Wellcome Film Project

Mathematics and Physics for Anaesthetists at the Institute of Basic Medical Sciences at the Royal College of Surgeons of England: Programme 5: Laminar and Turbulent Flow and the Anomalous Viscosity of Blood

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University of London Audio-Visual Centre, 1976.

Film extract by courtesy of Dr JT Wright, Bio-engineering and Medical Physics Unit, University of Liverpool.

Produced by David R Clark; Michael Tomlinson.

Made for British Postgraduate Medical Federation.

Black-and-white
Duration: 00:28:29:02

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<**Opening titles**>

<**DW Hill to camera**>

This videotape sequence has been designed to recapitulate the main points of the lecture on the subject given on the primary FFARCS course at the Institute of Basic Medical Sciences of London University. It's intended to be viewed after you have worked through your own lecture notes.

The pattern of gas flow in the respiratory passages and the blood and the cardiovascular system affects the work cost of breathing and the production of aneurysms and vessel occlusions respectively. In contrast to the respiratory system, the flow of blood usually follows an orderly or streamline pattern but localised turbulence can be present above the heart valves, it can also arise at branchings and bifurcations. From the viewpoint of the fluid dynamicist the situation is extremely hard to model as it's influenced by the pulsatile nature of the flow, the elasticity of the vessels and the fact that the viscosity of blood depends upon the forces which act upon it.

<**Hill over film visualising blood flow through a heart valve**>

In this visualisation of the flow through a heart valve into a model aorta, you can see the extremely complicated flow patterns which arise and also, on the left, the localised turbulence occurring at the entry to a side arm.

<**Hill to camera**>

Hence, in this videotape it is only possible to outline the broad principles of a subject which is of practical importance to anaesthetists, intensivists and vascular surgeons. Obviously, in practice, the respiratory blood vessels and passages do not consist of rigid, fixed-length pipes. However, this gross simplification is of assistance in describing the various patterns of flow which are encountered in practice.

<**Unspecified narrator over film comparing fluid and electrical flow, then the flow of different kinds of liquid**>

Basically a fluid will flow from within a region of higher pressure to one of a lower pressure, the rate of flow depending on both the rate of pressure and the resistance to flow existing between the two regions concerned. This is analogous to Ohm's law in electricity as this picture shows. In the upper part you see force being applied by finger pressure on to the plunge of a syringe and this causes a pressure difference across the tubing connected to the syringe and out comes the fluid from the end, of course.

Below, you see the companion picture for the electrical case where now a constant electrical pressure from a 6 volt battery is applied to the simple circuit consisting of a light bulb. The current flows round from the battery, through the lamp, and causes it to glow. And there you see the well-known Ohm's law equation: E=IR, E is the applied voltage, R is the resistance of the circuit and I is the resultant current flow. R is in ohms and I is in amperes of course.

It can be shown that for a given pressure drop across a given pipe, the volume of fluid, gas or liquid, flowing through the pipe per unit time depends upon a property of the fluid known as its viscosity. This can be most simply confined as the resistance of the fluid to flow.

A freely flowing fluid such as water or plasma has a low viscosity in contrast to a thick oil or whole blood. You can see that the same gravitational force is being applied to each of the syringes; they hold different fluids and it's obvious that for a different pressure applied to the syringe plunger, the viscosity of the fluid is limiting the amount of fluid that comes out. Obviously, there's much more flowing in the case of a low viscosity fluid such as water than occurs in some of the higher viscosity fluids that are being used here.

The whole question of the study of the flow of matter and its deformation is included in the subject known as rheology which is becoming increasingly important.

The resistance to flow of a fluid will depend upon the intermolecular forces operating within the fluid. In a viscous fluid the velocity of adjacent layers of the fluid will occur. A slip occurs between adjacent layers as a result of the shear forces acting between them against the intermolecular forces. The viscosity of a fluid can now be defined as the resistance which the fluid exhibits to the flow of one layer over another, or more particularly, as the ratio of shear stress to shear strain.

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<**Unspecified narrator continues over animated model showing flow through a series of layers of molecules**>

This is an animated model of a series of layers of molecules in a vessel. Now, suppose that we want fluid to flow down this vessel from left to right, so we apply a pressure difference across the vessel – here you see the force F being applied, and the result is that the layers of molecules slip relative to one another, and you see that the lowermost layer, adjacent to the vessel wall, i.e. the boundary layer, stays relatively at rest and the others slip progressively faster and faster as we go away from the boundary layer towards the midline of the vessel.

<**Unspecified narrator continues over animated models demonstrating flow down a pipe**>

We now want to proceed to discuss the motion of a fluid flowing down a pipe, and as a homely example I'd like to talk about a stream flowing between banks because this is something you've probably all observed. The fastest motion of the water occurs along the centre of the channel, the motion falling to zero at the banks where there exists a stagnant or so-called boundary layer. This pattern of flow implies a slippage of water molecules between adjacent layers.

In the picture, the length of each arrow represents the magnitude of the water velocity at that particular point along the cross section of the stream. The locus of the tips of the arrows is a geometrical shape known as a parabola and we speak of a velocity profile – the parabolic velocity profile shown is characteristic of laminar or streamline flow. This is so called because adjacent sheets of molecules move along with parallel motions in laminae, but with different velocities. This is an efficient process for transporting the fluid since none of the energy applied to the system is used in other than the production of linear velocities.

<Unspecified narrator continues over animated model showing Newton's law of viscous flow>

A quantitative relationship can be deduced for the relation between the applied force and the resulting velocity difference produced between adjacent layers between laminar flow. Let two areas, A, be situated in the fluid at distances of X and X+x respectively in one wall of the pipe. The corresponding velocities of these areas are V and V+v respectively. Due to the internal friction in the fluid, as the layers slide relative to one another a tangential shear force, F, will exist between them. The shear stress on the fluid causing a relative movement of the layers will be equal to F divided by A, where F is the tangential force acting on the area A. The rate of shear of a particular layer, also known as the velocity gradient, is equal to v divided by x and has the dimensions of seconds^{-1}, that is 1/seconds. The shear stress in SI units has the dimensions of Newton's per square metre.

Newton's law of viscous flow states that the rate of shear produced is directly proportional to the shear stress; that is $F = \eta Av/X$. The constant of proportionality eta is termed the coefficient of viscosity of the fluid. For a so-called Newtonian fluid the viscosity is independent of the magnitude of the shear stress and of the shear rate. Its value depends both on the nature of the fluid and its temperature.

<Hill to camera then over still image showing different fluids>

Newton's law of viscous flow applies only to laminar or streamline flow with a fluid of constant viscosity. This is an important point and one which you should remember.

Examples of commonly encountered Newtonian fluids are water, plasma and urine. Whole blood is a non-Newtonian fluid owing to the presence of the erythrocytes. As we mentioned earlier, rheology is the subject which deals with the flow properties of substances. Haemorheology, the study of the flow properties of blood, has assumed a considerable importance in vascular surgery and in anti-coagulant therapy.

<Hill over Hagen-Poiseuille equation>

The well-known Hagen-Poiseuille equation gives the volume flow rate down a pipe as Q. volume flow rate = π (P_1-P_2), where P_1 and P_2 are the inlet and the outlet pressures respectively across the pipe, r^4 where r is the radius, all divided by 8 x η, where η is the viscosity of the fluid x L, where L is the length of the pipe. P_1-P_2 is the pressure drop across the pipe, expressed SI units now in N/m^2, r is the radius of the pipe in metres, L is its length in metres and η is the viscosity of the fluid in cPs and Q. the volume flow rate is in m^3/sec. All of these are SI units.

It can be seen that for a given pressure difference across the ends of the pipe, and the length of pipe, the quantity of fluid which can be passed per unit time will depend mainly upon the radius of the pipe since this occurs in the equation raised to the power of 4.

<**Hill over graph, then film showing different shots of endotracheal tubes**>

As might be expected, the volume throughput varies inversely as the viscosity of the fluid and the length of the pipe. If you imagine that the radius of the pipe is somehow halved, then 2 to the power 4 is equal to 16, so that for a constant pressure drop and length of pipe, halving the radius drops the volume flow rate through the pipe to 1/16th of its former value.

Particular care should be given to the possible kinking of endotracheal tubes and the consequent marked reduction in the effective radius of the tube, for this will greatly reduce the minute volume of respired air which can be drawn through the tube by the patient's respiratory efforts.

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<**Hill over rearranged Hagen-Poiseuille equation**>

Rearranging the Hagen-Poiseuille equation gives the pressure drop across the pipe P_1-P_2 = Q. the volume flow x $8\eta L \div \pi r^4$ – by analogy with Ohm's law, resistance to fluid flow of the pipe is equal to $8\eta L \div \pi r^4$. Any reduction in the radius of the pipe produces a large increase in this resistance to flow.

<**Hill to camera, the over film showing pneumotachograph**>

As long as laminar flow conditions obtain, the pressure drop across the resistance will be directly related to the volume flow through it. This principle is utilised in the pneumotachograph which is a device for measuring the volume flow rate of gasses. One version, due to Professor Fleisch of Lausanne, consists essentially of a large number of tubes in parallel so that the flow through each tube remains laminar. Four sizes of Fleisch head are available covering the full range from 60 – 1000 litres per minute full scale, which in each case produces a pressure drop of no more than 7mm of water across the head. Pneumotachographs are widely used in respiratory and lung function studies.

<**Hill over graph, then over animated model showing progression of molecules down a pipe, then graphs comparing types of flow**>

As long as the Hagen-Poiseuille equation holds, a plot of the volume flow through a pipe against the pressure drop across it will be a straight line. However, beyond a certain limit of pressure drop, a laminar flow pattern no longer obtains. The orderly streamline progression of molecules down the pipe becomes interrupted. Localised turbulence starts to occur with the molecules and the fluid swirling around in eddies and vortices. These can be triggered off by rough patches on the walls, projections, sharp bends, branches or changes in diameter. The local eddies are shed and tend to die out downstream when the flow pattern again becomes laminar. At still greater pressure gradients the flow becomes fully turbulent, all the fluid now swirling around.

In contrast to laminar flow with its characteristic parabolic flow profile, in fully turbulent flow the advancing flow profile is almost flat. A marked velocity gradient now exists close to the walls whilst across the bulk of the vessels diameter the flow is maximal and approximately constant as you can see in the illustration. Turbulent flow is a less efficient way of transporting the fluid since now energy is wasted in producing the turbulent motion. A substantially greater pressure drop across the pipe is required for a given volume flow than was the case with laminar flow.

When the flow through a pipe is laminar, the pressure drop across the pipe is proportional to the volume and flow rate, i.e. $P=BF$, where B is a constant. When the flow is fully turbulent, the volume flow is nearly proportional to the square root of the pressure drop, i.e. $P=CF^2$, where C is a constant. When both laminar and turbulent flows exist, these equations can be combined to give $P=BF + CF^2$. A more general form is $P=DF^n$, where D is another constant as is also n and n = 1 for laminar flow and n = 2 for fully developed turbulent flow. Taking logarithms of both sides of the equation gives $\log P=\log D+n\log F$. The departure from laminar flow can be seen when the logP against logF plot starts to depart from the straight line.

Between the fully streamlined and the fully turbulent flow patterns there exists a transitional region. The fluid velocity at which fully developed turbulent flow sets in is known as the critical velocity – V_c; this is given by the equation V_c = k÷d all multiplied by $\eta \div \rho$, where k is a constant known as Reynolds number, η is the viscosity of the fluid and ρ is its density; d is the diameter of the pipe.

For both whole blood and water, the onset of turbulence occurs at Reynolds numbers of approximately 2000. The ratio of viscosity to density, that is $\eta \div \rho$, is known as the kinematic viscosity, it is measured in cS.

00:16:26:00

<Hill over illustrations showing different flow patterns>

Now we have 5 pairs of illustrations showing flow patterns, kindly provided by Professor Taylor from the Physiology Department of the Royal College of Surgeons of England. These were taken with water flowing through a 2m long, 25mm diameter glass tube, with a dye injected about halfway along.

The first of each pair is due to a continuous injection of dye, and the second to a bolus injection or slug to show up the flow profile. Here in the first picture you see, with a Reynolds number of only 500, that first of all there is a well-defined laminar flow and now, in the second of that pair with the slug injection we see the parabolic flow profile present as we would expect at a low Reynolds number of only 500. Increasing the Reynolds number to 1000, we see that the same flow patterns occur and we have the same velocity profile which is parabolic. In the Reynolds number of 1500, some disturbance of the streamline is seen and also of the flow profile. At the Reynolds number of 2000, however, localised turbulence is seen to be developing with a very distorted flow profile on the bolus or slug injection. Finally, at a Reynolds number of 2500, we have fully developed turbulence with a continuous injection of dye, and with the slug or bolus injection you see that now there is no longer a streamline flow profile which is parabolic but a rectangular flow profile characteristic of the fully developed turbulence.

Typical values of Reynolds number which have been measured in the ascending aorta of man are from 5000 to 12,000, and in the pulmonary trunk 5000 to 10,000. These contrast with measurements made in the thoracic inferior vena cava of man where the Reynolds numbers were of the order of 1320 to 1980, very much lower.

<**Hill to camera**>

Turbulent flow can occur at low gas velocities in the tracheobronchial tree due to its hundreds of thousands of branchings and irregularities in the nature of the surfaces arising from the presence of mucus exudate of foreign bodies. In the cardiovascular system, the absence of a murmur is no indication that the flow is, in fact, laminar since fully developed turbulence can occur without any detectable sound. It would be expected that turbulent flow in the circulation would occur at the root of the aorta and possibly at the heart valves where it may be responsible for the sound of murmurs. Flow changes at vessel bifurcations can give rise to a weakening of the wall and an aneurysm. For example, intracranial, saccular aneurysms arise at the apex of the bifurcation. Localised turbulence can be responsible for the erosion of arterial walls and the deposition of debris in stagnant flow regions can lead to the build up of plaques – it is fairly generally agreed that atherosclerosis is a patchy disease which occurs mainly at bends and bifurcations.

<**Hill over animated models showing bifurcations**>

There are two primary kinds of bifurcations – the Y bifurcation and the side arm. These are totally different in terms of fluid dynamics. There is good evidence that many arterial diseases are related to local blood flow disturbances.

00:20:14:00

<**Hill to camera**>

As we mentioned earlier, whole blood is a non-Newtonian fluid and has some anomalous properties in regard to its viscosity. These are not very helpful in the case of patients in shock and an understanding of what happens is of importance in intensive care.

<**Hill over graph**>

If whole blood is defibrinated or washed red cells are suspended in saline solution, an almost Newtonian behaviour is observed; the fibrinogen and erythrocytes together produce intercellular bonding, multi-cellular aggregation and the anomalous flow behaviour which is seen with whole blood. For whole blood, deviations from a simple Newtonian flow behaviour arise in two ways: one due to the fact that at low shear rates viscosity of blood increases rather than remaining constant, and two – in small calibre blood vessels the apparent viscosity of all rates of shear is lower than that found in larger vessels.

<**Hill to camera and briefly over tables listing points made**>

Red cell aggregation is reversible. It is a maximum when the flow is zero, the aggregates breaking up as the flow rate is increased. But in slow flows, much of the sheer stress is used to dissipate the aggregates, while at the higher flows much of the shear stress is available to generate the velocity. The viscosity of whole blood is dependent not only upon the haematocrit but upon mechanical factors which operate upon the blood during flow. For example, a normal value for the viscosity of whole blood, above the aortic valve would be approximately 5cP, and this will have risen to about 50cP in the postcapillary venules. Similarly, typical viscosities in the aorta, arteriolar bed and microcirculation of man would be respectively 6, 10 and 800cP.

During haemorrhagic shock when low blood flow rates can occur, the situation may be exacerbated by the increase in blood viscosity which occurs due to a partial settling out of the cells. In order to thin the blood so that a better flow can be obtained with the reduced pressure available, an infusion of Hartmann's solution may be given. Another possibility is an infusion of a low viscosity dextran solution such as Rheomacrodex, a dextran fraction with an average molecular weight of approximately 40,000. In a patient with bile peritonitis, it was found that the infusion of 500ml of 15% Rheomacrodex reduced the whole blood viscosity by approximately one third.

The homely and somewhat extreme example of a substance with an anomalous viscosity is a thixotropic paint.

<**Hill over film showing the viscosity of Dulux gloss paint**>

In the paint can, of course, the material is very stiff and viscous and it won't drop if you put it on the brush. But once you apply a shear force by working the brush across the surface, then the force reduces the apparent viscosity of the paint which flows and covers the surface as you see.

<**Hill over model showing Fahraeus Lindqvist Effect**>

The Fahraeus Lindqvist Effect is concerned with a reduction in the apparent viscosity of whole blood which occurs when blood flows down a narrow tube; the radius of the vessel can be up to about 100 times that of a red cell diameter. In a 20 micrometre diameter vessel such as an arteriole, the effective viscosity of the blood is reduced to approximately 2 thirds of the value found in a large vessel. The reduction in viscosity arises because from the fact that in a narrow tube, the red cells tend to congregate along the narrow axis of the tube, leaving a layer of plasma along the walls. Since the plasma has a lower viscosity than the red cells, a greater flow results than would be expected from the Hagen-Poiseuille equation, based upon a conventional value for the blood viscosity. Thus, although the majority of the flow in the vascular system is streamlined, the blood viscosity depends markedly upon the magnitude of the shear force acting in the particular vessel, and in very small vessels the boundary layer of plasma acts so as to reduce the effect of the viscosity.

00:24:52:15

<**Hill to camera and repeats of earlier slides and graphs to summarise**>

In conclusion, I would like to summarise the main points of this videotape.

The most efficient way of transporting a fluid down a pipe is with a streamline flow pattern. This has a characteristic parabolic velocity profile. The Hagen-Poiseuille equation can be used to calculate the volume flow rate in a tube under laminar flow conditions assuming that the fluid is Newtonian.

A Newtonian fluid exhibits a constant viscosity which is independent of the magnitude of the shear stress. Whole blood does not have a constant viscosity, it is a non-Newtonian fluid. At low rates, the apparent viscosity is higher than normal. Due to the Fahraeus Lindqvist Effect it is lower in small vessels due to the layer of plasma on the walls. In laminar flow, the pressure drop is proportional to the volume flow rate, whereas in fully turbulent flow it is proportional to the square of the flow rate. When the flow is just fully turbulent, the velocity equals the critical velocity. The onset of full turbulence is usually quoted as occurring when Reynolds number is approximately 2000. On the SI system of units, viscosities are quoted in terms of cP: the viscosity of water at 20˚C is taken as 1cP, that is 100th of a poise. The viscosity of whole blood in large vessels is of the order 3 to 6cP but it can rise to some 50cP in the postcapillary venules, and even to 800cP in parts of the microcirculation.

<Hill over tables showing suggestions for further study>

Finally, I have some suggestions for further study. I'd like to draw your attention to two previous videotapes in this series aimed at the primary Fellowship in Anaesthesia. Programme 1, entitled Logarithms and Exponentials, deals, as the title suggests, comprehensively with the use of logarithms and logarithmic plots, while Programme 4, SI Units, covers the whole range of SI units which you are likely to encounter and also deals with cP. This is a short list of books which you will find of use if you wish to study this subject further.

<Table>

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