Symbols of Two-dimensional Symmetry Operators

Point symmetry (1st position)			Line symmetry (2nd and 3rd positions)		
Printed symbol	Graphical symbol	Operator	Printed symbol	Graphical symbol	Operator
2	⬮	2-fold (180°) point	*m*	———	Mirror-reflection line of symmetry
3	▲	3-fold (120°) point			
4	◆	4-fold (90°) point	*g*	------	Glide-reflection line with translation of one half the repeat distance along the line
6	⬢	6-fold (60°) point			

NOTE. The symbol 1 denotes the identity operation and hence the absence of any other (point or line) symmetry.

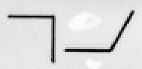
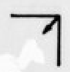
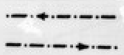
the origin, [illegible] edges (the x-, y-, z-axes) being [illegible]

Symmetry Symbols

In the diagrams of space groups, heights are indicated in the manner described above for equivalent gene[illegible] translation t along an axis always impl[illegible]s *not* apply to equivalent positions.

The graphical symbols used in the drawings are given in Tables 4.1.6 and 4.1.7, which also give the nature of the translations corresponding to the various symbols.

TABLE 4.1.6. **Symbols of Symmetry Planes**

Symbol	Symmetry plane	Graphical symbol		Nature of glide translation
		Normal to plane of projection	Parallel to plane of projection	
m	Reflection plane (mirror)	————		None (NOTE. If the plane is at $z=\frac{1}{4}$ this is shown by printing $\frac{1}{4}$ beside the symbol.)
a, b	Axial glide plane	- - - - - -		$a/2$ along [100] or $b/2$ along [010]; or along ⟨100⟩.
c	Axial glide plane	....	None	$c/2$ along z-axis; or $(a+b+c)/2$ along [111] on rhombohedral axes.
n	Diagonal glide plane (net)	-·-·-·-·-·-		$(a+b)/2$ or $(b+c)/2$ or $(c+a)/2$; or $(a+b+c)/2$ (tetragonal and cubic).
d	"Diamond" glide plane			$(a\pm b)/4$ or $(b\pm c)/4$ or $(c\pm a)/4$; or $(a\pm b\pm c)/4$ (tetragonal and cubic). See note below.

NOTE. In the "diamond" glide plane the glide translation is half of the resultant of the two possible axial glide translations. The arrows in the first diagram show the direction of the horizontal component of the translation when the z-component is positive. In the second diagram the arrow shows the actual direction of the glide translation; there is always another diamond-glide reflection plane parallel to the first with a height difference of $\frac{1}{4}$ and with the arrow pointing along the other diagonal of the cell face.

Symbols of Symmetry Axes

Symbol	Symmetry axis	Graphical symbol	Nature of right-handed screw translation along the axis	Symbol	Symmetry axis	Graphical symbol (normal to plane of paper)	Nature of right-handed screw translation along the axis
1	Rotation monad	None	None	4	Rotation tetrad		None
$\bar{1}$	Inversion monad		None	4_1	Screw tetrads		$c/4$
				4_2			$2c/4$
2	Rotation diad	(normal to paper)	None	4_3			$3c/4$
		(parallel to paper)		$\bar{4}$	Inversion tetrad		None
2_1	Screw diad	(normal to paper)	$c/2$	6	Rotation hexad		None
		(parallel to paper)	Either $a/2$ or $b/2$	6_1	Screw hexads		$c/6$
		Normal to paper		6_2			$2c/6$
3	Rotation triad		None	6_3			$3c/6$
				6_4			$4c/6$
3_1	Screw triads		$c/3$				
3_2			$2c/3$	6_5			$5c/6$
$\bar{3}$	Inversion triad		None	$\bar{6}$	Inversion hexad		None

In any space-group symbol the order in which the various parts of the symbol are written is, in general, the same as in the corresponding point groups (see Table 3.3.2).

The nomenclature described is sufficient to cover all but four of the 230 space groups. A convention is necessary to distinguish between the members of the pairs $I222$, $I2_12_12_1$ and $I23$, $I2_13$. This arises because in all four of these groups both 2-fold rotation and screw axes are present in each of [redacted] conventional symbol to distinguish between the [redacted] in which all three 2-fold rotation axes intersect in a point (the three screw axes intersecting in another point), as in the groups $I222$ and I[redacted] where this intersection of all three axes of the same kind [redacted]

TABLE I. THE 32 CRYSTAL CLASSES

(The eleven classes of distinct Laue symmetry are separated by vertical lines)

System	Axes and angles			Crystal classes (point groups)						
Triclinic	a α	b β	c γ	1 C_1	$\bar{1}$ C_i					
Monoclinic	a 90°	b β	c 90°	2 C_2	m or $\bar{2}$ C_s	$2/m$ C_{2h}				
Orthorhombic	a 90°	b 90°	c 90°				222 D_2	$mm2$ C_{2v}	mmm D_{2h}	
Tetragonal	a 90°	a 90°	c 90°	4 C_4	$\bar{4}$ S_4	$4/m$ C_{4h}	422 D_4	$4mm$ C_{4v}	$\bar{4}2m$ D_{2d}	$4/mmm$ D_{4h}
Trigonal (rhombohedral)	aaa $\alpha\alpha\alpha$ or a 90°	a 90°	c 120°	3 C_3	$\bar{3}$ C_{3i}		32 D_3	$3m$ C_{3v}	$\bar{3}m$ D_{3d}	
Hexagonal	a 90°	a 90°	c 120°	6 C_6	$\bar{6}$ C_{3h}	$6/m$ C_{6h}	622 D_6	$6mm$ C_{6v}	$\bar{6}m2$ D_{3h}	$6/mmm$ D_{6h}
Cubic	a 90°	a 90°	a 90°	23 T	$m3$ T_h		432 O	$\bar{4}3m$ T_d	$m3m$ O_h	

This gives rise to the seven well-known crystallographic systems, and the arrangement generally adopted is shown in Table I. Reference axes are also indicated, and these range from the triclinic system, w[illegible]clined at any angle, t[illegible]ually perpendicular.

While the 3-fo[illegible]rhombohedral system, it should be noted that this system cannot be based exclusively on the rhombohedral lattice (see Chapter III). When the space groups (Table III) are developed from these poi[illegible]that the sim[illegible]ases

$$2 \cdot \bar{1} = m, \qquad m \cdot \bar{1} = 2,$$

product sign indicating that the operations denoted by the
bols are applied in succession. Also

$$m^2 = 1, \qquad \bar{1}^2 = 1, \qquad 2^2 = 1.$$

may be noted that all the
tions of such a group
one point unmoved. In
e geometrical figure all
es of symmetry and any
es of symmetry must pass
ugh such a fixed point,
h is called the centre of
group, a group of this kind
called a *point group*. If
were two parallel planes of
etry, or two parallel

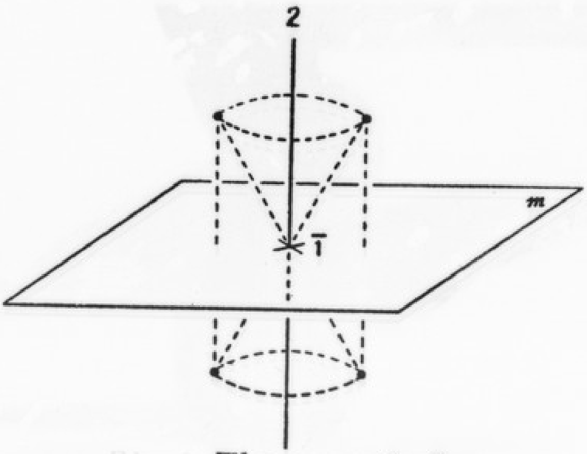


Fig. 2. The group $2m\bar{1}$.

of symmetry a fixed distance apart, the operation of one on the other, or of one axis on the other, would produce a and so on to infinity, the figure thus becoming indefinitely ded.

LIMITATION O

solid ... metrical figure c... constructed ...

o cubic gro

he introdu e n

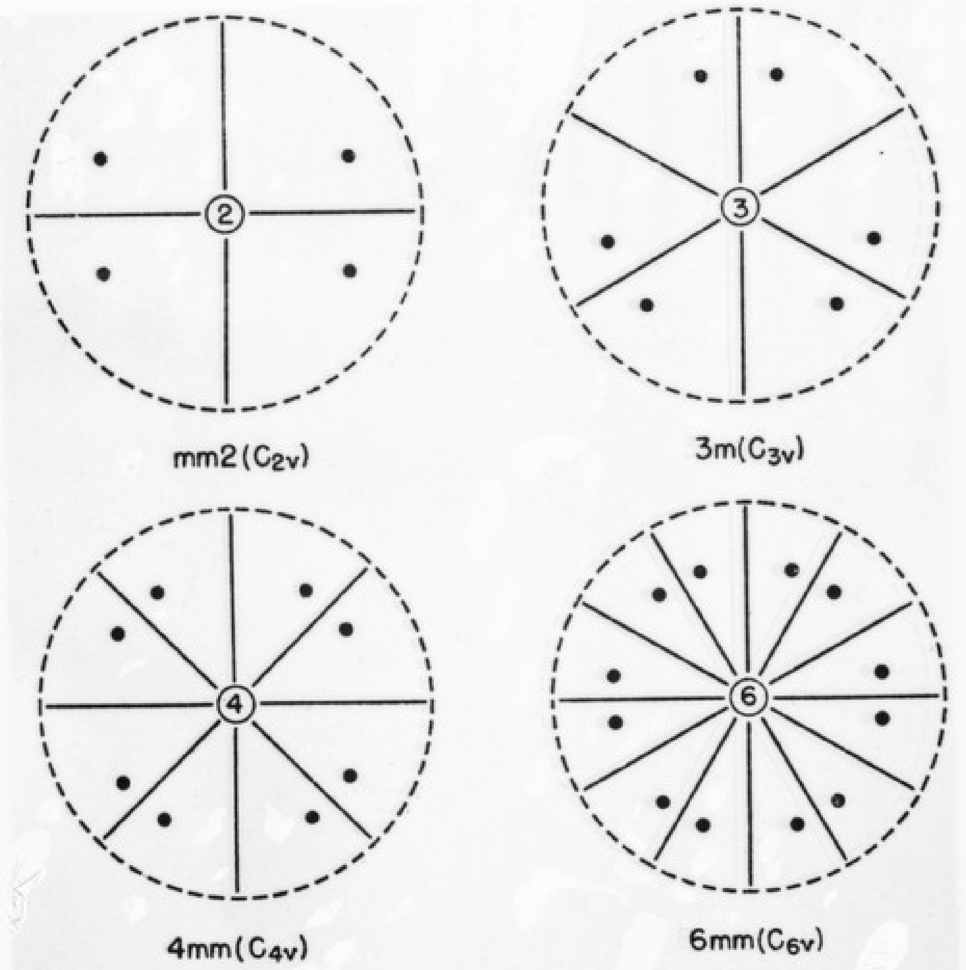
axis brings it into self-coincidence and gives rise to th

ps of Fig. 8 and one, $3/m=\bar{6}$ (C_{3h}), already mentioned abo

5). The introduction of a reflection plane *parallel* to t

d axis gives rise to a set of such planes equal in number

multiplicity of the axis. These groups are illustrated



Point groups, C_{2v}–C_{6v}.

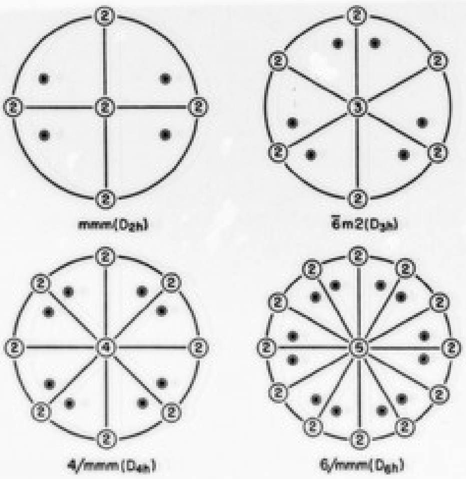
9, and are analogous to those of Fig. 6. A reflection pla

lso be introduced perpendicular to the n-fold axis of t

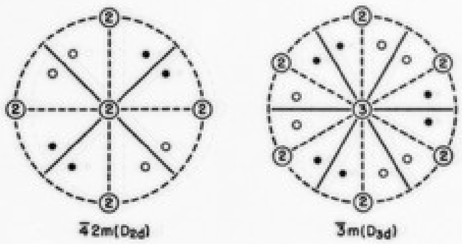
s D_2-D_6 (Fig. 6), giving rise to the four further groups

0. In the case of D_2 and D_3 (Fig. 6), reflection planes ca

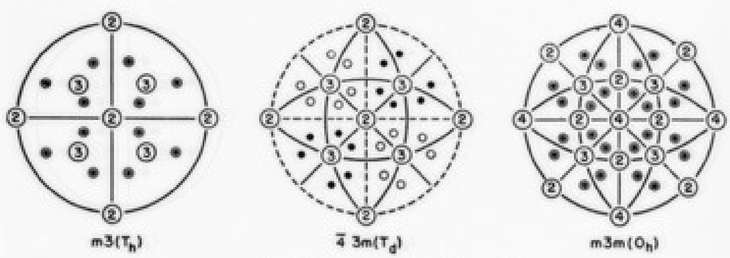
roduced bisect



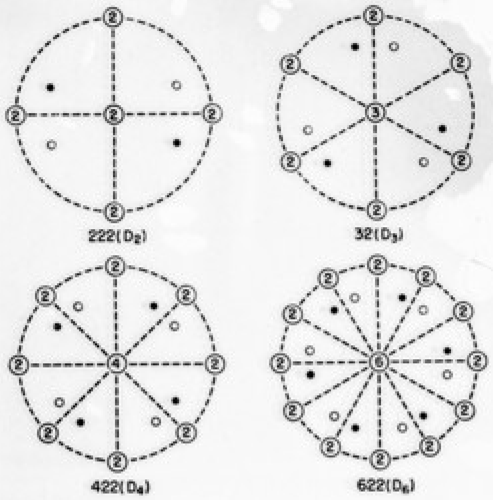
Point groups, D_{2h}–D_{6h}.



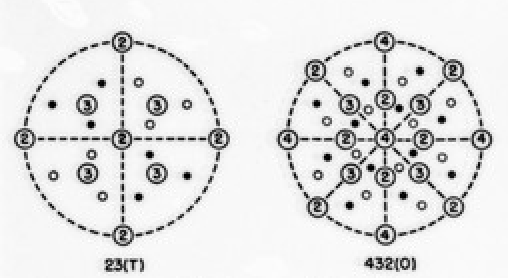
Point groups, D_{2d} and D_{3d}.



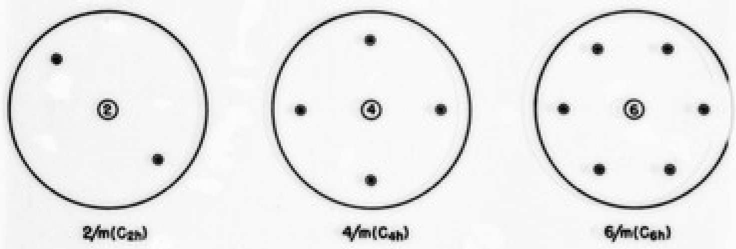
Point groups, T_h, T_d, and O_h.



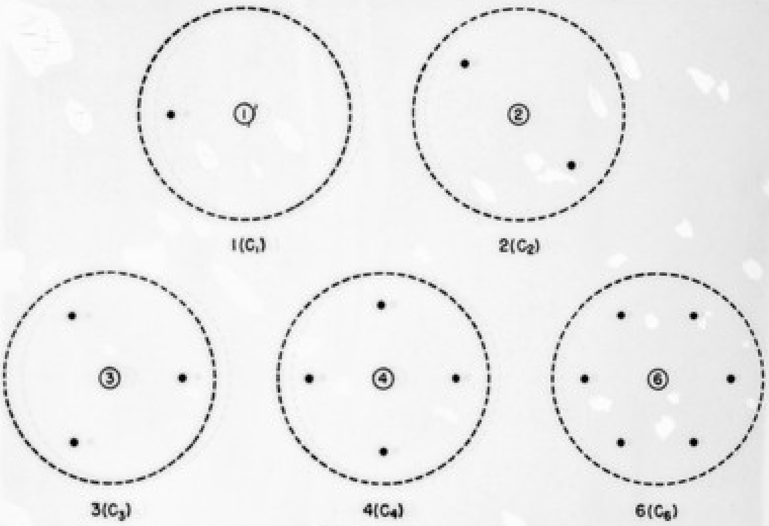
Point groups, D_2–D_6.



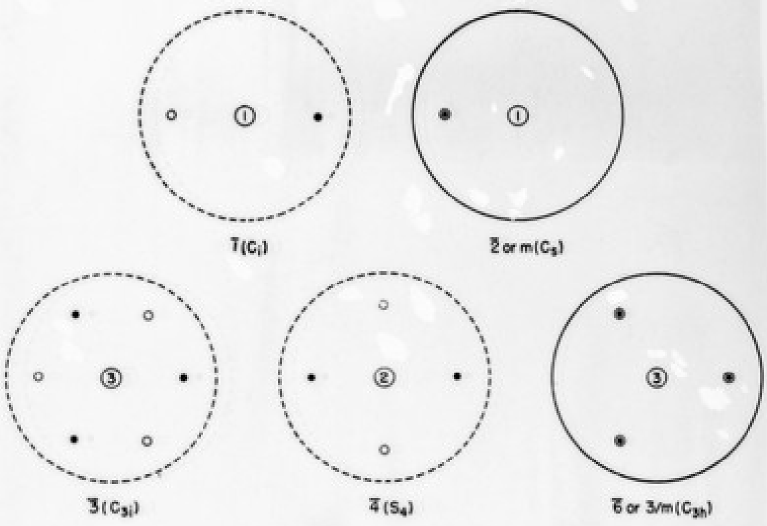
Point groups, T and O.



Point groups, C_{2h}–C_{6h}.



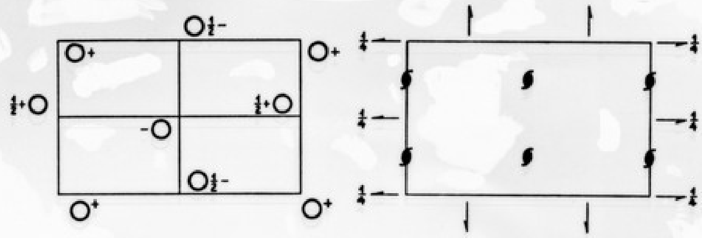
Point groups, 1–6.



Point groups, $\bar{1}$–$\bar{6}$.

Orthorhombic 222 $P\,2_1\,2_1\,2_1$ No. 19

$P\,2_1\,2_1\,2_1$
D_2^4



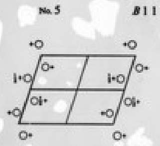
Origin halfway between three pairs of non-intersecting screw axes

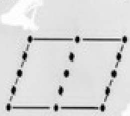
Number of positions, Wyckoff notation, and point symmetry			Co-ordinates of equivalent positions	Conditions limiting possible reflections
4	a	1	$x,y,z;\ \frac{1}{2}-x,\bar{y},\frac{1}{2}+z;\ \frac{1}{2}+x,\frac{1}{2}-y,\bar{z};\ \bar{x},\frac{1}{2}+y,\frac{1}{2}-z.$	hkl: $0kl$: $h0l$: $hk0$: No conditions $h00$: $h=2n$ $0k0$: $k=2n$ $00l$: $l=2n$

Symmetry of special projections

(001) pgg; $a'=a,\ b'=b$ (100) pgg; $b'=b,\ c'=c$ (010) pgg; $c'=c,\ a'=a$

$B2$ / C_2^3 — No. 5 — $B\,1\,1\,2$ — 2 Monoclinic





1ST SETTING

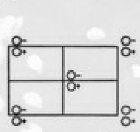
Origin on 2; unique axis c

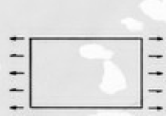
Number of positions, Wyckoff notation, and point symmetry			Co-ordinates of equivalent positions $(0,0,0;\ \tfrac{1}{2},0,\tfrac{1}{2})+$	Conditions limiting possible reflections
4	c	1	$x,y,z;\ \bar{x},\bar{y},z.$	General: hkl: $h+l=2n$; $hk0$: $(h=2n)$; $00l$: $(l=2n)$
2	b	2	$0,\tfrac{1}{2},z.$	Special: as above only
2	a	2	$0,0,z.$	

Symmetry of special projections

(001) $p2$; $a'=a/2$, $b'=b$ (100) $pm1$; $b'=b$, $c'=c/2$ (010) $c1m$; $c'=c$, $a'=a$

Monoclinic 2 — $C\,1\,2\,1$ — No. 5 — $C2$ / C_2^3





Origin on 2; unique axis b

2ND SETTING

Number of positions, Wyckoff notation, and point symmetry			Co-ordinates of equivalent positions $(0,0,0;\ \tfrac{1}{2},\tfrac{1}{2},0)+$	Conditions limiting possible reflections
4	c	1	$x,y,z;\ \bar{x},y,\bar{z}.$	General: hkl: $h+k=2n$; $h0l$: $(h=2n)$; $0k0$: $(k=2n)$
2	b	2	$0,y,\tfrac{1}{2}.$	Special: as above only
2	a	2	$0,y,0.$	

Symmetry of special projections

(001) $cm1$; $a'=a$, $b'=b$ (100) $p1m$; $b'=b/2$, $c'=c$ (010) $p2$; $c'=c$, $a'=a/2$

first results, obtained by M. L. Frankenheim,[3] were not entirely satisfactory, but rigid geometrical proofs were obtained soon afterwards by Auguste Bravais,[4] who demonstrated that only 14 distinct types of space [redacted] larly distributed in [redacted] nvestigation, we may proceed in three steps. A rectilinear system of equidistant points, extending indefinitely in both directions, is called a *row*

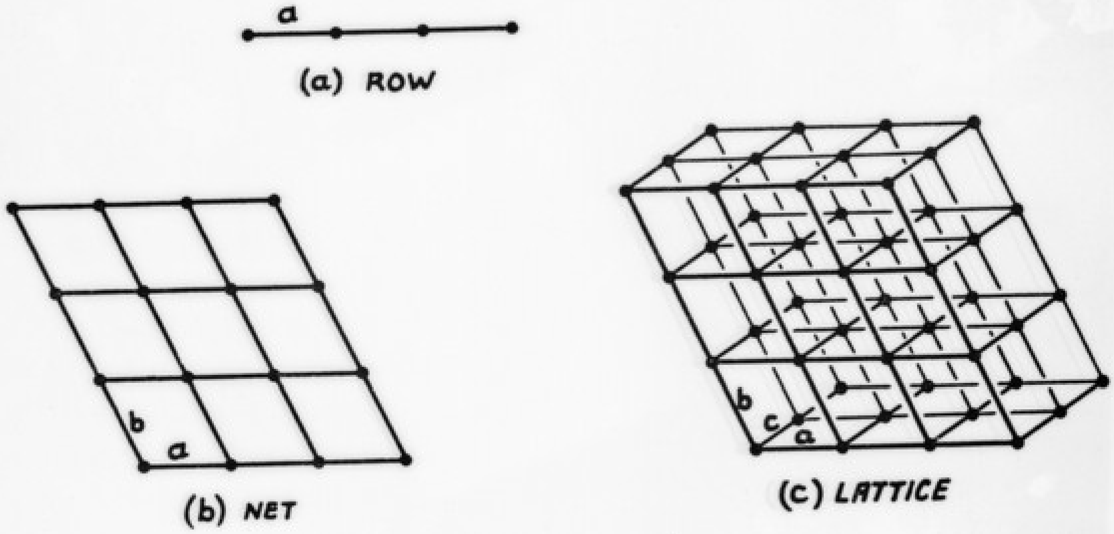


Fig. 15. Lattices and nets.

(Fig. 15a), and is fully described by one parameter, a, the fundamental interval separating two neighbouring points. A regular series of such parallel rows, lying in one plane, can be described by two fundamental intervals, a and b, and an arbitrary angle γ. This is called a *net* (Fig. 15b). The system of points regularly distributed in three dimensions consists of an indefinitely extended regular series of parallel nets, and can be described by three fundamental intervals, a, b, and c, together with three arbitrary angles, α, β, and γ. Such a system is called a *lattice* (Fig. 15c). The smallest parallelopiped which is identically

[3] M. L. Frankenheim, *Die Lehre von der Cohäsion*, Breslau, 1835; also *Nova Acta Acad. Caes. Leopoldino-Carolinae Nat. Cur.*, 1842, 19(2), 471–660.

[4] Auguste Bravais, "Mémoire sur les systèmes formés par des points distribués regulièrement sur un plan ou dans l'espace, *J. école polytéc*...