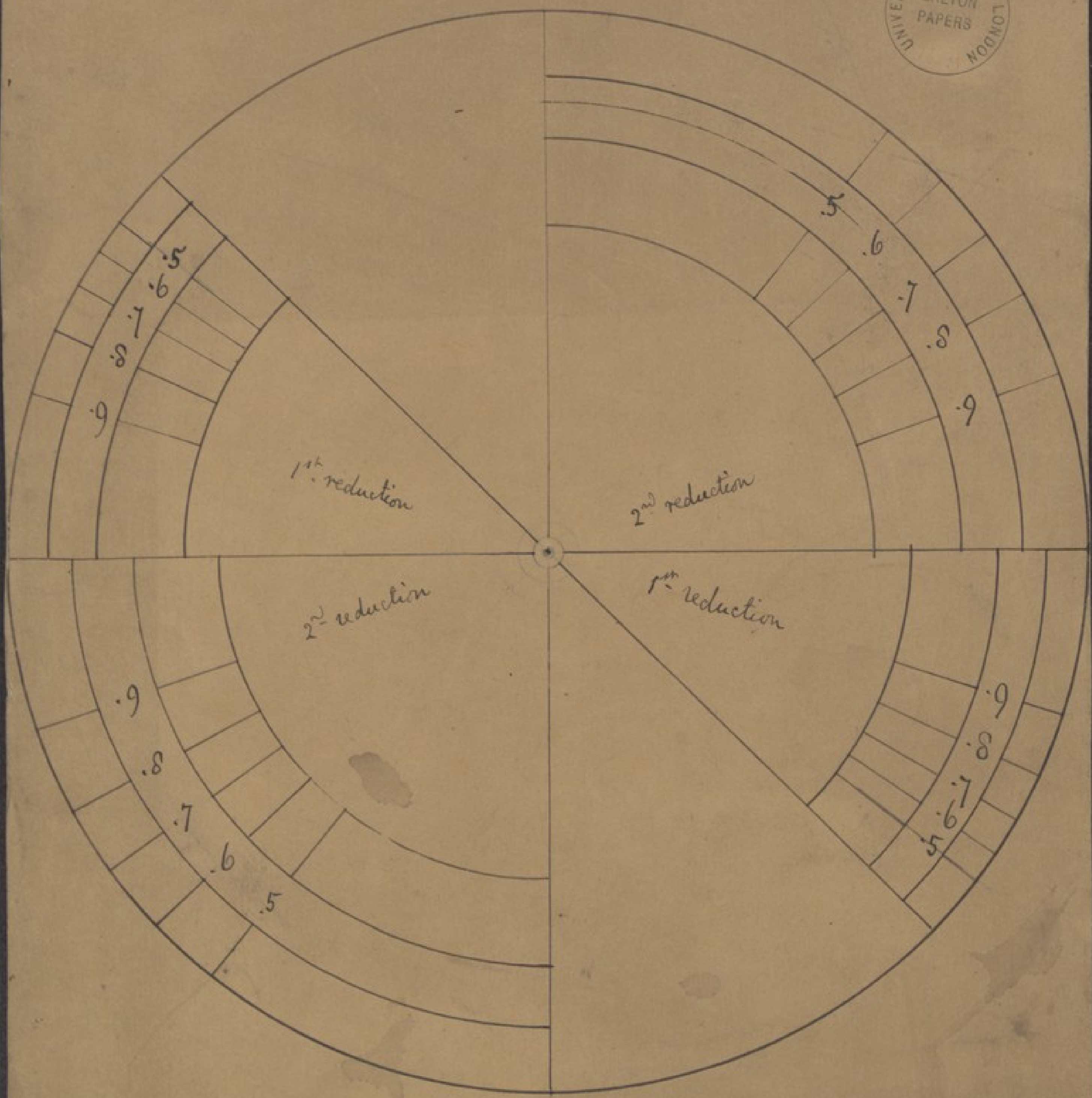
were mounted on the same as if had been plate x viewed from the same point E

x if the rotation were given to it through an angle φ in the opposite direction ~~as before~~ so that the longer side AF should be the farther from E, then the length as projected on the primary plane at a certain angle Δ'' wd. be reduced,

~~that it~~ A'F' would become equal to BO, x consequently A'B' wd. be horizontal x parallel to DO. But we have seen that A'B'F' ~~This being the case, as~~ AC must be a straight line, consequently the whole line AC must be parallel to the horizontal axis x ~~similarly from~~ again owing to the symmetry of the conditions, so must the lower line G''H''I''. In short, the square will have been turned into a narrow rectangle ~~of~~; unaltered in height, narrowed in breadth.

On working out the problem ~~to learn the~~ the relation of θ to φ, in order to produce this effect, proves to be extremely simple and elegant, it is $\tan\theta = \sin\varphi$, entirely independent of the distance d. ~~As negative values are inadmissible~~, The maximum rotation that φ can have is consequently $90°$, giving $\sin\varphi = 1$ ~~and consequently the maximum~~ and corresponding value of θ is $45°$. The problem is annexed to the paper. x tables are given, of the empirical θ x φ corresponding to various degrees of reduction.

It will easily be understood that the converse process of keeping the object to be photographed in the primary plane and of inclining the sensitive plate leads on the same principle x with similar results, to a broadening of the original picture without affecting its height.


1st. reduction
2nd reduction
2nd reduction
1st reduction
.5
.6
.7
.8
.9
.5
.6
.7
.8
.9
.5
.6
.7
.8
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.5
.6
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