Substituting this into (II, 6) and (II, 7) we obtain

$$
\begin{aligned}
4\pi^2\nu^2 m_1 x_1 &= k_{xx}^{11}x_1 + k_{xy}^{11}y_1 + k_{xz}^{11}z_1 + k_{xx}^{12}x_2 + \cdots + k_{xz}^{1N}z_N,\\
4\pi^2\nu^2 m_1 y_1 &= k_{yx}^{11}x_1 + k_{yy}^{11}y_1 + k_{yz}^{11}z_1 + k_{yx}^{12}x_2 + \cdots + k_{yz}^{1N}z_N,\\
4\pi^2\nu^2 m_1 z_1 &= k_{zx}^{11}x_1 + k_{zy}^{11}y_1 + k_{zz}^{11}z_1 + k_{zx}^{12}x_2 + \cdots + k_{zz}^{1N}z_N,\\
4\pi^2\nu^2 m_2 x_2 &= k_{xx}^{21}x_1 + k_{xy}^{21}y_1 + k_{xz}^{21}z_1 + k_{xx}^{22}x_2 + \cdots + k_{xz}^{2N}z_N,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
4\pi^2\nu^2 m_N z_N &= k_{zx}^{N1}x_1 + k_{zy}^{N1}y_1 + k_{zz}^{N1}z_1 + k_{zx}^{N2}x_2 + \cdots + k_{zz}^{NN}z_N.
\end{aligned}
\qquad \text{(II, 10)}
$$

This system of linear and homogeneous equations for x_1, y_1, z_1, x_2, y_2, z_2, $\cdots z_N$ cannot be solved for arbitrary values of the coefficients occurring therein but, as shown by the theory of linear algebraic equations, only if the determinant of the coefficients is equal to zero. Since the force constants k_{xy}^{il} are fixed for a given system the only way to fulfill this condition is by a suitable choice of the frequency ν. Thus *for certain frequencies defined by the condition*

$$
\begin{vmatrix}
k_{xx}^{11} - 4\pi^2\nu^2 m_1 & k_{xy}^{11} & k_{xz}^{11} & k_{xx}^{12} & \cdots & k_{xz}^{1N}\\
k_{yx}^{11} & k_{yy}^{11} - 4\pi^2\nu^2 m_1 & k_{yz}^{11} & k_{yx}^{12} & \cdots & k_{yz}^{1N}\\
k_{zx}^{11} & k_{zy}^{11} & k_{zz}^{11} - 4\pi^2\nu^2 m_1 & k_{zx}^{12} & \cdots & k_{zz}^{1N}\\
k_{xx}^{21} & k_{xy}^{21} & k_{xz}^{21} & k_{xx}^{22} - 4\pi^2\nu^2 m_2 & \cdots & k_{xz}^{2N}\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
k_{zx}^{N1} & k_{zy}^{N1} & k_{zz}^{N1} & k_{zx}^{N2} & \cdots & k_{zz}^{NN} - 4\pi^2\nu^2 m_N
\end{vmatrix} = 0. \qquad \text{(II, 11)}
$$

a simultaneous simple harmonic motion of all particles is possible. The determinant is of the $3N$th degree and therefore has $3N$ roots. Thus in principle the frequencies of the normal vibrations may be determined.

The *form of any one of the normal vibrations* may then be obtained by substituting the corresponding value of ν into the set of equations (II, 10) and solving for x_1, y_1, z_1, x_2, y_2, z_2, $\cdots z_N$. Of course, since these equations are homogeneous only the *ratios* of the x_1, y_1, z_1, x_2, y_2, z_2, $\cdots z_N$ can be determined. The ratio $x_1 : y_1 : z_1 : x_2 : y_2 : z_2 : \cdots : z_N$ is independent of the time for a given ν and gives, therefore, also the ratio of the components of the *amplitudes* of the different particles. It also gives the ratio of the *velocities* at any instant.

A closer examination of the determinantal equation (II, 11), taking account of (II, 8), shows that it has five or six roots that are equal to zero, depending on whether the system (in its equilibrium position) is linear or not. They correspond to *non-genuine normal vibrations* in which simply a translation along any one of the three