both terms are positive. Call the true attenuation D' and the correction C, then

$$D = D' + C \quad \text{or} \quad D' = D - C. \qquad \ldots\ldots(7)$$

Values of C for various values of S are given in Table 1.

Table 1.

S	C (db.)	S	C (db.)	S	C (db.)	S	C (db.)	S	C (db.)
0·6	−5·40	2·5	14·36	4·5	24·40	6·5	30·76	8·5	35·36
0·8	−2·81	3·0	17·46	5·0	26·21	7·0	32·02	9·0	36·37
1·0	0	3·5	20·08	5·5	27·84	7·5	33·21	9·5	37·29
1·5	6·02	4·0	22·36	6·0	29·36	8·0	34·20	10·0	38·20
2·0	10·68								

If the corrected attenuations D' are plotted against z we should expect to find a straight line of slope

$$40(\log e)\alpha = 17{\cdot}35\alpha. \qquad \ldots\ldots(8)$$

To illustrate the procedure results of an experiment in ethyl acetate at −19·7° c. and 7·47 Mc/s. are given in Figure 2, in which values of both corrected and uncorrected attenuation are plotted. Although in this case the attenuation caused by divergence is 2·22 times that produced by true absorption, the corrected readings lie close to a straight line.

The plot of corrected attenuation against z has been found to be linear in many other experiments in which the values of z, z_0, R and the correction varied widely, and the agreement is considered to justify the making of the correction.

For measurements in the Fraunhofer region the diameter of the reflector is important. We have assumed above that the intensity is constant all over the face of the reflector, but if the reflector is too big this is no longer true. Since the intensity of the echo is proportional to the area, it is usually desirable to use the largest permissible reflector and under these circumstances we can calculate the maximum allowable diameter as follows.

Suppose an experimental accuracy of 1% is aimed at. If the total attenuation to be measured is 50 db., we must not allow an error of more than 0·5 db. in the amplitude of the signal returned from a reflector of finite size. The error will obviously be greatest when the reflector is at minimum range and least at maximum range. The reflector is assumed to be circular and placed normal to the beam symmetrically on the axis of the crystal. Then it can easily be shown that if the intensity at the circumference of the reflector falls 0·75 db. below that in the centre, then the average reflected intensity will fall by 0·5 db. But the variation in intensity with angular distance β off the axis is given by equation (1) from which we find that 0·75 db. corresponds to $x = (2\pi R/\lambda)\sin\beta = 0{\cdot}8$. If we assume that measurements will not be made at shorter range than z_0, we have $\sin\beta = R_{max}/z_0$ where R_{max} is the largest permissible radius of the reflector. From these expressions it follows that $R_{max} = (0{\cdot}8\lambda/2\pi R)z_0 = 0{\cdot}8R/\pi \simeq R/4$.

If the radius of the reflector is allowed to become too large the measured value of the absorption will be low. For measurements in the Fresnel zone on the other hand, if the range of a large reflector is allowed to increase too far the measured absorption will be too large, owing to divergence of the beam. A comparison of the results of measurements made in the same liquid at the same frequency and temperature in the Fresnel and Fraunhofer regions will therefore show whether both errors have been avoided.

§3. THE CHOICE OF THE BEST EXPERIMENTAL CONDITIONS

(i) *Optimum Range of Intensities*

Suppose α is found from the relation

$$I_1 = I_2 \exp\{-\alpha(z_1 - z_2)\} \qquad \ldots\ldots(11)$$

and the proportional error $\epsilon = \Delta I/I$ in I_1 and I_2 is constant, then the most probable error in α is given by

$$(\Delta\alpha/\alpha) = (\epsilon_1^2 + \epsilon_2^2)^{\frac{1}{2}}/\ln(I_2/I_1). \qquad \ldots\ldots(12)$$

We may make $\Delta\alpha/\alpha$ as small as we please without diminishing ϵ, by making I_2/I_1 sufficiently large, but we are limited by two conditions: (*a*) I_2 so great that cavitation occurs, (*b*) I_1 so small that it approaches the noise level of the detecting system.

If pulses are used the ratio I_2/I_1 may be made larger than in any alternative method of measurement. In the experiments of Pellam and Galt the ratio corresponded to about 120 db. In the apparatus described below the maximum value of I_2 was restricted by overloading in the first stages of the receiver. In this case I_2/I_1 corresponded to about 60 db.

(ii) *Common Transmission and Reception with a Single Crystal*

By the use of a common transmitting and receiving crystal the attenuation is doubled for a given length of liquid column, there is only one crystal to be mounted and matched electrically, and the mechanical design is simplified since the reflector alone need be movable. On the other hand the electrical circuits may be more complicated. The apparatus to be described was designed so that the frequency could be easily changed and the number of separate tuned circuits to be switched was reduced to a minimum by the use of a common T and R system.

(iii) *Choice of Fresnel or Fraunhofer Regions for Measurement*

Whether the measurements are carried out in the Fresnel or Fraunhofer regions is dictated by considerations of practical convenience. As we have already seen in §2, if measurements are to be made in the Fresnel region the distance to the reflector must not exceed $R^2/4\lambda$. At low frequencies therefore, the Fresnel region may be so short that the total absorption in the region is too small to be measured accurately. In this case it is more convenient to work in the Fraunhofer region and to correct the observations for the divergence of the beam. The ratio of intensities I_2/I_1 may then be made as large as the apparatus will allow, even if α is quite small.

As a practical example consider water at 20° c. and let $\nu = 7{\cdot}5$ Mc/s., then $\alpha \simeq 0{\cdot}01$ cm^{-1} and $V = 1{\cdot}5 \times 10^5$ cm/sec. If $R = 1$ cm. the Fresnel region is limited to $z_c = R^2/2\lambda = 25$ cm. The corresponding attenuation is 2·17 db., which is clearly too small for accurate measurement. To increase the accuracy z_c must be increased to correspond to about 50 db., requiring the use of a crystal 4·8 cm. in radius, which would be most inconvenient and expensive to make. These considerations may explain why some measurements of absorption made at very low frequencies have given inaccurate results. What was measured as absorption was largely caused by divergence of the beam.

In the above example the critical distance z_0 (cf. §2) is $2R^2/\lambda = 100$ cm. It may be inconvenient to work in the Fraunhofer region beyond 100 cm. from the crystal because of the amount of liquid required, and accordingly z_0 may be reduced by limiting the area of plating on the crystal or by restricting the aperture externally by means of an iris (§4).

(iv) *Accuracy of setting the Reflector*

The criteria will be different in the case of measurements made in the Fresnel and Fraunhofer regions. In the Fresnel case the angular accuracy of setting is the only difficult criterion to meet. The error involved in treating the crystal and reflector as plane surfaces is assumed to be less than $\lambda/4$. In the case of a square crystal of side a with the reflector at very short range, the received echo falls to zero when the reflector is twisted from the setting for maximum signal through an angle $\Delta\theta \simeq \lambda/2a$. In the new setting the resultant signal received by one half of the crystal exactly cancels that received by the other. For a circular crystal $\Delta\theta$ will be modified by a numerical factor near to unity. In practice the signal must not fall by more than a small fraction of its amplitude, hence we have the condition $\Delta\theta \ll \lambda/2R$. For water at 75 Mc/s. $\lambda = 0{\cdot}002$ cm. and if $R = 0{\cdot}5$ cm. $\Delta\theta \ll 3{\cdot}6$ minutes of arc. Very accurate mechanical design is needed to preserve this accuracy of setting as the reflector is moved away from the source.

In the Fraunhofer case we have also to consider the transverse setting of the reflector in the beam. We have shown in §2 that a small reflector is necessary, and from the same considerations the transverse maladjustment permissible will be rather less than the radius of the reflector or roughly ±0·25 mm. The fine mechanical adjustment required is described in §4 and the methods of setting used in practice is described in §5. The additional adjustment of the angular position of the reflector need not be made as accurately as in the case of a large reflector in the Fresnel region, since the Fraunhofer diffraction pattern of the small reflector has a very broad central maximum.

(v) *Considerations in the Design of the Crystal*

The thickness of the crystal is fixed by the operating frequency but the radius may be chosen to suit the diffraction requirements. It is possible that the latter may conflict with electrical requirements. If the plated area of the crystal is made too small the effective parallel radiation resistance will be so large that it becomes difficult to match to the valve oscillator. Also a fixed external stray capacity will narrow the bandwidth more if the crystal is small.

The question of bandwidth has been discussed by Huntington, Emslie and Hughes (1948), who showed that for a crystal of acoustic impedance Z_0 in contact over one face only with a fluid of acoustic impedance Z_1, the effective Q of the crystal at resonance is given by

$$Q = (n\pi/4Z_1)\{2(2Z_0^2 - Z_1^2)\}^{\frac{1}{2}} \qquad \ldots\ldots(13)$$

where n is the order of the harmonic used to excite the crystal to resonance. For a quartz crystal with one face in water $Z_0 = 10Z_1$ so that, ignoring Z_1^2 in comparison with $2Z_0^2$, we find

$$Q = \frac{n\pi}{4}\,\frac{2Z_0}{Z_1} = 5n\pi. \qquad \ldots\ldots(14)$$

On the fundamental the Q is therefore about 16.

The width of the resonance curve is such that $Q = f/\Delta f$, where Δf is the difference between the frequencies exciting the crystal to half the maximum power. It can be seen from (14) that Δf is numerically the same on all the harmonics, although Q itself is proportional to n. This means that if the bandwidth is adequate to transmit pulses of a given width on the fundamental it will also be adequate on the harmonics. A Q of 16 will allow the transmission of pulses of duration about 1 microsecond at 15 Mc/s., or about 2 microseconds at 7·5 Mc/s. To obtain a pulse with a sensibly flat top, however, its duration must exceed the minimum value by a factor of 2 or 3. If the absorption is very high difficulties may be caused because the reflector must be placed extremely close to the crystal, with the result that the transmitted pulse and its echo become merged together; moreover under these conditions the effects of paralysis in the receiver are most severe. The bandwidth of the crystal has been sufficient to allow measurement of absorption up to 30 db/cm. To measure absorption higher than this one may use a quartz bar as a reflector which also delays the echo (Rapuano 1947); this technique makes it unnecessary to generate very short pulses. No attempt has therefore been made to generate pulses narrower than 1 microsecond.

(vi) *Effect of Pulses on experimental Accuracy*

If α varies with ν there must be some error involved in measuring with short pulses and Pellam and Galt (1946) showed that when α varies as ν^2 the error is about 1 part in 200 for one microsecond pulses at 15 Mc/s.

(vii) *Control of Temperature*

The importance of maintaining a uniform temperature in the liquid by stirring cannot be overstressed. This requires a more uniform temperature than is needed to remove errors caused by true variations in α with T, because local fluctuations cause refraction of the beam and "wandering" of the size of the echo. Experience shows that errors are likely if the temperature is not maintained uniform to ±1/10° c. throughout the liquid.

§4. EXPERIMENTAL EQUIPMENT

The requirements to be fulfilled by the apparatus were the following. An experimental accuracy of 1% was aimed at. Measurements were to be made on as many frequencies as possible and easy changing of the frequency was essential. Means had to be provided for varying controlling and measuring the temperature to about ±0·1° c. Measurements were to be made in water or in any organic liquid.

The preliminary measurements were made at 15 Mc/s. in water. On this frequency a large volume of liquid was required as the absorption is low, and in view of this it was decided to extend the measurements to higher frequencies. Use of the odd harmonics of the 15 Mc/s. crystal would leave a rather large gap between the fundamental and the third harmonic. A crystal cut for 7·5 Mc/s. and worked on odd harmonics up to the 9th would give a more even distribution distribution of frequencies if 15 Mc/s. also were included in the series. It was decided to construct a multi-channel transmitter-receiver using switching to