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Notes of Lectures
delivered by
Professor Foster B.A. F.R.S. etc.
on
Mechanics (Division A. Junior Class of Physics)
~~do~~ at
University College, London.
during the
Session 1877 - 1878 A.D.

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Mechanics.

Feb. 20th 1878.

The height to which the atmosphere would reach if the air was the same density throughout that it is at the surface can be found thus:-

1 c.cm. of air at $0^\circ C.$ & 76 cm. of mercury barometric pressure - weighs $\cdot 001293$ gms. $= a_0$

1 c.cm. of mercury at $0^\circ C.$ weighs 13·596 grammes $= m$

Let B = height of mercurial barometer

H_0 = height of atmosphere (uniform throughout) at $0^\circ C.$

$$\frac{H_0}{B} = \frac{m}{a_0} = \frac{13\cdot596}{\cdot001293} = \frac{13}{\cdot0013} \text{ approximately}$$

$$= 10000 \quad \text{(really rather more)}$$

$$H_0 = 10000\,B.$$

Now take into account the temperature say it is t.

$$a_t = \frac{a_0}{1+\cdot00366t}$$

$$H_t = B.\frac{m}{a_t} = B.\frac{m}{a_0}(1+\cdot00366t) \quad \text{----- (1)}$$

H increases with the temperature; ~~the~~ B varies but does not alter H. which is called the height of a homogeneous atmosphere: this can be seen thus:-

$$a_t = a_0 \frac{B}{76} \cdot \frac{1}{1+\cdot003665t}.$$

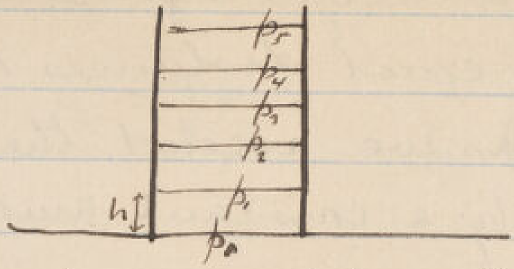
So if B increases a_t increases; but in equation (1) [see ~~the~~ last page] B thus occurs ~~on~~ both ~~side~~ as numerator & as denominator & so it vanishes.

$$\therefore \; \therefore H_t = \frac{m \times 76\,(1+\cdot003665t)}{\cancel{\cdot001293}\,a}$$

or expressed numerically

$$= \frac{13\cdot596 \times 76 \times (1+\cdot003665t)}{\cdot001293}$$

We can pass on from this to consider an atmosphere the density of which gradually diminishes as we ascend. Suppose it to be divided into layers of equal thickness; for simplicity let the temperature be 0°C: if it is t, we just multiply or divide as the case may be by $(1+\cdot003665t)$; in considering these strata suppose them to be ~~on~~ on a unit of area;



really the pressure diminishes continuously as we ascend; but if the layers are thin enough we ~~~~ can imagine it to diminish step by step. Let h be the vertical height of each stratum and p_0, p_1, p_2 etc the pressure in each as we ascend.

$$\left.\begin{array}{l} \text{Let } a = \text{mass of air} \\ a g = \text{weight} \end{array}\right\} \text{in each layer}$$

$$\left.\begin{array}{l} p_0 - p_1 = a\dfrac{p_0}{76}.hg \\ p_1 - p_2 = a.\dfrac{p_1}{76}.hg \\ p_2 - p_3 = a.\dfrac{p_2}{76}.hg \\ \quad\vdots \\ p_{n-1} - p_n = a.\dfrac{p_{n-1}}{76}hg \end{array}\right\}$$

these quantities of course alter as we ascend because g varies; this however is very small & may be neglected.

$$\frac{p_0 - p_1}{p_1 - p_2} = \frac{p_0}{p_1} \qquad \text{i.e. } p_1 = \sqrt{p_0 p_2}$$

Similarly

$$p_2 = \sqrt{p_1 p_3}$$

$$p_3 = \sqrt{p_2 p_4}$$

and so on.

i.e. each pressure is the geometrical mean of pressures at equal distances on each side of it; or, as we ascend the pressure diminishes not by a constant amount but by a constant ratio.

Let $\frac{p_0}{p_1} = \frac{p_1}{p_2} = \frac{p_2}{p_3} = \text{etc.} = \frac{1}{K}$

$$\therefore h = \frac{p_0 - p_1}{p_0} \cdot \frac{76}{ag}$$

$$= (1 - K) \cdot \frac{76}{ag}$$

also $h = \frac{p_1 - p_2}{p_1} \cdot \frac{76}{ag} = (1 - K) \cdot \frac{76}{ag}$

$$\underline{H} = nh = (1 - \underline{K}) \frac{76}{ag}$$

$$\frac{p_1}{p_0} = K \qquad \therefore p_1 = Kp_0$$

$$\frac{p_2}{p_1} = K \qquad \therefore p_2 = Kp_1 = K^2 p_0$$

Similarly $p_3 = Kp_2 = K^3 p_0$

$$\therefore p_n = K^n p_0$$

or $\frac{p_n}{p_0} = K^n$

ascend a known height (n) and observe pressures

$$K = \sqrt[n]{\frac{p_n}{p_0}}$$

so K can be found by logarithms.

Then ascend an unknown height (n')

$$K^{n'} = \frac{p_{n'}}{p_0}$$

and again observe pressures

$$\frac{\log p_{n'} - \log p_0}{\log K} = n'$$

so n' can be found.

A more accurate and convenient formula is

$$H = 2.3026 \left(\frac{m}{a}\cdot 76\right) \frac{p_n}{p_0}$$

[H = height to which we ascend (unknown)]

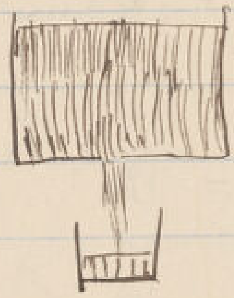
For these observations aneroid barometers are used; The essential parts ~~are~~ is a flat metallic box partially exhausted: this is squeezed in by the pressure of the air; which squeezes it in at various times with different forces: and the movement of the surfaces are transferred by mechanism to an index hand on a circular disc.

Feb. 25th 1878.

Motion of fluids.

Of this we shall only take a few special examples:

1. Through an orifice into a vacuum.
   First determine the rate of escape.



Let the liquid flow for a known time, measuring the quantity escaped and compare this with the size of the orifice. If the hole is circular, the water will flow down in the form of a cylinder, the sectional area of which = the area of the hole

Let l = length of cylinder

a = sectional area = area of the orifice

al = volume = v in a unit of time = say 1 second

l = Velocity = V

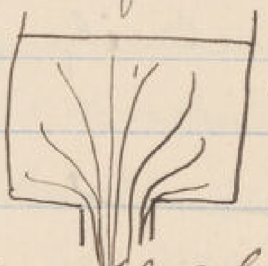
But the liquid must be collected in a graduated vessel

$$\frac{v}{a} = l = \nabla \quad \text{in 1 second}$$

or

$$\frac{v_c}{at} = \nabla \quad \text{in } t \text{ seconds.}$$

A short tube at the orifice will not impede the velocity, even if the tube is thinner at the bottom.



The above figure will show something of the direction in different parts of the liquid; i.e. all parts do not fall vertically; but only the vertical component is of effect. That very near the bottom will flow also horizontally, and when it reaches the orifice it has a certain horizontal velocity & so does not fall vertically, but slantingly

So the lower part of the tube is dry, and therefore it does not

Neglecting however those variations consider the velocity of escape as connected with the depth of the orifice below the ~~&~~ surface.

To do this, the energy of the water or power of doing work must be considered.

Let h = height of water above zero level

M = mass

Mhg = original energy

now imagine some water to have escaped: of that in the vessel to be divided into n strata of unit weight, say 1 lb of heights h_0, h_1, h_2 etc. — h_n above the zero level, whose thickness can be neglected, as in the above quantity h, we don't know what part of the mass of liquid to measure it to.

$$h_1 g + h_2 g + \text{-----} + h_n g$$

$$= (h_1 + h_2 + \text{-----} + h_n)\, g$$

$$nh = h_1 + h_2 + \text{-----} + h_n$$

h therefore is the height of the centre of gravity.

$\therefore hMg$ = original energy

$\frac{1}{2}mv^2$ = energy of velocity

then for a vessel of any shape (considering one stratum to be escaping)

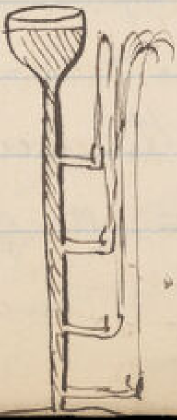
$$h_p Mg - h_1 mg + \frac{1}{2} m v^2 + h_n mg = h Mg \quad \text{(original energy)}$$

$$\frac{1}{2} v^2 = (h_1 - h_n) g$$

$$v^2 = 2Hg \qquad H = \text{~~the~~ depth of water in vessel}$$

the same velocity is that which is necessary to throw water to the same height in a fountain in theory; but two modifying influences are the resistance of the air, and the falling water meeting that which is rising. This may be shown experimentally; by having water in a long tube, with tubes sticking out of pointing upwards, at different places;

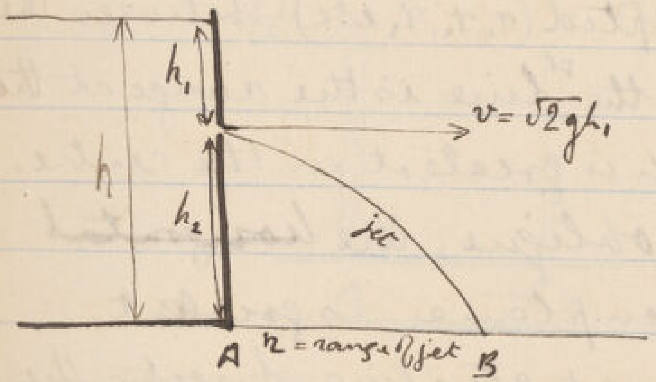
as in the figure the water which escapes from all rises practically to the same height viz: to ~~surface of the water.~~



Water with a horizontal velocity under the influence of gravity describes a parabola.

Feb 27. 1878

Suppose we have a horizontal jet



Let h_1 be its distance below the surface, & h_2 its above the bottom of the vessel, i.e. the distance it has to fall.

First find the time in falling:-

$$h_2 = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2h_2}{g}}$$

Now find the range of the jet (r) or the distance AB in the figure.

$$v = \sqrt{2gh_1}$$

$$r = vt = 2\sqrt{h_1 h_2}$$

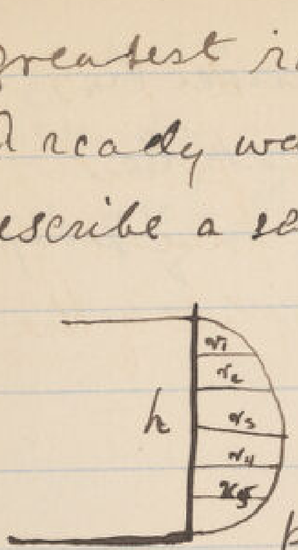
So if we interchange h_1 & h_2 we don't alter r; this is all leaving out the resistance of the air.

as $h_1 + h_2$ is ~~gi~~ constant $= h$

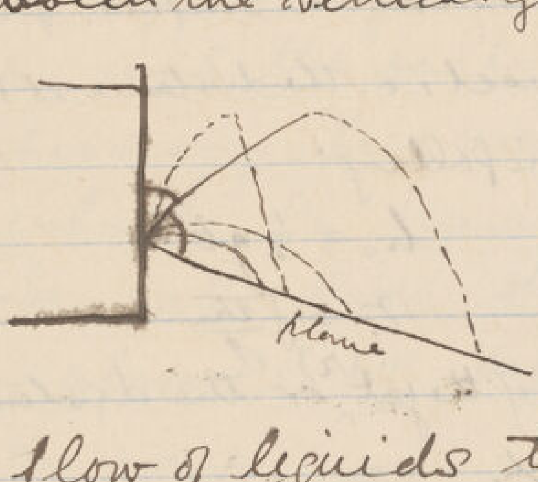
r is greatest when $h_1 = h_2$

greatest range $R = \frac{h^2}{2}$

A ready way to find the range is to describe a semicircle with diameter h; draw $\perp^{rs}$ from the side to it; the parts intercepted (r_1, r_2, r_3 etc) between the arc & the st. line is the range at those pts. which is greatest in the centre.



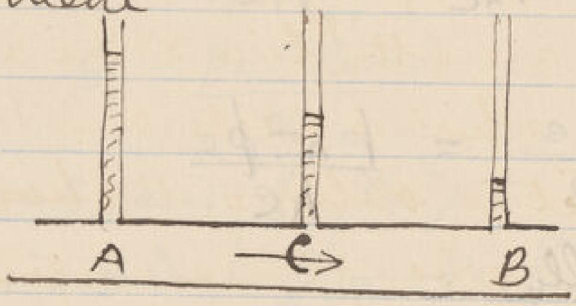
When the jet is oblique, the ~~horizontal~~ range on any given plane is greatest, when its direction at starting bisects the angle between the vertical & that plane.



2. The flow of liquids through tubes: taking as our example a horizontal tube of uniform bore.

To maintain a flow there must be a

difference of pressure at the two ends. To tell the pressure at different points; vertical gauge tubes must be placed & the pressure measured by the heights of the liquid in them



the force tending to move the water from A to B is the difference of pressure at A & B $(p_A - p_B)$ so we should get uniformly accelerated velocity, if the friction (r) were not equal to the excess of pressure: then we get a constant speed; let s = strength of stream.

$$p_A - p_B = r_{AB} S$$

$$S = \frac{p_A - p_B}{r_{AB}}$$

S is the same in all parts; take another point C:

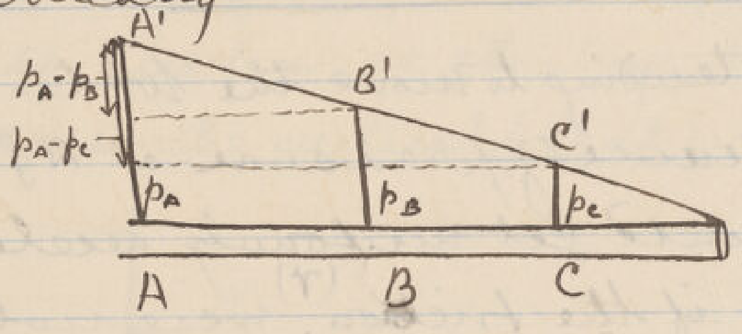
$$S = \frac{p_A - p_C}{r_{AC}}$$

& ~~r.t.~~ $\dfrac{r_{AB}}{r_{AC}} = \dfrac{AB}{AC}$ ∥

So $\dfrac{p_A - p_B}{p_A - p_C} = \dfrac{r_{AB}}{r_{AC}} = \dfrac{AB}{AC}$

or $\dfrac{p_A - p_B}{AB} = \dfrac{p_A - p_C}{AC}$

or geometrically



We get the same by considering similar Δs in the above figure. The line A'B'C' will always be found in the same straight line and passing through the orifice where the pressure is 0.

If the ~~speed~~ strength of the current increases, the slope of this line increases.

If the tube is wider; in order to get the same ~~stream~~, amount of water flowing out in the same time; the stream must be ~~more~~ less rapid.

$$s = \frac{p - p'}{r}$$

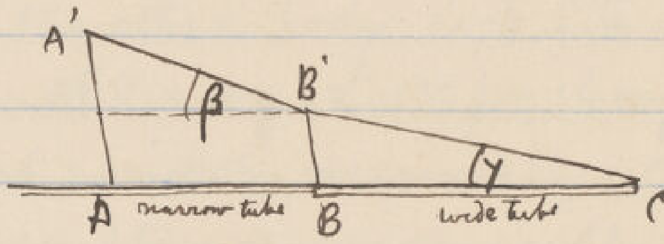
$$S_1 = \frac{p_1 - p_1'}{r_1}$$

then if $S = S_1$ and $r_1 > r$

$$\therefore \frac{p - p'}{r} = \frac{p_1 - p_1'}{r_1}$$

$$\text{i.e. } p_1 - p_1' > p - p'$$

Suppose we have the two tubes end to end.



the tan β & tan γ indicate the variation of pressure; i.e. they are proportion to the resistance in the tubes. The law of the passage of electricity along conductors is exactly analogous to the passage of water in tubes, & we get the same ~~x~~ result viz $S = \frac{p - p'}{r}$

In Electricity s = strength of current of electricity

$p - p'$ = electromotive force or difference of potential

r = resistance of conductor

It is called Ohm's ~~Law~~.

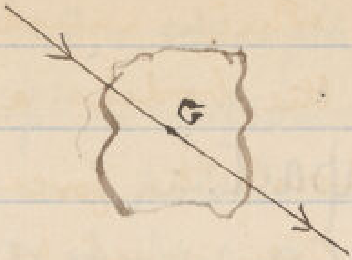
March 27. 1878

Impact.

The tendency of impact or a strike or collision is to diminish the motion of the striking body, and to increase that of the body struck; the force which acts on ~~both~~ each is the same in magnitude but opposite in direction i.e. action & reaction are equal & opposite; i.e. the changed momentum is the same in both bodies. if mv is the momentum of one before the blow &

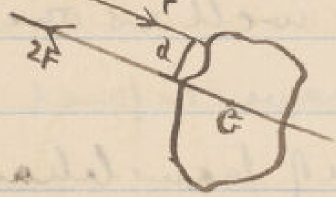
mv' - - - - - - - - - - - - - - after - - - - - ; the change of momentum is $m(v'-v) = ft$; the force which acts on the other body is $-f$ & t is the same in both i.e. $-ft = m_1(v_1'-v_1)$ = change of momentum

With impact at random, we generally get very complicated results: viz: we generally get a motion of rotation as well as one of translation; as in former cases we have confined ourselves to motion when it is one of translation, we shall still do so. To get this motion only in the struck body the forces must act through the centre of ~~g~~ gravity of the struck body.



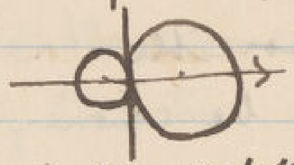
Suppose we had a force F acting through some other point.

this causes onward movement in the direction in which it acts; and rotation as well; that this is the case may be seen thus: the force F may be resolved into a force F' = F acting parallel to ~~but~~ and in the ~~opposite~~ same direction ~~to~~ as F, and at the same distance the other side of the centre of gravity; and a force = 2F acting through the centre of gravity: || to F and in the opposite direction: the force 2F may be



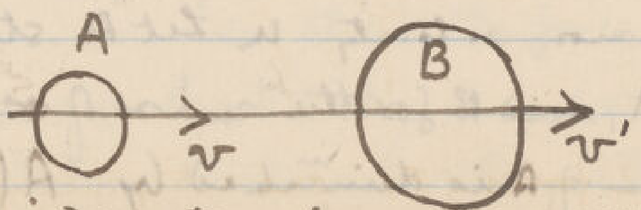
resolved into 2 forces = F: one acting through the c. g. of causing onward movement, and the other acting also through the centre of gravity, and making with F' a couple with arm d which causes rotation.

In order to make the bodies always strike so that the line of action of the force passes through the centre of gravity, they must be spherical



The force acts in the $\perp^r$ to the tangent plane; and passes through the two centres of the spheres which are the centres of gravity. If the bodies are smooth, this will be the case even if the force is not perpendicular to the tangent plane; for the force can be resolved into two components, one passing through the centres, and the other at right angles to it; the first is the only effective component.

Say we have two spheres moving in the same direction: the hinder ~~on~~ one must be moving most quickly, if it is to overtake the first



Say A the hinder one has a velocity V; and B a velocity v'.

when they strike v is diminished & v' is increased; even when in contact with one another; the centres must still be moving towards one another: because a sudden change of momentum is impossible as it requires an infinite force: this can be shown thus:

$$ft = m(v' - v)$$

$$\therefore f = \frac{m(v' - v)}{t}$$

so if t is infinitely small; $\frac{m(v'-v)}{t}$ i.e. f must be infinitely large. In order that the centres may approach one another after contact, the two bodies must be knocked out a shape: thus on an exaggerated scale

This goes on until the velocities are equal: call the common velocity u. Let A stand for mass of the body A, and B for the mass of ~~a~~ B.
the momentum of A is diminished by $A(v - u)$
" " " B " increased " $B(u - v')$

These must be equal to one another.

$$A(v-u) = B(u-v')$$

$$\text{or } u = \frac{Av + Bv'}{A+B}$$

Or we may get the same result in this way: the momentum of the two after collision must be equal to the ~~same~~ sum of the momenta before collision.

$$\text{i.e. } (A+B)u = Av + Bv'$$

$$\therefore \quad u = \frac{Av + Bv'}{A+B}$$

The action generally goes however beyond this: the bodies in most cases: in virtue of their ~~original shape~~ elasticity the bodies tend to regain their original shape: in fact it is as if we have a spring between the two balls: so they separate again. Generally the force with which the bodies return to their shape. is less than the force by which they are squeezed up. e.g. lead & clay are examples of bodies with hardly any tendency to return to their original shapes. when they are put out of shape.

Let -

V = final velocity of A after separation
V_1 = ————————— B ————————

change of momentum of $A = A(u - V)$

$= f't'$

$= B(V_1 - u)$

= change of momentum of B.

if $f't' = 0$, the bodies don't separate at all; i.e. they are inelastic.

if $f't' = ft$, the force of separation = force of deformation, i.e. the body is perfectly elastic; no body is so.

April 1. 1878.

$fs = \frac{1}{2}m(v^2 - u^2)$ is called the work done by a force i.e. the change of kinetic energy; the product ft which we have seen $= m(v-u)$ has also a name; it is called sometimes the effort and sometimes the impulse of the force; and forces like the blow of a hammer are called impulsive forces. The impulse of a force is divided into two parts:-

1. During the equalization of velocities
2. During the separation of the bodies.

The effect of the elasticity of the bodies is the same as if the bodies are at rest; for with regard to one another they are at rest; since they are moving with the same velocity;—

in most cases $f't' = e.ft$

e being some proper fraction i.e. between 0 & 1.

$$f't' = m(u-v')$$
$$= e.ft = e.m(v-u)$$

i.e. $m(u-v') = e.m(v-u)$ ......(1)

Similarly for the other body with mass $= m_1$

$$m_1(v_1'-u) = e.m_1(u-v_1) \ldots\ldots (2)$$

From these two equations v' and v_1' can be found v & v_1 being known, and also m & m_1; the values of v' and v_1' are the following:—

$$v' = \frac{mv + m_1v_1 - em_1(v-v_1)}{m+m_1}$$

$$v_1' = \frac{mv + m_1v_1 + em(v-v_1)}{m+m_1}$$

this includes the two special cases we have before considered: make $e = 0$ (i.e. body inelastic) & we get the expressions before got.

Now take the other extreme; i.e. $e = 1$; and let $m = m_1$; we then get:

$$v' = \frac{m(v+v_1) - m(v-v_1)}{2m}$$

$$= v_1$$

$$v_1' = m(v+v_1) + m(v-v_1)$$

$$= v$$

i.e. the bodies exchange velocities: this is a very important case. If there are several bodies of equal size in a row, and elastic, and the first be struck; the velocity is passed to the next ball, and that imparts it to the next & so on; until the last is reached, and that having nothing to impart it to, moves; leaving the intermediate bodies stationary; just as in the vibratory motion of sound.

Now let one of the bodies be infinitely great compared to the other; & the latter perfectly elastic:

i.e. $m_1 = \infty$

$e = 1$

$v_1 = 0$

$$v' = \frac{mv - m_1 v}{m + m_1}.$$

m is comparatively so small that it may be left out

$\therefore v' = -v$

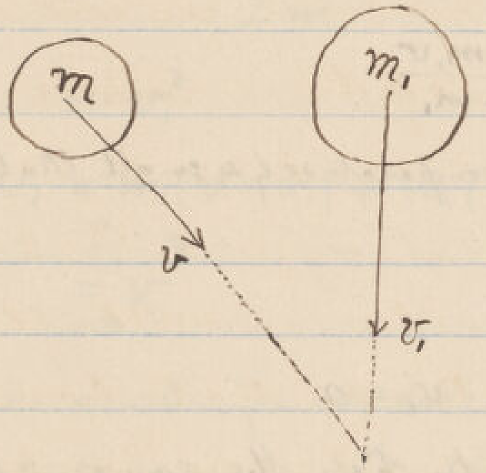
Similarly we get $v_1' = 0$
i.e. the small body has the same velocity before and after collision but in the opposite direction: while the large body remains at rest still: e.g. an elastic ball striking the earth rebounds to the height from which it fell, without moving the earth at all.

if $m_1 > m$, and m strikes m_1
v' will be negative

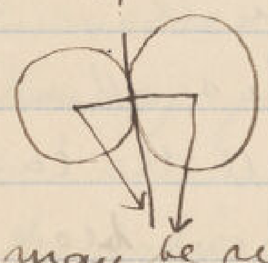
if the larger strike the smaller, both will go on in the same direction. All the cases we have hitherto discussed, have been cases of direct collision: we now go to cases of:—

Oblique Collision

Suppose two spherical bodies of masses m & m_1 to be moving in directions inclined to one another:



with velocities v and v_1 respectively. These will strike if they tend to pass the point where their directions meet, at the same instant: if one gets past before the other, they will never meet. Suppose they do meet.



Each of the forces may be resolve into two components: one along the line joining the centres, and the other ⊥r to that line i.e. tangential. When we have only the first of these we have direct collision; when we have only the second we have no collision at all; i.e. the first is the effective component, and the 2nd the non-effective component; i.e. when we have oblique collision it is just the same as if we had direct ~~&~~ collision with the 1st components, those which are tangential having no effect: of the laws of direct collision we have already studied. This of course ~~is~~ is only the case when the bodies are smooth.

April 3. 1878

We now go to more complex cases of motion than we have hitherto considered.

We have already got the following results

$$\left.\begin{aligned} v &= at \\ s &= \tfrac{1}{2}at^2 \\ v^2 &= 2as \end{aligned}\right\} \text{starting from rest}$$

$$\left.\begin{aligned} v - v_0 &= at \\ s &= v_0 t + at^2 \\ v^2 - v_0^2 &= 2as \end{aligned}\right\} \text{with initial velocity } v_0$$

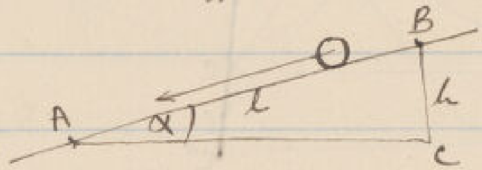
$$a = \frac{\text{resultant force in direction of the motion}}{\text{mass to be moved}} = \frac{f}{m}$$

also $$a = \frac{v - v_0}{t} \text{ i.e. rate of change of velocity.}$$

$$a = \frac{w}{m} = \frac{gm}{m} = g \quad (\text{free fall}$$

i.e. the numerical value of the acceleration is the same as the force = wt. of unit mass: though of course, the two cannot be physically equal; because an acceleration cannot be equal to a force.

Now take the case of a body falling down a <u>smooth</u> incline; the force tending to move the body down the incline is not the weight, but the component of the weight parallel to the surface; the other component which is $\perp^r$ to the plane is non-effective.



Take any two points in the plane A & B; from A draw a line horizontal, & from B a line vertical; these two lines intersect in a point C; the ratio $\frac{BC}{AB}\left(\frac{h}{l}\right)$ is always the same wherever the points A & B are taken in the plane.

$$f = w\frac{h}{l} = w\sin\alpha.$$

$$a = \frac{w\sin\alpha}{m} = g\sin\alpha$$

$$= \frac{w.\frac{h}{l}}{m} = g\cdot\frac{h}{l}$$

Consider the distance s' moved through in any given time

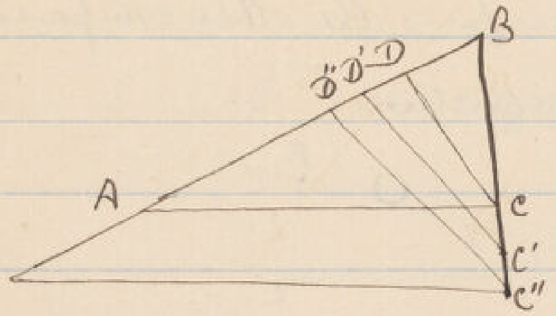
$$s' = \frac{1}{2}at^2$$

$$= \frac{1}{2}g\frac{h}{l}t^2$$

taking the case of free fall

$$s = \frac{1}{2}gt^2 \qquad \text{i.e. } \frac{s}{s'} = \frac{l}{h}$$

Let two bodies drop simultaneously: one down the plane and the other down the vertical; find where they are at any given instant:



suppose the body which has fallen vertically to be at a point C: i.e. CB is = s; we want to find what s' is; draw from C, a line $\perp^r$ to the incline i.e. to AB; cutting AB is D: from C draw also a horizontal line CA; from the similar Δ^s ABC & CBD; we get:

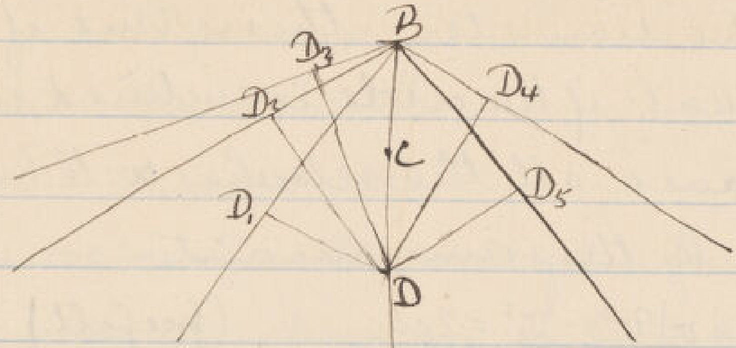
$$\frac{BC}{BD} = \frac{AB}{BC} = \frac{l}{h}$$

$$\text{i.e. } \frac{s}{BD} = \frac{l}{h}$$

$$\text{or } BD = s\frac{h}{l} = s'$$

So for any other points C' & C'' &c. we get points D', D'' etc: and C'D', C''D'' etc is ~~pa~~ always $\perp^r$ to the plane.

Suppose we have several inclines all starting from the Same point B; and bodies are let fall one down



each simultaneously: find where they will all be at the same instant; let BD be the vertical height fallen through: get the other points $D_1 D_2 D_3$ etc. by letting fall ⊥rs to the different planes, in the manner just described for one plane: the points D, D_1, D_2, D_3, D_4 etc. are the points where the bodies are at the same instant: the ∡s DD_1B, DD_2B, DD_3B etc. are all right angles standing on BD: hence they are the angles in a semicircle with ~~radius~~ diameter BD: ∴ all the points D, D_2, D_3 etc. all lie on a circumference of which C the middle point of BD is the centre, and $CD = CB = \frac{1}{2}BD$ $= \frac{1}{2}$ the ⊥r height fallen through, is the radius.

i.e. a body takes the same ~~f~~ time falling down any chord of a circle, as it does to fall down the diameter: this is true if the circle is vertical; if the circle is inclined it is also true. because all the velocities will be diminished by the same amount.

With initial velocity v_0 | $v^2 - v_0^2 = 2gh$ (free fall)

$$\frac{h'}{s} = \frac{BC}{AB} \quad \therefore h' = s.\sin\alpha.$$

$$s = h'\frac{\text{length}}{\text{height}} = h'.\frac{l}{h}$$

$$\therefore v'^2 - v_0'^2 = 2g\frac{h}{l}.h'\frac{l}{h}$$

$$= 2gh' \qquad \text{(down smooth incline)}$$

let $h = h'$

$$v^2 - v_0^2 = v'^2 - v_0'^2$$

i.e. the square of the velocity undergoes the same change whatever the steepness of the incline, and even if the fall is free.

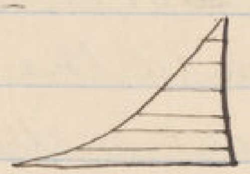
One particular case of this is

$$v_0 \;\&\; v'_0 = 0 \quad \text{i.e. let the bodies start from rest.}$$

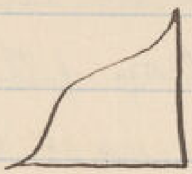
then $v^2 = v'^2$

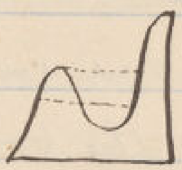
i.e. $v = v'$

i.e the velocity at the end of the time is the same on any smooth incline as in the case of free fall.
Or take a curve; -



the same is true here: ~~for~~ we may divide it into a number of minute inclines; or even we may take an ~~math~~ curve which rises: the same is true here: in

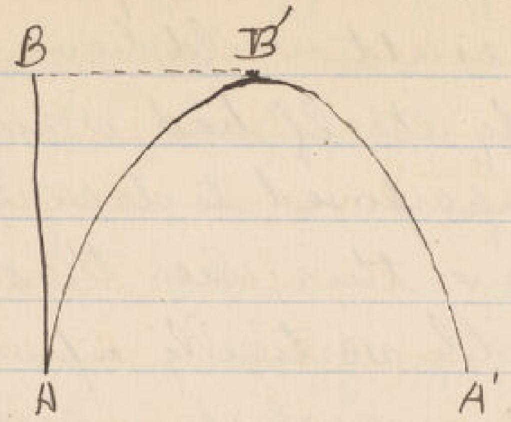




some cases of a curve rising a body falling down it, will often pass the same level 2 or 3 times: the velocity at the same level is always the same.

April 8. 1878.

We now come to consider cases of motion, in which gravity acts but not in the direction of the motion; although the rate of change in velocity i.e. the acceleration of a force is constant, the direction of the motion may change; e.g. if it acts in the opposite direction to the moving body; it may retard its motion, ~~or~~ or it may convert that motion into a motion in the opposite direction: the case of a body sent vertically upwards we have already considered; in that case the return path is the same as that by which the body travelled upwards; but we may have cases where we have in effect two motions: one in one direction & the other in ~~the~~ another; the body moves as a compromise between the two: suppose we throw a ball upwards, close to the side of the next page, so that it traces out a path for itself; say it is thrown from a point A; it



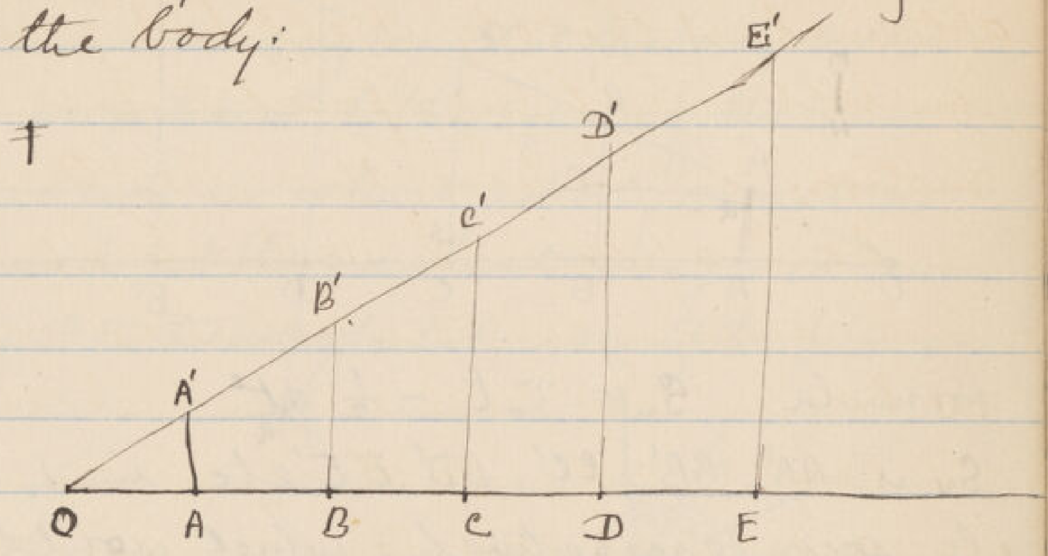
travels upwards vertically to a point B, marking ~~up~~ out a path AB; and falls down the same line again; but suppose that at the same time the page is moving from right to left; so that the point B' comes into the position B; the final position of the body will be B'; the mark left by the ball will be some line beginning at A and ending at B'; the real line will be a curve of something the shape in the figure; suppose now the body falls; while the same things are going on: in the same manner a curve B'A' will be left showing the path of the body: B'A' will be a reversed copy of AB'.

But instead of the page moving from right to left; the same result would have been obtained if the body itself had a motion from left to right superposed to its vertical motion; e.g. we have this; when throwing a ball apparently vertically upwards, in a railway carriage which is moving ~~for~~ horizontally: and so in fact it is in all cases; as the earth is always moving forwards; for a short time virtually horizontally: the curved path of the body is not noticed.
Suppose the horizontal velocity of a body to be uniform, and the vertical velocity that caused ~~f~~ by gravity:

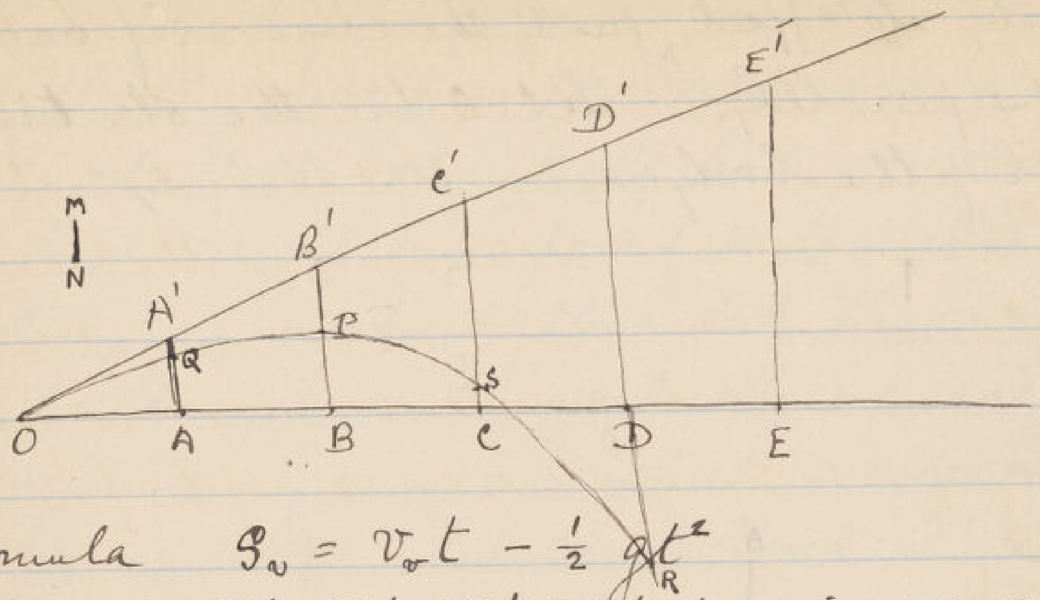
for uniform velocity $S_h = v_h t$

for vertical velocity $S_v = v_v t - \frac{1}{2} g t^2$

Suppose the two are superposed; this may be done by throwing a body in a horizontal direction; or in an inclined direction: gravity ~~acts~~ has its effect, the horizontal velocity its effect, and the combined effect is just the same as if each had acted separately: Let O be the starting point of the body:



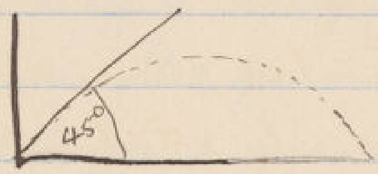
Let OA' be the direction in which the body is thrown; let OA' be the distance the body would have travel-led in a ~~s~~ unit of time say a second i.e. OA' is the velocity of the body: in each unit of time it would travel the same distance in that direction; if that was the only force acting; the horizontal distance is OA in the first second: AB = BC = CD = etc. = OA in succeeding seconds; but gravity is acting: if it were not the body would be at A', B', C', D', E' etc. in 1, 2, 3, 4, 5 etc. seconds respectively: the body will however be still at some point in the verticals at the points A, B, C, D, E etc. in 1, 2, 3, 4, 5, etc. seconds respectively: we have the



formula $S_v = v_v t - \frac{1}{2} g t^2$

S_v is AA', BB', CC', DD', EE', etc. in 1, 2, 3, 4, 5, etc., seconds respectively; which would be $= v_v t$ if gravity were not acting: this diminishes the vertical height by $\frac{1}{2}gt^2$ in each second: which is the same effect as if the body fell from rest; let MN be the measure of g: then in one second $h = \frac{1}{2}gt^2 = \frac{1}{2}g$ i.e. in one second the body has fallen by that amount; i.e. it is not at A' but at $\frac{1}{2}g$ below A': i.e. at Q; in two seconds $h = \frac{1}{2}gt^2 = 2g$; i.e. it is $2g$ below B'; i.e. at P; in 3 seconds $4\frac{1}{2}g$; i.e. at S; in 4 seconds $8g$ i.e. at R from, and we get a curve of something the shape in the figure, described, OQPSR etc; the result is the same as saying that the body falls a distance proportional to the square of the time:

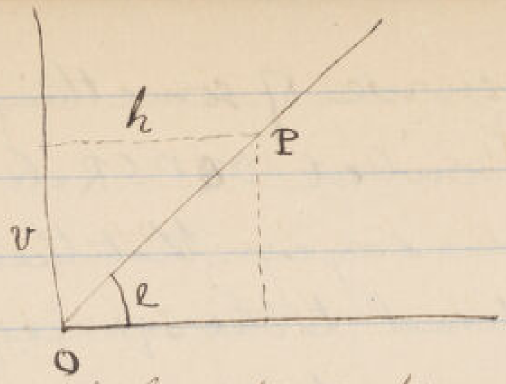
The direction in which a body must be thrown so as to get the greatest range is an angle of 45° to the horizontal: or if the plane is



not horizontal: the direction must bisect the angle between the plane and the vertical

April 10th 1878.

Suppose a body is thrown upwards from a point O. and it moves in the plane of the figure; draw through O a horizontal line and a vertical line; and let the body be thrown in a direction OP; i.e. if the body were thrown from a gun: it would have to be elevated along that line: so the angle e is called the angle of elevation: (for figure see next page)



call v the vertical component

V = the actual velocity

h = the horizontal component.

$$\frac{v}{V} = \sin e$$

$$\therefore v = V \sin e$$

$$\frac{h}{V} = \cos e$$

$$\therefore h = V \cos e$$

Let x be the horizontal distance moved through in the time t; and y the vertical distance in the same time; as h is constant:

$$\therefore x = ht$$
$$= Vt \cos e$$

Let $\bar{v}$ = average velocity during the time t; then

$$y = \bar{v}t$$

velocity at the beginning of the time $= v_0 = V \sin e$

- - - - - - - end - - - - - - - $= v_t = v_0 - gt$
$= V \sin e - gt$

$$\bar{v} = \tfrac{1}{2}(v_0 + v_c) = V\sin e - \tfrac{1}{2}gt$$

$$\therefore y = Vt\sin e - \tfrac{1}{2}gt^2.$$

Therefore to find the position of the point after the time t; find the values of x & y by the equations just given: measure off x along the horizontal line and draw a vertical at the point arrived at; the point must be somewhere in that line: measure off y along the vertical line and draw from the point arrived at a horizontal line: the point must be somewhere in that line: therefore the point of intersection of the two lines is the point where the body is. We may combine the two equations in one; thus:

$$x = Vt\cos e$$

$$\therefore t = \frac{x}{V\cos e}$$

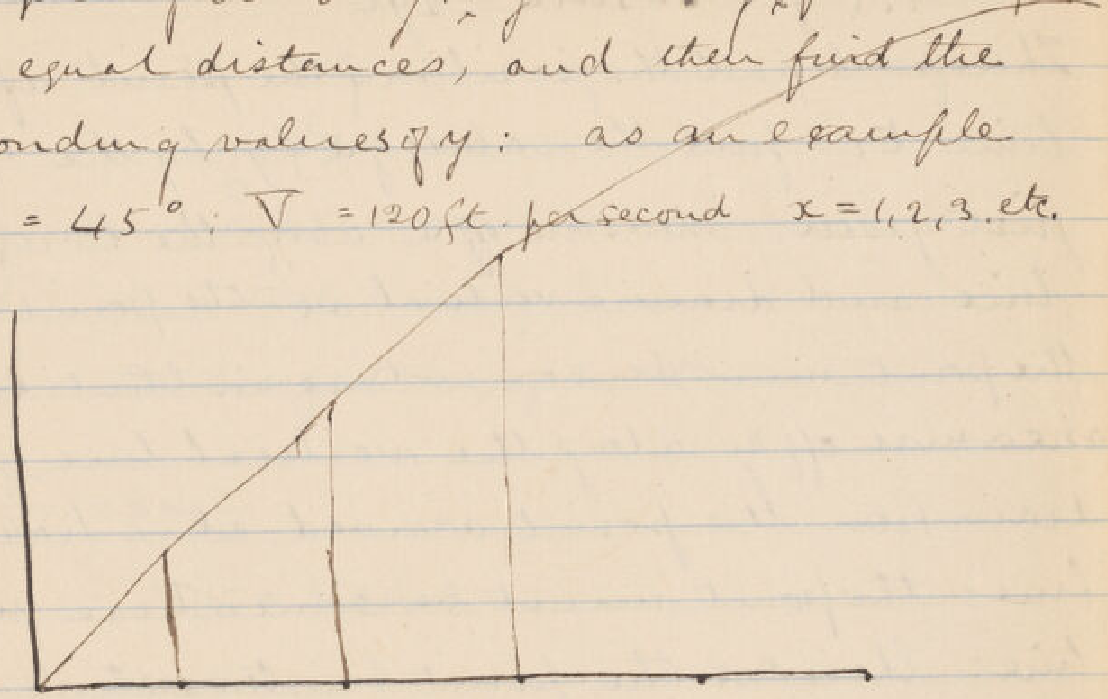
in the equation for y substitute this value for t.

$$y = V \cdot \frac{x}{V\cos e} \cdot \sin e - \frac{1}{2} g \cdot \frac{x^2}{V^2\cos^2 e}$$

$$= x\tan e - \frac{1}{2} g \cdot \frac{x^2}{V^2\cos^2 e}$$

This gives us the value of y in terms of the horizontal distance, and not of the time as before.

This last equation is the best for tracing out the actual path of the body; viz:- by taking a number of values for x say equal distances, and then find the corresponding values of y: as an example let $e = 45°$; $V = 120$ ft. per second $x = 1, 2, 3$. etc.



$x = 1$ $\quad y = 1 \times 1 - \frac{32 \times 1^2}{2 \times 120^2 \times \left(\frac{1}{\sqrt{2}}\right)^2} = 1 - \frac{1}{450} = \frac{449}{450}$

$x = 2$ $\quad y = x - \frac{1}{450}x^2$
$\quad = 2 - \frac{4}{450} = 1\frac{446}{450}$

~~y~~ $x = 3$ $\quad y = 3 - \frac{9}{450} = 3 - \frac{1}{50} = 2\frac{49}{50}$

& so on.

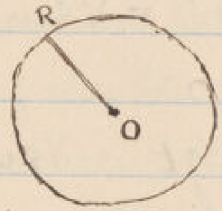
The curve which we get in all these cases is a parabola, for which the general equation is:

$$y = a + bx + cx^2$$

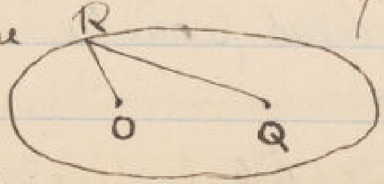
in the parabola we have just described

$$a = 0, \quad b = 1 \;\&\; c = \frac{1}{450}$$

The general properties of the parabola may be thus explained:



in the circle we have a figure in which every point of its circumference is equidistant from a fixed pt., viz: the centre. Let O be the centre and R any point on the circumference of a circle; measure from O to R, and then back again from R to O; we get a constant quantity = 2ce the radius: an ellipse is a sort of ~~centre~~ circle with two centres called foci: O & Q; let R be any point on the circumference; measure from O to R, & back again R to Q: this



distance is again constant; if O & Q coincide we get a circle: i.e. a circle is an extreme case of an ellipse:

let one of the foci say Q: be at infinity; we then get the other extreme case of the ellipse: viz: the parabola.

How long will a body rise, thrown in an oblique direction? this is given by the formula;

$$v_t = v_0 - gt = V\sin e - gt$$

at the summit $v_t = 0$

$$\text{i.e. } 0 = v_0 - gt = V\sin e - gt$$

$$\therefore t = \frac{V\sin e}{g}$$

the body is an equal time coming down;

$$\therefore \text{ the total time } T = \frac{2V\sin e}{g}$$

this includes the case of a body being thrown vertically upwards; for then $\sin e = \sin 90° = 1 \quad \therefore \quad T = \frac{2V}{g}$

What will be the horizontal distance the body will have traversed when it has reached its highest point, i.e. in the time t which is $= \frac{V\sin e}{g}$

$$x = ht$$

$$x = V\cos e . \frac{V\sin e}{g} = \frac{V^2 \sin e \cos e}{g}$$

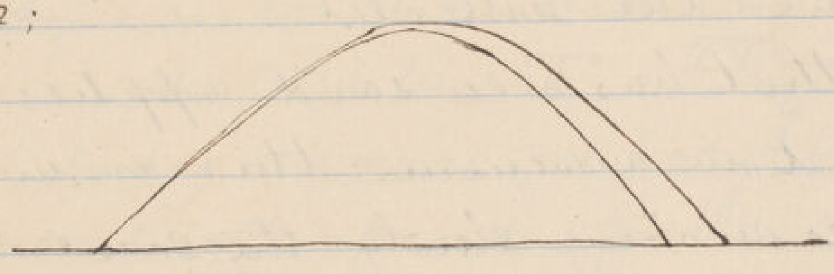
Therefore the total range $X = \frac{2V^2 \sin e \cos e}{g}$

$$= \frac{V^2}{g} \cdot 2 \sin e \cos e = \frac{V^2}{g} \sin 2e$$

The greatest value the sine can have is 1; when this is the case & $\sin 2e = \sin 90° = 1$ therefore $e = 45°$.

In a similar manner it can be proved that ~~not~~ the greatest range on an inclined plane is got, when the initial direction of the projectile bisects the angle between the plane and the vertical.

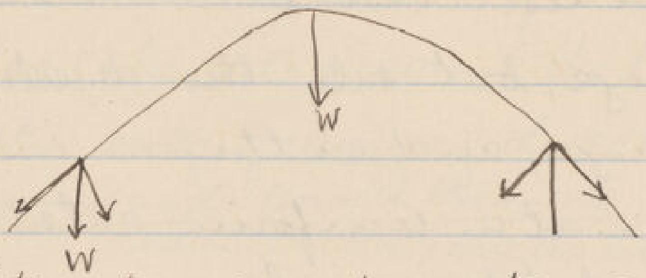
All that has been said applies only to movement in a vacuum; this is in the case of bodies moving slowly the same virtually in air; when moving quickly however; e.g. like the motion of a cannon ball the ~~air~~ resistance of the air must be taken into account as it makes very important differences: the effect is very complicated and not yet thoroughly understood: the greatest range of a gun is found therefore by practice. The general effect is this: the air in front of the ball gets compressed; the air behind it is rarefied; there is therefore a pressure backwards on the cannon ball equal to the difference of the pressures of the air in front and behind; by this means both the vertical and horizontal velocities are diminished i.e. the distance between the theoretical and the actual curve gets gradually more: something as in the figure:



in a ball or body moving slowly we have not this effect: because in that case the air has time to ~~to~~ settle down as the ball is moving, and thus we have the same pressures, (nearly), before & behind.

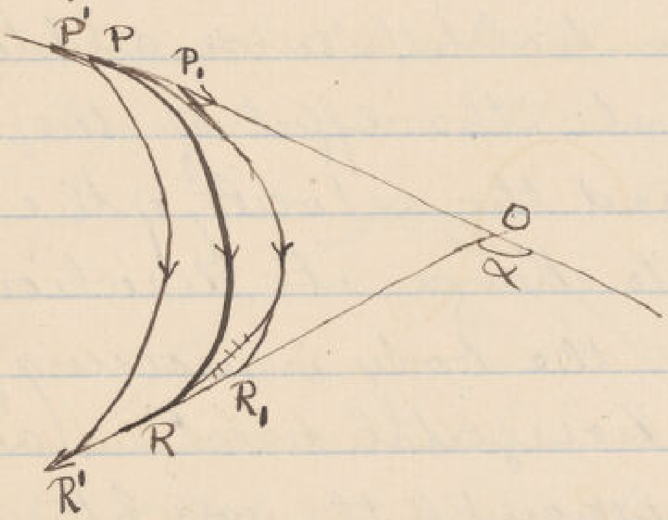
Centrifugal force. April 15th 1876

The effect of the weight on a body moving in a parabola is twofold: the weight may be re-



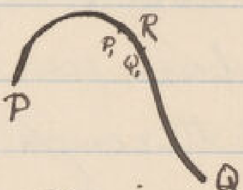
-solved into two components; one along the path; i.e. tangential to the curve, and the other at right angles to it: the effect of the first of these is to retard the velocity of the body, and of the second to change its direction; at the highest point, the body is moving for the moment in a horizontal direction, and so the horizontal component of the weight vanishes; i.e. there is no tendency to change in velocity but only in change of direction; on the downward path of the body, we have the two components, the tangential one increasing the velocity, & the one at right angles to this changing the direction of the body.

If the force were always in the condition we have at the highest point; i.e. the force must change in direction at every instant; we shall have a curve described, in which the velocity does not change, but only the direction of the motion change, and as the force is constant, would change at a uniform rate; this curve we shall find to be a circle



Suppose we have a body moving in the direction of the curve PR; at the point P it is moving in the direction of the tangent at that point i.e. PO; at the point R, in the direction OR; the angle α measures the amount of change of direction between the points P and R: but the same angle * measures the amount of change of direction for other curve $P'R'$, P_1R_1 etc. of which OP and OR are tangents: therefore we must also take into account the distance trav-ersed; the rate of change of direction is measured by the ratio $\dfrac{\text{angle turned through}}{\text{distance moved along the curve}}$

this rate may vary in different parts of a curve;



as in the curve PQ in the figure: the above ratio then expresses the average rate of change of direction between P and Q: if we want to find the actual rate ~~between~~ at any point R on the curve: two points P_1 and Q_1 must be taken on the curve at equal distances one on each side of ~~PfQ~~ R and very near to it, and the limiting value of the above ratio taken; the ratio expresses what is called the ~~limiting value~~ curvature of a curve: there is one curve where the curvature is constant; this is the circle; i.e. a body moving with the same change of direction and with no change of velocity must be moving in a circle: the constant force always being in the direction of the radius; another point yet to be discussed is the velocity: in describing a total revolution 360° is the distance moved through expressed in degrees; if this is done in the time t;

$\frac{360}{t}$ = distance passed through in a unit of time; a better way of expressing the velocity is the following: let r = radius;

$$2\pi r = \text{whole circumference.}$$

if we go along the curve a distance = radius.

$\frac{\beta}{4\pi L^3} = \frac{2}{2\pi r}$ $\frac{2\pi r}{r} \frac{1}{2\pi r} (4\pi L^3) = 2\pi$ ~~ex~~ expresses the angle (β) passed through $\beta\,2\pi$ when the whole circumference is gone along.

if this is done in the time t

$$\frac{2\pi}{t} = \omega = \underline{\text{angular velocity}}: \text{ which may be}$$

uniform or varying; for the average angular velocity

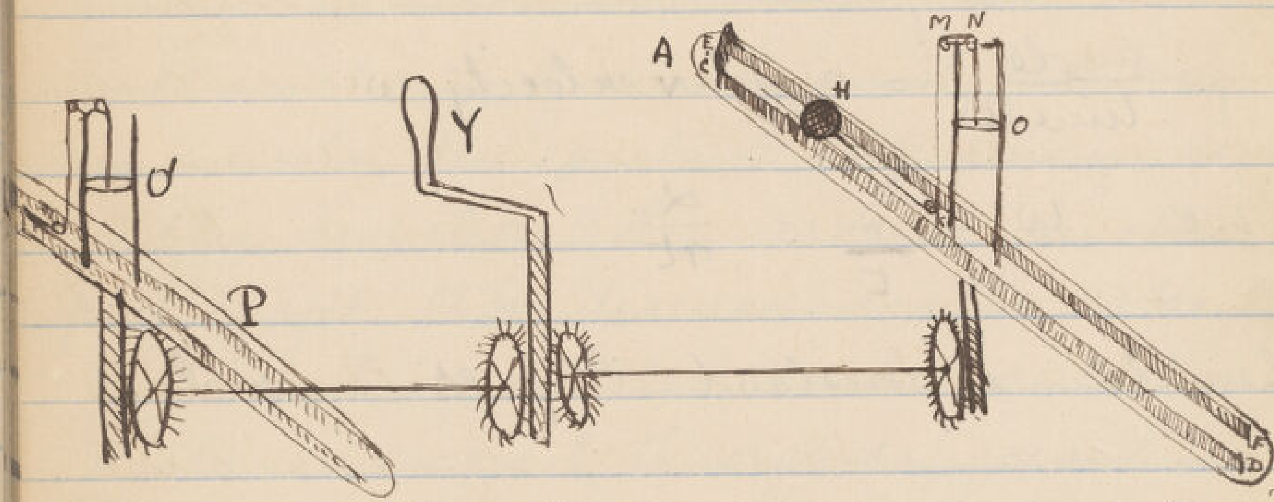
$$\frac{\text{arc}(\alpha)}{\text{radius}(r)} = \text{angle}(a)$$

$$\frac{\text{angle}}{\text{time}(t)} = \text{angular velocity } \omega$$

i.e. $$\omega = \frac{\frac{\alpha}{r}}{t} = \frac{\alpha}{rt}$$

when this is constant the path is a circle.

If a body is moving round a circle, and the force directed towards the centre be removed; the body flies off in the direction of the tangent at the point where the force breaks off; as in a sling: etc; the length of the radius, the force acting along it; the speed of rotation, and the mass of the body which is rotating are all connected with each other to determine when the force acting along the radius will be over-come and the body fly off at a tangent; the following apparatus is used to show experimentally this connection;



there is a long beam of wood AB; on which are placed two metal rails; CD & EF on this runs a metal disc H of known weight, and upon which other similar discs can be fastened according as a certain mass is required; to H is attached a string which passing under a pulley, K goes vertically upwards; and passing over two other pulleys, M & N at the top of an iron rod, passes vertically downwards; so as to be just over the middle pt. of the whole beam: the end of the string is fastened to a metallic disc O of known weight, to which others can be attached which moves up & down; if the disc H moves towards A; the disc O is drawn up; the whole beam is made to spin about its middle point by turning a handle Y; which works at the same time and therefore turns at the same speed another beam P of exactly the same construction as the one just described: on turning the handle, and gradually getting to a greater speed; the discs which move parallel to the motion overcome the resistance exerted by the central mass, and fly out along the rails pulling the weights O and O' up simultaneously: the speed at which this is done (the radius of the circle ~~at~~ which H and H' describe being known), may be taken as the unit of velocity: and one of the two beams kept in the first condition, while with the other the radius of the weights are altered and compare them with the first: i.e. find when the two fly up together.

The following are some results obtained by experiment:

M (mass of body on the rails)	R (radius)	S (speed)	W (weight lifted)
1	4	1	2
2	4	1	4
4	4	1	8
1	6	1	3
2	6	1	6

from these we get that $\frac{W}{M.R}$ is constant, in this case $= \frac{1}{2}$.

if $\frac{W}{MR}$ is constant $= A$

$$MR = AW$$

for another body,

$$M'R' = AW'$$

if $W = W'$

$$MR = M'R'$$

i.e. $\frac{M}{M'} = \frac{R'}{R}$

The Common centre of gravity of two bodies rigidly connected is a point which divides the line joining them into distances inversely as the masses.

April 17th 1878.

Now alter the speed of rotation; if one spindle rotates twice as fast as the other we get 4 times the force, other things being the same: and generally; the force is proportional to the square of the number of revolutions in a unit of time; or inversely proportional to the square of the period of one ~~number~~ of revolution.

$$n = \frac{1}{t} \qquad t = \frac{1}{n}$$

i.e. f is proportional to $\frac{mr}{t^2}$

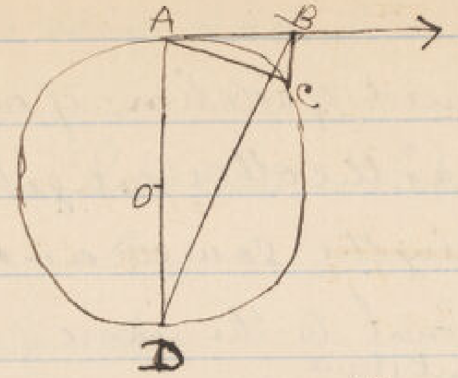
i.e. $f = K\frac{mr}{t^2} = Kn^2mr$

K is the factor of proportionality

$$\frac{f}{\frac{mr}{t^2}} = K$$

K can be found by making observations of the actual rate of rotation; or in the following manner from the consideration of the laws of motion.

Suppose a body going round a circle with uniform velocity $v = \frac{2\pi r}{t}$; what force must act on it so as to keep its path a circle? consider this question by conceiving the body to depart from its circular path an indefinitely short distance, continually, and being continually pulled back: the path it would then describe would in the limit be the same as a circle



Consider the body passing the point A; if left to itself it would get to B in ~~t~~ a certain time; but a force pulls it back to its circular path i.e. to C; (AC = AB) we shall have to find what force will bring it from B to C in the same time as it would get from A to B with velocity = $\frac{2\pi r}{t}$; we have

$$f = \frac{mv}{t}$$

$$s = \tfrac{1}{2}vt$$

$$v = \frac{2s}{t}$$

$$f = \frac{2ms}{t^2}$$

in the case we are considering $s = BC$, and let the ~~time~~ time = τ.

$$\text{i.e. } f = \frac{2m \cdot BC}{\tau^2}$$

but $\tau = \cancel{m}\frac{AB}{v}$

$$= \frac{AB.t}{2\pi r}$$

$$f = \frac{2m.BC}{\left(\frac{AB.t}{2\pi.r}\right)^2} = \frac{4\pi^2 r^2 2m.BC}{AB^2 t^2}$$

Join BD and AC; which being small will coincide with the arc AC : ABC and ABD are similar triangles

$$\therefore \frac{BC}{AB} = \frac{AB}{AD}$$

$$\text{i.e. } \frac{BC}{AB^2} = \frac{1}{AD} = \frac{1}{2r}$$

$$\therefore f = \frac{4\pi^2 r^2 2m}{2r.t^2} = \frac{4\pi^2 r m}{t^2}$$

this value for f may be expressed in several other ways; e.g. $v = \frac{2\pi r}{t}$ $\therefore v^2 = \frac{4\pi^2 r^2}{t^2}$

$$\frac{v^2}{r} = \frac{4\pi^2 r}{t^2}$$

$$\therefore f = \frac{v^2 m}{r \cancel{A^2}}$$

we now see that $k = 4\pi^2$

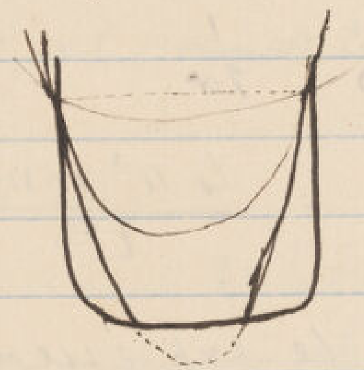
$$f = 4\pi^2 . \frac{\cancel{A} m r}{t^2} = 4\pi^2 m r n^2$$

Again we may express it in term of angular velocity ω

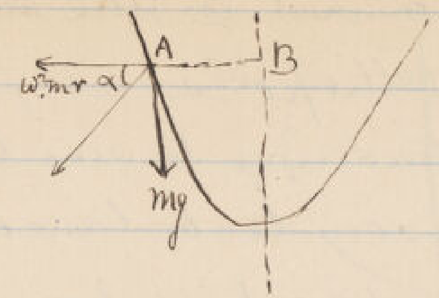
$$\omega = \frac{2\pi}{t}$$

$$\therefore f = \omega^2 mr$$

With a rotating bell jar of water, the flying away from the centre is seen; the water gradually becoming a hollow, and at last adhering ~~along~~ only to the sides; the curve in all cases is a parabola.



Suppose the ~~tr~~ volume of water to be rotating at a certain speed; the curve of the hollow will be constant: consider any point A: in the rotating liquid



on it acts a vertical force $= mg$
and a horizontal force $= \omega^2 m r$ which is
called the centrifugal
force.

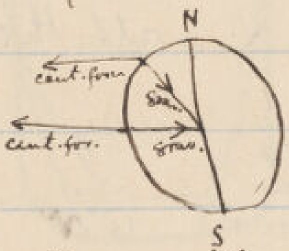
the line of action of the resultant of these two is a line between their lines of action, and making an angle α with the centrifugal force, such that

$$\tan\alpha = \frac{mg}{\omega^2 m r} = \frac{g}{\omega^2 r} \qquad r \text{ in this case } = AB$$

r varies in length in different parts of the curve.

The shape of the earth has doubtless become what it is in obedience to the centrifugal force: the radius ~~h~~ at the equator is greater than anywhere else, and hence the centrifugal force is greater: therefore there is a bulging out at the equator, or what is the same thing a flattening at the poles; all fluid on the earth has a tendency to run towards the equator; this is counterbalanced by a tendency to run downhill from the equator to the poles i.e. in the opposite direction.

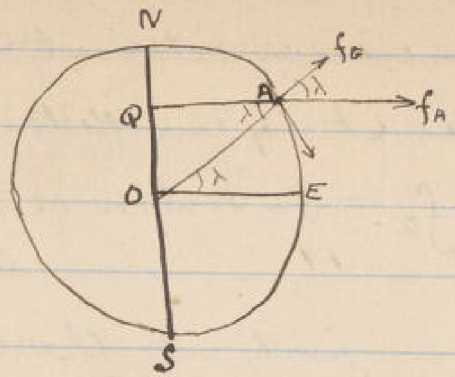
At the ~~centre of the earth~~ equator the centrifugal force is directly opposed in direction to the force of gravity: at other parts the two forces are differently inclined to each other:



hence the force of gravity i.e. g varies in different latitudes

April 24. 1878.

We now proceed to determine (approximately) the formula which expresses the value of g at any latitude λ. Let A be a point with that latitude



The angle QAO = AOE is λ.

$$\frac{AQ}{AO} = \cos\lambda$$

AO = radius of earth = R.

$$AQ = R.\cos\lambda.$$

The centrifugal force at A $\quad f_A = \omega^2 \, AQ$

$$= \omega^2 . R. \cos\lambda.$$

The component of the centrifugal force which is directly opposed to gravity $\quad f_g = f_A \cos\lambda.$

$$= \omega^2 R \cos^2\lambda$$

i.e. knowing the force which is opposed to gravity the gravity at that point can be found; in the following way; assuming the earth to be spherical; i.e. we suppose the gravity to vary only on account of the centrifugal force.

Let $\quad g_\lambda$ = intensity of gravity in latitude λ.

$\quad G$ = ~~intensity~~ of gravity at the pole.

G is the maximum intensity of gravity, because at the poles, it is unaffected by centrifugal force.

$$g_\lambda = G - f_G = G - R\omega^2 \cos^2\lambda.$$

The intensity of gravity at the equator g_E is

$$g_E = G - R\omega^2$$

$$\because \lambda = 0 \;\therefore\; \cos^2\lambda = 1.$$

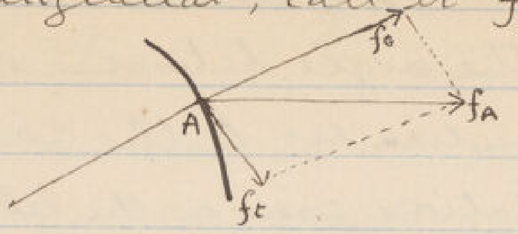
at the poles $\lambda = 90$ $\therefore \cos^2\lambda = 0$

$$\therefore\; G = G.$$

The earth however being not spherical, the intensity of gravity is really rather less than these values give. G has not been found actually by experiment, but measurements of the intensity of gravity have been made very near the pole, and from these ~~on~~ the actual value of G has been deduced; but as we go from pole to equator, we are really going up hill, and gravity varies from this reason also.

The other component of the centrifugal is at right angles to the one opposed to gravity: i.e.

it is tangential; call it f_t



$$\frac{f_t}{f_A} = \sin\lambda$$

$$\therefore f_t = f_A \sin\lambda$$
$$= R\omega^2 \cos\lambda \sin\lambda.$$

This component produces a tendency for bodies to roll from the poles towards the equator; it vanishes both at the equator and at the poles: because at the equator $\sin\lambda = 0$, and at the poles $\cos\lambda = 0$; it is greatest at 45°. And so matter gets heaped ~~at~~ up at the equator; ~~so~~ i.e. we have a flattening at the poles; why this does not go on, we shall see from the following consideration; if this heaping up at the equator takes place: the surface at any point is not perpendicular to the line joining that point to the centre of the earth; but is an inclined plane sloping from the equator towards the poles; the inclination (α) of this plane is the angle between it, and the plane perpendicular to the radius:

(N.B. In the equations given on the last three pages, unit mass has been considered; if we wish to apply them to bodies of m units of mass, we must multiply by m; mg being the weight of a body with mass m.)

The force urging ~~the~~ a body down this inclined plane is $g \sin\alpha$ for each unit of mass i.e. $mg \sin\alpha$ for the whole body. The heaping up at the centre goes on, therefore until it equals this force driving the body down the plane, when it must stop i.e. $f_c = g \sin\alpha$.

This however is only roughly true because neither R nor g are constant as we have seen.

In fact the equatorial radius of the earth is

20, 923, 596

or more roughly 20, 923, 600.

For a complete revolution $\omega = \frac{2\pi}{T}$

The period of rotation of the earth is not a day
i.e. $60 \times 60 \times 24 = 86400$ seconds.
as will be seen in this way; the earth besides rotating on its axis revolves round the sun i.e. roughly speaking goes 360 in 365 days; which is about a degree a day; let S be the sun

and E and E' the relative positions (exaggerated) of the earth on two successive days ~~at noon~~; let P be a point which is at noon on the first day; in 24 hours exactly, it will be noon at P on the second day; but P has really gone more than ~~so~~ once round viz; by about 1°; i.e. the quantity it has turned round the sun; so in the whole year the earth completes one extra revolution; counting the length of the day however by the fixed stars we get the true value: it is called a sidereal day and = 86164 seconds.

at the equator, the centrifugal force

$$f = R\omega^2 = \frac{20,923,596 \times (3.1416)^2 \times 4}{(86164)^2}$$

$$= 0.11126 \text{ ~~feet~~. units of force}$$

i.e. acting on a unit of ~~f~~ mass (a pound) for a unit of time (a second), the centrifugal force would give it the velocity of 0.11126 units of length (a foot; i.e. about an inch) in a unit of time, while gravity gives under the same condition gives in the opposite directions a velocity of 32 feet per second (about).

In order that the centrifugal force should just be equal to gravity, it would be found that the earth would have to rotate about 17 times as fast as at present i.e. the length of the day be $\frac{1}{17}$ of what it is now.

Another interesting point in connection with this is the centrifugal force of the moon; counting the moon and earth as ~~flattened~~ perfect spheres; the distance from the centre of the earth to that of the moon is about 60 times the radius of the earth: the correct number is 59.9644; the intensity of gravity on the moon at that distance is

$$g_M = \cdot 0089697 \quad \text{on each unit of mass of the moon.}$$

the centrifugal force $f = R\omega^2 = \dfrac{60 \times 20932596 \times 4\pi^2}{(2360591)^2}$

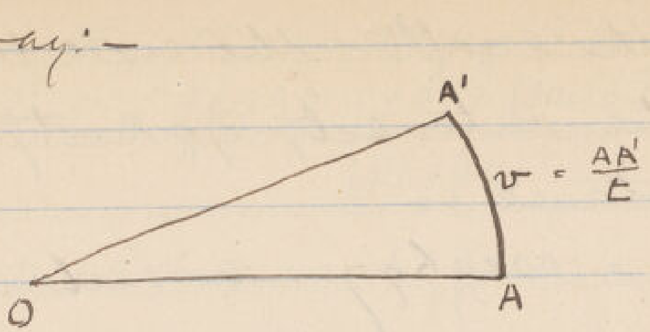
$$= \cdot 00889$$

which approximately $= g_M$

If done correctly (the moon & earth not being spherical etc.) the two would be found exactly equal, as they must be of course. This was the first verification Newton found of the law of gravitation.

April 29. 1878.

Any motion whatever can be considered as made up of a motion of translation, and a motion of rotation. Just as we have ~~a~~ acceleration in actual velocity, so we may have angular acceleration = the rate of change of angular velocity, in a motion of rotation. Actual velocity and angular velocity are connected in this way:—



$$\text{angle} = \frac{AA'}{OA} = \cancel{\ }$$

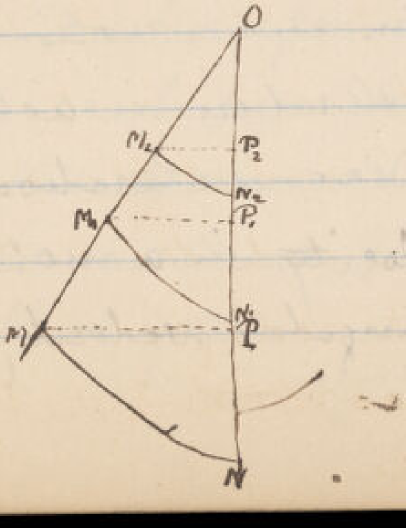
$$\frac{\text{angle}}{\text{time}} = \text{angular velocity i.e. } \cancel{\frac{v/r}{t}} = \cancel{\omega}\ \frac{\frac{AA'}{r}}{t} = \omega$$

$$\text{i.e. } \frac{v}{r} = \omega$$

$$\text{or } v = r\omega$$

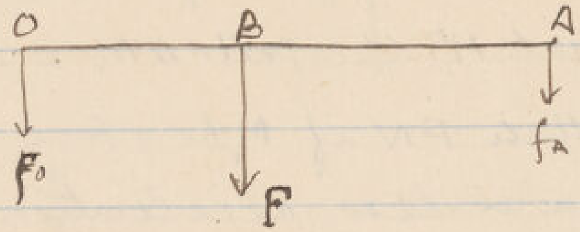
Acceleration $a = \frac{f}{m}$ just as in motion in a straight line.

Take as an example the case of a rod turning about a fixed point in its length.



Take a point M on the rod (which turns about O); it falls to N down a $\perp^r$ distance $= PN$ and acquires the same acceleration as if it fell down that $\perp^r$; take any other point M_1: this falls to N_1 a $\perp^r$ distance $= P_1N_1$ which is less than PN if M_1 be above M in the rod; it therefore acquires less acceleration; and so for any other particle M_2 etc. If these particles were distinct they would each get the different accelerations just indicated; but they are connected rigidly together in a rod, and therefore they have all the same angular velocity; which must be a compromise between the different angular velocities of the particles if separated; this can be seen experimentally by starting a solid pendulum, and one consisting of a thin cord with a heavy ball at the end, together: the latter can be made of different lengths, and vibrates the same as a single particle of the rod at the same distance: if the same length as the rod, the latter soon gets ahead; if shorter than the rod a good deal, it overtakes the rod: the compromise is ~~about~~ $\frac{2}{3}$ the length of the rod when they vibrate together.

Suppose we have a rod undergoing a motion of translation by a force F; let the two parallel component of this be F_0 and F_A



$OB = 1$
$OA = a$

then $f_A = F.\frac{1}{a}$

$$f_0 = F.\frac{a-1}{a}$$

for mass m

$$\frac{f_A}{m} = \frac{\frac{F}{a}}{m} = \frac{F}{am}$$

For angular motion, suppose the rod to turn about O. The same mass will require different forces to give it the same velocity at different distances from O. $\alpha = \frac{\text{acceleration}}{\text{mass}}$ In motion of translation the inertia of a body simply depends on its mass; but in rotation what corresponds to inertia depends not only on the mass, but on its distance from the centre of rotation: this quantity is called the Moment of Inertia; call this quantity I. a = ~~a.r.~~ radius

$$I = a^2 m$$

In linear motion (a = acceleration)

we have $a = \frac{f}{m}$

in angular (α = angular acceleration) motion

$$\alpha = \frac{\phi}{I}$$

ϕ = moment of the force i.e. force × radius.

f and ϕ correspond

m and I correspond.

For a body of any shape whatever: it must be divided into a number of thin spherical shells with centre the centre of rotation.

If m_1 m_2 m_3 etc. be the masses of these shells

$$I = a_1^2 m_1 + a_2^2 m_2 + \dots + a_n^2 m_n$$

$$= \Sigma(a^2 m)$$

When bodies are of complicated shapes, the finding of the moment of inertia is difficult and in some cases impracticable: therefore it is found by experiment.

May 1st 1878.

We have now

$$mr^2 = I \quad \text{for a single particle}$$

$$\Sigma(mr^2) = I \quad \text{of a mass.}$$

The moment of inertia of a body may be defined less geometrically: as the quantity of matter at unit distance from the centre which would replace the body, and have the same angular velocity.

For a body with mass = m

$$(\text{moment})\ \phi = fr$$

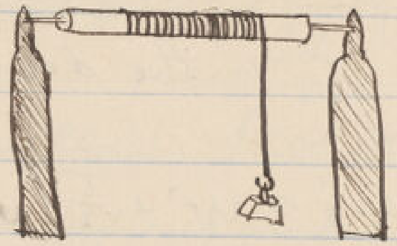
$$\text{angular acceleration } \alpha = \frac{\Sigma(\phi)}{\Sigma(mr^2)} = \frac{\Sigma(\phi)}{I}$$

In a straight line; the energy of a body with mass m is $\frac{1}{2}mv^2$. Suppose it to be moving about a centre: replace v by its value ωr

$$\text{energy} = \tfrac{1}{2}m(\omega r)^2 = \tfrac{1}{2}mr^2\omega^2 = \tfrac{1}{2}I\omega^2$$

Take now a few special examples.

Suppose a string twisted round an axle to which it is attached; attach a weight to the end of the string:



and let it fall; gravity it has to do work of three kinds

1. Give acceleration to the weight.
2. Wind the wheel round.
3. Do work against friction (rigidity of string etc).

~~Let us deter~~ The wheel has a quantity of ~~angular~~ energy of rotation given to it, sufficient to wind the wt part of the way up again. Let us determine the energy in this case

Let m = mass of the weight

then mg = weight of the weight.

mgh = work done on it while falling a distance = h.

This work we have seen is of three kinds:

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + \text{(work against friction)}$$

this last term is thus determined:

let μ = mass ~~which~~ = friction

then μg = the weight of such a mass

and μgh = work done against friction in the distance h.

i.e.

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + \mu gh$$

$$\omega = \frac{v}{r}$$

$$2(m-\mu)gh = mv^2 + I.\frac{v^2}{r^2}$$

$$h = \tfrac{1}{2}vt \quad \left\{ \begin{array}{l} v = \text{final velocity} \\ \tfrac{1}{2}v = \text{average velocity} \end{array} \right.$$

$$v = \frac{2h}{t}$$

$$2(m-\mu)gh = m\frac{4h^2}{t^2} + I\frac{4h^2}{t^2r^2}$$

from this we get

$$\frac{I}{r^2} = \frac{gt^2}{2h}m - m - \frac{gt^2}{2h}.\mu$$

The values r, h, m & t are determined by measurement and experiment; the same is done a second time

$$\frac{I}{r^2} = \frac{gt'^2}{2h}m' - m' - \frac{gt'^2}{2h}\mu$$

i.e.

$$\frac{It'^2}{r^2} = \frac{gt^2t'^2}{2h}m - mt'^2 - \frac{gt^2t'^2}{2h}\mu$$

$$\frac{It^2}{r^2} = \frac{gt^2t'^2}{2h}m' - m't^2 - \frac{gt^2t'^2}{2h}\mu$$

μ can be so eliminated, & we get

$$I = \frac{r^2}{t^2-t'^2}\left[(m'-m)\frac{g}{2h}t^2t'^2 + mt'^2 - m't^2\right]$$

The following are the results of actual experiment.

r	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	inches
t	12.8	9	18	11	seconds
m	1	2	1	2	ounces
h	52	52	52	52	inches

from these we get; in the first case

$$I = 96.5$$

In the second case $\qquad I = 98.45.$

May 6. 1878.

To determine the moment of inertia of a material line in terms of the mass & length of the line: when rotating about some pt in ~~the~~ its length:

1°. When the rod is rotating about an axis $\perp^r$ to it and passing thro' one extremity of it.

If m = mass of the line

l = length of the line

$\frac{m}{l}$ = mass of unit length.

conceive the line divided into very small portions of length δ; then for one of these $I = \frac{m}{l}\delta r^2$; for another $\frac{m}{l}\delta r'^2$ & so on: and for the sum of these i.e. for the whole line

$$I = \Sigma\left(\frac{m}{l}\delta r^2\right)$$

Let $\frac{m}{l}\delta\, a^2$ be the moment of inertia of the 1st of these

$\frac{m}{l}\delta\, b^2$ - - - - - - - - 2nd - -

$\frac{m}{l}\delta\, c^2$ - - - - - - - - 3rd - -

⋮

$\frac{m}{l}\delta\, z^2$ - - - - - - - - last - -

then $I = \frac{m}{l}\left(a^2\delta + b^2\delta + c^2\delta + \text{--------} + z^2\delta\right)$

$b = a + \delta$
$c = b + \delta$ $\therefore b^3 = (a+\delta)^3$
etc

$= a^3 + 3a^2\delta + (3a\delta^2 + \delta^3$ which may be neglected)

$$a^2\delta = \frac{b^3 - a^3}{3}$$

$$b^2\delta = \frac{c^3 - b^3}{3}$$

$$c^2\delta = \frac{d^3 - c^3}{3}$$

etc.

fill in these values

$$I = \frac{m}{3l}\left[(b^3 - a^3) + (c^3 - b^3) + (d^3 - c^3) + \text{--------} + (z^3 - y^3)\right]$$

$$= \frac{m}{3l}(z^3 - a^3) = \frac{m}{3l}(l_3 - 0)$$

$$= \frac{ml^2}{3}$$

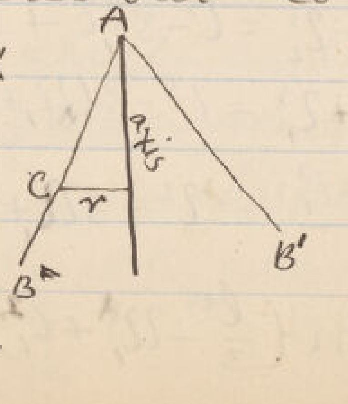
2°. The axis is not $\perp^r$ to the length of the rod.

e.g. when the rod marks out a cone then

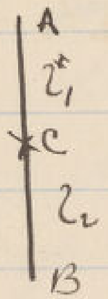
r for any pt C = AC sin α

do this for each & we get

$$I = \frac{1}{3} ml \sin^2 \alpha$$



3°. Axis $\perp^r$; but the centre of rotation some other point besides and the extremities.



This may be regarded as the rotation of two rods joined at C. $l = l_1 + l_2$

$$I = \frac{1}{3}\left(m_1 l_1^2 + m_2 l_2^2\right)$$

$$m_1 = \frac{m}{l} l_1$$

$$m_2 = \frac{m}{l} l_2$$

$$I = \frac{m}{3l}\left(l_1^3 + l_2^3\right)$$

$$l_2 = l - l_1$$

$$l_2^3 = l^3 - 3l^2 l_1 + 3ll_1^2 - l_1^3$$

$$l_2^3 + l_1^3 = l^3 - 3l^2 l_1 + 3ll_1^2$$

$$\frac{l_2^3 + l_1^3}{l} = l^2 - 3ll_1 + \cancel{3l_1}\, 3l_1^2$$

$$I = m\left(\frac{l^2}{3} - ll_1 + l_1^2\right)$$

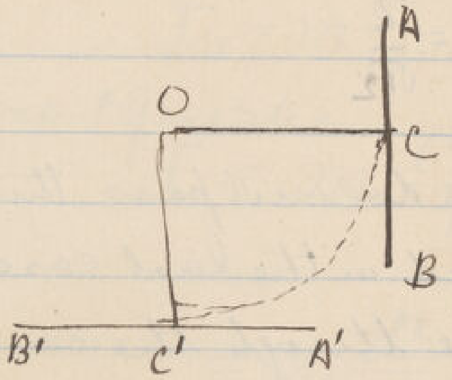
6°. When the centre of rotation is the mid pt.

$$I = m\left(\frac{l^2}{3} - l l_1 + l_1^2\right) \qquad l_2 = l_1 = \frac{1}{2}l$$

$$= m\left(\frac{l^2}{3} - \frac{1}{2}l^2 + \frac{1}{4}l^2\right)$$

$$= \frac{1}{12} m l^2$$

5°. For a point outside the line itself; say the line which joins is $\perp^{ly}$ to the line bisects the line.



i.e. the rod turns round ~~out~~ O; the figure denotes the rod in two position; one before, the other after ~~the whole~~ it has turned through a right angle; this may be regarded as compounded of two movements; a rotation of the rod about C its middle pt. & then a rotation of the rod concentrated at C about O; both with equal angular velocity; for the first of these

$$I_1 = \frac{1}{12} m l^2$$

Let OC = r : then for the second of these movements $I_2 = mr^2$

the total moment of inertia

$$I = m\left(\frac{l^2}{12} + r^2\right)$$

If a body be concentrated at a point, & R is the distance of it from the centre of rotation

i.e. $mR^2 = I$

~~i.e.~~ R is called the radius of gyration:

in this case

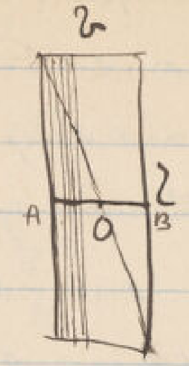
$$R = \frac{l}{\sqrt{12}}$$

6°. When the axis does not pass thro' the centre of gravity as it did in the last case; conceive a parallel one through the centre of gravity; calculate the radius of gyration about that; & add to it to ~~th~~ the square of the radius to the actual point C (which in the last case was the centre of gravity) i.e. $OC^2 = r^2$. into the mass i.e. mr^2

$$I = m\left(R^2 + r^2\right)$$

7° This may be extended to a rectangular plate; rotating about its centre of gravity O.

conceive it cut up into an indefinite number n of lines or narrow rods: each rotating about its middle pt.



let l = length
b = breadth

for each of these $I_1 = \frac{1}{12} l^2 \mu$ where $\mu = \frac{m}{n}$

now conceive the whole concentrated in the line AB which rotates about O

$$I_2 = m\frac{b^2}{12}$$

the total moment of inertia

$$\underline{I} = m\,\frac{l^2+b^2}{12}$$

let d = diagonal

$$= m\frac{d^2}{12}$$

May 13th 1878.

We now come to methods of determining the moment of inertia of any body experimentally; we have had already one example of this: viz:- in the wheel & axle. The following is a very good method: suspend by means of a wire a body whose moment of inertia can be found from its dimensions say a cylinder: twist it, it untwists, getting angular velocity: its time of vibration can be found with great accuracy by means of a telescope with cross wires: by watching every time a certain line drawn on the cylinder passes their pt. of inter-section; a complete vibration is 2 of what are generally called vibrations: i.e. one to the right & one to left: call the time of vibration t.

We must take for granted: that:—

$$t = 2\pi\sqrt{\frac{I}{K}}$$

K being the force which urges the body back to its position of equilibrium when moved through unit angle; it depends on the material, length, breadth etc of the wire; but for the same wire it remains constant. The moment of inertia of a cylinder turning about its geometrical axis is

$$I = \cancel{A}\tfrac{1}{2} m r^2$$

Now suspend any other mass, by the same wire: and observe the time:

$$t' = 2\pi\sqrt{\frac{I'}{K}}$$

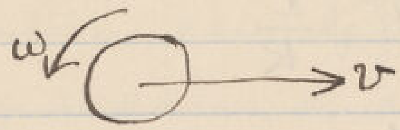
i.e. $\dfrac{t'^2}{t^2} = \dfrac{I'}{I}$ from which I' can be found.

If K is known to begin with; the second experiment only need be performed

Another way is; to add to a vibrating body, another whose moment of inertia is known, observing the time of vibration in both cases; the best thing to add is a narrow ring, vibrating about its geometrical axis; as this expresses (if the ring is thin) the definition of moment of inertia: here $I = mr^2$. This is useful in determining magnetic moments: in the first case $t = 2\pi\sqrt{\frac{I}{K}}$ with the ring added with moment of inertia (i) $t' = 2\pi\sqrt{\frac{I+i}{K}}$

i.e. $\dfrac{t'^2}{t^2} = \dfrac{I+i}{I}$ from which i can be found

We may now apply these laws; if we hit a billiard ball below the centre, it will get a forward motion of translation (v) & a backward one of rotation (ω): and one of three things



may happen: let ρ = coefficient of friction
m = mass of ~~ball~~
g = wt. of unit mass
ϕ = force of friction

$$\phi = mg\rho$$

$$mv = \phi t$$
$$I\omega = \phi r t'$$

$$\left(I = \tfrac{2}{3} m r^2\right)$$

$$\tfrac{2}{3} m r^2 \omega = \phi r t'$$

$$\frac{\frac{2}{3} r\omega}{v} = \frac{t'}{t}$$

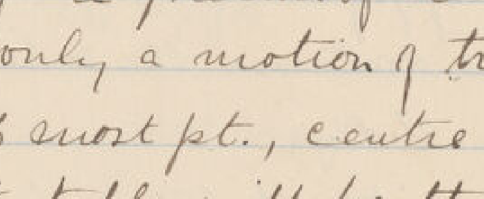
$$\frac{v}{\omega} = \frac{2}{3} r$$

or $v = \frac{2}{3} r\omega$; when this is the case the two velocities will come

to an end simultaneously: when $v > \frac{2}{3} r\omega$
ω dies out before v, ~~and~~ i.e. after sliding
a certain distance it begins to roll: if $v < \frac{2}{3} r\omega$
v dies out first, and the ball after rolling
a certain distance rolls back again.

May 15. 1878.

Suppose a ball struck in a level with its centre: it will first slide, and then in consequence of the friction of the table will roll; if we have only a motion of translation, the velocity of top most pt., centre and point in contact with table will be the same viz: v: if we had only a motion of rotation; the velocities of the same three points respectively will be: $v, 0, -v$: superposing these: i.e. when the ball is rolling: the ~~svelocities~~ velocities are respectively the sum of the ~~the~~ separate velocities in the two first cases; viz: $2v$, v, & 0.



When a ball is struck, ~~fro~~ ~~in~~ the way just described: find how far the ball will go before it begins to roll, and what velocity it will have; when that occurs.?.

$\frac{1}{2}mv^2$ = original energy of ball.

let v_1 = velocity of centre, when ball is rolling: this we want to find

$\frac{1}{2}mv_1^2$ = energy when rolling

$\frac{1}{2}I\omega^2$ = energy of rotation

let l = distance the ball slides ~~in~~ while sliding.

ϕl = force of friction between ball & table in that distance

$$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega^2 + \phi l = \frac{1}{2}mv^2$$

Momentum is lost $M(v - v_1)$ = change of momentum this is lost on account of the friction:

$$\text{i.e. } M(v - v_1) = \phi l \qquad (x)$$

and a momentum of rotation is imparted to the ball.

$$\text{i.e. } I\omega = \phi r.l \qquad (x)$$

$$v_1 = \omega r$$

$$I = \frac{2}{5}mr^2 \text{ (for sphere)}$$

put in these values

$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} m v_1^2 + \tfrac{1}{2}.\tfrac{2}{5} M r^2 \frac{v_1^2}{r^2} + \phi l$$

$$\phi = \rho m g$$

i.e. $\frac{1}{2} m v^2 = \frac{1}{2} m v_1^2 + \frac{1}{5} m v_1^2 + \rho m g l$

or $\frac{1}{2} v^2 = \frac{1}{2} v_1^2 + \frac{1}{5} v_1^2 + \rho g l$

$$\tfrac{1}{2} v^2 = \tfrac{7}{10} v_1^2 + \rho g l$$

or in decimals

$$\cdot 5 v^2 = \cdot 7 v_1^2 + \rho g l \quad \text{------- (1)}$$

from equations (x) (x) we get

$$\tfrac{2}{5} v_1 = v - v_1$$

$$\text{i.e. } v_1 = \tfrac{5}{7} v \quad \text{------- (2)}$$

we thus have the value of v_1 which is required.

Square equation (2) $v_1^2 = \frac{25}{49} v^2$

put this in in equation (1)

$$\left(\cdot 5 - \cdot 7 \times \tfrac{25}{49}\right) v^2 = \rho g l$$

$$\tfrac{1}{7} v^2 = \rho g l$$

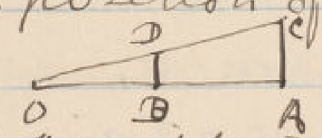
$$l = \frac{v^2}{7 \rho g}$$; which gives us the required value of l.

so that at the equator where g is least, l will be ~~least~~ greatest.

The Pendulum.

The simple pendulum consists of a single heavy particle suspended to a fixed point by a line of no mass, but of invariable length. The compound pendulum is generally a bar.

Take first the simple pendulum; ~~to~~ to which of course, only an approximation can actually be got; let the vibrations be small so that virtually the path is a straight line. Let O be the position of equilibrium; suppose



it displaced to A; it tends to return to O. Let $OA = a$; and K = restoring force at unit distance:

force at $A = K.OA = K.a$. Imagine some point B which it passes when returning from A to O; let $OB = b$

force at $B = K.OB = K.b$.

Draw lines AC and BD to represent the forces at A & B respectively; then

$$\frac{DB}{OB} = \frac{AC}{AO}$$

Join OD and OC; OAC and OBD are therefore similar triangles i.e. ODC is a st. line:
the mean energy = ½ the sum of first & last:

$$\text{i.e. } \tfrac{1}{2}mv^2 = (a-b)\,\tfrac{1}{2}(a+b)K$$

$$= \frac{K}{2}(a^2-b^2)$$

$$v^2 = \frac{K}{m}(a^2-b^2)$$

May 20th 1878.

I.e. the restoring force is directly proportional to the distance from the position of equilibrium.
We have already got

if v = velocity at a certain point

x = distance of that point from the position of equilibm

K = restoring force at unit distance.

$\therefore \frac{K}{m}$ = acceleration

a = extreme distance from position of equilibrium

$$v^2 = \frac{K}{m}(a^2-x^2)$$

Take a few special values of x.

P. T. O

If $x = 0$ $\quad v = a\sqrt{\frac{k}{m}}$ = maximum value of v
$= V$ i.e. velocity when passing position of equilibrium

$x = a \qquad v = 0$

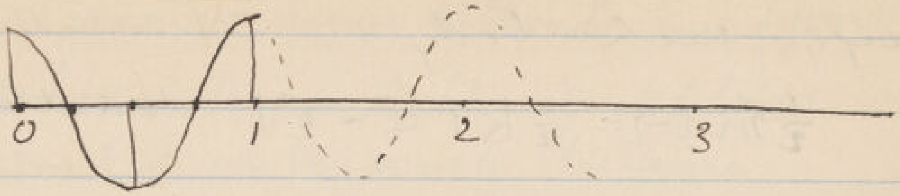
$x = -a \qquad v = 0$

$x = \frac{a}{\sqrt{2}} \qquad v = \frac{a}{\sqrt{2}}\sqrt{\frac{k}{m}} = \frac{V}{\sqrt{2}}$

$x = -\frac{a}{\sqrt{2}} \qquad v = \frac{a}{\sqrt{2}}\sqrt{\frac{k}{m}} = \frac{V}{\sqrt{2}}$

but $v = \pm\sqrt{\frac{k}{m}(a^2 - x^2)}$
i.e. no matter whether ~~x~~ be positive or negative the same value is obtained in both cases; but the sign of v may be + or −: that is, though different in direction the value is the same for any given point.

This given of vibration may be represented by a curve; mark off equal distances along a horizontal line; and suppose time to be measured by a point moving uniformly along it



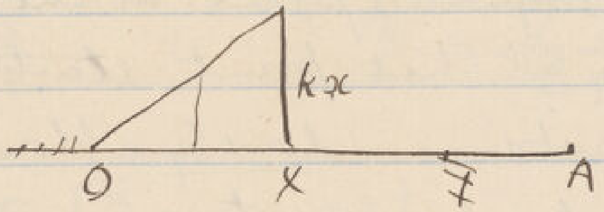
Let 01 = 12 = etc be the time for a complete cycle of vibration; imagine the body to start from its position of equilibrium; let vertical lines drawn to the curve from any point on the line of time be the velocity at that point; starting from the position of equilibrium (done by a blow) we have the greatest value for v; to one end i.e. to where v = 0 is a ¼ of the cycle: so through the other 3/4; then at the beginning of 2nd cycle we have the same conditions as at the commencement of the first & so the same thing takes place over again & so on; & so a body would go on moving for ever if it were not for three disturbing causes.

1. Resistance of the air.
2. friction at point of support, or inflexibility of thread.
3. Want of perfect fixity in point of support.

Neglecting these however, consider the energy of such a body.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}k(a^2 - x^2) \quad \text{kinetic energy}$$

Let OX = x; through X draw a vertical to represent the magnitude of the restoring force at that point = kx



a vertical through the middle point of OX will = the average restoring force

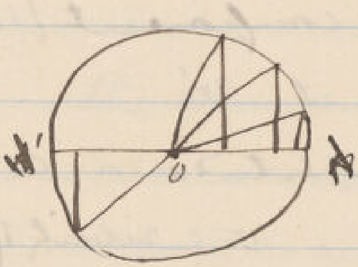
i.e. $\tfrac{1}{2}kx^2$ = potential energy

$$\tfrac{1}{2}k(a^2 - x^2 + x^2) = \tfrac{1}{2}ka^2 = \text{total energy}$$

= kinetic energy at position of equilibrium.

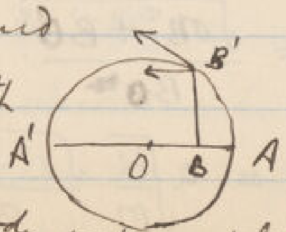
the quantity OA = a is called the <u>Amplitude</u> of vibration; and the above result may be expressed by saying that the energy of a vibrating body is proportional to the square of its amplitude. The expression we have hitherto used for v gives us its value in terms of the amplitude, we now want it in terms of the time.

We may have any no. of rt. ∠d triangles with the hypotenuse of the same length; for instance all the radii of a circle may have right angled Δs described on them, by drawing vertical lines to a horizontal diameter or in any other way;



in all these sq. on hypot = sum of sqs. on 2 sides. This may be expressed by the above equation; viz $v^2 = \frac{k}{m}(a^2 - x^2)$

describe a circle with radius = a, and imagine a body going round the circumference with uniform velocity

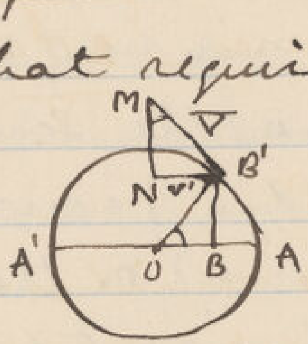


= velocity of a vibrating body with amplitude OA at O; i.e. $= V = a\sqrt{\frac{k}{m}}$

Take any pt. B' on the circumference; the component of the velocity at B' parallel to AO = velocity of ~~body~~ vibrating body at B the projection of B'.

May 22nd 1878

The statement that the component of the velocity of the revolving particle // to the diameter at any pt B', is the same as that of the vibrating particle at B the projection of B'; is a statement that requires proof.



B'N is the resolved component // to the diameter call it v'

$OB' = OA = $ amplitude $= a$

$OB = x$ $\qquad$ $v =$ velocity of vibrating particle at B

$B'M =$ uniform velocity of revolving particle

$=$ velocity of vibrating particle at centre O

$= V = a\sqrt{\frac{K}{M}}$ $\qquad$ We have to prove $v = v'$.

Join OB': the Δ^s BOB' and B'NM are similar.

$$\frac{v'}{V} = \frac{B'N}{B'M} = \frac{OB}{B'O} = \frac{\sqrt{OB^2 + B'O^2}}{B'O} = \frac{\sqrt{a^2+x^2}}{a}$$

$$v' = V.\frac{\sqrt{a^2+x^2}}{a} = a\sqrt{\frac{K}{M}}\,\frac{\sqrt{a^2+x^2}}{a}$$

$$= \sqrt{\frac{K}{M}(a^2+x^2)} = v \quad \text{Q.E.D}$$

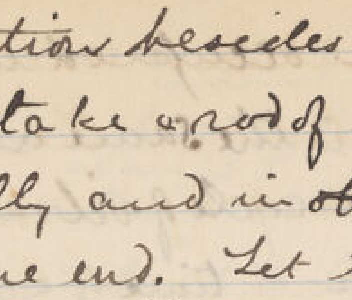
i.e. the time occupied in describing a complete circumference is the same as that occupied in completing one cycle of vibration; let T = time of one revolution = time of a cycle of vibration:

$$T = \frac{\text{circumference}}{\text{velocity}} = \frac{2\pi a}{V} \; \cancel{\frac{2\pi a}{\sqrt{\frac{K}{M}(a^2+x^2)}}} = \frac{2\pi a}{a\sqrt{\frac{K}{M}}}$$

$$= 2\pi\sqrt{\frac{M}{K}}$$

i.e. the amplitude vanishes: in other words all vibrations whatever the amplitude, are if small isochronous: i.e. occupy the same time. The pendulum is the commonest case of of vibration; as the vibrations are isochronous, it is used as a timekeeper; a clock is an instrument which (1) supplies to a pendulum small impulses to keep it going; three retarding causes ~~being~~ having been already mentioned (see p. 89), and (2) to count the number of vibrations it makes and record them on a dial. We shall see presently that the length of the pendulum is the important factor: it must therefore be kept of the same length by counteracting effect of moisture & temperature by combinations of different materials (See heat Vol. I)

§ Other forms of vibration besides the pendulum are the following: take a rod of wood, place it in water vertically and in order to keep it so load it at one end. Let the position of



equilibrium be at A; pull it up to B: the downward force predominates over the upward force by AB. S (S being = the s.g. of the liquid, & the cross section of the rod being unity); push now the rod down to B', AB being = AB': the upward force now predominates by AB'. S; the restoring force is thus proportional to the displacement from its ordinary position, and is the same though in opposite directions whichever way the body is moved, up or down.

Let K = restoring force at unit distance

α = area of section of rod. let it = 1

S = s.g. of liquid   let it be water i.e. = 1

x = distance displaced ~~but~~ which = 1. by def of K

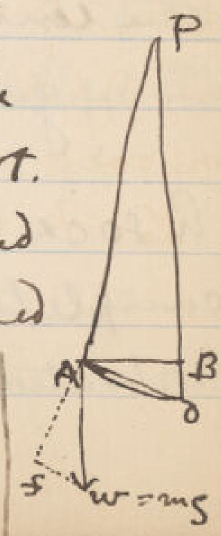
$K = \alpha S \times 1 = \alpha S$ & $T = 2\pi\sqrt{\frac{m}{\alpha S}} = 2\pi\sqrt{m}$.

i.e. we have rectilinear oscillations up & down which are isochronous for the time they last, which is short on account of friction.

There are the same conditions with water in a U tube;

push down the water one side, there is a restoring force proportional to the displacement; and we get isochronous vibrations as long as they continue.

The case of the pendulum which is an instrument of such great importance must be discussed more specially. Let P be its point of attachment; PO its position of equilibrium; & A the point to which it is displaced. The arc AO very nearly = chord AO for small vibrations; ~~as~~ let the pendulum be a simple one, & resolve the wt. w into two components, one effective f, and the other non-effective; the triangle so formed is similar to the $\triangle PAB$.



K = restoring force at unit distance

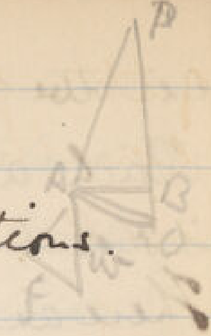
then f = ... distance AB

= K. AB

$$f = K \cdot AB$$

$$K = \frac{f}{AB} = \cancel{\ }$$

$AB = OA$ for small vibrations.



$$K = \frac{f}{AB} = \frac{mg}{l} \text{ (by similar } \Delta^s\text{)}$$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{\frac{mg}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

i.e. T varies with the square root of the length we now see how important the length of the pendulum is.

Another use of the pendulum, as important as its time keeping propensities, is the following; if we know T we can by the above equation find g i.e. the intensity of gravity in different parts of the earth

$$g = \frac{4\pi^2 l}{T^2}$$

A so called seconds pendulum is one that completes one swing in one second, or its cycle of vibration (T) in two seconds; if the length of this pendulum be found at any pt. on the earth's surface, g can be found by multiplying its length l by the square of π

$$g = \frac{4\pi^2 l}{4} = \pi^2 l.$$

in London it is 39.13908 inches; it get less and less as we approach the equator and consequently g less & less; greatest at the poles. least at the equator.

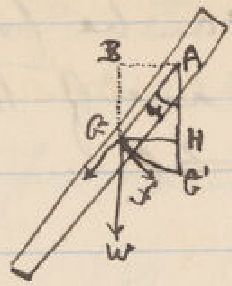
May 27th 1878.

Instead of taking a single point vibrating take a body like a rod; instead of the mass the moment of inertia (i) must be taken; and instead of the restoring force (k), the moment of that force (K); the formula then for the time of vibration (T) of such a pendulum, called the Physical Pendulum is:–

$$T = 2\pi\sqrt{\frac{i}{K}}$$

Let G be the center of gravity of a uniform rod, g

A the point about which it vibrates:



Let G' be the position of the centre of gravity at rest; the weight w will act through the c.g. this can be resolved into a non effective component, & an effective one f.

$$\frac{f}{w} = \frac{GH}{AG} \qquad GH = GG' \text{ if the displacement small.}$$
$$\text{angular distance} = \alpha = \frac{GG'}{AG}$$
$$f = w.\frac{GH}{AG}$$
$$k = \frac{f}{\alpha} = \frac{w.GH.AG}{AG.GG} = w = mg$$

$$K = k.AG. = mg.AG$$

$$\frac{mg.GH}{\alpha} = \text{angular displacement for unit distance}$$
$$= mg.AG$$
$$T = 2\pi\sqrt{\frac{l}{K}} = 2\pi\sqrt{\frac{l}{mg.AG}}.$$

AG being the distance from the point of suspension to the centre of gravity

Another way to deduce the same is the following.

$$w.AB = \text{moment of restoring force.}$$
$$= w.AG \sin\alpha$$
$$\therefore AB = AG.\sin\alpha$$

$$K = \frac{w.AG.\sin\alpha}{\alpha} = w.AG \quad \because \alpha = \sin\alpha \text{ for small angles.}$$
$$= mg\,AG$$

i.e. $T = 2\pi\sqrt{\frac{i}{K}} = 2\pi\sqrt{\frac{i}{mg.AG}}$

Let $AG = a$

R = radius of gyration

$i = mR^2$ when suspended at the centre of grav.

$i' = m(R^2+a^2)$ . . . . a point a distance a from the C.G.

The latter is the more general case.

For the Physical pendulum:-

$$T = 2\pi\sqrt{\frac{m(R^2+a^2)}{mga}} = 2\pi\sqrt{\frac{\frac{R^2}{a}+a}{g}}$$

For the Simple pendulum

$$T' = 2\pi\sqrt{\frac{l}{g}}$$

If $T = T'$, the pendulums are equivalent.

i.e. $\frac{R^2}{a} + a = L$

If supported at its c.g. L would be ~~a~~ ∞; i.e. there is an infinitely long period of vibration; i.e. the pendulum would be stationary in any position.

The moment of Inertia of a line rotating about its c.g. is (see p.77) $\frac{1}{12} m l^2$

$$I = mR^2 = \frac{1}{12} m l^2$$

$$R^2 = \frac{1}{12} l^2$$

Length of equivalent simple pendulum

$$L = \frac{l^2}{12a} + a$$

Let the pendulum be suspended ~~at~~ about one extremity

$$a = \frac{1}{2} l$$

$$L = \frac{l^2}{\frac{12}{2} l} + \frac{1}{2} l$$

$$= \frac{1}{6} l + \frac{1}{2} l = \frac{2}{3} l$$

I.e. the length of a simple pendulum equivalent to a bar or rod vibrating about one extremity is $\frac{2}{3}$ of the latter; this was assumed page 67.

a point like this i.e. $\frac{a}{3}$ down the rod is called the axis of oscillation; the axis of oscillation and the axis of suspension are interchangeable; i.e. whichever ~~be~~ the rod be suspended by it has the same period of vibration. This gives us the most accurate means of ~~to~~ determining the length of the simple pendulum equivalent to any compound pendulum; viz: find two points at which it has the same period of vibration and measure the distance between them: that distance is the length of the equivalent simple pendulum.

May 29th 1878.

The axis of oscillation may be defined as an axis parallel to the axis of suspension, and at a distance from it, measured through the centre of gravity, equal to the length of the equivalent simple pendulum. Therefore in a uniform rod there are 4 pts. at which the time of vibration is the same: viz: the two extremities and their corresponding axes of oscillation $\frac{1}{3}$ of the length of the rod from each.

l is always $> a$; so the axis of oscillation is always on the other side of the centre of gravity to the axis of suspension. In a simple pendulum, the size of the ball makes a difference as the following calculation will show; take a seconds pendulum ($T = 2$ seconds): ~~to~~ approximately: viz: the string be 39 inches, and the diameter of the ball one inches: making 40 inches in all.

$$l = \frac{R^2}{a} + a$$

$$a = 40$$
$$r = 1 \text{ inch}$$

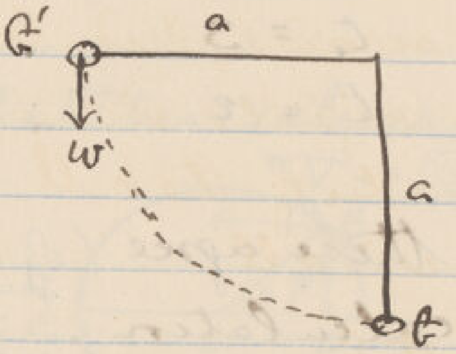
$$= \frac{4r^2}{a} + a$$

$$= \frac{\cdot 4}{40} + 40 \qquad = 40\cdot 01$$

i.e. the dimension of the ball adds 1 part in 4000; which would become important in a time keeper: viz: a loss of about 10 seconds a day. We have seen

$$T = 2\pi\sqrt{\frac{m(R^2+a^2)}{aw}}$$; the numerator of

the radical expresses the moment of inertia of the mass about the axis of suspension; & the denominator the moment of the mass when at the ~~b~~ same level as the axis of suspension.



Take a pendulum like the following to prove experimentally some of the results we have got.

a rod loaded equally at each end, vibrating upon a knife edge about its middle pt; i.e. its

centre of gravity; when this is the case the pendulum is stationary in any position; but load the lower end with extra weights: the time of vibration will then be definite.

Call the weight at each end P, & the small wt. added to the lower end p.

By experiment for different values of p, the following values for t are obtained:

$p_1 = \frac{2P}{15}$ $\qquad t_1 = 4$ (seconds)

$p_2 = \frac{2P}{8}$ $\qquad t_2 = 3$ "

$p_3 = \frac{2P}{3}$ $\qquad t = 2$ "

Let us see if these agree (of course only roughly) with calculation.

$l = \frac{1}{2}$ total length of pendulum. = 5 inches

Pl^2 = moment of inertia of one ball (neglecting wt. of rod itself)

$2Pl^2$ = moment of inertia of two together "

$(2P+p)l^2$ = - - - - - - - - - - - + added wt.

$$\frac{(2P+p)l^2}{pgl} = \frac{(2P+p)l}{pg}$$

$$T = 2\pi\sqrt{\frac{(2P+p)l}{pg}} = 6\cdot3\sqrt{\frac{(2P+p)l}{pg}} \text{ roughly}$$

for 1st case $= 6\cdot3\sqrt{\dfrac{P(2+\frac{2}{15})l}{P\frac{2}{15}g}}$

$g = 384$ inches

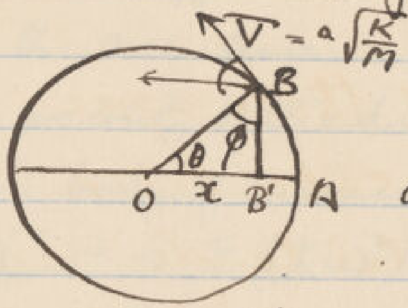
$$= 6\cdot 3\sqrt{\frac{\left(2+\frac{2}{15}\right)10}{\frac{2}{15}\times 384}}$$

$$= 6\cdot 3\sqrt{\frac{160}{384}} = 6\cdot 3\sqrt{\frac{5}{12}} = 6\cdot 3\times\tfrac{2}{3} \text{ nearly}$$

$$= 4\cdot 2 = 4 \text{ nearly}$$

as found by exp.

And so for the other values found.



t = time describing θ; or what is the same thing AB'

OA = a OB' = x

$$\frac{t}{T} = \frac{\theta}{2\pi} \quad \therefore \quad \theta = \frac{2\pi t}{T}$$

v = velocity of B'; i.e. the resolved velocity of B.

$$\cos B'BO = \cos\phi = \sin\theta$$

$$v = a\sqrt{\frac{k}{m}}\cos\phi = a\sqrt{\frac{k}{m}}\sin\theta$$

$$= a\sqrt{\frac{k}{m}}\sin\frac{2\pi t}{T}$$

$$x = OB' = OB\cos\theta$$

$$\text{i.e. } x' = a\cos\frac{2\pi t}{T}$$

These formulae show the variation of velocity in course of time, v varying as the sine, & x as the cosine of the same angle.

i.e. when the sine is a maximum, the cosine is a minimum; i.e. the displacement is greatest when velocity = 0, and = 0 when velocity greatest.

This shows also the periodic movement: when the sine is of more than 2 rt. ∠s it is negative i.e. velocity in opposite direction & so on.

We now go on to see how the principles of moment of Inertia etc. can be applied to actual cases of machinery: a machine consists for most part of a number of cogged wheels, levers etc. & worked by one large wheel called the fly wheel, which is worked by the motive power, hand, steam etc.

The energy of any revolving part is $= \frac{1}{2} I_1 \omega_1^2$
The total energy of the machine is $\Sigma(I\omega^2)$
Let ω' = angular velocity of the driving pt.
Determine a single body of moment of inertia I'
so that $\frac{1}{2} I' \omega'^2 = \frac{1}{2} \Sigma (I\omega^2)$
If the wheels are cogged, their angular velocities will be inversely as the number of teeth (n)

$$\therefore \frac{\omega_2}{\omega_1} = \frac{n_1}{n_2}$$

$$\text{or } \omega_2 = \omega_1 \frac{n_1}{n_2}$$

Similarly ω_3 etc. can all be expressed in terms of ω'.
Let s_1, s_2 etc. be the spaces moved through by points where forces F_1, F_2 etc. act
then $F_1 s_1 + F_2 s_2 + \ldots = \Sigma(Fs)$

Now consider the forces which resist motion &

Let a_1, a_2, etc. be the radii of the different axles;

$2\pi a_1$ = circumference of axle$_1$.

$2\pi a_1 f_1$ = force of friction at circumference of axle$_1$.

$2\pi a_1 f_1 + 2\pi a_2 f_2 + \ldots\ldots$ = total force of friction

But the axles do not make the same number of revolutions in the same time, so n_1, n_2 etc must be factors of the expression;

$$2\pi(n_1 a_1 f_1 + n_2 a_2 f_2 + \ldots\ldots)$$

$$= 2\pi\,\Sigma(nfa)$$

Let $\Sigma(R\sigma)$ = useful work done

Then total work done by driving power = total work done against it

i.e. $\Sigma(Ts) = \frac{1}{2}\Sigma(I\omega^2) + 2\pi\,\Sigma(naf) + \Sigma(R\sigma)$

If the action is steady the term $\frac{1}{2}\Sigma(I\omega^2)$ vanishes

Sometimes the inertia of machines is made purposely great, especially when a continuous action is not wanted as in punching, stamping machines; a small force acts in the machine for a good while; during the time the machine is idle; the heavy flywheel aquires greater & greater velocity, and has an immense store of energy thus put into it, to do work when required; when it does the work e.g. punches a hole, some of this ~~ene~~ energy is given up; & its speed abates; but by the time it has to do work next, it has acquired its former velocity

Friction.

The effect of friction is to diminish the difference of velocity between two contiguous surfaces

When velocity is small velocity decreases as the velocity increases; but friction here will be treated as if it were uniform.

Let P = intensity of pressure of total pressure p over area a

then $P = \frac{p}{a}$

Let μ = coefficient of friction

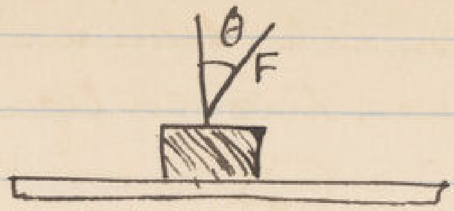
$f = \mu P$ for each unit of area

$= a\mu P$ for $\underline{a}$ units of area.

$= \mu p.$

If two surfaces in contact are perfectly smooth, the slightest possible force in any direction but the vertical will produce sliding; but with rough surfaces, the force may be inclined to the vertical ~~between~~ within a certain angle θ

without causing motion;



Leave the wt. of the ~~body~~ out of account, & only take into account the external pressure F.
the vertical component of F: $p = F\cos\theta$
— component $\parallel$ to the surface: $q = F\sin\theta = f$.
~~Wh~~ We had on the last page

$$f = \mu p$$
$$= \mu F\cos\theta$$
$$F\sin\theta = \mu . F\cos\theta.$$
$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

Determine the greatest value θ can have without causing motion; θ is then called the limiting angle of friction;
between oak & wrought iron ~~this~~ $\theta = 31°50'$
& $\mu = \tan\theta = .62$.
Another way to determine θ, is to tilt the lower surface and note when sliding begins; θ is the angle at which sliding begins

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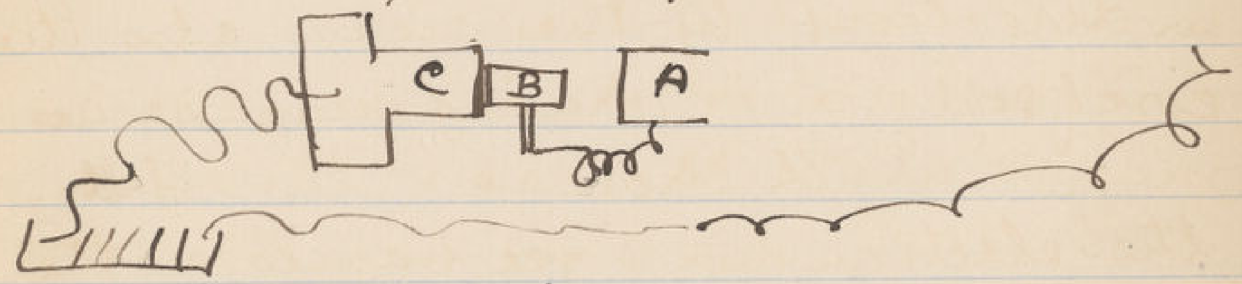
caused by (the sum of) all these), a spark can be got longer by this than by any other method; in the most powerful induction coils: i.e. with the greatest length of secondary wire the spark is 2 feet long.

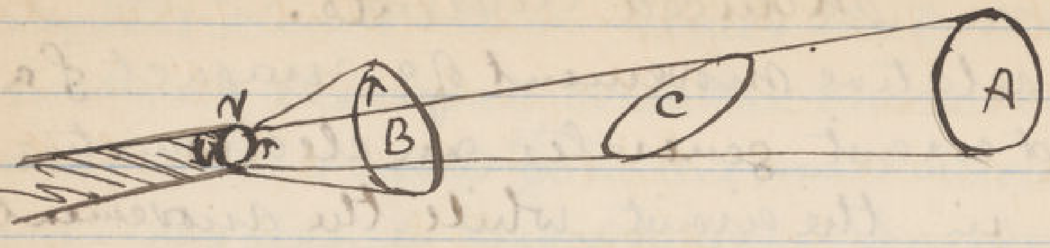
The contrivance by which the current is alternately made & broken is this:-



The electromagnet A as soon as it is made attracts to itself a small movable piece of iron B (the hammer) causing it to leave another fixed piece C (the anvil), & thus breaking the circuit; the magnet being thus unmade, B is released falling against C thus closing the circuit again. The hammer is thus kept in a state of ~~const~~ rapid vibration; the discharges ~~of~~ alternate in direction keeping time with its movements

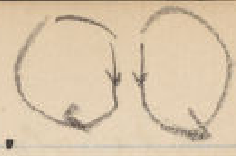
This is the principle of the <u>Induction Coil</u>; a thick coil of wire is wound round a core of soft iron; round this is a great length of thin wire; the current in the primary wire is alternately stopped and set going; this produces an ~~alternate~~ current in the secondary wire, alternating in direction; by this means also the core of soft iron is made & unmade a magnet which increases the effect; the electromotive force varies as the rate of change of strength in the primary current; so if this is fast enough a spark can be got between the ends of the secondary current; this can be done by a contrivance which will be described ~~in~~ a few lines hence, to such a degree that, (as the induced current takes place in every turn of the secondary wire, the spark between the ends being ~~equal to~~

effect is just the opposite to that when it is brought nearer to the pole; viz: diminishes the current opposite to that by which the pole may be represented, and causing one in the same direction; thus tending to ~~cause~~ attraction between the pole & circuit; i.e. in each case the force between the pole & circuit is opposite to that by which they are moved in relation to one another. Such currents are called <u>Induced Currents</u>; we have seen that in all cases a coiled current can be replaced by a magnet; and so it can here; so if one current be put inside another, we get a current in the second while the current in the first is being increased or diminished: putting in a magnet & taking it out again produces the same effect; & so if both are done together the effect is doubled: making & unmaking an electromagnet is the same as putting in & taking out a magnet alternately.



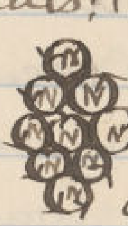
e.g. a change from A to B would set up a current in the circuit; but the circuit might be moved obliquely say to C, so that no alteration of angle takes place; then no current takes place; an increase of the angle, increases in the circuit a current opposite to that by which the pole can be represented (see arrows in figure); the electromotive force is proportion-al to rate of change of ~~strength~~ subtended angle; it also depends on the strength of the pole; this increase in the current opposite to that by which the pole can be represented tends to cause repulsion between the pole & the circuit. As the circuit is removed farther off again, the

Induced Currents.



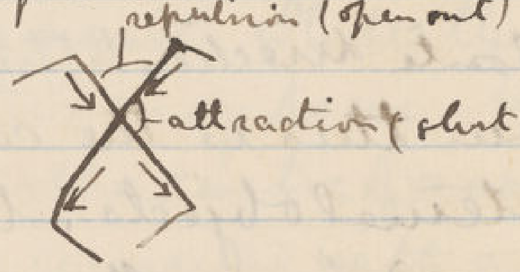
Any relative movement of a magnet & a closed circuit generates an electromotive force in the circuit while the movement continues; this may be seen thus; put a magnet into the centre of a coil, the ends of which are connected with a galvanometer. the effect will be seen on the galvanometer while the magnet is being put in, and the opposite effect when it is being taken out; putting in opposite poles also gives opposite effects. The current lasts only while change of relative position is taking place; or more properly the fact may be stated thus; imagine a single pole of a mag-net; it subtends at the conducting circuit a certain angle; ~~we~~ imagine the magnet ~~&~~ or circuit displaced so as to change this angle; as long as any alteration of this angle is taking place a current is set up in the circuit; (see figure next page)

magnets should all be turned that the direction of the closed circular current round each should flow in the same direction: viz in the opposite direction to the hands of a watch: when this is the case an effect is got on external objects: the same effect can be conferred by another magnet: i.e. a bar of iron in which all the closed molecular circular currents flow the same way. A north pole we may represent thus: (a)


a

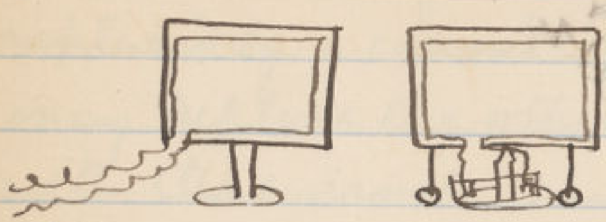

b

at the parts where the molecules touch a north current in one neutralises a south current in the other; so practically there is simply a north current round the periphery. (b); i.e. in a magnetic pole we have just the same as in a coil of wire through which a current passes: viz: a current flowing in a definite direction another; the angle where there is attraction tends to shut up, & the angle where there is repulsion to open out.

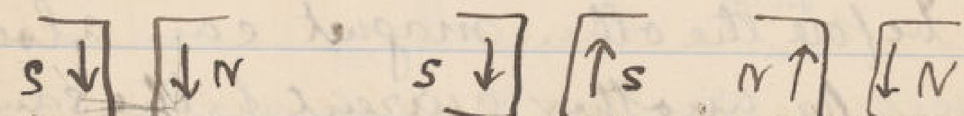


And two coils like that on page 186, produce the same effects as described with one coil & a magnet.

The explanation of all this is the following:- In a magnet, we have seen that it is such owing to ~~its~~ the arrangement of its molecular magnets one way; further, we may conceive that each molecular (magnet) owes its magnetisation to a circular closed current round it; and thus by another circular closed current these can all have a definite arrangement conferred on them; viz a north pole is got by acting (as we saw two coils acted on one another) so that when the molecular



When the poles are opposite they will attract and when ~~th~~ similar they will repel; the poles of opposite, i.e. there will be attraction when one current is going in the direction of the hands of a watch (south pole) & the other in the opposite direction (north pole) i.e. when the directions of the parts opposite each other are the same; repulsion when different:

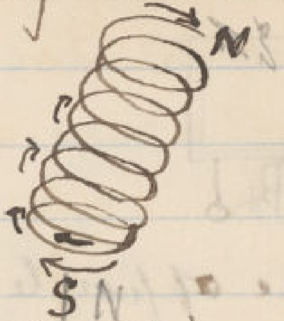


attraction repulsion repulsion

by inverting the direction of the current just as the revolving frame comes round rotation can be got.

When the currents are inclined to one

current circulating according to the direction



of the arrows in the figure; looking towards one ends frontways, the current is seen to go in the direction of the hands of a watch; this is the south pole; while looking to the front of the other end, ~~they~~ the current is seen to go to the left; i.e. in the opposite direction to the hands of a watch: this is the north pole.

As before said the other magnet can also be replaced by another current & the same effect is got between one current & another as between a magnet & a current, & that again as between a magnet & a magnet. The most convenient way of showing this is to take two rectangular circuits, one fixed & one able to rotate. as in the next figure

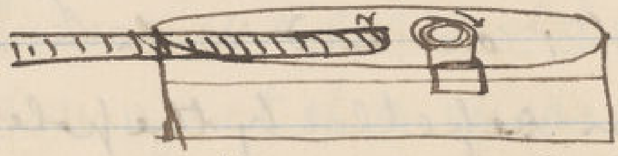
Thus we see that a magnet can be replaced by a current; and so another current may replace the other magnet hitherto used with the current; and the action between current & currents is found to be exactly the same as between magnets & currents.

June 6th 1878.

That a coil of wire through which a current is passing can ~~be ~~ act as a magnet, can be seen thus: take a coil of wire, twisted as in the figure & suspend it;

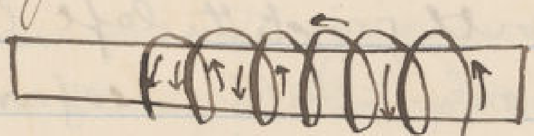
connecting the end with a battery so as to get a current; one end with be attracted and the other repelled by the pole of a magnet; take a ~~~~ coil such as was just used, & examine it to find the direction which represents N & S respectively: suppose the in another way; take a coil of wire; ~~which~~

the current in which has the direction of the arrows in the figure; place just within it a bar magnet; the north pole upwards; this goes to the left of the current; i.e. is sucked up into the coil. The same may be seen by having the magnet fixed & the coil movable; take a floating cell; the poles in which are connected by a coil of wire passing at the top of the cell; put it floating on water; place a pole of a magnet over one side of the basin; say the N pole.



the coil of wire will run on to the magnet obeying the law of direction viz N pole to left; if the south pole is put over the edge of the basin; the coil will turn round and then fit itself on to the bar.

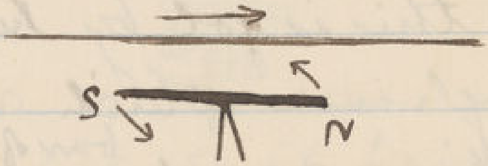
were carried all ~~all~~ over the ~~magnet~~ bar: the all molecular magnets of the ~~m~~ bar would be affected the same way and the bar so turned into a ~~x~~ magnet: this is got by twisting a coil of wire through ~~wire~~ which a current is passing many times round a ~~magnet~~ bar of soft iron; this



has a very strong effect; when each particle is affected in the same way; total magnetisation is obtained; directly the current ceases, the bar of iron ceases to be a magnet; if this is done alternately very quickly, the bar of iron being alternately a magnet & not one, will attract & leave go of a piece of iron alternately; this can be so arranged as to produce rotation in the piece of attracted iron, and in fact exceedingly fast motion can be got.

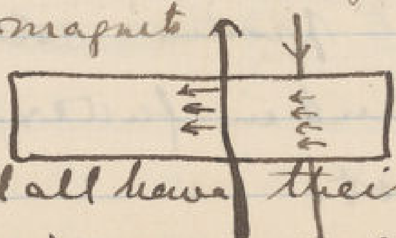
The same effect viz: that a magnetic pole tends to got to the left can be seen thus,

to obey the law before stated: the effect of a mag-
-net on a compass needle is simply a special
case of this; the same law is obeyed by both



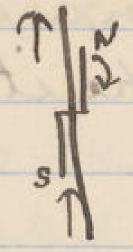
poles, the north going to the left, & south to the right;
but this can only take place so far that the direction
of the needle is at right angles to that of the
current; for if more than that took place the
law would be & disobeyed.

We have already seen that the process of
magnetisation is simply a process of arranging
similar poles in the same direction; suppose a
bar of iron, and a current going ~~past~~ say upwards it;
the molecular magnets
at the part the



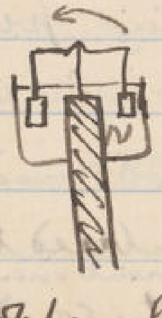
current passes will all have their north poles
turned to the left hand: a current going down the
other side would have the same effect: and if
currents like this up one side & down the other

This can be shown thus; take a conductor



going upwards as in the figure; bend a magnet as in the figure & suspend it so that the current passes through the middle; set the current flowing and it revolves as in the figure.

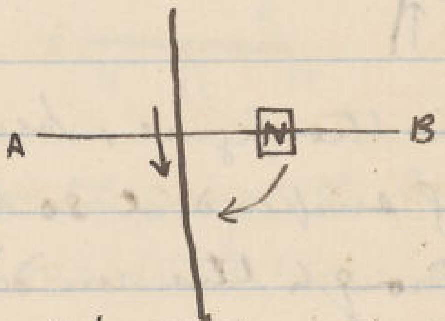
Or if the pole is fixed the current revolves around it; this can be shown thus; take the pole of a magnet; with a vessel to hold Hydric Sulphate at the end; dip into



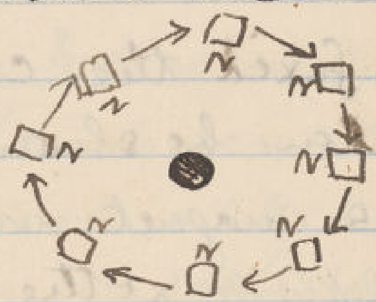
this a plate of ~~co~~ copper, & one of zinc; connect them by a wire, which is suspended on the magnetic pole; this then revolves round it in the direction

Electro-magnetism.

The effect of a current on a magnetic pole free to move is to make it revolve round the current



or in cross section through the line AB, the north pole would have the following positions;



the force always being proportional to the plane containing the current and the pole, & so it moves in a circle round the current. The direction, can be remembered thus: imagine a human figure inside the conductor, current coming in at its feet & going out of head, with its face to the magnet, the N pole is always urged to the ~~right~~ left hand

and can be detected by forming ammonium amalgam from it. the equation to represents the action is:-

$2HKO + nH_2O = K_2 + H_2O + O + nH_2O$.

Or with Sulphate of copper; $CuSO_4$;
Copper (Cu) is deposited at the negative pole; & (SO_4) at the 1st instant at the other: this however acts with the water in contact with it;
thus $SO_4\ H_2 | O$ & oxygen comes off.
With Sodic Sulphate $NaSO^4$;
the primary action is

$$\overset{-}{Na} \qquad \overset{+}{SO_4}$$

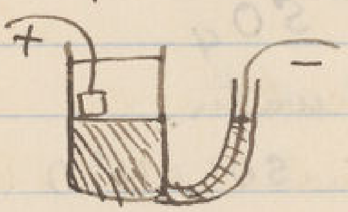
each of which reacts with water

$2H_2O + Na^2$ $\qquad$ $SO^4 + H^2O$
$= 2HNaO + H_2$ $\qquad$ $= SO^4H_2 + O$

the visible effect is that of gas coming off.
H_2 from alkaline solution $\qquad$ O from acid solution.

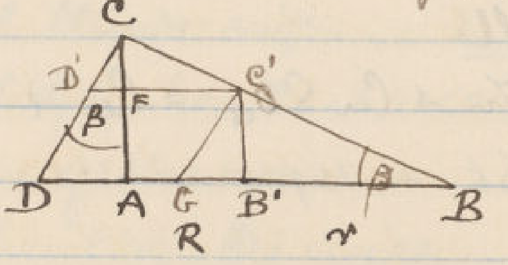
or just the same as with water simply.

When a conductor is a compound liquid it undergoes decomposition; the commonest way to get this is by having platinum terminals in the water or liquid to be decomposed: the constituent elements come off at these terminals and if gases can be collected by placing tubes as in a pneumatic trough over them. The evolu-tion of gas is proportional to the strength of the current measured by a magnetic needle.
Another method is employed to ~~detect~~ decompose potash and was that by which Davy discovered the element potassium; a vessel with a tube



~~to~~ be a dip from the bottom on one side is $\frac{1}{2}$ filled with mercury: above the mercury in the larger vessel is put solution of potash; into this the ~~negative~~ positive terminal of platinum is put; while into the mercury in the tube the negative (by comparison) end is placed; the potassium comes off at that end

The resistance may be divided into two parts the resistance of the battery itself (R), and that of the conducting wires (r). The relation of these may be shown by the same sort of figure as before.



Divide the total resistance represented by AB, into two parts, one AB' proportional to R, & the other $B'B$ proportional to r; at B' erect a $\perp^r$ to AB cutting BC in C'; through C' draw $C'D'$ parallel to BD cutting CA & CD in F & D' respectively; through C' also draw a // to CD cutting AB in G; then

$D'F$ represents the heat in the battery

& $B'G$ ... ... external conductor

& $D'F + GB' = AD$

therefore smaller proportion AB' is made of AB i.e. the smaller proportion the resistance in the battery is made of the total resistance, the greater $B'G$ or the resistance in the external conductor become.

Hydric Sulphate:

action - $Zn + H_2SO_4 = H_2 + Zn\,SO_4$; i.e. one atom
$65 + 98 = 2 + 161$
of Zinc is consumed for every two atoms of Hydrogen liberated.

(2) Daniell's

$$Zn + Cu\,SO_4 = Cu + Zn\,SO_4$$
$$65 + 159.5 = 63.5 + 161$$

(3) Grove's

$$Zn + H^2SO_4 + HNO_3^4 = H^2O + Zn\,SO^4 + HNO^2$$
$$65 + 98 + 63 = 18 + 161 + 47$$

The quantity of Zinc consumed in all is the same, but the chemical action & therefore the heat evolved is different: the total quantity of heat evolved during the consumption of 65 grains of zinc in each case is;

in Smee's battery	40,100	grm. degrees of heat
— Daniell's —	52,350	
— Grove's —	90,160	

This quantity is spread equally throughout the circuit; & may be made to appear in certain parts: e.g. in a Pt. wire.

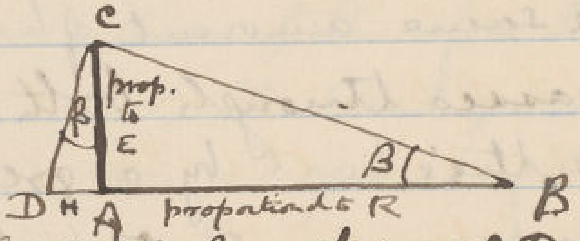
The heat produced is proportional to the resistance of the conductor: but the actual change of temperature which takes place in a conductor as a current passes through it depends on several things: viz:-

the quantity of heat it receives:
the mass of the wire.
Specific heat of the wire
Amount of surface presented for cooling.

e.g. through a thick copper wire & ~~Pt~~ a thin Platinum wire of the same length both receive the same amount of heat when one current passes through both; the Platinum wire shows this heat by a greater rise in temperature becoming white hot. The heat which is evolved however is spread through the whole circuit, battery included.
Now take three kinds of battery and compare the heats evolved:
(1) Smee's: Zinc & Platinum plates in dilute

the current is doubled, the heat produced is quadrupled and so on, in a unit of time; the quantity C^2R may be expressed otherwise, viz: $C^2R = C.CR = C.E. =$ strength of the current into the electromotive force. This applies to the whole circuit.

The above facts may be expressed diagramatically thus; take a line AB proportional to the resistance; & at right angles to it at the point A, draw AC propor-tional to the Electromotive force.



call the angle CBA, β; draw CD making with ~~CD~~ CA an angle $= \beta$; produce BA to meet CD in D: AD represents the heat produced per second $= H$ & $\frac{AC}{AB} = \tan\beta =$ strength of current. $\therefore C = \frac{E}{R}$

$H = E^2 C.E = C^2R = \frac{E^2}{C}$; i.e. E is a mean prop. between H & C; which AC is between AD & AB.

$AD = AB \tan^2\beta$

$$K = \frac{1}{R} \text{ or } R = \frac{1}{K}$$

Resistance is the better term of the two, because the word conduction is misleading; it gives one the idea that some inherent property of a body aids the propagation of a current of electricity through it, whereas it resists the passage of electricity; some bodies resist more than others, i.e. conduct less.

From the equations $\frac{E}{C_1} = R_1$ & $\frac{E}{C_2} = R_2$ etc. we get

$$C_1 = \frac{E}{R_1}$$

$$C_2 = \frac{E}{R_2}$$

The quantity of heat produced by an electric current depends on the strength of the current and the resistance of the conductor.

The heat produced in a unit of time is proportional to the resistance of the conductor through which the current passes, and to the square of the strength of the current; i.e. to $C^2 . R$; i.e. if the strength of

June 4th 1878.

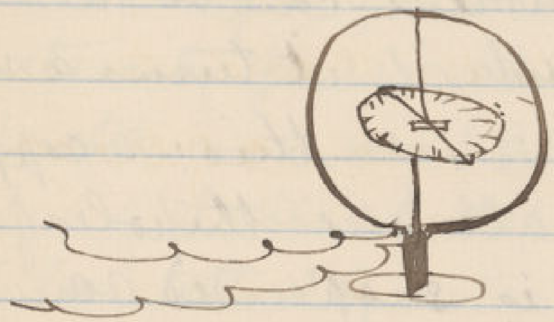
What has been hitherto called electrical difference between the ends of a conductor is what is known as Electromotive force; the strength of a current depends on this and on the nature of the conductor; let C_1, C_2 etc represent the strengths of currents in different conductors; & let E represent the Electromotive force at work; then

$\frac{E}{C_1}$ is constant for the 1st conductor, and is called the resistance of the conductor, $= R_1$

Similarly $\frac{E}{C_2} = R_2$

$\therefore \frac{C_2}{C_1} = \frac{R_1}{R_2}$

i.e. resistances are to one another inversely as the strength of the current for the same electromotive force. Resistance is only another way of expressing what hitherto has been called conduction; non-conducting bodies are those with an infinitely great resistance; if K = conducting power, & R the resistance of a conductor interposing different lengths of wire through which the current is made to pass can also be seen; and it is found that the greater the length of wire, the less the deflection of the needle, i.e. the less the strength of the current; the simplest form of the instrument is in the figure: other forms are also used.



Another effect difficult to obtain by an electrical machine is easily got & is got continuously by a galvanic current; e.g. a piece of thin platinum wire between the poles of a battery can easily be rendered white hot or even fuse it, by letting the current flow.

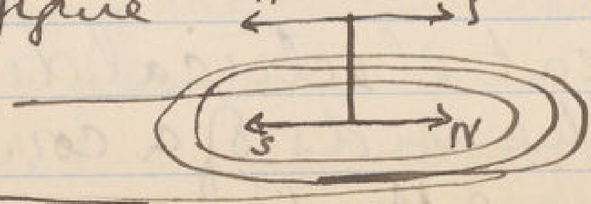
proportional inversely to its length, and directly to the sectional area of the conductor.

The Electrical difference required to produce a current of unit strength in a conductor of unit section & unit length = specific resistance of the material of the conductor.

The strength of a current can be measured by the force ~~at~~ with which it turns a needle: the instrument is like this: a copper circle to which are attached the poles of the battery in question is suspended round a short needle with an aluminium pointer attached to it, in the centre of the circle: the distance from the needle to the current is the radius of the circle; the current is allowed to pass, this causes the needle to be deflected: and its deflection is measured on a scale (see figure next page): the effect of we have been describing, viz Galvanic Currents, after Galvani one of the earliest investigators of its phenomena. There ~~are~~ is a great variety of Galvanometers the greater number of times the wire passes round the needle, and the lighter the needle, the more sensitive the instrument.

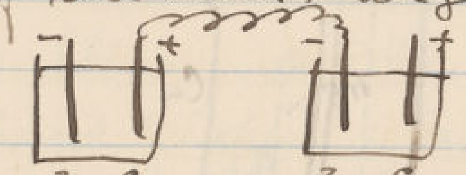
The Strength of a Current is the quantity of electricity passing each section of the conductor in a unit of time. The rate of the flow of electricity depends on conditions similar to that of water flowing through a pipe; the greater the difference of pressure at the two ends the greater the flow: it depends also on the nature of the pipe, and also of the conductor.

For a great electrical difference between the ends of a conductor; the strength of the Current is arranged the same way, and we get a distinct effect on the electroscope; an effect on an electroscope ~~we~~ is much more easily produced by the effect of an electrical machine, and generally speaking effects produced easily by the electrical machine are produced with difficulty by currents of electricity; and hardly by the electrical machine with ease by the other method. E.g. the effect on a magnetic needle by allowing a current to flow ~~&~~ through coils of wire round it is as we saw difficult to cause by an electrical machine, while by means of a battery it is produced with the greatest ease. The effect is increased by having two magnetic needles, fixed N to S poles of each other, and one (the lower) a little stronger: as in the figure



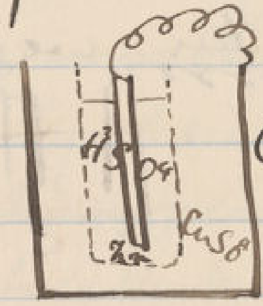
Instruments like this are called galvanometers since they measure ~~of~~ the effect of what

If we replace the Platinum by Carbon, we get what is known as Bunsen's Battery.
If we take two cells (taking the simple Copper & Zinc battery) and connect the Zinc of one with the Copper of the other; we get double the

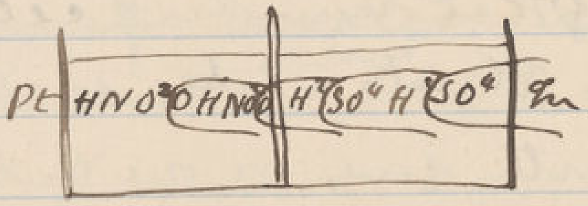


electrical difference between the end plates than with one; if we put three in the same order, three times as great.
Electrical difference it should be remarked does not necessarily mean one +, & the other −; but one less + or less − than the other.
A very great number of cells, about 3000 can be got into a tube: each cell consists of paper with zinc on one side, Manganic dioxide on the other: the paper being damp acts the part of Hydric sulphate in the cells just described: these must all be

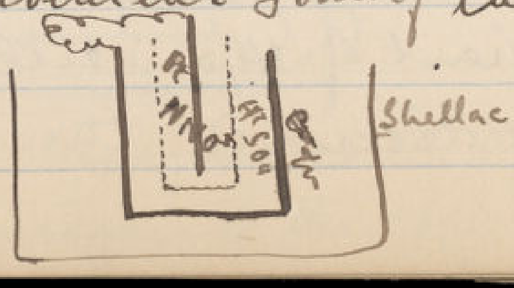
but does not impair its chemical action & hence not its electrical effect. This battery is called after its inventor Daniell's battery: the usual form of this is in section:



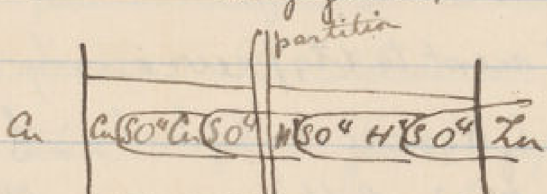
There are many other different arrangements: Grove's battery consists of two liquids Hydric Nitrate & Hydric Sulphate: the poles are of Zinc and some metal which cannot be attacked by Hydric Nitrate; Platinum is the best. & the chemical action is:



and the most convenient form of the battery;

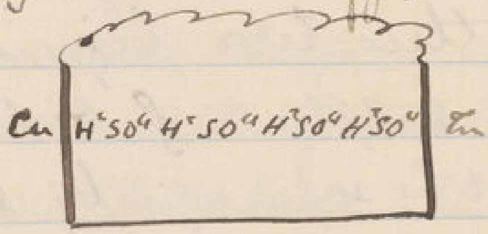


either to a mechanical or chemical ~~or~~ combination, or both, of the Copper & Hydrogen, all the latter does not come off as it ought to thus impairing the strength of the current. There are many ways devised of remedying this; one is to use a silver plate instead of Copper; but a better way still is to counteract it by chemical ~~the~~ action; in this two liquids must be used; copper sulphate & Hydric Sulphate; the two must be separated by something to prevent their mixing, and yet not to prevent molecular combination; unglazed earthen-ware ~~or~~ felt is the best to use; the chemical action is denoted in the figure:

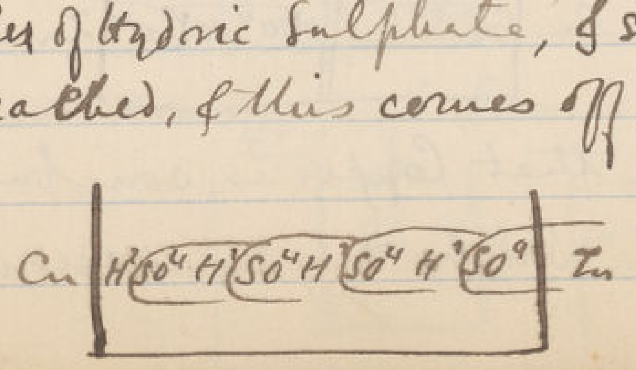


we thus see that Copper is combined with the Copper plate, which only increases its thickness

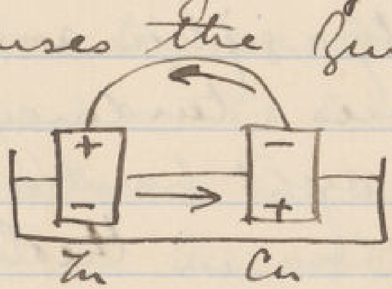
In actual practice different materials & different liquids are used but the principle of all is the same. A common form of cell is with Hydric Sulphate (dilute) instead of water in the cell just described; the current causes the decomposition of the liquid for every two atoms of Hydrogen which comes off from the Copper plate, one atom of Zinc is converted into Zinc Sulphate & dissolved off.



The above represents the arrangement of molecules in the liquid before the action begins: the decomposition takes place thus: the Zn unites with the SO^4 next to it leaving the next H^2 to combine with the SO^4 next to it, previously belonging to the 2nd molecule of Hydric Sulphate, & so on till the final H^2 is reached, & this comes off; but owing



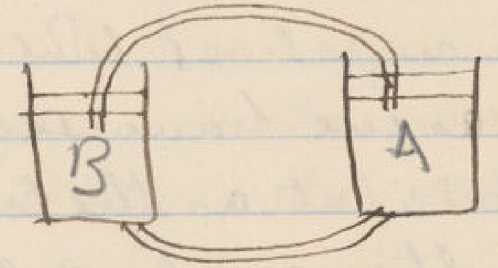
the circuit; so it is with electricity, take a Copper & a Zinc Plate, and connect them by a copper wire; this may be regarded simply as a prolongation of the copper plate; the two are thus in metallic contact; connect them also below by a liquid say water; the first causes the Zinc to be positive



and the Copper negative; While the liquid causes as we have seen exactly the opposite effect; but as the two cannot take place simultaneously, a condition of dynamical equilibrium or a continuous flow is obtained. We saw on page 158 that with the most powerful electrical machine the same can be got, but in a very small degree; so this self acting method is much better.

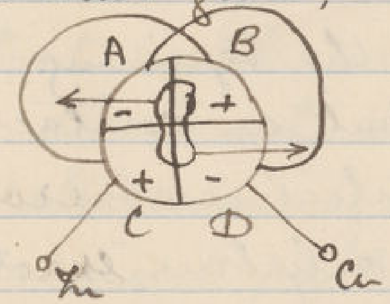
opposite effect to when placed in metallic contact. If the Copper be replaced by Platinum the same effect is increased.

In order to explain the important effect which follows from this a supposed analogy must be invented: suppose two vessels of water connected by a siphon, and imagine that this causes a tendency for the liquid in one, say the right hand one A to be in at a higher level than in the other; but suppose

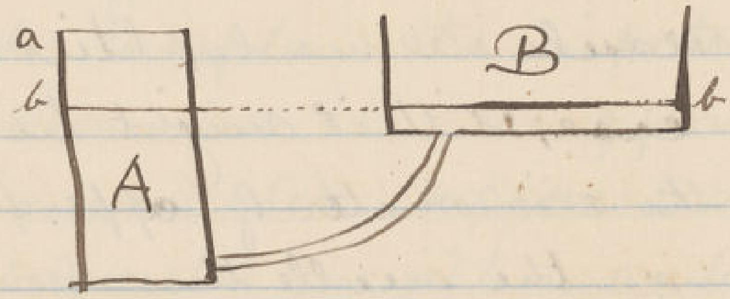


that when connected by a tube beneath, the effect is just the reverse i.e. the level tends to be higher in the left hand vessel: now suppose the two connections to be applied simultaneously; there will be no condition of statical equilibrium, but a continuous flow throughout

It is as if in the analogy of water pressure quoted on the last page, the equality of level depended on the materials of which the vessels are made; this however is not the case; still it might be imagined. The effect on the electrometer of Copper & Zinc is the following: the needle turns from ~~left~~ right to left: i.e. the quadrants A and D to which the Copper is attached are negative, while the plates to which the ~~cop~~ zinc is attached are positive: i.e. positive electricity flows from the Copper to the Zinc; but suppose



now instead of putting the plates in metallic contact, we connect them by a conducting liquid like water; the zinc then becomes the more negative; i.e. just the in a vessel A up to the level a, and it is allowed to communicate by a pipe to an empty vessel B; water will run from A to



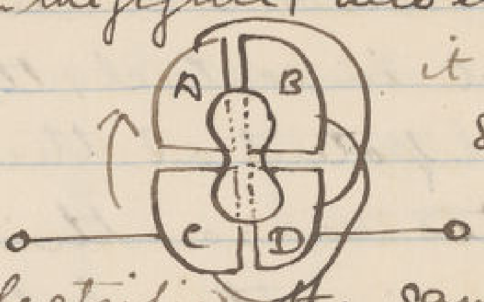
B till both have the same level bb; but here the quantity of water, and the pressure on the bottom in the two vessels is different, and the level and to this corresponds electrical potential is the same.

It was said at the begining that when two conductors are put in contact the same electrical potential is observed throughout; this however is only true exactly when the conductors are of the same material; for different materials, e.g. different metals like Copper & Zinc there is a slight electrical difference; which however can only be detected by the quadrant electrometer as described on page 159.

May 28th 1878.

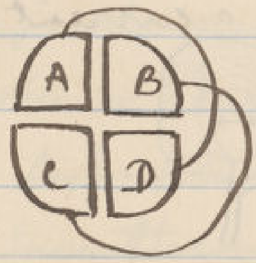
When conductors, one or both charged with electricity are put in contact, there is the same electrical potential through the virtually united mass; this is not the same as saying they both have equal quantities of electricity: for a large body will have more than a small one; if the conductors are spheres, in the same ratio as their radii: nor is it the same thing as saying that the electri-city is spread throughout the surface equally: for that we have seen not to be the ~~c~~ case: electricity accumulating most at the extremities of a conductor, or on the more prominent part; but it means that a sort of electrical pressure or level between the two, is equal in all parts: we may compare it to the following case of fluid pressure; suppose we have water

Over these is hung a plate of aluminium of shape as in the figure; and electrified +, by attaching it to inside coating of small leyden jar.

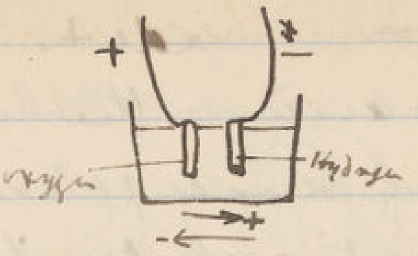


all being electrified the same way; this does not tend to move; but if one pair say A & D are electrified ever so little say -ly: the plate of positively electrified platinum will be made to turn in the direction of the arrow: this can be done by touching the knob fastened to D. Attach a ~~pl~~ mirror to the plate of aluminium, this acts as before described. Now attach to D a plate of zinc; to C a plate of copper; there will be no effect: when together ~~make~~ separate the two plates ~~touch~~ the spot of light is immediately deflected; showing that the Copper & Zinc are in a different Electrical state before & after contact.

If one end of a wire ~~becom~~ is made +, the whole is at once + also; if one end −; the whole −. But if one end is continuously being supplied with positive and the other with negative electricity; there can be no statical, but a dynamical condition of electrical equilibrium. By an electrical machine however powerful we ~~can~~ can get very small electrical difference between the two ends of a wire; but there are other methods of increasing this: to exhibit the principle a very delicate instrument called the quadrant electrometer must be used. It consists of four conductors A, B, C, D arranged in a circle; each being nearly a quadrant



A & D; B & C are connected so that each member of each pair is electrified similarly & to the same amount.



Hydrogen coming off at the negative, & oxygen at the positive sides, in the same proportion as they occur combined in water (viz.; 2:1). by such an arrangement however the bubble which come off can only be seen by throwing a magnified image of the apparatus onto a screen.

An electric current has also effects on a magnetic needle, tending to put it at right angles to the conductor; to get any visible effect however the following plan must be adopted: place a very fine needle within a coil of wire through which a current is passing: attach

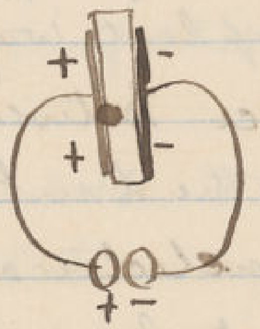
to it a small mirror on which a light is thrown, & receive the reflected ray on a screen; this is seen to move when the current passes.

Heat also takes place in the conductors, but a single flash is of such short duration that no instrument is sufficiently delicate enough to detect it; but if we keep up a succession of sparks, with so short an interval between each that it appears continuous, we get a permanent light called the electric light.

There are a number of different processes to do this; the simplest way of keeping up the electrical difference between the two wires: is to attach one to the + rubber the other to the conductor of an electrical machine, powerful enough to leave no interval between each spark: in rarefied air the spark is far longer: and in different gases of different colours. Instead however of letting the discharge take place through the air; let it take place through a compound liquid, say water; and have platinum plates instead of knobs; some of the water is decomposed principle: one charge having been given to the machine; by the circular motion of a glass plate is made to act inductively over & over again indefinitely

May 23rd 1878

Suppose we were to connect the two plates like we have in a Leyden jar very nearly say by wire ending in knobs put close together:



the electricity would pass from one to the other ~~accompanied by a~~ even through a non-conducting material; this passage would be accompanied by a flash: this is from the air being incandescent, and metallic particles at a great heat flying from one to the other; the heat makes the air expand suddenly: it as suddenly contracts & hence the sound of an explosion

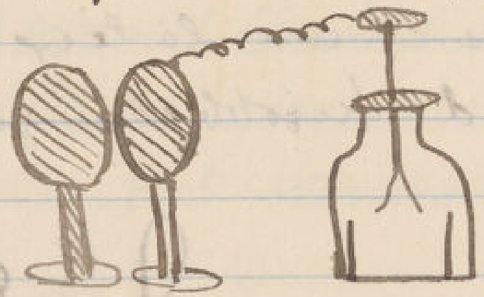
By means of the electrophorus, we can get an unlimited supply of electricity from one primary charge: it consists of a sheet of vulcanised india rubber: on the top of which is a metallic plate with two handles, one of glass, i.e. insulating, the other of metal and knobbed to get sparks from:

if the vulcanite be rubbed say by the hand it is permanently electrified negatively: put the other plate on the top of it; this is made by induction to give $\mp$ electricity to other bodies, so itself becoming ~~negative~~ positive: touch it: it gives $\mp$ electricity to the earth: lift it up by the glass handle: it is + electrified and will give a spark: this can be repeated any number of times.

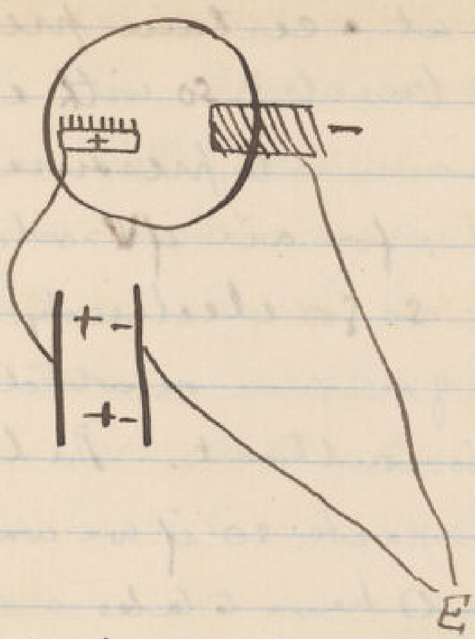
Holtz's machine is the same as this in

or held in the hand is connected by the earth to the amber.

The Same principle is applied in the electrical condenser; & used to detect small quantities of electricity; the top of an electroscope is connected



with an insulated metallic disc, by the side of which stands one not insulated; if the top of the electroscope be touched by a body feebly electrified there will be no effect on the gold leaves; but when the uninsulated disc is removed, the leaves diverge; this is because the feebly electrified body say a leyden jar with apparently no electricity inside, is connected ~~with~~ by the earth to the uninsulated metallic disc; electricity of one kind say + passes into the electroscope & fixed plate, and ~~neg~~ electricity of the opposite kind i.e. negative into the movable plate; so when the movable plate i.e. the − electricity is removed we get the full effect of the positive electricity



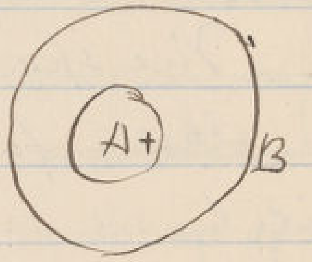
these two bodies have the same effect as the bodies A & B just described: the closer they are the better; then by connecting them we get a loud and large spark; the capacity for small distances being inversely as the distance between them.
The commonest application of this is the Leyden Jar the two bodies being a two sheets of tin foil, one pasted inside the other outside a bottle; the inner one is connected by a wire from the conductor of a machine to a knob above the lid from which passes a chain to the bottom of the jar: the outer ~~jar~~ lining, if the rubber is not insulated and if the jar be standing on the table

weak place; at a certain pressure of the air within it, it bursts; so with electricity: the factor corresponding to pressure in air is called the Potential; for air if V = volume & P = ~~volume~~ pressure PV is constant: so for electricity: if P = potential and C = amount of surface on which the electricity is collected CP is constant. The larger the surface the louder the spark; so if we want to get a large spark, we should have to take a body of inconveniently large dimensions: instead of this, though, the following plan is adopted. Suppose a positive body A is near another positive body B; the presence of A increases the power of B to give positive electricity to other bodies; if A is negative: the power of B to give positive electricity to other bodies is diminished; so more positive electricity can be put into ~~A~~ B; and for the same reason; more negative electricity can be put into A. This is applied to getting a ~~[illegible]~~ large quantity of electricity into a small space; one body ~~is~~ receives + by connecting it with the conductor of an electrical machine; the other, by the earth or otherwise to the rubber both outwards and inwards collect it and it passes on to the brass framework ~~of~~ or prime conductor of the machine, which is insulated and should have all its points rounded, and from which by touching it sparks can be got. The sparks are the louder the larger the surface which collects the electricity i.e. with a conductor of greater capacity.

May 21. 1878.

Take a Leyden jar, which is an apparatus for collecting electricity and will be explained soon, and count the number of turns of the machine necessary to fill it to overflowing as it were, & so make it give a spark between two knobs of metals, with one of which it is connected; say it takes 6 turns; this measures in a rough way what is known as the Potential; we may compare the putting of electricity into a receiver to pumping air into a vessel in which is a

If we have a conductor completely surrounded ~~by~~ in any way, by ~~the~~ conducting materials, we get an induced charge on all the surrounding enclosure: if not completely enclosed some escapes into the air etc.



Suppose A is completely enclosed by B. Suppose A to be +; connect B to the earth B becomes ~~negative~~ neutral and A has no further effect on it while inside of it; take away the charge of A, B becomes negative again and the negative charge of B is equal to the positive charge A first had. This we can prove by connecting them together and the whole is neutral once more.

The Electrical machine is the same in principle as rubbing a glass rod by ~~glass~~ silk: only the silk rubbers are fixed and glass cylinder or discs rubbed against them by turning a handle, & making it rotate: points which facilitate the escape of electricity by the same sort of action, it accumulates in any point; and the sharper the point the greater the accumulation of electricity there; and when accumulated in large quantities it can escape; so to prevent leakage in an electrical machine, all parts of the prime conductor have to be rounded, except the points which collect the electricity.

We find by experiments with hollow conductors, and the effect of their inside on electroscope, that no electricity ever gets inside; we may infer from this that in solid conductors the same is the case.

May 16th 1878

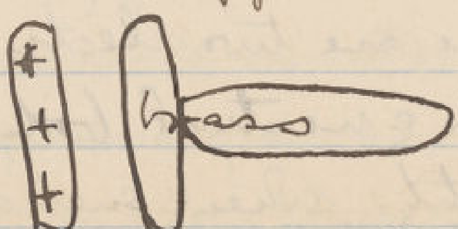
The force between any two portions of electricity is proportional to the inverse square of the distance between them; this can be found experimentally by means of a torsion balance, or proved from the fact that all the ~~e~~ electricity on a body is on the exterior, and that it is equally distributed over a sphere.

there is more of it: & vice versa; also the sum of electricity ~~of~~ in a body is constant so if + passes out, − passes in, in equal amount. (2) One fluid only: in neutral bodies there is a normal amount; in −, less than the normal, in + more.

May 14th 1878.

When the two bodies are brought nearer together, the inductive action increases; in the limit, when they touch we cannot suppose the inductive action to cease; i.e. the near parts of the second body are made to give their electrification to the farther parts; so that in a longitudinally length-ened conductor we get the electrical action (which we shall call electricity for short; just as we so use the term quantity of heat) exhibited at the ends chiefly; if we take a spherical conductor i.e. one with no ends we get the same amount of electricity in all parts of the sur-face; as we get in a long conductor by inductive action, the electricity manifested at the extremities it is found to be negatively electrical. How is this? the first cylinder while the glass is by it is for the time + electrical: this it shared by the other cylinder: i.e. the first cylinder gives up some + electricity to the second: when the glass rod is removed: the total sum of electricity in the two cylinders is the same as before: but the 1st has given up some + electricity to the second: i.e. is itself less + (when the 2 are separated): i.e. more negative: if instead of letting a second cylinder share the original electricity of the first, we let the earth share it: say by touching it when the glass rod is by it: we get it strongly −, when the finger & the rod are removed. There are two ways of accounting for this: (1) There are two electric fluids, opposite in character; in a neutral body they are present in equal quantity: when such a body loses − ~~nega~~ electricity, the + in it gets the supremacy ∴

electrified make any body in its neighbourhood electrical also; it is called Induction. If we electrify a body negatively, it makes everything near it negatively electrical also; suppose we hold a glass rod rubbed with silk near an insulated conductor; both act in the same way towards an electroscope thus showing both to be electrical in the same way; taking away the glass rod, the brass cylinder which we suppose to be the conductor used, returns to its previous condition; if not insulated we have a different effect; Suppose another cylinder to be placed touching the first; and the glass rod to be placed by the first as in the figure; take away the



glass rod; & separate the brass cylinders; test the 1st ~~glass~~ brass cylinder by an electroscope

the other destroys the divergence. In consequence; one is called positive, the other negative: which is + & which − is of course agreed on by all beforehand: viz:-
glass rubbed by silk is electrified positively
the silk itself ———— negatively.
No explanation has yet been given why the glass rather than the silk should be +.; a very small difference in the materials makes a difference in the kind of electrical effect: e.g. rub ground & smooth glass together: both being chemically the same: the ground glass gets − electrical properties, the smooth glass +
We can get electrical effects without conduction: viz:- we get an effect produced on a gold leaf electroscope at a great distance, though air is an insulator; just as a magnet makes everything in its neighbourhood magnetic; so a body

May 9th 1878.

~~The~~ conductors can be electrified by insulating them and then rubbing them: their electrical properties are best tested by the gold leaf electroscope; we have seen that when a body is electrified one way, the rubber is electrified the other way. When a body is electrified, and other conductor ~~ft~~ put so as to touch it, both share the electricity which is therefore less strong; so when a conductor ~~pas~~ is put into communication with the earth say by touching it which is in itself taken as a whole a conductor: the electricity gets spread over the whole earth which is virtually infinitely great: so the amount of electricity the body has is infinitely small i.e. none.

We have seen that electrical effects are of two kinds: they are also opposite: if the rubber attracts a third body, the rubbed body ~~other~~ repels it: if one makes the gold leaves of an electroscope be summarized by saying: bodies similarly electrified repel each other. A way to get a larger quantity of electricity is by rubbing a glass disc against amalgamed silk in a machine; the brass framework collects the electricity and can be conducted by means of brass cylinders to any part where it is wanted. Some bodies like ~~glass~~ brass transmit electricity, others like glass do not; they are called conductors, & non-conductors or insulators respectively; from one to the other is a continuous series of bodies, so the difference is rather one of degree than of kind. Air is a nonconductor; but a thin sheet of glass, or a thin stratum of air will be pierced by it in the form of a spark: When a quantity thus pierces the air, the latter comes together again with a noise, called thunder. Pure water is a non-conductor; ordinary water a good one: so much so that electrical experiments are difficult in moist air

Electricity.

May 7. 1878.

This is analogous to magnetism. Electrical effects were first observed in amber hence the name: When rubbed the amber attracts light bodies; this attraction is followed by repulsion. Similar properties are imparted to the body e.g. flannel against which the amber is ~~used~~ ~~rubbed~~. the same is observed when glass is ~~obser~~ rubbed with silk or better with some metal; the best to use is ~~to~~ an amalgam ~~alloy~~ of tin which is semi solid ~~fluid~~, & can be rubbed on silk; this silk so covered is what is usually used to impart electrical properties to glass. Two glass tubes rubbed in the same way repel one another, while the rubber attracts the glass tube which it rubs; the amber rubbed by flannel also attracts rubbed glass; while the flannel itself repels the glass; again ebonite rubbed by silk ~~attr~~ repels, rubbed by flannel attracts the glass; sulphur rubbed by flannel attracts very strongly the glass; two pieces of ebonite rubbed by the same thing repel each other. This ~~results~~ can

And so generally when the force is $\frac{mm'}{r^n}$ the ~~force~~ total effect in the 2nd case is n times that in the first.

The magnetic meridian is the line joining the poles of a magnet at rest.

A needle suspended in the magnetic meridian about a horizontal axis dips on account of terrestrial magnetism; the dip of the needle like the declination varies with time and place: a bar of iron can be made temporarily magnetic (i.e. will attract a needle) by the earths magnetism by holding it in the direction of the dipping ~~n~~ needle; permanently by striking it several blows sharply at one end while in that position

resultant f of these two force can be found; let
m = strength of one pole m' = strength of the other.

AD

$$\frac{AB}{AB} = \frac{NS}{SA}$$

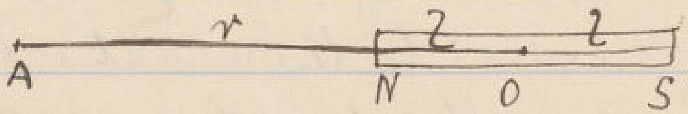
$$\frac{f}{\frac{mm'}{r^2}} = \frac{2l}{r} \qquad f = \frac{2l\,m\,m'}{r^3}$$

$$2lm = M = \text{magnetic moment.}$$

$$f = \frac{Mm'}{r^3}$$

Now suppose A to be in continuation of the line NS.

$AO = r$



force of one pole $\dfrac{mm'}{(r-l)^2}$ } total force = difference of these two

— other — $\dfrac{mm'}{(r+l)^2}$ } $= F.$

Let l be very small as compared to r.

$(r^2-l^2)^2$ = common denominator

$$F = mm'\,\frac{4rl}{(r^2-l^2)^2}$$

$$= mm'\,\frac{4rl}{r^4} = \frac{2lmm'}{r^3}$$

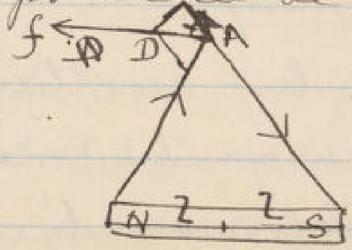
i.e. $F = 2f$ i.e. the force in the 2nd case is 2ce that in the first upper end is turned through a certain angle: say the upper end is turned through 100°; & the lower end ~~tw~~ follows through 5° then 100° − 5° = 95° = twist i.e. degree of torsion

$$\frac{95}{5} = 19$$

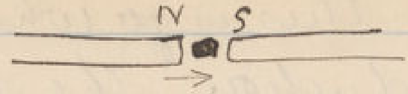
i.e. 1° of torsion will displace the fibre 19°: this is allowed for in the former computation. By this means Coulomb found that the force between two magnetic poles varies inversely as the square of the distance.

A unit pole is one which placed at unit distance from an equal pole exerts on it a unit of force.

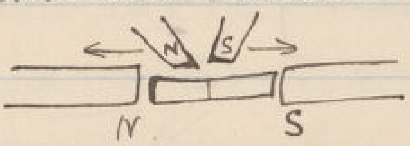
Accepting Coulomb's law the effect of a whole magnet on a pole can be found



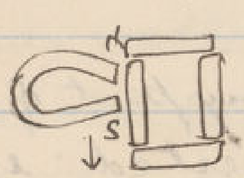
let A be the pole, placed so that the perpendicular to the magnet bisects it; the arrow heads denote the direction of the action on it; by the ▭^m of forces the following is an outline of his experiment; a steel magnet is suspended by a glass fibre so that when exactly untwisted the needle points directly along the magnetic meridian (i.e. its natural direction). This first magnet is contained ~~int~~ a cylindrical glass case, round which is a scale graduated with degrees. Another magnet is put thro' a hole in this case the opposite pole to face one pole of the 1st magnet; they repel each other, but the second ~~it~~ is fixed, and so the suspended one moves twisting the fibre; the amount it is twisted can be measured on the scale, when the ~~moving~~ the needle comes to rest; then by a screw at the top of the instrument twist the fibre back again so as to force the two needles near together again; observe again the distance they separate when at rest; the total twist = sum of the two displacements; the magnetic force = amount of twist; but another point to be taken into account is the magnetic force exerted by the earth. This can be determined by noticing how much the lower end of the fibre i.e. that on which is suspended the needle, turns when the



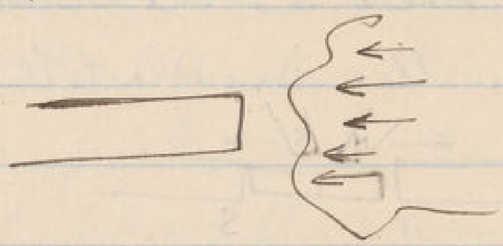
acts from left to right: the closer together they are the better; but for a large piece of steel as the two magnets cannot get close together; two other magnets should also be employed: which are moved as the arrows in the figure indicate from the middle to the ends



Another way to magnetise several pieces of steel is to arrange them in a square & pass a horse shoe magnet round them in one direction



The force exerted between two poles diminishes as the distance between them increases; the first experiments to determine the ratio in which this took place were made by Coulomb; the southpoles to it; this is ~~fa~~ what takesplace in magneticinduction: the molecular magnets in the same way as magnetic ~~&~~ needles all point the same way: viz: with their opposite poles to the pole of the large magnet, hence the body becomes a magnet itself.



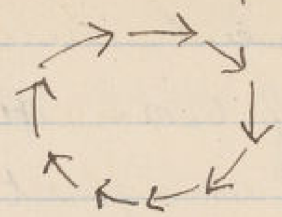
As the distant portions are less strongly attracted; the magnet ~~fa~~ should be rubbed in one direction over all parts of the body.

May 2nd 1878.

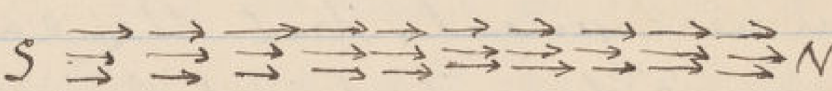
A magnetic field is any part of space where magnetic force is exerted; everywhere near the earth we have a magnetic field since the earth is itself a magnet. To make a piece of iron magnetic: place it between the two opposite poles of two magnets: in the figure the north pole force

That magnetic power is simply a matter of arrangement may be seen thus: take a tube full of iron ~~filings~~ filings & magnetise it; it will act as a magnet; take the iron filings out and mix them up anyhow together; replace them in the tube; it will no longer act as a magnet.

In soft iron the particles are more free to move, hence more likely to ~~follow the~~ be affected by an external force than in steel; so it is easier to confer on the particles of soft iron an arrangement; but they easily lose it again as just described; while in steel there is what we may called molecular friction; hence it is difficult to arrange the particles of steel in a particular way, but when they are arranged it is just as difficult to dis~~x~~arrange them. Suppose we had a number of magnetic needles round the pole (say North) of a large magnet; they would all point their or they might be arranged in a ring; of this we



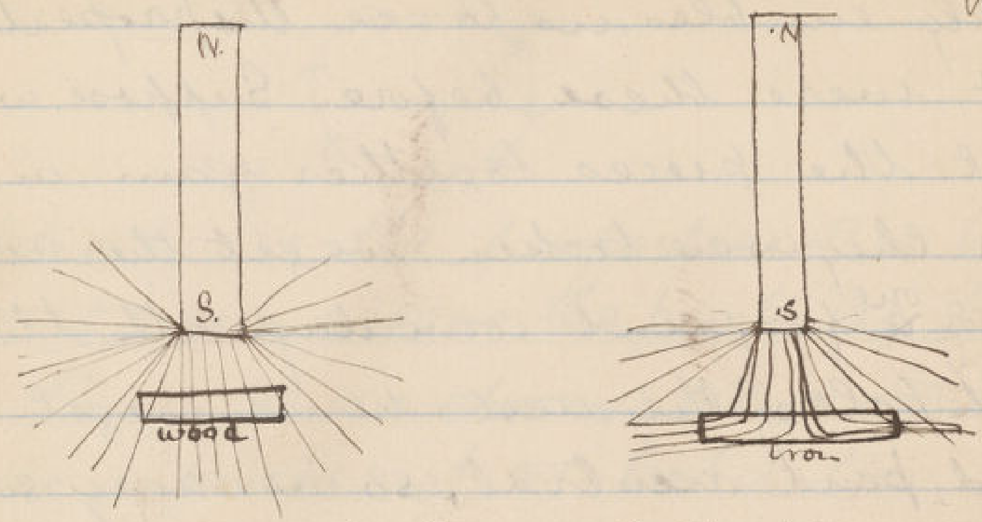
should only see the properties when broken. We can't break across the particles, but be-tween them so we always get magnetic surfaces exposed; and neutralisation is com-plete except at the ends. If we have a number of these rows we get a still more pow-erful effect at the ends; the neutralisation



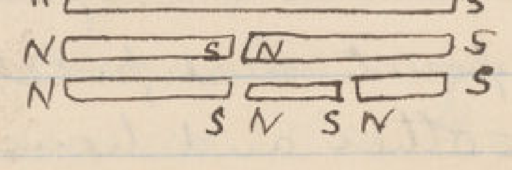
is not complete at the ends, and a number of similar poles being in juxtaposition they will repel each other if the particles are free to move, and hence the magnetic power of the ends will dissapear when the force which ar-ranged them in the proper order is removed; the ends being demagnetised, nothing prevents the next row going also and so up to the middle of the bar breaking confers on the steel properties; but simply enables us to see the properties that were there before. Suppose we were to put the pieces together again in the order they were broken; we get the north pole of ~~a~~ one fragment coinciding with the south pole of the next; hence we get the joined part neutral, so we may suppose that each particle of the bar is a complete magnet; in any bar of iron these molecular magnets are arranged indi~~finitely~~scriminate-ly; so on taking any part of the surface at random we get an equal number of N. & S. poles exposed and hence no effect at all. But if we were to arrange all the north poles one way & all the south poles another; we get at any point more poles of one kind exposed than ~~the~~ other and hence magnetic force exhibited.

S → → → → → → → → → → N

wood etc; but they will not pass through a



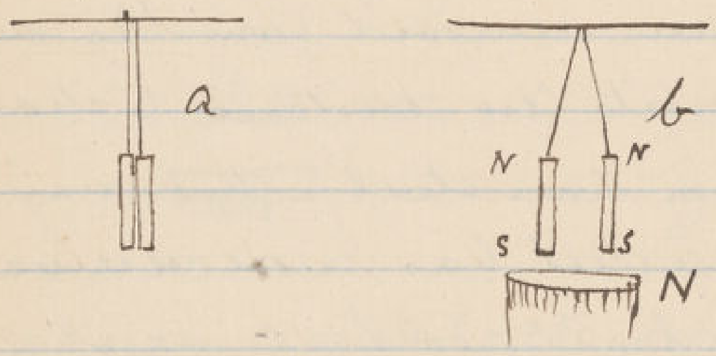
piece of iron etc. but are deflected as in the figure to the edges. We now consider the Constitution of Magnets. An experiment which throws great light on this question is to break a magnet in two: the two broken surfa--ces are found to be opposite poles i.e. both fragments are complete magnets; breaking one of



the fragments we find the same thing again & so on indefinitely. It cannot be supposed that the induced magnetism is lost when the magnet is removed. The induced magnetism which steel and hard iron undergo is less marked, but it retains more of its magnetic properties ~~th~~ when removed from the magnet; i.e. soft iron exhibits less resistance to change of magnetic condition than steel; this may be expressed by saying steel has more coercive force. Iron when at a bright red heat is not all magnetic. A magnet is weakened by heating it; but on cooling it comes back nearly to the condition it was before.

We may denote the magnetism from the pole of a magnet, as lines of force radiating out from the pole in all directions: they may be seen by allowing iron filings to fall on a sheet of paper underneath which is one of the poles of a magnet; the filings will arrange themselves along the lines of force. They are capable of passing through a piece of

The repulsion of like poles may be seen thus: if one pole of a strong magnet is held under two bars of iron suspended so as to touch (a): they stand apart as in b.



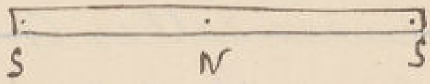
If the ~~south~~ one pole of one magnet have attached to it a bar of iron, and the opposite pole of another magnet be brought near it the effect of the first will be neutralised and the bar will drop: while if a like pole of a second magnet be brought near the cohesion will be strengthened.

April 30th. 1878.

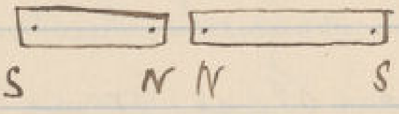
The conversion of iron into a magnet while in the presence of a magnet is called Magnetic Induction; and the iron is said to possess induced magnetism. If the iron is pure i.e. is soft iron, the permanent magnet is removed from its neighbourhood it loses all magnetic properties: nickel and many other substances but none in so great a degree as iron have similar properties; they are called Magnetic Substances, and are mostly substances chemically analogous to iron. Bismuth, copper and many other substances (but none so much as bismuth) are repelled by a magnet; they are called Diamagne-tic. All substances are either magnetic or diamagnetic. A method of getting a very strong magnet is to pass an electric current through a coil of wire, which becomes a magnet for the time: if a bar of soft iron is placed through the coil the effect is much increased.

Substances like wood, glass, brass, and almost anything except iron and magnetic substances are ~~magne~~ transparent to ~~x~~ magnetism: if however we used a plate of iron we find it opaque; ~~for~~ and we find the magnetic power at its edges: it having itself become converted into a magnet.

minutes per annum. One pole is thus seen to be always pointing more North than East or West; and is therefore called the North pole; and the other for a similar reason is called the South Pole. Likes poles ~~attract~~ repel; unlike ~~repel~~ attract; i.e. the two poles of a magnet are opposite in property. We may however have in a bar of a magnet more than two poles: say three; they



always occur alternately: if we break such a magnet at the middle poles; we get two complete magnets; and in all cases where we have in a

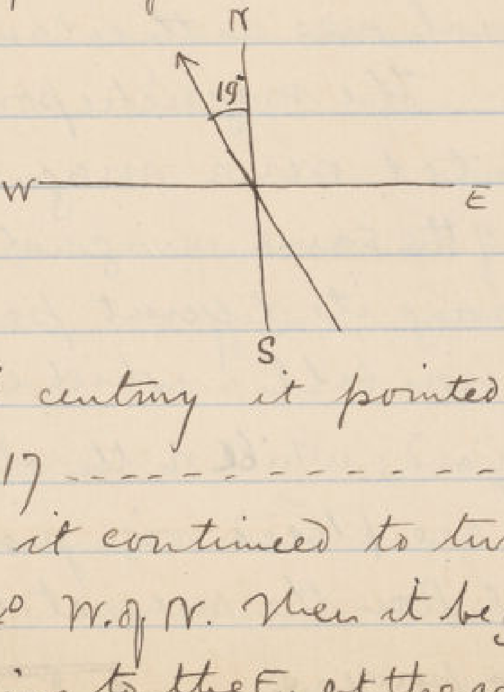


bar more than two poles, we may regard it as containing more than one magnet.

A magnet attracts iron ~~not because~~ for the same reason than the unlike poles of two mag-nets attract each other; for the iron is converted for the time being into a magnet, and may be made to attract another piece of iron; but immediately the

The point where the maximum attraction exist and which are at least two in number are called the poles of the magnet.

Another property of magnet is that when suspended freely; one end always points in a definite direction and the other of course in the opposite direction. This direction varies with position on the earth's surface, and time: at the present time in London: the line joining the poles of a magnet points 19° W. of N. & E. of S. } (approx)

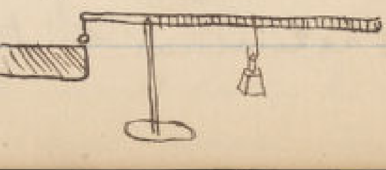


in the 16 century it pointed 11° E. of N. (approx)
early in the 17 - - - - - - - - - - - - exactly N & S.
Till 1820 it continued to turn to the W: still about 24° W. of N. Then it began to return & is now returning to the E. at the rate of about seven

Magnetism.

April 25. 1878

Magnetism is the study of the properties and phenomena of those substances which possessing those properties are called Magnets. These are found natural in the form of magnetic oxide of iron (Fe_3O_4) or Loadstone; their most striking property is that of attracting pieces of iron: this property can be transferred to steel etc. and so we get artificial magnets which are even better than natural ones as they can be got in convenient shapes. The magnetic power is not uniformly distributed over a magnet; the power in different parts of the same magnet may be compared; by allowing its different parts to attract a small iron ball; which is attached to one end of a steelyard; while on the other arm suitable weights are placed: viz: just sufficient to raise the ball from the magnet, and so comp-are the attracting powers.



the treble notes have shorter and thinner wires than the bass: they are made of steel: if the bass notes were made totally of steel they would be to thick to vibrate properly; so they are made of steel loaded with copper by wrapping copper wire round it; the tuning i.e the final adjustment of the pitch is done by altering the tension i.e. $\frac{\text{Mass of string}}{\text{Length of string}}$; then we can get the quantity

M expressed in various ways: e.g.

$$M = \rho v \quad = \text{density} \times \text{volume}$$

of the volume $v = l\alpha$ = length × area of section.

and the sectional area $\alpha = \frac{1}{4}\pi d^2$ d being = diameter.

substituting these values

$$v = l\,\tfrac{1}{4}\pi d^2$$

$$M = \rho l\,\tfrac{1}{4}\pi d^2$$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Tl}{M}} = \frac{1}{2}\sqrt{\frac{T}{Ml}} = \frac{1}{2}\sqrt{\frac{T}{\rho l^2 \frac{1}{4}\pi d^2}}$$

$$= \frac{1}{\cancel{2}ld}\sqrt{\frac{T}{\rho\pi}}$$

In musical stringed instruments; all properties of the string (length, thickness, material etc) are all taken advantage of to get sounds of different pitch: the piano has its strings stretched on a frame the shape of a harp; the piano being a harp played by mechanism;

n	$\frac{1}{2}\lambda$	t	$\frac{\lambda}{\sqrt{t}}$	
3.2	$12\frac{3}{8}$	$\frac{1}{2}$ oz	$12\frac{3}{8}$	
"	$\boxed{25\frac{1}{4}}$	2	$12\frac{5}{8}$	
"	36	$4\frac{1}{2}$	12	
64	$18\frac{3}{4}$	$4\frac{1}{2}$	$37\frac{1}{2}$	
32	$\boxed{10\frac{1}{2}}$	2		thicker cord.

The ratio $\frac{\lambda}{\sqrt{t}}$ is thus seen to be nearly constant for the same strip and same pitch.

Then with audible vibrations of wires we get similar results; e.g. with the following lengths and tensions we get the same note

Tension	Length
16 lbs	100
25 "	80

$\sqrt{t} \times l = 400$ in both cases

We may express n in many ways;

e.g. in place of m we may put $\frac{M}{2}$

the vibrations are transverse not longitudinal. the string has tension given to it by stretching it with weights: this is virtually the same as elasticity of air in pipes

e = elasticity
d = density
t = tension
m = mass of unit length of string

$$V = \sqrt{\frac{e}{d}} = \sqrt{\frac{t}{m}}$$

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}} = \frac{1}{\lambda}\sqrt{\frac{t}{m}}$$

$$\lambda = \frac{1}{n}\sqrt{\frac{t}{m}}$$

for another $\lambda' = \frac{1}{n}\sqrt{\frac{t'}{m}}$

$$\frac{\lambda}{\lambda'} = \sqrt{\frac{t}{t'}}$$

The truth of these 3 results is shown by the following experiments; a tuning fork making a string attached to it to vibrate: the length from node to node ($\frac{1}{2}\lambda$) being measured; and the weights stretching the string being also altered.

April 16th 1878

The wave in an open pipe is reflected; the reflection being a wave of the same kind.

Vibration of Strings.

In a string fastened at each end, we have a vibration just the opposite to that in an organ pipe:

In Organ pipe we have Antinode | Node. | Antinode
-- a string ------ Node | Antinode | Node.

the distance from node to node = ½ wavelength.
the reflection is of the opposite kind, to the wave first propagated.

Suppose we have a string vibrating in its simplest way; viz: in one loop; i.e. as above

l

Let λ = wave length $l = \frac{1}{2}$ wave length

V = velocity of ~~vibration~~ wave

n = number of vibrations in unit of time

$V = n\lambda = 2nl$

$$n = \frac{V}{2l}$$

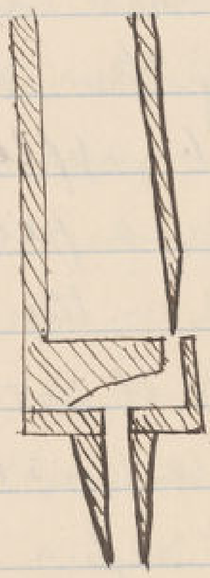
sheet of air to which it corresponds, and it is consequently silent; & alterations have to be made in the pipe e.g. in the shape of the lip, width of mouth etc. Organ pipes are made of wood, ~~when~~ in which case they are made square; and round, when made of metal. The pipes may be open at ~~both~~ the ends; i.e. it communi-cates with the air at the top, and the bottom also by means of the mouth; these correspond to the two ends of the rods, and are the positions of maximum movement and minimum change of density; while in the middle i.e. half way between the two; we have no movement, and the greatest change of density; so as in the case of the rod, we may put a partition across the middle i.e. hence a closed pipe of half the length and get a note of the same pitch; though of different quality.

~~So in an organ pi~~ If several tuning forks all in vibration a held over the mouth of an organ pipe; the latter will pick out the one to which it corresponds & answer it; this may explain the manner in which the fluttering sheet of air at the mouth of an organ pipe sets the whole column of air in vibration: the motion of this quivering sheet of air is very complicated; & if the column of air can pick out one vibration to which it corresponds it will answer to it; this vibration is very small but by repeated impulses, its effect is accumulated till at last the whole column is set in vibration, this reacts on the fluttering sheet of air increasing its vibration in the direction most favourable to itself, and so we get an increased effect. Often in newly constructed organ pipes, the pipe finds no vibration in the fluttering rate of vibration of the bridge: the accumulated effect will be so great that almost no structure could stand against it; for this reason soldiers are ordered to fall out of step when crossing a bridge. Another example of the same thing is the drawing of a rosined fiddle bow across any sounding object: e.g. a bell, a string, a tuning fork etc. the stickiness of the particles of rosin draw the string particle aside very little, but the combined effect of the whole number of them is very great. Another example is to set a tuning fork in vibration, and to hold it over the mouth of an organ pipe to which the number of its vibrations correspond; the small impulses of the tuning fork to the air in the pipe accumulate and the pipe sounds: if a tuning fork not corresponding in the number of its vibrations be used in the same way, the pipe will not answer to it (or at least very little); for the same reason that a number of random blows on a pendulum set it in motion very little.

explained:- small impulses following each other at definite periods, applied to a body in vibration, and at the same rate and in the same direction as the swing of the body, accumulate and cause a very great effect: instances of this are very numerous; if a small impulse be applied to a heavy pendulum at the same period as the vibration of the pendulum and in the same direction; e.g. a* gentle push each time it swings from right to left; will, although one push would produce no marked effect, get up a very large swing in time: if however the impulses be applied at random the chances are equal that it retards and helps on the motion of the pendulum: another example is that of soldiers marching over a bridge or platform to test it; the small impulses of the putting down of the foot set the bridge in vibration, and if the impulses coincide in rate with the

Vibration of columns of air

in organ pipes for example. Air is forced from the wind chest of an organ into the stalk of the pipe; and then into the pipe itself: which is of this shape.



When the air comes to the lip of the pipe it is broken up; and we get a fluttering or quivering sheet of air just by the lip; which sets the air in the pipe in the same sort of motion as in a rod; i.e. it stretches out & contracts altern-ately; the middle portion of the column of air being stationary; this is at a definite rate. The way in which the vibrating sheet of air sets the column of air in motion may be thus

Vibration of Rods.

If we rub a rod: of almost any kind wood, glass, metal etc holding it in the middle a certain musical sound is emitted: if it be held in any other place a different note is obtained; take however the case in which the rod is held in the middle; the motion consists of the particles of the rod squeezing up to the middle and then lengthening out again alternately; at the middle therefore which is stationary there is a state of compression and expansion alternately; while at the ends where there is the most move--ment there is least change of density. We have the middle point fixed; so we may cut off ~~the lower~~ one half, and leave fixed as one end what was formerly the middle; exactly the same takes place with one half as did before with the two halves, and we get a sound of the same pitch as before.
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Vibration of columns of air

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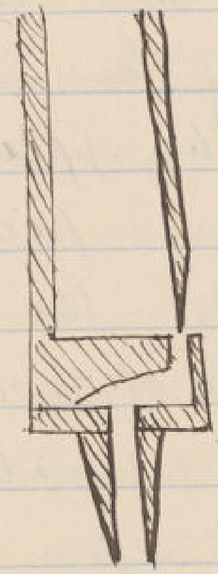

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$$V_t = \sqrt{\frac{B\mu g \frac{c'}{c}}{.001293}(1+\alpha t)\frac{76}{B}}$$

$$= \sqrt{\frac{\mu g \frac{c'}{c}(1+\alpha t)\,76}{.001293}}$$

April 9th 1878

$e = p\frac{c'}{c}$; therefore the elasticity of all gases at the same pressure is the same; let v_1 be the velocity of sound in one gas & v_2 that in another:

$$v_1 = \sqrt{\frac{e}{d_1}} \qquad v_2 = \sqrt{\frac{e}{d_2}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{d_2}{d_1}}$$

We now come to consider some special cases of vibrations, which are used to get sound: we take first to illustrate another case, the vibration of rods: though that method is not used practically to obtain sound:–

So by a sudden expansion a diminution of temperature take place; this may be shown by pumping a large quantity of air into a globe; and allowing it to escape suddenly on one side of a thermo electric pile: to which is attached a galvanometer; the deflection of the needle is very apparent. For a sudden expansion

Let c' = specific heat at constant pressure

& c = - - - - - - - - - - - - - - volume

then $E' = P \frac{c'}{c}$

for air $\frac{c'}{c} = 1.405$

the formula $P\frac{c'}{c} = E$ is the correct one for sound: for in the propagation of sound the compressions & rarefactions are very sudden.

Let B = hight of barometer in centimetres at 0°C. ~~say 76~~

μ = mass of 1 c.c. of mercury = 13.596

g = weight of unit mass i.e. the measure of gravity

then $P = B\mu g$

$E = B\mu g \frac{c'}{c}$

$D_t = \cdot 001293$ grammes $\div (1 + \cdot 003665\,t) \times \frac{B}{76}$

But if we increase the pressure, we affect both E & D in the same proportion, & therefore V remains in-variable.

Elasticity $= \dfrac{\text{change of pressure (increase)}}{\text{compression (decrease)}}$ if both are small:

compression $= \dfrac{\text{decrease in volume}}{\text{original volume.}}$

By Boyle's law:-

$$PV = (P+p)(V-v) \qquad P = \frac{(P+p)(V-v)}{V}$$

$$E = \frac{p}{\frac{v}{V}} = P \quad \text{when } p \,\&\, v \text{ are both small.}$$

If we have a sudden change of pressure $E = P$ is no longer true; because there is an evolution of heat;

if we take a quantity of air & decrease its volume we increase its temperature, keeping the pressure constant

if we take a quantity of air & increase the pressure, we increase its temperature keeping the volume constant.

if we do both i.e. increase pressure & decrease volume, we get still more heat given off: this may be shown by pressing a tightly fitting piston into a cylinder with a little piece of tinder at the end: if the compression of the air be sudden enough the heat evolved will be sufficient to fire the tinder.

By direct experiment: the numbers ~~26 were got~~: obtained were the following: two blows were struck at the same instant: one at each end; for the same reason as described in the case of air; the length of pipe was 951.25 ~~yards~~ metres, and the ~~distance~~ time elapsed between that heard through the air & through the ~~s~~ iron was ·26 second.

$$\frac{951.25}{.26} = 3496.5 \text{ metres}$$

The velocity of sound in any medium, can be calculated & by the following formula in this way:

$$V = \sqrt{\frac{\text{elasticity}}{\text{density}}} = \sqrt{\frac{E}{D}}$$

We see from this why temperature affects velocity of sound and not pressure:-

if we increase the temperature - we increase E in confined air

.... & - decrease D in unconfined air

i.e. in whatever way we raise the temperature we increase the value of the fraction $\frac{E}{D}$; & hence increase V.

observed from the other boat; and the sound then listened for by an ear trumpet let down into the water; the ~~tt~~ time which elapses between the two gives the time the sound has taken to travel a known distance & hence the velocity per second can be found.

The velocity of sound in iron was found experimentally by Biot in 1808: the iron he used was that of pipes laid down for the purposes of conveying water in Paris: the length of pipe was 1040 yards: a blow was struck at one end at a preconcerted instant & listened for at the other: the sound was heard double: once as conducted through the air, & once through the iron: the velocity in air of sound is known: and hence if the difference between the two sounds is noted; that in iron can be deduced: the number got were;

time through air 2·79 seconds
difference 2·5
·29 = time through iron.

sound of the other; the mean of the two will give the true ~~sve~~ velocity of the sound. The most accurate of these experiments were made by Moll & Van Beck at Amsterdam; the distance between the stations was 57840 feet i.e. about eleven miles; the observation was made at some temperature t; and the velocity at 0°C. was got thence & by the formula just given: viz:-

$$v_0 = \frac{v_t}{\sqrt{1+\alpha t}}$$

The velocity is the same whatever the pressure of the air: this was found from experiment as expected, to be the case by experiments made in 1840 among the mountains of Switzerland.

The Velocity of Sound in water was found experimentally by Colladon & Sturm on the Lake of Geneva: two boats were moored about eight miles apart: from one a bell is let down into the water; at the same time as this is rung a quantity of gunpowder is fired: this is

This ~~foll~~ formula gives us the following results.

at ~~0~~ 1°C. $v = 1093.8$

at 10°C. $v = 1111.6$

at 20°C. $v = 1131.1$

at 30°C. $v = 1150.2$.

The manner in which the velocity of sound was found was this: observers are situated on distant hills, the distance between them being known; a cannon is fired at one station at night, and the flash is visible at the other virtually instantaneously; the sound is heard afterwards; noting the time between the flash & the sound we get the time the sound takes to travel a known distance & hence its velocity. A precaution however has to be taken: the air is never perfectly still; & the wind helps or hinders the motion of the sound as the case may be: therefore at the same instant cannon are fired at the two stations, & the observation made at each: the wind helps the sound of one to exactly the same amount as it hinders the

April 4. 1878.

In free air we have the same thing going on in all directions that we have described for one only; i.e. the sound will always be heard at any pt. on a spherical surface at the same instant, which is continual-ly growing larger; this is expressed otherwise by saying that the wave of sound is spherical.

Velocity of Sound.

The following are results of experiments,

In	temperature	Velocity per second metres	feet
Air	0°C.	332·77	1092
Water	8·1°C.	1435	4708
Iron	11°C.	3496·5	11472

The temperature of a medium ~~&~~ makes considerable difference in the velocity of sound; take the case of air:-

Let v_0 = velocity at 0°C.

v_t = —"— — — t°C.

& α = coefficient of expansion of air.

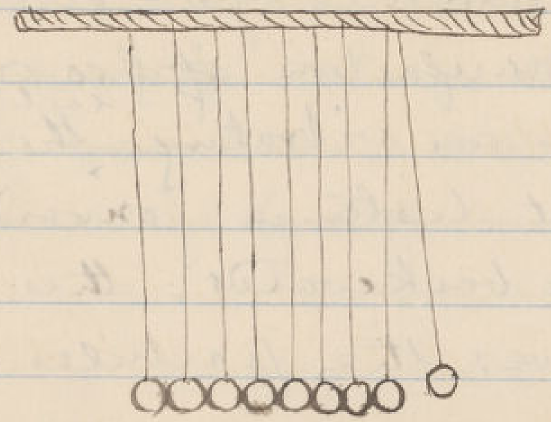
$$v_t = v_0 \sqrt{1 + \alpha t}$$

$$= 333 \sqrt{1 + \cdot 003665 t} \text{ metres}$$

$$= 1092 \sqrt{1 + \cdot 003665 t} \text{ feet}$$

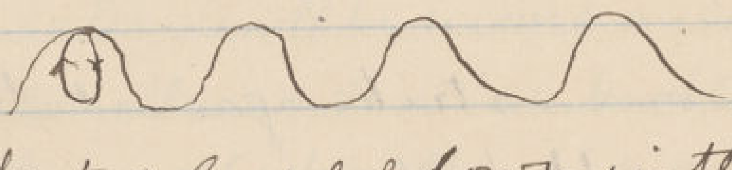
along the tube; or if struck downwards a downward-motion; some of the motion passes on into the less mobile body, and the rest returns in the opposite state: i.e. if a down-ward motion is sent, an upward motion returns: this may be instanced again by ivory balls of unequal size: if the smaller strike the ~~latter~~ larger: the latter will be moved on somewhat while the smaller will move backwards: if however the rope be attached to a body less mobile than itself e.g. suspended in the air; the free end instead of being a fixed point, will move more than the rest of the rope: while the return motion will be of the same kind as that sent: as, if a large ~~ivory~~ ball strike a smaller one; both will go on moving in the same direction: by continued up & down motion a series of waves will be got; the lengths of which can be regulated by the rapidity of the motion of the hand: and between each will be a fixed pt.

so that they are all in a row, & touching each other



let ~~one~~ the end one strike against the rest; the motion will be passed on until it gets to the last which having nothing to pass it on to moves, while the intermediate ones remain stationary.

To show the wave motion (up & down) take a long rope or india rubber tube; attach one end firmly to the wall, and stretch it out tightly holding the other end in one hand; strike it upwards with the other hand bent in the form of a fist; an upward motion will be seen to pass the tube; a condensation is followed ~~the crowding~~ by a rarefaction of the particles which is again followed by a crowding & ~~to~~ rarefaction ~~if~~ & so on if the tuning fork goes on vibrating; the each particles moving a short distance forwards & then the same distance backwards; this corresponds to a water wave: the particles do not move

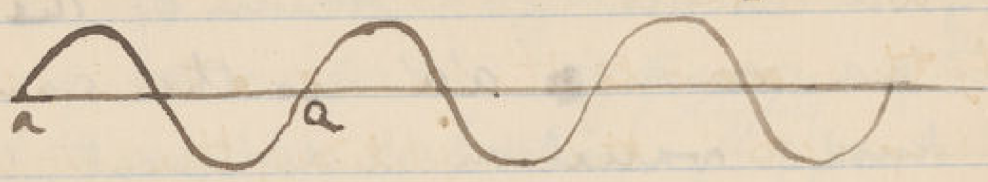


forwards but only up & down in the form of an ellipse; though the wave itself moves forwards; the top of the waterwave corresponds to the crowding of the particles, the distance from crest to crest is called ~~the~~ a wave length. This is determined by the rate of vibration of the body producing the wave; the way in which a condensation is transmitted, ~~to~~ the particles of air, may be shown on a large scale: by suspending a number of ivory balls, of the same size by threads of the same length, as in the figure

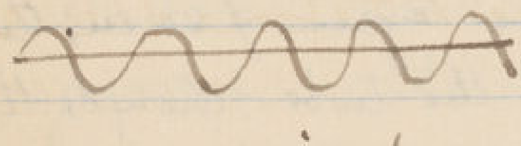
The time of each complete vibration in the figure at the top & bottom of the last page aa and a'a' is the same yet the kind of variation is different; this is what is meant by difference in character.

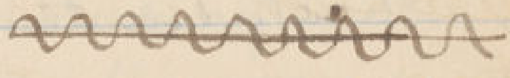
April 2 1878.

We now go to the propagation of sound:- first take the propagation of sound in one direction only, as by the air in a tube; a vibrating body say a tuning fork is placed at one end and set in vibration: a move- -ment towards the tube, causes a compression of the air next to it, which at last parts with all its motion to the air next to it, which does the same again & so on; thus a compression passes along the tube though the particles do not progress but a little way: the prong of the tuning fork now moves in the opposite direction, causing the air next to it, to move towards it, this in a like manner is propagated through may be expressed by a curve in this way:

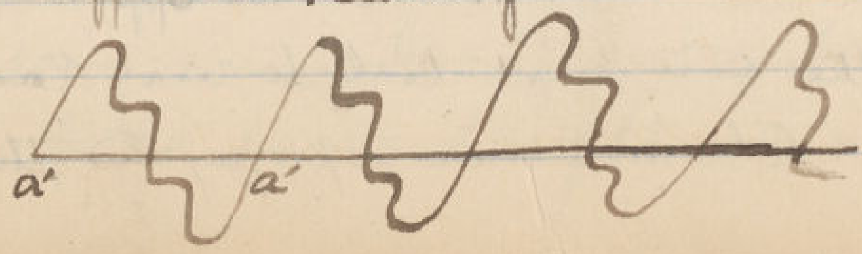


the horizontal line expresses lapse of time; and the amplitude may be expressed at any moment by the distance of the curve from the horizontal line, which is greatest at the turning points: distance in one direction being expressed by distance measured upwards, & in the other by distance measured downwards; or we might have double the no. of vibration in the same time, or treble; (and so on) having as well less amplitude





or we might have these compounded i.e. we have a different kind of variation



then $\frac{f}{W} = \frac{CD}{AC}$

apply this now to the case of a tuning fork vibrating; CD is the amplitude of the vibration (a): & call AB, l

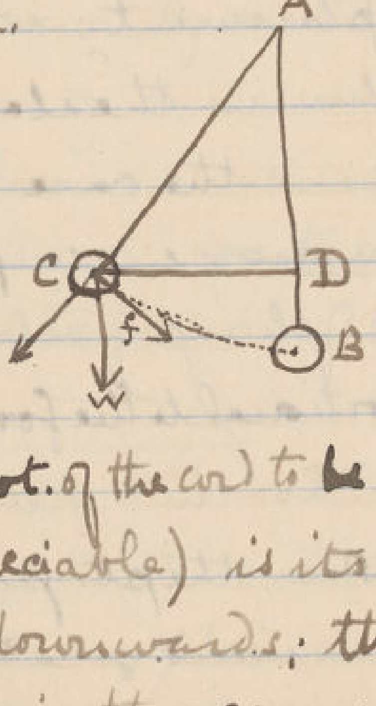
then $f = \frac{a}{l} . W$

i.e. $= a . \frac{W}{l}$

W & l remain the same: so the restoring force is proportional to the displacement i.e. to a; what represents the Wt. in the pendulum is the elasticity of the prongs; when this is the case, the vibrations are said to be isochronous i.e. occupying the same time. The rate of change of velocity i.e. the acceleration is proportional to the force & hence to the displacement; as in a pendulum the acceleration is 0 when the pt. passes its position of equilibrium; and greatest at the extremities of the swing i.e. when its motion is slowest. These changes of velocity particular pitch, the disc's rotations are counted by its action on an index finger on a clock face & so the number of vibrations increased.

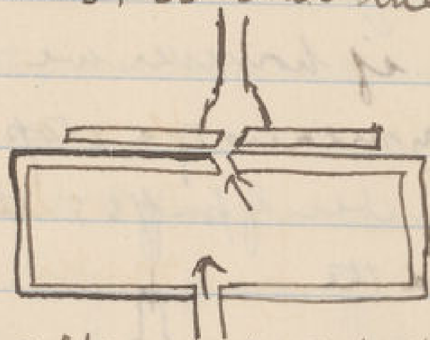
There is still another difference between sounds viz: that between the sounds of different instru-ments say between of a violin & piano, or between different voices etc; this is called the <u>Quality of Sound</u> or in French Timbre.

Take again to illustrate this the vibration of a pendulum.



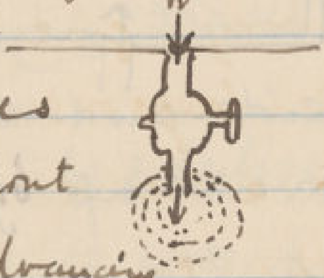
Suppose the pendulum to be in a position C, it falls to B: the force tending to make the ball fall (sup-posing the wt. of the cord to be relatively so small as to be inappreciable) is its weight which acts vertically downwards; this may be resolved into a force in the same line as the string viz AC and one f at right angles to it; which acts along the tangent of the circle which the ball describes: draw the horizontal line CD, which if the vibration is small = arc BC

what is done by the instrument we are describing, & which is called the Siren.
Instead of having one hole only there are 25 which are in the top of a circular box into which air is blown by bellows: above these is a plate containing 25 holes, just over the other 25: the holes are not vertical but of this shape taking a section through one of them: viz: slanting in the opposite directions: so that the air impinges



against the side and sets the disc in motion: the holes get covered up by the disc rotating, & open again when the next hole comes round, when the same takes place; and so the rate of rotation increases and at the same time the pitch increases: keeping it at some distance OA = $\frac{1}{2}$ the whole length of the path is called the amplitude of vibration.

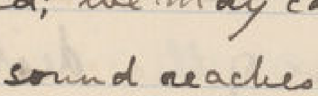
The smaller a body is (say a tuning fork) the more quickly it vibrates: just as in a pend-ulum, a short pendulum swings more quickly than a long one. The more quickly a body vibrates the higher is the pitch of the sound it produces. An instrument to count the number of vibrations is of this nature: air is escaping by an opening; if it escaped freely we should only hear a con-tinuous puff: if however we alternately close and open it say by means of a stop cock, we break it up into a number of puffs: these set the air in motion around the opening, producing waves which act on the air in front of them & so we obtain an advancing



wave of alternate compressions & rarefactions, just as is produced by a sounding body say a tuning fork, in fact a musical note is produced. This is open air, so we dont get much diminution of intensity in the sound even after a long distance.

March 25th 1878.

The loudness of sound besides depending on the amount of surface exposed, depends also on the amplitude of vibration, which term will be now explained; we may call it the vigorousness of vibration: the sound reaches the ear sooner because it gives off its motion more rapidly which is the same reason why we hear a sound better in a dense than a rare gas; viz: more of heavier particles have to be moved, and so the motion of the sounding body is given off more rapidly. Each vibration whether large or small takes the same time; just as in a pendulum a long or short swing takes the same time; let O be the



position of a particle at rest; when sounding it vibrates to A & back again to A' & so on: the

than a light shadow: this is because the undulations in producing sound are the larger. After a certain time a sound has travelled equally in all directions i.e. a sphere ~~with~~ of that radius will include all the points at which it is audible at that distance; in twice the time, the
2 same is true of a sphere of twice the radius or 4 times the size; i.e. the intensity varies inversely as the square of the distance. Although in most cases the ear can tell whether one sound is ~~lo~~ louder than another, no one has the means of saying how much louder it is e.g. whether it is twice as loud, or $\frac{1}{2}$ as loud etc. This difficulty is increased between sounds of different kinds. Sound may also be conducted in one direction only by a rod or a wire or the air in a tube etc. for a very long distance; as ~~the~~ one portion of the air which is in motion (taking the case of a tube) has ~~to~~ impart that motion to the next portion of air which is of the same size as the 1st, not continually increasing as in the

Or a solid body may be employed instead. If we had an electric bell striking in a perfect vacuum we should hear nothing: if we connect with it a rod which passes to the exterior of the receiver of the air pump, we hear it; or if we let air in we hear it.
If the sounding body is of small surface, it will perhaps be ~~two~~ too small to impart the vibrations to the air; so connect it with a larger body (e.g. with stringed instruments, we connect the strings with a sounding board): that has the vibrations of the small body imparted to it, and is large enough to impart them to the air. Loudness depends not only on the amount of motion, but on the size of the surface exposed also.
Sound is given off in all directions: bodies in the way present obstacles to its progress: i.e. there is such a thing as a sound shadow: but it is much less well defined.

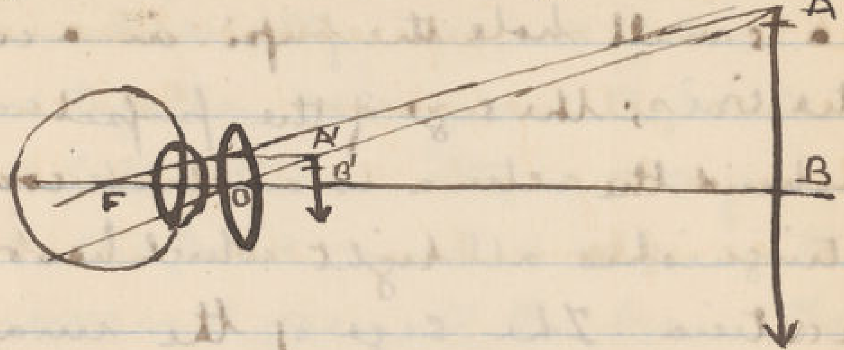
Sound

March 26 - 1878

All methods of producing sound are methods by which mechanical disturbance is produced; the bodies which emit sound all have a vibratory or un-dulatory motion; on an exaggerated scale this motion may be typified by that of a pendulum; it is in motion but still it has the same average motion. These fact may be shown in many way: e.g. make a glass bell emit a sound by drawing a rosined fiddle bow across it; the vibrations are there but are not visible: if light pieces of cork be suspended by threads at the edge of the bell, they will be struck out. If the same be done to a metallic plate over which sand is strewn, the sand will settle in certain lines forming a figure on the plate; these lines are the parts of the plate which are not moving. The figure differs for different sounds, the deeper the sound the simpler the figure. The vibration of wires are visible. The body which is vibrating imp-arts these vibrations to the air, which in its turn imparts them to the ear i.e. we hear them.

for a long time, the eye gets weak & is not so apt to receive impressions: this is owing to the nerves getting fatigued. When a certain colour is long looked at, and the eye removed, the complementary colour is seen

By employing a convex lens we get a better result

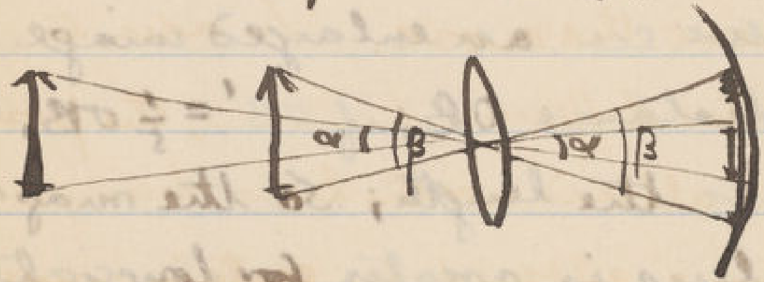


Let A'B' be the object placed at a very short distance OB from the eye (or more accurately from the lens): we see an enlarged image of it AB at a distance OB; if $OB' = \frac{1}{5} OB$, the image is 5 times the length; so the magnifying effect of a lens is greater for longsighted than for short sighted eyes.

Images leave impressions on the eye; on the average for about $\frac{1}{20}$ of a second; so if one object disappears & another take its place within that time we see a mixture of the two: e.g. in a rapidly rotating disc composed of different coloured sectors, we see a mixture of the colours. When used

A

focus behind the retina. To admit the light is a small hole the pupil in a coloured screen the iris; the size of the pupil can be varied. Behind the retina is a black coating which extinguishes all light which has once fallen on the retina. The size of the image varies inversely as the distance of the object from the eye as is seen from this figure: the angles α and α : or β & β being always equal.



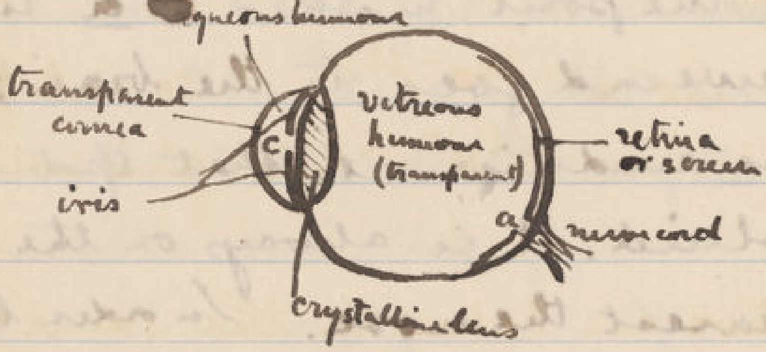
So by bringing an object very near to the eye we should get a very large image: but with the naked eye we get it blurred; the microscope is the name of the instrument which enables one to get large images by virtually bringing objects near to the eye. The simplest form of microscope is a hole in a blackened card, this contracts the pencil of light which enters the eye, but by this ~~x~~ very little light is admitted.

ramifications of the optic nerve, the extremities of which are sensitive: they are not placed infinitely close together, so very small points, the images of ~~which~~ might fall between them are invisible: at the point marked a in the figure the nerve end goes to the brain: there are no nerve endings at that spot which is therefore blind; it is always on the side of the eye nearest the nose. In order to adjust the sight for different distances, the position of the lens is not altered nor the lens changed, but its curvature altered by certain muscles; it can be so adjusted so as to see from infinity to a distance of 5 or 7 inches on an average; the last quantity differing in different people; shortsighted people are those in which parallel rays are brought to a focus before A they reach the retina; ~~so a~~ blurred image is got on the retina. Long sighted people are those in which the rays are brought to a

March 12. 1878

The Eye Considered as an Optical Instrument.

The principle of the eye is the same as that of the camera obscura. The general shape of the eye is that of a large sphere intersected by a smaller one,



horizontal section of right eye

which acts the part of a lens: the front part is transparent and is called the cornea, it is continuous with the external covering of the rest of the eye which is opaque: between this & a double convex lens called the crystalline lens is a space containing the aqueous humour: between the crystalline lens & the screen at the back of the eye called the retina which receives the images is the vitreous humour. From the front to the back of the eye the index of refraction increases, so that the general effect of the continuation of lenses in front is that of a converging lens. The retina is formed by the for a prism of another material

$$\delta' = A'(\mu'_H - \mu'_B)$$

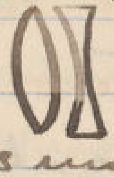
if $\delta - \delta' = 0$: the final direction of the rays will be parallel to their 1st direction.

if not

$$\frac{A}{A'} \text{ must } = \frac{\mu_H - \mu_B}{\mu'_H - \mu'_B}$$ so as to get no dispersion.

For lenses: two are taken in the same way:

Cr. Flint

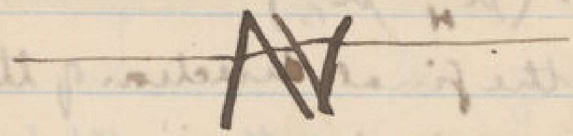


& the focal lengths must be inversely proportional to their dispersive powers

$$\text{or } \frac{f}{f'} = \frac{\phi'}{\phi}$$

A

first



But if we take them of different materials, we must take them of different sizes; the thinner being made of the more dispersive material will undo all the dispersion by the other, without undoing the bending

These combinations are called <u>achromatic</u> & are important in the construction of optical instruments. So with lenses, Chromatic Aberration may be corrected by having combinations of lenses of different kinds of glass say crown & flint glass, so making an achromatic combination.

This expressed numerically is for a prism

for the B line $D_B = A(\mu_B - 1)$

for the H line $D_H = A(\mu_H - 1)$

total bending $- \delta = D_H - D_B = A(\mu_H - \mu_B)$

is this quantity taking E as the middle of the spectrum is:-

$$\frac{\mu_H - \mu_B}{\mu_E - 1} = \phi$$

The Dispersive powers of the same five sub--stances are the following:-

Water	0·0396
Crown Glass	0·0390
Flint Glass	0·0667
Oil of Turpentine	0·0489
Sulphide of Carbon	0·1366

The Dispersive powers are thus seen not to be proportional to the indices of refraction; so by combining prisms of different materials we can undo the dispersion, without undoing the bending: if the two prisms are of the same material: they must be equal in size, and then the final direction is parallel to the

The following are the indices of refraction for some of the chief lines of a few important bodies:

	B (in red)	D	E (in green)	F	H
Water	1·3309	1·3336	1·3359	1·3378	1·3442
Crown Glass	1·5258	1·5296	1·5330	1·5361	1·5461
Flint Glass	1·6236	1·6306	1·6374	1·6435	1·6661
Oil of Turpentine	1·4705	1·4744	1·4784	1·4817	1·4939
Sulphide of Carbon	1·6207	1·6333	1·6463	1·6584	1·7091

Taking the difference of the extremes of these we get the relative length of the spectra i.e. $\mu_H - \mu_B$. This quantity is called the <u>coefficient of dispersion</u> of the substance: those of the substances above are

Water	0·0133
Crown Glass	0·0208
Flint Glass	0·0425
Oil of Turpentine	0·0234
Sulphide of Carbon	0·0883

The greater the total bending the greater the coefficient of dispersion. <u>The dispersive power (ϕ) of a substance</u>

March 7th 1878

The property of absorbing the more refrangible rays of the spectrum & giving out rays less refrangible was first observed in fluor spar: hence it is called Fluorescence.

The property of remaining luminous after being exposed to light is observed more or less in every substance: it is called Phosphorescence.

Every vapour absorbs the same sort of light which it gives out, if it is cooler than the source of light.

The Separation of the rays of light is known as the Chromatic Dispersion of light.

The Dark lines in the solar spectrum are always permanent, and hence they furnish good land marks in any spectrum; & they have names e.g. the Sodium line is called D; up to H is all that is visible in a spectrum, tho' it really reaches 2 or 3 times as far.

A

A good imitation of this is got by letting the light pass through red nitrous fumes.
If we burn sodium in an electric light & get its spectrum, we see two yellow lines, but if we get a full spectrum & let the light shine through vapour of sodium, we see dark lines on the coloured where the yellow lines were before; so the composition of the sun's atmosphere can be found
Beyond the violet rays are others which are too refrangible to be seen, they are called the ultraviolet rays; certain substances have the power of making them less refrangible & so visible; the substances themselves appearing self luminous. Uranium glass does this, so does sulphate of quinine &c.
Certain ~~others~~ substances notably the oxy-sulphides of the alkaline earths, appears to have the power of absorbing light & remaining luminous when removed to the dark.

e.g. brass is yellow, because it reflects the yellow light with more intensity than red or blue; and not by unequal absorbtion or transparency.

All vapours produce light of some one sort: if we heat a solid, it gives out, first red, then orange, then yellow, & then white (all rays of spectrum); with a vapour this is different; take the vapour of sodium; at a certain point it gives out yellow rays, & it keeps giving out the same rays however high the temperature is raised; before it gives out the yellow rays, it gives out none at all. So substances in state of vapour can be detected by their own special spectrum.

March 5. 1878.

We get for different substances bright lines in diff. parts of the spectrum; in the solar spectrum, we get these reversed, & see dark lines on a coloured ground.

Feb. 28. 1878.

Some substances only let one kind of light go through them, the others are converted into heat; e.g. red is opaque to yellow light etc. The appearance of opaque objects is the same: light penetrates a short way, only some say red, being able to get through; this is reflected back to the eye passing again through part of the medium, and so it appears red.

The superposition of coloured glasses is therefore the same as the mixing of pigments: each producing its own absorbed effect. e.g. blue & yellow pigments give green: but blue & yellow light are nearly complementary; so red & green & so on.

A Colour is called saturated when only 1 sort of light of the spectrum reaches the eye; it is unsaturated when diluted with white or its intensity diminished by black.

Colours are also produced by unequal reflection pairs are;-

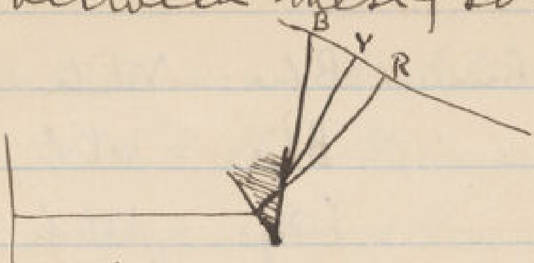
Red + Greenish Blue = White
Orange + Blue = White
Yellow + Indigo = White
Yellowish Green + Violet = White
Extreme Red + Green + Extreme Violet = White.

If we take all these together, we shall get white in greater quantity

If we split up light this way by a prism, we can recollect it by another prism turned the inverse way, & so white light once more.

When we say a surface is black we mean it gives back none of these rays of light; when white, all; when blue, the blue rays only; when red the red rays only & so on; so if we view a substance red in sunlight, by light which has no red rays in it, say the blue part of the spectrum, it can send back no red rays & so appears black; and so to a blue substance appears black in red light & so on.

Yellow is between these & so on



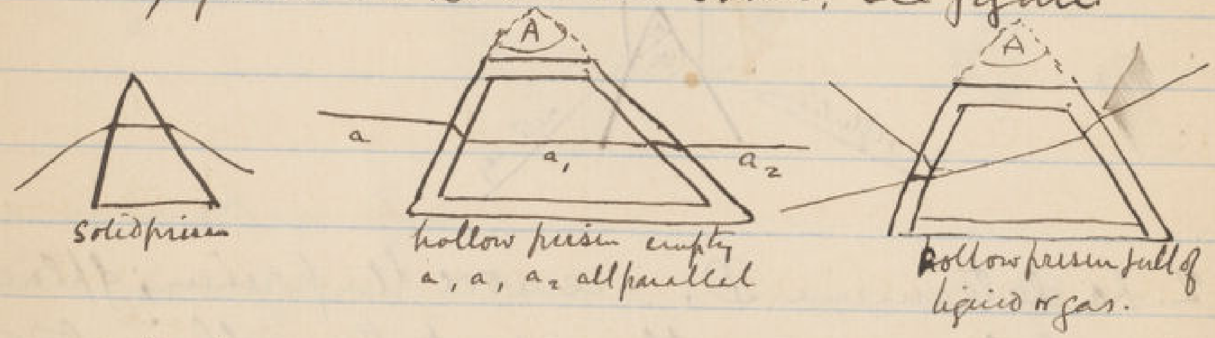
in White light we have all these together & we get a succession of images on the screen of different colours; each overlapping the next; unless the source of light (say a slit in a dark screen) is ~~a~~ very small: then we have the disadvantage of the light being in very small quantities.
The list of colours beginning at the least refrangible is

Red
Orange
Yellow
Green
Blue
Indigo
Violet.

shading off gradually one into the other.

If the red rays only come to the eye, we have the impres-sion red: if the blue, blue & so on. Certain pairs called complementary colours form white; these

With these hollow prisms however the ray is bent towards the refracting angle (A) not away from it as in the others: see figure.



$\frac{\mu - 1}{d}$ is found to be constant

for air $\mu = 1.000293$

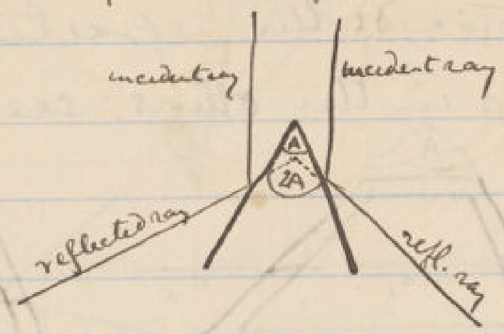
for all gases it is very slightly over unity.

Prismatic Spectrum.

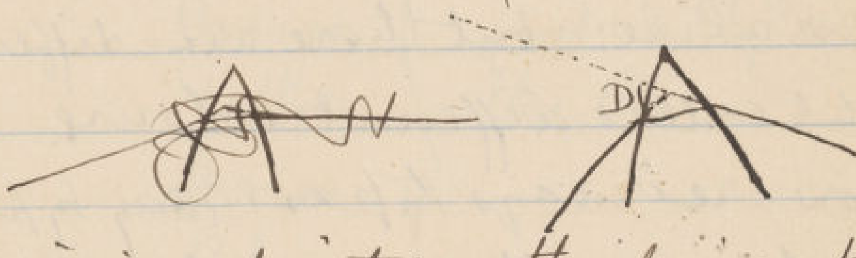
So far we have considered ~~as~~ all light as being alike; but there are different kinds of light called different colours. If we only allow red rays to pass (say by putting a piece of red glass over the source of light); we get an image red in colour at a certain point R; do the same for the blue; we get a blue image B farther off; i.e. the rays have been more refracted

Feb 26th 1878

This is seen by this figure

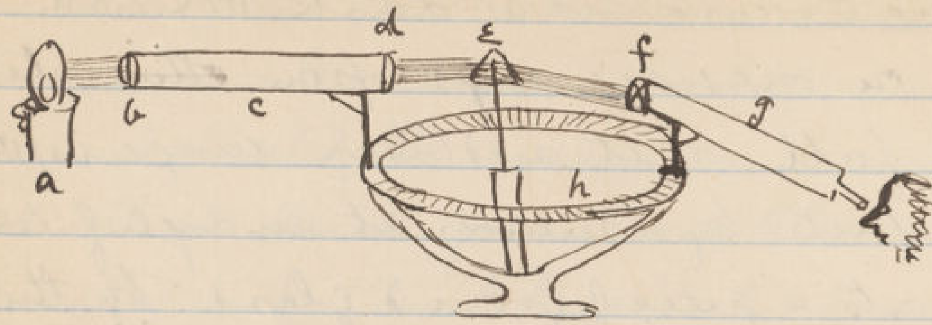


2. To determine D; remove the prism; & place the telescope exactly opposite the collimator so as to get this effect ⊛; then put the prism on the path of the rays; they are bent; move the telescope to get the same effect (⊛) again and measure the angle through which the telescope has turned, (which is D) placing the prism in the



direction of the minimum deviation; the figure, above, shows this.

For liquids, put them in a hollow prism of glass, & so for gases; in the latter case the ∠ of the prism must be very large to get any perceptible effect.



a is a strong light.
b a piece of glass with a line across it ⦶
c is a collimator
d is a lens ~~will~~ which throws the image of b on
e which is one of the surfaces of a ~~lens~~ prism, whence it ~~goes~~ is reflected to
f, the lens of a telescope g, which has cross wires ⊗;
Focus the telescope so that the line on b just passes through the intersection of the cross wires in this way ⊗; mark the point on the circular scale h; move the telescope round to get the same effect from one of the other surfaces of the ~~lens~~ prism, and measure on h the ∠ ~~the~~ it has moved through; half that ∠ is the angle ~~of the~~ between the two surfaces which we have called A.

The Camera Obscura is a dark box with a lens on one side; rays from ~~other~~ outer objects fall on it and their image is formed and reflected by a mirror at an angle of 45° to the vertical, onto a piece of ground glass; by this means the ~~object~~ image is erect. Without the mirror the ~~rays are~~ image is received inverted on a piece of ground glass at the back; this is the Photographic camera; after fixing up the image on the glass; a sensitive surface (collodion) is put in its place and receives & retains the image.

We now come to the mode of measuring the index of refraction of a substance, by its refraction through a prism of the substance. On page 47 is given the necessary formula.

$$\mu = \frac{\sin \frac{1}{2}(A+D)}{\sin \frac{1}{2}A}$$

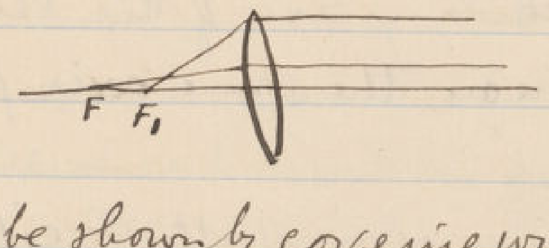
the angles A and D we can measure thus;

1. to measure A or the angle of the prism.

An instrument like that in the next figure is used;

P. T. O.

This is different for thick lenses; there the margins and the centre has different principal foci, and that of the remainder varies between these; that which goes through the centre is refracted to F, that through the margins ~~further off~~ nearer to F_1



this may be shown by covering with a screen first the middle then the margins; the image will get nearer the lens on the second.

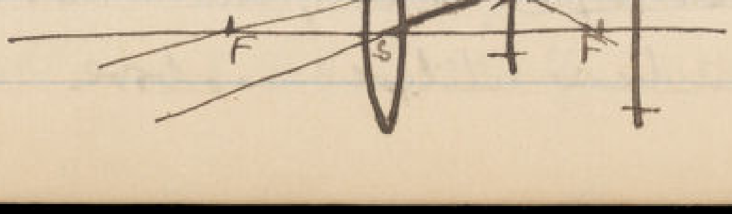
The Magic lantern, Consists of a powerful lens for throwing images of pictures, which as they are strongly magnified i.e the light spread over a large space, must be strongly illuminated.

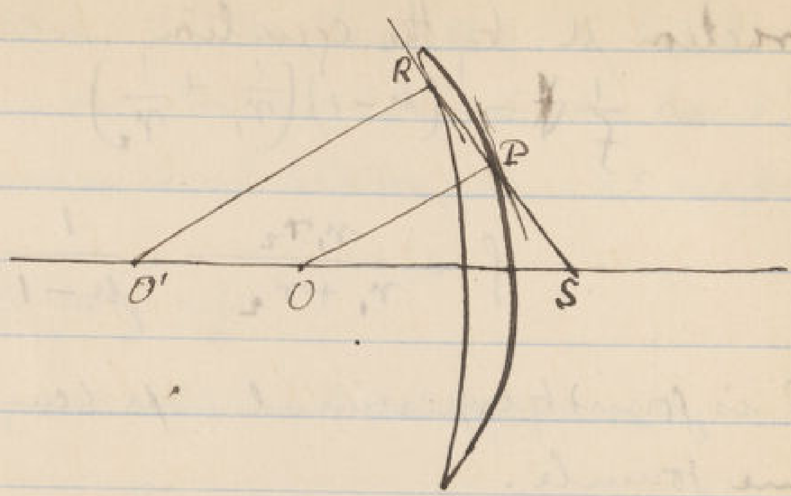
Feb 21st 1878.

The Solar microscope is very much the same, the rays of the sun being utilised as above.

optical centre undergoes no refraction; in the case we have taken the pt. is outside the lens; in a double convex lens it is inside; & if the two curvatures are equal, it is the midpt. of the lens. Going back to the 2nd figure on p. 64 we know also the line AA'. A line like AA' is called ~~the~~ a secondary axis of the lens. Any 2 conjugate foci are on the same axis principal or secondary.

The linear dimensions of the object & image are therefore directly as their distances from the lens. As long as the object is outside the principal focus we get a real & inverted image; when the object is at F, the image is at infinity; if then the image is behind the object i.e. it is virtual & erect i.e. there is really no image there but the rays on leaving the lens, leave it as if they came from that image as in the figure.





draw radii of each parallel to each other O'R & OP; join RP and produce to meet the principal axis in S. S is called the optical centre of the lens; it has this property: any ray passing through it enters and leaves the lens in directions which are ∥ to each other. This is seen thus: draw tangents at R & P; these will be parallel: the ray SP then enters as it were a plate with parallel faces but very small; and so it leaves it in a direction parallel to its ~~self~~ former direction. There is a lateral shifting but in a thin lens this may be neglected, and then it may be said, that a ray passing through the of refraction μ by the equation

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$f = \frac{r_1 r_2}{r_1+r_2}\cdot\frac{1}{\mu-1} = 8\cdot 9 \text{ in the former case}$$

If f is found by experiment, μ can be found by the same formula.

To find the image of the object, we must find the images of its extremities and join them.

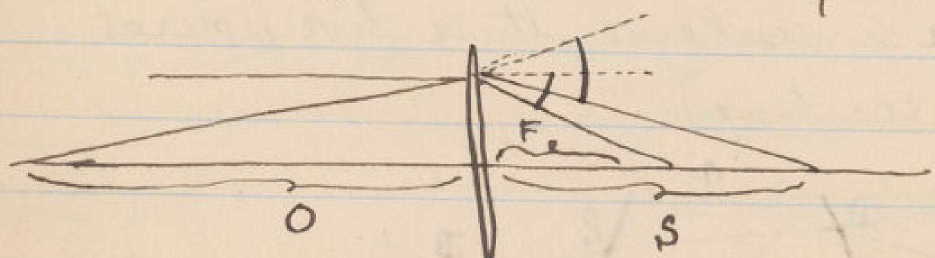
A ray from A ∥ to the principal axis goes through on refraction the principal focus on the farther side; a ray from A through F emerges on refraction ∥ to the principal axis; these two rays meet at A′ which is the image of A; similarly find B′; join A′B′; which is the image of AB.

Optical centre of a lens. This is another pt. in a lens to be determined; take for a change a meniscus lens. Let O & O′ be the centres of the two spherical surfaces; P.T.O.

If the lens is thin; PP_1 or the path of the light inside the lens can be neglected; the bending then may be considered to take place at one point and then

$$\frac{1}{F} = \frac{1}{O} + \frac{1}{S}$$

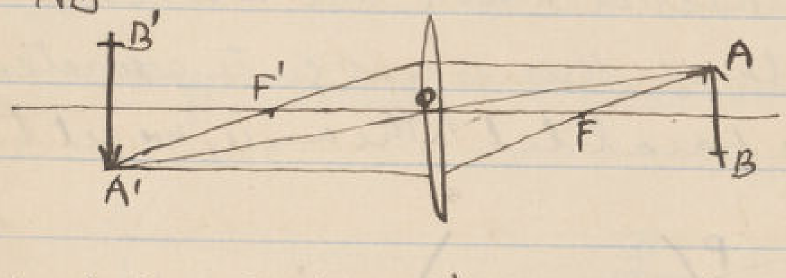
which is a geometrical consequence



$$\frac{1}{F} = \frac{1}{O} + \frac{1}{S}$$

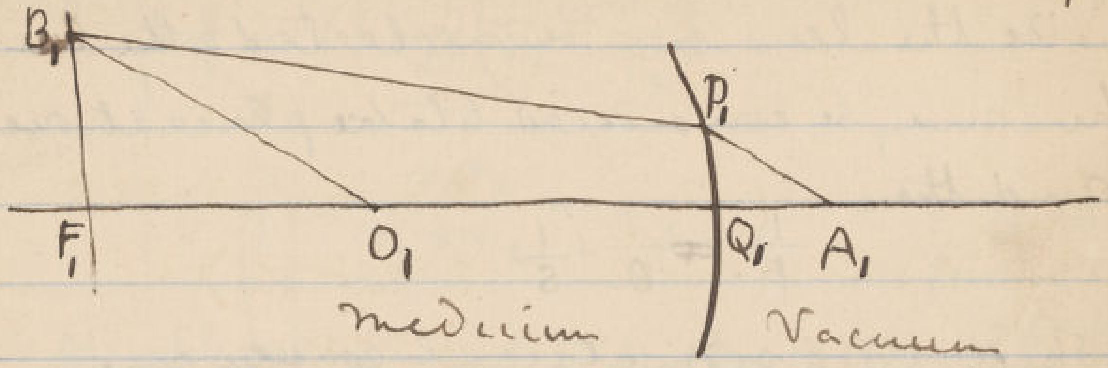
Feb. 19. 1878.

In a lens find the position of the image of an object AB

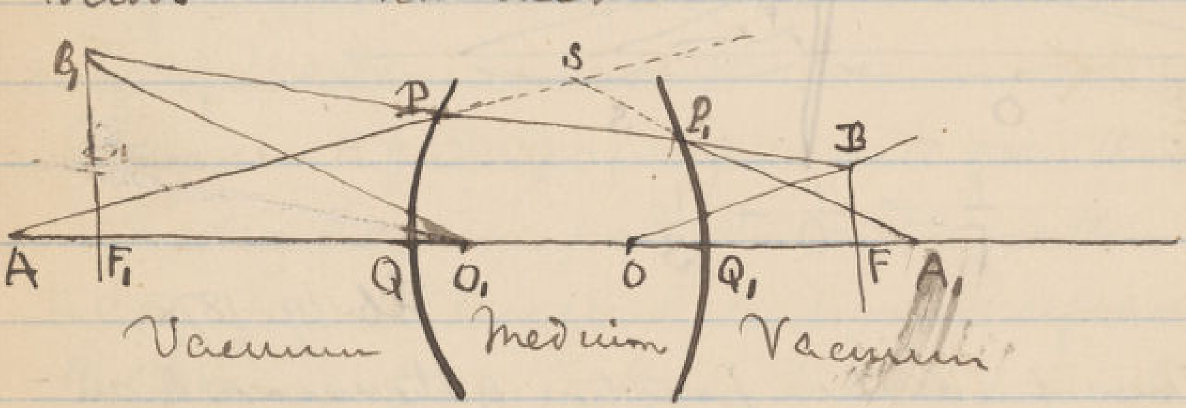


A The principal foci (ff) must first be found either by experiment or calculation from the radii of the spherical surfaces & the index

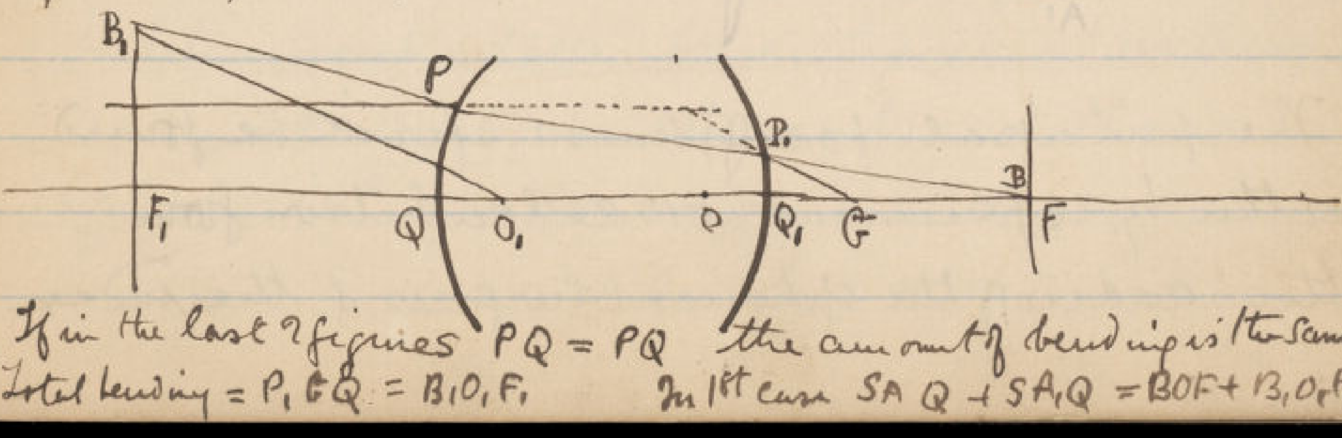
From medium to Vacuum we have for concave surface



For a double convex lens these two figures must be combined.



This applies however thick the lens is. When the angle PAQ is large this is not quite accurate.
If the rays are parallel to the axis, we get this;—



If in the last 2 figures $PQ = PQ$ the amount of bending is the same;
Total bending $= P_1GQ = B_1O_1F_1$ In 1st case $SAQ + SA_1Q = BOF + B_1O_1F_1$

$$F = \frac{OS}{O+S}$$

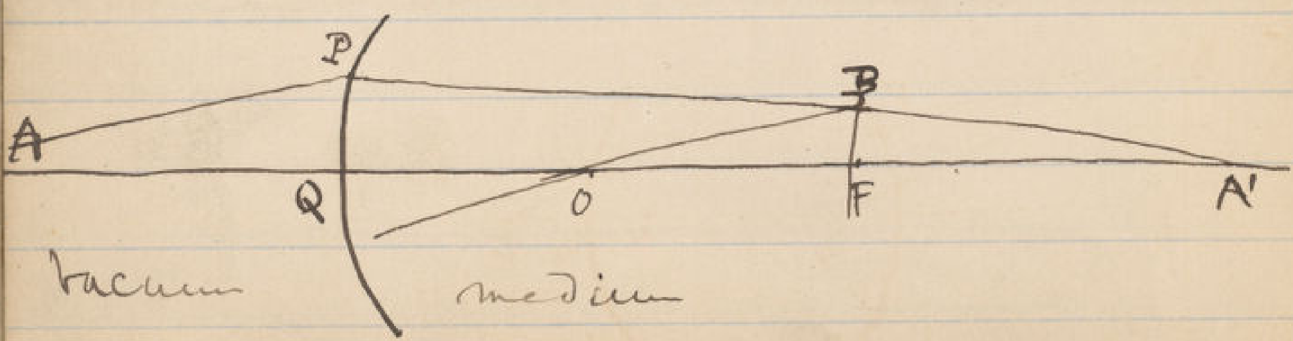
$$\frac{1}{F} = \frac{1}{S} + \frac{1}{O}$$

when O is infinite	$S = F = 8.928$
when $O > 2F$ i.e. > 17.86	$S < F$ & $> 2F$
when $O = 2F$	$S = 2F$
when $O < 2F$ & $> F$	$S > 2F$
when $O = F$	S is infinite
when $O \not< F$	$S > O$ & negative

i.e. must be measured in the same direction as O

These same results may be got by applied the laws of refraction at curved surfaces.

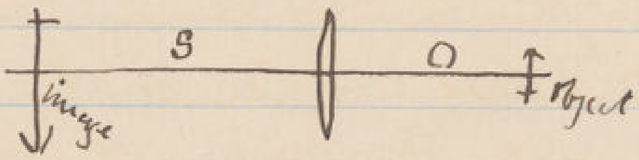
From Vacuum to medium we have for convex surface.



then $\frac{OS}{O+S}$ is constant.

Feb 14. 1878.

This in these experiments is not quite constant but is very nearly so; the reason of the slight variation is the roughness of the experiments; the mean however is 8.928.



by looking at the numbers, it will be seen that when either O or S becomes great, the other S or O respectively approaches nearer to 8.928. Suppose S to be infinitely great;

$$8.928 \quad \frac{OS}{O+S} = \frac{OS}{S} = O$$

i.e. when S is very great O is = the constant number; by saying that S is very great is meant that the rays are $||^l$; and by saying the O = 8.928 means that $||^l$ rays are brought to a focus at that distance from the lens or at the principal focus; F = 8.928 = principal focal length size as the object.

If we have a double concave lens of air or vacuum surrounded by water, we have light passing through exactly the same surfaces of the medium but in the ~~th~~ opposite order; this however does not make the slightest difference. But with a double concave water or glass lens in air we get the opposite result, namely divergence increased; and therefore no such thing as a real image.

The two distances at which an image may be obtained by varying the position of the lens is exactly the same as changing the positions of screen (i.e. image) and object; i.e. they are conjugate foci. The distances of Object to Lens and Lens to Screen (i.e. Image) are seen to go inversely nearly; in more accurate experiments they do exactly.

Call the distance Object to Lens O

and — — — — — Lens to Screen or Image S

formed on one side of objects on the other showing similar intensity-of-light & colour. These images can be received on a screen.

First let us see the results of an experiment as to images; the apparatus is a lantern which shines through some object such as a slit in the form of an arrow; a convex lens and a screen; & measure the distances when the screen is moved to different places.

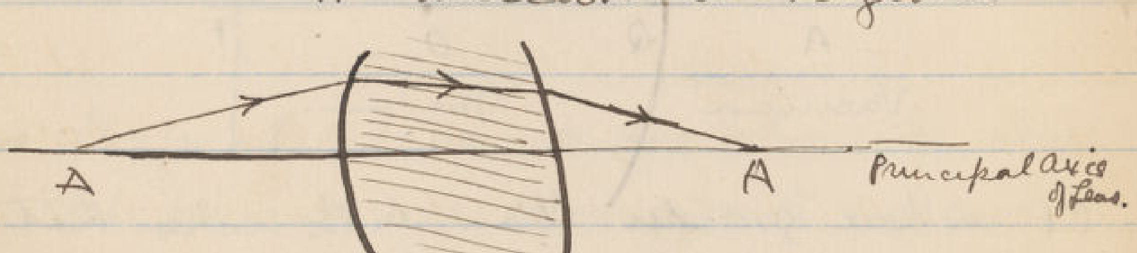
Object to Screen (i.e. to Image)	Object to Lens (we get two images for each position of Screen)		Lens to Screen (∴ we get 2 distances here)	
120	$9\frac{3}{4}$	$110\frac{1}{4}$	$110\frac{1}{4}$	$9\frac{3}{4}$
100	9·9	90·05	90·01	9·95
80	10·25	69·75	69·75	10·28
60	11·05	49·1	48·95	11·05
40	13·35	26·7	26·65	13·3
35·4	17·4			

after this no image.

Taking the second series of numbers viz: object to lens; in the 1st position the image is magnified, & in the 2nd diminished each in the same proportion both being inverted; except in the last case, where the image is of the same

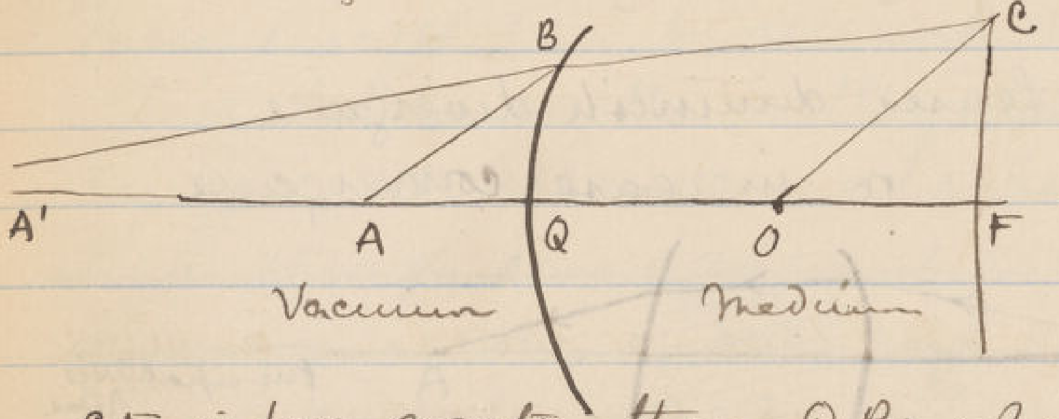
Lenses have two spherical surfaces, and the same sort of action takes place ~~x~~ at each.

Convex Lenses diminish divergence or increase convergence.



the ray in the vacuum meets a convex surface of the medium; it is consequently bent towards the principal axis of the lens i.e. the line passing the two centres of curva-ture (see p. 56); then the ray in the medium meets a concave surface; this has the same effect (see p. 54). i.e. both the refractions bend the ray to the principal axis i.e. rays proceeding from a point A on one side of the lens, are brought to meet in a point A' on the other side of a lens; we thus get images

To show that this is so by a figure, take AQ less than f'.



CF is here greater than QB: when that is the case, A' is behind A.

Feb. 19. 1878

A and A' in all these cases are conjugate foci

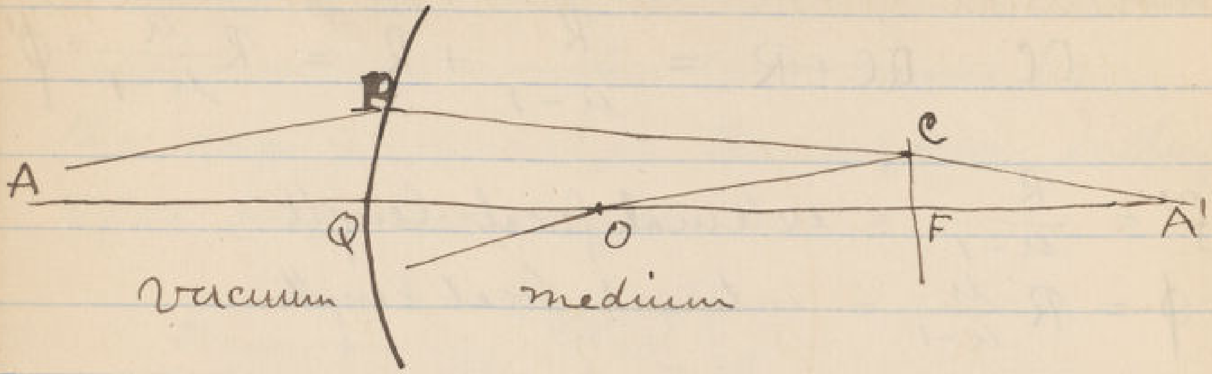
$$QF = f = R\frac{\mu}{\mu-1}$$
$$OF = \phi = \frac{R}{\mu-1}$$

Let $QA = a$
$QA' = a'$

$$\frac{1}{f} = \frac{1}{a'} + \frac{1}{a\mu}$$

Let $OA = \alpha$
$OA' = \alpha'$

$$\frac{1}{\phi} = \frac{1}{\alpha'} + \frac{\mu}{\alpha}$$



draw the focal plane or the plane in which all parallel pencils are brought to a focus say through F. Then through ~~A~~ O, draw a ray // to AP; this cuts the focal plane in C; hence AP and OC being parallel meet in C; PC is therefore the course of AP in the medium; this meets AQ in A'.
A' is the focus of A.

As long as $AQ > f'$ or external focal length $\frac{R}{\mu - 1}$
A' is a real focus.

When $AQ = f'$ A' is at an infinite distance. i.e. the rays are //

" $AQ < f'$ A' is a virtual focus, or negative i.e. on the same side from which the light comes.

$$OC = QC + R = \frac{R}{\mu - 1} + R = R\frac{\mu}{\mu - 1} = \phi'$$

$f' = \frac{R}{\mu - 1}$ = external focal length

$\phi = R\frac{\mu}{\mu - 1}$ = internal focal length.

We have here a real focus C.

Reversing the names medium & vacuum & the direction of the light; i.e. we still have it // in the medium but falling on a convex surface; the same focus C acts as a virtual focus, and the rays as if from it in direction CP.

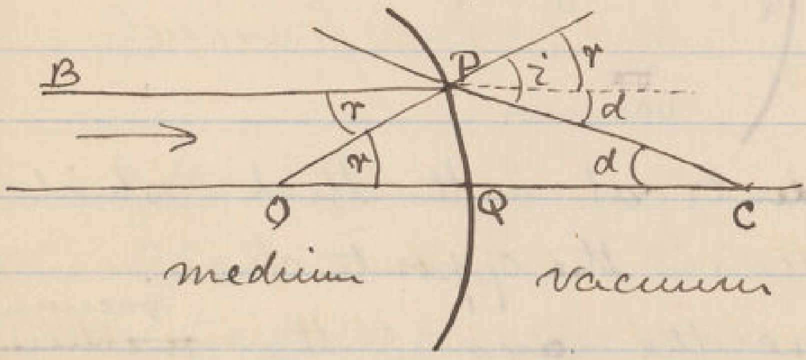
So far we have treated of parallel rays only; in vacuum

Now take rays divergent from a point A; one of these will pass through the centre of curvature O; cutting the surface in Q; take any other ray AP;

P. T. O.

i is small

So suppose ~~the figure turned round in the same way as in the last case; i.e.~~ the rays parallel in the medium falling upon a concave surface



BP is // to OQ, and is the incident ray: this is bent from the normal OP, and meets OQ in C.

Consider the triangle POC

$$\frac{PC}{PO} = \frac{\sin POC}{\sin PCO} = \frac{\sin r}{\sin d} = \frac{\sin r}{\sin i - r} = \frac{r}{i-r}$$

when i is small

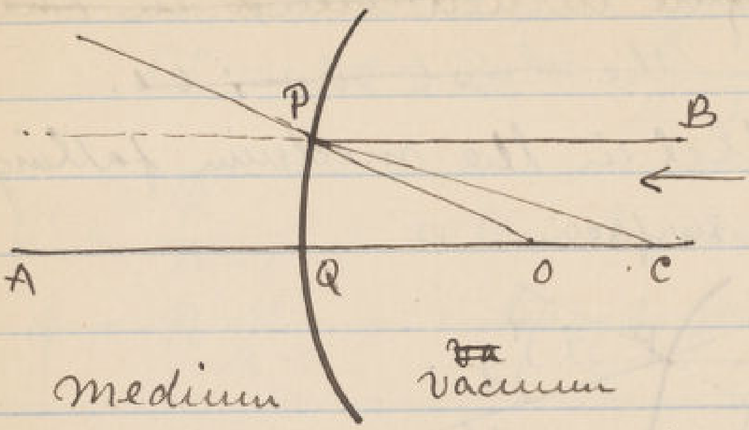
when i is small $i = \mu r$

$$PC = QC$$

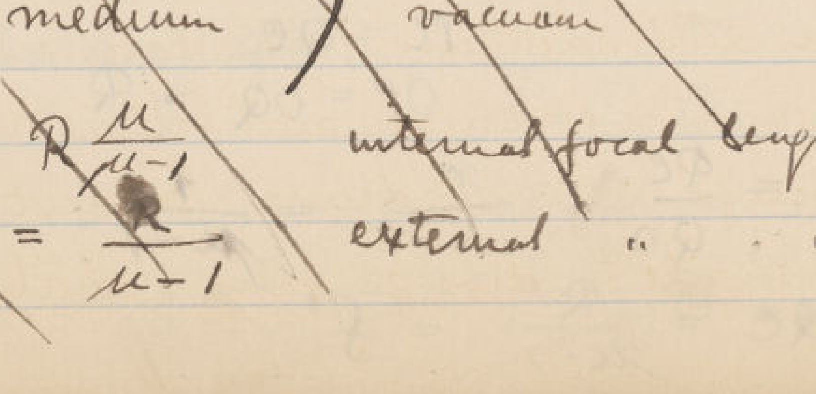
$$OP = OQ = R$$

$$\frac{PC}{PO} = \frac{QC}{QO} = \frac{r}{\mu r - r} = \frac{1}{\mu - 1}$$

$$QC = \frac{R}{\mu - 1} = f'$$



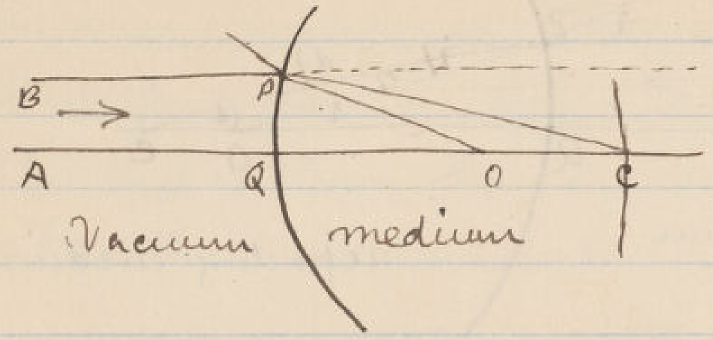
focus C identical with that which was the real focus in the opposite case.
Now suppose the rays in the ~~medium~~ vacuum in the first case diverging from C; they fall on the convex and concave in the second surface of the medium (~~see next~~ ~~last~~ ~~next~~ figure), and emerge in it parallel to line joining OQ.



$f = R\dfrac{\mu}{\mu-1}$ internal focal length

$\phi = \dfrac{R}{\mu-1}$ external " "

C is a focal point, and QC the focal

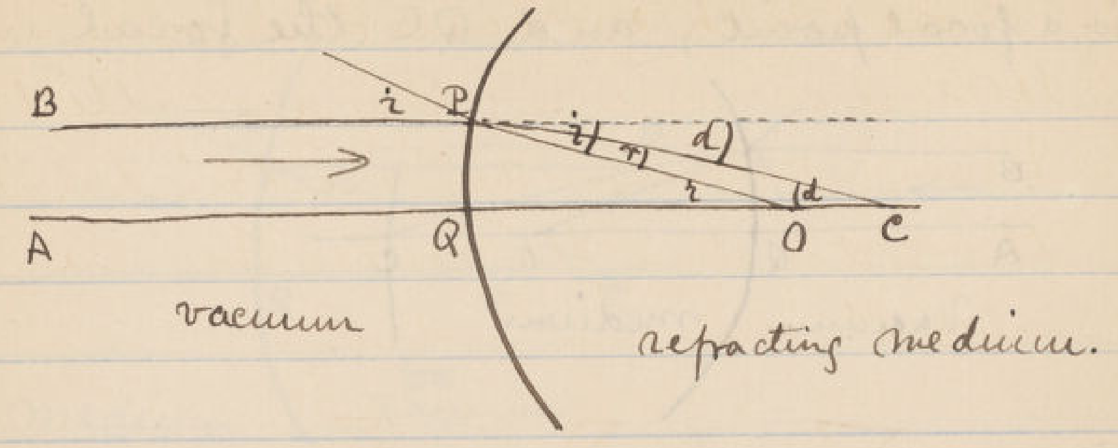


distance of the surface. $= f$.

$$OC = QC - R = R\frac{\mu}{\mu-1} - R = \frac{R}{\mu-1} = \phi \left(\begin{matrix}\text{ext} \\ \text{focal} \\ \text{length}\end{matrix}\right)$$

If the direction of the parallel rays is different, the figure must be turned round, and C will always lie upon a sphere with radius ϕ & centre O; when however the ~~m~~ refracting surface it small, this will be virtually a plane; it is called the focal plane; the focus in this case is real.

Now reverse the figure i.e. change the position of vacuum and medium; the rays are still parallel in the vacuum and falling on ~~the~~ a concave surface of a medium (see next figure); they are made to diverge from a virtual



if the angle i is small or what is the same thing PQ small.

$$d = i - r$$

Consider the triangle OPC

$$\frac{PC}{PO} = \frac{\sin POC}{\sin PCO} = \frac{\sin BOQ}{\sin PCO} = \frac{\sin i}{\sin d} = \frac{\sin i}{\sin(i-r)}$$

$$= \frac{i}{i-r} \text{ nearly when } i \text{ is small.}$$

$$\text{and } \mu r = i \quad \text{" " " " "}$$

$$\frac{PC}{PO} = \frac{\mu r}{\mu r - r} = \frac{\mu}{\mu - 1}$$

when i is small $PO = QO$ & $PC = QC$. Call OP _ R

$$\frac{QC}{R} = \frac{PC}{R} = \frac{\mu}{\mu-1} \qquad \text{i.e. } QC = R.\frac{\mu}{\mu-1} \text{ (int. focal length)}$$

with water then $QC = R\dfrac{\frac{4}{3}}{\frac{4}{3}-1} = 4R.$

with glass $QC = R\dfrac{\frac{3}{2}}{\frac{3}{2}-1} = 3R$

2. Those which increases divergence and decrease convergence; they are called Divergent Lenses; they are thicker at the sides than the centre. They are also three in number.

Biconcave Concavo-Convex Planoconvex.

Feb. 7. 1878.

We now proceed to study the laws of refraction applied to light when it enters a medium at a spherical surface

First suppose the rays parallel, in a vacuum, and they enter a convex surface of a refracting medium; one ray goes through the centre of ~~curvature~~ figure Q and the centre of curvature O; a ray BP parallel to it is bent towards the normal PO and meet AQ pro- -duced in a point C; this point can be found

In the first case

$$\mu = \frac{OA}{OA'},$$

in the second case

$$\mu = \frac{OA'}{OA}$$

in each case A appears raised.

Lenses.

In discussing lenses or the portion of a medium within two curfed surfaces, the only curves which will be taken into account will be spherical.

There are two classes of Lenses:

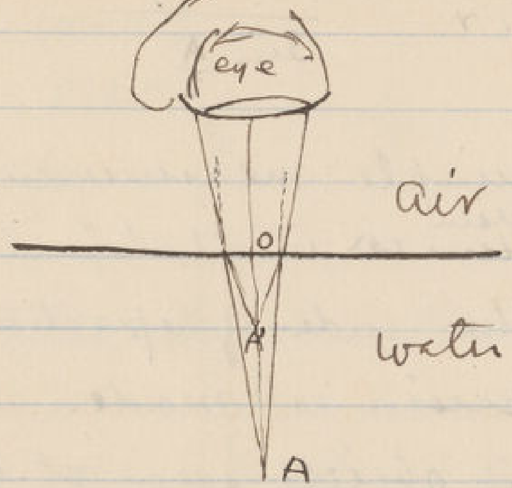
1. Those which increase convergence, or decrease divergence, called convergent lenses: they are all thicker at the middle than the sides. They are 3 in number.

Biconvex

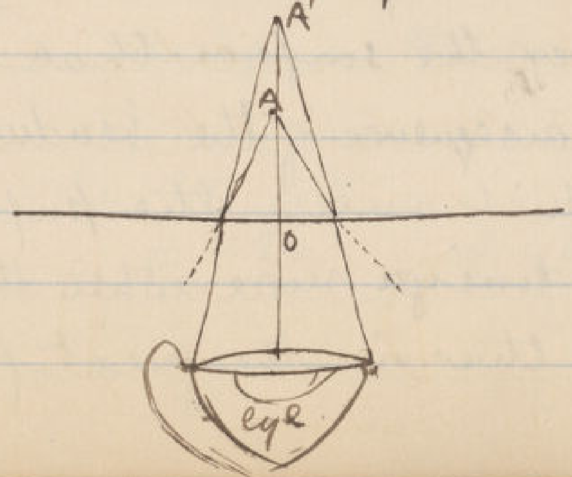
Meniscus

Plano Convex
(one of the spheres of ∞ radius)

rays which come from a point A under water,



(the figure of course being much exaggerated)
appear from the divergence of the rays caused by the water to come from A', i.e. nearer the surface; in the case of water is A is four feet deep, it appears to be only 3.
If the eye is under water and A in the air it appears at A' farther off



$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{d+A}{2}}{\sin \frac{A}{2}}$$

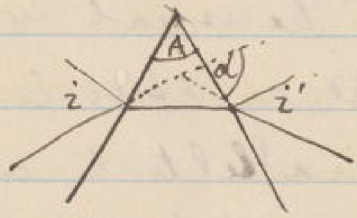
hence by a simple measurement of angles when the ~~lens is~~ prism in that particular position, we get find the index of refraction of the material of which the prism is made.

The position of objects seen ~~at~~ through a refracting medium appears different, for all the rays from the object except the one $\perp^r$ to the surface of the medium, are bent at that surface, and the object appears to be in a different position.

Looking at an object, the rays from which come through a refracting medium, $\perp^{ly}$ it appears nearer the surface than it really is: this is in consequence of the bending of the cone of rays which enters the pupil: they are made to diverge more when they leave the medium; thus in the next figure the

$$d = i + i' - (r + r')$$
$$d = i + i' - MA'N$$
$$= i + i' - A.$$

If the path of the light within the medium is $\perp^r$ to the bisector of the angle A i.e. if the angles of entering & leaving the medium be equal, we have the least deviation.



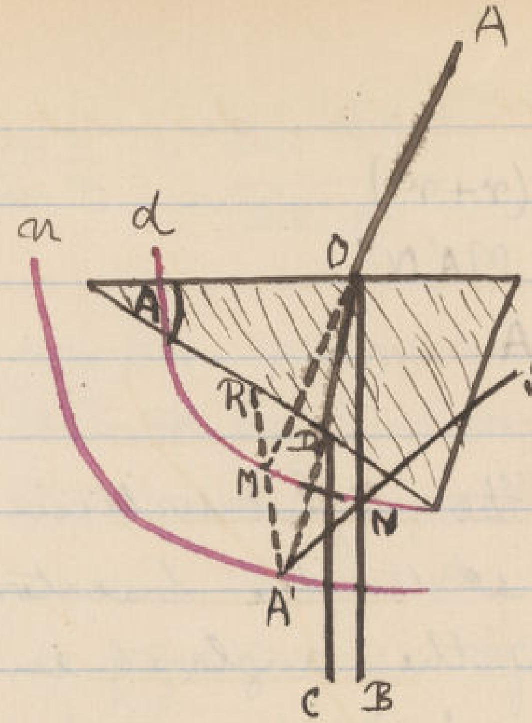
then $d = 2i - A$

$A = r + r'$

and therefore in this case $= 2r$

so

$$d = 2i - 2r$$
$$i = \frac{d + 2r}{2} = \frac{d + A}{2}$$
$$r = \frac{A}{2}$$



AO is the incident ray; the path of the ray (OD) inside the prism is found in the usual way; OB is the direction of the refracted ray after leaving the second surface, and DC parallel to it is the refracted ray: produce the $\perp^r$ A'M to meet the prism in R; join A'N & produce to S.

$\angle MON = d$ (deviation)

$\angle OMR = i$ (∠ of ~~incident ray~~)

$\angle OA'R = r$ (∠ of refraction)

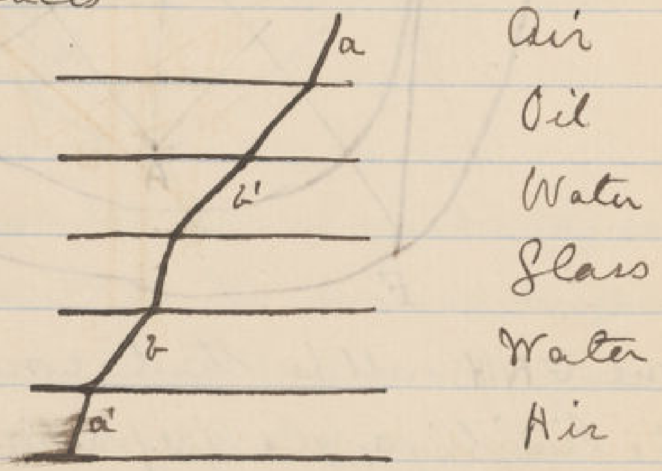
$i - r = MOA'$

~~A~~ $ONS = i'$ $\qquad i' - r' = NOA'$

$OA'S = r'$

back again thus giving diamond great brilliancy.

Suppose we have a number of media with parallel surfaces

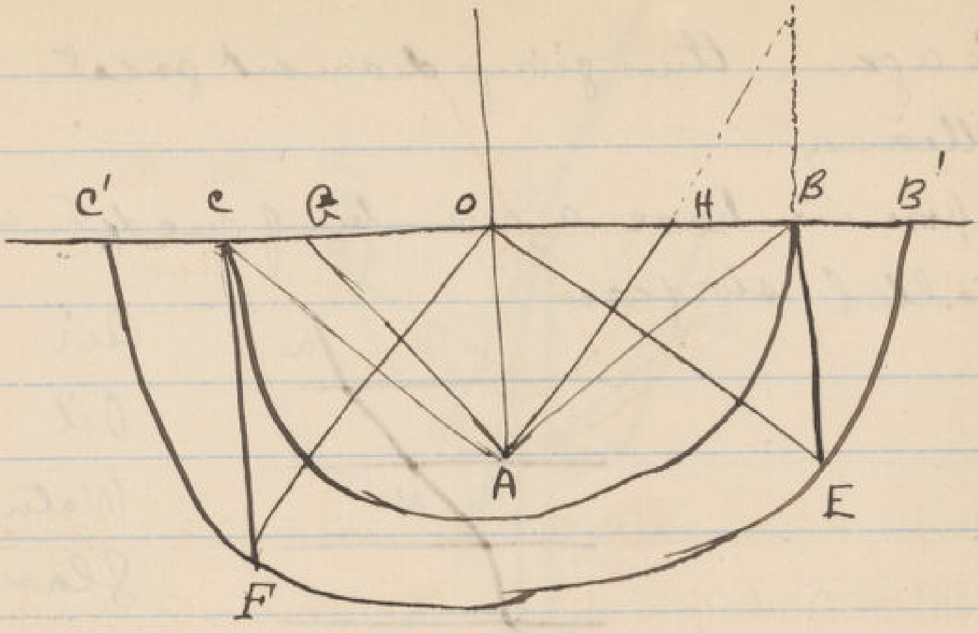


the paths in ~~similar strata~~ are always parallel ~~to one another; in the figure a || a', b || b'~~ etc.

The index of refraction of air to a vacuum is very small viz .000243.

Feb. 6th 1878.

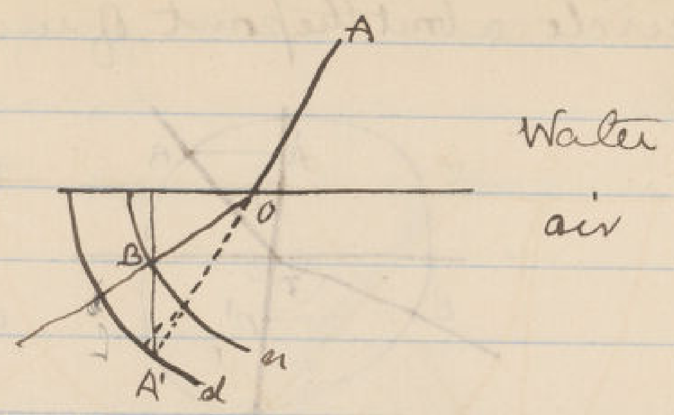
When the sides of a medium are not parallel we get a permanent refraction; suppose we have a triangular prism of water: the figure shows the path of the ray AO.



the cone GAH will be that cone: the angle OAH is the limiting angle of refraction, which is that angle whose sine is the index of refraction from a denser to a rarer medium (i.e $\frac{3}{4}$)

with water this angle is $48°35'$

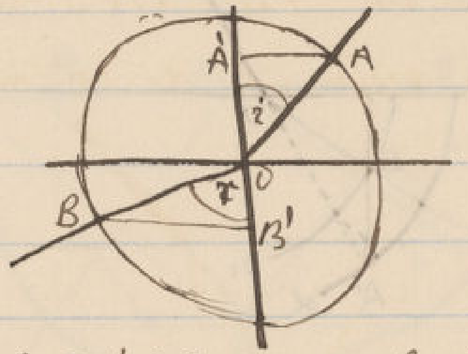
When we speak of index of refraction μ, that from a rarer to a denser medium is meant (i.e. $\frac{4}{3}$); i.e. Limiting angle of refraction is the angle whose sine is $\frac{1}{\mu}$: i.e. with water $\frac{1}{\frac{4}{3}} = \frac{3}{4}$; with glass $\frac{1}{\frac{3}{2}}$ i.e $\frac{2}{3}$: with glass the angle is $40°50'$; with sulphide of Carbon $36°31'$; with diamond whose index of refraction is $2\frac{1}{2}$, $23°53'$; i.e. very little light that gets in is refracted but most reflected



produce AO to meet the denominator (d) circles in A': from A drop a $\perp^r$ on the bounding surface; cutting the numerator circle (n) in B; join OB OB is the direction of the refracted ray.

If AO has more than a certain amount of obliquity the construction will fail: at a certain point A'B becomes a tangent, and then doesn't cut the n circle; when A'B is a tangent, the refracted ray lies along the ~~the~~ surface; when the obliquity is still greater, no refraction takes place but the whole of the light ~~passes~~ is reflected back by the surface of the water; i.e. when the ray is within a certain cone it gets into the air; when outside that cone it is reflected.

draw a circle about the point of incidence O.



water

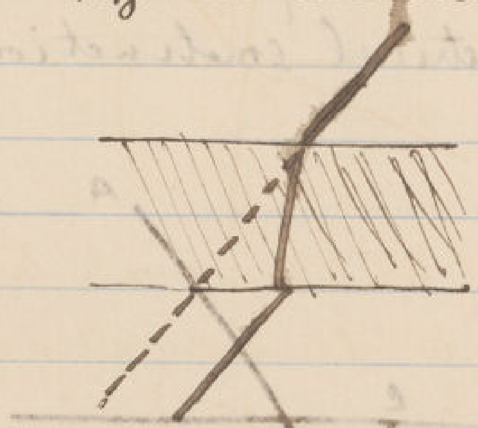
air

and draw $\perp^{rs}$ AA' & BB' to the normal ~~to~~ from the points where the circle cuts the incident & refractive rays respectively: $\frac{A'A}{BB'} = \frac{\sin i}{\sin r} = \frac{3}{4}$

If we know the incident ray & the index of refraction we can find the refracted ray; thus:- taking the case of light from water into a vacuum ~~or~~ or air which is for all purposes the same thing; about the point of incidence draw concentric circles whose diameters are in the proportion of the numerator (3) and denominator (4) of the index of refraction $(\frac{3}{4})$:

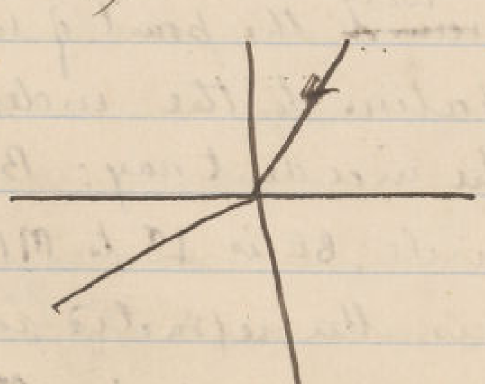
P.T.O

When light passes through another substance with // faces; it emerges in a direction parallel to the original direction.



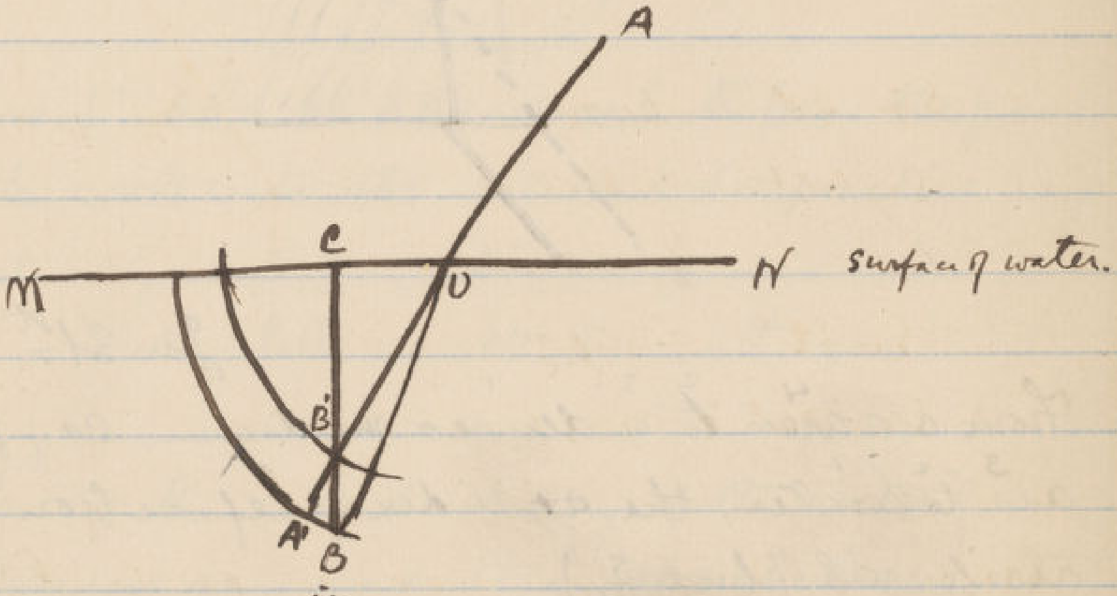
Jan 31st 1878.

From a rarer to a denser medium, say from air to water, the ~~ar~~ index of refraction is the reciprocal (here $\frac{3}{4}$)



i.e. the ray is bent from the normal.

When the ~~i~~index of refraction, & direction of incident ray, are known, the direction of the refracted ray can be found by calculation or by the following geometrical construction. (say for water)



draw circles ~~round~~ with the point of incidence as centre in proportion to the index of refraction i.e. 4:3; AO is the incident ray: B' is where it cuts the inner circle, BC is $\perp^r$ to MN (surface of water) through B'; OB is the refracted ray.

Proof $i = OA'C$ $\qquad \sin i = \frac{OC}{OA}$

$r = OBC$ $\qquad \sin r = \frac{OC}{OB}$

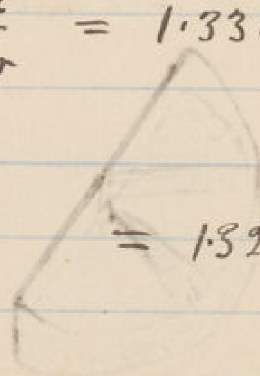
$$\frac{\sin i}{\sin r} = \frac{OB}{OA} = \frac{4}{3} \text{ (by construction) } = \mu .$$

In this we see nothing constant except for small angles when $\frac{i}{r}$ is constant.

Now take the sines

$\sin 10^\circ = \cdot 1736$ $\qquad \frac{\sin i}{\sin r} = 1{\cdot}330$

$\sin 7\frac{1}{2}' = \cdot 1305$



$\sin 40^\circ = \cdot 6428$ $\qquad = 1{\cdot}326$

$\sin 29' = \cdot 4848$

$\sin 82^\circ = \cdot 9903$ $\qquad = 1332$

$\sin 34^\circ = \cdot 7431$

$\frac{\sin i}{\sin r}$ is here very nearly constant; if the experiments had been more accurate they would have been quite constant. When angles are small, the sines vary as the angles; $\therefore \frac{i}{r}$ is constant for small angles as we saw. This constant quantity is different for different substances; for glass this is $\frac{4}{3}$, and is called the index of refraction.

air, and half through water: the degrees are numbered on the semicircle and the angles of incidence and refraction can be read off.

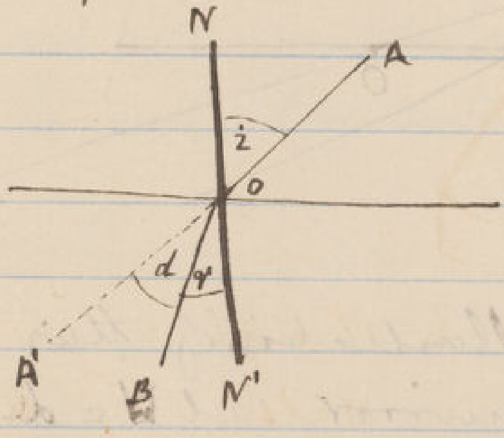
Angle through

Air i	Water r	d $i-r$	$\frac{i}{r}$
0°	0°.	0	–
10°	$7\frac{1}{2}$°	$2\frac{1}{2}$	1·333
20°	15°	5	1·333
30°	22	8	1·364
40°	29	11	1·380
50°	$35\frac{1}{4}$	$14\frac{3}{4}$	
60°	$40\frac{1}{2}$	$19\frac{1}{2}$	etc.
70°	$44\frac{3}{4}$	$25\frac{1}{4}$	
80°	$47\frac{3}{4}$	$32\frac{1}{4}$	
82°	48	34	1·709

Jan. 29th 1878

We now proceed to study the light which enters the surface: when light enters a surface it undergoes change of direction; this is called

Refraction

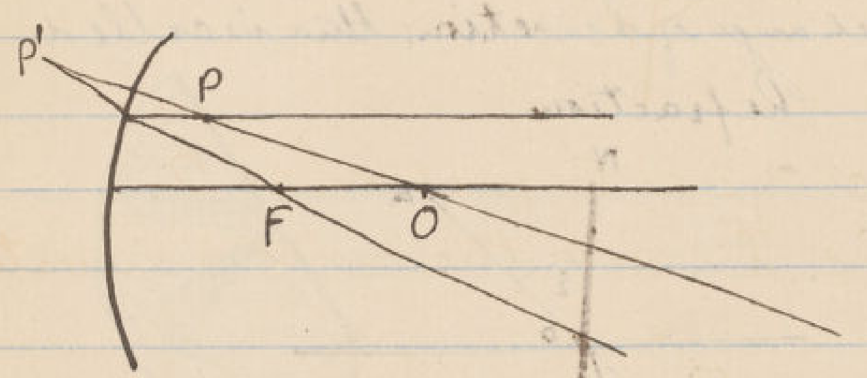


BOA' is the angle of deviation d
AON ——— incidence i
NOB ——— refraction r

$$d = i - r$$

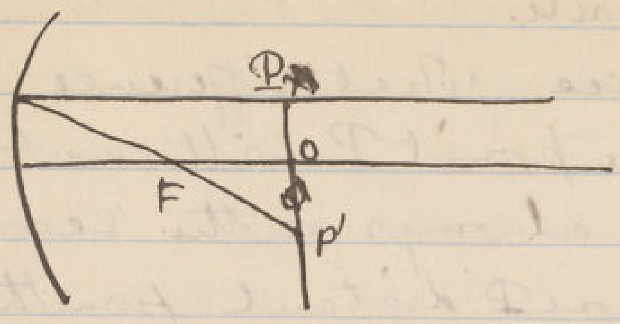
When the ray fall $\perp^{rly}$, it undergoes no refraction.

Compare the angles of refraction and incidence experimentally, by a semicircle half full of water: the light enters by a slit; ½ goes through on the other and when P is within the principal focus, the image is behind the mirror or virtual

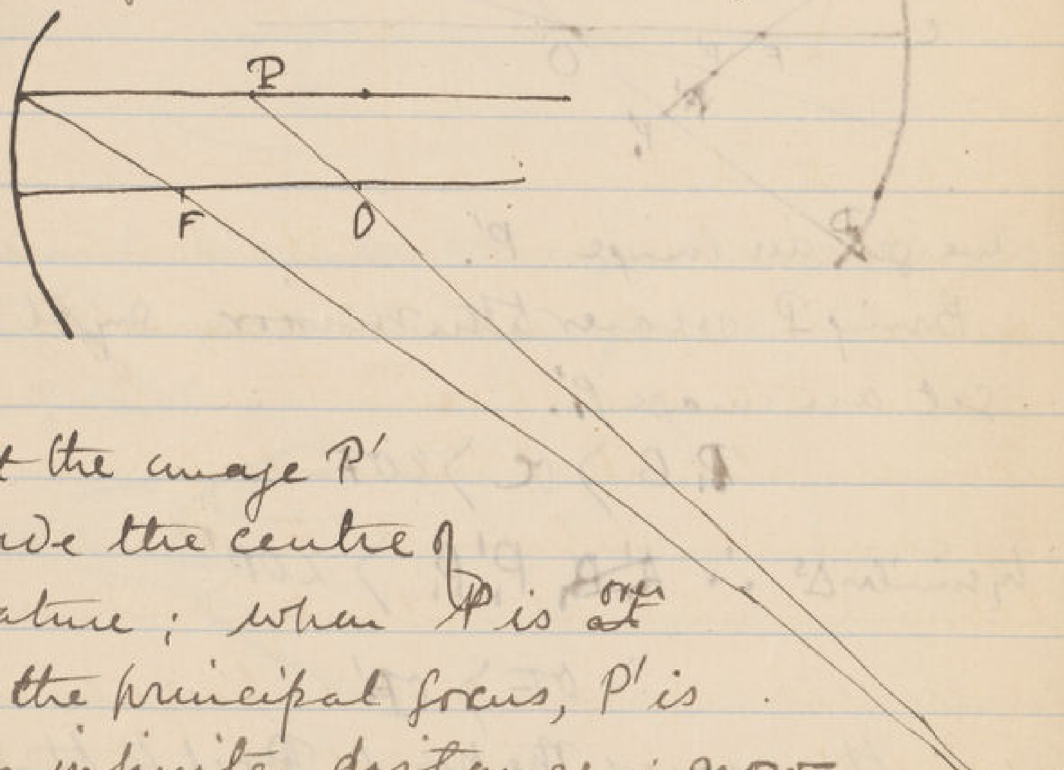


A practical illustration of this is to walk up to a spherical mirror; at a distance the image is real, inverted & diminished; walking nearer to the centre of curvature, the image advances also, till at the centre of curvature the two coincide: between O & F, the image is behind: at F at infinity, and within F it is real, upright and enlarged:
The same may be got from the formula

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

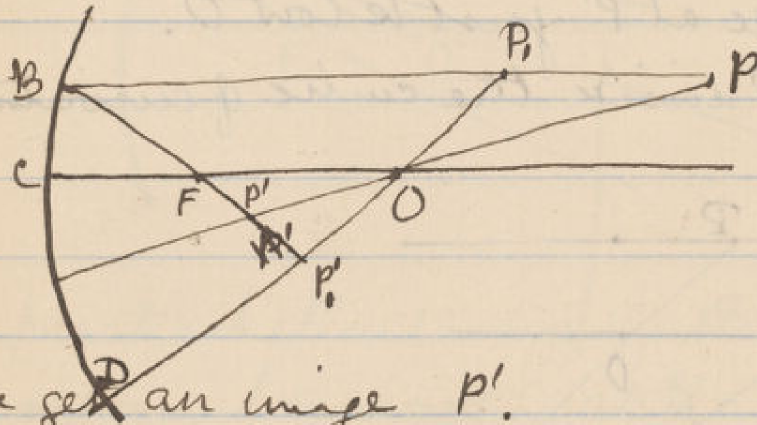


we get the image at P' just below O.
Now bring P inside the centre of curvature



we get the image P'
outside the centre of
curvature; when P is ~~at~~ over
~~the~~ the principal focus, P' is
at an infinite distance; now
suppose P to be within the principal
focus; infinity on one side changes to infinity

curves and a circle.
Let us now see what difference difference of position of the point P will make. The image will be always on the secondary axis: suppose P distant from the mirror



we get an image p'.
Bring P nearer to the mirror say at P_1; we get an image P_1'.

$$P_1B > OC > 2OF$$

by similar Δ^s ∴ $P_1'P_1 > 2OP'$

$$\therefore OF > OP'$$

i.e. the nearer the point P get to the mirror the nearer the image get to the centre O.
Suppose it be be just over O. (P.T.O)

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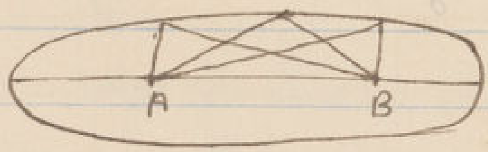
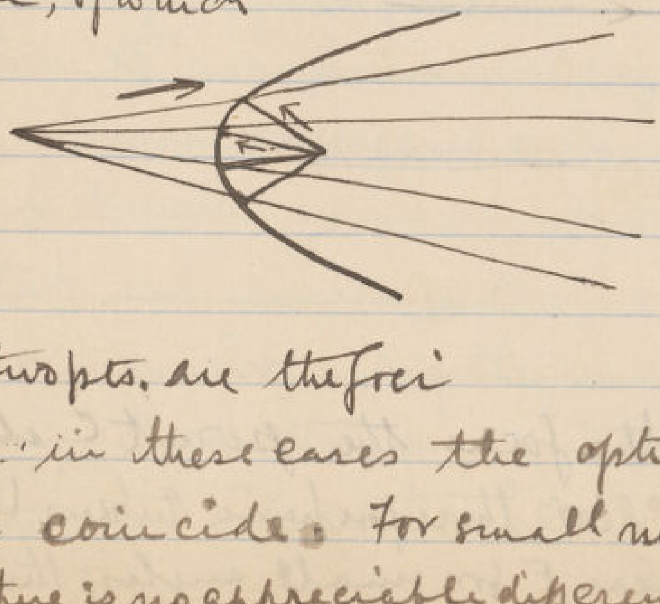
To have no aberration the curve must be more rapid: the exact curve required to bring all parallel rays to the same focus is a parabola. If we want to make rays diverging from a point A to converge to any other pt B; an ellipsal mirror must be taken of which A and B are the two foci.

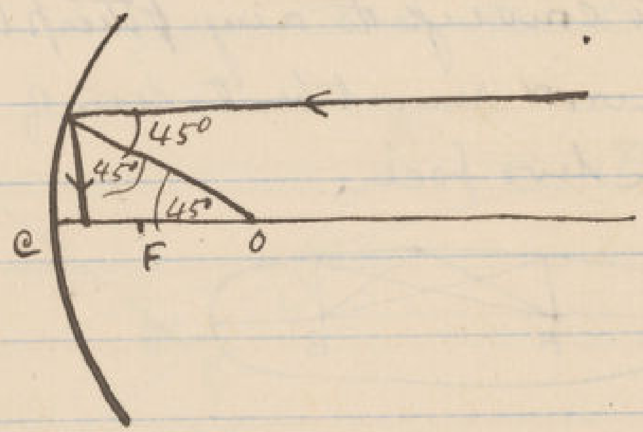


If we want to make convergent rays from a pt diverge from a focus: the hyperbola is the curve, of which

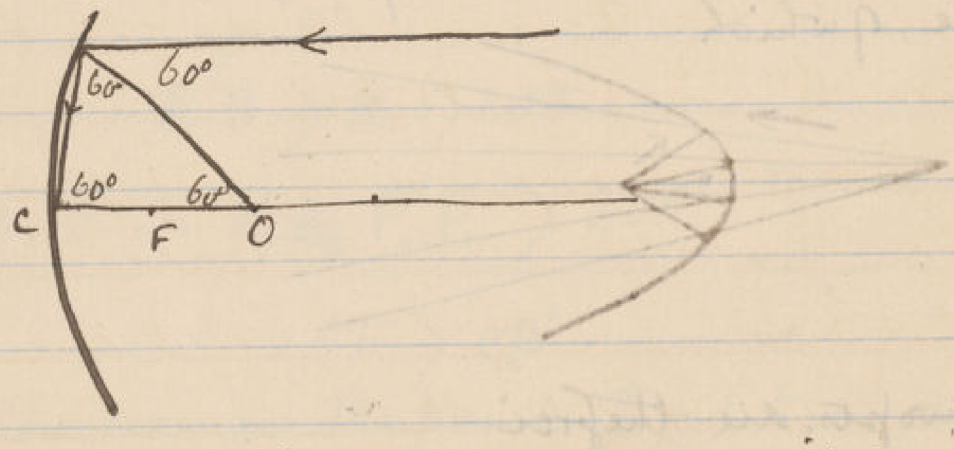


the two pts. are the foci
I.e. in these cases the optical & actual foci coincide. For small mirrors, in each case there is no appreciable difference between these

The formula $\frac{1}{q} + \frac{1}{p} = \frac{2}{r}$ we said was only true for small angles: this can be shown by taking a large angle - say 45°; the rays being parallel.



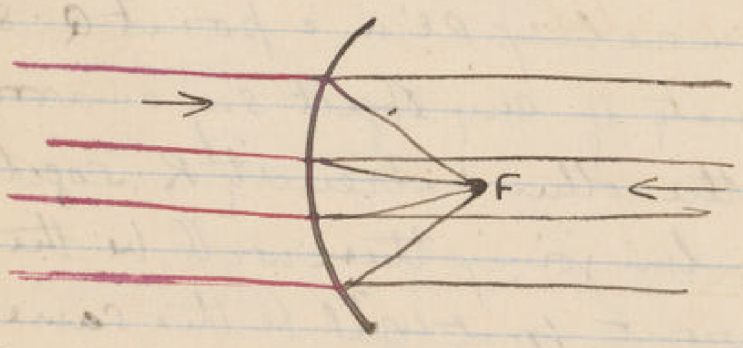
The figure shews the construction, and how far out the formula is: now take 60°



here we get for the focus the point C itself. Angles between 0° & 60° therefore have the focus between C & F; being very near F for small angles: this is called Spherical aberration.

Jan 24 · 1878.

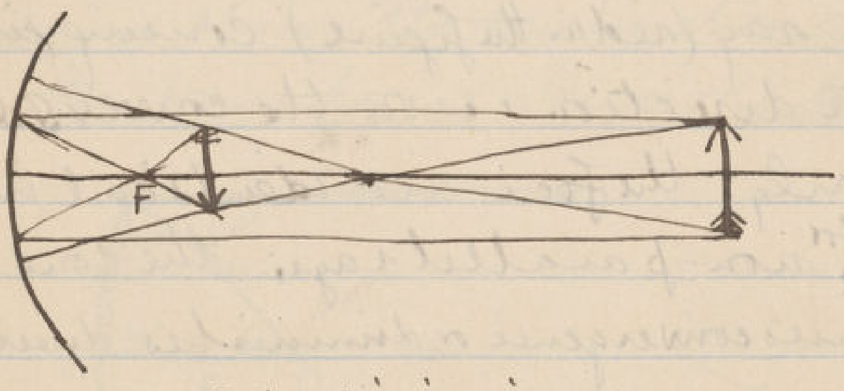
Now suppose the mirror to be convex: suppose we have parallel rays; we get a real



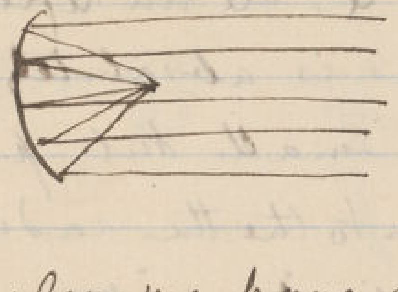
focus F for rays falling on the concave side; the focus will be the same but virtual for ray (red in the figure) coming in the opposite direction i.e. on the convex side. And similarly the foci are identical in position for non-parallel rays; the concave mirror increases convergence or diminishes divergence: the convex decreases ——— increases ——— ; so that we never get a real focus with a convex mirror: for if they are already diverging their divergence is increased; so that there are only virtual foci.

or geometrically thus:

through P draw a line representing a ray // to the principal axis; this is reflected through the principal focus, meeting PC in a point Q: suppose P to be the extremity of any object say an arrow; do the same for the other extremity R, so getting a point S: the line joining these will be the image of the arrow, for it will be the same as if we had done the same to each point of the arrow;

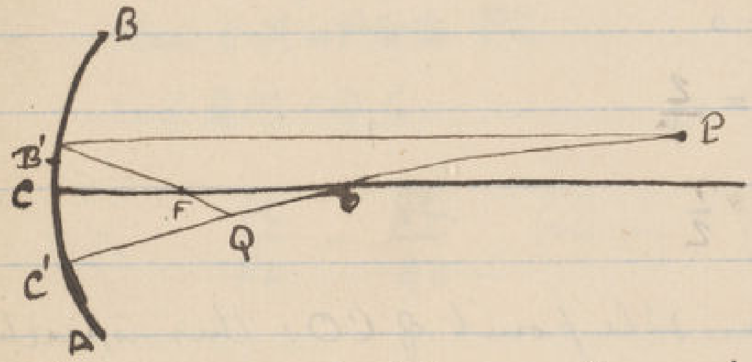


the image will be diminished and inverted. Here the light not only appears to come from the image as in a plane mirror but really does, and the image can be received upon a screen: it is therefore called a real image: while that on a plane mirror is called a virtual image if we put a luminous point at the principal focus, they a reflected parallel-ly; and meet at infinity



So that here also we have conjugate foci.

Suppose now P is a point outside the principal axis; we ~~p~~ want to find its focus



Join PO and produce it to meet the mirror; take C'B' = C'A then AB acts with regard to PC' as the AB to the principal axis OC; i.e. it is reflected back along the same straight line; the focus Q conjugate to P can be found by the equation

$$q = \frac{pr}{2p-r}$$

2 of the reflected rays we have seen pass thro' Q: therefore all do; conversely if the luminous point be put at Q; all the reflected rays would pass through P; this is absolutely true only if the mirror is infinitely small, but practically true when MC is less than $\frac{1}{6}$ to the the radius OC.

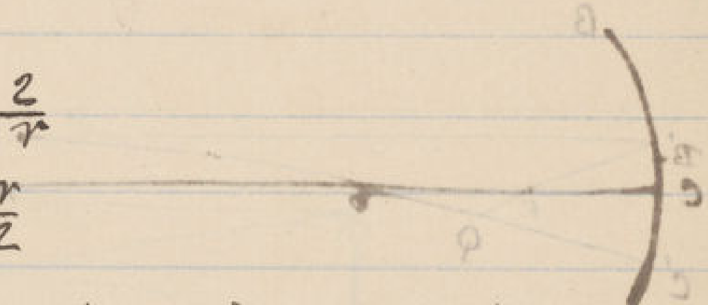
If P is at an infinite distance: $\frac{1}{p}$ is infinitely small

then $\frac{1}{q} + \frac{1}{p} = \frac{2}{r}$

is the same as

$$\frac{1}{q} = \frac{2}{r}$$

$$\text{or } q = \frac{r}{2}$$



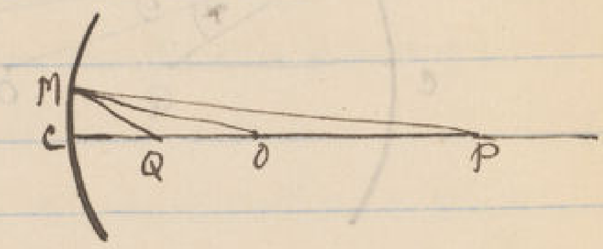
i.e Q is at the middle point of CO: this is practically true for the light from the heavenly bodies: i.e. when the rays of light are parallel. Q is the called the focus QC the focal length, and P and Q are conjugate foci because interchangeable. If Q is the middle pt: Q is called the principal focus, QC the principal focal length (f)

$$\frac{1}{q} + \frac{1}{p} = \frac{2}{r} \cdot \frac{1}{\frac{1}{2}r} = \frac{1}{f}$$

$\therefore$

$$\frac{\sin POM}{\sin i} = \frac{\sin QOM}{\sin r}$$

$$\therefore \quad \frac{PM}{PO} = \frac{QM}{QO}$$



Calc PC, p.
QC, q.
OC, r.

Suppose the angle MPC to be very small

then $PM = \cancel{PQ} \; PC$

& $QM = QC$

i.e. $\frac{PC}{PO} = \frac{QC}{QO}$ (nearly)

or $\frac{p}{p-r} = \frac{q}{r-q}$

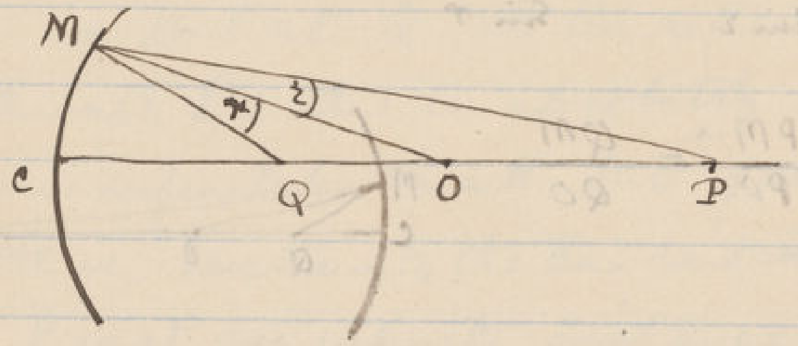
i.e. $pr - pq = pq - qr$

or $pr + qr = 2pq$

divide by pqr

then $\frac{1}{q} + \frac{1}{p} = \frac{2}{r}$

Suppose there is a luminous pt P on the principal axis



Suppose a ray from P to fall direct from P in a direction PC on the mirror, it will be reflected back along the same line; suppose ~~any~~ a other ray to fall on any other pt M of the mirror; OM will be the normal; PMO will be the angle of incidence make an angle OMQ = PMO: OMQ will be the angle of reflection; all the lines are in the same plane therefore Q is a pt. in the principal axis: we want to find its position.

in the Δ PMO $\qquad \dfrac{PM}{PO} = \dfrac{\sin POM}{\sin PMO} = \dfrac{\sin POM}{\sin i}$

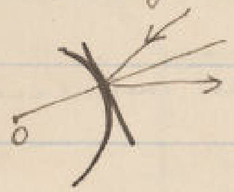
in the Δ OMQ $\qquad \dfrac{QM}{QO} = \dfrac{\sin QOM}{\sin QMO} = \dfrac{\sin QOM}{\sin r}$

and $\sin i = \sin r$ by hypothesis

and $\sin POM = \sin QOM$ $\angle$ POM & QOM being supplementary

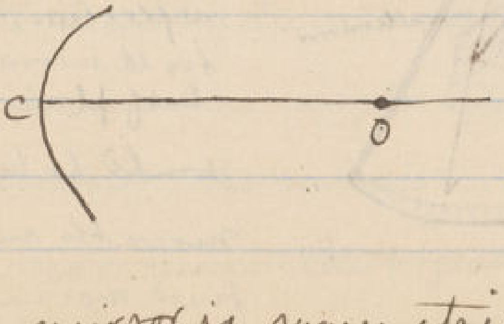
Jan 22. 1878.

We now proceed to study the reflection of light from curved surfaces; taking first the spherical mirror i.e. a mirror whose surface is part of a sphere; when a ray falls on a pt. on a spherical surface, it is ~~p~~ reflected as if it fell on a flat surface with that point in it; i.e. a surface tangent



to the sphere; the normal will be the production of the radius of the sphere passing through that point.

Take a section of a concave mirror with centre of figure at ~~Q~~ C, and the centre of the sphere at O



then the mirror is symmetrical about CO, which is called the principal axis.

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here $n = 90°$

$\frac{360}{n} = 4$ (i.e. once direct and 3 reflections)

if $n = 180°$ i.e. the mirrors be in a continuous plane

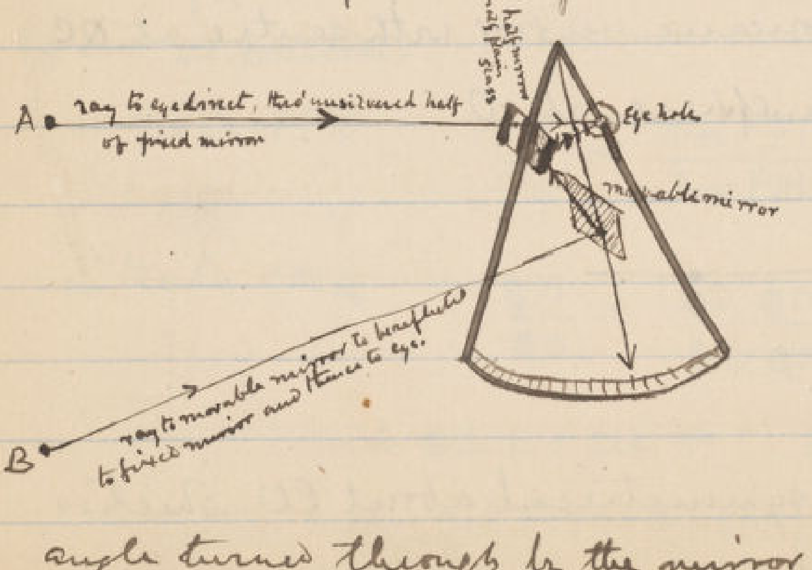
$\frac{360}{n} = 2$ (i.e. once direct and 1 reflection)

if ~~360°~~ 0°; i.e. if the mirror are parallel to one another

$\frac{360}{0} = \infty$; i.e. we ~~see~~ an immense number

the number only being finite by the images getting gradually dimmer and dimmer.

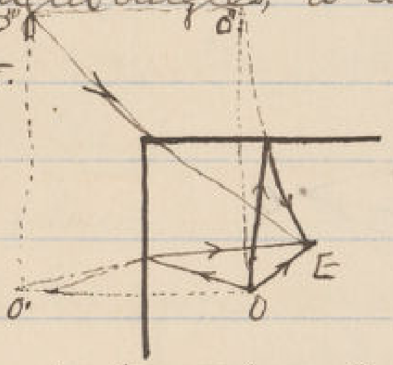
These principles ~~or~~ are applied to the Sextant or instrument for measuring the angle between distant objects.



The object is to get the two objects in the same horizontal st. line, one being seen direct, and the other by reflection; by means of a fixed mirror, ½ mirror and half plain glass; it would be better to put the movable mirror above the fixed one instead of below as in the figure. The angle turned through by the mirror will ½ as great turned through by the reflected ray.

This point O' is called the image of O; and doing the same for every point of any object, we get an image of that object:

With combinations of two mirrors various results are arrived at; e.g. suppose we have two mirrors at right angles, a luminous pt. at O, and an eye at E.



light could get to E from O in 4 ways; (1) direct; and from three image: (1) the reflection of O in the vertical mirror (2) the reflection of O in the horizontal mirror (3) the reflection in the vertical mirror of the reflection of O in the horizontal mirror, or what is the same thing, the reflection in the horizontal mirror of the reflection of O in the vertical mirror. Making a smaller angle between the mirrors we get more reflections: the rule is:-

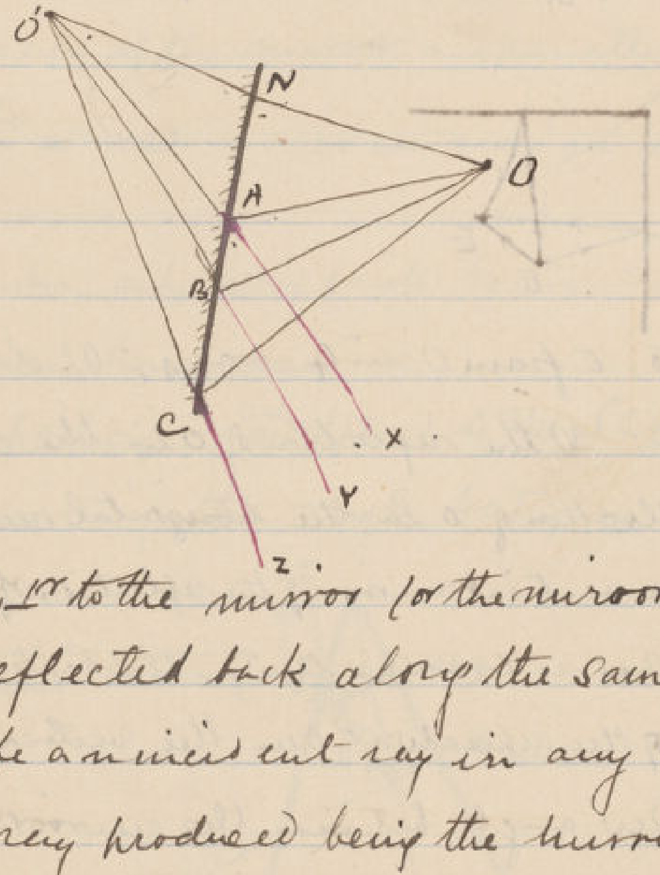
$\frac{360}{n}$ = no. of times the object is seen

n being the no of degrees ~~do~~ in the angle between the mirrors.

to come from one point the same distance behind the mirror as ~~as~~ the real luminous point is in front; and the line joining these two points in perpendicular to the mirror.

Jan. 17. 1878.

This can be shown thus

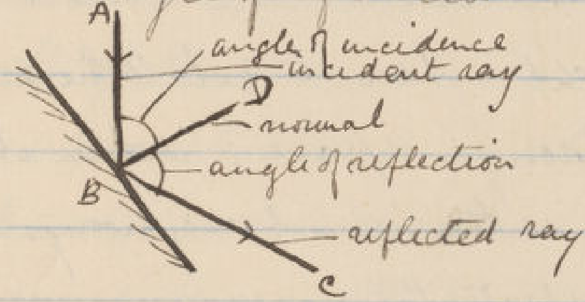


The ray ⊥r to the mirror (or the mirror produced) from a pt. O, is reflected back along the same straight line; let OA be an incident ray in any other direction; the reflected ray produced being the mirror will meet ON produced in a point O'; and by the law of reflection ∠CAX = ∠XAO = ∠NAO'; hence the Δs O'AN and OAN are equal i.e. O'N = ON. and so for all other rays: i.e. they are appear to proceed from O' after reflection.

The higher the polish of a surface, the less is the diffused light and the more reflected light; as a proof that light is diffused take this: if we only have one source of light in a room, we can see any object in the room from all parts of the room.

Reflection of Light.

The incident ray, the reflected ray and the normal at the point of incidence are all in the same plane; and the angle of incidence is equal to the angle of reflection



If the incident ray is DB, the angles of incidence & reflection = 0 i.e. the light is reflected back along the same path; in a ~~revolving~~ mirror rotating about its axis, as a consequence of this law the reflected ray moves twice as fast as the incident ray; after reflection from ~~in~~ a plane mirror, the light appears

$$\tan 20''.45 = \frac{E}{L}$$

$$L = \frac{E}{\tan 20''.45} = \frac{E}{\frac{1}{1088}} = 1088.E$$

circumference of the earth's orbit $= 2\pi \times 92,250,000$ miles

$$E \text{ or velocity of earth} = \frac{2\pi \times 92,250,000}{365 \text{ days. } 6 \text{ hrs. } 9 \text{ min. } 10\cdot388 \text{ seconds}}$$

$$L \text{ or velocity of light} = \frac{2\pi \times 92,250,000}{365 \text{ days. } 6 \text{ hrs. } 9 \text{ m. } 10.388 \text{ seconds}} \times 1088.$$

We now have to consider the question, what happens to light when it gets to the earth, after travelling through space; when it gets to a surface, the following is the number of ways it is divided.

I. Part penetrates the surface

A. Some transmitted thro' surface.

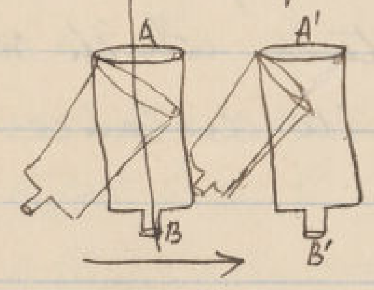
B. Some absorbed, i.e. extinguished as such.

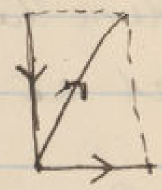
II. Part is thrown back.

A. Some bears a definite relation to the incident rays. Reflected light.

B. Some in all directions Diffused Light

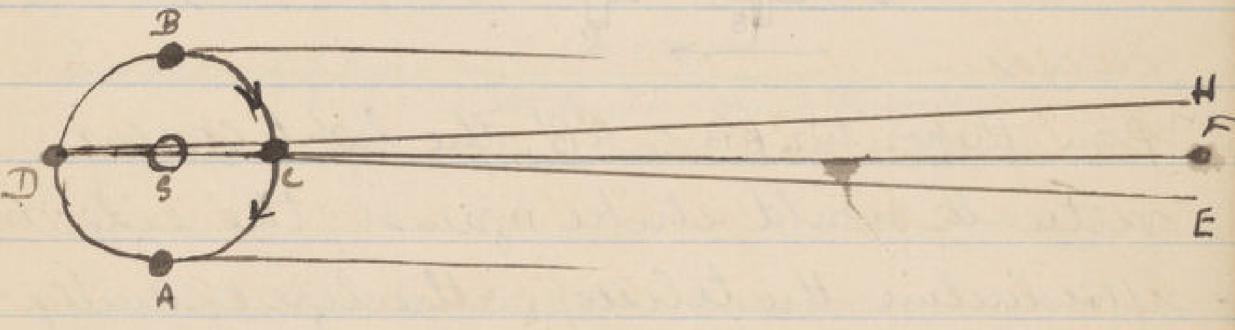
This is the same as when running against a vertical shower of rain, the drops appear coming in one's face; i.e. if we move the telescope





from the position AB to A'B', the light coming vertically would strike against the side; but if we incline the telescope the light coming vertically will go the length of the telescope not striking against the side; the light will thus appear to come in an oblique direction, or what is the same thing, the luminous pt will appear shifted in the direction in which the telescope is moved: the angle at which the telescope must be inclined is the same for all fixed stars and is 20".45. The tangent of this angle is the ratio between the velocity of the earth E, and and the velocity of light (L)

Bradley, the English Astronomer also determined the velocity of light by astronomical observations, in 1725. The observations were made on the fixed stars which may be considered at an infinite distance.



When the light from a fixed star F to the earth does not cross the earth's orbit round the sun; ~~eg.~~ ~~at~~ i.e. is at a tangent to it, the telescope at B or A must be pointed straight at the star; but at any point between B and A supposing the earth to move in the direction of the arrows; the star appears shifted in the direction in which the earth is moving; at C & D this shifting of the telescope will be the greatest; and this is called the angle of aberration; FCE & FDH in the figure getting nearer to the satellite, till when it is at A again, the eclipse is seen 16 minutes 26·4 seconds sooner than if the earth had remained at C; it is a year doing the whole revolution, and it takes 6 months therefore to get from C to A or from A to C. ~~tt~~ The reason of this lateness is of course that the light takes 16 m. 26·4 sec. to cross the earth's orbit; which though not circular is nearly so and may be taken as such; the question involves the distance of the earth from the sun which is not settled: it may be however taken in round numbers as

92,,250,000 miles

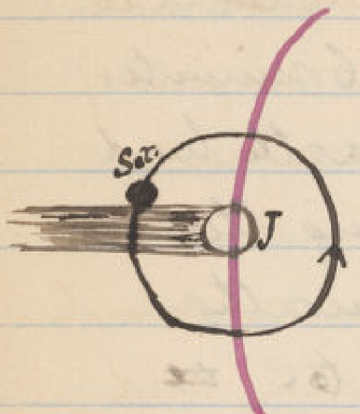
then twice this viz:

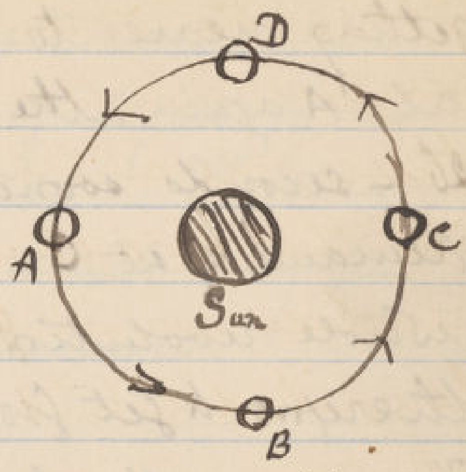
184,500,000 miles

is the diameter of the earth's orbit; and this divided by the time taken to cross it

$$\frac{184,600,000}{16\text{m. } 26{\cdot}4\text{ sec.}}$$

will give the velocity of light in miles & seconds.





The red line is the orbit of Jupiter round the sun: J represents Jupiter with its shadow which a satellite is just about to enter: A, B, C & D are various positions of the earth on its orbit round the sun. Suppose the Earth to be stationary at A; the time of revolution of the Satellite measured from the pt. when it enters the shadow of Jupiter will be always the same as the light from the satellite always has to travel the same distance; but the earth moves from A to C through B; the distance from the satellite is increasing consequently the light takes longer to travel or seen to the earth: and when the earth is at C; it is 16 minutes 26·4 seconds later than if the earth had stopped at A; the earth then travels through D to A from C gradually

1675-6; the observations were based on the eclipses of Jupiters satellites: there are four of these; one only need be taken into account, say the one nearest to the planet; this revolves round Jupiter in 42 hrs odd minutes; he wished to find the time of revolution; and let the point from where he counted the revolution to begin be the pt where the satellites began to be concealed by the shadow of the planet Jupiter; in one time of year he found it to be a certain time, and then for 6 months, the indication of the completion of the revolution arrived too late; this lateness increased as the 6 months progressed, and then for the next 6 months began to decrease until the lateness of the previous half year was just balanced; and for the whole year he got for his result what he had calculated. The following figure will show how this is:-

The light starts from a luminous body L, and falls upon a rapidly revolving mirror; from here it is reflected to another mirror slightly curved from there to another and so on for about half a dozen; it is reflected back along the same path, which is of known length; if the time it takes to travel is instantaneous, the light which falls on its return on the revolving mirror will find it in exactly the same place and will reflect the light back to L, but if it has moved through ever so small an angle during the time the light has been travelling, say in the position of the red line, the light will be reflected to a different place, in the direction coloured red.

The result he obtained was:-

298, 000 kilometres per second.

The velocity of light has also been ascertained by astronomical observations; some of them a very long time ago. The earliest were made by Römer, the Danish astronomer in

Jan 10th 1878

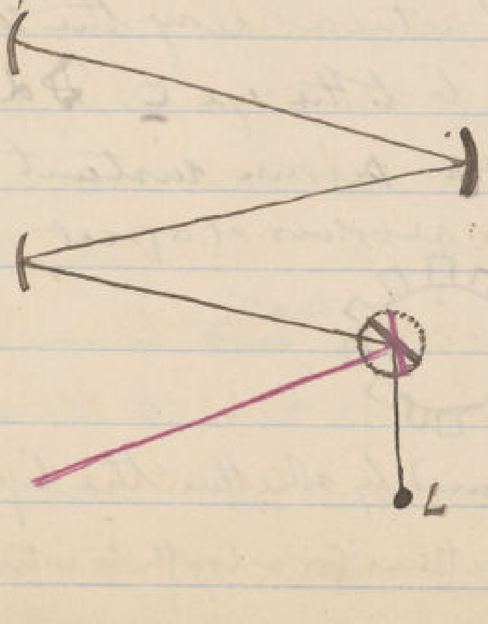
absent.

This is called Fizeau's method
The result was

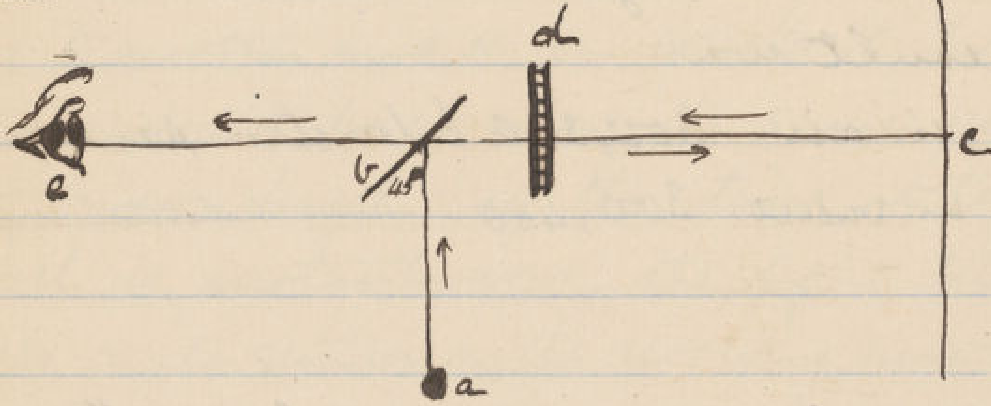
in air 300,330 kilometres per second
in vacuo 300,400

Jan 15th 1878.

Further experiments on a different plan were made by Foucault in 1862; the following is the principle of his experiment



a certain known instant, to let it fall on a mirror and thence come to the eye; this plan has been adopted



a is a powerful source of light; from it light falls on a sheet of plate glass, b placed at an angle of 45°; some light passes through and some is reflected to the mirror c, placed far away where it is reflected & returns along the same path some of it getting through b to the eye e. D d is the screen to take away at a known instant; this is a toothed wheel; which revolves at a great rate

and the velocity is measured by whether the light takes so much time to travel as there is time for a tooth to intercept it again.

Gas and candle, because the shadows are of different colours; with the grease spot this is less apparent.

We have got ~~that~~

$$I = \frac{L}{S}$$

Let B be the brightness of the source of Light

D = distance from it to illuminated object (screen)

$$I = \frac{B}{D^2}$$

$$B = I \cdot D^2$$

Of another source of light

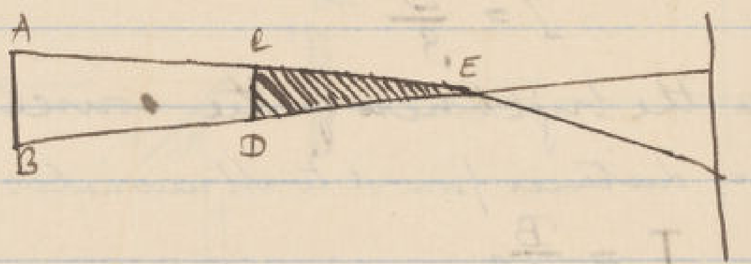
$$I' = \frac{B'}{D'^2}$$

If $I = I'$

$$\frac{B}{B'} = \frac{D^2}{D'^2}$$

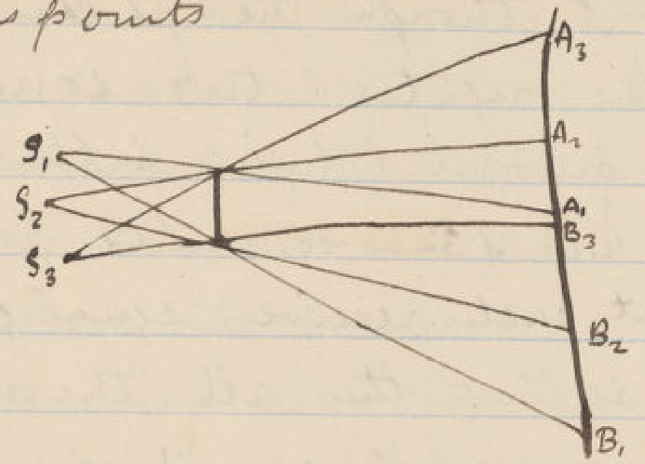
The propagation of light appears instantaneous but it is not really so. Measurements ~~of~~ have been made of its velocity. The most obvious way to measure ~~time~~ rate at which light travels ~~it~~ is to measure the time it takes to get from a luminous body to a screen; an improvement on this is to let the light from a luminous body be uncovered at

Suppose we have a source of light AB, and an object CD; the region of complete darkness will be a cone CDE



To measure the relative intensities of sources of light there are two chief ways: one is to cast shadows on the screen, and get them exactly of the same degree of darkness by moving the source of light to proper positions, and then measure the distance of each from the ~~screen~~ opaque object, and squaring the result to get the intensity; e.g. if we have a candlestick with 1 candle & another with 4 candles in it; to give the same depth of shadow the latter candle must be twice as far off from the opaque object as the one candle. Another way is to a grease spot on a paper screen invisible from either side and then measuring in the same way. There is one difficulty in different sorts of light, e.g.

its own effect. Suppose for example we take three luminous points



$A_1 B_3$ will be completely dark, and we pass from ~~E~~ complete darkness to complete light in three intermediate steps; if we increase the number of points we increase the number of intermed-iate stages; but if ~~an in~~ any source of light there is an infinite number of points, the number of intermediate stages is ~~great~~ infinite also i.e. we pass gradually from complete light to complete darkness; the nearer we move the opaque object to the screen, the larger proportion of the whole the completely dark portion forms, and the narrower the margin: and vice versa.

There is no absolute means of measuring quantity of light; though the eye can tell which is the brighter of two sources of light. Let L be the amount of light falling on surfaces 36, 144, & 324 sq. inches, placed in such positions that each receives equal quantities of light: i.e. so that they all throw the same sized shadow: (these positions will be at distance in proportion 1, 2, 3 from the source of light.

$$\left.\begin{aligned} I_1 &= \frac{L}{36} = \frac{L}{6^2} \\ I_2 &= \frac{L}{144} = \frac{L}{12^2} \\ I_2 &= \frac{L}{324} = \frac{L}{18^2} \end{aligned}\right\} \therefore \begin{aligned} \frac{I_1}{I_2} &= \frac{4}{1} \\ \frac{I_2}{I_3} &= \frac{9}{4} \end{aligned}$$

$$\text{i.e. } I_1 : I_2 : I_3 = 9 : 4 : 1$$

which is in accordance with the rule on the last page. The nearer an object is to a source of light the sharper is the outline; this is because the source of light is not a mere luminous point, and each point produces vary as their distances from O the luminous point; and therefore as the squares of their areas.

This may be shown by taking opaque screens whose areas are in the proportion 1, 4, 9, 16 etc, and placing them respectively at distances 1, 2, 3, 4, etc from the luminous body, and it will be seen that they throw shadows of the same size; i.e. they cut off equal quantities of light; i.e. ~~the~~ equal quantities of light fall upon them; the light being spread over areas which get gradually larger in the proportion of the square of their distance from the luminous point; the intensity of the light must vary in the inverse proportion.

Jan 8th. 1878

Let I = degree of illumination

L = amount of light incident on illuminated surface.

S = area of surface,

then $I = \frac{L}{S}$

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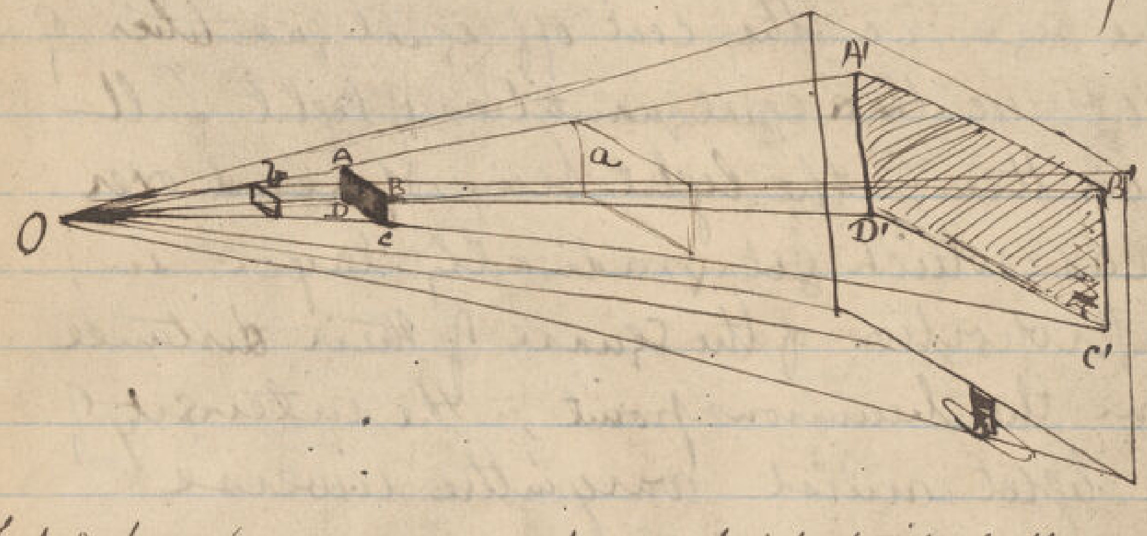
If an opaque object is put between the source of light, and a surface which would be illuminated if the opaque body were not there; we get what is called a shadow of the body; the shadow being the unilluminated portion. This is one of the best proofs that light proceeds from a luminous point in all directions equally:



Let O be a luminous pt. from which light falls on every part of a screen: place in the way, an opaque object ABCD, this cuts off a quantity of light, and forms a shadow A'B'C'D' on the screen; but a larger object at a, or a smaller one at b would cut off the same quantity of light and form a shadow of the same size. The linear dimensions of those objects would

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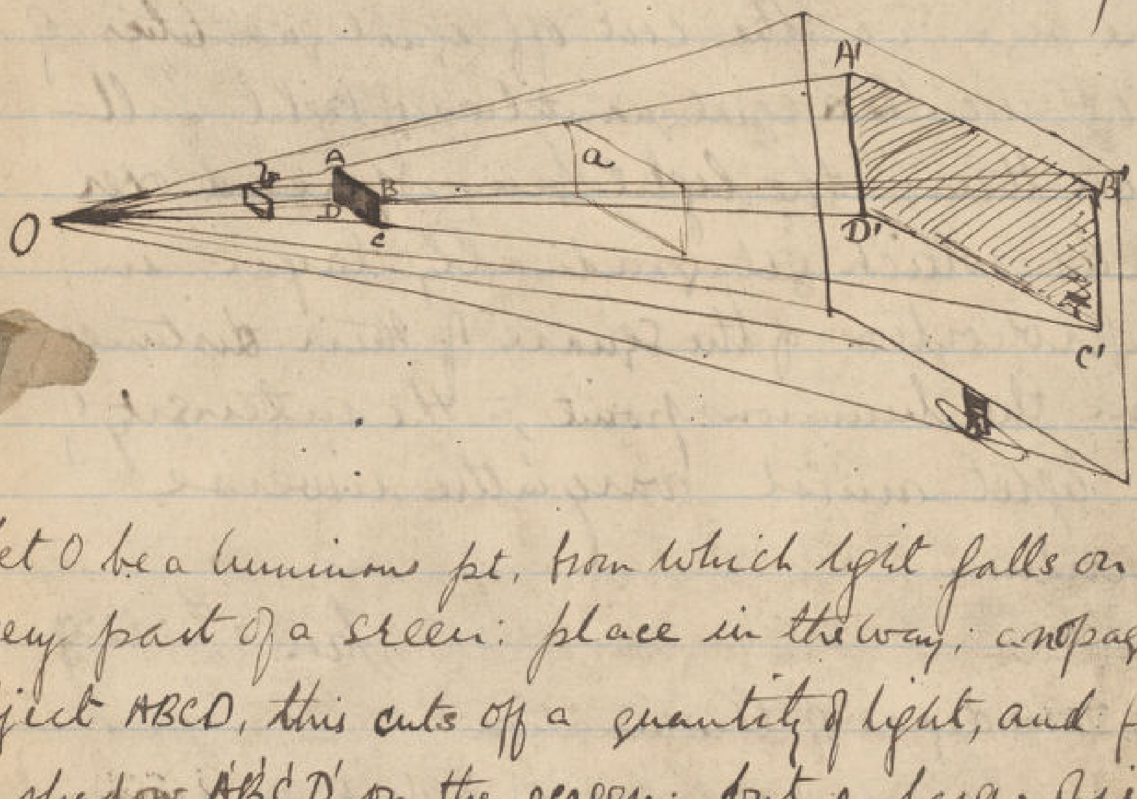

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PHYSICS