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1796 The great French classical work by
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1808 The first prof[illegible] of Med. Jurisp. made
[illegible] D. Andrew Duncan at Edinburgh.
1820 great work of M. Paris and M. Fon-
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1830 Beck's work published in America.
1836 Taylor's work published
1837 First [illegible] published in England.
D. Dixon [illegible] College, London.
that the [illegible] subject

MS. 2677

Notes of Lectures
delivered by
Professor F. Gavey Foster, B.A., F.R.S.,
Fellow of University College, London.
at
University College
London
on Mechanics (Division A. Junior Physics Class)
during the
Session 1877-1878 A.D.

W. D. Halliburton
University College
London, W.C.

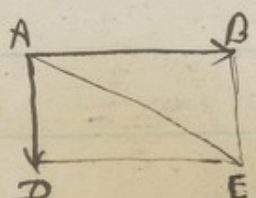
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Mechanics.

October 8th 1877.

A body of itself has no power to move, or if moving no power to stop or change its direction or speed. These effects are produced by forces. A force is that which tends to cause, alter or stop the motion of matter. Two of the commonest forces are gravity, the force which causes a body to fall to the earth, & muscular force. Force can be measured by the velocity it causes on a unit mass in a unit time.

Change of direction is effected by the same causes as change of speed or velocity. When two force act together on a body the effect of the two is the same as if each acted separately upon the body at rest. For instance if a force tends to move a body in the direction AB & another in the direction AD, it will move as if first AB & then AD (or vice versa) acted upon it



And be at the end of the time during which the force has acted ~~at~~ at E having moved in a curve AE; i.e. it had tendencies to move in two directions at once, and the result is the same as if it had first moved along AB & then along BE, or first along AD & then along DE. The fact that a body falls to the ~~ear~~ earth is not an inherent quality of the body but the effect of the force of gravity. That it is not so is seen from the fact that in different parts of the earth's surface bodies fall in ~~exactly~~ the different directions; e.g. at the antipodes in exactly the opposite direction. The speed also varies in different parts of the earth's surface; e.g. after one second of falling the speed at

The North Pole is 32.23 feet per second
at London 32.19 ——— ———
at the Equator 32.08 ———————
etc.

At different elevations also this speed varies.
When a body is tendipto move in two different directions, one is simply superadded & the other acts quite independently of it; i.e. if from the same pt.; two bodies be allowed to dispto the earth, one directly and the other having a horizontal velocity as well, they will both reach the earth at the same instant.

Velocity or speed implies space traversed & time taken to traverse the space. It varies in direct proportion as the distance and inversely as the time in which that distance is travelled.

Example velocity 1 = 1 mile in 1 hour
or in more convenient and smaller units
= 1 foot in 1 second.

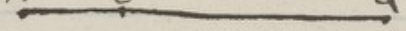
2 miles in 1 hour is a velocity twice as great
1 mile in 2 hour - - - - - - - - half - - - .

Expressing this fact otherwise we have

$$v = \text{velocity} = \frac{\text{distance or space}}{\text{time}} = \frac{S}{t}$$

{ taking the units of distance & time.

When $\frac{s}{t}$ is the same for great or small distances, the velocity is constant. Velocity may however change as the body is moving; and this change may be regular as in the case of a body falling to the earth or moving down an inclined plane etc.



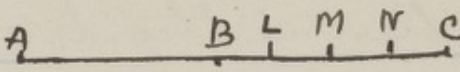
Suppose a body to move a distance = AO in the first interval of time & = AQ in the second interval of time. The change of velocity is a distance = OQ

$\frac{\text{change of velocity}}{\text{time}}$ = rate of change of velocity

Velocity = rate of change of distance

$\frac{\text{velocity at Q - velocity at O}}{\text{time}} = \frac{v_Q - v_O}{t}$ = rate of change of velocity = what is called <u>acceleration</u>.

Let a body move along the st. line AD,



at one point of time it will be at B, and at another at C: the change of distance $= AC - AB = BC$, and the velocity $= v = \frac{AC - AB}{\text{time}} = \frac{BC}{t}$ = the rate of change of distance. This is the average velocity: which may be shown thus. Divide BC into a number of ~~equal~~ parts BL, LM, MN, NC and let

$$t_{BL} = t_{LM} = t_{MN} = t_{NC} = T$$

$$v_1 = \frac{BL}{t_{BL}}, \quad v_2 = \frac{LM}{t_{LM}}, \quad v_3 = \frac{MN}{t_{MN}}, \quad v_4 = \frac{NC}{t_{NC}}$$

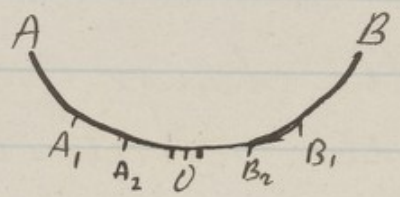
$$\frac{BL + LM + MN + NC}{T} = \frac{BC}{T} = v_1 + v_2 + v_3 + v_4$$

$$\frac{t}{4} = T \quad \text{i.e.} \quad t = 4T$$

$$v = \frac{BC}{t} = \frac{BC}{4T} = \frac{v_1 + v_2 + v_3 + v_4}{4} = \text{average velocity.}$$

Average velocity must be distinguished from velocity at any instant.

The difference between the two is seen in the case of the pendulum.



Let AB be the path of the pendulum. At the highest pts A & B the velocity is ~~greatest~~ least, at O it is ~~least~~ greatest; varying the whole time getting faster on the downward & slower on the upward course. Taking points nearer & nearer to O; we get that the limit of $\frac{AB}{t}$ as t diminishes without limit is the average velocity.

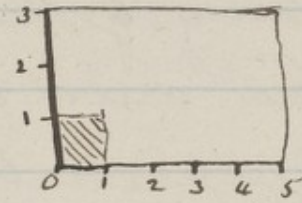
(in all cases) average velocity × time = space in ~~f~~uniform velocity

velocity × time = space

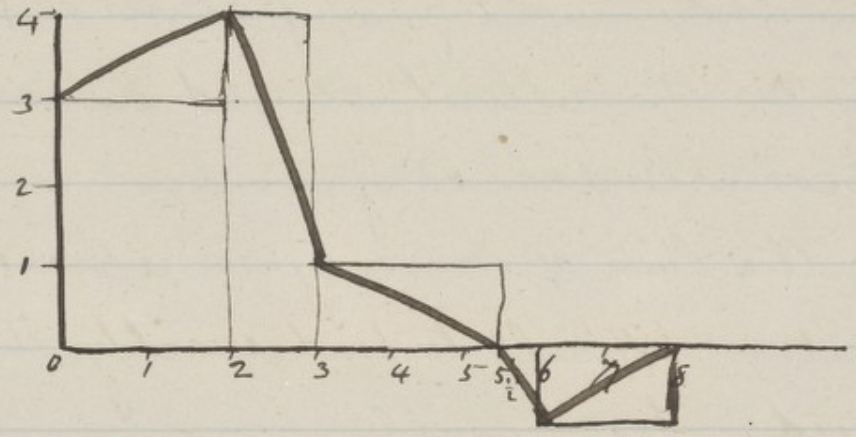
or $S = vt$.

Let $v = 3$ & $t = 5$, $S = 15$; this can be

represented graphically by a rectangle with sides respectively equal to v & t; the whole area of the rectangle being equal to the space.



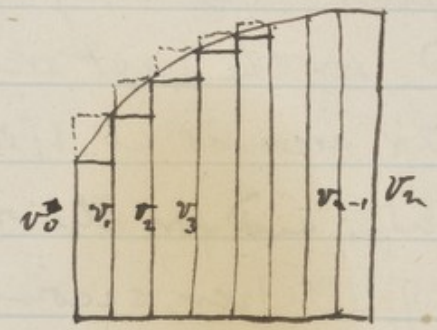
S = the area of the rectangle is 15 times the area of the unit of space or the square when both v & t = 1.



Suppose for three seconds a body moves at the rate of 3 ft. per second; for the next second at 4 ft per second, for the next 2½ seconds at 1 foot per sec., then for ½ a second no velocity; and for the next two seconds backwards at the rate of 1 ft. per second; the above figure can represent all this; and total space found by adding the areas of the rectangles together & subtracting those below

the line. Space can only be represented by rectangles in this way When the velocity is ~~changing~~ constant; when the velocity varies uniformly, i.e. gradually getting greater or less the upper lines must be sloping not parallel to line representing time; that is we get a figure represented by the slanting thick lines. If not uniformly increasing they might go up & down or be curved according as the velocity varied in each successive infinitesimal period of time.

When the line of velocity is represented by a curve, it is difficult to find exactly the space traversed. For example



Suppose v_0 & v_n to be the lines representing the initial and final velocities, and the curve line the line of velocity; we have to find the space traversed which is represented by the area of the figure. Divide the time line into n equal parts, and from the points of division draw the L^{rs} to represent the velocities at those particular points; complete rectangles with altitudes respectively equal to $v_0, v_1, v_2, \ldots\ldots v_{n-1}$ and also rectangles with altitudes respectively $v_1, v_2, v_3 \ldots\ldots v_n$; add the areas of the first rectangles together; and call $\frac{t}{n} \cdot \tau$

$$v_0\tau + v_1\tau + v_2\tau + \ldots\ldots + v_{n-1}\tau$$

$$= (v_0 + v_1 + v_2 + \ldots\ldots + v_{n-1})\tau$$

$$= (v_0 + v_1 + v_2 + \ldots\ldots + v_{n-1})\frac{t}{n}$$

add together the areas of the other rectangles

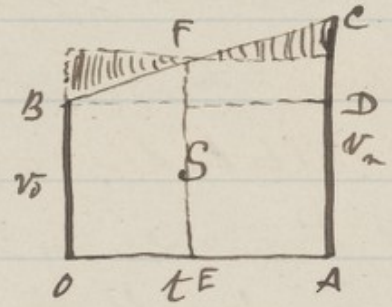
$(v_1 + v_2 + \ldots\ldots + v_n)\frac{t}{n}$; the true value is between these two, and by increasing the ~~values~~ value of n, we can get as near a result as possible.

Or the same result another way. The average velocity $= \frac{v_1 + v_2 + v_3 + \text{---} + v_n}{n}$

and S = av. velocity × time

$= \frac{v_1 + v_2 + \text{---} + v_n}{n} t.$

When however the velocity varies uniformly we get a simpler result.



Let OA represent the time, OB the velocity at the beginning of the time and AC the velocity at the end of the time; then supposing the velocity to increase uniformly the distance between the top line & the line of time must increase uniformly i.e. BC must be a straight line. OBCA represents the space ~~Act~~ traversed which we have to find. Draw through B

a line parallel to OA cutting AC in D. Then the whole figure = rectangle + a triangle

Area of rectangle = $OB \cdot OA$

---- triangle = $\frac{1}{2} DC \cdot BD$

$S = OB \cdot OA + \frac{1}{2} DC \cdot BD$ but $BD = OA$

$\therefore S = OB \cdot OA + \frac{1}{2} DC \cdot OA$

$= v_0 t + \frac{1}{2}(AC - AD) OA$

$= v_0 t + \frac{1}{2}(AC - OB) OA$

$= v_0 t + \frac{1}{2}(v_n - v_0) t$

$= \frac{1}{2} v_0 t + \frac{1}{2} v_n t$

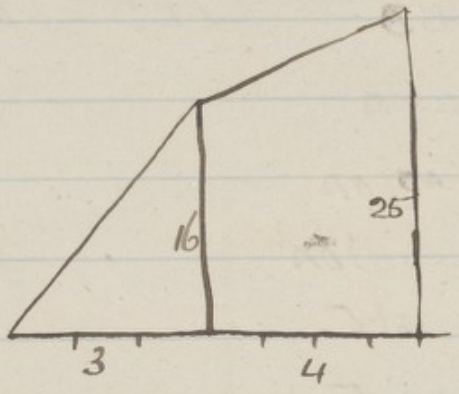
$= \frac{1}{2}(v_1 + v_n) t$

Or another way;

average velocity = velocity at middle of time; bisect OA in E; and draw the perpendicular EF which represents the average velocity. Draw through F a parallel to AO meeting OB produced. The shaded triangles are equal and the space = rectangle with base OA and altitude = EF.

$S = EF \cdot OA$ = aver. vel. $\times t = \frac{1}{2}(v_0 + v_n) t$.

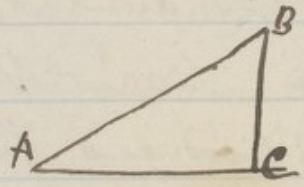
A train starts from rest, and in three minutes ~~gradually~~ uniformly acquires a velocity of 16 miles an hour; in the next four minutes ~~has a vel~~ increases uniformly to 25 miles an hour; find the space traversed?



$$S = \frac{3 \times 16}{2} + \frac{16+25}{2} \times 4$$

$$= \frac{48}{2} + \frac{41}{2} \times 4$$

$$= 24 + 82 = 106.$$



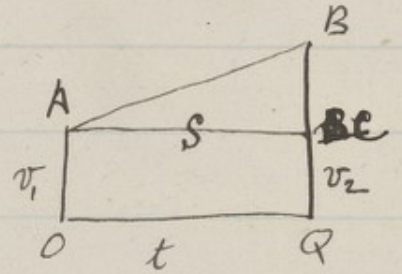
The change of velocity is represented by the slope of the line joining the initial & final velocities: steep if the velocity

increases fast and gradual if slowly.

~~rate~~ ~~slope~~ slope $= \frac{BC}{AC}$ ~~= change of velocity~~

~~change of velocity~~ ~~time~~ = ~~acceleration~~.

October 17. 1877



If we move from A to B we move from left to right the same distance as if we had moved from A to C.

If we move from A to B we move upwards the same distance as if we had moved from C to B

$\frac{CB}{AC}$ = slope (which ratio is independent of the position of A & B in the line AB)

acceleration = rate of change of velocity

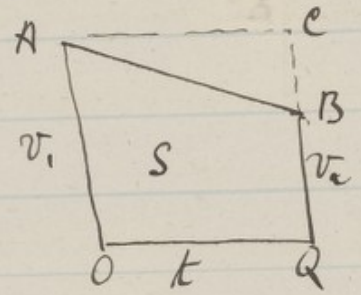
slope = rate of change of level.

$v_2 - v_1$ = change of velocity

$a = \frac{v_2 - v_1}{t}$ = rate of change of velocity

$= \frac{QB - OA}{OQ} = \frac{QB - QC}{AC} = \frac{CB}{AC}$ = Slope.

Which may be up or down according as the acceleration is increasing or diminishing.
Suppose the acceleration to be diminishing



we have a figure of this sort
A convention used here is one with regard to signs.
When distance is measured upwards the sign is +
---------- downwards ---------

here we have

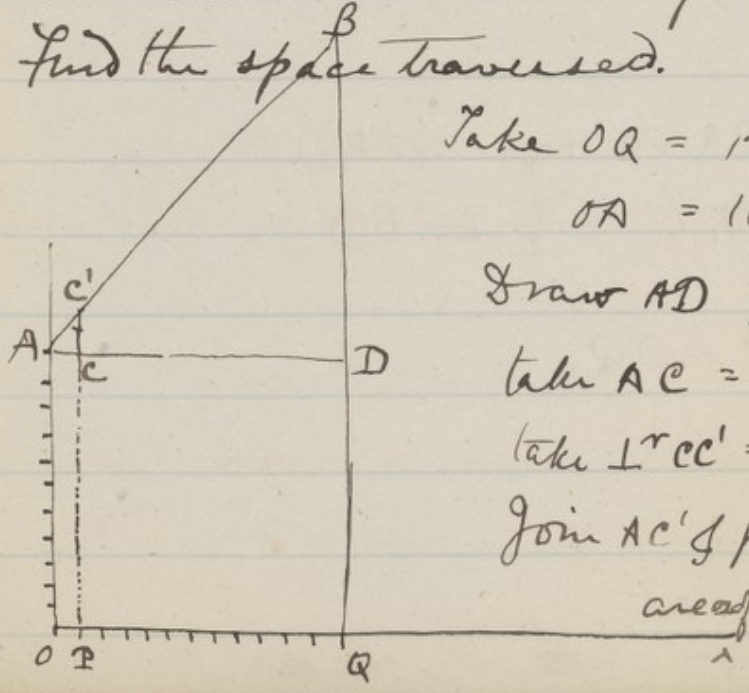
$-\frac{CB}{AC}$ = a neg. slope ∴ negative acceleration or a retardation.

or $a = \frac{v_2 - v_1}{t}$ $v_2 > v_1$ ∴ a is negative.

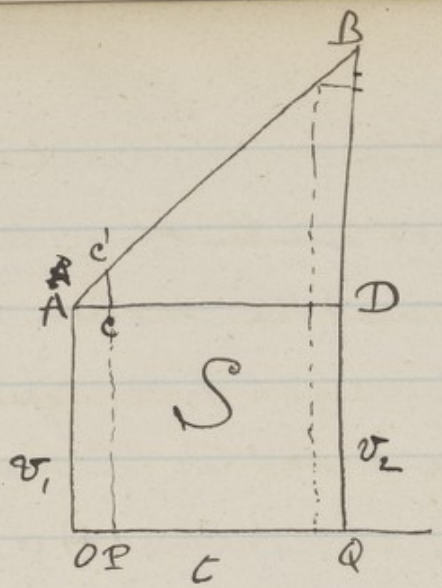
Suppose in each case (i.e. acceleration positive & negative) the final velocities are the same, the initial velocities also the same; and the motion to take place for the same time: we get two equal figures;
∴ the space traversed is the same whether acceleration is increasing (+) or diminishing (−).

Examples.

Suppose the acceleration of a body to be 2 (i.e. the velocity increases 2 units of length per unit of time in a unit of time, say 2 feet per second per second) the time to be 12 seconds
and the initial velocity 11 feet per second.
Find the space traversed.



Take OQ = 12 units
OA = 11 units
Draw AD parallel to OQ
take AC = 1 units
take ⊥r CC' = 2 units
Join AC' & produce to meet ⊥r from Q
area AOQB = space.



OP = AC represents 1 second

PC' − PC = CC' = change of velocity in 1 seconds

$\frac{CC'}{AC} = \frac{CC'}{1} = CC'$ = rate of change of velocity = slope = 2.

∴ the slope of the line AB is correct; i.e. the area of the figure AOQB represents the space

area of figure AOQB = $OA.OQ + \frac{1}{2} OQ.BD$

$S = v_1 t + \frac{1}{2} t.BD$

$\frac{DB}{CC'} = \frac{AD}{AC} = \frac{OQ}{AC} = \frac{OQ}{1} = OQ$

$DB = CC'.OQ$

$= a.t$

$S = v_1 t + \frac{1}{2} t.at$

$= v_1 t + \frac{1}{2} at^2$

in this example

$$S = 11 \times 12 + \tfrac{1}{2} . 12 . 2 . 12 .$$

$$= 132 + 144 = 276.$$

We have then these two expressions for S

$$S = v_1 t + \tfrac{1}{2} a t^2$$

$$S = \tfrac{1}{2} (v_1 + v_2) t$$

Suppose the final velocity (v_2) was given.

In this case

final velocity = DQ + DB = 11 + 24 = 35

Suppose the data in the question had been

Final velocity = 35

Acceleration = 2

Time = 12

We get the slope in the same way: measure 2 units down BD, and then at right angles 1 units & join. The proof is the same. & we get

$$S = v_2 t - \tfrac{1}{2} a t^2$$

i.e. if velocity increases the connecting sign is +

& ------ decreases ------ ------

let $\overline{V}$ stand for average velocity

$$\frac{S}{t} = \frac{1}{2}(v_1 + v_2) = \text{av. vel.} = \overline{V}$$

i.e.

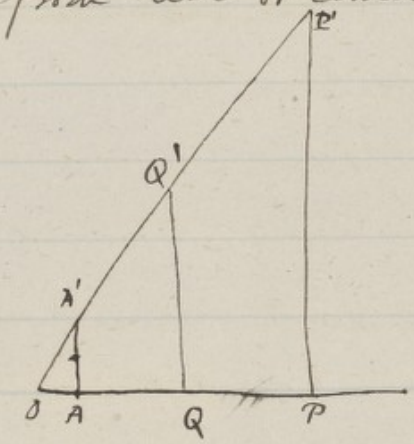
$$\overline{V} = \frac{v_1 + v_2}{2}$$

$$= v_1 + \tfrac{1}{2}at$$

$$= v_2 - \tfrac{1}{2}at$$

$$v_2 = v_1 + at.$$

The simplest case of uniform acceleration is when the body starts from rest or comes to rest.



Let OP represent the time, OA represent 1 second, AA' a velocity of 2 ft per second, then OA' produced will represent an acceleration of 2 ft per second, per second, and OPP' will represent the space traversed.

Let OP = 6 i.e. represent 6 seconds

$$\frac{PP'}{AA'} = \frac{OP}{OA} = \frac{6}{1}$$

$PP' = 6\,AA' = 12$ ft. per second = final velocity

$$S = \tfrac{1}{2} \times OP \times PP'$$
$$= \tfrac{1}{2} \times 6 \times 12 = 36 \text{ ft.}$$

If the body starts with a velocity of 12 ft per second, and comes to rest in 6 seconds, the space passed over will be the same viz 36 ft.

Let it now only move half the time; i.e. bisect OP in Q & erect the perpendicular QQ'.

$$S = \tfrac{1}{2} \times OQ \times QQ'$$

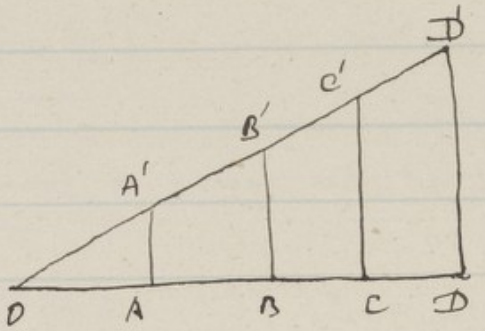
$OQ = \frac{1}{2} OP = 3$

$QQ = \frac{1}{2} PP' = 6$

$\therefore S = \tfrac{1}{2} \times 3 \times 6 = 9$ or $\frac{1}{4}$ of the former Space.

i.e. we have the space diminished by a half twice, viz: (1) because it only moves half the time, (2) because it only moves with $\frac{1}{2}$ the average velocity. i.e. the Space varies as the sq. of the time, or as the average velocity. The areas of similar figures are in the same proportion as

the squares of their linear dimensions.



In the case of triangles this is evident.

Let $OA = AB = BC = CD$

$$\frac{S_B}{S_A} = \frac{OB^2}{OA^2} = \frac{4}{1}$$

$$\frac{S_C}{S_B} = \frac{OC^2}{OB^2} = \frac{9}{4}$$

$$\frac{S_D}{S_C} = \frac{OD^2}{OC^2} = \frac{16}{9}$$

i.e. $S_A : S_B : S_C : S_D = 1 : 4 : 9 : 16.$

This law only applies when a body starts from rest. The simplest case of this, is that of a body falling

$$S = \tfrac{1}{2} a t^2$$
$$S' = \tfrac{1}{2} a t'^2$$

$$\frac{S}{S'} = \frac{\frac{1}{2} a t^2}{\frac{1}{2} a t'^2} = \frac{t^2}{t'^2}$$

This can be illustrated, either by allowing a body

to roll down an inclined plane, and measuring the distances passed in successive intervals, (as the force acting on it is the same all through) when they will be found in this proportion;

Times 1, 2, 3, 4, 5 ----

Spaces 1, 4, 9, 16, 25 ----

Or by Atwoods machine, when the same will be seen to take place.

Oct. 24th 1877

The Motion of a body while falling freely is uniformly accelerated. This may be proved experimentally thus: let a body fall at different heights, measure the times of falling by an electrical arrangement, by which a spark can be obtained at the instant of starting and at the instant of reaching the bottom of the given height; the time between these can be measured by the no. of vibrations of a tuning fork, made visible by a waved line ~~~~ on a blackened cylinder.

~~22~~

The following is the result obtained from actual experiment.

Heights	Times (vibrations)	Diff. of times	Average velocity	Diff. of av. vel.
12 inches	15·8 15·9		$\frac{12}{15·85} = ·76$	
		6·6		·31
24 inches	22·4 22·5		$\frac{24}{22·75} = 1·07$	
		9·3		·44
48 inches	32·0 31·5		$\frac{48}{31·75} = 1·51$	

$$\frac{\text{Increase of Av. Vel.}}{\text{Increase of time}} = \frac{·31}{6·6} = ·047 \text{ in 1st case}$$

$$= \frac{·44}{9·3} = ·047 \text{ in 2nd case}$$

i.e. the acceleration is uniform = 2 × ·047 = ·094

Final velocity = 2 ce the average velocity.

The acceleration of a falling body is generally stated in terms of feet and seconds, not in inches of vibrations. To convert one into the other is the next thing then.

1 vibration = $\frac{1}{64}$ second, 1 inch = $\frac{1}{12}$ foot.

$$a = \frac{v}{t} = \frac{2\frac{s}{t}}{t} = \frac{2s}{t^2}$$

$$= 094 \div 12 \times 64 \times 64$$

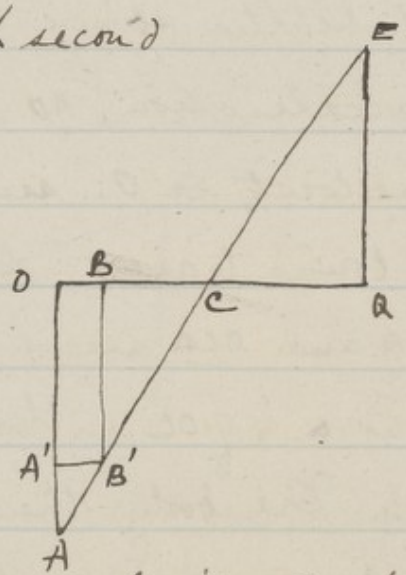
$$= \frac{\cdot 094 \times 64 \times 64}{12} = \frac{\cdot 094 \times 64 \times 16}{3} = 32 \cdot 1$$

R as most accurate determinations have found 32·2.

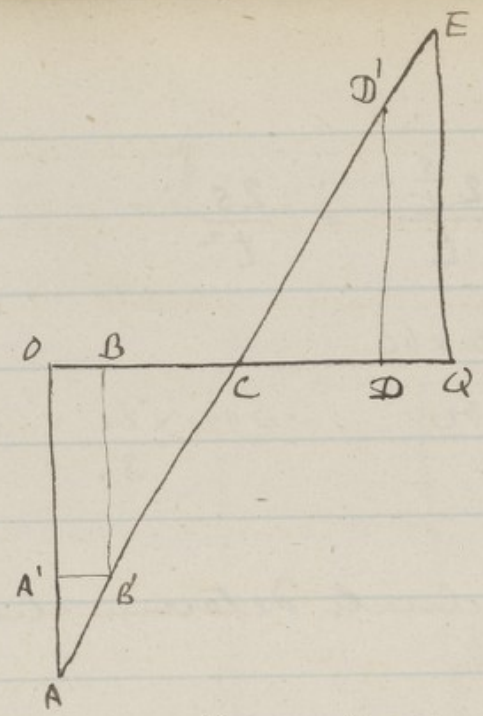
Oct. 25th 1877

Suppose one gives to a body an upward velocity to what height will it rise: etc.

Suppose the initial velocity to be 96 ft. per seconds; then there will be a negative acceleration of 32 ft. per second in each second



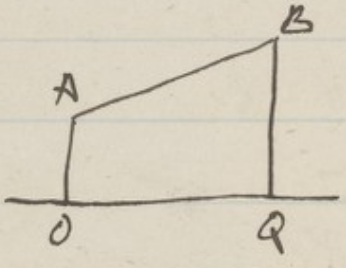
Let OA represent a velocity of 96 ft. per second:



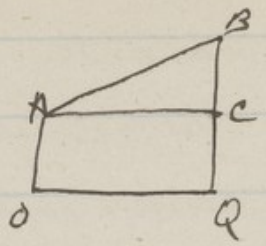
and let OB represent 1 second; take AA' = 32
and completed the ||m OB B'A'; then B'B will be the
~~position~~ velocity of the particle at the end of 1 second.
Join AB' and produce it to meet the line of time
in C. AOC will be the space described; since AC
represents the acceleration, AO the initial vel.
and the final velocity is 0: since when a body
is thrown up it comes to rest; then OC is the time.
but the Δs A'B'A and OCA are similar; and AA' is
= ⅓ of OA, ∴ OB or A'B' is = ⅓ of OC; therefore the time of
rising is 3 seconds. The body then begins to fall.
And as the rate of variation of velocity is the same

going up as coming down: produce the line AC which represents the acceleration; the distance gone through both ways is also the same: therefore make the Space, viz. the $\triangle$ CQE = OCA ; then EQ = OA; i.e. the velocity with which it ~~comes down~~ reaches the ground = initial velocity, and OC = CQ i.e. the times of ascent and descent are equal; also the velocity at equal times from the turning pt C, are equal i.e. if BC = DC, BB' = DD'; or in other words, the body passes every pt. with the same velocity on ~~the~~ its upward as on ~~the~~ its downward course.

Now suppose a body to be thrown downwards, in addition to its initial velocity, it will have an acceleration of 32 ft. per second, and the space will be represented by a figure like this:



OA represents the initial velocity
OQ ------ the time
area OABQ ——— space.



We have to deal with the following five quantities S, t, a, v_1, v_2; and if any 3 are known the rest can be found. Suppose S, v_1 & a are known. We have to find v_2 & t.

Let $S = 200$
$v_1 = 48$ ft. per sec.
$a = 32$ ft per sec. per sec.

S = av. vel × time

$= \frac{1}{2}(OA + BQ) \times OQ$

$= \frac{v_1 + v_2}{2} \cdot t$ ——————— (1)

Then we have

$\frac{BC}{AC} = a$

$QB = OA + BC$

$= OA + \frac{BC}{AC} \times OQ$

i.e. $v_2 = v_1 + at$

or $a = \frac{v_2 - v_1}{t}$ ——————— (2)

Multiply together equation (1) & (2): & we get:—

$$aS = \frac{v_2^2 - v_1^2}{2}$$

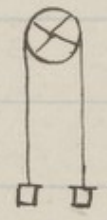
or in this case

$$32 \times 200 = \frac{v_2^2 - (48)^2}{2}$$

from which v_2 can be found, & by substituting its value in equation (1) or (2), t can be found.

—

In an Atwood's machine, if the weights on



each end of the string are equal, there will be no tendency to movement either way. If a small addition be made to either side, this will be acted upon by gravity & will fall uniformly accelerated, moving the other two masses. Now make this addition to equal weights of a much greater size, the same will take place, only the acceleration will not

be nearly so great as the same force has to move a much greater mass.

i.e. as we increase the mass, a diminishes (the acting force being the same.)

——— diminish ——— a increases

When a body moves down an inclined plane, it may be said to be moving in two directions, vertically and horizontally; but the moving force will vary when the inclination of the plane is varied, & the mass W remains the same, but the acceleration varies as the moving force f.

$$\frac{f}{W} = \frac{AC}{AB}$$



Velocity thus depends on the force exerted and the mass moved.

Bodies when moved exert a resistance, or when moving exert a resistance to be stopped. The measure of the resistance of a body to change of velocity is called its <u>inertia</u> which is proportional to its mass, or the quantity of matter it contains.

~~$f = m.a$~~.

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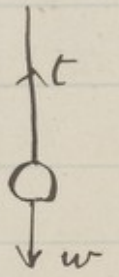
We have seen that f acceleration is directly proportional as the force exerted, and inversely as the mass of the body

i.e. $a = \frac{f}{m}$

or multiplying both sides by m

$$f = a.m.$$

i.e. force must be directly proportional as the resistance to change of velocity (i.e. mass), and also to the acceleration it causes. When two or more forces act on a body f means the resultant or effective force. For example, suppose a ball held up by a string, which is allowed to unroll on a cylinder;



There will be two forces acting on the ball; 1st its weight, w, acting downward, and the tension of the string (t)

acting downwards.

Then $f = w - t$

and $f = a.m$

$\therefore \quad a = \frac{w-t}{m}$

or $w - am = t$

Suppose a cage is going down a mine, and the rope is paid out with ever increasing velocity; the tension of the rope will be seen by the above to be less than the wt. of the cage. In pulling the cage up the mine;

$$w - (-a)m = t$$

$$w + am = t$$

that is an acceleration in pulling in the rope causes the tension of the string (t) to be a greater than the weight of the cage.

When there is no acceleration at all; $am = 0$

or $w = t$.

We have before obtained the fact, that in the

case of a body falling from rest;

$$S = \tfrac{1}{2} a t^2$$

{ Where S = Space, a = acceleration, & t = time } in one body

or $a = \frac{2S}{t^2}$

$$S' = \tfrac{1}{2} a' t'^2$$

{ Where S' = Space, a' = acceleration, t' = time } in another body

or $a' = \frac{2S'}{t'^2}$

if it is found by experiment ~~that~~ when $S = S'$, that $t = t'$; then $a = a'$

or $\frac{f}{m} = \frac{f'}{m'}$

f and f' in the case of falling bodies are simply the weights w & w' of those bodies.

$$\frac{w}{m} = \frac{w'}{m'}$$

or $$\frac{w}{w'} = \frac{m}{m'}$$

i.e. the weights are proportional to the masses.

Suppose the weight of 1 lb is allow to fall;

we have $m = 1$ & $a = 32$ ft. per. sec-per-sec.

$f = 32 \times 1 = 32$

meaning $\frac{1}{32}$ times the unit of force. The unit of force is when $a = 1$ & $m = 1$

$$f = am = 1 \times 1 = 1.$$

That is to say, the unit of force is that force which will produce on the unit of mass, the unit of acceleration.

This again depends, how we measure mass, & how we measure acceleration;

The measurement of acceleration again depends on the measurement of velocity & time;

And the measurement of velocity on how we measure space and time.

Suppose mass, space, and time to be measured in lbs, feet, & seconds respectively.

We then get that,

The unit of force is that force which causes the velocity of a mass of 1 lb. to vary at the rate of 1 ft. per second in every second.

Or taking, grammes, centimetres and seconds; we have

The unit of force is that force which causes the

velocity of a mass of 1 gramme to vary at the rate of 1 centimetre per second, in every second.

This unit of force is called the Dyne.

One unit of force can be converted into the other, by the knowledge that 1 inch = 2·54 centimetres

1 gramme = ·0022 lbs.

We had also for the acceleration a the expression

$$a = \frac{v_2 - v_1}{t}$$, which expresses the definition of acceleration, while the expression $a = \frac{f}{m}$ expresses the physical conditions to make it either greater or less.

$$f = a.m = \frac{v_2 - v_1}{t} . m.$$

$$= \frac{m.v_2 - m.v_1}{t}$$

mv_1 or the product of mass and velocity is called the quantity of motion in a body. If we have a railway train and a cricket ball moving at the same velocities, which would be said to have the most

motion in it? The train would as there is more mass in it. If the train be 6000 times greater than the ball, while the velocities are the same, the train will have 6000 times more motion in it. And similarly with velocity.

m = quantity of matter in a body = mass

mv = quantity of motion in a body = momentum.

The expression we first obtained

$$f = \frac{mv_2 - mv_1}{t}$$

expresses $\frac{\text{change of momentum}}{\text{time}}$

= rate of change of momentum.

Which is another way of expressing what force is; i.e. force can be estimated by the rate at which it is able to change momentum; or, the unit of force is the force which can produce (or destroy) the unit of momentum in the unit of time.

1 lb moving at the rate of 1 ft. per second, may be taken as the unit of momentum.

and substituting this we get the same expression for the unit of force as before; or the unit of momentum may be expressed in grammes of centimetres instead of lbs, of feet, and then we get the dyne for the unit of force as before.

In the case of 1 lb. falling for 1 second, we get $f = 32 \times 1 = 32$, i.e. the weight of one lb falling for one second produces 32 units of force.

Nov. 5. 1877

We have seen that

$$f = ma$$

and that

$$f = \frac{M}{t}$$

$M = mv$
$a = \frac{v}{t}$
$= \frac{f}{m}$

or $f = \frac{v.m}{t}$

i.e. $ft = v.m$

$$v = \frac{f.t}{m} = at.$$

This can be proved experimentally by means of Atwood's Machine. Put on the extra wt. in the form of a bar, and a certain instant let it be taken off by passing a ring in its path, through

which the bar cannot go, and therefore is left behind; the weight then goes on uniformly with the velocity it had acquired at the end of the time when the bar was removed. Place the ring at the mark 4 on the scale, the mass with the bar, will traverse this distance in two second. The velocity acquired at the end of this time will be 4 per second. Consequently it will be seen that on the removal of the bar, 4 is describing in each succeeding second.

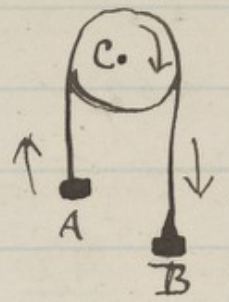
$$v = at$$
$$4 = 2a \quad \therefore a = 2$$

Let it now go 3 seconds with the bar; it will then describe 9 divisions on the scale, and by that time have acquired a velocity of 6 divisions per second. Consequently it will go 6 divisions in each succeeding second.

$$v = at$$
$$6 = 3a \quad \therefore a = 2$$

Time	Velocity
2 seconds	4 divisions
3 — —	6 — — —

We shall see if this is true



We have $a = \frac{f}{m}$

32·19 we have also found to be the wt. of 1 lb. expressed in units of force, in London; this number is generally called g.

if g is the wt. of the unit mass;

the wt of the mass A = Ag

" " " B = Bg

The force which causes motion is $Bg - Ag$

$= (B-A)g = f$

The total quantity of matter moved = $A+B+C = m$; it will be seen however that the mass of C must be estimated differently from that of A & B because different parts of it are moving at different velocities. ∴ imagine the wheel to be a mere rim with no wt. & the wt. of it equally divided between A & B.

we ~~that~~ then have $a = \frac{f}{m} = \frac{(B-A)g}{A+B+C}$; in the

experiment done $A = 6$

$B = 6\frac{1}{4}$

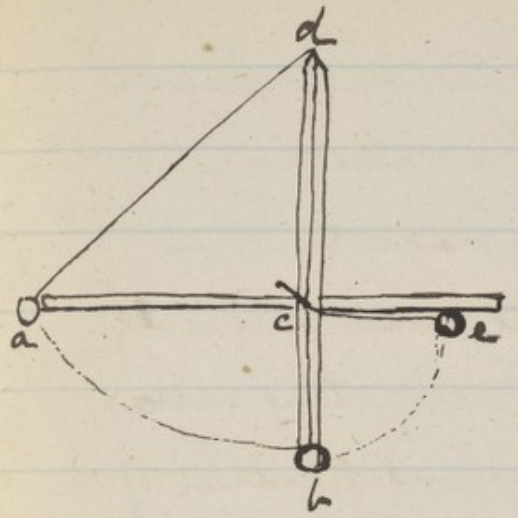
$C = 6$

$$\therefore a = \frac{(B-A)g}{A+B+C} = \frac{\frac{1}{4}g}{\frac{73}{4}} = \frac{1}{73}g;$$

but the divisions on the scale were 3 inches each or quarter ft : ∴ multiply 32 by 4 : then $g = 128$

$$a = \frac{128}{73} = \frac{16}{9} = 1\frac{7}{9}$$ which is a little less than 2; which was the result in the experiment. The velocity produced by gravity does not depend on the actual path a body describes while under its influence, but only upon that part of the path parallel to the force, i.e. the vertical height. An illustration of this is the following.

Take a pendulum, and let it fall from the position a in the figure, a horizontal bar marking the height from which it fell. Let it fall till it is in a position b, it will then have



acquired exactly the same velocity as if it had fallen from e to b; only the vertical height affecting the velocity, not the path of the ball; at c. let a peg be placed; the ball will not stop at b but go on through in the arc of a different circle, viz: the circle with centre c, instead of that with centre d. Or to be we said it had acquired a velocity equal to that it would have acquired if it had fallen from c to b, or what is the same thing what would carry it from b to c; and we have said that the path does not matter, therefore we should expect that the ball would rise to the same height as c, or to the same height as that from which it fell, and this is exactly what does take place, for the ball rises to e exactly the same height as a and c, though it has moved in a different path; but the vertical c b is the same, and this is all we have said that affects the velocity of a body under

the action of gravity, i.e. the part of the path parallel to the line of action of the force. The same is illustrated by the experiment on page 5: when two balls are allowed to descend, one vertically & the other having a horizontal force acting also upon it: the action of gravity is independent of this superposed force, and the line joining the centres of the balls at any instant in their motion is always horizontal & they reach the ground at the same instant.

We have before proved that

$$f = \frac{mv}{t}$$

and that $s = \frac{1}{2} v t$

Multiply these together;

$fs = \frac{1}{2} m v^2$ = the energy of motion.

By s is not meant the actual space traversed, but only that part of it which is parallel to the line of action of the force f; as an example we can again take the pendulum;

the force then acting and of which we want to take account is gravity; the part of the space traversed which is parallel to the direction of the force is the vertical; and not the absolute length of the arc of the circle, i.e. the absolute space, but the vertical be is what or the part of the space parallel to the line of action of the force must be taken as s in the calculation of the energy of motion of the ball of the pendulum.

Nov. 7. 1877

We have now got the following results

$$ft = mv$$
$$fs = \tfrac{1}{2}mv^2$$

The distance, s, through which a mass moves in the direction of a force f, multiplied by the force f is called the work done by the force.

$$fs = \text{work.}$$

The simplest example of work is the work done in lifting a heavy body a certain height. here the weight (which is the same as the force necessary to overcome it) multiplied

by the height is called the work done.
This example of work has the same ~~as~~ meaning as the usual application of the term. E.g. a workman carrying bricks up on to a scaffold; he does a certain amount of work, if he carries say a score of bricks a height of 100 ft; and he does twice as much work, if he carries two score the same height, or one score to twice the height, 200 ft. Suppose now instead of carrying them straight up he wheel them up an incline: to the same height; the actual distance is far greater, but the vertical is still the same, and that is all that affects the work done: viz: the distance in the direction of the force, which is in this case the vertical.
The result of doing work on a body is called the energy of motion generated

$$\frac{1}{2} mv^2 = \text{energy of motion.}$$

That is to say the work done by a force

on a body is equal to the energy generated.
There are however two different results of doing work; in the example of carrying bricks up to a scaffold; when the bricks arrive at the top they are still at rest.
Suppose them to be thrown up; the same work is still done; suppose the bricks thrown up 10 ft, and the force which throws them to act during one ft: it must generate enough velocity in the body during that time to carry the bricks up the remaining 9 ft;
i.e. the force must be equal to ten times the weight of the bricks, as a force just equal to their weight will if it act during the whole 10 ft carry them to the scaffold just as well;
i.e. a force 1 acting through a distance 10, generates the same energy as a force 10 acting through a distance 1.

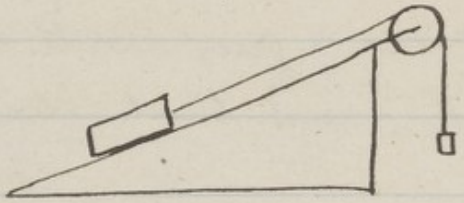
$$\underset{10\times1}{f.s} = \underset{1\times10}{f.s}$$

We have thus velocity generated or weights raised; this is only one ~~sec~~ example of work, and the most usual. But the work may be done against any other force such as a magnetic force, by pulling a piece of iron or steel away from a magnet. We have in each of these instances of doing work, the result that the body tends to move back again; if we left a body, and leave go of it, it fall again; if we drew a body from a magnet and leave go, it returns to the magnet. A resistance like this is called an active resistance; suppose now we drag a wt. along a table, considerable force is exerted to do so; but it has no tendency to return to its former position; the resistance is entirely due to friction; if the two bodies, the wt. & the table were perfectly smooth, one could be moved over the other without the action of the

smallest possible force; this is an example of Passive Resistance; i.e. it is not capable of producing motion in the opposite direction; it acts equally in opposition to motion in all directions; as much from right to left, as from left to right or any other direction. With gravity or magnetic force this is different, the body returning to its former position with the action of no force at all. We then can get the difference between the two resistances. (what constitutes)

Active resistances acts in definite directions
Passive resistances —— all directions equally

Take the following as ~~the~~ an example of this.



One body is dragged up an inclined plane by another one attached to it by a string which passes over a pulley and hangs down ~~thus~~ in a vertical direction as in the figure

The result is; (1) velocity is produced in the masses, (2) a rot is raised, and (3) friction is overcome. i.e. the following work is done;

1. Work against active resistance
2. ------ ------ passive ------.
3. ------ of acceleration.

If we throw a ball, work is chiefly done of the last kind; to a small extent there is also work done against a passive resistance viz: that of the air, and to a small extent also, which varies with the declination at which the ball is thrown, against an active resistance viz: gravity. Suppose a ball leaves a gun with a velocity of 1500 ft. per second, and that the mass of the ball is 80 lbs.

energy of ball $= 80 \times (1500)^2$

divide by s or the space during which the powder acts, i.e. the length of the muzzle which say is 4 ft.

and we get $\frac{(1500)^2 \times 80}{4}$.

and we get the force exerted or f; in this case however the force exerted for the whole length of the muzzle is different at different part; we get as the result therefore the average force or F.

we have already $V = \frac{s}{t}$ average velocity

and we have just got $\left\{ \begin{array}{l} F = \frac{mv}{t} \\ F = \frac{\frac{1}{2}mv^2}{s} \end{array} \right\}$ " force

The average force, like the average velocity, may or may not be the same as the force at any instant according as the force is uniform or not.

The expression $fs = \frac{1}{2}mv^2$ supposes that only one force acts, and therefore only work of acceleration done but this is very rarely obtainable; it is obtained in the falling of a body in a vacuum, when gravity is the only force. Let f therefore stand for the total force acting upon a body; and r stand for the total resistance; and s' be the space in its direction thro' which it acts.

We get then

$$fs = r'S' + \tfrac{1}{2}mv^2$$

but the resistance may be both passive R_p and active R_A; then

$$r = r_p + r_A$$

Then the expression stands

$$fs = r_A S_A + r_p S_p + \tfrac{1}{2} mv^2$$

supposing S_A and S_p to be the spaces respectively thro' which r_A & r_p act.

We have however so far supposed the body to start from rest; it may however have an initial velocity; the effect of the force then is simply to change its velocity, and may be expressed (Vide page 34.) by the rate of change of the momentum of the body.

i.e. $$f = \frac{mv' - mv}{t}$$

and

$$S = \tfrac{1}{2}(v_1 + v)t$$

Multiply them together

$$fS = \tfrac{1}{2}(v'^2 - v^2)m$$ i.e. the change of energy

Substituting this we get as the final result

$$\int S = r_A S_A + r_p S_p + \tfrac{1}{2} m (v'^2 - v^2)$$

This of course includes the case of starting from rest, by making v and therefore v^2 ~~equal to~~ infinitely small.

This equation enables us to see the condition of equilibrium of forces.

N.B. It must be for the present supposed in all cases which we shall consider, that the passive resistance of friction is either negligible or very small in proportion to the other resistances. For the present then the term $r_p S_p = 0$.

A body is in equilibrium if its velocity does not change. I.e. several forces are acting on a body: the test whether they are in equilibrium or not is whether the velocity of a body is changeable or not. Rest is one example of equilibrium; i.e. the

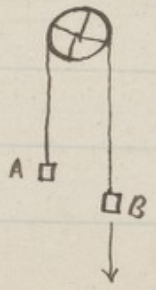
velocity of a body at rest is infinitely slow and remains infinitely slow. Another example is the case of uniform velocity. Then when a body is in equilibrium v' & v are the same, as we have said the velocity of the body does not change. The term $\frac{1}{2} m (v'^2 - v^2)$ therefore $= 0$; and the condition for equilibrium then is (leaving out of account, as before stated, friction)

$f s = v_A s_A$ or expressing it now in other terms $= v s'$

We get then

$$\frac{f}{v} = \frac{s'}{s}$$

The best example of this is an Atwood's Machine.



If the mass A = the mass B, the two will remain at rest until touched. Let B be the wt. of the force tending to cause motion and A the wt. of A to be the force tending to resist motion:

B is the force

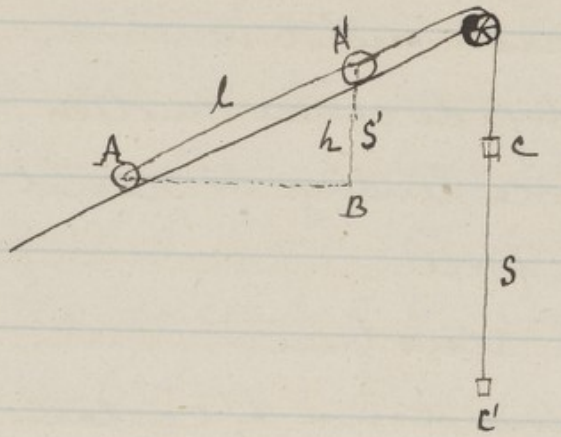
A —— resistance.

If B move down a certain distance, A will move up the same distance; i.e. $S = S'$; and that $\frac{S}{S'} = \frac{f}{r}$; f must also be equal to r; & i.e. to have equibrium the masses of A & B must be equal to one another. If they are then at rest they will remain so; and if made to move will continue to do so at a uniform speed.

Nov. 12. 1877

The conditions of equilibrium of a body resting on an inclined plane are these: We take as the formula to work upon —

$$\frac{f}{r} = \frac{s'}{s}$$



Suppose we have a body A connected by a string which passes over a pulley to another body C; we want to find what relation must exist between the two so that they may be in equilibrium; the force acting on C is gravity which pulls it down to C'; this then is the quantity S in the equation. When C moves a distance CC', A moves an equal distance AA'; but AA is not = S', because it is not measured in the direction of the force viz: gravity; the vertical distance BA' = S; we have then

$$\frac{f}{r} = \frac{BA'}{CC'} = \frac{BA'}{AA'}$$

Take a particular case; AA' = 24 : height of A = 6 } ∴ A'B = 8
height of A' = 14 }

i.e. $\frac{f}{r} = \frac{8}{24} = \frac{1}{3}$ or $f = \frac{1}{3} r$

If r i.e. the body A = 36 lbs; for the body C = 12 lbs;
i.e. the weight is to the resistance, as the vertical or height of the plane (h) is to the length of the incline (l).

$$\frac{f}{r} = \frac{h}{l}.$$

Suppose we make C greater than 12, say 14; the body C will fall and the body A rise with uniformly accelerated velocity; suppose we have to find the velocity (v) at the end of the movement. Let g as before stand for the force which acts on 1 lb while under the influence of gravity.

$fs = fr s' + \frac{1}{2} m v^2$ Supposing the bodies to start from rest.

$g.14 \times 24 = g.48 \times 8 + \frac{1}{2} m v^2$

$m = 48 + 18 = 66$

$g = 32$

$32 \times 14 \times 24 = 32 \times 48 \times 8 + \frac{1}{2} \times 66 \times v^2$; an equation from which v can be found.

Another case of this may be taken.



The figure represents the arrangement in section; suppose we have a wheel round whose edge is a string with a weight at the end; and on its axle, another string likewise with a weight at the end; supposing the circumferences (or diameters) of the wheel and of the axle to be known, we want to find what relation between the two weights there must be to cause equilibrium; as the string unwraps off the wheel, it will wrap on to the axle and vice versa; suppose the string to undo just one circumference's length of string; just the circumference length of the axle will wrap on to the axle. Suppose the diameters of the wheel and of the axle to be in the ratio, $g : 1$; and calling the weight attached to the wheel f, and the other weight r; we have

$$\frac{f}{r} = \frac{s'}{s} = \frac{1}{g} \quad ; \text{ this is the condition of equilibrium.}$$

One force does work against gravity and the other has work done against it by gravity: one force is g times the other; and the other moves through a distance g times as great as the one; so that the work

done by one, is equal to the work done by the other; only they are done in different directions, and must therefore be of opposite signs; so add up them together to get the total work done we get 0. That is to say, when bodies are in equilibrium (at rest or in motion) no work is done.

Suppose in Atwood's machine we take one force greater than the other, say one 40 of the other 41; $S = S'$ = (say) 21 quarters of a foot = $5\frac{1}{4}$ ft; let the weight of the wheel = 23;

then $m = 41 + 40 + 23 = 104$, and $g = 32$

$$(41+40)32 \times 5\tfrac{1}{4} = \frac{41+40+24}{2} v^2$$

A body with power to do work is said to possess energy. Let h be the height through which m is raised; then mgh = the work which it can do in the unit of time

$= \frac{1}{2}mv^2$ = energy of motion.

We have seen that when work is done against an active resistance, the body has the power to return again e.g. the power of a body to fall when work has been done on it in lifting it;

This power is called <u>potential energy</u> and is equal to the work done on it = fs.

(Nov. 14 : 1887)

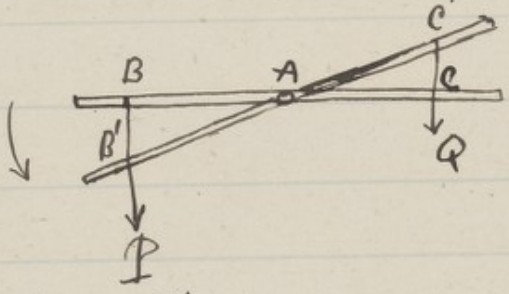
We shall now examine the conditions of equilibrium of a ~~fixed~~ bar turning about a fixed pt; and different weights hang at different parts of the bar.



Suppose we have a bar, with a weight on each side of the fixed pt. as in the figure; we require to find the relations between these weights and the distances of their pts. of application from the fixed pt., so that they may be in equilibrium..

Suppose the bar is moved, ~~the~~ say the left-hand arm downwards, then the right hand weight moves upwards.

In the case of ~~But~~ the first weight work is done by gravity, in the right hand weight we have, work done by it against gravity. The condition of equilibrium must be as in all other cases, that no work is done; i.e. the work done by one force must be equal & opposite in direction to the work done by the other force. If one weight is greater than the other, the effect will be to do work, in causing it to descend with uniformly accelerated velocity.



Say the bar moves about the fixed pt A, so that the pt. of application B of the force P, moves down to B'; and C consequently moves up to C'.

Then we have

$BB' : CC' = AB : AC$ { i.e. as the distances of their pts. of application from the fixed pt.

we must take the motion however in the direction of the force which is gravity i.e. the vertical; we can however neglect the curvature, because very nearly the same occurs on both sides, and is the more nearly true, the ~~means~~ smaller angle we take through which the bar is to turn.

P. BB' represents the work done by P when it moves through a certain angle BAB'

Q. CC' represents the work done by Q when it moves through an equal angle.

To have equilibrium, we must have

$$P.BB' = Q.CC'$$

which is the same as

$$P.BB' + (-Q.CC') = 0.$$

i.e. $P.BB - Q.CC' = 0$

from this we get

$$\frac{P}{Q} = \frac{CC'}{BB'} = \frac{AC}{AB}$$

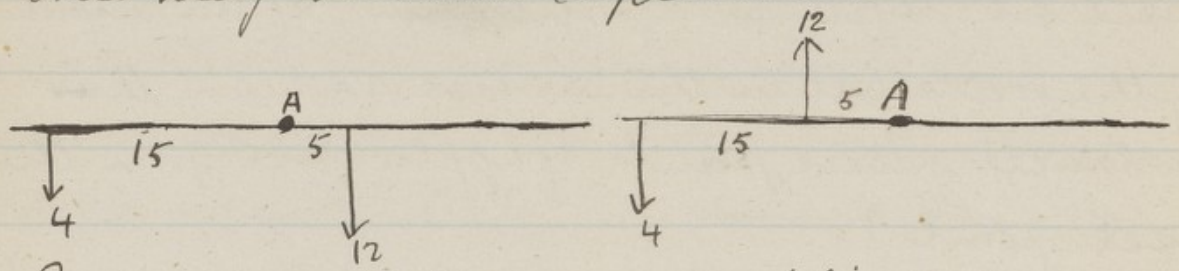
This expressed in words is, that two forces acting on a rigid bar capable of turning about a fixed pt, will be equilibrium, if the magnitudes of the forces are in the inverse ratio of ~~this~~ distances of their points of application from the fixed point.

If we take forces of four & twelve; and the distances 5 & 15; we have equilibrium: because

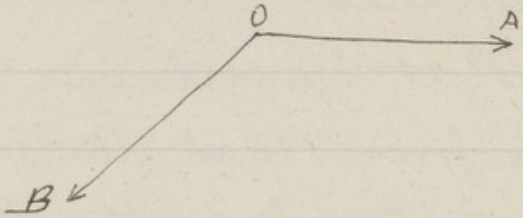
$$\frac{4}{12} = \frac{5}{15} \quad \text{or } 4.15 = 5.12$$

Instead of the two forces being on opposite sides of the fixed point, they may both be on the same ~~so~~ side, and still equilibrium ensue. Suppose we keep the force 12 at the same pt; viz 15 from the fixed pt. & bring the other force of 4, to the same distance on the same side of the fixed pt. as the 12 is; and at the same distance 5 from the fixed pt. as it was on the other side, and apply the force in an upward direction, we shall have equilibrium because still the work done by each would be done in opposite

directions, and be equal in value though opposite in sign; i.e. be equal to 0.
This may be thus represented



And so for all other more complicated arrangements, the condition of equilibrium is that no work shall be done.
We shall now consider the conditions of equilibrium when two or more forces act on a point.

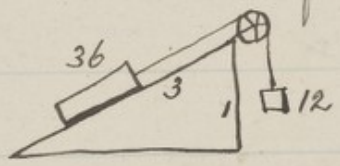


Suppose the two forces whose directions are represented by OA and OB act on a point O, what will be the combined effect of the two.
We have seen that if one alone acts, acceleration takes place in the direction of the force, if the body is at rest or in motion at the time.

but if two forces act, the acceleration cannot take place in both directions, as they are different; the question then is in what direction will it take place, if it does take place at all. One would naturally expect it to take place along some direction between the two.

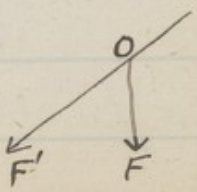
The efficiency of a force to cause motion in a direction other than its own, is less than its efficiency to cause motion in its own direction.

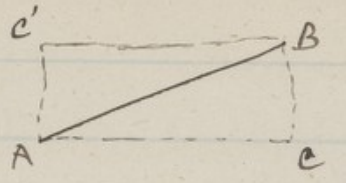
We can take as an example, the case of the inclined plane we had before (Page 52)



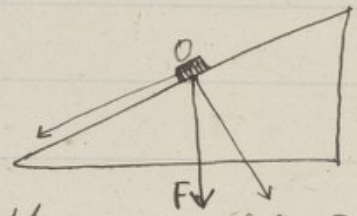
The weight of 36 when moving down the plane, is not so effective as when moving freely down vertically under the influence of gravity. In this example its efficiency is only $\frac{1}{3}$ as great.

We get from this the general case for the inclined plane; it moves not in a direction OF, but in a direction OF' & $\frac{F'}{F} = \frac{h}{l}$



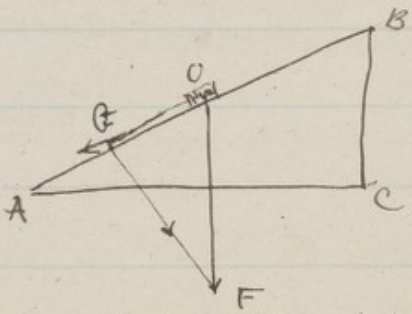


We have seen before that if we go from A to B; the result is the same as if we went from A to C and then from C to B; or from A to C' & then from C' to B. As we can decompose in this way a movement into two movements at right angles to one another; so we can do to a force.



Returning to the example of the inclined plane; If the force is parallel to the incline, we shall get the full effect of the force, if at right angles to the plane, there will be no effect, in causing motion downwards. Therefore the force of gravity OF, being between these viz:- vertical is partly effective and partly non-effective.

In representing the movement from B to A;
BC & AC at rt ∠
we can decompose this into 2 other movement;
BC represents the effective part of the movement, & ^
CA ——————— non effective ——— ——————.
And so forces can be represented by straight lines.
A force has a direction of its own, and a magnitude of its own: the direction of the line & the length of it correspond to these two properties of a force. But we must know also some pt. in the line, as we might have any number of lines with the same direction & magnitude; so we must know a pt. in the direction of the force.
And as we can decompose a movement into an effective and a non effective component, we can do the same to a force; do this to the force rep. by



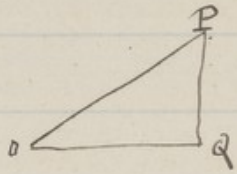
OF: ~~the~~ a line // to the plane of will be the

effective component, and OF, the l^r to it non effective.

But the triangles ABC and FOG are similar:

$$\therefore \frac{OG}{OF} = \frac{BC}{AB} = \frac{h}{l}$$

And so for the general case; suppose we have a force OP; and we want to find



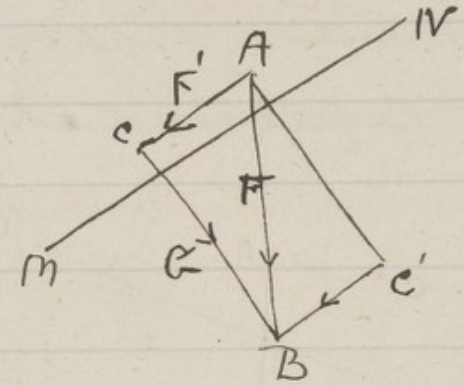
the efficient part of it in causing motion in say a horizontal direction; draw through O, a horizontal line, and through P a line l^r to this; they meet in the pt. Q; OQ is the the effective of PQ the non effective component of OP in causing horizontal motion.

This is called <u>Resolving of force</u> into an effective and non effective component; or the Resolution of forces

Nov. 19th 1877

Suppose we have a force ~~AA~~ F represented

in magnitude and direction by AB, and we want to find what part of it is effective in causing motion in a direction parallel to MN



Draw through A a line parallel to MN, and through B, a line ⊥ to it; or vice versa; then the force F' (or the effective component of F) will be represented in magnitude & direction by AC or C'B; while the non-effective component G, is represented by CB or AC'.

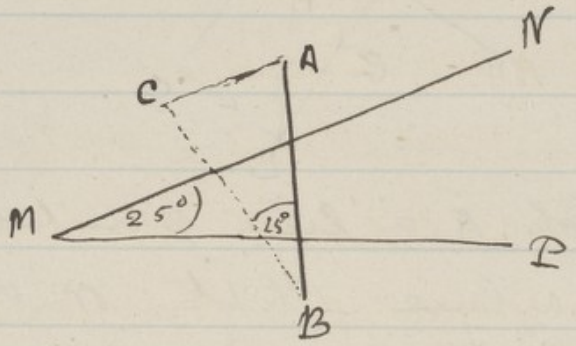
$$\frac{F'}{F} = \frac{AC}{AB} = \frac{C'B}{AB};$$

we have also

$$F'^2 + G^2 = F^2$$

Thus if any two of these quantities are known the third can be found.

Sometimes we have the force given in magnitude and direction, and only the direction of one component given: we want to find its value. Suppose we have a force given represented by AB, and we



want to find the value of the component acting parallel to MN say at an angle of 25° to the horizon: the simplest case of this kind, is what ~~weight~~ force would support another the force of whose weight is represented by AB on an inclined plane, at an angle of 25°. Do the same construction as before. AC is then the value we want to find; the angle ABC (= ∠ NMP) being 25. There are tables in which if AB (the hypotenuse) = 1

the values of AC and BC are given for every value of the angle which in this case is 25°. The calculation of these quantities is not possible in any elementary method.

If AB = 1, AC is called Sin ∠B

& AB = Cos ∠ B.

if AB has any other value than 1, the two must be multiplied together.

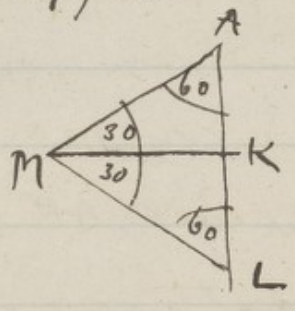
If F' = eff. component G = non effective comp.

& α = ∠ between F & F'

$F' = F \cos\alpha$

$G = F \sin\alpha$

In the cases of a few angles, these can be calculated. Suppose it to be 30°



AK = AM. Sin 30°

Take two right ∠d Δs, and put the two ∠s 30° together to make 60°; we thus get an equilateral Δ AML

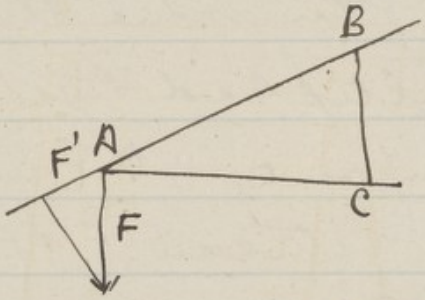
$\sin 30 = \frac{AK}{AM} = \frac{\frac{1}{2}AM}{AM} = \frac{1}{2}$

Similarly we get $\sin 45^0 = \frac{1}{\sqrt{2}}$

$\sin 60^\circ = \frac{MK}{AM} = \frac{\sqrt{AM^2 - (\frac{1}{2}AM)^2}}{AM} = \frac{\sqrt{3}}{2}$

Similarly it is possible to get the Sines of 15^0 & 75^0

We have seen that fs = work.

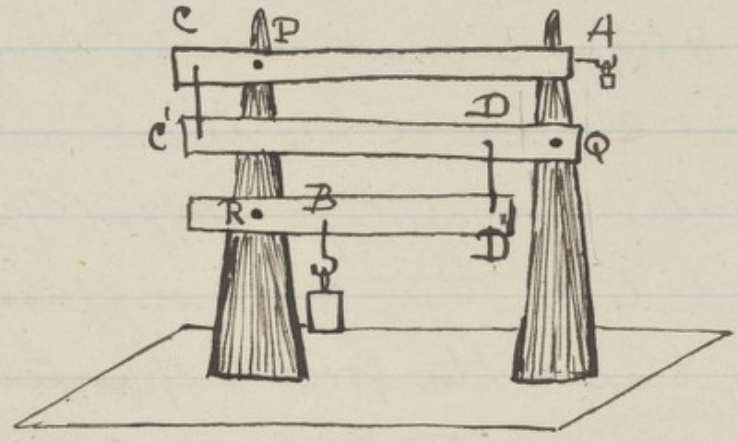


this may be represented ~~in this~~ (as in the above figure: Where F' is the effective component of F in the direction BA:) either by the effective component multiplied by its distance, or by the whole force multiplied by the distance measured in the direction of the force; i.e. either by F'. AB or F. CB.

This may be proved thus: the $\triangle$s in the figure are similar $\therefore \frac{F}{F'} = \frac{AB}{BC}$

i.e. F. BC = F'. AB Q. E. D

We have already studied the cases of parallel forces acting on a rigid bar, or lever, the inclined plane & the wheel & axle. Levers may be still further complicated as in the arrangement in the figure.



A very little weight at A will keep in equilibrium a very great weight at B; the distances moved through by the weights being inversely proportional. Taking each lever separately, the weights supported at each end will be inversely proportional to the distance they move through (so that no work may be done) which again is proportional to their

points of application from the fixed pt.
Suppose PA to be 8 times as long at PC; then a weight 8 times as great can be supported at C than ~~the~~ at A; because the distances moved through is greater: then uniting two together; C' & C move thro' the same distance; and supposing QC' = 8.QD; we get that a force 8 times as great can be supported at D than at C', hence 64 times as that @ at A; still further advantage is got from uniting the third to the other two; suppose RD' = 6RB we shall have the force supported at B 6 x 8 x 8 = 384 greater than at A.
This ratio between a force exerted and the force applied is called the mechanical advantage of a machine; in this case it is 384. In the inclined plane, the mechanical advantage is the ratio between the weight on the plane & the force required to keep it in equilibrium; which is the same as the proportion $\frac{h}{l}$

in the Wheel & axle, the ratio between the weight on the axle and the wt. on the wheel to keep it in equilibrium, which is as we saw the diameter of the wheel : the diam. of the axle: in the lever we have just seen it is the ratio of the two wts. which are in inverse proportion as their distances from the fixed pt.

i.e. Mech. Adv = $\frac{\text{Force Exerted}}{\text{Force applied}}$

which is generally spoken of in technical terms as $\frac{\text{Weight}}{\text{Power}}$.

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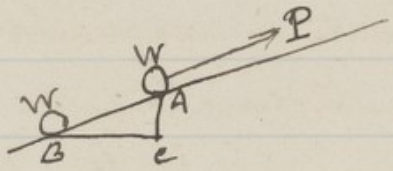
We have now

Mechan. Advantage = $\frac{\text{Wt.}}{\text{Power}} = \frac{S_p}{S_w}$

i.e $\underset{\text{Work done in lifting wt.}}{W.S_w} = \underset{\text{work done by Power.}}{P.S_p}$

or $\frac{W}{P} = \frac{S_p}{S_w}$

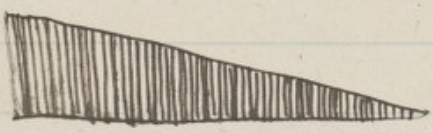
the simplest case of this is the inclined plane



$$AC.W = AB.P$$

$$\text{or } \frac{W}{P} = \frac{AB}{AC}$$

An inclined plane can be converted into a screw by wrapping it round a cylinder. The figure represents one turn of the screw.



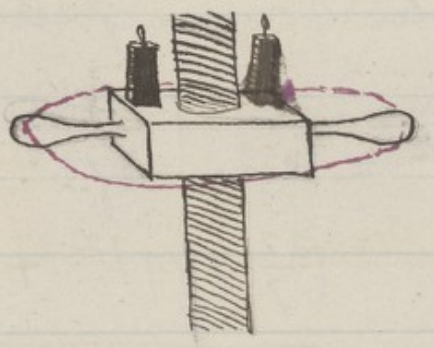


The mechanical advantage will be the same in each case, as to get to the same height, it makes no difference whether we move along a straight incline or up

a spiral as the ~~cord~~ space moved thro' in each case will be the same; the distance between the threads is called the pitch: the mechanical advantage will be:

$$= \frac{\text{the circumference measured along the spiral}}{\text{pitch.}}$$

as for each turn of the screw, a distance is moved up = the pitch. The screw however is never used alone, but with a nut or handle of some sort. It may be as



in the figure, where the nut is movable, & weights may be placed upon it; and raised by turning the handle: then for each complete turn, the power will move along a spiral

circumference, ~~to~~ at or near the end of the handle while the weights will be raised through the pitch.

Here then

$$\text{Mech. Adv} = \frac{\text{path of the power}}{\text{path of the weight}}$$

$$= \frac{\text{spiral at end of handle}}{\text{pitch.}}$$

$$(\text{spiral})^2 = (\text{circumference})^2 + (\text{pitch})^2$$

if the diameter of a circle = 1

the circumference = $\frac{22}{7}$

Let the pitch = $\frac{5}{8}$ inch.

diameter of circumference = 19 inches.

then

$$(\text{Spiral})^2 = (\text{circumf.})^2 + (\text{pitch})^2$$

$$= \left(\frac{22}{7} \times 19\right)^2 + \left(\frac{5}{8}\right)^2$$

$$\text{Spiral} = \sqrt{\left(\frac{22}{7} \times 19\right)^2 + \left(\frac{5}{8}\right)^2}$$

$$\text{Mech. Adv} = \frac{\sqrt{\left(\frac{22}{7} + 19\right)^2 + \left(\frac{5}{8}\right)^2}}{\frac{5}{8}}$$

This kind of screw is used for holding things together; e.g. when two piece of wood a screwed together, the upper one acts as the nut.

The screw however might be used differently; viz: in this manner:- the nut moves in one plane only, in a circumference, drawing the screw up; the mechanical advantage here = $\frac{\text{circumf.}}{\text{pitch}}$

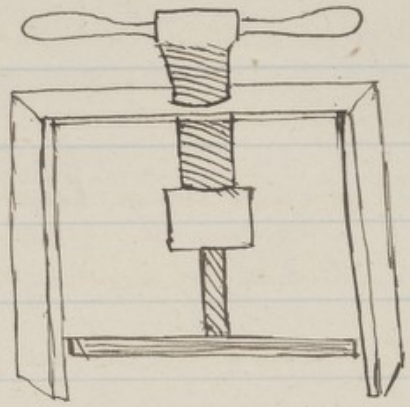
which in the case we have taken = $\frac{19 \times \frac{22}{7}}{\frac{5}{8}}$

Generally we have then, that the mechanical advantage of the screw

$$= \frac{\text{circumference (measured according as it is used)}}{\text{pitch.}}$$

To increase the mechanical advantage then the handles should be increased in length, and the pitch diminished; this is limited 1st; by not letting the machine get too cumbrous, 2nd; by not making the pitch so small as to have no strength.

Another way of increasing the mechanical advantage is to have the nut itself a screw; which works in a fixed frame; as in the figure;



when one screw is raised the other is lowered, and the distance that the weight moves through for each turn of the handle is the difference of the two pitches

Example Pitch (1) = 0·35

Pitch (2) = 0·28

difference = 0·07

diameter of circle moved through by power = 12 inches.

Mechanical Adv. = $\frac{22}{7} \times \frac{12}{0 \cdot 07} = \frac{22 \times 12}{\cdot 49} = 528$

Another application of the principle of the inclined plane is the wedge;

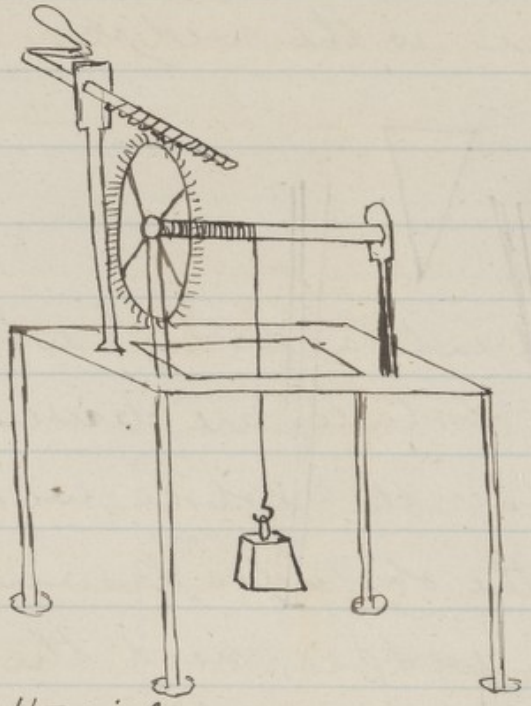


It is hammered into a body, which is split by it; or obstacles are driven aside by it; suppose the whole wedge to be drove in ~~into~~: the obstacles are driven aside the width of the wedge, and the power has been applied the length of it;

$\therefore$ Mech. Adv. $= \frac{\text{length}}{\text{width}}$ i.e. the sharper the wedge is, the more effective it is.

The following is a machine by which an immense mech. Advantage can be obtained.

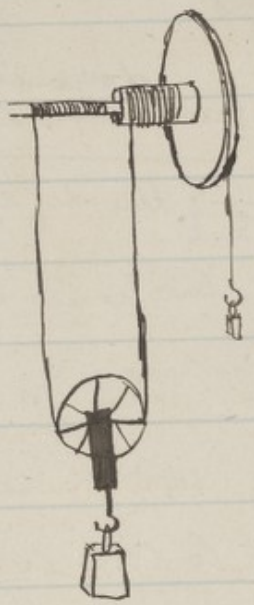
P. T. O.



The diameter of the circle which the power describes is 10; and in a hundred turns the weight is only raised 3 3/4 inches; this gives a mechanical advantage of about 900.

The machine can be divided into three parts, each of which gives ~~additional~~ multiplied mechanical advantage: 1st; the handle which works the screw; 2nd the screw which work the cogged wheel; 3rd the cogged wheel which is part of a wheel & axle, on which hangs the weight.

Another way of getting a large mechanical advantage is by another application of the Wheel & axle; the axle is composed of



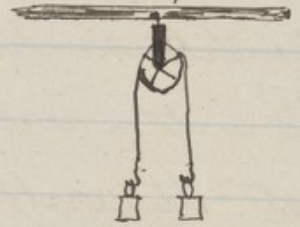
two parts, one thicker than the other; as the string winds off one part it winds on to the other: suppose now a complete turn of the wheel to take place: the power moves through a distance = the circumference of the wheel; and the wt. through a distance equal (half the) to the difference of the circumferences of the two parts of the axle; it only moves through half, because of the presence of the pulley; so that

the length the string gains for one turn is equally divided, one half going down and the other up again round the pulley; so we get a double mechanical advantage.

$$\text{Mech. Adv} = \frac{\{\text{circumference}\}\text{ of wheel}}{\frac{1}{2}\{\text{the difference of }\{\text{circumferences}\}\text{ of the two points of the axle.}}$$

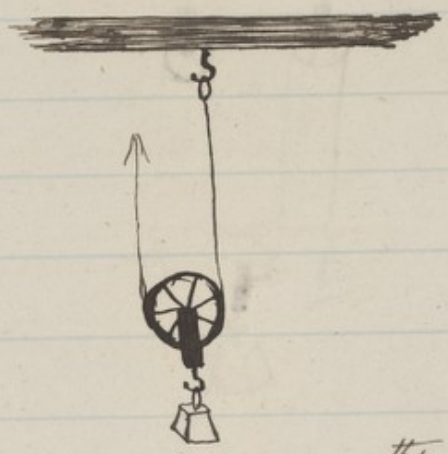
We now come to the case of pulleys; here again, the work done by the power must be equal to the work done against the resistance in order to have equi-librium i.e. $P.s_p = W.s_w$

When only one fixed pulley is employed;



we have no mechanical advantage; because as the distances moved through by the weight and the power are equal, the weight must be equal to the power

A single pulley may however be employed another way; as in the figure: i.e. the

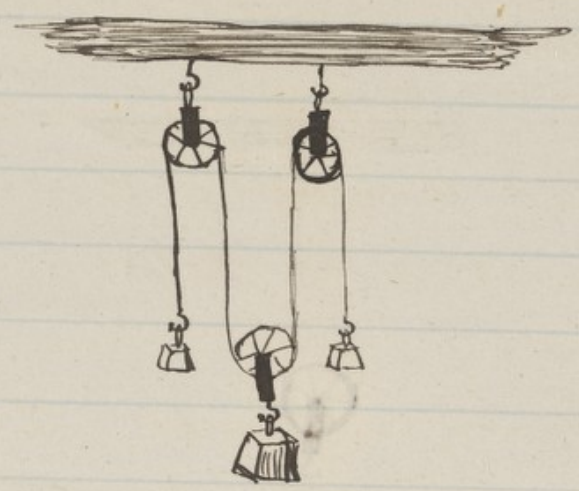


pulley is suspended in ~~the~~ the loop of a string of which one end is made fast to a beam and the other the power is applied. Here if the weight is raised a certain distance the power must move through double that distance; because each side of the loop must be raised that distance and as one side of the loop is fixed, the other must move through double that distance. Therefore the mechanical advantage = 2.

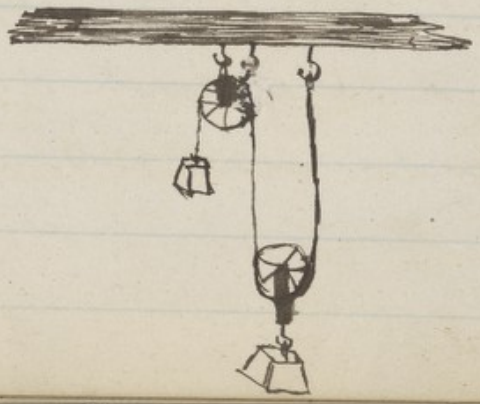
$$P \times S_p = W \times S_w$$

$$1 \times \text{[illegible]} = 2 \times \tfrac{1}{2}$$

we see this better thus;

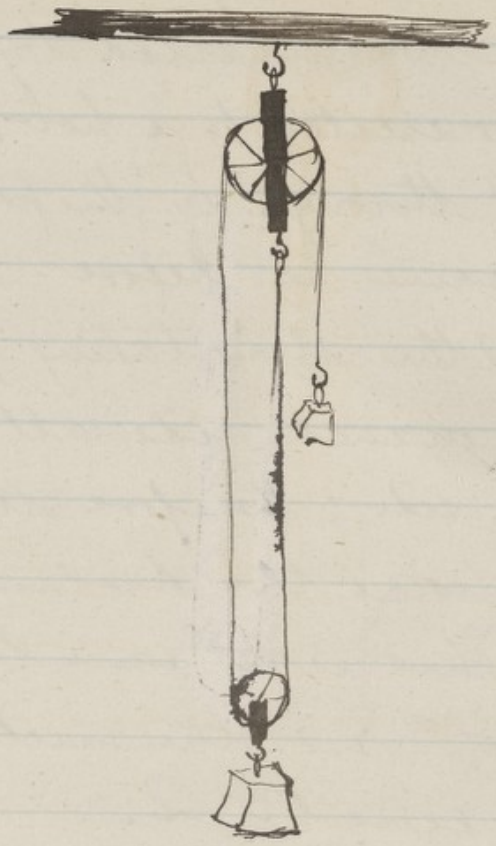


To raise the weight 1 foot; each of the other wts. must descend 1 ft; but as there are two of them each must be half the wt. of the largest one in the middle, so as to get the whole amount of work done = 0. Now suppose instead of having two fixed pulleys; we have only one as in the next figure, & the other end of the cord fixed;

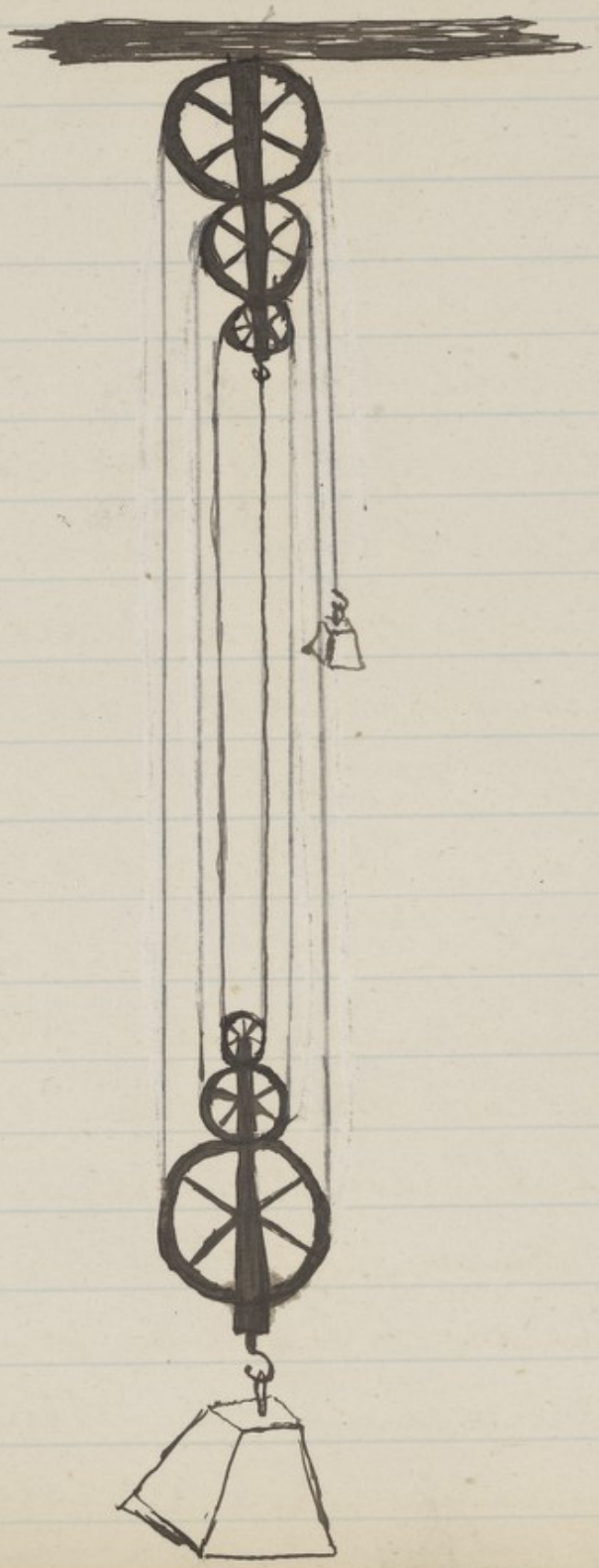


the arrangement is virtually the same: only the force that before was exerted by a wt. over a pulley, is now exerted by a hook: the distances moved through by the power & wt. remain the same as before: only instead of the shortening of the string taking place over two pulley, one on each side; all the shortening is done on one side: therefore the power moves through twice the distance that the wt. does; or what is the same thing: the power is ½ the weight: i.e. the mechanical advantage is equal to 2.

In cases of pulleys we must remember that any one string is pulled equally in all parts: i.e. the tension is the same throughout: tension meaning the force tending to break the string: the same arrangement as on the last page is put in a more convenient form in the figure on the next: - P.T.O.



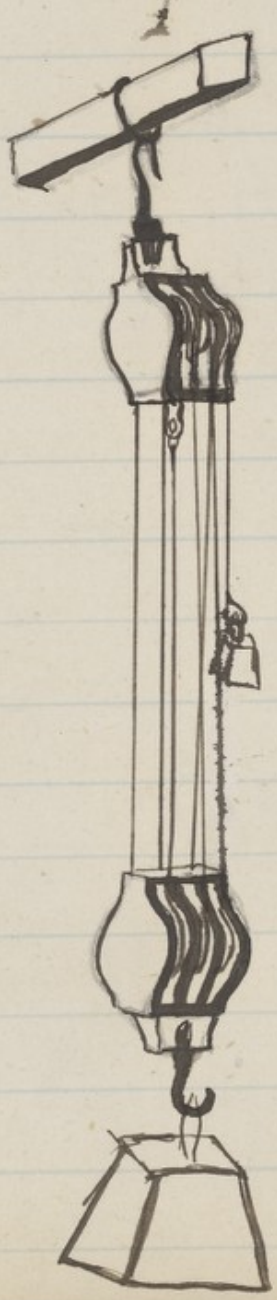
We may have a repetition of the same thing several times over; as in the figure on the next page, where we have a block of three fixed pulleys, and a block of three movable pulleys:-

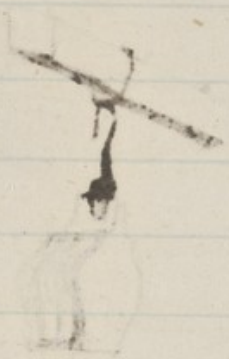


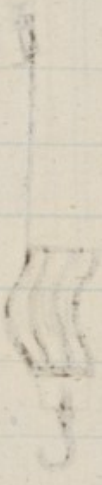


Here the weight hangs in three loops: and if the weight be lowered 1 ft: each side of each loop must rise one ft: and as the string is ~~only~~ all in one piece, the string passing round the sixth pulley must move through 6 ft: i.e. the Mech. Adv = 6. The middle string is fastened and therefore remains fixed: therefore when the weight is raised 1 ft: the string round the first pulley will rise 1 ft: round the next 2 ft: round the 3rd 3 ft: round the 4th 4 ft: round the fifth 5 ft & round the 6th i.e. the one to which the power is fastened, 6 ft: the strings round the bottom pulleys move in the ratio 1, 3, 5, (uneven numbers): round the top pulleys 2, 4, 6 (even nos.)

another and less bulky; & therefore more useful form of the same thing, is to put the pullays side by side, in one block.

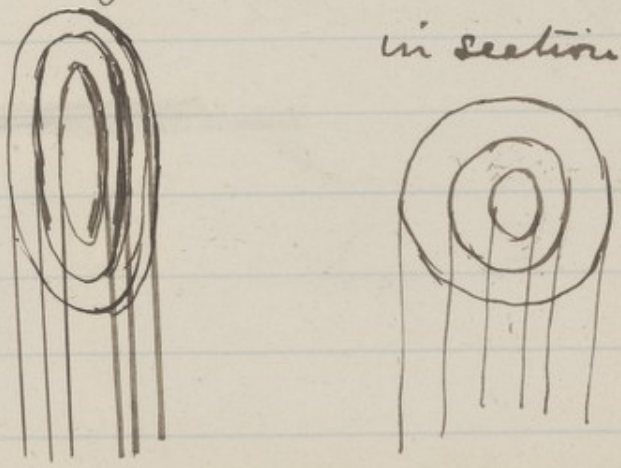






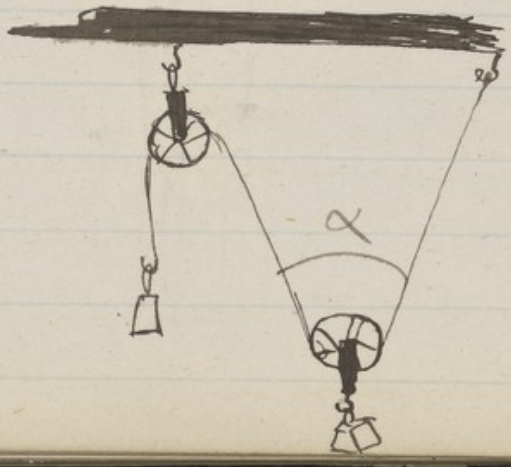
And if we go on increasing the number of pulleys, we get an enormously great mechanical advantage; one drawback to increasing the number of pulleys is that the friction increases also.

In the former arrangements, some of the pulleys move faster than others, because more string has to go over them; but by making the diameters of the pulleys different in the right proportion; they can be fastened together on one ~~block~~ axle & move at the same rate; a block of three pulleys would then be of this form.



this form of pulley can however only be used with one size of string; as the diameters have to be measured to the centre of the string: so if we alter the size of the string, we alter the diameters, and the whole arrangement is put out. For this reason, this form of pulley is of little practical value.

In the cases we have hitherto considered, the strings have all been parallel; but it is possible for the strings not to be parallel; e.g. in the first case we took; that of a single movable pulley

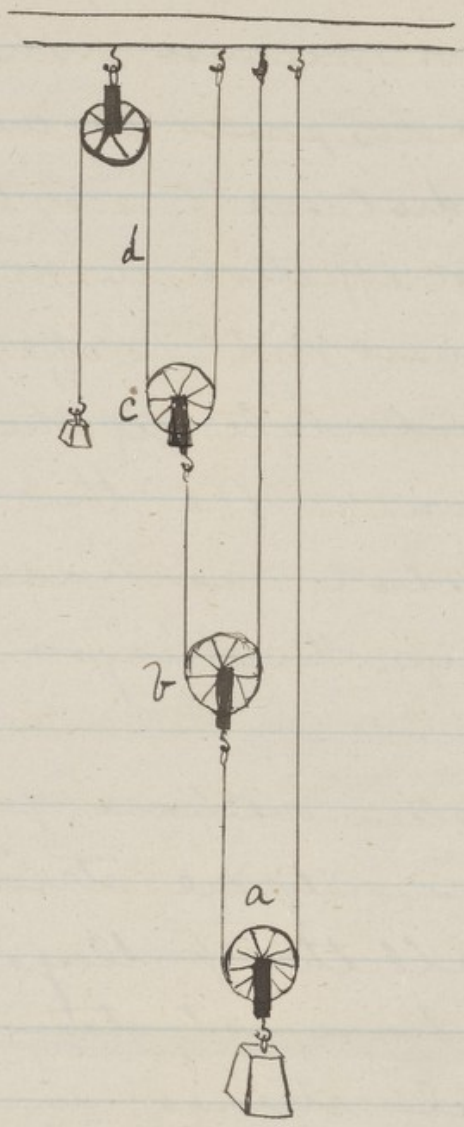


the mechanical advantage is now not so much as two: because when the wt. rises 1 ft.; the power descends less than double that distance; i.e. the whole of the power is not effective in producing vertical motion; we must find its effective component; thus we can calculate if the angle α in the figure is known. We thus see that to get the greatest mechanical advantage from pulleys, the strings must be parallel.

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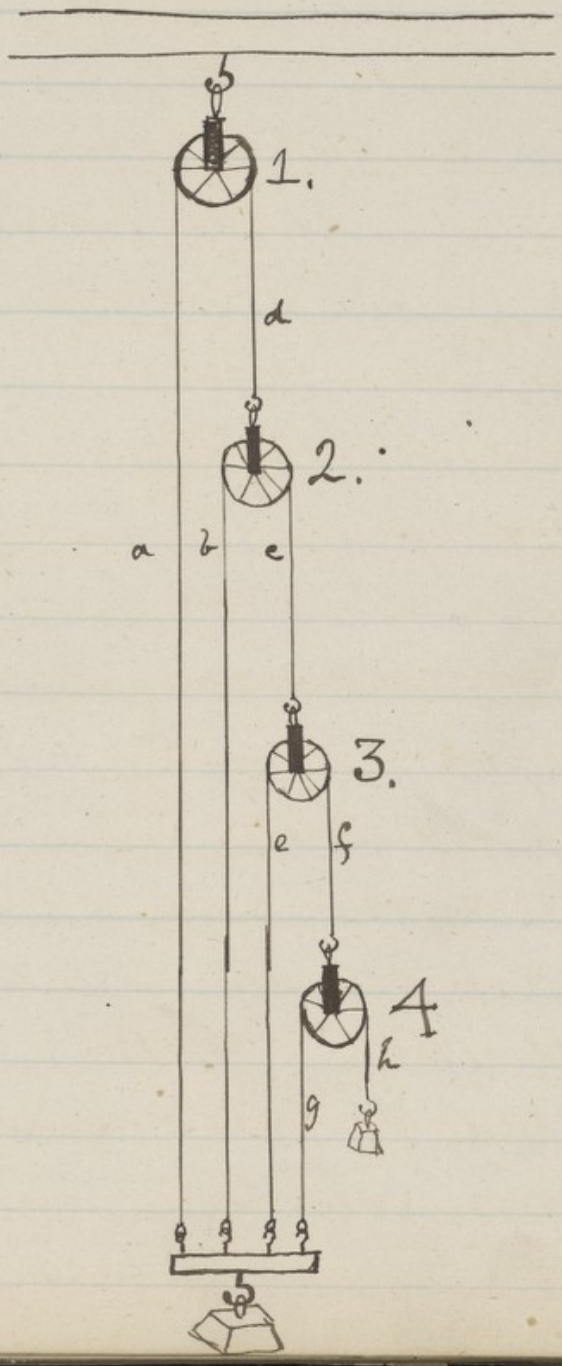
In the system we have just considered, there was only one string passing round all the pulleys: we may however have systems where there are several strings; in these of course there may be a different tension in each; the tension would not be the same throughout as in the case just considered; the following is one arrangement:—

P.T.O.


d
c
b
a

We here again get the mechanical advantage by considering simultaneous movements of the wt. & power. If the wt. rises 1ft, each side of the loop a must rise one ft; and as one side is fixed the other must rise 2ft: i.e. the pulley b rises 2ft: if this rises two ft, each side of the loop in which it is suspended must rise 2ft; i.e. the each side of pulley c must rise 4ft; and consequently the loop in which it is suspended must rise i.e be shortened by 4ft: i.e. the side d will rise 8ft; which is the same as the power falls: the fixed pulley making no difference; Therefore if the power moves eight times as fast as the weight in the same times: i.e. eight times as fast: the power will only be $\frac{1}{8}$ of the wt., in order that no work may be done: hence for 3 movable pulleys the mechanical advantage is 8: i.e. 2^3: for n movable pulleys, the mechanical advantage $= 2^n$; that is without taking into account the wts. of the pulleys.

We may have as another arrangement, the last system turned upside down:-



We get the mechanical advantage in the same way; Suppose the wt. to rise 1 inch; then all four strings attached to the beam from which it hangs must rise 1 inch also.

a rises one inch; and consequently d falls 1 inch, being the continuation of the same string over a fixed pulley.

Therefore pulley 2 falls 1 inch; therefore b & c being the side of the loop which hangs round it must each fall 1 inch; but b is fixed, therefore C descends 2 inches: to this must be added the fact that b rises 1 inch & therefore C falls 1 inch; the total fall of c is thus 2 + 1 = 3 inches.

Therefore pulley 3 falls 3 inches: therefore e & f must be shortened by 3 inches; & e is fixed so f fall 6 in. to which must be added the rise of 1 inch in e: so that the total fall of f is 7 inches

Similarly h falls 14 owing to the fall of pulley 4 of 7 inches, and 1 extra; i.e. total fall of h = 15.

To h is fastened the power: therefore the power

moves 15 times as fast as the weight; i.e. the mechanical advantage with 4 pullys = 15.
By fastening the power to any other of the strings: we get:

no. of pulleys.	1	2	3	4	5
Distances	1	3	7	15	31
Differences		2	4	8	16

The differences are doubled each time; and the distance moved through is one less than the difference between its distance & that of the next pulley

Suppose there are n pullys.

$$S_p = S_w \times 2^n - 1$$

Let $S_w = 1$

$$S_p = 2^n - 1$$

Let us see if this agrees with our results.

For one pulley $S_p = 2^1 - 1 = 1$

— two —— s $S_p = 2^2 - 1 = 3$

— three —— $S_p = 2^3 - 1 = 7$

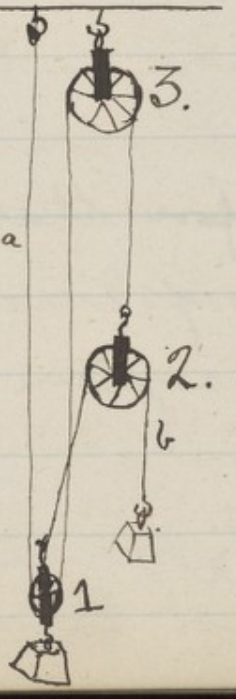
— four —— $S_p = 2^4 - 1 = 15$

& so on.

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{S_p}{S_w} = 2^n - 1$$

There are an innumerable no. of ways in which pulleys can be arranged; in all the cases we have considered each string was arranged in the same way: the next case is an example of where the strings can be ~~combined~~ arranged in two of these systems combined in one arrangement; each string being arranged according to a different system.

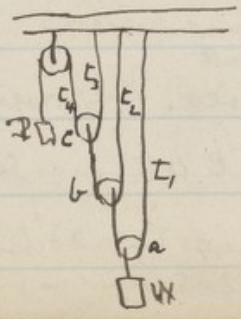
Need a is arranged as the system on p. 84, and b as the one on page 94. With a alone the Mech. Adv is 2; Pully 2. falling twice as fast as 1 (or the wt) rises. The power would thus fall 4 times as fast but the wt. rises in addition, therefore b falls 5 times as fast



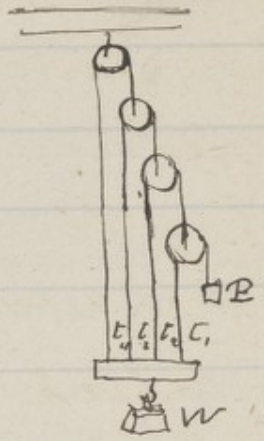
that is the mechanical advantage is 5.
Pulley 2 also balances some of the wt. & increases the mechanical advantage. Where the wt. raised is small, the mechanical advantage is either much increased or diminished by the wts. of the pulleys; in the last system & the one on p. 94, the pulleys act as an additional power; in the one on page 92, the pulleys act as an additional weight; i.e. When we left the weight, the pulleys have also to be lifted; but in the subsequent arrangement the pulleys move in the same direction as the power.

We said before that the tensions in the separate strings was not the same.
In the page 92 system

$t_1 = \frac{1}{2}$ the wt
$t_2 = \frac{1}{2} t_1 = \frac{1}{4}$ the wt.
$t_3 = \frac{1}{2} t_2 = \frac{1}{8}$ the wt.
$t_4 = t_3 = \frac{1}{8}$ the wt



In this system turned upside down;
the sum of all the tensions (t_1, t_2, t_3, t_4)
must be equal to the weight:



$$t_1 = P$$
$$t_2 = 2P$$
$$t_3 = 2t_2 = 4P$$
$$t_4 = 2t_3 = 8P$$

$t_1 + t_2 + t_3 + t_4 = P + 2P + 4P + 8P = 15P = W$ as we found.

In the first of these two, the pull on the beam = sum of the tensions of the 3 cords = Wt + Power + Wt of the Pulleys.

Let us now estimate the effect of the weight of the pulleys on the arrangement of one system, say the first: let each be of the same wt say 13 oz; & the wt. 14 lbs.; we want to find the power.

Let the wt. rise 1 ft, a rises 1 ft also (see fig. page 97)
b rises 2 ft, c 4 ft,

Work done in lifting $= \left[\left\{14 + \frac{13}{16}\right\} \times 1 + \left\{\frac{13}{16} \times 2\right\} + \left\{\frac{13}{16} \times 4\right\}\right]$

The power descends & the work done by it must = the work done in lifting. Work done by power = $8 \times P$

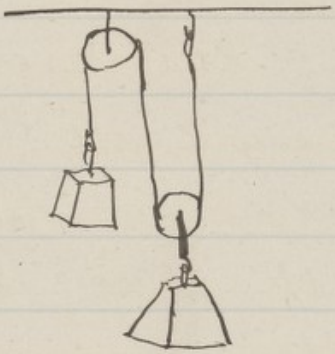
$14\frac{13}{16} + 1\frac{11}{16} + 3\frac{4}{16} = 8P$

$18 + 1\frac{11}{16} = 19\frac{11}{16} = 8P$

$$P = \frac{19\frac{11}{16}}{8} = 2\frac{59}{128} = 2\frac{1}{2} \text{ (approximately)}$$

The mechanical advantage of a system of pulleys can be found experimentally taking into account the weight of the pulleys without knowing their weight in this manner.

Suppose we have the simple movable pulleys:



Let W = weight

P = power

p = wt. of pulleys

then $\frac{W+p}{P}$ = Mech. Advantage.

Find experimentally what weights will balance each other; say the Wt is found to be 11, if the power is 9

$$\frac{11 + p}{9} = \text{Mech. Adv.}$$

Do this again; say 25 & 16 Balance each other

$$\frac{25 + p}{16} = \text{Mech. Adv.}$$

Eliminate p between these two equations and the Mech. Adv. will be then got.

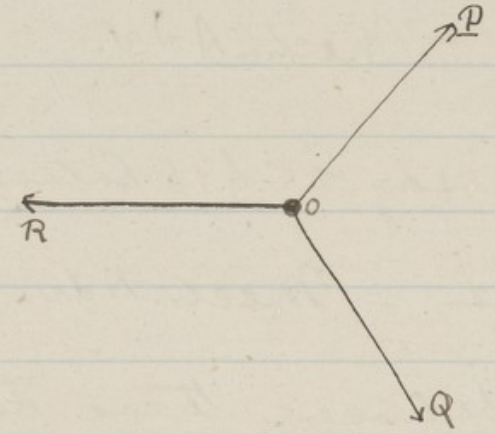
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We now come to consider the action of two or more forces on a point, more particularly When they act in the same st. line, the effect is the same as if the algebraic sum of them acted alone.

When they do not act in the same straight line, we have seen that the condition of equilibrium is: that the work done by the force is equal to the work done against the resistance

or $fs = rs'$ | See pp. 49-50

Suppose we have three cords attached to a mass, and that forces are exerted by means of these cords



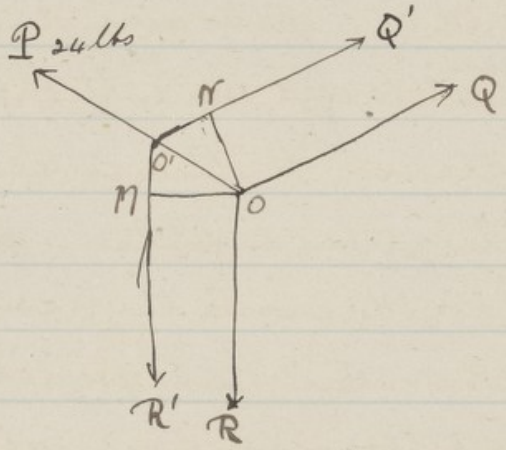
Each tends to move the body in a direction parallel to its own line of action, but it cannot move in three directions at once; therefore if it moves at all, it will be a sort of compromise being all three. Suppose that there are only two forces acting on it to begin with; the motion will take place between the two; and if the two forces are equal it requires no proof to see that it will move just as much in one direction as the other i.e. its line of motion bisects the angle between the lines of action of the two forces.

If one of the forces is greater than the other, it will move more nearly in the direction of the greater force. If no change of motion results by the action of 3 or more forces, it follows that each one of the three or more forces is able to neutralise the combined effect of all the rest.

When a single force can produce the same effect as two or more forces acting together, it is called their Resultant.

Therefore when we have any number of forces acting on a point, and in equilibrium each one is equal and acts in the same straight line as, though in the opposite direction to the resultant of all the rest; this gives us the means of finding by experiment or otherwise the resultant of a number of forces acting on a point; we find by experiment or otherwise what force will keep them in equilibrium acting simultaneously and then we have only to turn it round to get the resultant.

The simplest case possible is to find the resultant of two forces acting on a point; or, what is the same thing, we have three forces in equilibrium, and we have to find what magnitudes they must have to be so; the magnitude of one being known & the direction of the other two. To show the connection of this part with the rest of the subject, we shall do it by the principle of work



Suppose we have acting along P a force equal to the weight of 24 lbs.

Imagine O to be displaced in the direction OP, to O'.

work done by P = P. OO'

Q and R are supposed to be still acting: put in their lines of action R' & Q'; how much work has been done against them?

Thro' O draw a line OM ⊥r to O'R', also a line NON, ⊥r to QQ'; the displacement OO' may be considered to be made up of two displacements OM and MO'; the distance in which O has been displaced in the direction of R is MO, a displacement from OM would neither hinder nor help the displacement MO: consequently MO is all that has to be taken into account in speaking of the work done against the resistance R. Similarly NO is what has to be taken into account when talking of the work done against resistance Q.

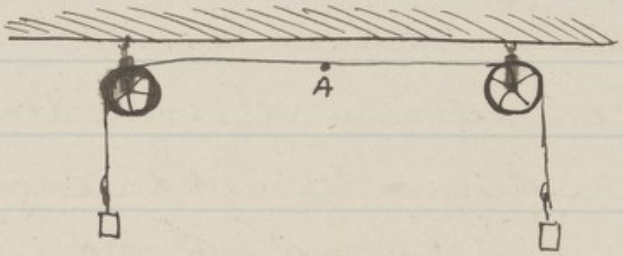
If there is equilibrium; the work done by the force causing the motion = the work done against the resistance to the motion. i.e. P. OO' = R. MO' + Q. NO'

P & OO' and the directions of Q & R are known, i.e. the angle between them; MO & NO can easily be found, and thence the magnitudes of R & Q.

Again if any two forces are known in magnitude and direction, the third can be found in magnitude and direction by the same sort of method.

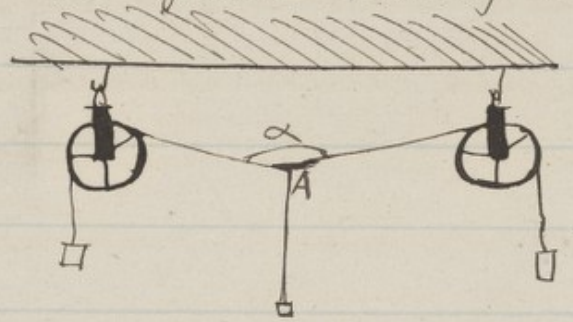
But this is not the method which we shall use in finding the resultant of two or more forces: it being more complicated & not so well suited for the purpose as the one we shall use: what we have done is only to shew how the problem could be dealt with by the principle of work and so keep up the connection of this with the rest of the subject.

To begin with take a few concrete examples.

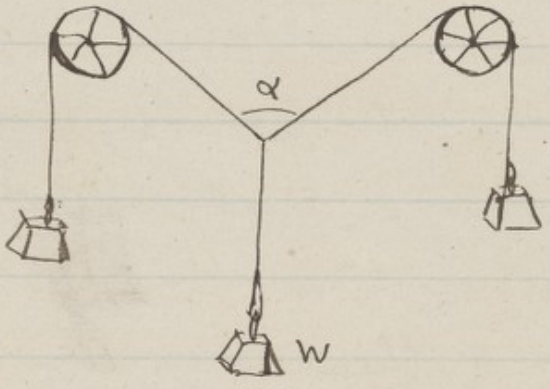


Suppose we have two equal and opposite forces acting on a pt. A, by means of pulleys, as in the

figure; there will be equilibrium; suppose now a small force acts vertically on the pt. A in addition, the first string will be no longer

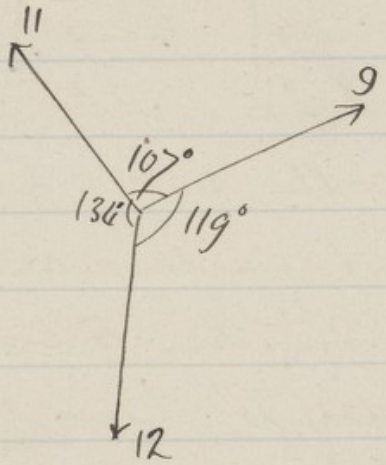


horizontal; but the angle α thus formed will get smaller & smaller according as A is increased; till when the B forces are equal ~~they~~ all three strings shave the same angle between them viz 120°.



As the force W increases, ∠α diminishes, but the other two angles are ~~the same~~ equal, provided the other 2 forces are.

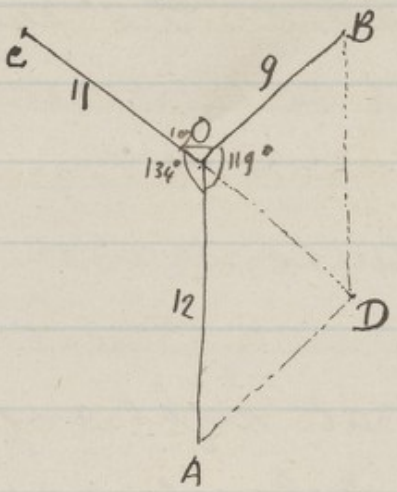
Now make all three forces unequal: say take wts. of 9, 11 & 12: the angle between 9 & 11 will be measured and found to be 107°; between 11 & 12, 134°; and the remaining angle 119°.



The smallest force is thus opposite the greatest angle; the greatest force opposite the smallest angle; and the intermediate force opposite the intermediate angle.

Take lines proportional to the forces & in the same direction: we thus get a representation of them; i.e. mark off 12 units of length on a vertical line; then draw line at angles on each side of it ~~at~~ of 134° & 119°,

measuring the distances 11 and 9 on them respectively ~~from~~ the ~~pt~~ O.



That is, Let OA = 12, OB = 9 & OC = 11: through A draw a line parallel to OB, meeting a line parallel to ~~BD~~ OA drawn through B; and draw the diagonal OD ~~with reference~~; OD will be found to be CO produced; and = OC i.e. = 11. And so for all other cases: Whatever the forces be ~~each one~~ they will be found to be in equilibrium, when each one is equal to and a continuation of the diagonal of a parallelogram, under the lines of the line of action of two others.

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This result is called the Parallelogram

of Forces; the theorem may be stated thus;
If one represents by straight lines any
two forces acting on the same pt; their
resultant will be represented in magnitude & direction by the
diameter of the parallelogram under the
two straight lines, drawn ~~through~~ from that
point.

If the forces act at right angles; the two
diameters of the rectangle are equal in
magnitude, but different in direction; ~~and~~ but
in any other parallelogram, they are unequal
also.

In drawing the ~~rectangle~~ parallelogram
both forces must act concurrently; i.e. either
both from or both to the point; and the
diameter must be concurrent with the forces.

The converse is also true:-

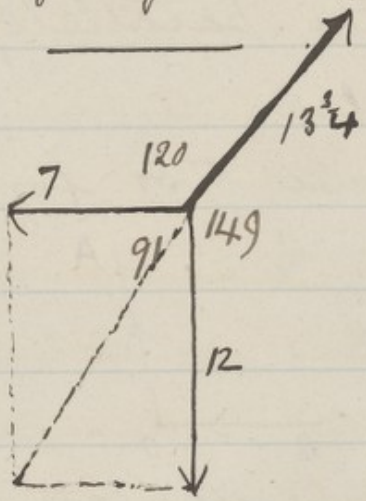
If two forces acting on a point be represented
in magnitude and direction by the two
sides of a parallelogram, and the third

by the diameter drawn through the point in the opposite direction, they will be in equilibrium.

This is because it will be equal & opposite to the resultant of the 2 forces.

This can thus be proved experimentally:-

Take a line representing a force 7 & another representing a force 12, and put them together at any angle say 91°, complete

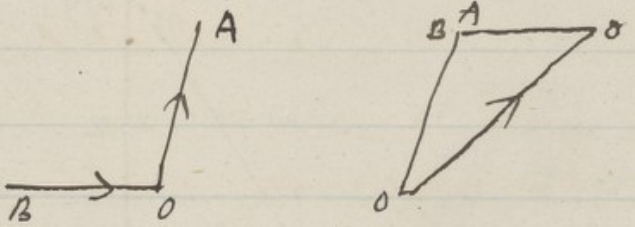


the parallelogram and measure the diameter; this will be found to be 13¾ and drawn in the opposite direction will be found to be the equilibrating force. Measure the angles, they will be found to be respectively 120° & 149°

That these are in equilibrium will be seen by hanging the wts 7, 12 & 13 3/4 by strings over pulley as on page 107; and they will come to rest in the position just indicated

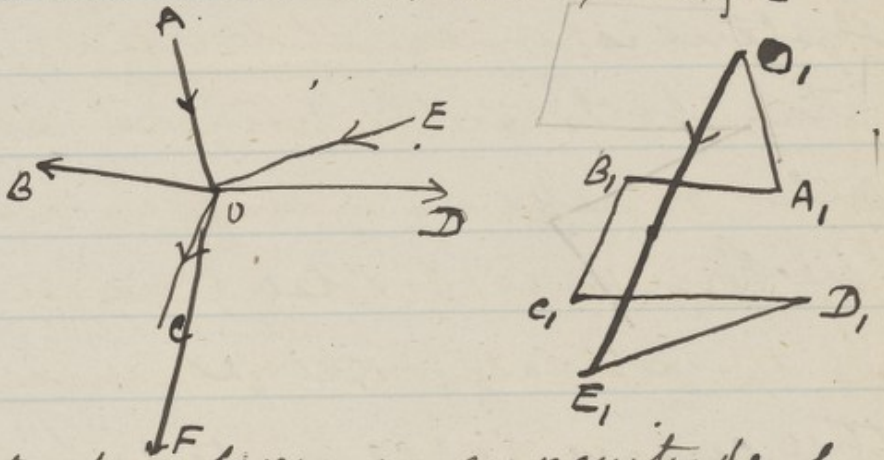
To find the resultant of more than two forces, the triangle and not the parallelogram of forces is used; it can also be applied to two forces; as we get the diameter in the same way: all we wanted with the parallelogram ~~for~~, being to get its diameter

Say we have two forces BO & OA acting



on a point O; we get the triangle which is half the parallelogram, by drawing a line to represent the force OA, in mag. & direction and from its extremity do the same for BO; and join the pt. we start from to the pt. we

get to; the line joining these two points representing the resultant.
This can be extended to any no. of forces,
e.g. suppose the lines OA, OB, OC, OD & OE



represent five forces in magnitude & direction; take any pt. O_1; draw from it a line O_1A_1 equal and parallel to OA; from the pt. we get to draw a line equal & parallel to the next force OB, and so on, till you have done so for all the forces; the resultant is (represented by) the line joining the starting point to the pt. we get to.
Suppose we had as the force OF; then the line joining these two pts. would be of no length; i.e. the resultant = 0 : i.e. the system is in equilibrium.

That is to say;-

If any number of forces acting on a pt can be represented by the sides of a closed polygon taken in order, that system is in equilibrium; this is true both for a polygon in one plane only, or askew, i.e. in several planes.

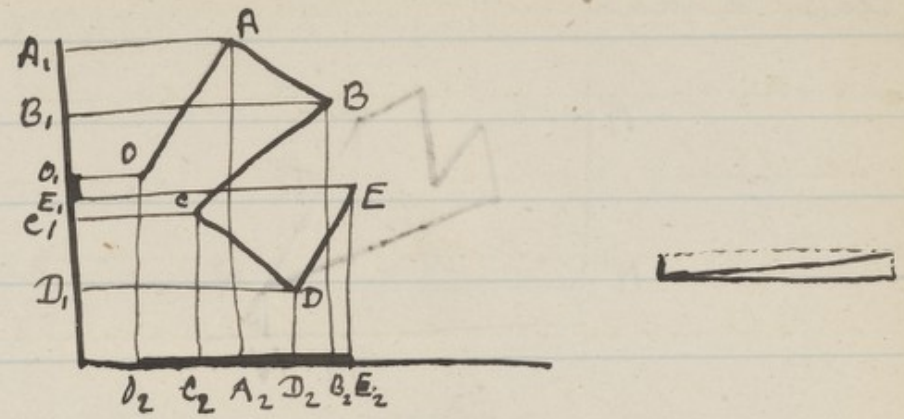
The converse is also true;-

If a system of forces acting on a pt. is in equilibrium they can be represented by the sides of a closed polygon taken in order

If we ~~we~~ have two forces A & B at right angles their resultant R $= \sqrt{A^2 + B^2}$

Another way of finding the resultant of a number of forces is ~~th~~ to ~~represent~~ reduce them all to two forces at right angles, and find it by this method.

E.S. suppose we get by drawing a polygon as just now, forces represented

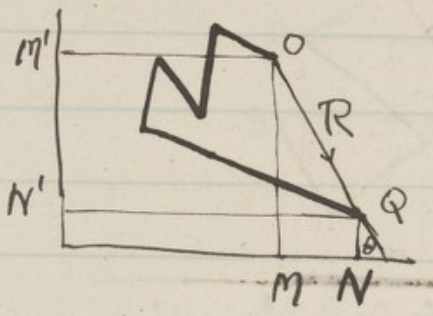


by the lines OA, AB, BC, CD, DE; draw two lines at right angles and project each force on the horizontal line; this gives us the horizontal component of each; and we get the vertical component of each similarly; then O_1E_1 & O_2E_2 are the components of the resultant, at right angles, from which we get the resultant; instead of projecting each line we need only project the starting pt & the pt. we get to i.e. O & E on each of the two lines; this of course gives us the components of the resultant.

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We can get the direction of ~~to~~ the resultant in

this way:



we know $R = \sqrt{\overline{MN}^2 + \overline{M'N'}^2}$

$$\text{Sin } \theta = \frac{M'N'}{R}$$

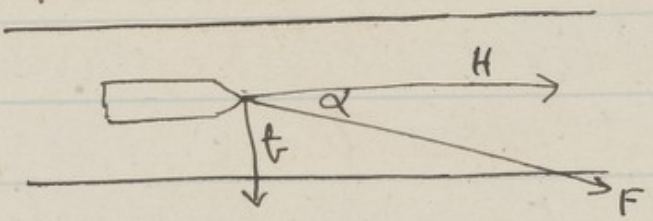
$$\text{Cos } \theta = \frac{MN}{R}$$

Suppose we get $\frac{M'N'}{R} = .678$

then $\theta = 42° 42'$

We saw some time back, that if we have a force applied in a different direction to that in which motion is to take place, the force can be resolved into an effective component, in the direction of the motion, and a non-

effective component at right angles to it; (see pp. 60-68). For instance, suppose we have the common case of a barge being drawn along a canal by a horse on the bank,

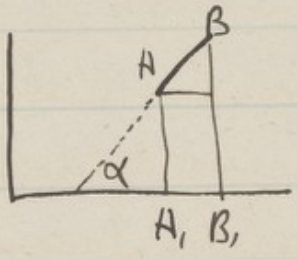


the force is applied in the direction F; the effective component called H & the non-effective, G.

then $H = F \cos \alpha$

$G = F \sin \alpha$

Let us see the result we get by projection:-



Suppose we have a force AB, project it on one line to a position A_1B_1, then $A_1B_1 = AB \cos \alpha$; therefore we find one component by multiplying $\cos \alpha$ by AB:

From this, we get a rule of universal application; -

Multiply any force by the cosine of the angle, its line of action makes with any given direction, & the product is the component in that direction.

Multiply any force by the sine of the angle its line of action makes with any given direction, and the product is the component at right angles to that direction.

We have hitherto to only considered forces acting on the same pt; they might however act on different points; if their lines of action intersect, the forces may be considered to act on the point of intersection; we may consider a string of particles to

transmit the force to that point.

But their lines of action may not intersect if produced: these then are either forces acting along parallel lines, or in different planes: if the latter case, one of the forces may be resolved into a component parallel to the line of action of the other forces, and one at right angles to it; we have then to consider the effect of the parallel forces, and then just add the effect the other force to it: thus in consid-ering forces whose lines of action do not intersect, we have only to consider

Parallel Forces.

The resultant of a system of parallel forces is found, by considering their efficiency to produce or prevent rotation about a fixed pt; and by the definition of resultant, that force must have the same efficiency.

The best way of considering is to let the forces act on a rigid bar, of which one line is fixed viz: that parallel to the direction of the forces.

The efficiency of a force to produce or prevent rotation about a fixed line depends on two things:-

1. It is proportional to the force itself: e.g. a force 2 would have twice as great an effect as a force 1.
2. The relative position of the line of action of the point and the fixed line.

If we take equal forces and put them at equal distances from the pivot, we have equilibrium; but if on one side we halve the force and keep the distance the same and on the other side, keep the force the same, and halve the distance,

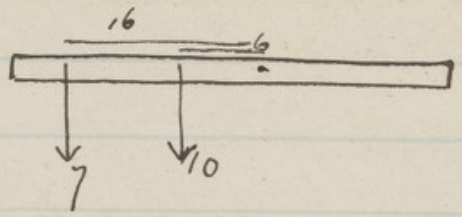
we still have equilibrium; as the product of the force & distance is the same on each side;

The efficiency of a force to cause rotation about a fixed line, is measured by the product of the force & its distance (its turning action) from that line, and is called the moment of the force ~~about the~~ with regard to that line. (something similar to work)

The resultant must have the same moment, as the forces; or the moment of the result with regard to that line, must be equal to the sum of the moments of the components.

We may take the moments about any pt. in the line, but it is most convenient to take the pivot as the pt.

Suppose we have the forces of 7 & 10 parallel to one another, and at distances respectively of 16 & 6 from the pivot;

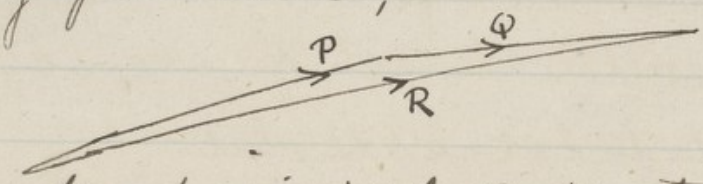


We want to find their resultant

F	×	D	=	M
10	×	6	=	60
7	×	16	=	112

Sum of moments = 172

Their resultant must have the a moment = 172 ; we can at once determine the magnitude of the resultant by the following consideration; Suppose we have two forces P & Q very nearly parallel,



then by the principle of the triangle of forces, the line R in the figure represents their resultant. The more

nearly P & Q are parallel, i.e. in the same straight line, the more nearly is R = P + Q: till ~~it~~ in the limit, i.e. when they are parallel, R = P + Q

In the case before us, then R = 17

We have still to determine its distance from the pivot: let x be this quantity;

then $R \times x = 172$

$$17 \times x = 172$$

$$x = \frac{172}{17} = 10\frac{2}{17}$$

That is a force of 17 applied at a distance of $10\frac{2}{17}$ from the pivot would have the same effect as the other two; or what is the same thing: is that force which if applied in the opposite direction would just keep the other two in equilibrium.

Thus for any number of parallel forces, F_1, F_2, F_3 ---- ; $R = F_1 + F_2 + F_3 + \text{-----}$

meaning the algebraic sum: let a, f_1, f_2, f_3

etc. stand for the distances of the lines of action of the forces R, F_1, F_2, F_3 etc. respectively from the fixed line, and we have

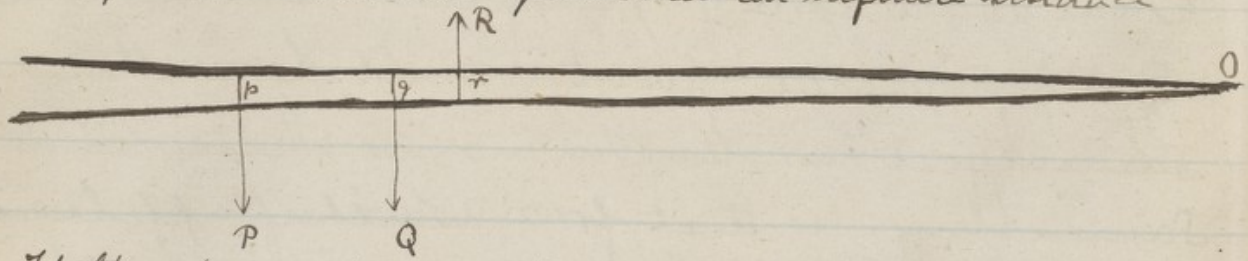
$$Rr = F_1 f_1 + F_2 f_2 + F_3 f_3 + \text{etc.}$$

$$r = \frac{F_1 f_1 + F_2 f_2 + F_3 f_3 + \text{etc}}{R}$$

$$= \frac{F_1 f_1 + F_2 f_2 + F_3 f_3 + \text{etc}}{F_1 + F_2 + F_3 + \text{etc.}}$$

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Suppose we have a bar pivoted at an infinite distance



If the bar rotates so that a force P moves through the distance p, Q through q, and R, the resistance through r. Then the work done by the forces must = the work done against the resistance, if the system is in equilibrium; i.e.

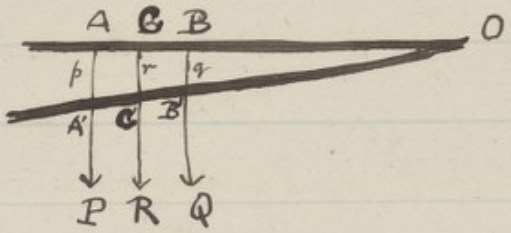
$$Pp + Qq = Rr.$$

But if the point O is at an infinite distance, the bar moves parallel to itself ∴ $p = q = r$

$$\therefore P + Q = R$$

i.e. in a system of parallel forces R', the resultant is a force equal ~~and opposite~~ to the sum of the components, and is equal and opposite to R, the force which keeps them in equilibrium.

We must now find where R is to act: Suppose the bar to turn about a point O. Suppose R to be the resultant of



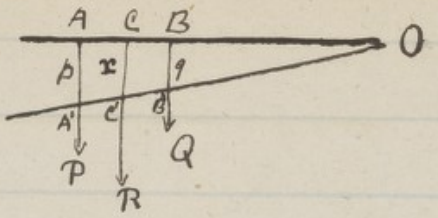
P and Q: knowing the lengths AO and BO in the figure we want to find CO: Let:—

$AA' = p$	$AO = a$
$BB' = q$	$BO = b$
$CC = r$	$CO = c$

then $Pp + Qq = Rr$

i.e. $Pp + Qq = (P+Q)r$

or $$r = \frac{Pp + Qq}{P + Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$



By similar triangles

$$\frac{AA'}{AO} = \frac{BB'}{BO} = \frac{CC'}{CO}$$

or $$\frac{p}{a} = \frac{q}{b} = \frac{r}{c} \qquad \text{.....(2)}$$

$$\therefore c = \frac{ar}{p}$$

and we have $r = \dfrac{Pp + Qq}{P+Q}$

$$c = \frac{a}{p}r = \frac{P\frac{a}{p}p + Q\frac{a}{p}q}{P+Q}$$

from (2) we get $\frac{a}{p}q = b$

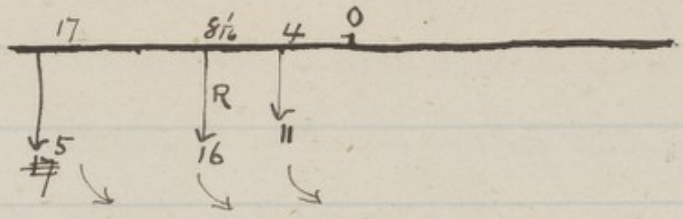
$$C = \frac{Pa + Qb}{P+Q}$$

Q.E.F.

expressed in other words

$$c = \frac{\text{sum of moments of components}}{\text{Sum of components.}}$$

It does not matter about what point we take the moments; to show this take an example, and we shall see that we get the same point, as the point of application of the result, wherever we take the moments.



Suppose we have a force 5 acting at a distance 17 from the pivot

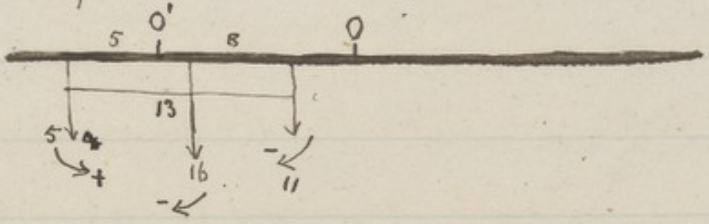
and ..... 11 ..... 4 .....

the resultant = 11 + 5 = 16 in magnitude.

we want to find c or the distance of the point of application of R from the pivot; take moments about the pivot:-

$$c = \frac{5\times 17 + 11\times 4}{5+11} = \frac{85+44}{16} = \frac{129}{16} = 8\tfrac{1}{16}$$

Now take the moments about anyother point, say, a point 12 from the pivot; that will bring us to a point between the two forces; the moments will then be in ~~equal~~ opposite directions; we must therefore call one positive and the other negative; the forces have the same sign as they act in the same direction:



$$c = \frac{5\times 5 + (-8)\times 11}{16} = \frac{25+(-88)}{16} = \frac{-63}{16} = -3\tfrac{15}{16}$$ i.e. a distance $12 - 3\tfrac{15}{16} = 8\tfrac{1}{16}$ from the pivot which is the same as before.

And so for any number of parallel forces, treat them in exactly the same way; viz: when they are in equilibrium the work done by the forces must = the work done against the resistance, which is a force equal and directly opposite to the resultant:

i.e. $Rr = Pp + Qq + Ss + \text{etc.}$

Then supposing the bar to move parallel to itself:

$r = p = q = s = \text{etc.}$

Then $R = P + Q + S + \text{etc.}$ i.e. we get the magnitude of R

this is the algebraic sum: some of the forces acting in the opposite direction, which we call negative.

& then by the principle of moments:-

$$Rc = Pa + Qb + Sd + \text{etc.}$$

$$c = \frac{Pa + Qb + Sd + \text{etc.}}{P + Q + S + \text{etc.}}$$; (remembering the proper signs)

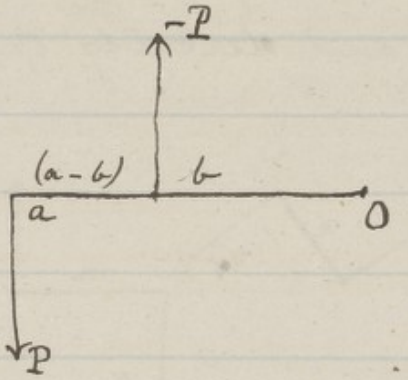
i.e. we get the position of R.

There is still one case of the resolution of forces which we have not studied; viz: when we have two forces, equal and in opposite directions:

i.e. $Q = -P$

then the resultant $R = P + Q = P + (-P) = 0$

to get its position; let a = the distance of the point

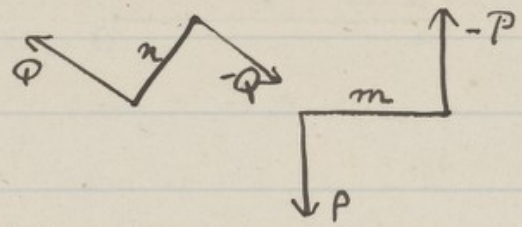


of application of P from the pivot O, and b = that of $-P$; and c, that of the resultant:—

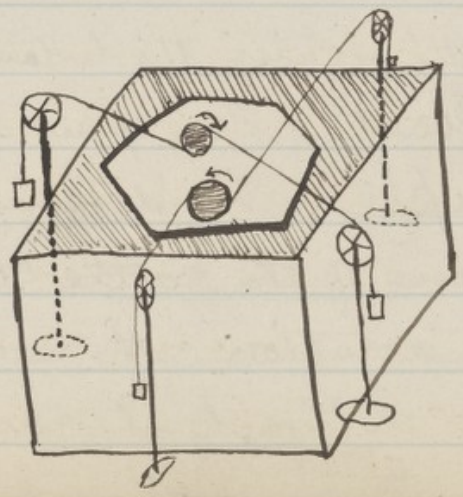
$$c = \frac{Pa - Pb}{0} = \frac{P(a-b)}{0} = \infty$$

i.e. the resultant is a force infinitely small, applied at an infinitely great distance; in other words they have no* resultant, but the smaller we take the force and the greater the distance, the nearer we get to a resultant. This pair of forces is called a <u>Couple</u>. The distance $(a-b)$ is called the <u>arm of the ~~cuttle~~ couple</u>; ~~its~~ the effect of a couple is to produce rotation; the moment of the couple being $P(a-b)$. Although it has no resultant, and

therefore no single force can neutralise it; yet it can be neutralised by a couple whose moment is equal to it and ~~directly~~ in the opposite direction;



The two couples in the figure ~~ss~~ tend to produce rotation in the opposite direction; i.e. their moments are opposite if they are equal and both applied to the same body: i.e if $Qn = -Pm$, the forces will be in equilibrium. This may be shown experimentally thus;



Take a tank full of water, and on its surface float a flat piece of wood of any shape; on the upper surface of the wood put two pulleys, round which strings are passed; let the pulleys ~~be~~ for this simple example have their diameters in the ratio 2:1; and let them be fixed so that they cannot rotate without the piece of wood to which they are attached, rotating also; so that in letting forces act upon them, we let them act on the floating piece of wood; to the smaller attach two strings one wrapped round one way, and the other the other; let forces act on these strings by means of the strings passing over pulleys, and weights being attached to the ends; ~~in the figure~~ let the weights on each be equal, and the directions of the strings directly opposite; in the figure if this couple acted alone, rotation of the piece of wood would take place from right (3) to left (2); do the same with the other so that rotation would take place from right to left; To have equilibrium ~~if~~ we must fulfil the equation. $Q_R = -P_R$; and here one arm of one couple is double that of the other; i.e. the weights on the larger-armed couple must be half that of the smaller. We then have equilibrium; these being the only conditions, that the moments must be equal and opposite; so that we might move the pulleys to any other part of the board; and still have

equilibrium; or we might alter the actual direction of any ~~pair~~ couple (still keep one member of the couple parallel to the other member), by moving the pulleys and we should still have equilibrium; if only the tendency to rotate were opposite. And so we might make the ratio of the arms other than that of 2:1; say $m:n$; then the weights P & Q must be in the opposite ratio i.e. $\frac{P}{Q} = \frac{n}{m}$, and we have equilibrium.

The principles of parallel forces are applied to finding what is called the centre of gravity of a body: ~~sup~~ each particle of the body is acted upon by the force of gravity, we thus have a system of parallel forces, the points of application of which are the particles of the body; the point of application of the resultant of these forces, is called the Centre of Gravity of the body; the processes of finding the centres of gravity of ~~the~~ bodies are simply Geometrical problems, depending on the principles of parallel forces.

Jan 7th 1878.

We now come to

Fluids.

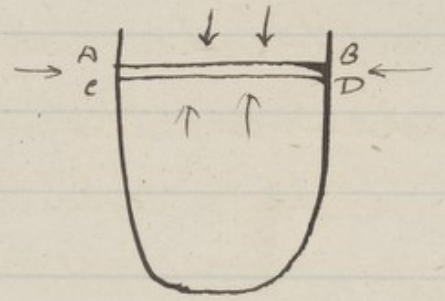
The characteristic property of a fluid is that a change of position of the particles of any portion of that fluid can be effected by the smallest possible force: this is the definition of a perfect fluid which does not exist. Fluids differ from solids moreover in having no definite shape of their own; they take the shape of the vessel containing them; there is another class of fluids which have neither a definite shape nor a definite volume; taking the shape & volume of the containing vessel: into these two classes fluids can be divided. the first are liquids, and the second gases. In order to keep a solid in equilibrium, Equal and opposite forces must be applied to it; but if we apply equal & opposite forces to a liquid, they exert no pressure upon it, because it escapes between them: forces must be applied equally on every side of a given

portion of fluid, this is a most important property of a fluid; a very great pressure on a liquid will affect an infinitesimal change of volume, while on a gas a very great change. This is an important difference between liquids and gases, which although only one of degree is a very marked one; for this reason liquids are sometimes spoken of as incompressible fluids, and gases as compressible.

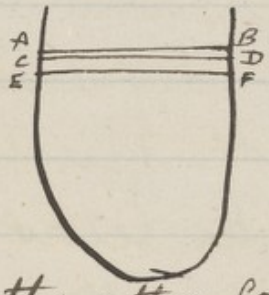
Liquids.

The property that a liquid exerts forces equally in all directions, can be seen in a syringe with opening in all directions; on pressing the piston water escapes not only in the direction in which the force is applied but with equal facility in all other directions. All particles on a liquid are acted upon by the force of gravity; on the surface also is the pressure of the atmosphere which also presses on every thing else but is not per-

-ceived as it is equal in all directions, and this may therefore be neglected. Suppose we have a bell jar filled with water; consider the top

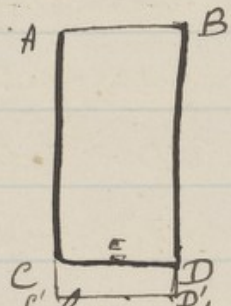


stratum of the liquid ABCD; on it is acting the wt. of the particles composing it, downwards; this force is exerted upon CD, which reacts with an equal and opposite force: the sides of the vessel also act with the same force, to keep that stratum in equilibrium; now take the next stratum CDEF



its weight and the other forces act upon it in the same way; but in addition there is the wt. of the top stratum; all this acts on the surface EF

Now take any other horizontal surface in the liquid; it supports the weight of the liquid above it; or beginning with the lowest stratum and going upwards, we get that as the depth increases, the pressure on the bottom increases; take a cylindrical vessel ABCD. full of water ABCD; the bottom CD supports



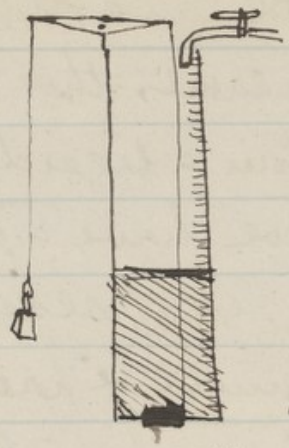
the weight of the liquid above it, and one tenth of the bottom supports the wt. of one tenth of the liquid and soon. Imagine a small plate E at the bottom, and the depth of the vessel to be increased to C'D'; the pressure on E will not be affected by the water below it, and the pressure on it remains the same as if it were part of the bottom of a vessel of that depth; put the plate which is supposed to be very

small in an oblique position and at the same depth and the result is the same; this is what is meant by pressure within a liquid; now suppose instead of a plate we have a small cube of the liquid we saw that if it is pressed by a force in one direction it must be pressed in all directions equally to have equilibrium; consequently if the cube is very small, it must be pressed on each side by an ~~equal~~ force equal to its depth below the surface; Let P be the pressure on a pt i.e. an infinitesimally small surface, and a the area of a unit surface say a sq. inch, sq. centimetre, etc., the

$$\frac{P}{a} = p$$

p being the intensity of pressure

This may be illustrated thus by experiment: Take a cylinder graduated, and with a valve at the bottom held shut by a bot, as in the figure (P. T. O) :-



Pour in water and mark the height to which it must rise so as to overcome the pressure on the valve; altering the wt. from time to time: the following are some results obtained in this way: let W stand for the wt. & D for the height of the column of water;

W	D	$\frac{W}{D}$
4	7.5	.53
6	11.2	.53
8	14.9	.53
9	16.45	.54
13	24.	.54
14	26.2	.53

We thus see that the quotient $\frac{W}{D}$ is nearly constant.

Jan 9th 1878

Let a = area of the valve
h = height of the water
& S = Specific gravity of the water = wt. of unit vol.

ahS = total pressure on the valve = force applied to lift it.
$= W$

and a and S are constant

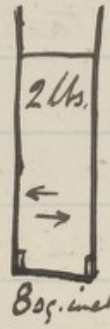
$$\therefore aS = \frac{W}{h}$$

i.e. $\frac{W}{h}$ is constant also, as we saw in the example.
Pressure in a liquid takes place not only vertically but also in all other directions equally. This may be shown for the upward pressure, by putting



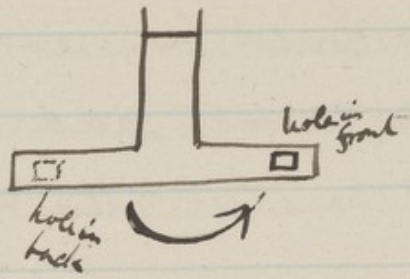
an empty cylinder with a loose plate at the bottom into water; this plate will support a wt. just equal to the wt of the water displaced. When the plate is removed the portion of water in its place has exactly the same forces acting upon it; i.e. the upward force = the downward force. The pressure obliquely may be shown by immersing the cylinder obliquely; & the same takes place. The lateral pressure may be shown in this way. Take a cylinder containing say

two lbs of water and of 8 sq. inches horizontal section



Then the pressure on each square inch is ¼ lb. The pressure on the sides each way is the same or else the cylinder if suspended by a cord would move in the direction of the greatest pressure. Take a sq. on the side near the bottom; the pressure would be nearly ¼ lb; not quite because not quite at the same depth as the bottom; cut away that sq. inch & the water will flow out, and the vessel would then move away from the hole if suspended: because the pressure on the opposite side would be greater; this may be shown better thus; pour water into a vessel with two horizontal arms, one with a hole in the back and the other one in the front; water escapes

and the vessel rotates away from the holes.



The pressure P on any surface of area a, depends only on the height of the liquid above it h, and the specific gravity of the liquid.

$$P = a h s.$$

Take three vessels are these shapes; of the



same height, and with a bottom of the same area; fill with water: the pressure on the bottom of each is equal; (this may be shown by a valve arrangement like that on page 138.) Though in one case the pressure is less than the wt. of the water, and in another greater.

Whenever a fluid is at rest, the pressure exerted by it on a surface in contact with it, is exerted ⊥r to that surface.

If were not so, the pressure would be oblique to that surface; this could be resolved into two pressures one $\perp^r$ to the surface, and the other paralled to it; the component paralled to the surface would cause motion.

Imagine the case of the cone of water, as composed of two portions, one cylindrical & the other surrounding it

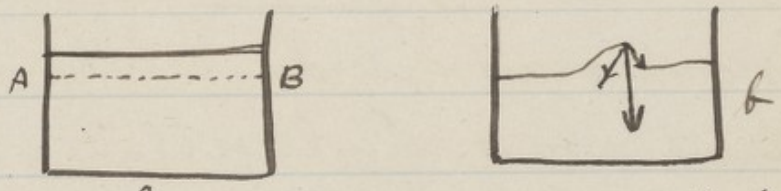


The pressure exerted by the side portion would be in the direction of the arrows, viz: $\perp^r$ to the cylinder and to the side of the vessel; this would make no difference to the pressure exerted by the cylinder itself, vertically therefor the pressure on the bottom would be just as if there was only the cylinder of water & no side portion. Take the other case.



in all parts the pressure exerted by the vessel itself is equal to the pressure of the water on it Lr to it. This pressure exerted by the vessel may be resolved into components vertical and horizontal; the vertical would be added to the pressure exerted Lr by on the bottom by the water itself and so be just the same as if the whole cylinder were there.

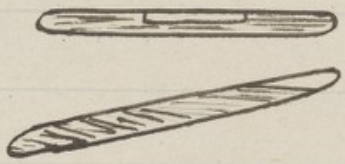
The free surface of a liquid at rest is horizontal. We have seen that the pressure at the same depth is the same in a liquid at rest



Say at AB; let this level be taken higher & higher: and this can only be the case if the surface is horizontal. Or another way let the surface be not horizontal (Fig b); the pressure which is horizontal could be resolved in 2 directions, one Lr to & the other ||e to the uneven surface: the latter would cause motion until

the surface was ⊥r to the pressure exerted on it, viz gravity; this acts vertically, and therefore the surface must be horizontal.

This property gives one the means of determining whether a solid surface is level or not; put the liquid in a tube leaving a bubble of air: this bubble will be in the middle if horizontal, and if sloping go to the higher end; this is



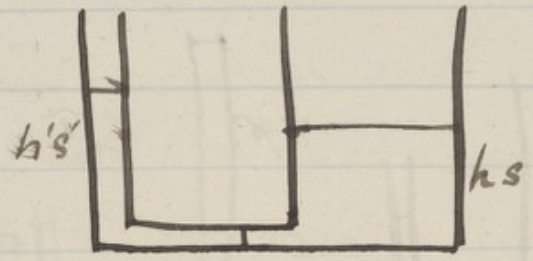
called the Spirit level, of which there are several forms according to the use for which they are required. In levelling a surface, it must be levelled in two directions at right angles to one another.

The surface of a liquid may be divided into various partitions, but the same level is observed in all.

Jan 14th 1878.

Saying that the surface of a liquid at rest is horizontal is the same as saying that it is perpendicular to the pressure exerted on it viz: gravity; this is only true in one liquid; if the liquids in different partitions are of different natures then the levels may or may not be the same.

Suppose we have two vessels connected by a tube at the lower part



The condition for equilibrium is that the pressure at any pt is the same in opposite directions; if not it will flow in the direction of the greatest pressure. Imagine a thin membrane stretched across the tube: let

$P = ahs$ = pressure from right to left

& $P' = ah's'$ = pressure from left to right; i.e. we are supposing different liquids in the two partitions.

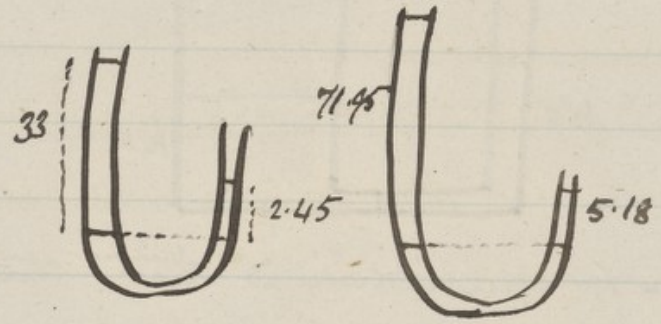
Then if there is equilibrium

$$P = P'$$

i.e. $$hs = h's'$$

or $$\frac{h}{h'} = \frac{s'}{s}$$

i.e. the heights of the columns vary inversely as their specific gravities; the heights being measured from the common surface. Water and Mercury as two good liquids for an example as they dont mix.



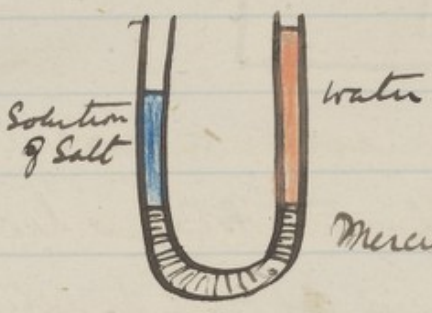
Experiment

$$\frac{h_m}{h_w} = \frac{33}{2{\cdot}45} = 13{\cdot}3$$ s.g. of mercury taking that of water as 1.

Do the same again

$$= \frac{71{\cdot}95}{5{\cdot}18} = 13{\cdot}9$$ which is not the same from inaccuracy of measurement.

Any other liquid may be taken instead of water and compared through mercury to water; but this is done better thus; say we want to find



the s.g. of solution of common salt; do as in the figure pour in ~~coal~~ quantities in each side of water and salt water so that the mercury may be at the same level in each side and compare the heights of the two columns as before

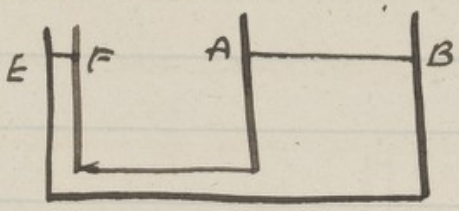
$$\frac{S_{salt}}{S_{water}} = \frac{h_w}{h_s}$$

This is applied to the measurement of the expansion of mercury (See page 28 Heat)

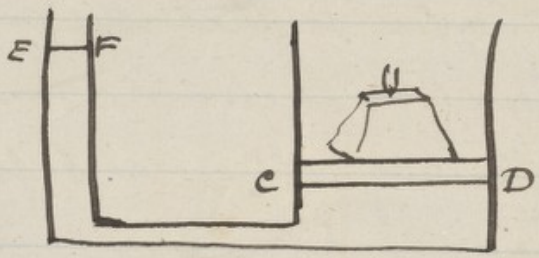
$$\frac{h_{100}}{h_0} = \frac{S_0}{S_{100}} = \frac{S_0}{\frac{S_0}{1+100\delta}} \qquad \delta = \text{coeff of exp. of mercury}$$

$$\delta = \frac{h_0 - h_{100}}{h_0 \times 100} \qquad \frac{h_{100} - h_0}{100 h_0}$$

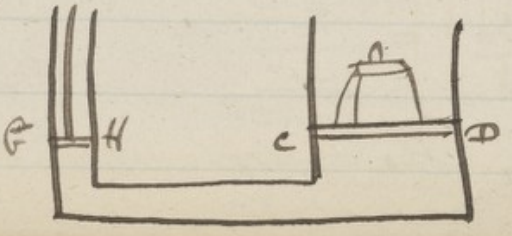
The Same principle is applied to the
Hydraulic Press.



The essential part consists of two cylinders connected by a tube; the level E F A B is horizontal, the pressure is the same in opposite directions everywhere; replace some of the water in the larger by a ~~cylinder~~ piston C D and wts = the wt. of water removed.

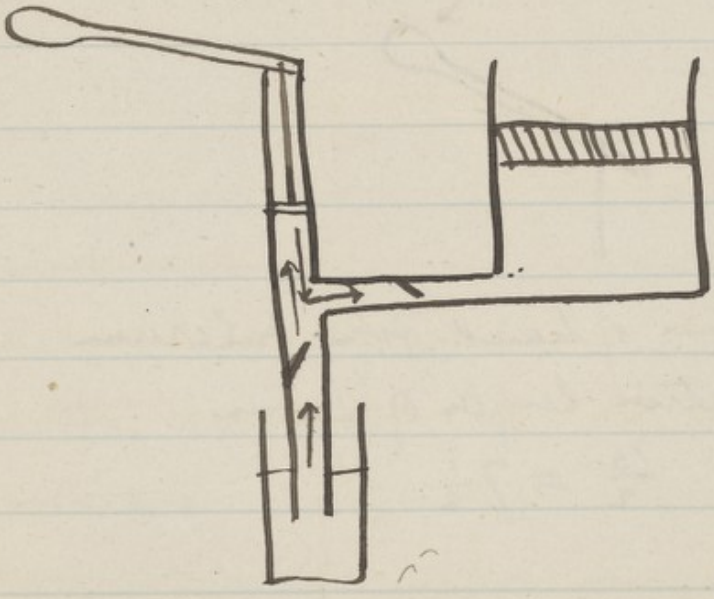


and thus a very small quantity of water can support a very great wt; if the larger vessel be very wide. Replace part of this small quantity of water by a piston G H



press down this small piston and the large one will be driven up as the water between cannot be compressed; suppose the area of small tube = 1 sq. inch. and of the large tube 100 sq. inches. Then if the small piston be pushed down one inch the large one will rise $\frac{1}{100}$ inch: or if the small piston be pushed down 100 inches, the large will rise 1 inch: i.e a.l. = A.L.

This may be improved by making the small cylinder & piston a pump also, and working it by a lever with a large mechanical advantage.



The position of the valves & course of water is shown in the figure.

Let d & D be the diameters of the pistons; the following show the enormous mechanical advantage of a small hand model.

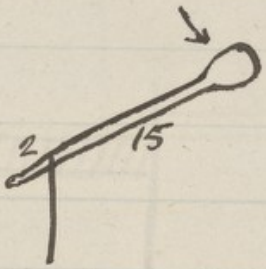
$$\frac{d}{D} = \frac{\frac{3}{8}}{1\frac{1}{2}} = \frac{1}{4} \quad \text{inches.}$$

$$a = \frac{\pi}{4} \cdot d^2$$

$$A = \frac{\pi}{4} \cdot D^2$$

$$\frac{a}{A} = \frac{d^2}{D^2} = \frac{1}{16}$$

Then there is the mechanical advantage of the lever as well.



15 inches = distance of hand from fulcrum
= effective length of Lever.

Mech. Adv = $\frac{15}{2} = 7\frac{1}{2}$

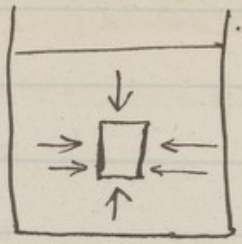
Total mechanical advantage of press = $7\frac{1}{2} \times 16 = 120$

Suppose we exert a pressure of 100 lbs; the force exerted by the large piston $= 120 \times 100$ lbs.
$= 12000$ lbs
$= 5$ tons

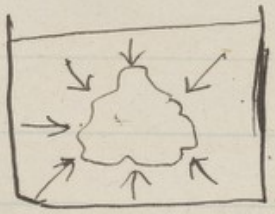
The equation $P = ahs$ is only true if the surface on which the pressure is exerted, be horizontal; if it is not horizontal h varies; and the equation will be true if by h one means the average depth of the surface, which is the same as the depth of the centre of gravity of the surface.

Jan 16th 1878

Let us now consider the pressure on bodies entirely immersed in a liquid; suppose first we have a body of regular shape such as a cube or a cylinder, and that the lower and upper surfaces are horizontal; there will be a pressure downwards on the upper surface equal to the weight of the column of water it supports: there

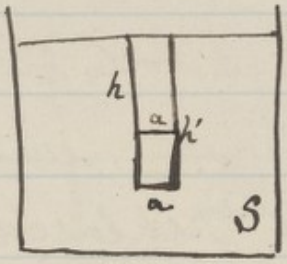


upon the lower surface there will be an upward pressure equal to the wt. of a column of water ~~at~~ of the same depth as the surface is. The pressure upwards will be greater because the surface is at a greater depth: the horizontal pressures cancel; the wt. of the body acts downwards. This holds equally good for a body of any shape; the parts on which the



upward pressure acts are deeper than the parts on which the downward pressure acts; and the pressures in all other directions may be ~~as~~ resolved into horizontal and vertical components; the horizontals all cancel; and

thus only the vertical have to be taken into account, when the same truth holds. We have now seen that there is in all bodies immersed in water a resultant upward pressure. What is the magnitude of this pressure: going back to the case of the cube; let the pressure on



the upper surface be represented by P, its area by a and the S.G. of the liquid S.

then $P = ahS$ = downward pressure:

Similarly $P' = ah'S$ = upward pressure

then $P' - P$ = resultant upward pressure:

$P' - P = (h' - h)aS$

but $h' - h$ = length of cube
$= l$

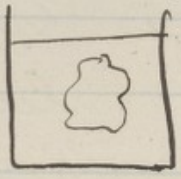
$= laS$

la = volume of cube
$= v$

$= vS$ = weight of a quantity of the liquid of the

Same bulk as the solid = w.

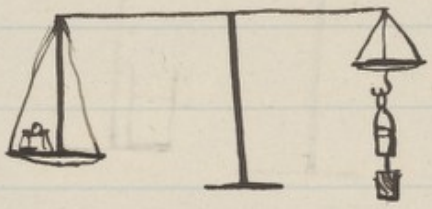
This truth is applicable in all cases: it may be seen in a different manner for a body of irregular shape:



Suppose we have a body of any shape in the liquid as in the figure; imagine this suddenly with drawn, and replaced by ~~water~~ the liquid; the pressure on this liquid exerted by the rest of the liquid will be of course the same as in the previous solid; let us see what forces are acting upon it; first there is the weight of the liquid acting downwards, and we have seen a resultant pressure acting upwards; but the liquid is in equilibrium therefore these two must be equal: i.e. the resultant upward pressure is equal to the wt. of the liquid: now put in the solid again; the only alteration

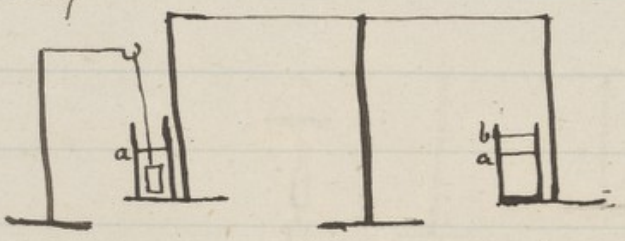
in the forces acting is the weight of the solid: the resultant pressure upwards is still the wt. of the liquid of the same bulk of the solid; i.e. when a solid is immersed in water it is pressed upwards by a force equal to the weight of the liquid displaced: i.e. it appears to lose that weight.

These principles may be shewn experimentally thus:



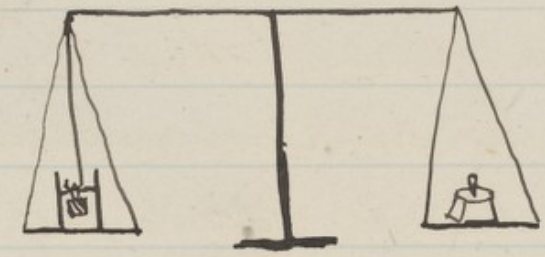
Take a balance of the ~~weight~~ shape above: hang to one beam a solid cylinder, and a bucket into which it fits: counterpoise this by suitable weights on the other side: then immerse the solid cylinder in water, it appears to lose weight; this weight is equal to the wt. of liquid of the same bulk as the cylinder, which may be shown by filling the bucket with water, when equi-librium is at once restored.

It is a universal law that if there is a pressure in one direction, there must be an equal and opposite pressure somewhere else: therefore as the ~~solid~~ liquid presses the ~~water~~ solid; the solid must press the liquid equally and in an opposite direction; this may be shown by the same apparatus arranged differently



Counterpoise on a balance two beakers of water i.e. one in each pan filled up to levels a a; we then have equilibrium: then hang in one the solid cylinder by a thread as in the figure; the water is pressed by this downwards and that side of the balance falls; the downward pressure must be equal to the weight of liquid displaced; which may be shown by adding water to the other beaker = the volume of the solid; i.e. fill the before mentioned bucket with water and empty it into the

beaker, up to the level b, and equilibrium is restored; these two experiments may be combined thus: ~~hang~~ first counterpoise



a beaker of water and the solid cylinder both put into the same pan by suitable weights in the other pan; then hang the cylinder in water by a thread attached to the beam; this is pressed upwards by the water, and presses the water downwards equally; hence there is no alteration at all in the equilibrium of the balance.

—

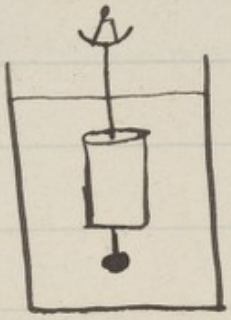
These results have important bearings on the conditions of floating: the resultant upward pressure may be either greater or less than the ~~resul~~ wt. of the body; if less, the solid sinks i.e. falls with speed diminished by the upward force;

if greater the body will rise and float with some part of it out of water; if the two are equal the body will be in equilibrium at any point in the liquid. We have said what is the same thing as, if a body is bulk for bulk heavier than a liquid it sinks; if lighter it rises; when it rises part projects, the part immersed depends on the principle above stated viz: that the downward force or wt. of the body, must be equal to the upward force or weight of liquid displaced; and the floating body will rise until the weight of the water it displaces is equal to its own weight. This may be shown by an instrument called the Hydrometer or Areometer; put it in



water it rises to a certain pt. Where the above

conditions are fulfilled; put it them in a liquid bulk for bulk ~~heavier~~ lighter them water: it of course sinks to a greater depth: put it in a heavier liquid and it rises. By this means specific gravities of liquids are determined. An other instrument of the same nature is called Nicholson's Hydrometer; load the cup at the



top of the stem it sinks to a greater depth; & as the stem is so fine: a very small difference in the weight will make a perceptible difference in the depth the stem sinks: it is therefore used as a delicate weighing instrument: Suppose we want to find x the unknown wt. of a body: we determine the load sufficient to make the stem sink to a certain pt say its middle pt: Say this is 500 grms: Let W = weight of instrument:

V = volume of portion immersed up to mid. pt. of stem.

then W + 500 grains = upward force = Vs ×

but in the body of which we want to find x, and determine how much we must add to make it sink to the same point;

Say this is 50½ grains;

then W + x + 50½ grains = upward pressure
= Vs

then $x = 500 - 50\frac{1}{2} = 449\frac{1}{2}$ grains.

Jan 21st 1878.

We have seen that $p = w = sv$; or expressed in words, the resultant upward pressure (p) is equal to the weight of the same a volume of the liquid as the solid (w) = volume of the solid or what is the same thing the volume of liquid displaced (V) multiplied by the specific gravity of the liquid i.e. the wt of unit volume (s).

then $v = \frac{p}{s}$ i.e. to find v we must know s;

or $s = \frac{p}{v}$ i.e. ---- s ——————— v;

Take for the solid some body of a regular shape

say a sphere:

For Example: let radius of sphere = 1·915

then volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3{\cdot}1416 \times (1{\cdot}915)^3$

Weigh the ball in air: the air presses against it the same as a liquid: i.e. it loses weight in air equal to the volume of air displaced: this however is generally so small that it may be neglected: then weigh it in water and take the difference:

Weight in air = 862½ grains

weight in water = 400 grains

difference = 462½ grains: this difference is the upward pressure of the water = p in our formula

then $S = \frac{p}{v} = \frac{462\frac{1}{2}}{\frac{4}{3} \times 3{\cdot}1416 \times (1{\cdot}915)^3}$ = the wt in grains of a cubic centimetre of water.

this quantity is at 4°C. i.e. the temperature of its maximum density or minimum volume = 15·43235 grains.

then knowing S we can find v; if however the

solid is soluble in water, the same must be done in a liquid of known spec gravity in which it is not soluble: if it is lighter than water there is another process.

By the same method we can find the s. gravity of solids also: first notice the definition of S. gravity viz = wt. of unit volume of substance:

for instance the following may be the specific gravity of water:

expressed in cubic centimetres	of grains	15·4
----------	grammes	1
---------- cubic inches	grains	252·7
---------- feet	pounds	62·3

and so for any other body. To find the s.g. of a solid we must know the weight therefore of the volume. The readiest way is to take a unit volume of the solid and weigh it: this of course is impracticable: the next most direct process, is to take a solid of regular

shape, say cube or parallelliped, cylinder etc & weigh that: for example of zinc take a rect-angular solid

one side 4·8
another 4·7
the third 4·78
} cubic centimetres

volume of block = 4·8 × 4·7 × 4·78 in cubic centimetres.

Then weight it, say it is 11726 grains

then S. gravity of zinc in grains and cubic centimetres = $\frac{\text{weight}}{\text{volume}} = \frac{W}{V} = \frac{11726}{4·8 \times 4·7 \times 4·78}$

Then having found the s.g.'s of water (S_w) & zinc (S_z) find the ratio of one to the other: and it will be found then

$$\frac{S_z}{S_w} = \text{about } 7$$

This is the number we should find in a table of so called specific gravities: as far as we have gone, the spec. gravity we have been talking about

is the absolute specific gravity; in tables we find what is a relative specific gravity i.e. all things are compared to water; e.g. if we find it stated that the S.G. of Platinum is 21, it means that bulk for bulk, Platinum is 21 times heavier than water; the great advantage of using the metrical system is that the absolute and relative specific gravities are ~~relatively~~ the same numerically; this is perhaps one of the greatest advantages of that system.

But returning to what we were discussing: in most cases we cannot measure as in the case of a cube the volume of the solid we are determining the spec. gravity of: therefore find the volume by the formula $v = \frac{p}{s}$, as just explained: and then

$$\text{S.G of solid} = \frac{\text{weight of solid}}{\text{its volume}} = \frac{w}{v} = \frac{w}{\frac{p}{s}} = \frac{ws}{p}$$

and in relative s. gravities we take S of water = 1 $\therefore$ S.G of solid $= \frac{w}{p}$

There are certain modifications of this process for solid soluble in water or lighter than it.

Jan 27th 1878

For substances lighter than water (say wood). A heavy substance must be added to sink it: take the following:-

Wt of wood in air ---- 170 grains.

Wt of wood in air + sinker in water } ---713 grains.

Wt. of both in water ----- 323 grains.

713 − 325 = 390 = upward pressure on wood.

volume of wood in cubic centimetres = $\frac{390}{15.4}$

15.4 being the wt in grs. of a cubic centimetre of water.

$\frac{390}{15.4} = 25.3$

wt. in grs. of 1 cubic. cm. of wood = $\frac{170}{25.3} = 6.72$

If we want to get the relative specific gravity. it is $\frac{170}{390}$

$= .436$

or $\frac{6.72}{15.4} = \frac{6.72x}{15.4x} = \frac{170}{390} = .436.$

For substances soluble in water; a fluid in which they are not soluble must be employed;

Let S' = wt. of 1 cubic centimetre of other liquid

w = wt. of substance in air

w' = wt. of substance in this other liquid of s.g. S'.

v = volume of substance

Specific gravity of substance = $\frac{w}{v} = S$

S' is found by weighing a solid body of known bulk in the liquid, as was done with water.

w & w' are found by actual weighing

$w - w'$ = wt. of volume v of liquid.

$= v S'$

$\therefore \frac{w - w'}{S'} = v$

$$S = \frac{w}{v} = \frac{w}{\frac{w - w'}{S'}} = \frac{w}{w - w'} \cdot S'$$

Suppose we want the relative specific gravity; if the liquid itself is water: relative s. gravity = $\frac{w}{w - w'}$

If we weigh it in another liquid:-

$\frac{w}{w-w'}$ = relative s.g. with regard to that liquid.

& $\frac{w}{w-w'} \cdot \frac{s'}{s}$ = relative s.g. as compared with water;

if s be taken as 1, and σ be the relative s.g. of the other liquid as compared with water.

$\frac{w}{w-w'}\sigma$ = relative specific gravity of the substance.

Let us apply these principles to floating bodies!

Let S = specific gravity of floating solid.

s = specific gravity of the liquid.

V = volume of solid

v = volume of the part of the solid immersed.

To have equilibrium the upward pressure must be equal to the downward pressure = wt. of solid.

i.e. $vs = VS$

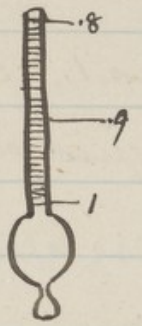
$v = V\frac{S}{s} = V\sigma$

Let $\sigma = \frac{S}{s}$ or relative s.g. of solid as compared with the liquid;

$\sigma = \frac{v}{V}$ or $v = V\sigma$

Apply this to the aréometer;

Suppose in water (relative S.G. = 1) the instrument sinks to the bottom of the scale: suppose in spirit (relative S.G. = ·8), it sinks to the top;



at first sight it would appear that in a liquid relative S.G = ·9; it ought to sink to half way between the two. But this is not so as will be seen thus:-

$S_1 = 1$ $\quad v_1 s_1 = \nabla S$ = upward force in first liquid

$S_2 = \cdot 9$ $\quad v_2 s_2 = \nabla S$ = ——— 2nd ———.

$S_3 = \cdot 8$ $\quad v_3 s_3 = \nabla S$ = ——— 3rd ———

The point we want to find is

$$\frac{v_2 - v_1}{v_3 - v_2}$$

if the point is half way $v_2 - v_1 = v_3 - v_2$ which they are not.

Jan 28th 1878.

By Nicholson Hydrometer, three things can be done:

1. find the s.g. of the liquid
2. —— —— a solid.
3. ~~of~~ wt. ——————.

Sykes's Hydrometer is similar to the glass one, except that it is made of brass, and can be suited for different liquids by means of different wts. attached to the bottom.

To determine the s.g. of liquids more directly; small flasks are used of accurately measured volume up to a certain mark; the shape of the neck of the bottle is:



fill this and weigh it putting in a stopper to prevent evaporation, subtract the wt. of the bottle when empty, and all data are obtained for the determination of the s.g. of the liquid; another kind has a hollow stopper by which the level can be adjusted better.

Suppose we want to find the S.G. of a solid obtainable only in small fragments. The same sort of bottle is used; put in the fragments, and fill up the bottle with water, and weigh: then if we know the wt. & volume of the water, the S.G. of the solid can be calculated thus;

Let Wt. of water filling up bottle = W
Wt. of solid = S
Wt. of water filling bottle containing solid = w
} subtracting the absolute wt. of the bottle.

S.G. of water = σ.

capacity of bottle = $\frac{W}{\sigma}$

Vol. of solid = $\frac{S}{x}$ x = S.G. of solid (unknown quantity)

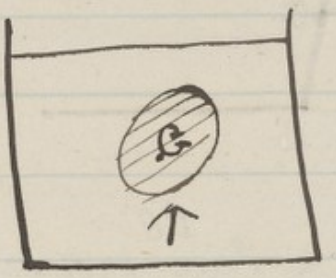
Volume of added water = $\frac{w}{\sigma}$

Then

$$\frac{S}{x} + \frac{w}{\sigma} = \frac{W}{\sigma}$$

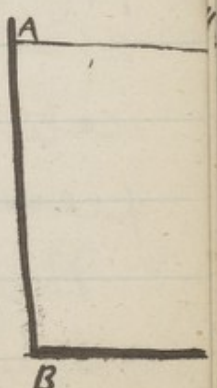
From which x can be found: or if the relative S.G. is wanted, $\sigma = 1$, or the absolute S.G. in grms. & cubic cm.

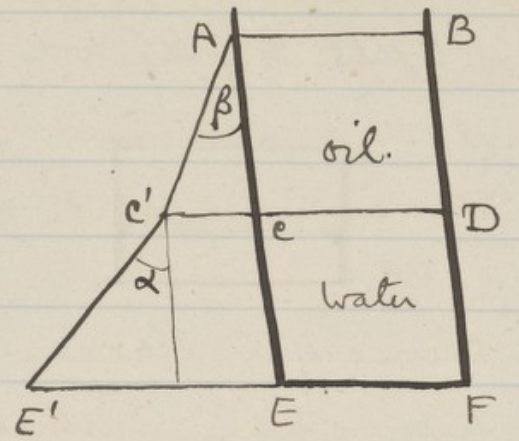
Another point to be considered is the point of application of the resultant upward pressure: it acts through the centre of gravity (G) of the displaced liquid,



because if the space filled by the solid, is filled by the liquid, there is equilibrium, there being an upward & a downward force; the downward force acts through the c. of g. of the liquid (vertically); therefore the upward force must do the same in the opposite direction.

Now consider the pressure on the side (AB) of a vessel; this is subject to horizontal pressure which varies as the depth; the pressure at B is = the wt. of a column of liquid of height AB: at A the pressure is infinitely small, half way between half the pressure at B (P. I. O.)





The pressure on the bottom is the sum of the vals of the two liquids. Let AC' represent the rate of increase of pressure in oil; then AC'C will represent the total pressure on AC; then the lower liquid being the heavier and the pressure on the top of the lower liquid being also represented by CC'; C'E' will represent the rate of increase pressure for that liquid, α being greater than β. Then C'CE'E will represent the pressure on CE; and the sum of AC'C and C'CE'E will represent the total pressure on the whole side AE.

If between the two liquid a solid be put of s.g. intermediate between those of the liquids

it will float between them; and the proportion in each, is determined by the same rule as before: viz: that the wt of the body = wt of fluid displaced.

Let ∇ = volume of solid Sg. = .9

v = ---------- in lower } liquid. Sg. = .75

v' = ---------- upper } Sg. = 1

then $\nabla = v + v'$

and $\nabla s = v s_1 + v' s_2$

equal in this case

$\nabla .9 = .75 v' + 1 v$

from which two equations v & v' can be found.

Jan 30th 1878.

Gases.

The chief difference between Gases & Liquids is that Gases are compressible i.e. readily change their volume with change of pressure; the following are instances of the great difference between the densities of liquid and gas.

1 cubic inch of water at 15°C. weighs 252·7 grains.
1 cubic inch of air at 15°C and at atmospheric pressure } " ·31 grains

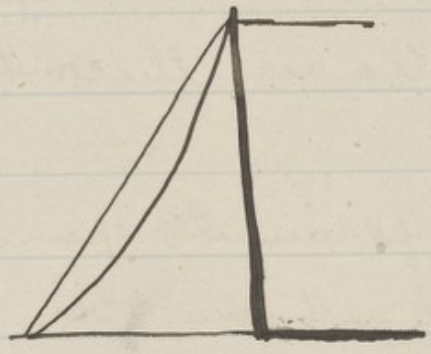
$\frac{252.7}{.31}$ = 815 nearly

i.e. there is 815 times as much water in the same space; or the spec. gravity of air compared to water is $\frac{1}{815}$.

1 cubic centimetre of water at 4°C. weighs 1 gramme
1 cubic centimetre of air at 0°C and at atmospheric pressure } " ·001293 grms.

$\frac{1}{.001293}$ gives as about the same quotient as before: not quite because the temperatures are not the same.

with liquids as we descend in them, the pressure increases at the same rate; with gases pressure increases at an increasing rate as we descend. this is because, as we descend the density of the gas increases also; c.g. the density of the atmosphere is greater near the earth than high up on a mountain: in the figure the slant of the straight line will represent the increasing of pressure of a liquid, the curved line, c.c. with the pressure ever increasing, of a gas;



the pressure in a liquid we have seen = a.h.ω with a gas, ω increases as we descend; that air expand when pressure is removed may be shown by putting a bladder with a little air in it under the receiver of an air pump; remove the exterior air, and the bladder swells out. The Pressure in a space

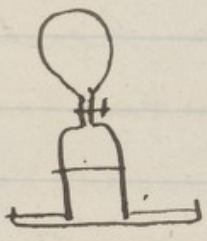
filled with air, and communicating with the outer air is the same as the pressure of the outer air: for if it were not air would move one way or the other till the pressures were equal in opposite directions were equal on any point of the air. That air has weight may be thus shown;

weigh a globe filled with air — 43½ grains.
take out the air the counterpoise
is too heavy; and the weight then is — 18 grains

difference 25½ grs.

Pump in extra air, the counterpoise is too light.

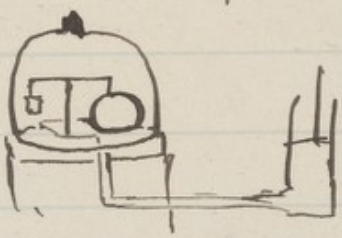
In the first experiment, open the globe over a bell jar filled with air over water; the air



goes in and the water rises; measure the bulk of air which goes in: in this experiment 96 cubic inches

then $\frac{25\frac{1}{2}}{96} = .26$ grs = wt. of one cubic inch of air: which allowing for roughness of the experiment is nearly what was given on page 176 (.31)

Objects immersed in air lose weight by the weight of the volume of air displaced; this may be shown by taking two bodies apparently of equal weight in the air, but one of them of a small specific gravity; this therefore occupies more space, and loses therefor more weight so that the balance when put beneath the receiver of an air pump and the air removed it appears to gain more weight and that side of the balance falls



Let W = true weight of one substance & w its loss of wt in air
Let W' = ---------- another ---------- w' ----------
then W - w = W' - w'
Let V & V' be the volumes of W & W' respectively
& S & S' ---- Spec. gravities ----------
then $V = \frac{W}{S}$ and $V' = \frac{W'}{S'}$

Let a = wt of unit volume air

$$w = Va = \frac{Wa}{S}$$

$$w' = V'a = \frac{W'a}{S'}$$

Substitute in the equation $W - w = W' - w'$

$$W\left(1 - \frac{a}{S}\right) = W'\left(1 - \frac{a}{S'}\right)$$

The pressure exerted on a gas is equal & opposite to the pressure exerted by that gas; let the pressure exerted by or on each unit of surface be p and the volume of the gas v

then pv is constant.

This can be seen by altering p, we get different values for v

$$p_0 v_0 = p_1 v_1 = p_2 v_2 = \text{etc.}$$

expressed otherwise this is

$$\frac{p_0}{p_1} = \frac{v_1}{v_0}$$

i.e. pressure is inversely as the volume.

mercury column, and S_m for the S.G. of mercury;

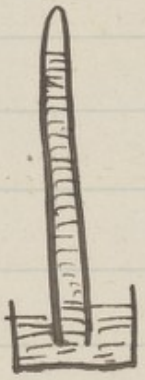
$$h S_m = A$$

A varies from day to day; So, S_m remaining the same, (which is does except for temperature which must be allowed for) h varies also

$$h' S_m = A'$$

$$\text{or } \frac{h}{h'} = \frac{A}{A'}$$

instead of this form of instrument, a straight tube open at one end which dips under mercury is a more convenient form.



We have here exactly the same conditions as before.

volume of mercury = section of tube × its height

mass of mercury = volume × density

* weight of mercury = volume × specific gravity

‡ pressure per unit of surface

= weight ÷ area pressed upon.

Let W = weight of mercury

p = pressure per unit of surface

h = height of mercury column.

S = specific gravity of mercury

a = area pressed upon

Expressing the above relations by letters we have:

* $W = ahS$

‡ $p = \frac{W}{a} = \frac{ahS}{a} = hS$

i.e. the pressure per unit of surface is independent of the section of the tube; if the section of the tube is greater, the absolute wt. of mercury is greater but this is spread over a greater surface, so p remains the same.

The weight of anything = mass × intensity of gravity

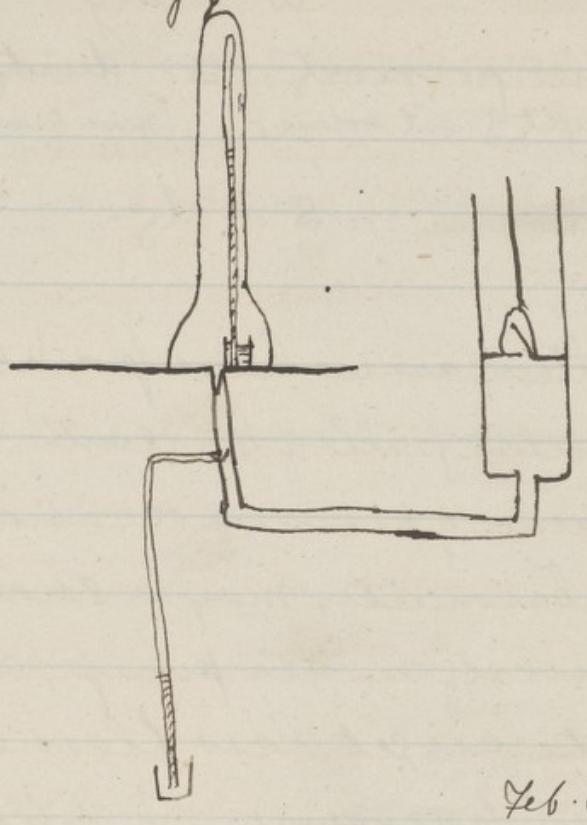
$$w = mg$$

Specific gravity (weight of unit volume) = density (mass of unit volume) × intensity of gravity:

$$s = dg$$

In this case we have $p = hs = hdg$: g varies in different parts of the earth so p varies also: that difference of pressure occasions a rise or fall in the barometer, may be shown by putting under the receiver of an air pump, and exhausting the air; as the air get less & less dense i.e. exerts less & less pressure, p falls also i.e. h is less; till when the air is quite exhausted (which is not possible) the mercury stands at the same level in the tube as in the vessel. The same thing may be shown differently, by having a barometer tube filled with air over mercury: exhaust the air from it & the mercury rises: in these two experiments: if at the same time we add the ~~two~~ heights of the two columns together, we shall get a constant quantity, i.e. if both are done by

the same air pump as may be by an arrange-
-ment like this in the figure

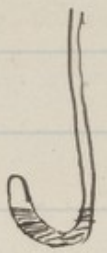


Feb. 6. 1878.

Any other liquid might be used instead of water mercury for a barometer, but most would be of an un-
-manageable height (e.g. with water, about 34 ft. for atmospheric pressure); if however very small changes in pressure have to be noticed; a long column is better.

An apparatus for measuring pressures on a

gas and the change of volume ensuing is this: a long tube bent and closed at one end is taken and mercury poured down so as to stand at the same level



in both limbs, enclosing air in the closed limb; more mercury is poured down and the volume of the air there observed, and so on: the pressure of the air however must be added to the pressure of mercury: the pressure of the atmosphere being = 77·5 centimitres of mercury in this case: the following are the results obtained by experiment.

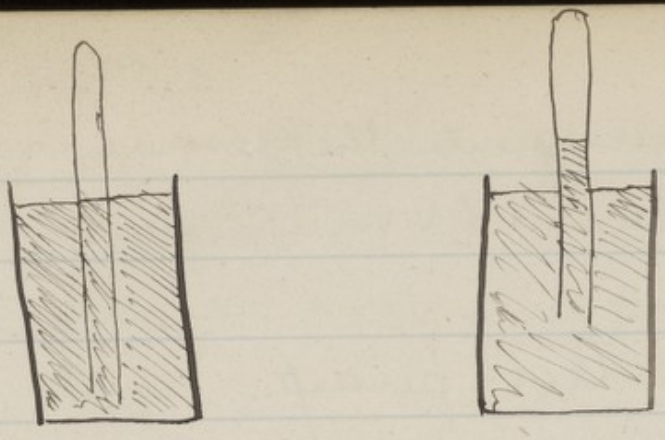
Volume of air	Height of mercury in closed limb	in open limb	Difference.	Total Pressure
20·4	2·4	3·4	1·0	78·5
18·0	4·6	16·8	12·2	89·7
16·4	6·2	27·9	21·7	99·2
13·0	9·0	56·3	47·3	124·8
10·7	11·2	86·3	75·1	152·6
8·5	13·1	125·2	112·1	189·6
6·9	14·6	173·2	158·6	236·1

Multiplying volume of air by pressure upon it; we get a

quantity which would be virtually constant if the experiments were delecate; but even here the product PV is nearly constant; take a few instances

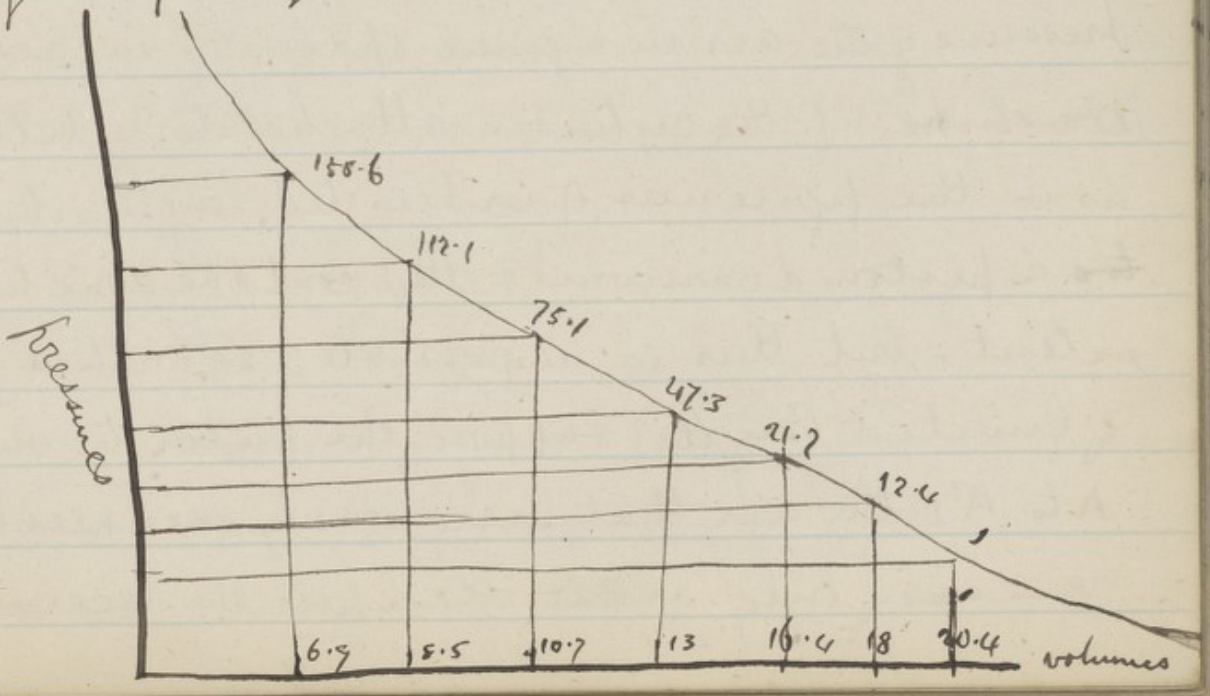
122·8	78·5	236·1
13	20·4	6·9
3744	3140	21249
1248	15700	14166
1622·4	1601·40	1629·09

This is the same for every known gas; the same truth may be expressed thus:- Volume is indirectly proptional to pressure & density directly so. The same is true for pressures less than that of the atmosphere: have a tube in a deep dish with mercury in both, at the same level: then raise the tube and measure the height of mercury in it above the mercury in the dish, and subtract that from the atmospheric and that will give the pressure on the air inside; measure also the volumes of the air ~~and~~ as the pressure is made to change, and PV will be



We may found constant.

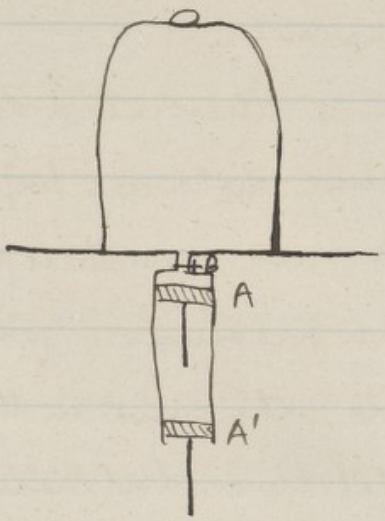
This law may be illustrated diagrammatically; take along a vertical line distances proportional to the volumes of a given mass of gas; and from these points measure vertical lines proportional to pressures: join the summits of these: & we get a hyperbola; the curve in the limit coincides with the vertical & horizontal lines, but never really so; & completing the rectangles they are equal in area i.e. PV is constant.



If in these experiments the pressure is kept constant the volume increases ∝ V. v.

Feb. 18. 1878.

The Air pump.



The Air pump is an apparatus for reducing the pressure of the air in a given space, by increasing its volume; if the cylinder attached to a bell jar as in the figure was of unlimited length; by drawing ~~too~~ a piston downwards this could be done to any extent: but this is impossible; so with a cylinder of limited length, suppose the piston to move from A to A' the air that previously occupied the receiver only now occupies the receiver + the

cylinder: now cut off the portion in the cylinder by a stopcock at B; get rid of the air somehow from the cylinder and return the piston to its former position A, and repeat the process; instead of stopcocks, a valves or self acting stop cocks is used: the pressure of air inside the receiver opens it when the piston is being pulled down and shut when being pushed up; besides this there is another valve in the piston for the air to escape by. its opening and shutting is the reverse of the other valve.

Let V = volume of the receiver

v = ------ cylinder

P_0 = original pressure in receiver

P_1 P_2 P_3 P_n etc = pressures after 1st, 2nd, 3rd nth etc. strokes.

then

$$P_0 V = P_1 (V+v)$$

$$P_1 = P_0 \frac{V}{V+v}$$

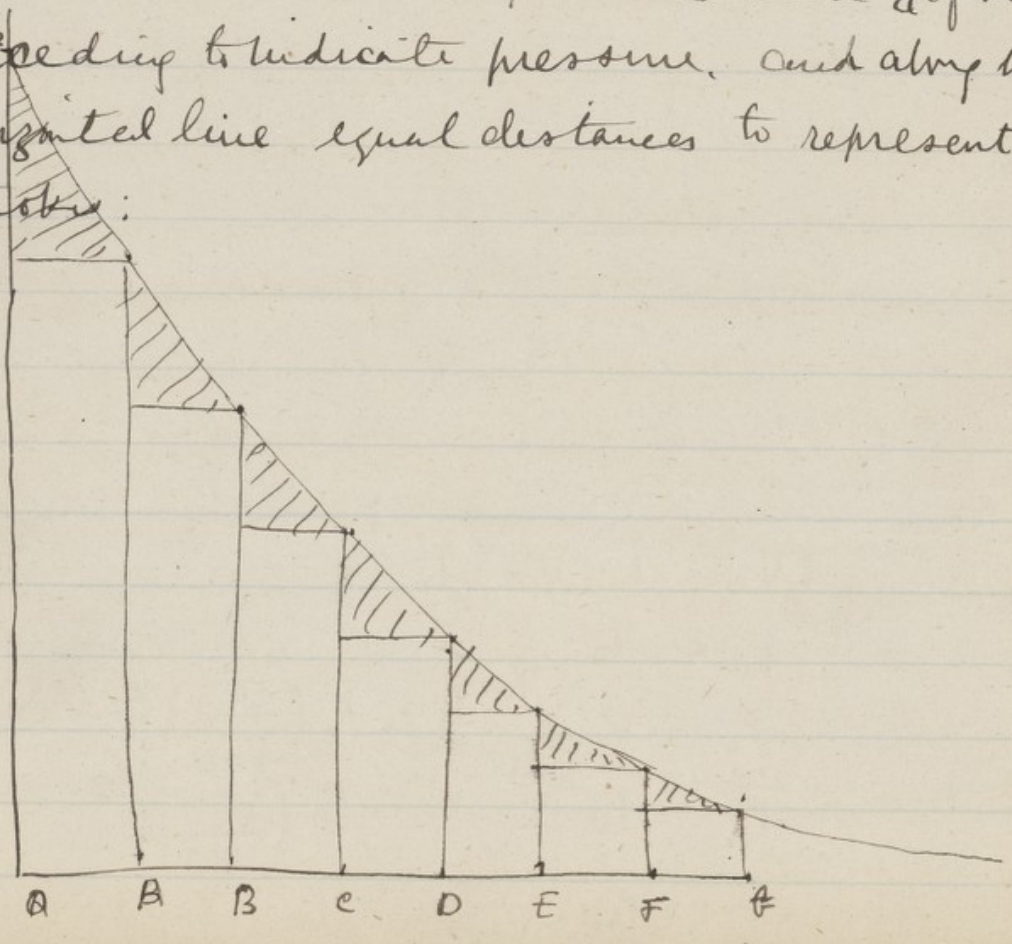
$$P_2 = P_1 \frac{V}{V+v} = P_0 \left(\frac{V}{V+v}\right)^2$$

$$P_3 = P_2 \frac{V}{V+v} = P_0 \left(\frac{V}{V+v}\right)^3$$

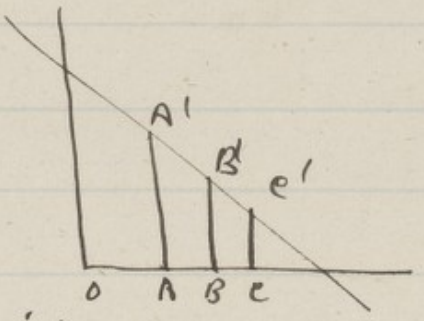
$$\vdots$$

$$P_n = P_{n-1} \frac{V}{V+v} = P_0 \left(\frac{V}{V+v}\right)^n$$

By making n large enough P_n can be made as small as one pleases. The change in pressure at each stroke however becomes less and less as it is a fraction of a ~~fixed~~ continually decreasing quantity of air. This can be shown by a figure; suppose initial pressure is 32 and each stroke reduce the pressure in the receiver by $\frac{1}{4}$; along the vertical lines take distances each $\frac{3}{4}$ of the preceding to indicate pressure, and along the horizontal line equal distances to represent strokes:



The Shaded triangles represent the changes in pressure, which are seen to get smaller & smaller, the curve continually gets more nearly horizontal. It is called a logarithmic curve. With a straight line have a similar construction taking



any points A, B, and C in the horizontal, and erecting the $\perp^{rs}$ AA' BB' & CC'

$$OB - OA = AB \qquad \text{and} \qquad OC - OB = BC$$

$$\frac{AA' - BB'}{OB - OA} = \frac{B'B - CC'}{OC - OB}$$

$$\text{or} \quad \frac{AA' - BB'}{AB} = \frac{BB - CC'}{BC} = \text{constant quantity}$$

Now it with this curve.

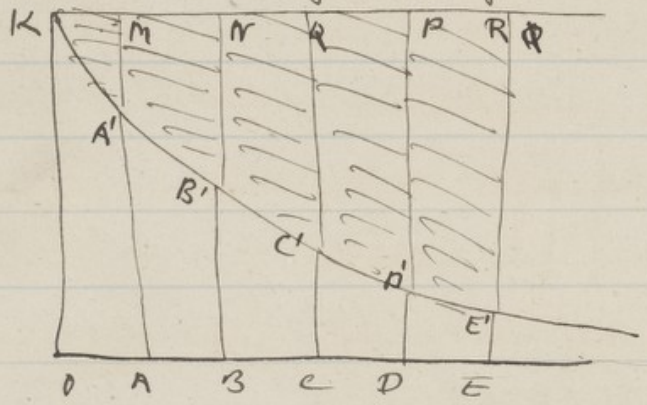
$$\frac{\log AA' - \log BB'}{AB} = \frac{\log BB' - \log CC'}{BC}$$

if the horizontal distances, as in this case, are equal

$$\log AA' - \log BB' = \log BB' - \log CC'$$

$$\text{i.e.} \quad \frac{AA'}{BB'} = \frac{BB'}{CC'}$$

A better sort of air pump is with two cylinders ~~piston~~; then by a lever, work can be done by both up & down strokes. In the single barrelled air pump work has to be done against atmospheric pressure on one stroke; repeating the figure on page 192: draw



through K a horizontal line and produce the vertical lines to cut it in M, N, Q, P, R, etc; the shaded portions represent the increase of atmospheric pressure at successive strokes. But in the 2 barrelled air pump, while atmospheric pressure is worked against by one piston, it is doing work on another: and these two neutralise one another. So all that has to be done by it is to overcome friction.

{Note (the spaces in the figure can be easily calculated if A'B', B'C', etc. be taken as straight lines).}
The valves hitherto we have considered as having no resistance but even the most delicate exerts some force which has to be overcome;

let a = area of opening over which valve closes
(which must be small for it to close with accuracy)

Let P = pressure below valve
P' = ------ above ------
} Let P' > P' which it will be when the piston is being pulled up (fig. P. 190)

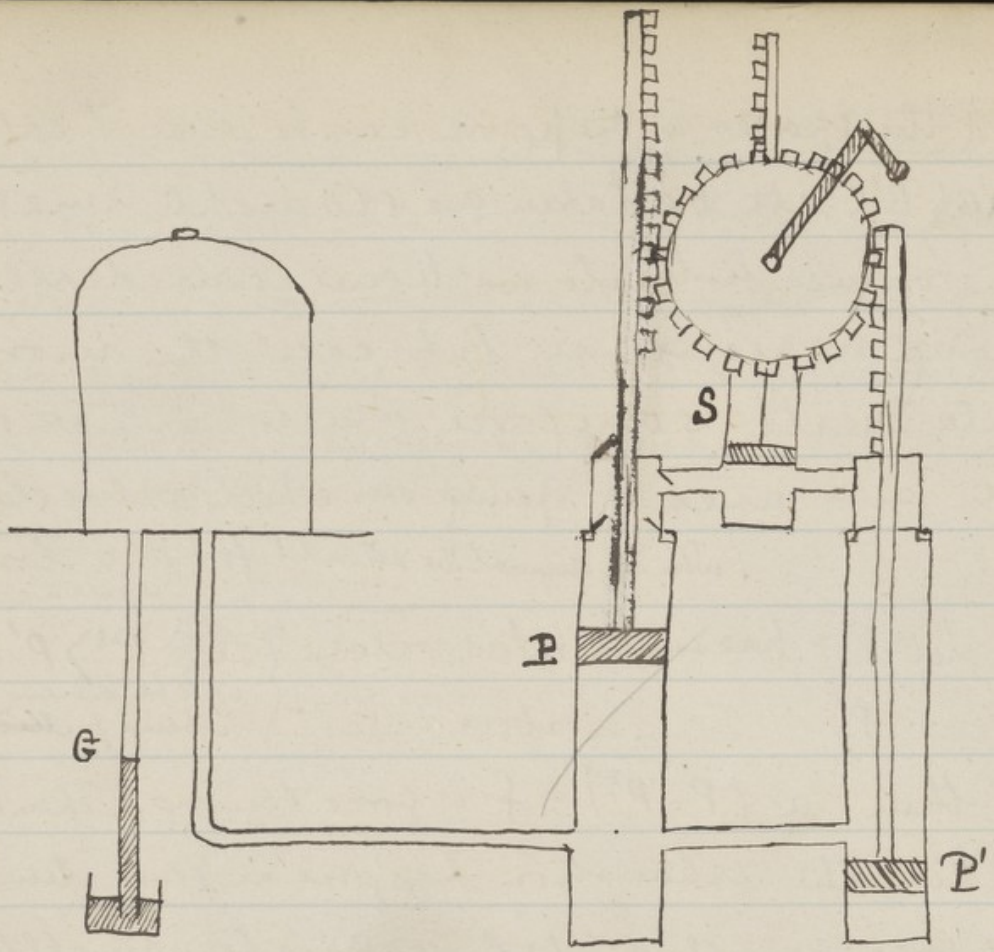
then a (P - P') = f = force tending to open valve

When the exhaustion has gone on some time P - P' get very small & at last f is too small to open the valve.

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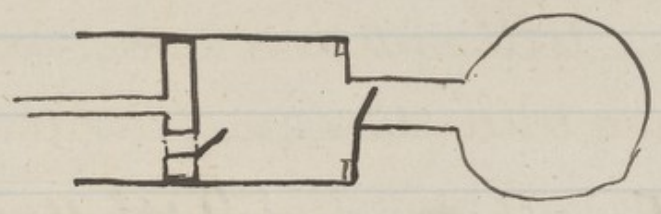
On the next page is represented in section one kind a double barreled air pump, there being many kinds.

P. T. O.



G is a gauge to show how much the receiver is exhausted; when quite empty which it never is of course, the mercury will be at the barometric height. The pistons are solid. S is a small pump to exhaust the air from the top of the other cylinders, when the action has gone on for some time. The condition of the pump in the figure is P rising and P' falling.

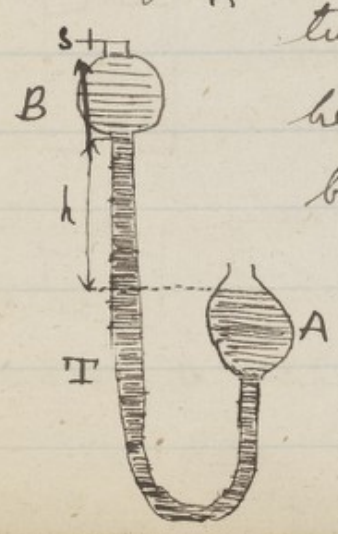
In the Condensing Pump i.e. for pumping air into a space, the valves open the opposite way



One great difference between this and the exhausting pump is, that the same quantity of air is pumped in each time viz: a cylinder-full at atmospheric pressure, not a gradually decreasing quantity.

In these air pumps, the pistons cannot go down to the bottom or top of the cylinders for fear of injuring the valves, so we get residual air at atmospheric pressure this is overcome by having a liquid piston.

One form of this is; two bulbs are connected by a flexible tube T: filled nearly with mercury: the height h must be greater than that of the barometer: the vessel to be exhausted is put on at S where there is a stopcock; A is raised to A': the stopcock opened and the liquid in B fall, and filling B: turn stopcock, set free of air of B: bring A' to A and repeat process.



there are a great numbers & varieties of mercury pumps; Sprengel's Air Pump is one; and the most perfect.

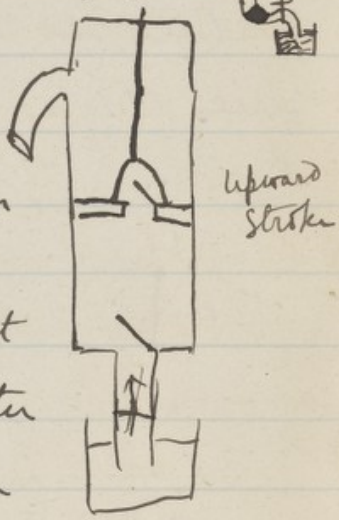
It consists in letting fall down a very narrow tube a number of little plugs of mercury; the sum of the lengths of which must at least equal the height of the barometer; each carries down with it a portion of the air in the Receiver R.

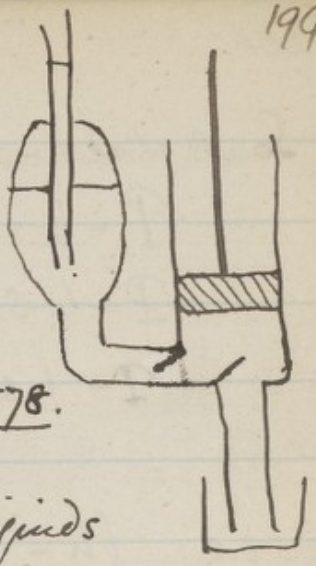


We can pass on now to the

Water Pump.

This is virtually an air pump only made stronger & larger: at first it acts as a ~~water~~ air pump, & then the water follows it. The length of the column of water must not be greater than the height of the water barometer, or the average about 34ft. When however the water once gets above the piston, it goes on. For heights greater than the water barometer the force pump is used; as in the next figure: A is an air chamber to keep up a more continuous flow.





The fire engine is a combination of two force pumps worked something the same way as a two barrelled air pumps.

Feb. 18. 1878.

The Siphon.

The Siphon is a bent tube for carrying liquids from a high to a low level: the arms may be of equal length or unequal; but the level of the liquid in the vessel which is being emptied must be above that of the aperture of the tube from which the water flows; the rate at which it flows being proportional to the difference of these levels. Both ends may be immersed or not.

Take the case where both ends are immersed.

P = pressure of atmosphere S = s.g. of liquid

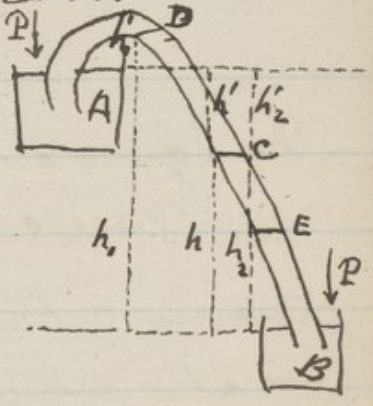
Let A & B be upper & lower vessels respectively; imagine partitions stretched across at various places; imagine one at C; pressure on one side = $P - hS$: on the other side $P - h'S$

imagine one at D: pressure one side $P - h_1 S$

pressure on other side = $P - h'_1 S$; and so for E or any other place;

but $h - h' = h_1 - h'_1 = h_2 - h'_2$ etc. for every other point;

(sometimes one of the heights being negative; negative being one direction & positive the other)



So it does not matter what point we consider

$P - hs$ = pressure on one side of any partition

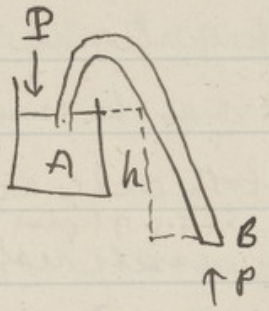
$P - h's$ = ... other side — — — — — .

$P - hs - P - h's = f$ = force tending to displace any partition in the tube.

i.e. $(h - h')s = f$.

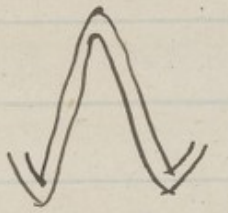
In the siphon consider the partitions to be made of water and we get a flow from higher to lower levels. f vanishes when $h = h'$.

Now if one limb is uncovered



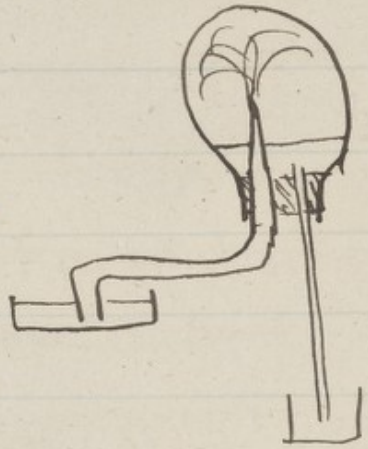
Imagine a membrane at B; pressure on one side is $P + hs$ which is greater than P the pressure of the air on the other side; so if the membrane is too weak we get a flow of water; imagine the membrane of water and we always get a flow, when h is any quantity > 0.

A better form of siphon is of this shape;

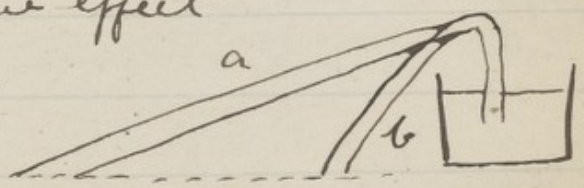


because it can be stopped at any moment by taking it out, without it running dry.

The same principle is applied to making a miniature fountain



The absolute length of tube does not matter; only the difference of level is of account; e.g. in the figure we should get in theory the same effect for the tubes a and b, the level being the same in each. Really we don't get so great effect in a as the friction is increased.



and the liquid on that side descends as in the figure

Dec. 13. 1881

The following are the results of experiments to show the different radiation of different substances, the sources being different; the numbers denote the number of divisions on a scale, through which a spot of light is deflected by the action of a galvanometer, (which is acted upon by a thermoelectric pile, carefully screened, and to which the heat can only get by a small hole in the screen), in half a minute.

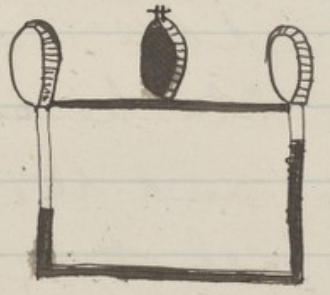
	Source of Heat			
	Copper at 400°	Luminous flame through glass lens	Naked luminous flame	Lampblack at 100°
Through air	233	80	150	81
——— glass 1 millim. thick	59	58	70	0
——— rocksalt 1·74 cm. ———	157	58	135	57
——— alum 1·78 cm. ———	0	5	2	0

While all vary, it will be noticed that the proportion which passes through the rock salt is nearly constant; with more delicate experiments, it would be still more so.

these have a bulb similar to them, and filled with hot water: if this bulb be turned the same way as the other two, no alteration in the levels of the liquid will take place: ~~but if~~ because, on one side a great deal is being radiated from the lamp black side of the middle bulb, and a little absorbed by the polished surface of the right hand bulb: and on the other side exactly the opposite takes place in the same proportion: but if we turn the middle bulb round: we have the two blackened surfaces facing each other and the two polished surfaces facing each other: thus on one side we have, much heat radiated & much absorbed, and on the other side, little radiated and little absorbed: thus the left hand bulb gets the hotter

Take a thermometer, and hang it up in a room; it will soon come to a constant temperature; i.e. it is losing and gaining heat at the same rate; cover the bulb of a thermometer with gold leaf; we diminish its radiating power in the proportion 90:12; but still its temperature is the same, because we have diminished its rate of absorption in the same proportion.

This principle can be shown better thus



Take a thermoscope, whose bulbs are flattened and one side of each made of polished metal, the other coated with lamp black; between

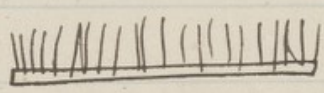
The shape of the surface makes no difference, with materials like ~~to~~ stone or marble; rough or polished stone radiating the same.

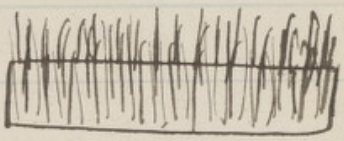
With metals however, a polished surface diminishes the radiating power: as with metals, polishing consists in rubbing down projections and thus thickening the surface; while in polishing stone, the projections are simply rubbed off.

Suppose we radiate heat upon a cube like that just described; most gets in through the lampblack surface, and least through the tin; thus showing that heat can pass with equal facility each way; i.e. good radiators are good absorbers, and bad radiators are bad absorbers.

greater

A thin metallic plate radiates just as much heat as a thick one: but a thick plate of bodies transparent to heat radiates more than a thin one: because as each particle radiates heat; the more particles there are the more heat is radiated, as the heat from the particles behind can get through the front ones.





If we coat a surface with solution of isinglass, the thicker the coat, the greater the radiating power, as to a certain limit, when it diminishes. because the heat is conducted away, isinglass being a good conductor of heat.

Supposing each body to be at 100°C, the following numbers express the relative radiating powers of those bodies

Lampblack	100
White lead	100
Paper	98
Resin	96
Glass	90
Plumbago	75
Tarnished Lead	45
Mercury	20
Iron	15
Tinplate, Gold, Silver, Copper	12

These numbers however alter for other temperatures than 100°C; e.g. at a higher temperature, lampblack & plumbago (White lead?) no longer have the same radiating power: but lampblack has the

opposite the thermopile; it will be found that the tin has nearly no effect; the other three surfaces have much more effect, the smoked surface having the most, then the paper, then the resin; very little difference however is observed being the paper and the smoked surface.

Fill two pots with water at the same temperature; let them be of the same size, one of bright metal, and the other smoked, put in each the bulb of a thermoscope (like that on page 47): the blackened pot will fall in temperature first, and consequently the liquid that side will fall. ~~Showing that heat can pass with equal facility or difficulty either from or into a body; or the same both ways; or a body which radiates heat quickly, absorbs it quickly; & slowly, then slowly.~~

Every thing gives out heat, and if the temperature of thermoelectric pile were below the temperature of the air; it would be affected by the rays of heat from the bodies in the room at the temperature of the air: rays which are not perceptible to the eye being perceptible to the pile.

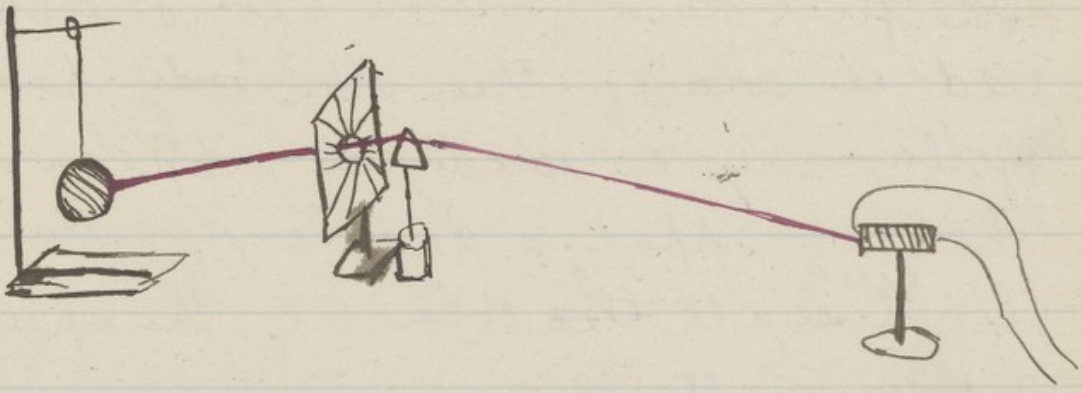
The first kind of visible heat, or light is red in colour; then the body has yellow rays as well, as appear orange, then it appears dazzlingly white. because all the colours of the spectrum are coming off.

Different materials however radeate heat more than others: e.g. take a cube, one side tin, another papanned (i.e. coated with resin); a third smoked, and the last of white paper; fill it with hot water, and put each side in turn

refracted something in this way:

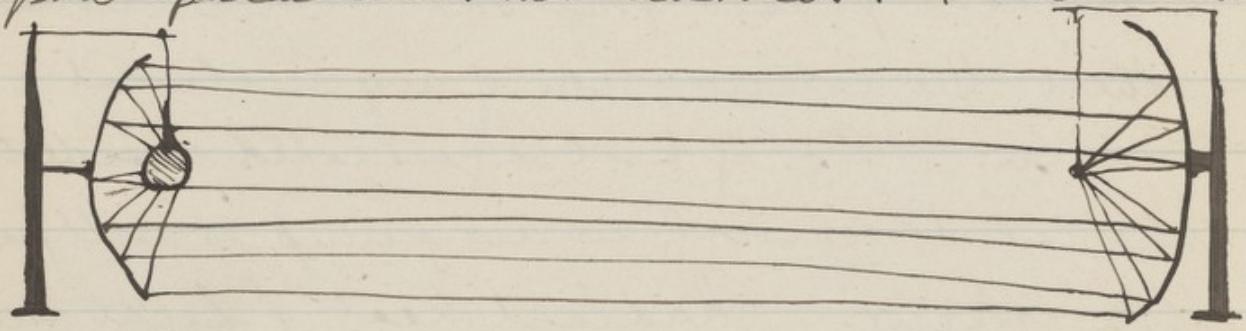


That the rays of heat do this may be thus shown: —



Let the rays of heat from a hot body pass thro' a hole in a screen and then through a prism of rock salt (not glass, as glass is not transparent to heat of low intensity), the rays will be deflected as in the red line of the figure, and will affect a thermoelectric pile, as will be seen from a galvanometer attached to it.

joining the centre of the sphere with the centre of the mirror is called the focus; all rays passing from it, after reflection pass parallel to one another. Place in the



focus of one of them a red hot cannon ball; rays ~~which~~ that pass from it in all directions; but a great many are collected by the mirror and sent in one direction parallel to each other: these are reflected by the other mirror back to its focus, and will ignite a piece of guncotton, or any other inflammable material placed there. Rays of heat can also be refracted.

Dec. 11. 187

Rays when passing through a prism are

Radiation of Heat.

Radiation of heat is said to take place when heat passes from one body to another without the intervention of any material medium. E.g. if heat be generated inside the exhausted receiver of an air pump it will pass to the exterior. Radiant heat & light are essentially the same; if a body is very hot the rays from it can affect the nerves of the eye, if not hot enough to do this, it will affect the nerves of touch; it is therefore a difference in us, and not between the rays of heat & light: these rays both obey the same laws; we can show that they obey the laws of reflection, (which will be studied in connection with light), thus Take two concave spherical mirrors, & place them opposite each other at ~~a certain~~ a good distance from one another; the middle point of the line

at which the earth is losing heat in any period of time, say in a year or in a century etc.; determine the coefficient of conductivity of the rocks; we can thus find it: letting

t' be the temperature at a certain depth.

δt ---- ---- ---- ---- lower ----.

These depths must be taken low enough that the season of the year makes no difference in their temperature.

δe, be the difference of depth.

The following are some coefficients of conductivity; if we take that of

Silver = 100

that of	Copper is	= 73·6
-----	Gold ...	= 52·2
------	Brass ..	= 23·1
----	Tinc ...	= 19·0
----	Iron ..	= 11·0
----	Lead ..	= 8·4
-----	Platinum ..	= 8·5
---	Bismuth ..	= 1·8

in contact with the air; this is very difficult to find, as part of the thermometer is in contact with the air; and if we cover it up with cotton wool, we prevent the heat escaping from that part; but say it is 115°.

Find the quantity of heat lost per hour each square foot of the outer surface; the thickness of the boiler being $\frac{3}{8}$ of an inch;

1 sq. foot being equal to 929 sq. cm.

1 linear inch = 2·54 c.m.

$$Q = (t - t') . T . \frac{a}{e} . K$$

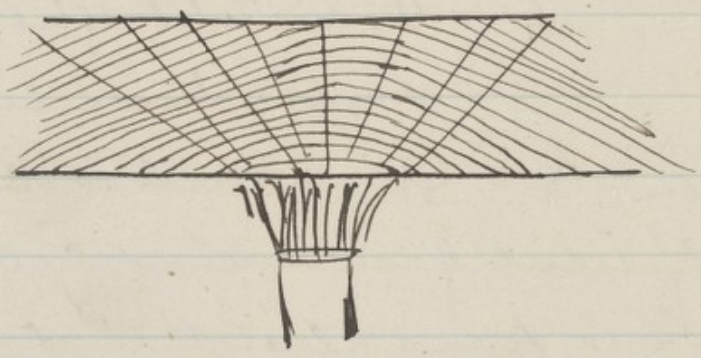
$$= (120° - 115°) . 3600 . \frac{929}{\frac{3}{8} \times 2.54} . \cdot 2$$

$$= 5 \times 3600 \times 929 \times \frac{8}{3} \times \frac{1}{2.54} \times \cdot 2$$

$$= 3716000 \text{ Approximately.}$$

As we bore down into the earth, it gets hotter & hotter, and the temperature varies proportionately to the depths; if we want to calculate the rate

lines it is necessary that all parts of the same face, be of the same temperature, and also all parts of all sections parallel to the faces; the flow of Heat being always perpendicular to the surfaces of uniform temperature something as in the figure.



Therefore, it is best ~~it is heat~~ to have the two faces in contact with liquids of different temperatures, and then they must be kept well stirred about.

Example. For iron K = ·2 (cm. & second)

Suppose we have a boiler; the temperature of the inner surface is 120°; and that of the outer surface will be lower, because it is

i.e. $$K = \frac{Q.\ell}{(t-t').a.T}$$

Let T = 1 second

$(t-t')$ = 1 degree

a = 1 square centimetre

ℓ = 1 centimetre

Q_1 = the quantity of heat which passes under these conditions.

then $K = Q_1$, which is called the coefficient of conductivity of the substance.

It is found also that it depends somewhat upon the absolute temperature of the plate, also; i.e. the coefficient of conductivity is greater at a low part of the scale than at a higher part. But as this would lead to very complicated results, it may be left out of account; still it is a matter of fact.

In order that heat may travel in straight

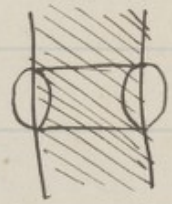
Q first depends on the difference of temperatures of the two plates

$$Q = (t - t') \ldots ;$$

it depends also on the time (T) during which the passage of heat continues

$$Q = (t - t')/T \ldots ;$$

it depends also on the area of the surfaces heated on each side; suppose these to be equal on each side; we have thus



heat passing along a cylinder of length e, and the area of the ends a; Q varies directly as a, and inversely as a.

$$Q = (t - t') \cdot T \cdot \frac{a}{e} \ldots$$

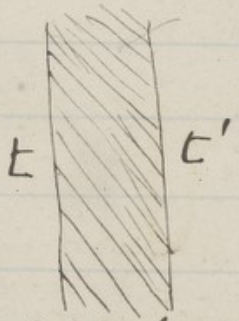
We have still to consider the material of which the plate is composed: let K be the factor depending on this;

$$Q = (t - t') \cdot T \cdot \frac{a}{e} \cdot K.$$

Therefore when we want to heat a liquid, the heat must be applied below; when the lower portions are heated, expand & rise to the surface so establishing what are called Convection currents, in which way the whole gets heated.

Dec. 6th 1877

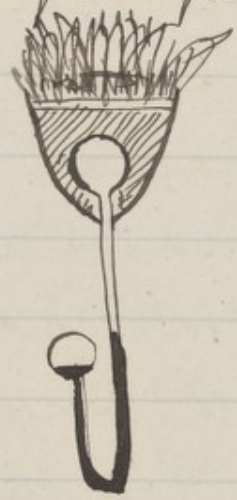
Let us see what we have to take into account when finding the conducting power of any body: Suppose we have a plate, of which the figure is a section



Suppose the temperature of one side is t, of the other t'; and let t be greater than t'; Let Q be the quantity of heat which passes from one side to the other:

That metals are good conductors can be seen in a simple manner thus: wrap a piece of paper round a metallic cylinder; and put it in the flame of a lamp; it will be a long time being even scorched, the metal conducting away all the heat given to it.

The conducting power of liquids is almost as small as that of gases; in fact it is not perceptible to an ordinary thermoscope.



Put some water in a funnel round the end of the bulb of a thermoscope, and pour some ether on the top & ignite, ~~and~~ hand & lamp heat will be sent through the water to the bulb & no alteration of the level of the liquid in the thermoscope will take place.

In the case we took v was the same in all

$$t_1 = \frac{Q_1}{v_1 \rho_1 c_1}$$

$$t_2 = \frac{Q_2}{v_2 \rho_2 c_2}$$

$$\frac{t_1}{t_2} = \frac{Q_1 \rho_2 c_2}{Q_2 \rho_1 c_1}$$

Take the case of iron & bismuth;

Of iron $c = .112$ $\rho = 8$ $c\rho = .896$

— bismuth $c = .031$ $\rho = 11$ $c\rho = .341$

i.e. bismuth requires about $\frac{1}{3}$ (3 times) as much heat as iron does to raise it to the same temperature and if they both conducted heat equally well, the bismuth would be three times hotter than the iron, when the same amount of heat had been transmitted in each case; the fact that bismuth was hotter than iron would not then prove anything; if anything just the opposite to the truth for it happens that bismuth conducts heat worse than iron; ~~but it is quite possible that it might not be so.~~

the results of ~~heat~~ the experiments on the last page might lead one into error; it is not a proof of itself that heat passes more quickly; it only shows that a rise of temperature takes place more quickly in one body than in another; we have to take into account the quantity of heat required to raise each body a certain temperature i.e. their specific heat.

Let m = mass of substance

t = no. of degrees its temperature is raised.

c = specific heat of substance.

Q = Quantity of heat, to raise m, t degrees in temperature

Then $Q = ctm$ (see page 43)

$$\therefore t = \frac{Q}{mc}$$

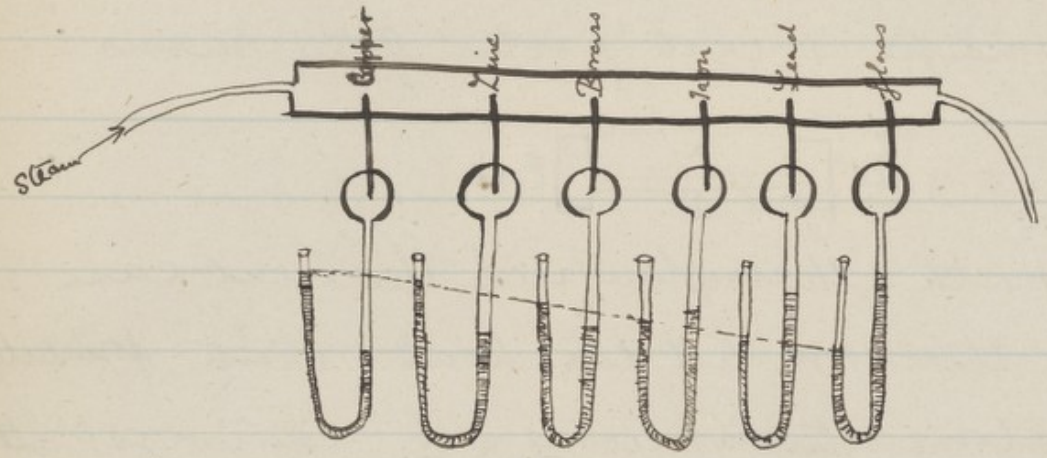
$m = vp$ when $\begin{cases} v = \text{volume} \\ p = \text{density} \end{cases}$ of the mass

i.e. $$t = \frac{Q}{vpc}$$

air within the bulb. Let the heights of the liquids be the same in the tube of each thermoscope, before Operations are commenced; examine their condition after a few minutes that the steam has been passing ~~it~~ in, and the appearance will be that presented in the figure; the liquid being most displaced ~~by the~~ in the thermoscope into which the end of the copper rod is passed, and not all displaced ~~by it~~ in that with the glass rod.

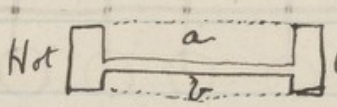
Metals are the best conductors of heat. Wool, feathers and loose materials generally are the worst: this is because air is confined between the different parts, and gases hardly transmit heat ~~at~~ all. Woollen is warm not because of any inherent quality of the material but because, being a bad conductor, loss of heat from the body is prevented.

The difference between good & bad conductors can be shown thus.



Steam is passed through a tube into which one end of each of six rods of equal length & thickness are put, as in the figure and all thus raised to the same temperature; the materials of which the rods are made are written above them; the other end of each tube is put into the bulb of a delicate thermoscope, and if any rise of temperature takes place in it by conduction of heat from the other end, the liquid in the thermoscope will be pushed down by the expansion of the

the equalization would take place more quickly in a body of that shape than in a body of this shape; each portion of the mass



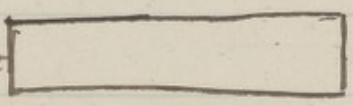
being a path through which the heat can travel; therefore all the heat that formerly passed along the spaces a & b cannot go; all the heat that can go has to get through the narrow part that is left: like water in a vessel, the more we contract the path, or neck of a bottle, through which the water has to pass, the more slowly does it pass.

Another consideration is the material of which the body is composed; e.g. the equalization is longer taking place in stone than in a metal; those in which it takes place readily are called Good Conductors; when slowly they are called Bad conductors

II. When heat passes from one body to another without requiring any intervening material medium, radiation is said to take place

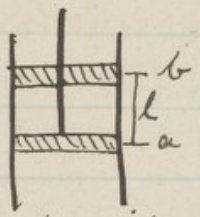
Conduction of Heat.

It is well known that when one part of a body is hot & the other cold; the hot part gets cold, and the cold hot, till there is an equalisation of temperature in all parts; this equalization takes place by a passage of heat from the hot to the colder part; it is called conduction; this equalization takes place most quickly in the parts nearest to one another; and the more intervening matter there is the more quickly does it take place also; suppose we have a body of the



shape in the figure one end hot & the other cold;

Suppose the piston in the working cylinder



to rise from a position a to position b:
l in the equation al (P - p) = work done, is the length ab.

Let v = volume the volume swept through by the piston, and w the effective pressure P - p

$$al\,(P-p) = vw$$

i.e. work done = volume × effective pressure.

Heat can pass from one body to another in two ways:

I. When heat passes from one part of a body to another; or from one body to another in contact with it; <u>conduction</u> of heat is said to take place.

To find how much work is being done by an engine in any part of its motion; an indicator is put at one end of the working cylinder



This is a small cylinder the piston in which is pressed down by a spring, so that when the pressure on its lower surface is zero, the piston is at the bottom of the cylinder; and by observing the distance up which it is forced, the effective pressure $(P-p)$ is got, and this being the only unknown term of the equation we just got for the Horse Power of an engine at any time, by substituting we get, what that Horse Power, or rate at which work is being done, is.

Let n = number of revolutions per minute

then $2n$ = number of strokes " "

& $2n \cdot a \cdot L \cdot (P-p)$ = work done per minute

Let d = diameter of cylinder

then $a = \frac{\pi \cdot d^2}{4}$

then $2n \cdot \frac{\pi \cdot d^2}{4} \cdot L \cdot (P-p) = \frac{1}{2} n \pi d^2 L (P-p)$ = work done per minute.

If P and p are measured by pounds avoirdupois and d & L in feet; we get the number of foot pounds of work done per minute;

$\frac{\text{force} \times \text{space}}{\text{time}} = \frac{FS}{t}$ = rate at which work is done

= Power.

If 33000 foot-lbs. of work are done by an engine in one minute (e.g. raising 33000 pounds thro' a distance of one foot in a minute); the engine is said to be one Horse Power.

therefore ~~$\frac{n}{}$~~ $\frac{n \pi \cdot d^2 \cdot L (P-p)}{2 \times 33000}$ = Horse Power of Engine.

the piston rods M and N in the figure on page 102; these move up & down alternately; imparting to the pts of the arrangement which correspond ~~with~~ to the pts. B & B' in the last figure a circular motion: this drives a wheel called the fly wheel: round which straps are put connecting it with, and working machinery.

In the year 1829, steam was first employed for locomotive purposes: when the "Rocket" of George Stephenson came into existence

Dec 4th 1877

To estimate the work done by an engine:

Let P = pressure of the steam

" p = ... in the condenser

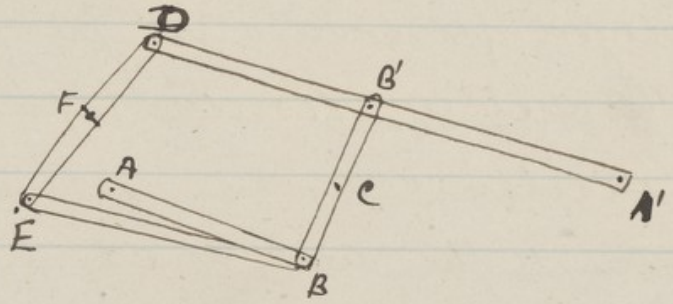
" a = area of the piston

then a (P - p) = total pressure on the piston

Let l = length of stroke

a (P - p) l = work done per stroke

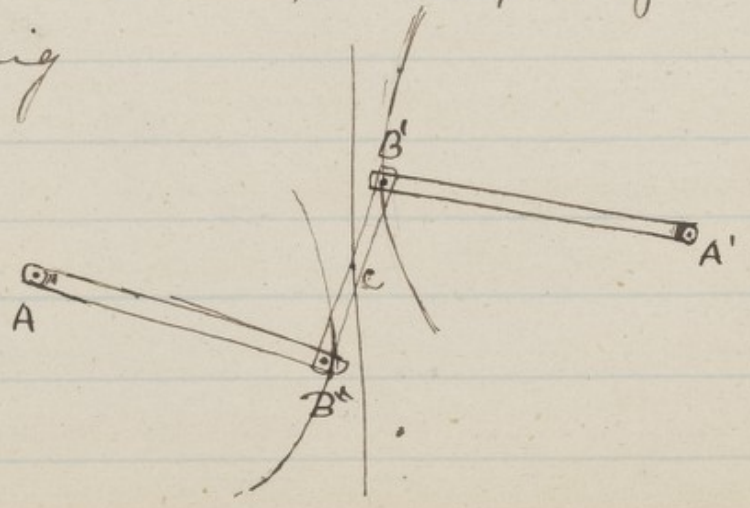
Three rods are jointed together and B & B' as in the figure: AB being equal to A'B'; A and A' are kept fixed and if B & B' move, they will move in a circle; but the middle pt. C of BB' will move up and down in what is really an elongated figure of eight, but virtually is a straight ~~st~~ line. Another pt. moving in a st. line can be got thus:-



Continue the rod AB' to D; D will move in a circle: to B fix a rod BE; E will move in a circle: then & F the middle pt. of DE moves in what is for a short distance, a st. line. These two pts. correspond to the ends of

but in a circle: viz the figure of the machine in page 102; the handle of the pump was fastened by a chain to the piston rod; this however would only pull, not communicate a thrust: the piston rod was therefore continued above the pt where this chain was fastened, and another chain fixed to pull the handle in the opposite direction i.e. up. An arc of a toothed wheel was also used.

What is called parallel motion was substituted for this in the year 1784 by Watts. The simplest form of this is, the following



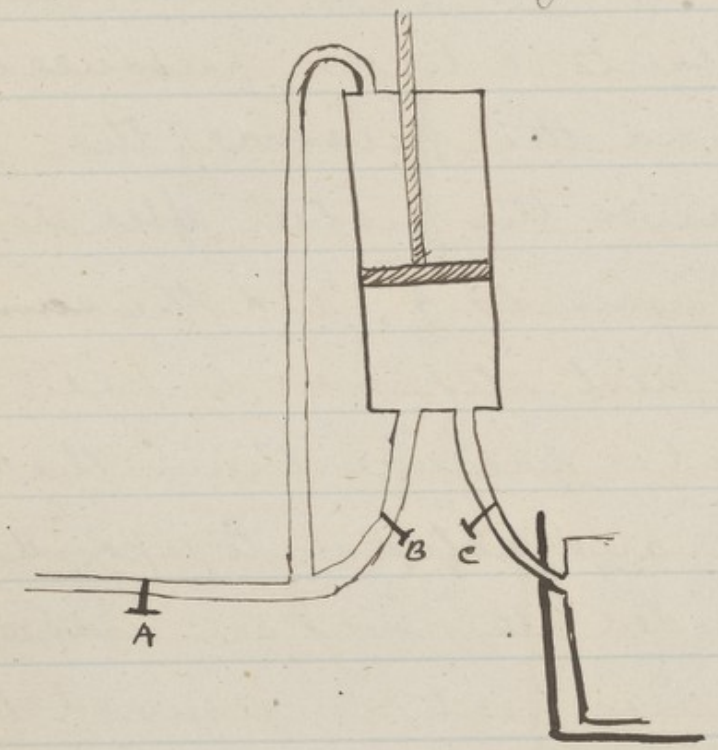
i.e. just the opposite to the first case.

This will occur when the piston is at the bottom of the cylinder; the steam presses against the lower surface of the piston and exerting greater pressure than the now condensed steam above it, will drive the piston up again; this is then repeated over again & so on.

Still however work was only done on the downstroke; the next improvement was to allow each end of the cylinder to communicate alternately with the condenser. This double acting arrangement was invented in the year 1778.

This led to the use of the steam engine to other purposes than pumping; it was first used to drive machinery. It was necessary that the piston should impart motion not in a straight line

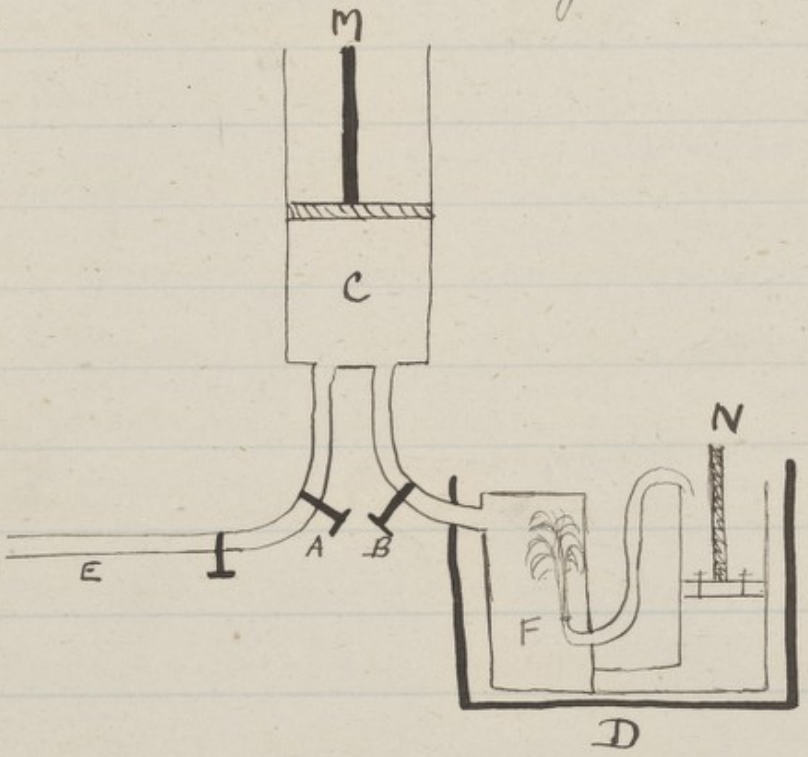
Watt's remedied by covering in the top of the cylinder and admitting steam alternately to the top & bottom.



To begin with, suppose A open, B shut, and C open; the steam will go to the cylinder above the piston, and as it there exerts greater pressure than the steam in the condenser, the piston will be driven down. Then suppose A shut; B open and C shut

goes into the condenser E which is kept cool by being in a vessel of water D, and a spray of water worked by another piston inside it; the steam is here condensed and consequently the steam that exerts a higher pressure in C, rushes down, and the pressure of the atmosphere drives the piston after it. Steam is let in again by A, and the same is repeated. The next step was to use steam at a greater pressure than that of the atmosphere and allow it to expand inside the cylinder; this was an improvement because less heat is required to produce steam at - say two atmospheres pressure, than to produce double that volume of steam at atmospheric pressure. This improvement was made in the year 1776. The piston was still however driven down by atmospheric pressure: this,

way, there was a great deal of fuel consumed in proportion to the work done. Watts remedied this by using a separate condensing vessel; this was in the year 1769.



A and B which are represented as stopcocks in the figure are really valves which work automat-ically; ~~when~~ the pipe E leads to the boiler: When A is opened the steam rushes in driving the piston up; when B is opened the steam

Suppose the pressure of the atmosphere to be 76 cm. and the pressure of the condensing steam 6 cm. the pressure on the piston will be 76 − 6 = 70 cm.
Let be diameter of the piston by 30 cm. (1st nearly)

$$\text{Area} = \frac{22}{7} \times \frac{30^2}{4} = \frac{22 \times 900}{7 \times 4} = \frac{19800}{28} = 707 \text{ sq. cm.}$$

13·6 = pressure on a sq. cm.

13·6 × 707 = — — the piston.

These engines were called Atmospheric engines. There were several minor improvements but the next great improvement was made by James Watts, this was the foundation of the modern steam engine. It was in repairing a model at the Glasgow university where he was a student that he first came into notice (1763-4)
In Newcomin's pump the cylinder had to be alternately filled with steam and cooled; & in this way a good deal of the heat of the steam went to heat up the cylinder, in this

turned on again; and it presses the surface of the water in C, and drives it back into the cylinder h; but as it cannot return by d, it pushes open g, and ascends the tube: meantime the other pump has been working in the alternate order in the same way.

The next improvement was made by Savery & Newcomin; the latter was the chief inventor; the date is 1705; this did not allow the steam & water to come in contact.

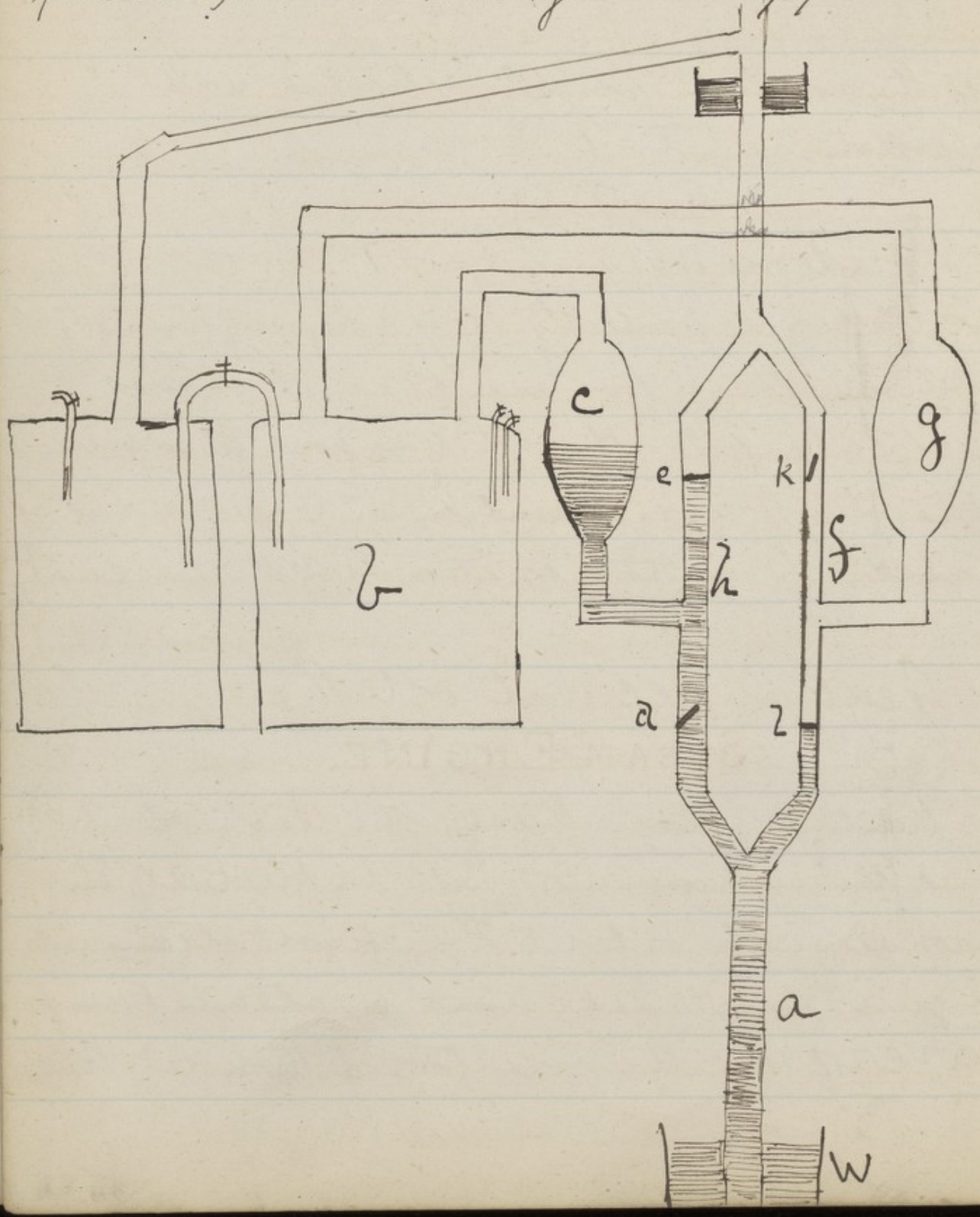
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In this pump it was so arranged that the steam was sent into a cylinder and drove up a piston the rod of which was fastened to the handle of the pump: when the cylinder was allowed to cool, the steam contracted & the pressure of the atmosphere drove the piston down.

The figure on page 98 represents in section the essential parts of the pump; it is simply a common force pump, only instead of having a solid piston, steam is used. In the figure two pumps are represented which work alternately; in one (S) the downstroke & in the other (h) the upstroke is seen; all the ~~of~~ valves open upwards.

Steam is formed in the boiler b; this escapes into the egg shaped vessel e, and into the cylinder h, shutting the valve k and opening e; (or in the other pump, opening k & shutting l.) The steam is then turned off from one pump & let into the other where the same goes on; the steam then cools & contracts, the pressure of the atmosphere on the surface of the well W, forces the water up the suction tube a, into k & c, to fill up the vacuum thus formed: leaving of course on its way d; through which it cannot return; the steam is then

patent for it in the year 1697.



degree of humidity $= \frac{P_t}{P_T}$

if the air is thoroughly saturated with moisture $T = t$

$\therefore P_T = P_t$

i.e. degree of humidity $= 1$

If the air is perfectly dry : degree of humidity $= 0$

Between these two quantities, it varies.

To calculate the wt. of vapour in a given quantity of air : equation on page 91 must be used, h in that equation being the same as P_t

General account of the STEAM ENGINE.

In treating of this subject, the fact about it and the improvements will be treated of in chronological order. The first steam engine ever used was a steam pump. Savery was the inventor, & took out the

leads from B to an aspirator. When water is allowed to run slowly, air is drawn into the ether, which is made thus to evaporate and the temperature of B is lowered, and the moisture when deposited is readily visible; the thermometer in B will indicate the dew point & in A the temperature of the air: if the operation be continued long enough, a temperature below zero can be reached, and hoar frost deposited.

By the term Degree of Humidity of the air is meant the ratio of the vapour present to the greatest quantity of vapour which could be present in the air at that temperature:

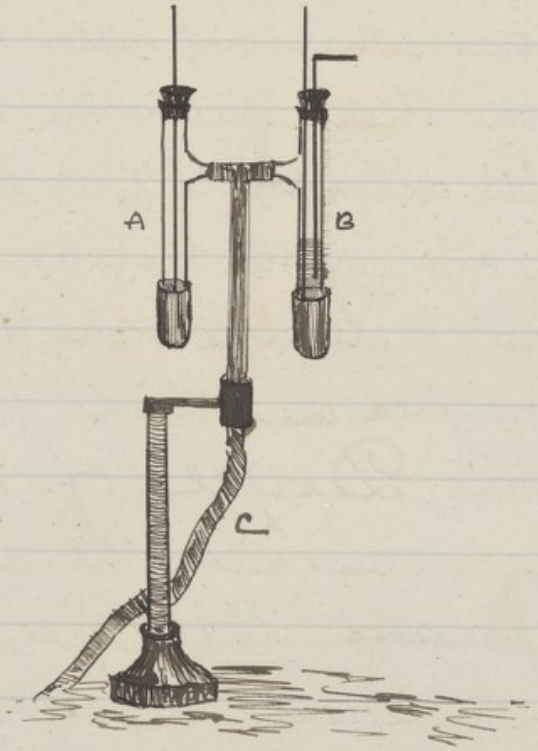
Let T = temperature of the air

t = ... —— - — .. dew point.

P_T = greatest pressure that the vapour could exert at T.

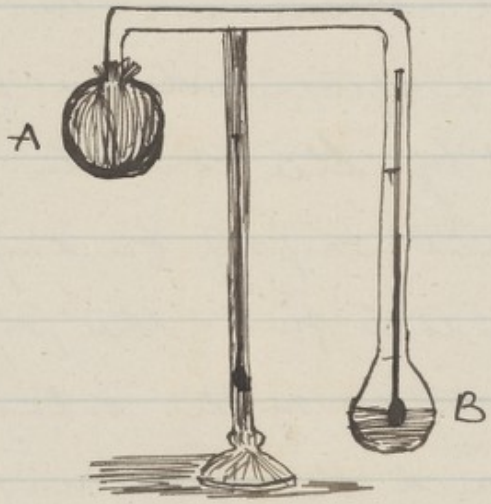
P_t = greatest pressure that it could exert at t.

Regnault; so that the ether can be evaporated without exposure to the air.



It consists of two tubes A & B, each fixed in a bright silver cup: a thermometer is fixed in the cork of each; B is also provided with a long glass tube open at both ends, the lower end being beneath some ether; an indiarubber tube C

It consists of a closed glass tube, with a bulb at each end, and bent twice at right angles as in the figure, and supported on a



stand; A is empty & surrounded with linen; B contains some ether, in which is a thermometer; Pour some ether on A; it evaporates cooling the bulb; the vapour within it condenses, and more is evaporated from B; this goes on and B is cooled, until dew is deposited on it; this is done several times to get the exact pt; which the thermometer in B tells.

An improvement on this was made by

state of vapour at the dew pt.; so that below that temperature a deposit of dew is formed. E.g. if we find the dew pt. is 10°C; then the pressure of the vapour in the air is .914, and the wt. in a given volume of air can thence be calculated by the formula on page 91.

There are several ways of finding the dew pt: a rough way is to find the pt. at which dew is deposited on the outside of a glass of cold water; and then warming the water & observing the temperature at which the dew disappears; then the dew pt. will be between these two; and on doing it several times with care the exact pt can be found. This however is not a good way as more vapour is evaporating into the air from the glass of water.

An instrument called Daniell's Hygrometer is often used to determine the dew pt.

The rate at which evaporation takes place depends partly on the quantity present in the atmosphere and partly on the difference between the quantity present in the atmosphere, and the greatest quantity which could be present; i.e. the temperature: the point on the thermometer at which the quantity present is the greatest quantity which could be present in the form of a vapour is called the **Dew Point**: so that the drier the air is and the warmer, the more rapidly does evaporation take place; therefore the more rapidly is heat absorbed, and therefore the more quickly a thermometer which is kept wet, cools.

This can be better done by cooling air, so as to get to the temperature which is the temperature at which the air can hold the quantity of moisture in it is the

But this is for pure & dry air; air however is generally moist; we must determine the tension of i.e. the pressure exerted by aqueous vapour in the atmosphere it supports a column of mercury of height h.

$$\therefore p = v.a.\frac{B-h}{76}.\frac{273}{273+t}$$

We must now take into account the weight of the moisture; this is similarly

$$v.a.623.\frac{h}{76}.\frac{273}{273+t}$$

aqueous vapour weighing 623 times as much as air at the same temp. & pressure.

These two added together give p; i.e.

$$p = v.a.\frac{273}{273+t}.\frac{1}{76}(B-h+623h)$$

a = wt of 1 cub. cm. of dry air at 0°. & 76 cms. barometric pressure = 0·001293 grms.

B = height of barometer

h = pressure of aqueous vapour

t = temperature.

it freezes.

Instruments are made on this principle to determine the quantity of moisture in the air.

Nov. 27. 1877

We now come to the methods of obtaining the quantity of water vapour in the air. The results are important in meteorological investigations, and also sometimes in delicate weighings in air;

if the real wt. of a body = P

& its loss of wt. in air = p,

its apparent weight is = P − p

To find p, several things must be taken into consideration:

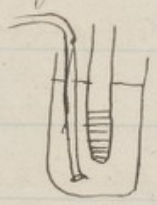
if v = volume of air, and a = the wt. of the unit vol. at 0°C. & 76 cm. press.

$$p = v.a$$

But if the barometer is B & the temperature t°

$$p = v.a.\frac{B}{76}.\frac{273}{273+t}$$

This has many important effects: to get very low temperatures, evaporation must be allowed to go on; i.e. the heat is given out in giving velocity to the particles thrown out by a liquid, & at last it gets so cold as to freeze. The liquid most generally used is ether; if we



blow air through ether surrounding a test tube of water, it is made to evaporate very quickly & as it requires heat to do so, it takes it from the materials with which it is in contact and thus heat is drawn away from the water & it freezes.

Another way of getting water to freeze is to make it evaporate quickly: put some under an air-pump, and put also ~~and~~ under the receiver a flat open vessel of oil of vitriol; and pump away: the water evaporates & the vapour is absorbed by the oil of vitriol; more evaporates & so on, and at last

$C = 1 + \cdot 00002\, t + \cdot 0000003\, t^2$

therefore W. being equal to Ct

$= t + \cdot 0002\, t^2 + \cdot 0000003\, t^3$

Let L stand for latent heat of water at t°.

$L = Q - W$

which is very roughly

$607 - \cdot 0708\, t°$

Say at 200°

$L = 607 - 141 \cdot 6° = 465 \cdot 4$

We have thus the following latent heats of water:-

at 0°C.	at 100°C.	at 200°C.
606·5	536·5	465·4

From this expression it seems, that if t is sufficiently great we get L = 0

But the equation is only trust worthy up to 250° (about); and when the temperature is sufficiently high there is no distinction between a liquid & vapour at that temperature.

Let the total heat needed to raise one gramme of water to $t°$ & evaporate it at that temperature be represented by Q:

$$Q = 606.5 + 0.305\,t°.$$

Say at 100°

$$Q = 606.5 + 0.305 \times 100$$

$$\begin{array}{r} 606.5 \\ 30.5 \\ \hline = 637.0 \end{array}$$

To get the latent heat of ~~water~~ steam at 100° we must subtract from this, the heat necessary to raise one gramme of water to 100° i.e. 100.5 gramme degrees of heat.

$$\text{Latent Heat} = 637.0 - 100.5$$

$$\begin{array}{r} 637.0 \\ 100.5 \\ \hline = 536.5 \end{array}$$

Let w = ~~weight of~~ this quantity: i.e. the quantity necessary to raise the temperature of a gramme of water from θ to $t°$, & c = the sp. heat:

then $w = ct$.

$81 \cdot 8 \times 30 \cdot 5 = 2495$ grain degrees of heat produced by the cooling of 30.5 gr. of water, from 100° to 18.2°.

$18405 - 2495 =$ heat evolved by condensation of 30.5 grains of steam to water.

$= 15910$

$\frac{15910}{30 \cdot 5} = 522$ (approximately) = latent heat of evaporation of steam at 100°.

The more correct number = 536.5

The sources of error; are, that the steam is not free from water spray, & that the calorimeter gives off some of the heat it receives from the steam to the air. Regnault was the man who made the experiments to determine latent heats, & he took great care & allowed for these sources of error.

This mode of proceeding is only applicable to water at 100°; for other liquids the vapour should be passed into another vessel within the calorimeter



The following is an experiment to determine the latent heat of vaporisation of water.

Weight of brass calorimeter 3100 grains

Sp. Heat of Brass = .094

3100 × .094 = 291.4 (i.e. the quantity of water brass is equivalent to)

Weight of water in it = 3080 grains.

Boil some water in flask & let the steam pass into the water in the calorimeter, where it condenses; then observe the increase of wt. (which will be the weight of the steam) & the change of temperature.

At first, temperature = 12.75°C.

Steam passed in = 30.5 grains

Final temperature = 18.2 C.

Rise in temperature = 5.46°C.

3080 + 291 = 3371 = virtual wt. of water

3371 × 5.46 = 18405 grain degrees of heat = total quantity of heat employed.

This is obtained: 1st from the condensation of steam

2nd from the cooling of the water got by condensation from 100° to 18.2° i.e. 81.8°

heat was thought to be an imponderable substance; and that the three states of a substance, solid, liquid & gas were three stages in the chemical combination of the substance with the imponderable matter called heat. Now however heat is believed to be a movement of the particles of the substance. The heat therefore that is required ~~to~~ cause of change of state cannot be spoken of as heat, because it does not exist as such; but it is expended in the effect it produces; and is reproduced when the effect is undone. But the term Latent Heat is so well known & understood, that it would be a work of supererogation to invent a new one. The latent heat of ~~a liq~~ vaporisation is different at different temperatures; we have seen that evaporation takes place at all temperatures (within certain limits), but that the melting of a solid takes place at one temperature only. The hotter a liquid is, the less heat must be given to it to change it into a gas; i.e. the less is its latent heat.

vapour at the same temp. & pressure: the 45 cm. of mercury
pressure exerted by water vapour at the temperature of 60°C being 14·9 cm. of mercury.

45 − 14·9 = 30.1 cm. = pressure of the air

If at 86 cm. pressure the vol. of dry air is 1265 c. cm.
then at 1 ——— will be 1265 × 86
and at 30·1 ——— 1265 × $\frac{86}{30.1}$
= Answer.

A practical application of this is in chemical operations, when we wish to raise the temp of water to over 100°C. all we have to do is to increase the to pressure.

Nov. 22. 1877

The latent heat of vaporisation is the number of units of heat necessary to change the unit wt. of a liquid into a gas; or the heat evolved when the gas changes to a liquid. The term latent heat was founded on a wrong idea: it was thought to be hidden away in some manner;

Say, we have 256 c.cm. of moist air at 25°C & pressure of 74·5 cm. of mercury. Required the wt of the dry air in the mixture; the wt. of 1 c.cm. of dry air measured at 0°C & 76 cm. mercurial pressure = ·00129 grams; and the pressure exerted by aqueous vapour at 25° being = 2·4 cm. (approximately)

256 × ·00129 = wt. at round temp. & pressure.

But the pressure is not 76 cm. but 74·5 cm. − 2·4 cm.
= 72·1 cm.

∴ therefor wt. of dry air at pressure = 72·1 & temp. 0°C

$$= 256 \times \cdot 00129 \times \frac{72 \cdot 1}{76}$$

but the temperature is not 0°C. but 25°C.

Therefor vol. of dry air at pressure = 72·1 & temperature = 25°C

$$= 256 \times \cdot 00129 \times \frac{72 \cdot 1}{76} \times \frac{273}{273+25}$$

(see page 41)

A similar Example is:—

We have 12 65 c.cm. of dry air at 60°C. under pressure = 86 cm. of mercury; what will be the volume of the air saturated with water

Atmospheric pressure = 75

each division on the scale being = ·138 cm.

by some peculiar way we get the pressure on the moist air in the tube = 65 + ·138 × 25·7

Then remove the moisture in the air, by putting up into the tube a piece of pumice stone ~~soaked~~ moistened with hydric sulphate (oil of vitriol)

We then get

Vol. of dry air = 240 c. cm.

Temperature = 13·1° C.

Pressure on dry air = 65 + ·138 × 22·5

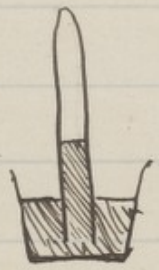
We thus find if we remove the vapour the volume and the pressure are both diminished

The following are examples of calculations founded on this:—

We have 256 c. cm. of moist air (meaning air saturated with water vapour i.e. the addition of more vapour to it, will not increase the quantity of vapour in it but will become liquid: saturated vapour is vapour when it will hold no more liquid in vapour at that temperature

When two vapours (which do not acts chemically upon one another) like water & benzene, are mixed we get as the total tension, the sum of the tensions of the two vapours. Or a gas and a vapour (such as air & ~~steam~~ water vapour) have just the same result. But if we take two which do act chemically upon one another (such as alcohol & water vapours), we get for the tension of the compound thus formed, a tension between the tensions of the two. This can be proved by experiments with the vacuum of two barometer tubes.

Put some damp air up a tube over mercury;



Volume of moist air put up = 243·2 c. cm.
Temperature = 18°C.

same time the boiling ~~increases~~ gets higher & this can be measured by a thermometer in the retort.

Nov. 20. 1877

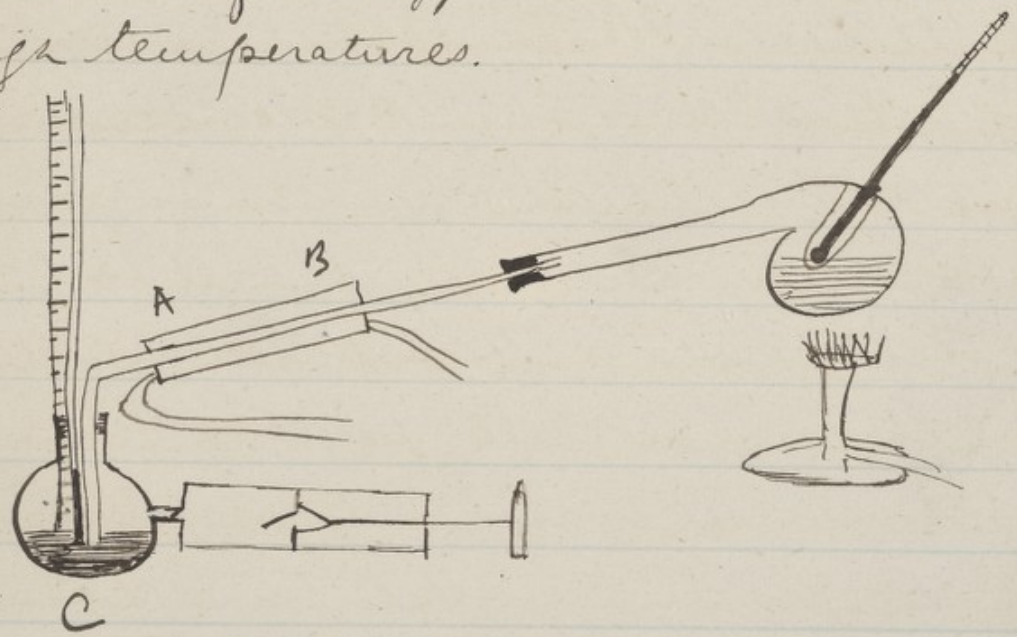
In the figure there is a mistake; the whole thing should slant the other way; so that the steam should be condensed, by passing cold water through the surrounding tube AB, and the water thus formed may run back to the bulb of the retort; the water thus never gets into the flask C.

The following are numbers the results of a rough experiment with the apparatus which shew the truth of the theory:

Height of Barometer = 75 cm.	boiling pt	Barometric Pressure
	100.2°	75 + 1
	107. 6	75 + 28
	109.7	75 + 31.

N.B. The bulb of the thermometer must be protected in some way so that the pressure inside the retort does not affect it.

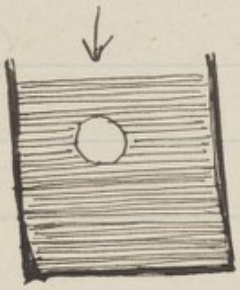
Regnault made experiments & determined the pressures of steam, and constructed a table giving its tension from low to very high temperatures. The following is a model of the apparatus he used in high temperatures.



The apparatus is so arranged that when the water boils, at 100° the surface of the mercury in the tube and in the flask shall be level. The pressure is increased by a force pump, and can be measured by the height of the mercury in the tube; at the

only no visible bubbles exist in the liquid; there are however very minute bubbles adhering to the bottom and sides of the vessel; when the liquid has once commenced boiling the process goes on more rapidly:
if all the air in water be taken out its temperature can be raised to several degrees above the boiling pt., but if then the smallest bubble of air be introduced the liquid will boil almost explosively. The separation of a gas from a liquid is in every part comparable to boiling; when soda water or champagne is flat, the gas can be again made to separate from it, by inserting a piece of bread, paper etc; because these convey a quantity of minute air bubbles to the liquid
When the pressure exerted by a vapour is higher than that of the atmosphere, a barometer tube cannot be used to determine it;

The process of boiling may be thus de-
-scribed.

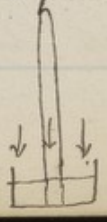


Suppose we have a babble of air in the middle of a vessel of water large enough to be visible as in the figure. The pres-
-sure of the air is transmitted to it by the liquid. Liquid is evaporating into this bubble in the manner formerly described, but it is condensed again when its strikes the opposite wall of the bubble. Now suppose the temperature of the liquid to be raised, more will evaporate into the bubble, and will push back the liquid walls making the bubble larger; it gets lighter and therefore rises to the top; this is the process of boiling;

The temperature of a boiling liquid is the temperature at which the vapour of water exerts a pressure equal to the pressure of the atmosphere. When the ~~the~~ atmospheric pressure is 76 cm.; then the temperature of boiling water is 100° C.; if we go up a mountain, the atmospheric pressure gets gradually less, therefore the temperature at which water boils diminishes; this can be easily seen by putting ~~some~~ some lukewarm water into a vessel under the receiver of an air pump; exhaust the air, and the water soon boils. Whenever the boiling pt. of water is spoken of as being fixed; it must be understood that the height of the mercurial barometer is supposed to be 76 cm. at 0° Centigrade at the time. Speaking approximately; for each 1/10 of an inch increase in the height of the barometer, the boiling pt. is diminished by 1/10 of a degree centigrade.

Take three barometer tubes; into the vacuum of one introduce a drop of different liquids: say water in one, alcohol in another, ether in a third. These will evaporate, and will exert pressure on the column of mercury; the ether most (over halfway down); the alcohol next, and the water least, though perceptibly.

Take the one with the water vapour in it, and heat it to 100°C by passing steam round it; the mercury will fast descend, shewing that the pressure of a gas increases with its temperature; When it is raised to the temperature of the steam, the mercury will not be any distance up the tube but level with that in the cistern, thus shewing that at that temperature the atmospheric pressure and the pressure of the vapour are equal.



vessel of ~~water~~ liquid be heated; the motion of the particles ~~more~~ is more violent; consequently more particles are thrown out; and more than strike the surface, at first; but soon a balance is established again, or a state of dynamical equilibrium.

The number of particles in a given space depends on the temperature. Different liquids will produce vapour at different rates, or throw out different numbers of particles (i.e. more vapour) for the same temperature.

Nov. 15th 1877.

The particles in different liquids are thrown out, with greater force in some than in others, in different sizes ~~at~~ and at different rates; we therefore get various degrees of pressure in the vapours of these different liquids. That this is so will be seen in this way.

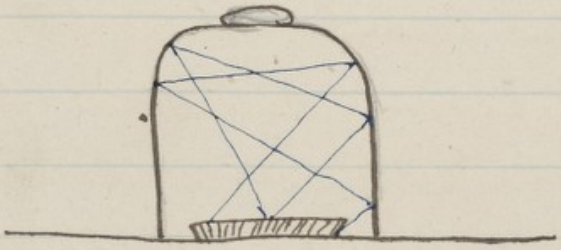
liquid, which then (being vapour) flys about the vessel from striking the sides; if it happens to strike the liquid again it will be incorporated with the liquid once more. But we have more particles leaving the liquid than returning to it; Consequently the number of particles flying about the liquid will be increasing; at last however a balance is established. i.e. as many return to the liquid as are driven out from it, in the same time. That is the evaporation ceases. But if the vessel is exposed to the Open air the particles leave it & never return; hence the evaporation is unlimited; but in the closed space it is limited.

Coming back again to the first example, and supposing equilibrium to be established between evaporation & liquefaction; let the

a piece of some salt, say Chloride of Sodium at the bottom of ~~some~~ a jar of ~~the liquor~~ water; the salt will first become liquid and soon will be found equally diffused through the liquid;

In a gas, the particles are moving about quite freely, and flying about in all directions, the pressure of a gas being causing by the particles striking against the sides of the vessel to get free.

Suppose we have a Pen vessel containing a liquid under a bell jar as in the figure.



At the free surface of the liquid the particles are jostling one another about, and every now and then a particle is shot out from the

of this is the familiar fact that any amount of water exposed to the open air, dries up, as it is called, after a time. But if the space is limited, the evaporation soon comes to an end: for instance, water in a stoppered bottle does not dissapear in time. We must remember that the vapour of a liquid has all the properties of a gas.

When a body is heated the particles of it are in motion is some way or another. In a solid body, the particles move about, without changing their place: e.g. like the leaves of a tree in a wind.

In a liquid the particles move about among one another, jostling each other about; like the people in a crowd. That this is so can be seen in a liquid by making a chemical difference in part of it. Put

temperature at which the fusion takes place. If water solidifies at 0°, it gives out as much heat as is taken up in the Opposite process, if it freezes below 0°, it gives out less.

We now pass to the phenomena of Evaporation, i.e. the passage of a body from a liquid to a ~~solid~~ gaseous state; and Liquefaction, or the Opposite process. There is a difference between the passage from a solid to a liquid state, and the passage from a liquid to a gaseous state. Take as an example ice; ice remains solid till 0°C.; i.e. it does not become liquid till that point; but if we expose water at any temperature to the air, it evaporates, or changes to the gaseous state. If the space in which it is exposed is unlimited, the passage from the liquid to the gaseous state goes on without limit. An example

$3100 \times .094 = 291.4$ (an equivalent quantity of water)

$291.4 + 3520 = 3811.4$ (i.e. as if both brass & water were water)

$3811.4 \times 1.96 = 7470.4$ = total quantity of heat given out.

this has besides melting the ice, raised the temperature of 90 grains of water produced from melting the ice from 0° to 11.48°.

The quantity of heat required to do this latter thing

$= 11.48 \times 90 = 1033.2$

$7470 - 1033 =$ quantity of heat used in melting the 90grs of ice!

$= 6437.$

$\frac{6437}{90} = 71.5 =$ quantity of heat required to melt one grain of ice

= latent heat of ice

. The experiment however if done with greater care would have given not 71.5 but 80.4

The latent heat of fusion depends partly on the

be 0°.

i.e. Ice at 0° becomes water at 0°

Water at 80° ---- ---- ---- 0°

i.e. a ~~grain~~ given mass of ice requires to melt it as the same mass of water gives out in cooling from 80° to 0°, or as it would require to raise it from 0° to 80°; or 80 times the quantity would require to raise it from 0° to 1°.

i.e. 80 is the latent heat of ice.

To find the latent heat of a liquid say water, take as an example the following actual experiment which was done roughly.

Weight of brass vessel 3100 grains

Weight of water inside it 3520 "

Temperature of water 13·44° C.

Put in some ice to be melted by the water, a quantity = 90 grains

And the temperature after is 11·48°

Specific heat of brass ·094

Specific heat of water 1

Difference of temperature = 13·44 - 11·48 = 1·96°

to take place in a calorimeter; the heat is taken up by the calorimeter and measured in the same way as Specific heat was. The heat taken up by the Calorimeter however is of three parts.

1. The heat given to it by the liquid put in.
2. The heat produced by solidification
3. The heat given to it by the solid formed.

To find 2. or the ~~specific~~ latent heat of solidification, we must know both 1. and 3., as well as the specific heats of the liquid put in, and the ~~solid formed~~ containing vessel.

Nov 13th 1877

As a simple experiment to show the meaning of latent heat, and to show that the latent heat of water is 80°; take equal weights of ice at 0° & water at 80°, and mix them together; When the ice has all melted the temperature of the mixture will be found to

at the ordinary temperature; this remains in a ~~solid~~ liquid state however at the ordinary temperature, on bringing into contact with it, any solid body like the bulb of a thermometer, it immediately becomes solid; ~~this~~ giving out heat which will raise the temperature of the thermometer a considerable distance; in the experiment done, 30° C.

One of the commonest methods to separate different solids in one solution, is by crystallisation, each solid crystallizing in its own particular system. E.g. if coloured water be allowed to freeze, the crystals of ice will contain no colouring matter, the latter having crystallised separately.

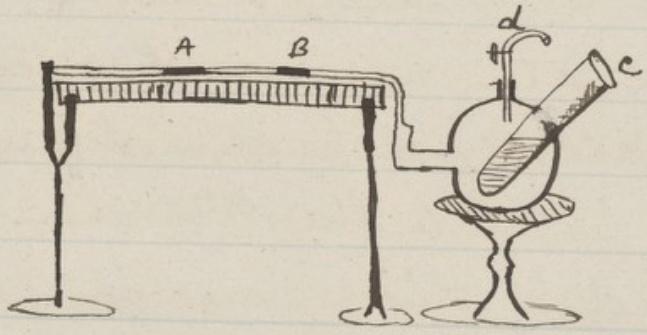
Methods of finding Latent heats of different substances

These experiments are on the whole very much like those, to find the Specific heats of substances. The solidification must be allowed

In the testube C, put some water, and then the salt; which on becoming liquid withdraws heat from the surrounding air. Which in consequence contracts, and draws the plug A into a position B nearer to the bulb. This same instrument, it may be here said, can be used as a measurer of heat (not temperature,) by filling it with mercury to the zero mark on the scale; and putting different substances of heat given off which are hot into the testube; and the amount by them can be measured by the expansion of the mercury caused; which simply measures the heat which is employed in heating the known mass of mercury.

That a liquid on solidifying becomes hot, may be thus shown. Dissolve Sodic Sulphate in water at a high temperature: & more is dissolved than would be

Crystallisation from solution is a very slow solidification of the liquid and heat is given off during its procedure. If we melt salt in water, heat is drawn away from the ~~liquid~~ water to liquefy the solid salt; that a perceptible cooling does take place may be seen from the following experiment.



A bulb like the one in the figure with three openings is taken; one is prolonged into a long tube, which runs by the side of a graduated rod; in the second is inserted a testtube C; and the third is furnished with a stopcock and tube d. In the long tube is put a plug of water, which can be put in any position A, by blowing in, or withdrawing air from the apparatus by means of the tube d.

Mercury has a much less latent heat (about 2½); in fact to some degree the latent heats of solids are pretty much in the same order as the specific heats. When on the other hand a liquid is solidified, this latent heat is given back again, in fact reproduced.

To get very low temperatures in bodies, certain mixtures are used, whose great cold depends on this property of latent heat; i.e. they are made to liquefy, without heat being supplied to them, and as heat is required for their ~~solid~~ liquefaction, it is drawn away from the bodies in contact with the mixtures. The commonest of these freezing mixtures, is that of ice and common table salt; the two liquefy one another mutually producing great cold about −20°C.

From what has been said, will be seen that the only temperature at which the liquid and solid states of the same substance can remain in contact with another, is the melting pt. of the solid. For example, water and ice can only remain in contact with one another is the temperature of 0°C

To effect the change from the solid to the liquid state, heat is required; the heat applied to a solid like ice, ~~for example~~ at its melting pt, does not raise the temperature of the ice, but goes to form water at the same temperature. The heat thus consumed appears to be hidden in the liquid, and is called the latent heat of the solid. Ice has the greatest latent heat, it is about 80; that is to say that 80 grammes units of heat are required to melt {the unit wt one gramme} of the solid; twice the quantity to melt 2 grammes & so on.

Another way to prevent the solidification of a liquid at its freezing point, is to keep it out of contact with any solid body: this is done by suspending the liquid in another liquid of the same specific gravity; e.g. a mixture of alcohol & water can be made in such proportions, as to be of exactly the same specific gravity as oil; globules of oil of considerable size, can then be suspended in this mixture at any pt, and so keep the oil out of contact with any other solid. Water can be similarly suspended in a similar mixture of Chloroform, a liquid of greater s.gravity than water, and Benzine, a liquid whose specific gravity is less than that of water. In cases like this, the liquid can always be cooled below its freezing point, or the highest pt. at which it becomes solid.

The melting pt. of a solid is a fixed pt; but the freezing pt of the corresponding liquid is not fixed, and the melting pt. of the solid is the highest point at which it can freeze, but not always does freeze. When a liquid is in a finely divided state, as water is in a fog, it will remain a long time floating in the air, though heavier than it, like a cloud of dust; and in this state, it seldom if ever freezes at the melting pt. of the corresponding solid, which is in this case viz: that of ice 0°C. Whenever a liquid however is below its freezing point proper, in temperature, it solidifies immediately on contact with the smallest bit of the solid. P.S. in the case of a fog below 0°C. in temperature, it begins to solidify on the twigs of trees, and this contact of ice with the surrounding portion of the fog, causes it to solidify directly, forming a thick coating of ice round the branches etc. of the trees.

temperature; between this two classes of bodies there is no hard and fast line; the lowest melting point ~~known~~ ascertained is that of nitrous oxide, which is -115°C.; while the highest ascertained is that of Platinum, 1776°C.

While the change from a solid to a liquid state is going on, there is usually a change of volume; in most cases an increase of volume. The case of ice is one of the most remarkable exceptions to this rule; a diminution of nearly 10 per cent going on; paraffin is the most markedly a case opposite to water.

When a liquid is cooled; it becomes solid; generally at the temperature at which the corresponding solid melts. But often it may be cooled much below the melting pt. of the corresponding solid.

in proportion to their atomic wts and it will be found that the quantity of heat necessary to raise any one of them to any definite temperature is the same as the quantity of heat necessary to raise ~~the same~~ any other of them to the same temperature. The explanation of this is that all atoms have the same specific heat, no matter of what material the atom is composed. A similar relation is observed in other bodies such as compounds and gases.

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Solid bodies when heated melt i.e. become liquid; this change may take place at a certain pt. of the thermometric scale, as is the case with ice; or it may take place gradually as it does in glass, or iron; the body becomes soft at a comparatively low temperature, (400°C. about, in glass) and continues to get softer & softer only becoming liquid at an exceedingly high

principle of all determinations of specific heat; certain modifications have to be made in the cases of liquids and gases, and sometimes different fluids are used in preference to water to heat the vessel, but these modifications are too numerous to enter upon here.

A fact worth noting about specific heats is the following; - Arrange the metals in the ascending order of their atomic wts; and it will be found that they are also in the descending order of their specific heats: besides that, the two vary proportionately. I.e. multiply the two together for any metal, and the same product is obtained, as of any other metal; i.e. the product of the atomic weight of a metal and its specific heat is constant. Or expressing the same another way: take wts. of the metals

only one unknown quantity in the whole equation, and that of course can be found.

Mercury we have before seen expands less than water; but this expansion takes place more quickly than with water; this is simply because its specific heat is less; i.e. it take less heat than water to raise it to the same temperature and therefore it receives all its heat in a much less time, and thus its expansion is ver fast: the sp. heat of mercury is about $\frac{1}{30}$: suppose we have equal volumes of Water & Mercury, their weights will be in the proportion 1: 13·6; and the heats required by them to raise the same temperature will be in the proportion $1 : \frac{13 \cdot 6}{30}$ or 1 : ·45: or. less than $\frac{1}{2}$ as much heat is required by a certain volume of mercury, as is required by the same volume of water, in order to rise to the same temperature.

The apparatus described on page 48, shows the

The following general equation is thus derived; supposing z to be the sp. heat of the matter under examination, and b the sp. heat of the material of calorimeter:

$$W(\theta - t)^\circ + V(\theta - t)^\circ b = M(T - \theta) z$$

or what is the same thing

$$M(T - \theta)^\circ z = (W + Vb)(\theta - t)^\circ \quad \text{----(1)}$$

Vb is sometimes called the <u>thermal capacity</u> or <u>water value</u> of a body; obtained by multiplying its mass V by its specific heat b.

We have however supposed that b is known in the above equation (1), and the sp. heat of the substance of the calorimeter can be easily found by substituting a known wt. of it for the piece of zinc in the foregoing experiment; we then get b instead of z on the left hand side of the equation, and we have it already on the left: so there is

If these two numbers

$3900 \times (18.6 - 13.1)^\circ + 3100 \times (18.6 - 13.1)^\circ b$

is the total quantity of heat which raises the calorimeter from 13.1° to 18.6°; and supposing all the heat given out by the zinc on cooling from 100° to 18.6 also to the calorimeter this sum must be equal to the heat given out by the zinc; which is by a similar method of reasoning (supposing z to be the specific heat of the zinc)

$2886 \times (100 - 18.6)^\circ z.$

We thus get the following equations:-

$3900 \times (18.6 - 13.1)^\circ + 3100 \times (18.6 - 13.1)^\circ b$

$= 2886 \times (100 - 18.6)^\circ z.$

Substituting for b the number .094: we find if the experiment has been done carefully enough that z or the specific heat of zinc is .0956. Which means that .0956 of a unit of heat is sufficient to raise the temp^{re} of the unit wt. of zinc one degree.

Weight of brass vessel	3100 grains	V
——— water in it	3900 "	W
——— piece of zinc	2886 "	M
Temp.^{re} of water	13·1°C.	t
" of zinc	100° C	T
Final temperature	18·26° C	θ

We have 2886 grs. of zinc cooled from 100° to 18·26°C.
We have also 3900 grs. of water and 3100 grs. of brass heated from 13·1°C. to 18·26°C.
To raise the temperature of 3900 grs. of water 1°, we require 3900 heat units; ∴ to raise 3900 grs. of water from 13·1° to 18·26° we require

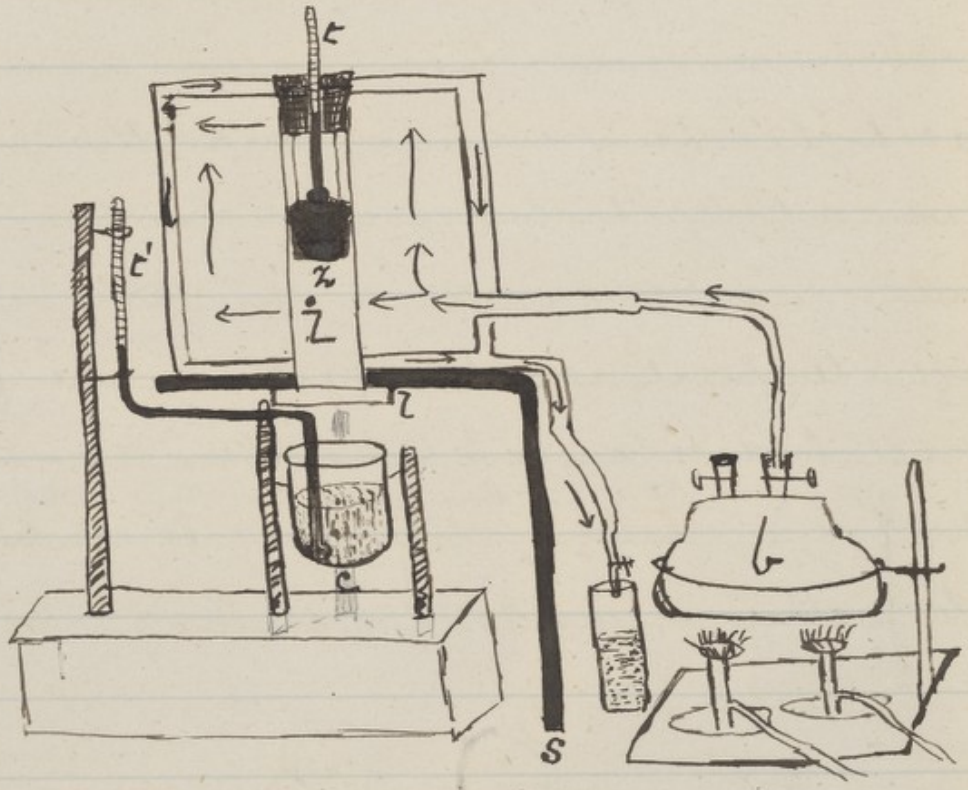
$$3900 \times (18{\cdot}26 - 13{\cdot}1)^{\circ}$$

Let b be the specific heat of the brass;
To raise 3100 grs. of brass one degree, we require 3100 b units of heat: therefore to raise 3100 grs. of brass from 13·1° to 18·26°, we require $3100 \times (18{\cdot}26 - 13{\cdot}1)^{\circ} b$; the sum

contains the material Z under investigation, which may be suspended by a silk thread or in a ~~wood~~ silk basket ~~as in the figure~~; the vessel i also contains a thermometer, mounted thro' the cork; When this thermometer stands at the required temperature in this case 100°C., the lid L, is removed, and Z is quickly dropped into a vessel, C, made of brass or some polished material, and containing a known wt. of water at a known temperature; this vessel also contains a thermometer graduated most carefully and best of the shape in the figure; observe the temperature of this, after the contact of Z and the water and we have all the necessary data; S is a screen of water or polished metal to prevent the radiation of heat from parts of the apparatus to the calori-meter as C is called. The following are the results obtained in a actual experiment, Z being zinc, and C being made of brass.

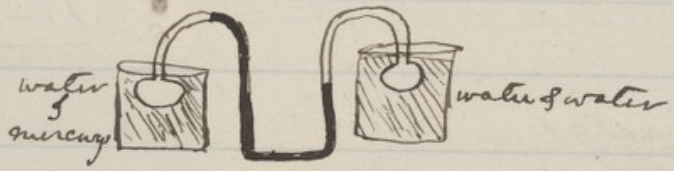
November 6. 1877.

The following is one way of finding the specific heat of a substance.



Water is heated in a boiler b to the boiling pt., and the steam formed ~~its es~~ is allowed to escape in the direction of the arrows, circulating round two outer vessels, thereby raising the temperature of the inner vessel i to the temperature of 100°C. The inner vessel i

mercury has required much less heat than the water; and therefore will give out less on cooling: let the quantity given out by the water be used in heating a known quantity of water in a beaker, to a higher temperature; and the quantity of heat given out by the mercury be used in heating an equal quantity of water in another beaker to a higher temperature; the first mixture of water & water will be of a much higher temperature than the mixture of water & mercury: i.e. the sp. heat of mercury is less than that of water. this may be rendered visible by a thermoscope of this shape;



the liquid in the tube will be pushed down by the expansion of the air being greater in the hotter bulb viz: that immersed in water & water, than in the other bulb, as in the figure. The same experiment may be done using equal wts. of iron and zinc; when the zinc will be found to have the greater specific heat. Or of iron and bismuth: the iron is the hotter even though equal volumes be taken.

In an actual experiment with a piece of iron, the following were the numbers obtained.

$92 = i$

$80 = h$

$718 = m$

$100 = t.$

$$c = \frac{92 \times 80}{718 \times 100}$$

The specific heat of a body we have said is the quantity of heat which ~~it~~ one gramme of it requires to raise its temperature one degree; this is the same thing as saying that the specific heat of a body is the quantity of heat it gives off when 1 gramme of it cools one degree.

That different bodies have different specific heats may be visibly proved thus; take equal weights of water and mercury, and heat them in a water bath to the temperature 100°C; that is we have equal wts. of 2 bodies at the same temperature. But these have not required the same quantities of heat to do this; the

the inside wiped quite dry; a hot body of known temperature was then placed inside: and allowed to cool, thus melting ice; the water measured off at the end of the experiment and what was before known give the data for calculating the Specific heat.

Let i = quantity of ice melted, or what is the same thing the quantity of water formed.

h = quantity of heat necessary to melt the unit wt of ice (which is 80 units of heat).

t = temperature of body.

m = its mass

& c = its specific heat.

then $i.h = m.t.c.$

or $c = \frac{i.h.}{m.t.}$

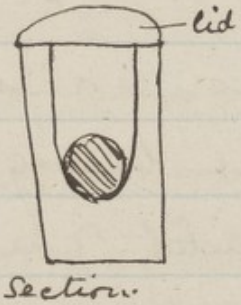
In more elaborate experiments, by Lavoisier & Laplace, two layers of ice were used, the outer of which only could be affected by the atmosphere; and the inner was melted by the hot body; the water thus formed being allowed to escape by a stopcock.

From 0°C. to 1°C. is said, because it does not follow that the same quantity of heat is capable of raising one gramme of water 1 degree of temperature at all parts of the scale say from 90°C. to 91°C. or 100°C to 101°C, etc, and therefore from 0°C. to 1°C. is the standard.

The <u>specific heat</u> of a body then is, = the quantity of heat required to change the temp^{re}. of {the unit mass / one gramme} of the substance by 1 degree of ~~th~~ temperature.

Therefore it follows that the specific heat of water is 1.

The plan of calculating specific heats from the power of heat to melt ice was carried out by Drs. Black, and Wilke.



A vessel of the above shape was made of ice; of

If Q stand for the quantity of heat required for the change of temperature $t' - t$, we have

$$Q = m(t' - t)c$$

c being an unknown quantity depending on the nature of the body.

$$\frac{Q}{m} = (t' - t)c$$

$$\frac{Q}{m(t' - t)} = c;$$

this number c is called the Specific Heat of the substance. We have spoken of Quantity of heat, therefore we must have such a thing as a unit of heat. This may be measured by the melting of ice; the quantity of heat which is necessary to melt {the unit mass / one gramme} of ice. It is now however taken by the power heat has to raise temperature, some standard substance being taken viz:- water. The unit of heat then is; the quantity of heat necessary to raise the temperature of {the unit mass / one gramme} of water from 0°C. to 1°C.

A cubic centimetres of dry air at 0°C. and barometer 76 millimetres weighs .00129 grammes.
Find the weight of 124 cubic centimetres at 57.6°C. and atmospheric pressure 27.5 cm.

$$pv = p_0 v_0 \left(1 + \frac{1}{273} t^0\right)$$

$$v = v_0 \frac{p_0}{p} \left(1 + \frac{1}{273} t^0\right)$$

$$= 124 . \frac{76}{27.5} \left(1 + \frac{1}{273} 57.6\right)$$

from which the weight can be found.

Nov. 1. 1877

It is found that the facility for change of temperature is different in different bodies; i.e. some bodies require more heat to raise their temperature a certain number of degrees than others. The facility for heat, a body depends on three conditions;

1. Mass of body = M
2. Change of temperature = $t^1 - t^2$
3. Nature of substance, the body is made of.

to show small differences of temperature. There are many forms of these; one of the commonest is this:-



If the temperature is the same in both bulbs, the liquid stands at the same height in both; but if one be heated more than the other, say by placing a warm hand on it, the liquid is immediately pushed up the other tube by the expansion of the air inside.

Examples.

Suppose there is a room containing 1670 cubic ft. of air at 0°, and barometer at 29·2 inches; find the volume of the air at 24° & barometric pressure 30·6 inches.

$$\frac{\text{Vol. at } 24^\circ}{\text{Vol. at } 0^\circ} = \frac{273+24}{273} = \frac{297}{273}$$ { reckoned from absolute zero.

$$\text{Vol. at } 24^\circ = 1670 \times \frac{297}{273} \quad \text{---------- (1)}$$

$$\frac{\text{Vol. at } 30.6}{\text{Vol. at } 29.2} = \frac{292}{306} \quad \text{---------- (2)}$$

multiply (1) by (2)

$$\text{Vol. at } 24^\circ \text{ bar. } 30.6 = 1670 \times \frac{297}{273} \times \frac{292}{306} = \text{Answer.}$$

diminishes the velocity of these particles diminishes or the pressure of the gas diminishes; till at −273 or the Absolute Zero, we have no pressure at all.

Reckoned from the Absolute Zero

0°C = 273

100°C = 373.

---

temp^re.	Volume	Increase of Volume
0°C.	273	$\frac{10}{273}$
10°C.	283	$\frac{10}{283}$
20°C.	293	$\frac{10}{293}$
30°C	303	$\frac{10}{303}$

When finding the coefficient of expansion of gases, they must be dry, because the moisture in a gas expands also and makes the gas expand more than it would be itself.

The expansion of gases is used for Thermoscopes

This constant quantity is ·3665, or more roughly $\frac{1}{273}$ or $\frac{11}{3000}$

i.e. $p_0 v_0 = 273$

$p_{100} v_{100} = 373$

=

$p_2 v_2 = 271$

$p_1 v_1 = 272$

$p_0 v_0 = 273$

$p_{-1} v_{-1} = 274$

$p_{-2} v_{-2} = 275$

and if we go low enough viz: to −273, we have $pv = 0$

this does not mean that $v = 0$, but that the product $pv = 0$; which is also satisfied by p being $= 0$; i.e. we have a solid body. The pressure of a gas is due to its particles flying about in all directions; if the temperature increases, the velocity of the particles increase, and if the temp^{re}.

Let p stand for pressure at any temperature
& v ------ volume " "
pv is a constant quantity.

E.S.

Temperature	Volume (pressure being constant)	Pressure (Volume being const.)
0°	10000	10000
100°	13665	13665

Oct. 30. 1877.

Meaning by $p_0, p_{100}, v_0, v_{100}$, the pressures and volumes at 0° & 100C. respectively, we have

$$p_{100} v_{100} = 1.3665\, p_0 v_0$$

supposing $v_{100} = v_0$

we have $p_{100} = 1.3665\, p_0$

supposing $p_{100} = p_0$

we have

$$v_{100} = 1.3665\, v_0$$

Expansion gets more rapid as temperature rises. Water is a peculiar exception; if water gets cool, it contracts till it gets to a temperature of 4°C. when it begins to expand again, (by keeping it quiet, water can be cooled as much below 0°C.) This point 4°C. is called the point of maximum density or ~~minimum vol~~ minimum volume.

Temperature	Volume
−10	1,001,858
+4	1,000,000
+18	1,001,348.

Water by pressure can be prevented from boiling at 100°.

+100	1,043,120
+157	1,101,600.

Unlike water, the expansion of mercury is nearly regular.

In gases however, if the pressure be kept constant the volume increases regularly on a rise of temperature; and the pressure increases regularly if the volume be kept constant.

apparent coefficient of expansion $\alpha' = \dfrac{P_0 - (P_0 - p_{100})}{(P_0 - p_{100})\,100}$

$$= \frac{p_{100}}{(P_0 - p_{100})\,100}$$

If the instrument be again filled and heated to a temperature x which we want to find, and the quantity driven out be weighed as before;

As before we get;

$$\alpha' = \frac{P_0 - (P_0 - p_x)}{(P_0 - p_{100})\,100} = \frac{p_x}{(P_0 - p_{100})\,100}$$

$$\alpha' = \frac{P_0 - (P_0 - p_x)}{(P_0 - p_x)\,x} = \frac{p_x}{(P_0 - p_x)\,x}$$

We have therefore

$$\frac{p_x}{(P_0 - p_x)\,x} = \frac{p_{100}}{(P_0 - p_{100})\,100}$$

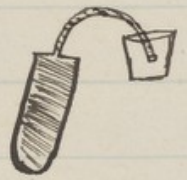
$$x = \frac{(P_0 - p_{100})\,100}{p_{100}} \cdot \frac{p_x}{P_0 - p_x}$$

In this experiment the temperature $t = 15°C$; and the apparent height of the barometer $N = 74 \cdot 115$ centimetres.

$$H' = 74 \cdot 115 \, (1 - \cdot 00017 \times 15)$$
$$= 73 \cdot 936$$

—

In some thermometers the mercury which expands is weighed instead of measured. These are called Weight Thermometers.



Weight the instrument, which is often of the form above, when full of mercury at 0°C. Heat it to 100°C. some of the mercury flows out; weight this;

Let P_0 = wt. of mercury which fills the instrument at 0°C.

p_{100} = wt. ------ driven out at 100°.

then $P_0 - p_{100}$ = wt ------ remaining in, neglecting the exp. of glass.

Generally however m & m_0 may be left out of account; then

$$H' = H\frac{1+dt}{1+\mu t}$$

A rough manner of dividing this fraction is;— since d & μ are such very small quantities, it may be subtracted from both numerator & denominator without much altering the fraction; then we have

$$H' = H(1+dt-\mu t)$$
$$= H[1+(d-\mu)t]$$

or expressing it otherwise: $d-\mu$ is negative

$$= H[1-(\mu-d)t]$$

Supposing the scale to be brass, of which the coefficient of expansion (d) is ·0000188; and μ = ·00018

we have $\mu - d$ = ·00018 − ·0000188 = ·0001612

$$H' = H[1-0001612\,t)$$

then at any other temperature t, let the length be L; and its coefficient of expansion d.

$$L = L_0(1+dt)$$

Let H be the apparent height of the liquid at $t°$
& H' ..... true height at $0°$

$$H' = H(1+dt) \quad \text{---------- (1)}$$

Let m_0 = s.g. of mercury at $0°$ = 13.596.
i.e. 1 cubic centimetre of mercury at $0°$
contains 13.596 grammes.

μ = coefficient of exp. of mercury.
& m = s.g. at mercury at any temp $t°$.

then

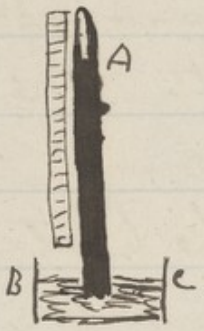
$$m_0 = \frac{m}{1+\mu t} \quad \text{---------- (2)}$$

multiply together equations (1) & (2)

$$H'm_0 = H(1+dt)\frac{m}{1+\mu t}$$
$$= Hm\frac{1+dt}{1+\mu t}$$

this is called the corrected height of the barometer.

at the side of the tube. Supposing the figure below to be a barometer tube.



Since the whole apparatus is in equilibrium the pressure on all parts of the mercury in the cistern at the level BC, must be the same: i.e. the atmospheric pressure, is equal to the pressure of the column of liquid in the tube, reaching to a level A.

Let H be the height of the liquid in the tube,

m = its specific gravity

Hm = pressure of the column of liquid

The temperature makes the apparent height ~~less~~ more than it really is, by the scale expanding, and m diminishes; these act in opposite directions.

Let the scale be at the temp. of 0°C., $= L_0$ in cm

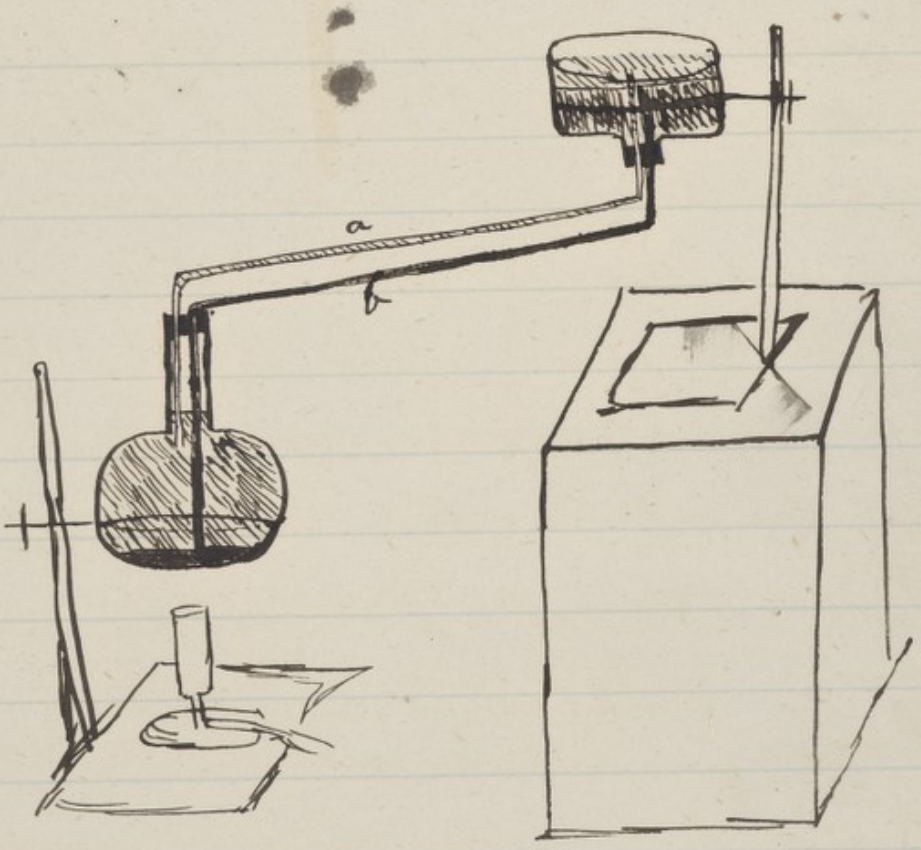
through the cork of an inverted bell jar. a, going nearly to the surface of the water, and b just into it. The water in the bell jar is then coloured, and that in the flask is heated. The heated water is lighter than the cold water, and therfore rises to the surface and passes along the tube a into the bell jar. To fill up the space left by the heated water, cold water falls down b and collects at the bottom of the flask, while the colourless hot water collects on the surface of the coloured water in the bell jar; thus there is a constant circulation going on: a being filled with hot colourless water moving upwards; & b with cold coloured water, falling to the flask.

Oct 25, 1877

Temperature

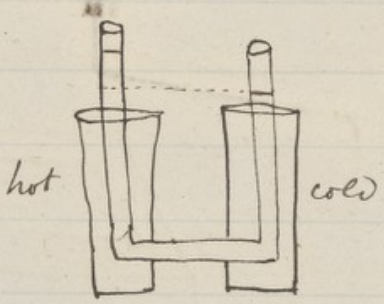
" has an effect upon the barometer, since a rise of temperature causes expansion of the mercury in the tube, and also in the scale

Another effect of this difference of specific gravities between hot & cold liquids is seen thus;

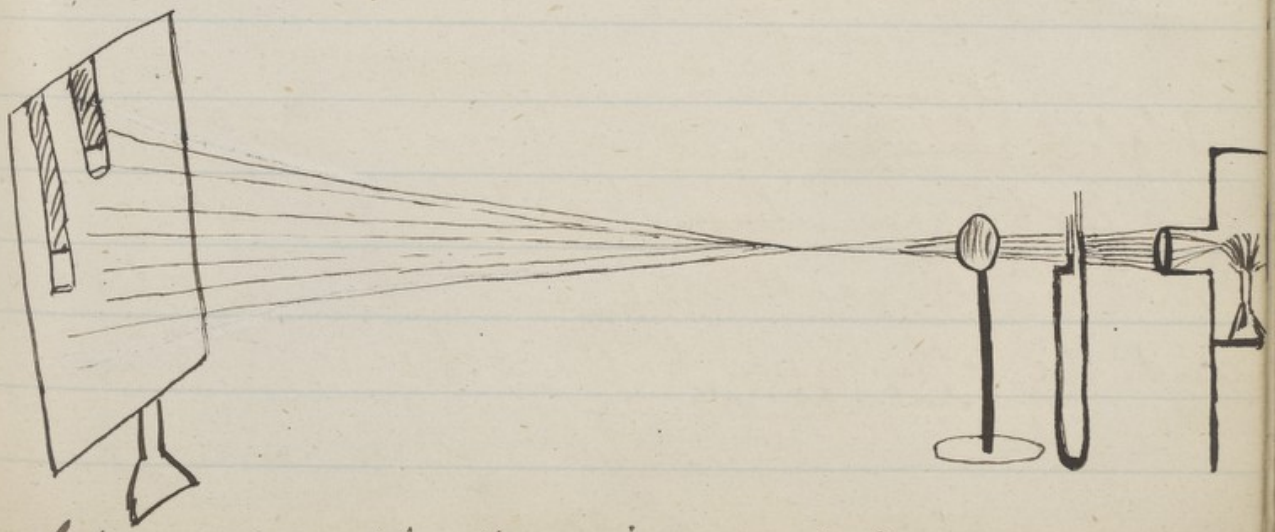


A glass ~~globe~~ flask is filled with water; through its cork pass ~~three~~ two tubes, one (a) going just underneath the surface of the water and the other (b) going to the bottom of the flask. The tubes which should be filled with water at their upper extremity pass

tube at the required temperature should be; to enclose one arm in a vessel of hot water, & the other in a vessel of cold water and ice; the temperature of each being ascertained by thermometers in them



This may be made visible by an arrangement like this; viz: by throwing a strong light on a similar small



tube, and catching the inverted shadow on a screen.

in a tube as in the figure; the condition of equilibrium of these two liquids is

$$ah\,s = ah's'$$

or $hs = h's$

i.e. the breadth of the tube does not matter.

or $\frac{h}{h'} = \frac{s'}{s}$ i.e. the heights vary inversely as the specific gravities. Let h & s, h' & s' be the depths & sp. gravities respectively of the cold & hot liquids respectively (the temperatures being known) The S.G.s decreased as the volumes v & v' increase.

$$\frac{h'}{h} = \frac{s}{s'} = \frac{v'}{v}$$

subtract unity from each quantity

$$\frac{h'-h}{h} = \frac{s-s'}{s'} = \frac{v'-v}{v}$$

$$\frac{h'-h}{ht} = \frac{v'-v}{vt} = d.$$

The arrangement for keeping each branch of the

vessel occupied at 0° by the liquid.

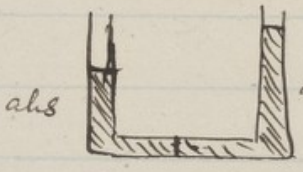
$V_0(1+dt)$ = Vol. of the liquid at t°

$V(1+kt)$ = capacity of portion of vessel filled at t°

but $V_0(1+dt) = V(1+kt)$.

All liquid do not expand the same; take equal bulbs & filled with mercury & water (as an example); and plunge both into the same vessel of hot water; the water will be found to expand most though the mercury <u>at first</u> expands a little more quickly.

The expansion of liquids may be found without regard for the expansion of the containing vessel.



The following fact must here be taken for granted Pressure on area <u>a</u> due to depth of liquid <u>h</u> of $s.g. = s$ $= ahs$; on another liquid of depth h' of $s,g,s' = ah's'$

This however is not quite correct; it would be correct if the containing vessel were not heated also: but this is impossible: therefore the expansion of the containing vessel must be added on.

Let k = coeff^{t} of expansion of the material of vessel.

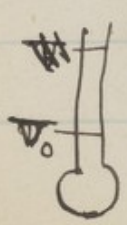
and d = ———————— of liquid.

$$\frac{kt\,V + (b-a)v}{(V+av)t} = \frac{kt\,n + (b-a)v}{(n+a)vt} = \frac{kt\,n + b - a}{(n+a)t} = d$$

i.e. $kt\,n + b - a = (n+a)t\,d$

i.e. if k is known d can be found, & vice versa.

If the bulb be plunged suddenly into hot water, the water in the bulb will first descend, because the containing vessel is heated first & therefore expands first. It then of course rises.



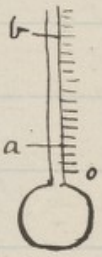
V_0 = volume of liquid at 0°

V = capacity at 0° of portion of

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Expansion of Liquids.

The only expansion of liquids which can be studied is the expansion of volume; because liquids possess no definite shape of their own; they can be made to expand in only 1 direction by enclosing it in a bulb with a long tube, which should be graduated.



Let v = capacity of 1 division of stem

V = capacity of bulb and stem up to the ~~mark~~ 0°

Volume of liquid at 0° = $V + av$

" " " $t°$ = $V + bv$

$(V + bv) - (V + av)$ = expansion in $t°$.

$$\frac{(V + bv) - (V + av)}{(V + av)\,t} = \text{coefficient of expansion.}$$

$$= \frac{(b - a)\,v}{(V + av)\,t} \qquad \text{Let } V = nv$$

$$= \frac{(b - a)\,v}{(n + a)\,vt} = \frac{b - a}{(n + a)\,t}$$

Let A be the linear dimension at original temperature

A^2 ... - superficial - ——— - ——— ~~~

A^3 ——— cubical ——— - ——— - ———.

and let $\underline{a}$ be the coefficient of linear expansion

	At Original temperature	At final temperature
linear	A	$A+a$
superficial	A^2	$(A+a)^2$
cubical	A^3	$(A+a)^3$

$$(A+a)^2 = A^2 + 2Aa + a^2$$

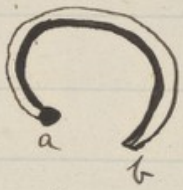
$$(A+a)^2 = A^3 + 3A^2a + 3Aa^2 + a^3$$

$\underline{a}$ is always very small; therefor a^2 & a^3 may be thrown away as inappreciable.

$$(A+a)^2 = A^2 + 2Aa = A(A + \underline{\underline{2a}})$$

$$(A+a)^2 = A^3 + 3A^2a = A^2(A + \underline{\underline{3a}})$$

thermometers. It may be made of the same



two metals viz. iron & brass; a is fixed: when the temperature rises, the coil twists up: and b work upon some machinery, which shows the movement on a dial; when the temperature falls, the coil of course untwists.

A curious exception to the rule about expansion on subjection to heat is, the case of vulcanised india rubber which contracts when heated; also iodide of silver at certain temperature

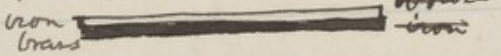
If d_l be the coefficient of expansion in length or linear c. of e.
and d_s ---------- superficial expansion
— d_v ---------- cubical ----

$d_s = 2 d_l$ } very nearly.
and $d_v = 3 d_l$ } which may be seen thus.

very often iron & brass are used, where $\frac{d_i}{d_b} = \frac{2}{3}$; and often iron and some mercury in a vessel at the end of it.

The property of the expansion of ~~solids~~ metals must be taken into account in any continuous lengths of pieces of the metal: e.g. in railway lines, or water pipes, where each piece must not be firmly fixed to another.

The unequal expansion of metals, caused change of shape. E.g. take a bar, of which one side is of iron and the other brass

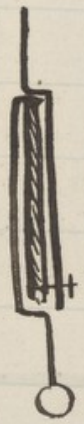


When heated, the brass expands most, and therefore bends the bar over into this shape



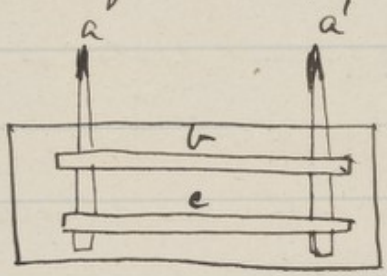
The same principle is employed in metallic

the number of beats of a pendulum on a dial. The length of the beat of the pendulum is varies with its length; therefore it is most important to have it of the same length always. Being made of metal, it expands with heat & contracts with cold; therefore this expansion must be counteracted. A common form of compensated pendulum is this:



The two outer bars are made of iron, and the inner one of zinc. The iron expands downwards, and the zinc upwards; zinc however expands more than iron in the proportion nearly as 3:1. consequently the lengths of the bars must ~~also~~ be in the inverse proportion.

directions of two metals. In Trigonometrical Surveys it is important to preserve measures; the following ~~is~~ is one of the expedients adopted

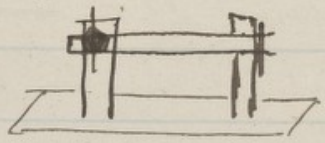


the points a a' are kept in their proper positions but connecting them by two bars of metal, the upper one b of iron & the lower c of brass. Brass however expands more than iron; when the brass expands, it pushes a to the right when the iron expands ——— a ——— left. Therefore the distances of the points of junction of each metal with the measuring bar, ~~must be~~ from their respective pivots must be in the ratio of their Coefficients of ~~friction~~ expansion

The pendulum is the true measurer of time, a clock is simply an instrument for recording

Applications of Exp. of Solids.

The expansion of solids is used to exert great force over a small distance; the force it exerts is that force which would be necessary to pull out the bar the same distance when cold, or to contract it the same distance when hot. Wheel tires are put on hot and then fit tolerably tight. But when they cool they clasp the wheel with immense force. If a hot bar of iron be taken and fixed in a stand by a smaller bar passed at right angles to the first through a hole in the first, ~~and~~ at one end and by a nut at the other end, and the bar be allowed to cool; it will contract & soon break the smaller bar in two



Although Expansion cannot be prevented it can be counteracted by the expansion in different

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We have here

$l = 1500$ m.

$t = 12°$

$t' = 100$

$l' - l = 2.5$ millimetres (Brass)

1.68 —— —— (Iron

$\frac{2.5}{1500} \times \frac{1}{88} = .0000195$ = Coeff. of exp. of brass

$\frac{1.68}{1500} \times \frac{1}{88} = .0000127$ = —— —— of iron.

$$\frac{l' - l}{l_0} \times \frac{1}{t - t} = \alpha$$

When l_0 is the length at 0°C.

$$l = l_0(1 + \alpha t)$$

$$l' = l_0(1 + \alpha t')$$

or $$l_0 = \frac{l}{1 + \alpha t} = \frac{l'}{1 + \alpha t'}$$

This result is called the coefficient of expansion of ~~[illegible]~~ Brass

Suppose l to be the initial length
t ---------- temperature
l' ---------- final length
t' ---------- temperature.

in general terms d or the coefficient of expansion

$$d = \frac{(l'-l)}{l} \times \frac{1}{(t'-t)}$$

from this we get

$$dl(t'-t) = l'-l$$
$$l' = l[1 - d(t'-t)]$$

expansion = e

$$e = l'-l = dl(t'-t)$$

let $t'-t$ be a unit of temperature = 1
& l ---------- length = 1

$$e = 1 \times 1 \times d$$
$$= d.$$

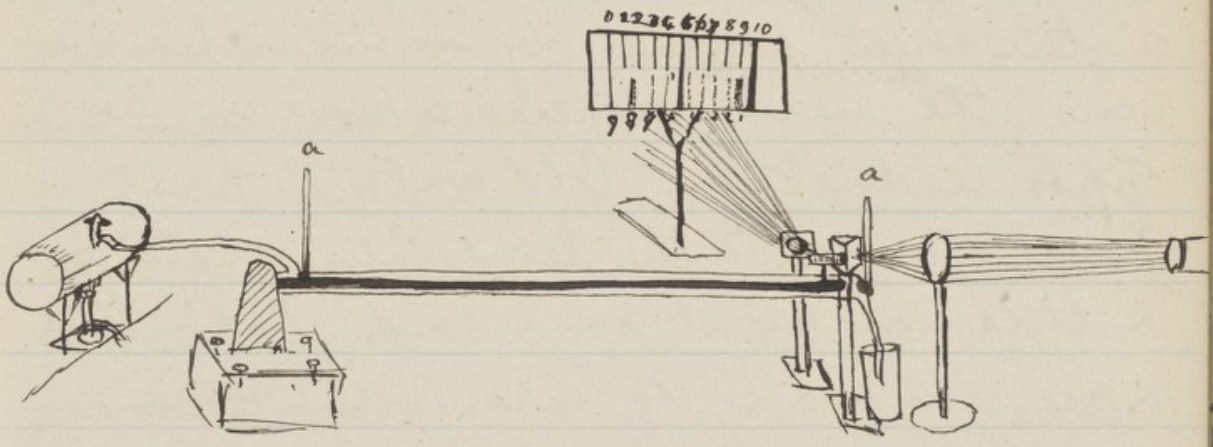
i.e d. of the coefficient of expansion is the change of length that a unit of length undergoes when heated a unit of temperature.

Expansion takes place from left to right; on the screen however by the action of a lens, the shadows of the marks appears to move in the opposite direction. Cold water is first passed in, and the temperature noted by a thermometer at each end. When these agree the rod is of course of the same temperature as the water. Steam is then passed in and the temp. thus raised to 100°. The shadows move showing the expansion to tenths of millimetres. In an actual experiment the length of the rod at 12° (temp. of water passed in) was 1½ metres = 1500 millimetres
it expanded 2·45 millimetres, in 88°, or
length at 100° = 1502·45

$\frac{2{\cdot}45}{1500}$ = expansion in each unit of length

and expansion in 1°

$$\frac{2{\cdot}45 \times \frac{1}{88}}{1500} = \frac{2{\cdot}45}{1500 \times 88}$$



In this experiment a sort of microscope is employed, the expansion being magnified;
A rod of metal say ~~zinc~~ brass, is ~~fixed~~ placed in a hollow tube open at both ends; with one end of the rod fixed tightly so that expansion may take place in one direction only. At one end is fixed a millimetre scale
A strong light (limelight) is passed through this and the shadows of the marks on the glass received on a screen, on which there is ~~only~~ also a scale, of ~~to~~ which the shadow of nine of the optical divisions = 10 of the scale on the screen.



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Expansion

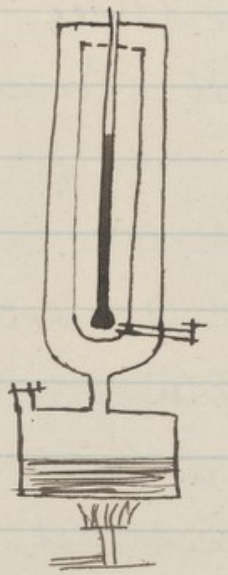
Expansion of solids. Though solids expand all ways on exposure to heat, the increase of length is more perceptible than ---- width or depth. This therefor we will take first.

Suppose we take a rod of metal, the change of length depends on three things:-

1. How much we warm it - Elevation of temperature
2. Original length.
3. Material of which it is made

Of solids metals expand most, & of metals zinc. if for instance the length of a rod of zinc in melting ice is 1000, in boiling water it is 1003. Platinum expands least; at 0°C. 1000; at 100° 1000.9 Glass expands a little less than platinum. A more accurate ~~---~~ meanner of ~~found~~ the expansion of solids in regard to length, than the apparatus in the 1st Lecture is this.

correct for all ordinary purposes. When the thermometer tube has a convenient amount of mercury in it; put it into melting ice and mark the point at which it stands and then put it into a vessel like this and pass steam over and again mark the point at which the mercury stands

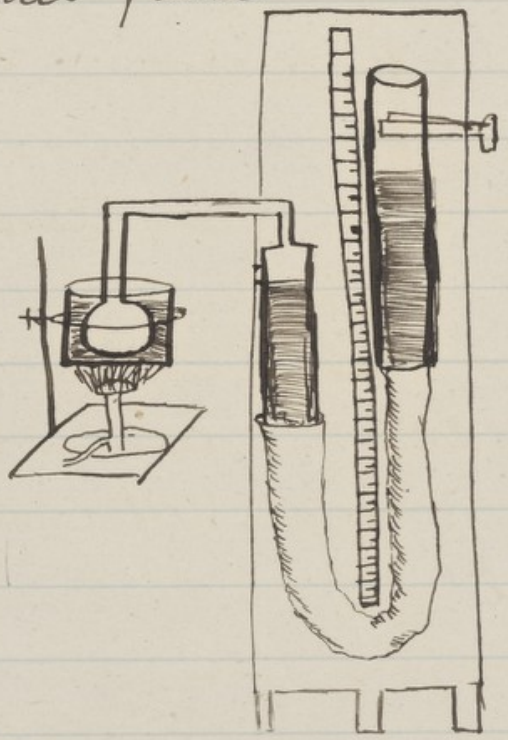


Section.

The temperature in melting ice is called 0°
——— boiling water ——— 100°

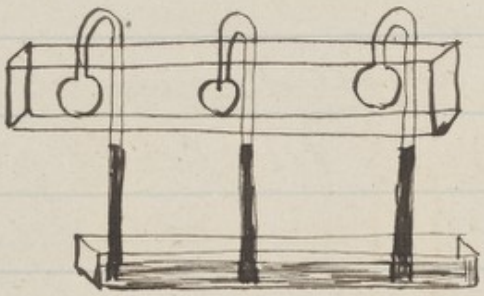
bulb and delivery tube; mark the difference between the two levels accurately and put a ✓ mark on the tube at the point at which the mercury stands in the left hand branch; say that the difference is 8·8 and the pressure of the air is 75·95 in centimetres then 84·75 is the pressure on air, or the pressure which it exerts; heat the air to the boiling pt. by putting the bulb in boiling water; the air expands and drives the mercury down; reduce it to its former volume by lifting the right hand branch, and then mark the difference between the levels, say it is 36·8 - total pressure = 112·75. This is a form of air thermometer which is the most correct one. Air and mercury expand however in very nearly the same proportions, i.e. mercury expands pretty regularly, and is sufficiently

as they are proportional one to the other. The 1st of the two is the one generally chosen to make an air thermometer: which may take this form



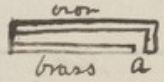
Take a bulb with a delivery tube which passes into a thicker tube; to this is connected by an india rubber another glass tube; and a measure is between the two as in the figure: partly fill with mercury, and let air remain in the

the same level in each; fill one with



coal gas, another with oxygen and another with air; heat all to the same temperature by plunging all ~~the~~ three at once into a vessel of hot water, and the gases will be seen to expand equally driving the mercury in each down regularly, at the same rate in each, and to the same pt. in each. This is true for all permanent gases: it may be expressed thus - The change of pressure of a gas when its volume is kept constant, or the change of volume of a gas when its pressure is kept constant, may either be taken to mark changes of temperature

	air	mercury	water
0°	1·0000	1·000000	1·0000
at 100°	1·3665	1·018153	1·0430

Therefore we should get different results ~~for~~ with water mercury thermometers. We should get no better results ~~for~~ with solids; e.g. brass expands more than iron; which may be proved by taking rods of iron and brass fastened firmly at the end and of this shape 

; the brass expands till it touches the air closing up the space a forming a complete way for an electric current which acts upon a bell when the bars are hot enough to expand enough to touch.

Air is more trustworthy than either liquids or solids, because it expands regularly. Besides this all permanent gases expand the same as air, an example of each may be roughly seen thus; take 3 bulbs all of the same size, attached to tubes filled to

used, because although it does not expand so much as alcohol or water, it is easily seen because opaque & moreover it does not wet glass. Temperature like distance must be measured from some fixed point, and in intervals between bodies of gradually getting higher temperature. As a standard, the fixed points are the melting point of ice or the freezing point, and the boiling point of water or the boiling pt as it is called.

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When the temperature rises in water and mercury they do not expand to the same amount, not regularly, nor in the same proportion for a similar rise of temperature in each material E.S. take equal volumes of air mercury & water at 0° and heat to one hundred

If a piece of metal and a piece of wood be both of the same temperature, and both either hotter or colder than the hand; and the hand be placed in contact with both; the metal will feel the ~~hottest~~ coldest because its conducting power is greater than that of wood.
If 3 basins be taken, one filled with hot water, one with lukewarm and one with cold water and the right hand be placed in the hot water & the left in the cold, and then both together into the lukewarm, the effect will be that the lukewarm water feels hot to the left hand & cold to the right.
An instrument to measure temperature is called a thermometer; the way it is measured is by the expansion it causes in a liquid: a liquid is used because it can be contained in some vessel and ~~only~~ be made to expand in one direction only not in all; for this purpose a tube with a bulb at the end is used. Mercury is the liquid generally

When we say that a body has a high temperature we mean that it is hot i.e. it has the power of making other bodies hot.

Temperature is the measure of a body's power to impart heat to other bodies placed in contact with it. If A is of high temperature & B low, A gives heat to B; till they are both of the same temperature i.e. the power to part with heat is equal in both. Two bodies in equilibrium of temperature with a third are in equilibrium of temperature with each other.

Let A, B & C be bodies of different temperatures, A hotter than B, and B hotter than C. If A cools down to C, it must pass through the temperature of B; i.e. it is comparable to a st. line being of one dimension only.

of heat. These bodies which have been made hot acquire in like manner the power of imparting heat to other bodies, which in turn get the same property. This gives one the idea that heat is a thing which can be transferred; though whether it is a transference such as the transference of velocity when one moving body strikes another that is stationary, or like the transference of oxygen from oxide of lead to carbon, is not known.

Suppose we have two pieces of iron of the same mass & put them into a store of heat, ~~both~~ each acquires the same amount of heat, and the two together acquire twice as much as one alone; this gives us the idea of quantities of heat expressed numerically.

A change of temperature is caused by heat

Continuing the effects of heat.

3rd. Change of chemical condition; e.g. paper wood, etc. undergo combustion i.e. are oxidised when heated in the air. Compounds also are generally resolved into simpler bodies or into their elements.

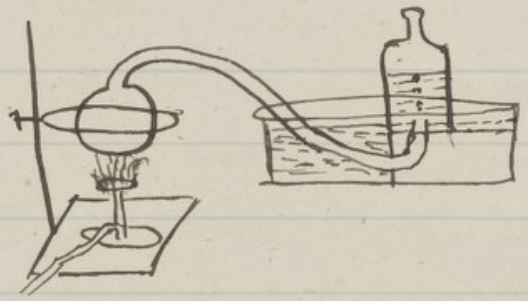
4th Change of colour – in some few substances.

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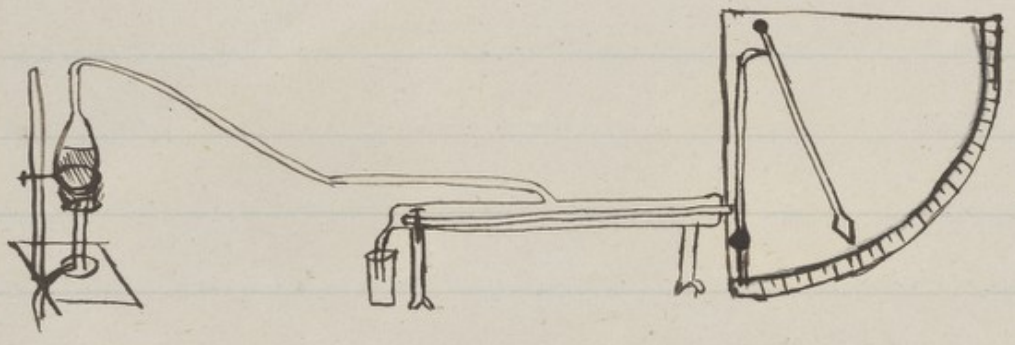
Taking as two examples of producers of heat, a flame and a vessel of hot water: we find an essential difference between the two; the flame imparts heat to other bodies but does not get colder in so doing; it is called a source of heat; while the hot water, by giving up its heat to other bodies becomes less hot; in other words its power of producing heat becomes exhausted as it produces heat in other bodies; it is called a store

being filled to about ½ way up the tube with coloured liquid; on heating the liquid, it expands visibly as it rises in the tube, and sinks to its original level on cooling.

For gases an example is the expansion of air;



Take a glass bulb or flask filled with air and fitted with a delivery tube which passes under a bell jar filled with water & placed on the shelf of a pneumatic trough. Heat the bulb, the air within it expands, & fills, besides the bulb, part of the bell jar driving the water down.



A bar of zinc is heated by passing steam over it; it expands in length: but not enough to be visible to the eye directly; one end is therefore fastened ~~to a string~~ by a firm screw while the other on expanding pushes a lever which in turn pushes another; this latter lever is an index which points on a scale, thus showing the movement that the pushing or expanding of the rod has caused.
For liquids a simple apparatus is used, consisting of a flask fitted with a long tube of narrow bore; the whole

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Heat.

Oct 4th 1877

Heat is measured by temperature; there may be a great quantity of heat in a body but its temperature may be low, & vice versa; temperature is the intensity of heat.

Sources of heat.

1st & most common - Combustion which may be feeble or intense as in the case of a small flame but an intense heat & light in the oxy hydrogen blowpipe.

2nd Friction. i.e. by rubbing

3rd Electrical action.

Effects of heat

1st. Elevation of temperature

2nd Change of bulk. Almost without exception bodies expand when heated. This may be illustrated in the case of solids thus;

P.T.O.

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Notes of Lectures
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at
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London
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Experimental Physics (Division B. Junior Class Physics)
during the
Session 1877-1878 A.D.

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