Copy of J M M Pinkerton's paper, "On the Pulse Method of Measuring Ultrasonic Absorption in Liquids" in Proceedings of the Physical Society, Section B, Volume 62, Number 5, 1949.

London (PR1 8573).
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On the Pulse Method of Measuring Ultrasonic Absorption in Liquid

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Communicated by Sir Lawrence Bragg; MS. received 22nd September 1948

ABSTRACT. This paper deals with the experimental problems involved in accurate measurement of the absorption of ultrasonic waves in liquids. Reasons are given for preferring a method employing pulses of ultrasonic energy. The errors likely to be introduced by diffraction are discussed and it is shown that reliable measurements may be made in both the Fresnel and Fraunhofer regions. An account is given of a convenient method of correcting for divergence of the beam in the Fraunhofer region. The choice of the optimum conditions for accuracy is discussed and illustrated by practical examples. A description is given of the essential features of an apparatus working on six frequencies between 7·5 and 67·5 Mc/s. using the pulse technique.

§1. INTRODUCTION

The aim of this paper is to discuss some of the details involved in the measurement of the absorption of ultrasonic waves by the pulse method, which has already been described (Pellam and Galt 1946, Teeter 1946, Pinkerton 1947, Huntingdon, Emslie and Hughes 1948, Arenberg 1948). The discussion is illustrated by a short description of an experimental apparatus which has been used to obtain results described elsewhere (Pinkerton 1947, 1948). Measurements made in the Fraunhofer region of the quartz crystal require correction for the divergence of the beam (Grossmann 1932, Leonard 1940); a simple method of applying this correction is given. For the measurement of velocity the pulse method does not offer any substantial advantage over the acoustic interferometer or methods using diffraction of light, and the discussion will therefore be restricted to the problem of measuring the absorption alone.

All methods of measuring the absorption involve measurement of the intensity at a variable distance, x, from the transmitting crystal. The absorption coefficient α is then obtained from the relation $I = I_0 e^{-2\alpha x}$. In the past the intensity has been measured by using the optical diffraction effects or the Rayleigh pressure of the sound. The inherent disadvantages of these two methods have been fully discussed by Fox and Rock (1941) and Willard (1941). Their conclusions will now be briefly summarized to show how the inherent inaccuracies of these methods may be avoided by using pulses of sound.

In order to detect the radiation more readily large powers have frequently been employed, as much as 100 watts being recorded (Sörensen 1936). Since all the sound is ultimately absorbed, this leads to an appreciable heating of the liquid. This heating in turn may alter the absorption coefficient, which varies quite rapidly with temperature in certain liquids. Local fluctuations in the temperature also cause refraction of the sound waves, which leads to further errors, especially if the path length is great. Moreover excessive intensities of sound cause cavitation, and the bubbles of gas released scatter and absorb the sound, so that the absorption is found to depend on the amplitude, and on whether or not the liquid was previously degassed (Sörensen 1936, Fox and Rock 1941). The intensity required to produce cavitation in water is only about 1 watt/cm².

Unless the total absorption between the source and the detector is great, standing waves set up in the intervening space may alter considerably the variation in intensity with distance, and cause large errors in α.

The method based on the Rayleigh pressure has certain special disadvantages. The liquid cannot be stirred unless the pressure balance is specially protected, but without stirring absorption of even feeble intensities of sound inevitably raises the temperature of the liquid locally. The pressure balance must also be protected against bodily motion of the liquid, since the absorption of the wave by the liquid gives it momentum and therefore drives it away from the source.

In principle these difficulties of former methods can all be overcome in suitable ways. In practice the low intensity that must be used means that the sensitivity of the detecting system is too low to give precise results.

If measurements are made using pulses these difficulties can readily be overcome. Stirring the liquid has no ill effect, and heating is reduced by a large factor. The heating is proportional to the mean power radiated, which is the peak power multiplied by the "mark to space" ratio of the pulses. In the author's measurements the mean power was about 1/500 of the peak power. The peak intensity may be kept low enough to avoid cavitation; at the same time detection of much lower intensities is possible. A considerable advantage is that intensities can be measured conveniently and accurately by an electrical method, over a very wide range. The useful range of intensities is 100 to 1,000 times that of the pressure balance. This is because a tuned amplifier can be used to magnify the received pulses of sound, so that discrimination against random thermal movements of the liquid, i.e. ultrasonic "noise", is greater than with a pressure balance which responds to sound of all frequencies.

§2. DIFFRACTION EFFECTS

If sound is emitted from a circular crystal many wavelengths in diameter, one might assume, neglecting absorption, that the intensity is uniform everywhere inside a cylinder based on the crystal, and zero outside. It has been recognized that this approximation is only valid exceedingly close to the crystal, and off the central axis (Biquard 1935, Willard 1941, H. Born 1942).

A well known argument using Fresnel half-period zones shows that, along the axis of a circular crystal, the intensity oscillates between zero and a constant maximum value, so long as the axial distance, z, is less than z_1, where $z_1 \simeq R^2/\lambda$; R is the radius of the crystal and λ the wavelength. Any point at a distance less than z_1 may be said to be in the Fresnel region, and any point at a greater distance, in the Fraunhofer region. Beyond z_1 the beam begins to diverge, and there are no more maxima and minima on the axis, until at sufficiently great distances the intensity falls off as $1/z^2$.

If measurements of the absorption of sound are to be exact these phenomena must be taken into account. In practice, for reasons which will become clear, the measurements must be made either entirely in the Fresnel region or entirely in the Fraunhofer region, and in the latter case the readings must be corrected to allow for the divergence of the beam. It will be convenient to consider separately the problems of making exact measurements in the two regions.

It is well known in optics that the Fresnel diffraction pattern of a circular aperture, in a plane normal to the axis, consists of a series of concentric circles. The number of circles increases with decreasing distance from the aperture. If the quartz crystal is assumed to be vibrating as a perfect piston, then its diffraction pattern will be the same as in the analogous optical case. Now suppose

the absorption is measured in the Fresnel region of the crystal with a detecting device whose diameter is small compared with that of the crystal. It is then obvious that the measured intensity will not fall off uniformly with the distance, but will oscillate about the exponential diminution caused by true absorption.

If a perfectly plane detecting system which is bigger than the crystal is set perpendicular to the axis, then the measured intensity will involve a summation taken over the whole diffraction pattern. If the detector responds to the Rayleigh pressure it will be insensitive to phase and the appropriate summation is the arithmetic sum of the intensities at all points. For a crystal used as detector we must sum the amplitudes at all points vectorially. A mathematical treatment is evidently complex and in neither case has the problem been investigated. It is apparent however that proportional errors in α due to variations in the average intensity received by a crystal detector will be minimized if (*a*) the transmitting and receiving crystals are made as large as possible, or (*b*) the absorption coefficient per unit length is very large. For the errors caused by divergence to be negligible the total distance from transmitting to receiving crystal may not exceed R^2/λ. Without a mathematical treatment it is not certain that even this condition will be sufficient to avoid error. The limiting range can be decided by experiment however, and from the author's results it has been found that errors from divergence are negligible if z is not allowed to exceed $R^2/2\lambda$. At shorter ranges the plot of $\log I$ against z is perfectly linear, but if $z > R^2/2\lambda$ there is noticeable departure from linearity, as illustrated by the experimental results of Figure 1, for which $R^2/2\lambda = 33 \cdot 5$ cm.

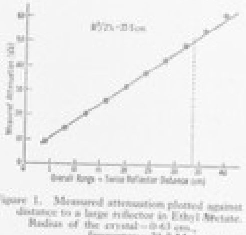

Figure 1. Measured attenuation plotted against distance to a large reflector in Ethyl Acetate. Radius of the crystal = 0·63 cm., frequency = 21·7 Mc/s., temperature = –6·0° c., $R_0^2/2\lambda = 33 \cdot 5$ cm. The graph becomes non-linear beyond 33·5 cm.

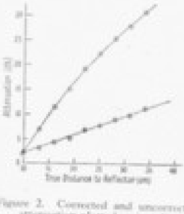

Figure 2. Corrected and uncorrected attenuation plotted against distance to a small reflector in the Fraunhofer region. Liquid, ethyl acetate; frequency = 7·47 Mc/s., temperature = –19·7° c., $z_1 = 10$ cm. Upper curve uncorrected, lower curve corrected points.

At ranges less than $R^2/2\lambda$ there is no systematic wandering of the points about the straight line. This fact shows that *the average* intensity received by a crystal, whose diameter is the same as the sender, would be uniform within the Fresnel region in the absence of absorption. A common transmit–receive crystal, acting as receiver, is equivalent to a second crystal at twice the distance of an infinite plane reflector normal to the axis. We conclude that the slope of the straight line in Figure 1 accurately represents the absorption.

We now consider the case of measurements made in the Fraunhofer region where precise correction for divergence is essential. Fortunately an exact theoretical treatment for points on the axis has been given by Backhaus and Trendelenburg (1926), who showed that the intensity at distance z is proportional to $\sin^2[\frac{1}{2}k\{(R^2+z^2)^{\frac{1}{2}}-z\}]$, where $k = 2\pi/\lambda$. This expression is also valid for axial points in the Fresnel region. The intensity at a point off the axis and well in the Fraunhofer region can be shown to be related to that on the axis by the expression

$$I_\beta/I_a = [\{J_1(x)\}/x]^2 \qquad \ldots\ldots(1)$$

where J_1 is a Bessel function of order one, $x = (2\pi R/\lambda)\sin\beta$ and β is the angle between the axis and a line through the point to the centre of the crystal.

To correct results of an experimental determination, which are observed directly in decibels, it is most convenient to calculate the correction also in decibels. $z_0 = 2R^2/\lambda$ is approximately the axial distance for which the aperture of the crystal is completely filled by a quarter-period Fresnel zone. Then if we express all the measured ranges in terms of the ratio $S = z/z_0$ we can show that the correction in decibels is determined only by S. A standard correction graph can therefore be plotted to find the correction if z_0 is known. Subtraction of the appropriate correction in decibels gives a set of readings of true absorption against distance. The measured intensity and therefore the corrected readings are relative to an arbitrary reference level, but since we are only interested in the slope of the graph of attenuation versus range this does not matter.

The calculation is made as follows.

Following H. Born (1942) we write the amplitude A_z in the form

$$A_z = A_0 e^{-\alpha z}\sin[\tfrac{1}{2}k\{(z^2+R^2)^{\frac{1}{2}}-z\}] \qquad \ldots\ldots(2)$$

where A_0 is the amplitude at $z = 0$. In practice $z^2 \gg R^2$, so we can write

$$A_z = A_0 e^{-\alpha z}\sin(\pi R^2/2\lambda z). \qquad \ldots\ldots(3)$$

We have also

$$A_{z_0} = A_0 e^{-\alpha z_0}\sin(\pi R^2/2\lambda z_0) = A_0 e^{-\alpha z_0}\sin(\pi/4). \qquad \ldots\ldots(4)$$

Dividing (3) by (2) and substituting for z in terms of S, we find:

$$A_{z_0}/A_z = e^{-\alpha(z_0-z)}[\sin(\pi/4)/\sin(\pi/4S)]. \qquad \ldots\ldots(5)$$

If we now define A'_z and A'_{z_0} to denote the amplitudes of signals received by the transmitting crystal from a very small reflector placed at z_0 and z respectively, then we have

$$A'_{z_0}/A'_z = e^{-2\alpha(z_0-z)}[\sin(\pi/4)/\sin(\pi/4S)]^2. \qquad \ldots\ldots(6)$$

We may write equation (6) in the form

$$20\log(A'_{z_0}/A'_z) = 40\alpha(z-z_0)\log e + 40\log[\sin(\pi/4)/\sin(\pi/4S)]. \qquad \ldots\ldots(7)$$

The left hand side of equation (7) is simply the observed uncorrected attenuation in decibels below the arbitrary reference level A'_{z_0}; let this be denoted D, which is a positive quantity for $z > z_0$. The first term on the right hand side represents the true loss due to absorption, and the second the correction. Again for $z > z_0$